Positron acceleration in plasma wakefields

PHYSICAL REVIEW ACCELERATORS AND BEAMS **27,** 034801 (2024)

Review Article

Positron acceleration in plasma wakefields

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Numerical simulations were performed using HPC resources from GENCI-TGCC (Grant No. 2020-A0080510786 and No. 2020-A0090510062) and using the open source quasistatic PIC code QuickPIC.

Outline

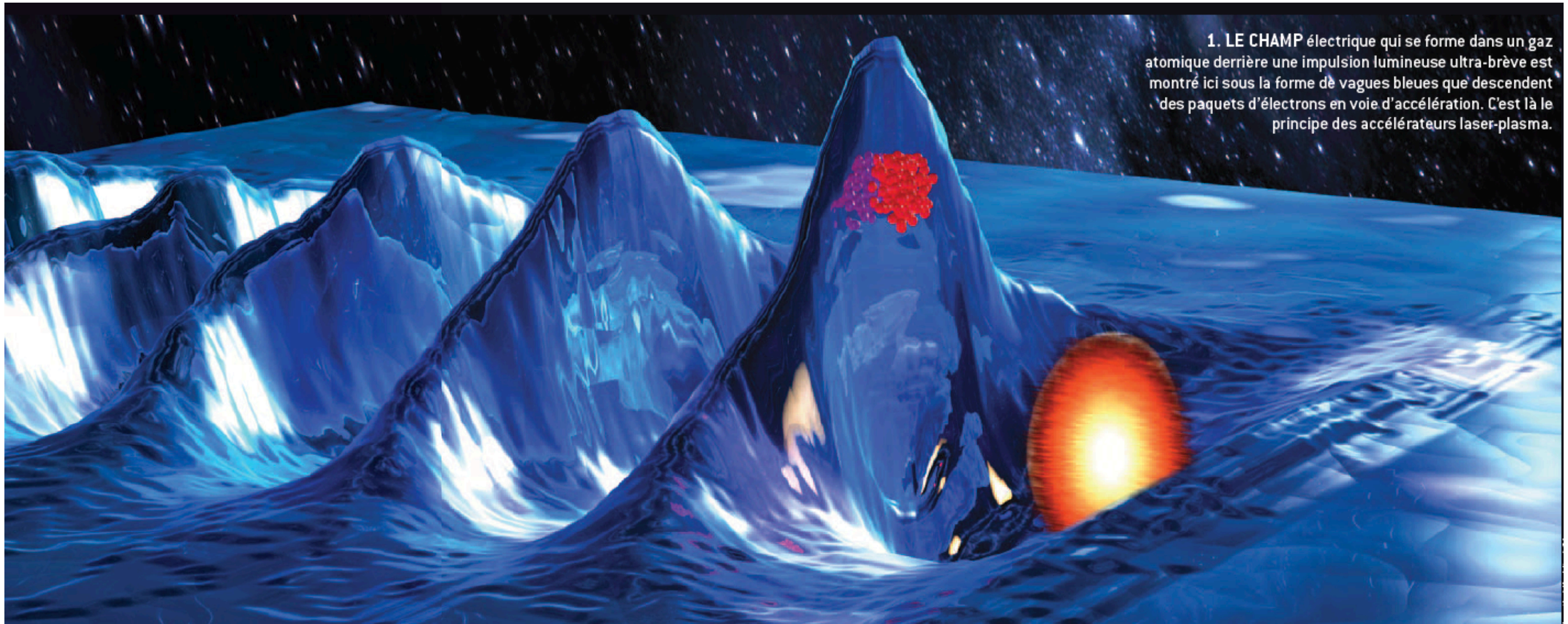
- Basics of plasma accelerator physics
- Scientific context for positron acceleration
- Preliminary considerations with positron-loaded quasilinear plasma wakefields
  - ➢ Efficiency
  - ➢ Evolution of transverse emittance
  - ➢ Uncorrelated energy spread
  - ➢ Energy efficiency vs beam quality tradeoff
- The positron problem
  - ➢ Luminosity-per-power
  - ➢ Electron motion
  - ➢ Strategies

Basics of plasma accelerator physics

Plasma-based accelerators

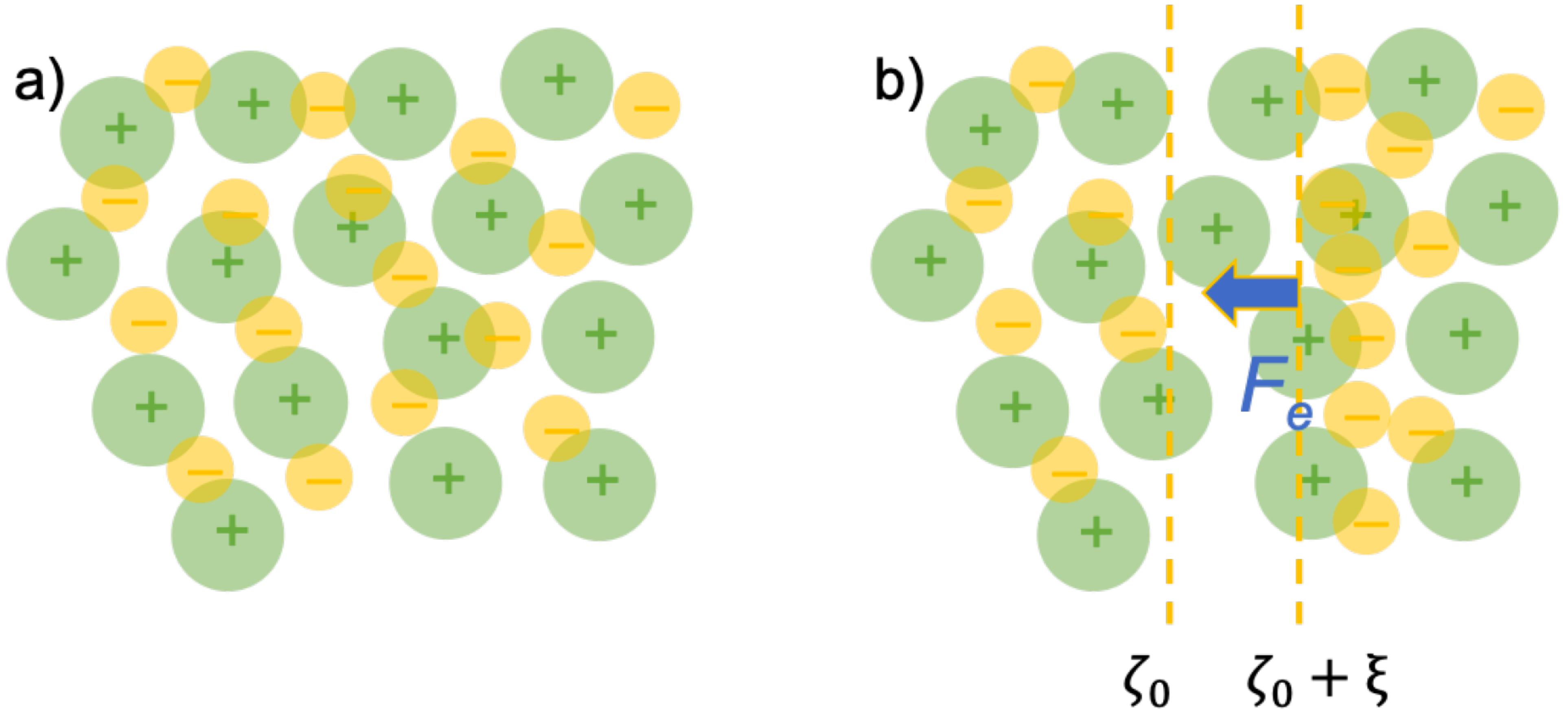
- Motivations, the obvious:
  - Plasma accelerator: already ionized medium, no breakdown limit for the field
  - Plasma photocathode: opportunities to generate brighter beams

- Two key physics area in plasma-based accelerators:
  - Exciting plasma wakefields with particle beams and laser pulses (plasma time scale)
  - Driver and particle beam evolution in plasma (driver and beam time scale)



Plasma-based accelerators: linear wakefield

- At the heart of plasma accelerators, the plasma oscillation:



a displaced plasma slice is an harmonic oscillator:

$$d_t^2 \xi = -\omega_p^2 \xi$$

with a characteristic frequency ω_p called the plasma frequency:

$$\omega_p = \frac{n_0 e^2}{m_e \epsilon_0}$$

- Using a cold fluid description for plasma electrons and immobile ions:

continuity $$\frac{\partial n_p}{\partial t} + \nabla \cdot (n_p \vec{v_p}) = 0$$

eq. of motion $$\left(\frac{\partial}{\partial t} + \vec{v}_p \cdot \nabla\right) \vec{p}_p = -e(\vec{E} + \vec{v}_p \times \vec{B})$$

Maxwell $$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}, \quad \text{etc.}$$

linearize $\longrightarrow$ $n_p = n_0 + n_1, \quad \text{etc.}$

$$\frac{\partial n_1}{\partial t} + n_0 \nabla \cdot (\vec{v_1}) = 0$$

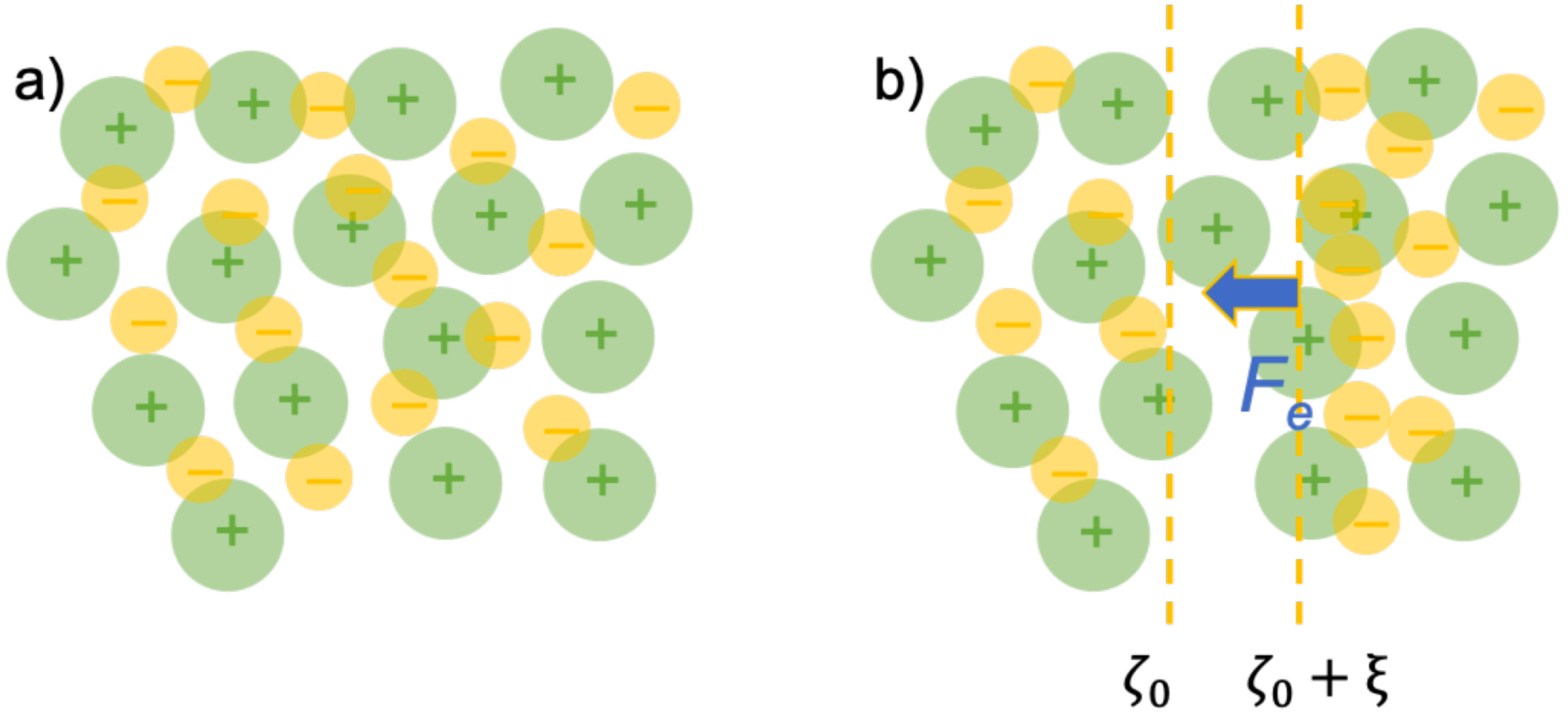
$$m_e \frac{\partial \vec{v}_1}{\partial t} = -e\vec{E}_1$$

$$\nabla \cdot \vec{E}_1 = -e\frac{n_1}{\epsilon_0} + q\frac{n_b}{\epsilon_0}, \quad \text{etc.}$$

external particle beam

Plasma-based accelerators: linear wakefield

➢ At the heart of plasma accelerators, the plasma oscillation:



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Maxwell $$\nabla \cdot \vec{E}_1 = -e \frac{n_1}{\epsilon_0} + q \frac{n_b}{\epsilon_0}, \quad \text{etc}.$$

⟶

$$\left(\frac{\partial}{\partial t^2} + \omega_p^2 \right) n_1 = \omega_p^2 \frac{q}{e} n_b$$

Plasma-based accelerators: linear wakefield

laser driver

If laser, ponderomotive term:

$$\left(\frac{\partial}{\partial t^2} + \omega_p^2\right) n_1 = \omega_p^2 \frac{q}{e} n_b + n_0 c^2 \nabla^2 \frac{a_0^2}{4}$$

drive beam and/or accelerated beam

VOLUME 54, NUMBER 7 PHYSICAL REVIEW LETTERS 18 FEBRUARY 1985

Acceleration of Electrons by the Interaction of a Bunched Electron Beam with a Plasma

Pisin Chen[a]
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305

and

J. M. Dawson, Robert W. Huff, and T. Katsouleas
Department of Physics, University of California, Los Angeles, California 90024
(Received 20 December 1984)

A new scheme for accelerating electrons, employing a bunched relativistic electron beam in a cold plasma, is analyzed. We show that energy gradients can exceed 1 GeV/m and that the driven electrons can be accelerated from $\gamma_0 mc^2$ to $3\gamma_0 mc^2$ before the driving beam slows down enough to degrade the plasma wave. If the driving electrons are removed before they cause the collapse of the plasma wave, energies up to $4\gamma\delta mc^2$ are possible. A noncollinear injection scheme is suggested in order that the driving electrons can be removed.

PACS numbers: 52.75.Di, 29.15.−n

VOLUME 43, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JULY 1979

Laser Electron Accelerator

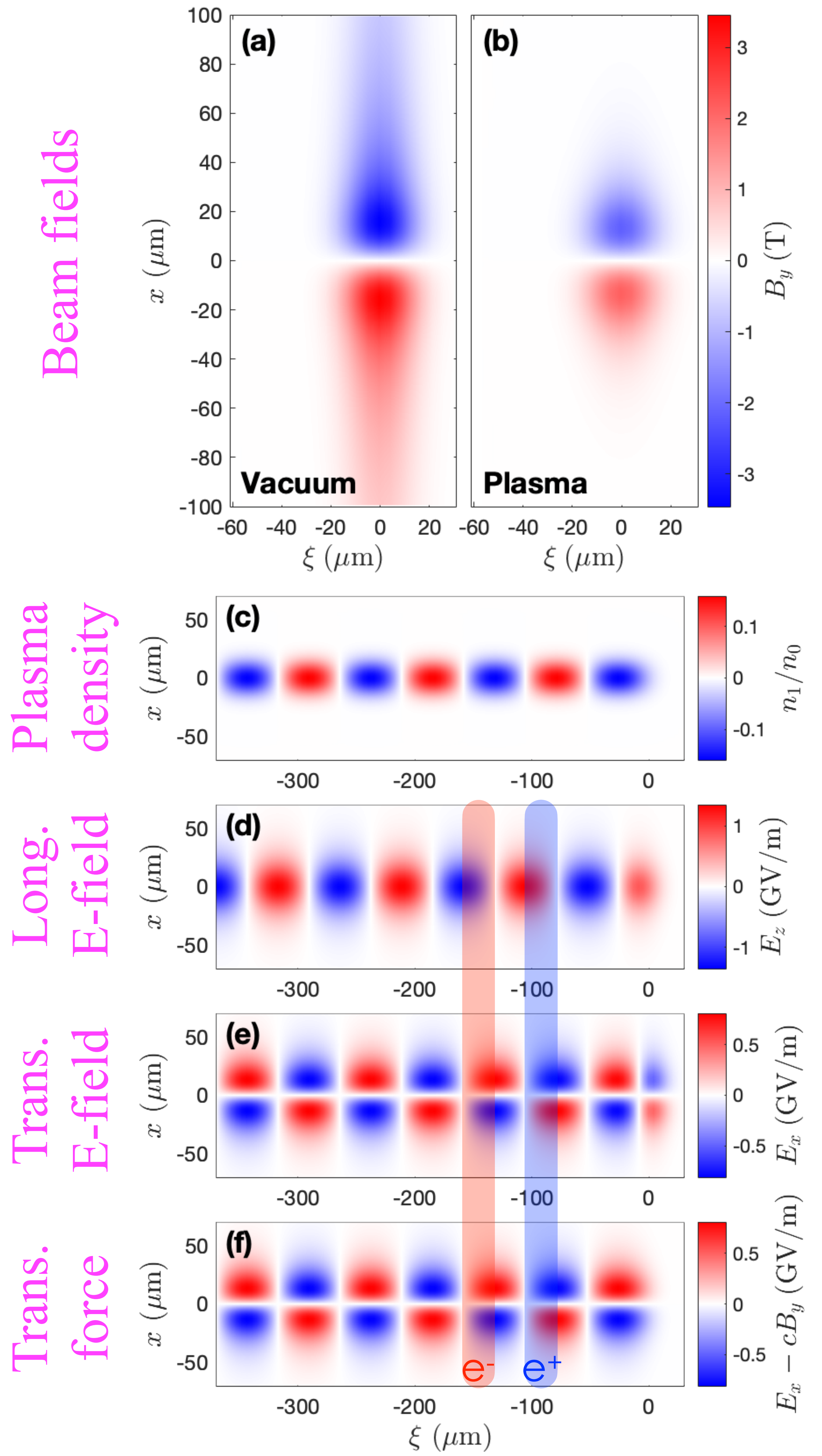
T. Tajima and J. M. Dawson
Department of Physics, University of California, Los Angeles, California 90024
(Received 9 March 1979)

An intense electromagnetic pulse can create a weak of plasma oscillations through the action of the nonlinear ponderomotive force. Electrons trapped in the wake can be accelerated to high energy. Existing glass lasers of power density 10^{18}W/cm^2 shone on plasmas of densities 10^{18} cm^{-3} can yield gigaelectronvolts of electron energy per centimeter of acceleration distance. This acceleration mechanism is demonstrated through computer simulation. Applications to accelerators and pulsers are examined.

So-called "beam-driven plasma accelerators"
Or Plasma WakeField Accelerator (PWFA)

So-called "laser-driven plasma accelerators"
Or Laser WakeField Accelerator (LWFA)

Plasma-based accelerators: linear wakefield



$\xi = z - ct$ is the co-moving coordinate, longitudinal position with respect to the drive beam center.

Common to laser-driven and beam-driven linear wakefields, we have behind the driver:

- E_r and E_z are sinusoidal functions of ξ that are 90° out of phase with each other
- $B = 0$
- Acceleration and focusing: a quarter of the plasma wave

Inside the beam (different from laser-driven case):

- E_r and B_θ are shielded by the plasma, and decays radially over a plasma skin depth $1/k_p$
- $B_\theta \neq 0$ due to beam current and plasma radial and longitudinal currents
- Long beam: $n_1 \simeq \dfrac{q}{e} n_b$ to shield E_r, return current to shield B_θ
- Short beam: there is an instantaneous inductive shielding driven by the fast variation of the plasma radial current

Plasma-based accelerators: nonlinear wakefield

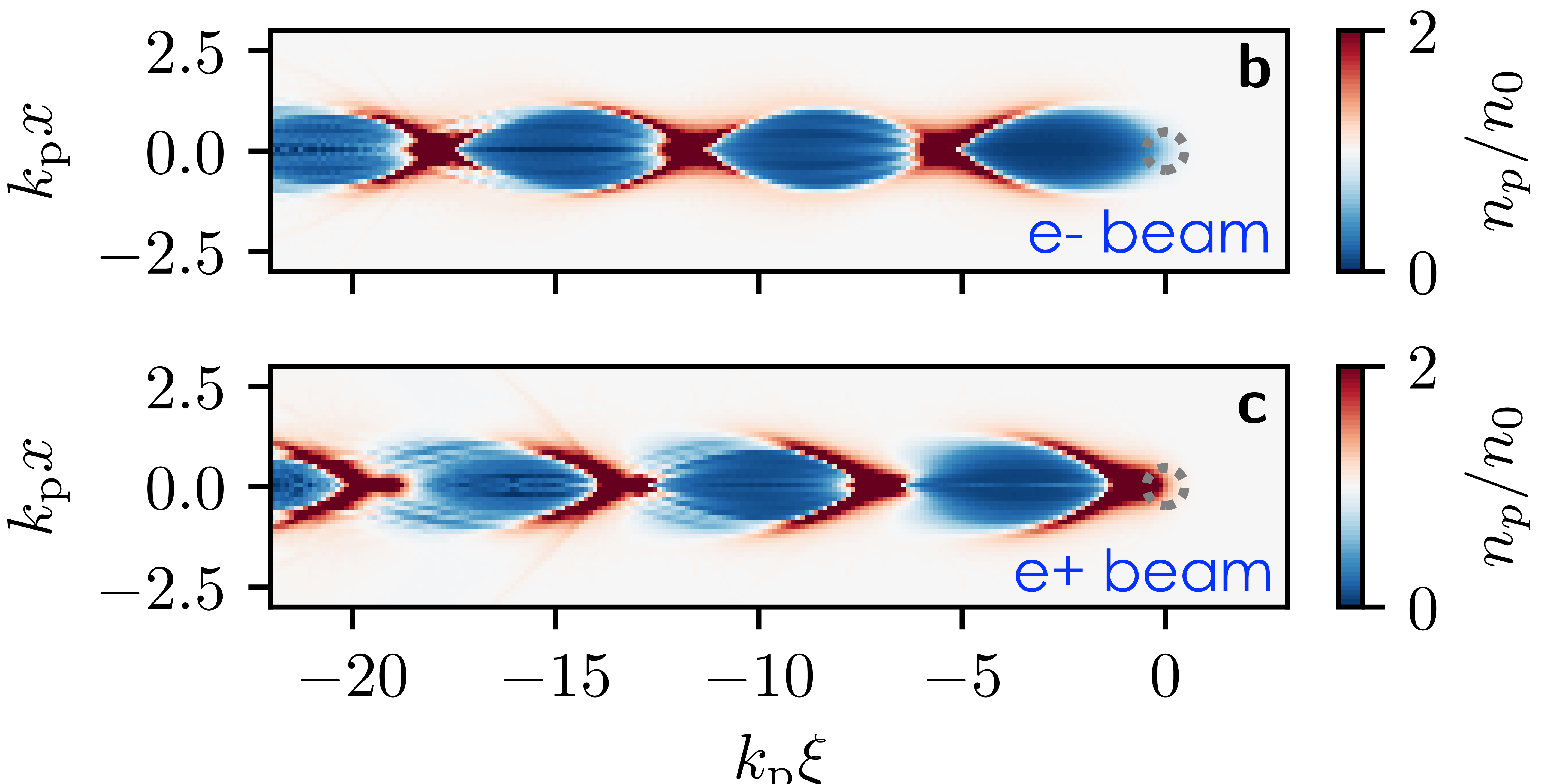
Electron or laser-driven nonlinear 3D wakes

- plasma electrons are expelled/blown out of the propagation axis, thus forming an ion cavity
- plasma electrons are then pulled back by plasma ions, overshooting the propagation axis and setting up a nonlinear plasma oscillation
- this is the so-called blowout regime, where the plasma wave takes the form of ion cavities surrounded by thin electron sheaths
- electron self-injection and controlled injection possible for LWFA

Positron-driven nonlinear 3D wakes:

- Plasma electrons are sucked in towards the propagation axis
- If bunch is short, plasma electrons overshoot and set up a nonlinear plasma oscillation with similar ion cavities as in the blowout regime
- Otherwise, more complicated

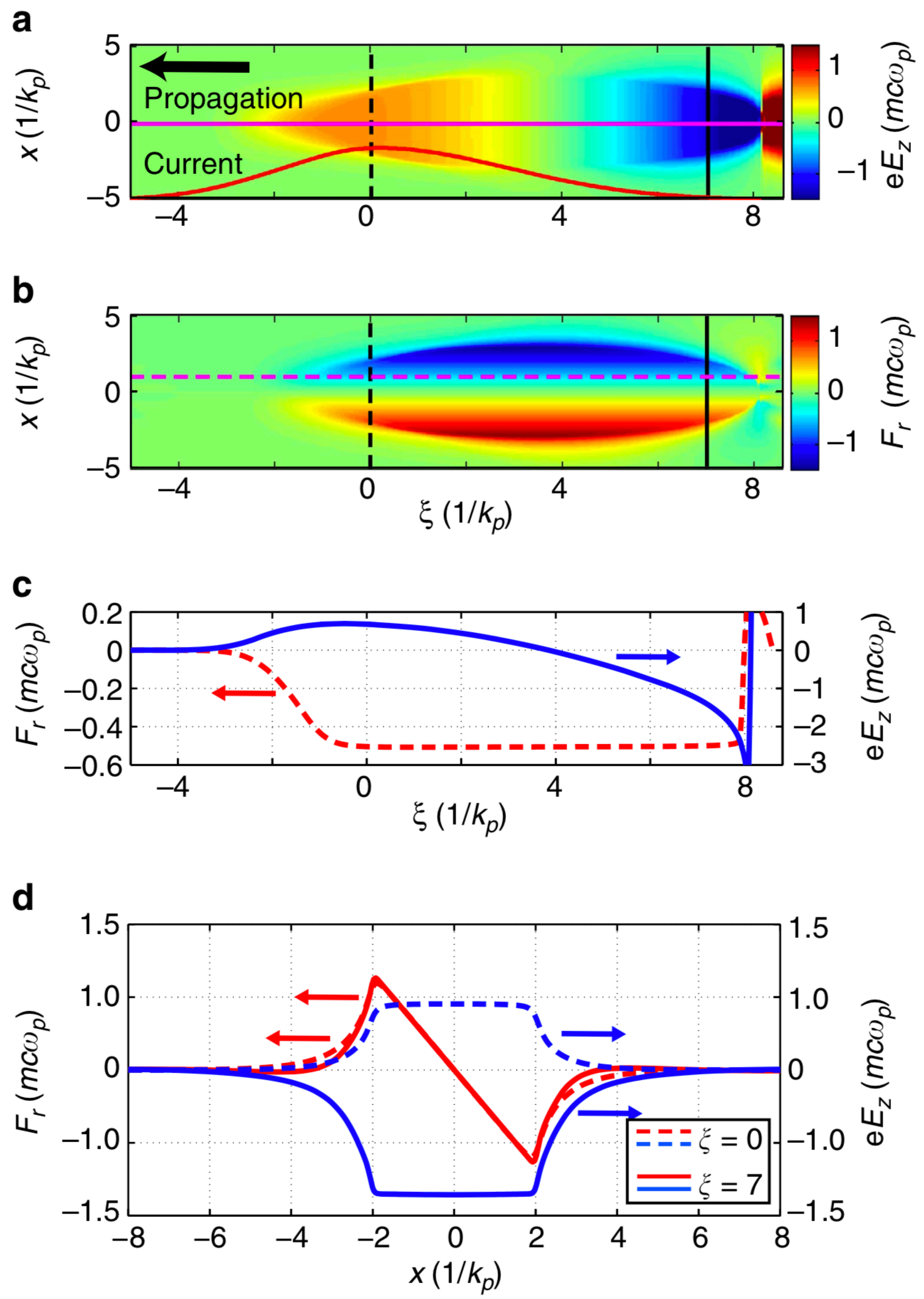
$n_b/n_0 = 3 \qquad k_p\sigma_r = k_p\sigma_z = 0.42$



Scientific context for positron acceleration

Plasma-based accelerators: ideal for e$^-$

Key properties of the blowout regime:



EM fields inside cavity:

$$\mathbf{E}/E_0 = \frac{1}{2}k_p\xi\,\mathbf{e}_z + \frac{1}{4}k_p r\,\mathbf{e}_r$$

$$c\mathbf{B}/E_0 = -\frac{1}{4}k_p r\,\mathbf{e}_\theta$$

Transverse force experienced by an e$^-$:

$$F_r = -e(E_r - cB_\theta) = -\frac{eE_0 k_p}{2}\,r$$

→ Focusing force linear in *r*

Additional properties:

$$\partial_\xi F_r = 0 \qquad \partial_r F_z = 0$$

The blowout regime has ideal field properties for e$^-$:

→ emittance preservation is expected to be achievable.

→ beam loading allow for high efficiency, flat E_z field and therefore low energy spread.

→ most studied regime for electron acceleration, in both LWFA and PWFA.

But:

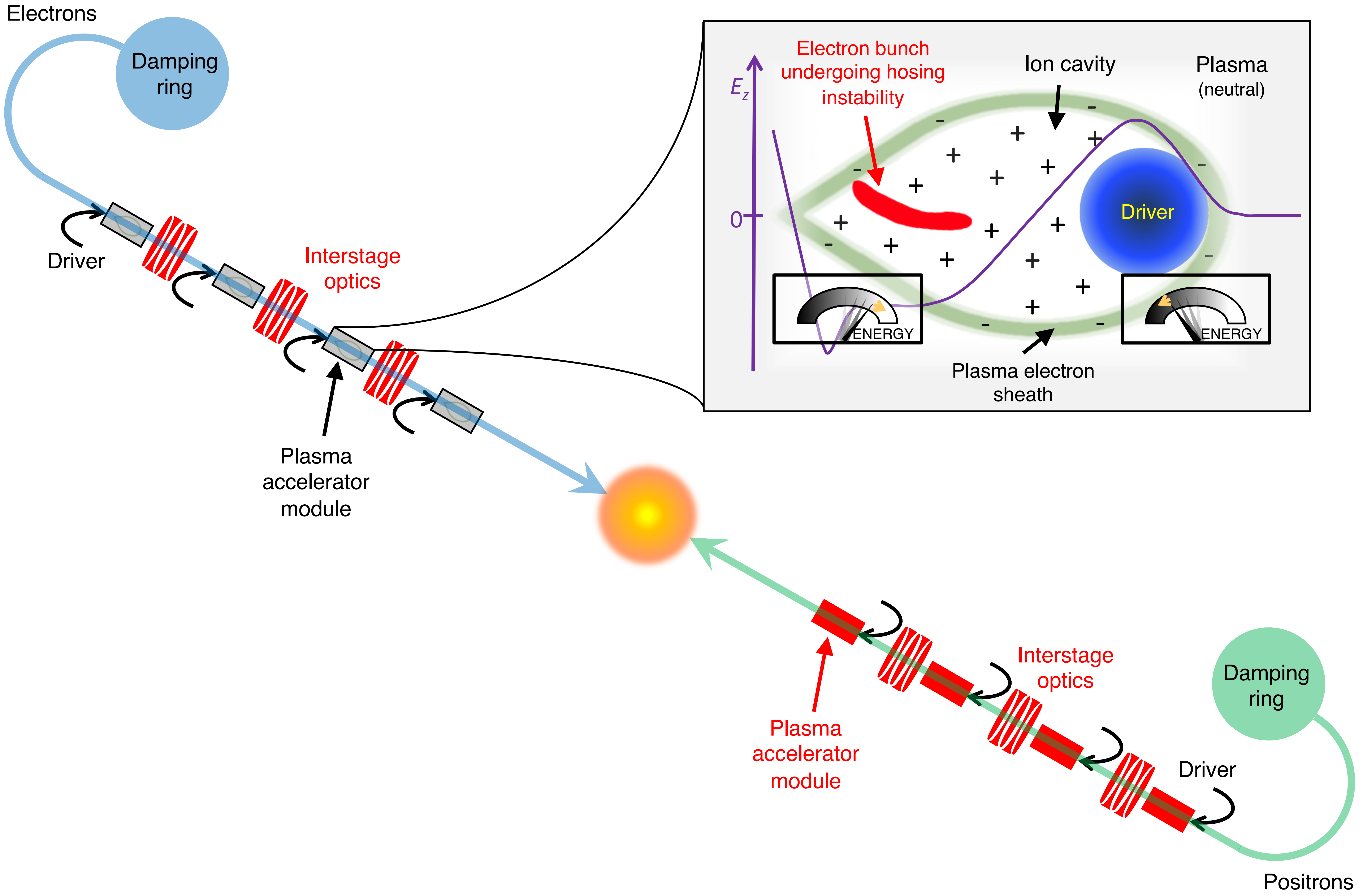
→ hosing instability may be an important limitation for collider beam parameters.

→ ion motion may lead to emittance growth.

→ what abut e$^+$?

Clayton et al., Nat Comm 7, 12483 (2016)

Plasma-based accelerators: challenges



The blowout regime has ideal field properties for e^-:

- emittance preservation is expected to be achievable.
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But:

- hosing instability may be an important limitation for collider beam parameters.
- ion motion may lead to emittance growth.
- what abut e^+?

Joshi, Corde and Mori, Phys. Plasmas 27, 070602 (2020)

Plasma-based accelerators: positrons

Accelerating positrons in plasma?

Linear plasma wakefields: symmetrical for e^-/e^+. Directly applicable to linear colliders?

Nonlinear plasma wakefields: NOT symmetrical for e^-/e^+. Blowout properties for e^- not achievable for e^+.

- mobile plasma electrons
- mostly immobile plasma ions

$$m_i \gg m_e$$

Wealth of advanced regimes varying beam and plasma geometries

- common ingredient: mobile plasma electrons flowing through the e^+ bunch → physics beyond idealised blowout

Plasma-based accelerators: positrons

What is the positron problem today?

Unloaded plasma wakefield suitable for e^+ acceleration (accelerating&focusing)?

NO

Loaded plasma wakefield with efficiency, beam quality, and ultimately competitive luminosity-per-power for e^+ arm?

YES

With loading comes plasma electron motion, basically ion motion with a much smaller mass

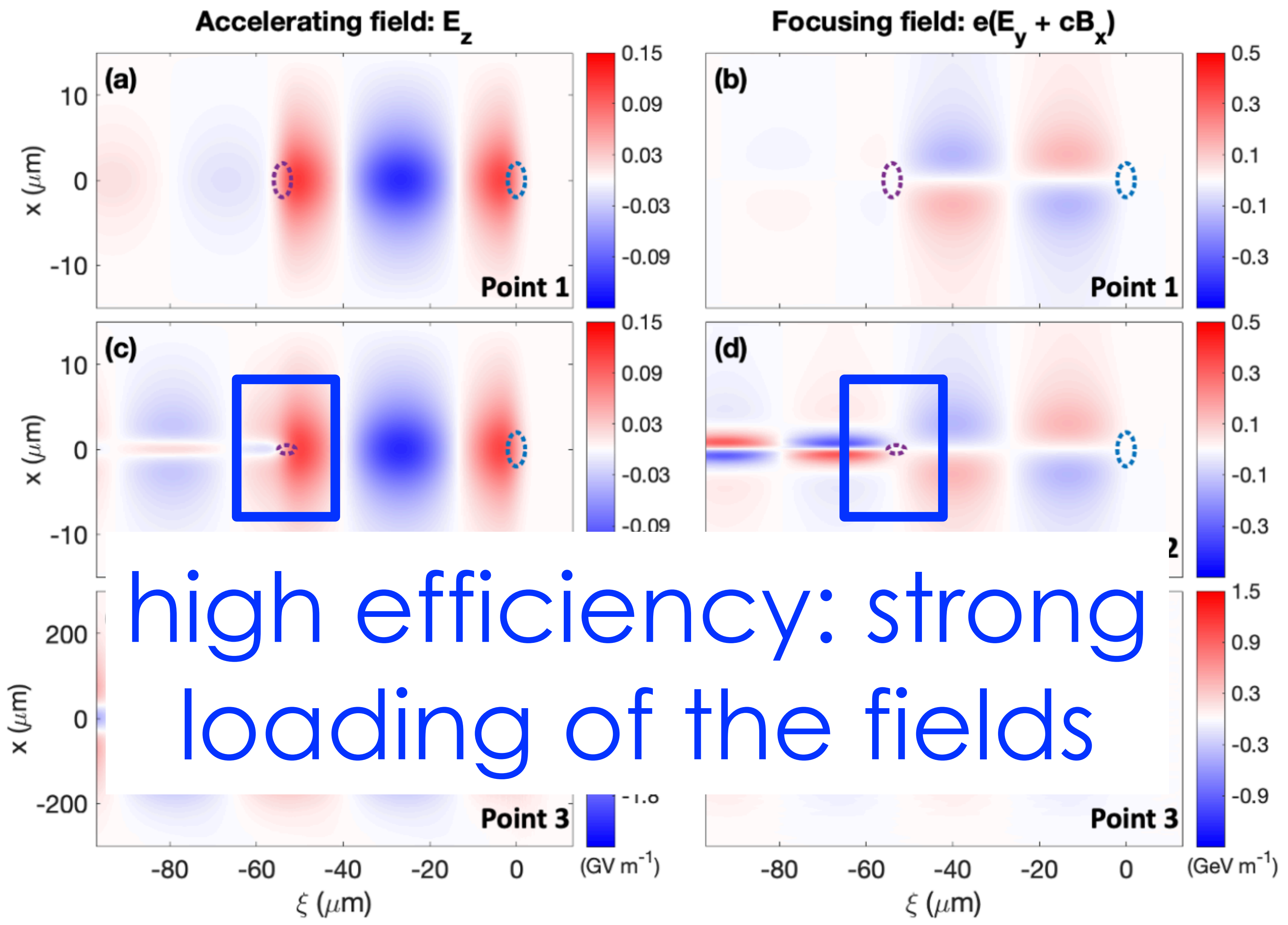
Preliminary considerations with e^+ loaded quasilinear plasma wakefields

Efficiency in quasilinear regime

Energy efficiency from plasma to accelerated trailing bunch

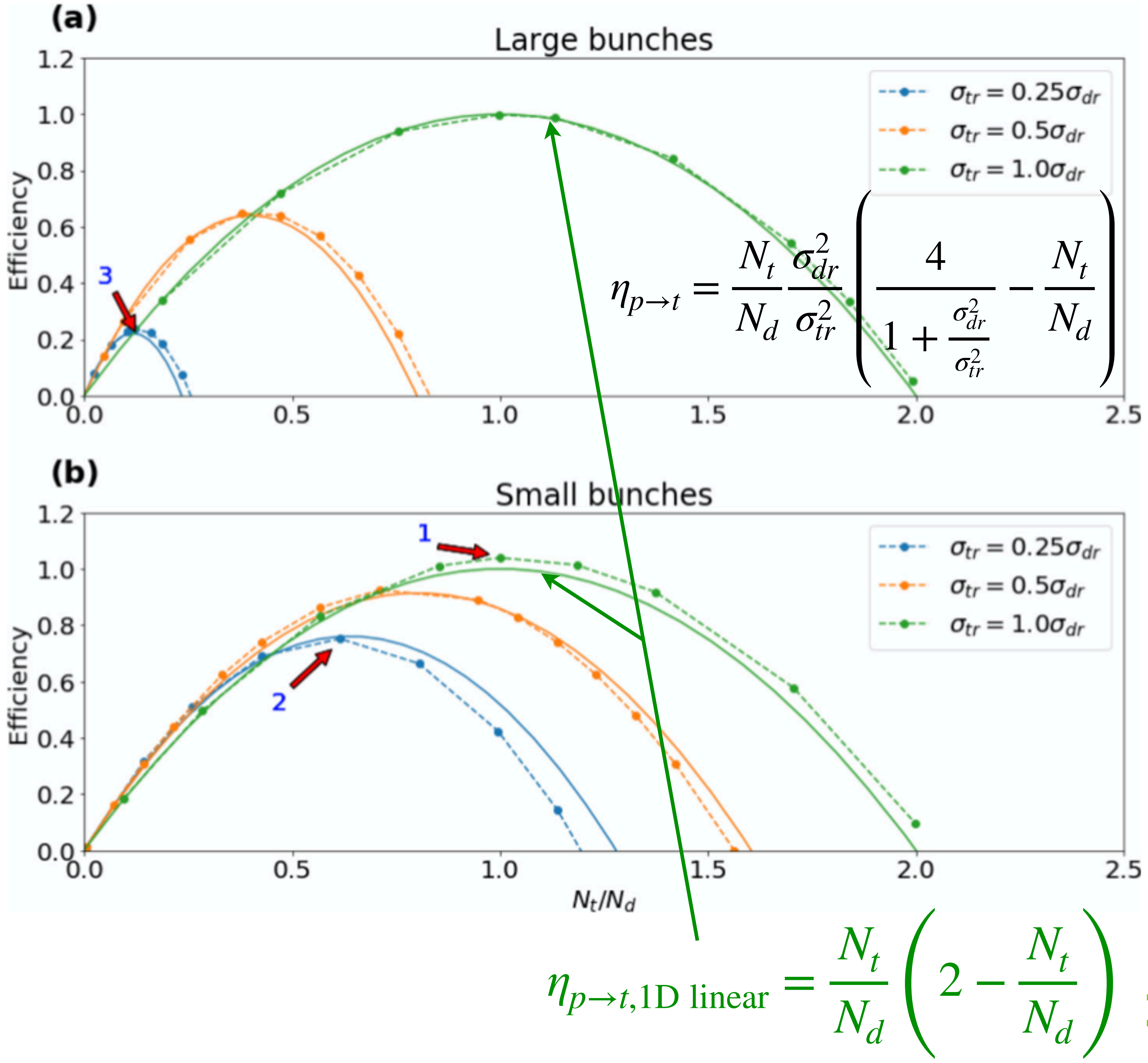
$$\eta_{p\to t} = \frac{W_{\text{gain}}}{W_{\text{loss}}} = \left|\frac{N_t\langle E_z\rangle_t}{N_d\langle E_z\rangle_d}\right| \quad \xrightarrow{\text{short bunches, linear and 1D}} \quad \eta_{p\to t,\text{1D linear}} = \frac{N_t}{N_d}\left(2 - \frac{N_t}{N_d}\right)$$

Linear 3D case:



high efficiency: strong loading of the fields

- Same shape for drive and trailing bunches: linear 3D = linear 1D.
- Highest efficiency: smallest fields left behind
- Small beams ($k_p\sigma_r \ll 1$) are much better because the fields extend over a plasma skin depth regardless of beam size



[Hue et al., PRR 3, 043063 (2021)]

Transverse emittance in quasilinear regime

Evolution of transverse emittance

Quasi-matching/transverse equilibrium:

$F_x \simeq -gx$ with g the gradient of the focusing force,

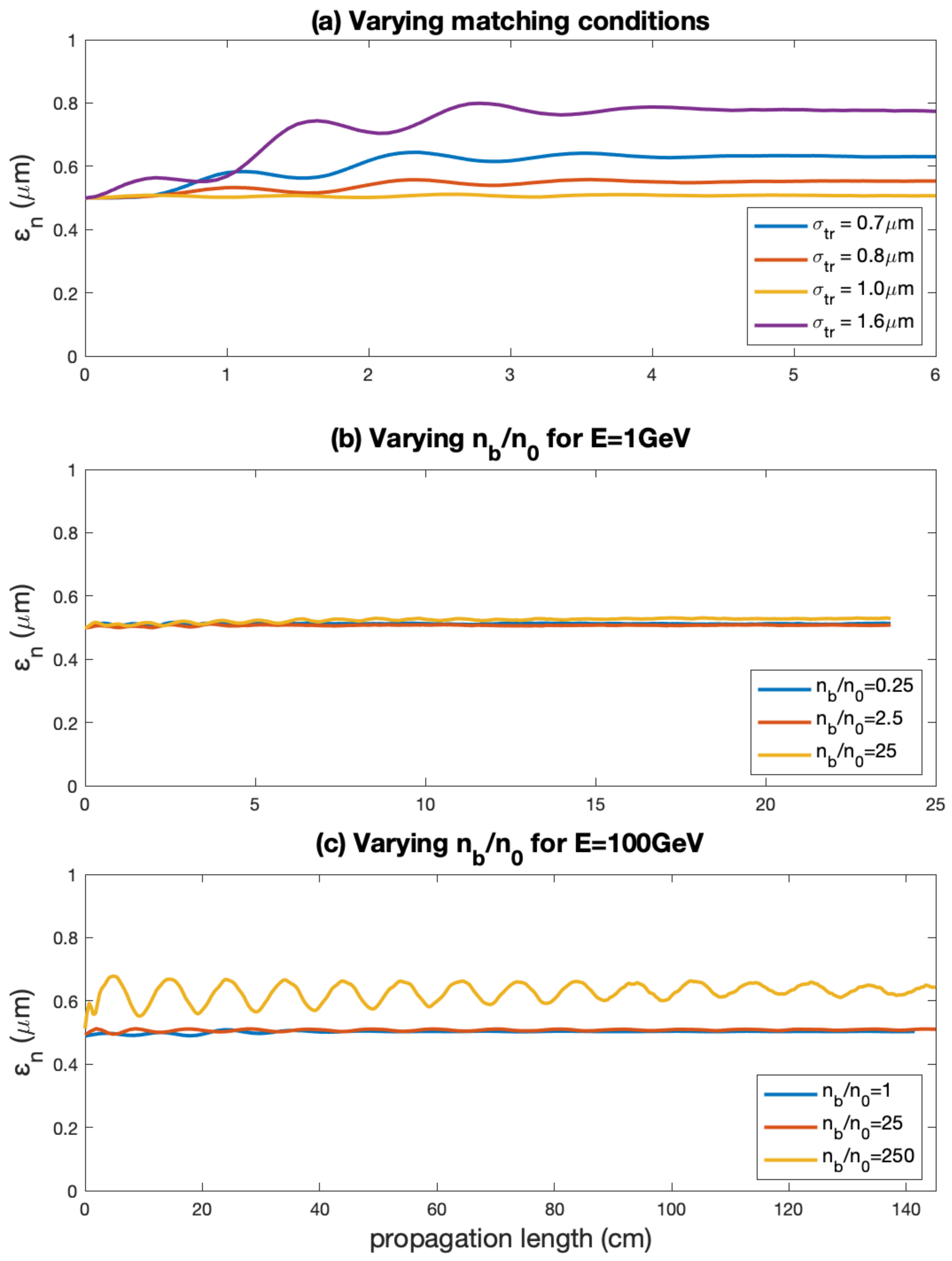
Enveloppe equation: $$\frac{d^2\sigma_x}{dz^2} = -k_\beta^2\sigma_x + \frac{\varepsilon^2}{\sigma_x^3} \quad \text{with} \quad k_\beta = \sqrt{g/\gamma m_e c^2}$$

$$\Longrightarrow \beta_{\text{matched}} = 1/k_\beta$$

- (a): quasi-matching is extremely important to minimize emittance growth at acceptable levels. Demonstrate that near transverse equilibrium is possible with Gaussian positron beams.
- (b): this is still valid for $n_b/n_0 \gg 1$, that is for a nonlinear positron load in a linearly-driven plasma wakefield.

electron motion

- (c): for $\Delta\phi_e \sim k_b\sigma_z > 1$, the situation qualitatively changes, and new ideas are needed to mitigate emittance growth $k_b = \frac{1}{c}\sqrt{\frac{n_b e^2}{m_e \epsilon_0}} = \sqrt{\frac{n_b}{n_0}} k_p$



	σ_{tr} (μm)	ε_n (μm)	β (cm)	σ_{tz} (μm)	n_b/n_0	$k_b\sigma_{tz}$	E (GeV)	η (%)	$\Delta\varepsilon_n$ (%)
	0.7	0.5	0.20	2.14	1	0.09	1	0.30	27.6
	0.8	0.5	0.26	2.14	1	0.09	1	0.39	11.6
Fig. 3(a)	1.0	0.5	0.40	2.14	1	0.09	1	0.61	1.74
	1.6	0.5	1.02	2.14	1	0.09	1	1.55	55.4
	1.01	0.5	0.41	2.14	0.25	0.045	1	0.16	1.74
Fig. 3(b)	1.00	0.5	0.40	2.14	2.5	0.14	1	1.52	2.64
	0.80	0.5	0.26	2.14	25	0.45	1	9.15	5.83
	0.327	0.5	4.28	2.14	1	0.09	100	0.07	2.73
Fig. 3(b)	0.288	0.5	3.33	2.14	25	0.45	100	1.63	3.67
	0.189	0.5	1.43	2.14	250	1.4	100	5.24	30.0

Uncorrelated energy spread in quasilinear regime

Evolution of longitudinal phase space

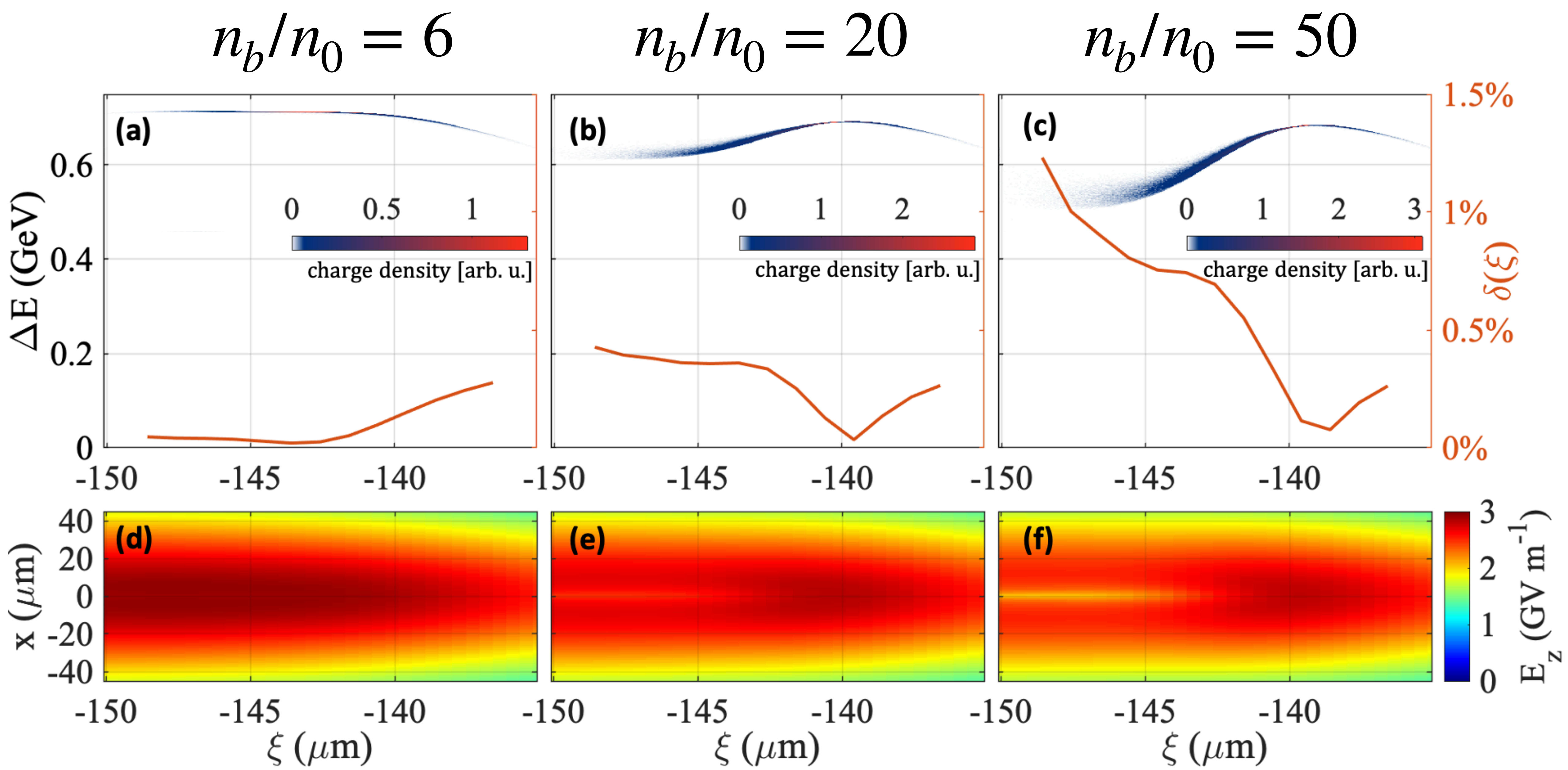
Two contributions to the energy spread:

- Correlated energy spread: very important but can potentially be removed by dechirping or beam loading
- Uncorrelated/slice energy spread: fundamental limit, it spoils the longitudinal emittance irreversibly

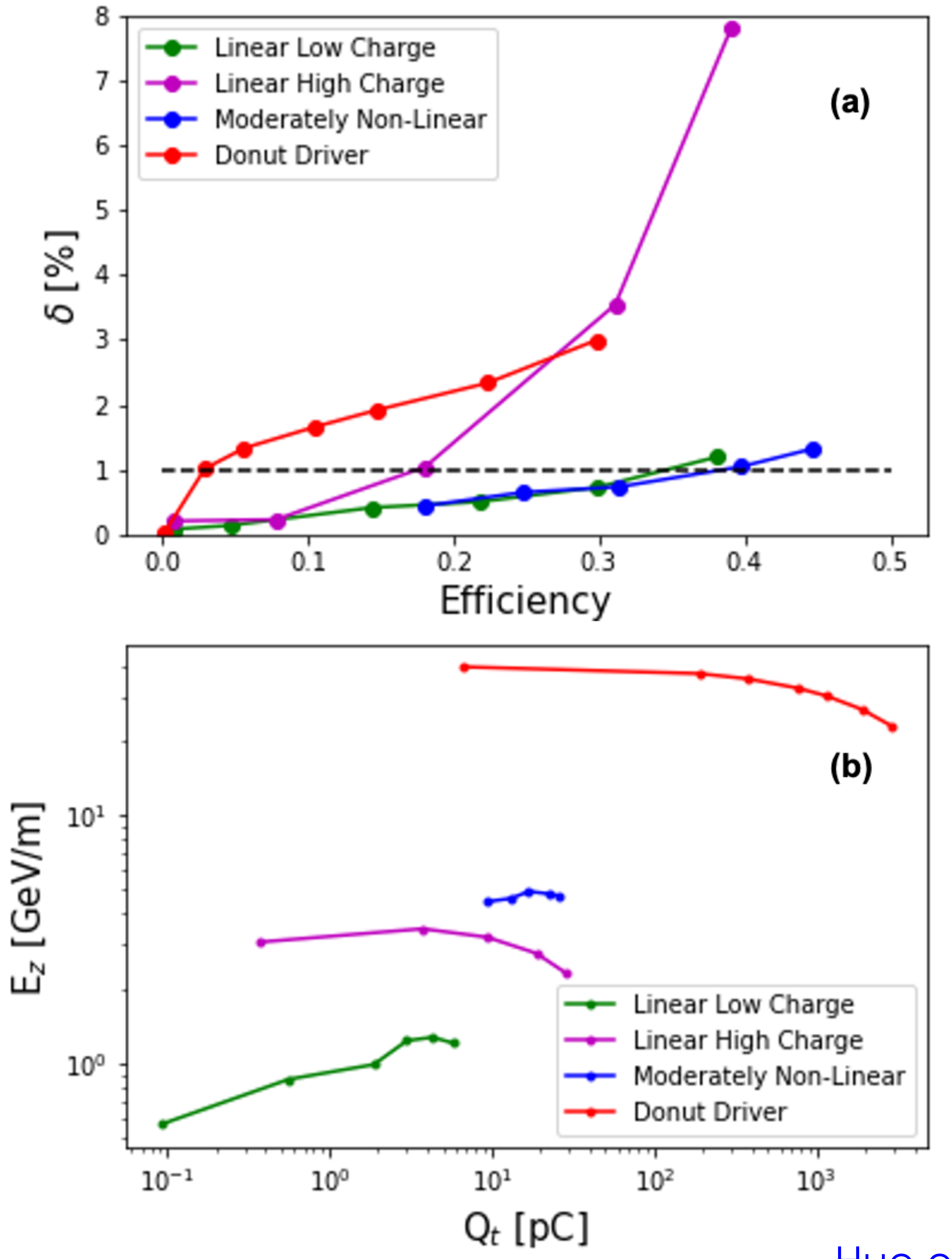
Uncorrelated energy spread as figure of merit:

$$\delta = \frac{1}{\langle E_z \rangle}\left[\frac{1}{N_b}\int [E_z(x,y,\xi) - \langle E_z \rangle(\xi)]^2 n_b dx dy d\xi\right]^{1/2}$$

slice energy spread



Energy efficiency η vs uncorrelated energy spread δ



Observation:

There is generally a **tradeoff between energy efficiency and beam quality** (here uncorrelated energy spread) when **e+ beam loading** is involved.

Hue et al., PRR 3, 043063 (2021)

The positron problem

Plasma electron motion and transverse beam loading

The positron problem

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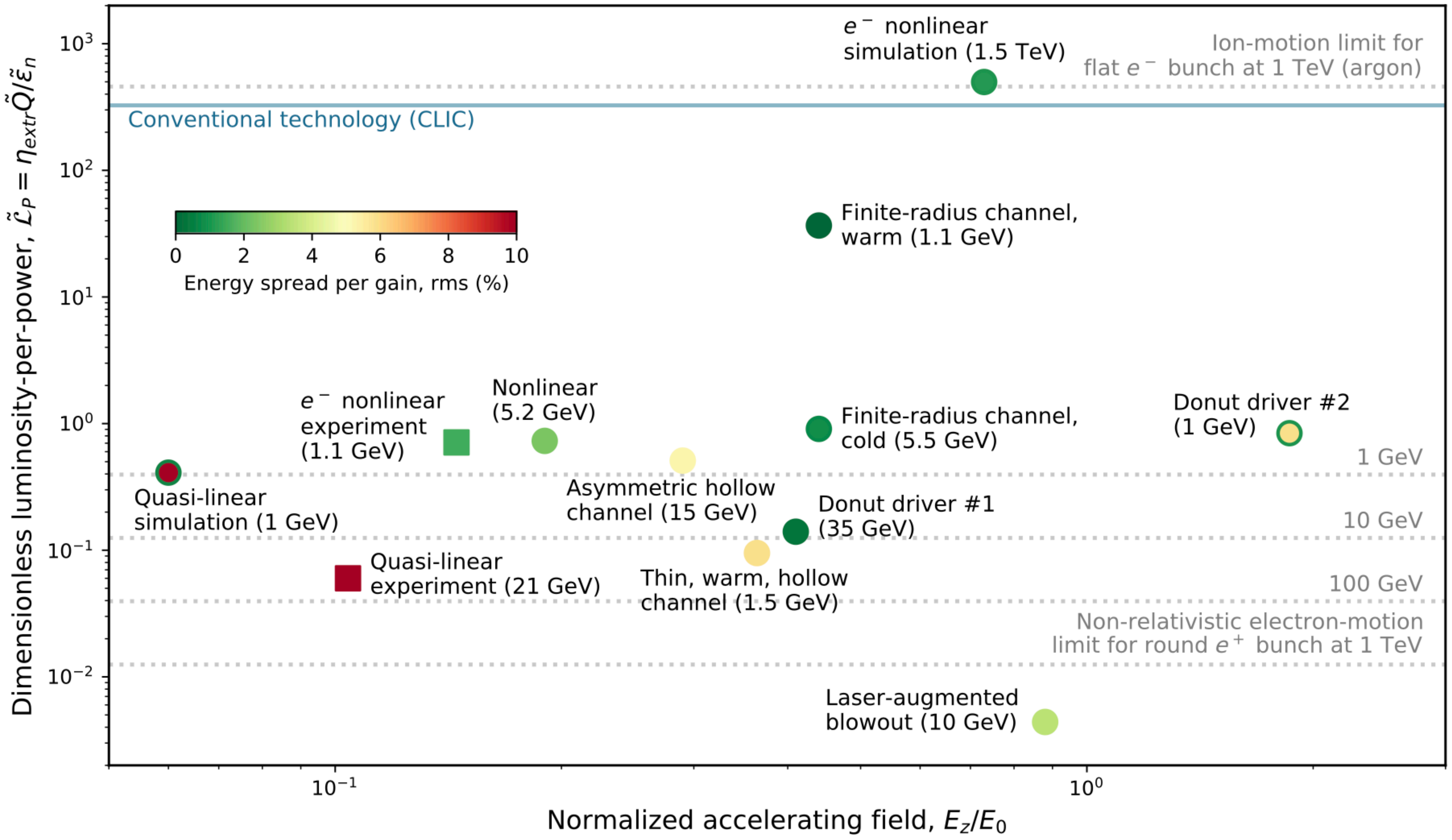
(Received 6 October 2023; accepted 5 February 2024; published 5 March 2024)

Plasma acceleration has emerged as a promising technology for future particle accelerators, particularly linear colliders. Significant progress has been made in recent decades toward high-efficiency and high-quality acceleration of electrons in plasmas. However, this progress does not generalize to the acceleration of positrons, as plasmas are inherently charge asymmetric. Here, we present a comprehensive review of historical and current efforts to accelerate positrons using plasma wakefields. Proposed schemes that aim to increase energy efficiency and beam quality are summarized and quantitatively compared. A dimensionless metric that scales with the luminosity-per-beam power is introduced, indicating that positron-acceleration schemes are currently below the ultimate requirement for colliders. The primary issue is *electron motion*; the high mobility of plasma electrons compared to plasma ions, which leads to nonuniform accelerating and focusing fields that degrade the beam quality of the positron bunch, particularly for high efficiency acceleration. Finally, we discuss possible mitigation strategies and directions for future research.



[Cao, Lindstrøm et al., PRAB 27, 034801 (2024)]

The positron problem

Figure of merit:
luminosity per power

$$\mathscr{L} \approx \frac{fN^2}{4\pi\sigma_x\sigma_y} \approx \frac{1}{8\pi m_e c^2}\frac{P_{\mathrm{wall}}}{\sqrt{\beta_x\epsilon_{nx}}}\frac{\eta N}{\sqrt{\beta_y\epsilon_{ny}}}$$

$$\frac{\mathscr{L}}{P_{\mathrm{wall}}} \approx \frac{1}{8\pi m_e c^2}\frac{\eta_{\mathrm{prod}}\eta_{\mathrm{depl}}}{\sqrt{\beta_x\beta_y}}\boxed{\frac{\eta_{\mathrm{extr}}N}{\sqrt{\epsilon_{nx}\epsilon_{ny}}}} = \alpha\tilde{\mathscr{L}}_P$$

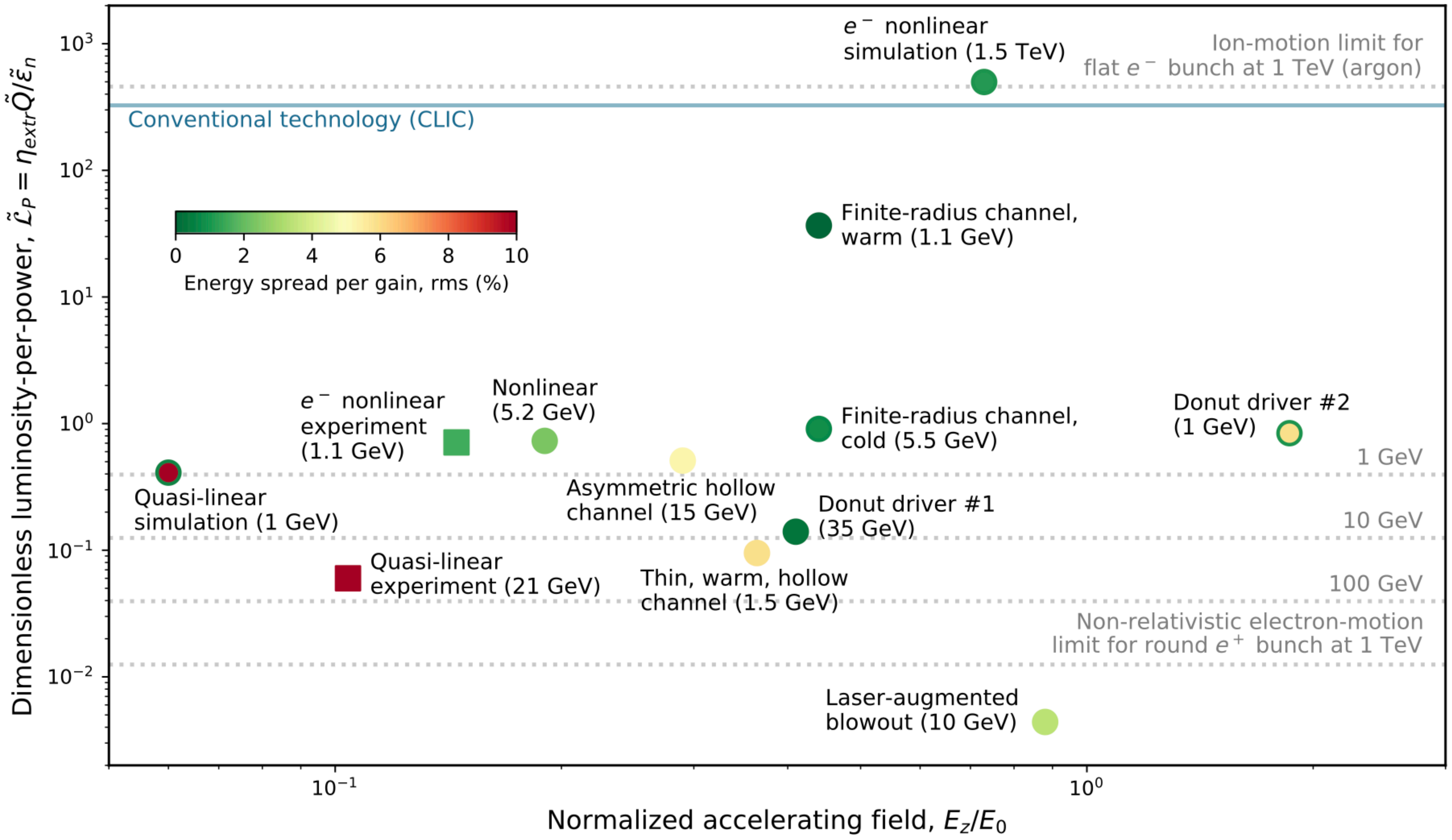
$$\boxed{\tilde{\mathscr{L}}_P = \frac{\eta_{\mathrm{extr}}\tilde{Q}}{\tilde{\epsilon}_n}}$$

independent of plasma density

with:

$$\tilde{\epsilon}_n = k_p\sqrt{\epsilon_{nx}\epsilon_{ny}}$$

$$\tilde{Q} = 4\pi r_e k_p N$$



Cao, Lindstrøm et al., PRAB 27, 034801 (2024)

The positron problem

Why such a gap between e⁻ and e⁺?

- Plasma electrons used for positron focusing are very light, much lighter than ions used for electron focusing in blowout:

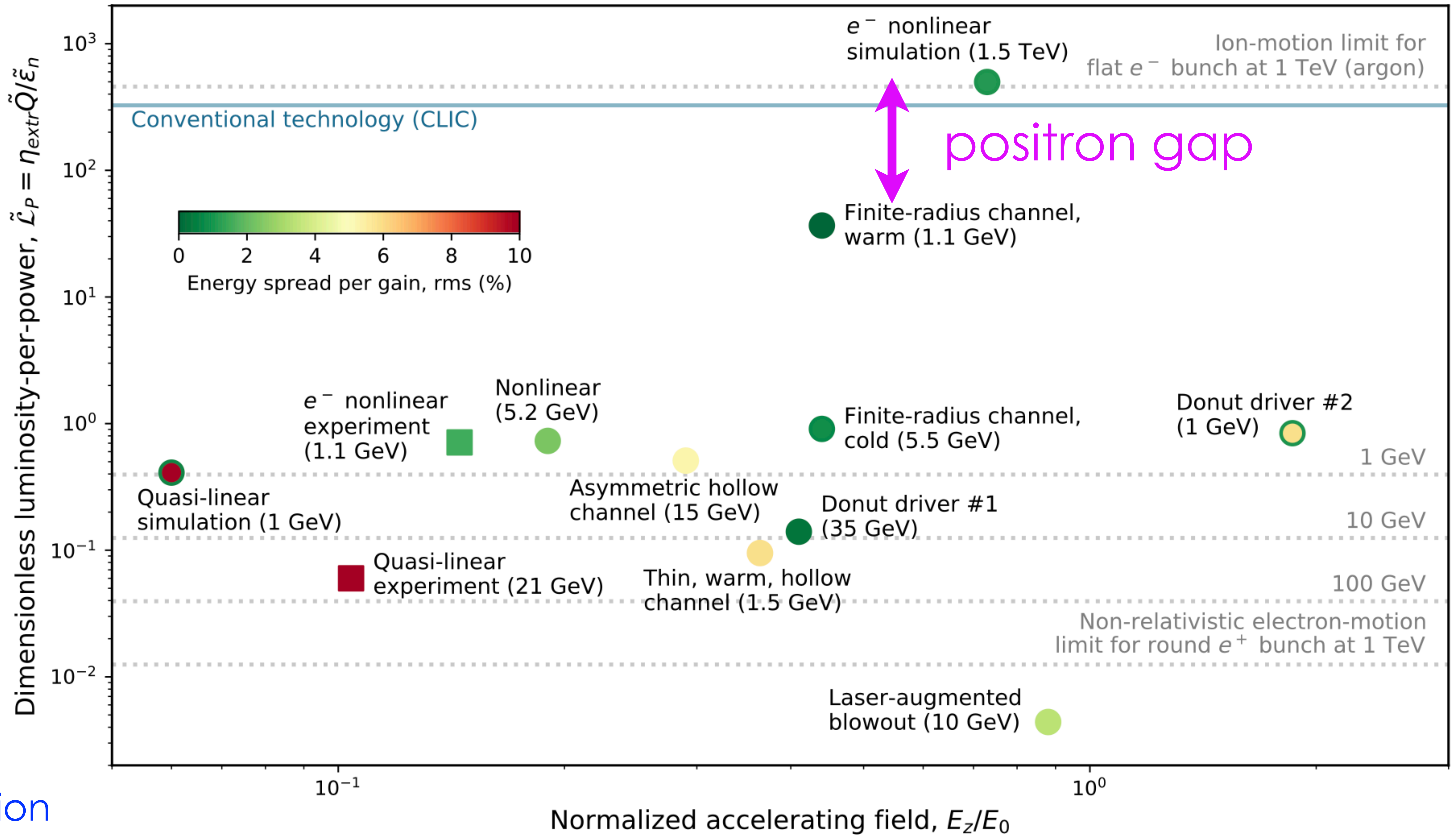
$$m_e \ll m_i$$

- Plasma electron motion similar to ion motion in blowout, and can be described by a phase advance in the bunch:

$$\Delta\phi_i \simeq k_i \Delta\zeta = \sqrt{\frac{\mu_0 e^2}{2}\frac{Z\sigma_z N}{m_i}\sqrt{\frac{r_e \gamma n_0}{\epsilon_{nx}\epsilon_{ny}}}}$$ ion motion

$$\Delta\phi_e \simeq k_e \Delta\zeta = \sqrt{\frac{\mu_0 e^2}{2}\frac{\sigma_z N}{\gamma_{pe} m_e}\sqrt{\frac{r_e \gamma \Delta n}{\epsilon_{nx}\epsilon_{ny}}}}$$ electron motion

electron motion limit: $\Delta\phi_e \lesssim \pi/2$



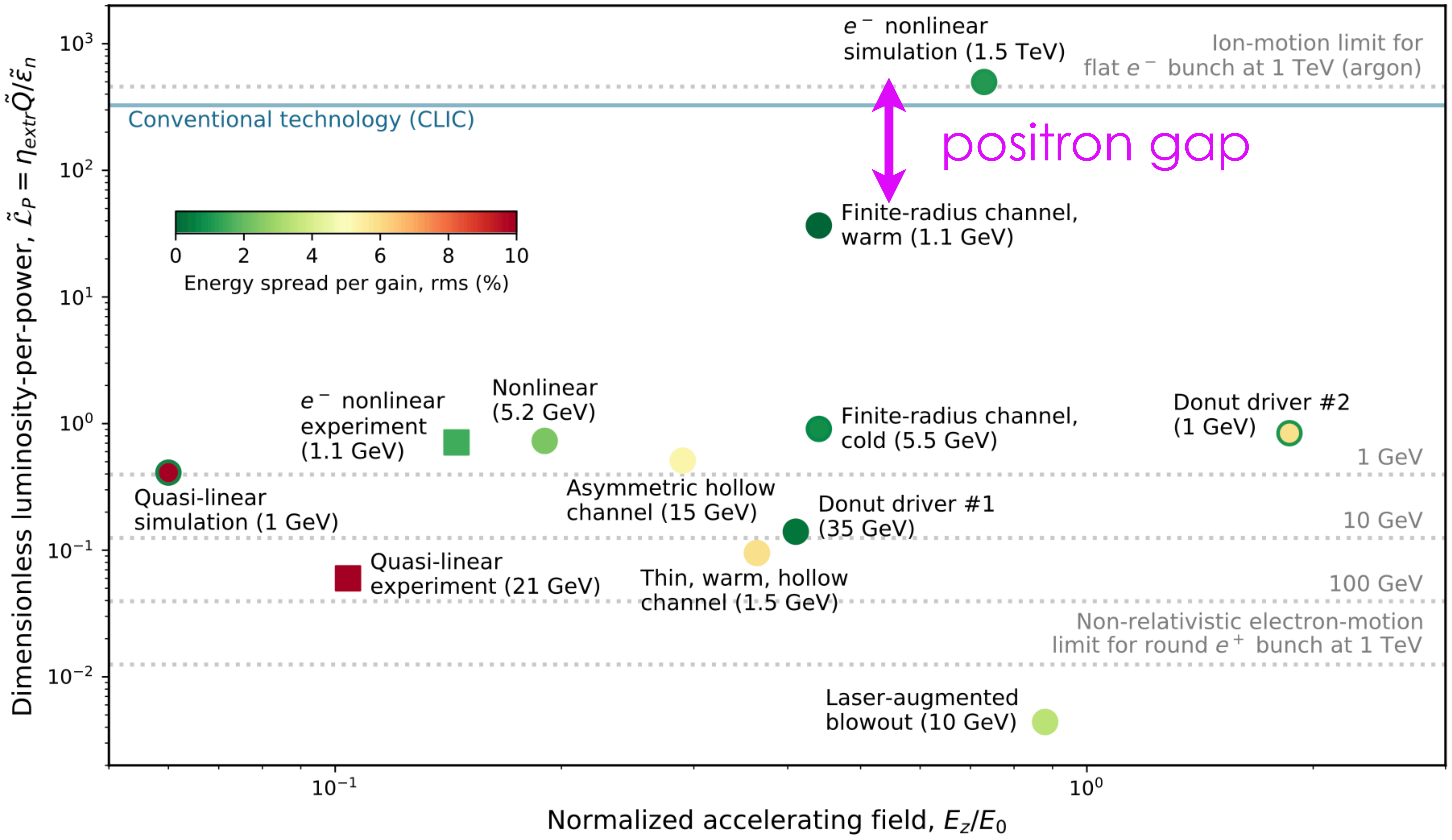
Cao, Lindstrøm et al., PRAB 27, 034801 (2024)

The positron problem

Electron motion limit embedded in luminosity-per-power

- Can rewrite $\tilde{\mathscr{L}}_P$ using $\Delta\phi_e$:

$$\tilde{\mathscr{L}}_P = \sqrt{\frac{16\pi}{\gamma}}(\Delta\phi_e)^2\left(\frac{\eta_{\mathrm{extr}}}{k_p\sigma_z}\right)\gamma_{pe}\sqrt{\frac{n_0}{\Delta n}}$$



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The positron problem

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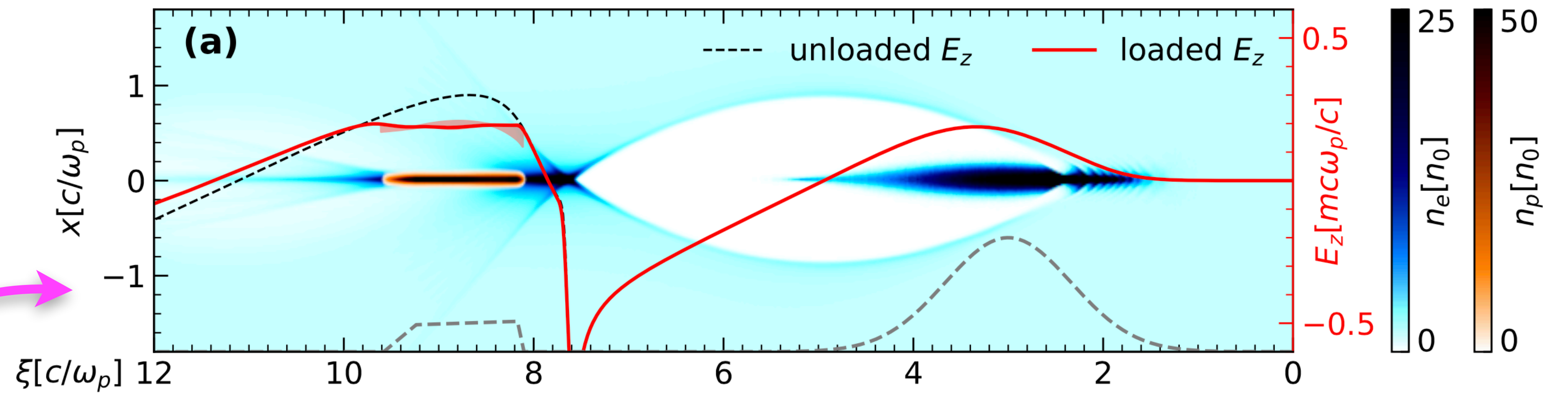
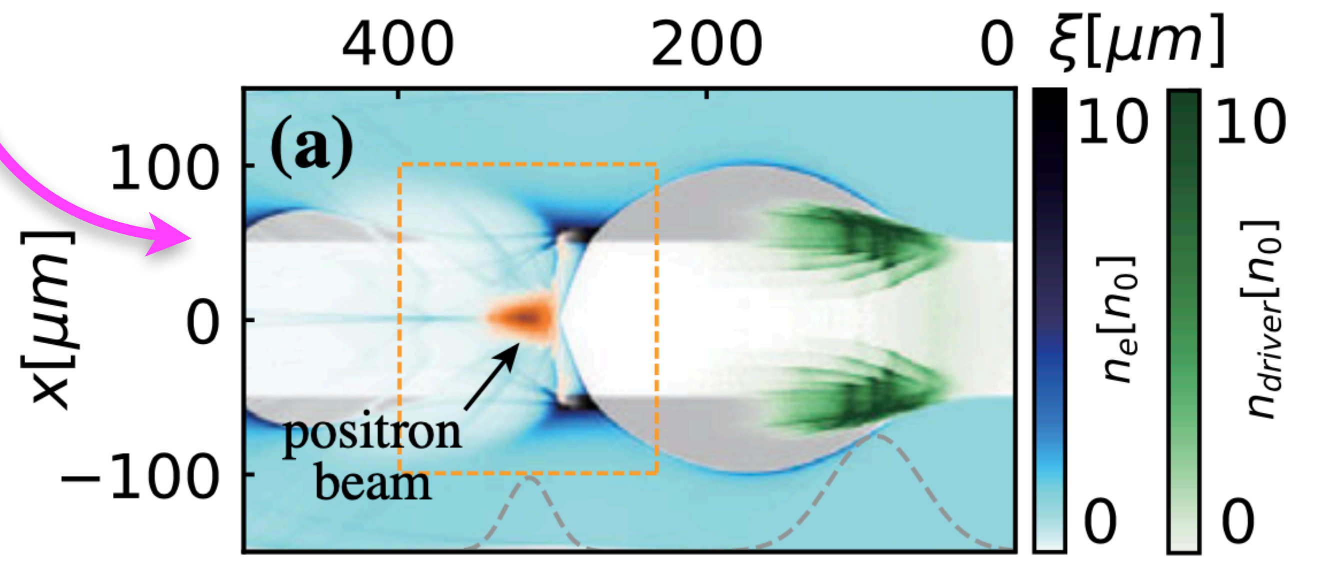
- What do we learn from e⁺ schemes:

Overcoming electron motion limit is a must

Charge and efficiency also important (favoring nonlinear regimes)



Scheme	Density (cm⁻³)	Gradient (GV/m)	Charge (pC)	Energy efficiency	Emittance (mm m rad)	Energy spread per gain	Uncorrected energy spread	Fin. energy (GeV)	$\Delta\phi_e$ [a]	Ref.
Quasilinear regime (simulation)	5×10^{16}	1.3	4.3	30%	0.64	~10% [b]	0.7%	1	0.77	[152]
Quasilinear regime (experiment)	1×10^{16}	1	85	40%	127 [c]	~14%	···	21	0.51	[87]
Nonlinear regime	7.8×10^{15}	1.6	102	26%	8	2.4%	···	5.2	7.6	[162]
Donut driver (No. 1)	5×10^{16}	8.9	13.6	0.17%	0.036	0.3%	···	35.4	0.50	[167]
Donut driver (No. 2)	5×10^{16}	40	189	3.5%	1.5 [d]	6%	1%	1	7.1	[152]
Finite-radius channel (cold)	5×10^{17}	30	52	3%	0.38	0.86%	0.73%	5.5	34	[180]
Finite-radius channel (warm)	5×10^{17}	30	84	4.8%	0.015	··· [e]	~0.01%	1.1	269	[181]
Laser-augmented blowout	2×10^{17}	20	15	5.5%	31	3.4%	···	~10	0.67	[187]
Thin, warm, hollow channel	1×10^{16}	3.5	100	4.7% [f]	7.4	6%	···	1.45	2.0	[188]
Asymmetric hollow channel	3.1×10^{16}	4.9	490	33%	67	5.3%	···	14.6	6.5	[189]
e^- nonlinear regime (simulation)	2×10^{16}	−10	800	37.5%	0.133 [g]	1.1%	≲1%	1500	292	[191]
e^- nonlinear regime (experiment)	1.2×10^{16}	−1.4	40	22%	2.8	1.6%	···	1.1	3.0	[192]
Conventional technology (CLIC)	Not applicable	0.1	596	28.5% [h]	0.11	0.35%	···	1500	Not applicable	[10]



Cao, Lindstrøm et al., PRAB 27, 034801 (2024)

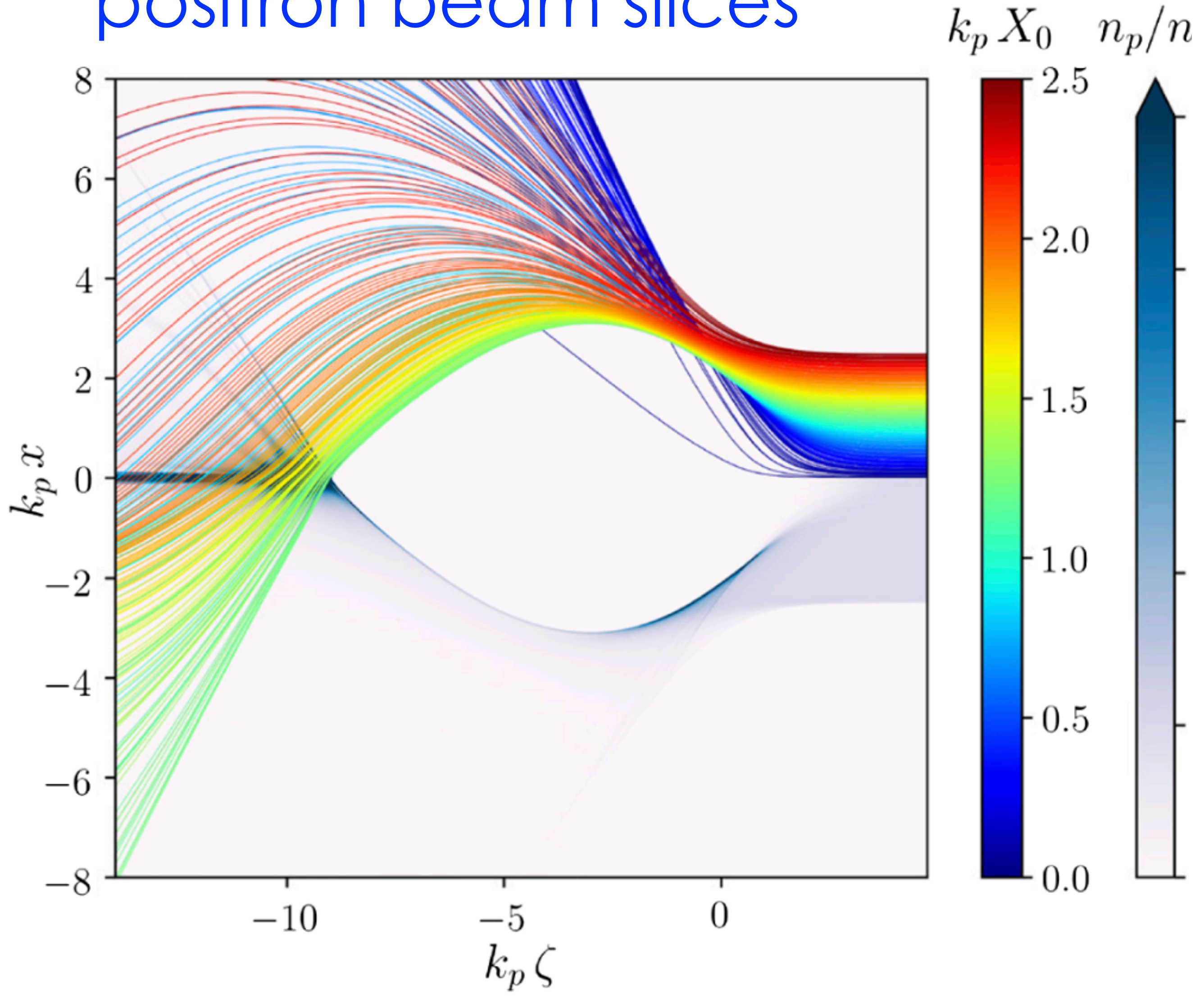
The positron problem

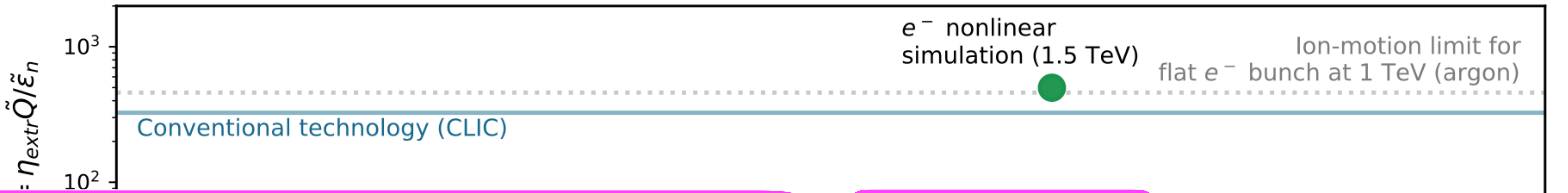
Strategies to fill the gap:

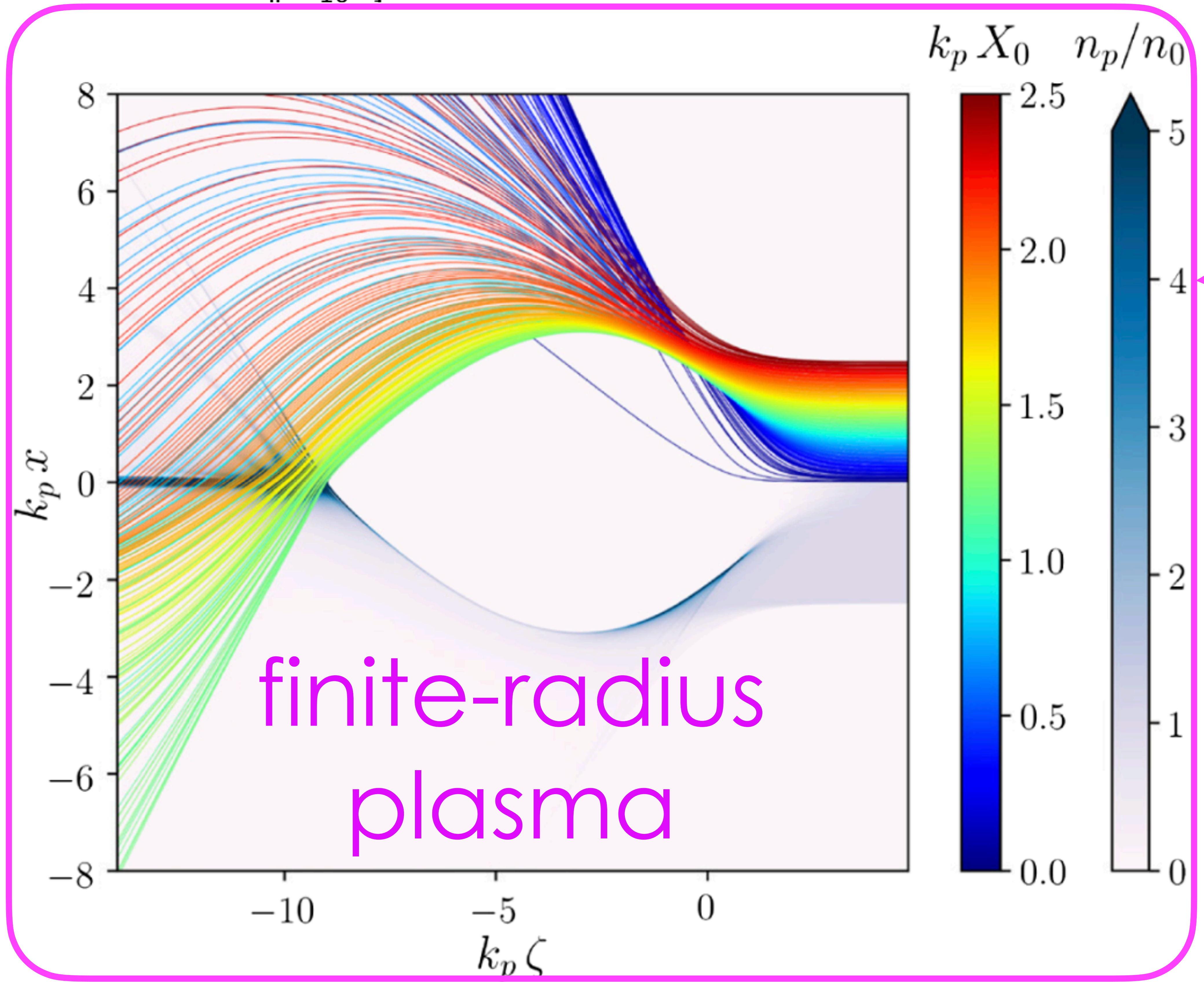
$$\tilde{\mathscr{L}}_P = \sqrt{\frac{16\pi}{\gamma}}(\Delta\phi_e)^2\left(\frac{\eta_{\mathrm{extr}}}{k_p\sigma_z}\right)\gamma_{pe}\sqrt{\frac{n_0}{\Delta n}}$$

$\Delta\phi_e$

- Slice-by-slice matching
- Plasma electron temperature
- Spread plasma electrons: differe[nt] plasma electrons to focus differ[ent] positron beam slices







The positron problem

Strategies to fill the gap:

$$\tilde{\mathscr{L}}_P = \sqrt{\frac{16\pi}{\gamma}(\Delta\phi_e)^2\left(\frac{\eta_{\text{extr}}}{k_p\sigma_z}\right)\gamma_{pe}\sqrt{\frac{n_0}{\Delta n}}}$$

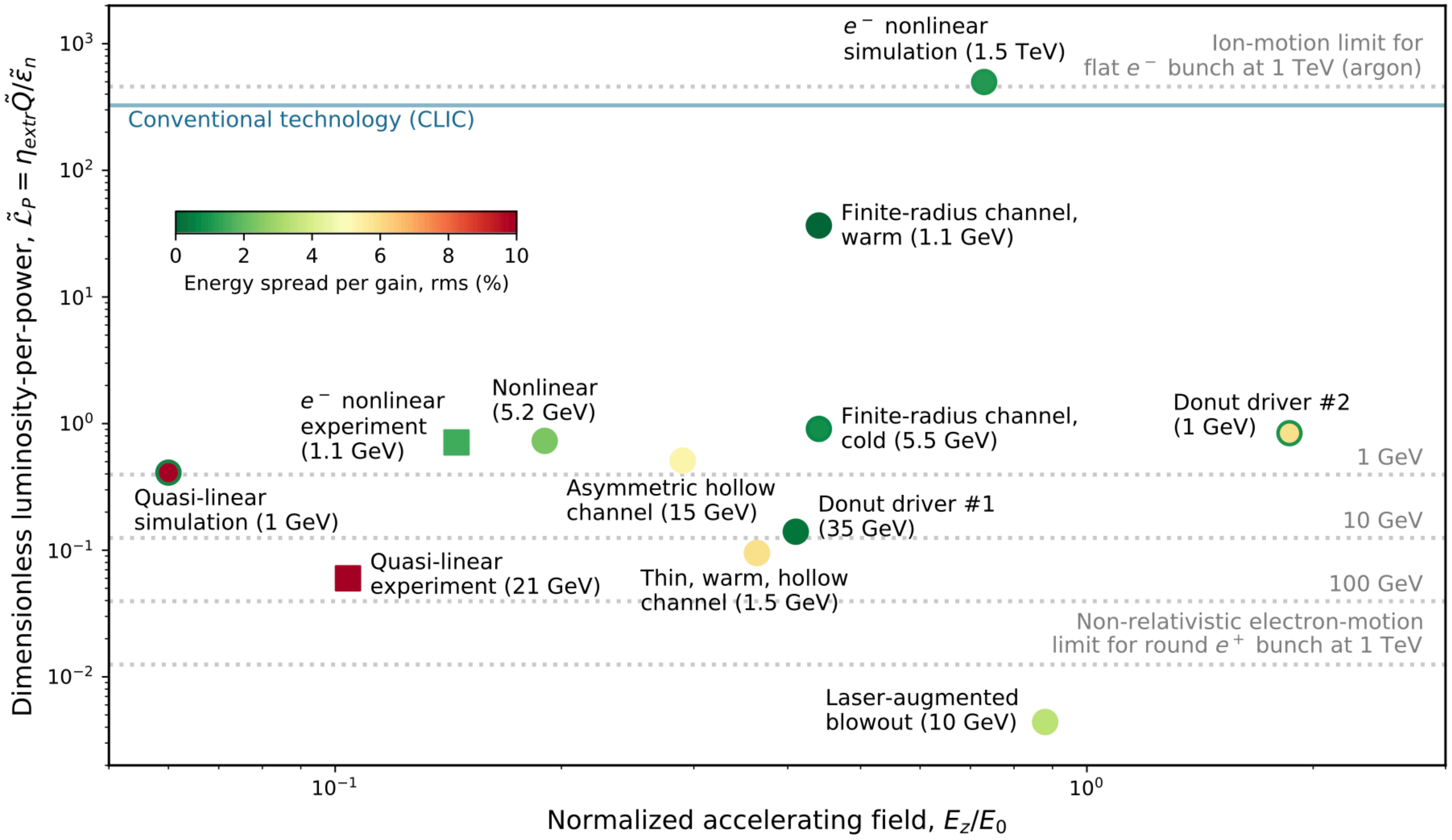
$\Delta\phi_e$

- Slice-by-slice matching
- Plasma electron temperature
- Spread plasma electrons: different plasma electrons to focus different positron beam slices

η_{extr}

- Energy recovery to improve efficiency
- Decrease emittance to compensate for low efficiency in $\tilde{\mathscr{L}}_P$

- Low focusing and large beta function
- High Lorentz factor for plasma electrons



Conclusion

- Energy efficiency and luminosity-per-power comes with a strong positron load, and thus with transverse beam loading and electron motion
- For most regimes, there is a tradeoff between energy efficiency and beam quality (e.g. emittance, uncorrelated energy spread)
- Luminosity-per-power scaling and electron motion highlights future directions

Thank you for your attention