First principle calculations for bound state problems in QCD

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5 Nov, 2024

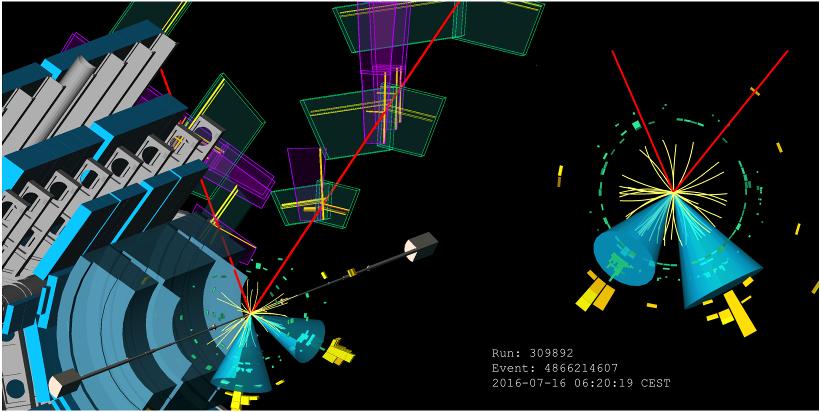
Outline

- Experimental motivation and Lessons form perturbative QCD
- Math of non perturbative QCD
- Applications

Experimental motivation and Lessons form perturbative QCD

- Experimental motivation
- Perturbative QCD
- Analytic structure of amplitudes
- Existing approaches to bound state physics

LHC physics:jets



JLab physics: low energy data, spectra(JAM collaboration), resonances

Perturbative QCD

- Jet physics
- DGLAP evolution, HERA data
- alpha-s evolution
- Contrast with first principle calculations of low energy quantities(e.g. masses)

Amplitudes

- $$J(p,m) = \int d^{dL} q q^m \prod ((q_i + p_i)^2 + m_i^2)^{\alpha_i} \qquad (1)$$
- The case $m_i = 0$: amplituhedron (N.Arkani-Hamed,...). (S): differential equation for the vertex.
- **Th:**(S)

$$\frac{\partial J_m}{\partial \{p_s p_t, m_i^2\}} = \sum_S \frac{A_{S,m,m',\{p_s p_t, m_i^2\}}(p_s p_t, m_i^2)}{\prod_{l \in S} L_l} J_{m'} \qquad (2)$$

- Analytic structure is determined by the intersection theory of Landau varieties(zeros of the Landau polynomials)
- Beyond $m_i = 0$: 1) the case of propagators solved(S). Deep work by S.Bloch, P.Vanhove,... 2) vacuum bubbles

Existing approaches to bound state physics

- chiral perturbation theory
- non local models. C.Ji,W.Melnitchouk,...
- Schwingder Dyson: the truncation,..., the problem: not structure equation
- Bethe-Salpeter: the problem: gauge invariance(path integral over quotient space, while wave functions are sections of the bundle)
- Light front methods(S.Brodsky,...)
- Lattice (mutilates the theory)

Math of non perturbative QCD

- Approaches to non perturbative QCD
- Functional differential equations
- Methods of solution, results
- Comparison with generalized SD hierarchies

The problem

- As. L. McLerran said: "hard to argue against QCD". But what "is" QCD? Lagrangian is not enough.
- Bethe-Salpeter equations are not generalizable beyond 2-particle functions in a gauge invariant way.
- Need basic dynamical equations, consistent with previous findings.
- Deal effectively with the tower of partons($\geq$ 90% of the proton mass comes from gluons). Need formalism powerful enough to deal with functions of any number of partons.

In this talk: analysis of possible equations, and methods of their solution

QED

- g-2: 6 or 7 loops, agreement with experiment of Gabrielse(geonium).
- Lamb shift: 40 years of work to get 1loop result beyond Bethe log(K.Pachucki 1991).
- Diagrammatic expression, consistent with the diagrammatic expression of Bethe-Salpeter. (U.Jentschura,S.Karshenboim,...)
- 2-particle wave functions never part of calculation(compare helium at high quantum number)

Functional Schrodinger equation

- $$i\frac{\partial}{\partial t}F(t)[a(\vec{p})] = \hat{H}(t)[a(\vec{p}), \frac{\delta}{\delta a(\vec{p})}]F(t)[a(\vec{p})] \qquad (3)$$
- $\hat{H}$ is the second quantized Hamiltonian(obtained by plugging in the second quantized field into the interaction part)
- Get back perturbation theory
- Covariance properties, gauge invariance
- Relationship with Bethe-Salpeter
- Methods of solution

Functional differential equations

- Functional Laplacian

$$\int dx\ dy K(x,y)\frac{\delta^2}{\delta a(x)\delta a(y)}F[a] = EF[a] \tag{4}$$

- $K(x,y)$ algebraic. E.g. rational. $a = a(x), x \in \mathbb{C}$
- The bound state problem is well posed: **Th:** In the case when the boundary conditions are given by $F[b] = F_1[a], \partial_n F[a] = F_2[a]$ for $a \in B$, the solution is unique.
- In practice: integrability conditions instead of boundary conditions

A class of solutions by perturbation theory

- $$\int (P(x,y))^{\alpha} f(y) d^1 y = \lambda f(x) \tag{5}$$
- In practice there are infinitely many singularities on unphysical sheets of analyticity(the solution is "resurgent")
- Prediction: generalization of Landau analysis.
- Can this be observed?

A class of solutions: ansatz choice

- Choose factorizable solutions:

$$F(p_1, ..., p_n) = \sum F_k(p_\parallel) L^{\alpha+k} \tag{6}$$

- L_i must form an A_∞ algebra
- L_i may be entire functions($=$ generalized polynomials)
- Encompasses PDF parametrization
- For TMD parametrization, need to generalize the anzats

Comparison with p-space hierarchies

- Any version of SD hierarchy in p-space

$$\hat{L}_n G_n(p_1, \dots, p_n) = \sum \int \prod G_{k_i}(p; q)(dq) \qquad (7)$$

- Branch loci are generated by the zeros of $L_n(p)$
- The set of singularities must be closed under the A_∞-algebra operations

Bound state equation

- Generalized hierarchy
- Functional differential equations
- Infinite dimensional Cayley Koszul complex
- Riemann Hilbert problem

Riemann-Hilbert approach

- Factorization
$$J_m = \sum J_k(p_{\|})L^{\alpha_m+k} \tag{8}$$
- $F_g(p)$, $g \in \pi_1$
-
$$F_{g*g_i}(p) = M_g(p)F_g(p) \tag{9}$$
- The functions $L_i(p)$ generalize the perturbative Landau polynomials. In PT, M_g are constant(see work on intersection pairing in homology S.Weinzier, S.Mizera,..)

Functional renormalization group

- Wilsonian approach taken to its extreme: partial integration over modes
- Polchinski and Wetterich equations
- Holomorphic Local RG: integrating out holomorphic families of modes.
- Physics: renormalization energy scale fixing in multiscale problem(simplest case: helium atom)

Resurgent QCD

- Functional differential equations
- Solutions in terms of functions with regular(p-space) or irregular(x-space) singularities along sets that depend on masses and coupling and RG scale(s)
- Multidimensional analogies of 1-dim resurgent functions(as e.g. in works on Painleve eqns., Schiappa,Dunne, Unsal,math: Shishikura,Dudko, Sauzin,...)
- Systematic ways to solve integral eqns with singular kernels

Applications

- Spectral data from JLab. Low energy scattering in the resonance region.
- Positron beam physics at JLab
- Muon g-2 puzzle

Positronium scattering

- "Physics with Positron Beams at Jefferson Lab 12 GeV"
- 2-photon exchange
- Analytic models of the form factor
- The usual approach: dipole model (i.e. effective vertex + propagator with resonance mass)
- GPD approach (A. Afanasev, E. Voutier, et. al.)
- Can it be used to probe the resurgent singularities? Yes, indirectly - through parameterization.

Muon g-2 puzzle

- Dispersion approach
- Relates the vacuum polarization to hadron production in e+e- scattering
- Model dependent calculations of exclusive hadron production
- Work in progress on more constrained anzats for the solutions of the functional PDEs

Conclusion

- JLab experimental program provides exciting opportunities for theoreticians
- Resurgent QCD program is underway
- Alternative to lattice
- Interesting from pure math point of view