UNIVERSIDAD DE GRANADA

Study of isotope chains in a mean field model with deformation

Marta Anguiano (mangui@ugr.es)

Dpto de Física Atómica, Molecular y Nuclear (UGR)

V Gogny Conference

Paris, December 11, 2024

Work in colaboration with

- Antonio M. Lallena (lallena@ugr.es)

  Universidad de Granada (Spain)

- Giampaolo Co' (gpco@le.infn.it)

  Università del Salento (Italy)

Outline

Outline

Motivation

The method

Results: even-even nuclei

Results: even-odd nuclei

Conclusions

Outline

Outline

Outline

Motivation

The method

Results: even-even nuclei

Results: even-odd nuclei

Conclusions

Overview

Motivation

The method

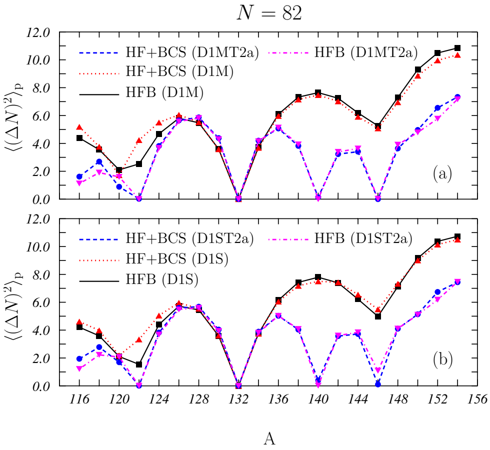
Results: even-even nuclei

Results: even-odd nuclei

Conclusions

Introduction

- HF+BCS using finite range interactions with tensor force



M.A *et al.* Eur. Phys. J. A (2016) 52: 183

Introduction

- Only for spherical even-even nuclei
- How to extend the model to study deformed even-even and odd nuclei?
- S.p. wave functions whose radial parts depend on the projection of the angular momentum on the quantisation axis $\Longrightarrow m$
- For a fixed value of j_k, states with smaller value of $|m_k|$ are more bound in case of prolate deformations, and the inverse happens in oblate nuclei.
- Slater determinant of the odd-even nucleus by adding one single nucleon on a specific s.p. level $\Longrightarrow$ blocking effect.

Introduction

- Only for spherical even-even nuclei
- How to extend the model to study deformed even-even and odd nuclei?
- S.p. wave functions whose radial parts depend on the projection of the angular momentum on the quantisation axis $\Longrightarrow m$
- For a fixed value of j_k, states with smaller value of $|m_k|$ are more bound in case of prolate deformations, and the inverse happens in oblate nuclei.
- Slater determinant of the odd-even nucleus by adding one single nucleon on a specific s.p. level $\Longrightarrow$ blocking effect.

Introduction

- Only for spherical even-even nuclei
- How to extend the model to study deformed even-even and odd nuclei?
- S.p. wave functions whose radial parts depend on the projection of the angular momentum on the quantisation axis $\Longrightarrow m$
- For a fixed value of j_k, states with smaller value of $|m_k|$ are more bound in case of prolate deformations, and the inverse happens in oblate nuclei.
- Slater determinant of the odd-even nucleus by adding one single nucleon on a specific s.p. level $\Longrightarrow$ blocking effect.

Introduction

- Only for spherical even-even nuclei
- How to extend the model to study deformed even-even and odd nuclei?
- S.p. wave functions whose radial parts depend on the projection of the angular momentum on the quantisation axis $\Longrightarrow m$
- For a fixed value of j_k, states with smaller value of $|m_k|$ are more bound in case of prolate deformations, and the inverse happens in oblate nuclei.
- Slater determinant of the odd-even nucleus by adding one single nucleon on a specific s.p. level $\Longrightarrow$ blocking effect.

Introduction

- Only for spherical even-even nuclei
- How to extend the model to study deformed even-even and odd nuclei?
- S.p. wave functions whose radial parts depend on the projection of the angular momentum on the quantisation axis $\Longrightarrow m$
- For a fixed value of j_k, states with smaller value of $|m_k|$ are more bound in case of prolate deformations, and the inverse happens in oblate nuclei.
- Slater determinant of the odd-even nucleus by adding one single nucleon on a specific s.p. level $\Longrightarrow$ blocking effect.

Overview

Our mean-field (HFBCS) approximation

(G. Co', M.A and A.M. Lallena, Phys. Rev. C104 (2021) 014313)

- We considere as effective nucleon-nucleon interaction a finite-range two-body force of the type:

$$V(\vec{r_1},\vec{r_2}) = \sum_{p=1}^{6} V_p(\vec{r_1},\vec{r_2}) O_p(1,2) + V_{\rm SO}(\vec{r_1},\vec{r_2}) + V_{\rm DD}(\vec{r_1},\vec{r_2}) + V_{\rm Coul}(\vec{r_1},\vec{r_2})$$

- $O_p(1,2)$ indicates $\mathbb{1}$, $\vec{\sigma_1}\cdot\vec{\sigma_2}$, $\vec{\tau_1}\cdot\vec{\tau_2}$, $\vec{\sigma_1}\cdot\vec{\sigma_2}\,\vec{\tau_1}\cdot\vec{\tau_2}$, S_{12}, $S_{12}\,\vec{\tau_1}\cdot\vec{\tau_2}$.
- $V_{\rm SO}$ and $V_{\rm DD}$, terms of zero-range.
- $V_p(\vec{r_1},\vec{r_2})$, finite range terms: Gaussians, Yukawians, etc.

Our mean-field (HFBCS) approximation

(G. Co', M.A and A.M. Lallena, Phys. Rev. C104 (2021) 014313)

- We considere as effective nucleon-nucleon interaction a finite-range two-body force of the type:

$$V(\vec{r_1},\vec{r_2}) = \sum_{p=1}^{6} V_p(\vec{r_1},\vec{r_2}) O_p(1,2) + V_{\rm SO}(\vec{r_1},\vec{r_2}) + V_{\rm DD}(\vec{r_1},\vec{r_2}) + V_{\rm Coul}(\vec{r_1},\vec{r_2})$$

- $O_p(1,2)$ indicates $\mathbb{1}$, $\vec{\sigma_1}\cdot\vec{\sigma_2}$, $\vec{\tau_1}\cdot\vec{\tau_2}$, $\vec{\sigma_1}\cdot\vec{\sigma_2}\,\vec{\tau_1}\cdot\vec{\tau_2}$, S_{12}, $S_{12}\,\vec{\tau_1}\cdot\vec{\tau_2}$.
- $V_{\rm SO}$ and $V_{\rm DD}$, terms of zero-range.
- $V_p(\vec{r_1},\vec{r_2})$, finite range terms: Gaussians, Yukawians, etc.

Our mean-field (HFBCS) approximation

(G. Co', M.A and A.M. Lallena, Phys. Rev. C104 (2021) 014313)

- We considere as effective nucleon-nucleon interaction a finite-range two-body force of the type:

$$V(\vec{r_1},\vec{r_2}) = \sum_{p=1}^{6} V_p(\vec{r_1},\vec{r_2}) O_p(1,2) + V_{\rm SO}(\vec{r_1},\vec{r_2}) + V_{\rm DD}(\vec{r_1},\vec{r_2}) + V_{\rm Coul}(\vec{r_1},\vec{r_2})$$

- $O_p(1,2)$ indicates $\mathbb{1},\ \vec{\sigma_1}\cdot\vec{\sigma_2},\ \vec{\tau_1}\cdot\vec{\tau_2},\ \vec{\sigma_1}\cdot\vec{\sigma_2}\,\vec{\tau_1}\cdot\vec{\tau_2},\ S_{12},\ S_{12}\,\vec{\tau_1}\cdot\vec{\tau_2}$.
- $V_{\rm SO}$ and $V_{\rm DD}$, terms of zero-range.
- $V_p(\vec{r_1},\vec{r_2})$, finite range terms: Gaussians, Yukawians, etc.

Our mean-field (HFBCS) approximation

(G. Co', M.A and A.M. Lallena, Phys. Rev. C104 (2021) 014313)

- We considere as effective nucleon-nucleon interaction a finite-range two-body force of the type:

$$V(\vec{r_1},\vec{r_2}) = \sum_{p=1}^{6} V_p(\vec{r_1},\vec{r_2}) O_p(1,2) + V_{\rm SO}(\vec{r_1},\vec{r_2}) + V_{\rm DD}(\vec{r_1},\vec{r_2}) + V_{\rm Coul}(\vec{r_1},\vec{r_2})$$

- $O_p(1,2)$ indicates $\mathbb{1},\ \vec{\sigma_1}\cdot\vec{\sigma_2},\ \vec{\tau_1}\cdot\vec{\tau_2},\ \vec{\sigma_1}\cdot\vec{\sigma_2}\,\vec{\tau_1}\cdot\vec{\tau_2},\ S_{12},\ S_{12}\,\vec{\tau_1}\cdot\vec{\tau_2}$.
- $V_{\rm SO}$ and $V_{\rm DD}$, terms of zero-range.
- $V_p(\vec{r_1},\vec{r_2})$, finite range terms: Gaussians, Yukawians, etc.

Our mean-field approximation: HFBCS

- We assume that the s.p. wave functions, $\phi_k(x)$, can be factorized:

$$\phi_k(x) \;=\; R_k(r)\,|\tilde{k}\rangle\,\chi_{t_k}$$

$x \Longrightarrow$ generalized coordinate, including **r**, spin and isospin.

- The radial part of the s.p. wave function,

$$R_k(r) \;\equiv\; R^{t_k}_{n_k l_k j_k,\, m_k}(r)\,,$$

- The part of the s.p. wave function depending on the angular coordinates, $\Omega_k \equiv (\theta_k, \phi_k)$, and on the spin third component, s_k,

$$|\tilde{k}\rangle \;\equiv\; |l_k \tfrac{1}{2} j_k m_k\rangle \;=\; \sum_{\mu_k s_k} \langle l_k \mu_k \tfrac{1}{2} s_k | j_k m_k\rangle\, Y_{l_k \mu_k}(\Omega_k)\,\chi_{s_k}\,,$$

- Time-reversal invariance $\Longrightarrow R^{t_k}_{n_k l_k j_k,\, m_k}(r) = R^{t_k}_{n_k l_k j_k,\, -m_k}(r) \Longrightarrow$ nucleus is an ellipsoid with the z axis as the symmetry axis.

Our mean-field approximation: HFBCS

- We assume that the s.p. wave functions, $\phi_k(x)$, can be factorized:

$$\phi_k(x) \;=\; R_k(r)\,|\tilde{k}\rangle\,\chi_{t_k}$$

$x \Longrightarrow$ generalized coordinate, including $\mathbf{r}$, spin and isospin.

- The radial part of the s.p. wave function,

$$R_k(r) \;\equiv\; R^{t_k}_{n_k l_k j_k,\, m_k}(r)\,,$$

- The part of the s.p. wave function depending on the angular coordinates, $\Omega_k \equiv (\theta_k, \phi_k)$, and on the spin third component, s_k,

$$|\tilde{k}\rangle \;\equiv\; |l_k \tfrac{1}{2} j_k m_k\rangle \;=\; \sum_{\mu_k s_k} \langle l_k \mu_k \tfrac{1}{2} s_k | j_k m_k\rangle\, Y_{l_k \mu_k}(\Omega_k)\,\chi_{s_k}\,,$$

- Time-reversal invariance $\Longrightarrow R^{t_k}_{n_k l_k j_k,\, m_k}(r) = R^{t_k}_{n_k l_k j_k,\, -m_k}(r) \Longrightarrow$ nucleus is an ellipsoid with the z axis as the symmetry axis.

Our mean-field approximation: HFBCS

- We assume that the s.p. wave functions, $\phi_k(x)$, can be factorized:

$$\phi_k(x) \; = \; R_k(r)\,|\tilde{k}\rangle\,\chi_{t_k}$$

$x \Longrightarrow$ generalized coordinate, including $\mathbf{r}$, spin and isospin.

- The radial part of the s.p. wave function,

$$R_k(r) \; \equiv \; R^{t_k}_{n_k l_k j_k,\, m_k}(r)\,,$$

- The part of the s.p. wave function depending on the angular coordinates, $\Omega_k \equiv (\theta_k, \phi_k)$, and on the spin third component, s_k,

$$|\tilde{k}\rangle \; \equiv \; |l_k \frac{1}{2} j_k m_k\rangle \; = \; \sum_{\mu_k s_k} \langle l_k \mu_k \frac{1}{2} s_k | j_k m_k\rangle \, Y_{l_k \mu_k}(\Omega_k)\, \chi_{s_k}\,,$$

- Time-reversal invariance $\Longrightarrow R^{t_k}_{n_k l_k j_k,\, m_k}(r) = R^{t_k}_{n_k l_k j_k,\, -m_k}(r) \Longrightarrow$ nucleus is an ellipsoid with the z axis as the symmetry axis.

Our mean-field approximation: HFBCS

- We assume that the s.p. wave functions, $\phi_k(x)$, can be factorized:

$$\phi_k(x) \; = \; R_k(r)\, |\tilde{k}\rangle \, \chi_{t_k}$$

$x \Longrightarrow$ generalized coordinate, including $\mathbf{r}$, spin and isospin.

- The radial part of the s.p. wave function,

$$R_k(r) \; \equiv \; R^{t_k}_{n_k l_k j_k,\, m_k}(r)\,,$$

- The part of the s.p. wave function depending on the angular coordinates, $\Omega_k \equiv (\theta_k, \phi_k)$, and on the spin third component, s_k,

$$|\tilde{k}\rangle \; \equiv \; |l_k \frac{1}{2} j_k m_k\rangle \; = \; \sum_{\mu_k s_k} \langle l_k \mu_k \frac{1}{2} s_k | j_k m_k\rangle \, Y_{l_k \mu_k}(\Omega_k)\, \chi_{s_k}\,,$$

- Time-reversal invariance $\Longrightarrow R^{t_k}_{n_k l_k j_k,\, m_k}(r) = R^{t_k}_{n_k l_k j_k,\, -m_k}(r) \Longrightarrow$ nucleus is an ellipsoid with the z axis as the symmetry axis.

Our mean-field approximation: HFBCS

- We solve, in coordinate space, a set of equations of the type:

$$\left[\langle\tilde{k}| - \frac{\hbar^2}{2m}\nabla^2|\tilde{k}\rangle + \mathcal{U}_k(r_1) + \mathcal{K}(r_1) \right] R_k(r_1) - \int \mathrm{d}r_2 r_2^2 \mathcal{W}_k(r_1,r_2)\, R_k(r_2) = \epsilon_k\, R_k(r_1)$$

- Hartree (Direct) term

$$\mathcal{U}_k(r_1) = \sum_{i=1}^{A} v_i^2 \int \mathrm{d}r_2\, r_2^2\, R_i^2(r_2)\, \langle\tilde{k}\tilde{i}|V(\mathbf{r}_1,\mathbf{r}_2)|\tilde{k}\tilde{i}\rangle$$

- Fock-Dirac term

$$\mathcal{W}_k(r_1,r_2) = \sum_{i=1}^{A} v_i^2 \left[R_i^*(r_2)\, R_i(r_1)\, \langle\tilde{k}\tilde{i}|V(\mathbf{r}_1,\mathbf{r}_2)|\tilde{i}\tilde{k}\rangle \right]$$

Our mean-field approximation: HFBCS

- We solve, in coordinate space, a set of equations of the type:

$$\left[\langle\tilde{k}| - \frac{\hbar^2}{2m}\nabla^2|\tilde{k}\rangle + \mathcal{U}_k(r_1) + \mathcal{K}(r_1) \right] R_k(r_1) - \int \mathrm{d}r_2 r_2^2 \mathcal{W}_k(r_1, r_2)\, R_k(r_2) = \epsilon_k\, R_k(r_1)$$

- Hartree (Direct) term

$$\mathcal{U}_k(r_1) = \sum_{i=1}^{A} v_i^2 \int \mathrm{d}r_2\, r_2^2\, R_i^2(r_2)\, \langle\tilde{k}\tilde{i}|V(\mathbf{r}_1, \mathbf{r}_2)|\tilde{k}\tilde{i}\rangle$$

- Fock-Dirac term

$$\mathcal{W}_k(r_1, r_2) = \sum_{i=1}^{A} v_i^2 \left[R_i^*(r_2)\, R_i(r_1)\, \langle\tilde{k}\tilde{i}|V(\mathbf{r}_1, \mathbf{r}_2)|\tilde{i}\tilde{k}\rangle \right]$$

Our mean-field approximation: HFBCS

- We solve, in coordinate space, a set of equations of the type:

$$\left[\langle \tilde{k}| - \frac{\hbar^2}{2m}\nabla^2|\tilde{k}\rangle + \mathcal{U}_k(r_1) + \mathcal{K}(r_1) \right] R_k(r_1) - \int \mathrm{d}r_2 r_2^2 \mathcal{W}_k(r_1,r_2)\, R_k(r_2) = \epsilon_k\, R_k(r_1)$$

- Hartree (Direct) term

$$\mathcal{U}_k(r_1) = \sum_{i=1}^{A} v_i^2 \int \mathrm{d}r_2\, r_2^2\, R_i^2(r_2)\, \langle \tilde{k}\tilde{i}|V(\mathbf{r}_1,\mathbf{r}_2)|\tilde{k}\tilde{i}\rangle$$

- Fock-Dirac term

$$\mathcal{W}_k(r_1,r_2) = \sum_{i=1}^{A} v_i^2 \left[R_i^*(r_2)\, R_i(r_1)\, \langle \tilde{k}\tilde{i}|V(\mathbf{r}_1,\mathbf{r}_2)|\tilde{i}\tilde{k}\rangle \right]$$

Our mean field approximation: HFBCS

- Density-dependent term:

$$\mathcal{K}(r_1) = \frac{1}{4\pi} \sum_{i,j=1}^{A} v_i^2\, v_j^2 \int \mathrm{d}r_2\, r_2^2 \left[R_i^*(r_1)\, R_j^*(r_2)\, \langle \widetilde{ij} | \frac{\partial V(\mathbf{r}_1, \mathbf{r}_2)}{\partial \rho} | \widetilde{ij} \rangle\, R_i(r_1)\, R_j(r_2) - R_i^*(r_1)\, R_j^*(r_2)\, \langle \widetilde{ij} | \frac{\partial V(\mathbf{r}_1, \mathbf{r}_2)}{\partial \rho} | \widetilde{ji} \rangle\, R_j(r_1)\, R_i(r_2) \right]$$

- Total energy of an even-even nucleus:

$$E(A,Z) = \sum_k v_k^2\, \epsilon_k - \frac{1}{2} \sum_k v_k^2 \int_0^\infty \mathrm{d}r_1\, r_1^2\, \left[\mathcal{U}_k(r_1) + 2\,\mathcal{K}(r_1)\right] R_k^2(r_1) + \frac{1}{2} \sum_k v_k^2 \int_0^\infty \mathrm{d}r_1\, \mathrm{d}r_2\, r_1^2\, r_2^2\, \mathcal{W}_k(r_1, r_2) \times R_k(r_1)\, R_k(r_2)$$

Our mean field approximation: HFBCS

- Density-dependent term:

$$\mathcal{K}(r_1) = \frac{1}{4\pi} \sum_{i,j=1}^{A} v_i^2\, v_j^2 \int \mathrm{d}r_2\, r_2^2 \left[R_i^*(r_1)\, R_j^*(r_2)\, \langle \widetilde{ij} | \frac{\partial V(\mathbf{r}_1, \mathbf{r}_2)}{\partial \rho} | \widetilde{ij} \rangle\, R_i(r_1)\, R_j(r_2) - R_i^*(r_1)\, R_j^*(r_2)\, \langle \widetilde{ij} | \frac{\partial V(\mathbf{r}_1, \mathbf{r}_2)}{\partial \rho} | \widetilde{ji} \rangle\, R_j(r_1)\, R_i(r_2) \right]$$

- Total energy of an even-even nucleus:

$$E(A,Z) = \sum_k v_k^2\, \epsilon_k - \frac{1}{2} \sum_k v_k^2 \int_0^\infty \mathrm{d}r_1\, r_1^2\, [\mathcal{U}_k(r_1) + 2\,\mathcal{K}(r_1)]\, R_k^2(r_1) + \frac{1}{2} \sum_k v_k^2 \int_0^\infty \mathrm{d}r_1\, \mathrm{d}r_2\, r_1^2\, r_2^2\, \mathcal{W}_k(r_1, r_2) \times R_k(r_1)\, R_k(r_2)$$

Our mean field approximation: HFBCS

- Nuclear density ⇒ Multipole expansion:

$$\rho^{\alpha}(\mathbf{r}) = \sum_k |\phi_k(x)|^2 = \sum_L \rho_L^{\alpha}(r)\, Y_{L0}(\Omega)$$

- Radii:

$$R_\alpha = \left[\frac{\int d^3r\, r^2\, \rho^{\alpha}(\mathbf{r})}{\int d^3r\, \rho^{\alpha}(\mathbf{r})}\right]^{\frac{1}{2}} = \left[\frac{\int dr\, r^4\, \rho_0^{\alpha}(r)}{\int dr\, r^2\, \rho_0^{\alpha}(r)}\right]^{\frac{1}{2}}, \quad \alpha \equiv p, n$$

- Nuclear deformation:

$$Q_{20} = \sqrt{\frac{16\pi}{5}} \int dr\, r^4 \rho_2(r) \quad \beta_2 = \sqrt{\frac{5\pi}{9}} \frac{1}{AR_0^2} Q_{20} \quad R_0 = 1.2\, A^{1/3}$$

Our mean field approximation: HFBCS

- Nuclear density $\Rightarrow$ Multipole expansion:

$$\rho^{\alpha}(\mathbf{r}) = \sum_{k} |\phi_k(x)|^2 = \sum_{L} \rho_L^{\alpha}(r)\, Y_{L0}(\Omega)$$

- Radii:

$$R_{\alpha} = \left[\frac{\int d^3r\, r^2\, \rho^{\alpha}(\mathbf{r})}{\int d^3r\, \rho^{\alpha}(\mathbf{r})} \right]^{\frac{1}{2}} = \left[\frac{\int dr\, r^4\, \rho_0^{\alpha}(r)}{\int dr\, r^2\, \rho_0^{\alpha}(r)} \right]^{\frac{1}{2}}, \quad \alpha \equiv \mathrm{p}, \mathrm{n}$$

- Nuclear deformation:

$$Q_{20} = \sqrt{\frac{16\pi}{5}} \int dr\, r^4 \rho_2(r) \quad \beta_2 = \sqrt{\frac{5\pi}{9}} \frac{1}{AR_0^2} Q_{20} \quad R_0 = 1.2\, A^{1/3}$$

Our mean field approximation: HFBCS

- Nuclear density ⇒ Multipole expansion:

$$\rho^{\alpha}(\mathbf{r}) = \sum_{k} |\phi_k(x)|^2 = \sum_{L} \rho_L^{\alpha}(r)\, Y_{L0}(\Omega)$$

- Radii:

$$R_{\alpha} = \left[\frac{\int d^3r\, r^2\, \rho^{\alpha}(\mathbf{r})}{\int d^3r\, \rho^{\alpha}(\mathbf{r})}\right]^{\frac{1}{2}} = \left[\frac{\int dr\, r^4\, \rho_0^{\alpha}(r)}{\int dr\, r^2\, \rho_0^{\alpha}(r)}\right]^{\frac{1}{2}}, \quad \alpha \equiv \mathrm{p}, \mathrm{n}$$

- Nuclear deformation:

$$Q_{20} = \sqrt{\frac{16\pi}{5}} \int dr\, r^4 \rho_2(r) \quad \beta_2 = \sqrt{\frac{5\pi}{9}}\, \frac{1}{AR_0^2}\, Q_{20} \quad R_0 = 1.2\, A^{1/3}$$

Numerical procedure

- The contribution of the two space coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$, separated by considering the Fourier transform of the effective nucleon-nucleon interaction.
- The radial HF differential equations are solved by using the plane wave expansion technique.
- After each HF iteration, the s.p. wave functions just obtained are used in BCS equations in order to modify their occupation probabilities.
- The iterative procedure starts by using the s.p. wave functions obtained by solving the Schrödinger equation for a deformed Woods-Saxon potential:

$$V_{WS}(r,\Omega) = \frac{U_0}{1+\exp(\frac{r-R_0}{a})} + \frac{U_{so}}{r}\frac{\exp(\frac{r-R_0}{a})}{\left[1+\exp(\frac{r-R_0}{a})\right]^2}\mathbf{l}\cdot\mathbf{s} + V_C - \Lambda Y_{20}(\Omega)$$

Numerical procedure

- The contribution of the two space coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$, separated by considering the Fourier transform of the effective nucleon-nucleon interaction.
- The radial HF differential equations are solved by using the plane wave expansion technique.
- After each HF iteration, the s.p. wave functions just obtained are used in BCS equations in order to modify their occupation probabilities.
- The iterative procedure starts by using the s.p. wave functions obtained by solving the Schrödinger equation for a deformed Woods-Saxon potential:

$$V_{\mathrm{WS}}(r,\Omega) = \frac{U_0}{1+\exp(\frac{r-R_0}{a})} + \frac{U_{so}}{r}\,\frac{\exp(\frac{r-R_0}{a})}{\left[1+\exp(\frac{r-R_0}{a})\right]^2}\,\mathbf{l}\cdot\mathbf{s} + V_C - \Lambda\, Y_{20}(\Omega)$$

Numerical procedure

- The contribution of the two space coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$, separated by considering the Fourier transform of the effective nucleon-nucleon interaction.
- The radial HF differential equations are solved by using the plane wave expansion technique.
- After each HF iteration, the s.p. wave functions just obtained are used in BCS equations in order to modify their occupation probabilities.
- The iterative procedure starts by using the s.p. wave functions obtained by solving the Schrödinger equation for a deformed Woods-Saxon potential:

$$V_{WS}(r,\Omega) = \frac{U_0}{1+\exp(\frac{r-R_0}{a})} + \frac{U_{so}}{r}\frac{\exp(\frac{r-R_0}{a})}{\left[1+\exp(\frac{r-R_0}{a})\right]^2}\mathbf{l}\cdot\mathbf{s} + V_C - \Lambda Y_{20}(\Omega)$$

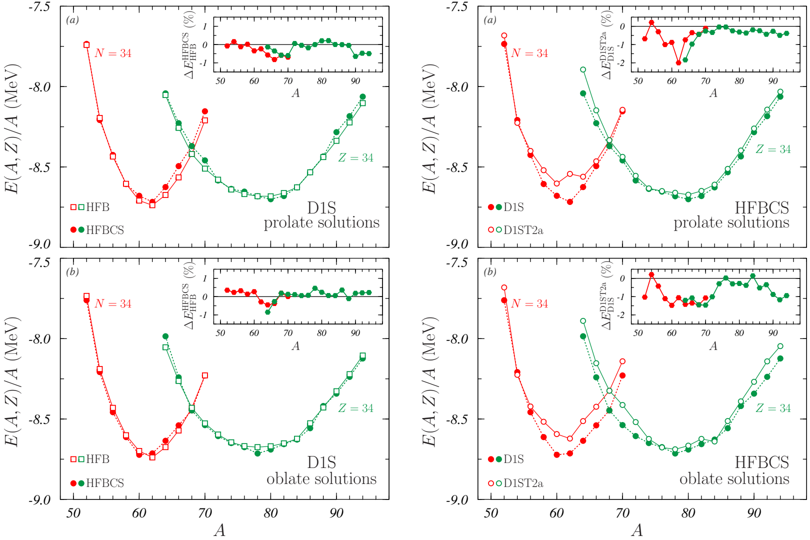
Numerical procedure

- The contribution of the two space coordinates $\mathbf{r}_1$ and $\mathbf{r}_2$, separated by considering the Fourier transform of the effective nucleon-nucleon interaction.
- The radial HF differential equations are solved by using the plane wave expansion technique.
- After each HF iteration, the s.p. wave functions just obtained are used in BCS equations in order to modify their occupation probabilities.
- The iterative procedure starts by using the s.p. wave functions obtained by solving the Schrödinger equation for a deformed Woods-Saxon potential:

$$V_{\mathrm{WS}}(r,\Omega) = \frac{U_0}{1+\exp(\frac{r-R_0}{a})} + \frac{U_{so}}{r}\,\frac{\exp(\frac{r-R_0}{a})}{\left[1+\exp(\frac{r-R_0}{a})\right]^2}\,\mathbf{l}\cdot\mathbf{s} + V_{\mathrm{C}} - \Lambda\, Y_{20}(\Omega)$$

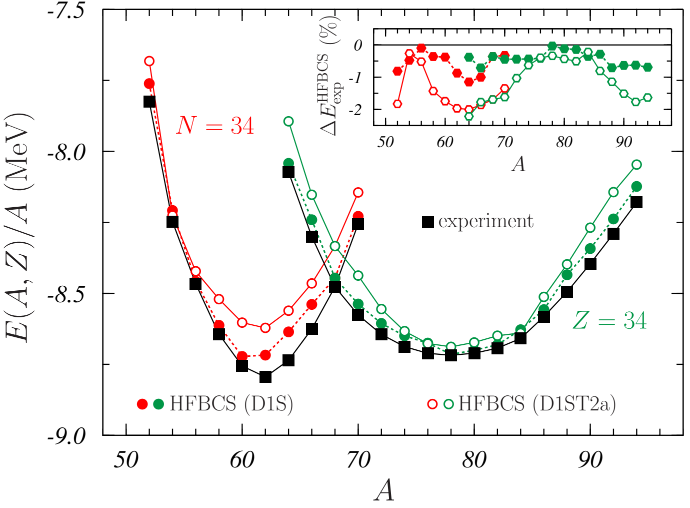
Overview

Binding energies



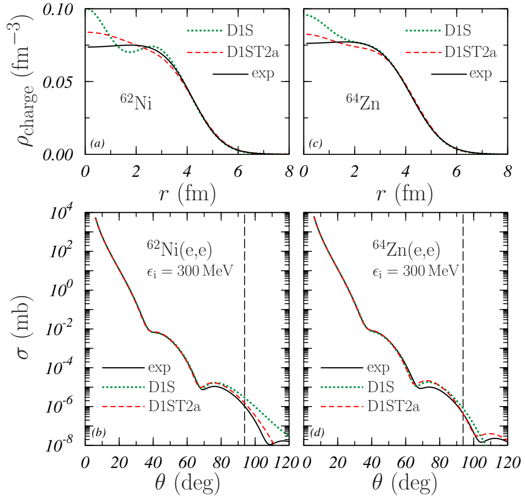
Comparison with experimental data

Brookhaven National Laboratory ⟹ **`http://www.nndc.bnl.gov/`**



Comparison with experimental data

H. De Vries, C.W. De Jager and C. De Vries, At. Data Nucl. Data Tables 36, 495 (1987).

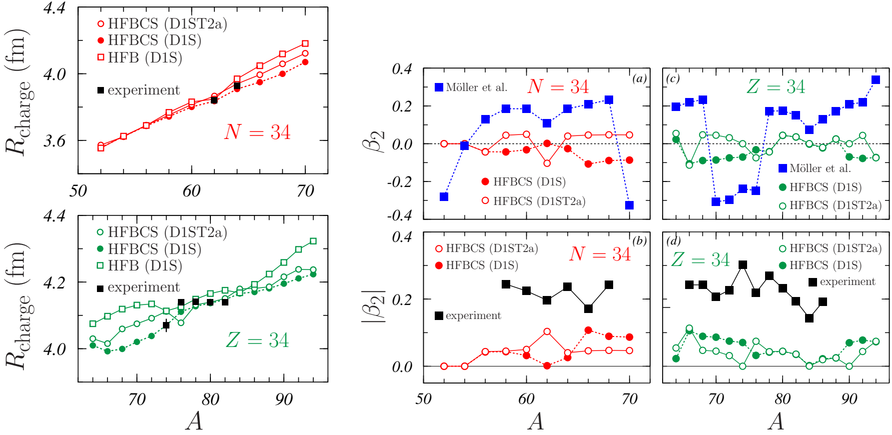


Comparison with experimental data

P. Möller *et al.*, At. Data and Nucl. Data Tab. 109 (2016) 1

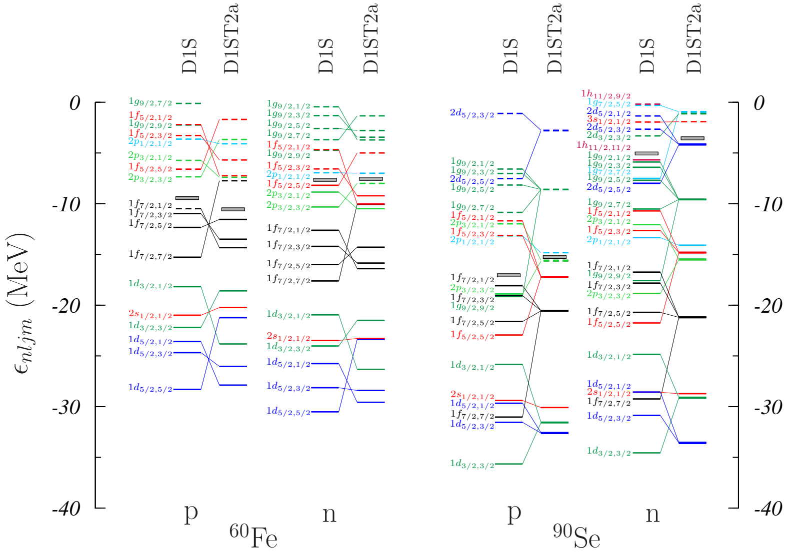
Brookhaven National Laboratory ⟹ `http://www.nndc.bnl.gov/`

I. Angeli, K. P. Marinova, At. Data and Nucl. Data Tab. 99 (2013) 69



CAUTION!!! Exp. $|\beta_2|$ considering first 2^+ excited state due to a rotation of the nucleus described by a liquid drop model ⟹ $\beta_2 = 0.353$ for ^{16}O: 2^+ state at 6.917 MeV

Single particle spectra



Overview

Approximation for even-odd nuclei

- Solving HF $\Longrightarrow$ obtaining s.p. wave functions.
- Blocking $\Longrightarrow$ forcing a full occupation of the odd nucleon s.p. wave function.
- Values of v_k^2 $\Longrightarrow$ by solving BCS equations after each HF iteration.
- BCS equations modify the v_k^2 of the other s.p. states.
- Preliminary results for some light nuclei isotopic chains: O, Ne, Mg, Si, Ar and Ca.

Approximation for even-odd nuclei

- Solving HF $\Longrightarrow$ obtaining s.p. wave functions.
- Blocking $\Longrightarrow$ forcing a full occupation of the odd nucleon s.p. wave function.
- Values of $v_k^2 \Longrightarrow$ by solving BCS equations after each HF iteration.
- BCS equations modify the v_k^2 of the other s.p. states.
- Preliminary results for some light nuclei isotopic chains: O, Ne, Mg, Si, Ar and Ca.

Approximation for even-odd nuclei

- Solving HF $\Longrightarrow$ obtaining s.p. wave functions.
- Blocking $\Longrightarrow$ forcing a full occupation of the odd nucleon s.p. wave function.
- Values of v_k^2 $\Longrightarrow$ by solving BCS equations after each HF iteration.
- BCS equations modify the v_k^2 of the other s.p. states.
- Preliminary results for some light nuclei isotopic chains: O, Ne, Mg, Si, Ar and Ca.

Approximation for even-odd nuclei

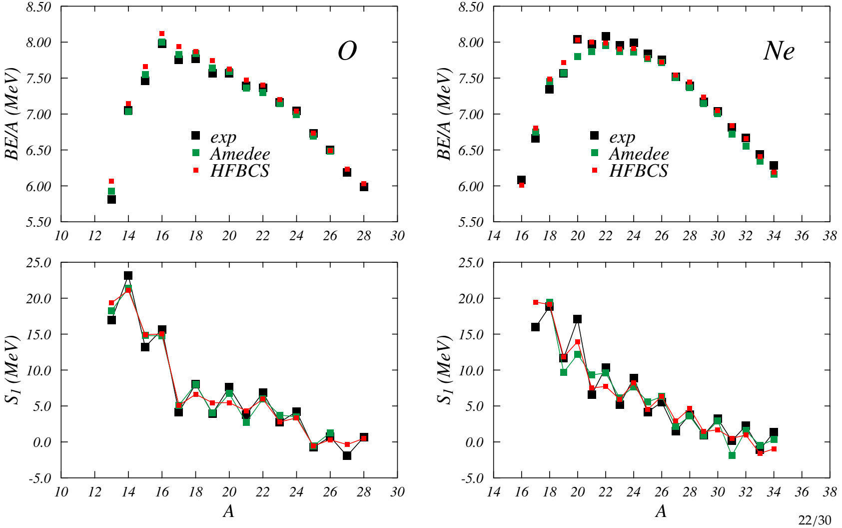
- Solving HF $\Longrightarrow$ obtaining s.p. wave functions.
- Blocking $\Longrightarrow$ forcing a full occupation of the odd nucleon s.p. wave function.
- Values of $v_k^2 \Longrightarrow$ by solving BCS equations after each HF iteration.
- BCS equations modify the v_k^2 of the other s.p. states.
- Preliminary results for some light nuclei isotopic chains: O, Ne, Mg, Si, Ar and Ca.

Approximation for even-odd nuclei

- Solving HF $\Longrightarrow$ obtaining s.p. wave functions.
- Blocking $\Longrightarrow$ forcing a full occupation of the odd nucleon s.p. wave function.
- Values of v_k^2 $\Longrightarrow$ by solving BCS equations after each HF iteration.
- BCS equations modify the v_k^2 of the other s.p. states.
- Preliminary results for some light nuclei isotopic chains: O, Ne, Mg, Si, Ar and Ca.

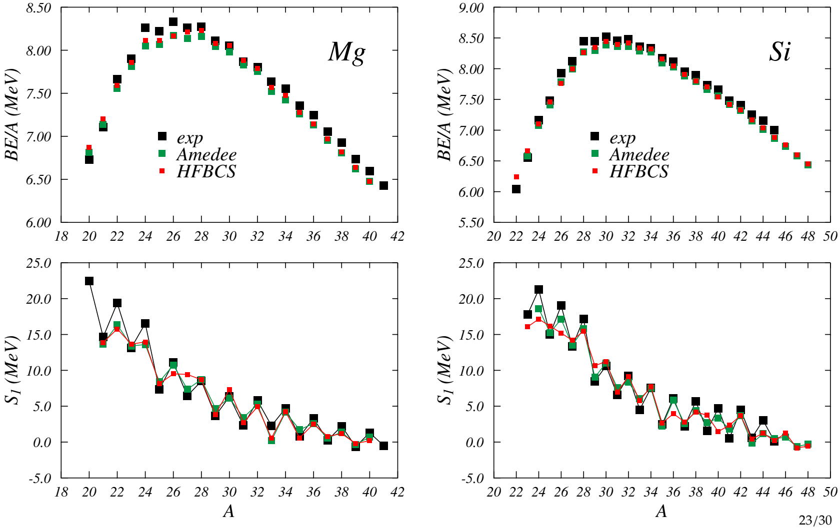
Binding and separation energies

D1S Interaction



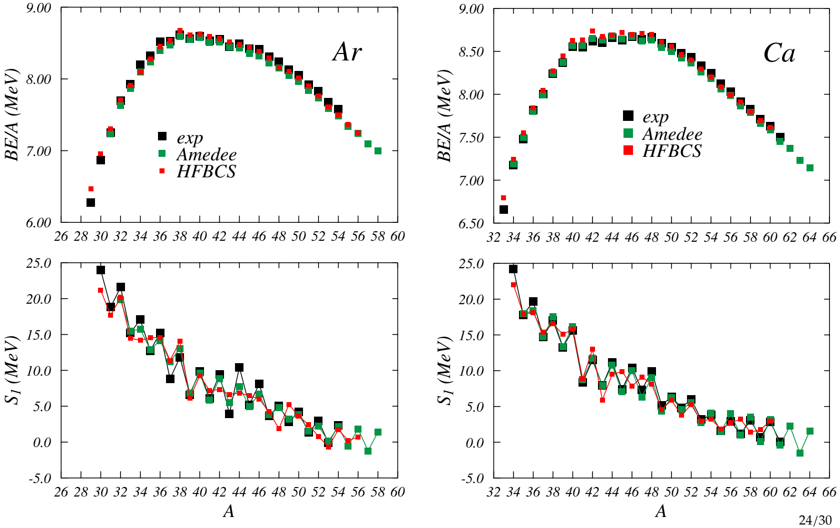
Binding and separation energies

D1S Interaction



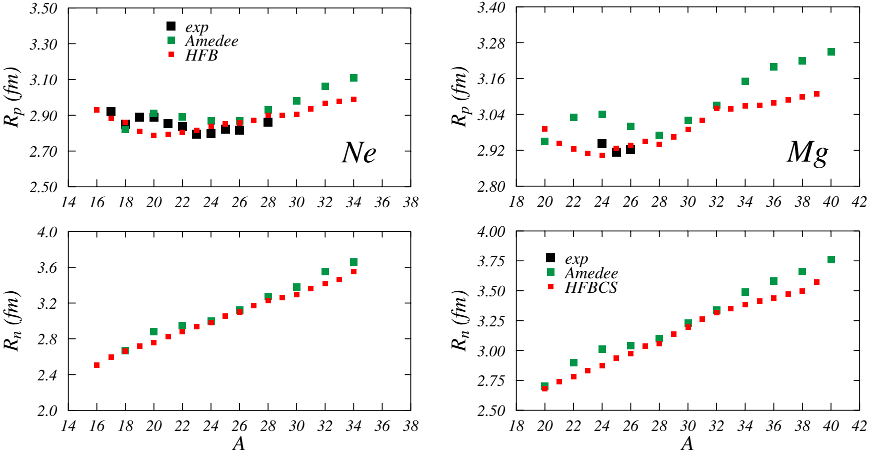
Binding and separation energies

D1S Interaction



Nuclear radii

D1S Interaction

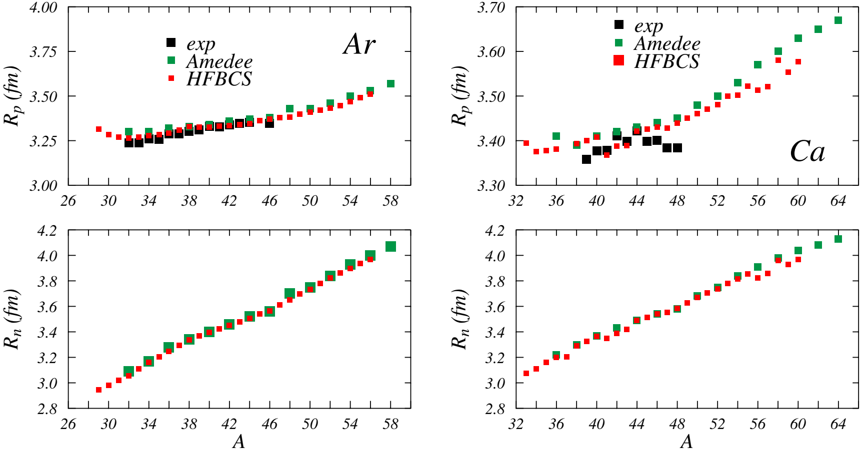


J.-P. Delaroche *et al.* Phys. Rev. C81 (2010) 014303:

$$R_p^2 = R_{ch}^2 - (0.8775\,\text{fm})^2 + 0.1148\,\frac{N}{Z}\,\text{fm}^2 - 0.033\,\text{fm}^2$$

Nuclear radii

D1S Interaction



Overview

Main results

- Model to describe open shell nuclei, even-even and even-odd ones, using finite range efective interactions including tensor terms.
- Variational principle + Slater determinants built with s.p wave functions whose radial part depends on $m \Longrightarrow$ deformation.
- Two steps in the iterative procedure of the method, **HFBCS**:
  - Solving HF equations to generate the s.p. wave functions.
  - Solving BCS equations to obtain the occupation probabilities of the s.p. states.
- For each nucleus, oblate and prolate solutions $\Longrightarrow$ **optimal solution**, the smallest energy value.
- **Optimal** solutions with tensor force are less deformed.
- Very small values of the deformation parameter $|\beta_2|$.
- Concerning binding energies, separation energies and radii, good results for even-odd nuclei.

Main results

- Model to describe open shell nuclei, even-even and even-odd ones, using finite range efective interactions including tensor terms.
- Variational principle + Slater determinants built with s.p wave functions whose radial part depends on $m \Longrightarrow$ deformation.
- Two steps in the iterative procedure of the method, **HFBCS**:
  - Solving HF equations to generate the s.p. wave functions.
  - Solving BCS equations to obtain the occupation probabilities of the s.p. states.
- For each nucleus, oblate and prolate solutions $\Longrightarrow$ **optimal solution**, the smallest energy value.
- **Optimal** solutions with tensor force are less deformed.
- Very small values of the deformation parameter $|\beta_2|$.
- Concerning binding energies, separation energies and radii, good results for even-odd nuclei.

Main results

- Model to describe open shell nuclei, even-even and even-odd ones, using finite range efective interactions including tensor terms.
- Variational principle + Slater determinants built with s.p wave functions whose radial part depends on $m \Longrightarrow$ deformation.
- Two steps in the iterative procedure of the method, **HFBCS**:
  - Solving HF equations to generate the s.p. wave functions.
  - Solving BCS equations to obtain the occupation probabilities of the s.p. states.
- For each nucleus, oblate and prolate solutions $\Longrightarrow$ **optimal solution**, the smallest energy value.
- **Optimal** solutions with tensor force are less deformed.
- Very small values of the deformation parameter $|\beta_2|$.
- Concerning binding energies, separation energies and radii, good results for even-odd nuclei.

Main results

- Model to describe open shell nuclei, even-even and even-odd ones, using finite range efective interactions including tensor terms.
- Variational principle + Slater determinants built with s.p wave functions whose radial part depends on $m \Longrightarrow$ deformation.
- Two steps in the iterative procedure of the method, **HFBCS**:
  - Solving HF equations to generate the s.p. wave functions.
  - Solving BCS equations to obtain the occupation probabilities of the s.p. states.
- For each nucleus, oblate and prolate solutions $\Longrightarrow$ **optimal solution**, the smallest energy value.
- **Optimal** solutions with tensor force are less deformed.
- Very small values of the deformation parameter $|\beta_2|$.
- Concerning binding energies, separation energies and radii, good results for even-odd nuclei.

Main results

- Model to describe open shell nuclei, even-even and even-odd ones, using finite range efective interactions including tensor terms.
- Variational principle + Slater determinants built with s.p wave functions whose radial part depends on $m \Longrightarrow$ deformation.
- Two steps in the iterative procedure of the method, **HFBCS**:
  - Solving HF equations to generate the s.p. wave functions.
  - Solving BCS equations to obtain the occupation probabilities of the s.p. states.
- For each nucleus, oblate and prolate solutions $\Longrightarrow$ **optimal solution**, the smallest energy value.
- **Optimal** solutions with tensor force are less deformed.
- Very small values of the deformation parameter $|\beta_2|$.
- Concerning binding energies, separation energies and radii, good results for even-odd nuclei.

Main results

- Model to describe open shell nuclei, even-even and even-odd ones, using finite range efective interactions including tensor terms.
- Variational principle + Slater determinants built with s.p wave functions whose radial part depends on $m \Longrightarrow$ deformation.
- Two steps in the iterative procedure of the method, **HFBCS**:
  - Solving HF equations to generate the s.p. wave functions.
  - Solving BCS equations to obtain the occupation probabilities of the s.p. states.
- For each nucleus, oblate and prolate solutions $\Longrightarrow$ **optimal solution**, the smallest energy value.
- **Optimal** solutions with tensor force are less deformed.
- Very small values of the deformation parameter $|\beta_2|$.
- Concerning binding energies, separation energies and radii, good results for even-odd nuclei.

Main results

- Model to describe open shell nuclei, even-even and even-odd ones, using finite range efective interactions including tensor terms.
- Variational principle + Slater determinants built with s.p wave functions whose radial part depends on $m \Longrightarrow$ deformation.
- Two steps in the iterative procedure of the method, **HFBCS**:
  - Solving HF equations to generate the s.p. wave functions.
  - Solving BCS equations to obtain the occupation probabilities of the s.p. states.
- For each nucleus, oblate and prolate solutions $\Longrightarrow$ **optimal solution**, the smallest energy value.
- **Optimal** solutions with tensor force are less deformed.
- Very small values of the deformation parameter $|\beta_2|$.
- Concerning binding energies, separation energies and radii, good results for even-odd nuclei.

Main results

- Model to describe open shell nuclei, even-even and even-odd ones, using finite range efective interactions including tensor terms.
- Variational principle + Slater determinants built with s.p wave functions whose radial part depends on $m \Longrightarrow$ deformation.
- Two steps in the iterative procedure of the method, **HFBCS**:
  - Solving HF equations to generate the s.p. wave functions.
  - Solving BCS equations to obtain the occupation probabilities of the s.p. states.
- For each nucleus, oblate and prolate solutions $\Longrightarrow$ **optimal solution**, the smallest energy value.
- **Optimal** solutions with tensor force are less deformed.
- Very small values of the deformation parameter $|\beta_2|$.
- Concerning binding energies, separation energies and radii, good results for even-odd nuclei.

Main results

- Model to describe open shell nuclei, even-even and even-odd ones, using finite range efective interactions including tensor terms.
- Variational principle + Slater determinants built with s.p wave functions whose radial part depends on $m \Longrightarrow$ deformation.
- Two steps in the iterative procedure of the method, **HFBCS**:
  - Solving HF equations to generate the s.p. wave functions.
  - Solving BCS equations to obtain the occupation probabilities of the s.p. states.
- For each nucleus, oblate and prolate solutions $\Longrightarrow$ **optimal solution**, the smallest energy value.
- **Optimal** solutions with tensor force are less deformed.
- Very small values of the deformation parameter $|\beta_2|$.
- Concerning binding energies, separation energies and radii, good results for even-odd nuclei.

Future work

- A global fit of the effective force containing tensor terms is needed to have a good accuracy of the experimental data on binding energies, charge radii and distributions.
- Our HFBC approach shows s.p. properties still well recognizable.
- Set of s.p. wave functions with their occupation probabilities $\Longrightarrow$ starting point to build up a Deformed Quasi-Particle Random Phase Approximation.
- Extension of the method to study odd-odd nuclei.

Future work

- A global fit of the effective force containing tensor terms is needed to have a good accuracy of the experimental data on binding energies, charge radii and distributions.
- Our HFBC approach shows s.p. properties still well recognizable.
- Set of s.p. wave functions with their occupation probabilities $\Longrightarrow$ starting point to build up a Deformed Quasi-Particle Random Phase Approximation.
- Extension of the method to study odd-odd nuclei.

Future work

- A global fit of the effective force containing tensor terms is needed to have a good accuracy of the experimental data on binding energies, charge radii and distributions.
- Our HFBC approach shows s.p. properties still well recognizable.
- Set of s.p. wave functions with their occupation probabilities $\Longrightarrow$ starting point to build up a Deformed Quasi-Particle Random Phase Approximation.
- Extension of the method to study odd-odd nuclei.

Future work

- A global fit of the effective force containing tensor terms is needed to have a good accuracy of the experimental data on binding energies, charge radii and distributions.

- Our HFBC approach shows s.p. properties still well recognizable.

- Set of s.p. wave functions with their occupation probabilities $\Longrightarrow$ starting point to build up a Deformed Quasi-Particle Random Phase Approximation.

- Extension of the method to study odd-odd nuclei.

Thank you for your attention!

Patio Arrayanes
Alhambra (Granada)

Charles Nègre, Le Stryge
©Musée Orsay (Paris)

...and we will see in the VI Gogny Conference