Consistently treating bound and scattering states in many-body methods

Mack C. Atkinson

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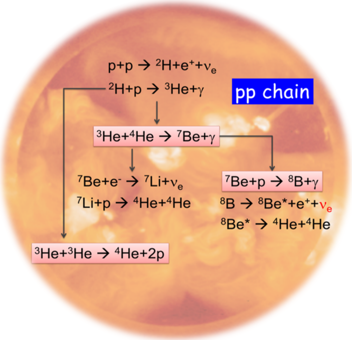
  1. No-core shell model with continuum (NCSMC) - *ab initio*
  2. Dispersive optical model (DOM) - phenomenological
- I will present different reaction calculations that benefit from their simultaneous bound/scattering states
- Results of these calculations warrant futher investigation of the nuclear force

^{3}He$(\alpha,\gamma)^7$Be important for solar-model predictions



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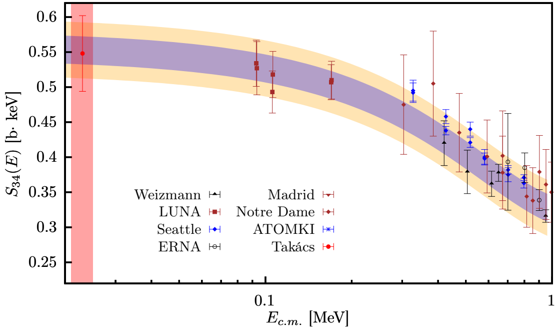

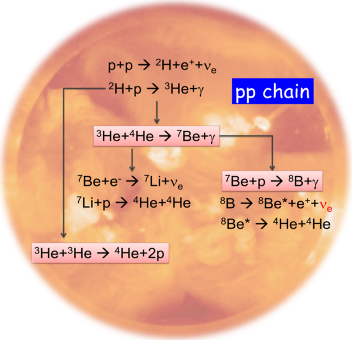

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Adelberger *et al.*, Rev Mod Phys **83** 195 (2011)

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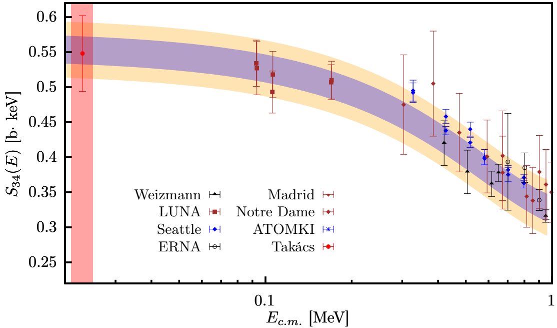

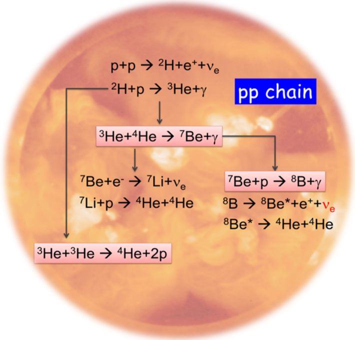

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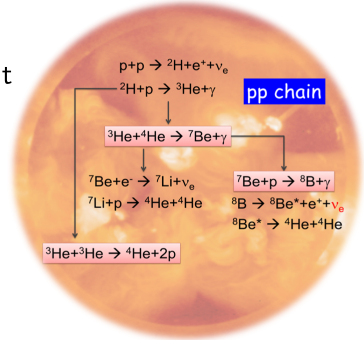
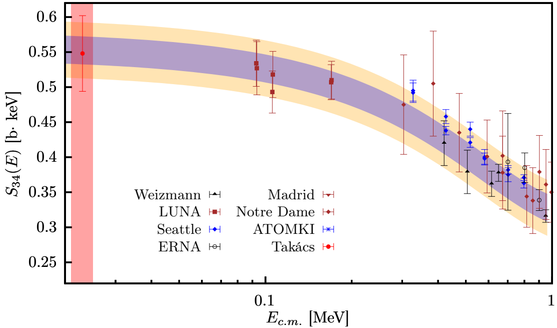
- Reaction rates too low at solar energies in the lab
- Current evaluations depend on both theory and experiment
- Ideally, theory will accurately predict $S_{34}(E)$





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The *ab initio* method: from NCSM to NCSMC

$$\left\langle \Psi_{bs}\left({}^{7}\mathrm{Be}\right) \middle| \hat{\mathcal{M}}_{\mathrm{EM}} \middle| \Psi_{sc}\left({}^{3}\mathrm{He}+\alpha\right) \right\rangle$$

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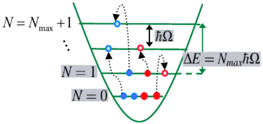

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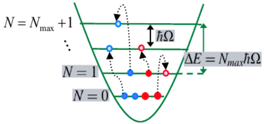

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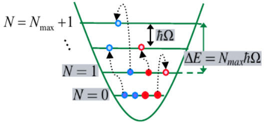

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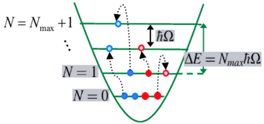

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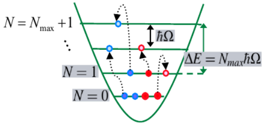

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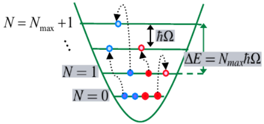

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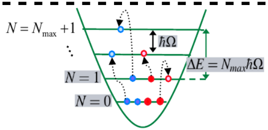

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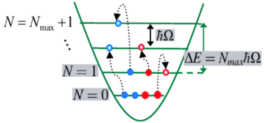

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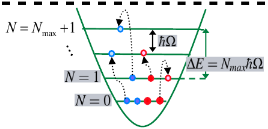

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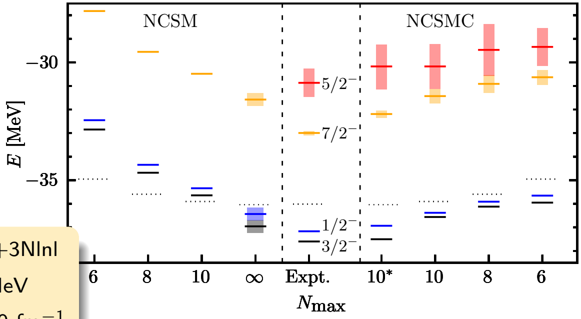
NCSMC Calculation of ^{3}He+^{4}He well-converged, levels need shifting



NN-N3LO+3NlnI

$\hbar\Omega = 20$ MeV

$\lambda_{\text{SRG}} = 2.0$ fm^{-1}

V. Soma *et al*, PRC **101**, 014318 (2020)

U. van Kolck, PRC **49**, 2932 (1994)

D.R. Entem and R. Machleidt, PRC **68**, 041001(R) (2003)

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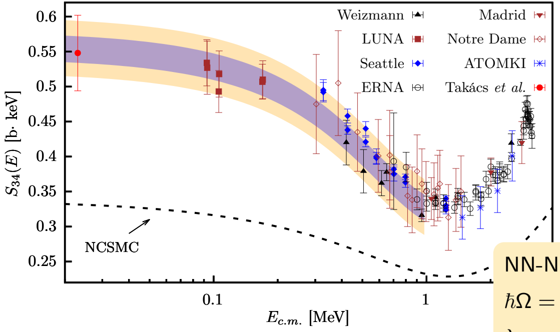

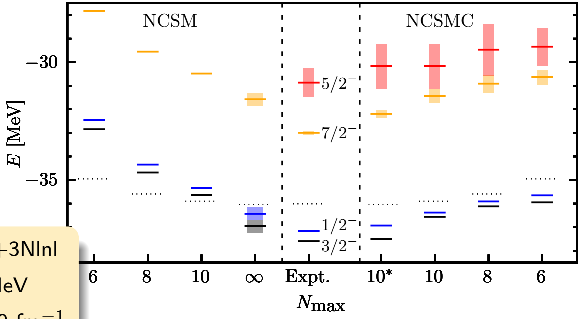

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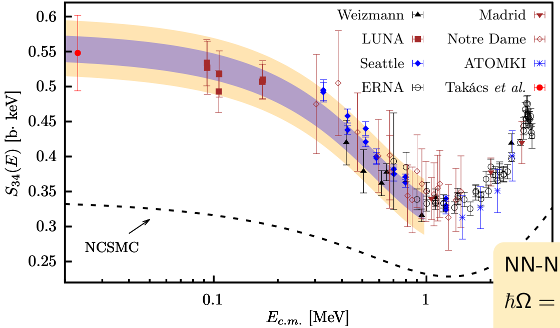

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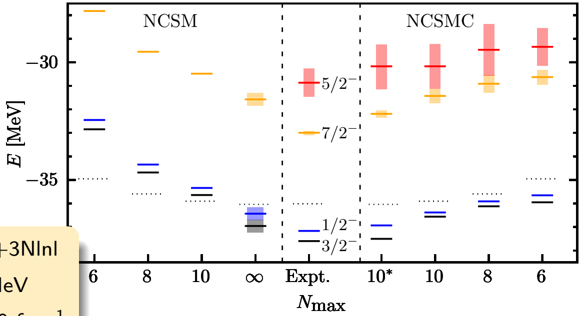

^{7}Be	NCSM	NCSMC	Expt.
$E_{3/2^-}$	0.261	-1.05	-1.587
$E_{1/2^-}$	0.563	-0.874	-1.16
$\mathcal{C}_{3/2^-}$	-	3.46	-
$\mathcal{C}_{1/2^-}$	-	3.44	-
r_{ch}	2.44	2.70	2.647(17)

- Capture rate accurate only if Expt. levels reproduced

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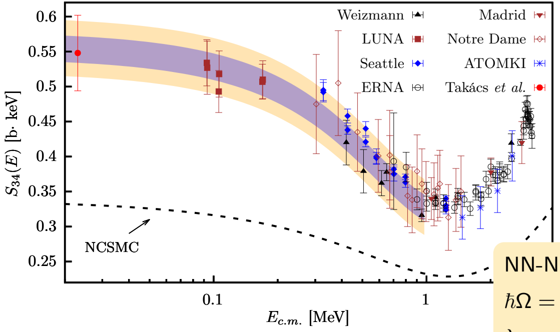

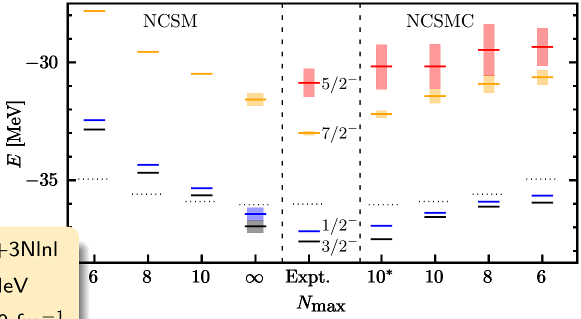

NN-N3LO+3NInl

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$$E_\lambda^{NCSM} \rightarrow E_\lambda^{NCSM} + \epsilon$$

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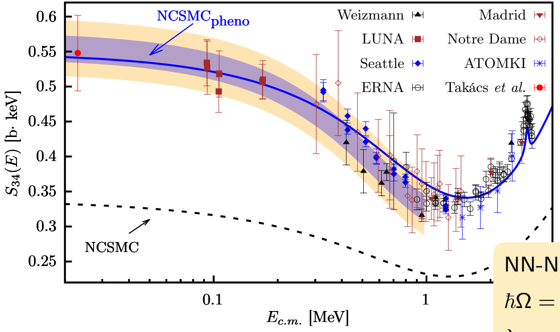

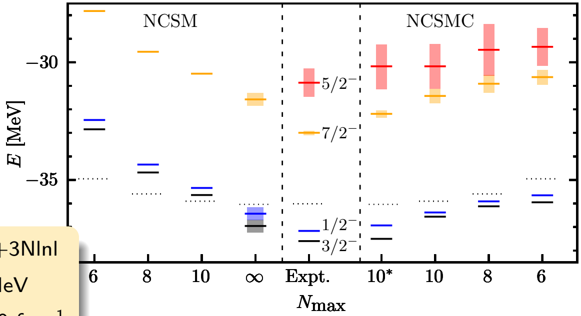

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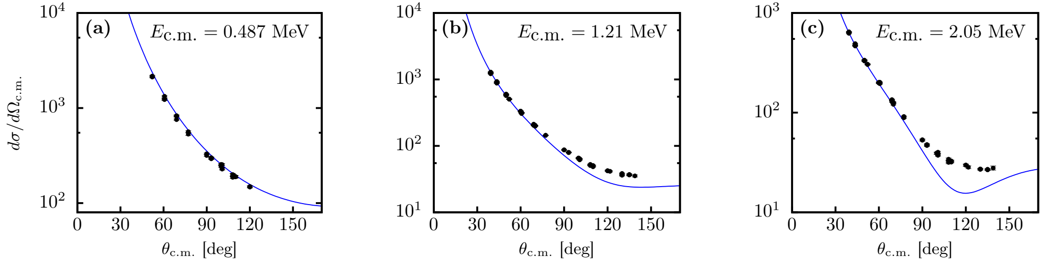

Paneru *et al.*, arXiv:2211.14641 (2022)

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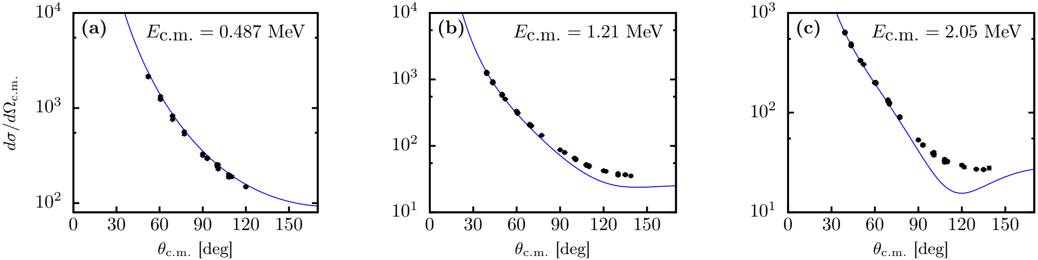

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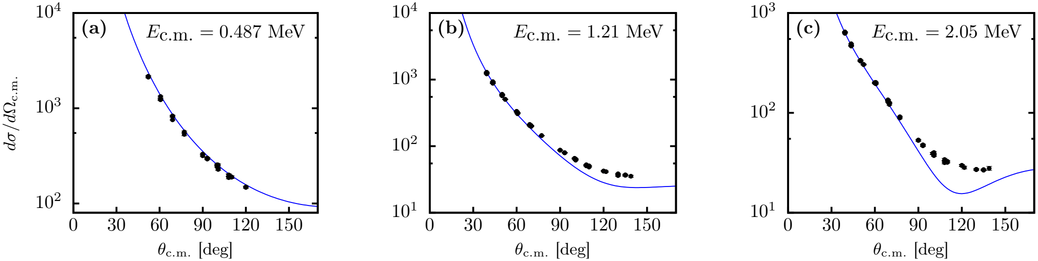

- What is the source of discrepancy at large angles?

Paneru *et al.*, arXiv:2211.14641 (2022)

The $1/2^+$ channel is responsible for this constant shift

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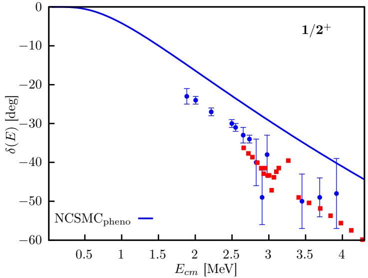

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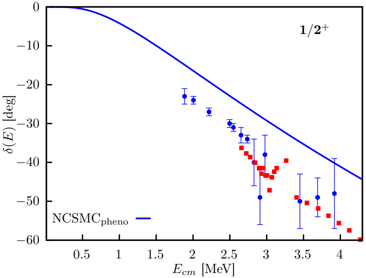
$$\mathcal{H}_{\nu\nu'}^{1/2^+}(r,r') \rightarrow \mathcal{H}_{\nu\nu'}^{1/2^+}(r,r') + V(r,r')$$



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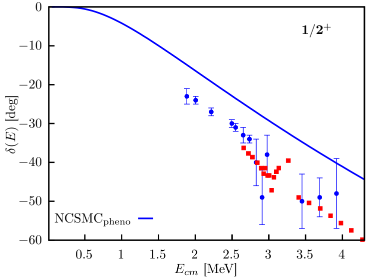
$$V(r,r') = \frac{V_0}{1+e^{(R-r_0)/a_0}} \times e^{(r-r')^2/a_0^2}$$



The $1/2^+$ channel is responsible for this constant shift

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- Explicitly add repulsion

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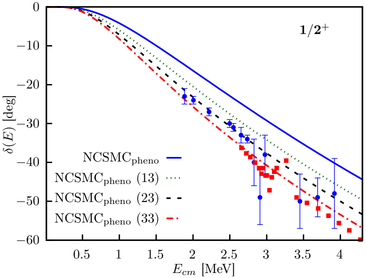

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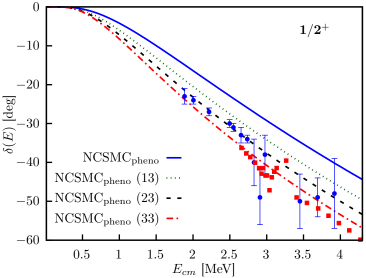

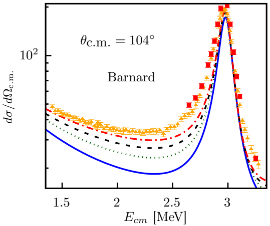

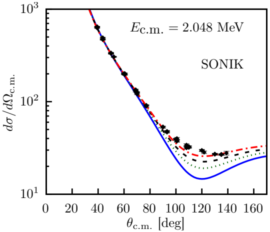

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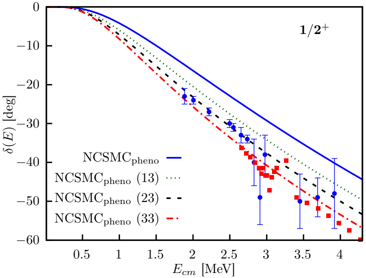

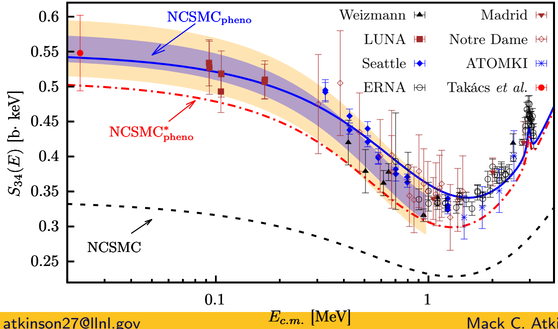

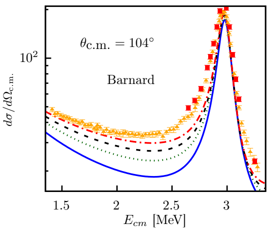

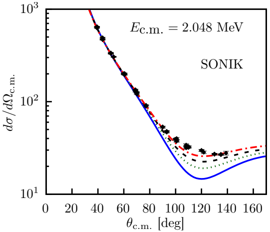

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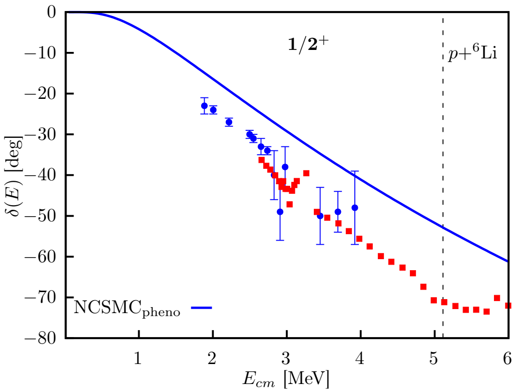

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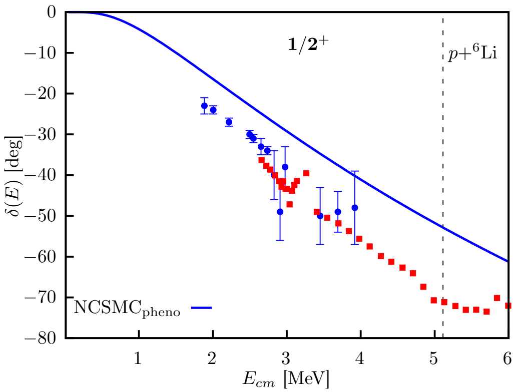
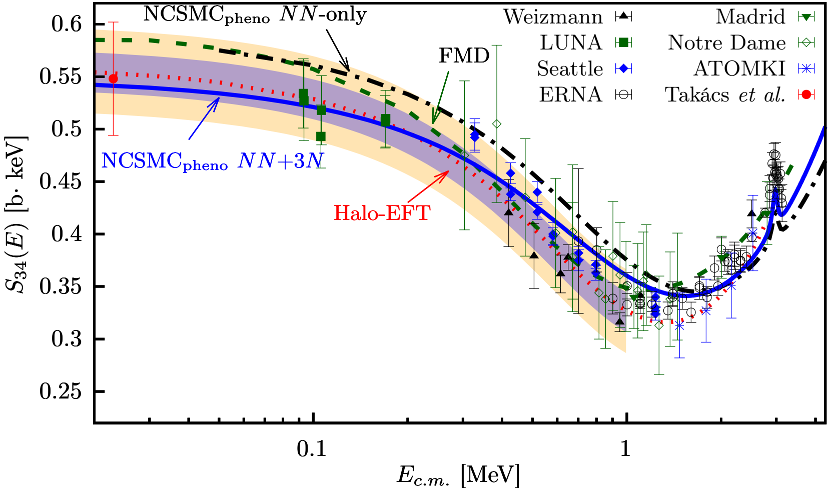
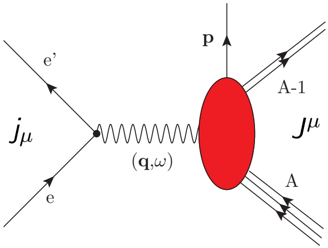
2. **Chiral interaction**
   - Phase shift shows some dependence on interaction
   - Need to compare more interactions



Comparison with other theories



The exclusive $(e, e'p)$ reaction



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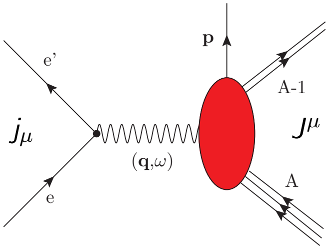

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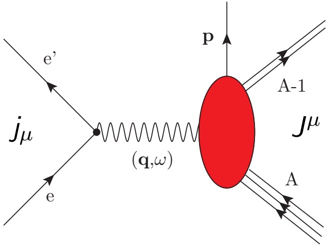

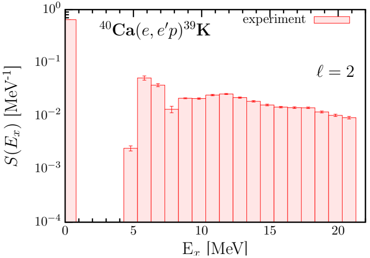

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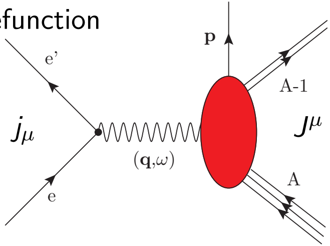

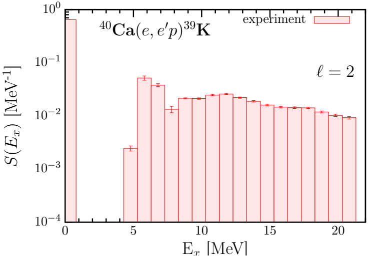

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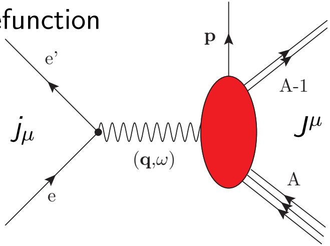

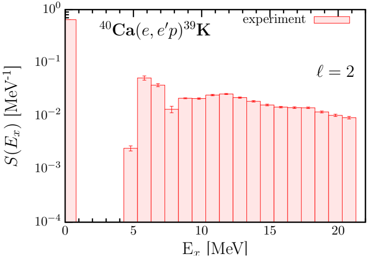

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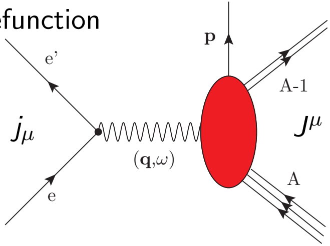

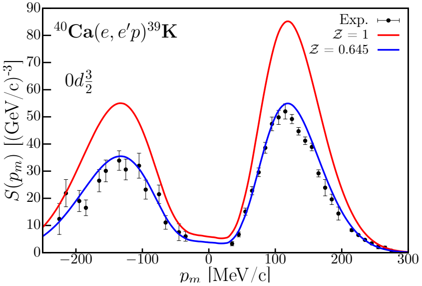

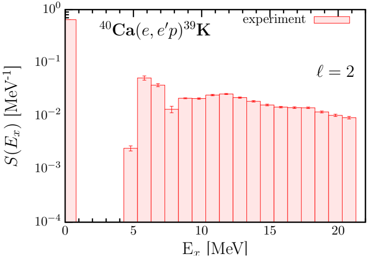

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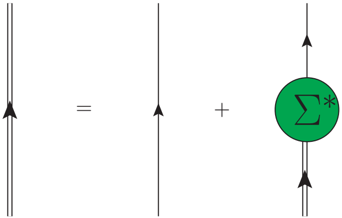

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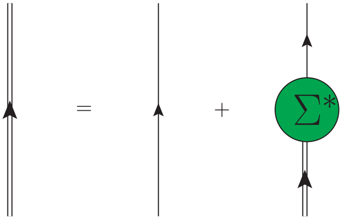

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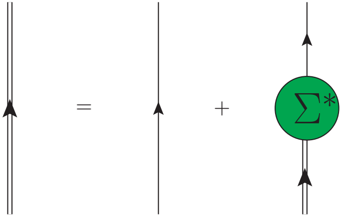

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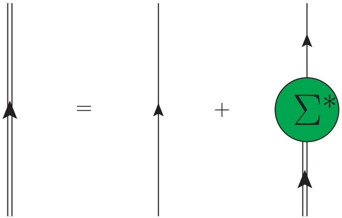

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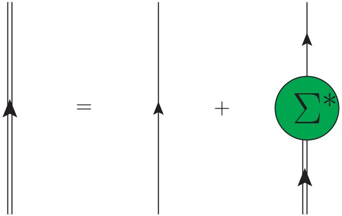

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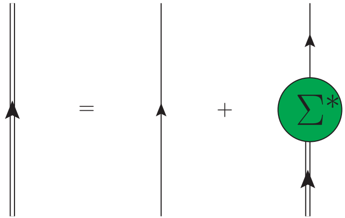

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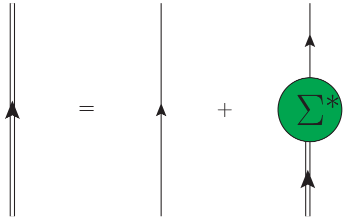

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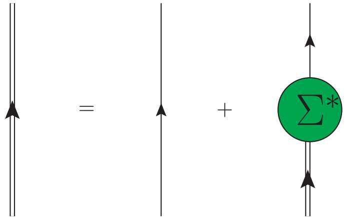

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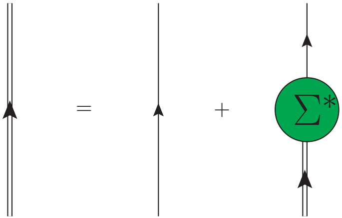

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Can this also describe negative energy observables?

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- The DOM makes use of complex analysis to formulate a consistent self-energy

1. Calculation could be missing channels

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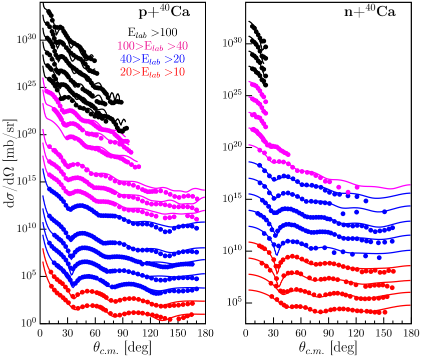

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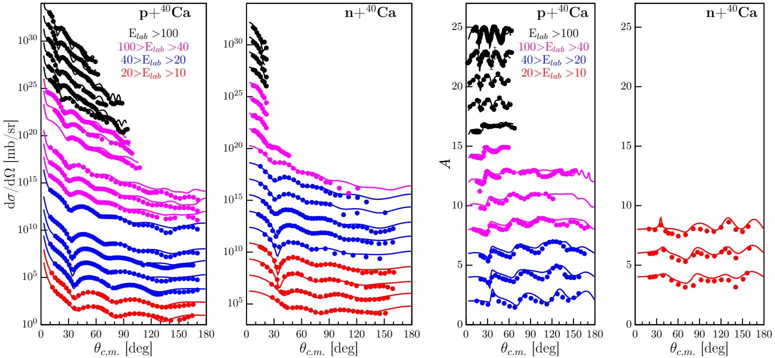

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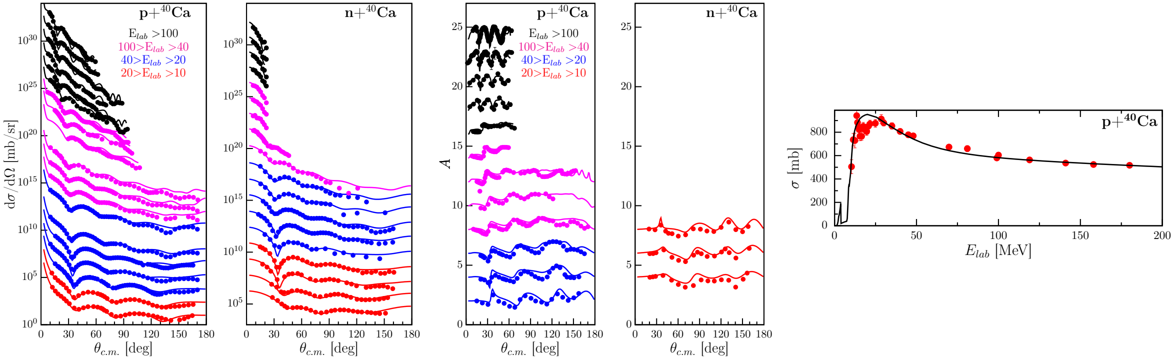

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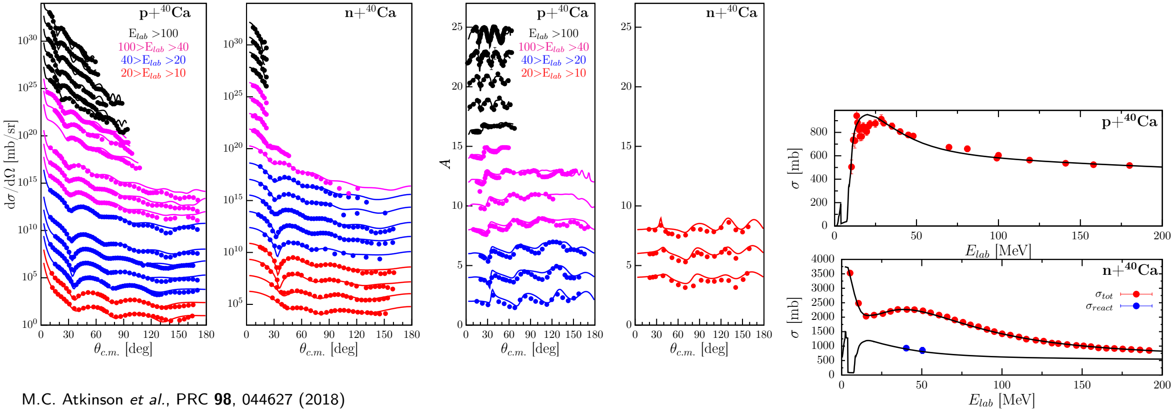

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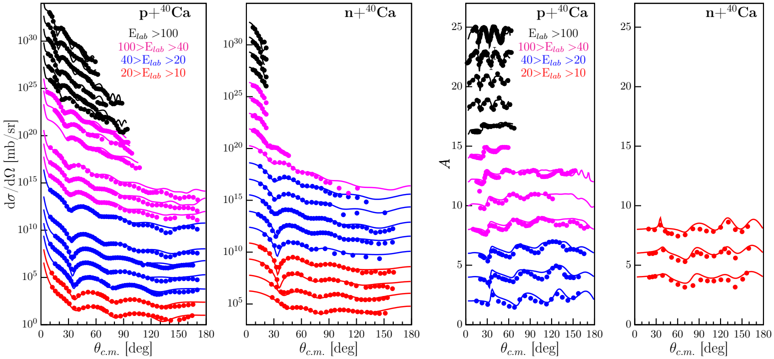

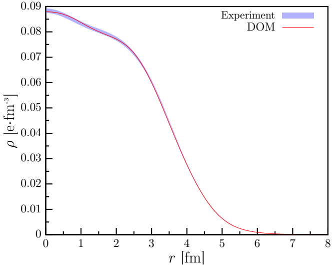

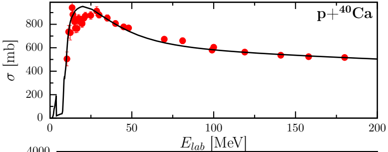

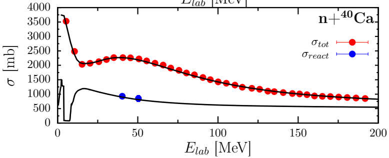

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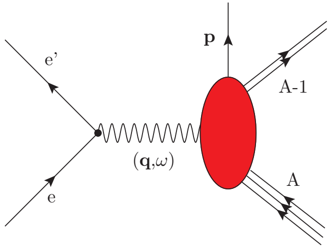

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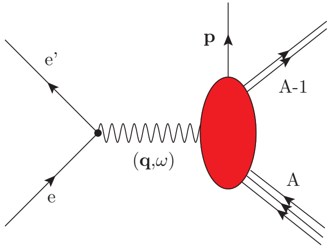

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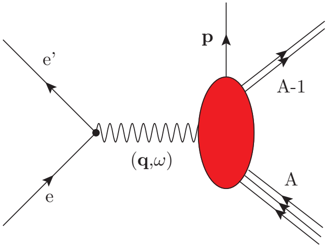
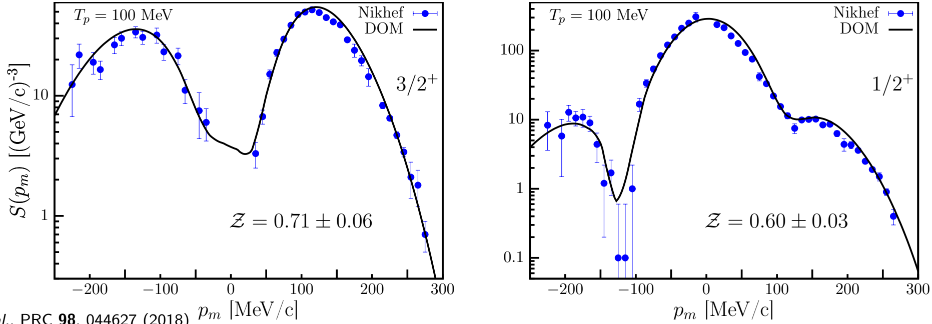
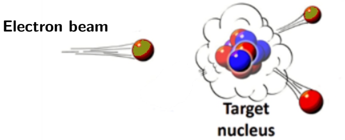




M.C. Atkinson *et al.*, PRC **98**, 044627 (2018)

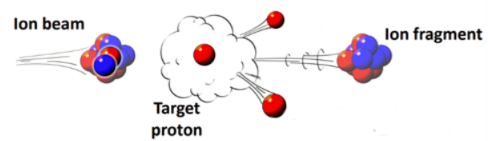
Want to study knockout in exotic nuclei too

Experimental sketch for **stable** nuclei

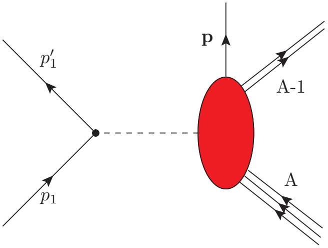


Reaction mechanism well-understood

Experimental sketch for **exotic** nuclei (RIB)



To get started, consider the stable ^{40}Ca$(p, 2p)^{39}$K reaction



K. Yoshida, M.C. Atkinson, K. Ogata, W.H. Dickhoff PRC **105**, 014622 (2022)

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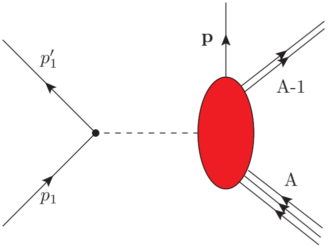

- Same DOM ingredients used

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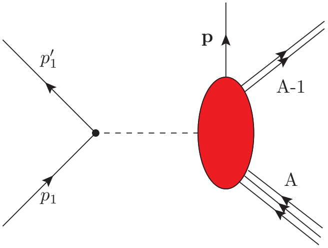

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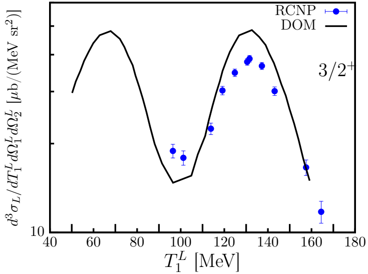

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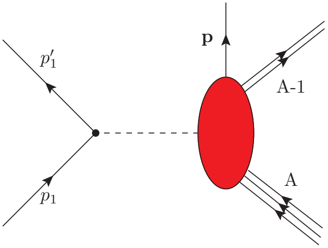

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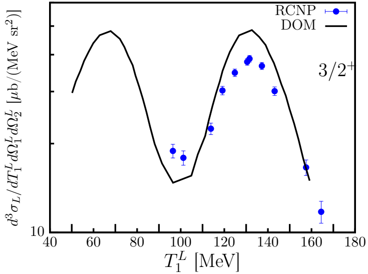
"S_F"	$(p,2p)$	$(e,e'p)$
DOM	0.560	0.71 ± 0.04



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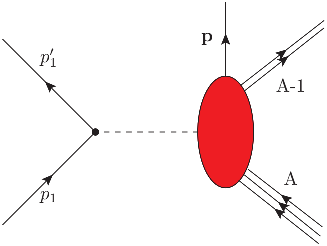

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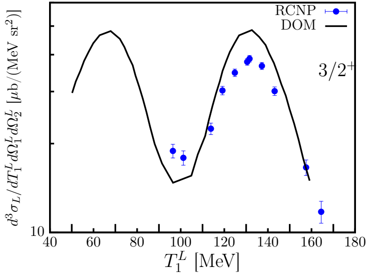
$$\mathcal{S}_{\mathcal{F}} = \left(1 - \frac{\partial \Sigma^*(\alpha_{qh}, \alpha_{qh}; E)}{\partial E}\bigg|_{\epsilon}\right)^{-1}$$

- Remember that S_F comes directly from Σ^*_{DOM}



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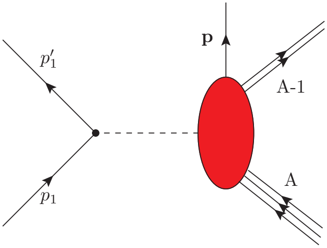

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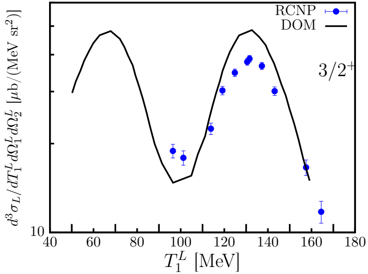
$$\mathcal{S}_{\mathcal{F}} = \left(1 - \frac{\partial\Sigma^*(\alpha_{qh},\alpha_{qh};E)}{\partial E}\bigg|_{\epsilon}\right)^{-1}$$

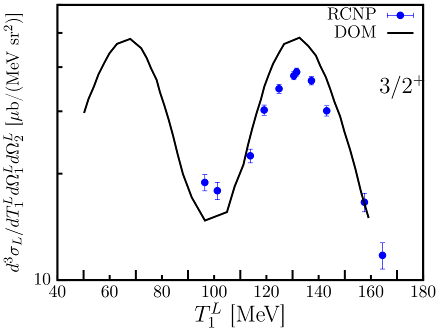
- Remember that S_F comes directly from Σ^*_{DOM}
- Main difference is the probe $\implies$ problem is likely V_{pp}



K. Yoshida, M.C. Atkinson, K. Ogata, W.H. Dickhoff PRC **105**, 014622 (2022)

Nucleus-informed *pp* interaction: $V_{pp} \to \Gamma_{pp}$

- Try varying V_{NN} to see effect on S_F



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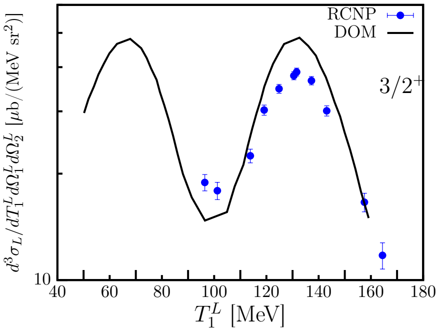

S_F	V_{NN}	$(p, 2p)$
DOM	FL	0.560 ± 0.05
DOM	Mel	0.489 ± 0.05
DOM	Mel (free)	0.515 ± 0.05

K. Yoshida, M.C. Atkinson, K. Ogata, W.H. Dickhoff PRC **105**, 014622 (2022)

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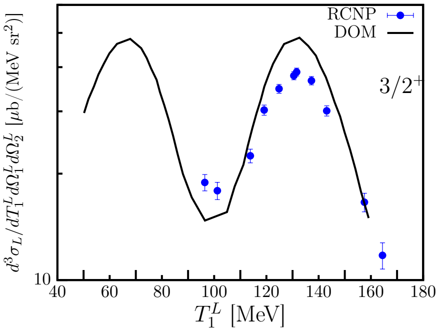
- Try varying V_{NN} to see effect on S_F
- Dependence of S_F on choice of V_{NN} is another sign the problem lies in V_{NN}



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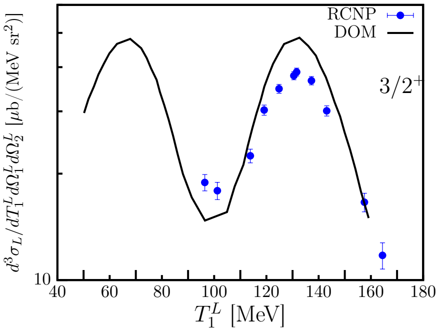
- Interactions need information about the nucleus



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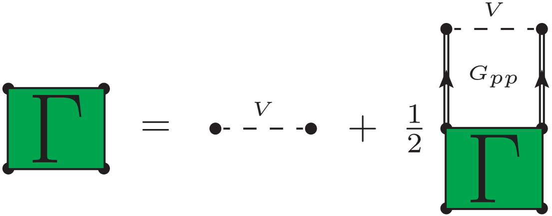

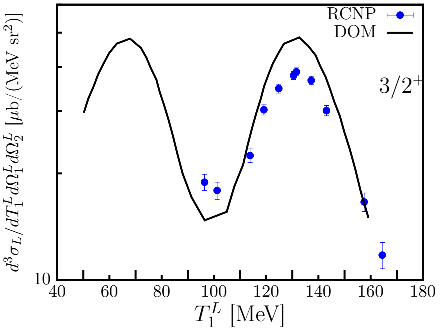

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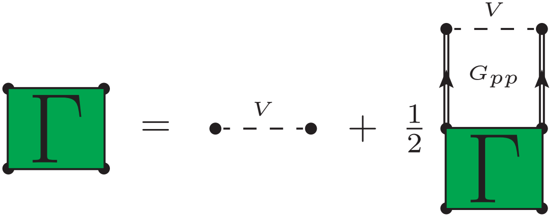
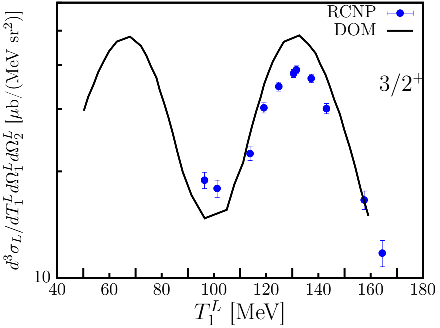


- $G_{pp} \approx \int G_{\mathrm{DOM}} \times G_{\mathrm{DOM}}$



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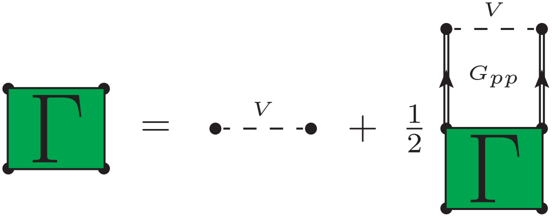
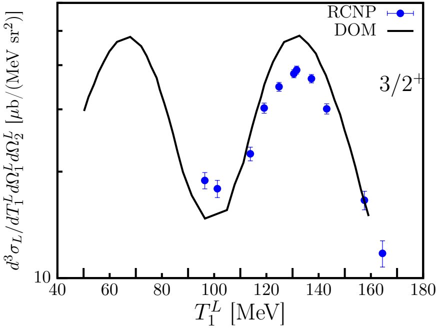


- $G_{pp} \approx \int G_{\mathrm{DOM}} \times G_{\mathrm{DOM}}$
- Similar to G-matrix, except this is calculated in finite nuclei



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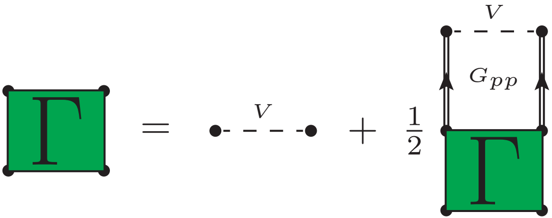

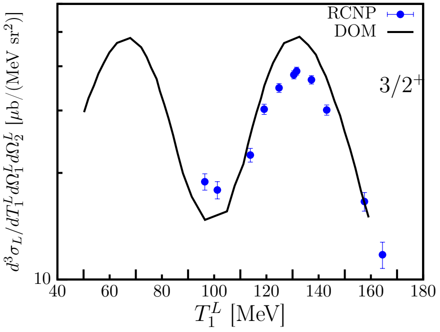

- $G_{pp} \approx \int G_{\mathrm{DOM}} \times G_{\mathrm{DOM}}$
- Similar to G-matrix, except this is calculated in finite nuclei
- Good approximation for typical $(p, 2p)$ energies

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Summary

1. *Ab initio* calculation of ^{3}He$(\alpha,\gamma)^{7}$Be capture reaction using the NCSMC
   - Simultaneous analysis of elastic and capture data reveals mild tension
   - Lack of repulsion in $1/2^{+}$ channel could be due to V_{NN} or the lack of $p+^{6}$Li channel
2. The DOM accurately predicts ^{40}Ca$(e,e'p)^{39}$K but not the very similar ^{40}Ca$(p,2p)^{39}$K reaction
   - Likely cause of discrepancy in these knockouts is the *pp* interaction used in the DWIA

- In both examples of the NCSMC and the DOM, the ability to simultaneously describe bound and scattering states helped isolate areas of improvement
- Resolution of these issues will lead to improved nucleus-induced reaction calculations

Thanks

- Willem Dickhoff
- Hossein Mahzoon

- Cole Pruitt
- Gregory Potel
- Sofia Quaglioni
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- Henk Blok

- Petr Navratil

- Kazuyuki Ogata

- Kazuki Yoshida