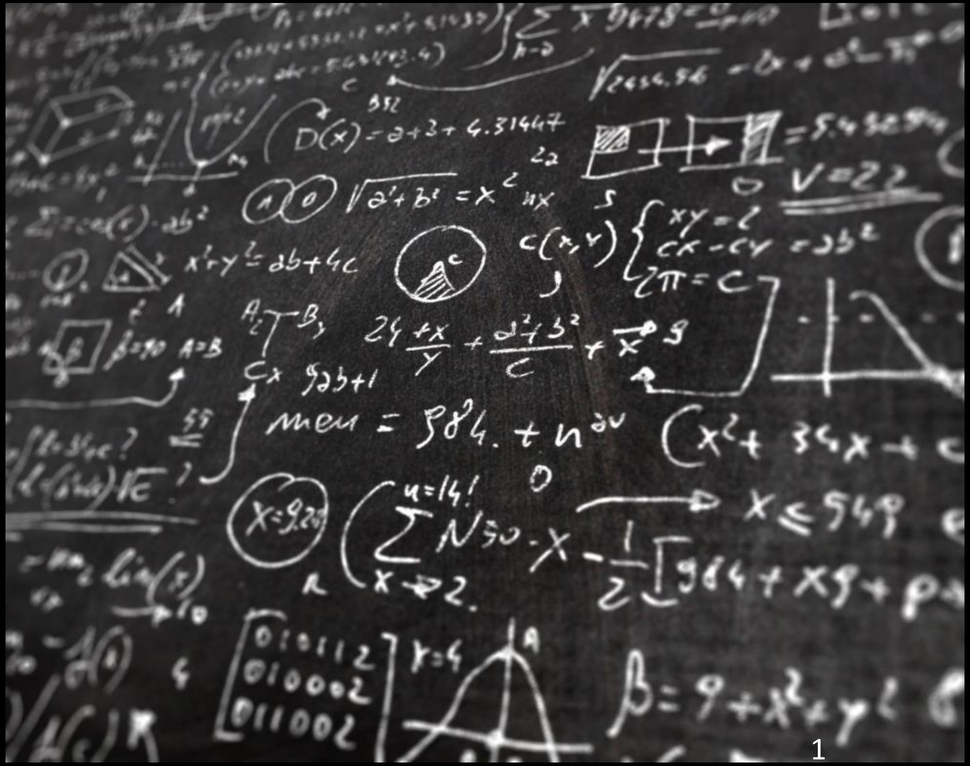
Beta-Delayed Neutron Branching Ratios calculated by proton-neutron QRPA and statistical model

Futoshi Minato
Kyushu University

Contents

1) r-process and Half-life measurements
2) Half-Lives Estimations
3) Delayed-Neutron & Fission
4) Summary



Fifth Gogny Conference, Paris, 10-13 Dec. 2024

r-process and half-live measurements

N = 82

127In 1.087 s β^-=100% β^-n≤0.03%	128In 0.84 s β^-=100% β^-n=0.038%	129In 629 ms β^-=100% β^-n=0.228%	130In 277 ms β^-=100% β^-n=0.93%	131In 261 ms β^-=100% β^-n=2.3%	132In 200 ms β^-=100% β^-n=12.3% β^-2n ?	133In 162 ms β^-=100% β^-n=90% β^-2n ?	134In 121 ms β^-=100% β^-n=89% β^-2n=9%	135In 97 ms β^-=100% β^-n>0% β^-2n>0%	136In 85 ms β^-=100% β^-n ? β^-2n ?	137In 65 ms β^-=100% β^-n ? β^-2n ?
126Cd 512 ms β^-=100%	127Cd 0.45 s β^-=100% β^-n<1.2%	128Cd 245.9 ms β^-=100% β^-n=1.9%	129Cd 151 ms β^-=100% β^-n=1.84%	130Cd 129 ms β^-=100% β^-n=3%	131Cd 98 ms β^-=100% β^-n=3.5% β^-2n ?	132Cd 84 ms β^-=100% β^-n=60% β^-2n ?	133Cd 61 ms β^-=100% β^-n ? β^-2n ?	134Cd 65 ms β^-=100% β^-n ? β^-2n ?	135Cd ?	
125Ag 176 ms β^-=100% β^-n=1.1%	126Ag 52 ms β^-=100% β^-n=3.8%	127Ag 89.1 ms β^-=100% β^-n=5.5%	128Ag 66.6 ms β^-=100% β^-n=9.3% β^-2n ?	129Ag 52 ms β^-=100% β^-n=17.9%	130Ag 41 ms β^-=100% β^-n ? β^-2n ?	131Ag 35 ms β^-=100% β^-n ? β^-2n ?	132Ag 28 ms β^-=100% β^-n ? β^-2n ?	133Ag ?		
124Pd 94 ms β^-=100% β^-n=0.89%	125Pd 64.2 ms β^-=100% β^-n=3.7%	126Pd 48.6 ms β^-=100% β^-n=4.9%	127Pd 38 ms β^-=100% β^-n=9% β^-2n ?	128Pd 36 ms β^-=100% β^-n=10%	129Pd 31 ms β^-=100% β^-n ? β^-2n ?	130Pd β^-=100% β^-n ? β^-2n ?	131Pd β^-=100% β^-n ? β^-2n ?	132Pd ?		
123Rh 42.2 ms β^-=100% β^-n=24.2% β^-2n ?	124Rh 32 ms β^-=100% β^-n=28% β^-2n ?	125Rh 26.5 ms β^-=100% β^-n ? β^-2n ?	126Rh 19 ms β^-=100% β^-n ? β^-2n ?	127Rh 20 ms β^-=100% β^-n ? β^-2n ?	128Rh β^-=100% β^-n ? β^-2n ?	129Rh ?	130Rh ?	131Rh ?		
122Ru 25 ms β^-=100% β^-n ? β^-2n ?	123Ru 19 ms β^-=100% β^-n ? β^-2n ?	124Ru 15 ms β^-=100% β^-n ? β^-2n ?	125Ru β^-=100% β^-n ? β^-2n ?	126Ru ?	127Ru ?	128Ru ?	129Ru ?	130Ru ?		
121Tc 22 ms β^-=100% β^-n ? β^-2n ?	122Tc β^-=100% β^-n ? β^-2n ?	123Tc ?	124Tc ?	125Tc ?	126Tc ?	127Tc ?	128Tc ?			

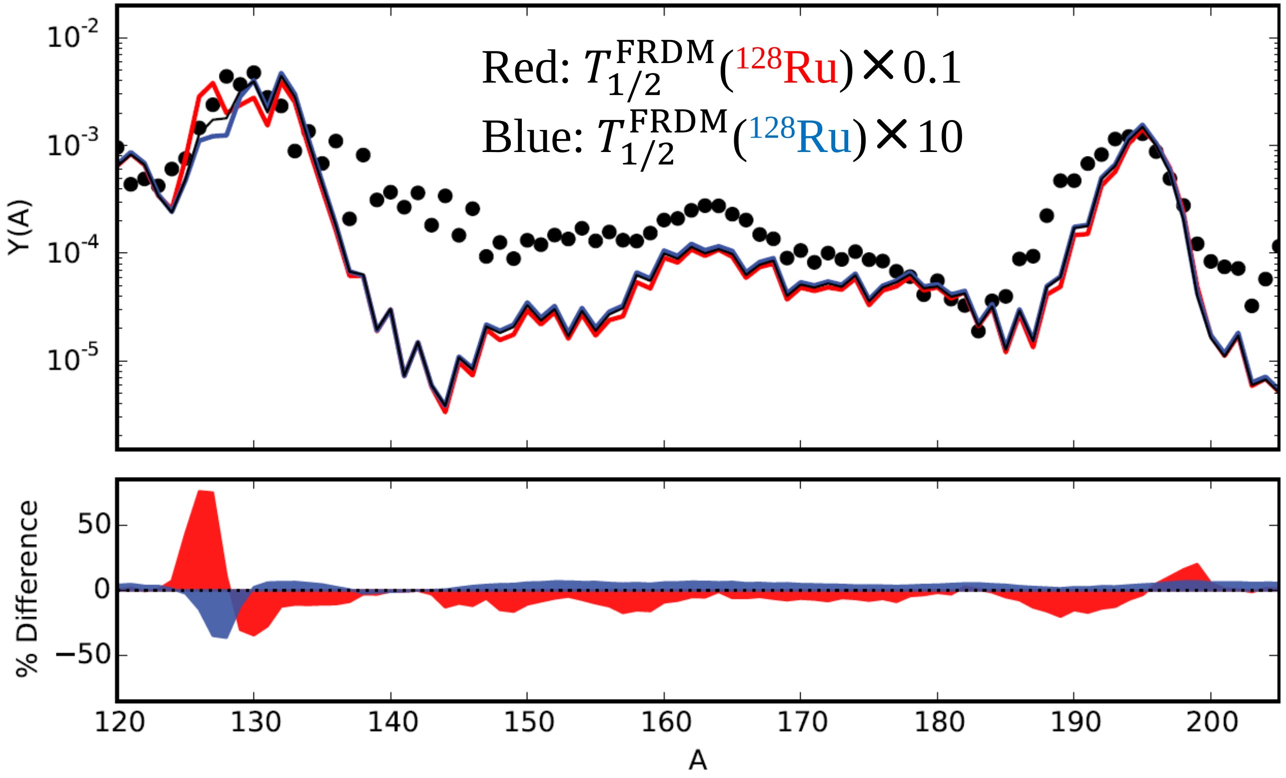
Measurements are approaching to r-process path..

NNDC Nudat 3

https://www.nndc.bnl.gov/nudat3/

Sensitivity of r-abundance to $T_{1/2}$ of ^{128}Ru

Figure: R-process r-abundance pattern



MR Mumpower, Prog. Part. Nucl. Phys. 86 (2016) 86-126

🔔 $T_{1/2}^{\mathrm{calc}}$ range from 5-25 ms.

→Unmeasured half-lives give uncertainties in r-abundance

In addition, we need β-Spectra, Delayed-Neutron and Delayed–Fission for various applications

Predicting β-decays and its subsequent decays

Half-Life Calculations

- FRDM+QRPA

P. Möller et al, At. Data Nucl. Data Tables **66**, 131 (1997).
P. Möller et al, Atomic Data and Nuclear Data Tables **125**, 1 (2019).

- HFB+QRPA/FAM

T. Marketin et al, PRC **93**, 025805 (2016).
I. N. Borzov, Phys. At. Nucl. **83**, 700 (2020).
M. Martini et al, PRC **89**, 044306 (2014).
M.T. Mustonen et al, PRC **93**, 014304 (2016)
Ney et al, PRC **102**, 034326 (2020).
K. Yoshida Phys. Rev. C **100**, 024316 (2019)

- Gross Theory

T. Yoshida, T. Tachibana, JNST**37**, 491 (2000).
T. Tachibana, RIKEN Review, **26**, 109 (2000).

- Configuration Interaction (CI)

T. Suzuki et al, PRC **85**, 015802 (2012); Astrophys. J. **859**, 133 (2018).
A. Kumar et al, PRC **109**, 064319 (2024).

- Interacting Boson Model

K. Nomura et al, PRC **101**, 044318 (2020).

- Systematics

K.-L. Kratz, G. Herrmann, Z. Phys., **263** (1973), 435

Delayed-Neutron/Fission Calculations

- Cutoff approximation by threshold energy

T. Marketin et al, PRC **93**, 025805 (2016).
P. Möller et al, At. Data Nucl. Data Tables **66**, 131 (1997).
I. N. Borzov, PRC **71**, 065801 (2005).
J-U. Nabi et al, EPJA **52**, 5 (2016).

- Systematics

E.A. McCutchan et al, PRC **86**, 041305(R) (2012).
K. Miernik et al, PRC **88**, 041301(R) (2013).

- Hauser-Feshbach Statistical Model (HFSM)

P. Möller et al, At. Data Nucl. Data Tables **125**, 1 (2019).
FM, T. Marketin, N. Paar, PRC **104**, 044321(2021).
M. Mumpower et al, PRC **106**, 065805 (2022).

Density Functional + QRPA + Hauser-Feshbach Model

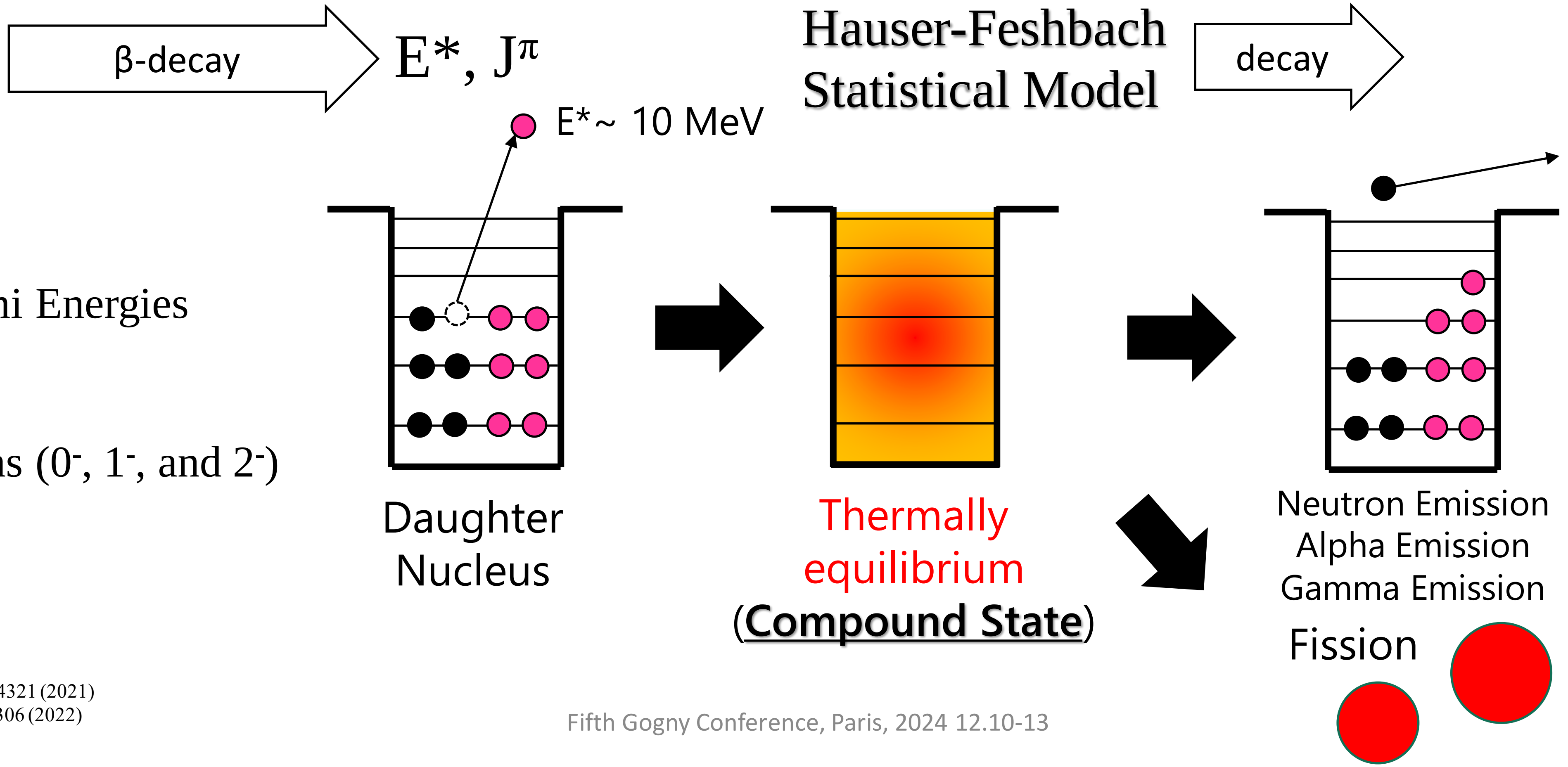
(1) Applicable to all nuclei in the nuclear chart with one effective force
(2) Various data (β, γ, delayed-neutron yields, etc.) are provided

T. Kawano, P. Möller, and W. B. Wilson, Phys. Rev. C **78**, 054601 (2008).

Theoretical Framework

DF+QRPA

Nuclear shape:
Spherical
Odd nuclei:
Adjusting Fermi Energies
β-decay type:
GT(1^+),
First Forbiddens (0^-, 1^-, and 2^-)



FM, T. Marketin, N. Paar, PRC**104**, 044321 (2021)
FM, ZM Niu, HZ Liang, PRC**106**, 024306 (2022)

Pairing correlations in Skyrme HFB+QRPA

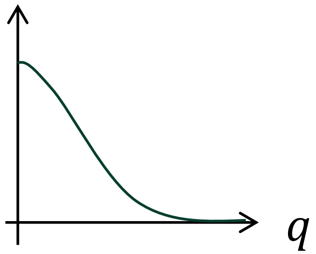
Gogny D1S for $T = 1$ pairing

$$V_{T=1}(r_1, r_2) = \sum_{i=1}^{2} (W_i + B_i P_\sigma - H_i P_\tau - M_i P_\sigma P_\tau)\, e^{-r_{12}^2/\mu_i^2}$$

Gogny force introduces a natural cut-off in pairing model space (momentum transfer q)

Fourier transform of Gaussian

$$e^{-\frac{r^2}{\mu^2}} = \frac{\mu}{\sqrt{2}} \int_{-\infty}^{\infty} e^{-\left(\frac{\mu q}{2}\right)^2} e^{-iqr}\, dq$$

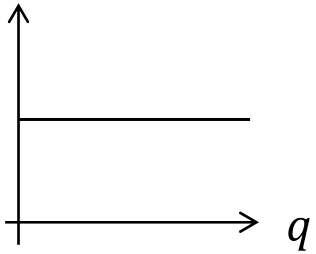


high-momentum transfer is quenched

→ benefits systematical calculations of nuclear properties

c.f. zero-reange force

$$\delta(r) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-iqr}\, dq$$



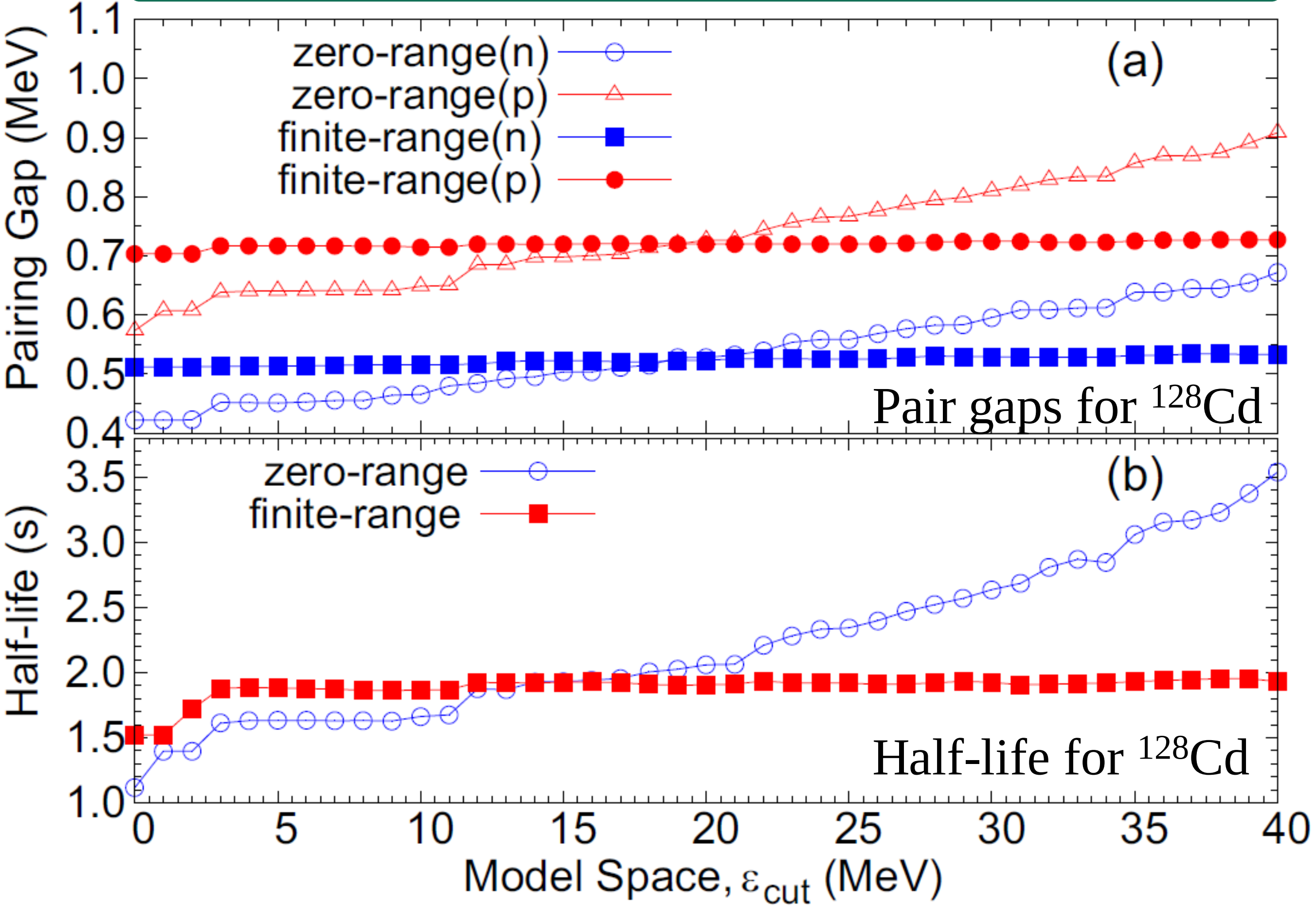
all momentum transfer q is equally considered

→lead to a divergence problem unless appropriate cutoff is introduced

Effects of Gogny force (T=1 pairing) on ^{128}Cd

Figure: Pairing gap and Half-life of ^{128}Cd

SkO' effective force for HFB and ph-channels



FM, ZM Niu, HZ Liang, PRC**106**, 024306 (2022)

Pairing correlations in Skyrme HFB+QRPA

For $T = 0$ pairing, Gogny-type force is used

$$V_{T=0}(Z,N;r_{12}) = V(Z,N)\left(e^{-r_{12}^2/\mu_1^2} - 2e^{-r_{12}^2/\mu_2^2}\right)$$

to be determined from expt. $T_{1/2}$

J. Engel et al, PRC**60**, 014302 (1999)
T. Marketin, L. Huther, and G. Martınez-Pinedo, PRC **93**, 025805 (2016)

FM, ZM Niu, HZ.Liang, PRC**106**, 024306 (2022)

Parameter Fitting for $V(Z,N)$

1. Collect $V = V_{\mathrm{opt}}(Z,N)$ optimized to $T_{1/2}$ in NUBASE2016 (950 nuclei)
2. Estimate $V(Z,N)$ of neutron-rich nuclei using $V_{\mathrm{opt}}(Z,N)$ & BNN
3. Predict $T_{1/2}$ with the estimated $V(Z,N)$

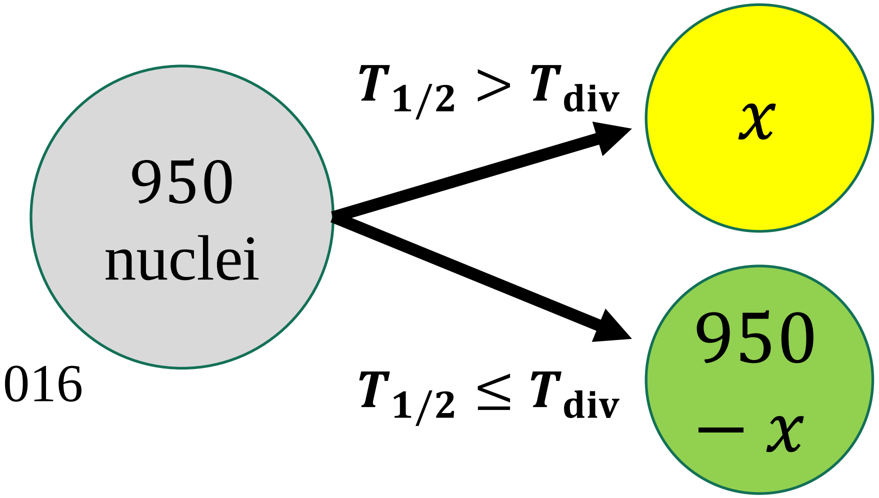
Bayesian Neutral Network (**BNN**)

$$\chi^2 = \sum_{n=1}^{N} \left[\frac{S(\boldsymbol{x};\boldsymbol{\omega}) - V_k}{\Delta V_k} \right]^2 \qquad S(\boldsymbol{x};\boldsymbol{\omega}) = a + \sum_{j=1}^{H} b_j \tanh\left(c_j + \sum_{i=1}^{I} d_{ji} x_i \right)$$

one hidden layer
H=30: number of neurons
I: number of inputs

Z. Niu and H. Liang, Phys. Lett. B **778**, 48 (2018).

Verification



NUBASE2016

Training Data

Test data

Verification of $V(Z, N)$

$$r_i = \log10\left(\frac{T_{calc}}{T_{exp}}\right)$$

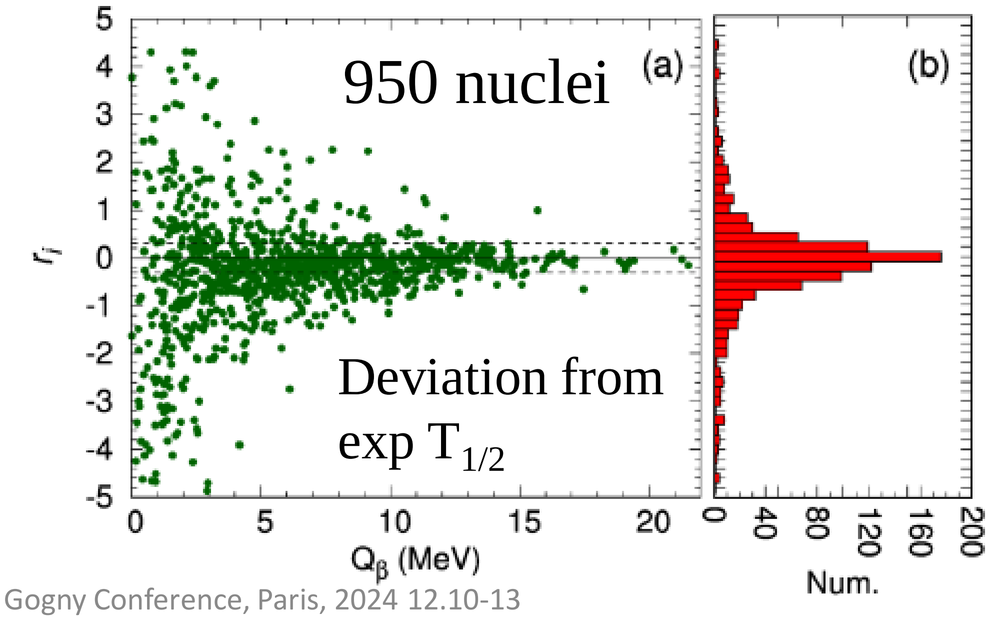
Setting	T_{div} (s)	Number of training data	Number of test data	$\bar{r}$	s
1	1.00	569	381	−0.080	0.478
2	0.50	626	324	−0.020	0.494
3	0.10	776	174	−0.085	0.335
4	0.05	841	109	−0.031	0.270

Mean Deviation $\bar{r} = \frac{1}{N}\sum_i r_i$

Standard Deviation $s = \sqrt{\frac{1}{N}\sum_i (r_i - \bar{r})^2}$

→7 % under-estimation on average
reproduced within a factor of 1.86

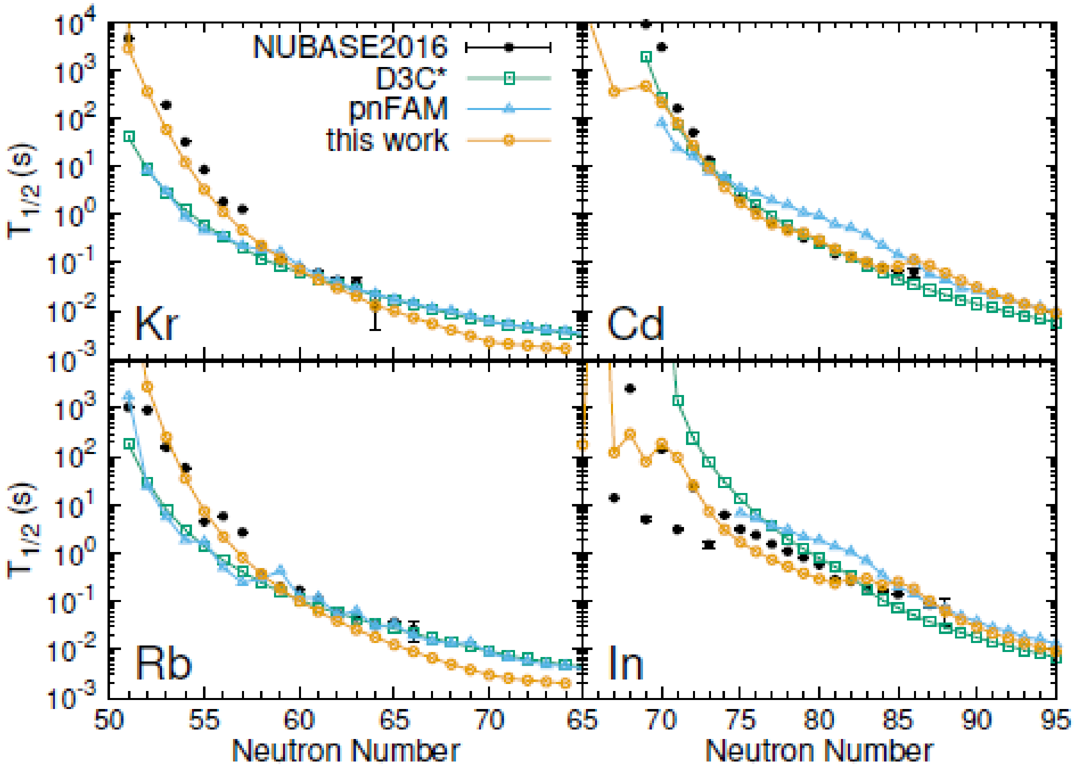
FM, ZM Niu, HZ.Liang, PRC**106**, 024306 (2022)

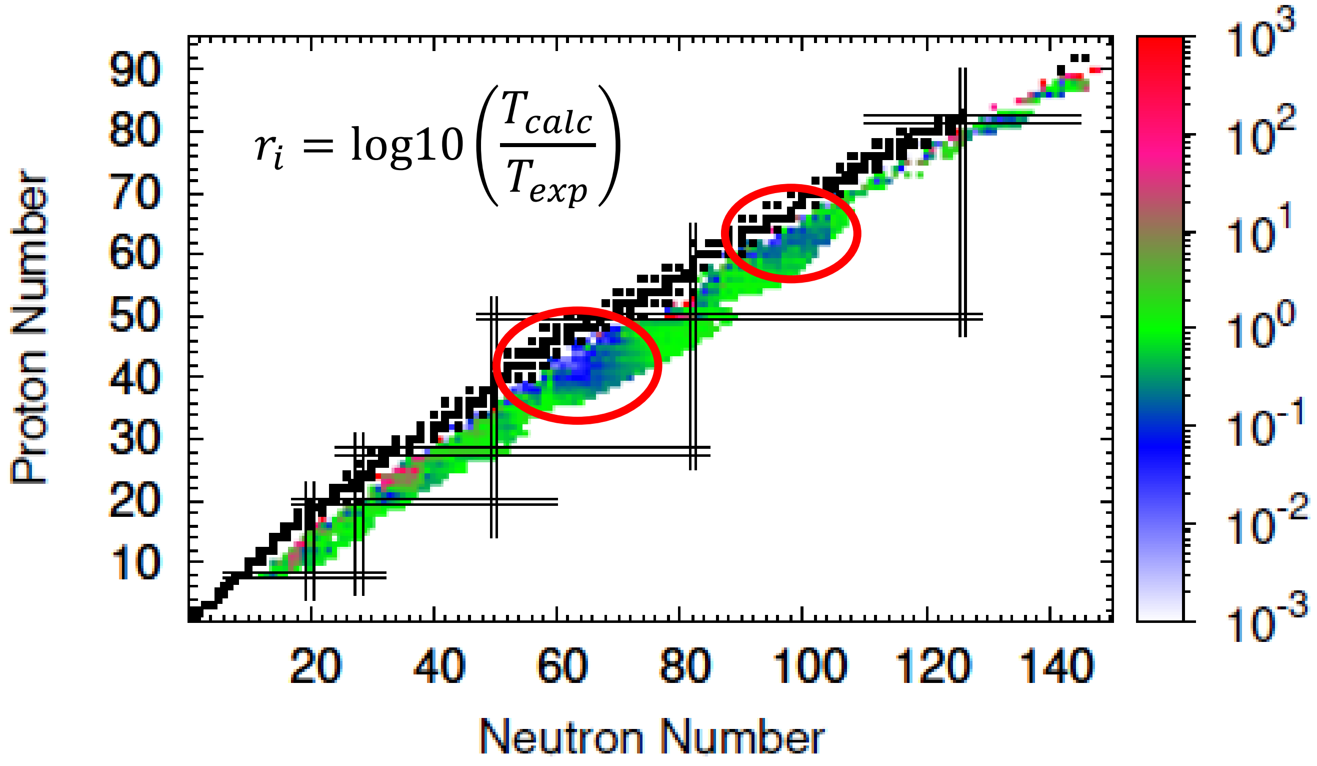


Comparison with expt. data and other models for Kr, Rb, Cd, In isotopes

FM, ZM Niu, HZ.Liang, PRC**106**, 024306 (2022)

Deviations from expt. data in N-Z plane



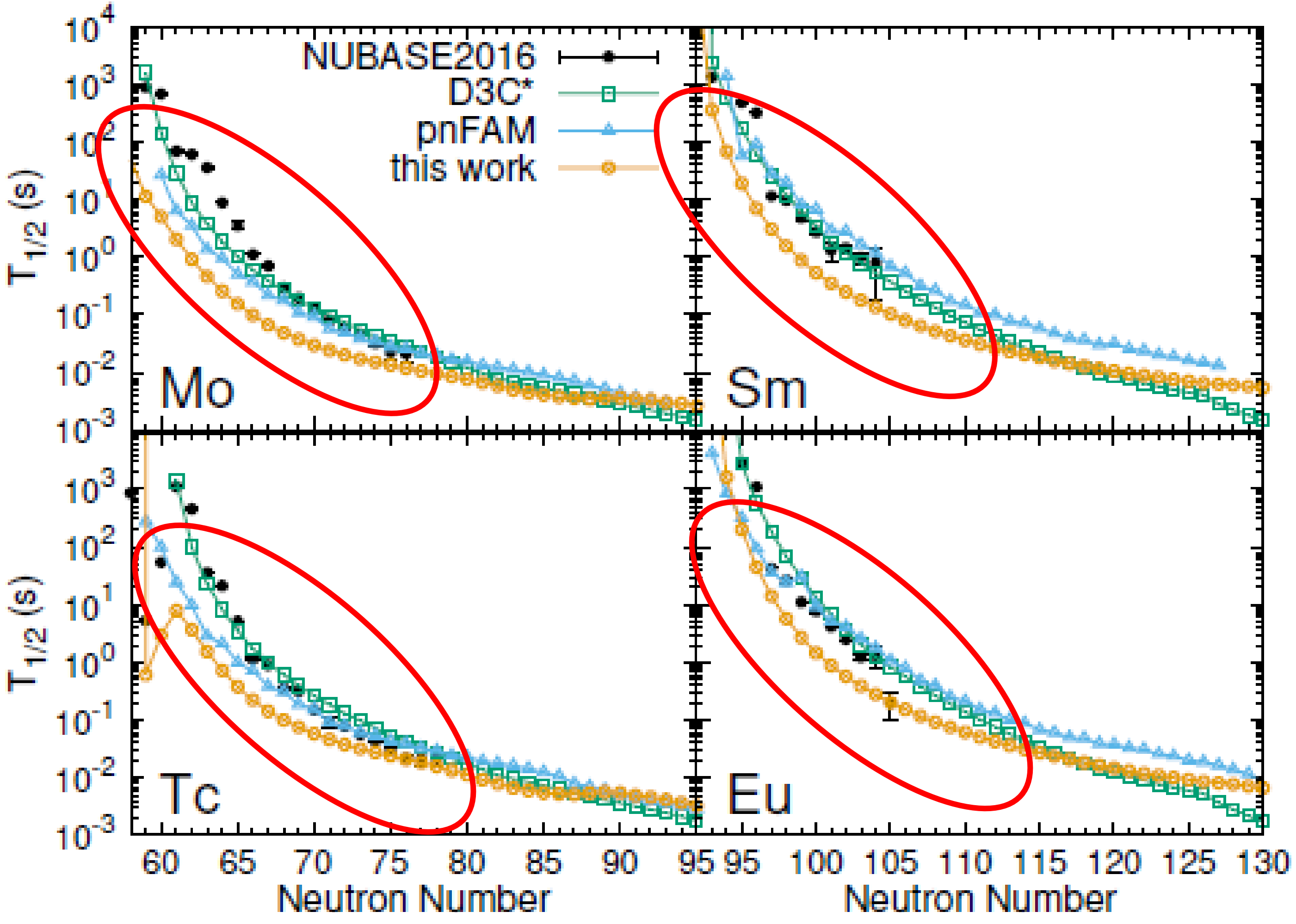


$T_{1/2}^{\mathrm{calc}}(^{128}\mathrm{Ru})$=9.9+/-1.0 ms

pnFAM: 23.7 ms
FRDM: 21.5 ms
D3C*: 11.8 ms
GT2: 4.86 ms

FM, ZM Niu, HZ.Liang, PRC**106**, 024306 (2022)

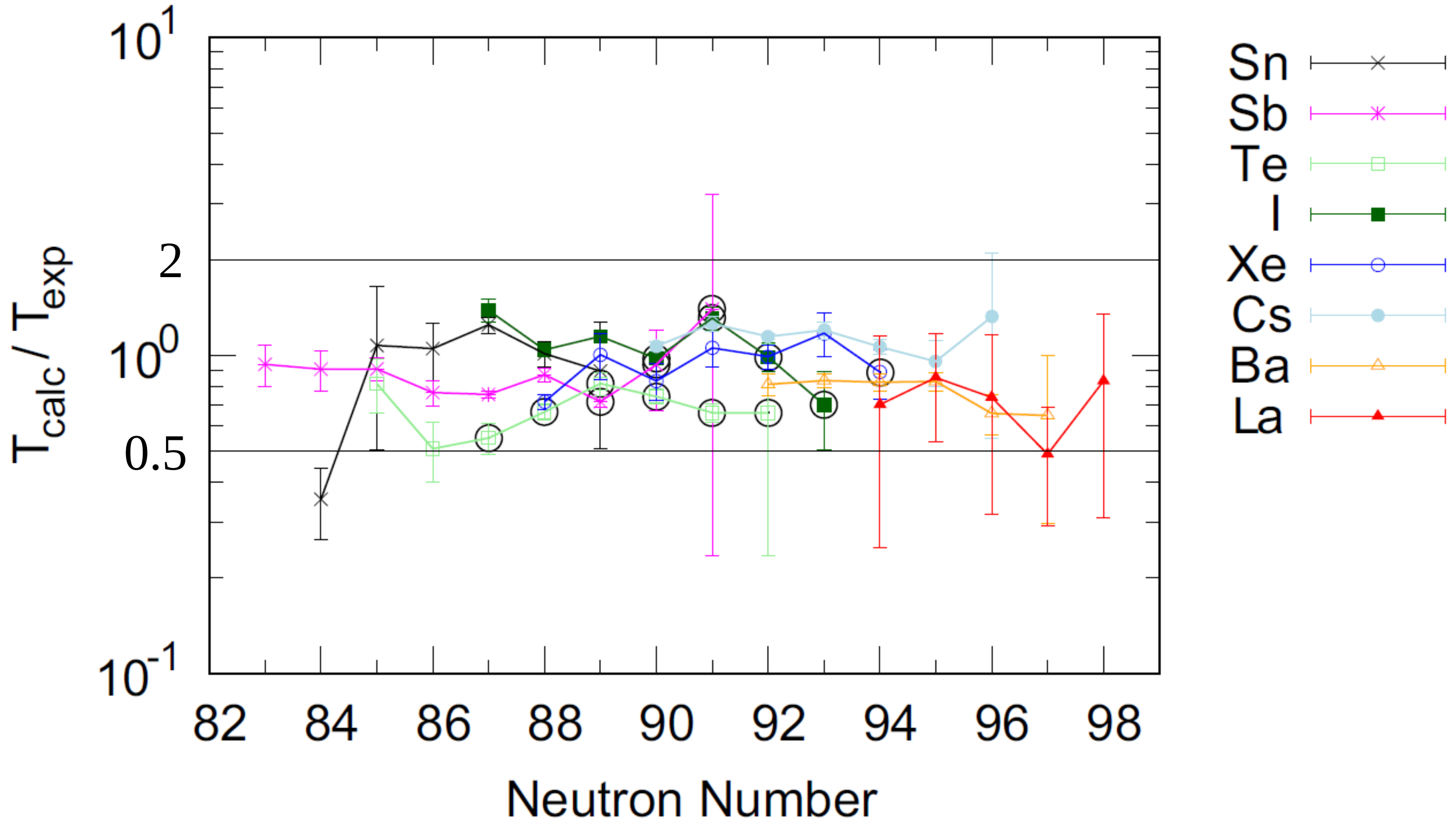
Comparison with expt. data and other models for Mo (Z=42), Tc (43), Sm (62) , Eu (63)



c.f. Deformation effect on $T_{1/2}$
K. Yoshida, JPS. Conf. Proc. **6**, 020017 (2015).

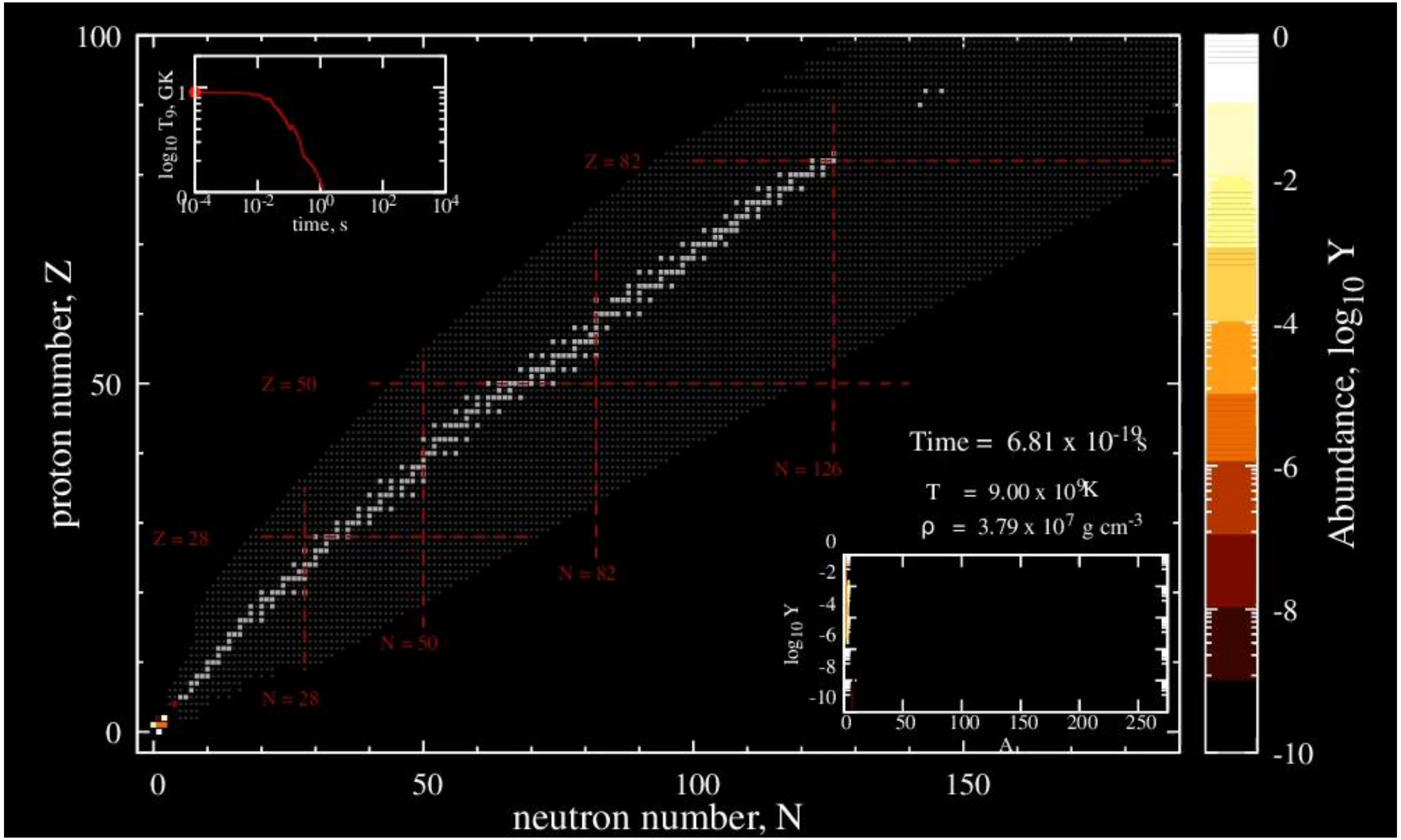
Underestimations arising from spherical shape assumption

Comparison with Recent Experimental Data



Expt: J. Wu, *et al.*, PRC **101**, 042801 (2020).

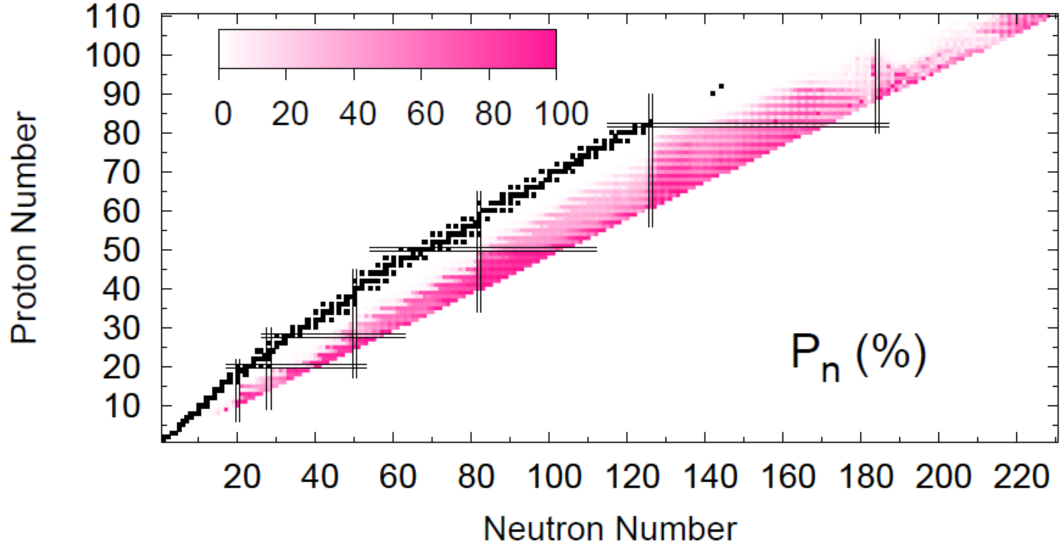
r-process simulation

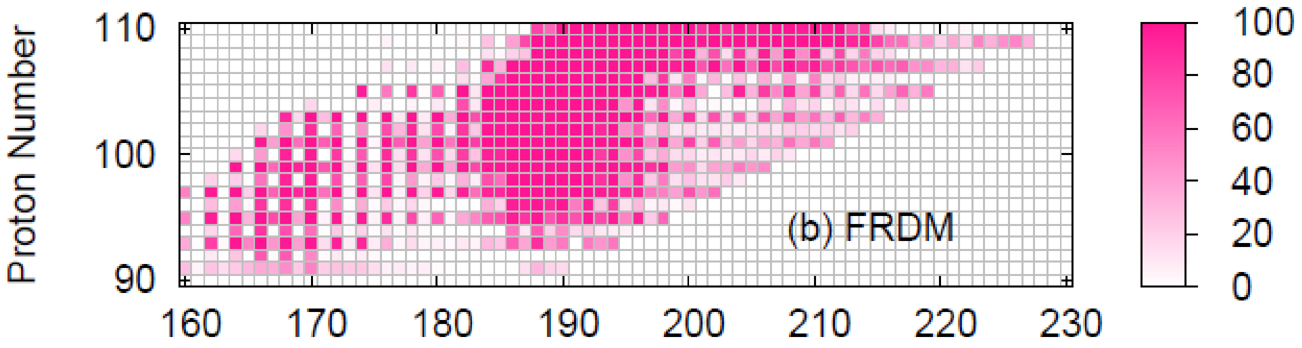


Courtesy of N. Nishimura (@CNS, U. Tokyo)

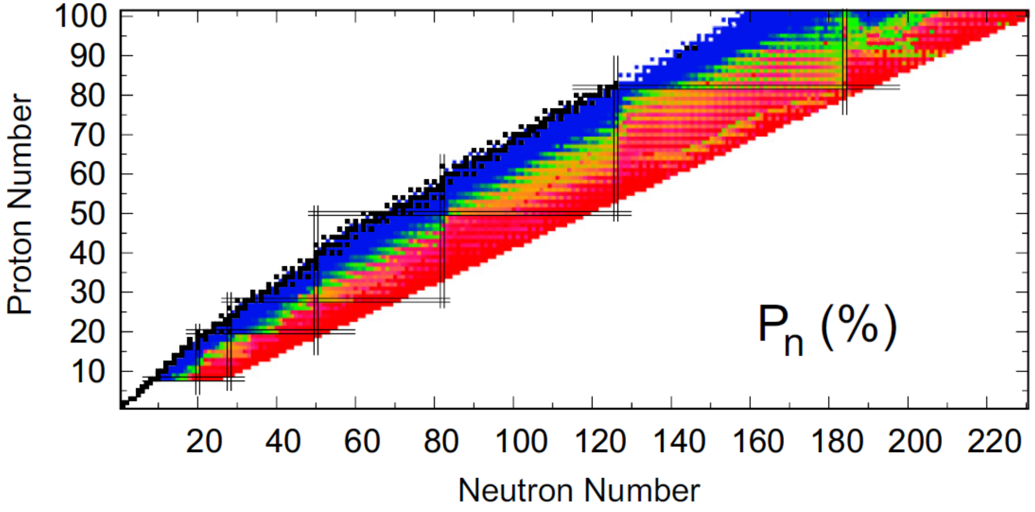
Delayed neutron branching ratios within QRPA+HFSM

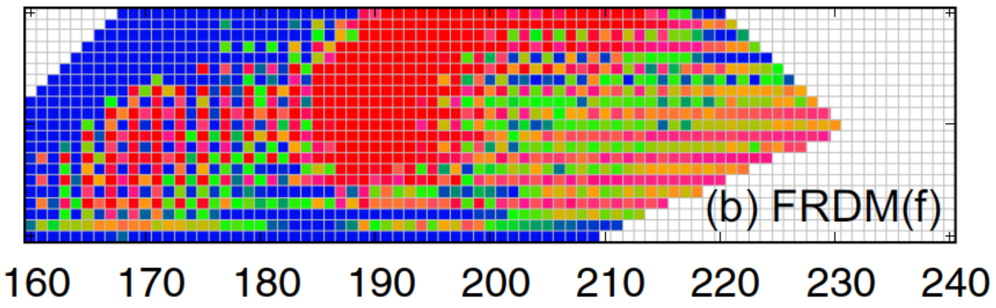
Skyrme (SkO’) +QRPA+HFSM





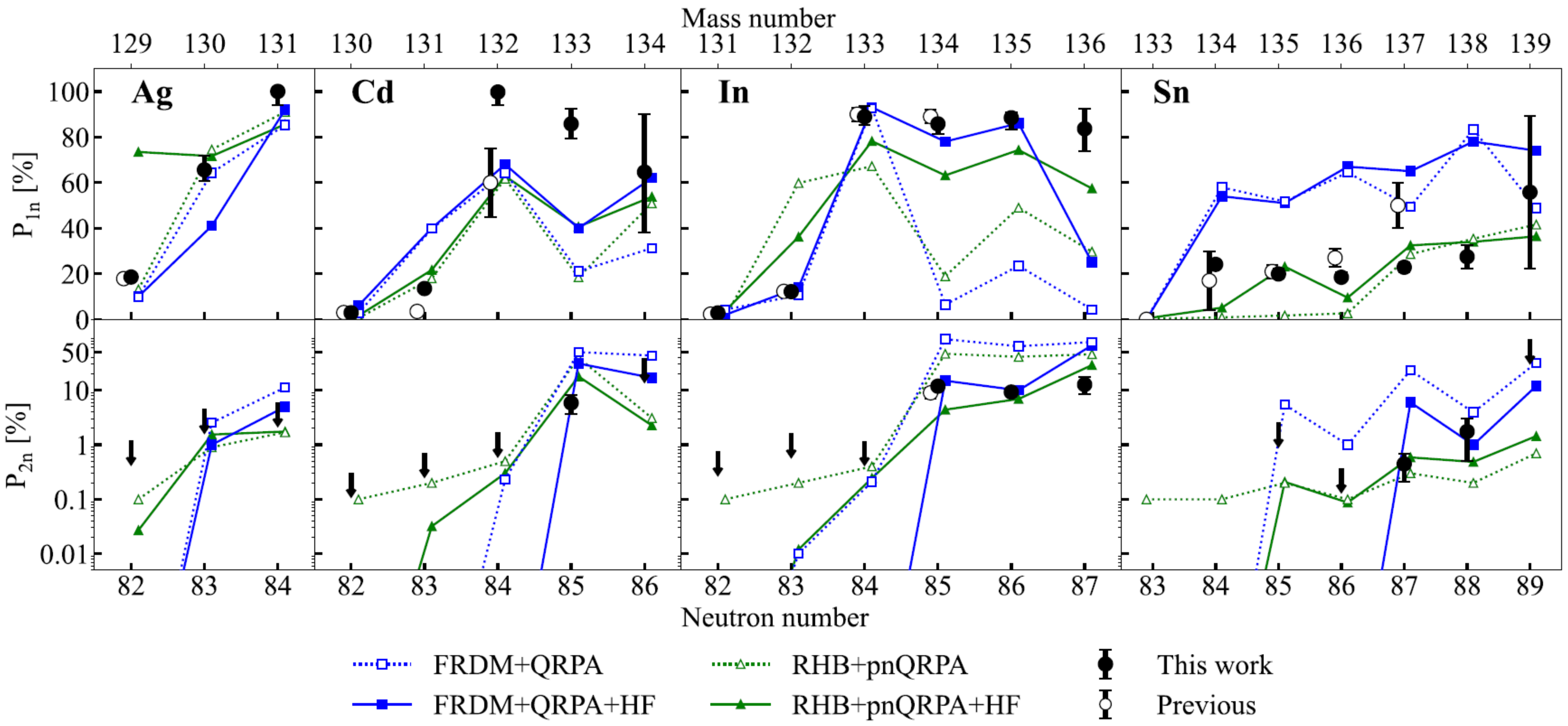
RMF(D3C*)+QRPA+HFSM





FM, T. Marketin, N. Paar, PRC**104**, 044321 (2021)

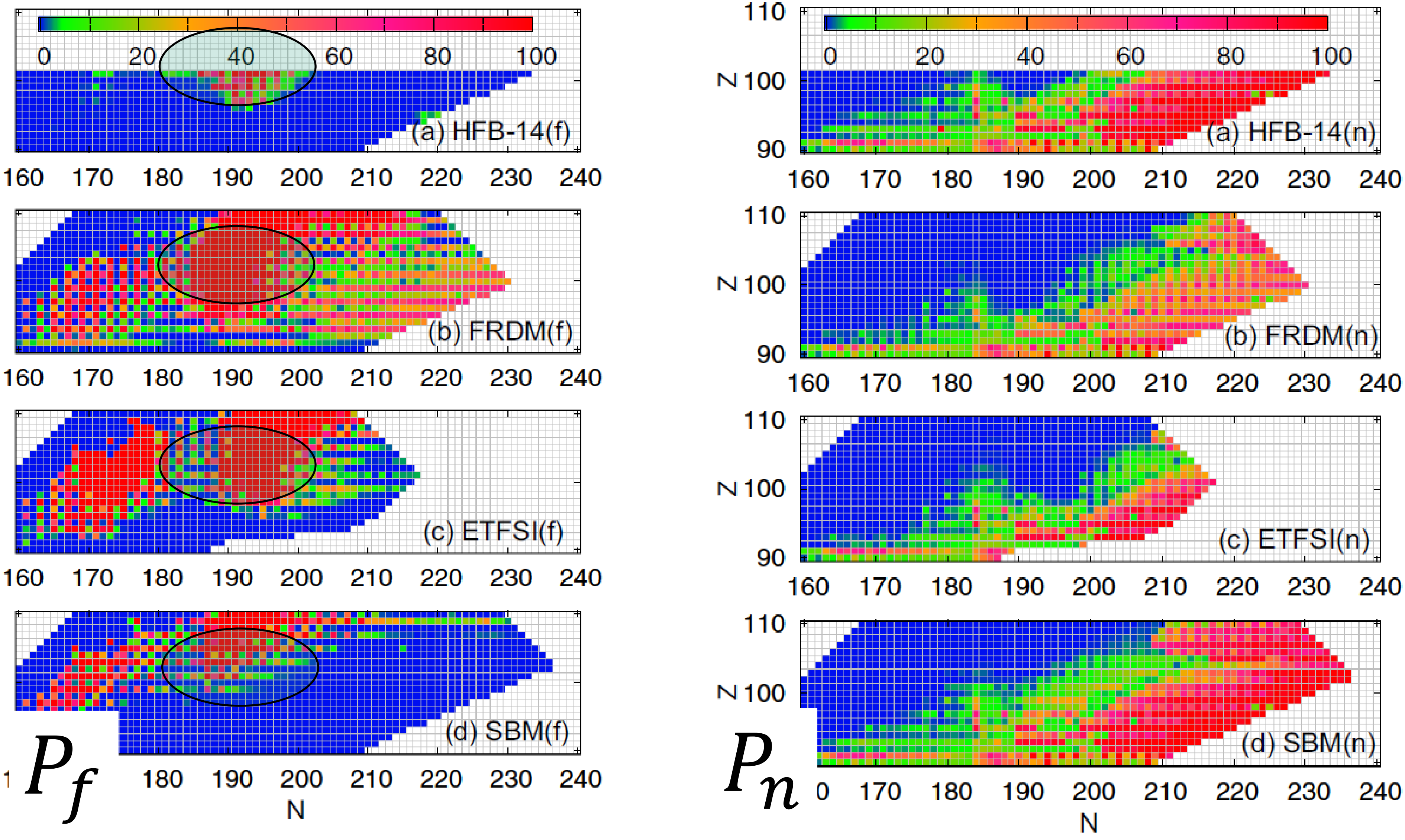
Comparison with Recent Experimental Data



V.H. Phong et al. PRL **129**, 172701 (2022)

Uncertainties coming from Fission Barrier

FM, T. Marketin, N. Paar, PRC**104**, 044321 (2021)



Summary

Predict $T_{1/2}$ within DF+QRPA, P_n by HFSM

Isoscalar pairing predicted by BNN

Newly measured $T_{1/2}$ are also reproduced

Adopted in JENDL-5 Decay Data & R-process simulation

https://wwwndc.jaea.go.jp/jendl/j5/j5.html

Non-relativistic, Relativistic DF **database** for Beta-Delayed Neutron

Please feel free to contact me for calculated data inquiry.

Further study is in progress

Deformation Effect → Project is ongoing

Muon Nuclear Data → Work on very hard!

Fission Barriers