

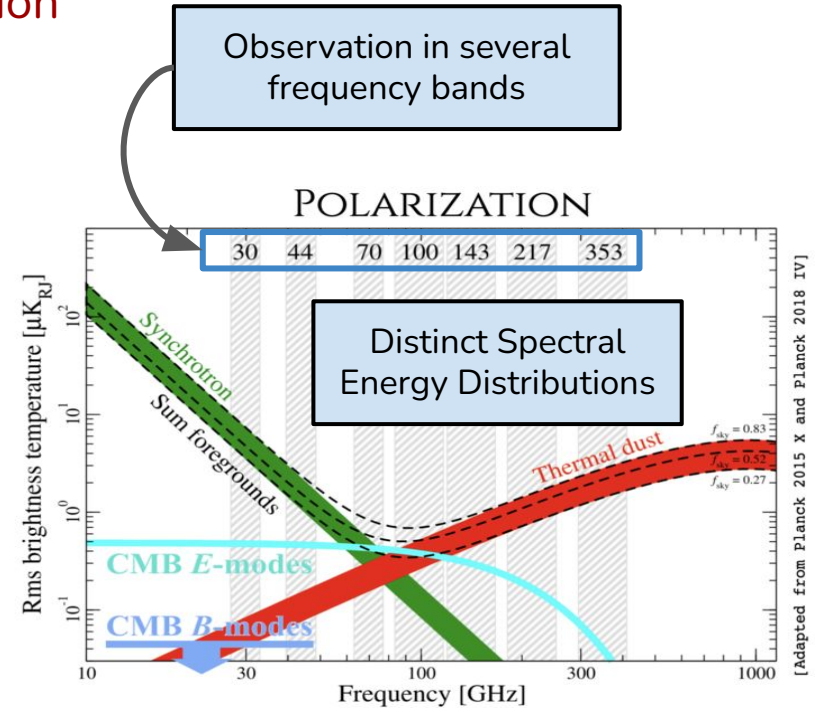
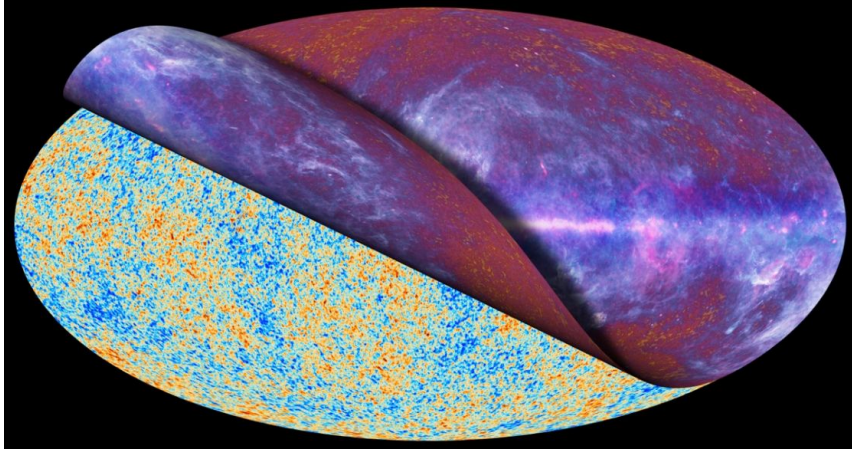


# Generative models of cosmological fields with scattering transforms on the sphere

Louise Mousset

E. Allys, M. Price, J. Aumont, J.M. Delouis, L. Montier, J. McEwen

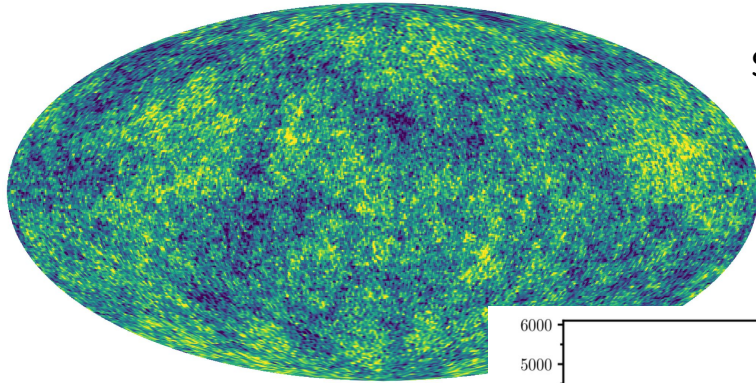
# Component separation using spectral information



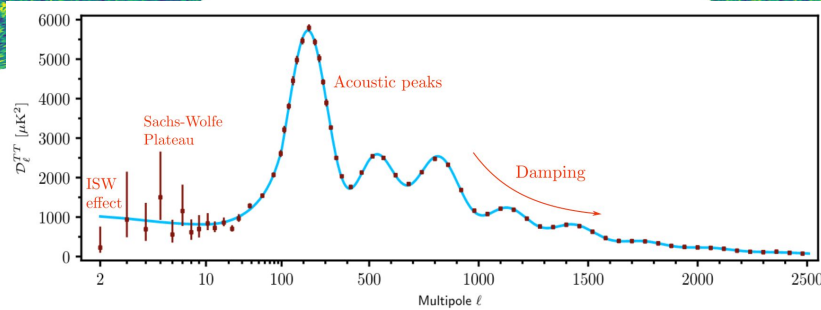
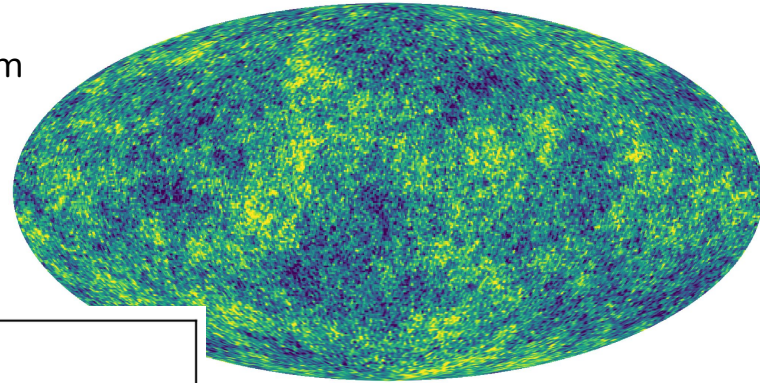
Credits: Planck

Can we also benefit from the fact that CMB and Galactic foregrounds have very different **spatial textures** ?

# CMB is a Gaussian field



Same power spectrum



Linear processes

Fourier modes are independent,  
no coupling  
between scales

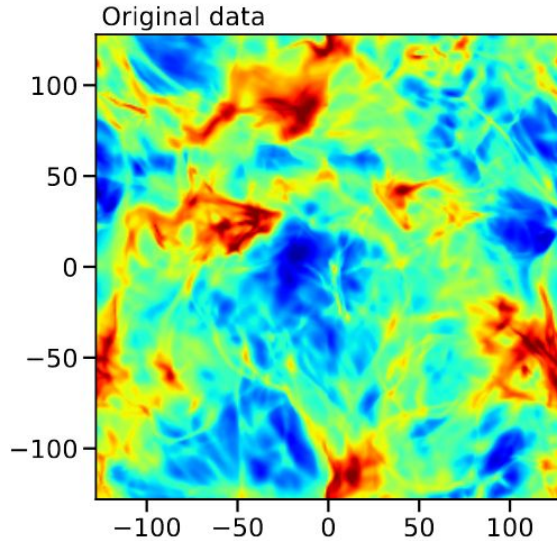
Power spectrum  
contains all the  
statistical information

# Non Gaussian field

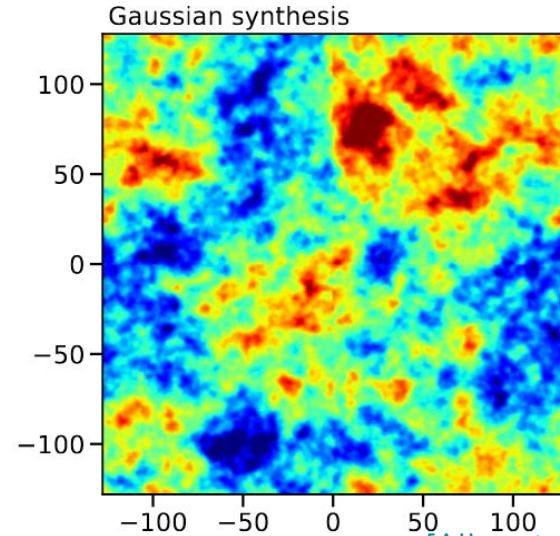
Non linear physical processes: gravity, magneto-hydrodynamics...

Coupling between scales, Non Gaussian structures (ex: filaments)

Power spectrum is not sufficient



Same power spectrum



[Allys et al. 2019]

=> Need higher order non-Gaussian statistics

# Scattering Transforms

**Scattering Transforms are** [\[Bruna et al. 2013\]](#)

- a set of non-Gaussian summary statistics
- constructed by successive wavelet transforms and non-linearities
- inspired from neural networks, but can be computed without a training stage, from a single image.

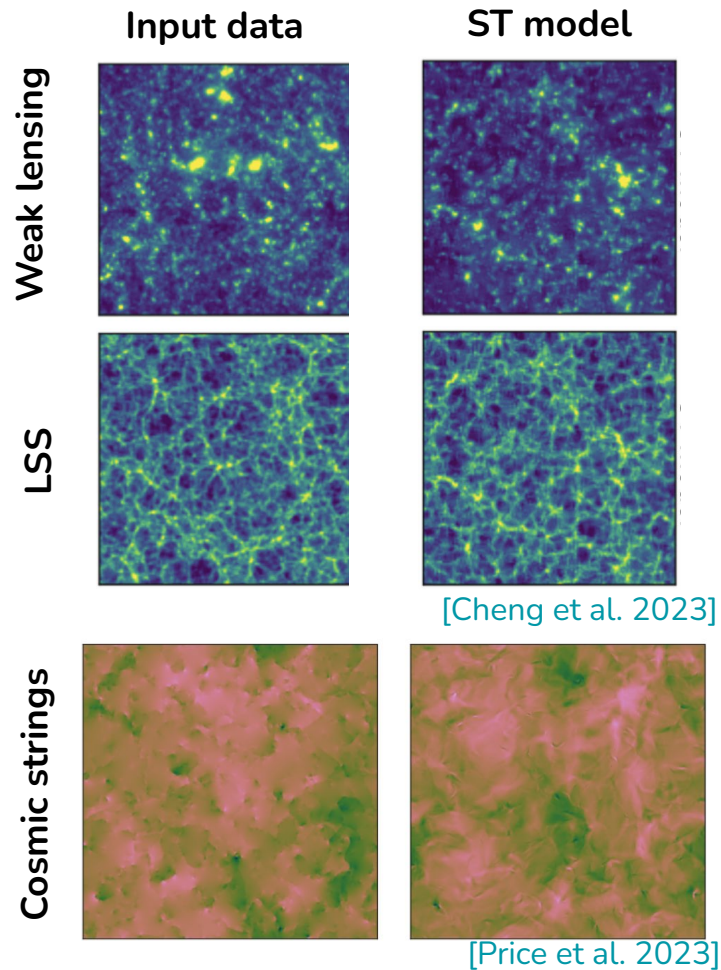
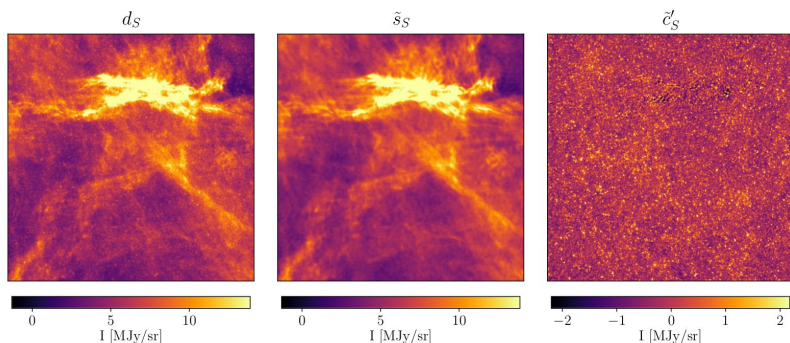
# Scattering Transforms

Scattering Transforms are [Bruna et al. 2013]

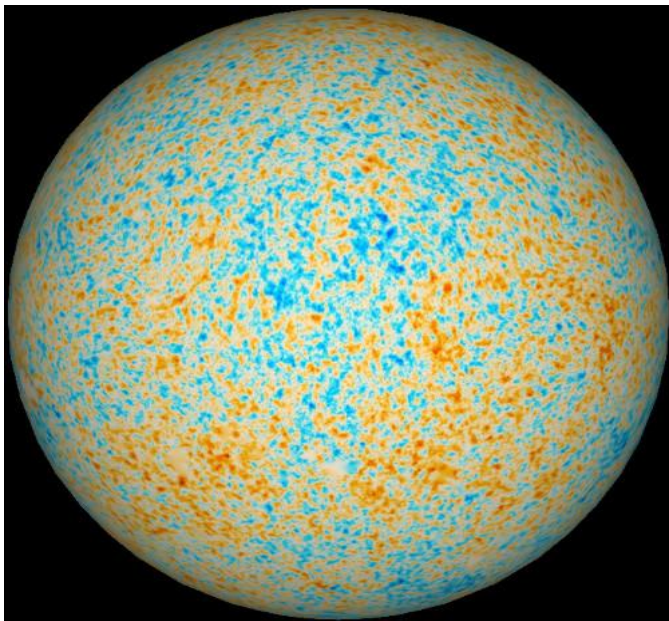
- a set of non-Gaussian summary statistics
- constructed by successive wavelet transforms and non-linearities
- inspired from neural networks, but can be computed without a training stage, from a single image.

Applications :

- Statistical description [Allys et al. 2019, Cheng et al. 2023]
- Generative model [Allys et al. 2020, Price et al. 2023]
- Component separation [Blancard et al. 2021, Auclair et al. 2023, Delouis et al. 2023]

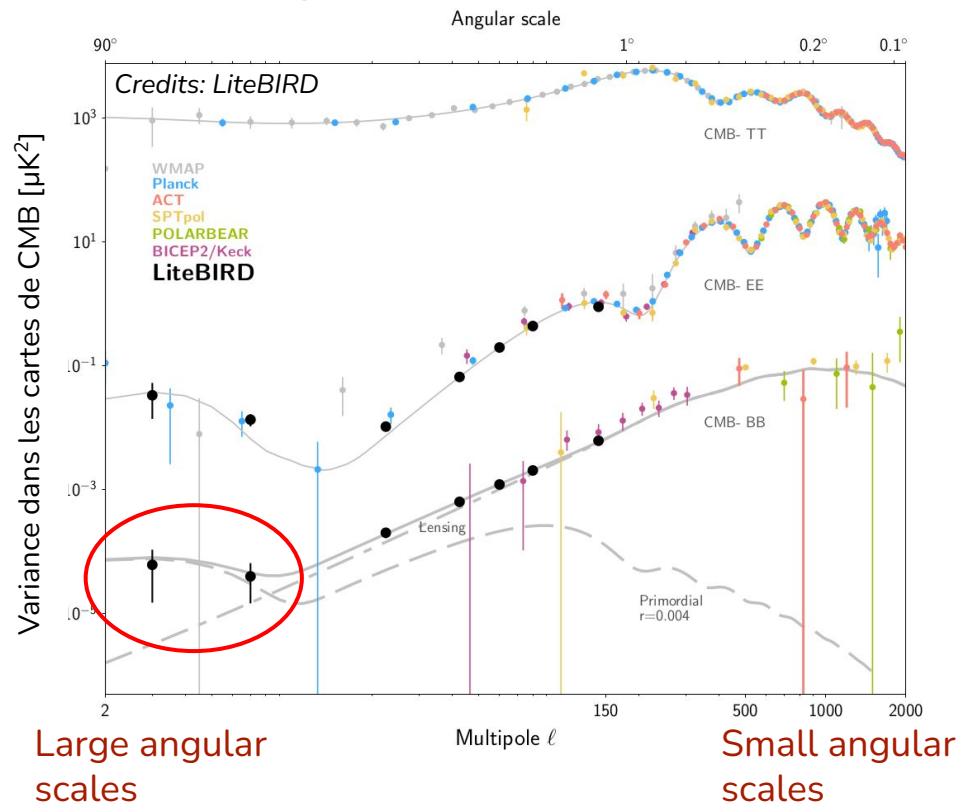


# Full-sky observation



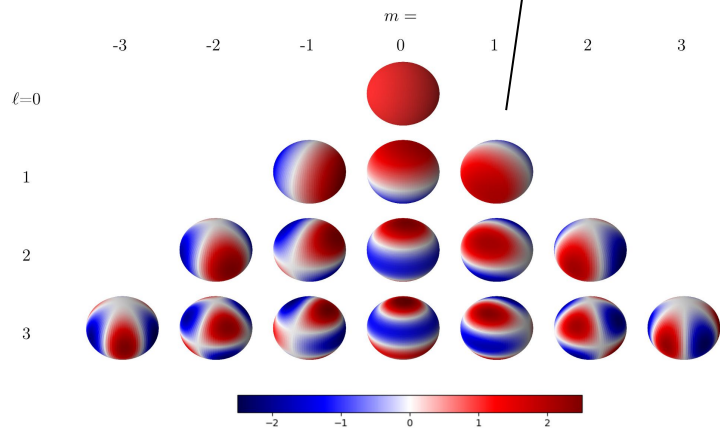
Spherical maps  
Planar approximation not valid

## Angular power spectrum



Spherical harmonic transform:

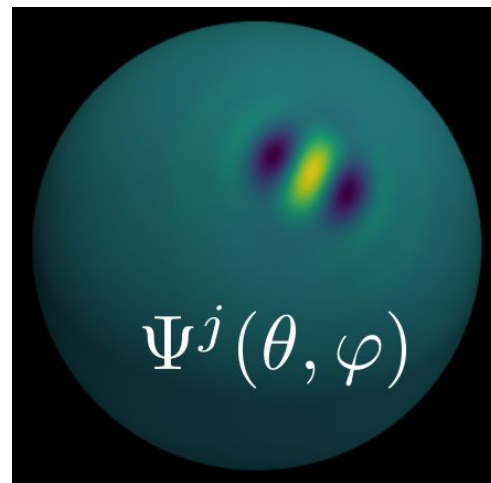
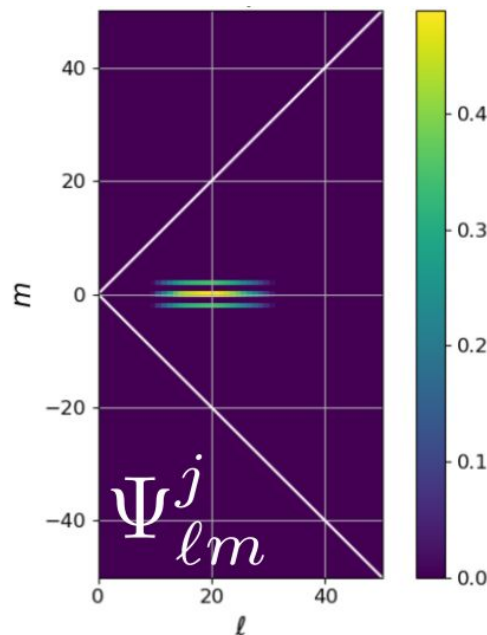
$$\Psi(\theta, \varphi) = \sum_{\ell m} \Psi_{\ell m} Y_{\ell m}(\theta, \varphi)$$



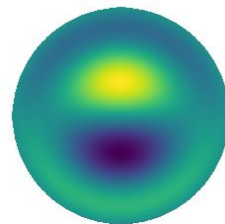
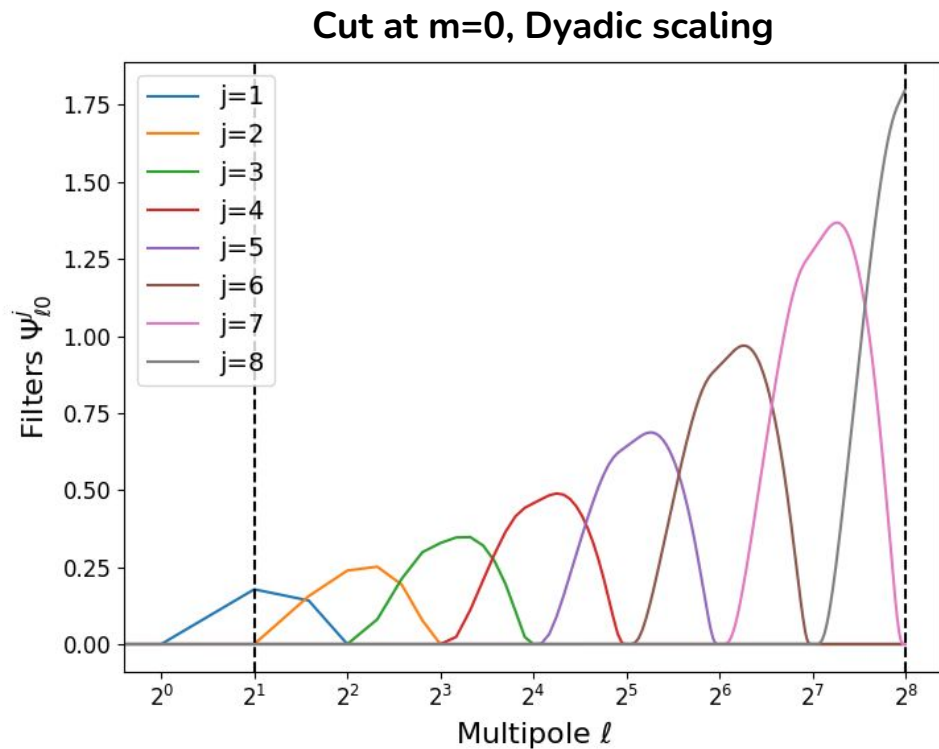
$$-\ell \leq m \leq \ell \quad \ell \sim \frac{\pi}{\theta}$$

Wavelet filters:

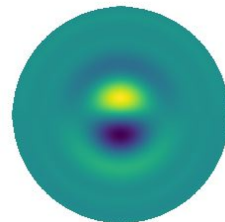
$$\Psi_{\ell m}^j = \sqrt{\frac{2\ell + 1}{8\pi^2}} \underbrace{\kappa_{\ell}^j}_{\text{Filter scale}} \underbrace{\xi_{\ell m}}_{\text{Directionality}}$$



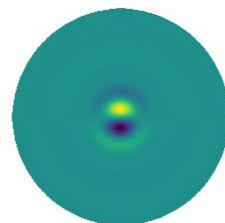
# Filter set scaling



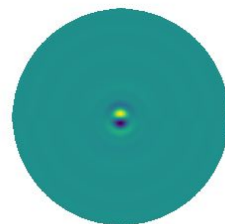
$$j = 1$$



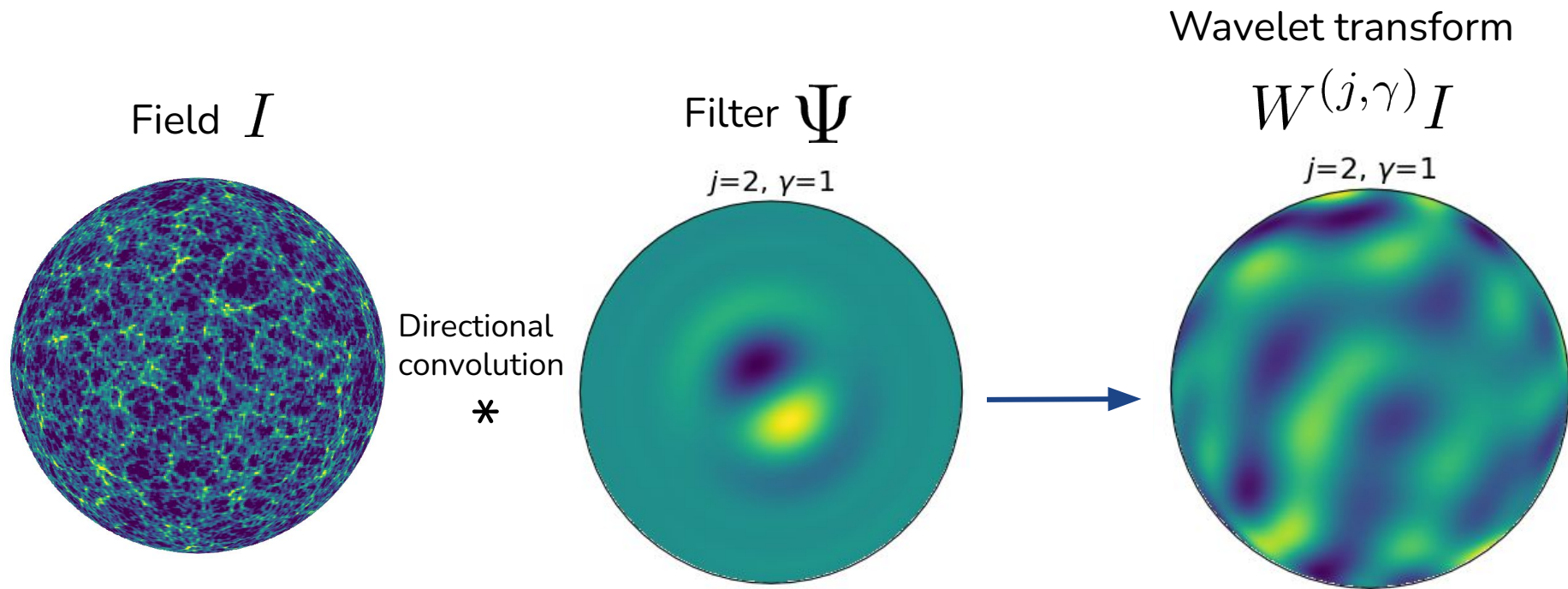
$$j = 2$$



$$j = 3$$



$$j = 4$$



**Coefficients associated to a single scale:**

$$\begin{aligned} S_1^\lambda &= \langle |W^\lambda I| \rangle \\ S_2^\lambda &= \langle |W^\lambda I|^2 \rangle \end{aligned}$$

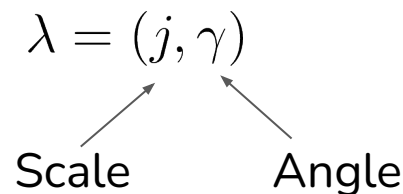
Averages over pixels

**Coefficients to probe the coupling between scales:**

$$\begin{aligned} S_3^{\lambda_1, \lambda_2} &= \text{Cov} [W^{\lambda_1} I, W^{\lambda_1} |W^{\lambda_2} I|] \\ S_4^{\lambda_1, \lambda_2, \lambda_3} &= \text{Cov} [W^{\lambda_1} |W^{\lambda_3} I|, W^{\lambda_1} |W^{\lambda_2} I|] \end{aligned}$$

For instance, with  $j = [1, 8]$  and  $\gamma = [1, 5]$   
 $\Rightarrow \sim 10^3$  coefficients.

$W^\lambda I$  the wavelet transform

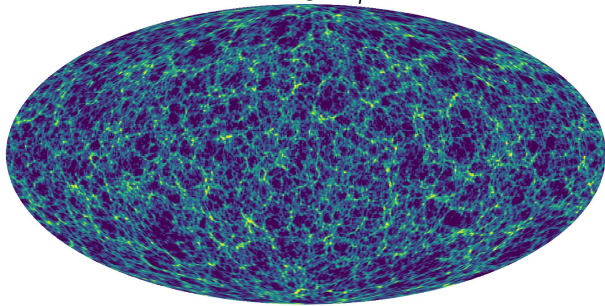


# Maximum entropy generative model

Summary statistics:

$$\phi = \{S_1, S_2, S_3, S_4, \langle I \rangle, Var(I)\}$$

Target  $\phi_t$

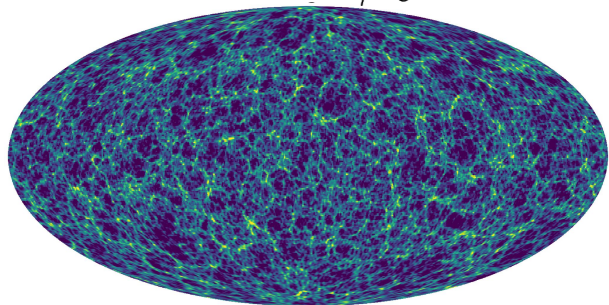


# Maximum entropy generative model

Summary statistics:

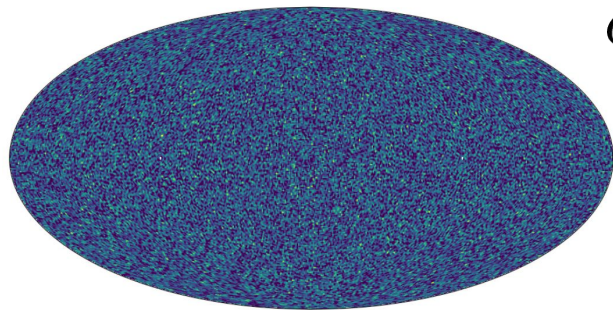
$$\phi = \{S_1, S_2, S_3, S_4, \langle I \rangle, Var(I)\}$$

Target  $\phi_t$



Start (white gaussian noise)

$\phi_{\text{start}}$

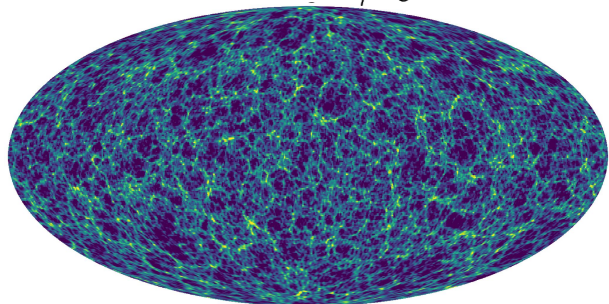


# Maximum entropy generative model

Summary statistics:

$$\phi = \{S_1, S_2, S_3, S_4, \langle I \rangle, Var(I)\}$$

Target  $\phi_t$



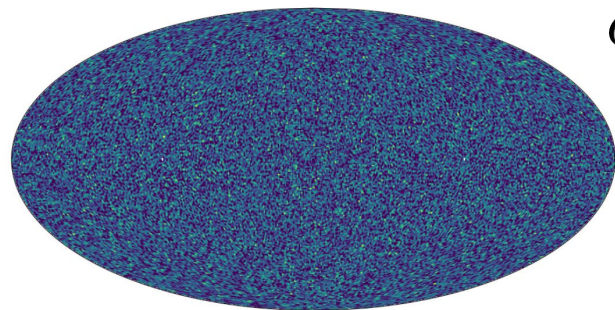
Loss function to be minimized:

$$\mathcal{L}(\phi_t, \phi_i) = |\phi_t - \phi_i|^2$$

Iteration i

Start (white gaussian noise)

$\phi_{\text{start}}$

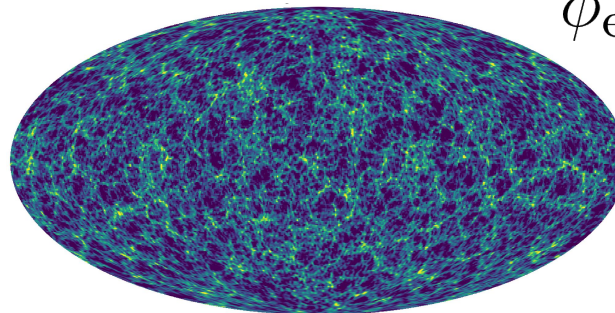


Iterate on the pixels or on  
the harmonic coefficients

Gradient descent  
Automatic differentiation (JAX)  
on GPU

Generated

$\phi_{\text{end}} \simeq \phi_t$



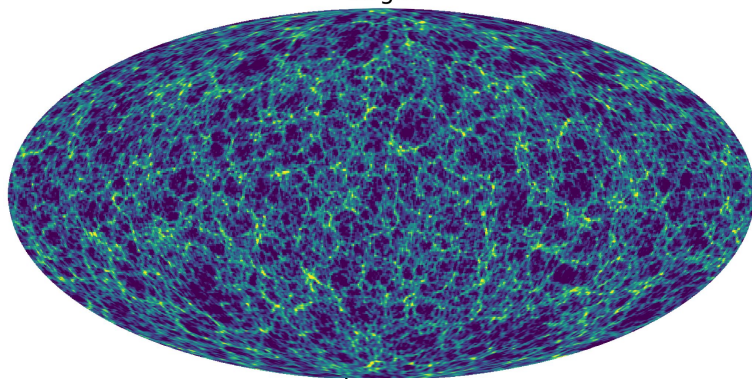
~5min at  
Nside=128

# Visual validation

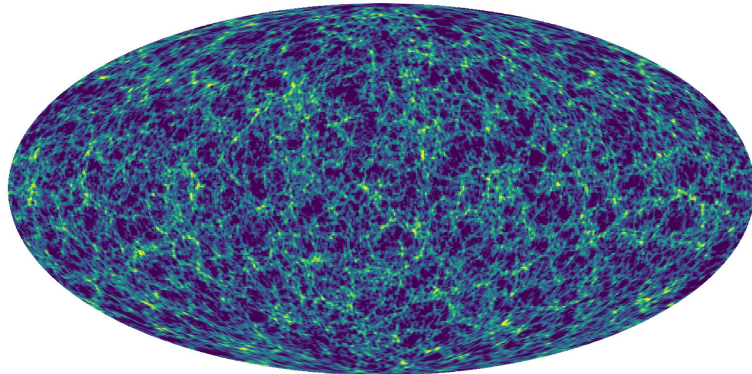
## Weak lensing

N-body simulation [Kacprzak et al. 2022]

Target

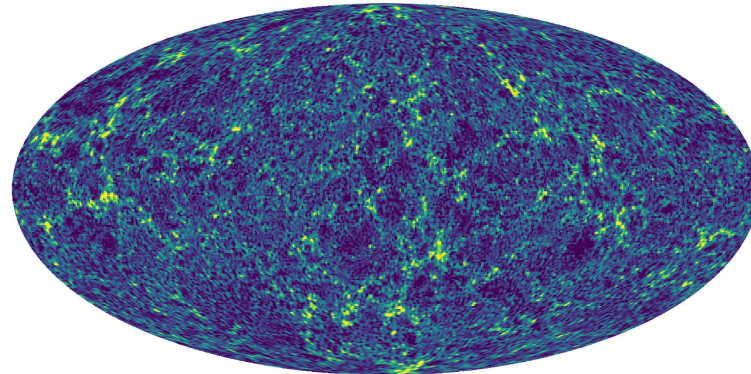
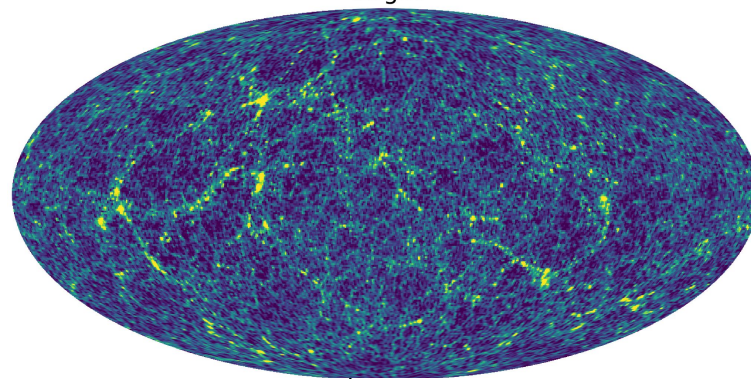


Generated



## Density field

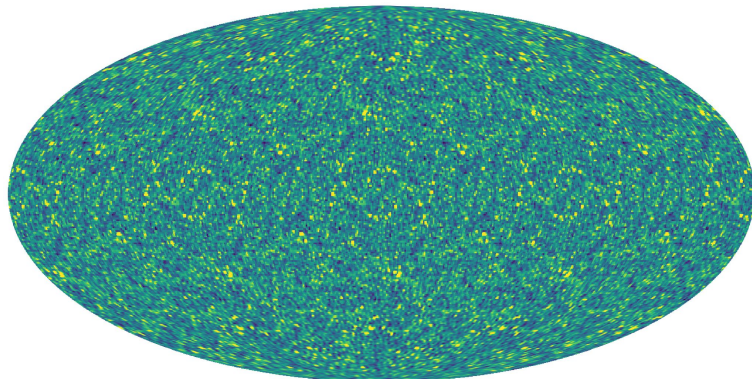
Quijote simulation [Anbajagane et al. 2023]



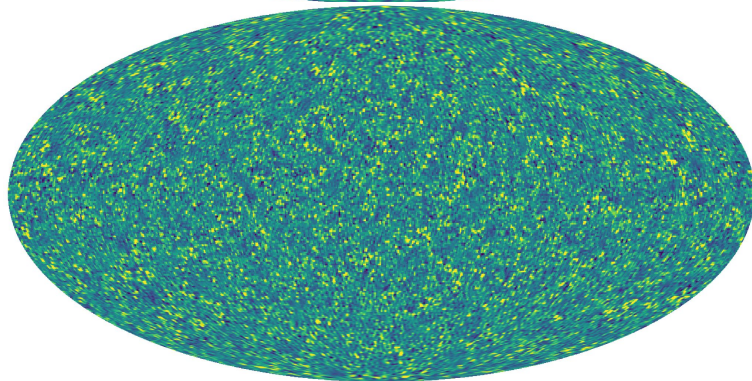
# Visual validation

Thermal Sunyaev-Zeldovitch  
effect (tSZ) [\[Ade et al. 2019\]](#)

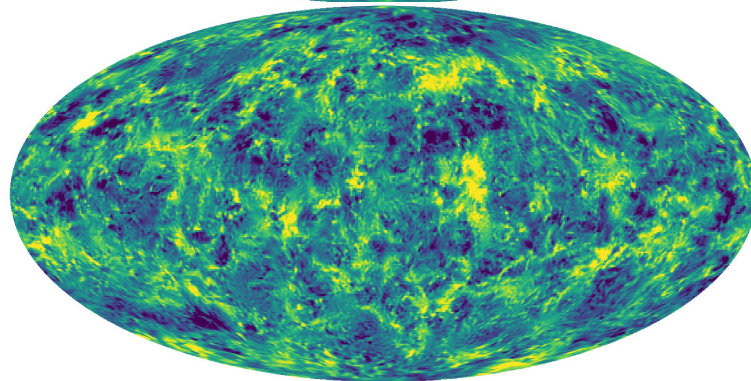
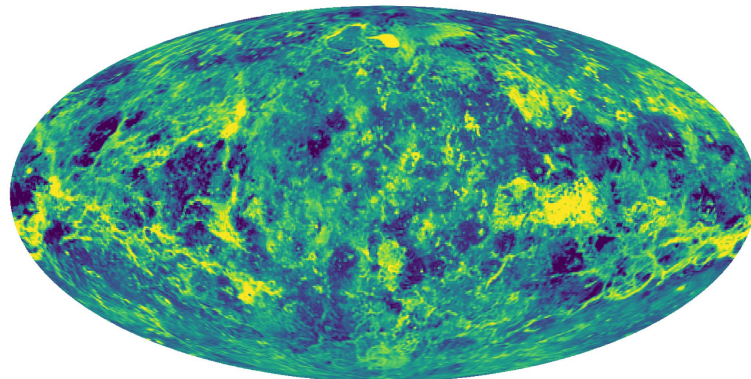
Target



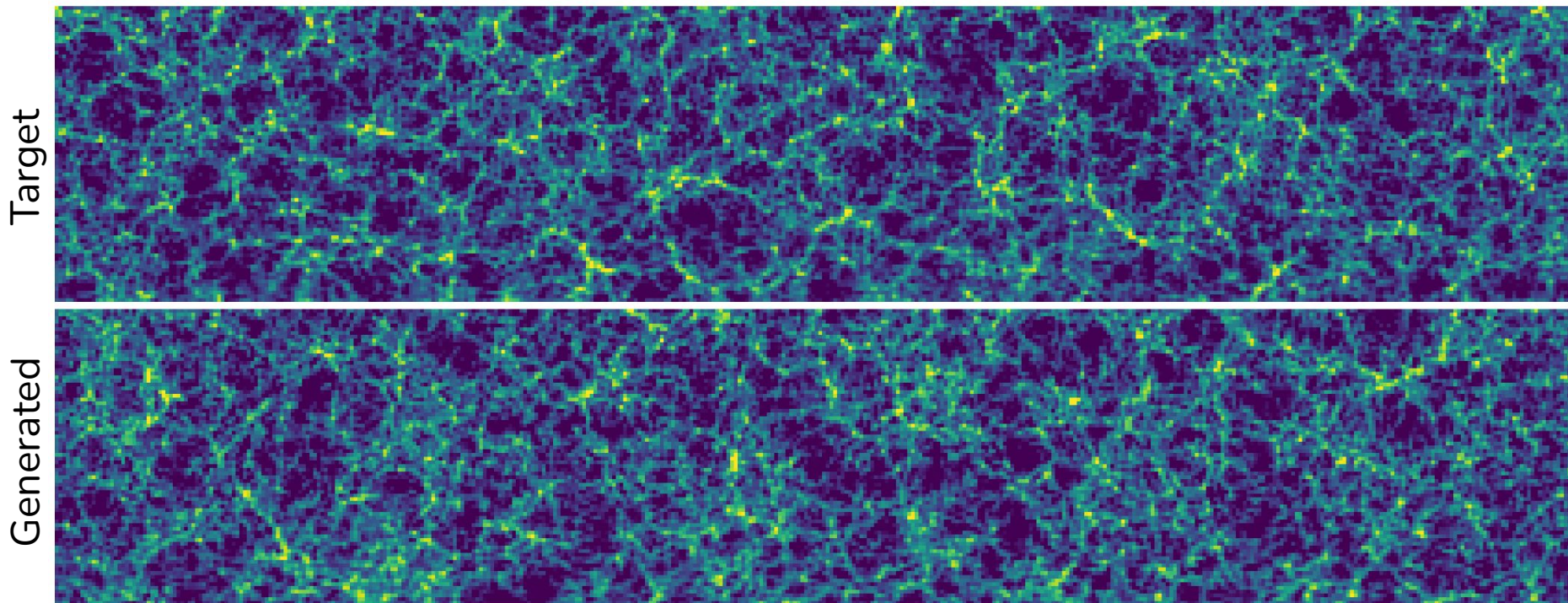
Generated



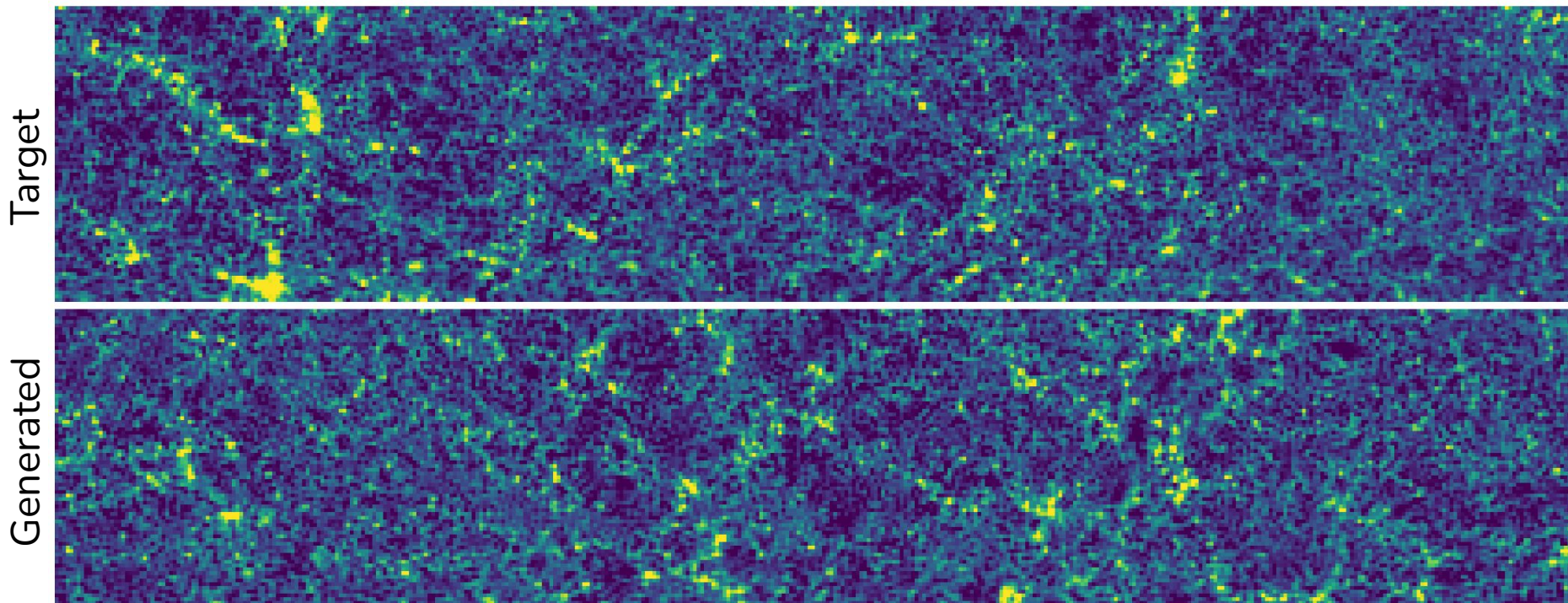
Venus surface



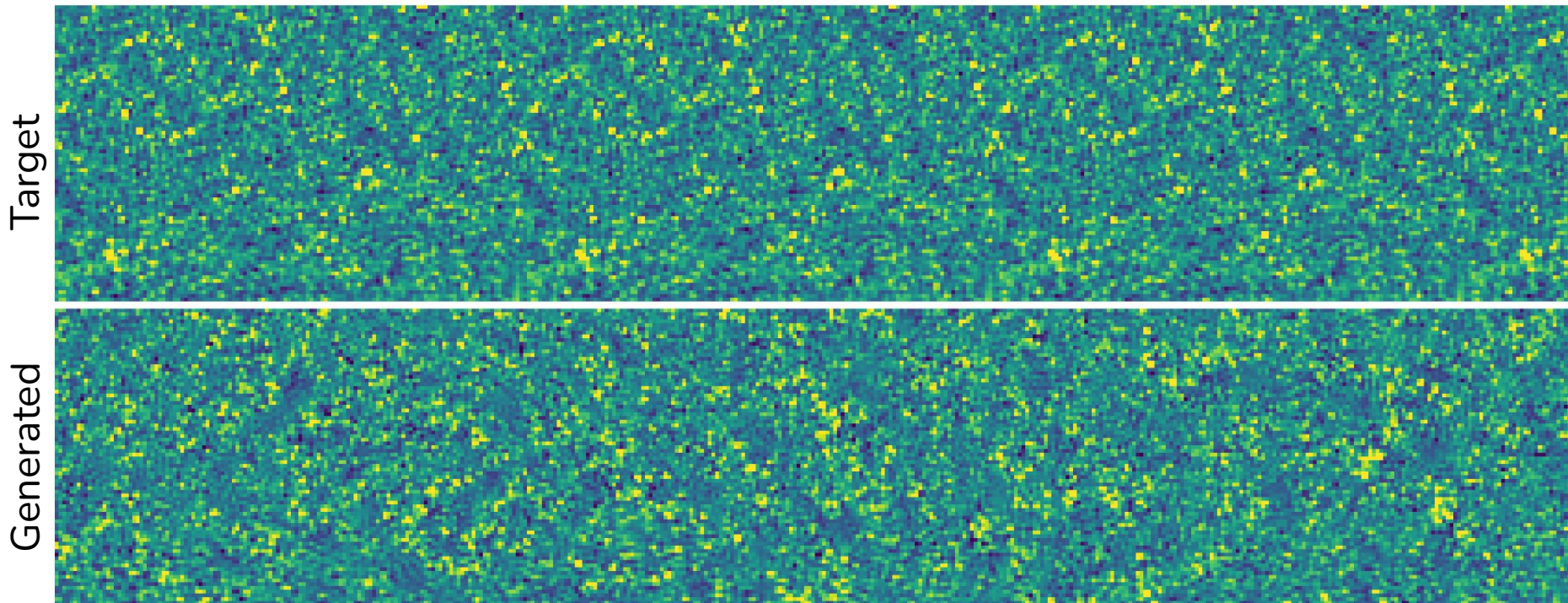
## Zoom on a patch - Weak lensing



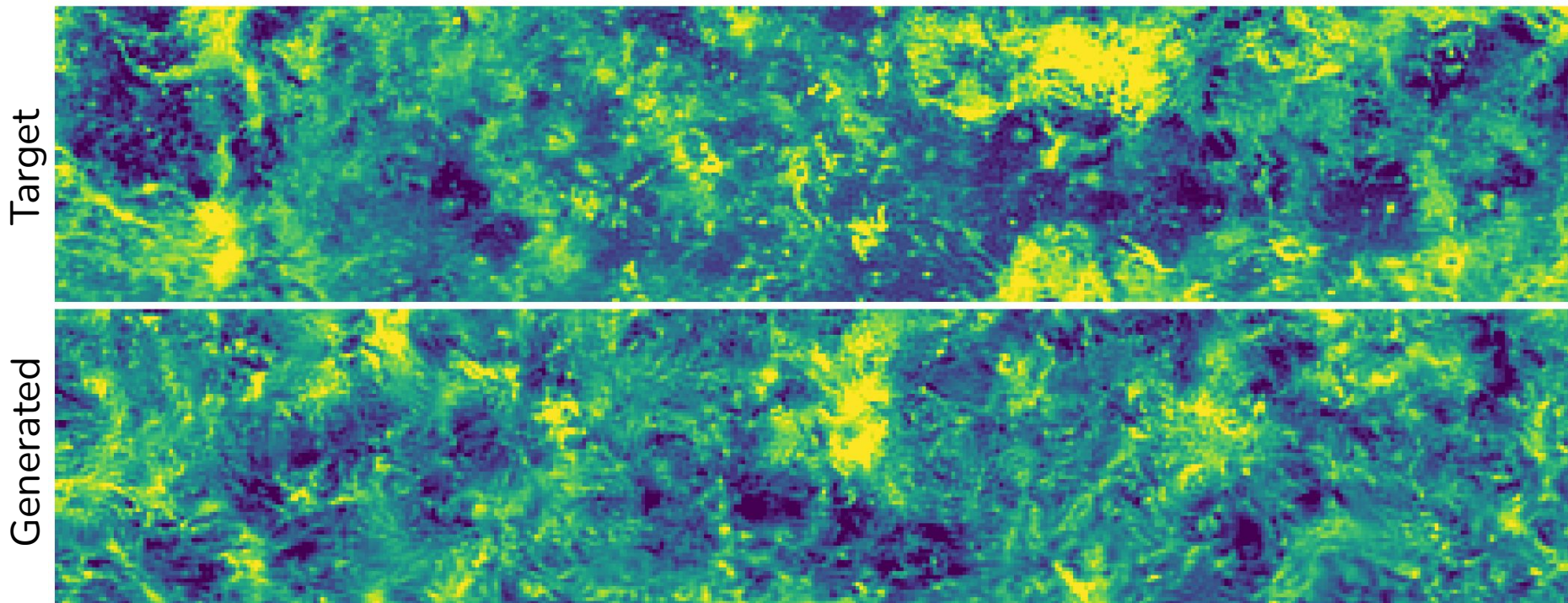
## Zoom on a patch - Density field



## Zoom on a patch - tSZ

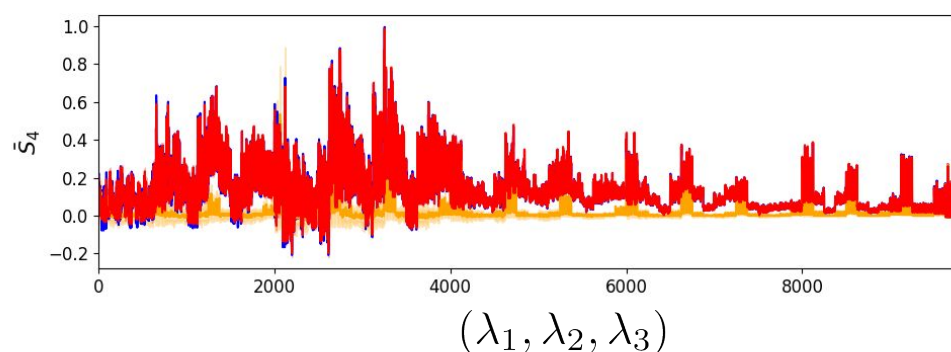
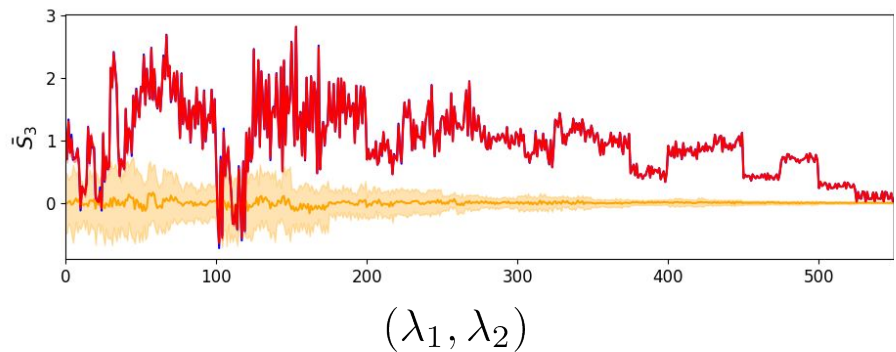
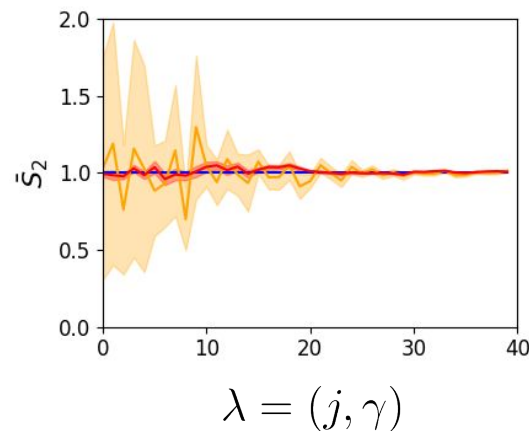
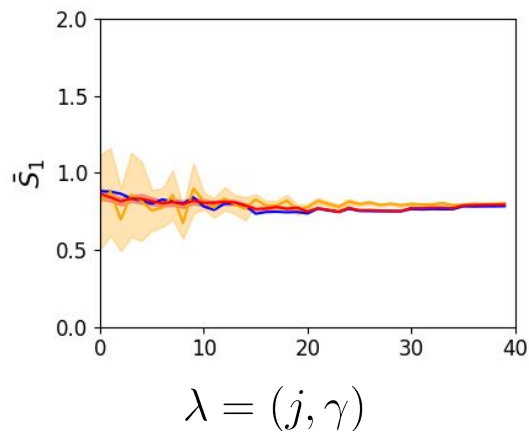


## Zoom on a patch - Venus



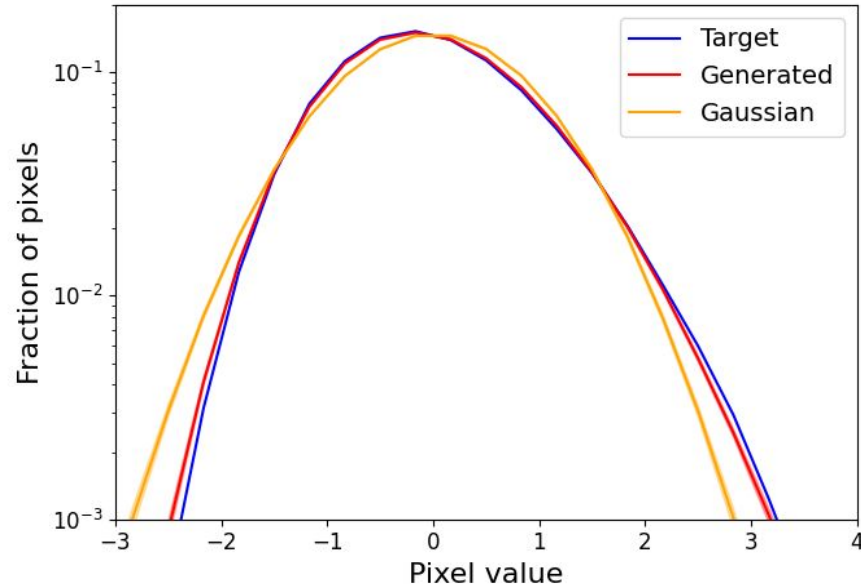
# Scattering covariances on the density field

— Target    — Gaussian    — Generated

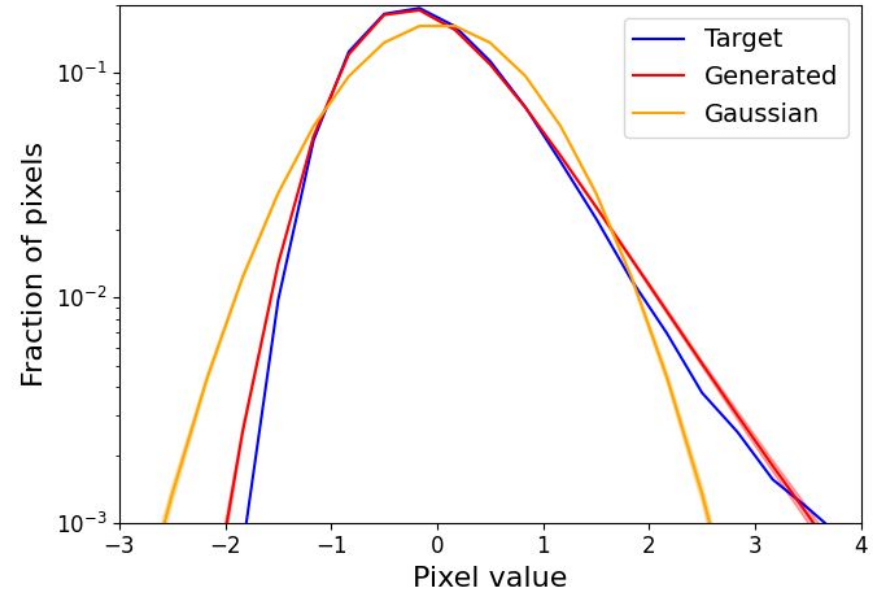


# Statistical validation - Probability Density Function (PDF)

## Weak lensing

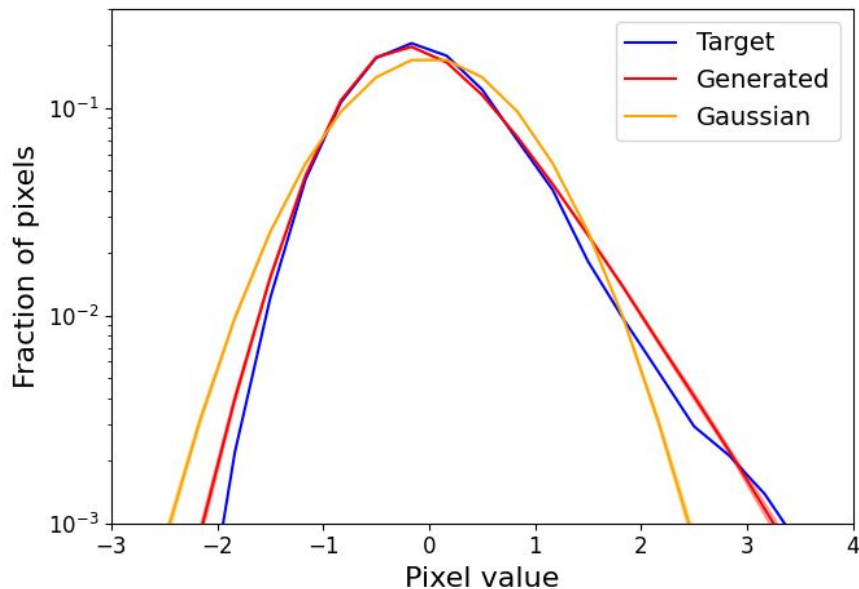


## Density field

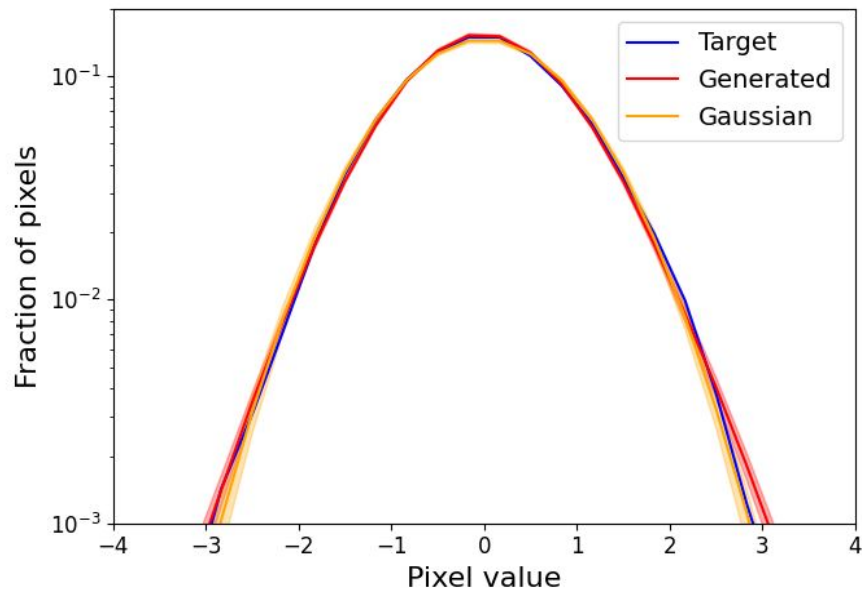


# Statistical validation - Probability Density Function (PDF)

tSZ

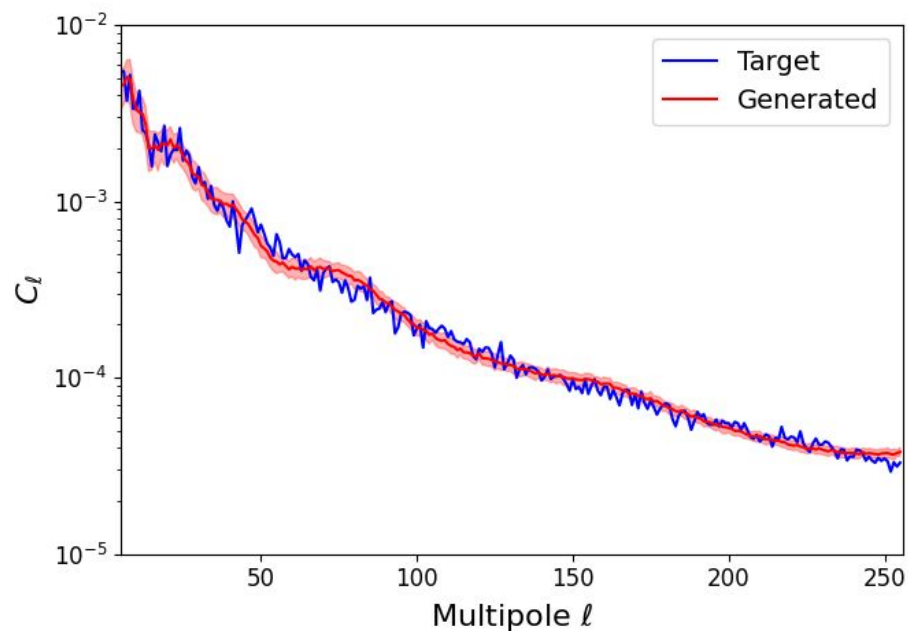


Venus

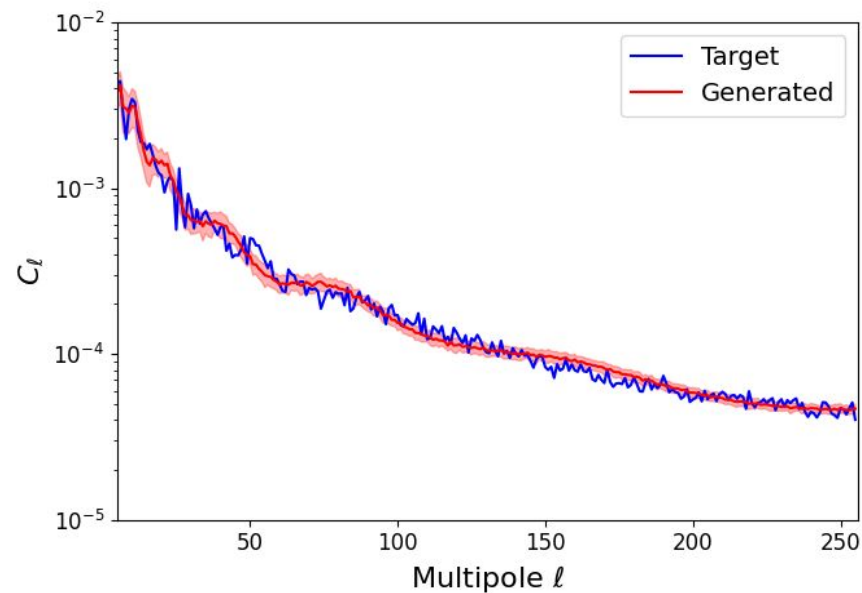


# Statistical validation - Angular power spectrum

## Weak lensing

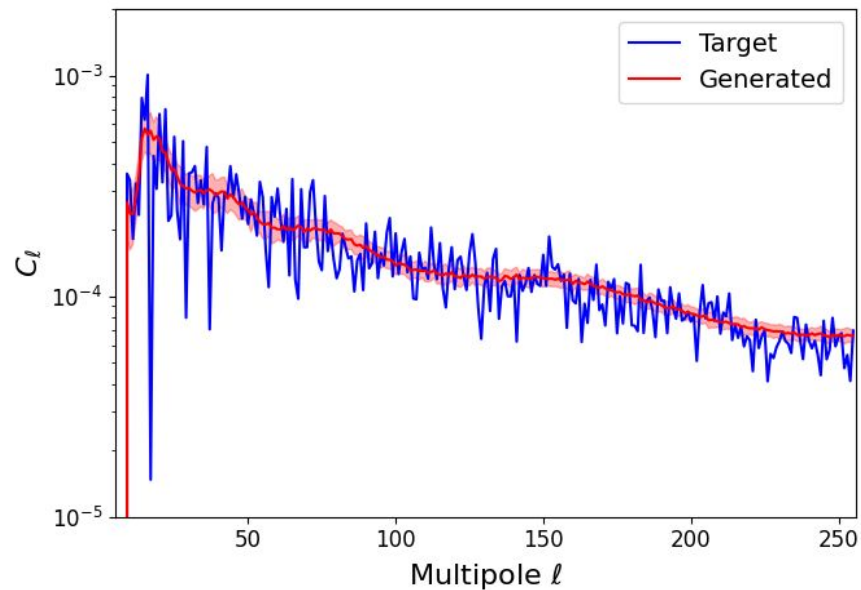


## Density field

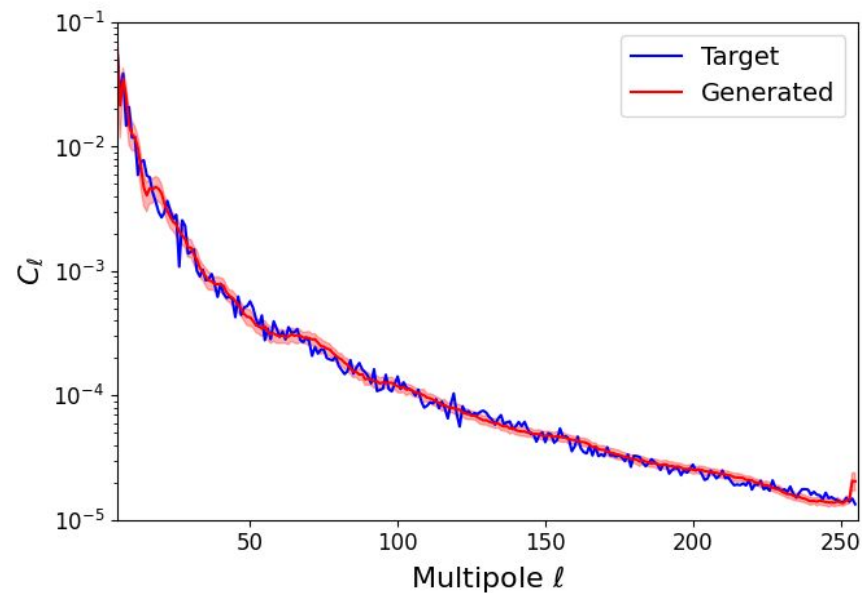


# Statistical validation - Angular power spectrum

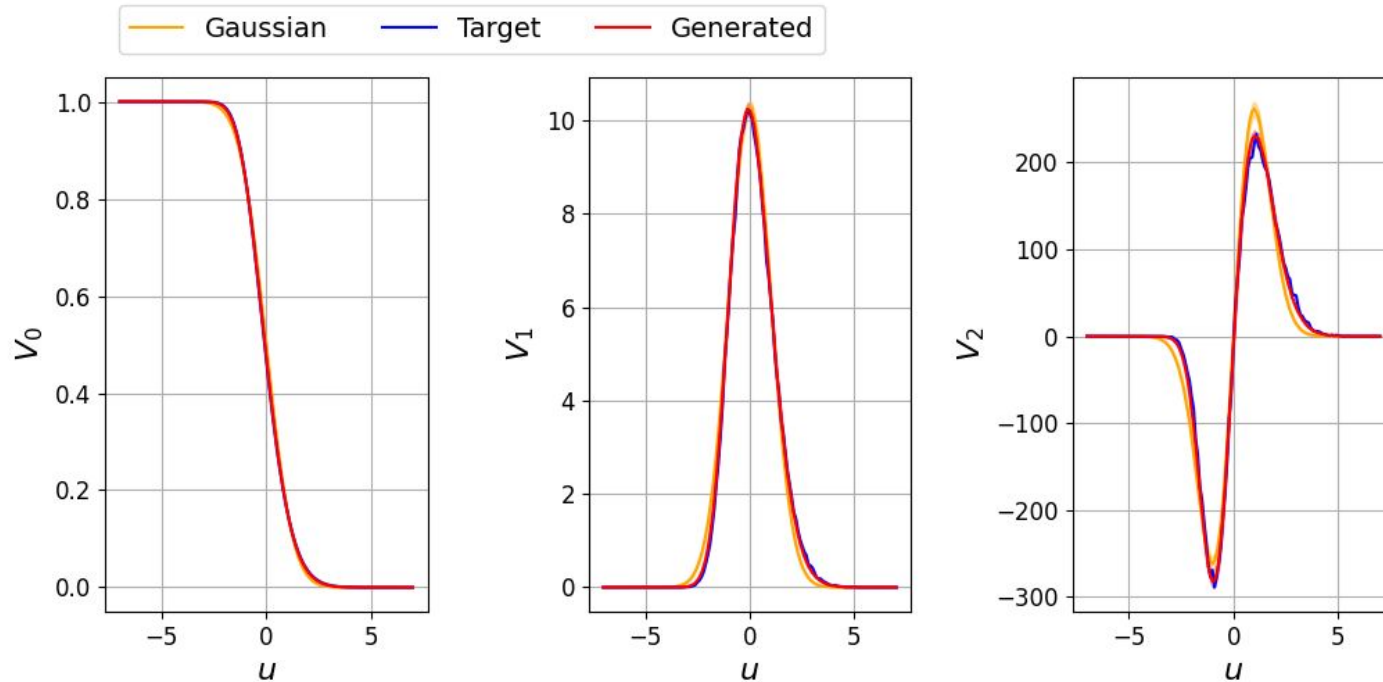
tSZ



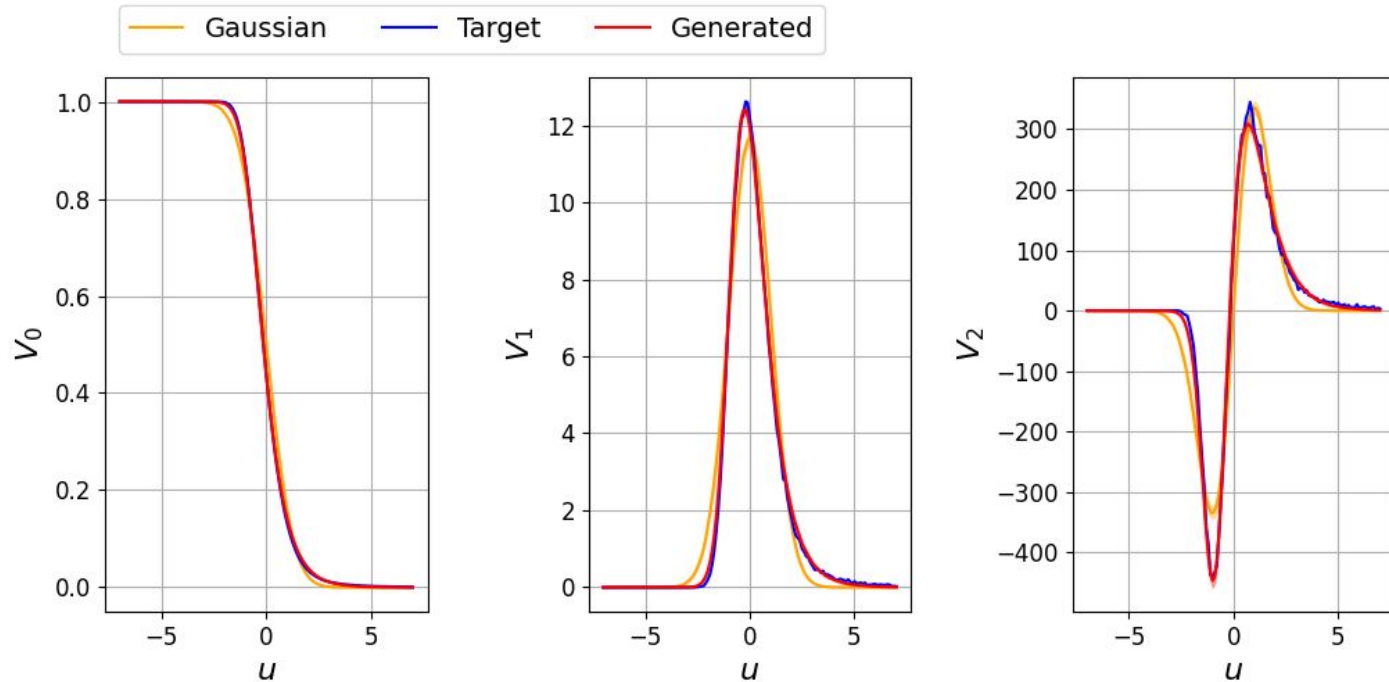
Venus



## Weak lensing

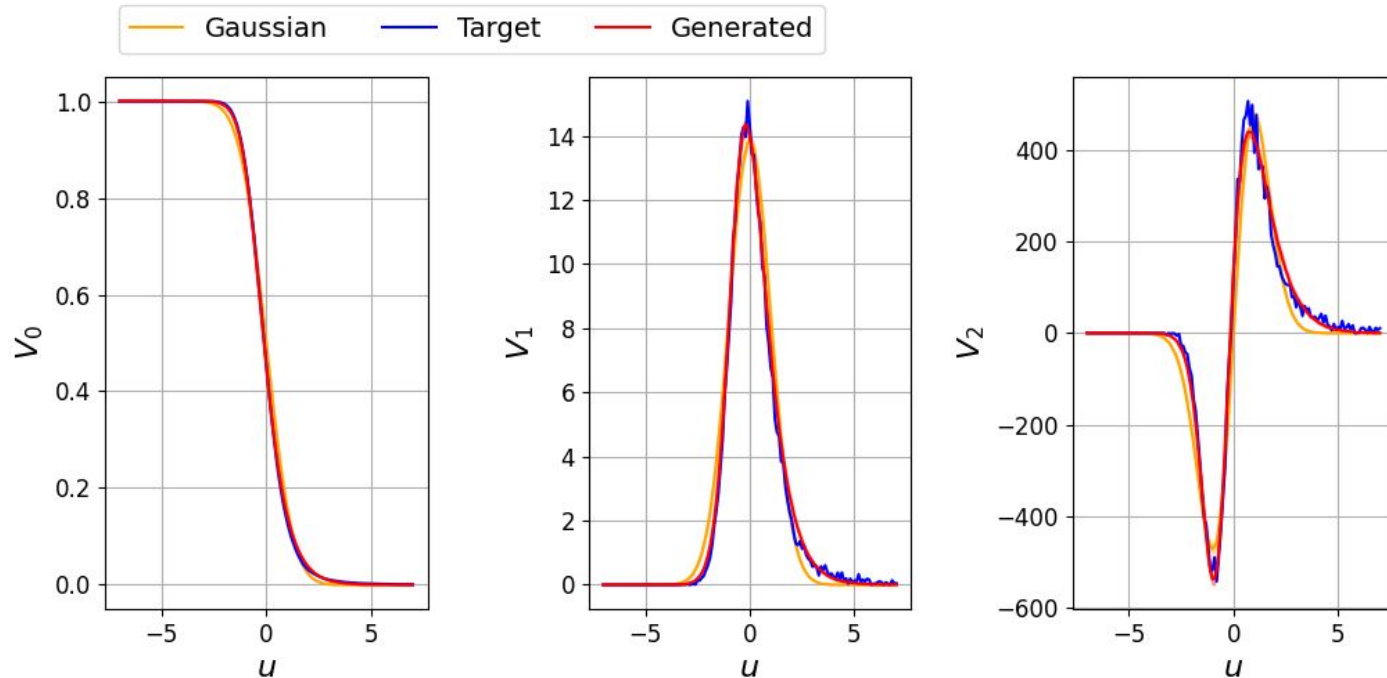


## Density field



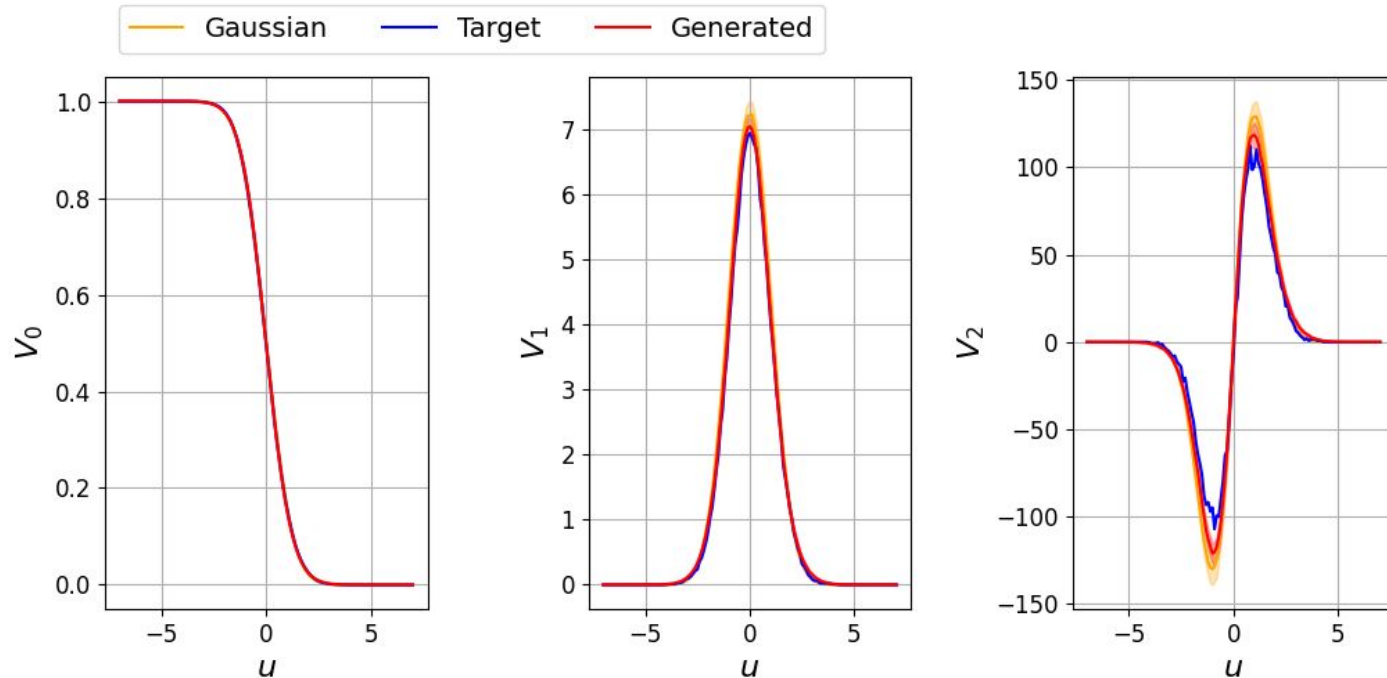
# Statistical validation - Minkowski functionals

tSZ



# Statistical validation - Minkowski functionals

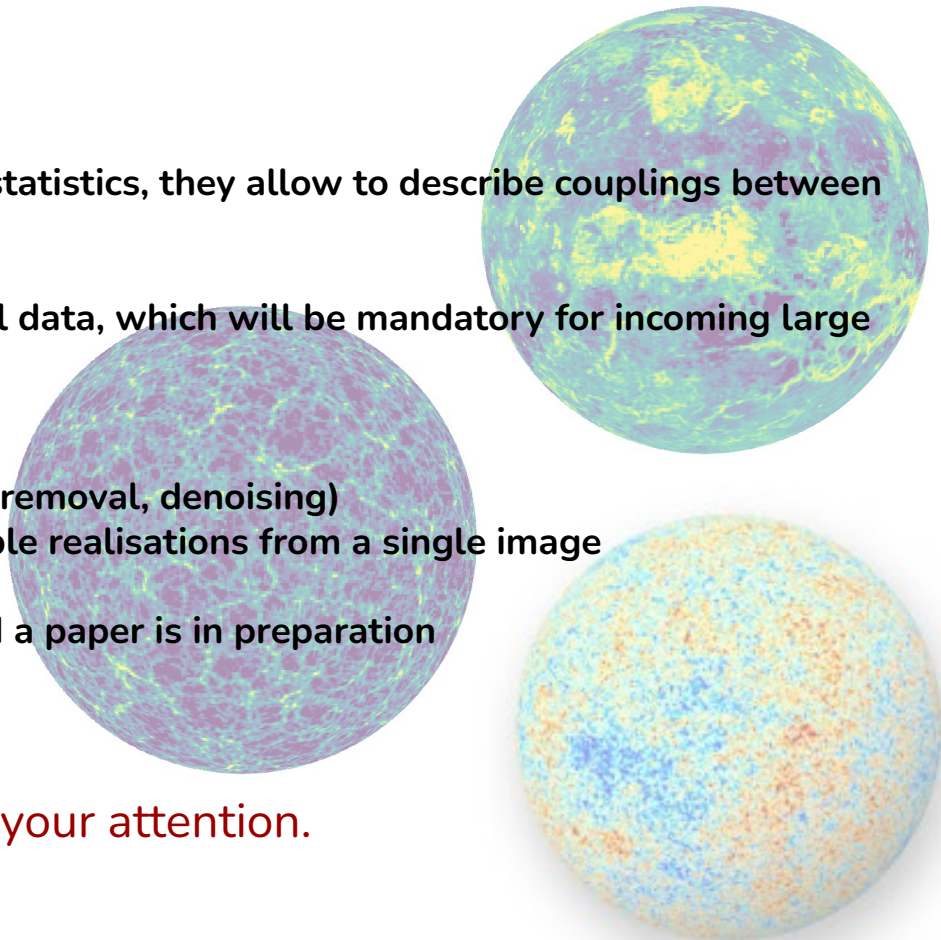
## Venus



# Summary

- Scattering transforms are non Gaussian statistics, they allow to describe couplings between scales inherent to non-linear physics.
- We have adapted these tools to spherical data, which will be mandatory for incoming large scale surveys.
- Two main applications for cosmology:
  - component separation (foreground removal, denoising)
  - generative model to produce multiple realisations from a single image
- The software will be soon on GitHub and a paper is in preparation

Thank you for your attention.



$$W_{\ell mn}^j = \frac{8\pi^2}{2\ell+1} I_{\ell m} \Psi_{\ell n}^{j*}$$

Inverse Wigner Transform

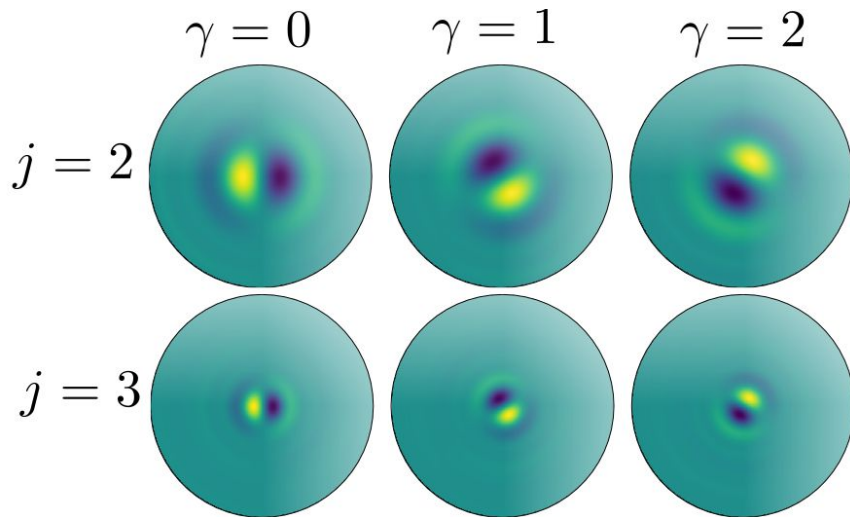
$$W^j(\alpha, \beta, \gamma) \rightarrow W^{j\gamma}(\theta, \varphi)$$

Euler angles

Scale

Angle

Convolution of a Dirac map:



# Microcanonical generative model

$$\Omega_\varepsilon = \{x : |\phi(x) - \phi(x_t)|^2 < \varepsilon\}$$

Microcanonique:  
 $\mathcal{P}(x_1) = \mathcal{P}(x_2) = \mathcal{P}(x_i)$

