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Decoding the early universe through primordial black hole abundance, dark matter, and gravitational waves

(Based on Phys.Rev.D **108** (2023) 6, 063523; Phys.Rev.D **109** (2024) 2, 023521; e-Print: 2403.16963; and JHEP 10 (2024) 142**)**

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Outline of the talk

Motivation:
Observational difficulty in the early Universe and introduction to the reheating phase

Goal:

- Constraining the reheating phase through scalar-induced secondary gravitational waves with NANOGrav 15-year data.
- Possibilities of PBH reheating
- Doubly peak-induced GWs associated with ultralight PBHs.

Conclusions

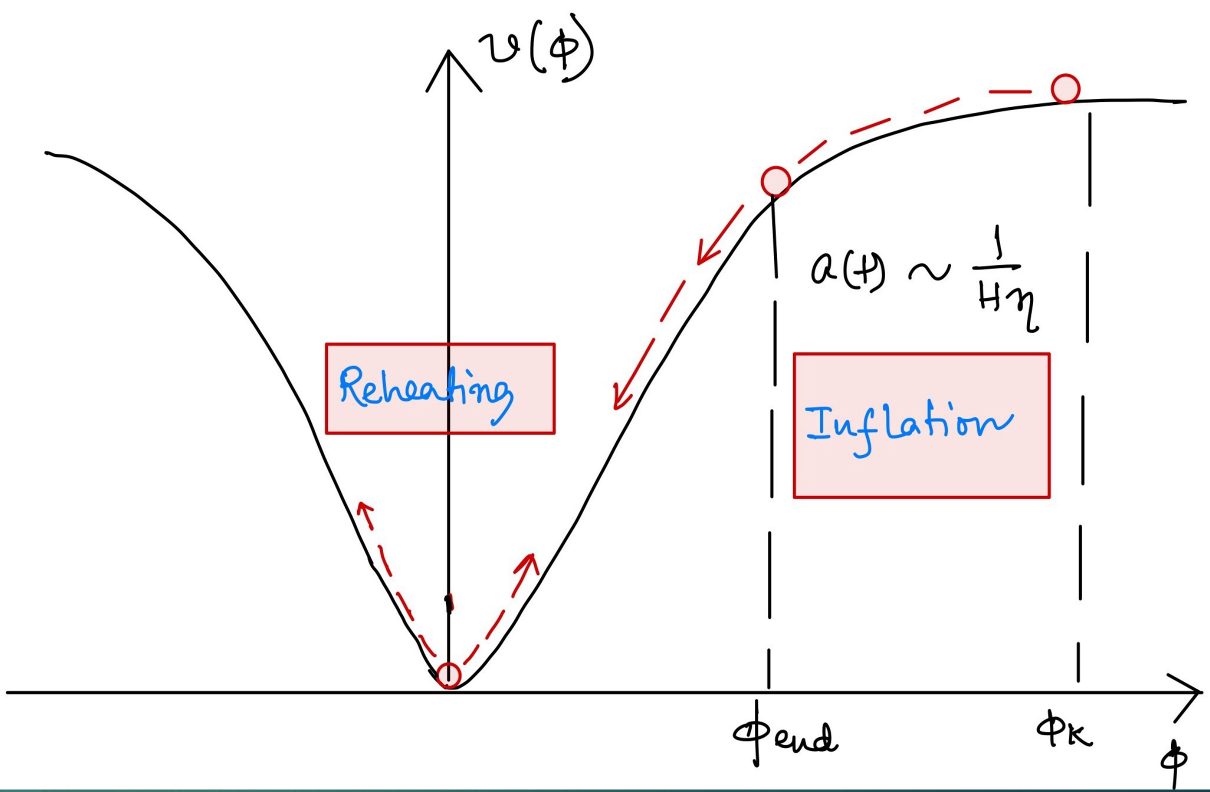
Why do we need reheating phase?

- ❑ The end point of inflation

  - ❖ The universe is cold, dark, and dominated by the homogeneous inflaton field.

- ❑ How does the Universe transition to a the hot, thermalized, radiation-dominated state after inflation, which is required for nucleosynthesis.

- ❑ Reheating!



- ❑ Natural consequence after inflation: fill the empty space with matter (**generate entropy**)

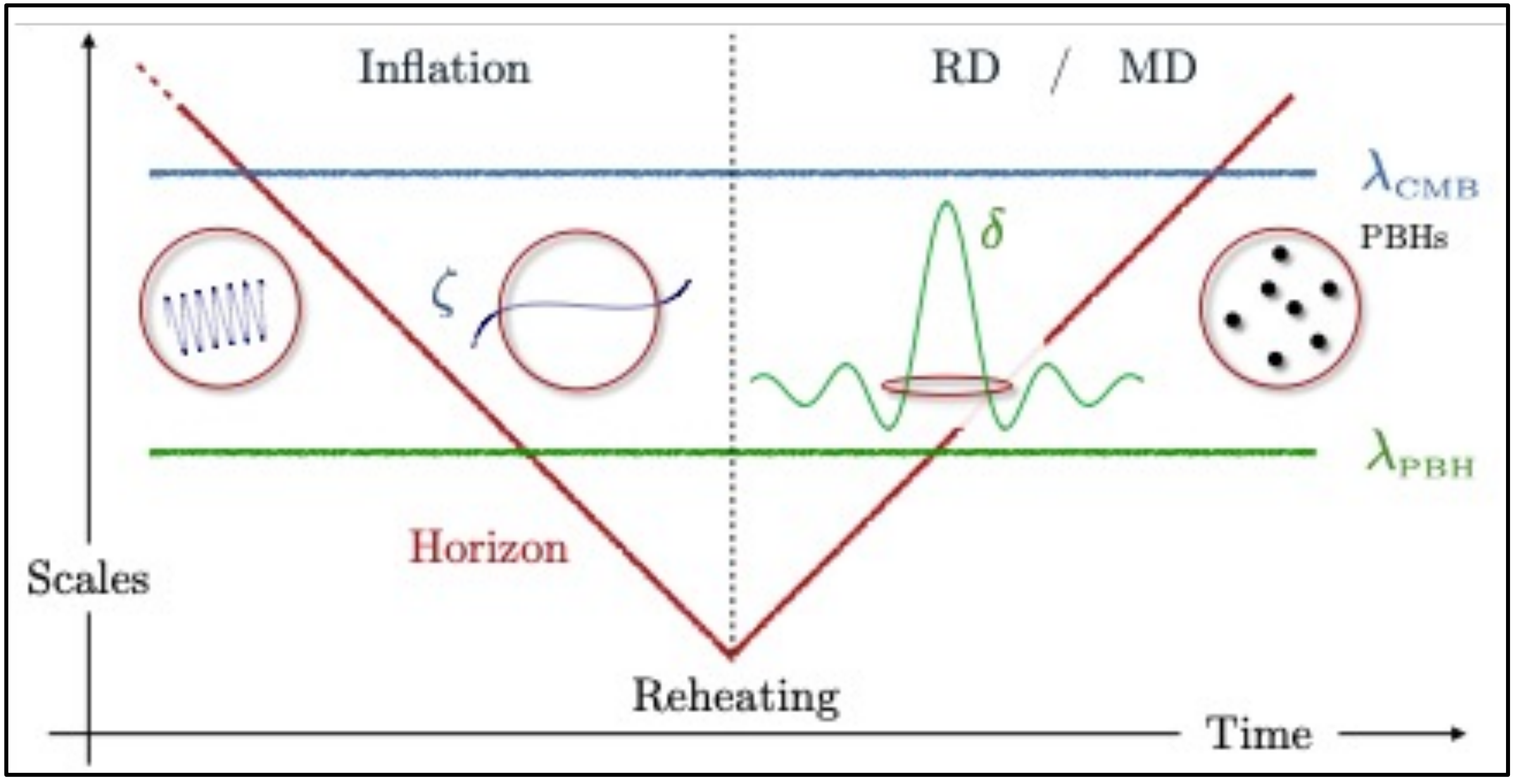
PBH formation during reheating : possibilities

- The production of PBHs from inflation usually requires the existence of a short period of ***ultra-slow-roll*** that produces a peak in the primordial power spectrum of scalar curvature perturbations.

- Perturbations that were generated during the late inflationary era can get resonantly amplified and collapse into black holes before the Universe is reheated. Depending on the reheating temperature, the PBH mass fraction can peak at different masses.

- Bubble collision during phase transition and in principle that can happen during reheating.

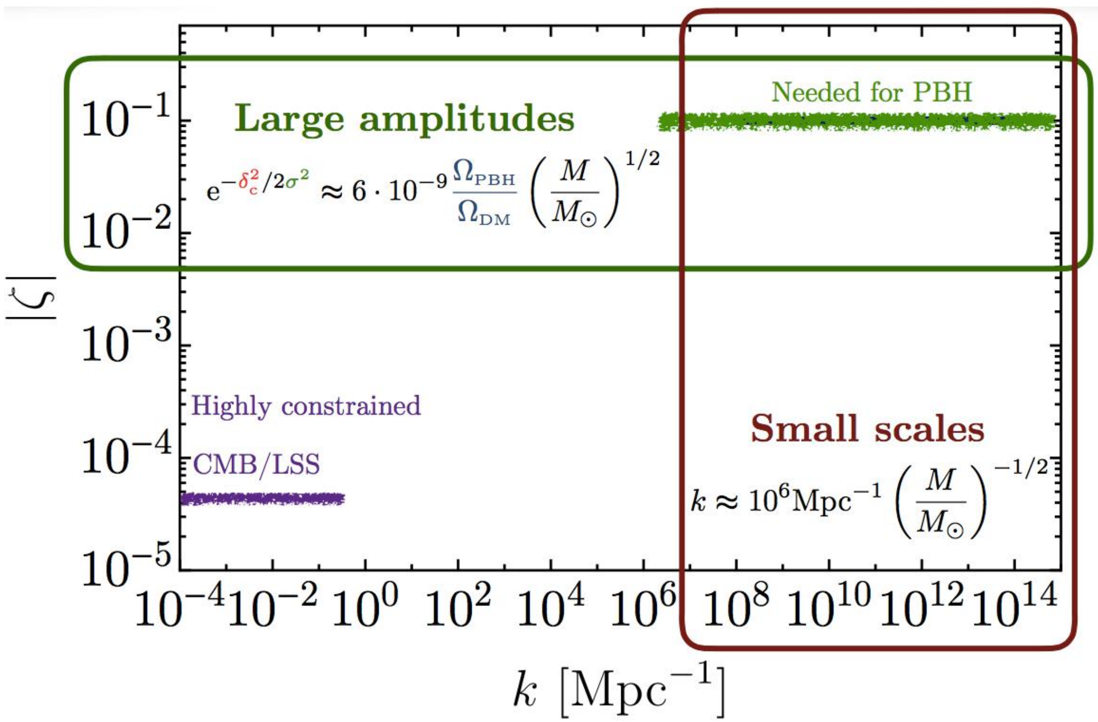
Formation of primordial black holes (PBHs) during reheating/radiation domination



A schematic representation of the standard PBH formation scenario. The green line indicates the comoving scale of perturbations generated during inflation responsible for the PBH formation, much smaller than the CMB scales indicated in blue.

G. Franciolini, arXiv:2110.06815 [astro-ph.CO].

Amplitude of the perturbation required to form PBHs



❖ In order to form significant number of black holes, the amplitude of the perturbations on small scales has to be large enough such that the dimensionless amplitude of the scalar perturbation is close to unity

G. Franciolini, arXiv:2110.06815 [astro-ph.CO].

Formation of the PBHs during reheating

❖ We assume that the inflationary scalar power spectrum with a broken power law is given by

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\rm S}\left(\frac{k}{k_*}\right)^{n_{\rm S}-1} + A_0 \begin{cases} \left(\dfrac{k}{k_{\rm peak}}\right)^{4} & k \le k_{\rm peak} \\ \left(\dfrac{k}{k_{\rm peak}}\right)^{n_0} & k \ge k_{\rm peak} \end{cases}$$

where $A_{\rm S}$ and $n_{\rm S}$ are the amplitude and spectral index of the power spectrum at the CMB pivot scale of $k_* = 0.05\,{\rm Mpc}^{-1}$.

❖ We shall assume that the threshold value of the density contrast for the formation of PBHs is given by following analytical expression

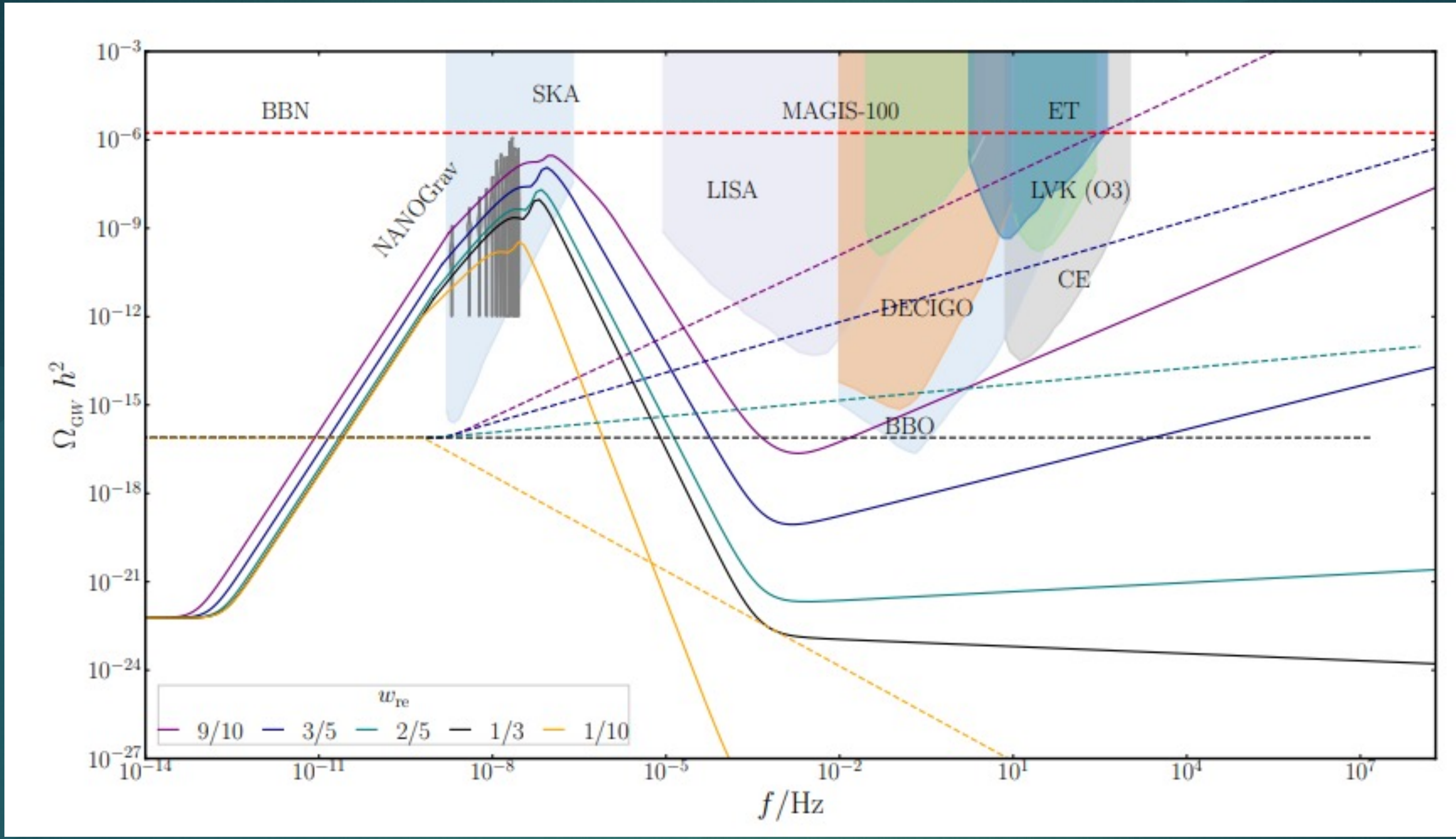
$$\delta_{\rm c}^{\rm an} = \frac{3\,(1+w_{\rm re})}{5+3\,w_{\rm re}}\,\sin^2\left(\frac{\pi\,\sqrt{w_{\rm re}}}{1+3\,w_{\rm re}}\right)$$

❖ Fraction of the dark matter contributed from PBH today

$$f_{\rm PBH}(M) = \beta(M)\,\frac{\Omega_{\rm m}\,h^2}{\Omega_{\rm c}\,h^2}\left(\frac{g_{s,{\rm eq}}}{g_{s,{\rm re}}}\right)\left(\frac{g_{\rm re}}{g_{\rm eq}}\right)^{\frac{1}{1+w_{\rm re}}}\left(\frac{T_{\rm re}}{T_{\rm eq}}\right)^{\frac{1-3\,w_{\rm re}}{1+w_{\rm re}}}\left(\frac{M}{\gamma\,M_{\rm eq}}\right)^{-\frac{2\,w_{\rm re}}{1+w_{\rm re}}}$$

T. Harada, C.-M. Yoo, and K. Kohri, Phys. Rev. D **88**, 084051 (2013).

Generation of scalar induced secondary GWs during the epoch of reheating



The dimensionless spectral energy density of primary and secondary GWs today have been plotted for a given reheating temperature and different values of the parameter describing the equation of state during reheating

S. Maity, N. Bhaumik, M. R. Haque, D. Maity and L. Sriramkumar, arXiv 2403.16963.

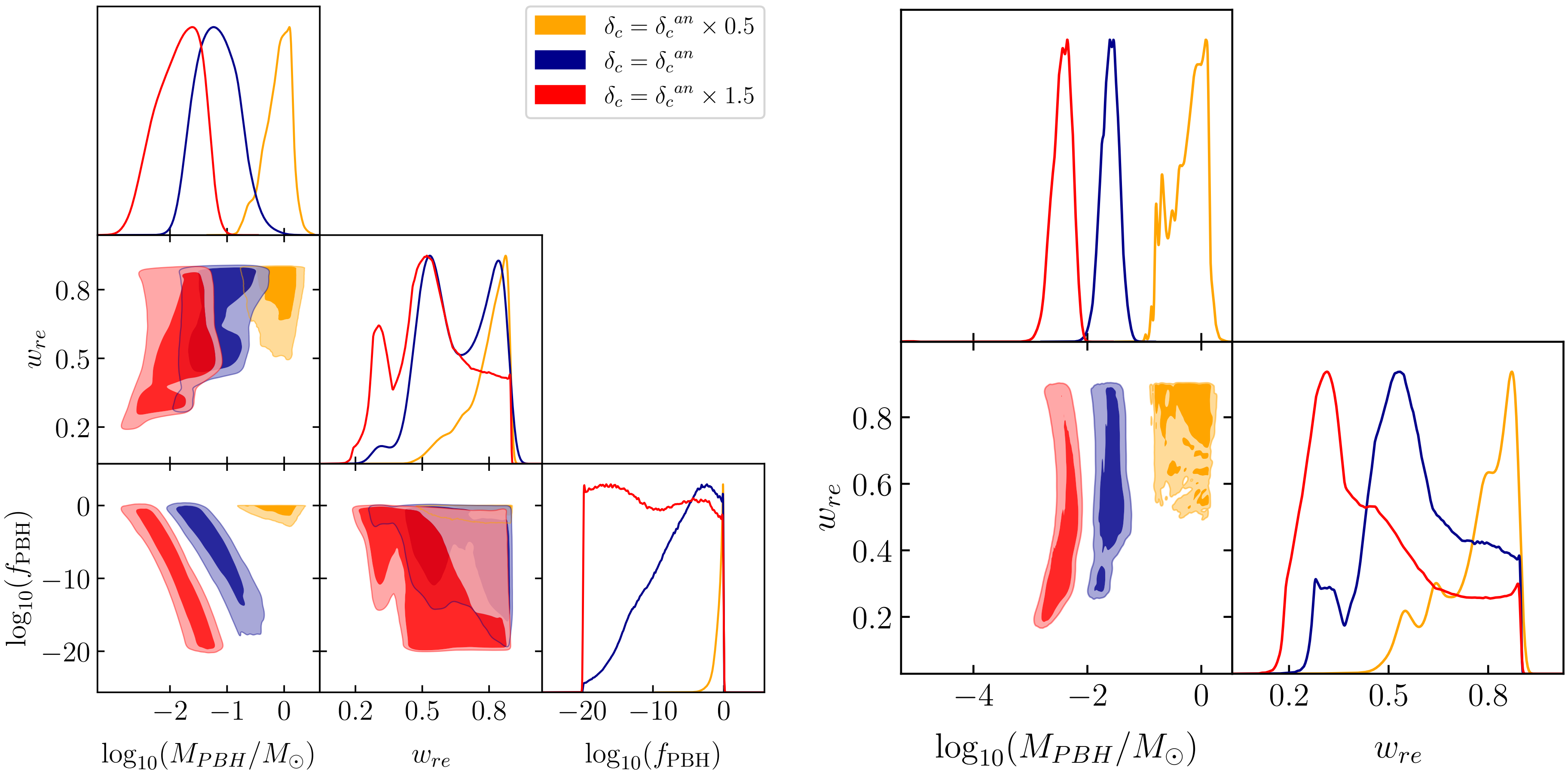
Best-fit values

Model	Parameter	Prior	Mean value		
R4pF	$\log_{10}\left(\frac{k_{\rm peak}}{\rm Mpc^{-1}}\right)$	$[6, 9]$	$7.62^{+0.35}_{-0.41}$		
	$\log_{10}(A_0)$	$[-3, 0]$	$-1.23^{+0.38}_{-0.66}$		
	$w_{\rm re}$	$[0.1, 0.9]$	0.52 ± 0.23		
	n_0	$[-3.0, -1.5]$	-2.26 ± 0.43		
R3pF	$\log_{10}\left(\frac{k_{\rm peak}}{\rm Mpc^{-1}}\right)$	$[6, 9]$	$7.54^{+0.36}_{-0.44}$		
	$\log_{10}(A_0)$	$[-3, 0]$	$-1.26^{+0.26}_{-0.64}$		
	$w_{\rm re}$	$[0.1, 0.9]$	$0.55^{+0.39}_{-0.14}$		
			$\mathbf{0.5\,\delta_c^{an}}$	$\boldsymbol{\delta_c^{an}}$	$\mathbf{1.5\,\delta_c^{an}}$
R3pB	$\log_{10}\left(\frac{M}{M_\odot}\right)$	$[-6, 3.5]$	$-0.12^{+0.28}_{-0.15}$	$-1.18^{+0.35}_{-0.39}$	$-1.85^{+0.49}_{-0.30}$
	$\log_{10}(f_{\rm PBH})$	$[-20, 0]$	$-0.67^{+0.68}_{-0.16}$	$-6.6^{+6.5}_{-1.9}$	$-10.2^{+8.2}_{-9.6}$
	$w_{\rm re}$	$[0.1, 0.9]$	$0.78^{+0.11}_{-0.030}$	$0.66^{+0.23}_{-0.19}$	0.55 ± 0.17
R2pB	$\log_{10}\left(\frac{M}{M_\odot}\right)$	$[-6, 3.5]$	$-0.24^{+0.38}_{-0.45}$	$-1.60^{+0.16}_{-0.14}$	$-2.45^{+0.20}_{-0.13}$
	$w_{\rm re}$	$[0.1, 0.9]$	$0.77^{+0.13}_{-0.038}$	0.59 ± 0.16	$0.464^{+0.095}_{-0.25}$

The best-fit values arrived upon comparison with the NANOGrav 15-year data.

S. Maity, N. Bhaumik, M. R. Haque, D. Maity and L. Sriramkumar, arXiv 2403.16963.

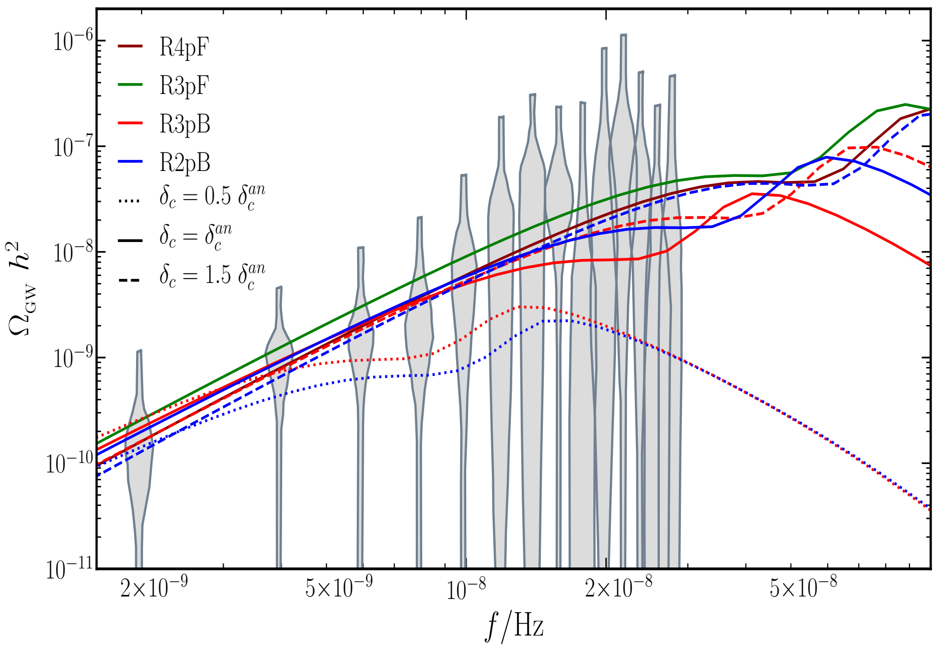
Constraints on the epoch of reheating

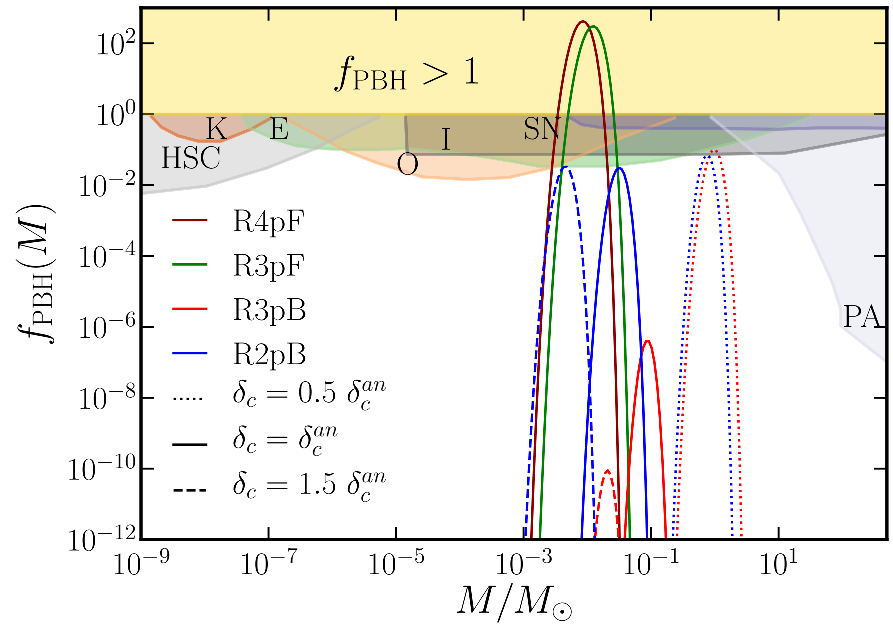


❖ we have plotted the marginalized posterior distributions of the parameters that have been arrived at upon comparing our model with the NANOGrav 15-year data.

S. Maity, N. Bhaumik, M. R. Haque, D. Maity and L. Sriramkumar, arXiv 2403.16963.

Spectrum of the secondary GWs and the formation of the PBHs with the best-fit values





- The dimensionless spectral energy density of the secondary GWs today Ω_{GW} (f) is plotted for a given reheating temperature and the best-fit values of the parameters in the different models.

- The fraction of PBHs that constitute the dark matter density today is plotted for a given reheating temperature T_{re}=50 MeV and the best-fit values of the parameters in the different models.

S. Maity, N. Bhaumik, M. R. Haque, D. Maity and L. Sriramkumar, arXiv 2403.16963.

Bayesian evidence

Model X	Model Y	$\mathrm{BF}_{Y,X}$		
		$\delta_c = 0.5\,\delta_c^{\rm an}$	$\delta_c = \delta_c^{\rm an}$	$\delta_c = 1.5\,\delta_c^{\rm an}$
SMBHB	R2pB	$1.7 \pm .06$	260.04 ± 19.21	350.61 ± 27.36

- We obtain the marginalized likelihood in support of model Y and utilize it to evaluate the Bayesian factor against a reference model X.

- When $\delta_c = \delta_c^{an}$ and $\delta_c = 1.5\,\delta_c^{an}$, our comparison with the NANOGrav's 15-year data finds strong Bayesian evidence in favor of the scenario wherein PBHs are formed during reheating, resulting in the generation of secondary GWs rather than the SMBHB model.

S. Maity, N. Bhaumik, M. R. Haque, D. Maity and L. Sriramkumar, arXiv 2403.16963.

Overview of PBH Reheating

Dependency on the evolution of the PBHs

1. PBH mass distribution

$$f_{\rm PBH}(M) = \delta(M - M_{\rm in})$$

2. Formation mass

$$M_{\rm in} = \gamma\, M_H = \gamma \frac{4\pi}{3} \frac{\rho(t_{\rm in})}{H^3(t_{\rm in})} = 4\pi\gamma \frac{M_P^2}{H(t_{\rm in})}$$

❖ Collapse efficiency : $\gamma = w^{3/2}$

3. Total energy falls into BH at the point of formation $\beta = \frac{\rho_{\rm BH}(t_{\rm in})}{\rho_{\rm tot}(t_{\rm in})}$

$$\rho_{\rm tot} = \rho_\phi + \rho_R$$

❑ Boltzmann equations :

$$\dot{\rho_\phi} + 3H(1+w_\phi)\rho_\phi = -\Gamma_\phi \rho_\phi\,(1+w_\phi)$$

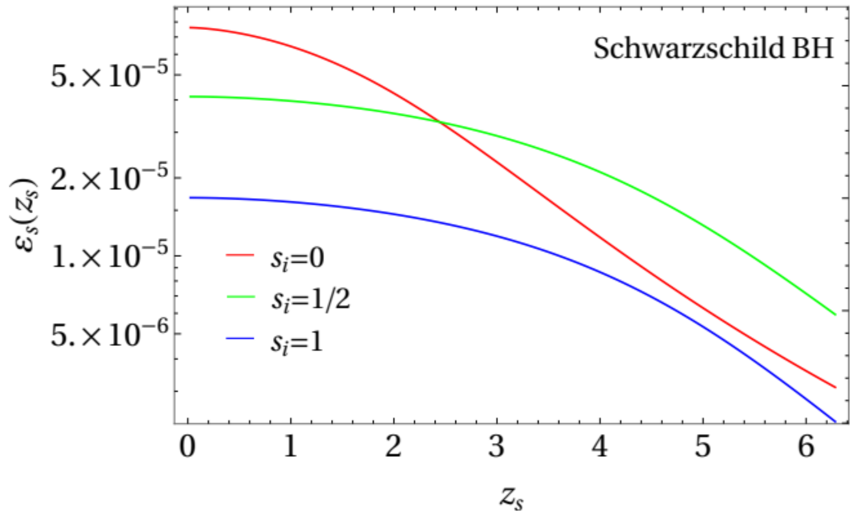
$$\dot{\rho}_R + 4H\rho_R = \Gamma_\phi\rho_\phi(1+w_\phi) - \frac{\rho_{\rm BH}}{M_{\rm BH}}\frac{dM_{\rm BH}}{dt}\,\theta(t-t_{\rm in})\,\theta(t_{\rm ev}-t)$$

$$\dot{\rho}_{\rm BH} + 3H\rho_{\rm BH} = \frac{\rho_{\rm BH}}{M_{\rm BH}}\frac{dM_{\rm BH}}{dt}\,\theta(t-t_{\rm in})\,\theta(t_{\rm ev}-t)$$

❑ Mass reduction:

$$\frac{dM_{\rm BH}}{dt} = -\epsilon \frac{M_P^4}{M_{\rm BH}^2} \Rightarrow M_{\rm BH} = M_{\rm in}\left(1 - \Gamma_{\rm BH}(t-t_{\rm in})\right)^{\frac{1}{3}}$$

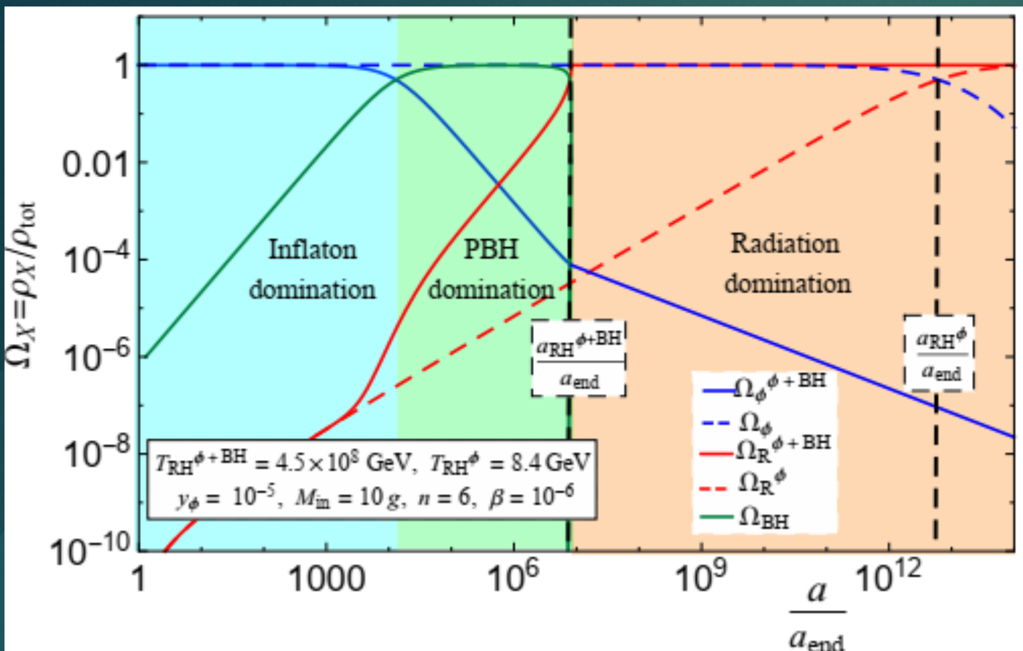
❑ Lifetime of the BH : $t_{\rm ev} = \frac{1}{\Gamma_{\rm BH}}$, $\Gamma_{\rm BH} = 3\epsilon \frac{M_P^4}{M_{\rm in}^3}$

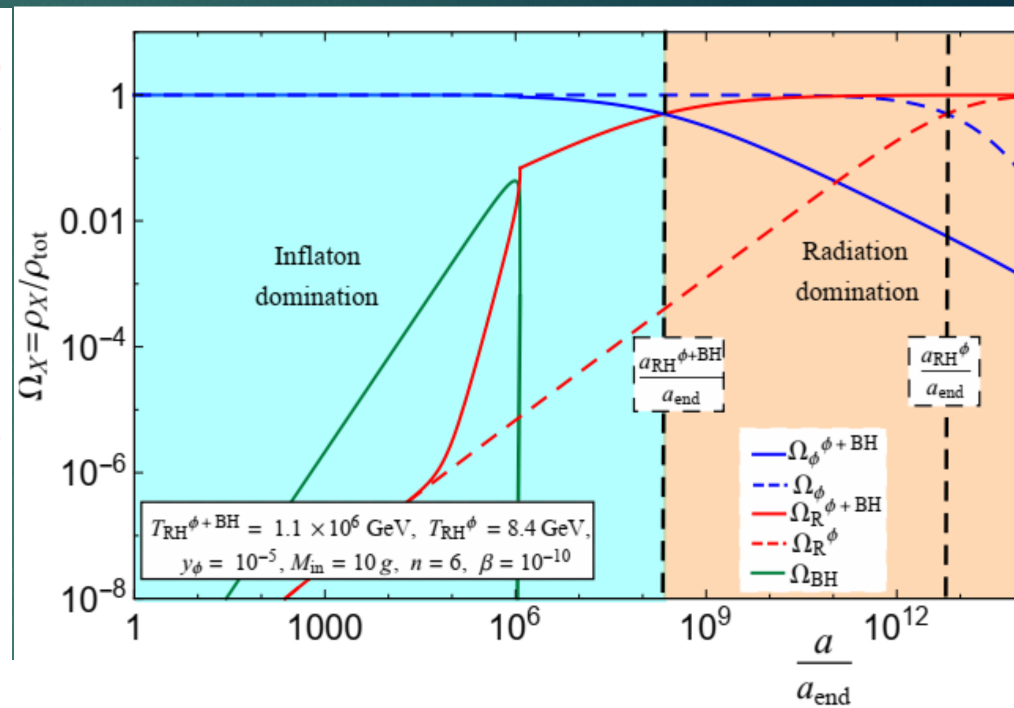


❖ Evaporation coefficient as function of $z = m_j/T_{BH}$

M. R. Haque, E. Kpatcha, D. Maity and Y. Mambrini, Phys. Rev. D 108, 063523 (2023).

Overview of my previous work:
Background dynamics of reheating: PBH Reheating





Condition for the PBH domination :

$$\beta_c = \left(\frac{\epsilon}{(1+w_\phi)2\pi\gamma}\right)^{\frac{2w_\phi}{1+w_\phi}} \left(\frac{M_P}{M_{in}}\right)^{\frac{4w_\phi}{1+w_\phi}}$$

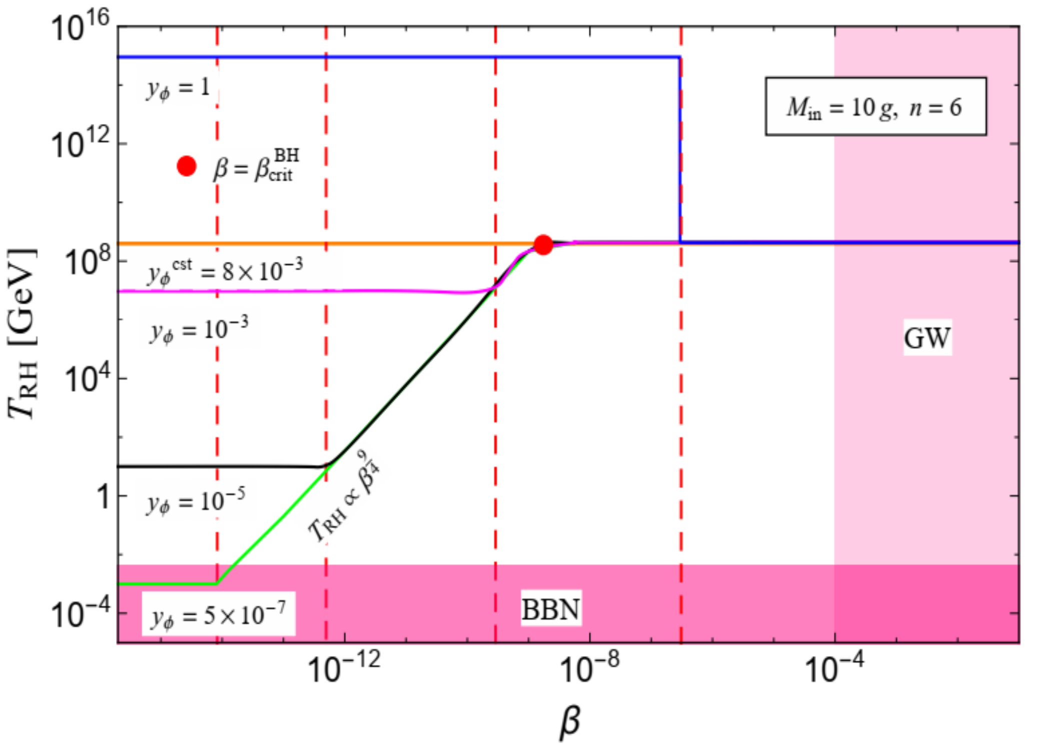
- Reheating temperature $$T_{RH} = M_P \left(\frac{3\,\epsilon^2}{\alpha_T}\right)^{\frac{1}{4}} \left(\frac{M_P}{M_{in}}\right)^{\frac{3}{2}}$$

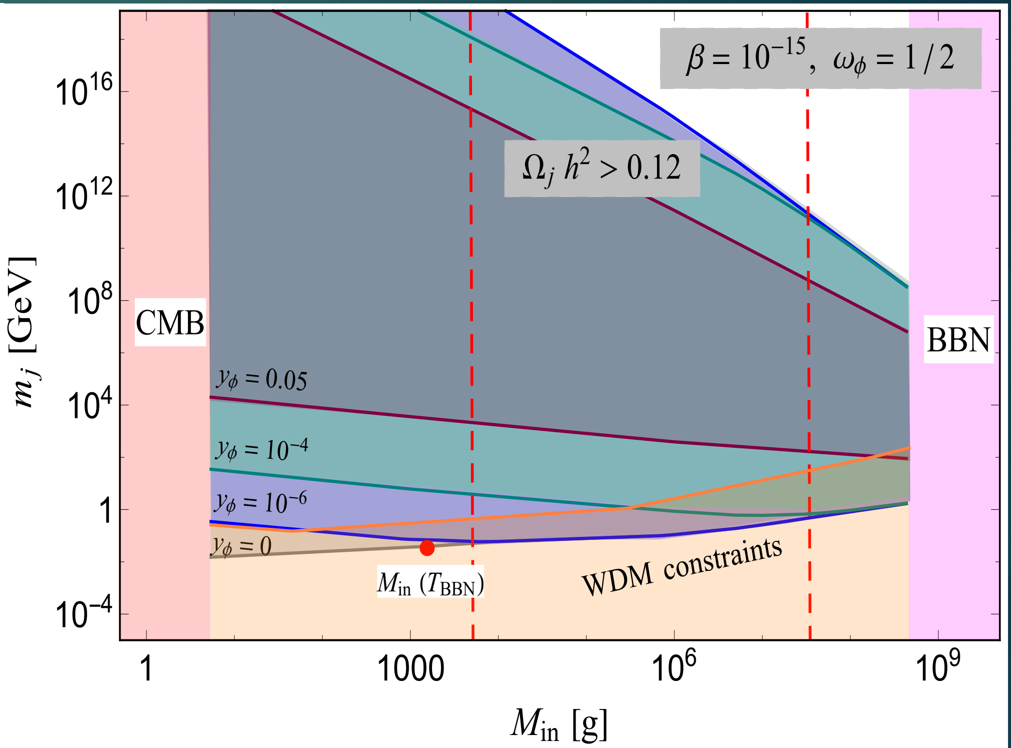
- Reheating temperature

$$T_{RH} \sim M_P \beta^{\frac{3}{4}\frac{1+w_\phi}{3w_\phi-1}} \left(\frac{M_{in}}{M_P}\right)^{\frac{3}{2}\frac{1-w_\phi}{3w_\phi-1}}$$

M. R. Haque, E. Kpatcha, D. Maity and Y. Mambrini, Phys. Rev. D **108**, 063523 (2023).

Reheating and DM parameter space from PBH evaporation



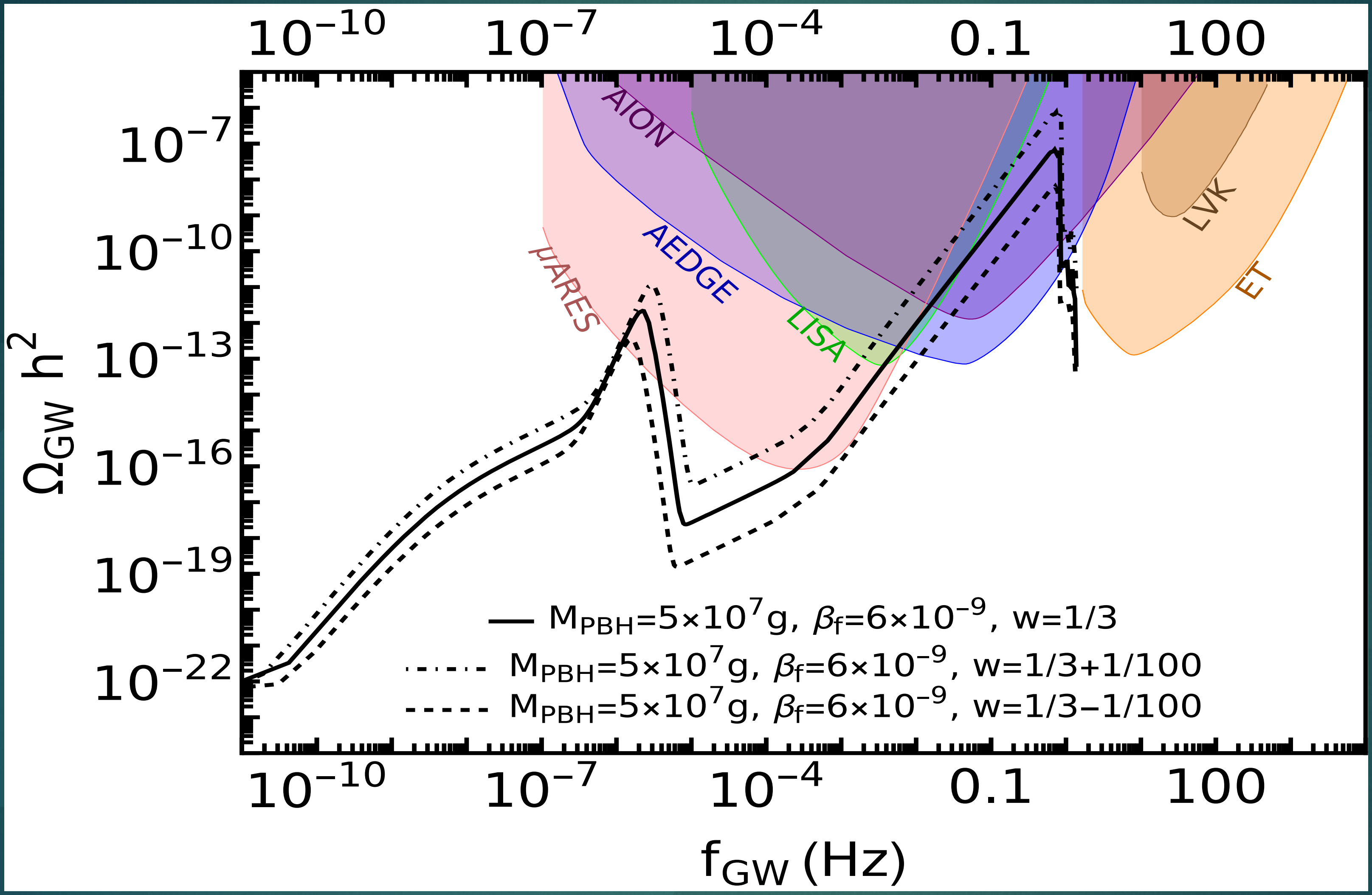


Inflaton reheating Vs PBH reheating

DM parameter space from evaporating PBHs

M. R. Haque, E. Kpatcha, D. Maity and Y. Mambrini, Phys. Rev. D 108, 063523 (2023).
M. R. Haque, E. Kpatcha, D. Maity and Y. Mambrini, Phys. Rev. D **109**, 023521 (2024).

Doubly peaked GWs for PBH reheating scenario



N. Bhaumik, M. R. Haque, R.K. Jain, and M. Lewicki, JHEP 10 (2024) 142.

Conclusions

- We assume a specific functional form for the primordial scalar power spectrum and examine the production of PBHs and the scalar-induced secondary GWs during the reheating phase. Specifically, we account for the uncertainties in the conditions for the formation of PBHs and ensure that the extent of PBHs produced remains within the observational bounds. We find that the scalar-induced SGWB generated during a phase of reheating with a steeper equation of state (than that of radiation) fit the NANOGrav 15-year data with stronger Bayesian evidence than the astrophysical scenario involving GWs produced by merging supermassive binary black holes.

- I have discussed the reheating and DM parameter space in the background of the reheating phase dynamically obtained from two chief systems in the early Universe: the inflaton ϕ and the primordial black holes. The DM is assumed to be produced purely gravitationally from the PBH decay, not interacting with the thermal bath and the inflaton.

- Ultra-low mass primordial black holes (PBH), briefly dominating the expansion of the universe, would leave detectable imprints in the secondary stochastic gravitational wave background (SGWB). Such a scenario leads to a characteristic doubly peaked spectrum of SGWB and strongly depends on the background where the ultra-light PBHs form.

Thank You