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# Updated Constraints on Hubble Tension solutions

With recent SPT-3G and SH0ES data

— Ali Rida Khalife —

[arXiv:2312.09814](https://arxiv.org/abs/2312.09814)

**JCAP 04(2024)059**

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**Collaborators:** Mariam Bahrami, Sven Günther and Julien Lesgourgues from **RWTH Aachen**

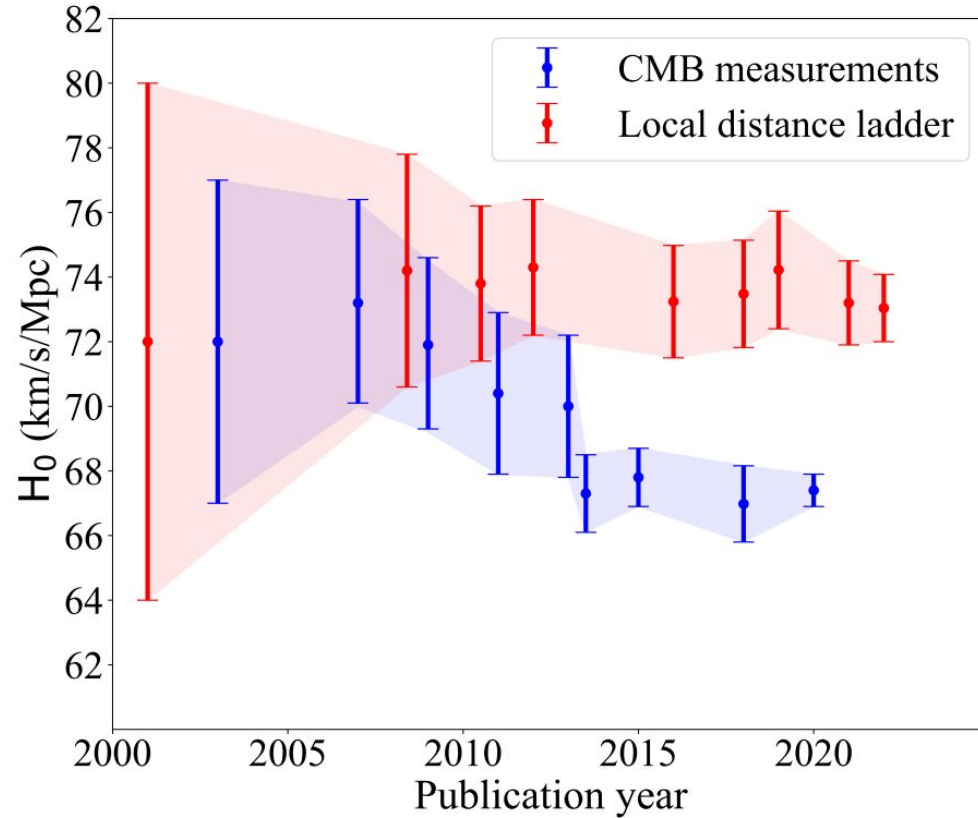
Silvia Galli and Karim Benabed from **IAP**

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Thanks to the great support from the IAP CMB team:

Federica Guidi, Aristide Doussot, Eric Hivon, Etienne Camphuis, Lennart Balkenhol and Aline Vitrier

# The Trouble with Hubble



Ref: Hubble Tension: The Evidence of New Physics([2302.05709](#))

# The Trouble with Hubble

## CMB with Planck

- Balkenhol et al. (2021), Planck 2018+SPT+ACT :  $67.49 \pm 0.5$
- Pogosian et al. (2020), eBOSS+Planck mH2:  $69.6 \pm 1.8$
- Aghanim et al. (2020), Planck 2018:  $67.27 \pm 0.60$
- Aghanim et al. (2020), Planck 2018+CMB lensing:  $67.36 \pm 0.54$
- Ade et al. (2016), Planck 2015,  $H_0 = 67.27 \pm 0.66$

## CMB without Planck

- Dutcher et al. (2021), SPT:  $68.8 \pm 1.5$
- Aiola et al. (2020), ACT:  $67.9 \pm 1.5$
- Aiola et al. (2020), WMAP9+ACT:  $67.6 \pm 1.1$
- Zhang, Huang (2019), WMAP9+BAO:  $68.36^{+0.53}_{-0.52}$
- Henning et al. (2018), SPT:  $71.3 \pm 2.1$
- Hinshaw et al. (2013), WMAP9:  $70.0 \pm 2.2$

## No CMB, with BBN

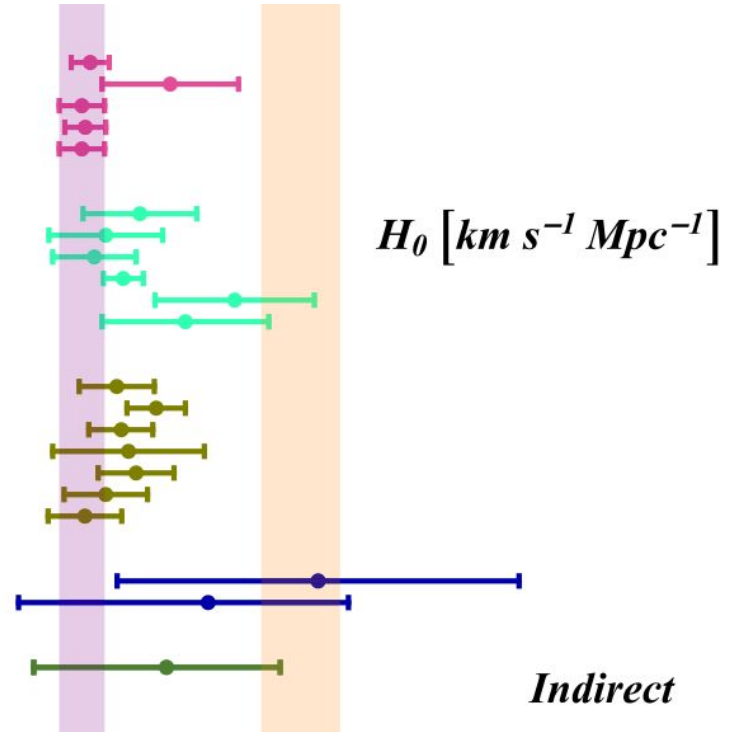
- Zhang et al. (2021), BOSS correlation function+BAO+BBN:  $68.19 \pm 0.99$
- Chen et al. (2021), P+BAO+BBN:  $69.23 \pm 0.77$
- Philcox et al. (2021), P+Bispectrum+BAO+BBN:  $68.31^{+0.83}_{-0.86}$
- D'Amico et al. (2020), BOSS DR12+BBN:  $68.5 \pm 2.2$
- Colas et al. (2020), BOSS DR12+BBN:  $68.7 \pm 1.5$
- Ivanov et al. (2020), BOSS+BBN:  $67.9 \pm 1.1$
- Alam et al. (2020), BOSS+eBOSS+BBN:  $67.35 \pm 0.97$

## CMB lensing

- Baxter et al. (2020):  $73.5 \pm 5.3$
- Philcox et al. (2020),  $P_T(k)$ +CMB lensing:  $70.6^{+3.7}_{-5.0}$

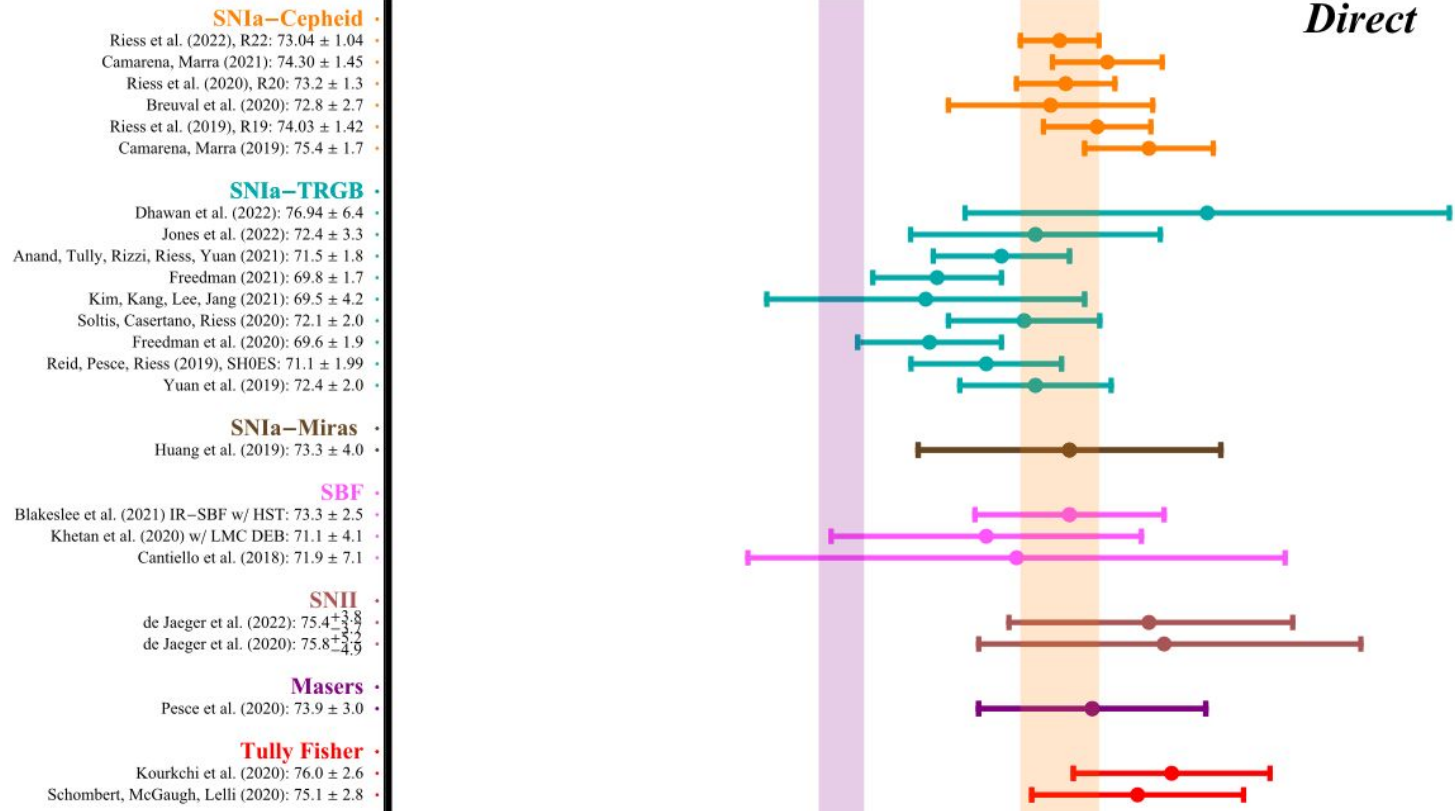
## LSS $\tau_{\text{eq}}$ standard ruler

- Farren et al. (2021):  $69.5^{+3.0}_{-3.5}$



Ref: [2203.06142](https://arxiv.org/abs/2203.06142)

# The Trouble with Hubble



Ref: [2203.06142](https://arxiv.org/abs/2203.06142)

# H<sub>0</sub> Olympics 2021

Model	$\Delta N_{\text{param}}$	$M_B$	Gaussian Tension	$Q_{\text{DMAP}}$ Tension		$\Delta\chi^2$	$\Delta\text{AIC}$		Finalist
$\Lambda\text{CDM}$	0	$-19.416 \pm 0.012$	$4.4\sigma$	$4.5\sigma$	X	0.00	0.00	X	X
$\Delta N_{\text{nr}}$	1	$-19.395 \pm 0.019$	$3.6\sigma$	$3.8\sigma$	X	-6.10	-4.10	X	X
SIDR	1	$-19.385 \pm 0.024$	$3.2\sigma$	$3.3\sigma$	X	-9.57	-7.57	✓	✓ 🟡
mixed DR	2	$-19.413 \pm 0.036$	$3.3\sigma$	$3.4\sigma$	X	-8.83	-4.83	X	X
DR-DM	2	$-19.388 \pm 0.026$	$3.2\sigma$	$3.1\sigma$	X	-8.92	-4.92	X	X
$\text{SI}\nu\text{+DR}$	3	$-19.440^{+0.037}_{-0.039}$	$3.8\sigma$	$3.9\sigma$	X	-4.98	1.02	X	X
Majoron	3	$-19.380^{+0.027}_{-0.021}$	$3.0\sigma$	$2.9\sigma$	✓	-15.49	-9.49	✓	✓ 🟡
primordial B	1	$-19.390^{+0.018}_{-0.024}$	$3.5\sigma$	$3.5\sigma$	X	-11.42	-9.42	✓	✓ 🟡
varying $m_e$	1	$-19.391 \pm 0.034$	$2.9\sigma$	$2.9\sigma$	✓	-12.27	-10.27	✓	✓ 🟡
varying $m_e + \Omega_k$	2	$-19.368 \pm 0.048$	$2.0\sigma$	$1.9\sigma$	✓	-17.26	-13.26	✓	✓ 🟡
EDE	3	$-19.390^{+0.016}_{-0.035}$	$3.6\sigma$	$1.6\sigma$	✓	-21.98	-15.98	✓	✓ 🟡
NEDE	3	$-19.380^{+0.023}_{-0.040}$	$3.1\sigma$	$1.9\sigma$	✓	-18.93	-12.93	✓	✓ 🟡
EMG	3	$-19.397^{+0.017}_{-0.023}$	$3.7\sigma$	$2.3\sigma$	✓	-18.56	-12.56	✓	✓ 🟡
CPL	2	$-19.400 \pm 0.020$	$3.7\sigma$	$4.1\sigma$	X	-4.94	-0.94	X	X
PEDE	0	$-19.349 \pm 0.013$	$2.7\sigma$	$2.8\sigma$	✓	2.24	2.24	X	X
GPEDE	1	$-19.400 \pm 0.022$	$3.6\sigma$	$4.6\sigma$	X	-0.45	1.55	X	X
DM $\rightarrow$ DR+WDM	2	$-19.420 \pm 0.012$	$4.5\sigma$	$4.5\sigma$	X	-0.19	3.81	X	X
DM $\rightarrow$ DR	2	$-19.410 \pm 0.011$	$4.3\sigma$	$4.5\sigma$	X	-0.53	3.47	X	X

Ref: [2107.10291](https://arxiv.org/abs/2107.10291)

# Goal of the Project

- Evaluate the potential of Cosmological models to solve the Hubble Tension.
- Include primary CMB data from [SPT-3G 2018](#), in combination with other data sets.
- Compare to recent [SH0ES analysis](#):

$$H_0 = 73.29 \pm 0.90 \text{ km/s/Mpc (Murakami et al., 2023; 2306.00070).$$

- Study 5 classical  $\Lambda$ CDM extensions + 3 Elaborate Models (+extensions).
- Assess these models with new Tension metrics.
- Update  $H_0$  Olympics paper (Schöneberg et al., 2021; [2107.1029](#)).

# How to Solve the Tension

- Solutions to the Hubble Tension include changing the Physics pre-recombination or in the late universe
- Note:  $100 \times \theta = 1.04075 \pm 0.00028$  (Balkenhol *et al.*, 2022; [2212.05642](https://arxiv.org/abs/2212.05642))

$$\theta_s = \frac{r_s}{D_A} = \frac{\int_{z_*}^{\infty} \left[ 3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right) \right]^{-1/2} \left[ \frac{8\pi G}{3} \Sigma_i \rho_i \right]^{-1/2} dz}{H_0^{-1} \text{sin}_K \left[ \int_0^{z_*} \left( \Sigma_i \Omega_i(z) \right)^{-1/2} dz \right]}$$

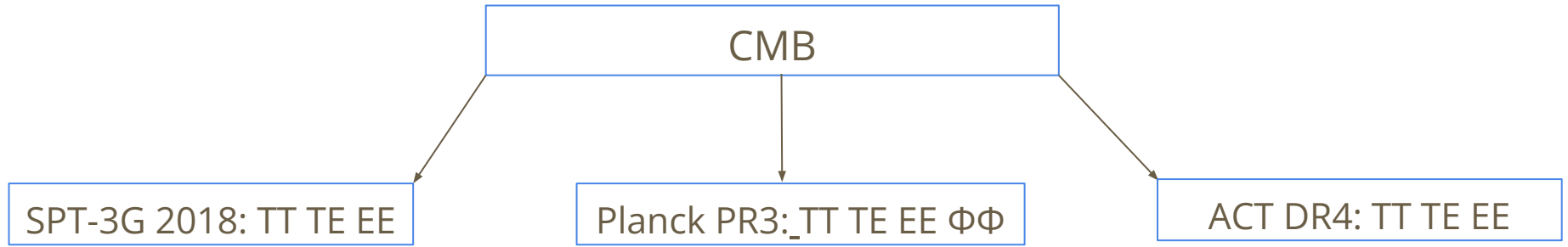
Sound Speed  $\rightarrow$  (points to the upper integral)

$H(z)$   $\rightarrow$  (points to the upper integral)

Flat, closed or open  $\rightarrow$  (points to the denominator)

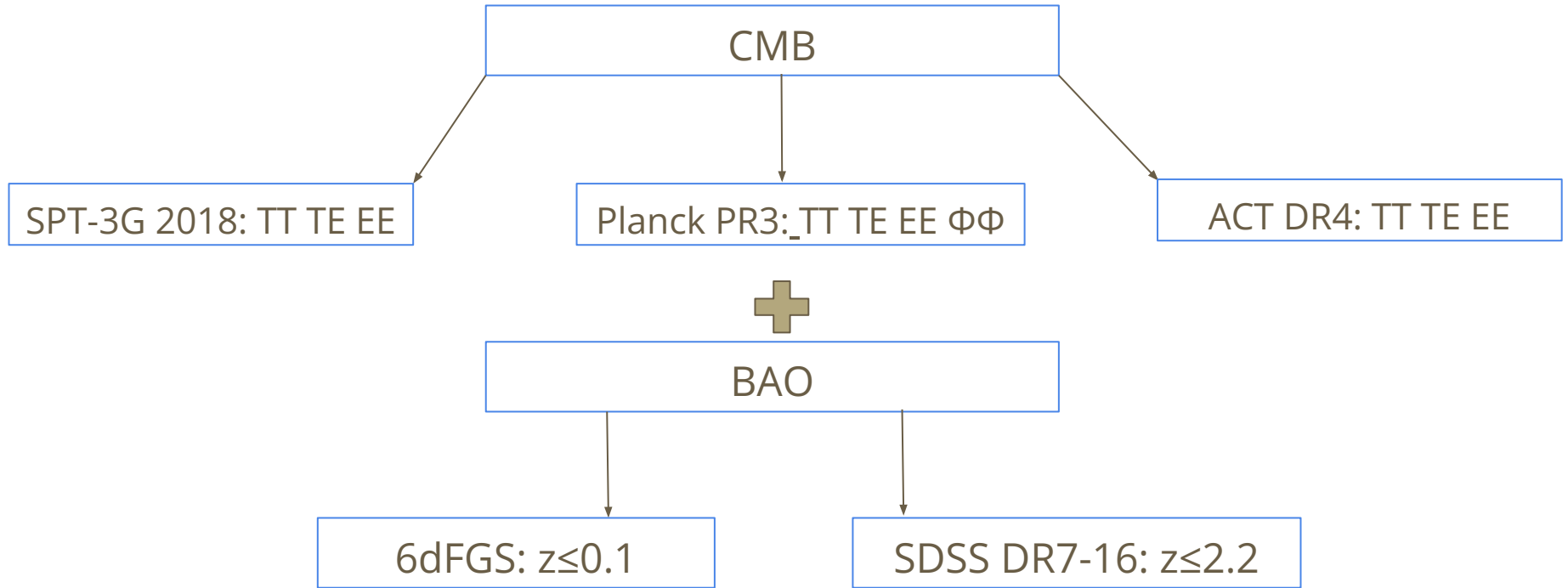
$H(z)/H_0$   $\rightarrow$  (points to the denominator)

# Data Sets

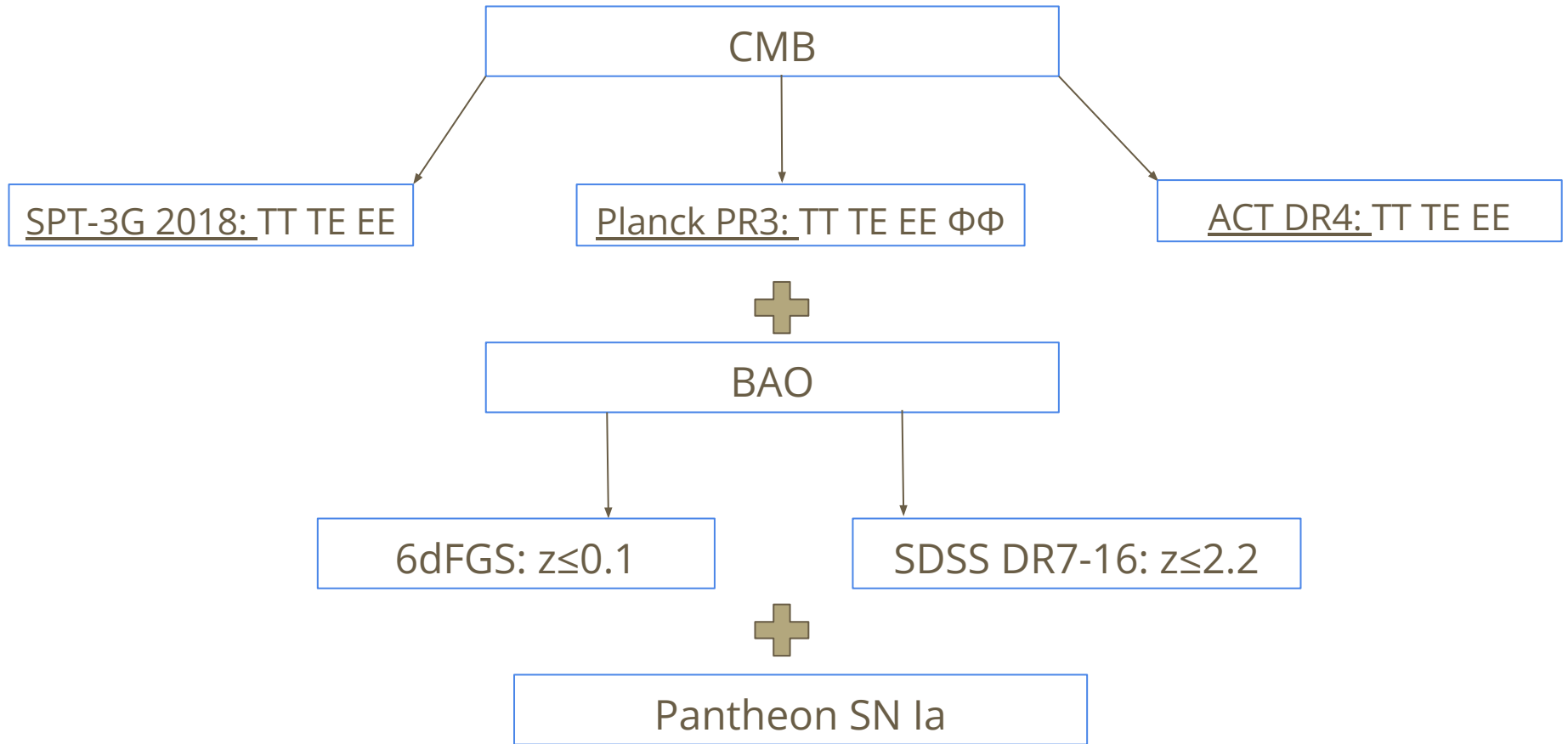




# Data Sets



# Data Sets



# $\Lambda$ CDM Extensions

Extending  $\Lambda$ CDM with 3 degenerate **massive neutrinos** ( $\Sigma m_\nu$ ) and:

Small scale CMB+BAO

- Chevallier-Polarski-Linder (CPL) Dark Energy ( $\omega(a) = \omega_0 + \omega_a(1-a)$ );  $a \equiv$  scale factor
- Spatial Curvature ( $\Omega_K$ )
- Free streaming Dark Radiation ( $N_{\text{eff}}$ )
- Self Interacting Dark Radiation ( $N_{\text{SIDR}}$ )

$$\theta_s = \frac{r_s}{D_A} = \frac{\int_{z_*}^{\infty} \left[ 3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right) \right]^{-1/2} \left[ \frac{8\pi G}{3} \Sigma_i \rho_i \right]^{-1/2} dz}{H_0^{-1} \sin_K \left[ \int_0^{z_*} \left( \Sigma_i \Omega_i(z) \right)^{-1/2} dz \right]}$$

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Large scale CMB + SN Ia

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Small scale CMB

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Model	$\Delta N_{\text{param}}$	$M_B$	Gaussian tension	$Q_{\text{DMAP}}$ tension		$\Delta\chi^2$	$\Delta\text{AIC}$		Finalist
SIDR	1	$-19.385 \pm 0.024$	$3.2\sigma$	$3.3\sigma$	X	-9.57	-7.57	✓	✓ 🏆

- Self Interacting Dark Radiation ( $N_{\text{SIDR}}$ )

(Schöneberg *et al.*, 2021; [2107.1029](#))

# Elaborate Models

- Varying electron mass: (Hart & Chulba, 2017; [1705.03925](#))
  - Motivated by higher dimensional theories, e.g. string theory
  - Changes the time (redshift) of hydrogen recombination.
  - Previously found to be an excellent reducer of the tension.
  - Must include BAO with large-scale CMB data.

$$\theta_s = \frac{r_s}{D_A} = \frac{\int_{z_*}^{\infty} \left[ 3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right) \right]^{-1/2} \left[ \frac{8\pi G}{3} \Sigma_i \rho_i \right]^{-1/2} dz}{H_0^{-1} \sin_K \left[ \int_0^{z_*} \left( \Sigma_i \Omega_i(z) \right)^{-1/2} dz \right]}$$



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varying $m_e$	1	$-19.391 \pm 0.034$	$2.9\sigma$	$2.9\sigma$	✓	-12.27	-10.27	✓	✓ 🏆

(Schöneberg *et al.*, 2021; [2107.1029](#))

# Elaborate Models

- Varying electron mass: (Hart & Chulba, 2017; [1705.03925](#))
  - $+\Sigma m_\nu$ : First to constrain this combination.

# Elaborate Models

- Varying electron mass: (Hart & Chulba, 2017; [1705.03925](#))

- $+\Sigma m_\nu$

- $+\Omega_K$

- Changing  $z_*$  changes  $D_A$ . Need to compensate with late universe parameter.
- Intermediate scale polarization data from SPT-3G was crucial.

$$\theta_s = \frac{r_s}{D_A} = \frac{\int_{z_*}^{\infty} \left[ 3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right) \right]^{-1/2} \left[ \frac{8\pi G}{3} \Sigma_i \rho_i \right]^{-1/2} dz}{H_0^{-1} \sin_K \left[ \int_0^{z_*} \left( \Sigma_i \Omega_i(z) \right)^{-1/2} dz \right]}$$

# Elaborate Models

- Varying electron mass: (Hart & Chulba, 2017; [1705.03925](#))
  - $+\Sigma m_\nu$
  - $+\Omega_K$ 
    - Intermediate scale polarization data from SPT-3G was crucial.
    - Even More promising than its ancestor.

Model	$\Delta N_{\text{param}}$	$M_B$	Gaussian tension	$Q_{\text{DMAP}}$ tension		$\Delta\chi^2$	$\Delta\text{AIC}$		Finalist
varying $m_e + \Omega_k$	2	$-19.368 \pm 0.048$	$2.0\sigma$	$1.9\sigma$	✓	-17.26	-13.26	✓	✓ 🏆

(Schöneberg *et al.*, 2021; [2107.1029](#))

# Elaborate Models

- Varying electron mass: (Hart & Chulba, 2017; [1705.03925](#))
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- Early Dark Energy: (Poulin *et al.*, 2023; [2302.09032](#))

- Motivated by higher dimensional theories.
- Scalar field reduces sound horizon around Matter-radiation equality.

$$\theta_s = \frac{r_s}{D_A} = \frac{\int_{z_*}^{\infty} \left[ 3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right) \right]^{-1/2} \left[ \frac{8\pi G}{3} \Sigma_i \rho_i \right]^{-1/2} dz}{H_0^{-1} \sin_K \left[ \int_0^{z_*} \left( \Sigma_i \Omega_i(z) \right)^{-1/2} dz \right]}$$


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EDE	3	$-19.390^{+0.016}_{-0.035}$	$3.6\sigma$	$1.6\sigma$	✓	-21.98	-15.98	✓	✓ 

(Schöneberg *et al.*, 2021; [2107.1029](#))

# Elaborate Models

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  - $+\Sigma m_\nu + \Omega_K$
- Early Dark Energy: (Poulin *et al.*, 2023; [2302.09032](#))
- The Majoron: (Escudero & Witte, 2021; [2103.03249](#))
  - Breaking symmetry in the early Universe produces interacting Dark Radiation.


$$\theta_s = \frac{r_s}{D_A} = \frac{\int_{z_*}^{\infty} \left[ 3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right) \right]^{-1/2} \left[ \frac{8\pi G}{3} \Sigma_i \rho_i \right]^{-1/2} dz}{H_0^{-1} \sin K \left[ \int_0^{z_*} \left( \Sigma_i \Omega_i(z) \right)^{-1/2} dz \right]}$$



# Elaborate Models

- Varying electron mass: (Hart & Chulba, 2017; [1705.03925](#))
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Model	$\Delta N_{\text{param}}$	$M_B$	Gaussian tension	$Q_{\text{DMAP}}$ tension		$\Delta\chi^2$	$\Delta\text{AIC}$		Finalist
Majoron*	3	$-19.380^{+0.027}_{-0.021}$	$3.0\sigma$	$2.9\sigma$	✓	-10.99	-4.99	X	✓ ②

(Schöneberg *et al.*, 2021; [2107.1029](#))

# Tension Metrics

- *Marginalised Posterior Compatibility Level* ( $Q_{\text{MPCL}}$ ):

- Generalises Gaussian Tension metric to non-Gaussian posteriors of  $H_0$ .
- Bayesian.

$$n = \frac{\bar{x}_2 - \bar{x}_1}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$$

- *Difference of the Maximum A Posteriori* ( $Q_{\text{DMAP}}$ ):

- Comparison of best-fit  $\chi^2$  for a model and data set, w/ and w/o SH0ES.
- Frequentist.

$$Q_{\text{DMAP, model}} \equiv \sqrt{\chi_{\text{min, model, } \mathcal{D}+\text{SH0ES}}^2 - \chi_{\text{min, model, } \mathcal{D}}^2}$$

- *Akaike Information Criterion* ( $\Delta\text{AIC}$ ):

- Comparison of best-fit  $\chi^2$  for a model, given a data set that includes SH0ES, with that of  $\Lambda\text{CDM}$
- Penalty for models with additional parameters.

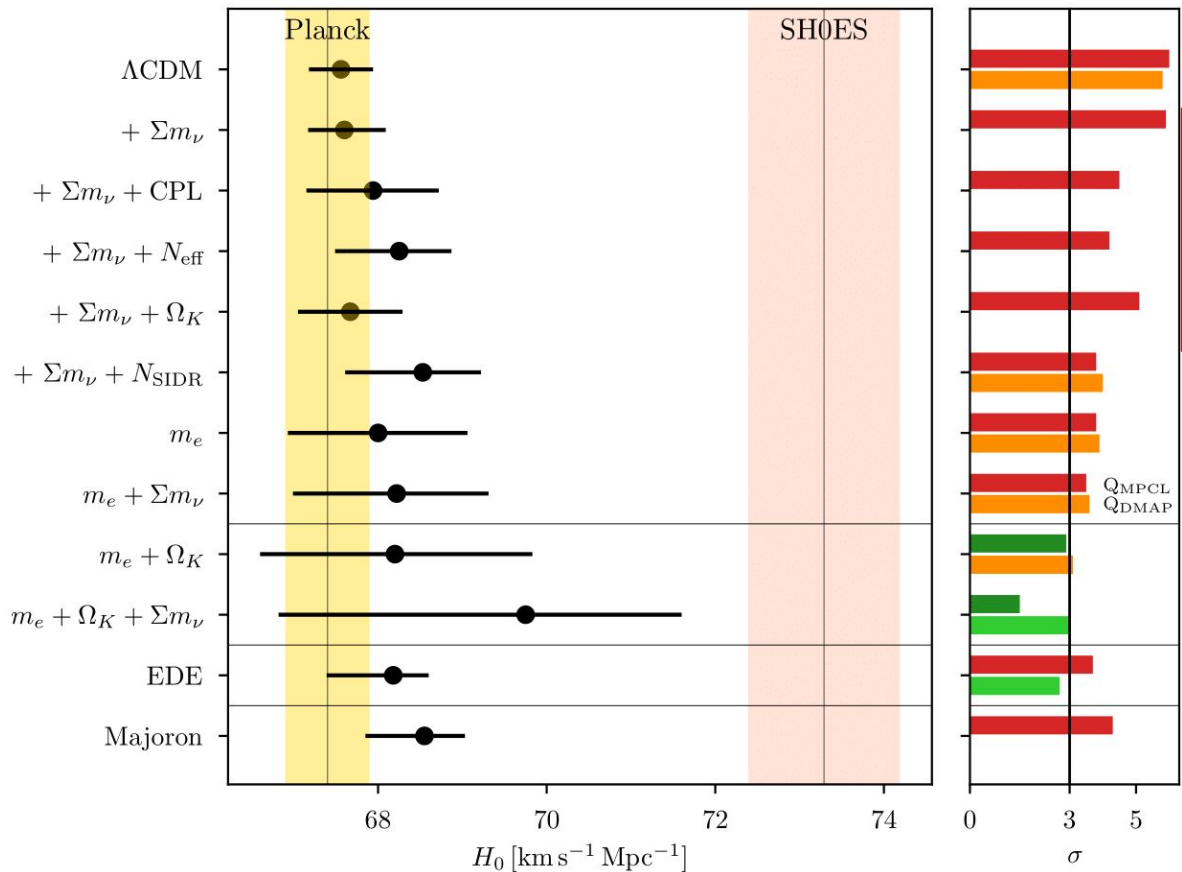
$$\Delta\text{AIC}_{\text{model}} = \chi_{\text{min, model, } \mathcal{D}+\text{SH0ES}}^2 - \chi_{\text{min, } \Lambda\text{CDM, } \mathcal{D}+\text{SH0ES}}^2 + 2(N_{\text{model}} - N_{\Lambda\text{CDM}}) .$$

- *$\Delta\text{AIC}$  without SH0ES*

# Numerical Tools

- Theory Codes: [CLASS](#), [AxiCLASS](#) and [CAMB](#)
- Monte Carlo Sampler: [COBAYA](#)
- Minimizing  $\chi^2$ : [Py-BOBYQA](#)
- New cosmological emulator (Günther, 2023; [2307.01138](#))
- Our reference data set: SPT+Planck+BAO+Pantheon (SPBP)

# Main Results



None of the models completely solve the tension.  
 Only  $m_e + \Omega_K$ ,  $m_e + \Omega_K + \Sigma m_\nu$  and EDE reduce it below  $3\sigma$ .

Data: SPT+Planck+BAO+Pantheon

# Main Results

Models	w/o SH0ES		w/ SH0ES	
	$\Delta\chi^2$	$\Delta\text{AIC}$	$\Delta\chi^2$	$\Delta\text{AIC}$
$\Lambda\text{CDM}$	0	0	0	0
$+\Sigma m_\nu$	—	—	—	—
$+\Sigma m_\nu + \text{CPL}$	—	—	—	—
$+\Sigma m_\nu + N_{\text{eff}}$	—	—	—	—
$+\Sigma m_\nu + \Omega_K$	—	—	—	—
$+\Sigma m_\nu + N_{\text{SIDR}}$	-0.1	3.9	-17.1	-13.1
$m_e$	0.0	2.0	-18.0	-16.0
$m_e + \Sigma m_\nu$	-0.9	3.1	-21.6	-17.6
$m_e + \Omega_K$	-1.0	3.0	-24.7	-20.7
$m_e + \Omega_K + \Sigma m_i$	-0.9	5.1	-25.8	-19.8
EDE	-4.6	1.4	-31.1	-25.1
Majoron	—	—	—	—

Without SH0ES, the models are not performing appreciably better than  $\Lambda\text{CDM}$ .

# Summary

- Update previous constraints on Hubble Tension solutions with:  
SPT-3G 2018, SH0ES and SDSS DR16.
- Introduced new tension metrics that improve the assessment.
- We used a Boltzmann code emulator, making the computations faster.
- SIDR, varying  $m_e$  and the Majoron models are no longer possible solutions to the Hubble Tension.
- None of the studied models actually solve the tension.

# Future Plans

- Further investigation of the still viable models is needed.
- Revisit these models, along with others, with upcoming SPT-3G 2019/2020 and ACT DR6 data.
- Incorporate improved numerical techniques.
- Perform forecasts for SO, CMB-S4 and future SPT surveys

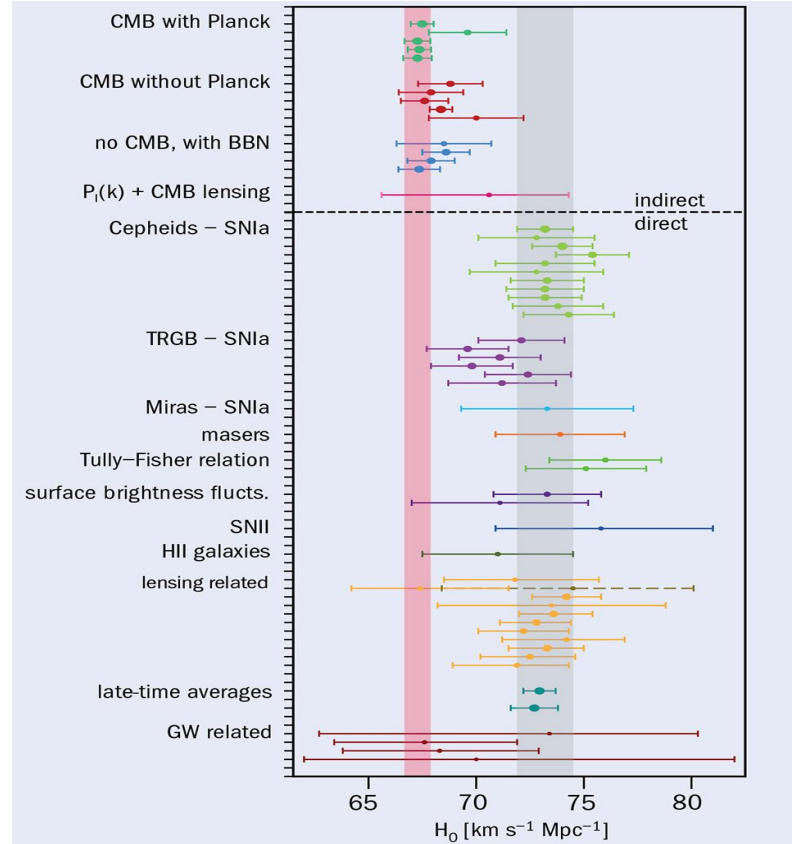
**Thank you!**

**Questions? Comments?**



# Back Up

# The Trouble with Hubble



Ref: In the Realm of the Hubble Tension ([2103.01183](#))

# Elaborate Models

- Varying electron mass:

Compactification in higher dimensional theories results in scalar fields that alter the effective mass of elementary particles, specifically electrons.

Recombination rate is affected  Recombination time changes

Additional parameter:  $m_{e,early}/m_{e,late}$

More details: Hart & Chulba, 2018([1705.03925](#)); *Planck* 2015([1406.7482](#))

$$\theta_s = \frac{r_s}{D_A} = \frac{\int_{z_*}^{\infty} \left[ 3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right) \right]^{-1/2} \left[ \frac{8\pi G}{3} \Sigma_i \rho_i \right]^{-1/2} dz}{H_0^{-1} \sin_K \left[ \int_0^{z_*} \left( \Sigma_i \Omega_i(z) \right)^{-1/2} dz \right]}$$

# Elaborate Models

- Varying electron mass ( $m_{e,early}/m_{e,late}$ )
  - $+\Sigma m_\nu$ : Study interplay between masses of the two species

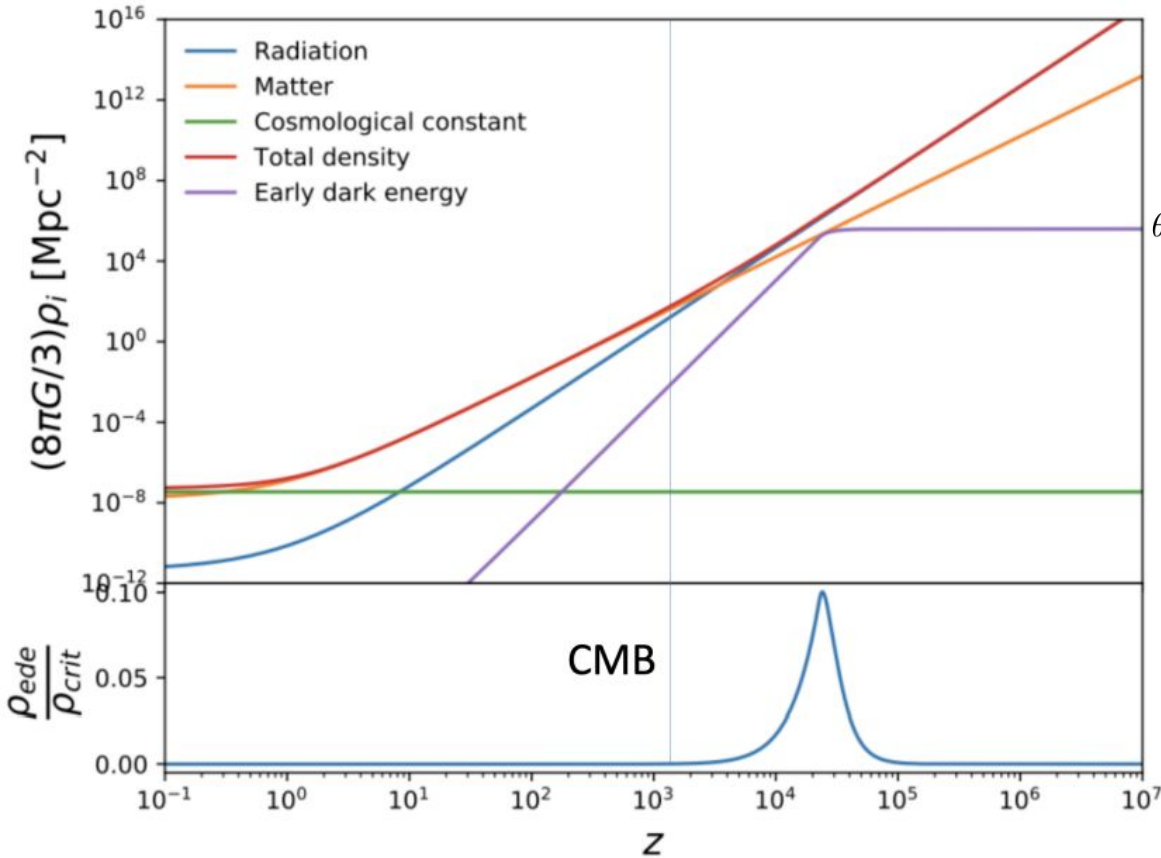
# Elaborate Models

- Varying electron mass ( $m_{e,early}/m_{e,late}$ )
  - $+\Sigma m_\nu$
  - $+\Omega_K$ : Changing the time of recombination changes the distance

$$\theta_s = \frac{r_s}{D_A} = \frac{\int_{z_*}^{\infty} \left[ 3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right) \right]^{-1/2} \left[ \frac{8\pi G}{3} \Sigma_i \rho_i \right]^{-1/2} dz}{H_0^{-1} \sin_K \left[ \int_0^{z_*} \left( \Sigma_i \Omega_i(z) \right)^{-1/2} dz \right]}$$

More details: Sekigushi & Takahashi (2020) ([2007.03381](#))

# Early Dark Energy



$$\theta_s = \frac{r_s}{D_A} = \frac{\int_{z_*}^{\infty} \left[ 3 \left( 1 + \frac{3\rho_b}{4\rho_\gamma} \right) \right]^{-1/2} \left[ \frac{8\pi G}{3} \sum_i \rho_i \right]^{-1/2} dz}{H_0^{-1} \sin_K \left[ \int_0^{z_*} \left( \sum_i \Omega_i(z) \right)^{-1/2} dz \right]}$$

# Early Dark Energy

- Also motivated by higher dimensional theories.
- A scalar field contributes briefly to the expansion rate around matter-radiation equality.
- Decrease in sound horizon, compensated by increase in  $H_0$ .
- References: Poulin *et al.*, 2018 ([1811.04083](#)), Smith & Poulin, 2023 ([2309.03265](#))

# Elaborate Models

- Varying electron mass ( $m_{e,early}/m_{e,late}$ )
  - $+\Sigma m_\nu$
  - $+\Omega_K$
  - $+\Sigma m_\nu + \Omega_K$
- Early Dark Energy:
  - $\Theta_i$ : Initial value of the scalar field
  - $Z_c$ : Critical redshift, i.e. the field becomes dynamical
  - $f_{EDE} = \rho_{EDE}/\rho_{tot}$



# Elaborate Models

- Varying electron mass ( $m_{e,early}/m_{e,late}$ )
  - $+\Sigma m_\nu$
  - $+\Omega_K$
  - $+\Sigma m_\nu + \Omega_K$
- Early Dark Energy ( $\theta_i, z_c, f_{EDE}$ )
- The Majoron:

Breaking lepton number symmetry produces a pseudo-scalar ( $\varphi$ ) that gives neutrinos their mass (like the Higgs). A particle Physics motivated SIDR.

Free parameters:  $m_\varphi, \Gamma_{\text{eff}}$  and  $N_{\text{DR}}$

More details: [Escudero & Witte, 2020](#) (1909.04044); [Escudero & Witte, 2021](#) (2103.03249)

# Tension Metrics

- *Marginalised Posterior Compatibility Level (MPCL):*  
What's the probability of getting 0 in the distribution of the difference between SH0ES and a model's  $H_0$  posteriors?

$$\mathcal{P}(\delta) = \mathcal{N} \int dH_0 \mathcal{P}_{\text{model}}(H_0) \mathcal{P}_{\text{SH0ES}}(H_0 - \delta) \simeq \mathcal{N}' \sum_i w_i \mathcal{P}_{\text{SH0ES}}(H_{0,i} - \delta)$$

Normalisation

Normalisation

Weights from chains

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$$q = \int_0^{\delta'} d\delta \mathcal{P}(\delta) .$$

Probability of finding  $\delta$  in  $[0, \delta']$ , such that  $\mathcal{P}(\delta') = \mathcal{P}(0)$

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Probability of finding  $\delta$  in  $[0, \delta']$ , such that  $\mathcal{P}(\delta') = \mathcal{P}(0)$

$$n = \sqrt{2} \operatorname{erf}^{-1}(q)$$

Tension in units of  $\sigma$ , denoted by:  $Q_{\text{MPCL}}$

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Assuming Gaussian posteriors



$$n = \frac{\bar{x}_2 - \bar{x}_1}{\sqrt{\sigma_{x_1}^2 + \sigma_{x_2}^2}}$$

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# Tension Metrics

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$$n = \sqrt{2} \operatorname{erf}^{-1}(q) \quad \text{Tension in units of } \sigma, \text{ denoted by } Q_{\text{MPCL}}$$

- *Difference of the Maximum A Posteriori (DMAP):*

$$Q_{\text{DMAP, model}} \equiv \sqrt{\chi_{\min, \text{model}, \mathcal{D}+\text{SH0ES}}^2 - \chi_{\min, \text{model}, \mathcal{D}}^2} ; \chi^2 = -2 \ln \mathcal{L} ; \mathcal{D} \equiv \text{data set}$$

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- *Akaike Information Criterion (AIC):*

$$\Delta \text{AIC}_{\text{model}} = \chi_{\min, \text{model}, \mathcal{D}+SH0ES}^2 - \chi_{\min, \Lambda\text{CDM}, \mathcal{D}+SH0ES}^2 \quad ; \quad N \equiv \# \text{ of parameters} \\ + 2(N_{\text{model}} - N_{\Lambda\text{CDM}}) .$$



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- *AIC without SH0ES*

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# Results

— Further Results —

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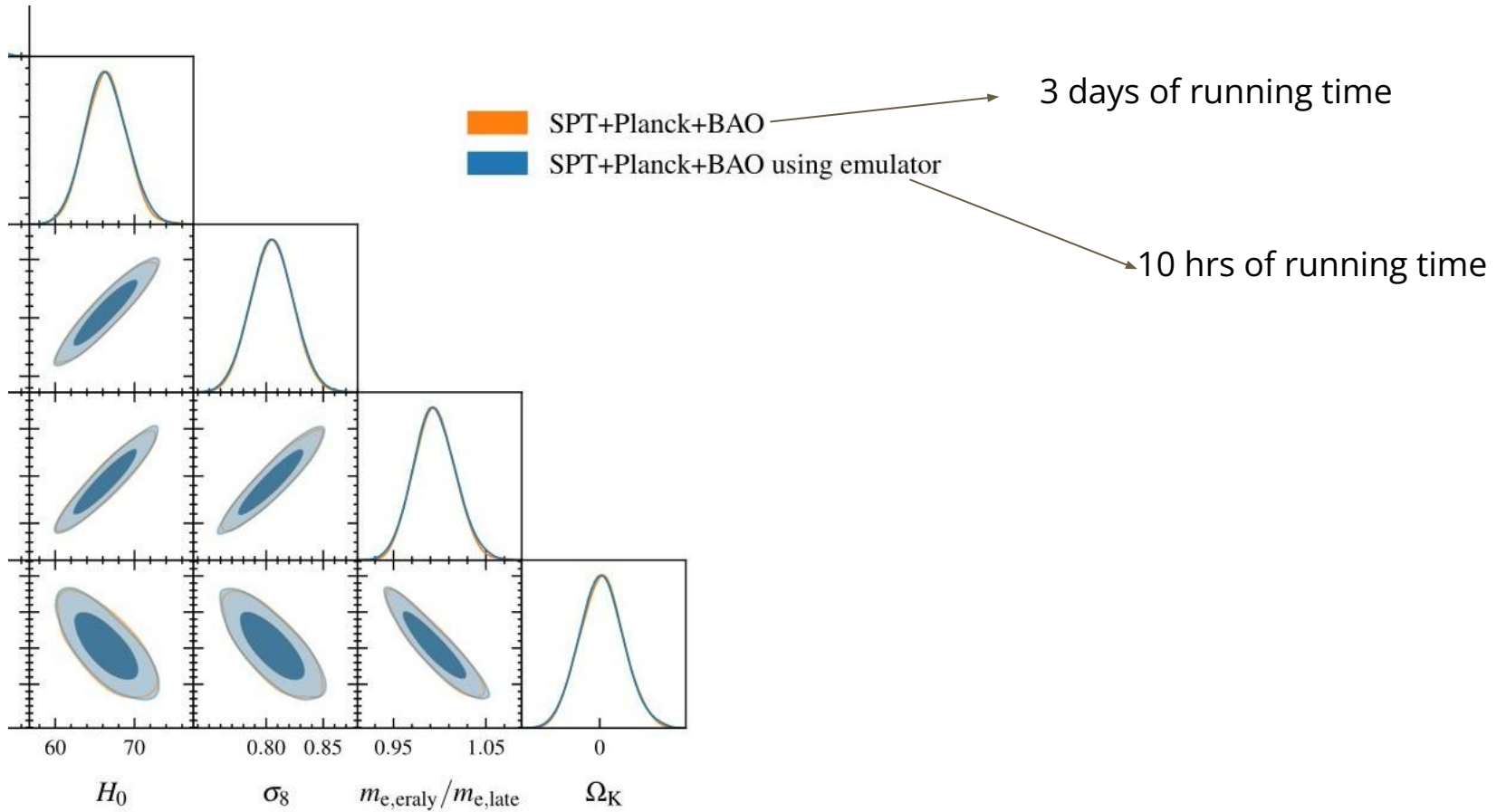
# Main Results

Models	$H_0(\text{km/s/Mpc})$	$Q_{\text{MPCL}}(\sigma)$	$Q_{\text{DMAP}}(\sigma)$	w/o SH0ES		w/ SH0ES	
				$\Delta\chi^2$	$\Delta\text{AIC}$	$\Delta\chi^2$	$\Delta\text{AIC}$
$\Lambda\text{CDM}$	$67.56(67.58)_{-0.38}^{+0.38}$	6.0	5.8	0	0	0	0
$+\Sigma m_\nu$	$67.60(67.01)_{-0.43}^{+0.49}$	5.9	—	—	—	—	—
$+\Sigma m_\nu + \text{CPL}$	$67.94(67.89)_{-0.79}^{+0.78}$	4.5	—	—	—	—	—
$+\Sigma m_\nu + N_{\text{eff}}$	$68.25(67.45)_{-0.76}^{+0.62}$	4.2	—	—	—	—	—
$+\Sigma m_\nu + \Omega_K$	$67.67(66.88)_{-0.62}^{+0.62}$	5.1	—	—	—	—	—
$+\Sigma m_\nu + N_{\text{SIDR}}$	$68.53(69.06)_{-0.92}^{+0.69}$	3.8	4.0	-0.1	3.9	-17.1	-13.1
$m_e$	$68.00(68.03)_{-1.07}^{+1.06}$	3.8	3.9	0.0	2.0	-18.0	-16.0
$m_e + \Sigma m_\nu$	$68.22(67.70)_{-1.23}^{+1.09}$	3.5	3.6	-0.9	3.1	-21.6	-17.6
$m_e + \Omega_K$	$68.20(67.42)_{-1.60}^{+1.63}$	<b>2.9</b>	3.1	-1.0	3.0	-24.7	-20.7
$m_e + \Omega_K + \Sigma m_\nu$	$69.75(67.75)_{-2.93}^{+1.85}$	<b>1.5</b>	<b>3.0</b>	-0.9	5.1	-25.8	-19.8
EDE	$68.18(68.55)_{-0.79}^{+0.42}$	3.8	<b>2.7</b>	-4.6	1.4	-31.1	-25.1
Majoron	$68.55(68.08)_{-0.70}^{+0.48}$	4.3	—	—	—	—	—

# Compare with Olympics Paper

Model	$\Delta N_{\text{param}}$	$M_B$	Gaussian Tension	$Q_{\text{DMAP}}$ Tension		$\Delta\chi^2$	$\Delta\text{AIC}$		Finalist
$\Lambda\text{CDM}$	0	$-19.416 \pm 0.012$	$4.4\sigma$	$4.5\sigma$	X	0.00	0.00	X	X
$\Delta N_{\text{nr}}$	1	$-19.395 \pm 0.019$	$3.6\sigma$	$3.8\sigma$	X	-6.10	-4.10	X	X
SIDR	1	$-19.385 \pm 0.024$	$3.2\sigma$	$3.3\sigma$	X	-9.57	-7.57	✓	✓ 🟡
mixed DR	2	$-19.413 \pm 0.036$	$3.3\sigma$	$3.4\sigma$	X	-8.83	-4.83	X	X
DR-DM	2	$-19.388 \pm 0.026$	$3.2\sigma$	$3.1\sigma$	X	-8.92	-4.92	X	X
SI $\nu$ +DR	3	$-19.440^{+0.037}_{-0.039}$	$3.8\sigma$	$3.9\sigma$	X	-4.98	1.02	X	X
Majoron	3	$-19.380^{+0.027}_{-0.021}$	$3.0\sigma$	$2.9\sigma$	✓	-15.49	-9.49	✓	✓ 🟡
primordial B	1	$-19.390^{+0.018}_{-0.024}$	$3.5\sigma$	$3.5\sigma$	X	-11.42	-9.42	✓	✓ 🟡
varying $m_e$	1	$-19.391 \pm 0.034$	$2.9\sigma$	$2.9\sigma$	✓	-12.27	-10.27	✓	✓ 🟡
varying $m_e + \Omega_k$	2	$-19.368 \pm 0.048$	$2.0\sigma$	$1.9\sigma$	✓	-17.26	-13.26	✓	✓ 🟡
EDE	3	$-19.390^{+0.016}_{-0.035}$	$3.6\sigma$	$1.6\sigma$	✓	-21.98	-15.98	✓	✓ 🟡
NEDE	3	$-19.380^{+0.023}_{-0.040}$	$3.1\sigma$	$1.9\sigma$	✓	-18.93	-12.93	✓	✓ 🟡
EMG	3	$-19.397^{+0.017}_{-0.023}$	$3.7\sigma$	$2.3\sigma$	✓	-18.56	-12.56	✓	✓ 🟡
CPL	2	$-19.400 \pm 0.020$	$3.7\sigma$	$4.1\sigma$	X	-4.94	-0.94	X	X
PEDE	0	$-19.349 \pm 0.013$	$2.7\sigma$	$2.8\sigma$	✓	2.24	2.24	X	X
GPEDE	1	$-19.400 \pm 0.022$	$3.6\sigma$	$4.6\sigma$	X	-0.45	1.55	X	X
DM $\rightarrow$ DR+WDM	2	$-19.420 \pm 0.012$	$4.5\sigma$	$4.5\sigma$	X	-0.19	3.81	X	X
DM $\rightarrow$ DR	2	$-19.410 \pm 0.011$	$4.3\sigma$	$4.5\sigma$	X	-0.53	3.47	X	X

# The Power of an Emulator

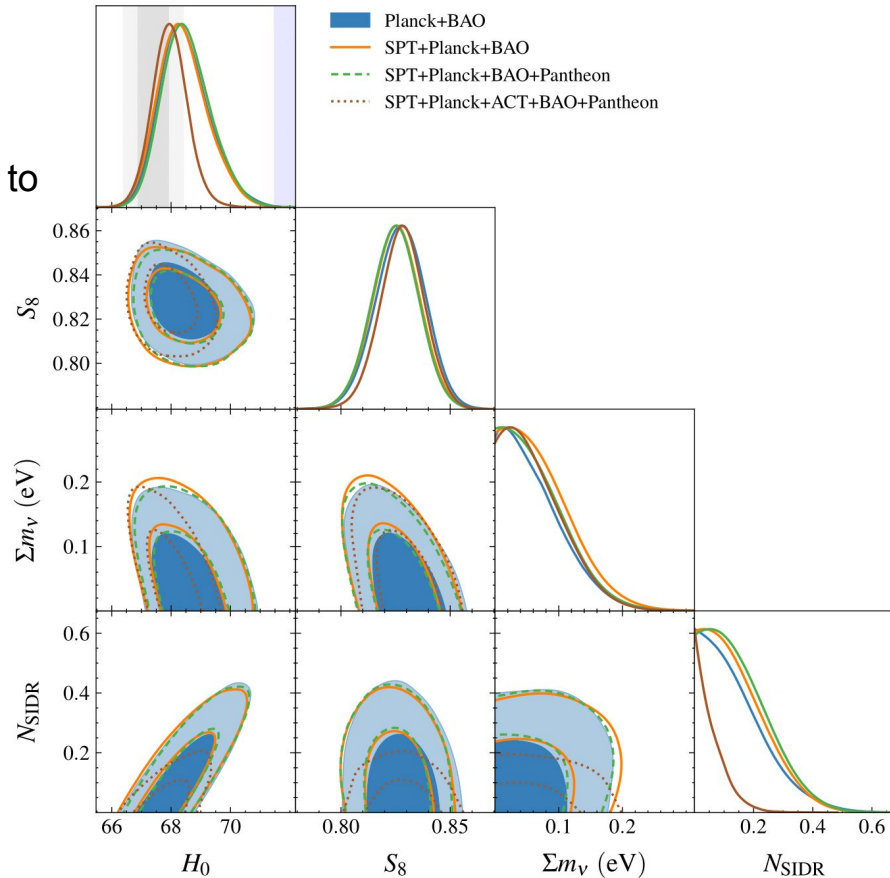


# $Q_{\text{MPCL}}$ for Each Model and Data-set

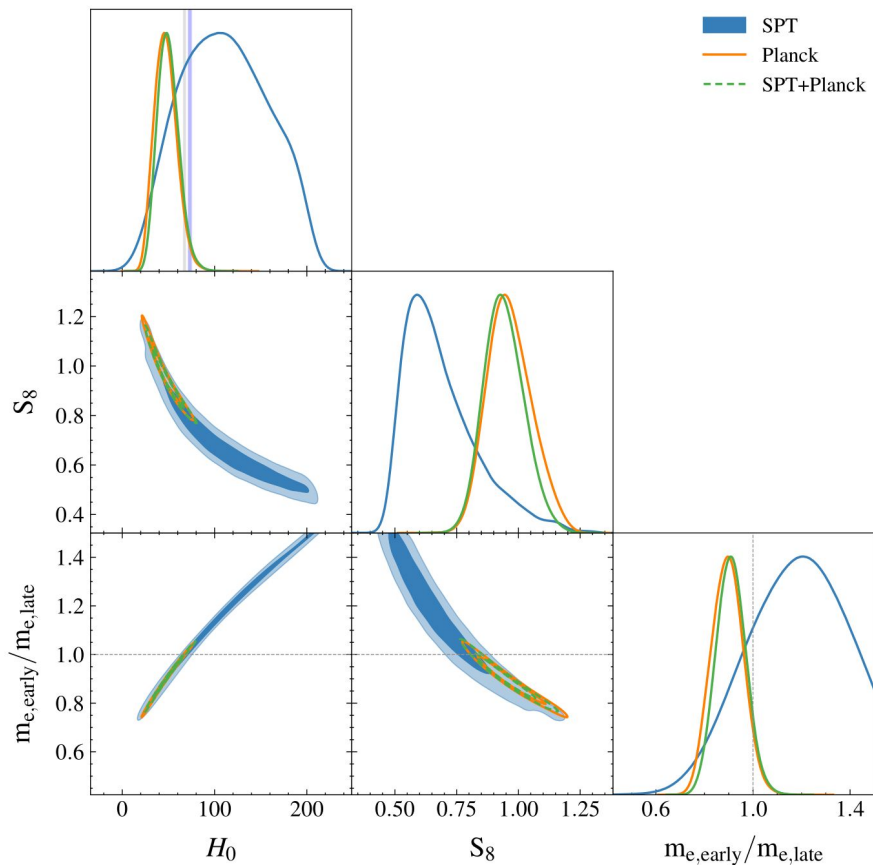
	$\mathcal{D}_S$	$\mathcal{D}_{\text{SP}}$	$\mathcal{D}_{\text{SB}}$	$\mathcal{D}_{\text{PB}}$	$\mathcal{D}_{\text{SPB}}$	$\mathcal{D}_{\text{SPBP}}$	$\mathcal{D}_{\text{SPAB}}$	$\mathcal{D}_{\text{SPABP}}$
$\Lambda\text{CDM}$	<b>2.7 <math>\sigma</math></b>	6.0 $\sigma$	5.4 $\sigma$	5.9 $\sigma$	6.3 $\sigma$	6.0 $\sigma$	6.3 $\sigma$	6.1 $\sigma$
+ $\Sigma m_\nu$	3.4 $\sigma$	5.4 $\sigma$	5.6 $\sigma$	5.7 $\sigma$	6.0 $\sigma$	5.9 $\sigma$	5.9 $\sigma$	5.9 $\sigma$
+ $\Sigma m_\nu$ + CPL	<b>0.5 <math>\sigma</math></b>	<b>0.0 <math>\sigma</math></b>	3.3 $\sigma$	3.1 $\sigma$	3.2 $\sigma$	4.5 $\sigma$	4.1 $\sigma$	4.5 $\sigma$
+ $\Sigma m_\nu$ + $N_{\text{eff}}$	<b>1.4 <math>\sigma</math></b>	4.0 $\sigma$	<b>1.3 <math>\sigma</math></b>	4.0 $\sigma$	4.3 $\sigma$	4.2 $\sigma$	5.0 $\sigma$	5.1 $\sigma$
+ $\Sigma m_\nu$ + $\Omega_K$		4.0 $\sigma$	5.2 $\sigma$	5.2 $\sigma$	5.2 $\sigma$	5.1 $\sigma$	5.3 $\sigma$	5.3 $\sigma$
+ $\Sigma m_\nu$ + $N_{\text{SIDR}}$	<b>1.7 <math>\sigma</math></b>	<b>3.0 <math>\sigma</math></b>	<b>1.8 <math>\sigma</math></b>	3.7 $\sigma$	3.9 $\sigma$	3.8 $\sigma$	4.8 $\sigma$	4.7 $\sigma$
$m_e$	<b>-0.1 <math>\sigma</math></b>	<b>1.4 <math>\sigma</math></b>	3.3 $\sigma$	3.8 $\sigma$	3.9 $\sigma$	3.8 $\sigma$	3.8 $\sigma$	3.8 $\sigma$
$m_e$ + $\Sigma m_\nu$	<b>0.0 <math>\sigma</math></b>	<b>1.9 <math>\sigma</math></b>	<b>0.4 <math>\sigma</math></b>	3.5 $\sigma$	3.4 $\sigma$	3.5 $\sigma$	3.7 $\sigma$	3.7 $\sigma$
$m_e$ + $\Omega_K$	<b>-0.7 <math>\sigma</math></b>	<b>1.8 <math>\sigma</math></b>	3.3 $\sigma$	<b>1.9 <math>\sigma</math></b>	<b>2.8 <math>\sigma</math></b>	<b>2.9 <math>\sigma</math></b>	<b>2.8 <math>\sigma</math></b>	<b>2.8 <math>\sigma</math></b>
$m_e$ + $\Omega_K$ + $\Sigma m_\nu$			<b>1.2 <math>\sigma</math></b>	<b>1.0 <math>\sigma</math></b>	<b>1.3 <math>\sigma</math></b>	<b>1.4 <math>\sigma</math></b>	<b>1.4 <math>\sigma</math></b>	<b>1.4 <math>\sigma</math></b>
EDE	<b>1.5 <math>\sigma</math></b>	4.2 $\sigma$	<b>2.2 <math>\sigma</math></b>	3.8 $\sigma$	3.7 $\sigma$	3.7 $\sigma$	3.1 $\sigma$	3.1 $\sigma$
Majoron	<b>-0.1 <math>\sigma</math></b>	3.7 $\sigma$	<b>1.4 <math>\sigma</math></b>	4.0 $\sigma$	4.2 $\sigma$	4.3 $\sigma$	4.0 $\sigma$	4.4 $\sigma$

# $\Lambda$ CDM Extensions

- $Q_{\text{MPCL}} \geq 3.1\sigma$  for all models with at least Planck+BAO.
- SPT & ACT marginally increase the tension compared to Planck+BAO.
- Expected degeneracies.
- ACT is slightly less compatible with larger  $N_{\text{SIDR}}$ .



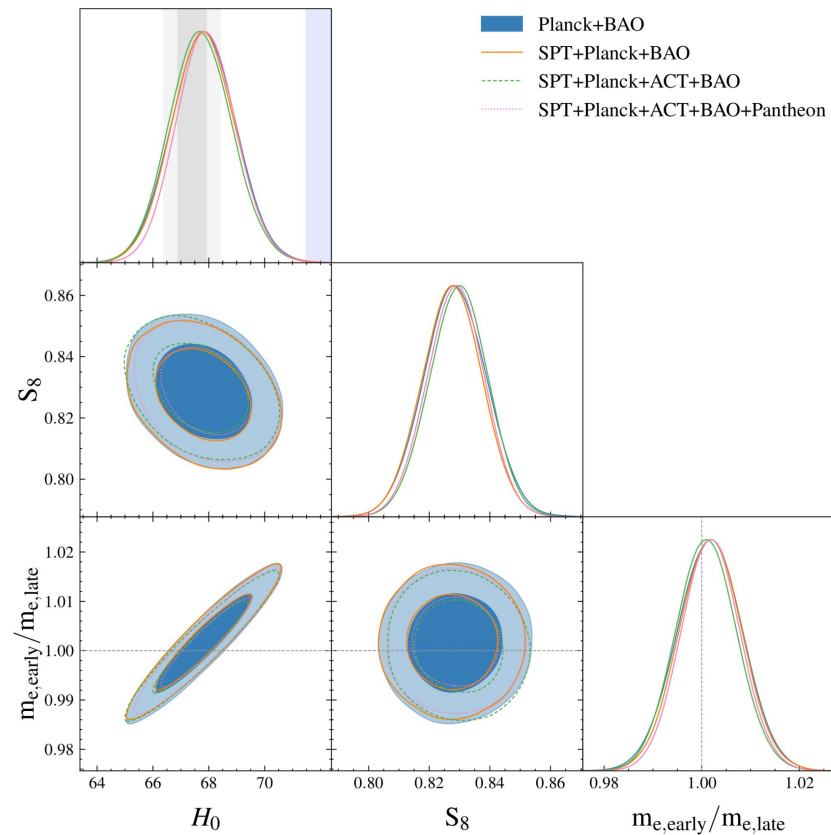
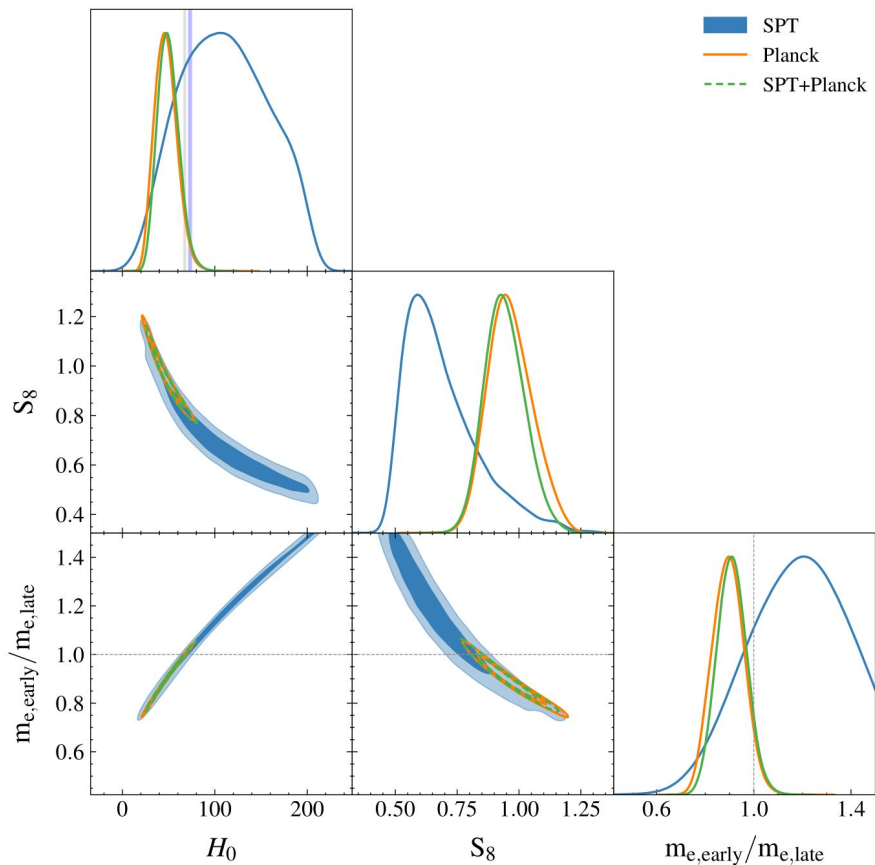
# Varying Electron Mass



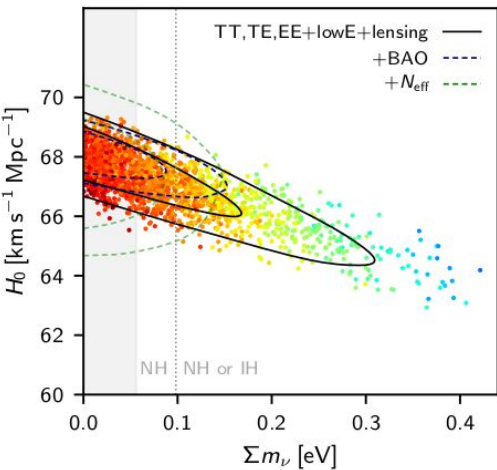
- No longer a potential solution to the tension.
- Planck is still more constraining than SPT.
- CMB alone cannot constrain this model.



# Varying Electron Mass

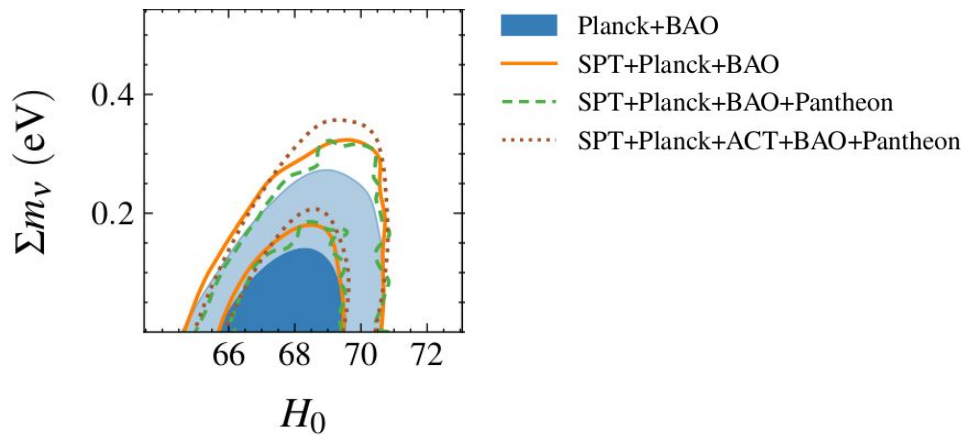


# Varying Electron Mass + $\Sigma m_\nu$

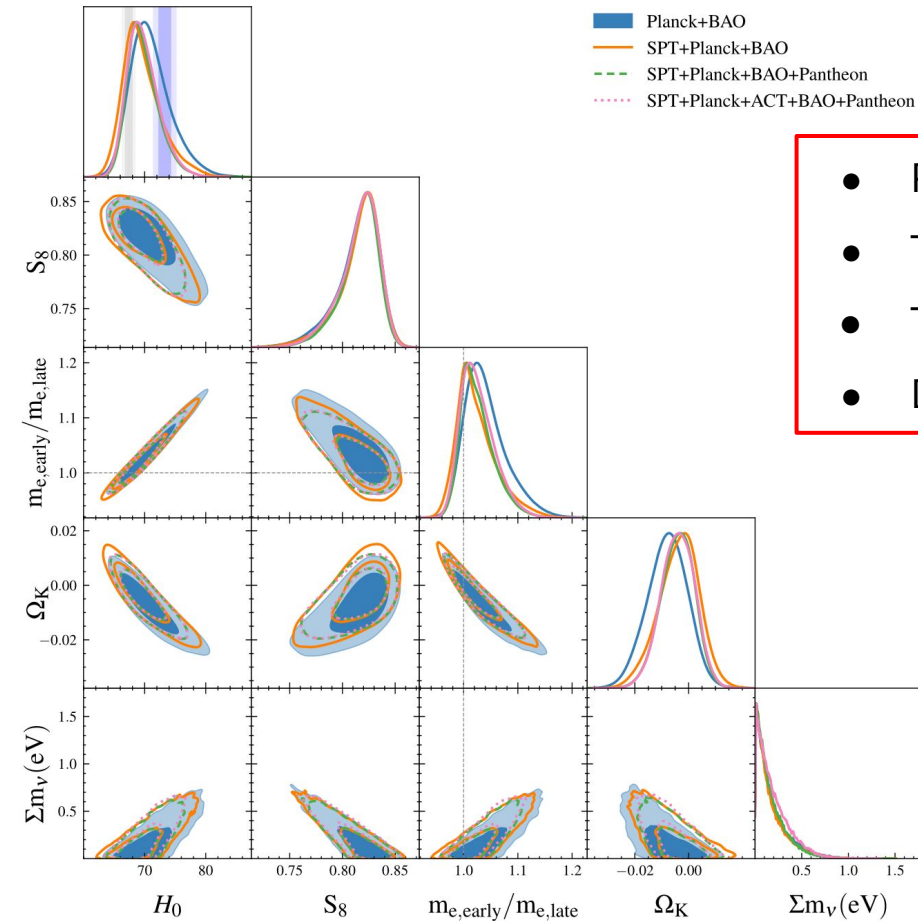


Planck 2018 ([Aghanim et al.](#))

- Allowing  $\Sigma m_\nu$  to vary doesn't help.
- Degeneracy direction in the  $\Sigma m_\nu$ - $H_0$  flips.

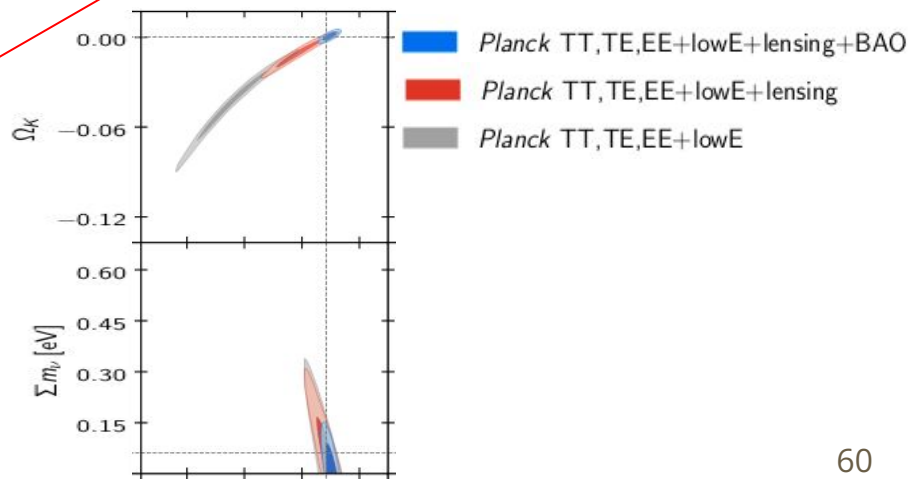
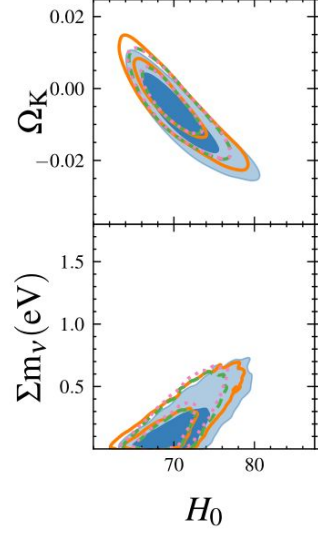
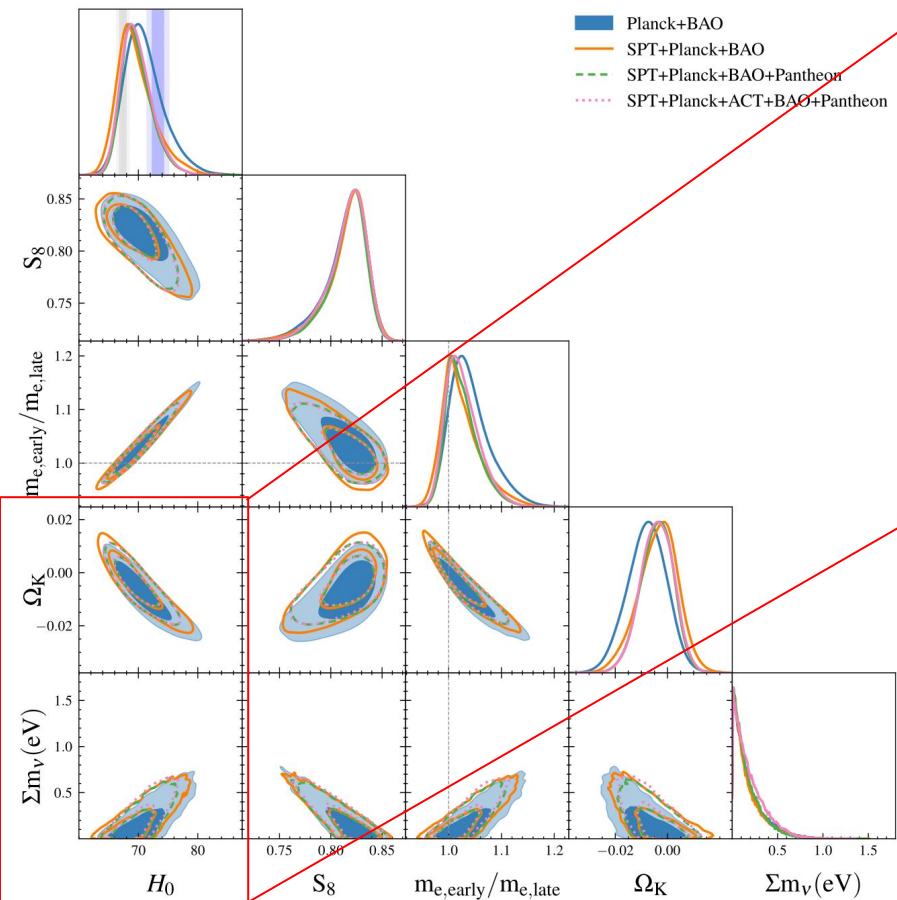


# Varying Electron Mass + $\Sigma m_\nu + \Omega_K$

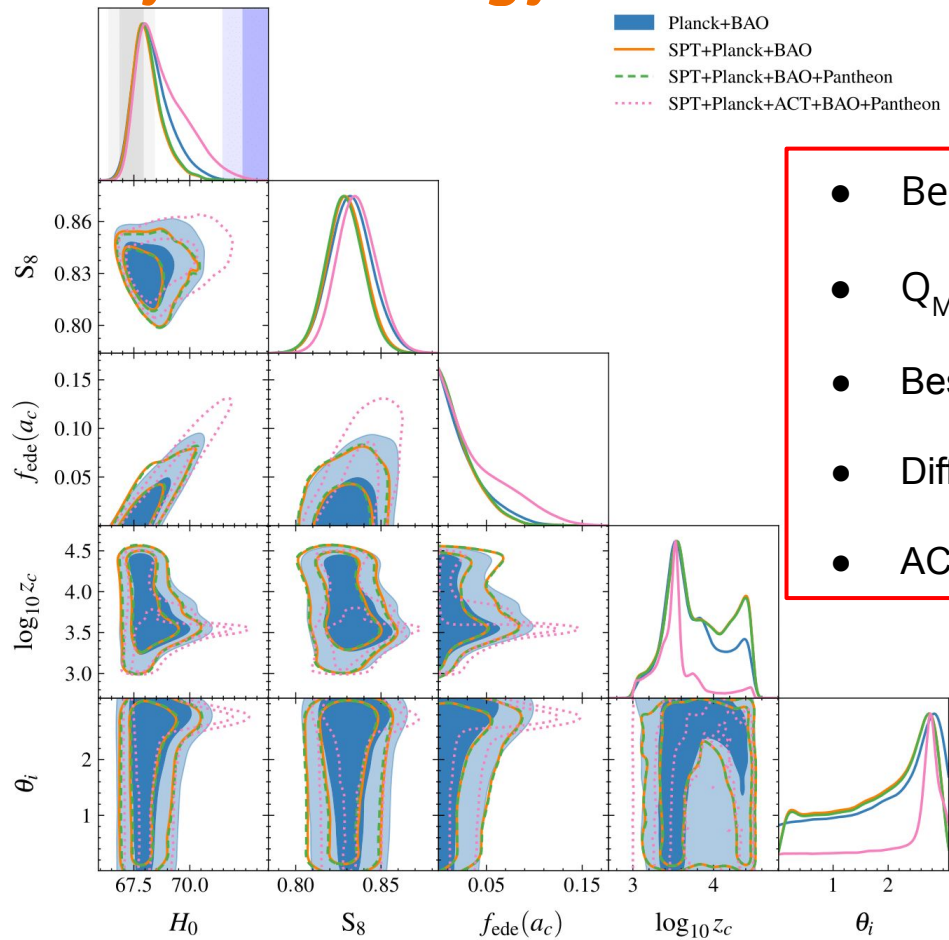


- Polarization data from SPT is particularly useful.
- The model that reduces the tension the most.
- The model with the largest error bars.
- Degeneracy direction also flips in the  $\Omega_K$ - $H_0$  plane.

# Varying Electron Mass + $\Sigma m_\nu + \Omega_K$

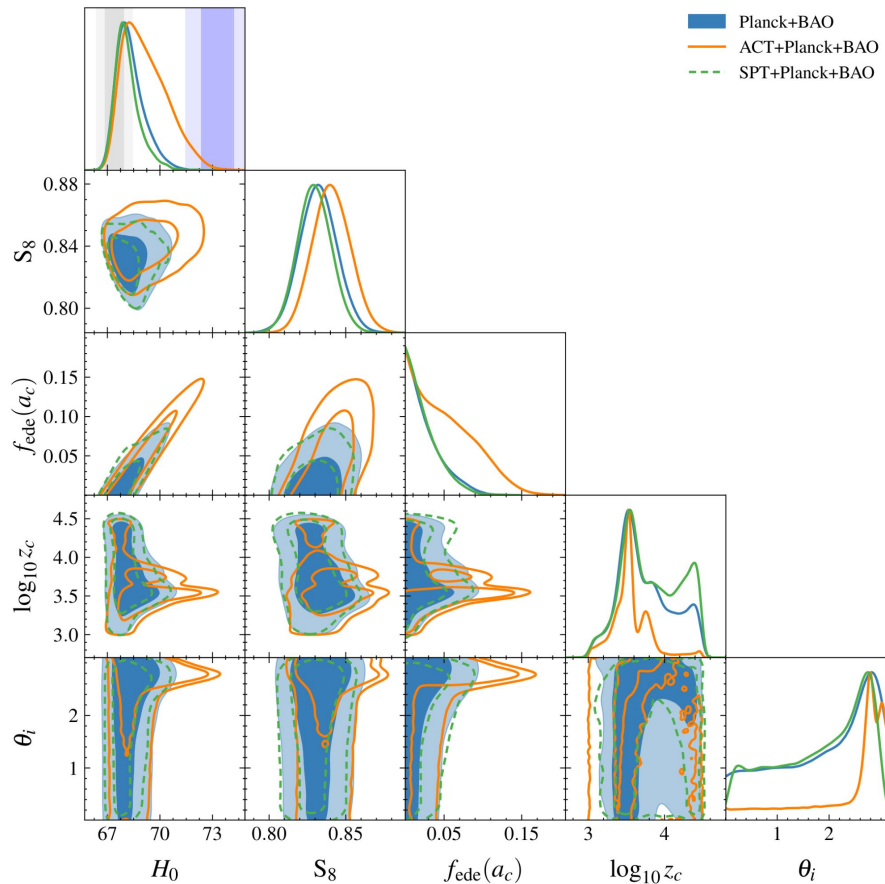


# Early Dark Energy

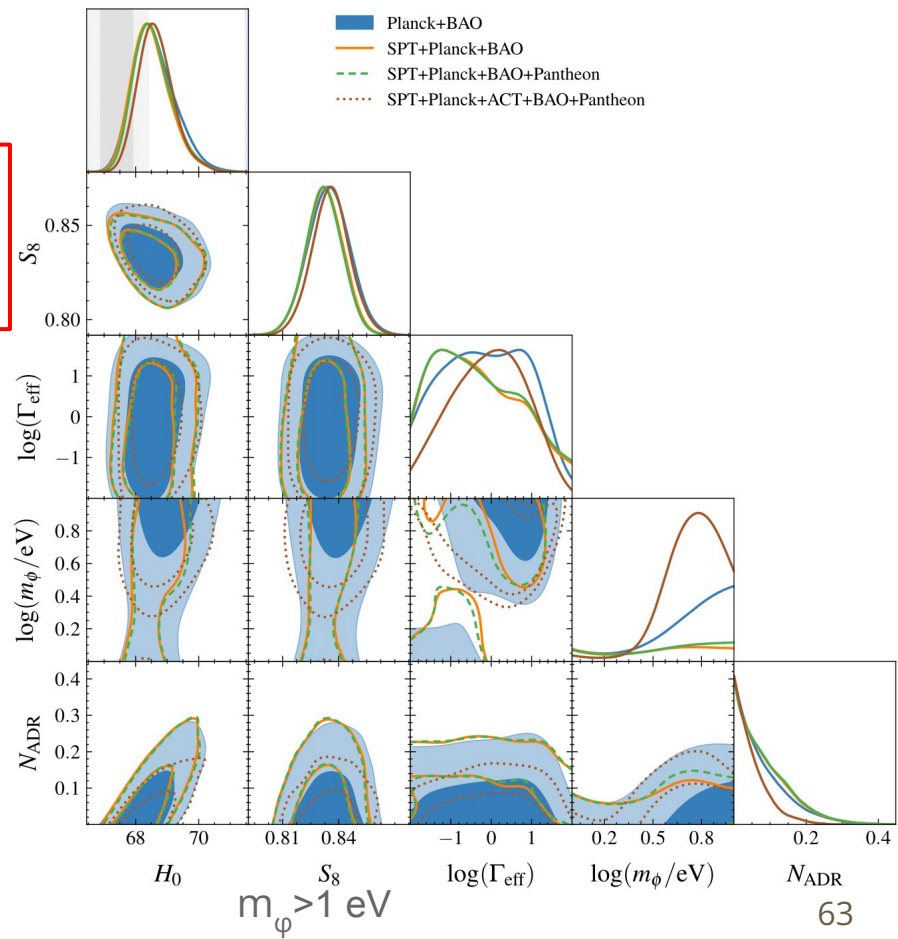
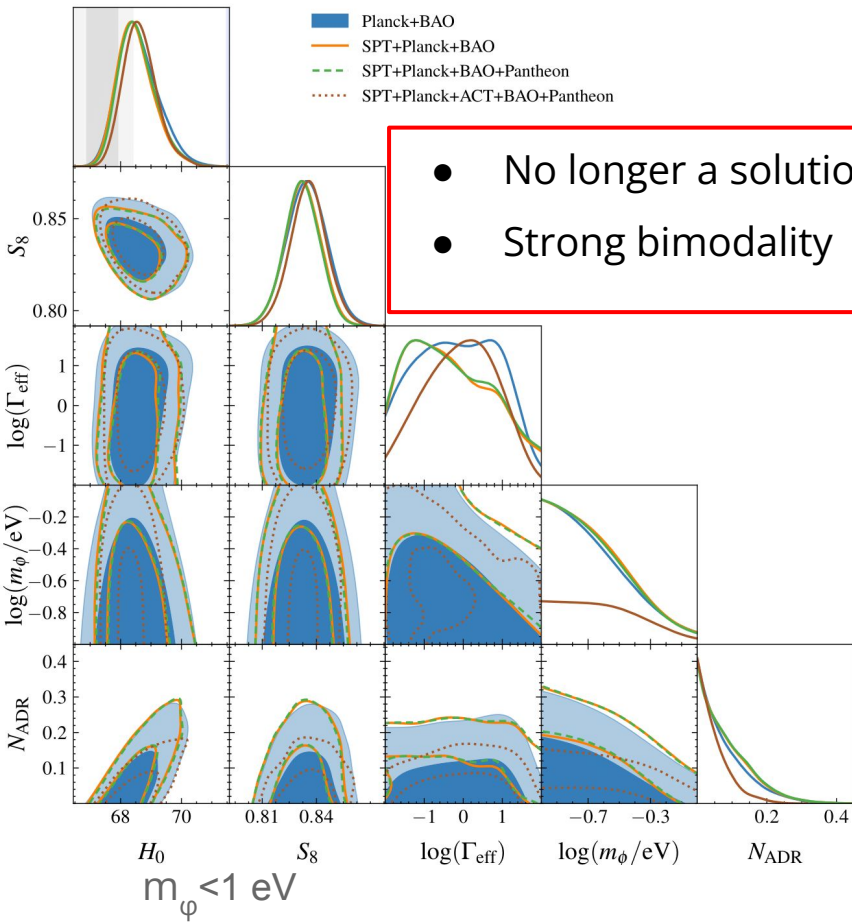


- Best constrained by CMB.
- $Q_{\text{MPCL}} = 3.7\sigma$  while  $Q_{\text{DMAP}} = 2.7\sigma$  for SPBP.
- Best-fit  $\chi^2$  compared to all models, w/ and w/o SH0ES.
- Difficult to constrain, with some bimodality.
- ACT DR4 is compatible with higher  $f_{\text{EDE}}$ .

# Early Dark Energy: SPT vs ACT



# The Majoron



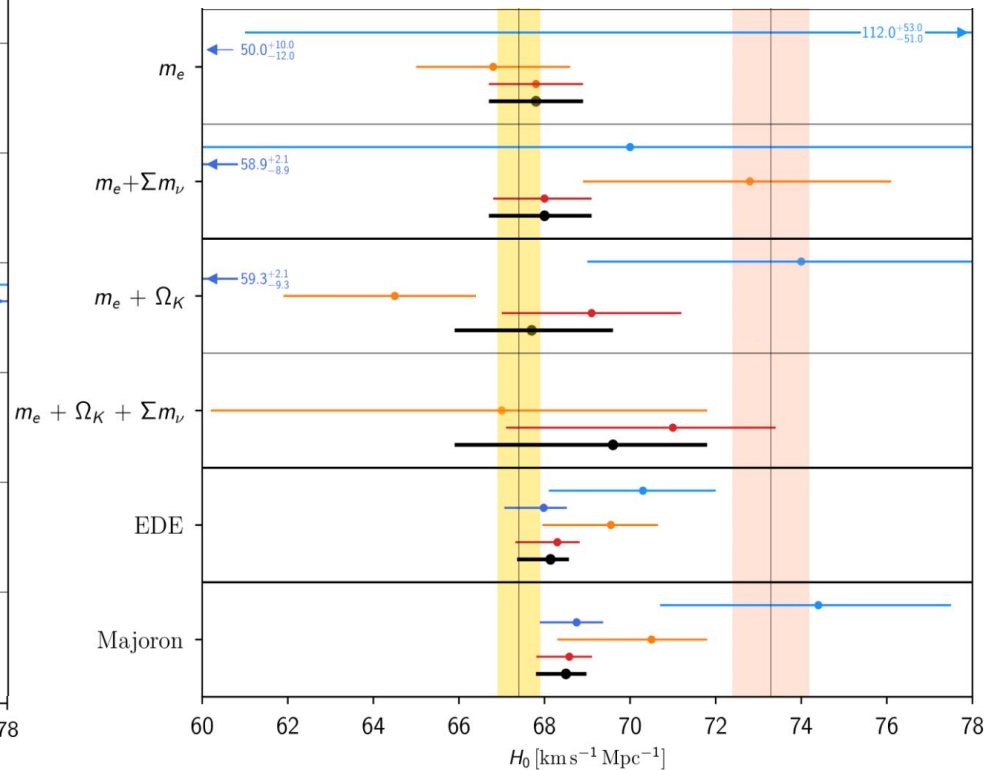
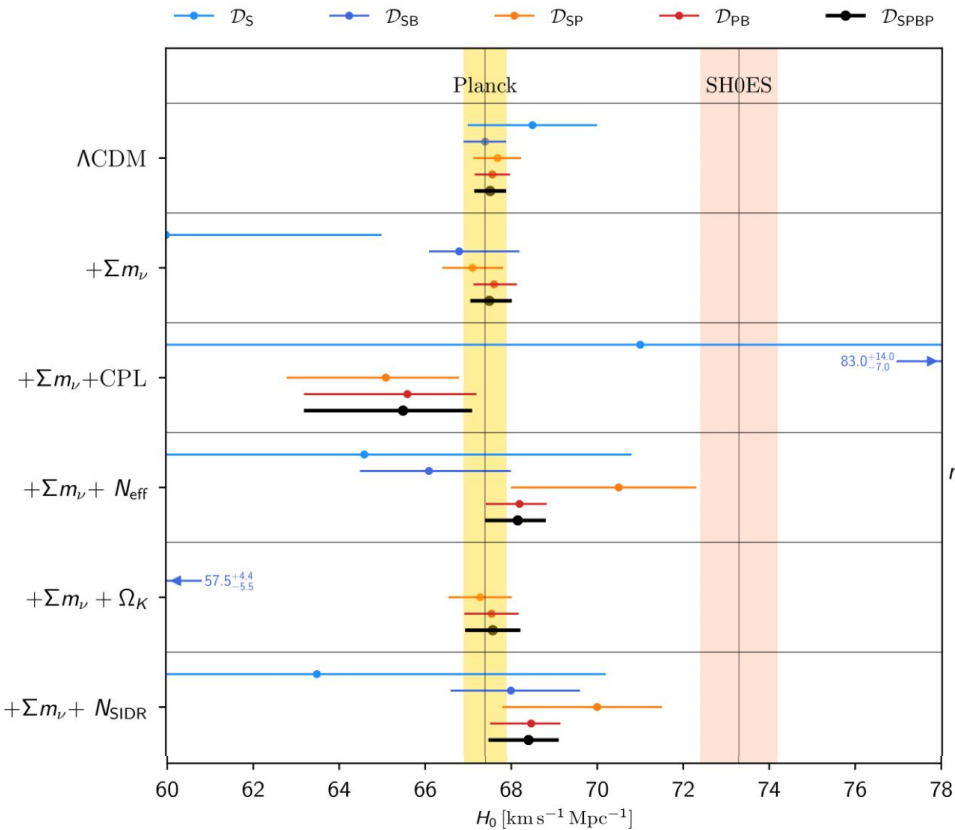
# The Power of an Emulator

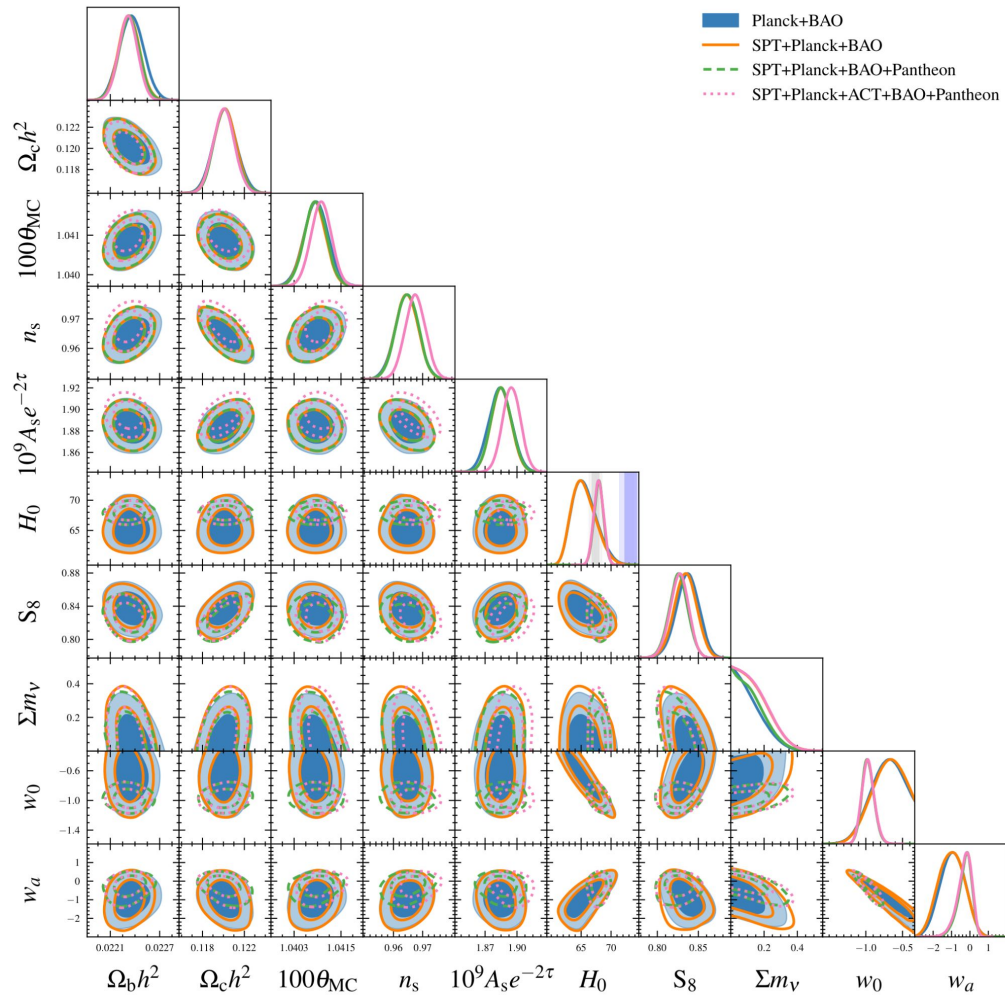
- Boltzmann codes are the tightest bottleneck of Bayesian analysis.
- To speed up the process, use neural-networks based emulators of Boltzmann codes.
- Classical emulators build on previously trained samples.
- The emulator we use builds its training data while running, i.e. online
- Stable results for minimizations
- Refs: [arXiv:2307.01138](https://arxiv.org/abs/2307.01138)

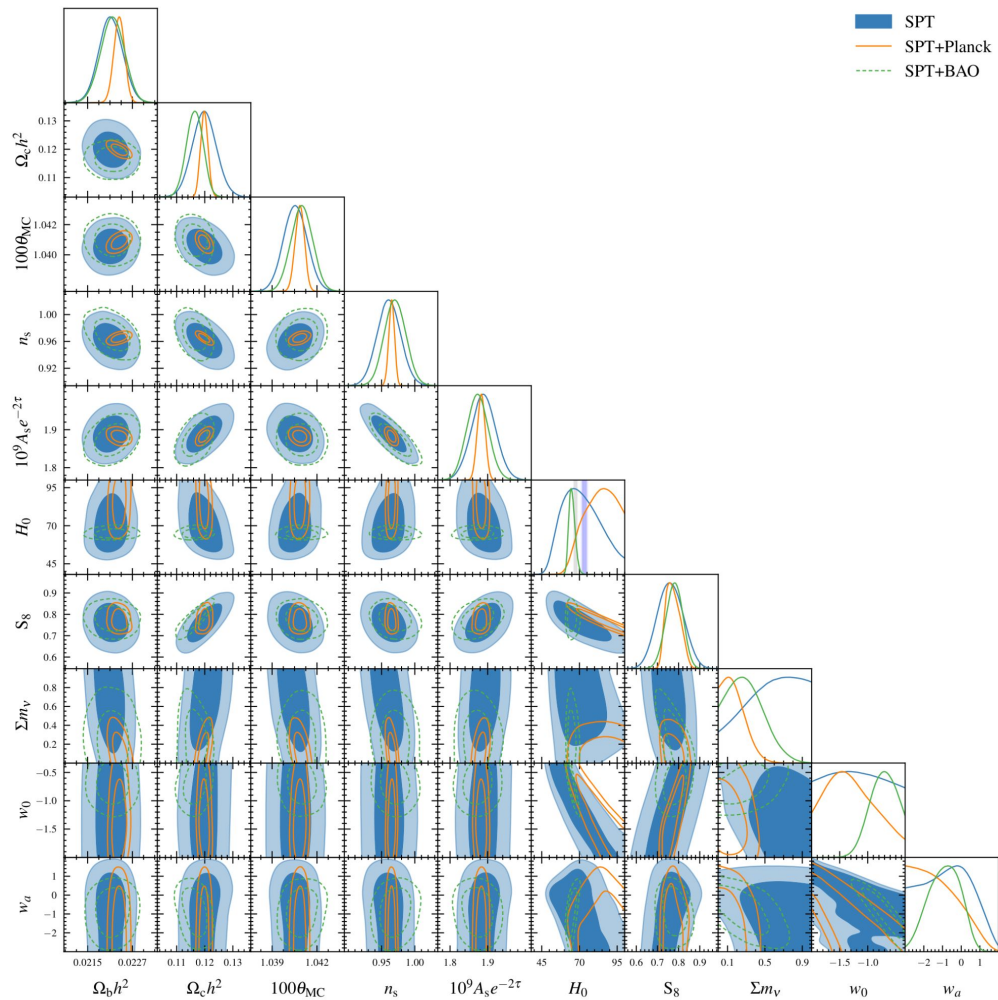
<https://github.com/svenguenter/cobaya>

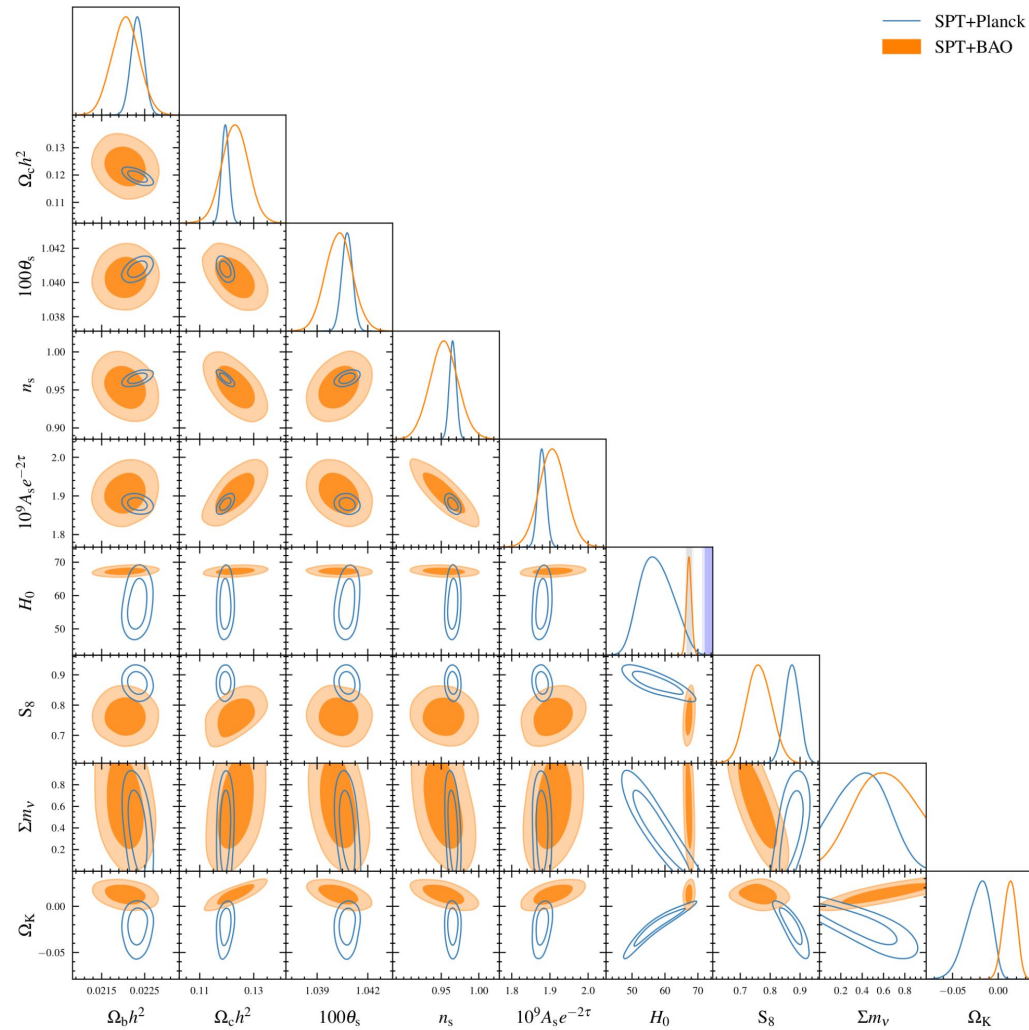


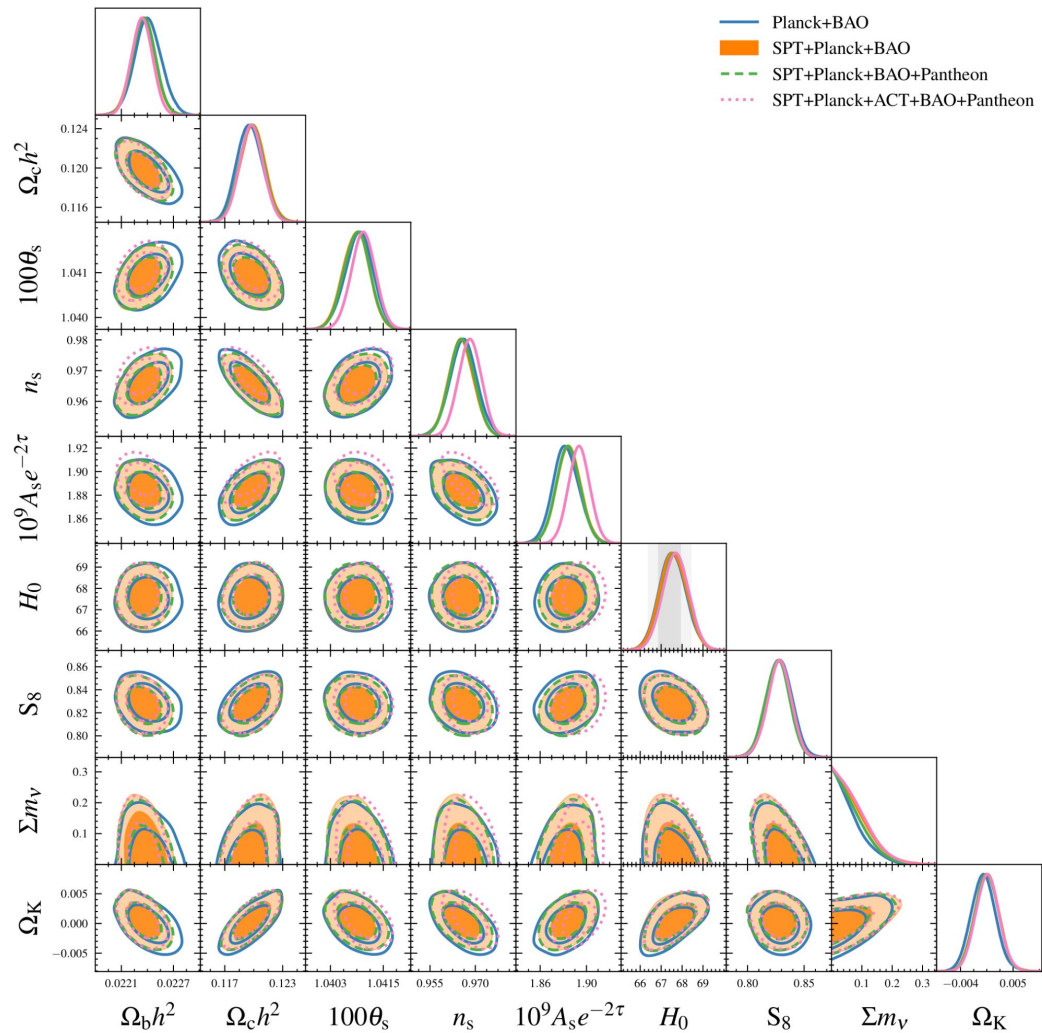
# $H_0$ for Each Model and Data-set

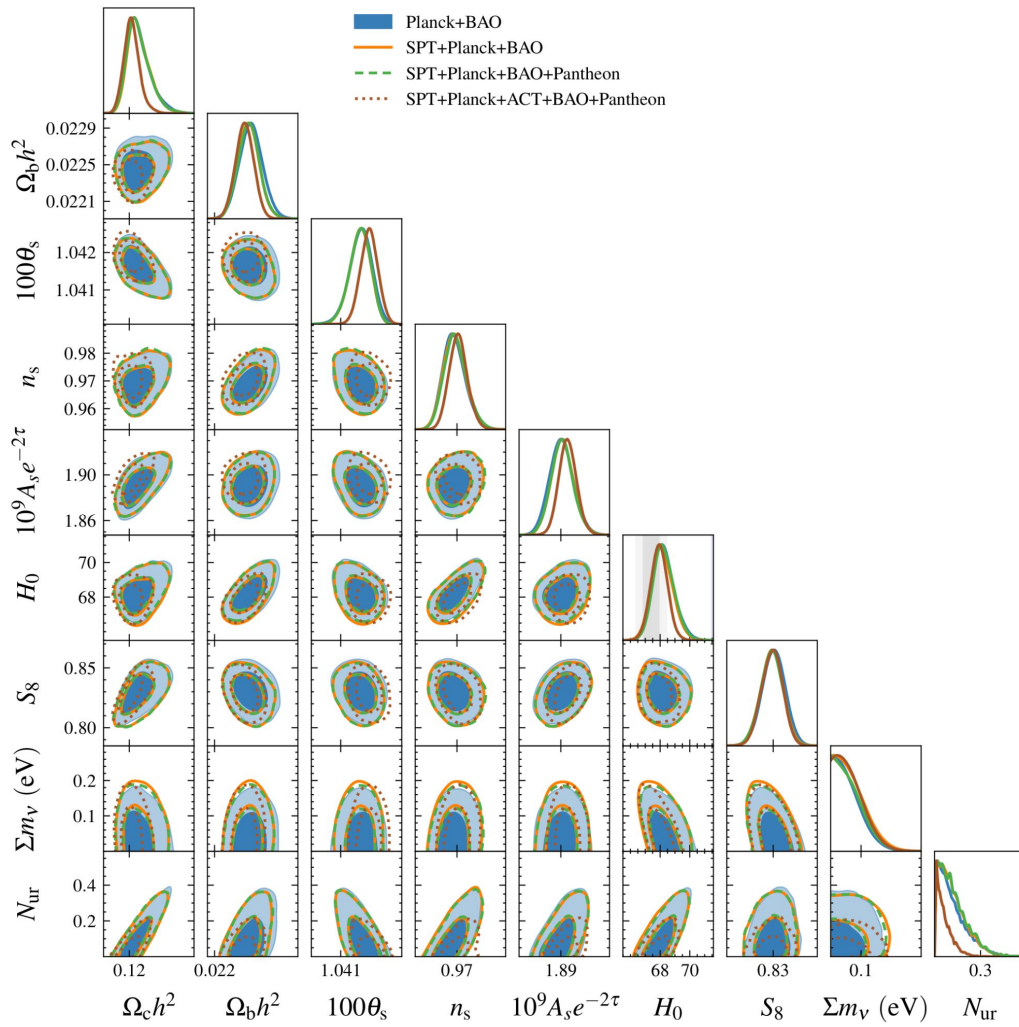










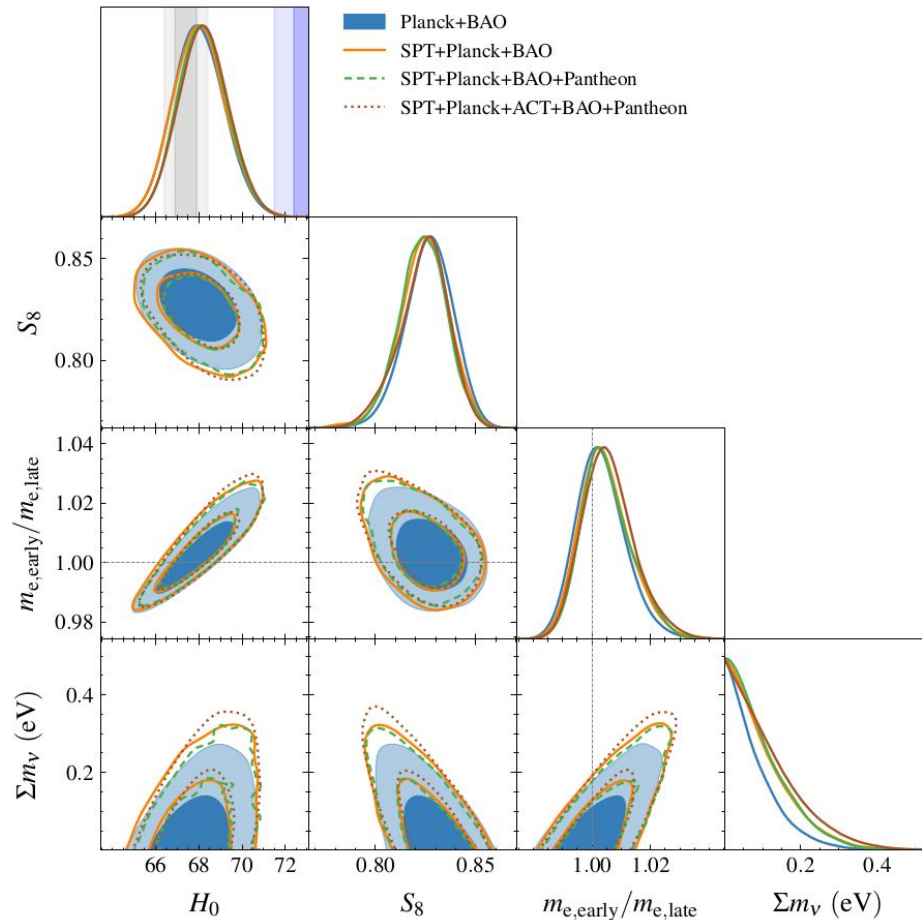


Models	$\mathcal{D}_S$	$\mathcal{D}_{SP}$	$\mathcal{D}_{SB}$	$\mathcal{D}_{PB}$	$\mathcal{D}_{SPB}$	$\mathcal{D}_{SPBP}$	$\mathcal{D}_{SPAB}$	$\mathcal{D}_{SPABP}$
$\Lambda$ CDM	$68.5^{+1.5}_{-1.5}$	$67.40^{+0.49}_{-0.50}$	$67.69^{+0.55}_{-0.56}$	$67.57^{+0.41}_{-0.41}$	$67.52^{+0.37}_{-0.37}$	<b><math>67.56^{+0.35}_{-0.38}</math></b>	$67.49^{+0.34}_{-0.39}$	$67.53^{+0.34}_{-0.37}$
$+\Sigma m_\nu$	$60.0^{+5.0}_{-5.6}$	$66.8^{+1.4}_{-0.7}$	$67.11^{+0.71}_{-0.70}$	$67.61^{+0.53}_{-0.48}$	$67.50^{+0.52}_{-0.44}$	<b><math>67.60^{+0.49}_{-0.43}</math></b>	$67.50^{+0.58}_{-0.44}$	$67.59^{+0.53}_{-0.42}$
$+\Sigma m_\nu + \text{CPL}$	$71^{+10}_{-15}$	$83^{+14}_{-7}$	$65.1^{+1.7}_{-2.3}$	$65.6^{+1.6}_{-2.4}$	$65.6^{+1.6}_{-2.4}$	<b><math>67.94^{+0.78}_{-0.79}</math></b>	$66.5^{+1.3}_{-1.7}$	$67.92^{+0.81}_{-0.81}$
$+\Sigma m_\nu + N_{\text{eff}}$	$64.6^{+6.2}_{-7.0}$	$66.1^{+1.9}_{-1.6}$	$70.5^{+1.8}_{-2.5}$	$68.20^{+0.63}_{-0.78}$	$68.16^{+0.65}_{-0.76}$	<b><math>68.25^{+0.62}_{-0.76}</math></b>	$67.83^{+0.58}_{-0.60}$	$67.93^{+0.57}_{-0.58}$
$+\Sigma m_\nu + \Omega_k$	—	$57.4^{+4.4}_{-5.5}$	$67.29^{+0.73}_{-0.74}$	$67.55^{+0.63}_{-0.63}$	$67.58^{+0.64}_{-0.64}$	<b><math>67.67^{+0.62}_{-0.62}</math></b>	$67.59^{+0.64}_{-0.64}$	$67.69^{+0.62}_{-0.62}$
$+\Sigma m_\nu + N_{\text{SIDR}}$	$63.5^{+6.7}_{-6.8}$	$68.0^{+1.6}_{-1.4}$	$70.0^{+1.5}_{-2.2}$	$68.47^{+0.68}_{-0.95}$	$68.41^{+0.70}_{-0.93}$	<b><math>68.53^{+0.69}_{-0.92}</math></b>	$67.86^{+0.60}_{-0.61}$	$67.96^{+0.57}_{-0.58}$
$m_e$	$112^{+53}_{-51}$	$50^{+10}_{-13}$	$66.8^{+1.8}_{-1.8}$	$67.8^{+1.1}_{-1.1}$	$67.8^{+1.1}_{-1.1}$	<b><math>68.0^{+1.1}_{-1.1}</math></b>	$67.7^{+1.1}_{-1.1}$	$67.9^{+1.1}_{-1.1}$
$m_e + \Sigma m_\nu$	$70^{+20}_{-20}$	$58.9^{+2.1}_{-8.9}$	$72.8^{+3.3}_{-3.9}$	$68.0^{+1.1}_{-1.2}$	$68.0^{+1.1}_{-1.3}$	<b><math>68.2^{+1.1}_{-1.2}</math></b>	$68.0^{+1.2}_{-1.2}$	$68.2^{+1.2}_{-1.2}$
$m_e + \Omega_k$	$74^{+16}_{-5}$	$59.3^{+2.1}_{-9.3}$	$64.5^{+1.9}_{-2.6}$	$69.1^{+2.1}_{-2.1}$	$67.7^{+1.9}_{-1.8}$	<b><math>68.2^{+1.6}_{-1.6}</math></b>	$67.5^{+1.9}_{-1.9}$	$68.1^{+1.6}_{-1.6}$
$m_e + \Omega_k + \Sigma m_\nu$	—	—	$67.0^{+4.8}_{-6.8}$	$71.0^{+2.4}_{-3.9}$	$69.6^{+2.2}_{-3.7}$	<b><math>69.8^{+1.8}_{-2.9}</math></b>	$69.5^{+2.3}_{-3.7}$	$69.8^{+2.0}_{-3.0}$
EDE	$70.3^{+1.7}_{-2.2}$	$67.98^{+0.54}_{-0.92}$	$69.6^{+0.9}_{-1.6}$	$68.3^{+0.52}_{-0.98}$	$68.12^{+0.43}_{-0.78}$	<b><math>68.18^{+0.42}_{-0.79}</math></b>	$68.7^{+0.6}_{-1.4}$	$68.8^{+0.6}_{-1.4}$
Majoron	$74.4^{+3.1}_{-3.7}$	$68.75^{+0.62}_{-0.86}$	$70.5^{+1.3}_{-2.2}$	$68.58^{+0.53}_{-0.77}$	$68.50^{+0.48}_{-0.70}$	<b><math>68.55^{+0.48}_{-0.70}</math></b>	$68.6^{+0.46}_{-0.64}$	$68.64^{+0.48}_{-0.61}$

Models	Additional Parameters
$\Lambda$ CDM	—
$+\Sigma m_\nu$	$\Sigma m_\nu < 0.16$ eV (95%)
$+\Sigma m_\nu + \text{CPL}$	$\Sigma m_\nu < 0.29$ eV (95%), $w_0 = -0.97 \pm 0.08$ , $w_a = -0.29 \pm 0.39$
$+\Sigma m_\nu + N_{\text{eff}}$	$\Sigma m_\nu < 0.15$ eV (95%) , $N_{\text{eff}} < 0.17$ (95%)
$+\Sigma m_\nu + N_{\text{SIDR}}$	$\Sigma m_\nu < 0.15$ eV(95%), $N_{\text{SIDR}} < 0.16$ (95%)
$+\Sigma m_\nu + \Omega_K$	$\Sigma m_\nu < 0.17$ eV (95%), $\Omega_K = -0.0005 \pm 0.0020$
$m_e$	$m_{e,\text{early}}/m_{e,\text{late}} = 1.003 \pm 0.006$
$m_e + \Sigma m_\nu$	$m_{e,\text{early}}/m_{e,\text{late}} = 1.0057 \pm 0.0090$ , $\Sigma m_\nu < 0.29$ eV(95%)
$m_e + \Omega_K$	$m_{e,\text{early}}/m_{e,\text{late}} = 1.0035 \pm 0.0164$ , $\Omega_K = -0.0005 \pm 0.0048$
$m_e + \Omega_K + \Sigma m_\nu$	$m_{e,\text{early}}/m_{e,\text{late}} = 1.03 \pm 0.03$ , $\Omega_K = -0.004 \pm 0.006$ , $\Sigma m_\nu < 0.48$ eV (95%)
EDE	$\theta_i = 1.8 \pm 0.9$ , $\log(a_c) = -3.8 \pm 0.4$ , $f_{\text{EDE}}(a_c) < 0.06$ (95%)
Majoron	$\log(m_\phi/\text{eV}) = 0.2950 \pm 0.6598$ , $\log(\Gamma_{\text{eff}}) = 0.0556 \pm 0.8846$ , $\Delta N_{\text{ADR}} < 0.15$ (95%)

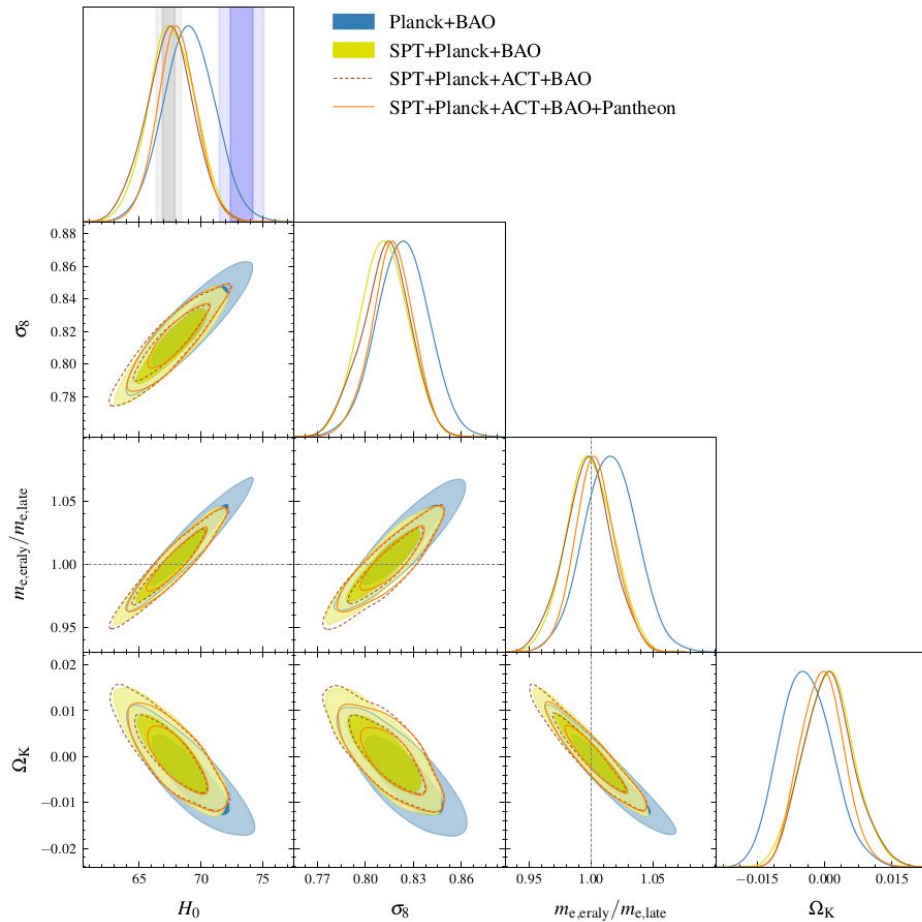


# Me+Mnu: Results

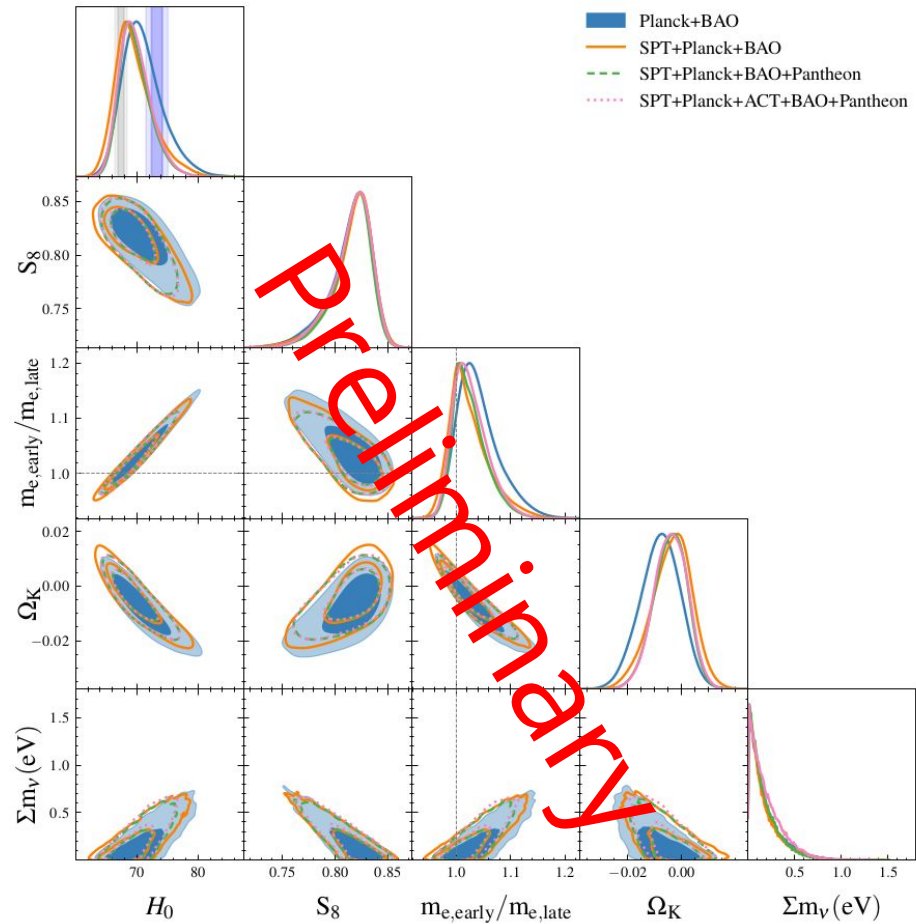


Grey Band: Planck 2018 LCDM  
Purple Band: SH0ES

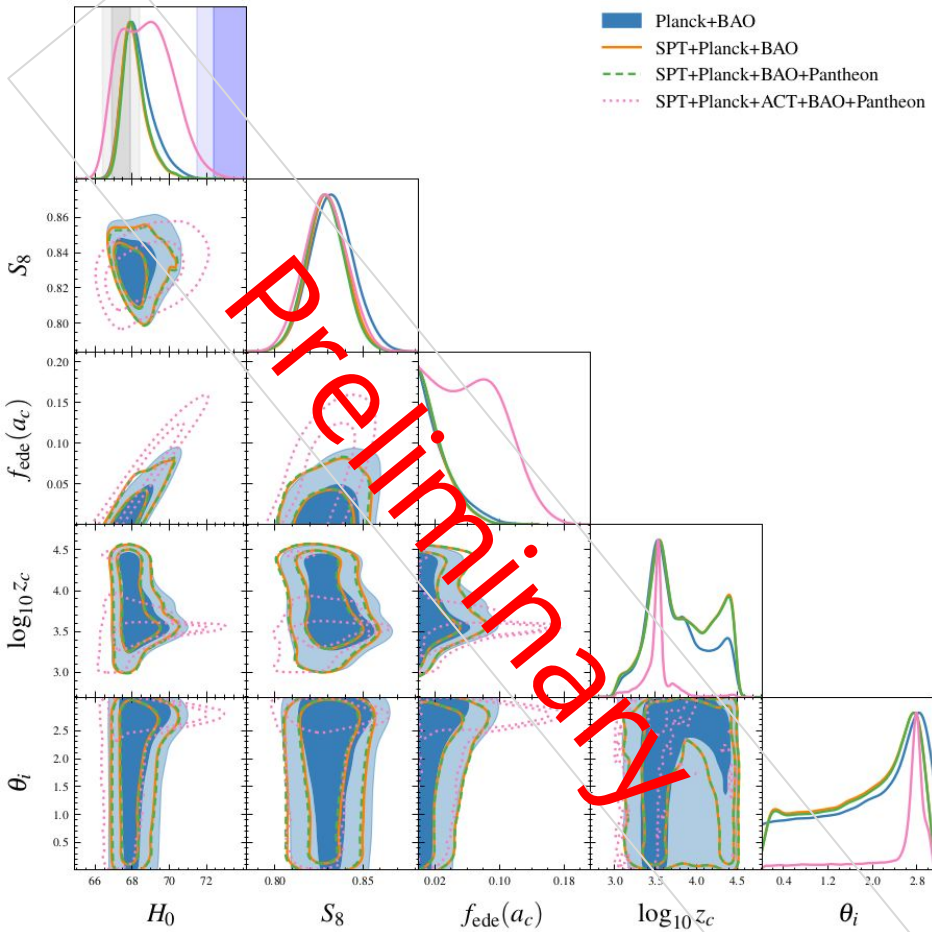
# Me+0mk



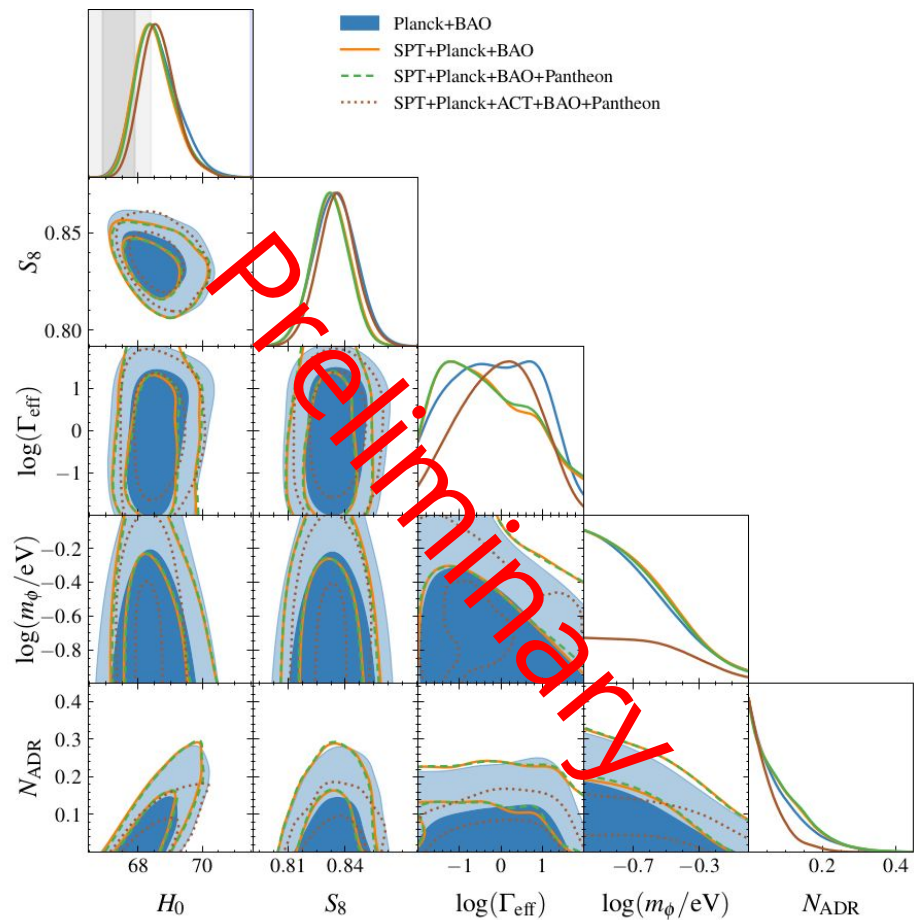
# Me+Mnu+Omk



# EDE

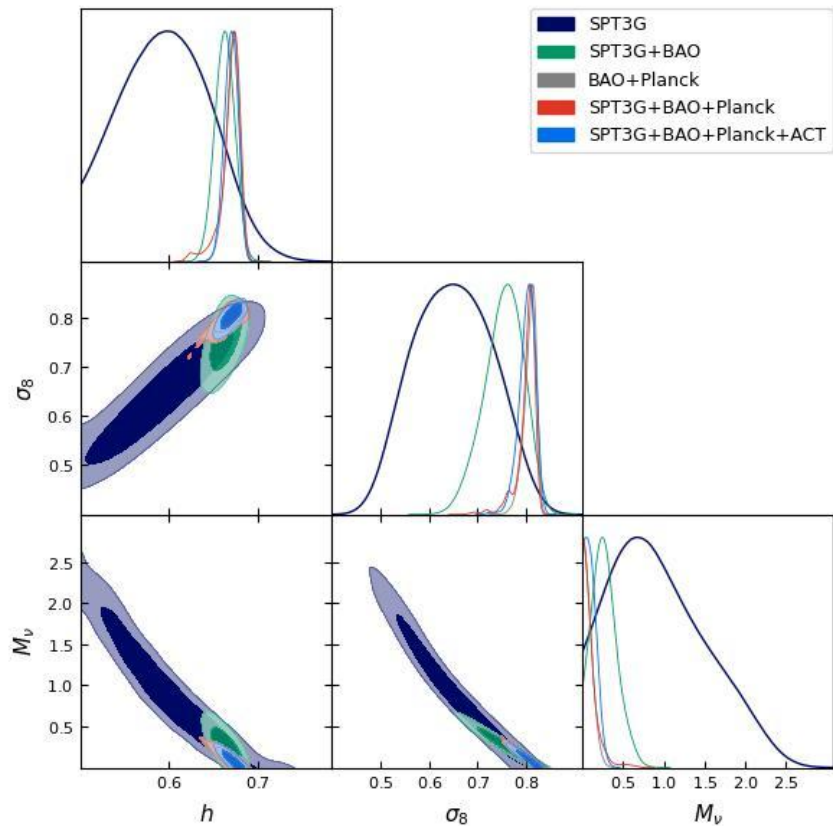


# Majoron



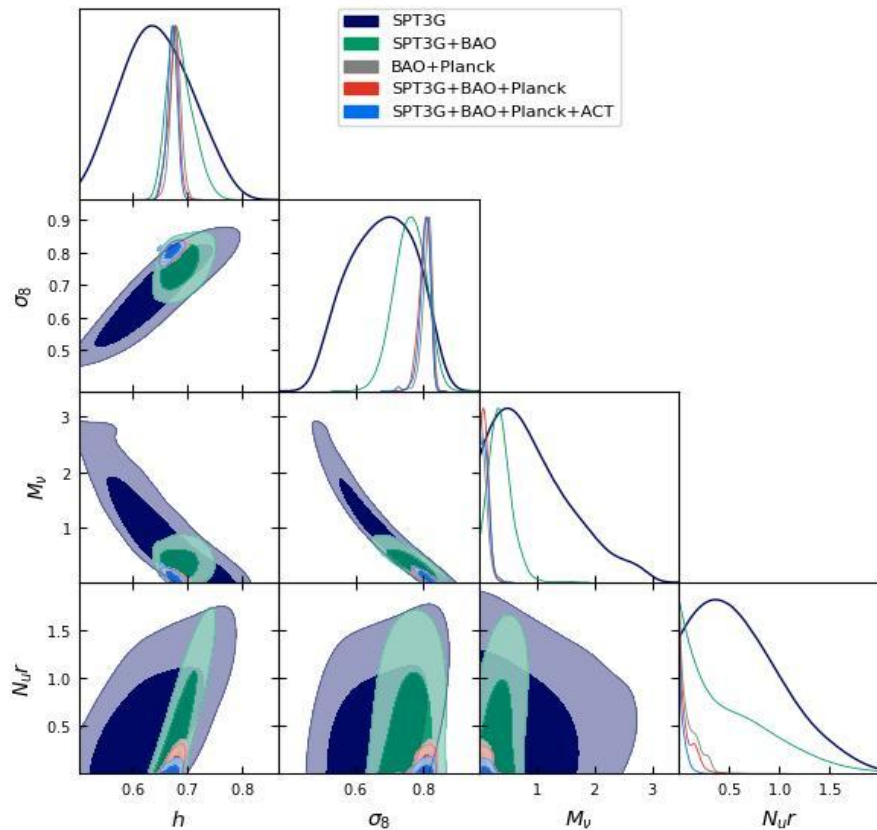
Sub eV mass

# $\Lambda$ CDM + $\Sigma m_\nu$

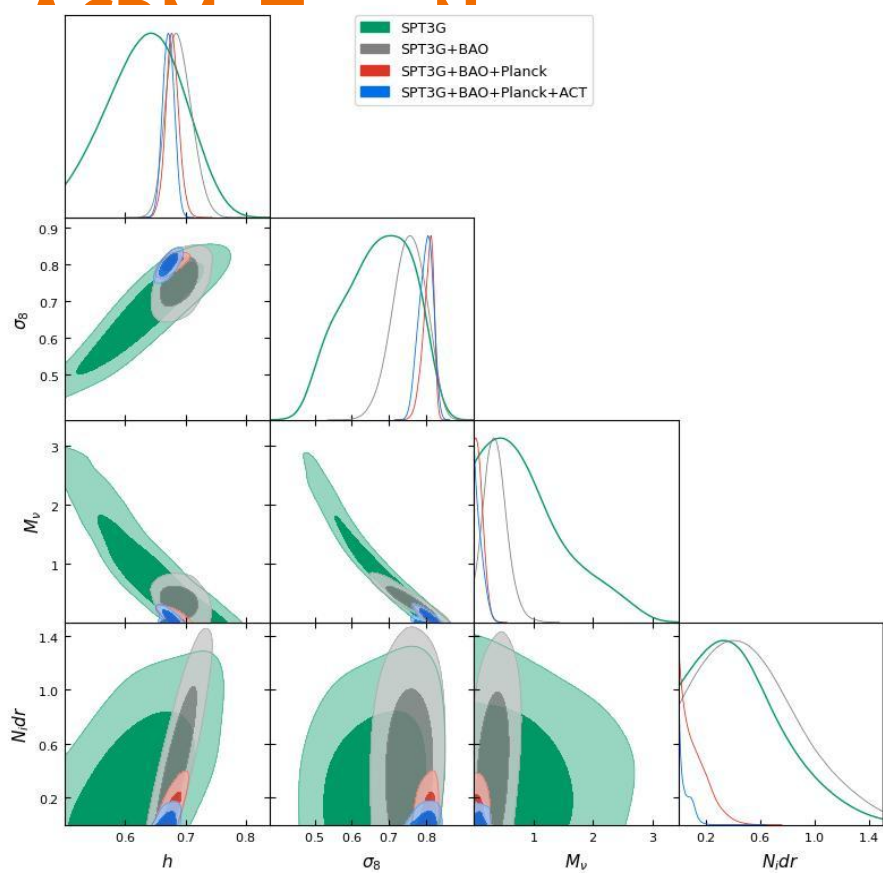


Parameter	SPT+ ACT+ Planck+ BAO
$H_0$	$67.00 \pm 0.82$
$\sigma_8$	$0.803 \pm 0.019$

# $\Lambda$ CDM + $\Sigma_m$ + $N_{eff}$

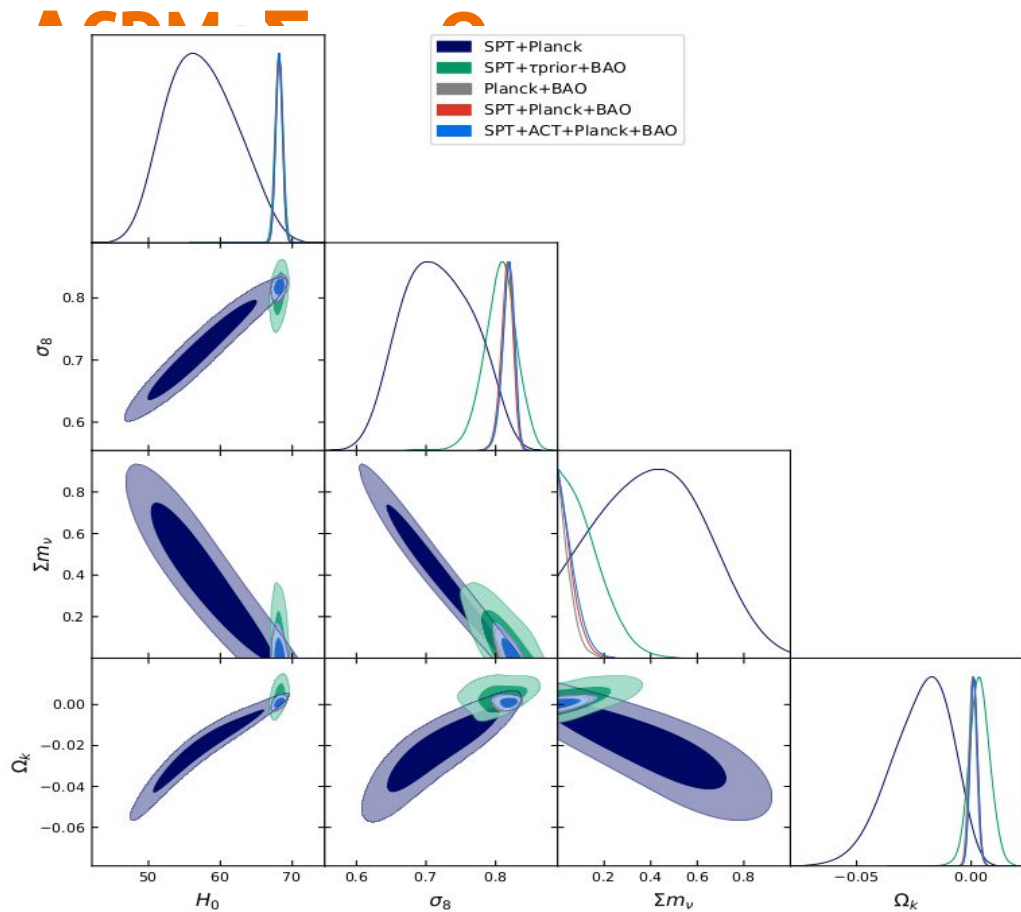


Parameter	SPT+ ACT+ Planck+ BAO
$H_0$	$67.10 \pm 0.85$
$\sigma_8$	$0.812 \pm 0.009$

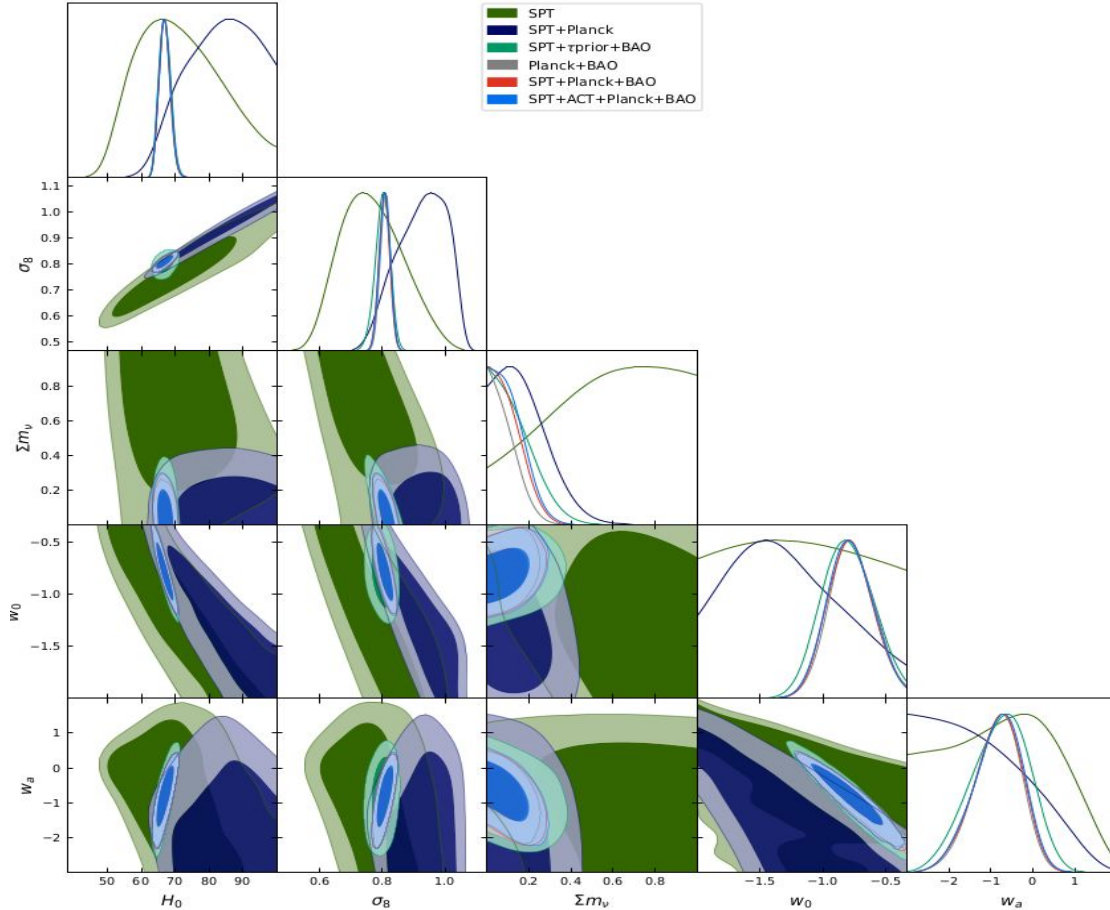


Parameter	SPT+ ACT+ Planck+ BAO
$H_0$	$67.22 \pm 0.91$
$\sigma_8$	$0.801 \pm 0.022$





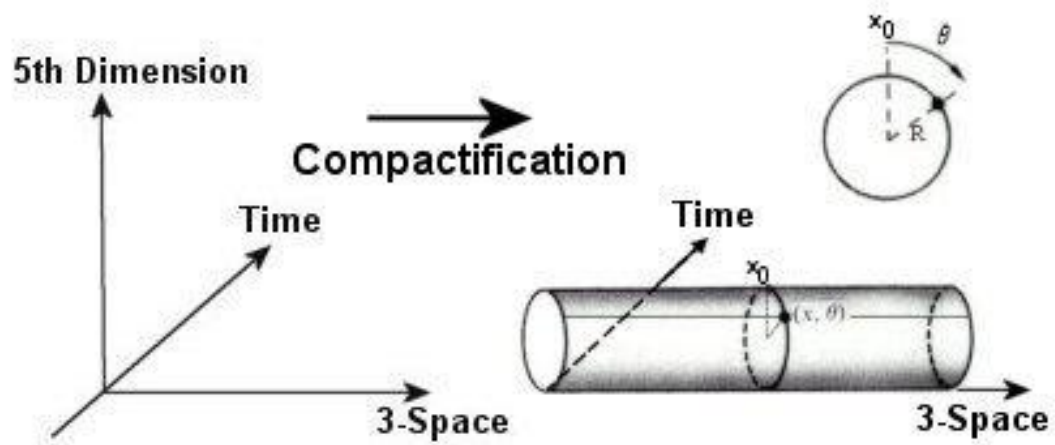
Parameter	SPT+ ACT+ Planck+ BAO
$H_0$	$68.16 \pm 0.46$
$\sigma_8$	$0.818 \pm 0.009$



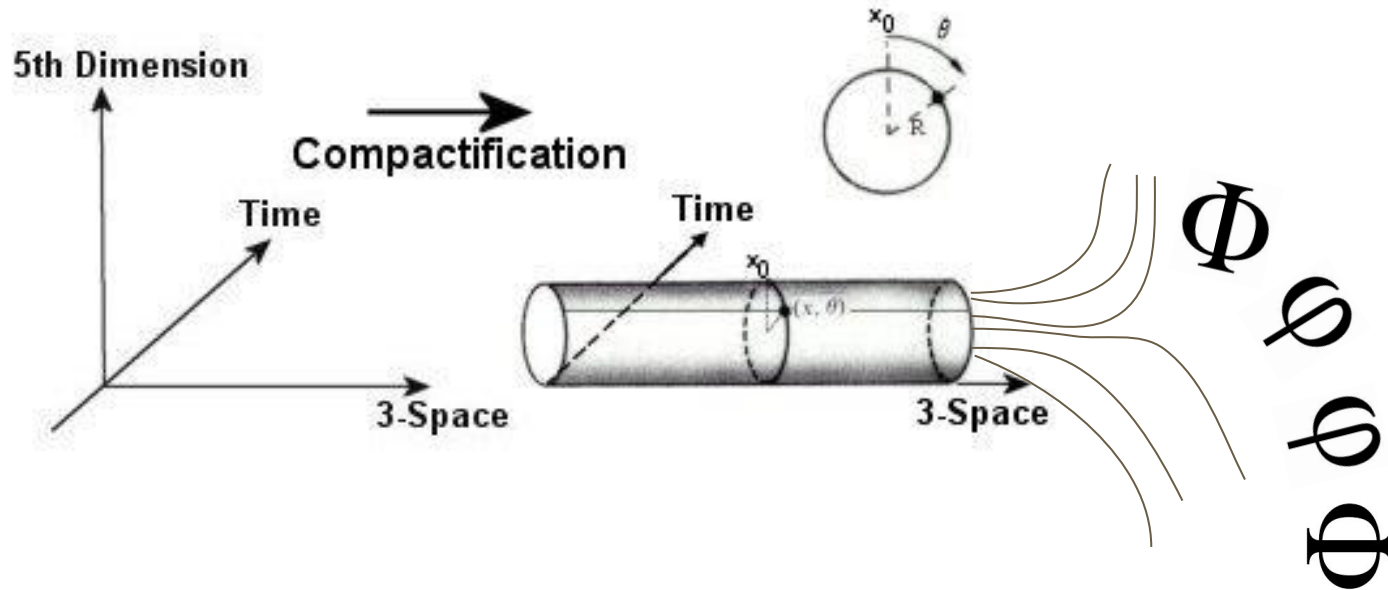
Parameter	SPT+ ACT+ Planck+ BAO
$H_0$	$66.89 \pm 1.62$
$\sigma_8$	$0.808 \pm 0.017$

# SLIDES FOR A GENERAL AUDIENCE TALK

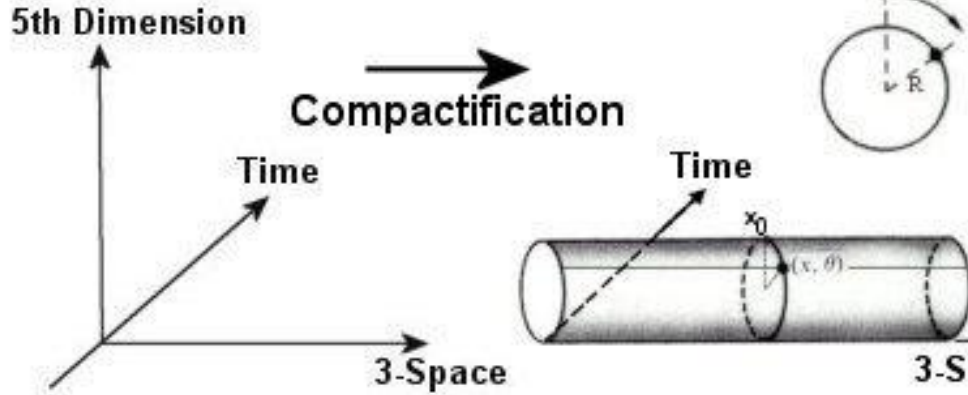
# Varying

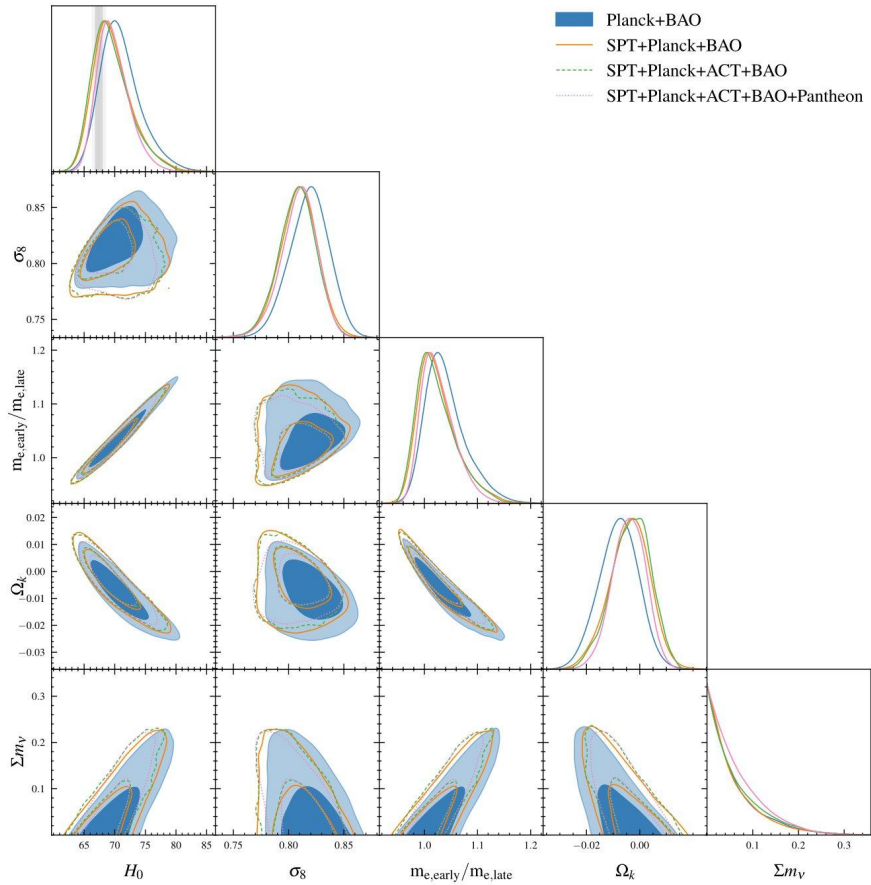


# Varying Electron Mass: Theory

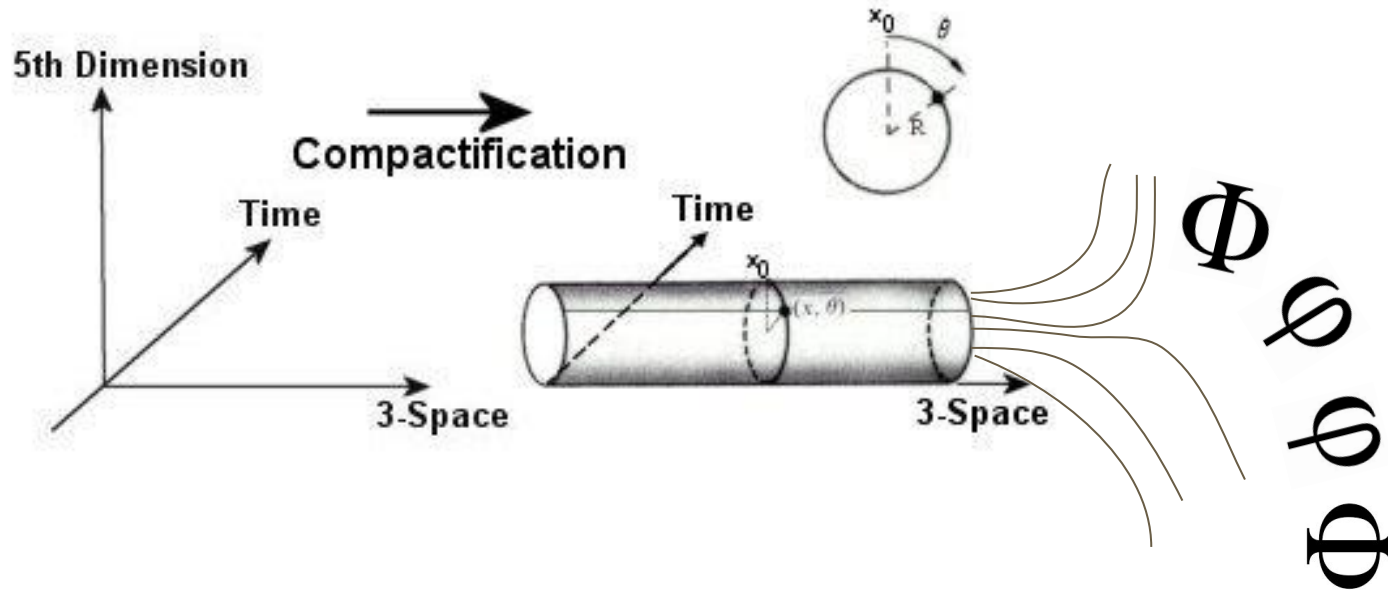


# Varying Electron Mass: Theory



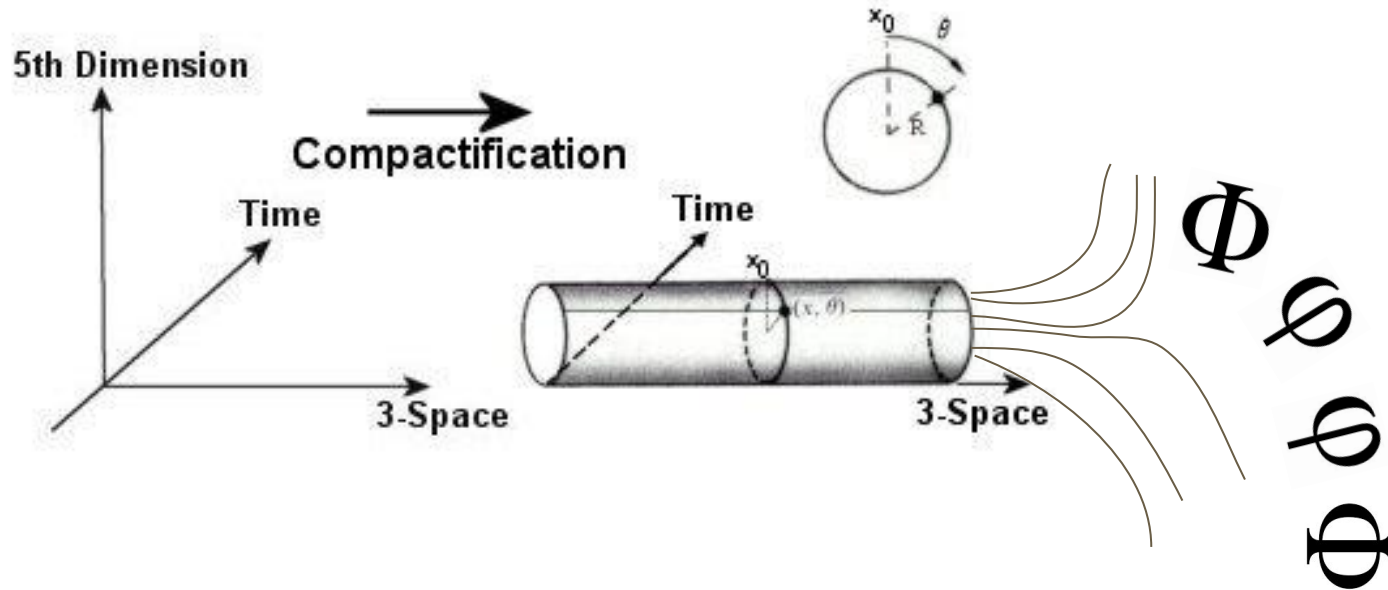


# Early Dark Energy Theory





# Early Dark Energy Theory



$$V(\phi) = \Lambda_{\text{ede}}^4 [1 - \cos(\phi/f_{\text{ede}})]^n$$

[Kamionkowski & Riess\(2022\)](#)