Computing GPDs in asymmetric frames: A New Perspective

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Joshua Miller (Temple U.)
Swagato Mukherjee (BNL)
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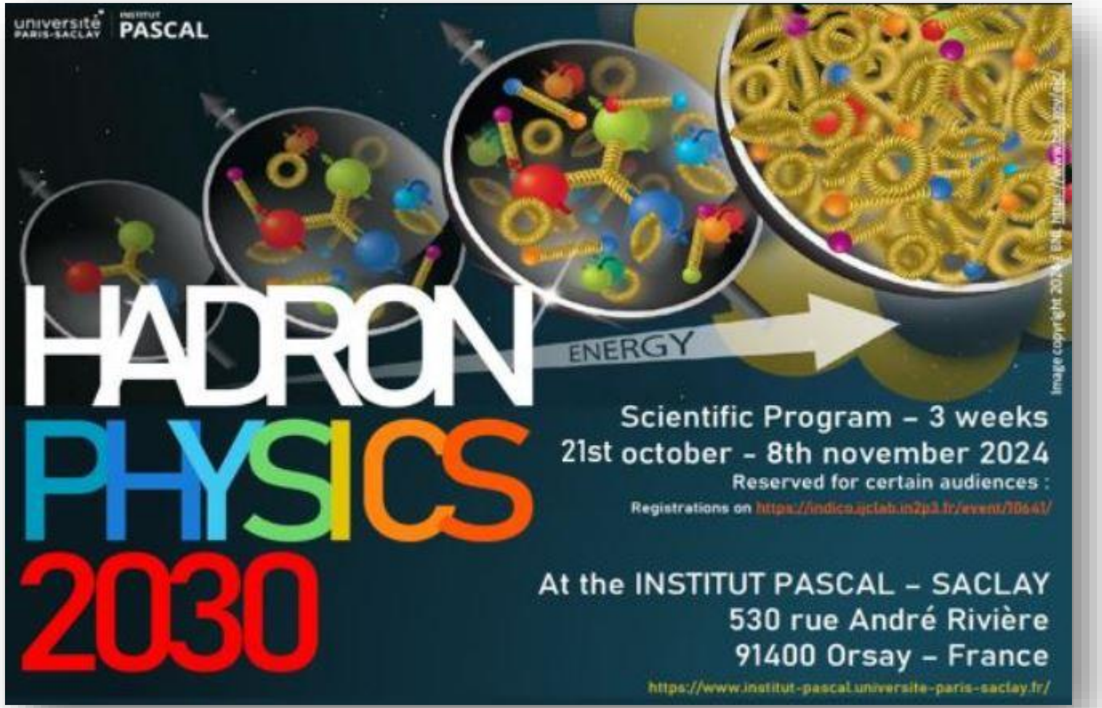
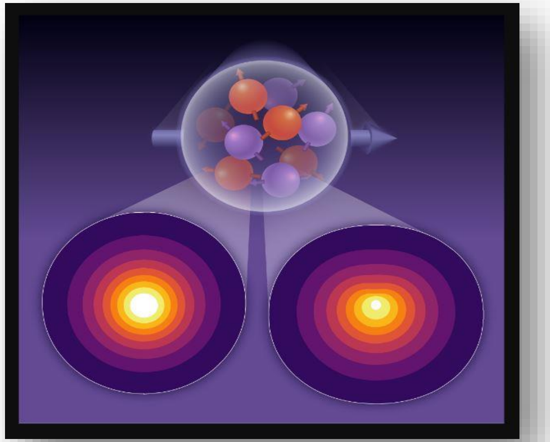
Yong Zhao (ANL)



Based on arXiv: 2209.05373, 2310.13114

Motivation for studying GPDs

1) **3D imaging** (Burkardt, 0005108 ...)



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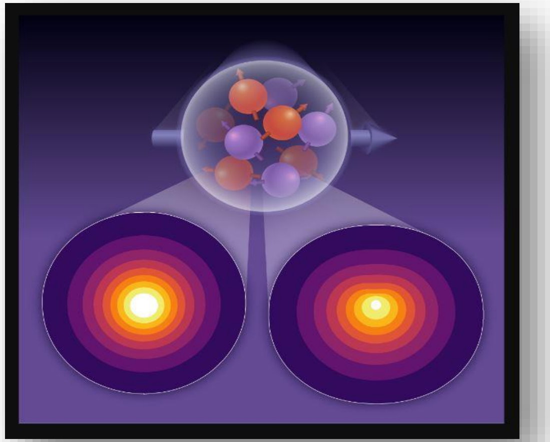
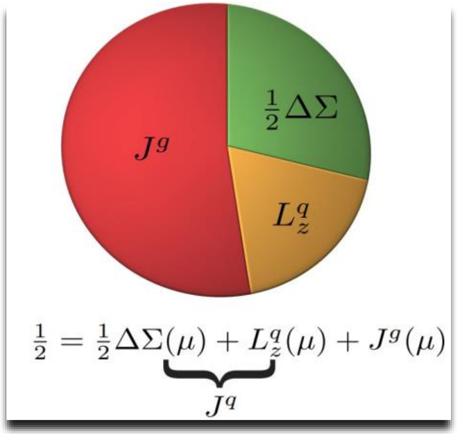


2) **Spin sum rule & orbital angular momentum** (Ji, 9603249)



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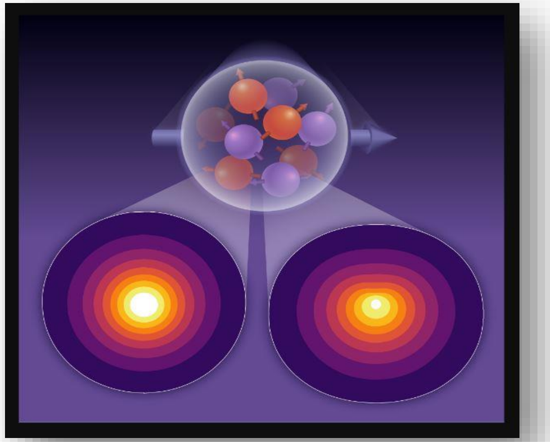

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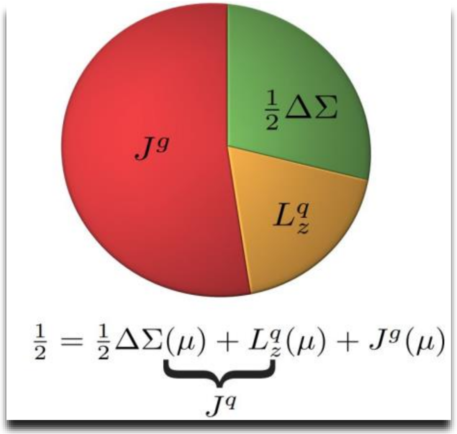

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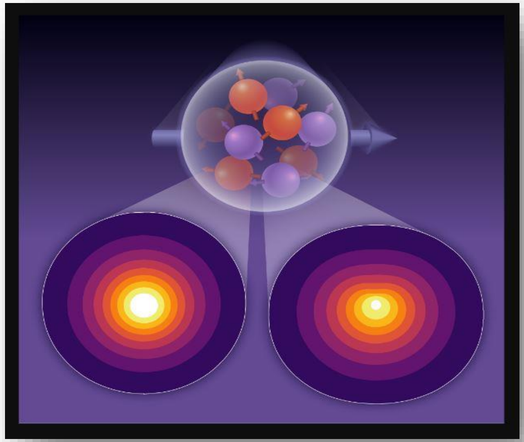

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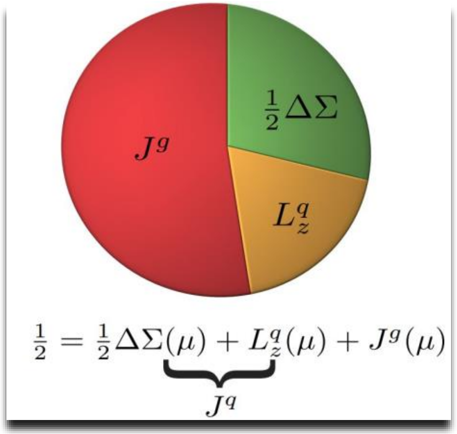
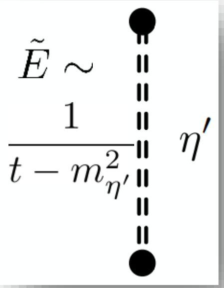
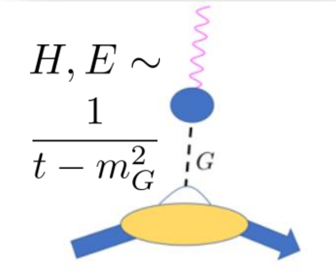


3) **Mechanical properties (pressure/shear) inside nucleon** (Polyakov, Shuvaev, 0207153 ...)

4) **Mass generations & chiral symmetry breaking**

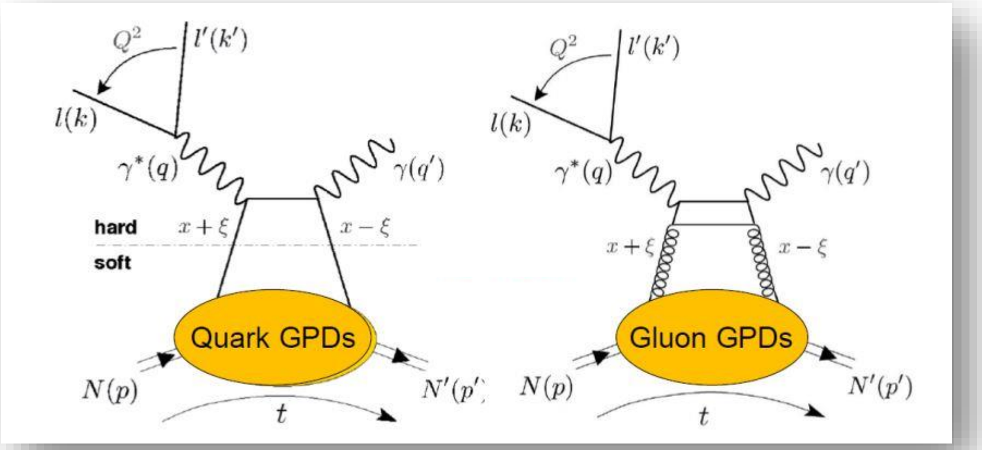
(SB, Hatta, Vogelsang, 2210.13419, 2305.09431)



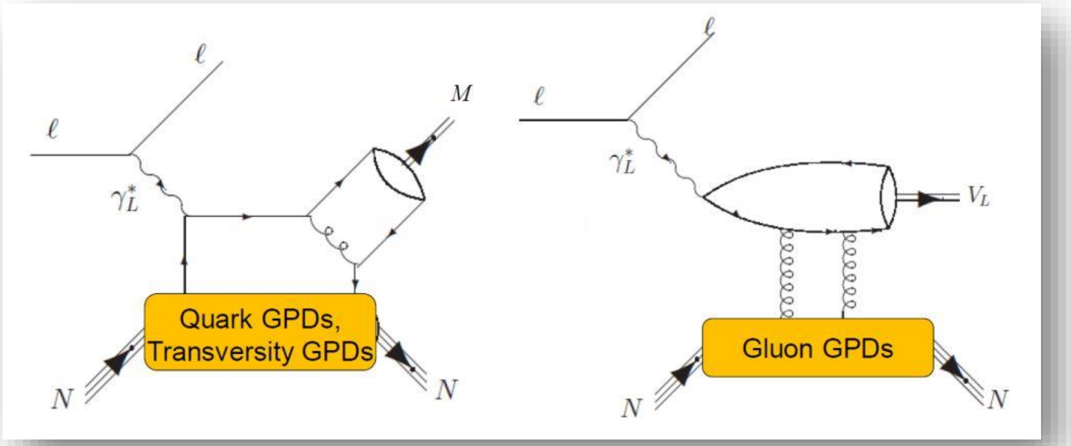


Physical processes sensitive to GPDs

Deeply Virtual Compton Scattering



Deeply Virtual Meson Production

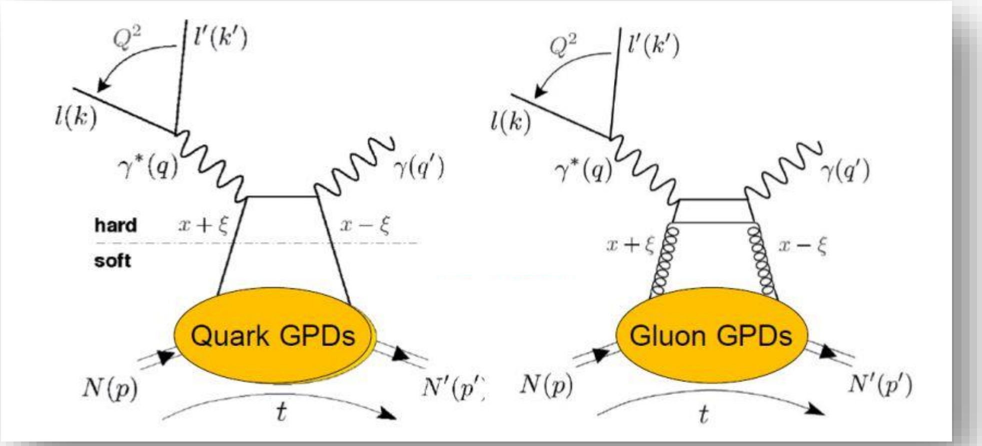


Courtesy: Hyon-Suk Jo, KPS Meeting

No access to x-dependence of GPDs

Physical processes sensitive to GPDs

Deeply Virtual Compton Scattering



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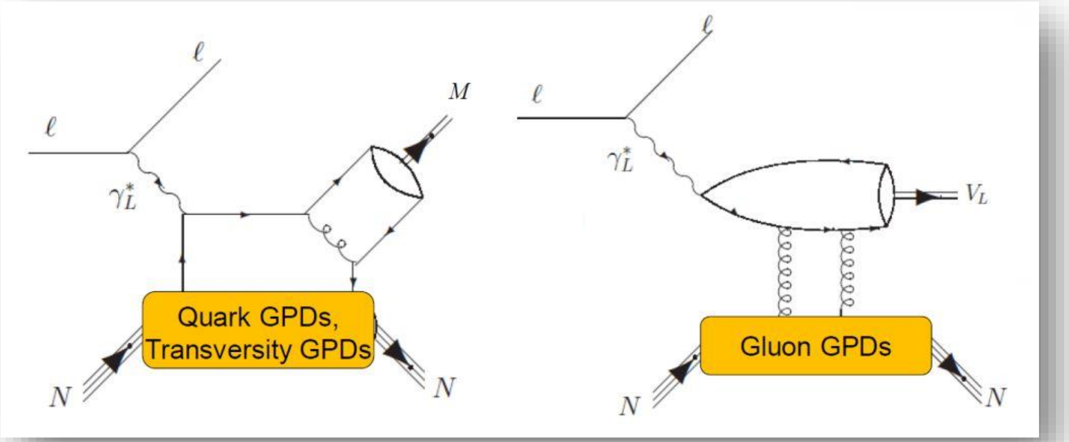

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No access to x-dependence of GPDs

Complementarity: Lattice results can be integrated into global analysis of experimental data

Physical processes sensitive to GPDs

Deeply Virtual Compton Scattering

Deeply Virtual Meson Production

KPS Meeting

Exclusive production of a pair of high transverse momentum photons in pion-nucleon collisions for extracting generalized parton distributions

Jian-Wei Qiu[a,b] Zhite Yu[c]

Hard photoproduction of a diphoton with a large invariant mass

A. Pedrak,[1] B. Pire,[2] L. Szymanowski,[1] and J. Wagner[1]

Access to x-dependence of GPDs

Physical processes sensitive to GPDs

Deeply Virtual Compton Scattering

Deeply Virtual Meson Production

We require complementary measurements of the GPDs using Lattice QCD

In recent years, significant breakthroughs have been made in our ability to access the x-dependence of GPDs

Ex... m... extracting generalized parton distributions

Jian-Wei Qiu[a,b] Zhite Yu[c]

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Access to x-dependence of GPDs

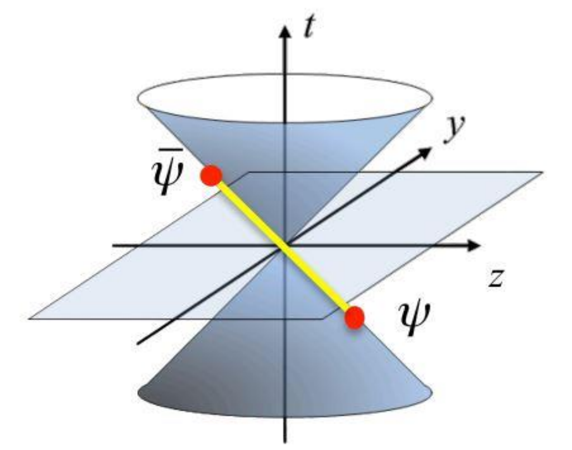
Calculating Parton Distributions in Lattice QCD

"Physical" distributions

Light-cone (standard) correlator $-1 \leq x \leq 1$

$$F^{[\Gamma]}(x, \Delta; \lambda, \lambda') = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ik\cdot z} \times \langle p'; \lambda' | \bar{\psi}(-\tfrac{z}{2})\, \Gamma\, \mathcal{W}(-\tfrac{z}{2}, \tfrac{z}{2}) \psi(\tfrac{z}{2}) | p; \lambda\rangle \Big|_{z^+ = \vec{z}_\perp = 0}$$

- **Time dependence :** $z^0 = \frac{1}{\sqrt{2}}(z^+ + z^-) = \frac{1}{\sqrt{2}} z^-$
- **Cannot be computed on Euclidean lattice**



Calculating Parton Distributions in Lattice QCD

"Physical" distributions

Parton Physics on Euclidean Lattice

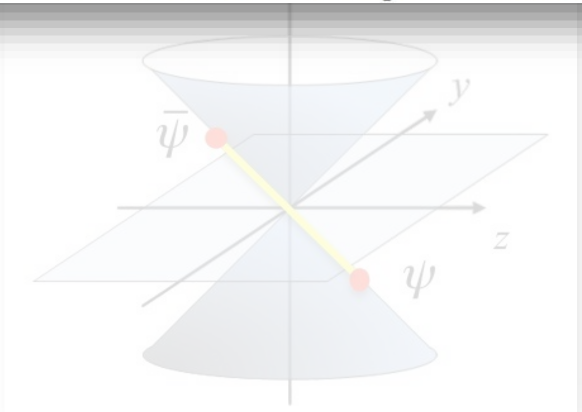
Xiangdong Ji[1,2]

[1] INPAC, Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai, 200240, P. R. China
[2] Maryland Center for Fundamental Physics, Department of Physics, University of Maryland, College Park, Maryland 20742, USA

(Dated: May 8, 2013)

Abstract

I show that the parton physics related to correlations of quarks and gluons on the light-cone can be studied through the matrix elements of frame-dependent, equal-time correlators in the large momentum limit. This observation allows practical calculations of parton properties on an

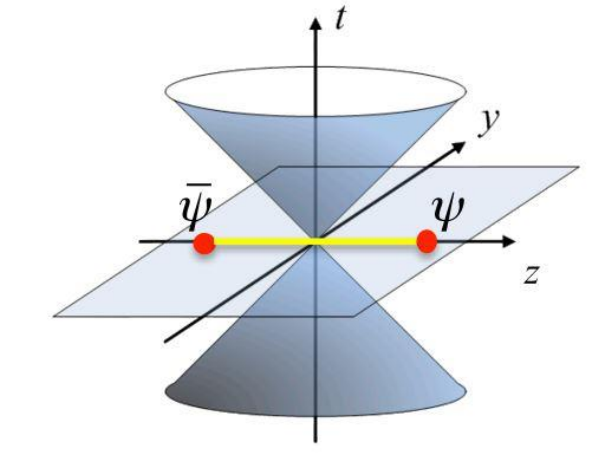


"Auxiliary" distributions

Correlator for quasi-GPDs (Ji, 2013) $-\infty \le x \le \infty$

$$F_Q^{[\Gamma]}(x,\Delta;\lambda,\lambda';P^3) = \frac{1}{2}\int \frac{dz^3}{2\pi} e^{ik\cdot z} \times \langle p',\lambda'|\bar\psi(-\tfrac{z}{2})\,\Gamma\,\mathcal{W}_Q(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p,\lambda\rangle\Big|_{z^0=\vec z_\perp=0}$$

- Non-local correlator depending on position z^3
- Can be computed on Euclidean lattice



Calculating Parton Distributions in Lattice QCD

Matching formula:

Matching coefficient

$$\tilde{q}(x,\xi,t,\mu,P^3) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\xi}{y},\frac{\mu}{P^3}\right) q(y,\xi,t,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(P^3)^2},\frac{M_N^2}{(P^3)^2},\frac{t}{(P^3)^2}\right)$$

GPD matching known up to one-loop order (non-singlet & singlet)

References: (not exhaustive)

Connecting Euclidean to light-cone correlations: From flavor nonsinglet in forward kinematics to flavor singlet in non-forward kinematics

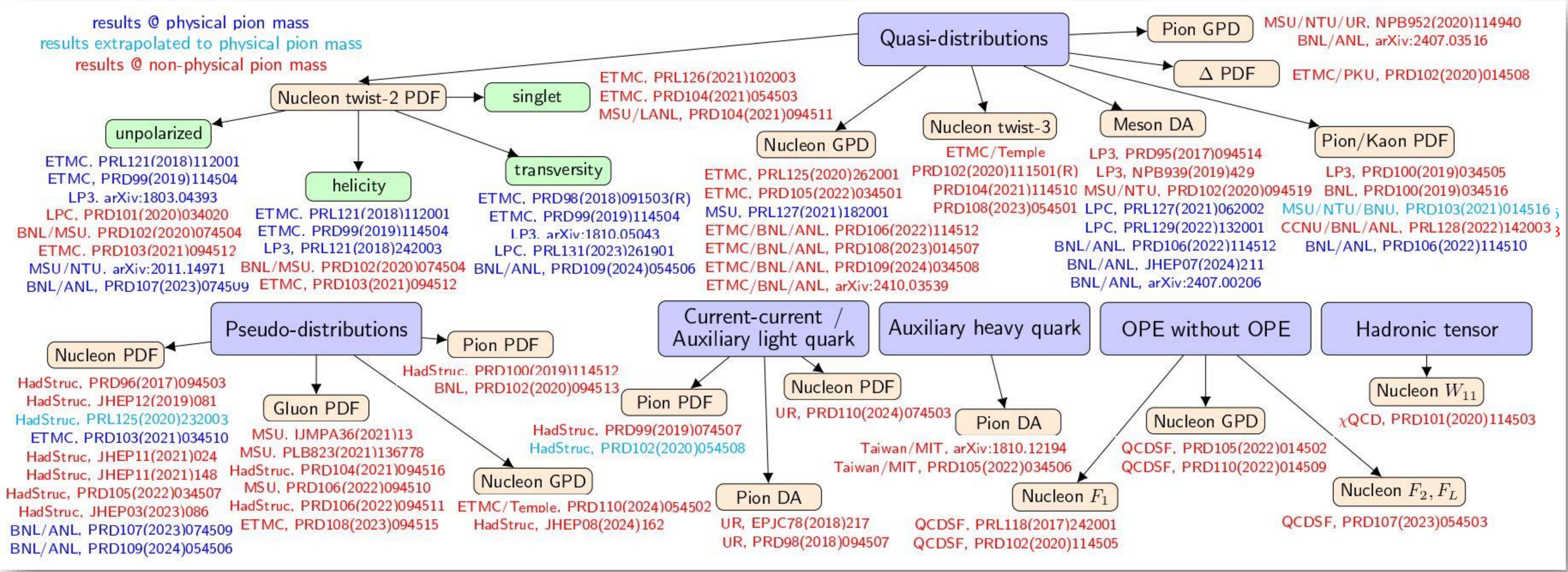
Yao Ji,[a] Fei Yao[b] and Jian-Hui Zhang[c,b]

One-Loop Matching for Generalized Parton Distributions

Xiangdong Ji,[1,2,3] Andreas Schäfer,[4] Xiaonu Xiong,[5,6] and Jian-Hui Zhang[1,4]

Dynamical Progress of Lattice QCD calculations of PDFs/GPDs

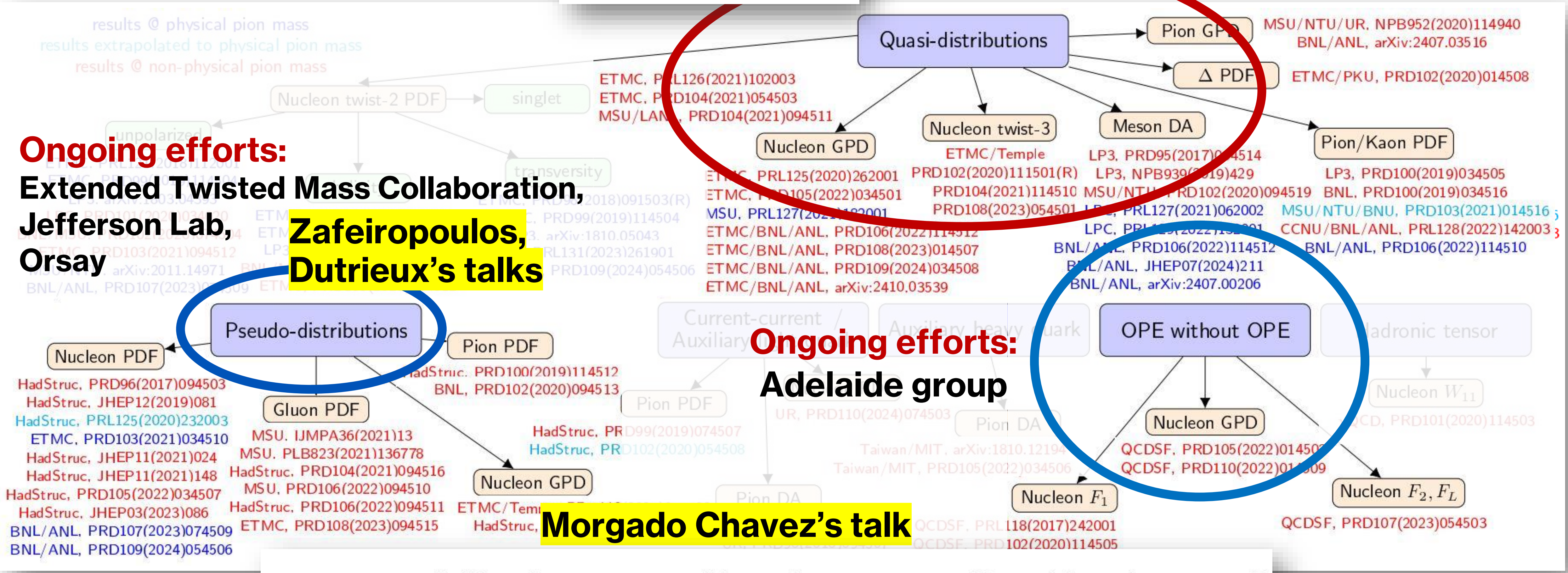
Lattice QCD calculations of x-dependence of PDFs & related quantities using Euclidean correlators:



Compilation by Cichy, 2110.07440

Dynamical Progress of Lattice QCD calculations of PDFs/GPDs

This talk



Ongoing efforts:
Extended Twisted Mass Collaboration, Jefferson Lab, Orsay

Zafeiropoulos, Dutrieux's talks

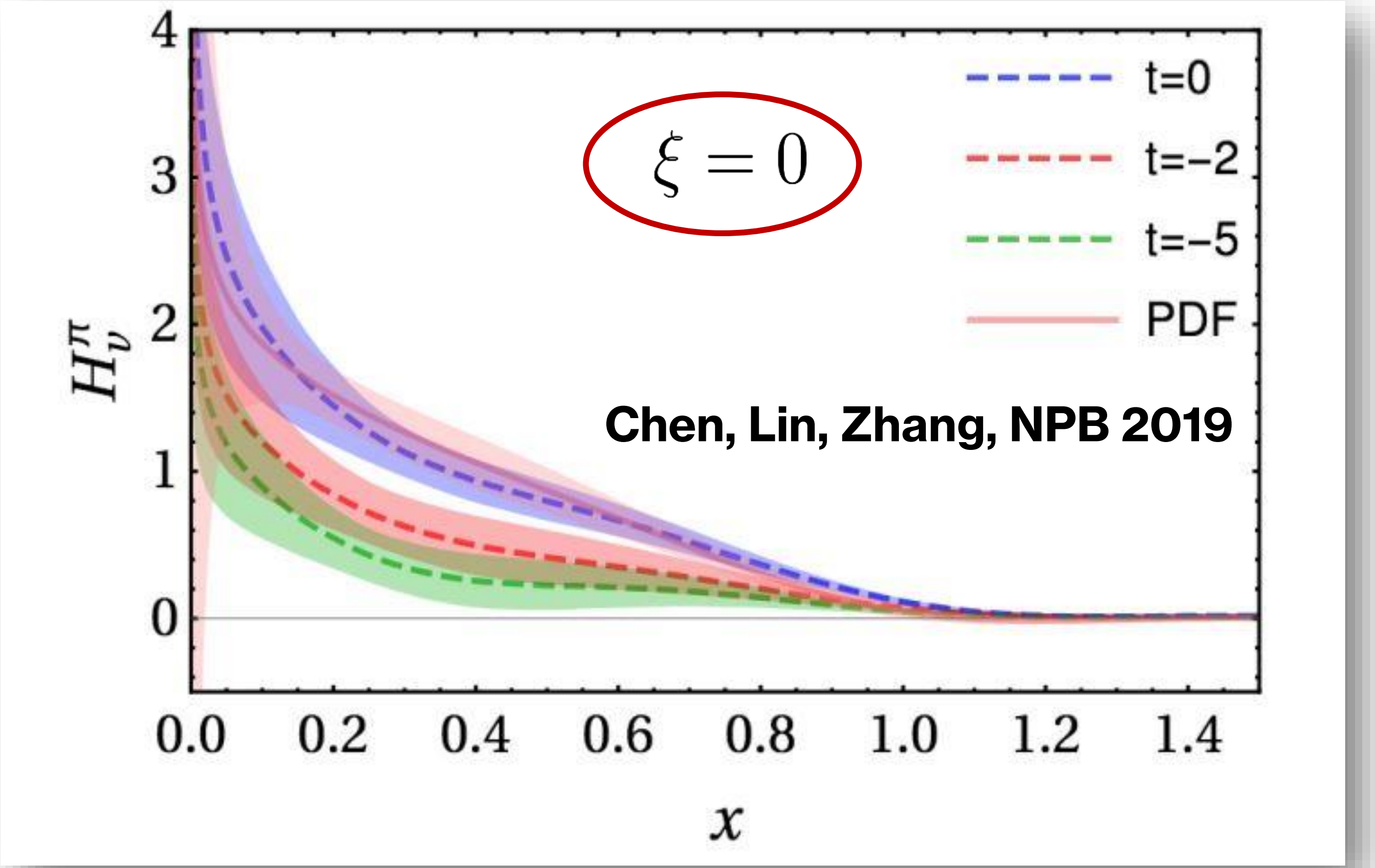
Ongoing efforts:
Adelaide group

Morgado Chavez's talk

Forward-limit generalized parton distributions of the η_c-meson

First Lattice QCD results of the x-dependent GPDs

Pion:

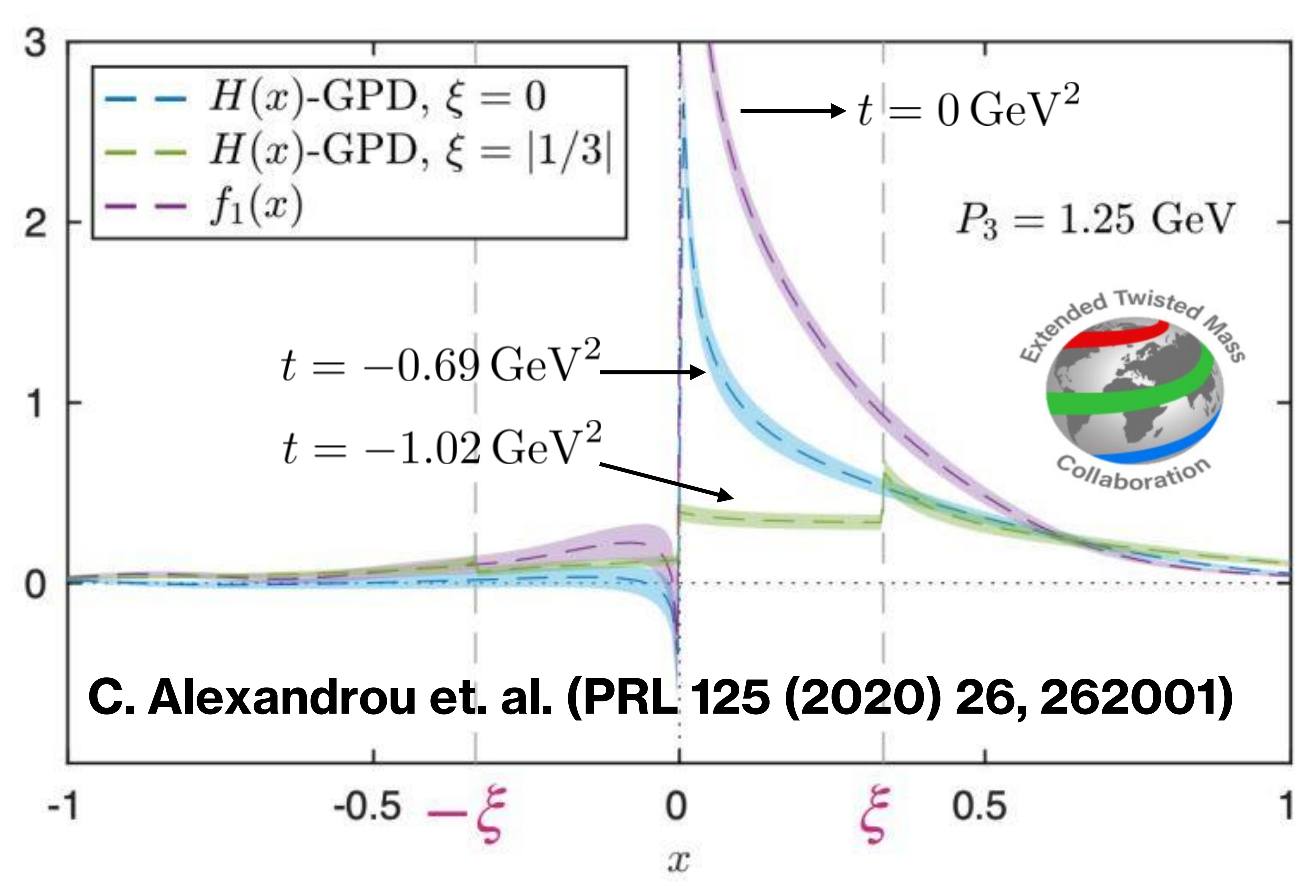


As t increases, the distribution flattens

First Lattice QCD results of the x-dependent GPDs

Cichy's talk

Proton:



ERBL/DGLAP: Qualitative differences

As $x \to 1$, qualitative behavior in agreement with power counting analysis

(F. Yuan, 0311288)

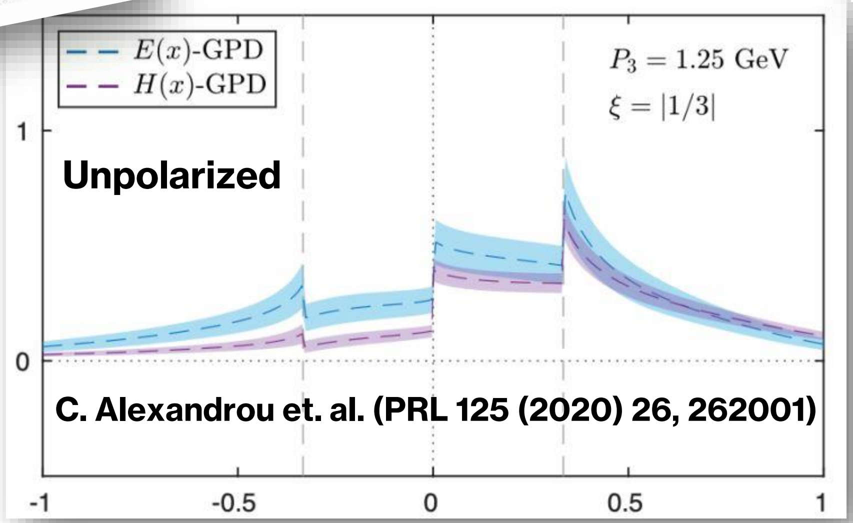
First Lattice QCD ... he x-dependent GPDs

Cichy's talk

Twist-2 GPDs

Pol \ Γ	γ^+	$\gamma^+\gamma_5$	$\sigma^{+j}\gamma_5$
U	H		E_T
L		$\widetilde{H}$	$\widetilde{E}_T$
T	E	$\widetilde{E}$	H_T $\widetilde{H}_T$

Proton:



C. Alexandrou et. al. (PRL 125 (2020) 26, 262001)

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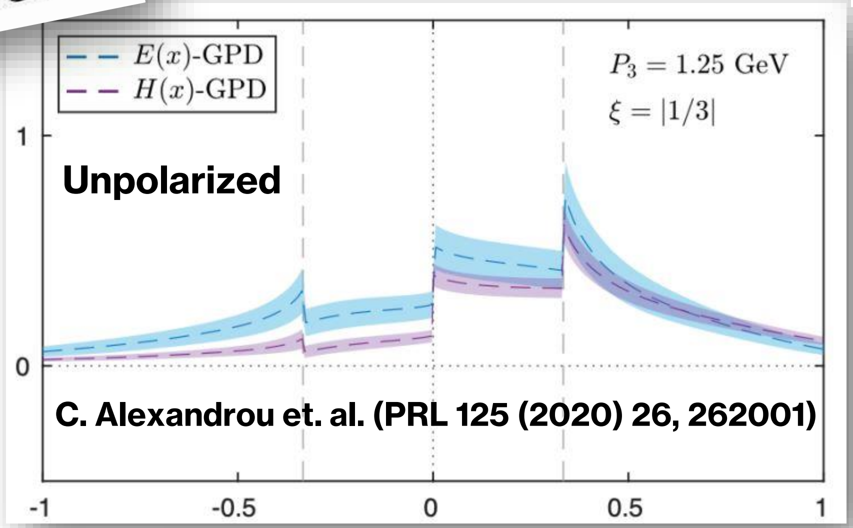

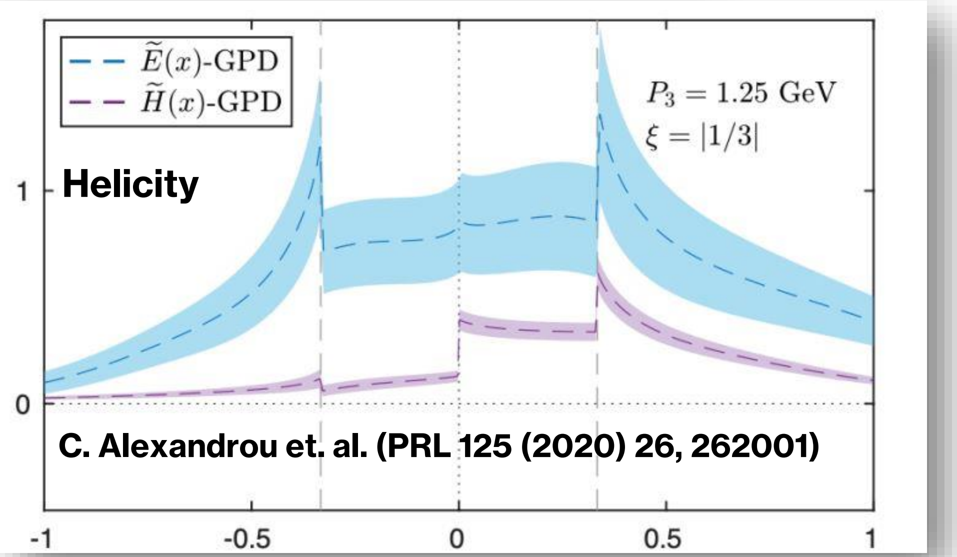

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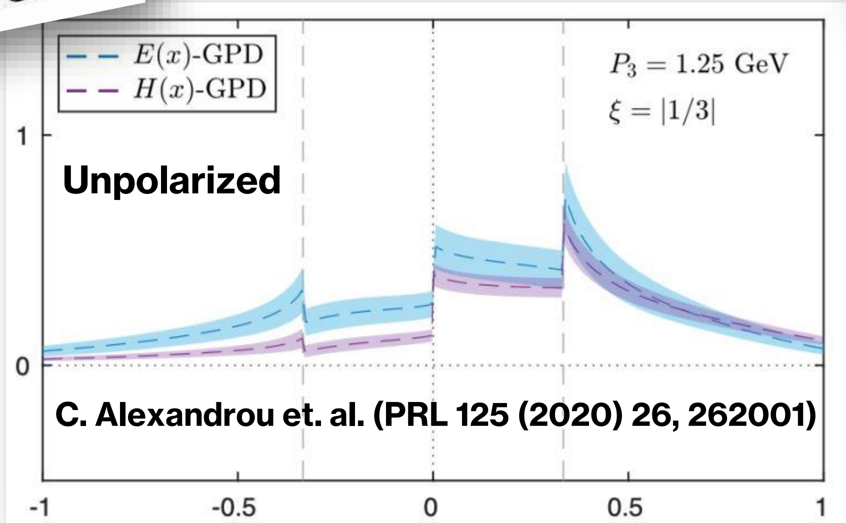

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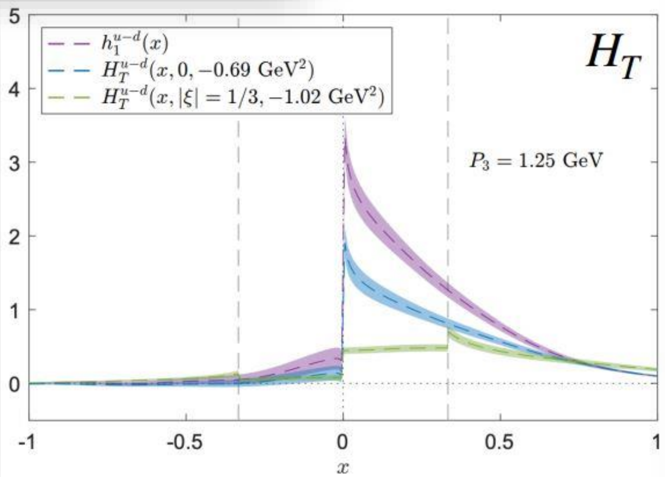
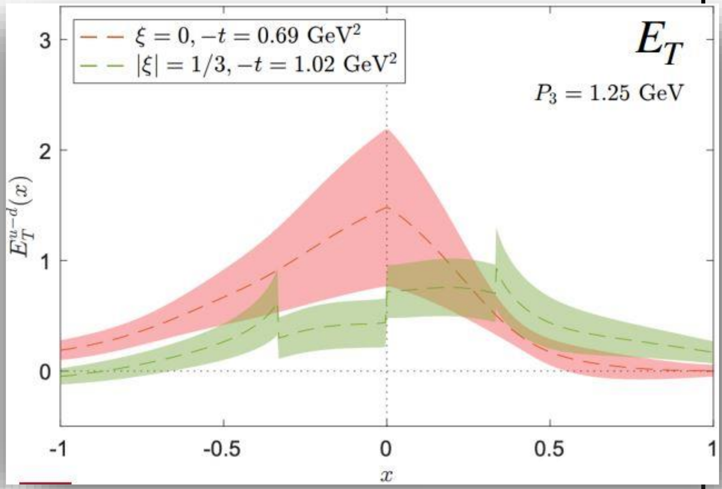
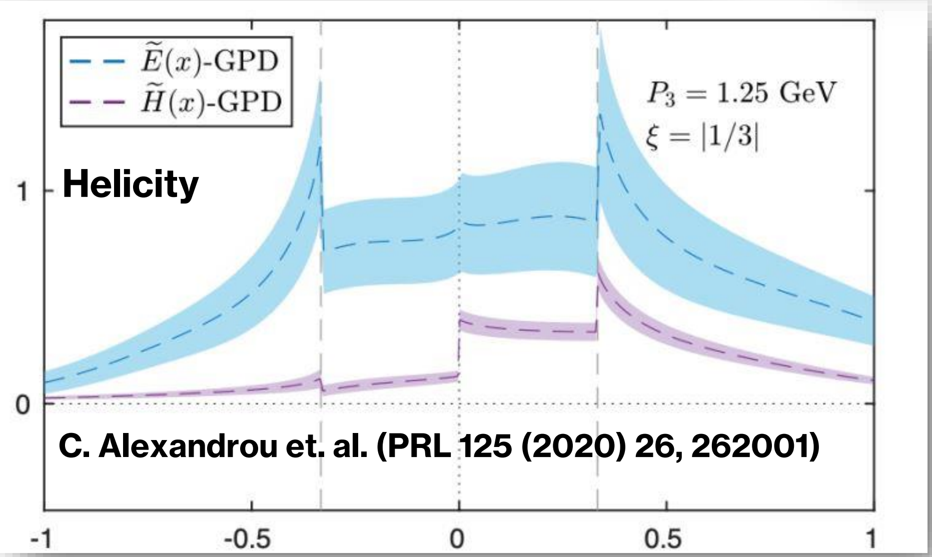
Transversity







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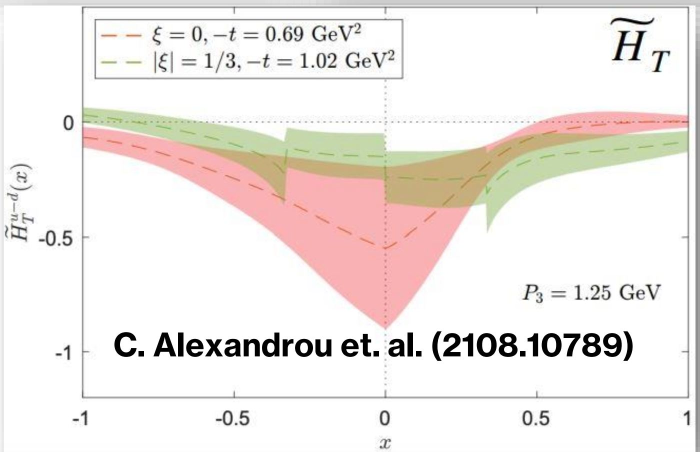


C. Alexandrou et. al. (2108.10789)

GPD $\widetilde{E}_T$ is small/zero within uncertainties (not shown)

First exploration of twist-3 GPDs

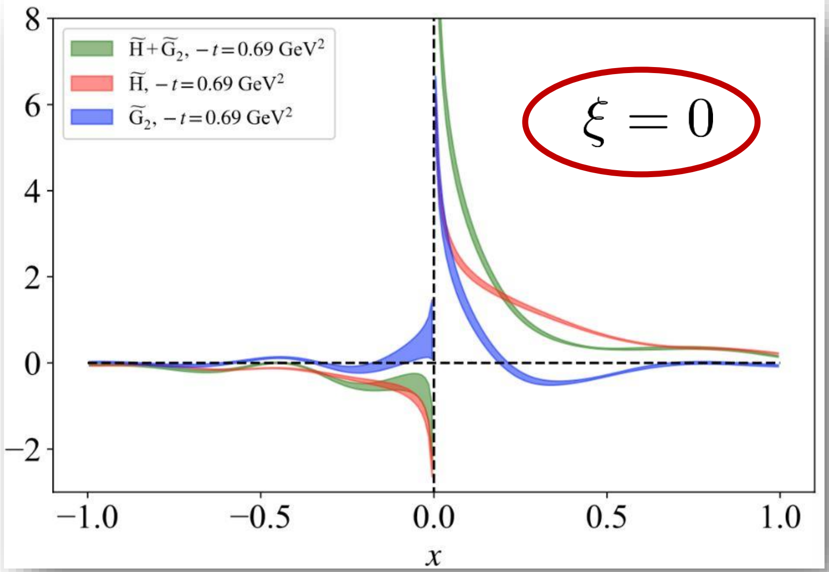
Cichy's talk

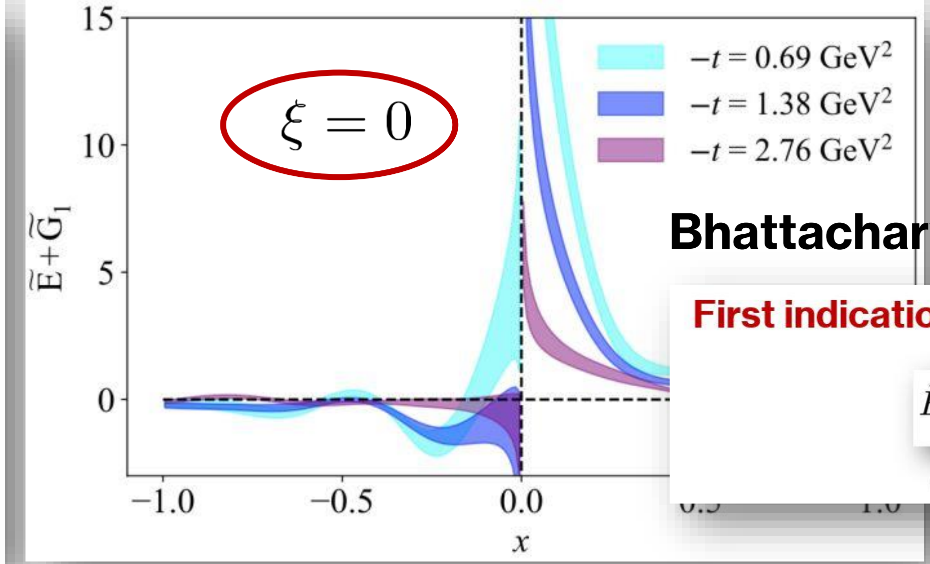
Proton:

$$F^{[\gamma^\mu\gamma_5]}(x,\Delta;P^3) = \frac{1}{2P^3}\bar{u}(p_f,\lambda')\Big[P^\mu\frac{\gamma^3\gamma_5}{P^0}F_{\widetilde{H}}(x,\xi,t;P^3) + P^\mu\frac{\Delta^3\gamma_5}{2mP^0}F_{\widetilde{E}}(x,\xi,t;P^3)$$
$$+ \Delta_\perp^\mu\frac{\gamma_5}{2m}F_{\widetilde{E}+\widetilde{G}_1}(x,\xi,t;P^3) + \gamma_\perp^\mu\gamma_5 F_{\widetilde{H}+\widetilde{G}_2}(x,\xi,t;P^3)$$
$$+ \Delta_\perp^\mu\frac{\gamma^3\gamma_5}{P^3}F_{\widetilde{G}_3}(x,\xi,t;P^3) + i\varepsilon_\perp^{\mu\nu}\Delta_\nu\frac{\gamma^3}{P^3}F_{\widetilde{G}_4}(x,\xi,t;P^3)\Big]u(p_i,\lambda)$$

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

[F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]





Bhattacharya et al, 2306.05533

First indication of pion pole from Lattice QCD!

$$\tilde{E}_u - \tilde{E}_d \sim \frac{1}{l^2 - m_\pi^2}$$

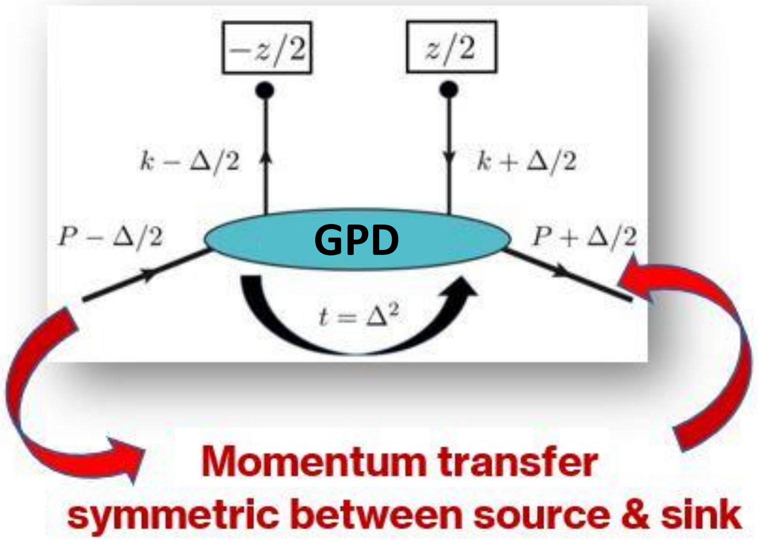
(Penttinen, Polyakov, Goeke)

But little hiccup ...

But little hiccup ...

Traditionally, GPDs have been calculated from "symmetric frames"

Practical drawback



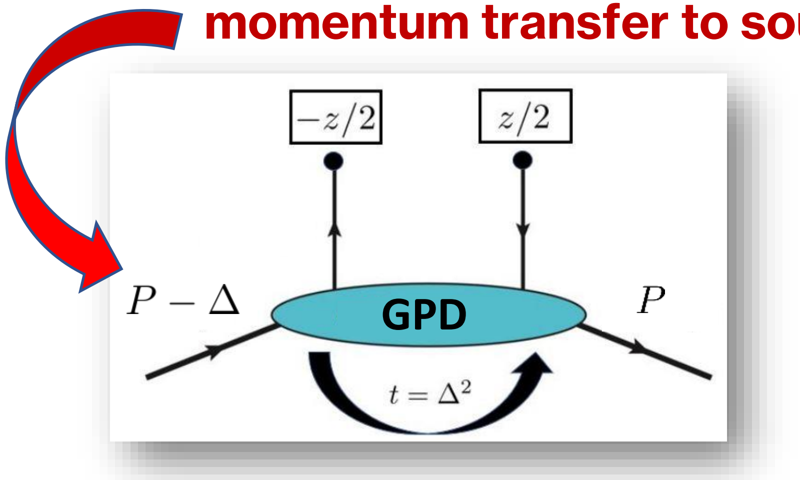
Lattice QCD calculations of GPDs in symmetric frames are expensive

In symmetric frame, full new calculation required for each momentum transfer (△)

GPDs from asymmetric frames

Resolution:

All
momentum transfer to source



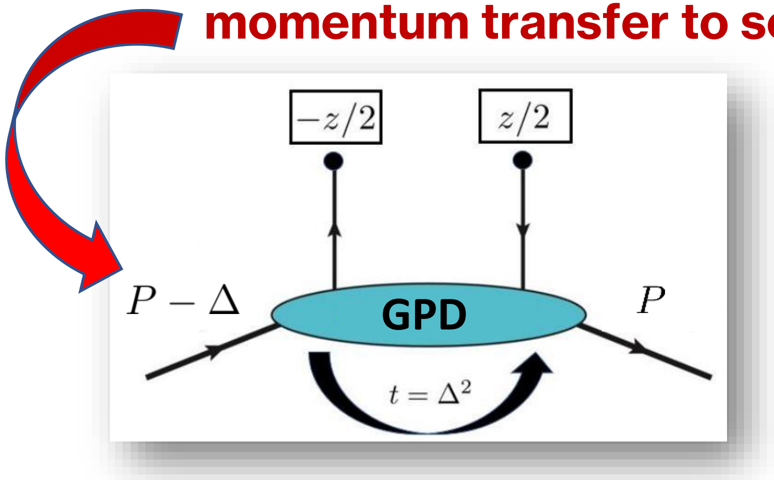
Perform Lattice QCD calculations of GPDs in asymmetric frames: Cichy's talk

- **Reduction in computational cost**
- **Access to broad range of t (enabling creation of high-resolution partonic maps)**

GPDs from asymmetric frames

Resolution:

All momentum transfer to source



Major theoretical advances (Bhattacharya et al, 2209.05373, 2310.13114):

- **Lorentz covariant formalism for calculating quasi-GPDs in any frame**
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs**

GPDs from asymmetric frames

Resolution:

All
momentum transfer to source

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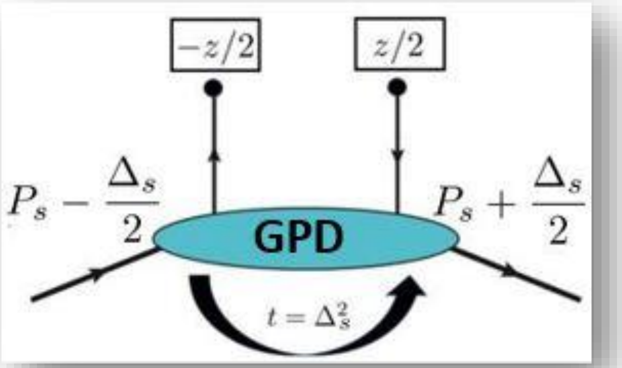
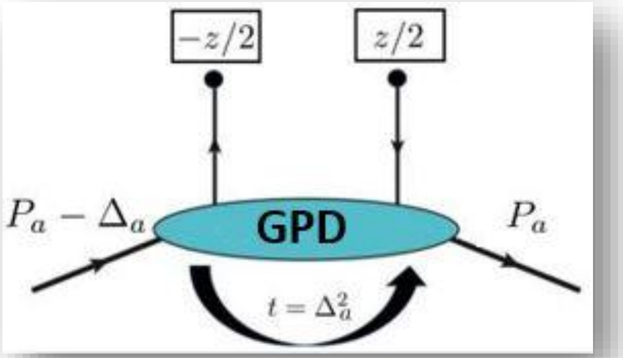
A tale of two frames

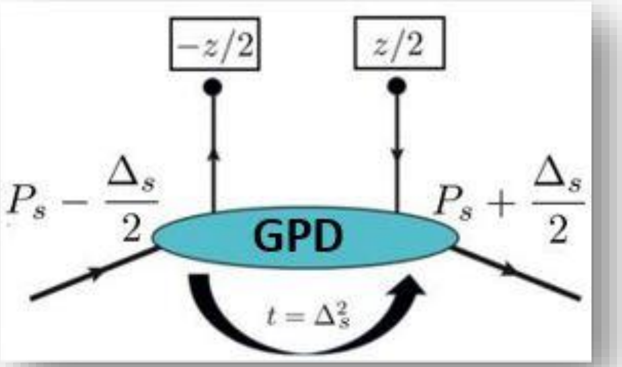
Symmetric & asymmetric frames

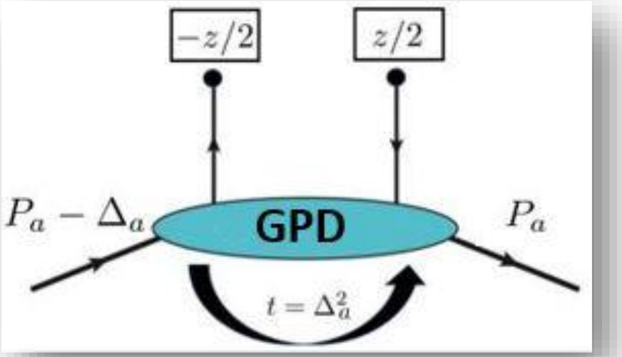




A tale of two frames

Symmetric & asymmetric frames

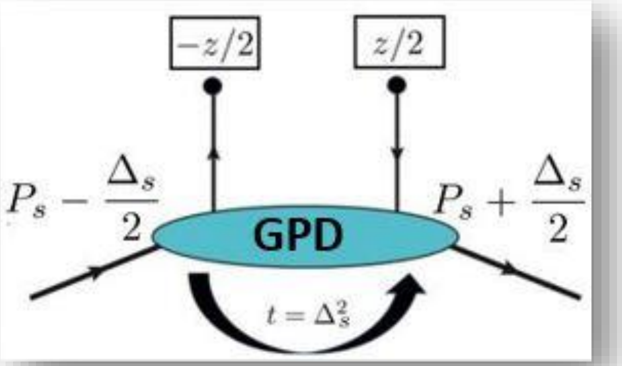




Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

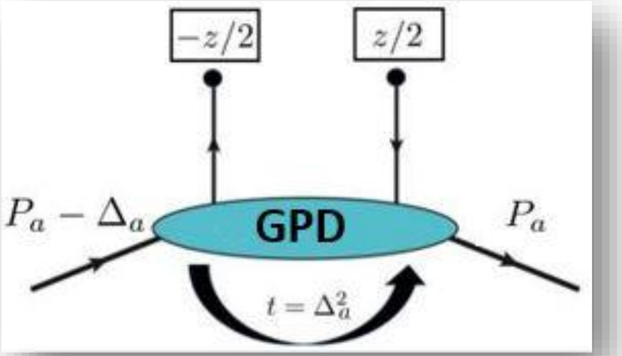
A tale of two frames

Symmetric & asymmetric frames



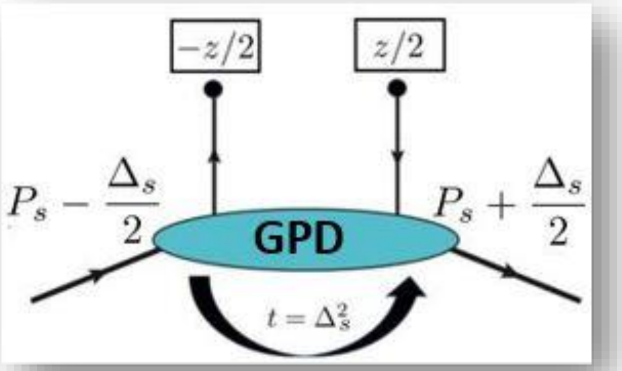
Related via
Lorentz transformation?

What kind?



A tale of two frames

Symmetric & asymmetric frames



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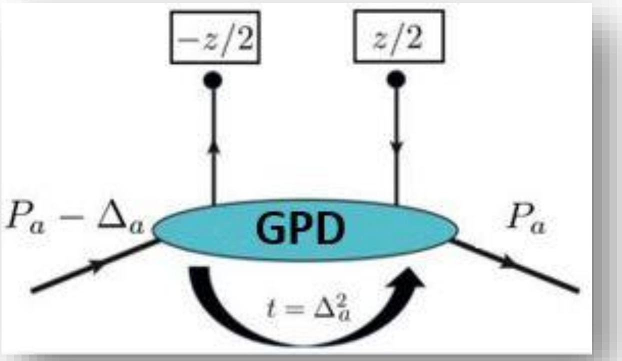
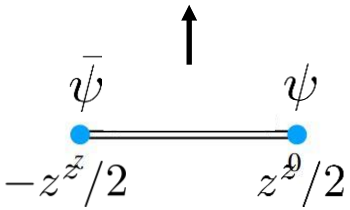


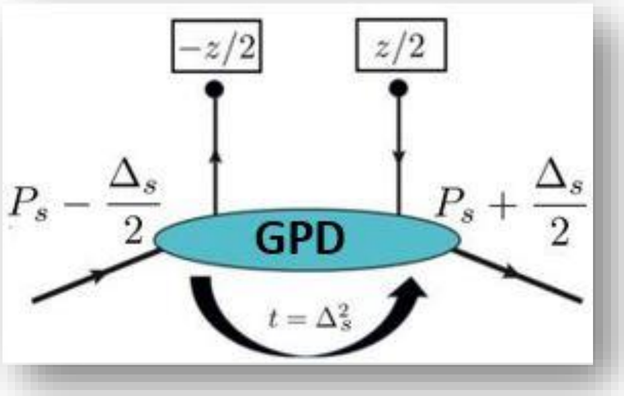
Case 1: Lorentz transformation in the z-direction

$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & 0 & -\gamma\beta \\ 0 & 1 & 0 \\ -\gamma\beta & 0 & \gamma \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$



A tale of two frames

Symmetric & asymmetric frames



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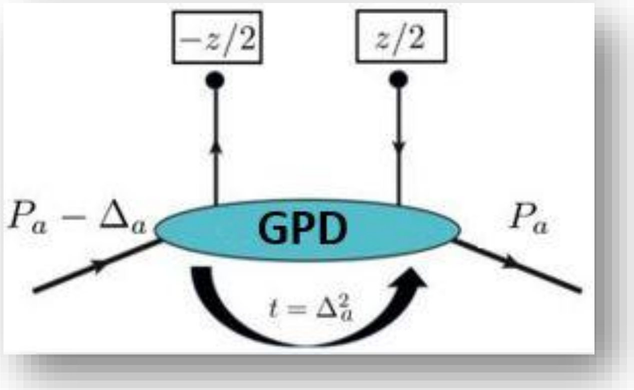
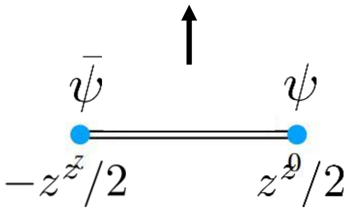


Case 1: Lorentz transformation in the z-direction

$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & 0 & -\gamma\beta \\ 0 & 1 & 0 \\ -\gamma\beta & 0 & \gamma \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$



Results:

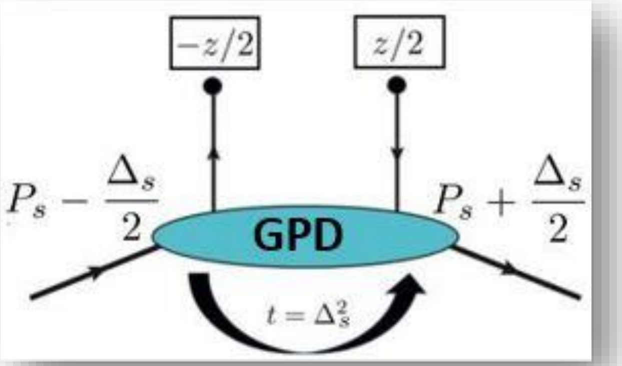
$$z_s^0 = -\gamma\beta z_a^z$$

$$z_s^z = \gamma z_a^z$$

Operator distance develops a non-zero temporal component

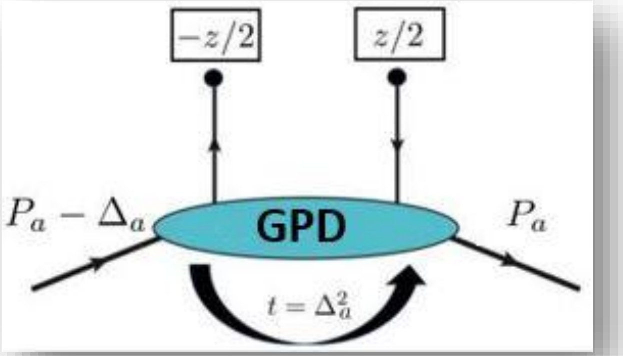
A tale of two frames

Symmetric & asymmetric frames



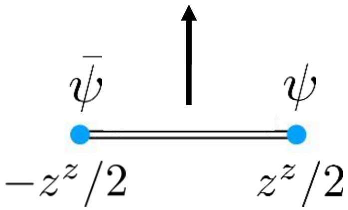
Related via Lorentz transformation?

What kind?



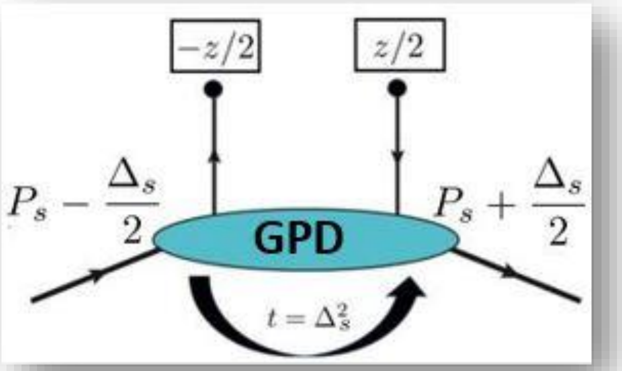
Case 2: Transverse boost in the x-direction

$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 \\ -\gamma\beta & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$



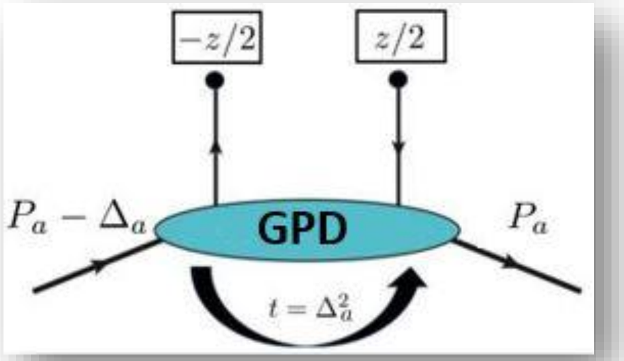
A tale of two frames

Symmetric & asymmetric frames



Related via Lorentz transformation?

What kind?



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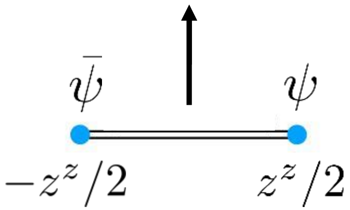
$$\begin{pmatrix} z_s^0 \\ z_s^x \\ z_s^z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 \\ -\gamma\beta & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ z_a^z \end{pmatrix}$$



Results:

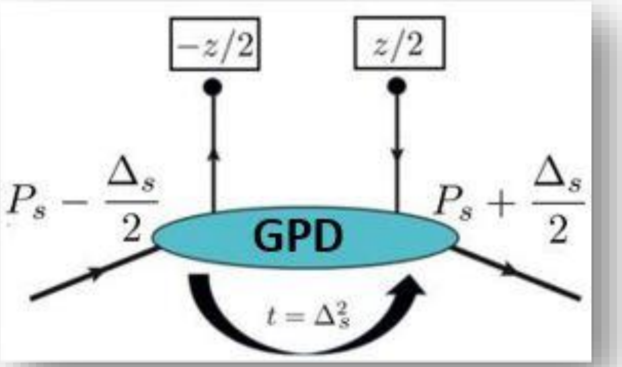
$$z_s^0 = 0$$

$$z_s^z = z_a^z$$

Operator distance remains spatial (& same)

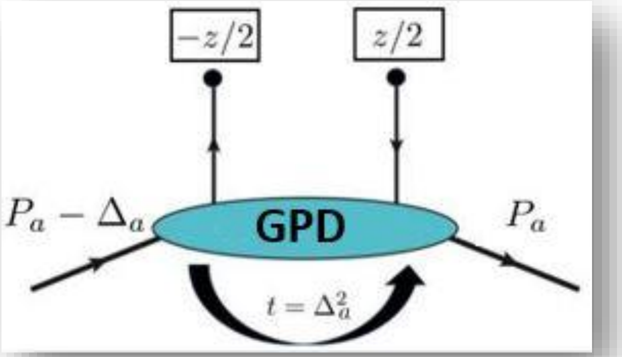
A tale of two frames

Symmetric & asymmetric frames



Related via
Lorentz transformation?

What kind?



Can we calculate a quasi-GPD in symmetric frame through an asymmetric frame?

Transverse boost allows for an exact calculation of quasi-GPDs in symmetric frame through matrix element of asymmetric frame

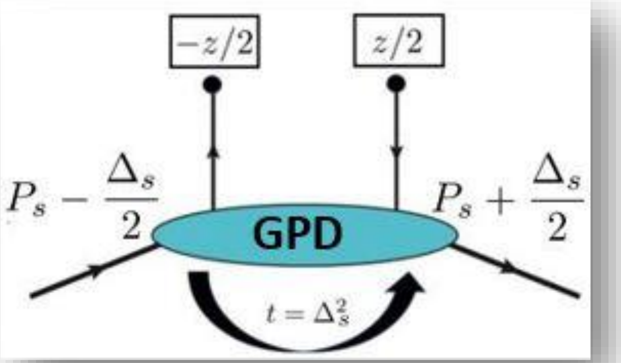
A tale of two frames

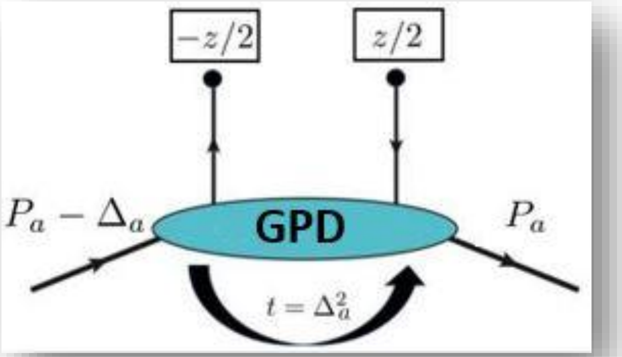
Symmetric & asymmetric frames

Why does it matter in which frame quasi-GPDs are calculated?

$$H(x,\xi,t) = \frac{1}{n\cdot P}\int \frac{d\lambda}{4\pi}\, e^{ix\lambda}\langle p_f|\bar{q}(-\tfrac{\lambda n}{2})\not{n}\,\mathcal{W}(-\tfrac{\lambda n}{2},\tfrac{\lambda n}{2})q(\tfrac{\lambda n}{2})|p_i\rangle$$

GPDs on the light-cone can be defined in a Lorentz invariant way





A tale of two frames

Symmetric & asymmetric frames

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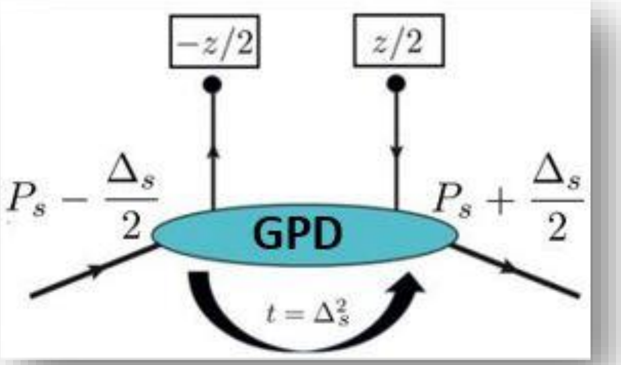
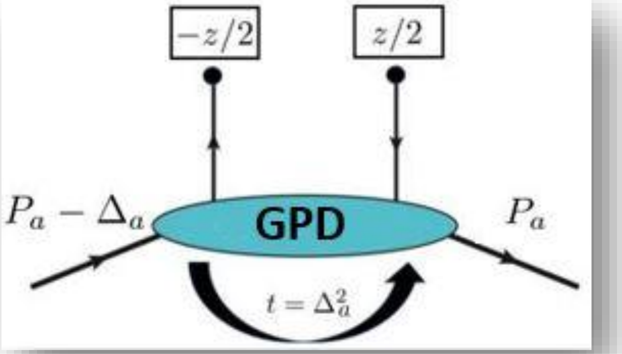




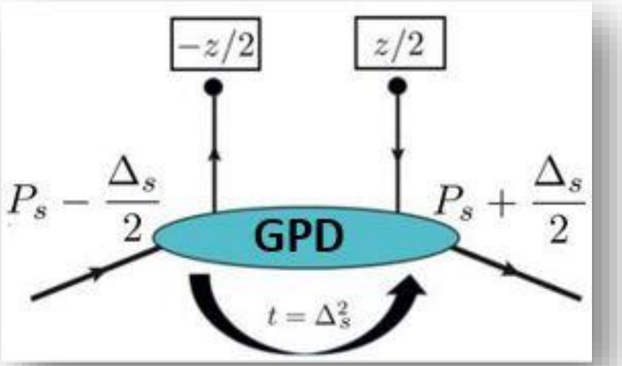
Are quasi-GPDs Lorentz invariant?

A tale of two frames

Definitions of quasi-GPDs

A tale of two frames

Definitions of quasi-GPDs



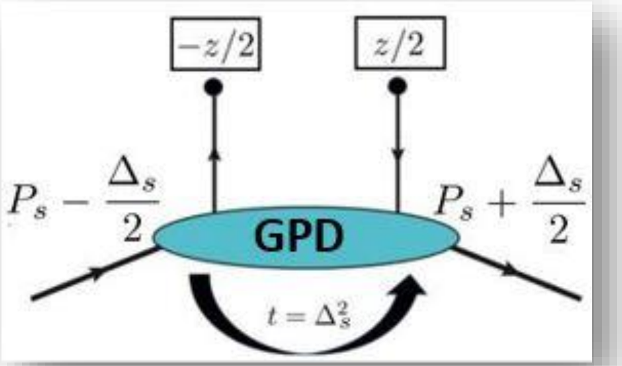
Definition of quasi-GPDs in symmetric frames: (Historical)

$$F^0_{\lambda,\lambda'}\big|_s = \langle p'_s, \lambda' | \bar{q}(-z/2)\gamma^0 q(z/2) | p_s, \lambda\rangle \Big|_{z=0, \vec{z}_\perp = \vec{0}_\perp}$$

$$= \bar{u}_s(p'_s, \lambda') \left[\gamma^0 H_{Q(0)}(z, P_s, \Delta_s)\big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M} E_{Q(0)}(z, P_s, \Delta_s)\big|_s \right] u_s(p_s, \lambda)$$

A tale of two frames

Definitions of quasi-GPDs



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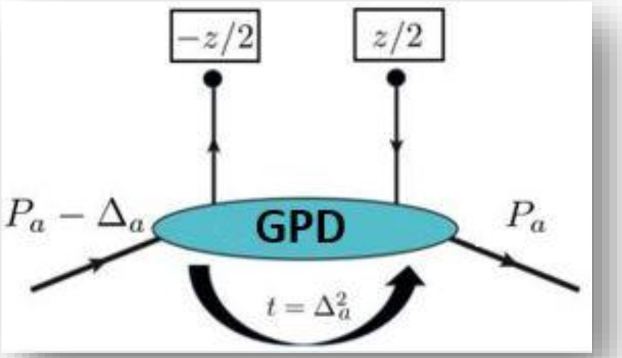


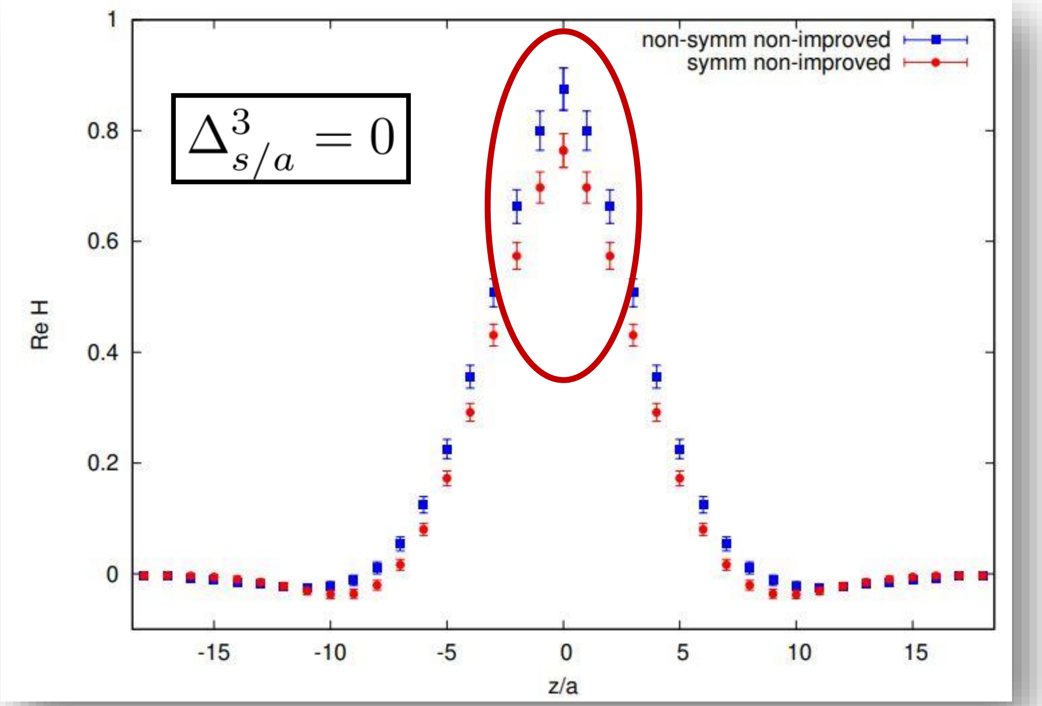
Definition of quasi-GPDs in asymmetric frames:

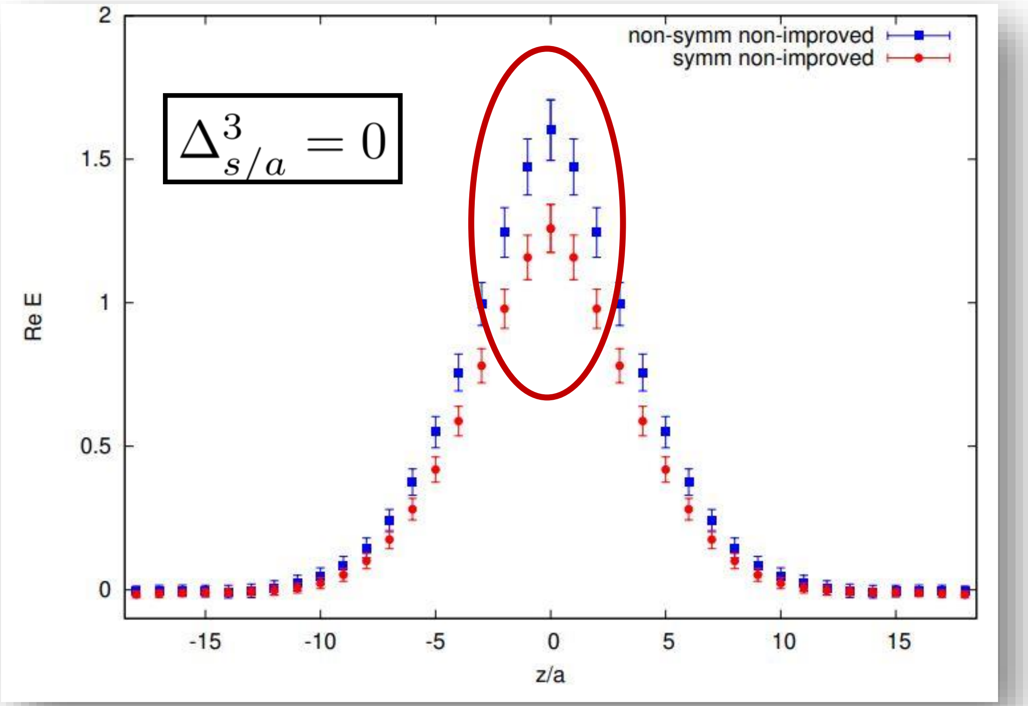
$$F^0_{\lambda,\lambda'}\Big|_a = \langle p'_a, \lambda' | \bar{q}(-z/2)\gamma^0 q(z/2) | p_a, \lambda\rangle \Big|_{z=0, \vec{z}_\perp = \vec{0}_\perp}$$
$$= \bar{u}_a(p'_a, \lambda')\left[\gamma^0 H_{Q(0)}(z, P_a, \Delta_a)\Big|_a + \frac{i\sigma^{0\mu}\Delta_{\mu,a}}{2M} E_{Q(0)}(z, P_a, \Delta_a)\Big|_a\right] u_a(p_a, \lambda)$$

A tale of two frames

Definitions of quasi-GPDs

Lattice QCD results:

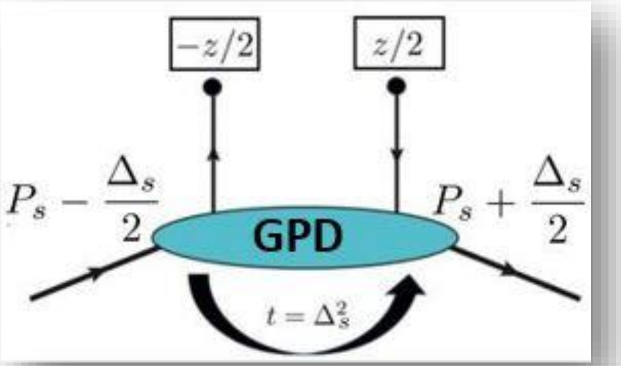




Frame dependence of quasi-GPDs

A tale of two frames

Definitions of quasi-GPDs

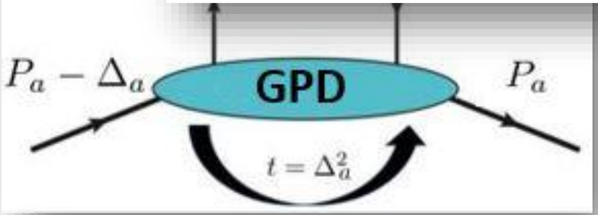


Definition of quasi-GPDs in symmetric frames: (Historical)

$$F^0_{\lambda,\lambda'}\Big|_s = \langle p'_s, \lambda' | \bar{q}(-z/2)\gamma^0 q(z/2) | p_s, \lambda\rangle \Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$
$$= \bar{u}_s(p'_s, \lambda')\left[\gamma^0 H_{Q(0)}(z, P_s, \Delta_s)\Big|_s + \frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M} E_{Q(0)}(z, P_s, \Delta_s)\Big|_s\right] u_s(p_s, \lambda)$$

Historic definitions of quasi-GPDs are not manifestly Lorentz invariant

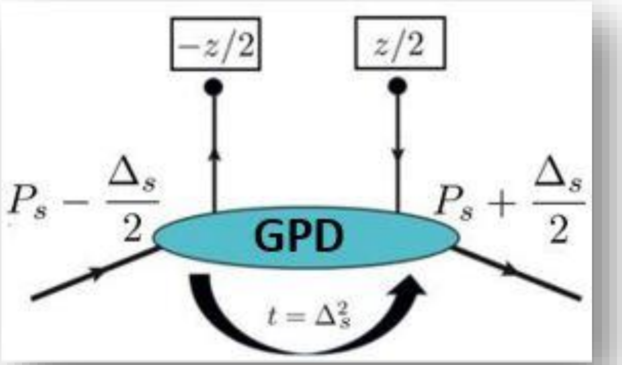
This means that the basis vectors $(\gamma^0, i\sigma^{0\Delta_{s/a}})$ do not form a complete basis for a spatially-separated bi-local operator at finite boost momentum

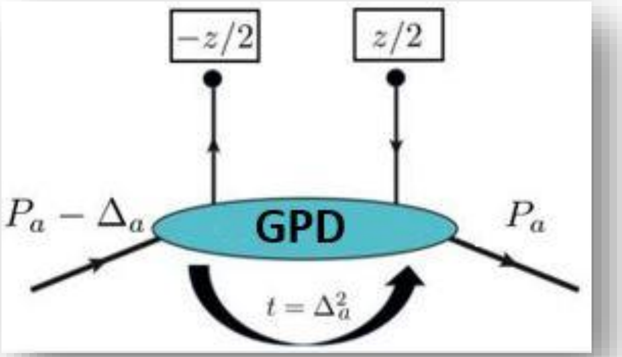


$$= \bar{u}_a(p'_a, \lambda')\left[\gamma^0 H_{Q(0)}(z, P_a, \Delta_a)\Big|_a + \frac{i\sigma^{0\mu}\Delta_{\mu,a}}{2M} E_{Q(0)}(z, P_a, \Delta_a)\Big|_a\right] u_a(p_a, \lambda)$$

A tale of two frames

Definitions of quasi-GPDs





Can we come up with a manifestly Lorentz invariant definition of quasi-GPDs for finite values of momentum?

GPDs from asymmetric frames

Lorentz covariant formalism

GPDs from asymmetric frames

Example

Lorentz covariant formalism

Novel parameterization of position-space matrix element:

$$F^\mu(z,P,\Delta)=\bar{u}(p_f,\lambda')\left[\frac{P^\mu}{m}\boldsymbol{A_1}+mz^\mu\boldsymbol{A_2}+\frac{\Delta^\mu}{m}\boldsymbol{A_3}+im\sigma^{\mu z}\boldsymbol{A_4}+\frac{i\sigma^{\mu\Delta}}{m}\boldsymbol{A_5}+\frac{P^\mu i\sigma^{z\Delta}}{m}\boldsymbol{A_6}+mz^\mu i\sigma^{z\Delta}\boldsymbol{A_7}+\frac{\Delta^\mu i\sigma^{z\Delta}}{m}\boldsymbol{A_8}\right]u(p_i,\lambda)$$

Vector operator $F^\mu_{\lambda,\lambda'}=\langle p',\lambda'|\bar{q}(-z/2)\gamma^\mu q(z/2)|p,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$

Features:

- **8** linearly-independent Dirac structures
- **8** Lorentz-invariant (frame-independent) amplitudes $\boldsymbol{A_i\equiv A_i(z\cdot P,z\cdot\Delta,t=\Delta^2,z^2)}$

GPDs from asymmetric frames

Example

Lorentz covariant formalism

Novel parameterization of position-space matrix element:

$$F^{\mu}(z,P,\Delta)=\bar{u}(p_f,\lambda')\left[\frac{P^{\mu}}{m}\boldsymbol{A_1}+mz^{\mu}\boldsymbol{A_2}+\frac{\Delta^{\mu}}{m}\boldsymbol{A_3}+im\sigma^{\mu z}\boldsymbol{A_4}+\frac{i\sigma^{\mu\Delta}}{m}\boldsymbol{A_5}+\frac{P^{\mu}i\sigma^{z\Delta}}{m}\boldsymbol{A_6}+mz^{\mu}i\sigma^{z\Delta}\boldsymbol{A_7}+\frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}\boldsymbol{A_8}\right]u(p_i,\lambda)$$

Validating frame-independence of amplitudes from Lattice QCD

Cichy's talk

GPDs from asymmetric frames

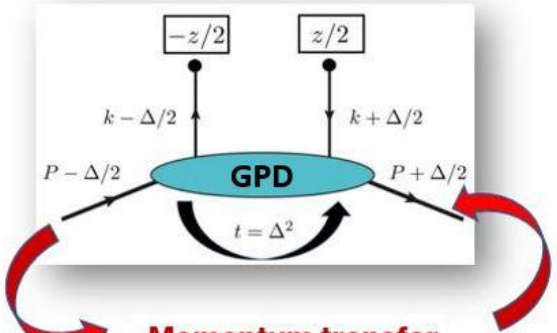
Example

Lorentz covariant formalism

Novel parameterization of position-space matrix element:

$$F^{\mu}(z,P,\Delta)=\bar{u}(p_f,\lambda')\left[\frac{P^{\mu}}{m}\boldsymbol{A_1}+mz^{\mu}\boldsymbol{A_2}+\frac{\Delta^{\mu}}{m}\boldsymbol{A_3}+im\sigma^{\mu z}\boldsymbol{A_4}+\frac{i\sigma^{\mu\Delta}}{m}\boldsymbol{A_5}+\frac{P^{\mu}i\sigma^{z\Delta}}{m}\boldsymbol{A_6}+mz^{\mu}i\sigma^{z\Delta}\boldsymbol{A_7}+\frac{\Delta^{\mu}i\sigma^{z\Delta}}{m}\boldsymbol{A_8}\right]u(p_i,\lambda)$$

Traditional definition (symmetric frame):



Momentum transfer symmetric between source & sink

$$F^{0}_{\lambda,\lambda'}\Big|_s=\langle p_s',\lambda'|\bar{q}(-z/2)\gamma^0 q(z/2)|p_s,\lambda\rangle\Big|_{z=0,\vec{z}_\perp=\vec{0}_\perp}$$

$$=\bar{u}_s(p_s',\lambda')\left[\gamma^0 H_{Q(0)}(z,P_s,\Delta_s)\Big|_s+\frac{i\sigma^{0\mu}\Delta_{\mu,s}}{2M}E_{Q(0)}(z,P_s,\Delta_s)\Big|_s\right]u_s(p_s,\lambda)$$

Quasi-GPDs are intrinsically frame-dependent

GPDs from asymmetric frames

Example

Lorentz covariant formalism

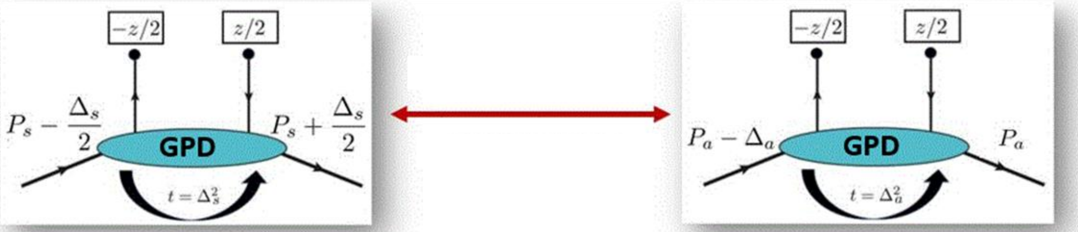
Nove **Main point:**

$$H^{s}_{\mathrm{Q}(0)} = \sum_i c_i\, A_i$$

$$F^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^\mu}{m}A_1 + m \ldots + m z^\mu i\sigma^{z\Delta} A_7 + \frac{\Delta^\mu i \sigma^{z\Delta}}{m} A_8\right] u(p_i,\lambda)$$

Main point:

Calculate quasi-GPD in symmetric frame through matrix elements of asymmetric frame



Symmetric frame   Asymmetric frame

- 8 Lorentz … $A_i(z\cdot P, z\cdot\Delta, t=\Delta^2, z^2)$

GPDs from asymmetric frames

Resolution:

All
momentum transfer to source

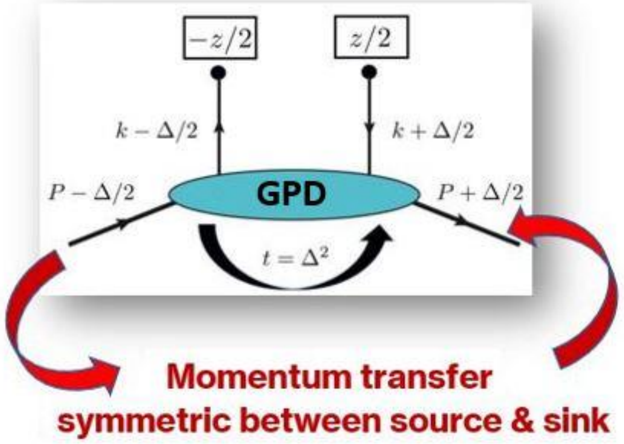
Major theoretical advances:

- Lorentz covariant formalism for calculating quasi-GPDs in any frame
- **Elimination of power corrections potentially allowing faster convergence to light-cone GPDs**

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Example: Symmetric frame



Quasi-GPD:

$$H^s_{\mathrm{Q}(0)}(z, P^s, \Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}} A_3 - \frac{m^2 \Delta^{0,s} z^3}{2P^{0,s} P^{3,s}} A_4 + \left[\frac{(\Delta^{0,s})^2 z^3}{2P^{3,s}} - \frac{\Delta^{0,s}\Delta^{3,s} z^3 P^{0,s}}{2(P^{3,s})^2} - \frac{z^3 (\Delta^s_\perp)^2}{2P^{3,s}}\right] A_6$$

$$+ \left[\frac{(\Delta^{0,s})^3 z^3}{2P^{0,s} P^{3,s}} - \frac{(\Delta^{0,s})^2 \Delta^{3,s} z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s} z^3 (\Delta^s_\perp)^2}{2P^{0,s} P^{3,s}}\right] A_8\,,$$

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Light-cone GPD: (Lorentz-invariant)

$$H(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3$$

Quasi-GPD: (Symmetric frame)

$$H^s_{\mathrm{Q}(0)}(z, P^s, \Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}}A_3 - \frac{m^2\Delta^{0,s}z^3}{2P^{0,s}P^{3,s}}A_4 + \left[\frac{(\Delta^{0,s})^2 z^3}{2P^{3,s}} - \frac{\Delta^{0,s}\Delta^{3,s}z^3 P^{0,s}}{2(P^{3,s})^2} - \frac{z^3(\Delta^s_\perp)^2}{2P^{3,s}}\right]A_6$$
$$+ \left[\frac{(\Delta^{0,s})^3 z^3}{2P^{0,s}P^{3,s}} - \frac{(\Delta^{0,s})^2\Delta^{3,s}z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s}z^3(\Delta^s_\perp)^2}{2P^{0,s}P^{3,s}}\right]A_8\,,$$

GPDs from asymmetric frames

Relations between GPDs & amplitudes

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$$H(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3$$

Contamination from additional amplitudes or explicit power corrections

Quasi-GPD: (Symmetric frame)

$$H^s_{\mathrm{Q}(0)}(z,P^s,\Delta^s) = A_1 + \frac{\Delta^{0,s}}{P^{0,s}}A_3 - \frac{m^2\Delta^{0,s}z^3}{2P^{0,s}P^{3,s}}A_4 + \left[\frac{(\Delta^{0,s})^2z^3}{2P^{3,s}} - \frac{\Delta^{0,s}\Delta^{3,s}z^3P^{0,s}}{2(P^{3,s})^2} - \frac{z^3(\Delta^s_\perp)^2}{2P^{3,s}}\right]A_6$$
$$+\left[\frac{(\Delta^{0,s})^3z^3}{2P^{0,s}P^{3,s}} - \frac{(\Delta^{0,s})^2\Delta^{3,s}z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s}z^3(\Delta^s_\perp)^2}{2P^{0,s}P^{3,s}}\right]A_8\,,$$

GPDs from asymmetric frames

Interlude: quasi-PDFs

GPDs from asymmetric frames

Interlude: quasi-PDFs

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

A. V. Radyushkin
Old Dominion University, Norfolk, VA 23529, USA and
Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar{\psi}(0)\,\gamma^\alpha\,\hat{E}(0,z;A)\psi(z)|p\rangle \tag{12}$$

type, where $\hat{E}(0,z;A)$ is the standard $0 \to z$ straight-line gauge link in the quark (fundamental) representation. These matrix elements may be decomposed into p^α and z^α parts:

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha \mathcal{M}_p(-(zp),-z^2) + z^\alpha \mathcal{M}_z(-(zp),-z^2)\,. \tag{13}$$

2 amplitudes

The $\mathcal{M}_p(-(zp),-z^2)$ part gives the twist-2 distribution when $z^2 \to 0$, while $\mathcal{M}_z((zp),-z^2)$ is a purely higher-twist contamination, and it is better to get rid of it.

GPDs from asymmetric frames

Interlude: quasi-PDFs

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

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2 amplitudes

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The $\mathcal{M}_p(-(zp),-z^2)$ part gives the twist-2 distribution when $z^2 \to 0$, while $\mathcal{M}_z((zp),-z^2)$ is a purely higher-twist contamination, and it is better to get rid of it.

If one takes $z=(z_-,z_\perp)$ in the $\alpha=+$ component of $\mathcal{M}^\alpha$, the z^α-part drops out, and one can introduce a

GPDs from asymmetric frames

Interlude: quasi-PDFs

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

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type, where $\hat{E}(0,z;A)$ is the standard $0 \to z$ straight-line gauge link in the quark (fundamental) representation. These matrix elements may be decomposed into p^α and z^α parts:

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha \mathcal{M}_p(-(zp),-z^2) + z^\alpha \mathcal{M}_z(-(zp),-z^2)\,. \qquad (13)$$

2 amplitudes

The $\mathcal{M}_p(-(zp),-z^2)$ part gives the twist-2 distribution when $z^2 \to 0$, while $\mathcal{M}_z((zp),-z^2)$ is a purely higher-twist contamination, and it is better to get rid of it.

If one takes $z = (z_-, z_\perp)$ in the $\alpha = +$ component of $\mathcal{M}^\alpha$, the z^α-part drops out, and one can introduce a

formula (6). For quasi-distributions, the easiest way to remove the z^α contamination is to take the time component of $\mathcal{M}^\alpha(z=z_3,p)$ and define

$$\mathcal{M}^0(z_3,p) = 2p^0 \int_{-1}^{1} dy\, Q(y,P)\, e^{iyPz_3}\,. \qquad (14)$$

Hence, γ^0 is better behaved than γ^3 (power corrections)

GPDs from asymmetric frames

Interlude: quasi-PDFs

arXiv: 1705.01488

Quasi-PDFs, momentum distributions and pseudo-PDFs

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Statement needs a qualifier for quasi-GPDs:

Contrary to quasi-PDFs, γ^0 operator for quasi-GPDs is plagued with (frame-dependent) power corrections

$$\mathcal{M}^\alpha(z,p) \equiv \langle p|\bar\psi(0)\,\gamma^\alpha \hat{E}(0,z;A)\psi(z)|p\rangle \quad (12)$$

type, where $\hat{E}(0,z;A)$ is the standard $0 \to z$ straight-line gauge link in the quark (fundamental) representation. These matrix elements may be decomposed into p^α and z^α parts:

2 amplitudes

$$\mathcal{M}^\alpha(z,p) = 2p^\alpha \mathcal{M}_p(-(zp), -z^2) + z^\alpha \mathcal{M}_z(-(zp), -z^2)\,. \quad (13)$$

The $\mathcal{M}_p(-(zp), -z^2)$ part gives the twist-2 distribution when $z^2 \to 0$, while $\mathcal{M}_z((zp), -z^2)$ is a purely higher-twist contamination, and it is better to get rid of it.

... + component ... drops out, and one can introduce a

formula (6). For quasi-distributions, the easiest way to remove the z^α contamination is to take the time component of $\mathcal{M}^\alpha(z = z_3, p)$ and define

$$\mathcal{M}^0(z_3,p) = 2p^0 \int_{-1}^{1} dy\, Q(y,P)\, e^{iyPz_3}\,. \quad (14)$$

Hence, γ^0 is better behaved than γ^3 (power corrections)

GPDs from asymmetric frames

Relations between GPDs & amplitudes

Light-cone GPD: (Lorentz-invariant)

Main finding

Schematic structure of (operator-level) Lorentz-invariant definition of quasi-GPD:

$$H_Q \to c_0 \langle \bar{\psi}\gamma^0\psi \rangle + c_1 \langle \bar{\psi}\gamma^1\psi \rangle + c_2 \langle \bar{\psi}\gamma^2\psi \rangle$$

Here, c's are frame-dependent kinematic factors that cancel additional amplitudes such that quasi-GPD has the same functional form as light-cone GPD (Lorentz invariant)

$$+\left[\frac{(\Delta^{0,s})^3 z^3}{2P^{0,s}P^{3,s}} - \frac{(\Delta^{0,s})^2\Delta^{3,s}z^3}{2(P^{3,s})^2} - \frac{\Delta^{0,s}z^3(\Delta_\perp^s)^2}{2P^{0,s}P^{3,s}}\right]A_8\,,$$

GPDs from asymmetric frames

New definition of quasi-GPDs

Light-cone GPD:

$$H(z \cdot P^{s/a}, z \cdot \Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a} \cdot z}{P^{s/a} \cdot z} A_3$$

$$A_i \equiv A_i(z^2 = 0)$$

Lorentz-invariant definition of quasi-GPD:

$$\mathcal{H}(z \cdot P^{s/a}, z \cdot \Delta^{s/a}, (\Delta^{s/a})^2, z^2) = A_1 + \frac{\Delta^{s/a} \cdot z}{P^{s/a} \cdot z} A_3$$

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Same functional forms

GPDs from asymmetric frames

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Features:

- **Lorentz-invariant definition of quasi-GPDs may converge faster**

GPDs from asymmetric frames

New definition of quasi-GPDs

Light-cone GPD:

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$$A_i \equiv A_i(z^2\neq 0)$$

Features:

- **Lorentz-invariant definition of quasi-GPDs may converge faster**
- **Caveat:** It is essential to acknowledge that the amplitudes themselves also contain implicit power corrections. (So, the presence of additional amplitudes could potentially mitigate the implicit power corrections.) Ultimately, the actual convergence of the different quasi-GPD definitions is determined by the underlying non-perturbative dynamics. **See Cichy's talk**

Summary

- **Tremendous recent activity in studying parton structure of hadrons in lattice QCD through Euclidean correlators**

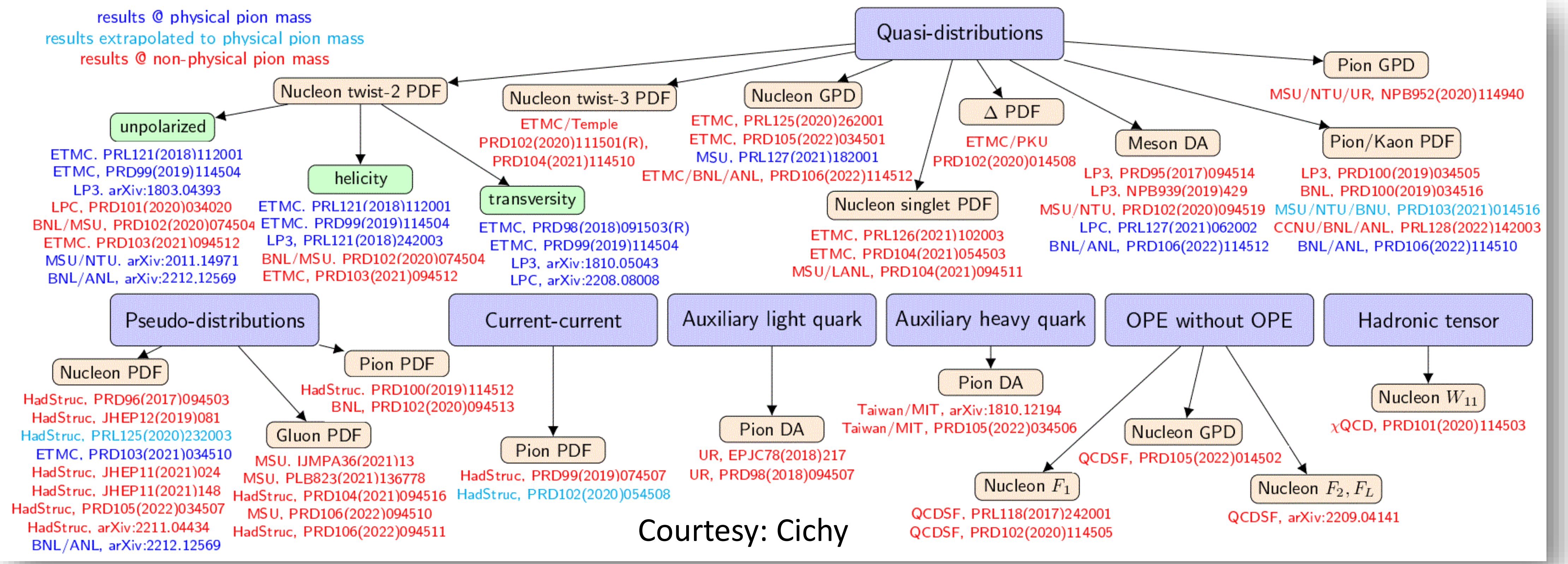
- **Impact of approach(es) largest where experiments are difficult → GPDs**

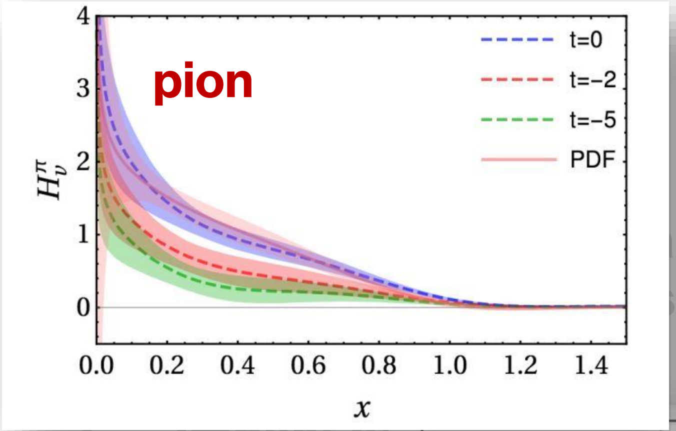
Summary

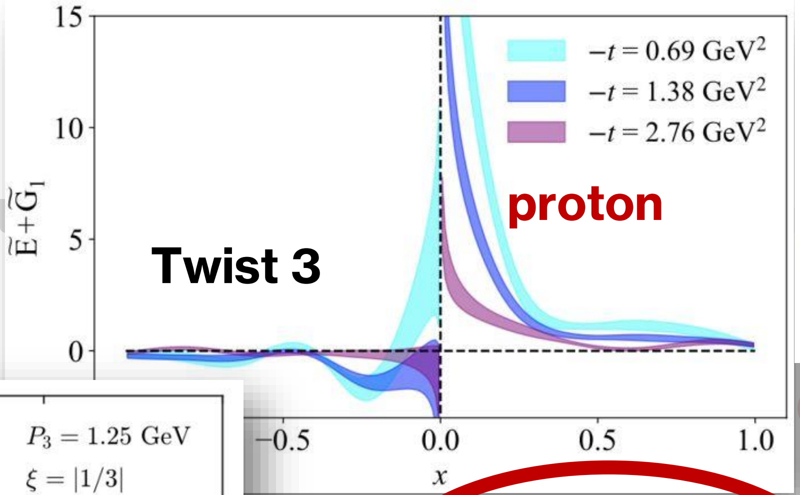
- Tremendous recent activity in studying parton structure of hadrons in lattice QCD through Euclidean correlators

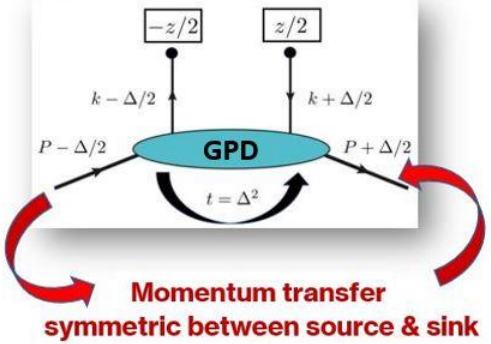
- Impact of approach(es) largest where experiments are difficult → GPDs

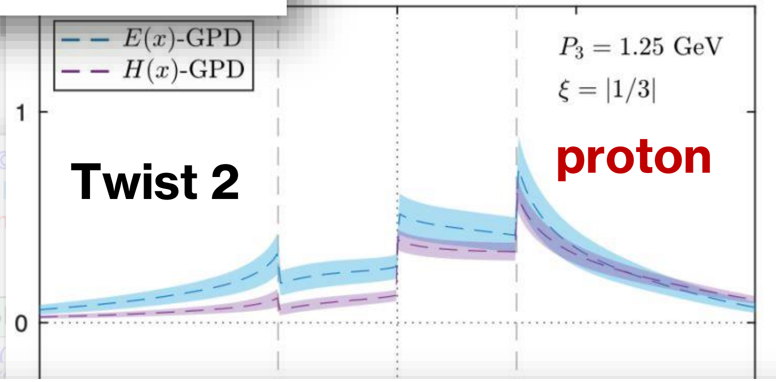
Overview of Euclidean-correlator approaches

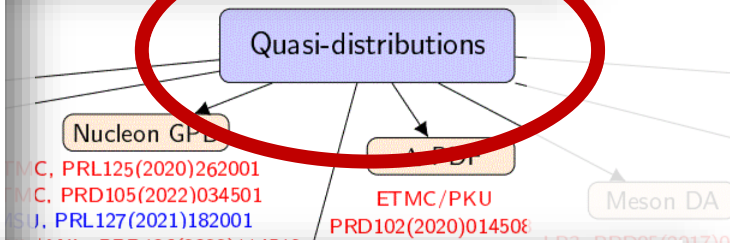


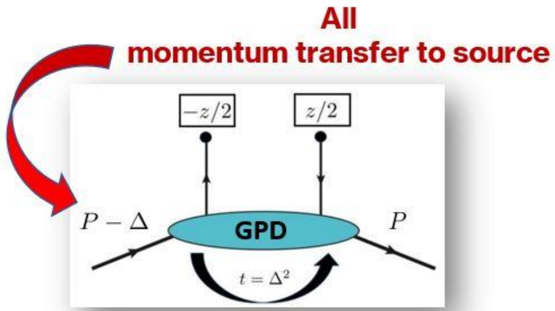




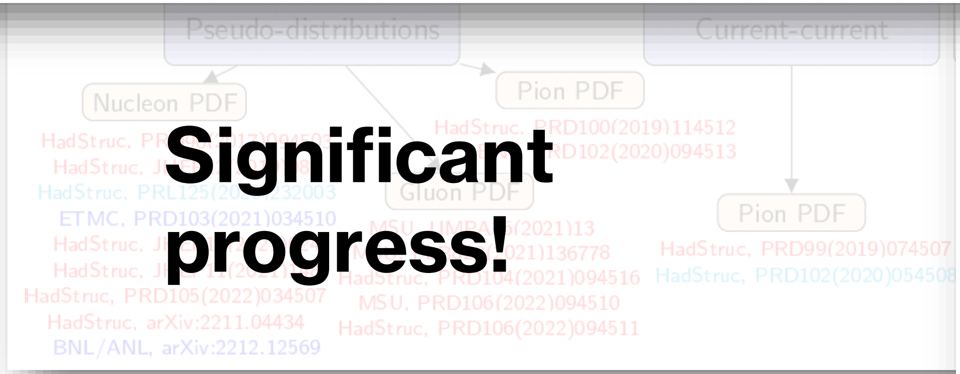








Lattice calculations can serve as a valuable complement to the ongoing efforts at the EIC



Significant progress!