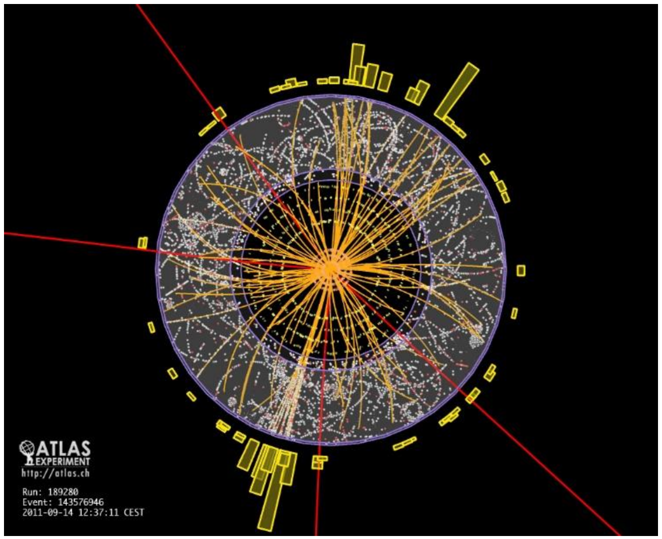
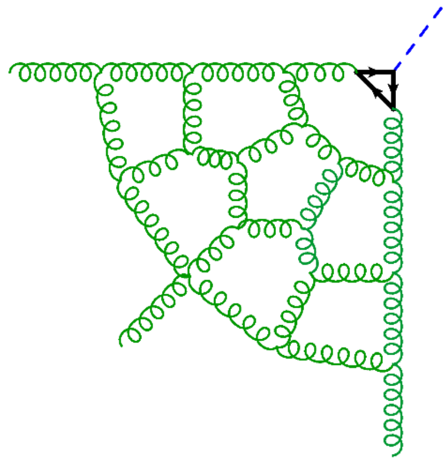
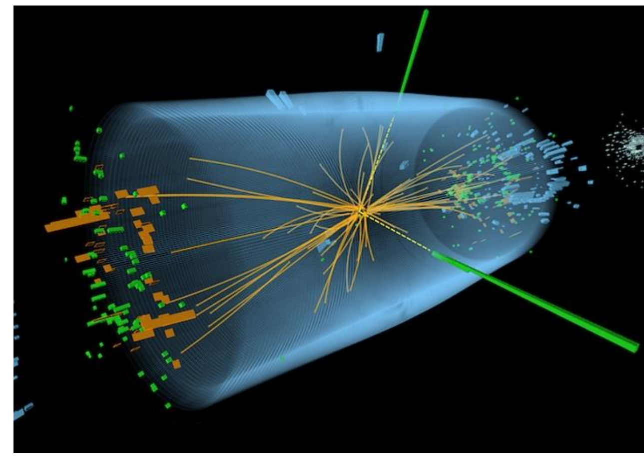
On-shell Visions for Higgs Physics







Lance Dixon

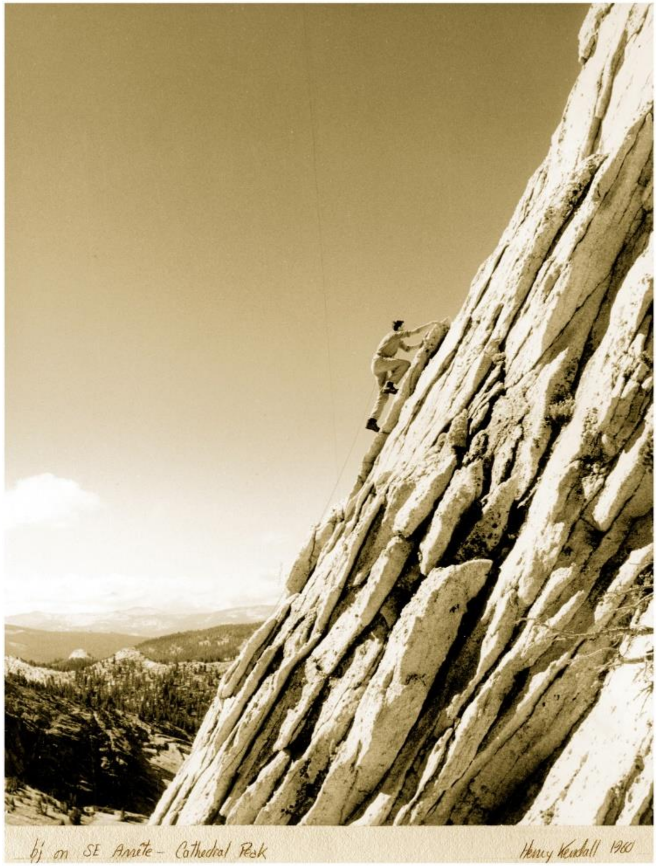
Higgs Hunting 2024

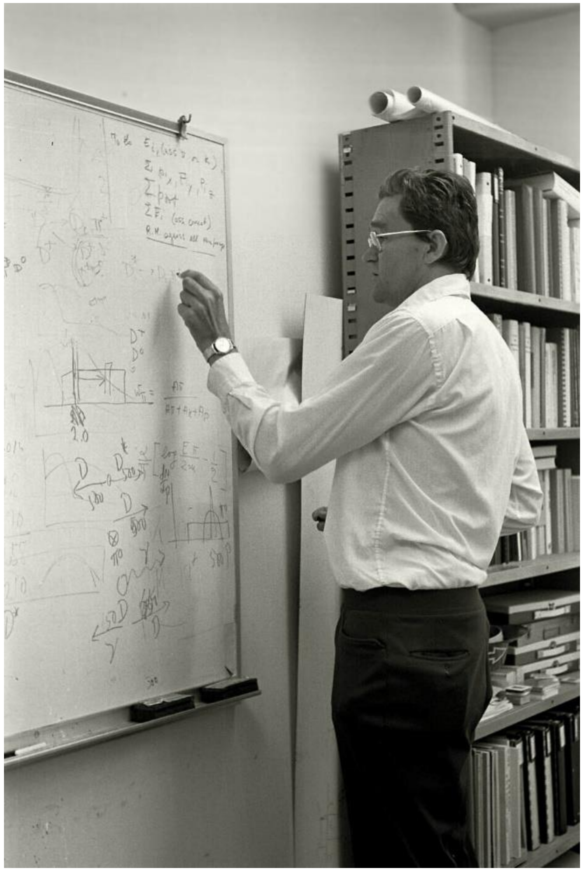
24 Sept. ’24

Orsay/Paris

NATIONAL ACCELERATOR LABORATORY

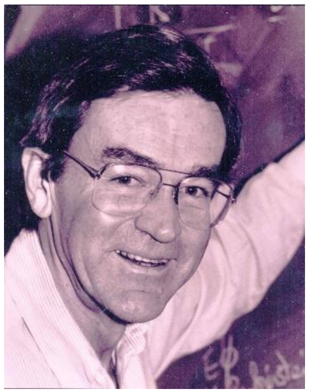
Bj Bjorken

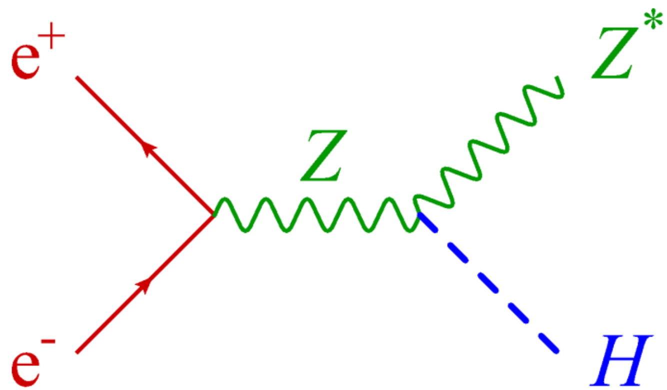


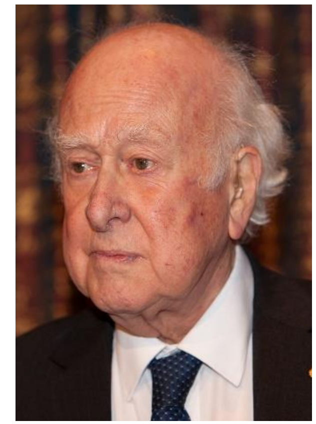


June 22, 1934 – August 6, 2024

Bjorken and Higgs

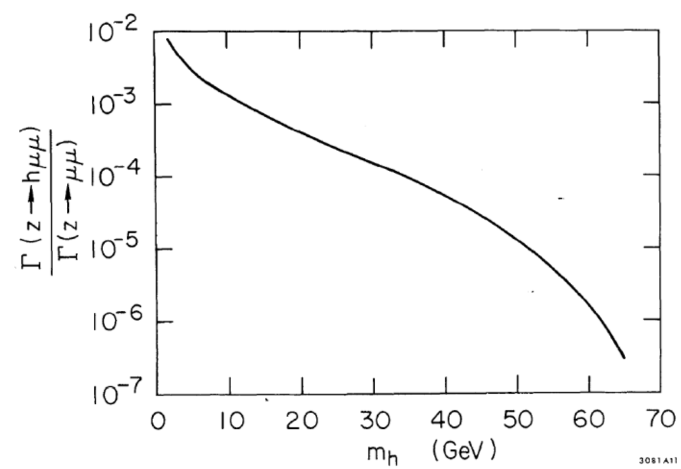






"Bjorken process" (1976 SSI Proceedings)

The signature evidently is very good; one looks at a peak in the mass recoiling against an energetic acoplanar dilepton pair. We must, however, point out that this estimate, as is any estimate which directly involves the Higgs sector, is very unreliable: the theoretical status is very poorly understood.[42] Indeed there is no certainty that $m_h \lesssim 40$ GeV; Higgs bosons could be ten times more massive.[59] And there could well be several.



On-shell philosophy

- Lagrangians are ugly
- To quantize (do loop calculations), must fix a gauge, introduce unphysical ghosts, check gauge invariance, …
- Lagrangians change under field redefinitions
- Lagrangians obscure relations between theories
- Working with on-shell *S* matrices avoids many of these issues

Standard Model Lagrangian

$$
\begin{gathered}
\mathbf{1}\quad -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \\
\tfrac{1}{2}ig_s^2(\bar{q}^\sigma_i \gamma^\mu q^\sigma_j) g^a_\mu + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
\mathbf{2}\quad M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \tfrac{1}{2}\partial_\mu H \partial_\mu H - \\
\tfrac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2 \phi^+\phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \tfrac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\tfrac{2M^2}{g^2} + \\
\tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \tfrac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - \\
W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \\
\tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + \\
g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \\
\tfrac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2] - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \tfrac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - \\
W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \tfrac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \\
\phi^+\partial_\mu H)] + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + \\
igs_w MA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + \\
igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \tfrac{1}{4}g^2 W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] - \\
\tfrac{1}{4}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-] - \tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^1 s_w^2 A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m_u^\lambda)u^\lambda_j - \\
\mathbf{3}\quad \bar{d}^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_w A_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)] + \\
\tfrac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1 + \gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - \\
1 - \gamma^5)u^\lambda_j) + (\bar{d}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 - \gamma^5)d^\lambda_j)] + \tfrac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1 + \gamma^5)e^\lambda) + \\
(\bar{u}^\lambda_j\gamma^\mu(1 + \gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \tfrac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1 + \gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1 + \\
\gamma^5)u^\lambda_j)] + \tfrac{ig}{2\sqrt{2}}\tfrac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1 - \gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1 + \gamma^5)\nu^\lambda)] - \\
\mathbf{4}\quad \tfrac{g}{2}\tfrac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \tfrac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}^\lambda_j C_{\lambda\kappa}(1 - \gamma^5)d^\kappa_j) + \\
m_u^\lambda(\bar{u}^\lambda_j C_{\lambda\kappa}(1 + \gamma^5)d^\kappa_j] + \tfrac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1 + \gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1 - \\
\gamma^5)u^\kappa_j] - \tfrac{g}{2}\tfrac{m_u^\lambda}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \tfrac{g}{2}\tfrac{m_d^\lambda}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \\
\tfrac{ig}{2}\tfrac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \\
\mathbf{5}\quad \tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \\
\partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y - \\
\partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) - \tfrac{1}{2}gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \tfrac{1}{c_w^2}\bar{X}^0 X^0 H] + \\
\tfrac{1-2c_w^2}{2c_w}igM[\bar{X}^+ X^0\phi^+ - \bar{X}^- X^0\phi^-] + \tfrac{1}{2c_w}igM[\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-] + \\
igMs_w[\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-] + \tfrac{1}{2}igM[\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0]
\end{gathered}
$$

Lagrangian standard model

Courtesy of T.D. Gutierrez

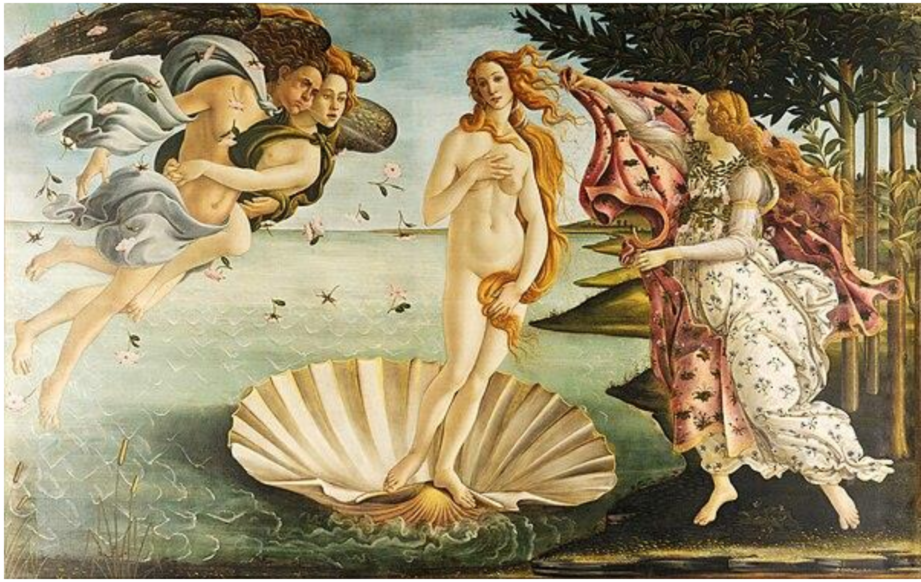
Symmetry magazine

Outline

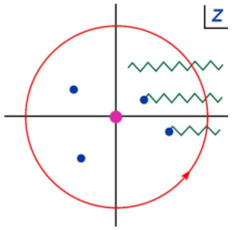
- On-shell for QCD @ LHC
- On-shell to relate theories, like gauge theory to gravity
- On-shell to describe theories unambiguously
- On-shell for SMEFT

"On-shell" philosophy

- Rather than relying solely on Feynman diagrams with off-shell virtual particles, $p^2 \neq m^2$, try to get all information from particles on their mass shell, $p^2 = m^2$
- Actually quite an old idea

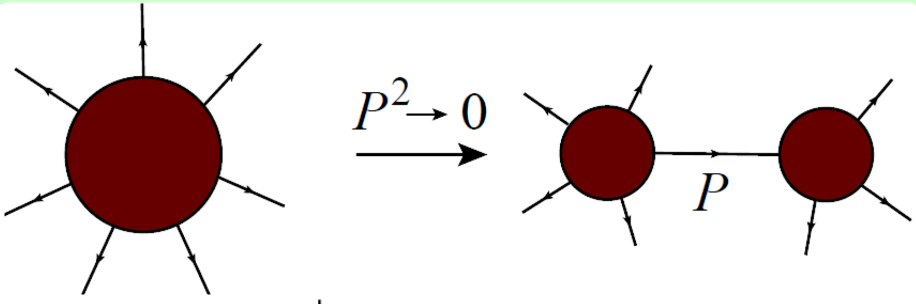


1960's Analytic *S*-Matrix



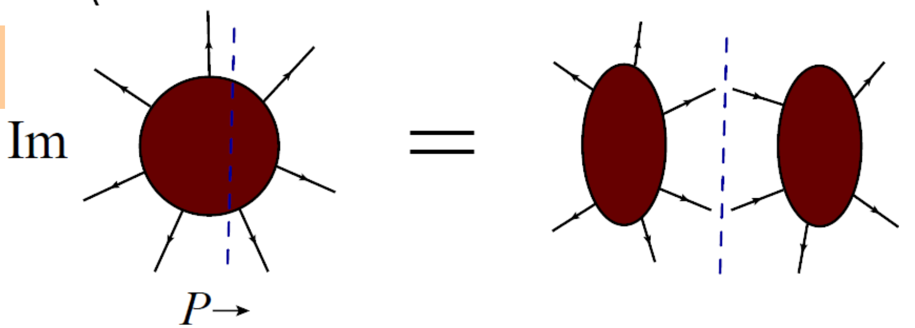
No QCD, no Lagrangian or Feynman rules for strong interactions.
Bootstrap program: Reconstruct scattering amplitudes **directly** from **analytic properties: "on-shell" information**

• Poles



Landau; Cutkosky; Chew, Mandelstam; Frautschi; Eden, Landshoff, Olive, Polkinghorne; Veneziano; Virasoro, Shapiro; … (1960s)

• Branch cuts

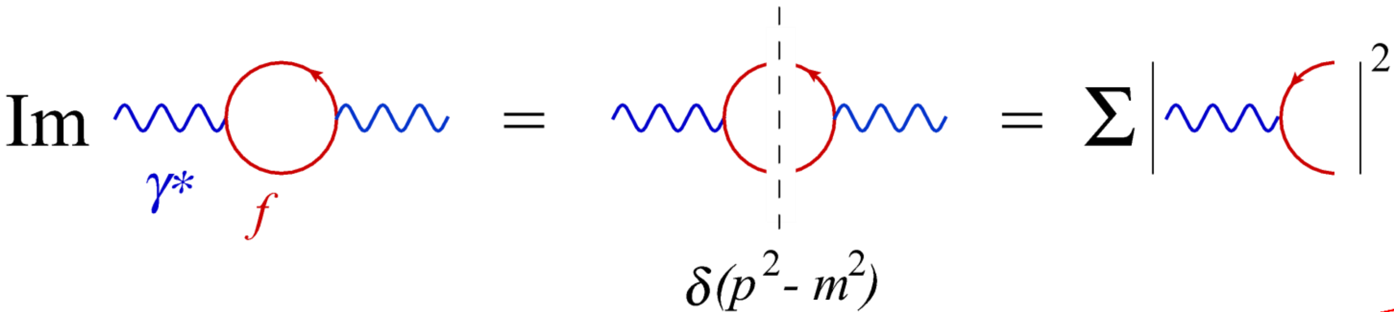


Analyticity fell out of favor in 1970s with the rise of QCD & Feynman rules

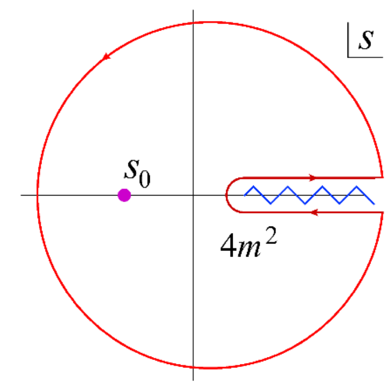
Resurrected for computing amplitudes in **perturbative** QCD
– as **alternative to Feynman diagrams!**
Perturbative information now assists analyticity

Simple example

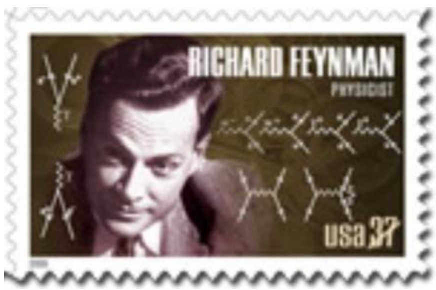
- Cutkosky rules or optical theorem (unitarity):

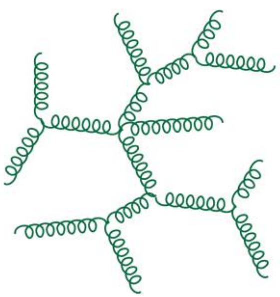


- Can recover real part nonperturbatively by a dispersion relation in $s = q^2$
- Or perturbatively by knowing what the relevant (one) loop integrals are, matching integrand to cuts (unitarity method)

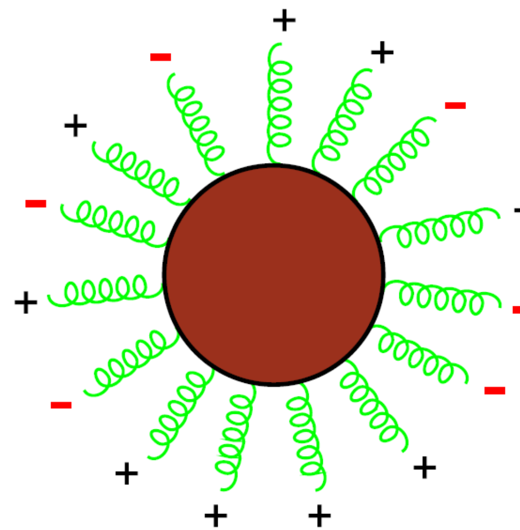


"On-shell" Methods at high orders:
Granularity vs. Fluidity





+ ...

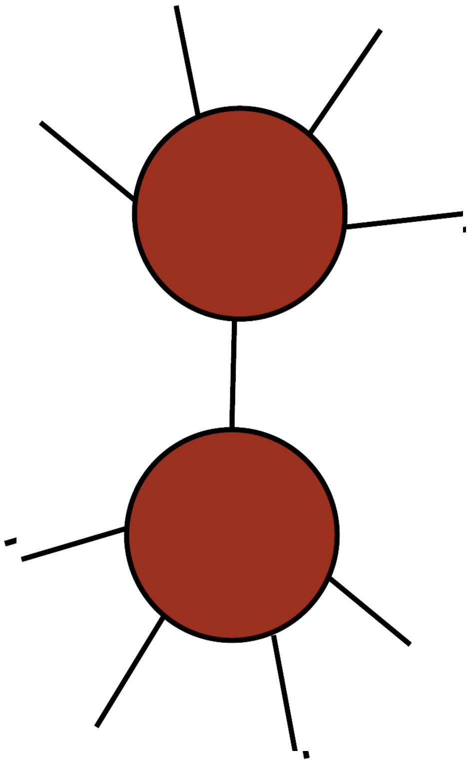


Recycling “Fluid” Amplitudes

Tree amplitudes fall apart into simpler ones in special limits – pole information

Leads to BCFW (“on-shell”) recursion relations
Britto, Cachazo, Feng, Witten (2005)

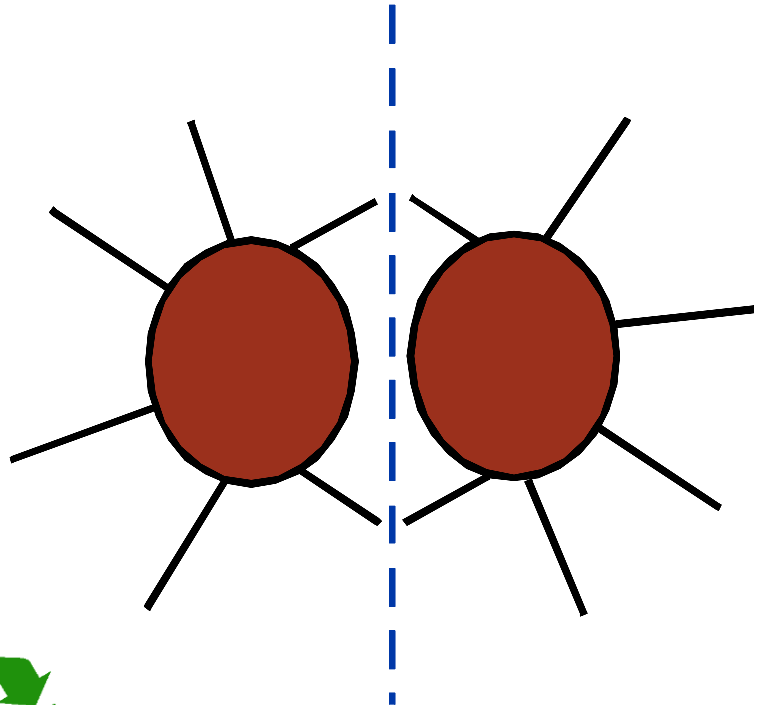
Trees recycled into trees

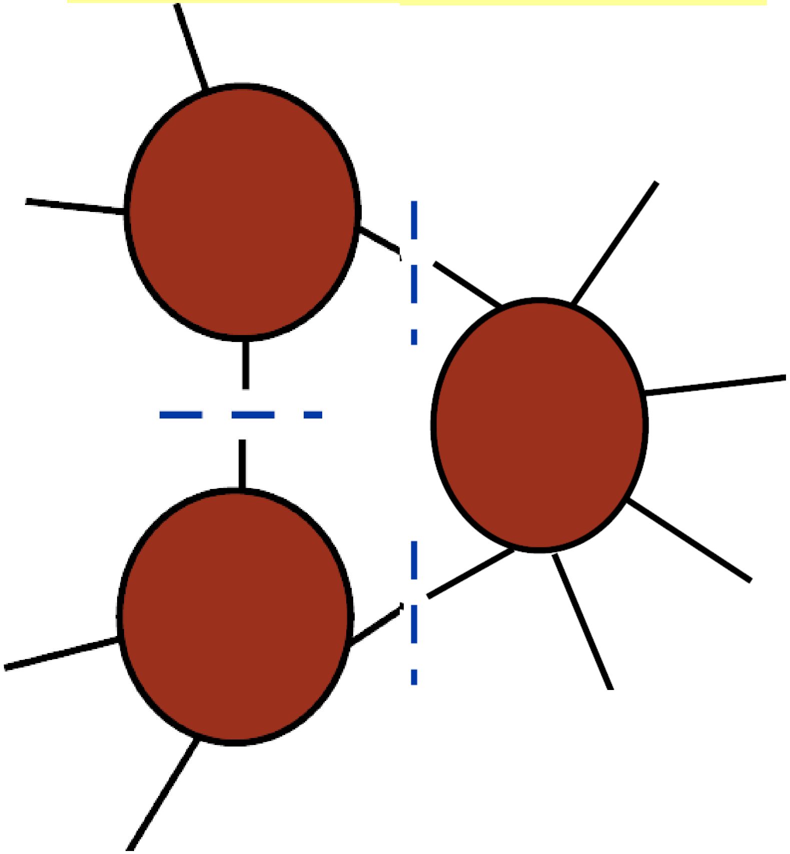


Branch cut information → Generalized Unitarity (One-loop Fluidity)

Ordinary unitarity:
put 2 particles on shell, with real momenta

Generalized unitarity:
put 3 or 4 particles on shell, complex momenta





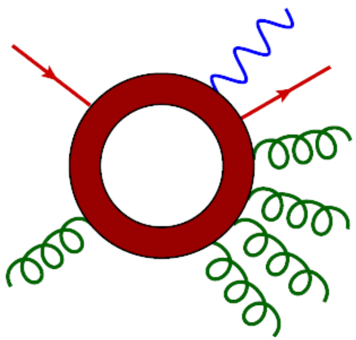
Trees recycled into loops!

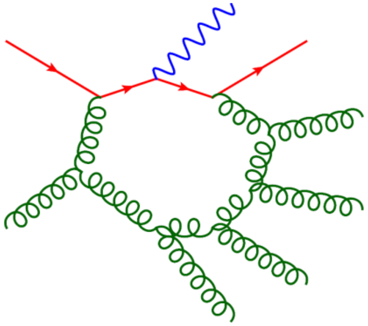
NLO needs 1 loop

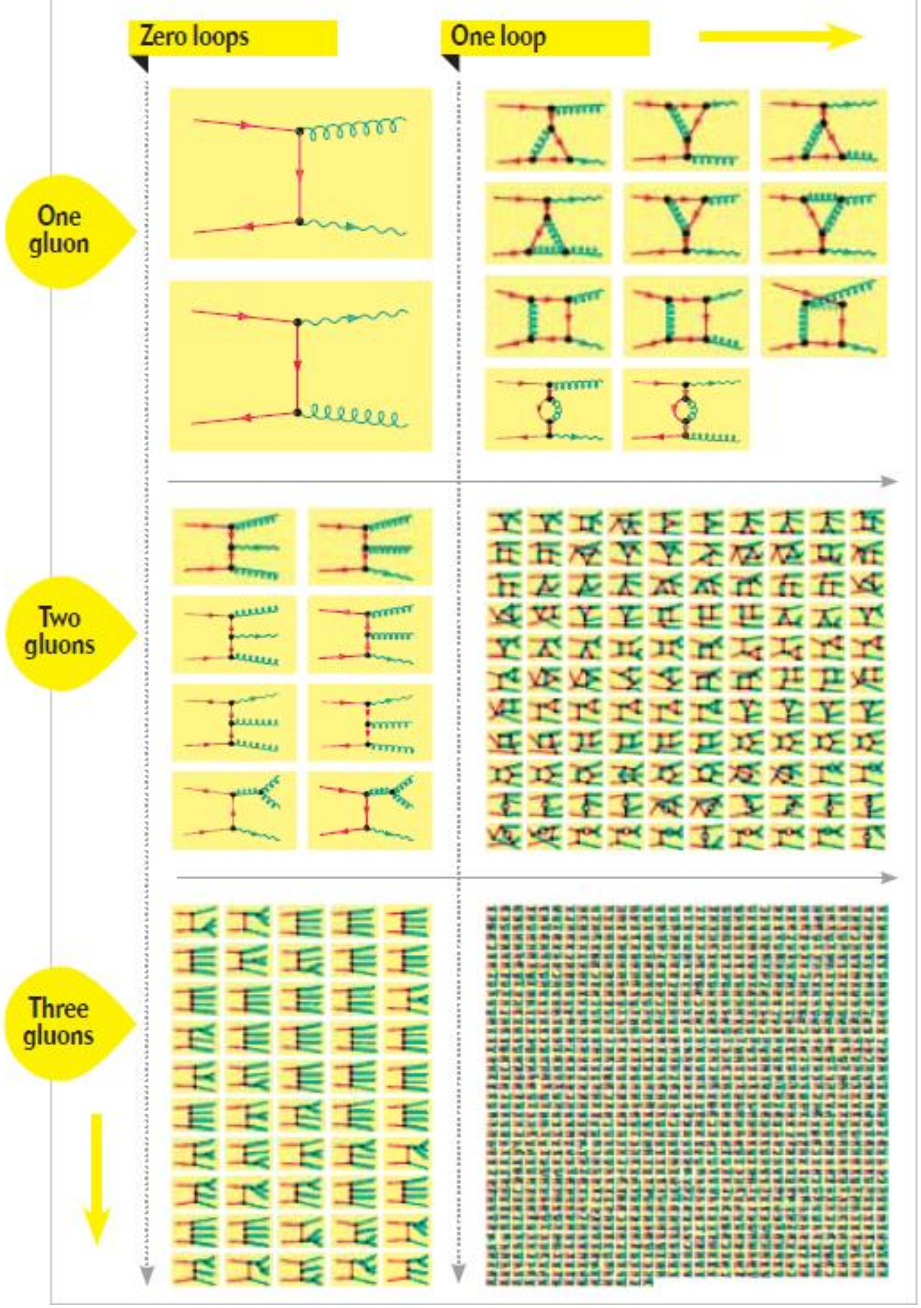
first quantum corrections

Challenging in QCD if many legs – depends on many variables

$q\bar{q} \rightarrow W + n$ gluons

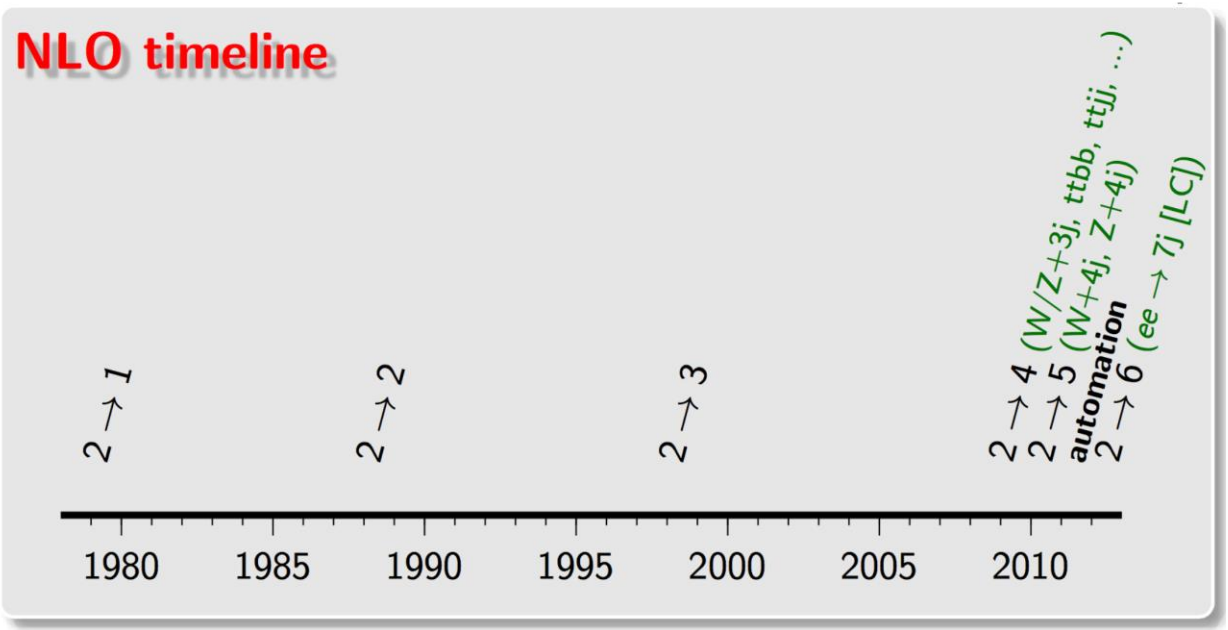


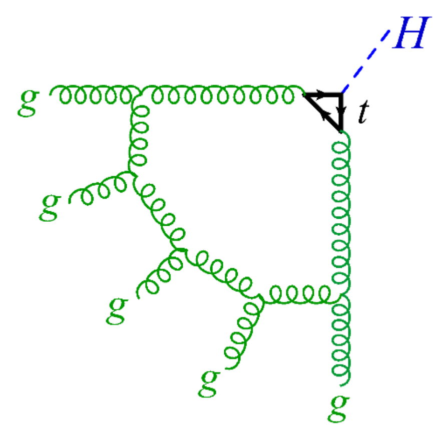
=  + 256,264 more



1-loop progress → NLO QCD @ LHC

G. Salam (2012)

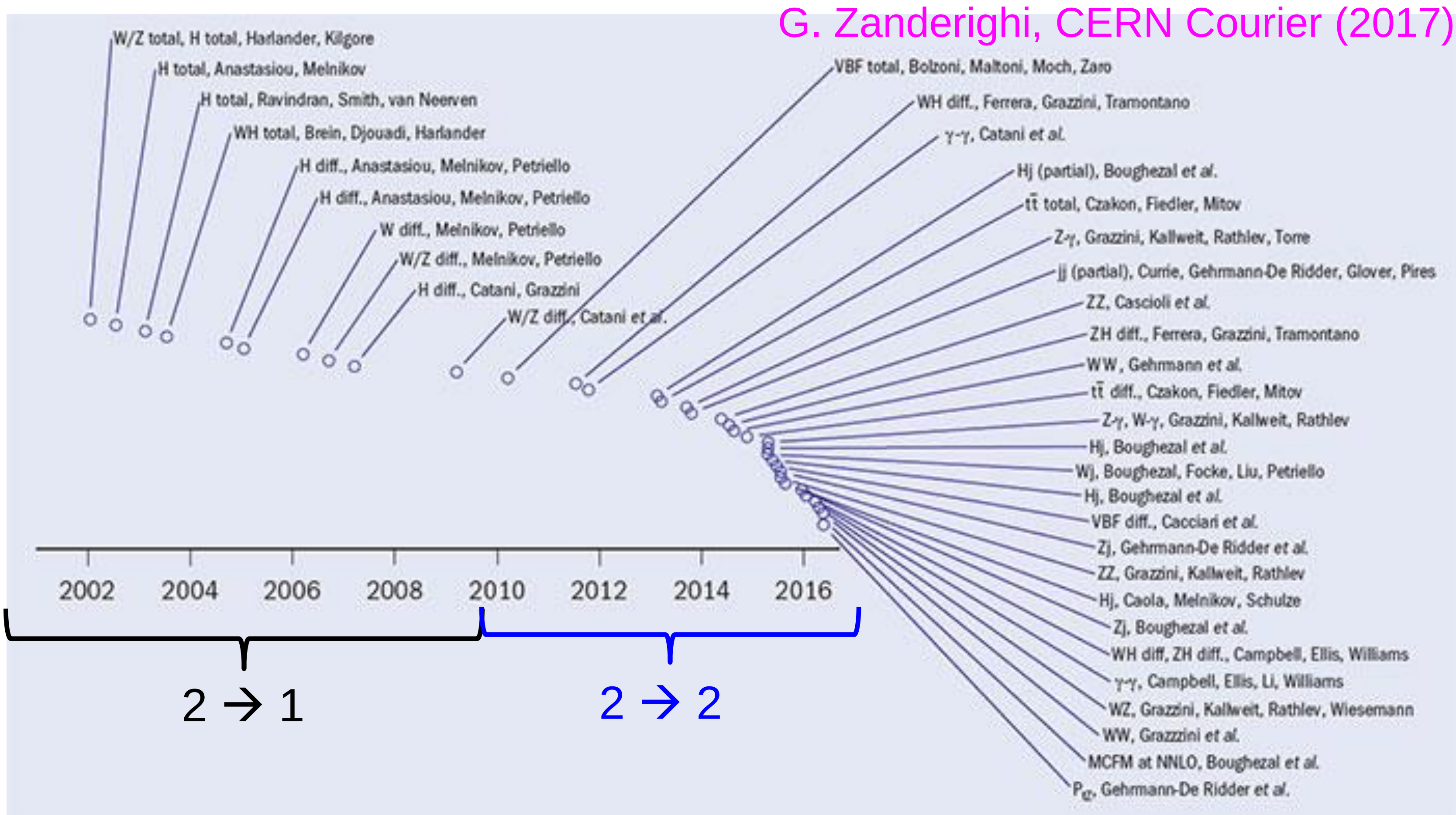




2010: NLO $W+4j$ [BlackHat+Sherpa: Berger et al] [unitarity]
2011: NLO $WWjj$ [Rocket: Melia et al] [unitarity]
2011: NLO $Z+4j$ [BlackHat+Sherpa: Ita et al] [unitarity]
2011: NLO $4j$ [BlackHat+Sherpa: Bern et al] [unitarity]
2011: first automation [MadNLO: Hirschi et al] [unitarity + feyn.diags]
2011: first automation [Helac NLO: Bevilacqua et al] [unitarity]
2011: first automation [GoSam: Cullen et al] [feyn.diags(+unitarity)]
2011: $e^+e^- \to 7j$ [Becker et al, leading colour] [numerical loops]

2013: NLO $H+3j$ in gluon fusion [GoSam, Sherpa, MadEvent: Cullen et al.]

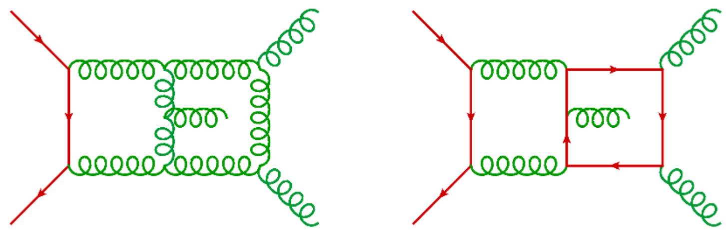
NNLO QCD @ LHC



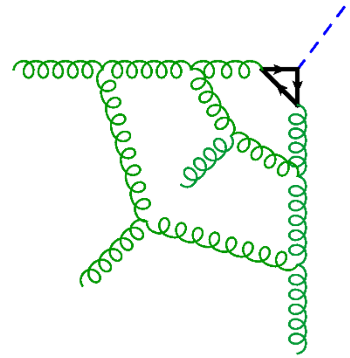
Additional challenge of 2 loop integrals → frontier is at lower multiplicity, so many results use Feynman diagrams for 2 loop virtual terms

NNLO QCD for 2 → 3

- Includes implementation of multi-loop unitarity method: Caravel Abreu, Dormans, Febres Cordero, Ita, Kraus, Page, Pascual, Ruf, Sotnikov, 2009.11957
- First applied to all-massless 5 parton scattering, e.g. fo NNLO 3 jet production:



- With recent advances in 2 loop integrals with one massive leg, NNLO H + 2 jets in gg fusion on horizon



Status of 2 loop amplitudes for NNLO QCD for $pp \to 3$ objects

Processes	Analytic Results	Public Codes	Cross Sections
$pp \to \gamma\gamma\gamma$	[3, 4]‡ **[5]**	[3]‡, **[5]**	[6, 7]‡
$pp \to \gamma\gamma j$	[8–10]† **[11]**	[8, 10]†	[12, 13]†
$pp \to \gamma j j$	**[14]**		**[14]**
$pp \to j j j$	[15]†**[16–18]**	[15]†**[18]**	[19, 20]†
$pp \to W b\bar{b}$	[21]⋆ [22–24]†	[24]†	[23, 25]†
$pp \to H b\bar{b}$	[26]⋆		
$pp \to W j\gamma$	[27]‡		
$pp \to W j j$	[22, 24]†	[24]†	
$pp \to (Z/\gamma^\star) j j$	[22, 24]‡	[24]‡	
$pp \to ttH$			[28]⋆

Table 1: Summary of known two-loop QCD corrections for five-point scattering processes at hadron colliders. † denotes calculations performed in l.c. approximation, where l.c. coincides with planar, while ‡ denotes planar computations that are not l.c. accurate. ⋆ denotes additional approximations, such as on-shell W, $m_b = 0$ but $y_b \neq 0$, or soft Higgs. Bold denotes non-planar, full-color results.

Table from De Laurentis, 2406.18374; see references therein; many (but not all) use on-shell methods

- One bottleneck was nonplanar loop integrals, but much recent progress, e.g. Abreu, Chicherin, Ita, Page, Sotnikov, Tschernow, Zoia, 2306.15431

Lagrangians & Field Redefinitions

- Path integral for QFT: $\int [D\phi] e^{i \int d^4x \, \mathcal{L}(\phi)}$
- ϕ is a dummy variable
- We can redefine ϕ without changing the physics!
- E.g. $\mathcal{L}(\phi) = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{\lambda}{4!}\phi^4$

  Let $\phi \to \phi + c\phi^3$
- $\mathcal{L} \to \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + 3c\phi^2\partial_\mu\phi\partial^\mu\phi + \frac{\lambda}{4!}\phi^4 + c\,\frac{\lambda}{3!}\phi^6 + \cdots$
- Looks non-renormalizable, but it is not

Off-shell Feynman Rules

- Due to field-redefinition and gauge ambiguities, Feynman rules can be very messy, and obscure relations between different theories.
- Case in point: Double copy / KLT relations between
  - non-abelian Yang-Mills theory (at heart of SM)
  - Einstein's theory of gravity

Gravity = [Yang-Mills]^2

Off-shell graviton vertices are complicated:

YM $\longrightarrow$ $g f^{abc}\Big[(p_1 - p_2)_\sigma \eta_{\mu\nu} + \text{cyclic}\Big]$

DeWitt [1967]

GR $\longrightarrow$

$$\begin{aligned}\text{Sym}\Big[&-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})\\&-\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda})+\tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+\tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho})+P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu})\\&-P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})\Big],\end{aligned}$$

On-shell amplitudes are remarkably simple:

$$A_3^{\mathrm{YM}} = f^{abc}(\epsilon_1\cdot\epsilon_2\,\epsilon_3\cdot p_1 + \text{cyc.})$$

$$A_3^{\mathrm{GR}} = (\epsilon_1\cdot\epsilon_2\,\epsilon_3\cdot p_1 + \text{cyc.})^2$$

Hayden Lee talk

Similar relations for any tree amplitudes!

Kawai, Lewellen, Tye (1985)

n-point amplitudes in gravity

n-point (color-ordered) amplitudes in Yang-Mills theory

$$M_4^{\text{tree}}(1,2,3,4) = -is_{12}A_4^{\text{tree}}(1,2,3,4)\,A_4^{\text{tree}}(1,2,4,3)\,,$$

$$M_5^{\text{tree}}(1,2,3,4,5) = is_{12}s_{34}A_5^{\text{tree}}(1,2,3,4,5)A_5^{\text{tree}}(2,1,4,3,5)$$
$$+\,is_{13}s_{24}A_5^{\text{tree}}(1,3,2,4,5)\,A_5^{\text{tree}}(3,1,4,2,5)$$

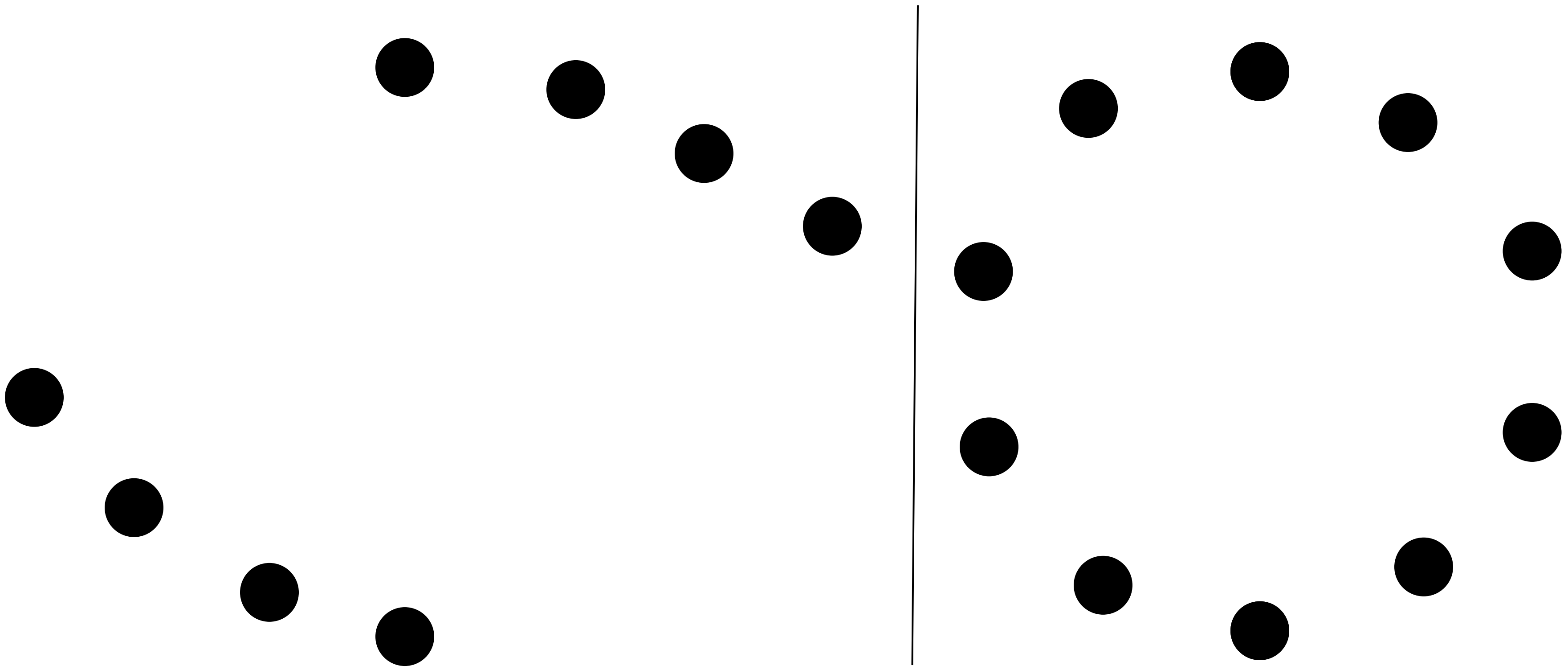
Mandelstam invariants $s_{ij} = (p_i + p_j)^2$

- Originally found by relating closed & open string theory amplitudes
- Now have alternate field-theory interpretations:
  - double-copy — Bern, Carrasco, Johansson (2008,2010)
  - CHY formalism — Cachazo, He, Yuan (2013)

From LHC to LIGO

- Gravitationally interacting particles can also be massive → black-holes
- Can use on-shell (quantum) methods, originally developed for QCD @ LHC, to compute classical black-hole (or neutron star) scattering
- Extract higher perturbative orders (in $G_N m_1 m_2$) in gravitational interaction during binary inspiral (bound orbit) for more accurate LIGO waveforms

Black hole scattering vs. inspiral



- Related by “analytic continuation around $r = \infty$”
- Accomplish with effective Hamiltonian, e.g. Cheung, Rothstein, Solon, 1808.02489
- Or more directly in terms of trajectories Kälin, Porto, 1910.03008, 1911.09130

Classical restrictions compatible with on-shell methods

$3\text{PM} = G_N^3 = 2\text{ loop}$ computation $(S_1 = S_2 = 0)$

Bern, Cheung, Roiban, Shen, Solon, 1901.04424, 1908.01493

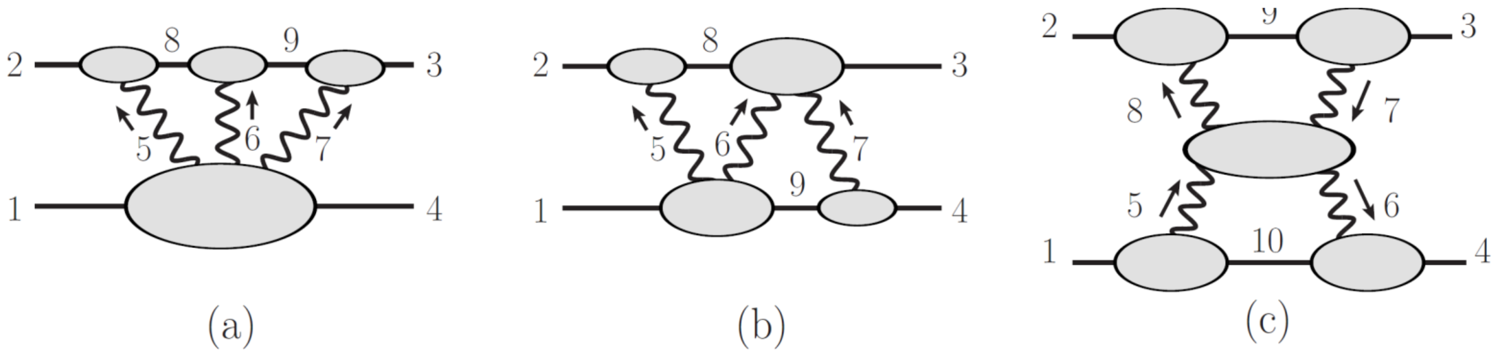
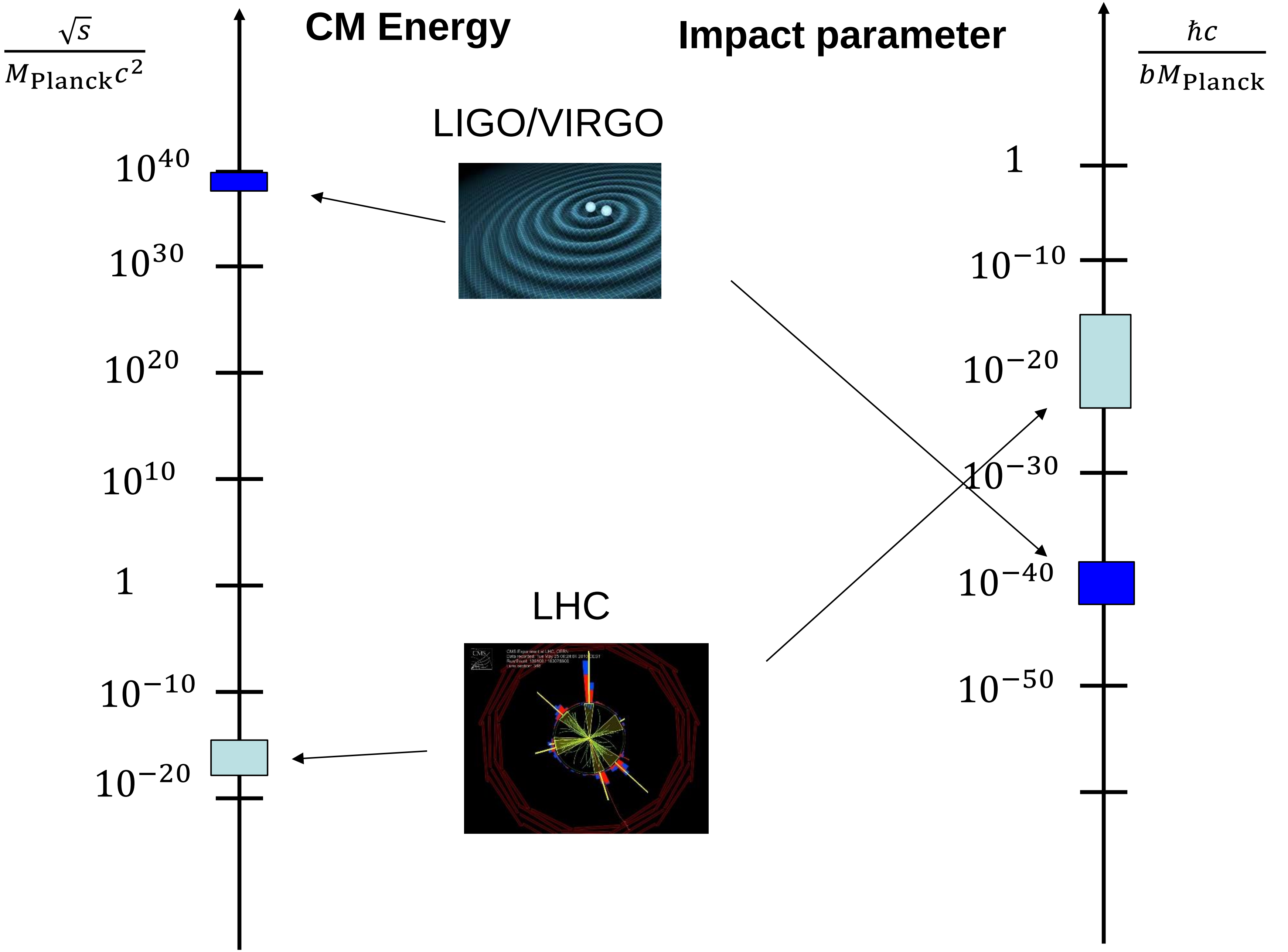


Figure 12: The independent generalized cuts needed at two loops for the classical potential. The remaining contributing cuts are given by simple relabeling of external legs. Here the straight lines represent on-shell scalars and the wiggly lines correspond to on-shell gravitons or gluons.

- More recently, $G_N^4 = 3\text{ loops}$ Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng (2021)
- Also many results including black hole spins $S_{1,2}$
- In progress: $G_N^5 = 4\text{ loops}$


CM Energy
Impact parameter
$\frac{\sqrt{s}}{M_{\text{Planck}}c^2}$
$\frac{\hbar c}{bM_{\text{Planck}}}$
LIGO/VIRGO
LHC
10^{40}
10^{30}
10^{20}
10^{10}
1
10^{-10}
10^{-20}
1
10^{-10}
10^{-20}
10^{-30}
10^{-40}
10^{-50}

Back to Higgs

- Studying Higgs properties includes determining (or bounding) coefficients of local higher-dimension operators in e.g. Standard Model Effective Field Theory (SMEFT)
- Need independent basis of operators, taking into account field redefinitions and total derivatives
- Use classical field equations of motion to eliminate redundancies
- E.g. at dim. 6 Buchmueller, Wyler (1986); Jenkins, Manohar, 0907.4763; Grzadkoswki, Iskrzynski, Misiak, Rosiek, 1008.4884;…
- In on-shell approach, associate independent operators with independent on-shell amplitudes, which are independent polynomials in momentum invariants

Scalar field example

- Q: How many independent local operators for a color- and flavor-less single scalar field ϕ?
- A: Start at 4-points, count number of Bose symmetric polynomials in s, t, u after imposing $s + t + u = 0$
- Dimension 4: 1 ϕ^4
- Dimension 6: $-$ (no $\phi^2(\partial\phi)^2$ for single scalar)
- Dimension 8: $\sigma_2 \equiv s^2 + t^2 + u^2$
- Dimension 10: $\sigma_3 \equiv s^3 + t^3 + u^3$
- By a theorem about symmetric polynomials, the ring is generated by σ_2 and σ_3, so the number at dimension $4 + 2N$ is d_N with the generating function:

$$d(t) = \sum_{n=0}^{\infty} d_N t^N = \frac{1}{(1-t^2)(1-t^3)} = 1 + t^2 + t^3 + t^4 + t^5 + 2t^6 + \cdots$$

SMEFT On-Shell

Ma, Shiu, Xiao, 1902.06752

- For massless spinning particles, use polynomials in spinor inner products $< ij >, [ij]$ that transform appropriately under Lorentz transformations.
- Define *primary amplitude* building blocks that are then combined with $SU(3) \times SU(2) \times U(1)$ group invariants
- At dim. 6, recover Warsaw basis

(n_ψ, n_A, h)	Primary amplitude	m_{min}	n_s	d_{min}
(0,0,0)	$f(\phi^{n_s}) = 1$	0	$n_s \geqslant 3$	3
(0,2,2)	$f(A^+A^+\phi^{n_s}) = [12]^2$	2		5
(0,3,3)	$f(A^+A^+A^+) = [12][23][31]$	3		6
(2,0,1)	$f(\psi^+\psi^+\phi^{n_s}) = [12]$	1		4
(2,0,0)	$f(\psi^+\psi^-\phi^2) = [1\|p_3\|2\rangle$	2	$n_s \geqslant 2$	6
(2,1,2)	$f(A^+\psi^+\psi^+\phi^{n_s}) = [12][13]$	2		5
(4,0,2)	$f(\psi^+\psi^+\psi^+\psi^+) = [12][34]^*$	2		6
(4,0,0)	$f(\psi^+\psi^+\psi^-\psi^-) = [12]\langle 34\rangle$	2		6

TABLE I: All classes of amplitude basis with $d \leq 6$. The $*$ for the $(4,0,2)$ case stands for multiple ways of spinor contraction.

Bottom-up EFT On-Shell

Aoude, Machado, 1905.11433; Durieux, Kitahara, Shadmi, Weiss, 1909.10551

- For amplitudes in Higgs (massive) phase, use little-group covariant massive spinor formalism
  Arkani-Hamed, Huang, Huang, 1709.04891
- Rederive properties of Higgs couplings from on-shell perspective, without SM Lagrangian and Higgs mechanism:
  - The coupling of a scalar h to two transverse vectors of equal polarizations, only arises at the non-renormalizable or loop level.
  - At the renormalizable level, the tree VVh amplitude is controlled by a single coupling. Vector bosons of opposite polarizations are involved.
  - The only renormalizable amplitudes that remain non-zero in the high-energy limit are the $(0{\pm}0)$ and $({\pm}00)$ ones, which involve one transverse and one longitudinal vector.

EFT hunting goes on shell

An EFT hunter's guide to two-to-two scattering: HEFT and SMEFT on-shell amplitudes

2301.11349

Hongkai Liu[a], Teng Ma[a,b], Yael Shadmi[a], Michael Waterbury[a]

[a] *Physics Department, Technion – Israel Institute of Technology, Technion city, Haifa 3200003, Israel*

[b] *IFAE and BIST, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona*

- Formulation of EFT analyses directly in terms of observable quantities

- [On-shell] vision: "EFT Hunting 20nn"
- In addition to "Higgs Hunting 20nn" of course!

In Conclusion: Bj’s vision in 1985, reminiscing on a decade since November 1974 revolution

I think that what is in our future is a new adventure in confusion. For a long time the evidence that can be uncovered about the nature of the Higgs sector is likely to be small compared to the number of hypotheses bandied about upon what it really is - too small for decisive conclusions. We will be forced back into the mode I remember so vividly in the 1960s - one with a great variety of hypotheses, a great variety of approaches, a great uncertainty as to which approach is going to win and which one isn't, and a great uncertainty as to which energy scale is going to provide the key to the solution. It may be as surprising as in 1974, when 3 GeV in the center of mass for e^+e^- was sufficient…

Two symposia at SLAC (and on Zoom)

- Friday, November 8, 2024:

"Symposium on the 50th Anniversary of the November Revolution (Jpsi50)"

https://indico.slac.stanford.edu/event/9040/

- Saturday, November 9, 2024:

"Remembering Bj: a Symposium in Honor of James Bjorken"

https://indico.slac.stanford.edu/event/9148/

Extra slides

3-point amplitudes: Gravity = YM^2

Completely dictated by symmetries!

- Only nonzero gauge-theory helicity amplitude (helicity ± 1):

gauge coupling

$$\mathcal{A}_3^{\mathrm{YM}}(1^-,2^-,3^+) = g_s \frac{<12>^3}{<23><31>} f^{a_1 a_2 a_3}$$

color factor

- For experts: $< ab >$ are inner products of Weyl spinors, would be $\sqrt{s_{ab}}$ if momenta were real

- Only nonzero **gravity** helicity amplitude (helicity ± 2) is:

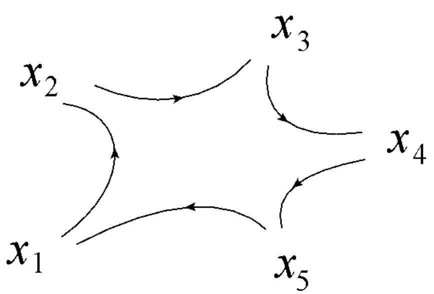
$$\mathcal{M}_3^{\mathrm{grav}}(1^{--},2^{--},3^{++}) = \frac{\kappa}{2}\left[\frac{<12>^3}{<23><31>}\right]^2 \propto \left[\mathcal{A}_3^{\mathrm{YM}}\right]^2$$

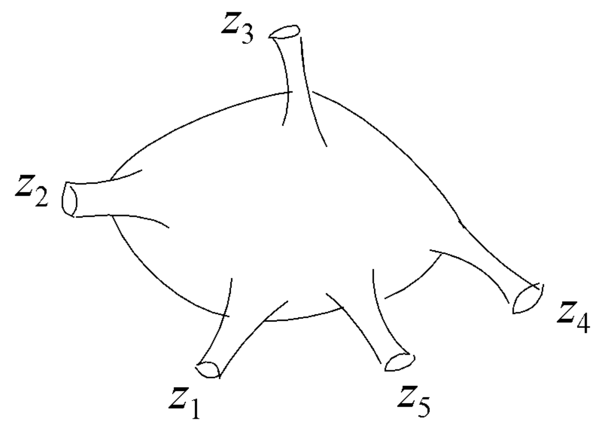
KLT relations

Kawai, Lewellen, Tye (1985)

1-dimensional string sweeps out a 2-dimensional world-sheet

open → with boundary (disk) closed → no boundary (sphere)





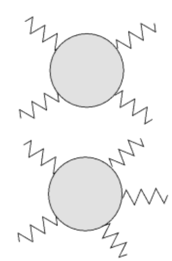
$$A_n^{\text{open}} \sim \int dx_a\, f(x_b, k_b) \qquad \mathcal{M}_n^{closed} \sim \iint dz_a d\bar{z}_a\, |f(z_b, k_b)|^2$$

deform integral contours, take low energy limit, ignore couplings and color factors

$$M_4^{\text{tree}}(1,2,3,4) = -is_{12} A_4^{\text{tree}}(1,2,3,4)\, A_4^{\text{tree}}(1,2,4,3)\,,$$

$$M_5^{\text{tree}}(1,2,3,4,5) = is_{12}s_{34} A_5^{\text{tree}}(1,2,3,4,5) A_5^{\text{tree}}(2,1,4,3,5) + is_{13}s_{24} A_5^{\text{tree}}(1,3,2,4,5)\, A_5^{\text{tree}}(3,1,4,2,5)\,,$$

…



…

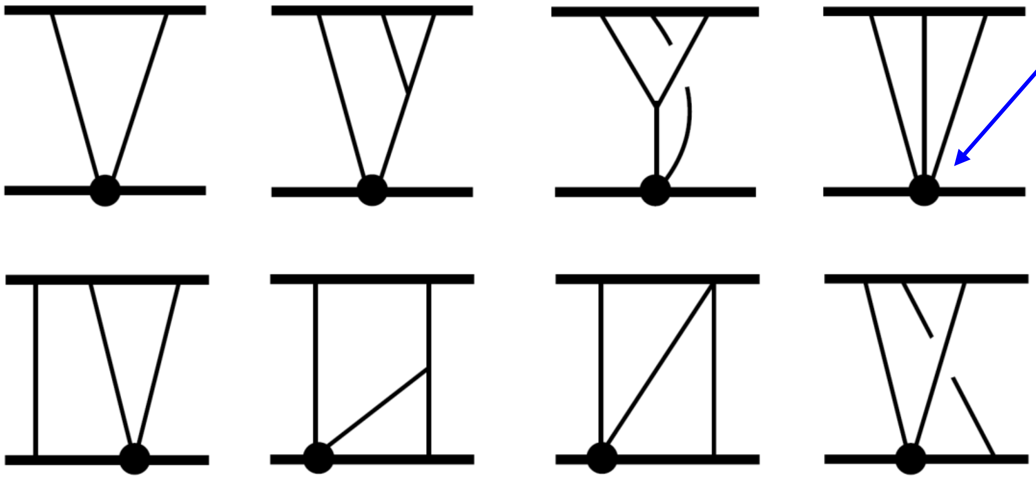
Spin and tidal effects also computable within similar framework

Bern, Luna, Roiban, Shen, Zeng, 2005.03071

Spinning Black Hole Binary Dynamics, Scattering Amplitudes and Effective Field Theory

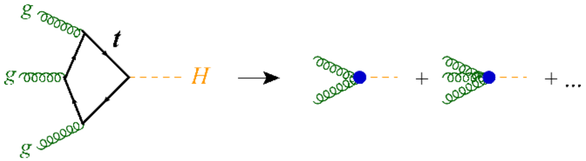
Cheung, Solon, 2006.06665

Tidal Effects in the Post-Minkowskian Expansion



operator(s) encoding multipole moments of neutron star

analogous to $HF_{\mu\nu}F^{\mu\nu} + \cdots$ operator(s) encoding couplings of gluons to Higgs boson at LHC



Spinless black hole example

- Scattering depends on both relative velocity v and strength of potential $G_N M_1 M_2 / r \equiv G/(\frac{r}{r_{Schw}})$ (deviation from Minkowski metric)
- In bound state, locked together by virial theorem:
- Kinetic energy ~ potential energy

$$v^2 \sim G$$

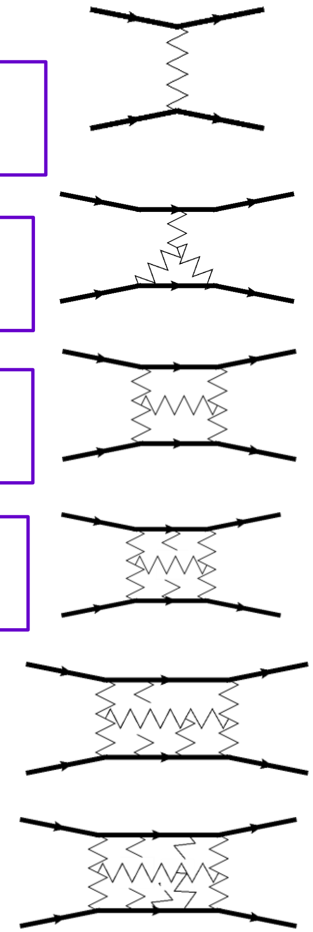
- Common parameter controls perturbative post-Newtonian approximation relevant for inspiral accuracy
- But in scattering one can compute separate orders in v^2 (or p^2) and G

$$H^{(0)}(r^2, p^2) \;=\; \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + \frac{G}{r} c_1^{(0)}(\boldsymbol{p}^2) + \left(\frac{G}{r}\right)^2 c_2^{(0)}(\boldsymbol{p}^2) + \mathcal{O}(G^3)$$

- Powers of G alone referred to as post-Minkowskian

Double expansion of spinless conservative Hamiltonian

	0PN (1687)	1PN (1938)	2PN (1972-1985)	3PN (2001)	4PN (2014)	5PN (2020, 2010.13672)	
1PM (1956)	$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ v^{12} + \cdots)\, G$
2PM (1960)		$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10} + \cdots)\, G^2$
3PM (2019)			$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8 + \cdots)\, G^3$
4PM (~2021)				$(1$	$+ v^2$	$+ v^4$ (2019-2020)	$+ v^6 + \cdots)\, G^4$
5PM					$(1$	$+ v^2$ (almost)	$+ v^4 + \cdots)\, G^5$
6PM						$(1$ (2019)	$+ v^2 + \cdots)\, G^6$



Many contributed to these advances, for 3-4PN notably T. Damour and collaborators

4PM new state of art for PM

Bern, Parra-Martinez, Roiban, Ruf, Shen, Solon, Zeng, 2101.07254

$$\mathcal{M}_4(\boldsymbol{q}) = G^4 M^7 \nu^2 |\boldsymbol{q}| \left(\frac{\boldsymbol{q}^2}{4^{\frac{1}{3}} \tilde{\mu}^2} \right)^{-3\epsilon} \pi^2 \left[\mathcal{M}_4^{\rm p} + \nu \left(\frac{\mathcal{M}_4^{\rm t}}{\epsilon} + \mathcal{M}_4^{\rm f} \right) \right]$$

$$\mathcal{M}_4^{\rm p} = -\frac{35\,(1 - 18\sigma^2 + 33\sigma^4)}{8\,(\sigma^2 - 1)} \qquad \mathcal{M}_4^{\rm t} = h_1 + h_2 \log\left(\tfrac{\sigma+1}{2}\right) + h_3 \frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2 - 1}}$$

$$\begin{aligned} \mathcal{M}_4^{\rm f} = {} & h_4 + h_5 \log\left(\tfrac{\sigma+1}{2}\right) + h_6 \frac{\operatorname{arccosh}(\sigma)}{\sqrt{\sigma^2-1}} + h_7 \log(\sigma) - h_2 \frac{2\pi^2}{3} + h_8 \frac{\operatorname{arccosh}^2(\sigma)}{\sigma^2 - 1} + h_9 \left[\operatorname{Li}_2\left(\tfrac{1-\sigma}{2}\right) + \frac{1}{2} \log^2\left(\tfrac{\sigma+1}{2}\right) \right] \\ & + h_{10} \left[\operatorname{Li}_2\left(\tfrac{1-\sigma}{2}\right) - \frac{\pi^2}{6} \right] + h_{11} \left[\operatorname{Li}_2\left(\tfrac{1-\sigma}{1+\sigma}\right) - \operatorname{Li}_2\left(\tfrac{\sigma-1}{\sigma+1}\right) + \frac{\pi^2}{3} \right] + h_2 \frac{2\sigma(2\sigma^2-3)}{(\sigma^2-1)^{3/2}} \left[\operatorname{Li}_2\left(\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) - \operatorname{Li}_2\left(-\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) \right] \\ & + \frac{2h_3}{\sqrt{\sigma^2-1}} \left[\operatorname{Li}_2\left(1 - \sigma - \sqrt{\sigma^2-1}\right) - \operatorname{Li}_2\left(1 - \sigma + \sqrt{\sigma^2-1}\right) + 5\operatorname{Li}_2\left(\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) - 5\operatorname{Li}_2\left(-\sqrt{\tfrac{\sigma-1}{\sigma+1}}\right) + 2\log\left(\tfrac{\sigma+1}{2}\right) \operatorname{arccosh}(\sigma) \right] \\ & + h_{12} \mathrm{K}^2\left(\tfrac{\sigma-1}{\sigma+1}\right) + h_{13} \mathrm{K}\left(\tfrac{\sigma-1}{\sigma+1}\right) \mathrm{E}\left(\tfrac{\sigma-1}{\sigma+1}\right) + h_{14} \mathrm{E}^2\left(\tfrac{\sigma-1}{\sigma+1}\right), \end{aligned}$$

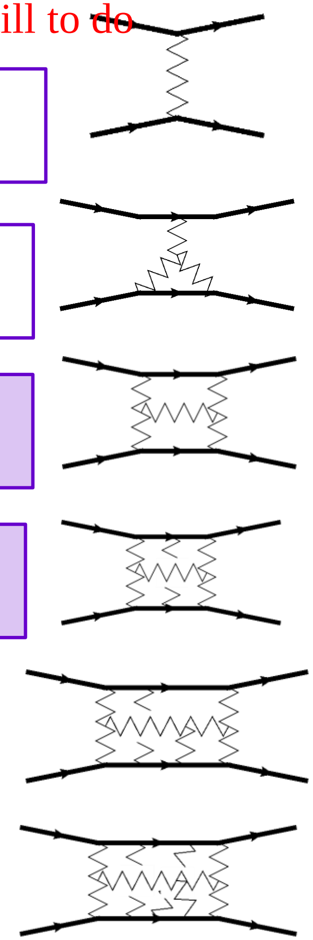
elliptic integrals

non-local “tail” still missing

First contributions of “amplitudes” to LIGO physics

	0PN (1687)	1PN (1938)	2PN (1972-1985)	3PN (2001)	4PN (2014)	5PN (2020, 2010.13672, Blümlein, Maier, Marquard, Schäfer; non-potential terms still to do)			
1PM 1956	$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ v^{12}$	$+ \cdots)$	G
2PM 1960		$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ v^{10}$	$+ \cdots)$	G^2
3PM 2019 Z. Bern et al., 1901.04404			$(1$	$+ v^2$	$+ v^4$	$+ v^6$	$+ v^8$	$+ \cdots)$	G^3
4PM 2021 Z. Bern et al., 2101.07254, 2112.10750				$(1$	$+ v^2$	$+ v^4$ (2019-2020)	$+ v^6$	$+ \cdots)$	G^4
5PM					$(1$	$+ v^2$ (almost)	$+ v^4$	$+ \cdots)$	G^5
6PM						$(1$ (2019)	$+ v^2$	$+ \cdots)$	G^6

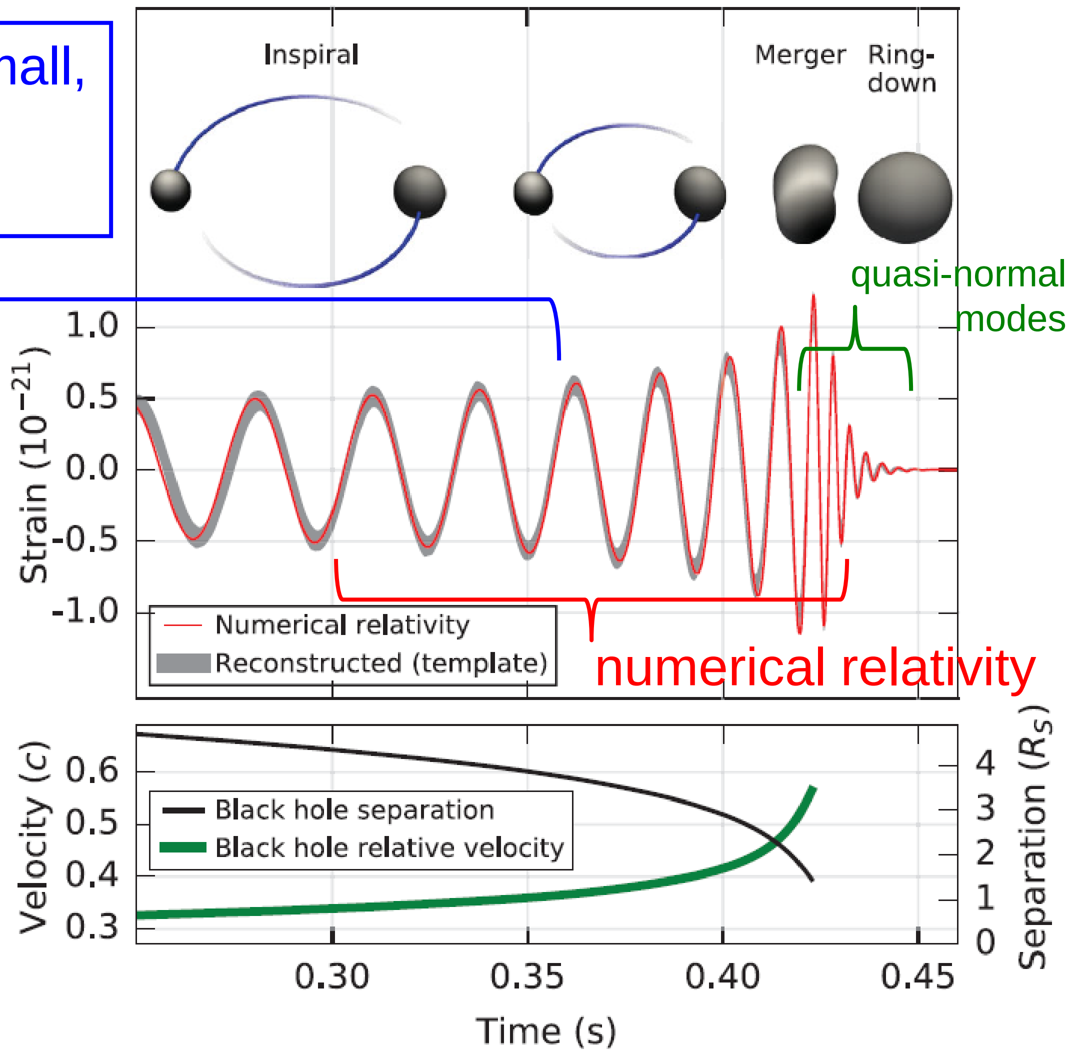
Bini, Damour, Geralico, 1909.02375



Typical LIGO Event

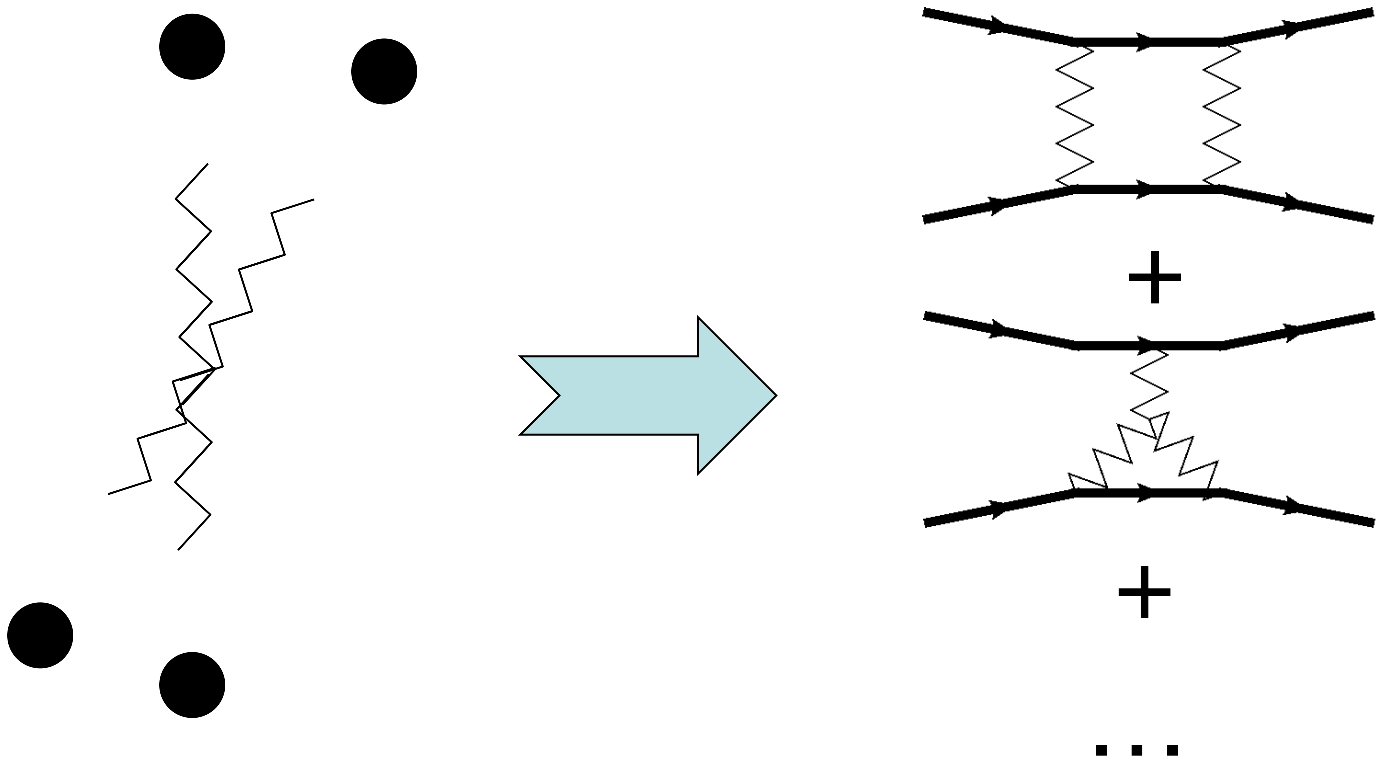
nonlinearities small, perturbatively calculable

NS-NS mergers @ LIGO/VIRGO/... or BH's @ LISA
→ many, many cycles
in perturbative regime
→ phase of orbit can be measured
very precisely



Loops contain classical pieces

- Especially if particles move slowly, lots of time for multiple exchanges of virtual gravitons, to build up smooth classical trajectory.



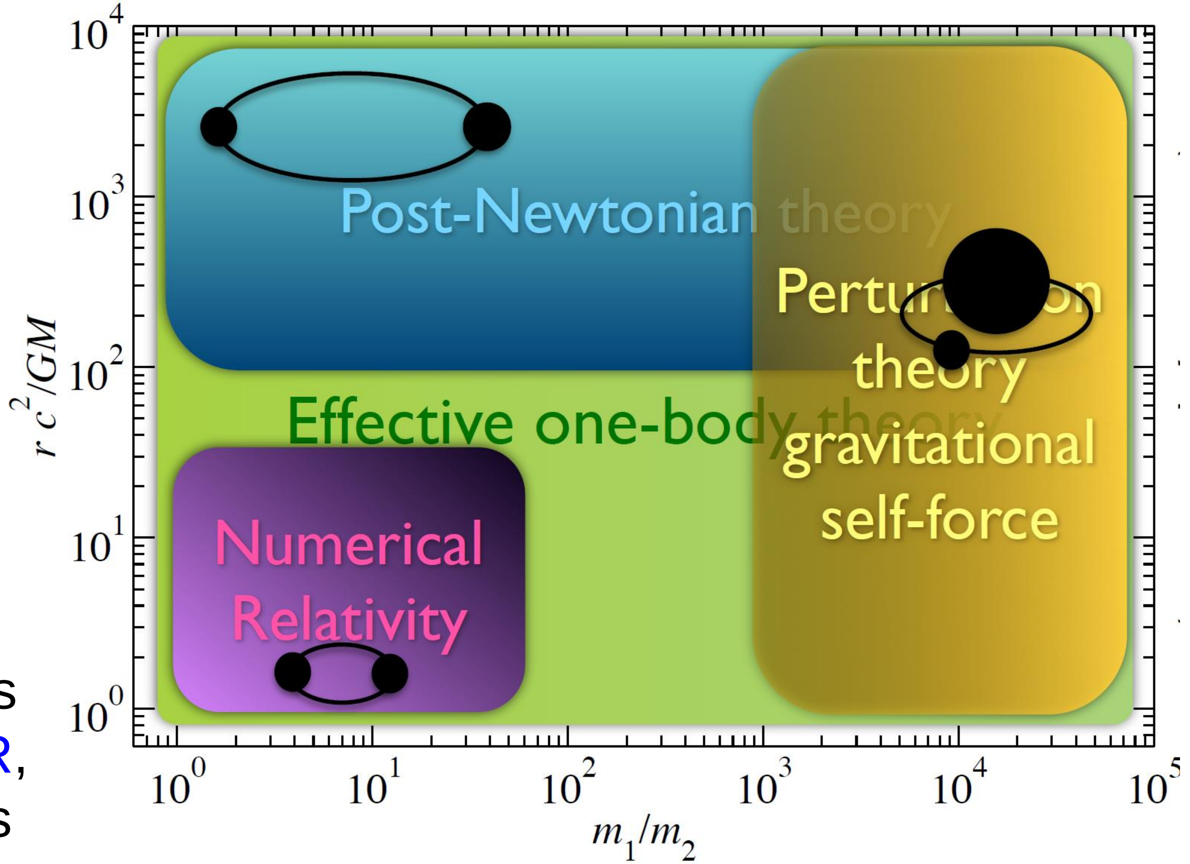
Effective one-body approach

Inspired by properties of bound states in QFT.

Interpolates information from various sources, including PN and PM expansions, test particle limit $m_1 \ll m_2$, and numerical relativity results.

Provides accurate gravitational wave templates very close in, faster than NR, allowing many combinations of initial masses and spins.

Buonanno, Damour, 9811091



Buonanno, Satyaprakash, 1410.7832

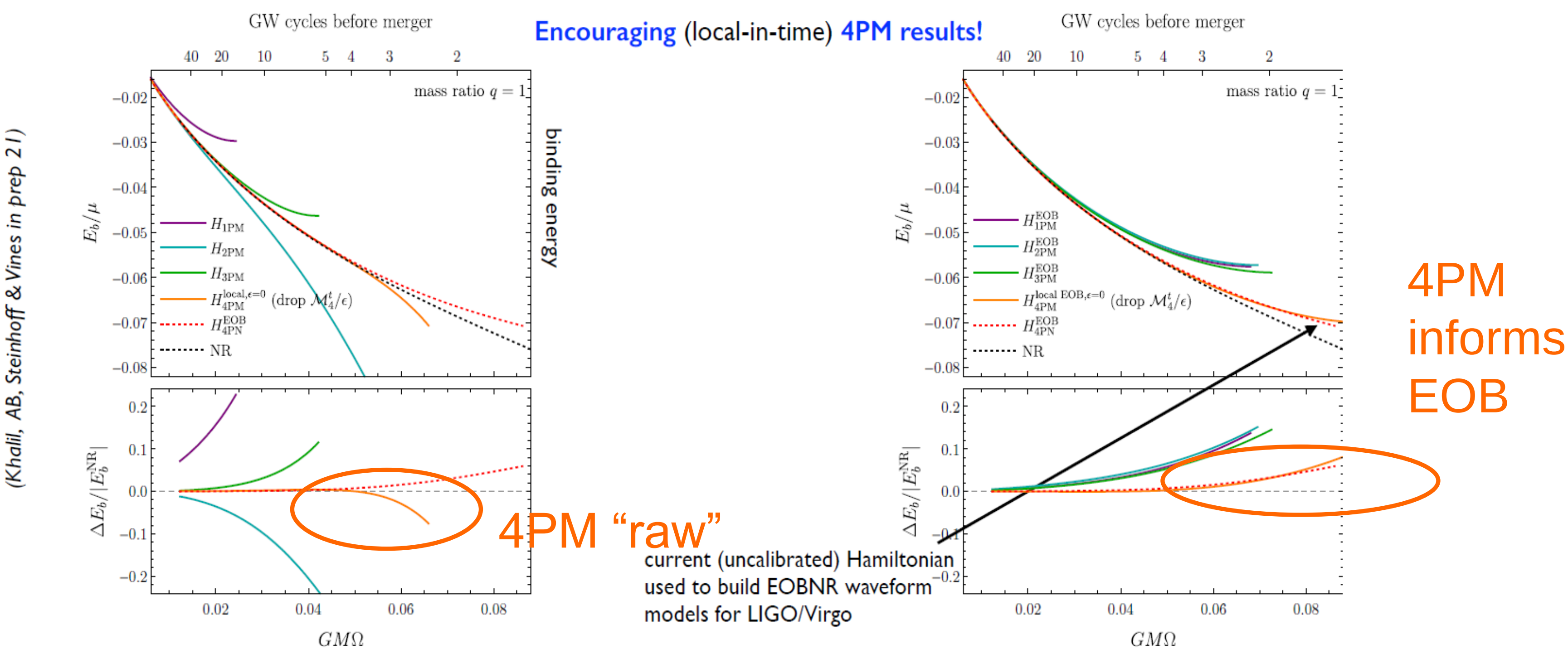
4PM (still missing non-local tail) now competes with previous 4PN EOB!

NEW! from Alessandra Buonanno's recent talk at GGI

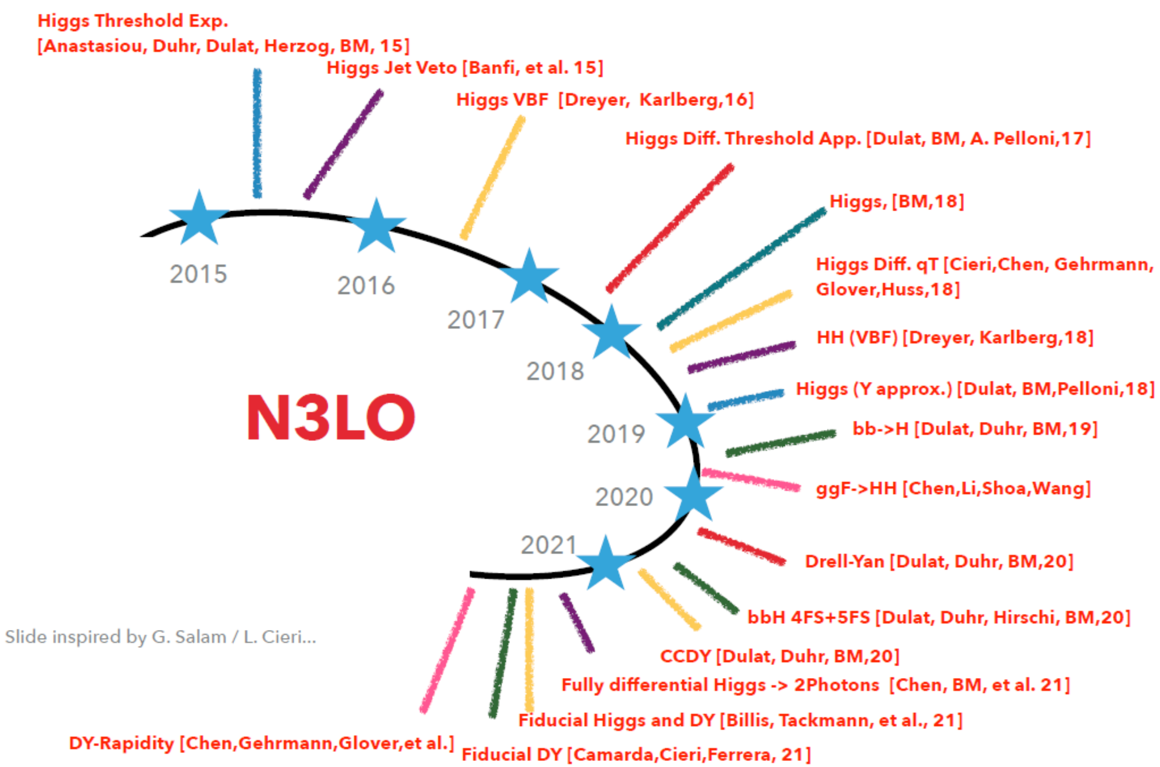
Crucial to **push PM** calculations at **higher order,** and **resum them** in EOB formalism.

(Damour 19, Antonelli, AB, Steinhoff, van de Meent & Vines 19, Khalil, AB, Steinhoff & Vines in prep 21)

Encouraging (local-in-time) **4PM results!**



N3LO revolution too!



- Work in progress to make N3LO more differential (i.e. implement actual experimental cuts)
- Enabling the next steps in the N3LO revolution

Many Automated Programs for One-Loop QCD

Blackhat: Berger, Bern, LD, Diana, Febres Cordero, Forde, Gleisberg, Höche, Ita, Kosower, Maître, Ozeren, 0803.4180, 0808.0941, 0907.1984, 1004.1659, 1009.2338…
+ **Sherpa** → NLO *W,Z* + 3,4,5 jets pure QCD 4 jets

CutTools: Ossola, Papadopolous, Pittau, 0711.3596
NLO *WWW, WWZ*, ... Binoth+OPP, 0804.0350
NLO $t\bar{t}b\bar{b}$, $t\bar{t}$ + 2 jets,…
Bevilacqua, Czakon, Papadopoulos, Pittau, Worek, 0907.4723; 1002.4009
MadLoop: Hirschi, Frederix, Frixione, Garzelli, Maltoni, Pittau 1103.0621
HELAC-NLO: Bevilacqua et al, 1110.1499

Rocket: Giele, Zanderighi, 0805.2152
Ellis, Giele, Kunszt, Melnikov, Zanderighi, 0810.2762
NLO *W* + 3 jets Ellis, Melnikov, Zanderighi, 0901.4101, 0906.1445
$W^+W^\pm$ + 2 jets Melia, Melnikov, Rontsch, Zanderighi, 1007.5313, 1104.2327

SAMURAI → GoSAM: Mastrolia, Ossola, Reiter, Tramontano, 1006.0710,…

NGluon: Badger, Biedermann, Uwer, 1011.2900,…

OpenLoops: Cascioli, Maierhofer, Pozzorini, 1111.5206,...