Non-minimal minimal dark matter

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based on: B. Grządkowski, **AS** 2411.07222

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Outline

Introduction

The action for a **massive, non-minimally coupled spin-1 spectator field X_μ** in the FLRW background metric $g_{\mu\nu}$ takes the form:

$$S_{\rm X} = \int d^4x\sqrt{-g}\left(\mathcal{L}_{\rm X}^{\rm M} + \mathcal{L}_{\rm X}^{\rm N}\right), \quad \mathcal{L}_{\rm X}^{\rm M} \equiv -\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}X_{\mu\nu}X_{\alpha\beta} + \frac{m_{\rm X}^2}{2}g^{\mu\nu}X_\mu X_\nu,$$

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Remarks

Gauge-breaking terms could arise from the generalized Stuckelberg action:

$$S_{\mathcal{S}} = \int d^4x\sqrt{-g}\Big\{ -\frac{1}{4}g^{\mu\alpha}g^{\nu\beta}X_{\mu\nu}X_{\alpha\beta} + \frac{1}{2}\left[g^{\mu\nu} - \xi_1\frac{R}{m_X^2}g^{\mu\nu} + \xi_2\frac{R^{\mu\nu}}{m_X^2}\right](\partial_\mu\Phi_X + m_X X_\mu)(\partial_\nu\Phi_X + m_X X_\nu)\Big\}$$

$$X_\mu(x) \to X'_\mu(x) = X_\mu(x) + \partial_\mu\lambda(x), \quad \Phi_X(x) \to \Phi'_X(x) = \Phi_X(x) - m_X\lambda(x).$$

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For the choice $\xi_1 = \xi_2/2$, the X_μ has only one non-minimal coupling. $\sqrt{-g}\xi_1 G_{\mu\nu}X^\mu X^\nu$

O. Özsoy et al. , arXiv: 2310.03862

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- X_0 does not have a kinetic term; it is an auxiliary field.

Action for the transverse modes

$$\tilde{S}_{\mathrm{T}} = \sum_{T=\pm} \int d\tau \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{2}|X_{\mathrm{T}}'(\tau,\vec{k})|^2 - \frac{1}{2}[k^2 + a^2 m_{\mathrm{eff,x}}^2]|X_{\mathrm{T}}(\tau,\vec{k})|^2 \right\},$$

Action for the longitudinal mode

$$\tilde{S}_{\mathrm{L}} = \int d\tau \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{2} \frac{1}{\mathcal{A}_{\mathrm{L}}^2(a,k)} |X_{\mathrm{L}}'(\tau,\vec{k})|^2 - \frac{1}{2} a^2 m_{\mathrm{eff,x}}^2 |X_{\mathrm{L}}(\tau,\vec{k})|^2 \right\},$$

$$\mathcal{A}_L^2 \equiv \frac{k^2 + a^2 m_{\mathrm{eff,t}}^2}{a^2 m_{\mathrm{eff,t}}^2},$$

The emergence of two effective masses

$$m_{\mathrm{eff,t}}^2 \equiv m_X^2 - \xi_1 R(a) + \frac{1}{2}\xi_2 R(a) + 3\xi_2 H^2(a),$$

$$m_{\mathrm{eff,x}}^2 \equiv m_X^2 - \xi_1 R(a) + \frac{1}{6}\xi_2 R(a) - \xi_2 H^2(a).$$

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$$\tilde{S}_{\rm T} = \sum_{T=\pm} \int d\tau \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{2} |X'_{\rm T}(\tau,\vec{k})|^2 - \frac{1}{2}[k^2 + a^2 m^2_{\rm eff,x}] |X_{\rm T}(\tau,\vec{k})|^2 \right\},$$

Action for the longitudinal mode

$$\tilde{S}_{\rm L} = \int d\tau \int \frac{d^3k}{(2\pi)^3} \left\{ \frac{1}{2} \frac{1}{\mathcal{A}^2_{\rm L}(a,k)} |X'_{\rm L}(\tau,\vec{k})|^2 - \frac{1}{2} a^2 m^2_{\rm eff,x} |X_{\rm L}(\tau,\vec{k})|^2 \right\},$$

Note that $\mathcal{A}^2_L$ is not necessarily positive for all k

$$\mathcal{A}^2_L \equiv \frac{k^2 + a^2 m^2_{\rm eff,t}}{a^2 m^2_{\rm eff,t}},$$

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Instabilities

Ghost instability

For an arbitrary choice of non-minimal couplings $m^2_{\mathrm{eff},\mathrm{t}}$ might be negative. Hence, we are looking for the values of ξ_1, ξ_2 for which

$$\boxed{s(a, \xi_1, \xi_2) = \mathrm{sign}\left[\frac{k^2 + a^2 m^2_{\mathrm{eff,t}}}{a^2 m^2_{\mathrm{eff,t}}}\right] > 0}$$

for all values of k.

C. Capanelli et al. , arXiv:2405.19390
C. Capanelli et al. , arXiv:2403.15536

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The effective mass has two sources of time-dependency

$$m^2_{\mathrm{eff,t}} = m_X^2 - 3\left[\left(\xi_1 - \frac{1}{2}\xi_2\right)(3w(a) - 1) - \xi_2\right] H^2(a),$$

$$w(a) = [-1, 1]$$

One can get rid of one of them by choosing $\xi_1 = \xi_2/2$.

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$$m_X^2/H^2(a) > -3\xi_2$$

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$$\rightarrow m_X^2/H^2(a) > -3\xi_2$$

In general case:

$$\rightarrow f(w, \xi_1, \xi_2) < \eta_e^{-1}, \quad f(w, \xi_1, \xi_2) \equiv 3\left[\left(\xi_1 - \frac{1}{2}\xi_2\right)(3w(a) - 1) - \xi_2\right]$$

$$\eta_e^{-1} \equiv m_X^2/H^2_{\mathrm{inf}} \in (0, 1)$$

EoMs for the non-minimally coupled vectors

$$\mathcal{X}_{\rm L}'' + \omega_{\rm L}^2(a)\mathcal{X}_{\rm L} = 0, \qquad \mathcal{X}_\pm'' + \omega_\pm^2(a)\mathcal{X}_\pm = 0$$

The angular frequency for two transverse modes:

$$\omega_\pm^2(a,k) \equiv k^2 + a^2 m_{\rm eff,x}^2 = k^2 + a^2\left\{m_X^2 - \left[3\left(\xi_1 - \frac{1}{6}\xi_2\right)(3w(a) - 1) + \xi_2\right]H^2(a)\right\}$$

The angular frequency for the longitudinal mode:

$$\omega_{\rm L}^2(a,k) \equiv k^2\frac{m_{\rm eff,x}^2}{m_{\rm eff,t}^2} + a^2 m_{\rm eff,x}^2 - \frac{k^2}{k^2 + a^2 m_{\rm eff,t}^2}\left[\frac{a''}{a} + \frac{m_{\rm eff,t}''}{m_{\rm eff,t}} + 2\frac{a'}{a}\frac{m_{\rm eff,t}'}{m_{\rm eff,t}} - 3\frac{(a' m_{\rm eff,t} + m_{\rm eff,t}' a)^2}{k^2 + a^2 m_{\rm eff,t}^2}\right]$$

For $k \in \{k_e, k_{\rm eff,m}\}$ and $m_X < H_e$ the angular frequency is negative

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For $k \in \{k_e, k_{\rm eff,m}\}$ and $m_X < H_e$ the angular frequency is negative

Tachyonic enhancement of both polarizations??

Instability of short-wavelength modes

In the limit, $k \to \infty$, one finds

$$\omega_{\pm}^2(a,k) \to k^2, \quad \omega_{\mathrm{L}}^2(a,k) \to k^2 \frac{m_{\mathrm{eff,x}}^2}{m_{\mathrm{eff,t}}^2}, \quad \text{as } k \to \infty.$$

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⇨ **During inflation**

$$\frac{m_{\mathrm{eff,x}}^2}{m_{\mathrm{eff,t}}^2} = 1, \quad \omega_{\mathrm{L}}^2(a,k) = k^2, \quad \text{as } k \to \infty.$$

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⇨ **However, for $w \neq -1, \xi_2 \neq 0$**

$$\omega_{\rm L}^2(a,k) \to -\infty, \quad \text{as } k \to \infty, \quad \text{and } m_{\rm eff,x}^2 < 0.$$

Uncontrolled tachyonic enhancement of short-wavelength modes!

C. Capanelli et al. , arXiv:2405.19390
C. Capanelli et al. , arXiv:2403.15536

Instability of short-wavelength modes

The credibility of the model might be restored if one imposes the positivity condition on $m^2_{\mathrm{eff,x}}$ analogously to $m^2_{\mathrm{eff,t}}$. Namely,

$$\tilde{f}(w,\xi_1,\xi_2) \equiv 3[3w(a)-1]\left(\xi_1 - \frac{1}{6}\xi_2\right) + \xi_2,$$

is required to meet the condition

$$\tilde{f}(w,\xi_1,\xi_2) < \eta_e^{-1}.$$

In addition, to avoid super-luminal propagation of short-wavelength modes, one demands

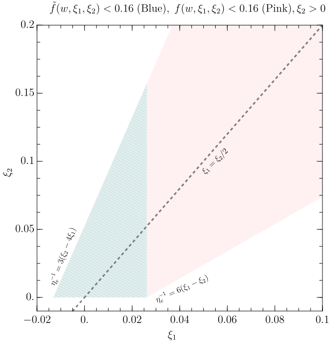
$$m^2_{\mathrm{eff,x}} \leq m^2_{\mathrm{eff,t}}, \qquad \xi_2 > 0.$$

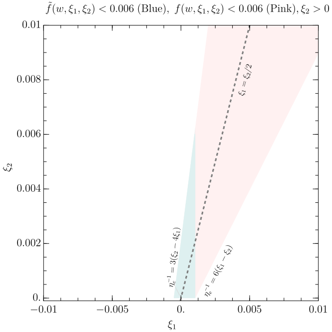
C. Capanelli et al. , arXiv:2403.15536
C. Capanelli et al. , arXiv:2405.19390

The model is well-defined in the region:

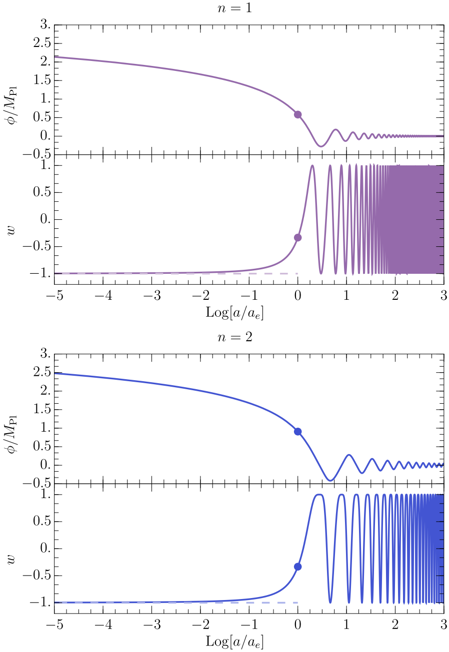
$$\xi_1 \in \left(-\frac{\eta_e^{-1}}{12}, \frac{\eta_e^{-1}}{6}\right),$$
$$\xi_2 \in \left[0, \frac{\eta_e^{-1}}{3} + 4\xi_1\right).$$

The viable parameter space shrinks as the ratio $m_X/H_{\rm ini}$ decreases. For $m_X \to 0$, it collapses to a point $(\xi_1, \xi_2) = (0, 0)$.





Inflationary and post-inflationary evolution



As an illustration, we consider the α-attractor T-model:

R. Kallosh et al., arXiv:1306.5220
R. Kallosh et al., arXiv:1311.0472

$$V(\phi) = \Lambda^4 \tanh^{2n}\left(\frac{|\phi|}{\sqrt{6\alpha}\,M_{\rm Pl}}\right)$$

The inflationary scale

$3 \cdot 10^{-3}\, M_{\rm Pl}$ $\qquad$ 1

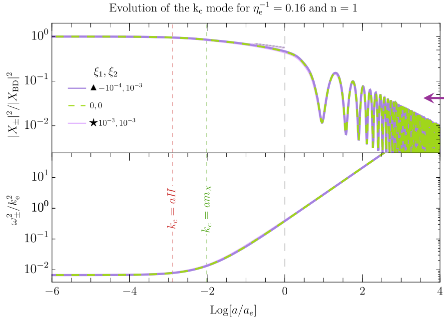
During reheating

$$V(\phi) \simeq \Lambda^4 \left(\frac{|\phi|}{M_{\rm Pl}}\right)^{2n}$$

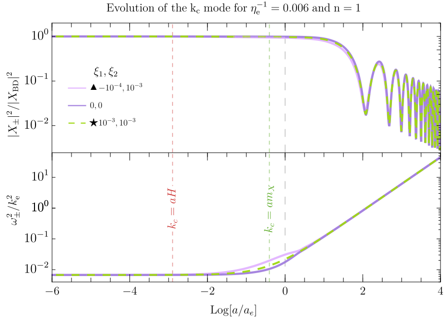
Free parameter

$n \in \{1, 2\}$

Evolution of transverse modes



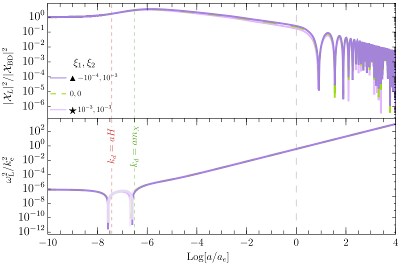
Transverse modes do not experience tachyonic growth.



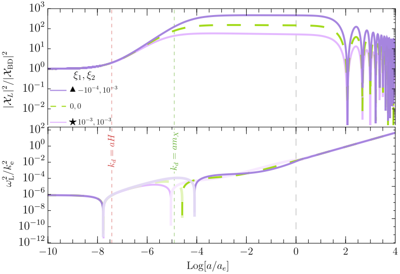
Frequency remains positive throughout the whole evolution.

Evolution of the longitudinal mode

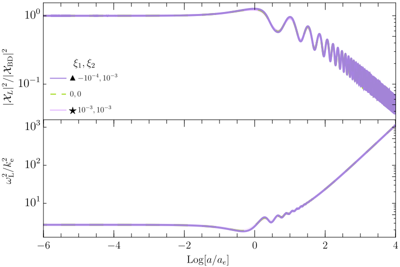
Evolution of the k_d mode for $\eta_e^{-1} = 0.16$ and n = 1



Evolution of the k_d mode for $\eta_e^{-1} = 0.006$ and n = 1



Evolution of the k_a mode for $\eta_e^{-1} = 0.16$ and n = 1



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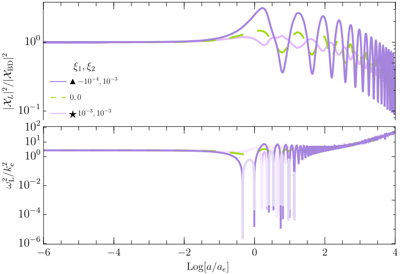
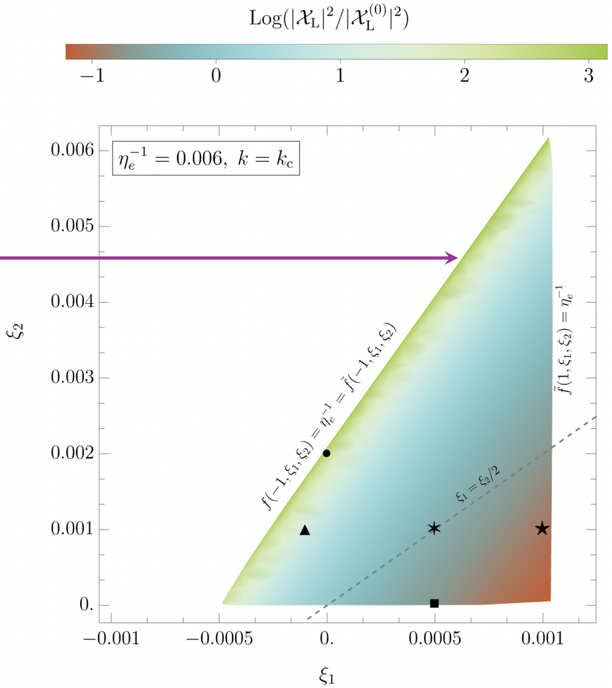


The most significant enhancement occurs along the curve

$m^2_{\rm eff,t} = 0 = m^2_{\rm eff,x}$



Energy density

The vacuum expectation value of the non-minimally coupled vector field energy density has several contributions...

$$\langle \hat{\rho}_X \rangle = \langle \hat{\rho}_\pm \rangle + \langle \hat{\rho}_\mathrm{L} \rangle$$

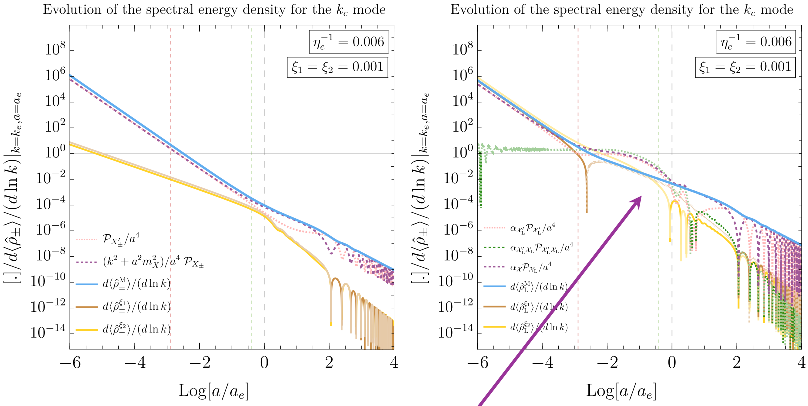
Transverse modes

$$\langle \hat{\rho}_\pm \rangle = \langle \hat{\rho}^\mathrm{M}_\pm \rangle + \langle \hat{\rho}^{\xi_1}_\pm \rangle + \langle \hat{\rho}^{\xi_2}_\pm \rangle,$$

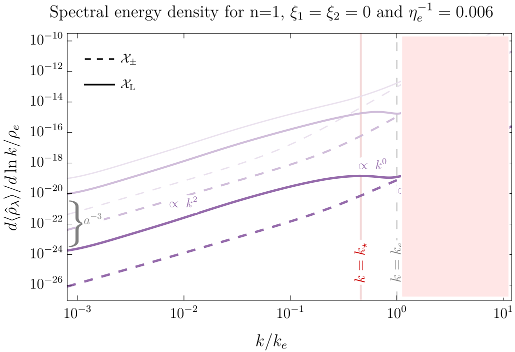
Redefined longitudinal mode

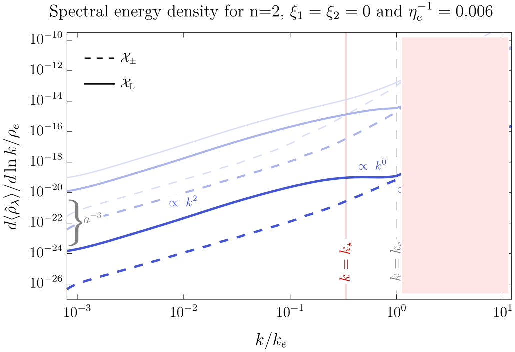
$$\langle \hat{\rho}_\mathrm{L} \rangle = \langle \hat{\rho}^\mathrm{M}_\mathrm{L} \rangle + \langle \hat{\rho}^{\xi_1}_\mathrm{L} \rangle + \langle \hat{\rho}^{\xi_2}_\mathrm{L} \rangle,$$

At late times: $\langle \hat{\rho}^{\xi_1,\xi_2}_{\pm,L} \rangle \ll \langle \hat{\rho}^\mathrm{M}_\pm \rangle \ll \langle \hat{\rho}^\mathrm{M}_\mathrm{L} \rangle$



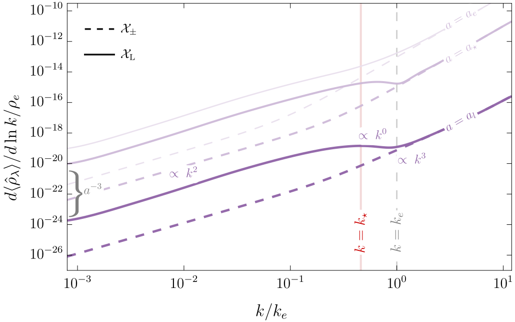
Tachyonic enhancement





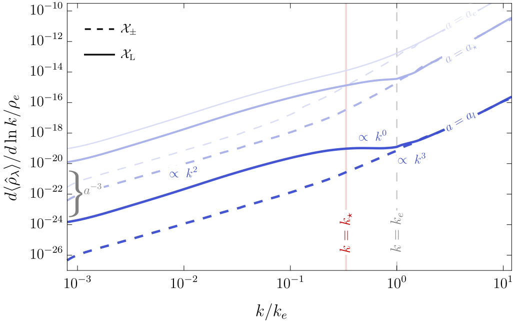
Energy density of the longitudinal polarization has a peak structure at $a \geq a_\star$.

Spectral energy density for n=1, $\xi_1 = \xi_2 = 0$ and $\eta_e^{-1} = 0.006$

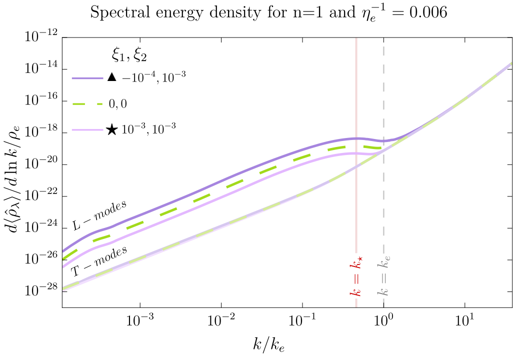


Energy density of the longitudinal polarization has a peak structure at $a \geq a_\star$.

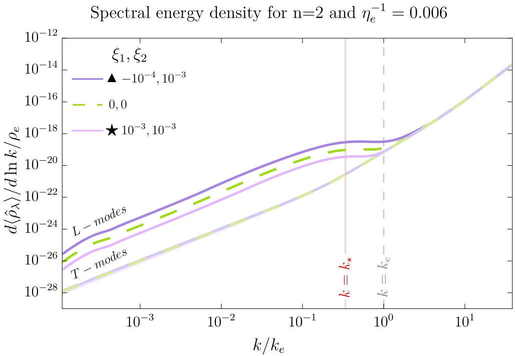
Spectral energy density for n=2, $\xi_1 = \xi_2 = 0$ and $\eta_e^{-1} = 0.006$



Divergent behavior for $k > k_e$ requires regularization.



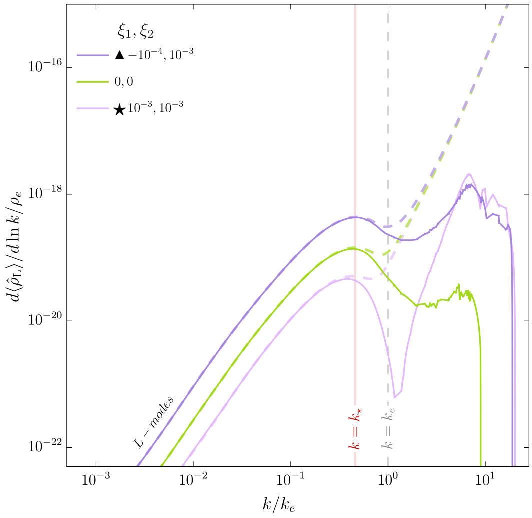
Qualitatively, the non-minimal spectral energy density resembles the minimal case.



Depending on the values of the non-minimal couplings spectral energy density of spin-1 field might exceed or fall behind the minimal case.

Regularization via normal ordering

Unregulated (dashed) and regulated (solid) spectral energy density



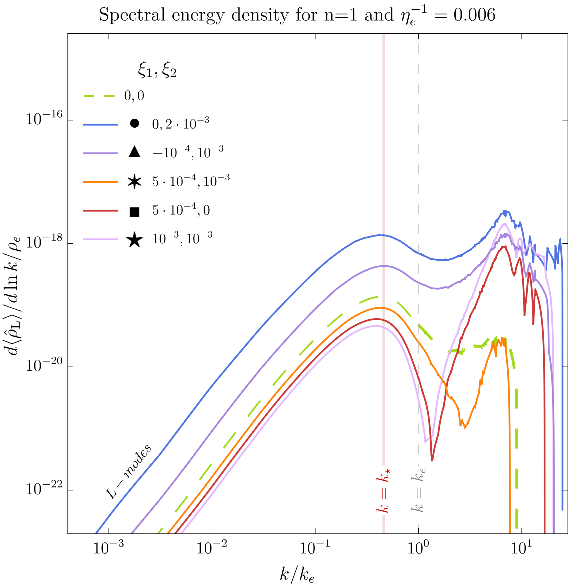
Applied at late times, in the adiabatic regime:

$$\langle\hat{\rho}_{\rm L}\rangle \approx \frac{1}{a^4}\int \frac{d^3k}{(2\pi)^3}\omega_{\rm L}|\beta_{\rm L}|^2,$$

$$\omega_{\rm L} \approx \left(k^2 + a^2 m_X^2\right)^{1/2},$$

$$|\beta_{\rm L}|^2 \approx \frac{1}{2\omega_{\rm L}}|\mathcal{X}_{\rm L}'|^2 + \frac{\omega_{\rm L}}{2}|\mathcal{X}_{\rm L}|^2 - \frac{1}{2}.$$

Emergence of the second UV peak!!

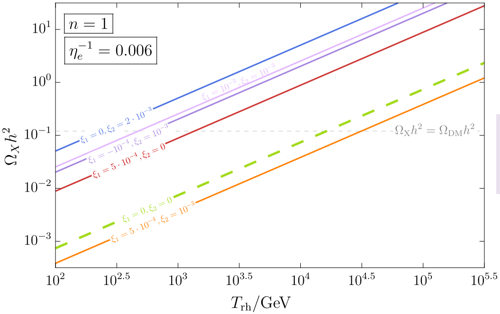


The strongest enhancement is observed for ξ_1, ξ_2 for which $m^2_{\rm eff,t} \approx 0 \approx m^2_{\rm eff,x}$.

The least significant enhancement is observed for ξ_1, ξ_2 for which $\xi_1 = \xi_2/2$.

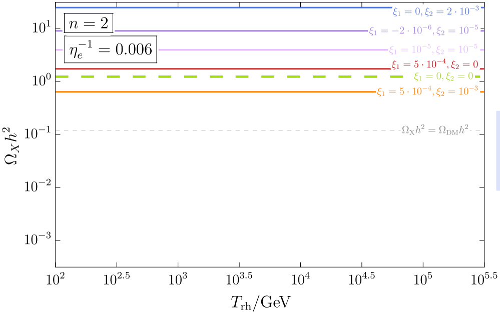
Relic abundance

Relic abundance



Quadratic reheating

$$\frac{\Omega_X h^2}{0.12} \simeq \frac{\mathcal{N}_X}{a_e^3 H_e^3} \frac{m_X}{H_e} \left(\frac{H_e}{10^{12}\,\mathrm{GeV}} \right)^2 \frac{T_{\rm rh}}{10^4\,\mathrm{GeV}}$$

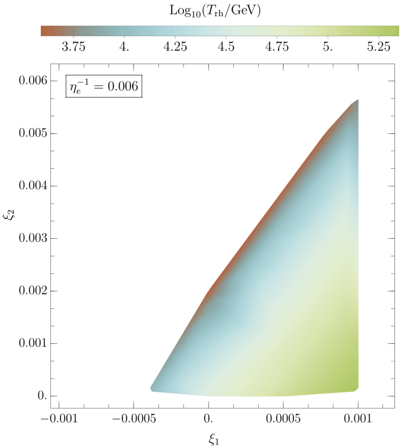


Quartic reheating

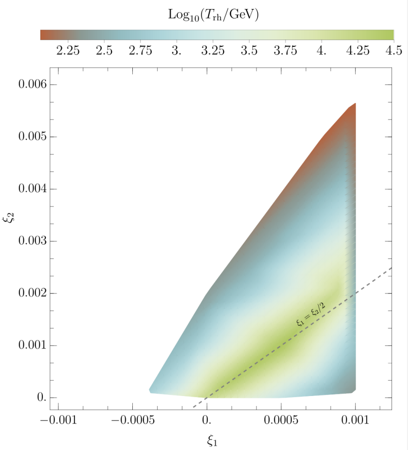
$$\frac{\Omega_X h^2}{0.12} \simeq \frac{1}{0.12} \frac{\mathcal{N}_X}{a_e^3 H_e^3} \frac{m_X}{H_e} \left(\frac{H_e}{10^{8}\,\mathrm{GeV}} \right)^{5/2}$$

Relic abundance

$$\Omega_X h^2 = \Omega_{DM} h^2$$



$\Lambda_{UV} = k_e$



$\Lambda_{UV} \sim 30 k_e$

Summary

Summary

- The inclusion of the non-minimal couplings leads to the emergence of two instabilities of the model: ghost instability and uncontrolled growth of short-wavelength modes.
- The viable parameter space of the model shrinks with $\eta_e^{-1} \equiv m_X^2/H_{\rm inf}$, and collapses to a single point as $m_X \to 0$.
- It has been established that the long-wavelength part of the spectrum has a peak structure, centered around the characteristic momentum scale $k_\star$.
- To cure the UV divergence of the energy density, regularization via normal ordering has been applied. This scheme reveals the existence of a second high-k peak, whose amplitude is sensitive to the values of ξ_1, ξ_2.
- We have demonstrated that accounting for the finite duration of reheating has a significant impact on the production of non-minimally coupled vectors.