

# Recent Advances in the Nuclear Matrix Elements for Neutrinoless Double-Beta Decay

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10/12/2024



Arthur B. McDonald  
Canadian Astroparticle Physics Research Institute



Introduction

Corrections to  $0\nu\beta\beta$ -Decay Nuclear Matrix Elements

Correlations with other observables to constrain the matrix elements

Summary and Outlook

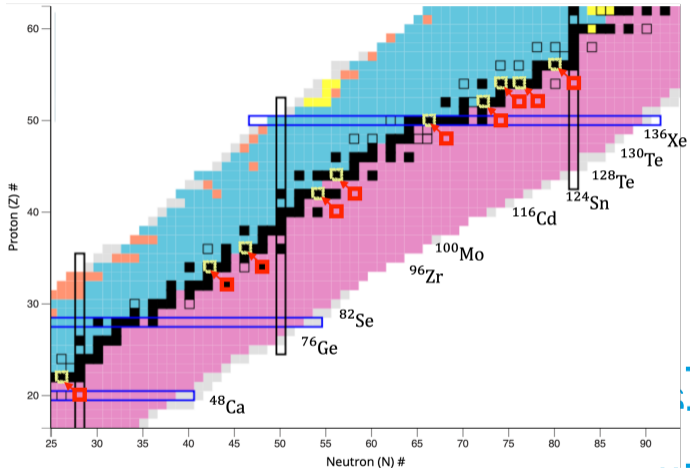
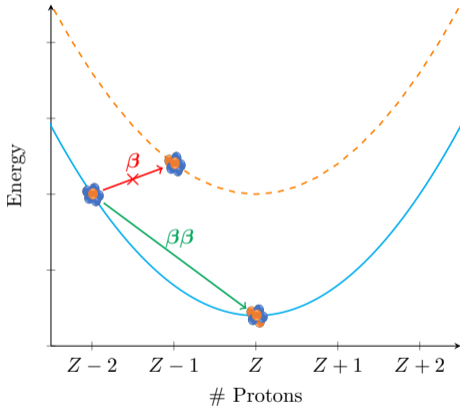
## Introduction

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# Double-Beta Decay



[nndc.bnl.gov](http://nndc.bnl.gov)

Disc accelerated



# Neutrinoless Double-Beta ( $0\nu\beta\beta$ ) Decay

- Violates lepton-number conservation

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\nu_e$$

*Maria Goeppert-Mayer*



$2\nu\beta\beta$

1935

*Ettore Majorana*



Majorana particles

1937

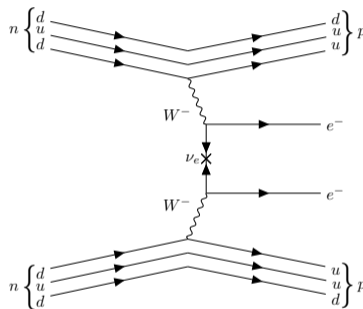
*Wendell H. Furry*



$0\nu\beta\beta$

1939

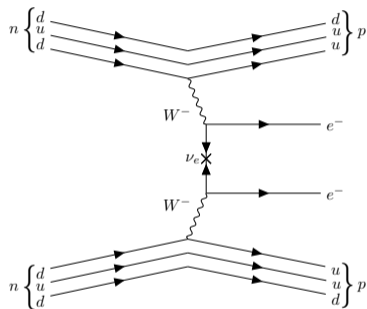
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# Neutrinoless Double-Beta ( $0\nu\beta\beta$ ) Decay

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- Requires that neutrinos are Majorana particles

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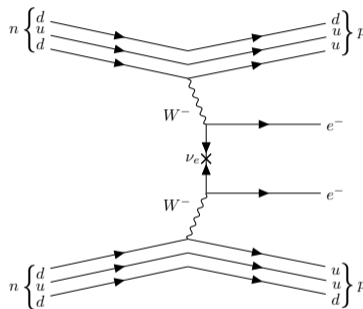
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# Neutrinoless Double-Beta ( $0\nu\beta\beta$ ) Decay

- Violates lepton-number conservation
- Requires that neutrinos are Majorana particles
- If observed,  $t_{1/2}^{0\nu} \gtrsim 10^{25}$  years

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# Neutrinoless Double-Beta ( $0\nu\beta\beta$ ) Decay

- Violates lepton-number conservation
- Requires that **neutrinos are Majorana particles**
- If observed,  $t_{1/2}^{0\nu} \gtrsim 10^{25}$  years  
 ( $t_{1/2}^{2\nu} \approx 10^{20}$  years,  
 age of the Universe  $\approx 10^{10}$  years)

Maria Goeppert-Mayer

Ettore Majorana

Wendell H. Furry



$2\nu\beta\beta$

Majorana particles

$0\nu\beta\beta$

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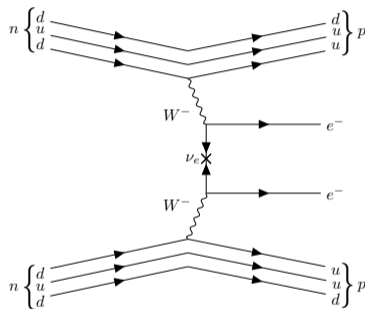


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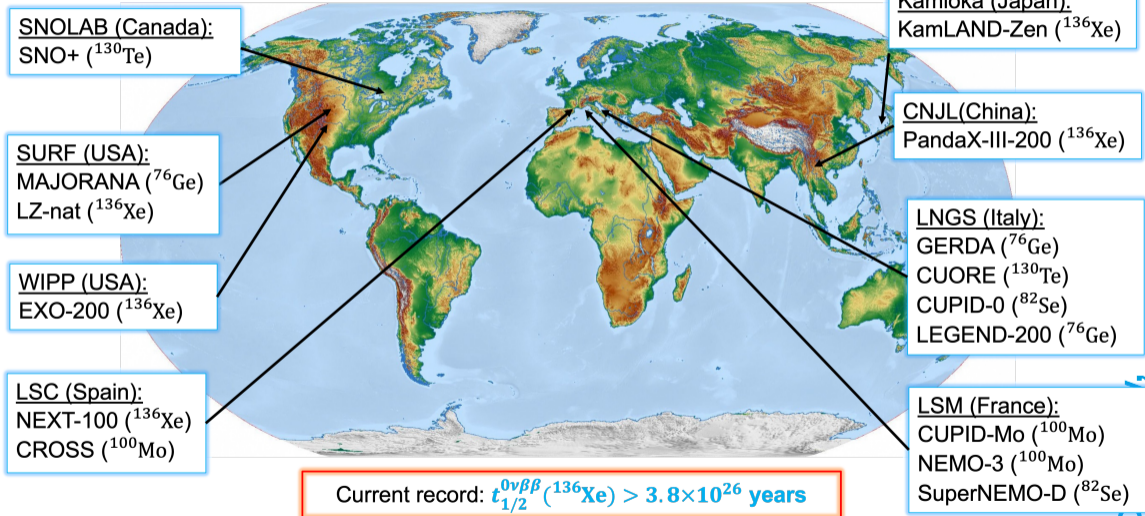


...

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\nu_e$$



# $0\nu\beta\beta$ -Decay Experiments



KamLAND-Zen, arXiv:2407.11438

accelerated

# Next-Generation $0\nu\beta\beta$ -Decay Experiments

SNOLAB (Canada):  
SNO+II ( $^{130}\text{Te}$ )

Kamioka (Japan):  
KamLAND2-Zen ( $^{136}\text{Xe}$ )

Yemilab (Korea):  
PandaX-III-200 ( $^{136}\text{Xe}$ )

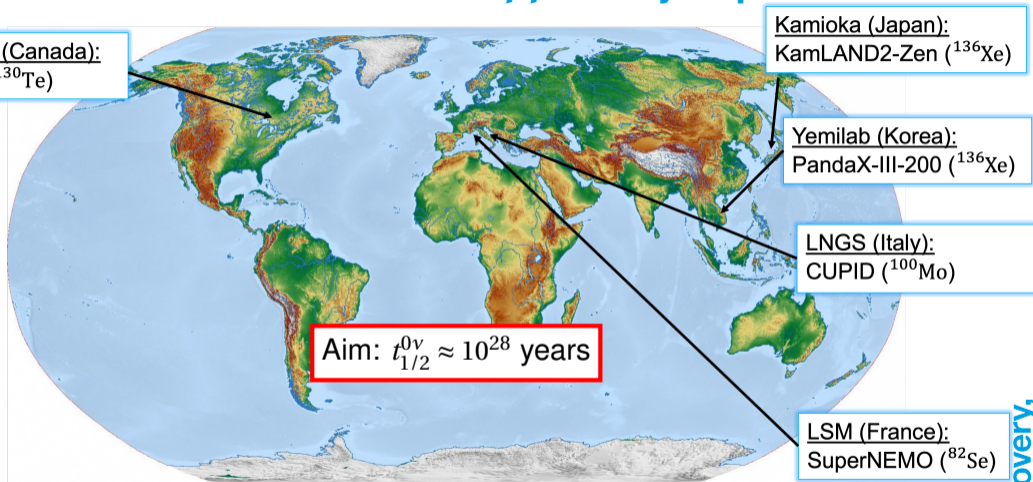
LNGS (Italy):  
CUPID ( $^{100}\text{Mo}$ )

LSM (France):  
SuperNEMO ( $^{82}\text{Se}$ )

+nEXO ( $^{136}\text{Xe}$ ), LEGEND-1000 ( $^{76}\text{Ge}$ ), NEXT-HD ( $^{136}\text{Xe}$ ), Darwin ( $^{136}\text{Xe}$ ), ...

*M. Agostini et al., Rev. Mod. Phys. 95, 025002 (2023)*

# Next-Generation $0\nu\beta\beta$ -Decay Experiments



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*M. Agostini et al., Rev. Mod. Phys. 95, 025002 (2023)*

*What would be  
measured*

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G_{0\nu} |M^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

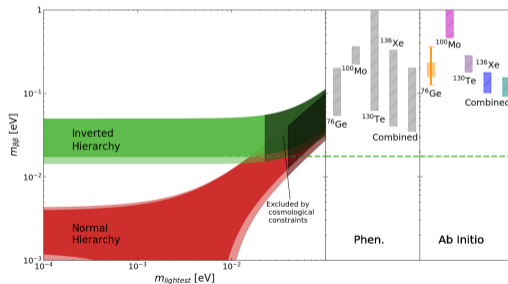


# $0\nu\beta\beta$ -Decay Half-Life

What would be measured

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G_{0\nu} |M^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

Majorana mass  
 $m_{\beta\beta} = \sum_k (U_{ek})^2 m_k$



T. Shickele, L.J. A. Belley, J. D. Holt, in preparation

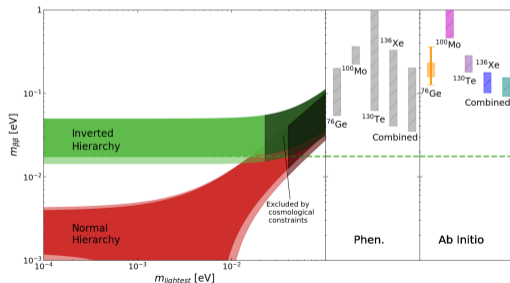
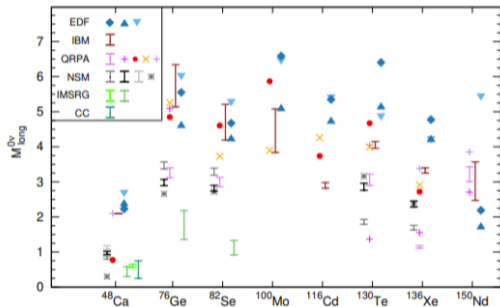
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Majorana mass  
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Nuclear matrix element



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**Corrections to  $0\nu\beta\beta$ -Decay Nuclear Matrix Elements**

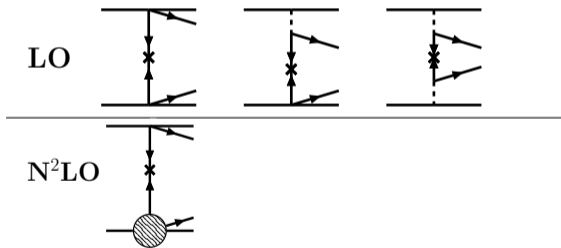
Correlations with other observables to constrain the matrix elements

Summary and Outlook

# Effective-Field-Theory Corrections to $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

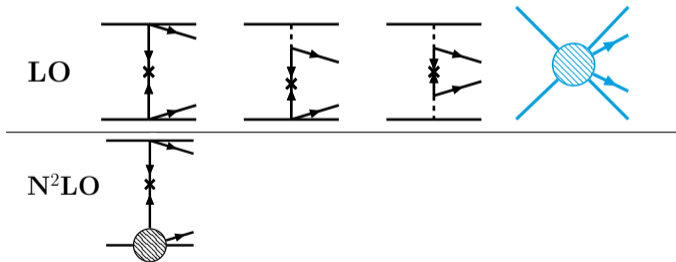
*V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)*



# Effective-Field-Theory Corrections to $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu} + M_S^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

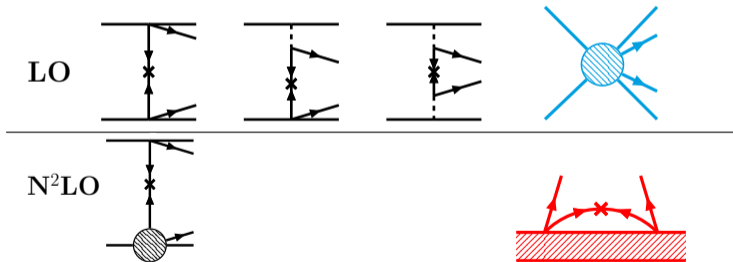
*V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)*



# Effective-Field-Theory Corrections to $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu} + M_S^{0\nu} + M_{\text{usoft}}^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

*V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)*





## Traditional $0\nu\beta\beta$ -Decay Operators

$$M^{0\nu} = \frac{R}{g_A^2} \int \frac{d\mathbf{k}}{2\pi^2} \frac{e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}}{|\mathbf{k}|} \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$



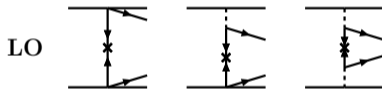
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- Traditionally, the nuclear current includes the leading-order (LO) transition operators

$$\mathcal{J}^0 = \tau [g_V(0)]$$

$$\mathbf{J} = \tau [g_A(0)\boldsymbol{\sigma} - g_P(0)\mathbf{p}(\mathbf{p}\cdot\boldsymbol{\sigma})]$$



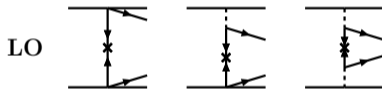
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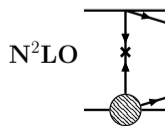
$$\mathbf{J} = \tau [g_A(0)\boldsymbol{\sigma} - g_P(0)\mathbf{p}(\mathbf{p}\cdot\boldsymbol{\sigma})]$$



- and next-to-next-to-leading-order (N<sup>2</sup>LO) corrections absorbed into **form factors** and **induced weak-magnetism terms**

$$\mathcal{J}^0 = \tau [g_V(\mathbf{p}^2)]$$

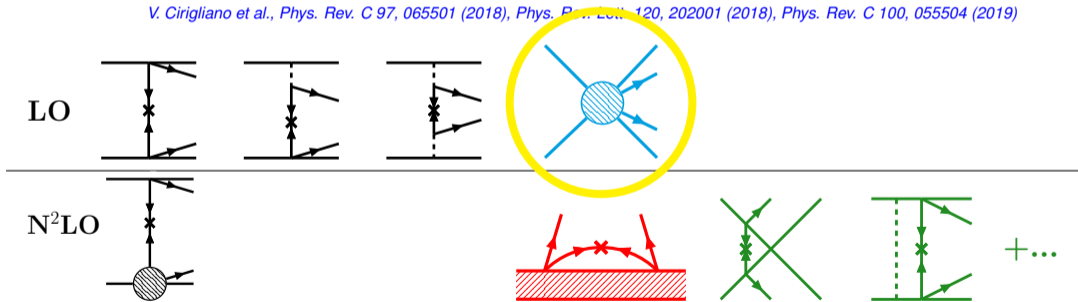
$$\mathbf{J} = \tau \left[ g_A(\mathbf{p}^2)\boldsymbol{\sigma} - g_P(\mathbf{p}^2)\mathbf{p}(\mathbf{p}\cdot\boldsymbol{\sigma}) + ig_M(\mathbf{p}^2) \frac{\boldsymbol{\sigma} \times \mathbf{p}}{2m_N} \right]$$



# Leading-order short-range contribution to $0\nu\beta\beta$ decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu} + M_S^{0\nu} + M_{\text{usoft}}^{0\nu} + M_{N^2\text{LO}}^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

V. Cirigliano et al., *Phys. Rev. C* 97, 065501 (2018), *Phys. Rev. Lett.* 120, 202001 (2018), *Phys. Rev. C* 100, 055504 (2019)



# Contact Term in pnQRPA and Nuclear Shell Model (NSM)

- The contact term reads

$$M_S^{0\nu} = \frac{2R}{\pi g_A^2} \langle 0_f^+ | \sum_{m,n} \tau_m^- \tau_n^- \int j_0(qr) h_S(q^2) q^2 dq | 0_i^+ \rangle$$

with

$$h_S(q^2) = 2g_{\nu}^{NN} e^{-q^2/(2\Lambda^2)}.$$

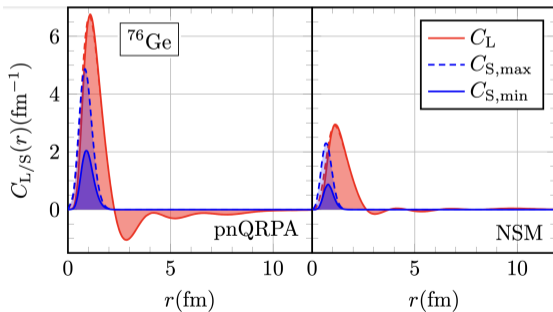
In pnQRPA:

$$M_S/M_L \approx 30\% - 80\%$$

In NSM:

$$M_S/M_L \approx 15\% - 50\%$$

$$\int C_{L/S}(r) dr = M_{L/S}^{0\nu}$$



LJ, P. Soriano and J. Menéndez, *Phys. Lett. B* **823**, 136720 (2021)



# Ultrasoft Neutrinos in pnQRPA and NSM

- Contribution of ultrasoft neutrinos ( $|\mathbf{k}| \ll k_F \approx 100 \text{ MeV}$ ) to  $0\nu\beta\beta$  decay:

*V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018)*

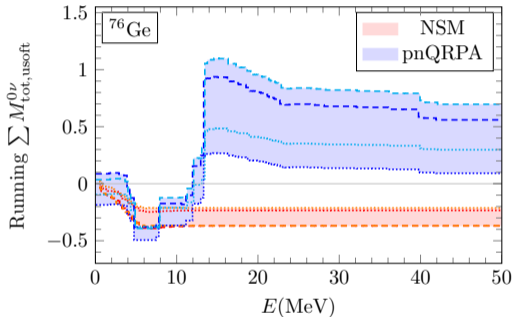
$$M_{\text{usoft}}^{0\nu} = -\frac{2R}{\pi} \sum_n \langle f || \sum_a \sigma_a \tau_a^+ || n \rangle \langle n || \sum_b \sigma_b \tau_b^+ || i \rangle \times (E_e + E_n - E_i) \left( \ln \frac{\mu_{\text{us}}}{2(E_e + E_n - E_i)} + 1 \right)$$

**In pnQRPA:**

$$|M_{\text{usoft}}^{0\nu} / M_L^{0\nu}| \leq 30\%$$

**In NSM:**

$$|M_{\text{usoft}}^{0\nu} / M_L^{0\nu}| \leq 10\%$$



*D. Castillo, LJ, P. Soriano, J. Menéndez, arXiv:2408.03373*



- The N<sup>2</sup>LO loop corrections read as

$$M_{\text{loops}}^{0\nu} = \frac{4R}{\pi g_A^2} \langle 0_f^+ | \sum_{a,b} \tau_a^- \tau_b^- \int e^{-\frac{q^2}{2\Lambda^2}} j_u(qr) V_{\nu,2}^{(a,b)} q^2 dq | 0_i^+ \rangle$$

$$\int C_{\text{N}^2\text{LO}}^{0\nu}(r) dr = M_{\text{loops}}^{0\nu}$$

with

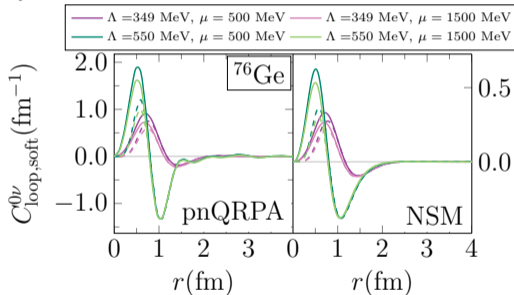
$$V_{\nu,2}^{(a,b)} = V_{\text{VV}}^{(a,b)} + V_{\text{AA}}^{(a,b)} + \ln \frac{m_\pi^2}{\mu_{\text{us}}^2} V_{\text{us}}^{(a,b)} + V_{\text{CT}}^{(a,b)}$$

**In pnQRPA:**

$$|M_{\text{N}^2\text{LO}}/M_{\text{L}}| \approx 2\% - 10\%$$

**In NSM:**

$$|M_{\text{N}^2\text{LO}}/M_{\text{L}}| \approx 4\% - 10\%$$



*D. Castillo, L.J. P. Soriano, J. Menéndez, arXiv:2408.03373*



Introduction

Corrections to  $0\nu\beta\beta$ -Decay Nuclear Matrix Elements

**Correlations with other observables to constrain the matrix elements**

Summary and Outlook



# $0\nu\beta\beta$ Decay vs Double-Charge-Exchange Reactions

$$M^{0\nu} = M_{\text{GT}}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_{\text{F}}^{0\nu} + M_{\text{T}}^{0\nu} + M_{\text{S}}^{0\nu} + M_{\text{N}^2\text{LO}}^{0\nu}$$

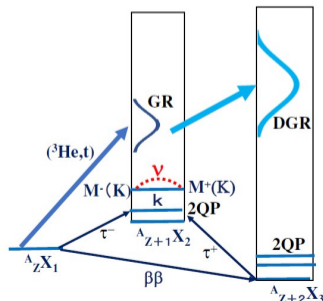
Leading contribution

$$M_{\text{GT}}^{0\nu} = \langle f || \sum_{jk} \tau_j^- \tau_k^- \sigma_j^- \sigma_k^- V_{\text{GT}}(r_{jk}) || i \rangle$$

- Double-Gamow-Teller (DGT) strength function

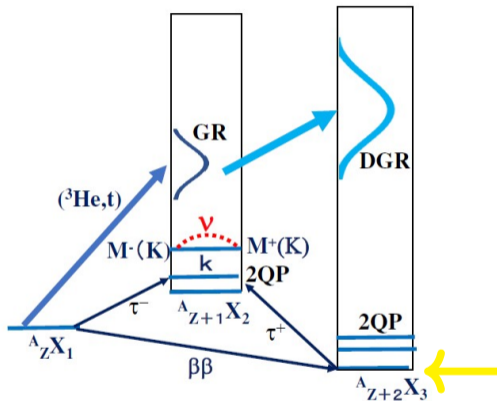
$$B(\text{DGT}; \lambda) = \frac{1}{2J_i + 1} |\langle f || [\sum_{jk} \sigma_j \tau_j^- \times \sigma_k \tau_k^-]^{(\lambda)} || i \rangle|^2$$

- Could we probe  $0\nu\beta\beta$  decay by DGT reactions?



# Correlations Between DGT and $0\nu\beta\beta$ Decay

$$M_{\text{DGT}} = -\langle 0_{\text{gs},f}^+ | [\sum_{jk} \sigma_j \tau_j^- \times \sigma_k \tau_k^-]^{(0)} | 0_{\text{gs},i}^+ \rangle$$

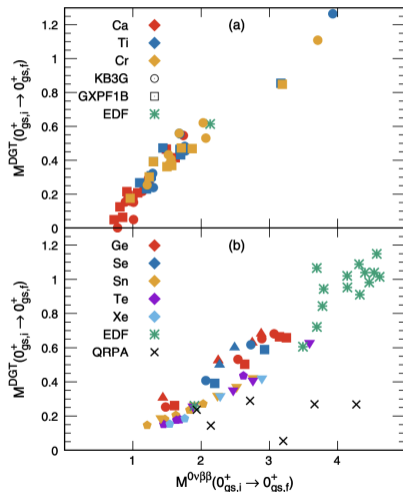


H. Ejiri, L.J. J. Suhonen, *Phys. Rev. C* 105, L022501 (2022)

# Correlations Between DGT and $0\nu\beta\beta$ Decay

$$M_{\text{DGT}} = -\langle 0_{\text{gs},f}^+ || [\sum_{jk} \sigma_j \tau_j^- \times \sigma_k \tau_k^-]^{(0)} || 0_{\text{gs},i}^+ \rangle$$

- Correlation between  $M^{0\nu}$  and  $M_{\text{DGT}}$  found in **nuclear shell model** and **EFT**

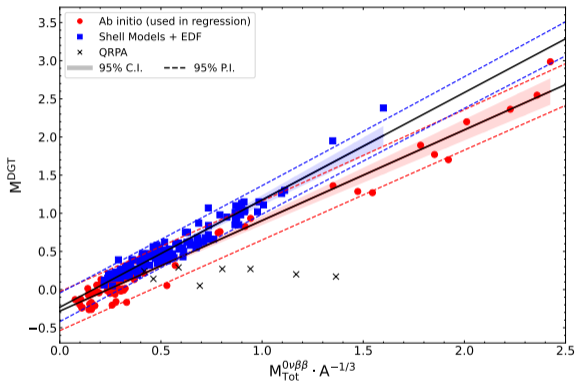


N. Shimizu, J. Menéndez, K. Yako, *Phys. Rev. Lett.* 120, 142502 (2018)

# Correlations Between DGT and $0\nu\beta\beta$ Decay

$$M_{\text{DGT}} = -\langle 0_{\text{gs},f}^+ | [\sum_{jk} \sigma_j \tau_j^- \times \sigma_k \tau_k^-]^{(0)} | 0_{\text{gs},i}^+ \rangle$$

- Correlation between  $M^{0\nu}$  and  $M_{\text{DGT}}$  found in **nuclear shell model** and **EFT**
- Correlation also holds in *ab initio* **VS-IMSRG**

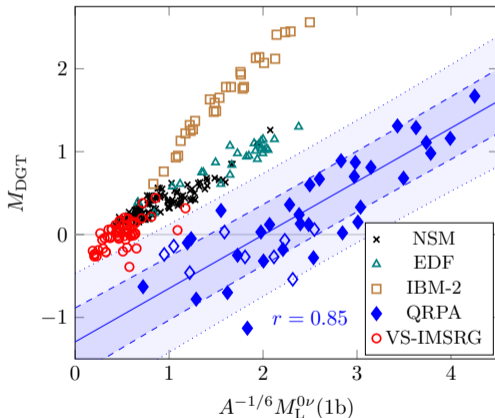


J. M. Yao, I. Ginnett, A. Belley et al., *Phys. Rev. C* 106, 014315 (2022)

# Correlations Between DGT and $0\nu\beta\beta$ Decay

$$M_{\text{DGT}} = -\langle 0_{\text{gs},f}^+ | [\sum_{jk} \sigma_j \tau_j^- \times \sigma_k \tau_k^-]^{(0)} | 0_{\text{gs},i}^+ \rangle$$

- Correlation between  $M^{0\nu}$  and  $M_{\text{DGT}}$  found in **nuclear shell model** and **EFT**
- Correlation also holds in *ab initio* **VS-IMSRG**
- ...and **QRPA**, when proton-neutron pairing varied
  - ▶ **Observation of  $M_{\text{DGT}}$  → constraints for  $M^{0\nu}$**



LJ, J. Menéndez, *Phys. Rev. C* 107, 044316 (2023)

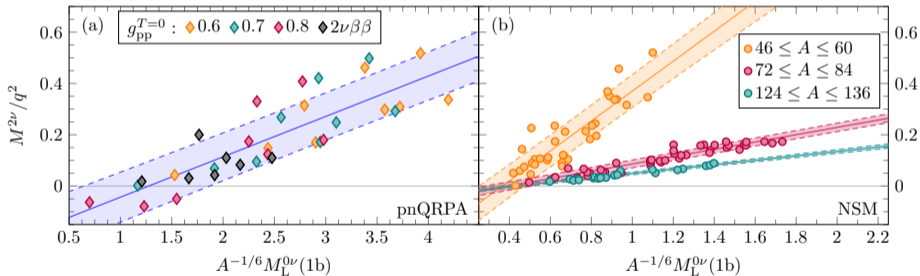
## Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

- *How about  $2\nu\beta\beta$  decay?*



# Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

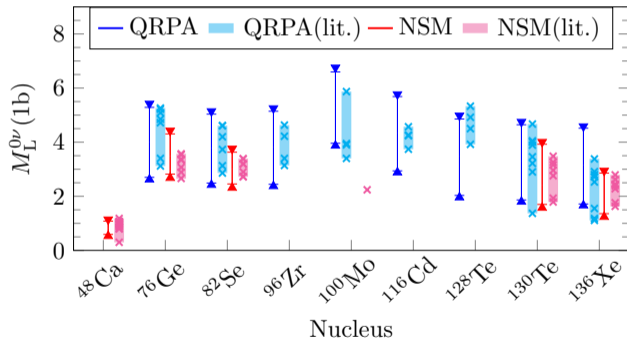
- *How about  $2\nu\beta\beta$  decay?*
- $2\nu\beta\beta$ -decay also correlated with  $0\nu\beta\beta$ -decay!



LJ, B. Romeo, P. Soriano and J. Menéndez, *Phys. Rev. C* **107**, 044305 (2023)

## Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

- **How about  $2\nu\beta\beta$  decay?**
- $2\nu\beta\beta$ -decay also correlated with  $0\nu\beta\beta$ -decay!
- We can use the existing data to estimate  $0\nu\beta\beta$ -decay NMEs!



LJ, B. Romeo, P. Soriano and J. Menéndez, *Phys. Rev. C* **107**, 044305 (2023)

Introduction

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Correlations with other observables to constrain the matrix elements

Summary and Outlook

- $\chi$ EFT corrections to  $0\nu\beta\beta$ -decay seem to respect the power counting, but N<sup>2</sup>LO corrections still significant
- Correlation between  $0\nu\beta\beta$  and  $2\nu\beta\beta$  decays helped us predict  $0\nu\beta\beta$ -decay NMEs with uncertainties
- Correlations with DGT and M1M1 transitions with future data can help us further constrain the NMEs

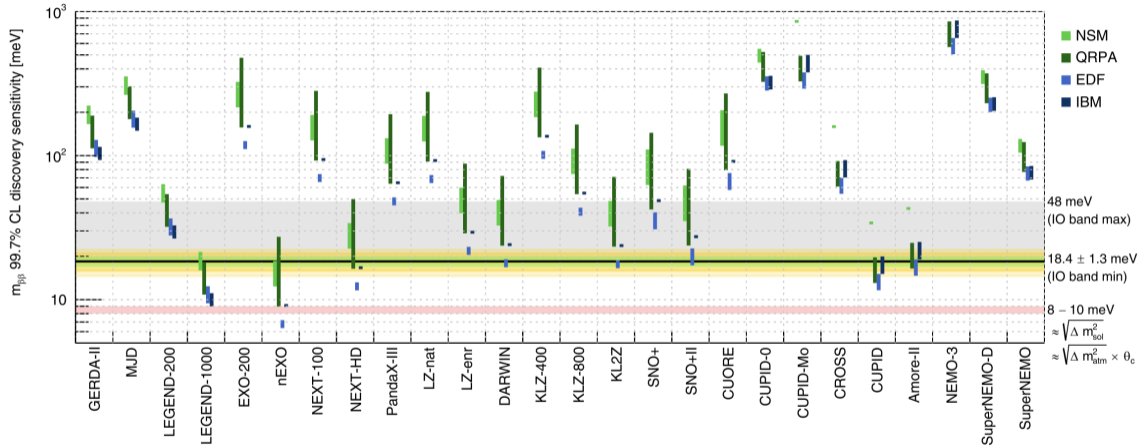
## For the Future...

- Test  $M_{\text{usoft}}^{0\nu}$  predictions with data from charge-exchange reactions (currently limited at  $\sim 5$  MeV)
- Study N<sup>2</sup>LO corrections to  $M^{0\nu}$  with consistent Hamiltonians in an *ab initio* framework
- Study correlation between  $0\nu\beta\beta$  and  $2\nu\beta\beta$  decay in an *ab initio* framework

Thank you  
Merci



# Next generation experiments



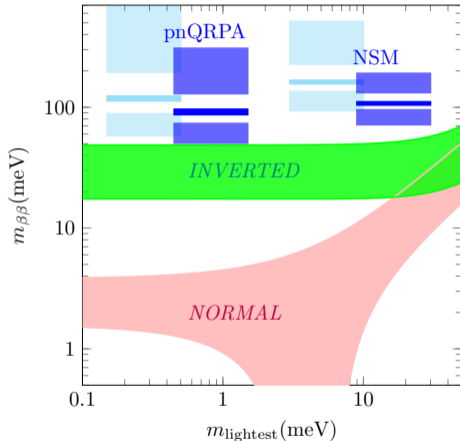
M. Agostini et al., Rev. Mod. Phys. **95**, 025002 (2023)

## Effective Neutrino Masses

- Effective neutrino masses combining the likelihood functions of GERDA ( $^{76}\text{Ge}$ ), CUORE ( $^{130}\text{Te}$ ), EXO-200 ( $^{136}\text{Xe}$ ) and KamLAND-Zen ( $^{136}\text{Xe}$ )

*S. D. Biller, Phys. Rev. D* **104**, 012002 (2021)

- Middle bands:  $M_L^{(0\nu)}$   
 Lower bands:  $M_L^{(0\nu)} + M_S^{(0\nu)}$   
 Upper bands:  $M_L^{(0\nu)} - M_S^{(0\nu)}$



*LJ, P. Soriano and J. Menéndez, Phys. Lett. B* **823**, 136720 (2021)

## Traditional nuclear matrix elements of neutrinoless double-beta decay

$$M^{0\nu} = \frac{R}{g_A^2} \int \frac{d\mathbf{k}}{2\pi^2} \frac{e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}}{E_\nu} \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{E_\nu + E_n - \frac{1}{2}(E_i + E_f) - \frac{1}{2}(E_1 - E_2)}$$

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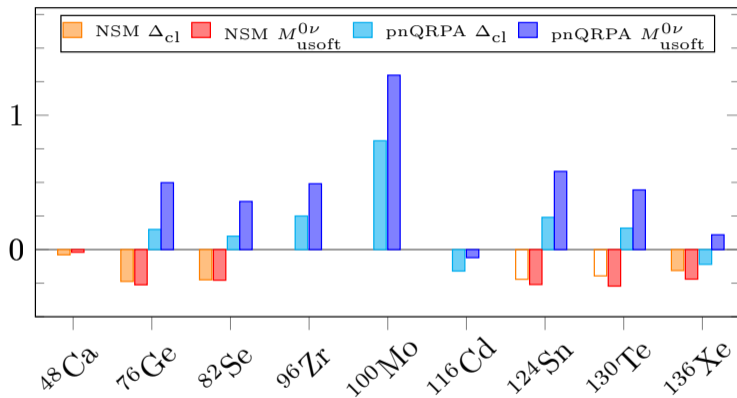
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# Ultrasoft Neutrinos as Closure Correction



$$\Delta_{cl} = M_{\text{non-cl}}^{0\nu} - M_{cl}^{0\nu}$$

*LJ, D. Castillo, P. Soriano, J. Menéndez, in preparation*

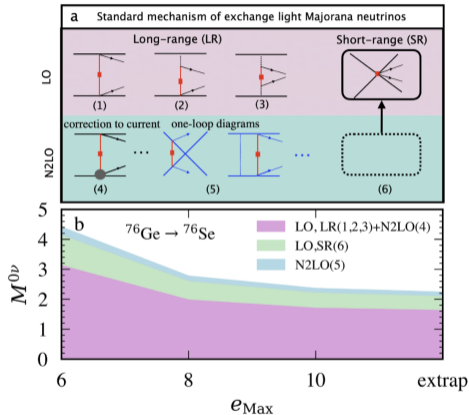
# Similar effects found in *ab initio* studies

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$$M_S^{0\nu} / M_L^{0\nu} \sim 40\%,$$

$$M_{N^2LO}^{0\nu} / M_L^{0\nu} \sim 5\%{}^a$$

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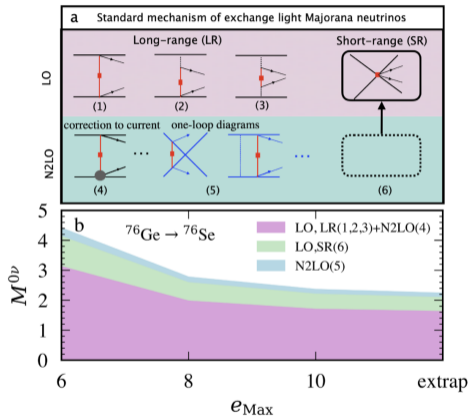
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- In  $^{130}\text{Te}$  and  $^{136}\text{Xe}$ :

$$M_S^{0\nu} / M_L^{0\nu} \sim 20\% - 120\%$$

A. Belley et al. arXiv:2307.15156 (2023)

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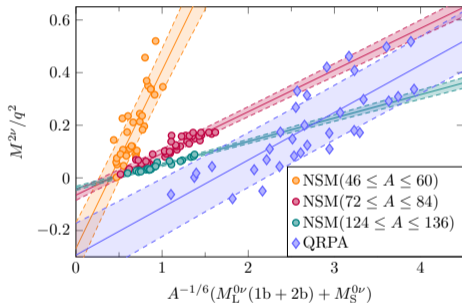


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LJ, B. Romeo, P. Soriano and J. Menéndez,  
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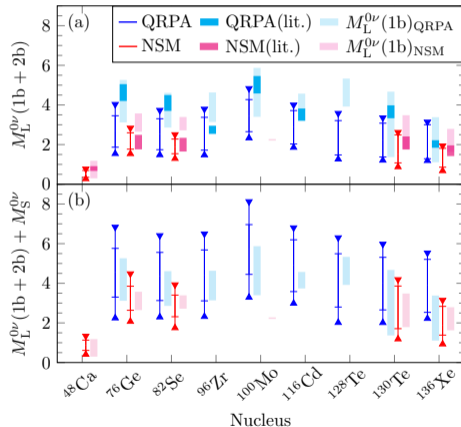
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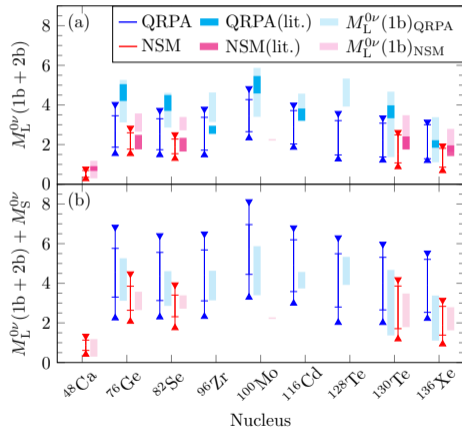
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- 2BCs and the contact term largely cancel each other



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