Halo EFT: an effective bridge between *ab initio* nuclear-structure calculations and nuclear-reaction modelling

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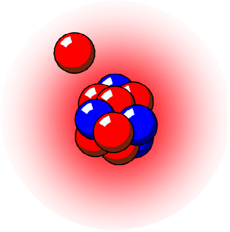
Halo nuclei

Halo nuclei are found far from stability

Exhibit peculiar quantal structure :

- Light, n-rich nuclei
- Low S_{n} or $S_{2\mathrm{n}}$



With large matter radius

due to strongly clusterised structure :

neutrons tunnel far from the core and form a diffuse halo

One-neutron halo

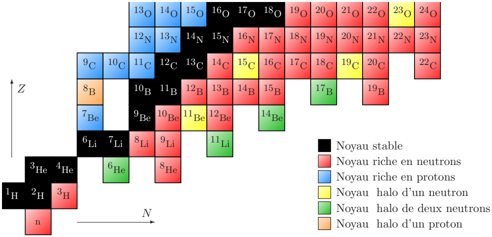
$^{11}\mathrm{Be} \equiv {}^{10}\mathrm{Be} + \mathrm{n}$

$^{15}\mathrm{C} \equiv {}^{14}\mathrm{C} + \mathrm{n}$

Two-neutron halo

$^{6}\mathrm{He} \equiv {}^{4}\mathrm{He} + \mathrm{n} + \mathrm{n}$

$^{11}\mathrm{Li} \equiv {}^{9}\mathrm{Li} + \mathrm{n} + \mathrm{n}$



⬛ Noyau stable

🟥 Noyau riche en neutrons

🟦 Noyau riche en protons

🟨 Noyau halo d'un neutron

🟩 Noyau halo de deux neutrons

🟧 Noyau halo d'un proton

This exotic structure challenges nuclear-structure models

Reactions with halo nuclei

Halo nuclei are fascinating objects

Some have been calculated *ab initio* [Calci *et al.* PRL 117, 242501 (2016)]

However difficult to study experimentally [$\tau_{1/2}(^{11}\mathrm{Be}) = 13$ s]

How can one probe their structure ?
test the *ab initio* predictions ?

⇒ require indirect techniques, like reactions :

- breakup : ^{11}Be + Pb/C → ^{10}Be + n + Pb/C
- transfer : ^{10}Be(d,p)^{11}Be
- knockout : ^{11}Be + Be → ^{10}Be + X

Need good understanding of the reaction mechanism
(i.e. a good reaction model)
to know what nuclear-structure information is probed

Here, we couple precise reaction models with Halo EFT
(For a short review, see [P.C. Few Body Syst 63, 14 (2022)])

We consider ^{11}Be, the archetypical one-neutron halo nucleus

Ab initio description of ^{11}Be

NCSMC calculation of ^{11}Be [Calci *et al.* PRL 117, 242501 (2016)]

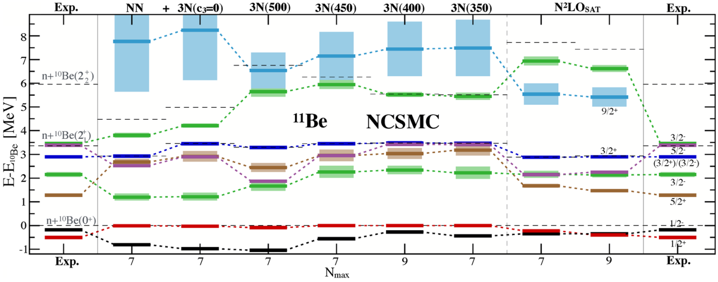


FIG. 2. NCSMC spectrum of ^{11}Be with respect to the $n + {}^{10}$Be threshold. Dashed black lines indicate the energies of the ^{10}Be states. Light boxes indicate resonance widths. Experimental energies are taken from Refs. [1,51].

- $\frac{1}{2}^{+}$ ground state :
  $\epsilon_{\frac{1}{2}^{+}} = -0.500$ MeV
  $C_{\frac{1}{2}^{+}} = 0.786\ \text{fm}^{-1/2}$
  $\mathcal{S}_{1s\frac{1}{2}} = 0.90$
- $\frac{1}{2}^{-}$ bound excited state :
  $\epsilon_{\frac{1}{2}^{-}} = -0.184$ MeV
  $C_{\frac{1}{2}^{-}} = 0.129\ \text{fm}^{-1/2}$
  $\mathcal{S}_{0p\frac{1}{2}} = 0.85$

Calci *et al.* also predict the ^{10}Be-n phaseshift

^{10}Be-n Halo-EFT potential

Replace ^{10}Be-n interaction by effective potential in each partial wave

Use Halo EFT : clear separation of scales (in energy or in distance)
⇒ provides an expansion parameter (small scale / large scale)
along which the low-energy behaviour is expanded

[C. Bertulani, H.-W. Hammer, U. Van Kolck, NPA 712, 37 (2002)]

[H.-W. Hammer, C. Ji, D. R. Phillips JPG 44, 103002 (2017)]

Use narrow Gaussian potentials @ NLO

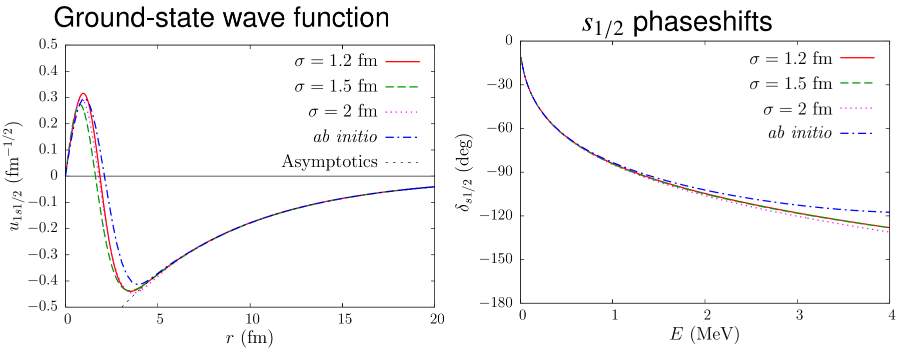
$$V_{lj}(r) = V_0^{lj}\, e^{-\frac{r^2}{2\sigma^2}} + V_2^{lj}\, r^2 e^{-\frac{r^2}{2\sigma^2}}$$

- In $s\frac{1}{2}$ and $p\frac{1}{2}$: fit V_0^{lj} and V_2^{lj} to reproduce
    - ϵ_{nlj} (known experimentally)
    - C_{nlj} (predicted *ab initio*) [Calci *et al.* PRL 117, 242501 (2016)]
- $V_{p3/2} = 0$ to reproduce *ab initio* $\delta_{3/2^-} \sim 0$
- For $l > 1$: $V_{lj} = 0$ @ NLO

σ = 1.2, 1.5 or 2 fm evaluates the sensitivity to short-range physics

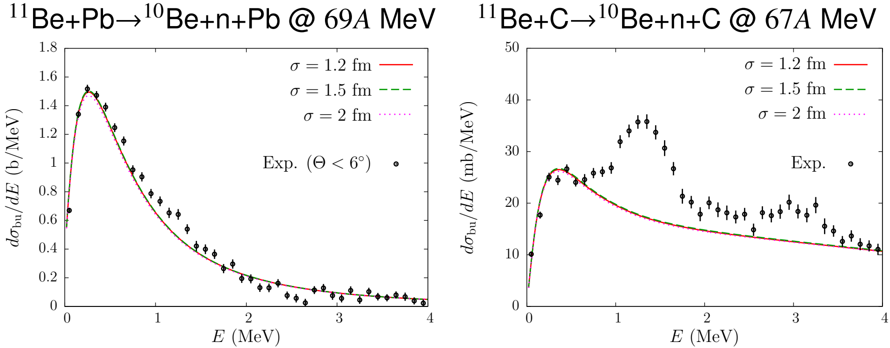
$s\frac{1}{2}$: @ NLO potentials fitted to $\epsilon_{\frac{1}{2}^+}$ and $C_{\frac{1}{2}^+}$

Potentials fitted to $\epsilon_{1s\frac{1}{2}} = -0.503$ MeV and $C_{1s\frac{1}{2}} = 0.786$ fm$^{-1/2}$



- Wave functions : same asymptotics but different interior
- $\delta_{s\frac{1}{2}}$: all effective potentials are in good agreement with *ab initio* up to 1.5 MeV (same effective-range expansion)
- Similar results obtained for $p\frac{1}{2}$ (excited bound state)

Breakup : ^{11}Be+Pb/C$\rightarrow$^{10}Be+n+Pb/C @ $\sim 70A$ MeV



Exp : [Fukuda *et al.* PRC 70, 054606 (2004)]
Th. : [P.C., Phillips & Hammer, PRC 98, 034610]

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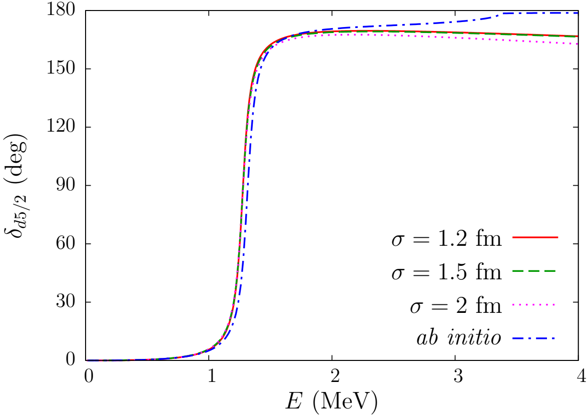
- All calculations provide very similar results $\forall\sigma$
  despite the difference in the internal part of the wave function
  $\Rightarrow$ reaction is peripheral [P.C. & Nunes PRC 75, 054609 (2007)]
- Excellent agreement with data on Pb (no fitting parameter)
  $\Rightarrow$ confirms *ab initio* ANC and phaseshift
- On C, breakup strength missing at the $5/2^+$ and $3/2^+$

$d\frac{5}{2}$: potentials fitted to $\epsilon^{\text{res}}_{\frac{5}{2}^+}$ and $\Gamma_{\frac{5}{2}^+}$

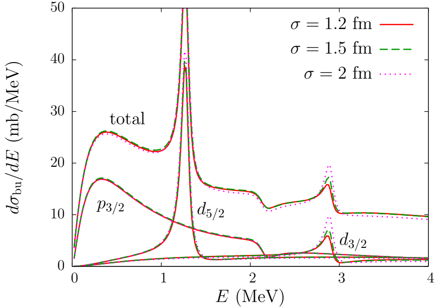


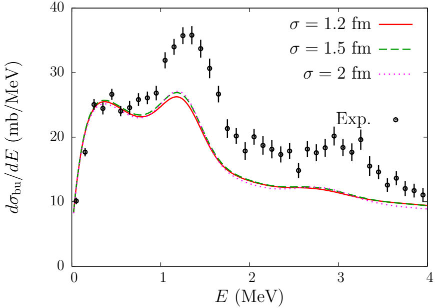
- Identical $\delta_{d\frac{5}{2}}$ up to 1.5 MeV
  up to 5 MeV for the narrow potentials (σ = 1.2 or 1.5 fm)
- Excellent agreement with ab initio results up to 2 MeV

^{11}Be+C→^{10}Be+n+C @ 67AMeV (beyond NLO)

Total breakup cross section and dominant contributions

Folded with energy resolution [Fukuda *et al.* PRC 70, 054606 (2004)]





- In nuclear breakup, resonances play significant role

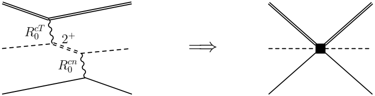
  [P.C., Goldstein & Baye PRC 70, 064605 (2004)]

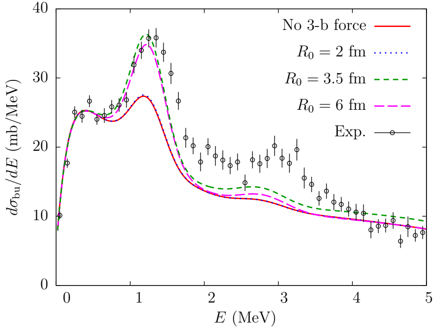
- Still, resonant breakup not correctly described degrees of freedom [^{10}Be(2^+)] missing in the effective model

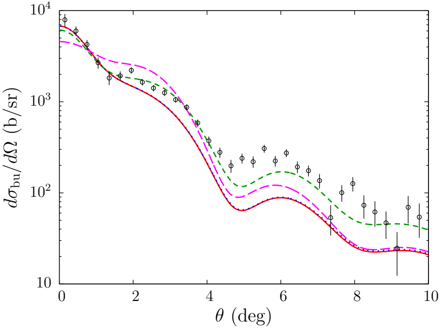
  [Moro & Lay PRL 109, 232502 (2012)]

Simulating core excitation with 3-b force

Virtual excitation of $^{10}\mathrm{Be}(2^+)$
can be simulated by 3 body force :







- 3-b force can efficiently simulate ^{10}Be excitation

  [P.C., Phillips & Hammer PLB 825, 136847 (2022)]

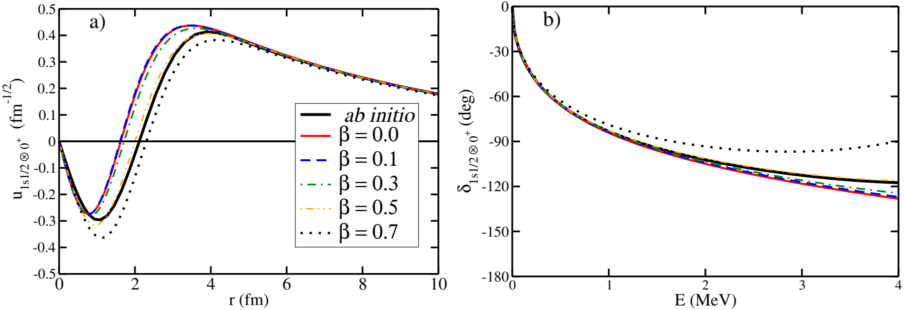
- Range in the c-T distance should equal that of V_{cT} $R_0 = 3.5$ fm
  - too small ($R_0 = 2$ fm) : no effect
  - too large ($R_0 = 6$ fm) : erroneous angular distribution

Including core excitation in Halo-EFT (PhD Kubushishi)

To account for core excitation within Halo-EFT :

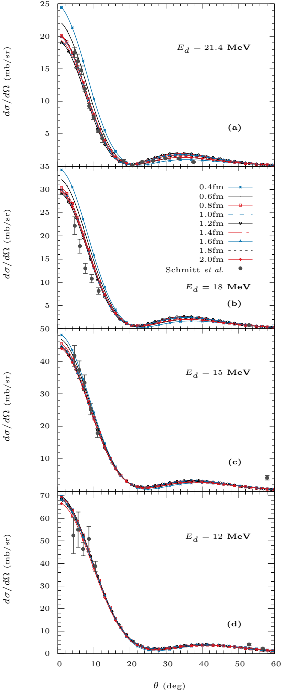
- ^{10}Be seen as deformed rotor [Nunes *et al.* NPA 596, 171 (1996)]
- deformation β treated perturbatively to couple 0^+ and 2^+_1 states
- equations solved with R-matrix using Lagrange radial mesh

[L.-P. Kubushishi & P.C. (in preparation)]



- $\beta \sim 0.5$ improves agreement with *ab initio* radial wave function
- improves $\delta_{1/2^+}$ up to 4 MeV (similar results $\forall\sigma$)
- Stay tuned for reaction calculations. . .

^{10}Be(d,p)^{11}Be

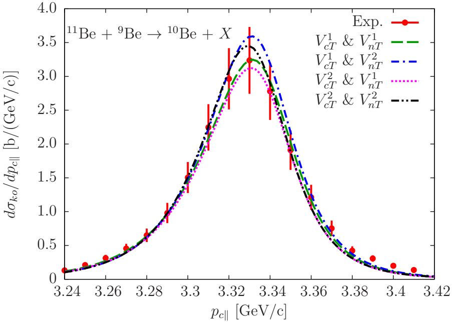


- This idea can be extended to transfer
  [Yang & P.C. PRC 98, 054602 (2018)]
- Various descriptions of ^{11}Be (@ LO) with $\sigma = 0.4 - 2.0$ fm show that ^{10}Be(d,p)^{11}Be is peripheral at fwd angle and low E_d
- This enables to reliably infer ^{11}Be ANC
  Provides a value identical to *ab initio*
- Excellent agreement with data
  Schmidt *et al.* PRL 108, 192701 (2012)]

^{11}Be+^{9}Be→^{10}Be+X @ 60AMeV

Using Halo-EFT within eikonal model of KO gives also good results

[Hebborn & P.C. PRC 100, 054607 (2019), *ibid* 104, 024616 (2021)]



- Excellent agreement with experiment [Aumann PRL 84, 35 (2000)]
- Wave functions with same ANC give same σ_{KO} ⇒ peripheral
- Insensitive to description of continuum ⇒ good probe of ANC
- For deeply bound projectile $\sigma_{\mathrm{KO}} \propto r^2_{\mathrm{rms}}$ ⇒ not SF...

[Hebborn & P.C. PLB 848, 138413 (2024)]

Summary and prospect

- Halo nuclei studied mostly through reactions
- Mechanism of reactions with halo nuclei understood
  How to relate *ab initio* calculations to reaction observables ?
  Halo EFT : [P.C., Phillips, Hammer, PRC 98, 034610 (2018)]
  Efficient way to include the significant degrees of freedom
- Using one Halo-EFT description of ^{11}Be, we reproduce
  - Breakup : [P.C., Phillips, Hammer, PRC 98, 034610 (2018)]
    - On Pb : only ANC and δ_{lj} matter
    - On C : effect of core excitation [Kubushishi, P.C. in preparation]
  - ^{10}Be(d,p) : [Yang & P.C., PRC 98, 054602 (2018)]
  - KO : [Hebborn, P.C., PRC 104, 024616 (2021)]
- Validate the *ab initio* predictions
- Same results on ^{15}C : [Moschini, Yang & P.C., PRC 100, 044615 (2019)]
- Future :
  - Include Halo EFT with core excitation in reaction models
  - Extend to other nuclei (e.g., ^{31}Ne)

Thanks to my collaborators

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