Nucleon and nuclear gravitational form factors

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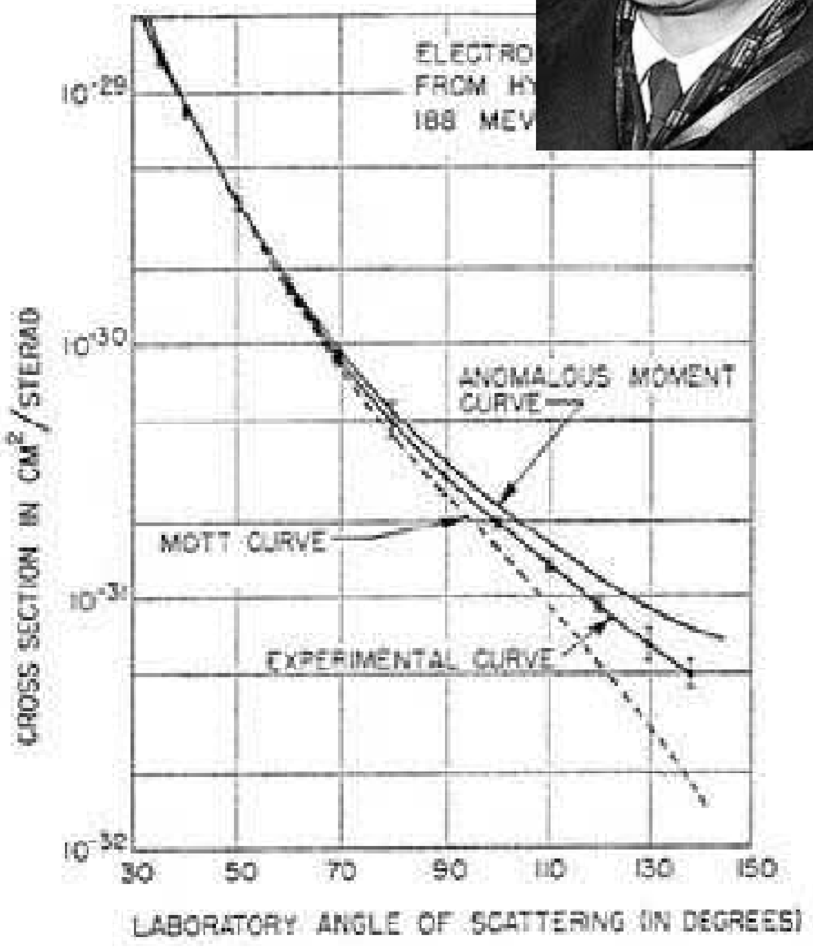
Proton electromagnetic form factors

EM form factors from elastic scattering

$$\langle p'|J^\mu(0)|p\rangle = \bar{u}(p')\left[\gamma^\mu F_1 + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}F_2\right]u(p)$$



Electric form factor

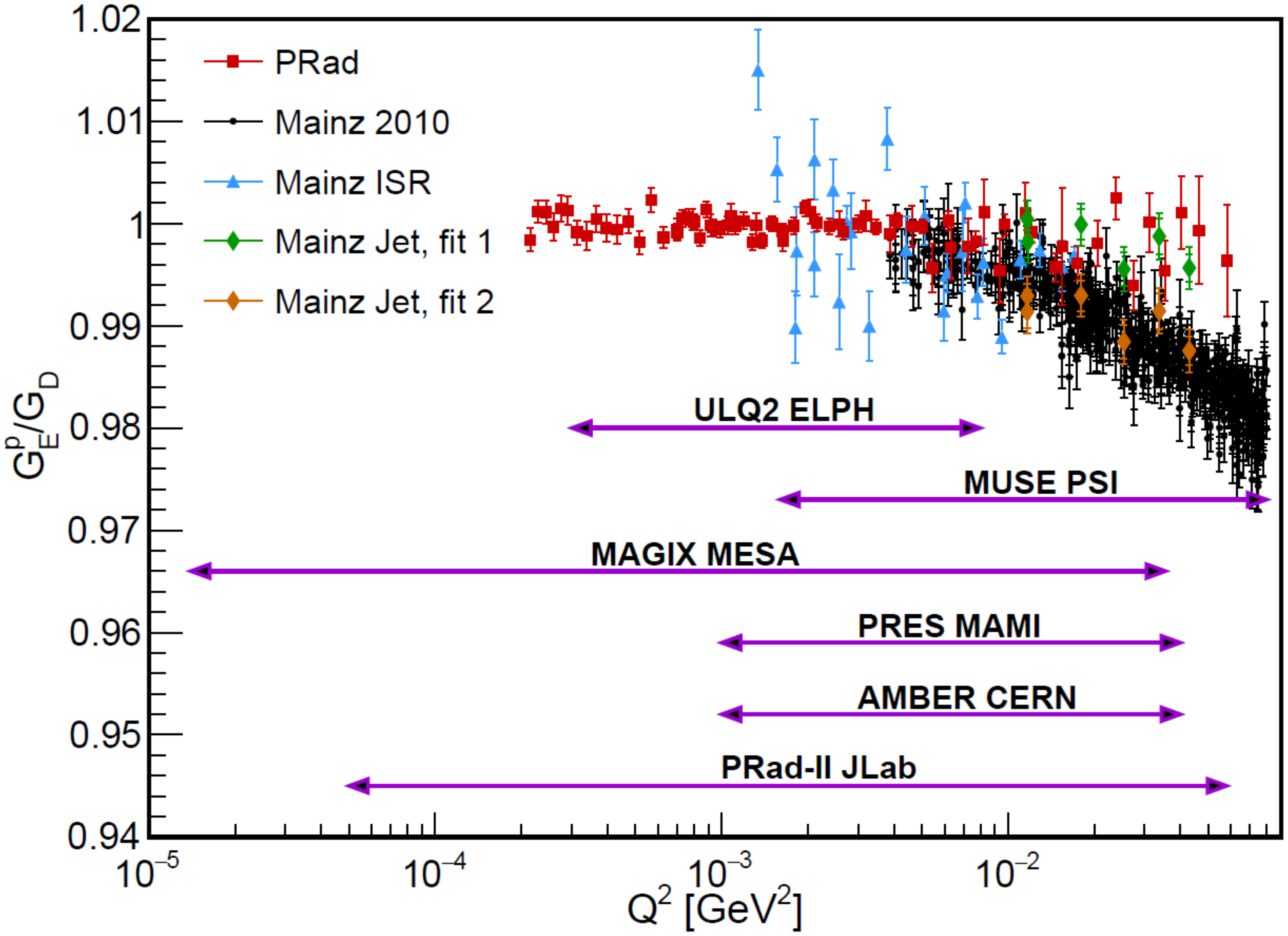
$$G_E(t) = F_1(t) + \frac{t}{4M^2}F_2(t)$$

Proton charge radius

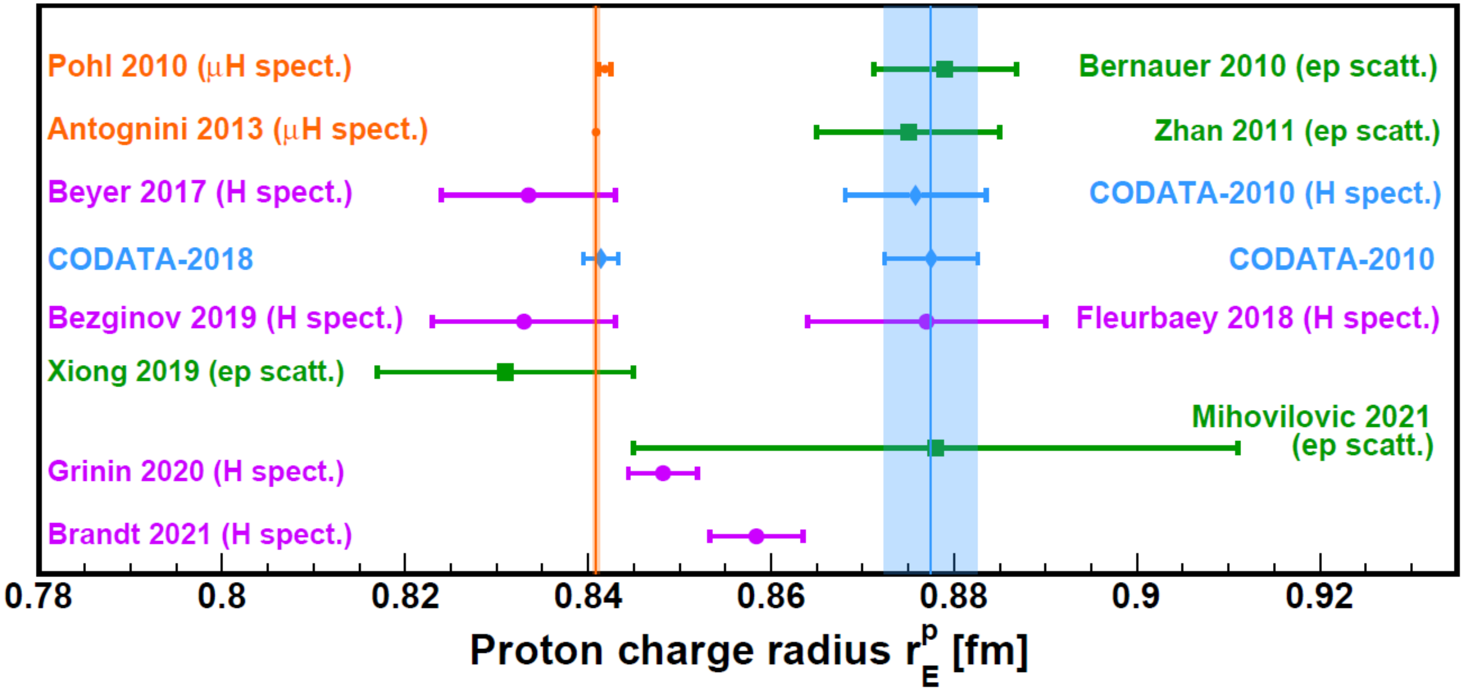
$$\langle r^2\rangle = 6\left.\frac{dG_E(t)}{dt}\right|_{t=0}$$

Elastic scattering 70 years later

Xiong, Peng, 2302.13818



Proton radius: Is there still a puzzle?



Both CODATA and PDG now recommend the smaller value ~0.84fm.

Several future experiment planned, aim for 1% precision

PRad (2019) $$r_p = 0.831 \pm 0.007_{\text{stat}} \pm 0.012_{\text{syst}}$$

Radius zoo

Charge radius $\langle r^2\rangle_c = \frac{\int d\boldsymbol{x} x^2 \rho_c(\boldsymbol{x})}{\int d\boldsymbol{x} \rho_c(\boldsymbol{x})} = \frac{6}{G_E(0)} \frac{dG_E(t)}{dt}\Big|_{t=0}$

Magnetic radius $\langle r^2\rangle_M = \frac{6}{G_M(0)} \frac{dG_M(t)}{dt}\Big|_{t=0}$

Baryon number radius $\langle r^2\rangle_B = \frac{\int d\boldsymbol{x} x^2 \rho_B(\boldsymbol{x})}{\int d\boldsymbol{x} \rho_B(\boldsymbol{x})}$

Mass radius $\langle r^2\rangle_m = \frac{\int d\boldsymbol{x}\, x^2 T^{00}(\boldsymbol{x})}{\int d\boldsymbol{x} T^{00}(\boldsymbol{x})} = 6\frac{dA(t)}{dt}\Big|_{t=0} - \frac{3D(0)}{2M^2}$

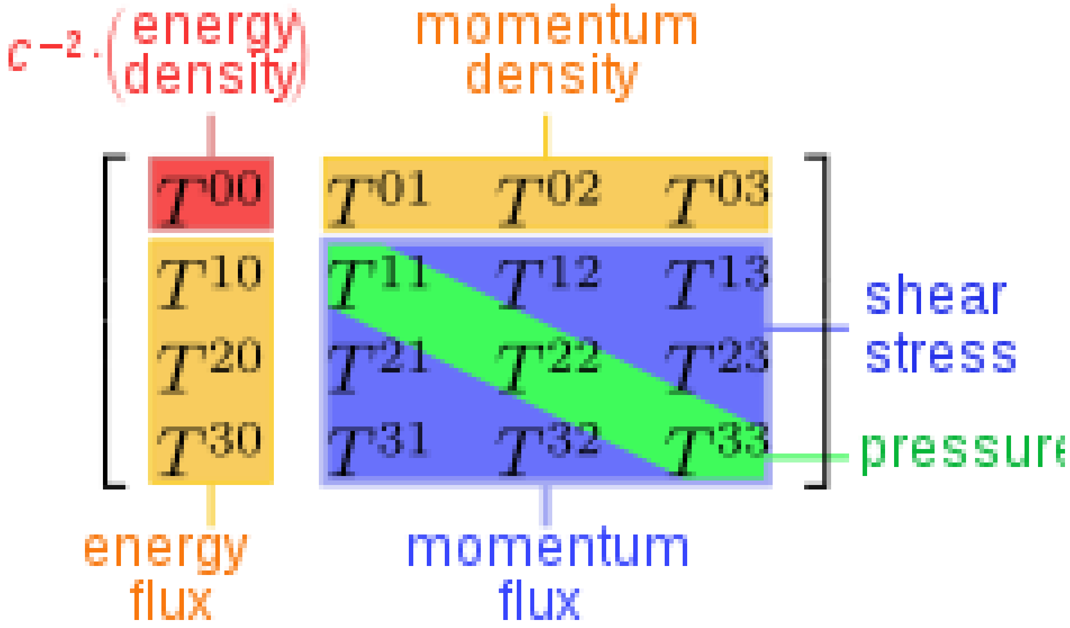
Scalar radius $\langle r^2\rangle_s = \frac{\int d\boldsymbol{x}\, x^2 T^{\mu}_{\mu}(\boldsymbol{x})}{\int d\boldsymbol{x} T^{\mu}_{\mu}(\boldsymbol{x})} = 6\frac{dA(t)}{dt}\Big|_{t=0} - \frac{9D(0)}{2M^2}$

Tensor radius $\langle r^2\rangle_t \equiv \frac{\int d\boldsymbol{x}\, x^2 \left(T^{00}(\boldsymbol{x}) + \frac{1}{2}T_{ii}(\boldsymbol{x})\right)}{\int d\boldsymbol{x}\, \left(T^{00} + \frac{1}{2}T_{ii}\right)} = 6\frac{dA(t)}{dt}\Big|_{t=0}$

Mechanical radius $\langle r^2\rangle_{mech} = \frac{\int d\boldsymbol{x} x^2 \frac{x_i x_j}{x^2} T_{ij}(\boldsymbol{x})}{\int d\boldsymbol{x} \frac{x_i x_j}{x^2} T_{ij}(\boldsymbol{x})} = \frac{6D(0)}{\int_{-\infty}^{0} dt D(t)}$

…

Gravitational form factors



QCD energy momentum tensor

$$T^{\mu\nu} = \sum_f \bar{\psi}_f \gamma^{(\mu} i D^{\nu)} \psi_f - F^{\mu\rho} F^{\nu}{}_{\rho} + \frac{g^{\mu\nu}}{4} F^{\alpha\beta} F_{\alpha\beta}$$

Associated form factors

$$\langle P' | T^{\mu\nu} | P \rangle = \bar{u}(P') \left[A(t) \gamma^{(\mu} \bar{P}^{\nu)} + B(t) \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2M} + D(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} \right] u(P)$$

Spin-0 → 2 form factors
Spin-1/2 → 3 form factors
Spin-1 → 6 form factors

GFFs for quarks and gluons

Separately defined for quarks and gluons (Ji 1996)

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{u}(P')\Big[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{C}_{q,g}M\eta^{\mu\nu}\Big]u(P)$$

hidden form factor

$\bar{C}_q + \bar{C}_g = 0$ because the total EMT is conserved.

$$\langle P|(T_{q,g})^\mu_\mu|P\rangle = 2M^2(A_{q,g} + 4\bar{C}_{q,g})$$

Connection to the trace anomaly and gluon condensate $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$ → Origin of hadron masses

Relation between $\bar{C}_{q,g}$ and $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$

1 loop, 2 loop: YH, Rajan, Tanaka (2018)

3 loop: Tanaka (2019)

4 loop: Ahmed, Chen, Czakon (2022)

$$\left\langle \mathrm{Tr}\left([\Theta_g]_R^{\overline{\mathrm{MS}}}\right)\right\rangle_{\mathrm{P}} = \langle [O_F]_R\rangle_{\mathrm{P}}\ (-0.437676\,\alpha_s - 0.261512\,\alpha_s^2 - 0.183827\,\alpha_s^3 - 0.256096\,\alpha_s^4)$$

$$+\langle [O_m]_R\rangle_{\mathrm{P}}\ (0.495149\,\alpha_s + 0.776587\,\alpha_s^2 + 0.865492\,\alpha_s^3 + 0.974674\,\alpha_s^4)\ ,$$

$$\left\langle \mathrm{Tr}\left([\Theta_q]_R^{\overline{\mathrm{MS}}}\right)\right\rangle_{\mathrm{P}} = \langle [O_F]_R\rangle_{\mathrm{P}}\ (0.079578\,\alpha_s + 0.058870\,\alpha_s^2 + 0.021604\,\alpha_s^3 + 0.013675\,\alpha_s^4)$$

$$+\langle [O_m]_R\rangle_{\mathrm{P}}\ (1\ +\ 0.141471\,\alpha_s - 0.008235\,\alpha_s^2 - 0.064351\,\alpha_s^3 - 0.065869\,\alpha_s^4)$$

D-term: the last global unknown

$D(0)$ is a fundamental constant of the proton!

The value, even the sign, is unknown at the moment.

Spatial components of the energy momentum tensor
→ May be interpreted as radial force (`pressure') exerted by quarks and gluons Polyakov (2003)

$$T^{ij}(\boldsymbol{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right)s(r) + \delta^{ij}\,p(r) \qquad D = M\int d^3r\, r^2 p(r)$$

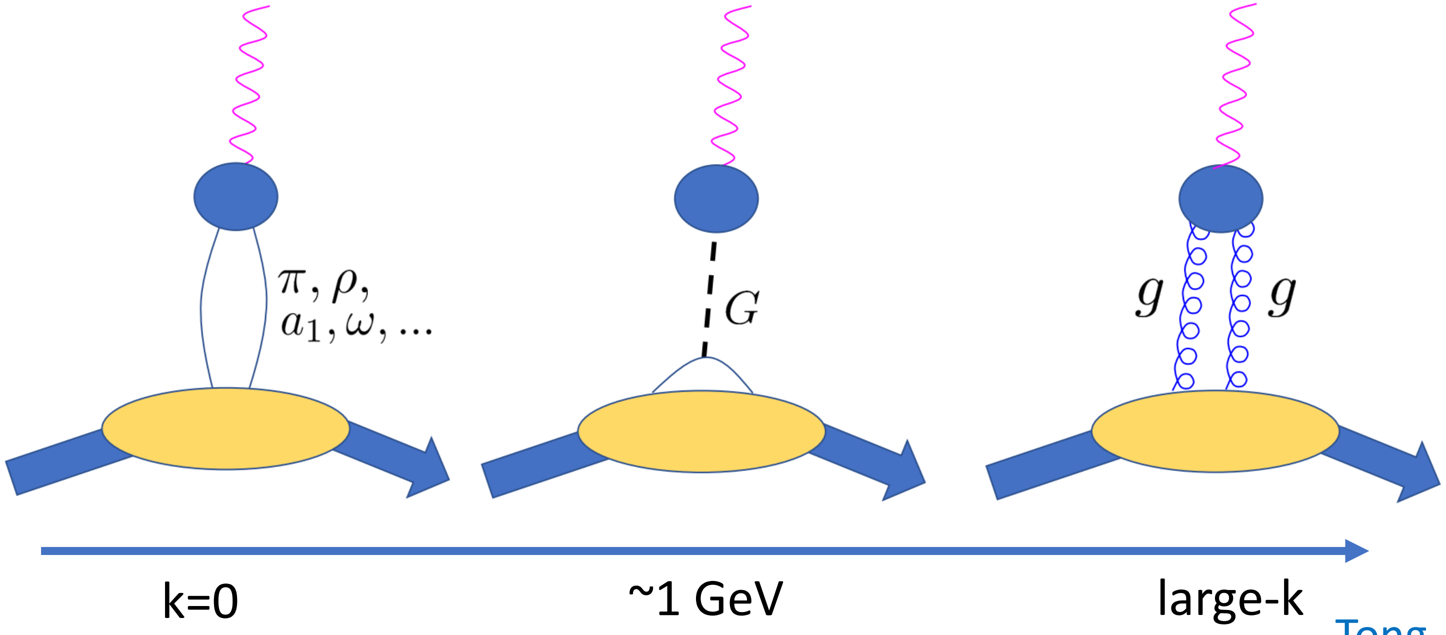
Conjecture: Stable hadrons must have a negative D-term $D(t=0) < 0$

D-term in the Sakai-Sugimoto model
Glueball dominance

Fujita, YH, Sugimoto, Ueda (2022)

D(k) form factor receives contributions from both spin-0 and spin-2 glueballs

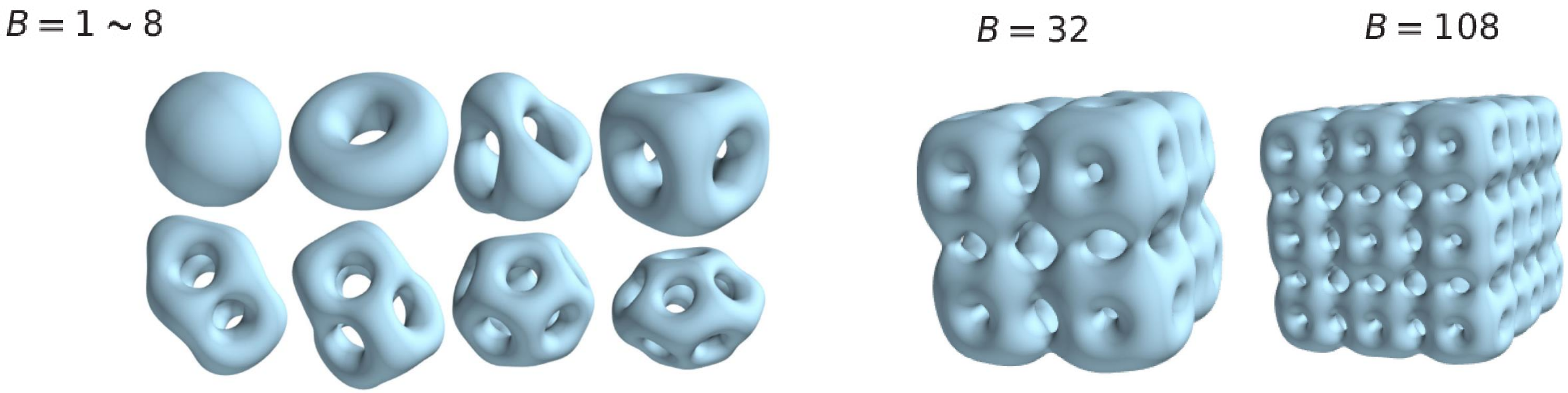
$$D(|\vec{k}|) \sim \sum_{n=1}^{\infty} \frac{c_n^{\mathrm{T}}(|\vec{k}|)}{\vec{k}^2 + (m_n^{\mathrm{T}})^2} + \sum_{n=1}^{\infty} \frac{c_n^{\mathrm{S}}(|\vec{k}|)}{\vec{k}^2 + (m_n^{\mathrm{S}})^2}$$



Tong, Ma, Yuan (2021)

D-term of atomic nuclei in the Skyrme model

Martin-Caro, Huidobro, YH 2304.05994



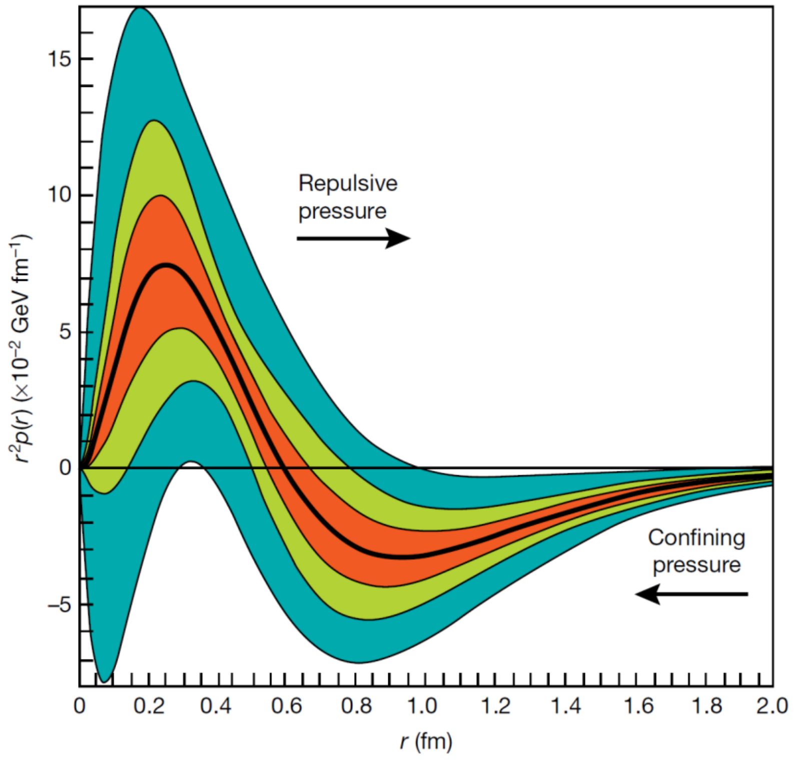
B	1	2	3	4	5	6	7	$8a$	$8b$	32	108
$D(0)$	-3.701	-13.126	-26.757	-43.304	-62.72	-85.95	-106.596	-128.368	-140.816	-1.874×10^3	-2.152×10^4

The value D(0) grows quickly with increasing B

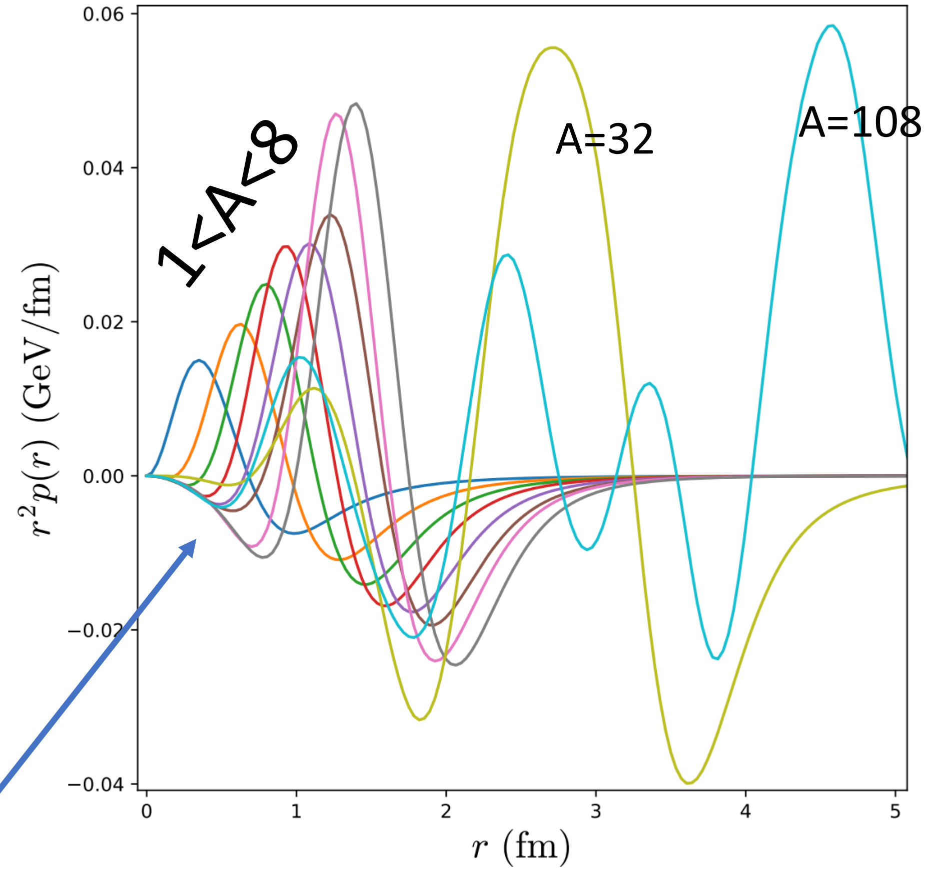
cf. Polyakov (2003); Liuti, Taneja (2005); Guzey, Siddikov (2005)

`Pressure' inside nucleon and nuclei

Burkert, Elouadrhiri, Girod (2018)



Martin-Caro, Huidobro, YH, 2312.12984

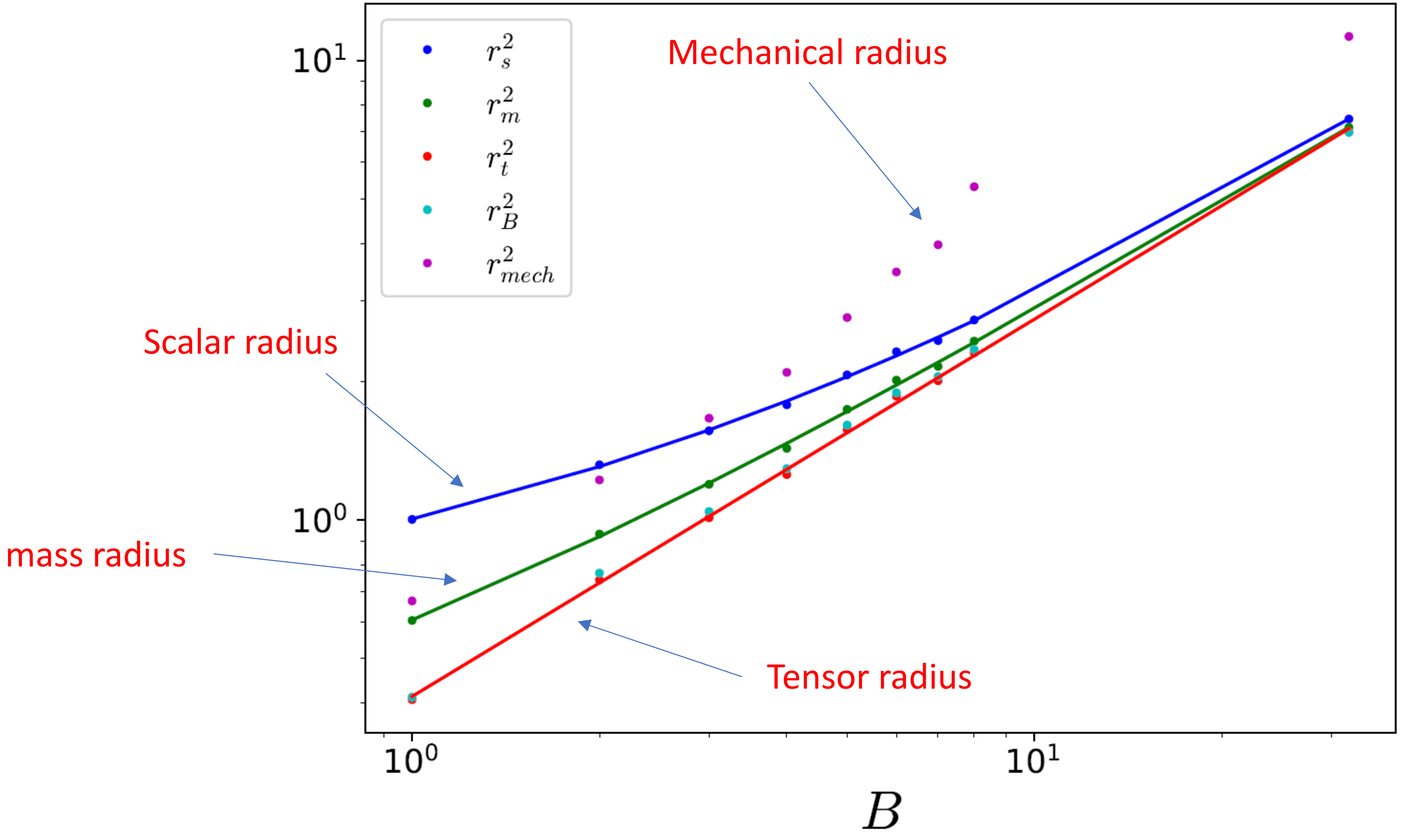


Negative pressure near the core for nuclei A>1

see also, Freese, Cosyn (2022), He, Zahed (2023)

Nuclear radii

2312.12984



$$\langle r^2 \rangle_s = \langle r^2 \rangle_m - \frac{3D(0)}{M^2}$$

$$\frac{D(0)}{M^2} \propto B^{\beta-2}$$

Experimental study of GFFs?

- Introduced theoretically in the 60s.
- Received far less attention than EM form factors, not because they are less interesting/important.
- The obvious reason: We cannot measure them directly!

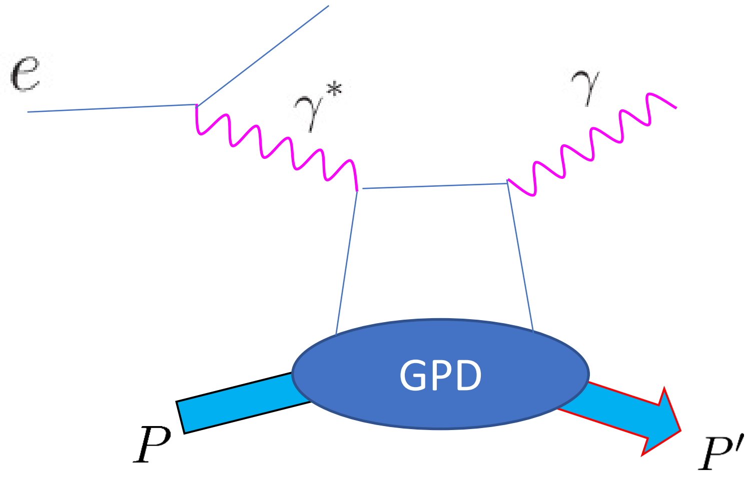
One-graviton exchange cross section $$\frac{d\sigma}{dt} \sim G_N^2 \frac{s^2}{t^2}$$

$$G_N \sim 1/M_P^2 \qquad M_P \sim 10^{19}\,\mathrm{GeV}$$

- There are, however, indirect ways to measure them.

Quark D-term from Deeply Virtual Compton Scattering (DVCS)

$$D = D_u + D_d + D_s + D_g + \cdots$$



$D_{u,d}$ related to the subtraction constant in the dispersion relation for the Compton form factor

Teryaev (2005)

$$\mathrm{Re}\mathcal{H}_q(\xi,t) \;=\; \frac{1}{\pi}\int_{-1}^{1} dx \mathrm{P}\frac{\mathrm{Im}\mathcal{H}_q(x,t)}{\xi - x} + 2\int_{-1}^{1} dz \frac{D_q(z,t)}{1-z}$$

$$\int_{-1}^{1} dz z D_q(z,t) = D_q(t)$$

1 graviton $\approx$ 2 photons

1+1=2

After all, 1 graviton $\neq$ 2 photons

$$\int_{-1}^{1} dz \frac{D_q(z,t)}{1-z}$$

what is measurable

$$\int_{-1}^{1} dz z D_q(z,t)$$

what we want

2-photon state couples to operators with arbitrary spin.
How can one isolate the spin-2 component?

1+1= anything

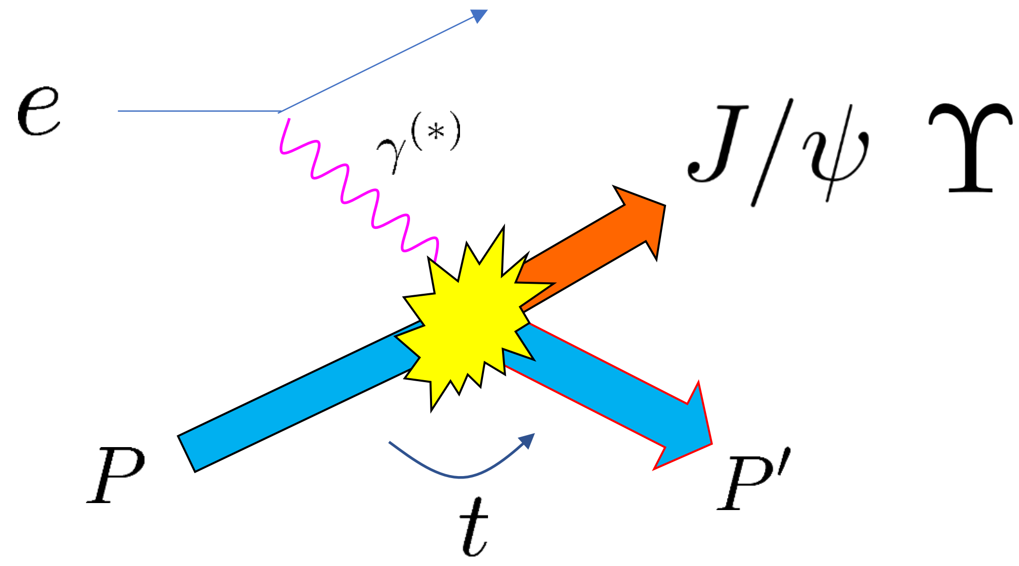
$$\frac{1}{1-z} = 1 + z + z^2 + z^3 + \cdots$$

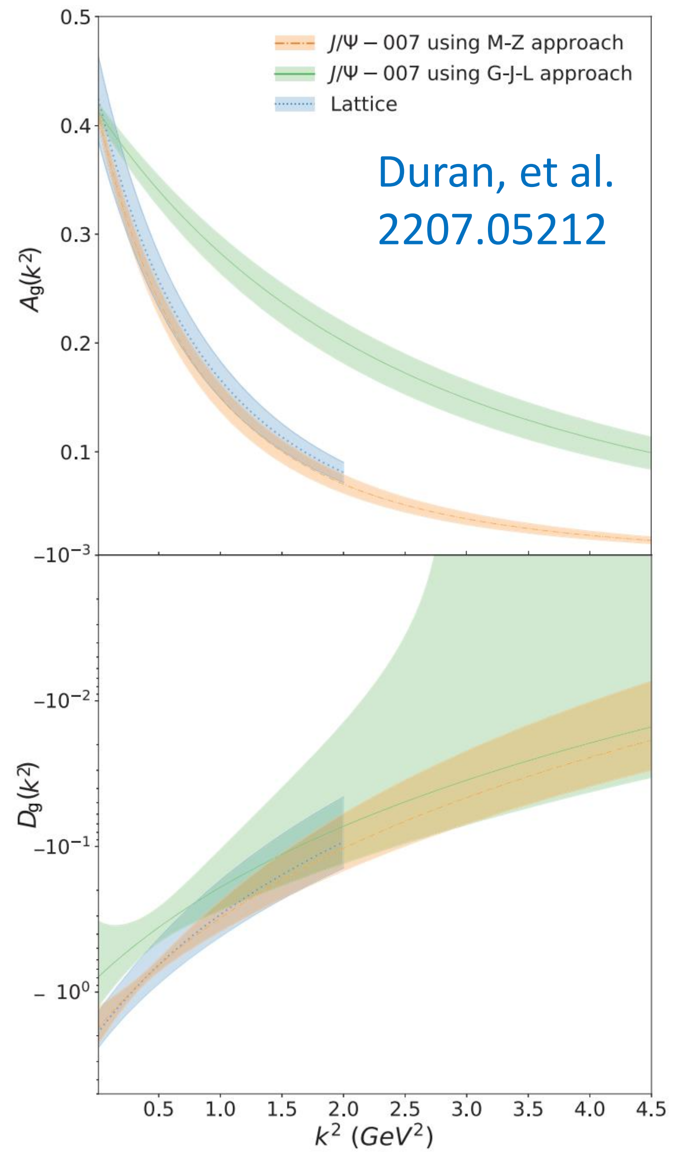
spin-2 (EMT)   spin-4

$$
\begin{aligned}
d_1^{uds}(t=0, 2\ \mathrm{GeV}^2) &= -2.1 \pm 26.6 \\
d_3^{uds}(t=0, 2\ \mathrm{GeV}^2) &= 1.5 \pm 26.5 \\
\hline
d_1^{g}(t=0, 2\ \mathrm{GeV}^2) &= -2.9 \pm 37 \\
d_3^{g}(t=0, 2\ \mathrm{GeV}^2) &= 0.2 \pm 4.1
\end{aligned}
$$

Dutrieux, Meisgny, Mezrag, Moutarde (2024)

Quarkonium photo-(electro-)production near threshold





Ongoing experiments at JLab, future measurement at EIC?

Originally proposed by Kharzeev, Satz, Syamtomov, Zinovev (1997) to probe the gluon condensate.

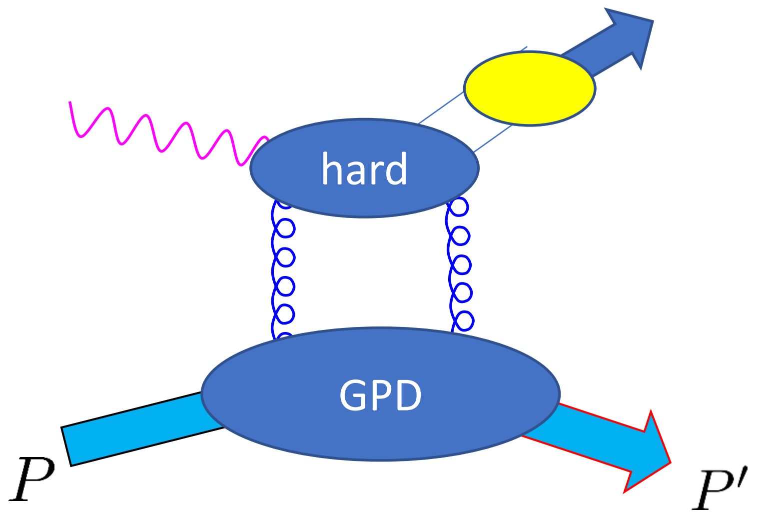
One can also study gluon GFFs in this process YH, Yang (2018)

GPD factorization

1 graviton $\approx$ 2 gluons

Light-cone dominance when $Q^2 \to \infty$ or $M_{QQ} \to \infty$

GPD factorization   Collins, Frankfurt, Strikman (1996)
Ivanov, Schafer, Szymanowski, Krasnikov (2004)
Guo, Ji, Liu (2021)



Amplitude proportional to   Compton form factor

$$\int_{-1}^{1} \frac{dx}{x} \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H_g(x, \xi, t)$$

Gluon GPD

Skewness $\xi = \dfrac{P^+ - P'^+}{P + + P'^+}$

Again, 1 graviton $\neq$ 2 gluons

what is measurable

$$\int_{-1}^{1} \frac{dx}{x} \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H_g(x, \xi, t)$$

what we want

$$\int_{-1}^{1} dx H_g(x, \xi, t)$$

Essentially the same problem as in the extraction of quark D-term from DVCS

A dilemma:

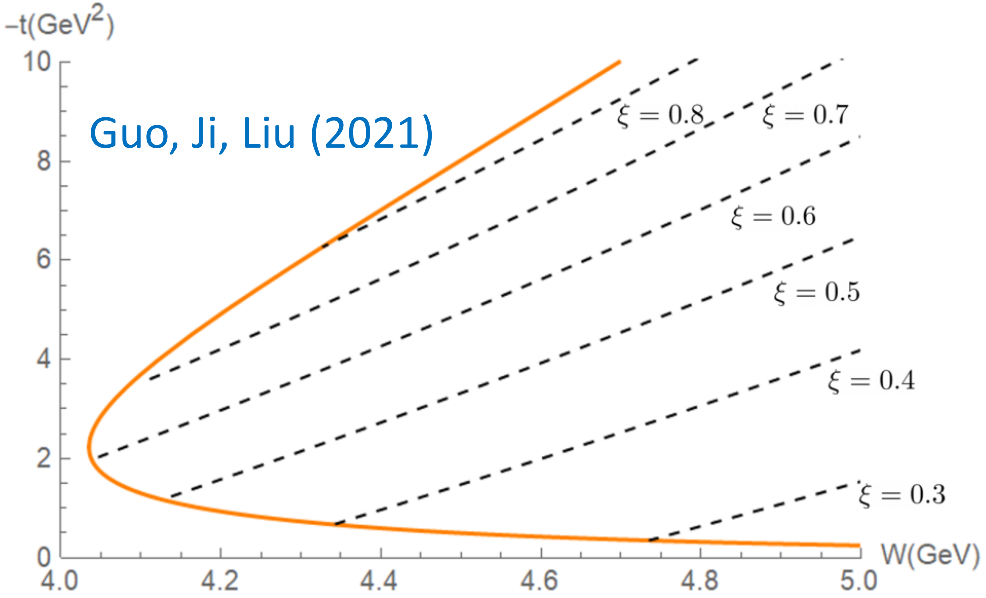
GPD factorization allows us to study this reaction from first principles.

But it also means that we are dealing with infinitely many operators with arbitrary spin.
How can one extract the spin-2 component?

Skewness!

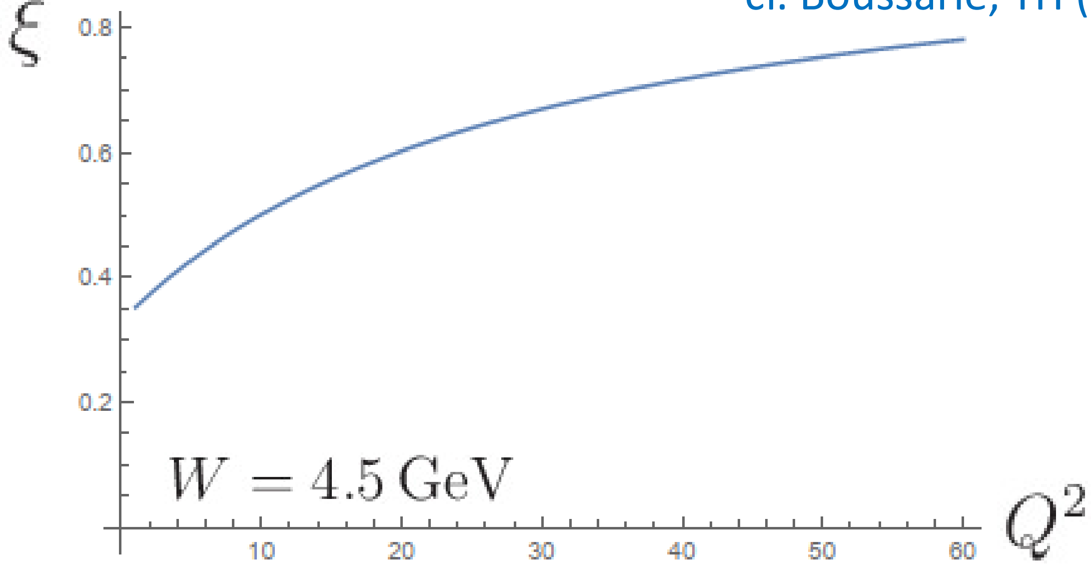
Threshold production characterized by large values of skewness YH, Strikman (2021)

J/ψ photo-production



J/ψ electro-production

cf. Boussarie, YH (2020)



$\xi \approx 1$ in the ideal limit $Q^2 \to \infty$ or $m_V \to \infty$

Energy momentum tensor strikes back

YH, Strikman 2102.12631
Guo, Ji, Liu 2103.11506

If $\xi \approx 1$, one can Taylor expand.

spin=2 (energy momentum tensor)

$$\int_{-1}^{1} \frac{dx}{x} \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H_g(x,\xi,t) \approx 2 \int dx (1 + x^2 + x^4 + \cdots) H_g(x,\xi,t)$$

spin=4 spin=6

Try $H_g(x,\eta=1) \approx (1-x^2)^2$

all spins $\int dx \frac{H_g(x,\xi=1,t)}{1-x^2} \sim \int_0^1 dx \frac{(1-x^2)^2}{1-x^2} = \frac{2}{3}$

spin-2 only $\int_0^1 dx (1-x^2)^2 = \frac{8}{15}$

80% of the total

When $\xi < 1$, expansion becomes an asymptotic series Guo, Ji, Yuan (2023)

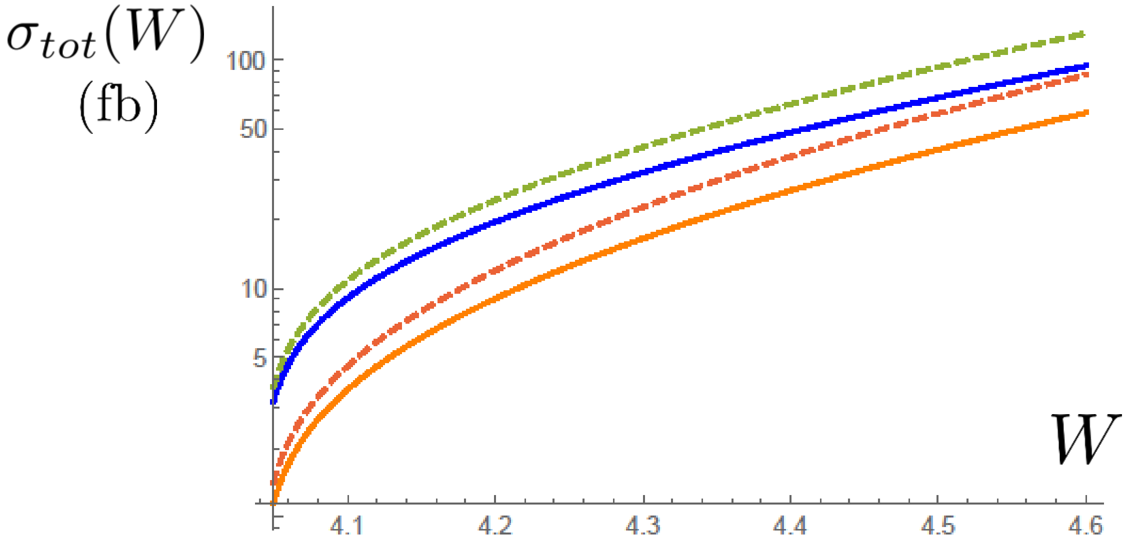
EMT dominates over all the other twist-2 operators combined!

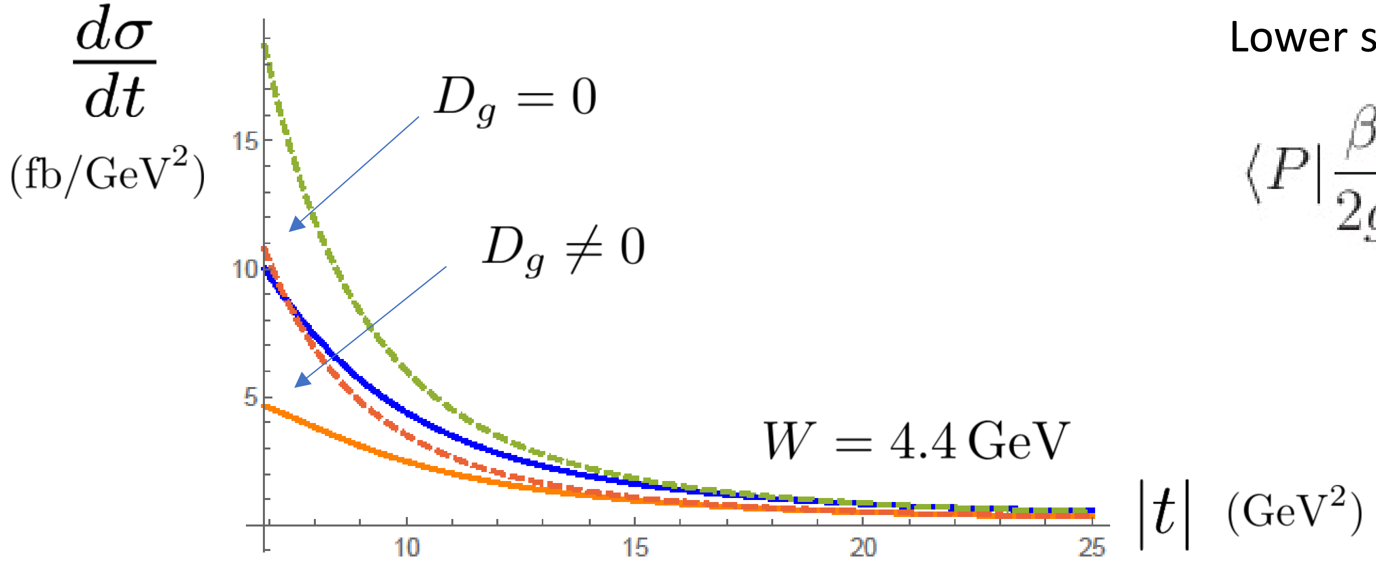
J/ψ electroproduction at the EIC

Boussarie, YH (2020)

$$Q^2 = 64\,\mathrm{GeV}^2$$

$$\sqrt{S_{ep}} = 20\,\mathrm{GeV}$$





Dashed curves:
without gluon D-term

Solid curves: with gluon D-term

Upper solid $b = 1$

Lower solid $b = 0$

$$\langle P|\frac{\beta}{2g}F^2|P\rangle = 2M^2(1-b)$$

Direct measurement of GFFs?

- Graviton exchange suppressed by the Planck energy $M_P \sim 10^{19}\ \mathrm{GeV}$

- But in some BSM scenarios, the effective Planck energy could be in the TeV region.
  e.g. extra dimension models.

- These models typically predict massive gravitons.

- Long history of tests of Newton's inverse-square law

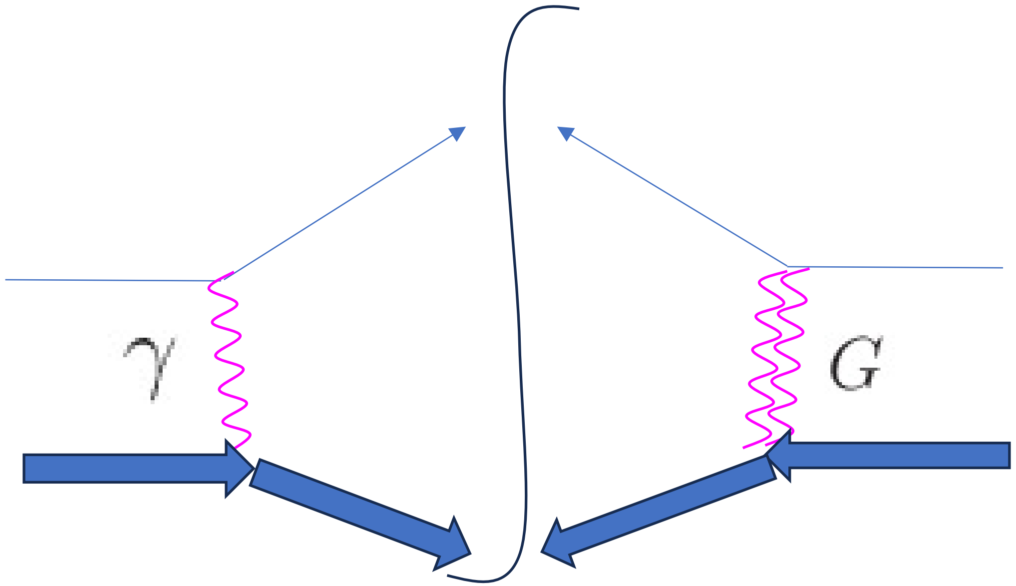
$$V(r) = -G\frac{m_1\ m_2}{r}\left[1 + \alpha\, e^{-r/\lambda}\right]$$

TeV-scale elastic ep, eA scattering

YH, 2311.14470

$$\delta \mathcal{L} = \kappa h_{\mu\nu} T^{\mu\nu}$$

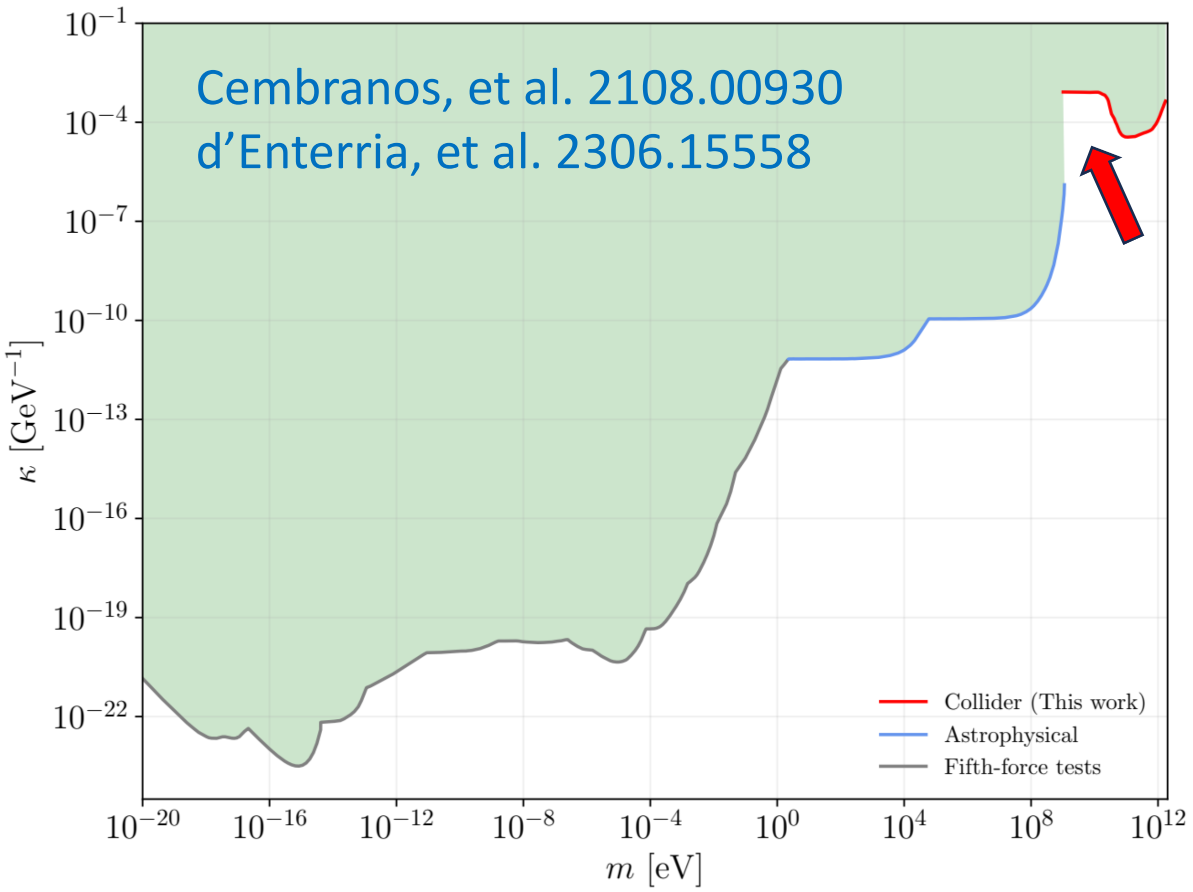
assume $\quad \kappa \sim 1\,\mathrm{TeV}^{-1}$

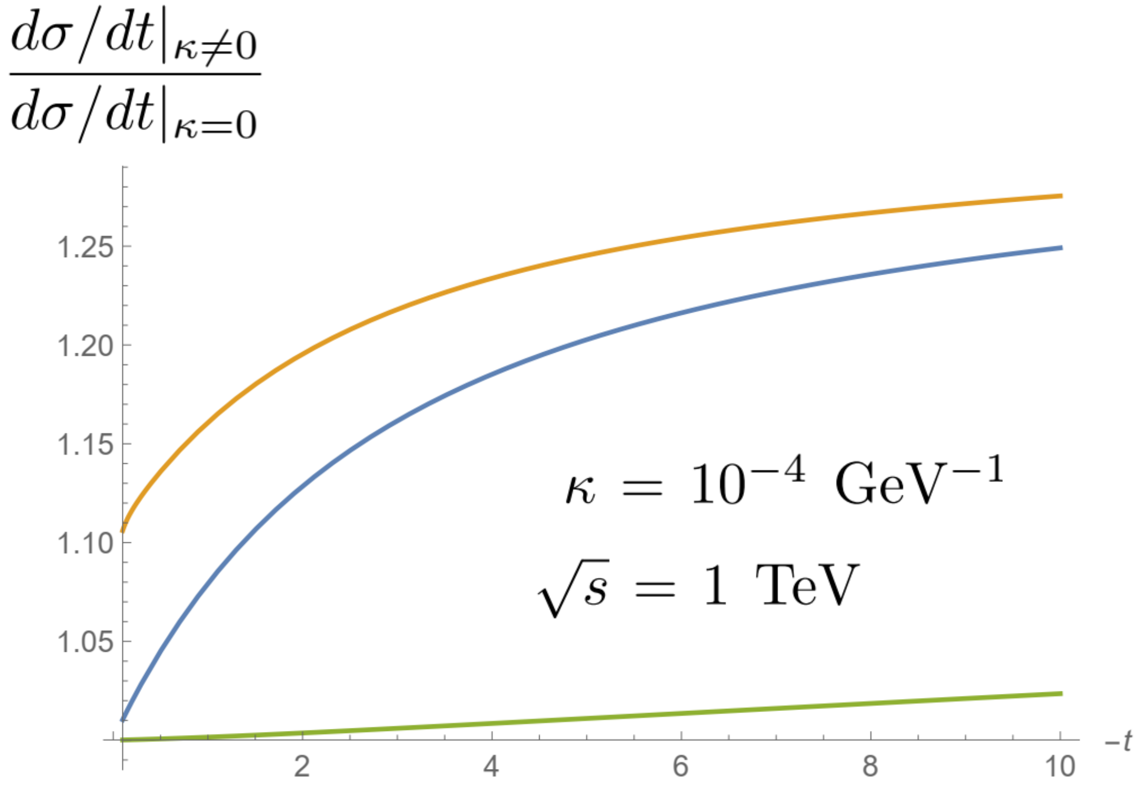


Rosenbluth

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha_{em}^2}{t^2}\left\{\left(1+\frac{t-2M^2}{s}+\frac{M^4}{s^2}\right)\left(F_1^2(t)-\frac{tF_2^2(t)}{4M^2}\right)+\frac{t^2}{2s^2}(F_1(t)+F_2(t))^2\right\}$$
$$+\frac{\alpha_{em}\kappa^2 s}{t(t-m^2)}\left\{\left(1+\frac{3(t-2M^2)}{2s}\right)\left(A(t)F_1(t)-\frac{tB(t)F_2(t)}{4M^2}\right)+\mathcal{O}(s^{-2})\right\}+\mathcal{O}(\kappa^4)$$

Evading the LHC constraints





Where to look for?

MuIC : a future Muon-ion collider at BNL Acosta, Li 2107.02073

Conclusions

- EM form factors: very active field even after 70 years, aiming for 1% precision
- GFFs: just the beginning!
- Indirect measurements from DVCS, quarkonium threshold production.
  Challenging to extract the spin-2 component. Large skewness can help.