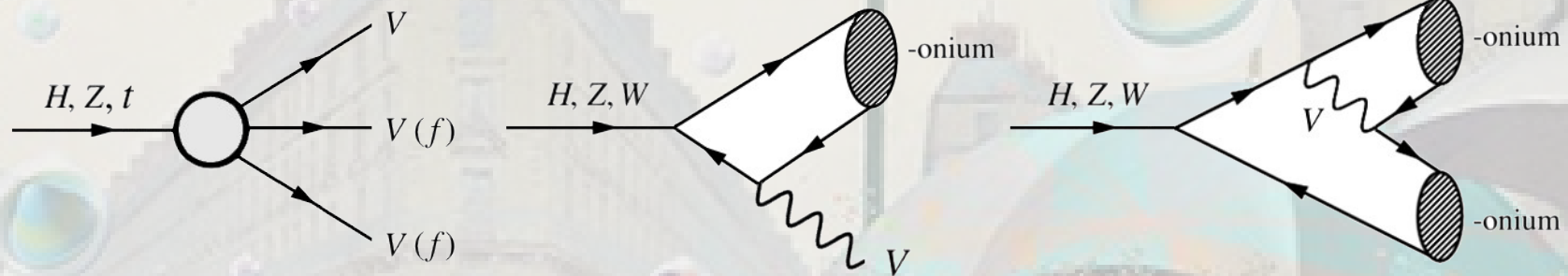


Rare and exclusive few-body decays of the Higgs boson

Higgs Hunting 2024
ICJLab/APC, Paris, 3rd Sept. 2024



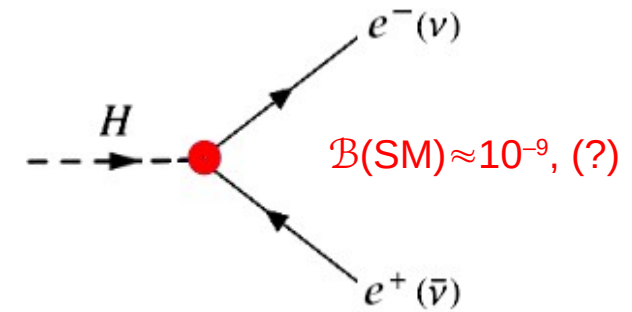
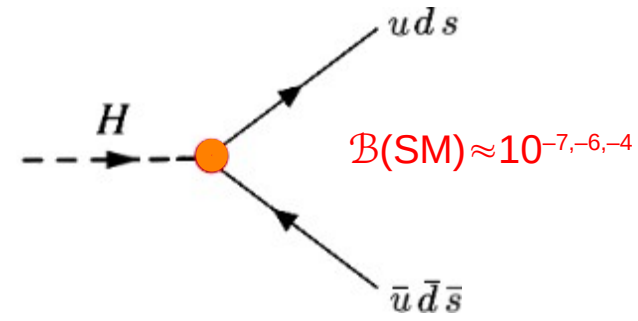
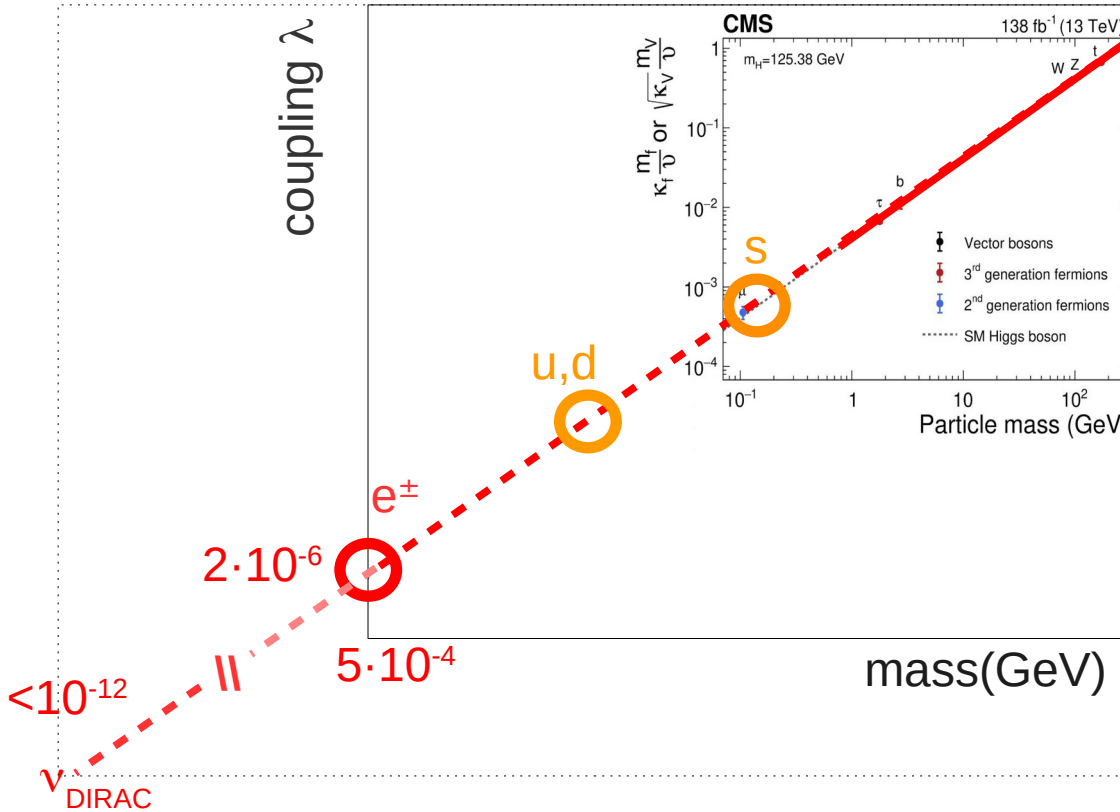
David d'Enterria (CERN)

Van Dung Le (Ho Chi Minh Univ)

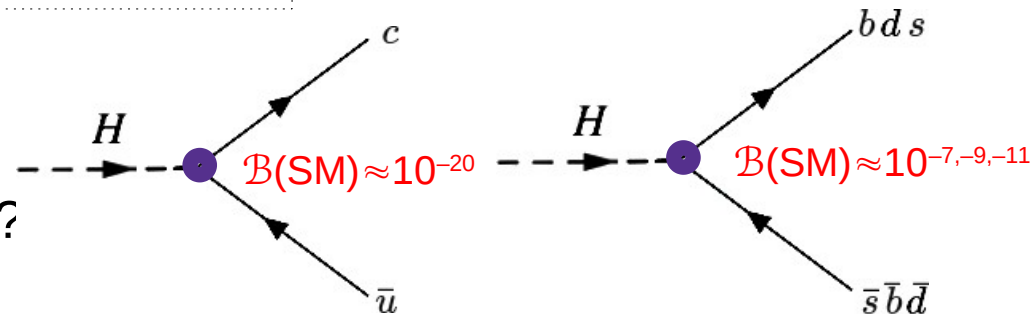
Details in: [arxiv:2312.11211](https://arxiv.org/abs/2312.11211), to appear in JPG

Many open questions in the Higgs sector...

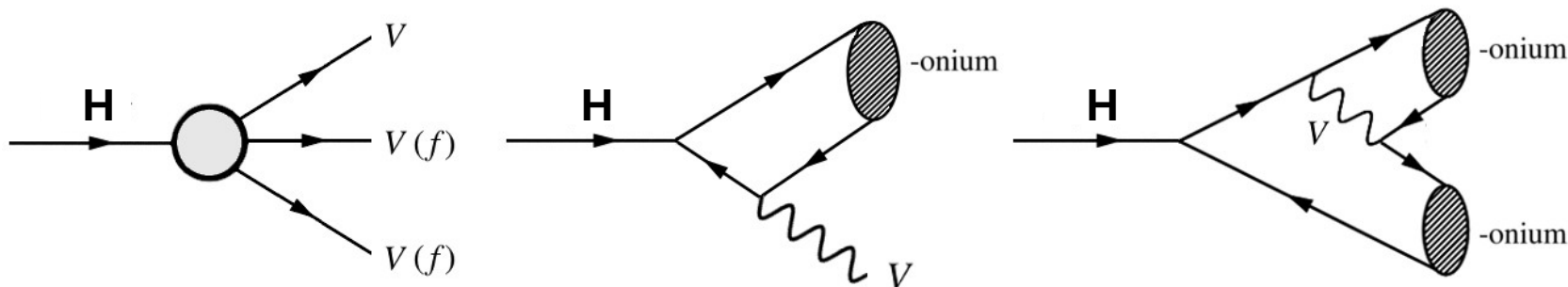
- Do the **lightest fermions** (u, d, s, e, ν = stable visible matter in universe!) acquire their masses through their Higgs (**Yukawa**) couplings?



- Are there **FCNCs** mediated by Higgs boson $H \rightarrow qq'$ at tree level?



Rare & exclusive few-body Higgs boson decays



■ Physics motivations:

- Search for suppressed/forbidden SM decays.
- Probe quark/lepton Yukawa couplings ($H \rightarrow cc, qq, \ell\ell$) via exclusive final states with charm mesons, light mesons, or leptonia.
- Search for FCNC Higgs decays.
- (Perform *precision tests of the QCD factorization formalism & constraint nonperturbative hadronic bound-state parameters*).

■ Work performed:

- Revisit theor. & exp. status of 2-,3-,4-body rare/exclusive decays (branching fractions: $\mathcal{B} < 10^{-5}$) of the SM Higgs boson.
- Collect all \mathcal{B} 's computed (41) and/or with exp. upper limits (20) to date. Update a few old TH \mathcal{B} 's. Identify missing decays & compute their \mathcal{B} 's (22).
- Provide (\mathcal{L}_{int} -based) projections for HL-LHC, FCC-ee, FCC-hh searches: Help guide and prioritize future experimental searches.

Theoretical methods

- For rare few-body decays into elementary particles (γ, ν): MG5@NLO
- For exclusive hadronic channels: 3 models based on pQCD factorization to describe decay of heavy object $\mathcal{O}(100 \text{ GeV})$ into much lighter quarks/mesons:

Decay width = hard ME (perturbative) \otimes quark with mom. x (nonpQCD)

1. **Light cone (LC)**: nonperturbative mesons described by LCDAs. Employed for light (uds) mesons. Decay amplitude $\propto f_M$ decay constant

$$A(X \rightarrow M + \gamma) \sim f_M \int_0^1 dx H_M(x, \mu) \phi_M(x, \mu) \quad f_M^2 = 4N_c \frac{|\phi_M(0)|^2}{m_M} \quad (\text{from meson leptonic width})$$

$\Phi_M(x) \approx x\delta(x)$ LCDA: Amplitude of q at mom. x

2. **Soft-Collinear Effective Theory (SCET)**: EFT multiple scale resum. + LCDAs. Mostly used for light (uds) mesons.

$$A = -if_M E \int_0^1 dx H_M(x, \mu) \phi_M(x, \mu) + \text{power corr}$$

$$\phi_M(x, \mu) = 6x(1-x) \left[1 + \sum_{n=1}^{\infty} a_n^M(\mu) C_n^{3/2}(2x-1) \right]$$

(a_n moments from latt.-QCD or sum rules)

(**Heavy-Quark Effective Theory, HQET**: for light+heavy-quark mesons)

3. **Non-Relativistic QCD (NRQCD)**: LDMEs for decays into charm/bottom mesons:

$$d\Gamma[X \rightarrow M(n) + Y] = \sum_n d\hat{\Gamma}[X \rightarrow (Q\bar{Q})_n + Y] \langle \mathcal{O}^M(n) \rangle$$

$$\sum_n d\hat{\Gamma}[X \rightarrow (Q\bar{Q})_n + Y] \propto \frac{1}{2m_X} |\mathcal{A}|^2 d\Phi_3$$

$$\langle \mathcal{O}^{\eta_c}({}^1S_0^{[1]}) \rangle = \frac{N_c}{2\pi} |R^{\eta_c}(0)|^2, \quad \langle \mathcal{O}^{J/\psi}({}^3S_1^{[1]}) \rangle = \frac{3N_c}{2\pi} |R^{J/\psi}(0)|^2$$

Theoretical parameters

- Numerical parameters used in calculations of rare/exclusive Higgs decay widths (MG5@NLO, LC, SCET/HQET frameworks):

masses and widths		couplings and CKM elements		meson masses and form-factor parameters (in GeV)			
m_{ee}	1.02991 MeV	$\alpha(0)$	1/137.036	m_{π^\pm}	0.13957	f_π	0.1304 [45]
$m_{\mu\mu}$	0.21126 GeV	$\alpha(m_Z)$	1/128.943	m_{K^\pm}	0.493677	f_K	0.1562 [45]
$m_{\tau\tau}$	3.5537 GeV	$\alpha_s(m_Z)$	0.1180			λ_K	0.255 [69]
m_c	1.50 GeV	$\sin^2 \theta_w$	0.2351	m_{ρ^\pm}	0.77526	f_ρ	0.212 [45]
m_b	4.18 GeV	G_F	$1.1664 \cdot 10^{-5} \text{ GeV}^{-2}$	$m_{K^{*\pm}}$	0.89166	f_{K^*}	0.203 [45]
m_t	172.69 GeV	$ V_{ud,cs} $	0.974	m_ϕ	1.0194	f_ϕ	0.223 [14]
m_W	80.377 GeV	$ V_{us,cd} $	0.225	m_{D^\pm}	1.86966	f_D	0.212 [70]
m_Z	91.188 GeV	$ V_{cb,ts} $	0.041			λ_D	0.354 [71]
m_H	125.25 GeV	$ V_{ub} $	0.00382	$m_{D_s^\pm}$	1.96835	f_{D_s}	0.2499 [70]
Γ_W	2.085 GeV	$ V_{tb} $	0.999	$m_{D^{*0}}$	2.007	$f_{D^{*0}}$	1.28 f_D [72]
$\Gamma_{W \rightarrow \mu\nu}$	0.1063 Γ_W	$ V_{td} $	0.0086	$m_{D^{*\pm}}$	2.01027	f_{D^*}	1.28 f_D [72]
Γ_Z	2.4955 GeV			$m_{D_s^{*\pm}}$	2.1123	$f_{D_s^*}$	1.26 f_{D_s} [72]
$\Gamma_{Z \rightarrow \ell\ell}$	0.0337 Γ_Z			m_{B^\pm}	5.27934	f_B	0.190 [70]
Γ_H	4.1 MeV [73]					λ_B	0.338 [74]
$\Gamma_{H \rightarrow \gamma\gamma}$	$2.5 \cdot 10^{-3} \Gamma_H$			$m_{B^{*\pm,0}}$	5.32471	f_{B^*}	0.941 f_B [75]
Γ_t	1.331 GeV [76]					f_{B_s}	0.2303 [70]
						λ_{B_s}	0.438 [77]
				$m_{B_s^{*0}}$	5.415	$f_{B_s^*}$	0.953 f_{B_s} [75]
				$m_{B_c^\pm}$	6.27447	f_{B_c}	0.427 [75]
				$m_{B_c^{*\pm}}$	6.32877 [75]	$f_{B_c^*}$	0.988 f_{B_c} [75]

Experimental limits projections

■ Number of Higgs bosons at different colliders:

$N(\text{LHC}, 13 \text{ TeV}) \approx 10\text{e}6 \rightarrow N(\text{HL-LHC}) \approx 350\text{e}6 \rightarrow N(\text{FCC-hh}) \approx 2.8\text{e}10$
 $\rightarrow N(\text{FCC-ee}) \approx 2\text{e}6$ (with ~ 0 backgds.)

Collider	W [±] bosons		Z bosons		H bosons		top quarks	
	$\sigma(\text{W})$	$N(\text{W})$	$\sigma(\text{Z})$	$N(\text{Z})$	$\sigma(\text{H})$	$N(\text{H})$	$\sigma(\text{t}\bar{\text{t}})$	$N(\text{top})$
LEP	4.0 pb	0.8×10^5	59 nb	2×10^7	$\sim 2, 1 \text{ fb}$	~ 5	–	–
FCC-ee	4.0 pb	5×10^8	59 nb	6×10^{12}	200, 30 fb	1.9×10^6	0.5 pb	3.8×10^6
<i>Increase factor LEP \mapsto FCC-ee</i>	1	6250	1	300,000	70, 30	400,000	–	–
Tevatron (1.96 TeV, 10 fb^{-1})	25.3 nb	2.5×10^8	7.6 nb	7.6×10^7	1.1 pb	1.1×10^4	7.1 pb	1.4×10^5
HL-LHC (14 TeV, $2 \times 3 \text{ ab}^{-1}$)	200 nb	1.2×10^{12}	62.5 nb	3.8×10^{11}	58 pb	3.5×10^8	1 nb	1.2×10^{10}
FCC-hh (100 TeV, 30 ab^{-1})	1300 nb	4.1×10^{13}	415 nb	1.2×10^{13}	0.93 nb	2.8×10^{10}	35 nb	2.1×10^{12}
<i>Increase factor Tevatron \mapsto HL-LHC</i>	8	4800	8.2	5000	52.7	31 800	141	86 000
<i>Increase factor HL-LHC \mapsto FCC-hh</i>	6.5	34	6.7	32	16	80	35	175

■ **Conservative** extrapolation of present (13 TeV) **B** limits for HL-LHC via:

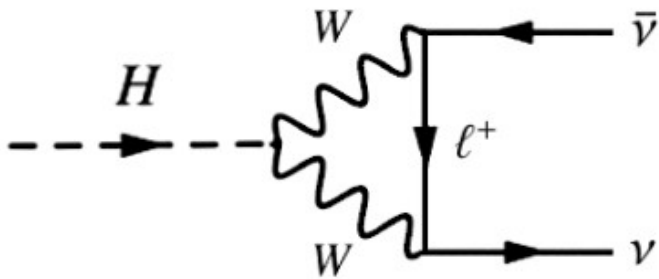
- Existing **dedicated CMS/ATLAS studies** for a few channels.
- Stat.-based projection of current bounds: $\sqrt{2 \times 3 \text{ ab}^{-1} / \mathcal{L}_{\text{int}}(13 \text{ TeV})} \sim 6.5\times$ **improvement**

■ Indicate whether **H decay will be producible at FCC-ee/FCC-hh** by simply checking the relation $[BR(X) \times N(X)] > 1$?

Very suppressed SM Higgs decays

Rare 2-, 3-, 4-body Higgs boson decays

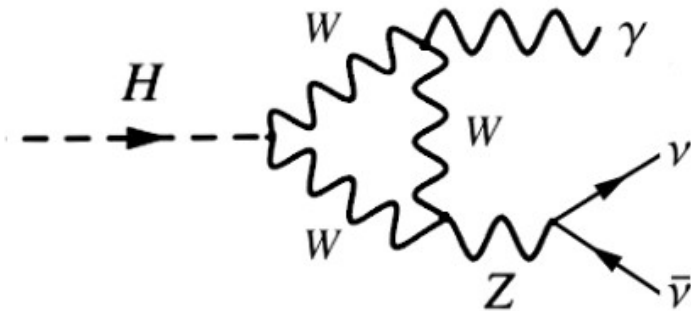
- Ultra-rare $H \rightarrow 2\nu$ invisible decay: Loop-induced for massless ν 's in the SM



$\mathcal{B} = 7e-36(!)$: Loops + double-chirality flip. Negligible compared to std. $H \rightarrow ZZ^* \rightarrow 4\nu$ invisible decay ($\mathcal{B} \approx 0.1\%$).

But **very dependent on actual nu-mass-generation mechanism** realized in nature.

- Rare $H \rightarrow \gamma + 2\nu$ (monophoton Higgs) decay:



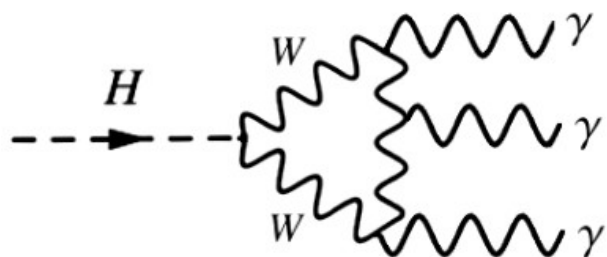
$\mathcal{B} = 3.7e-4$ (~20% larger than naive

$\mathcal{B}(H \rightarrow \gamma Z(2\nu)) = 7.7e-3 \times 0.2$, due to extra W-induced channels). It **should be visible at HL-LHC & FCC**.

	Branching fraction	Framework	Exp. limits		Producible at	
			2023	HL-LHC	FCC-ee	FCC-hh
$H \rightarrow \nu + \bar{\nu}$	7.2×10^{-36}	NLO SM, MG5_AMC (this work)	-	-	✗	✗
$H \rightarrow \gamma + \nu + \bar{\nu}$	$(3.74 \pm 0.01) \times 10^{-4}$	NLO SM, MG5_AMC (this work)	-	-	✓	✓
	3.4×10^{-4}	NLO SM [82]	-	-	✓	✓

Rare 2-, 3-, 4-body Higgs boson decays

- Ultra-rare $H \rightarrow 3\gamma$ (triphoton Higgs) decay: Test of fundamental symmetries:

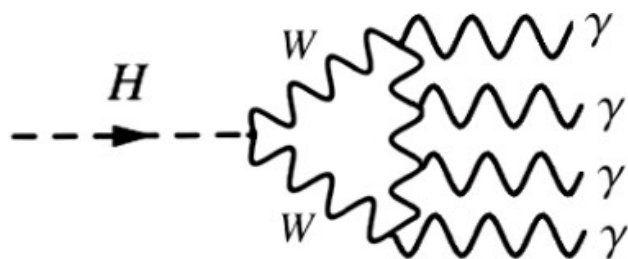


Forbidden in QED (C-violation):

$$C(H) = 0 \neq C(3\gamma) = (-1)^3 = -1$$

but not in full EW theory. However, final state must be composed of **3 spatially symmetric γ 's** with zero total ang. momentum: $\mathcal{B}=1e-40(!)$

- Ultra-rare $H \rightarrow 4\gamma$ (4-photon Higgs) decay:



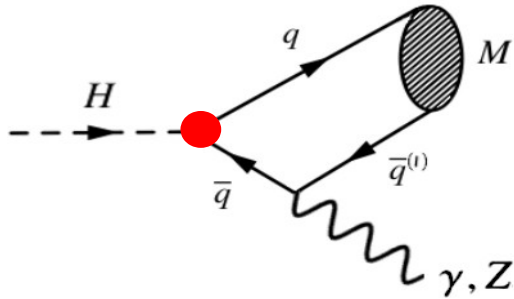
$\mathcal{B} = 4.6e-12$ (suppressed by heavy loops and $\alpha^4 \approx 3e-9$). **Not observable** at LHC/FCC (unless BSM), but limits could be set from existing $H \rightarrow a(\gamma\gamma) a(\gamma\gamma)$ searches.

	Branching fraction	Framework	Exp. limits		Producible at	
			2023	HL-LHC	FCC-ee	FCC-hh
$H \rightarrow \gamma + \gamma + \gamma$	1.0×10^{-40}	NLO SM, MG5_AMC (this work)	–	–	✗	✗
$H \rightarrow \gamma + \gamma + \gamma + \gamma$	$(4.56 \pm 0.01) \times 10^{-12}$	NLO SM, MG5_AMC (this work)	–	–	✗	✗

Higgs quark Yukawa couplings

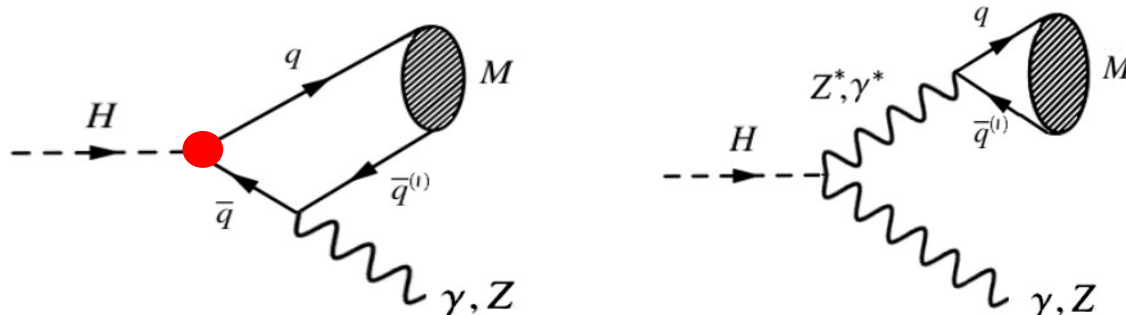
Higgs quark Yukawas via exclusive $H \rightarrow VM+V$ decays

- Higgs decays into a meson + EW boson offer the possibility to **constrain** (very) suppressed Yukawa couplings to valence quarks ($H \rightarrow cc, qq$):



Higgs quark Yukawas via exclusive $H \rightarrow VM+V$ decays

- Higgs decays into a meson + EW boson offer the possibility to **constrain** (very) suppressed Yukawa couplings to valence quarks ($H \rightarrow cc, qq$):



G. T. Bodwin et al. arXiv:1306.5770 [hep-ph]
 G. Isidori et al., arXiv:1305.0663 [hep-ph]
 A. L. Kagan et al. arXiv:1406.1722 [hep-ph]
 M. König, M. Neubert, arXiv:1505.03870 [hep-ph]
 G. Perez et al., arXiv:1505.06689 [hep-ph]
 ...

- However, **direct** (Yukawa-sensitive) contribution is much **smaller** (for SM g_{qH}) **than the indirect** $H \rightarrow Z, \gamma + Z, \gamma$ followed by $Z, \gamma \rightarrow VM$ conversion.

- Destructive interference** of dir./indir. amplitudes:

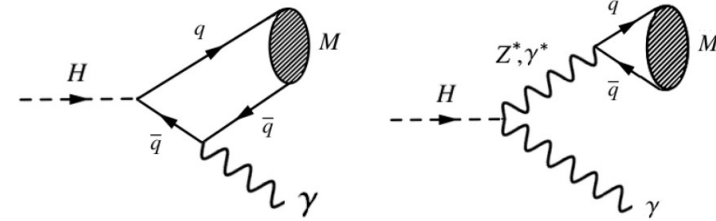
$$\Gamma(H \rightarrow V\gamma) = \frac{1}{8\pi} \frac{m_H^2 - m_V^2}{m_H^2} |\mathcal{A}_{\text{direct}} + \mathcal{A}_{\text{indirect}}|^2$$

$\mathcal{A}_{\text{direct}} \propto g_{qH}^2$: For SM Yukawas, the **dir. contribution** is very small (except for c,b-quark), but limits on **BSM-enhanced light-q** Yukawas can still be set.

	$\frac{\mathcal{A}_{\text{direct}}}{\mathcal{A}_{\text{indirect}}}$
$H \rightarrow \gamma + \rho^0$	} -0.002
$H \rightarrow \gamma + \omega$	
$H \rightarrow \gamma + \phi$	
$H \rightarrow \gamma + J/\psi$	} -0.06
$H \rightarrow \gamma + \psi(2S)$	
$H \rightarrow \gamma + \Upsilon(1S)$	} -0.8
$H \rightarrow \gamma + \Upsilon(2S)$	
$H \rightarrow \gamma + \Upsilon(3S)$	

Exclusive Higgs decays into vector-meson + γ

- Exclusive $H \rightarrow VM + \gamma$ decays computed by multiple groups in SCET (q), NRQCD (Q):

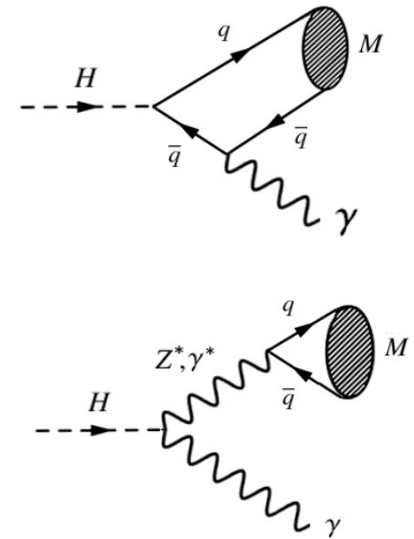
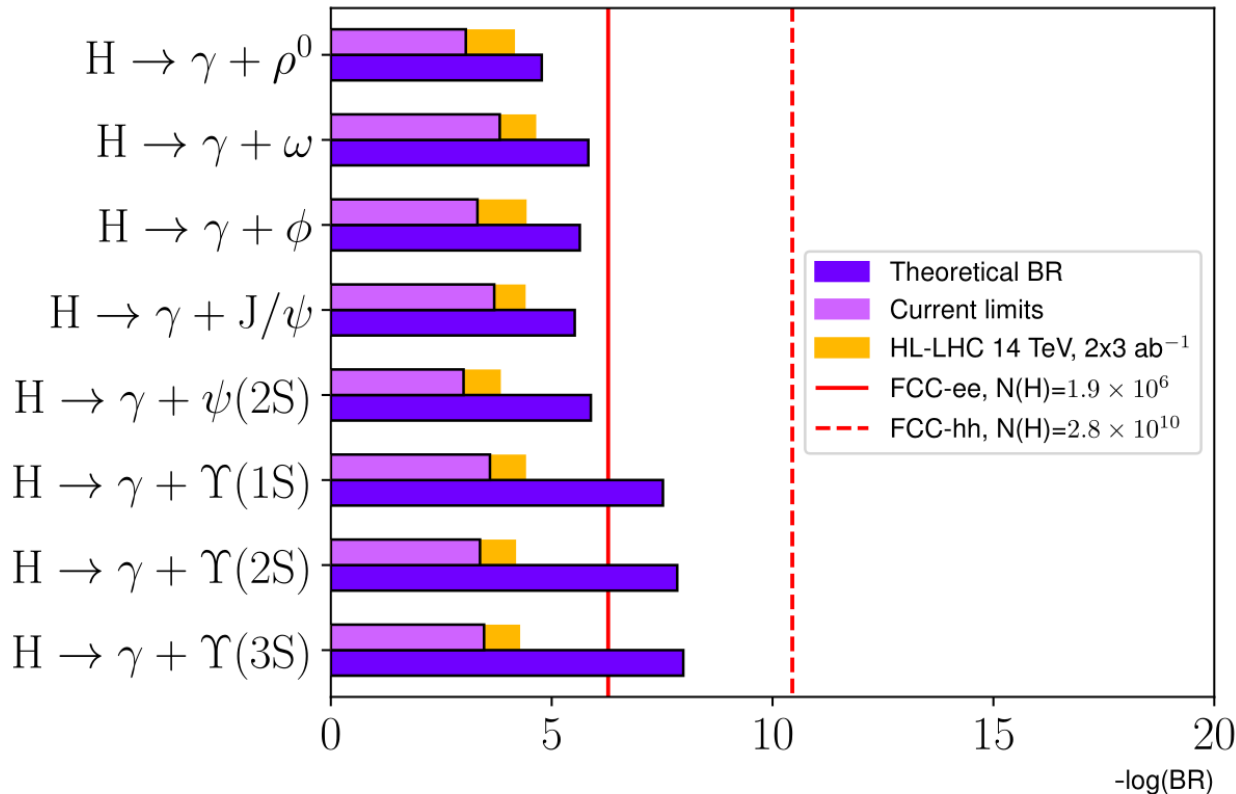


$H \rightarrow \gamma + M$	Branching fraction	Framework	Exp. limits		Producible at	
			2023	HL-LHC	FCC-ee	FCC-hh
ρ^0	$(1.68 \pm 0.08) \times 10^{-5}$	SCET+LCDA [14]	$< 8.8 \times 10^{-4}$ [88]	$\lesssim 6.8 \times 10^{-5}$	✓	✓
ω	$(1.48 \pm 0.08) \times 10^{-6}$	SCET+LCDA [14]	$< 1.5 \times 10^{-4}$ [91]	$\lesssim 2.2 \times 10^{-5}$	✓	✓
ϕ	$(2.31 \pm 0.11) \times 10^{-6}$	SCET+LCDA [14]	$< 4.8 \times 10^{-4}$ [88]	$\lesssim 3.7 \times 10^{-5}$	✓	✓
J/ψ	$(2.95 \pm 0.17) \times 10^{-6}$	SCET+LCDA [14]				
	$(3.01 \pm 0.15) \times 10^{-6}$	NRQCD (NLL)+LDME [94]	$< 2.0 \times 10^{-4}$ [93]	$\lesssim 3.9 \times 10^{-5}$ [62]	✓	✓
	$(2.99^{+0.16}_{-0.15}) \times 10^{-6}$	NRQCD+LCDA [95]				
$\psi(2S)$	$(1.3 \pm 0.0) \times 10^{-6}$	SCET+LCDA [14]	$< 9.9 \times 10^{-4}$ [92, 93]	$\lesssim 1.4 \times 10^{-4}$	✓	✓
$H \rightarrow \gamma + \Upsilon(1S)$	$(4.61^{+1.76}_{-1.23}) \times 10^{-9}$	SCET+LCDA [14]				
	$(9.97^{+4.04}_{-3.03}) \times 10^{-9}$	NRQCD (NLL)+LDME [94]	$< 2.5 \times 10^{-4}$ [93]	$\lesssim 3.8 \times 10^{-5}$	✗	✓
	3.0×10^{-8}	NRQCD (NLO)+LDME [96]				
	$(5.22^{+2.02}_{-1.70}) \times 10^{-9}$	NRQCD+LCDA [95]				
$\Upsilon(2S)$	$(2.34^{+0.76}_{-1.00}) \times 10^{-9}$	SCET+LCDA [14]				
	$(2.62^{+1.39}_{-0.91}) \times 10^{-9}$	NRQCD (NLL)+LDME [94]	$< 4.2 \times 10^{-4}$ [93]	$\lesssim 6.4 \times 10^{-5}$	✗	✓
	1.4×10^{-8}	NRQCD (NLO)+LDME [96]				
	$(1.42^{+0.72}_{-0.57}) \times 10^{-9}$	NRQCD+LCDA [95]				
$\Upsilon(3S)$	$(2.13^{+0.76}_{-1.12}) \times 10^{-9}$	SCET+LCDA [14]				
	$(1.87^{+1.05}_{-0.69}) \times 10^{-9}$	NRQCD (NLL)+LDME [94]	$< 3.4 \times 10^{-4}$ [93]	$\lesssim 5.2 \times 10^{-5}$	✗	✓
	1.1×10^{-8}	NRQCD (NLO)+LDME [96]				
	$(9.1^{+4.8}_{-3.8}) \times 10^{-10}$	NRQCD+LCDA [95]				

- Theory \mathcal{B} 's: $\mathcal{O}(10^{-5}-10^{-10})$. Factors 2–3 differences among SCET-NRQCD for $\Upsilon(nS)$
- Experiment: All 8 channels studied with limits: $\mathcal{B} < \mathcal{O}(10^{-3}-10^{-4})$

Exclusive Higgs decays into vector-meson + γ

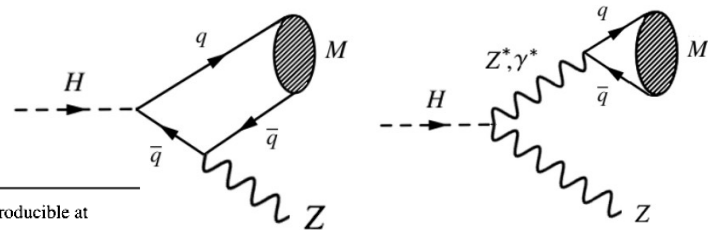
Most recent & future limits on exclusive $H \rightarrow VM + \gamma$ decays:



- **Theory \mathcal{B} 's:** $\mathcal{O}(10^{-5}-10^{-10})$. Factors 2–3 differences among SCET-NRQCD for $\Upsilon(nS)$
- **Experiment:** All 8 channels studied with limits $\mathcal{B} < \mathcal{O}(10^{-3}-10^{-4})$
- **HL-LHC:** $H \rightarrow \rho + \gamma$ may be observed (possible upper bound for u,d Yukawas?)
- **5 (all 8) producible channels at FCC-ee (FCC-hh).**

Exclusive Higgs decays into vector-meson + Z

- Exclusive $H \rightarrow VM + Z$ decays computed by multiple groups in EFT (q), NRQCD (Q):



$H \rightarrow Z + M$	Branching fraction	Framework	Exp. limits		Producible at	
			2023	HL-LHC	FCC-ee	FCC-hh
π^0	$(2.3 \pm 0.1) \times 10^{-6}$	EFT+NRQM [10]	-	-	✓	✓
	$(2.3 \pm 0.1) \times 10^{-6}$	EFT+LCDA [102]	-	-	✓	✓
η	$(8.3 \pm 0.9) \times 10^{-7}$	EFT+LCDA [102]	-	-	✓	✓
ρ^0	$(1.4 \pm 0.1) \times 10^{-5}$	EFT+NRQM [10]	$< 1.2 \times 10^{-2}$ [103]	$\leq 1.8 \times 10^{-3}$	✓	✓
	$(7.19 \pm 0.29) \times 10^{-6}$	EFT+LCDA [102]	-	-	✓	✓
ω	$(1.6 \pm 0.1) \times 10^{-6}$	EFT+NRQM [10]	-	-	✓	✓
	$(5.6 \pm 0.2) \times 10^{-7}$	EFT+LCDA [102]	-	-	✓	✓
η'	$(1.24 \pm 0.13) \times 10^{-6}$	EFT+LCDA [102]	-	-	✓	✓
ϕ	$(4.2 \pm 0.3) \times 10^{-6}$	EFT+NRQM [10]	$< 3.6 \times 10^{-3}$ [103]	$\leq 5.4 \times 10^{-4}$	✓	✓
	$(2.42 \pm 0.10) \times 10^{-6}$	EFT+LCDA [102]	-	-	✓	✓
η_c	$(1.0 \pm 0.1) \times 10^{-5}$	EFT+NRQM [10]	-	-	✓	✓
	$(1.0 \pm 0.0) \times 10^{-5}$	EFT+LCDA [104]	-	-	✓	✓
$H \rightarrow Z + J/\psi$	3.4×10^{-6}	NRQCD (NLO)+LMDE [105]	-	-	✓	✓
	3.2×10^{-6}	EFT+NRQM [107]	$< 1.9 \times 10^{-3}$ [106]	$\leq 2.1 \times 10^{-4}$ [64]	✓	✓
$\psi(2S)$	$(2.3 \pm 0.1) \times 10^{-6}$	EFT+LCDA [102]	-	-	✓	✓
	1.5×10^{-6}	EFT+NRQM [107]	$< 6.6 \times 10^{-3}$ [106]	$\leq 1.0 \times 10^{-3}$	✓	✓
η_b	$(2.69 \pm 0.05) \times 10^{-5}$	EFT+LCDA [104]	-	-	✓	✓
	$(4.739^{+0.276}_{-0.244}) \times 10^{-5}$	EFT (NLO)+LCDA [108]	-	-	✓	✓
$\Upsilon(1S)$	1.7×10^{-5}	NRQCD (NLO)+LMDE [105]	-	-	✓	✓
	1.7×10^{-5}	EFT+NRQM [107]	-	-	✓	✓
$\Upsilon(2S)$	$(1.54 \pm 0.06) \times 10^{-5}$	EFT+LCDA [102]	-	-	✓	✓
	8.9×10^{-6}	EFT+NRQM [107]	-	-	✓	✓
$\Upsilon(3S)$	$(7.5 \pm 0.3) \times 10^{-6}$	EFT+LCDA [102]	-	-	✓	✓
	6.7×10^{-6}	EFT+NRQM [107]	-	-	✓	✓
$\Upsilon(3S)$	$(5.63 \pm 0.24) \times 10^{-6}$	EFT+LCDA [102]	-	-	✓	✓

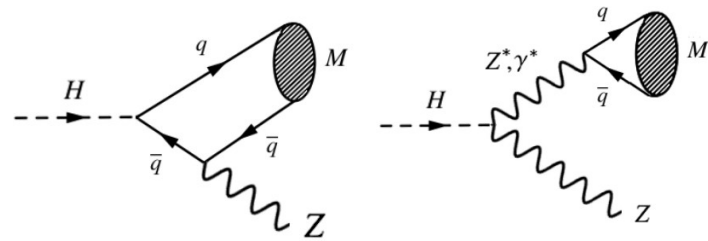
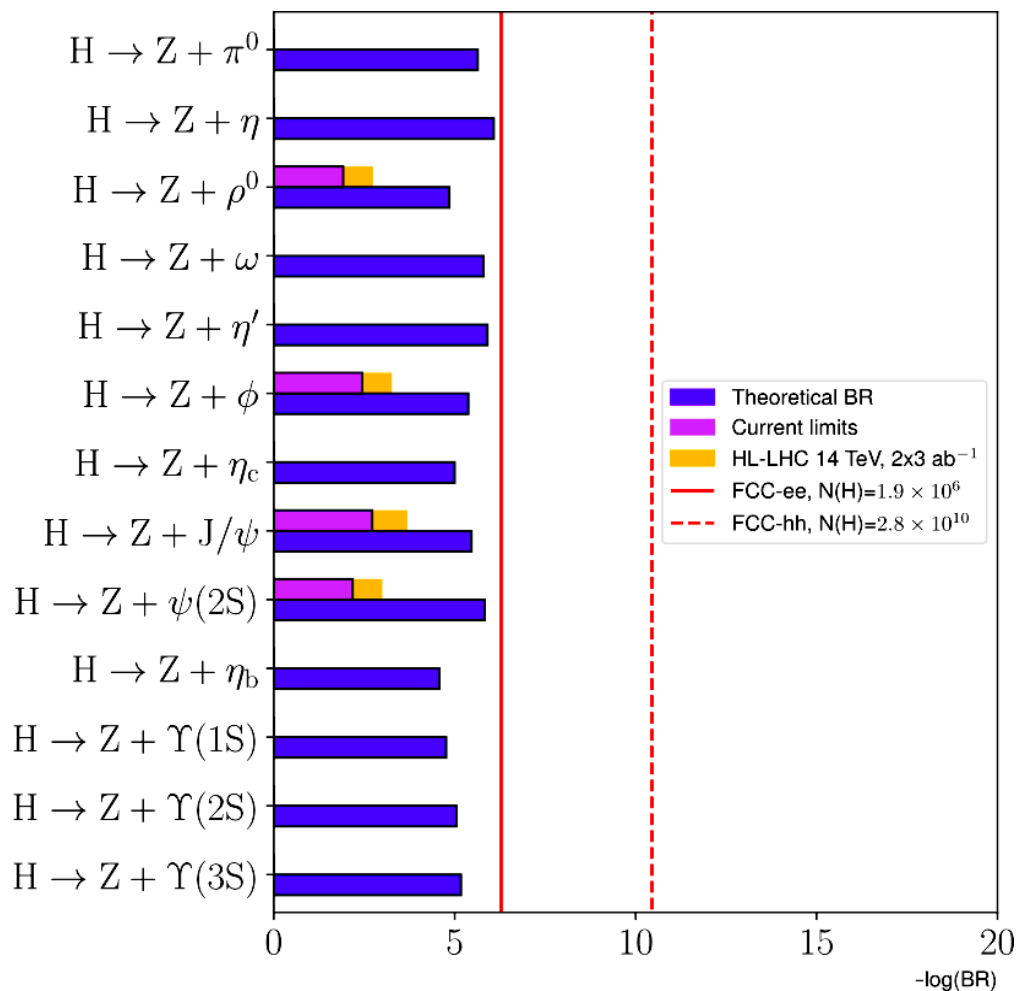
Less sensitivity to g_{qH} , but probe CP-even, CP-odd $H\gamma Z$ couplings:

$$\begin{array}{l}
 A_{\text{direct}} \\
 A_{\text{indirect}}
 \end{array}
 \left\{
 \begin{array}{l}
 H \rightarrow Z + \pi^0 \\
 H \rightarrow Z + \eta \\
 H \rightarrow Z + \rho^0 \\
 H \rightarrow Z + \omega \\
 H \rightarrow Z + \eta' \\
 H \rightarrow Z + \phi \\
 H \rightarrow Z + \eta_c \\
 H \rightarrow Z + J/\psi \\
 H \rightarrow Z + \psi(2S) \\
 H \rightarrow Z + \eta_b \\
 H \rightarrow Z + \Upsilon(1S) \\
 H \rightarrow Z + \Upsilon(2S) \\
 H \rightarrow Z + \Upsilon(3S)
 \end{array}
 \right.
 \begin{array}{l}
 < 0.0001 \\
 -0.014 \\
 +0.05
 \end{array}$$

- Theory \mathcal{B} 's: $\mathcal{O}(10^{-5}-10^{-7})$. Factors $\times 2$ max. differences among predictions
- Experiment: 4 channels studied with limits: $\mathcal{B} < \mathcal{O}(10^{-2}-10^{-3})$

Exclusive Higgs decays into vector-meson + Z

Most recent & future limits on exclusive $H \rightarrow VM + Z$ decays:

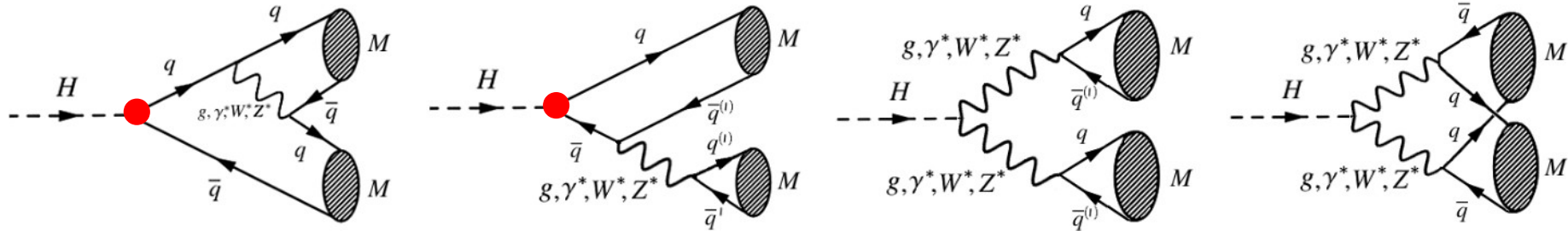


- Botomonium have largest \mathcal{B} 's, but no exp. bound set so far.
- No observable channel at HL-LHC
- All channels producible at FCC-ee

- Theory \mathcal{B} 's: $\mathcal{O}(10^{-5}-10^{-7})$. Factors $\times 2$ max. differences among predictions
- Experiment: 4 channels studied with limits: $\mathcal{B} < \mathcal{O}(10^{-2}-10^{-3})$

Exclusive Higgs decays into two mesons

- Can **exclusive double-meson** Higgs decay **constrain Yukawa couplings** ?



- **Challenges:**

- **Multitude of hard channels** contribute, **not well-controlled** theoretically (not all predictions include all diagrams plotted above...).
- Exclusive pair meson formation: **Doubly suppressed** \rightarrow very small rates.
- Calculations carried out in **multiple frameworks** (LC+LCDA, NRQCD/NRCSM, NRQCD+LDME,...) predict very small **rates: $\mathcal{O}(10^{-9} - 10^{-11})$**

Exclusive Higgs decays into two mesons

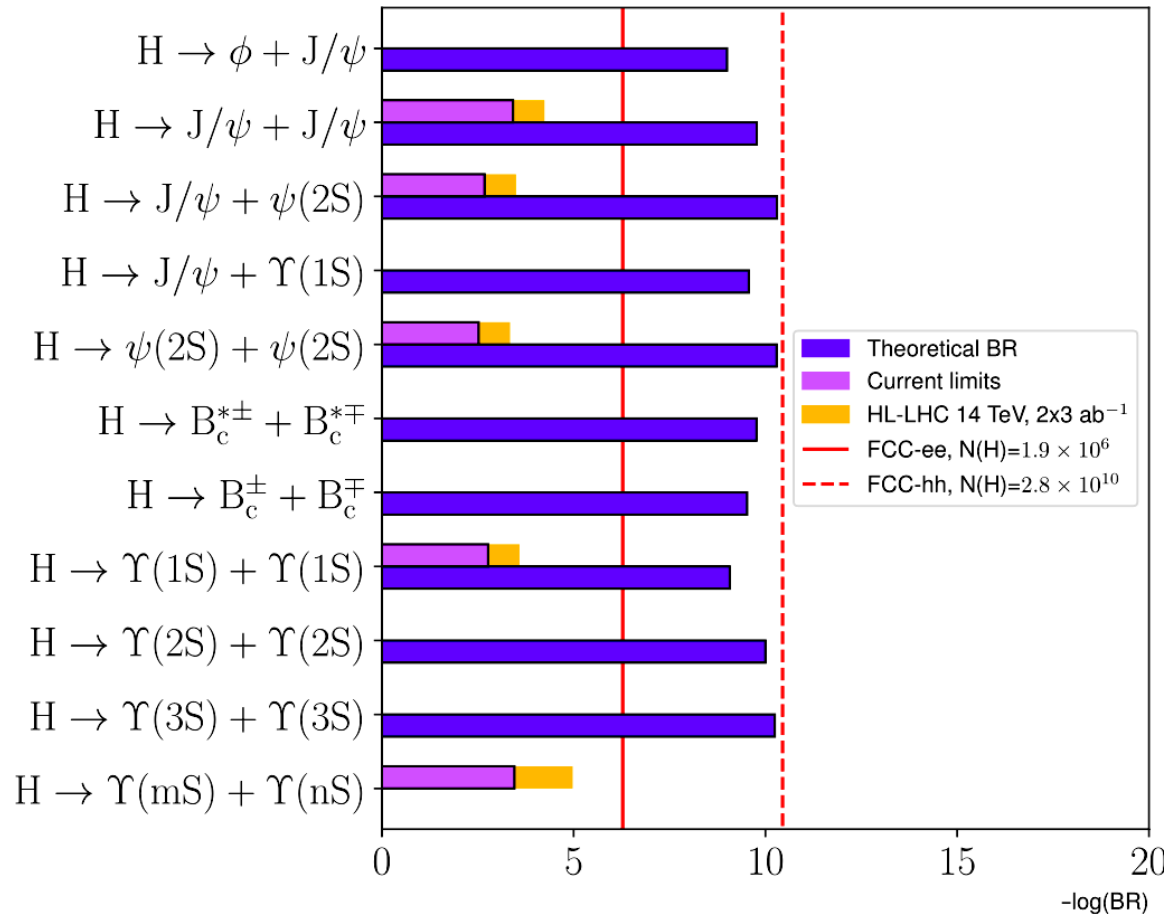
■ Exclusive $H \rightarrow VM + VM$ decay branching fractions:

H →	M	+	M	Branching fraction	Framework	Exp. limits		Producible at	
						2023	HL-LHC	FCC-ee	FCC-hh
H →	ϕ	+	J/ψ	1.0×10^{-9}	LC+LCDA [120]	–	–	✗	✓
				$(5.8 - 6.0) \times 10^{-9}$	NRQCD+LDME [125]				
	J/ψ	+	J/ψ	1.7×10^{-10}	RQM [122]				
				2.1×10^{-10}	RQM [124]	$< 4.7 \times 10^{-4}$ [109]	$\lesssim 7.1 \times 10^{-5}$	✗	✓
				$(5.9 \pm 2.3) \times 10^{-10}$	NRQCD/NRCSM [123]				
	$\psi(2S)$	+	J/ψ	$\mathcal{O}(5) \times 10^{-11}$	–	$< 2.6 \times 10^{-3}$ [109]	$\lesssim 3.9 \times 10^{-4}$	✗	✓
				$(5.1 \pm 2.0) \times 10^{-11}$	NRQCD/NRCSM [123]	$< 3.6 \times 10^{-3}$ [109]	$\lesssim 5.5 \times 10^{-4}$	✗	✓
	B_c^{\pm}	+	B_c^{\pm}	$(1.4 - 1.7) \times 10^{-10}$	RQM [121]	–	–	✗	✓
				$(2.0 - 3.0) \times 10^{-10}$	RQM [121]	–	–	✗	✓
	H →	J/ψ			$(2.7 - 3.6) \times 10^{-10}$	NRQCD+LDME [125]	–	–	✗
1.6×10^{-11}					LC+LCDA [120]				
$\Upsilon(1S)$		+	$\Upsilon(1S)$	$(8.5 - 9.2) \times 10^{-10}$	NRQCD+LDME [125]				
				1.8×10^{-10}	RQM [122]				
				2.3×10^{-9}	RQM [124]	$< 2.0 \times 10^{-3}$ [109]	$\lesssim 3.0 \times 10^{-4}$	✗	✓
$\Upsilon(1S)$				$(4.3 \pm 0.9) \times 10^{-10}$	NRQCD/NRCSM [123]				
				2.3×10^{-9}	LC+LCDA [120]				
$\Upsilon(2S)$		+	$\Upsilon(2S)$	$(1.0 \pm 0.2) \times 10^{-10}$	NRQCD/NRCSM [123]	–	–	✗	✓
$\Upsilon(3S)$		+	$\Upsilon(3S)$	$(5.7 \pm 1.2) \times 10^{-11}$	NRQCD/NRCSM [123]	–	–	✗	✓
$\Upsilon(mS)$		+	$\Upsilon(nS)$	–	–	$< 4.3 \times 10^{-4}$ [109]	$\lesssim 1.1 \times 10^{-5}$ [64]	✗	✗

- Theory BRs: $\mathcal{O}(10^{-9} - 10^{-11})$. Predictions within factor 10 (depending on diags. considered)
- Experiment: 5 channels searched-for with limits: $\mathcal{O}(10^{-3} - 10^{-4})$.
- **No (All) producible channels at FCC-ee (FCC-hh)**

Exclusive Higgs decays into two mesons

■ Most recent & future limits on $H \rightarrow VM + VM$ decays:

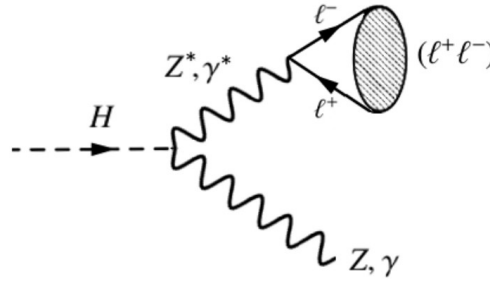
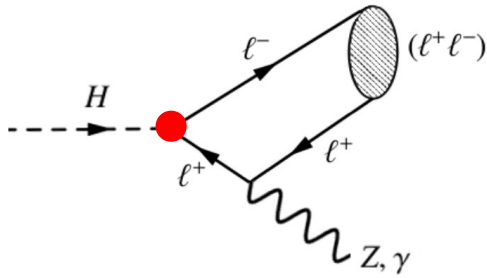


- Theory BRs: $\mathcal{O}(10^{-9}-10^{-11})$. Predictions within factor 10 (depending on diags. considered)
- Experiment: 5 channels searched-for with limits: $\mathcal{O}(10^{-3}-10^{-4})$.
- **No (All) producible channels at FCC-ee (FCC-hh)**

Higgs lepton Yukawa couplings

Exclusive Higgs decays into leptonium + γ, Z

- Can Higgs decays into leptonium + EW boson **constrain** the e, μ (and τ) Yukawas? Contributions from **direct+indirect mechanisms**. Never computed to date. Expected decay **width much smaller than for quarkonium** counterparts:



$$f_M^2 = 4N_c \frac{|\phi_M(0)|^2}{m_M}$$

Leptonium wavefunction at origin:

$$|\phi_{n,(\ell\ell)}(r=0)|^2 = \frac{(m_\ell \alpha(0))^3}{8\pi n^3}$$

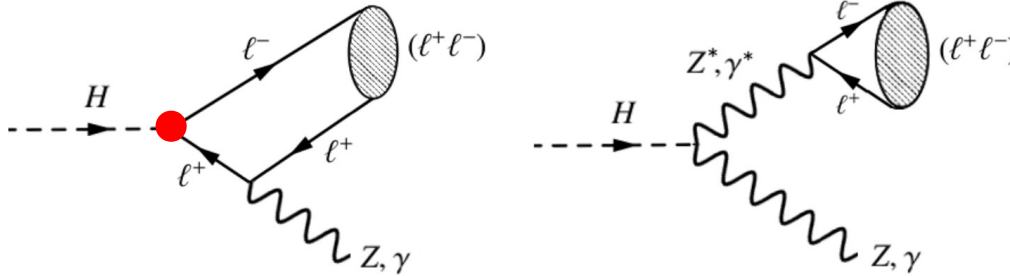
$$[\alpha(0)m_{\ell\ell}/(\alpha_s(m_{q\bar{q}})m_{q\bar{q}})]^3 \ll 1$$

Ortholeptonium: $VM = (\ell\ell)_1$ is the only state possible (CP conservation) **with γ** .

Paraleptonium: $PS = (\ell\ell)_0$ also possible (but suppressed) **with Z** (long. polarized)

Exclusive Higgs decays into leptonium + γ, Z

- Can Higgs decays into leptonium + EW boson **constrain** the **e, μ (and τ) Yukawas**? Contributions from **direct+indirect mechanisms**. Never computed to date. Expected decay **width much smaller than for quarkonium** counterparts:



$$f_M^2 = 4N_c \frac{|\phi_M(0)|^2}{m_M}$$

Leptonium wavefunction at origin:

$$|\phi_{n,(\ell\ell)}(r=0)|^2 = \frac{(m_\ell \alpha(0))^3}{8\pi n^3}$$

$$[\alpha(0)m_{\ell\ell}/(\alpha_s(m_{q\bar{q}})m_{q\bar{q}})]^3 \ll 1$$

Ortholeptonium: $VM = (\ell\ell)_1$ is the only state possible (CP conservation) **with γ** .

- $H \rightarrow (\ell\ell)_1 + \gamma$** decay widths computed using **similar approach as for $H \rightarrow (q\bar{q}) + \gamma$** :

$$\mathcal{B}(H \rightarrow \gamma + (\ell^+ \ell^-)_1) = \frac{1}{8\pi} \frac{m_H^2 - m_{\ell\ell}^2}{m_H^2 \Gamma_H} |\mathcal{A}_{\text{dir}} + \mathcal{A}_{\text{indir}}|^2 \quad (\text{destructive interference})$$

$$\mathcal{A}_{\text{dir}} = 2Q_\ell \sqrt{4\pi\alpha(0)} (\sqrt{2}G_F m_{\ell\ell})^{1/2} \frac{m_H^2 - m_{\ell\ell}^2}{\sqrt{m_H(m_H^2 - m_{\ell\ell}^2/2 - 2m_\ell^2)}} \phi_{n,(\ell\ell)}(0)$$

$$\mathcal{A}_{\text{indir}} = \frac{Q_\ell \sqrt{4\pi\alpha(0)} f_{\ell\ell}}{m_{\ell\ell}} (16\pi\Gamma_{H \rightarrow \gamma\gamma})^{1/2} \frac{m_H^2 - m_{\ell\ell}^2}{m_H^2} = -\frac{2Q_\ell \sqrt{4\pi\alpha(0)}}{m_{\ell\ell}^{3/2}} (16\pi\Gamma_{H \rightarrow \gamma\gamma})^{1/2} \frac{m_H^2 - m_{\ell\ell}^2}{m_H^2} \phi_{n,(\ell\ell)}(0)$$

$\mathcal{A}_{\text{dir}}/\mathcal{A}_{\text{indir}}$

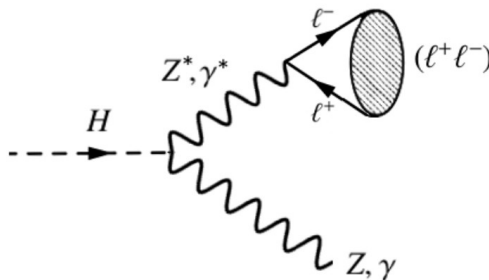
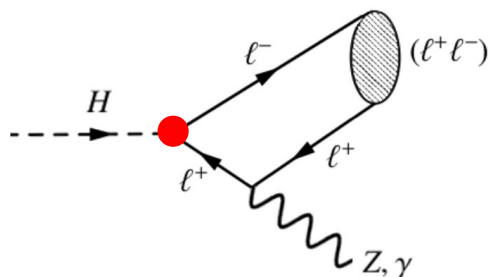
$$\sim 10^{-8} \quad H \rightarrow \gamma + (ee)_1$$

$$\sim 10^{-3} \quad H \rightarrow \gamma + (\mu\mu)_1$$

$$\sim 0.2 \quad H \rightarrow \gamma + (\tau\tau)_1$$

Exclusive Higgs decays into leptonium + Z

- Can Higgs decays into leptonium + EW boson **constrain** the **e, μ (and τ) Yukawas**? Contributions from **direct+indirect mechanisms**. Never computed to date. Expected decay **width much smaller than for quarkonium** counterparts:



$$f_M^2 = 4N_c \frac{|\phi_M(0)|^2}{m_M}$$

Leptonium wavefunction at origin:

$$|\phi_{n,(\ell\ell)}(r=0)|^2 = \frac{(m_\ell \alpha(0))^3}{8\pi n^3}$$

$$[\alpha(0)m_{\ell\ell}/(\alpha_s(m_{q\bar{q}})m_{q\bar{q}})]^3 \ll 1$$

Ortholeptonium: $VM = (\ell\ell)_1$ plus Z.

- $H \rightarrow (\ell\ell)_1 + Z$ decay widths computed using similar approach as for $H \rightarrow (q\bar{q}) + Z$:**

$$\Gamma(H \rightarrow Z + (\ell^+ \ell^-)_1) = \Gamma_1 + \Gamma_2 + \Gamma_3,$$

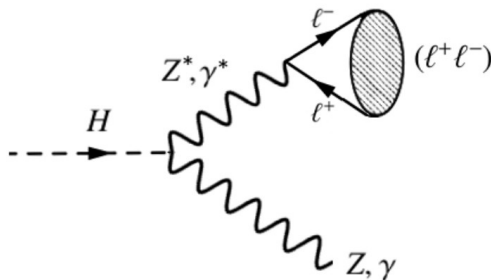
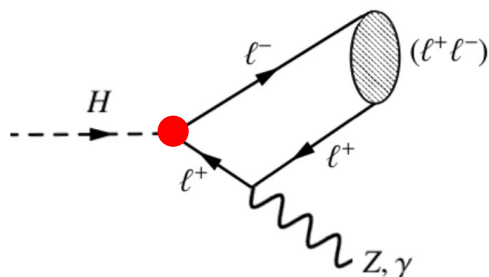
$$\Gamma_1 = \frac{m_H^3 (g_{\ell\ell} f_{\ell\ell})^2}{16\pi v^4} \frac{\lambda^{1/2}(1, r_Z, r_{\ell\ell})}{(1 - r_{\ell\ell}/r_Z)^2} [(1 - r_Z - r_{\ell\ell})^2 + 8r_Z r_{\ell\ell}],$$

$$\Gamma_2 = \frac{\alpha(0)^3 f_{\ell\ell}^2 Q_\ell^2 m_H^3 C_{Z\gamma}^2}{32\pi^2 v^2 \sin^2 \theta_W m_{\ell\ell}^2} \lambda^{1/2}(1, r_Z, r_{\ell\ell}) [(1 - r_Z - r_{\ell\ell})^2 + 2r_Z r_{\ell\ell}],$$

$$\Gamma_3 = \frac{3\alpha(0)^2 f_{\ell\ell}^2 g_{\ell\ell} Q_\ell m_H C_{Z\gamma}}{8\pi \cos \theta_W \sin^2 \theta_W v^2} \frac{\lambda^{1/2}(1, r_Z, r_{\ell\ell})}{1 - r_{\ell\ell}/r_Z} (1 - r_Z - r_{\ell\ell}),$$

Exclusive Higgs decays into leptonium + Z

- Can Higgs decays into leptonium + EW boson **constrain** the **e, μ (and τ) Yukawas**? Contributions from **direct+indirect mechanisms**. Never computed to date. Expected decay **width much smaller than for quarkonium** counterparts:



$$f_M^2 = 4N_c \frac{|\phi_M(0)|^2}{m_M}$$

Leptonium wavefunction at origin:

$$|\phi_{n,(\ell\ell)}(r=0)|^2 = \frac{(m_\ell \alpha(0))^3}{8\pi n^3}$$

$$[\alpha(0)m_{\ell\ell}/(\alpha_s(m_{q\bar{q}})m_{q\bar{q}})]^3 \ll 1$$

Paraleptonium: $PS = (\ell\ell)_0$ also possible (but suppressed) **with Z** (long. polarized)

- $H \rightarrow (\ell\ell)_0 + Z$ decay widths computed using **similar approach as for $H \rightarrow (q\bar{q}) + Z$** :

$$\Gamma(H \rightarrow Z + (\ell^+\ell^-)_0) = \frac{m_H^3}{4\pi v^4} \lambda^{3/2}(1, r_Z, r_{\ell\ell}) |F^{Z+\ell\ell_0}|^2 \quad \text{with } F^{Z+\ell\ell_0} = F_{\text{dir}}^{Z+\ell\ell_0} + F_{\text{ind}}^{Z+\ell\ell_0}$$

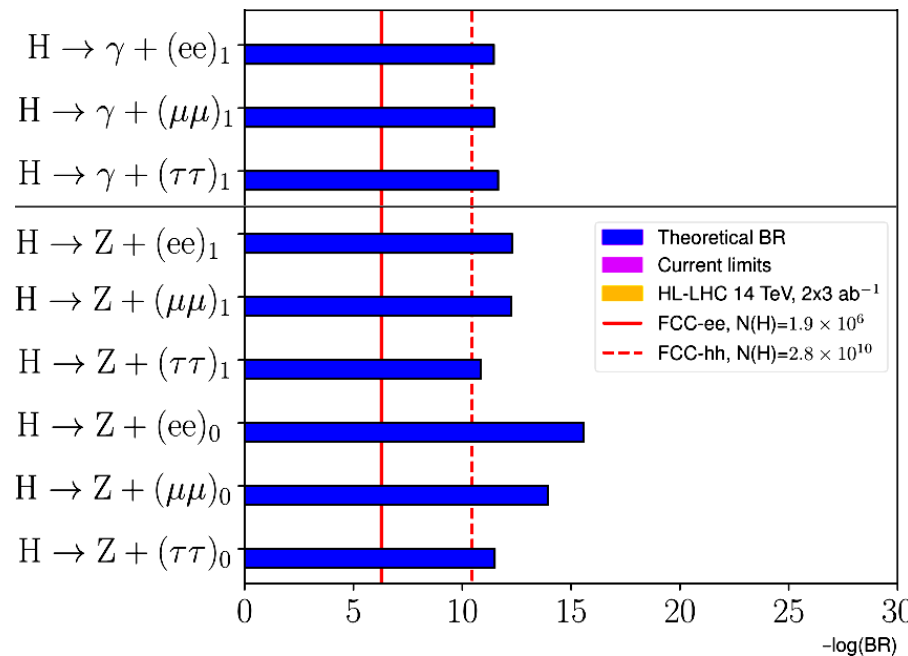
$$F_{\text{dir}}^{Z+\ell\ell_0} = -f_{\ell\ell} a_\ell \frac{m_\ell^2}{m_H^2} \frac{1 - r_Z^2 + 2r_Z \ln r_Z}{(1 - r_Z)^3}, \quad F_{\text{ind}}^{Z+\ell\ell_0} = f_{\ell\ell} a_\ell.$$

Exclusive Higgs decays into leptonium + γ, Z

■ Exclusive $H \rightarrow (\ell\ell)_{0,1} + \gamma, Z$ decay branching fractions:

$H \rightarrow V + (\ell\ell)$	Branching fraction	Framework	Exp. limits		Producible at		
			2023	HL-LHC	FCC-ee	FCC-hh	
$H \rightarrow \gamma + (\ell\ell)_1$	$(ee)_1$	3.5×10^{-12}	(this work)	–	–	✗	✗
	$(\mu\mu)_1$	3.5×10^{-12}	(this work)	–	–	✗	✗
	$(\tau\tau)_1$	2.2×10^{-12}	(this work)	–	–	✗	✗
$H \rightarrow Z + (\ell\ell)$	$(ee)_1$	5.2×10^{-13}	(this work)	–	–	✗	✗
	$(\mu\mu)_1$	5.7×10^{-13}	(this work)	–	–	✗	✗
	$(\tau\tau)_1$	1.4×10^{-11}	(this work)	–	–	✗	✗
	$(ee)_0$	2.7×10^{-16}	(this work)	–	–	✗	✗
	$(\mu\mu)_0$	1.1×10^{-14}	(this work)	–	–	✗	✗
	$(\tau\tau)_0$	3.2×10^{-12}	(this work)	–	–	✗	✗

- Tiny branching fractions: $\mathcal{O}(10^{-12}-10^{-16})$
- **No channel searched-for** to date. Leptonia are long-lived = **clear LLP signature!** (displaced $3\gamma, e^+e^-, \mu^+\mu^-$ vertices)
- **Unless BSM enhances lepton Yukawas, no SM channels producible at FCC-ee.**
 $H \rightarrow (\tau\tau) + Z$ may be reachable at FCC-hh



Higgs FCNC decays

FCNC Higgs decays

- **Flavour-changing** neutral currents (via H boson) are a **powerful probe of many BSM** scenarios. Only possible via suppressed **W loops in the SM**:

- Exclusive FCNC $H \rightarrow qq'$ decay:
 - **Very suppressed** SM FCNC vertex (negligible SM background).
 - **Well-controlled** elementary vertex (small theoretical uncertainty)

- SM $H \rightarrow qq'$ branching fractions:

$$\mathcal{B}(H \rightarrow qq') \equiv \mathcal{B}(H \rightarrow q\bar{q}' + q'\bar{q})$$

$$\mathcal{B}(H \rightarrow sb) = (8.9 \pm 1.5) \cdot 10^{-8}$$

$$\mathcal{B}(H \rightarrow db) = (3.8 \pm 0.6) \cdot 10^{-9}$$

$$\mathcal{B}(H \rightarrow ds) = 1.19 \cdot 10^{-11}$$

$$\mathcal{B}(H \rightarrow uc) = (2.7 \pm 0.5) \cdot 10^{-20}$$

[J.F.Kamenik et al., arXiv:2306.17520 [hep-ph].]

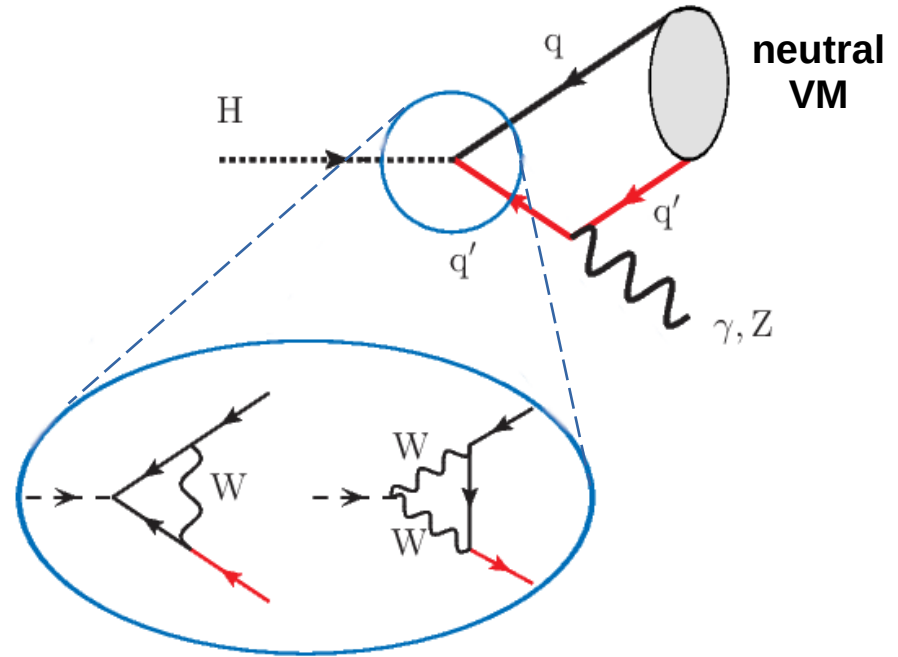
[I.Aranda et al., arXiv:2009.07166 [hep-ph].]

- $H \rightarrow VM^0 + \gamma, Z$ with exclusive **meson** formation from **LCDA-based expression**:

$$\mathcal{B}(H \rightarrow \gamma + VM(qq')) = \frac{\alpha(0)}{2 m_H} \left(\frac{f_{VM} m_{VM}}{2 \lambda_{VM}(\mu)} Q_q \right)^2 \frac{|\kappa_{qq'}|^2 + |\kappa_{q'q}|^2}{\Gamma_H},$$

With known theoretical ingredients: f_M , HQET λ_M, \dots

$$\mathcal{B}(H \rightarrow Z + VM(qq')) = \frac{9 m_H [f_{VM}^\perp(\mu_{HZ})]^2}{8 \pi v^2} v_q^2 \frac{|\kappa_{qq'}|^2 + |\kappa_{q'q}|^2}{2 \Gamma_H} \frac{r_Z}{(1 - r_Z)^3} (1 - r_Z^2 + 2 r_Z \ln r_Z)^2.$$

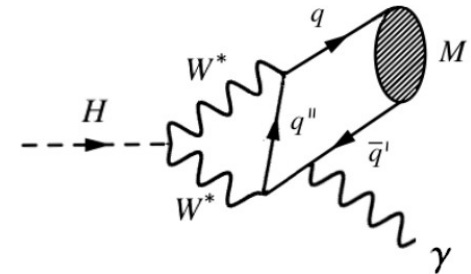
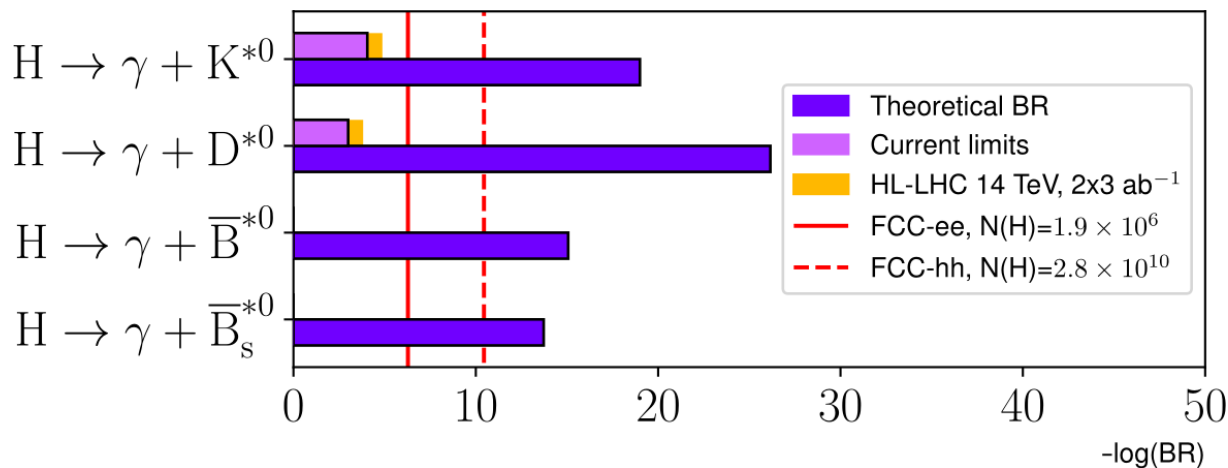


Exclusive Higgs decays into γ + flavoured-meson

■ Exclusive $H \rightarrow VM^0 + \gamma$ decays computed here within EFT+LCDA approach:

$H \rightarrow X + M$	Branching fraction	Framework	Exp. limits		Producible at	
			2023	HL-LHC	FCC-ee	FCC-hh
$H \rightarrow \gamma + K^{*0}$	1.6×10^{-19}	EFT+LCDA (this work)	$< 1.7 \times 10^{-4}$ [91, 93]	$\lesssim 2.6 \times 10^{-5}$	✗	✗
$H \rightarrow \gamma + D^{*0}$	6.7×10^{-27}	EFT+LCDA (this work)	$< 1.0 \times 10^{-3}$ [104]	$\lesssim 1.5 \times 10^{-4}$	✗	✗
$H \rightarrow \gamma + B^{*0}$	8.2×10^{-16}	EFT+LCDA (this work)	–	–	✗	✗
$H \rightarrow \gamma + B_s^{*0}$	1.8×10^{-14}	EFT+LCDA (this work)	–	–	✗	✗

■ Current & future experimental limits:



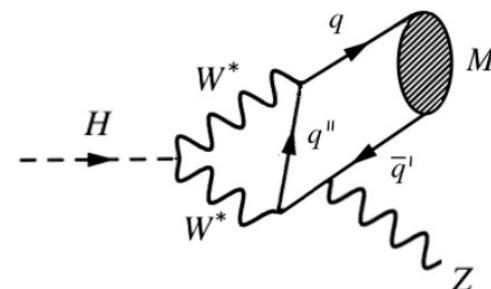
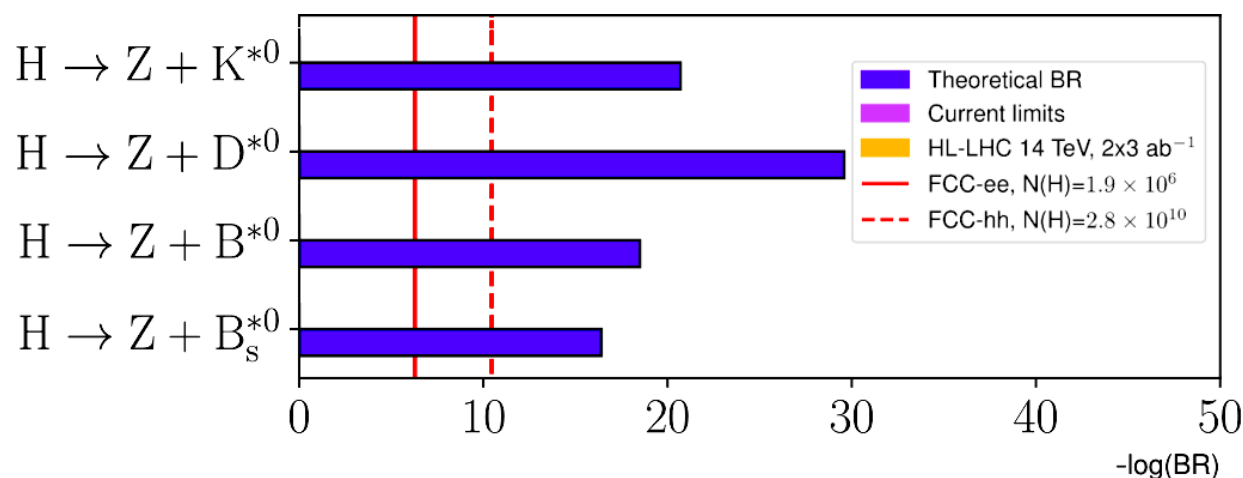
- Tiny theoretical SM branching fractions: $\mathcal{O}(10^{-14} - 10^{-27})$. Any visible signal is BSM!
- Experiment: 2 channels searched-for with limits: $\mathcal{O}(10^{-3} - 10^{-4})$
- **No channels** producible at FCC-ee (FCC-hh) either: **Very strong BSM FCNC limits.**

Exclusive Higgs decays into Z + flavoured-meson

■ Exclusive $H \rightarrow VM^0 + Z$ decays computed here within EFT+LCDA approach:

H → X + M	Branching fraction	Framework	Exp. limits		Producible at		
			2023	HL-LHC	FCC-ee	FCC-hh	
H → Z +	K^{*0}	1.4×10^{-21}	EFT+LCDA (this work)	–	–	✗	✗
	D^{*0}	1.8×10^{-30}	EFT+LCDA (this work)	–	–	✗	✗
	B^{*0}	2.4×10^{-19}	EFT+LCDA (this work)	–	–	✗	✗
	B_s^{*0}	2.9×10^{-17}	EFT+LCDA (this work)	–	–	✗	✗

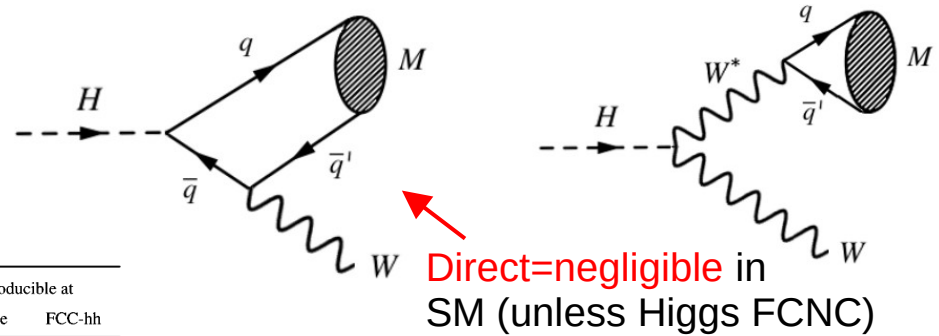
■ Future experimental limits:



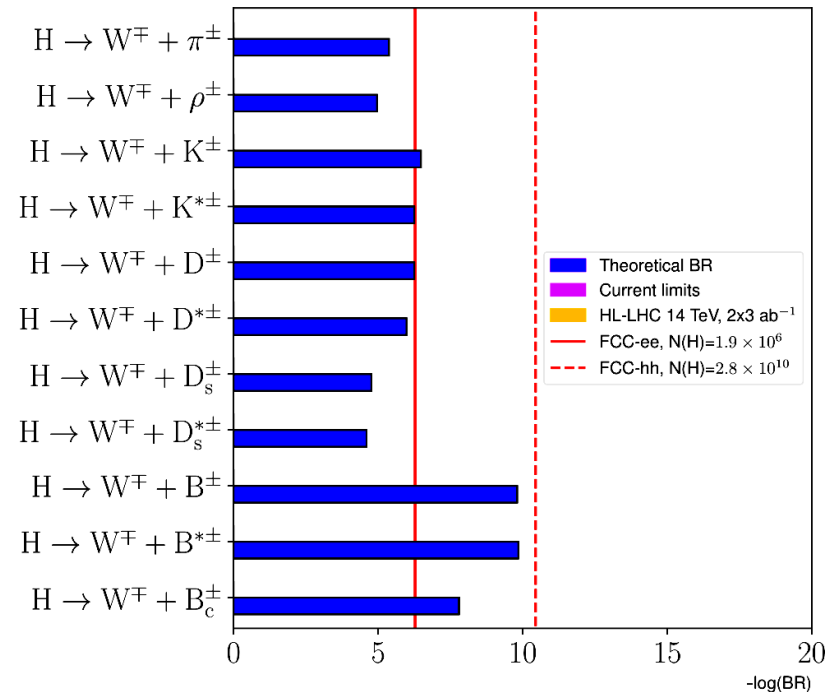
- Tiny theoretical SM branching fractions: $\mathcal{O}(10^{-17}-10^{-30})$. Any visible signal is BSM!
- Experiment: No channels searched-for to date
- **No channels** producible at FCC-ee (FCC-hh) either: **Very strong BSM FCNC limits.**

Exclusive Higgs decays into W + charged meson

- Exclusive $H \rightarrow M^\pm + W^\mp$ decays computed in EFT+LCDA/NRQM models:



$H \rightarrow W$	+	M	Branching fraction	Framework	Exp. limits		Producible at	
					2023	HL-LHC	FCC-ee	FCC-hh
$H \rightarrow W$		π^\pm	$(4.2 \pm 0.2) \times 10^{-6}$	EFT+NRQM [10]	-	-	✓	✓
			$(4.3 \pm 0.2) \times 10^{-6}$	EFT+LCDA [102]	-	-	✓	✓
		ρ^\pm	$(1.5 \pm 0.1) \times 10^{-5}$	EFT+NRQM [10]	-	-	✓	✓
			$(1.09 \pm 0.05) \times 10^{-5}$	EFT+LCDA [102]	-	-	✓	✓
		K^\pm	$(3.3 \pm 0.1) \times 10^{-7}$	EFT+NRQM [10]	-	-	✗	✓
			$(3.3 \pm 0.1) \times 10^{-7}$	EFT+LCDA [102]	-	-	✗	✓
		$K^{*\pm}$	$(4.3 \pm 0.2) \times 10^{-7}$	EFT+NRQM [10]	-	-	✗	✓
			$(5.6 \pm 0.4) \times 10^{-7}$	EFT+LCDA [102]	-	-	✗	✓
		D^\pm	$(5.8 \pm 0.6) \times 10^{-7}$	EFT+NRQM [10]	-	-	✓	✓
			$(5.6 \pm 0.5) \times 10^{-7}$	EFT+LCDA [102]	-	-	✓	✓
$H \rightarrow W^\mp$		$D^{*\pm}$	$(1.3 \pm 0.1) \times 10^{-6}$	EFT+NRQM [10]	-	-	✓	✓
			$(1.04 \pm 0.14) \times 10^{-6}$	EFT+LCDA [102]	-	-	✓	✓
		D_s^\pm	$(1.6 \pm 0.1) \times 10^{-5}$	EFT+NRQM [10]	-	-	✓	✓
			$(1.71 \pm 0.11) \times 10^{-5}$	EFT+LCDA [102]	-	-	✓	✓
		$D_s^{*\pm}$	$(3.5 \pm 0.2) \times 10^{-5}$	EFT+NRQM [10]	-	-	✓	✓
			$(2.51 \pm 0.19) \times 10^{-5}$	EFT+LCDA [102]	-	-	✓	✓
		B^\pm	$(1.6 \pm 0.4) \times 10^{-10}$	EFT+NRQM [10]	-	-	✗	✓
			$(1.54 \pm 0.40) \times 10^{-10}$	EFT+LCDA [102]	-	-	✗	✓
		$B^{*\pm}$	$(1.3 \pm 0.2) \times 10^{-5}$	EFT+NRQM [10]	-	-	✓	✓
			$(1.41 \pm 0.36) \times 10^{-10}$	EFT+LCDA [102]	-	-	✓	✓
	B_c^\pm	$(1.6 \pm 0.2) \times 10^{-8}$	EFT+NRQM [10]	-	-	✗	✓	
		$(8.21 \pm 0.83) \times 10^{-8}$	EFT+LCDA [102]	-	-	✗	✓	



- Theoretical branching fractions: $\mathcal{O}(10^{-5} - 10^{-10})$, enhanced if FCNC-Higgs couplings
- Experiment: No searches performed to date.
- Future: 7 (all 11) channels producible at FCC-ee (FCC-hh)

Summary: Rare & exclusive Higgs decays

- Revisited theory & exp. status of **2-,3-,4-body rare/exclusive decays** of the SM Higgs boson (branching fractions: $\mathcal{B} < 10^{-5}$):
 - \mathcal{B} 's predictions for **41 channels**, and exp. upper limits for **20 channels**.
- Physics motivations:
 - Probe light-quark & lepton Yukawa couplings.
 - Search for FCNC Higgs decays. Search for **suppressed/forbidden SM decays**.
- Explicitly **computed \mathcal{B} 's for 22 new channels**:
 - Neutrinos, multi-photons: $H \rightarrow 2\nu, \gamma+2\nu, 3\gamma, 4\gamma$ with SM rates $\mathcal{O}(10^{-4}-10^{-40})$
 - Exclusive leptonium: $H \rightarrow (\ell\ell)_{0,1} + \gamma, Z$ with SM rates $\mathcal{O}(10^{-12}-10^{-16})$
 - FCNC decays: $H \rightarrow VM^0 + \gamma, Z$ with SM rates $\mathcal{O}(10^{-14}-10^{-30})$
- Provided (\mathcal{L}_{int} -based) **projections for 63 Higgs decays at HL-LHC, FCC-ee, hh**:
 - **“Promising” decays at HL-LHC** (“easy” new limits, or observation at reach):

		Branching fraction	Exp. limits		
			2023	HL-LHC	$\mathcal{B}(\text{th})/\mathcal{B}(\text{exp})$
	$\gamma\gamma\gamma\gamma$	$(4.56 \pm 0.01) \times 10^{-12}$	–	–	–
H \rightarrow	$\gamma + \rho^0$	$(1.68 \pm 0.08) \times 10^{-5}$	$< 8.8 \times 10^{-4}$ [88]	$\lesssim 6.8 \times 10^{-5}$	$\sim 1/4$
	J/ψ	$(2.95 \pm 0.17) \times 10^{-6}$	$< 2.6 \times 10^{-4}$ [90, 92]	$\lesssim 3.9 \times 10^{-5}$ [62]	$\sim 1/10$
	$W^\mp + \rho^\pm$	$(1.5 \pm 0.1) \times 10^{-5}$	–	–	–
	$D_s^{*\pm}$	$(3.5 \pm 0.2) \times 10^{-5}$	–	–	–
	$Z + \rho^0$	$(1.4 \pm 0.1) \times 10^{-5}$	$< 1.2 \times 10^{-2}$ [106]	$\lesssim 1.8 \times 10^{-3}$	$\sim 1/100$
	$\Upsilon(1S)$	1.7×10^{-5}	–	–	–

Back-up slides