# Nonthermal heavy dark matter from a first-order phase transition

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Ref. G. Giudice, HML, A. Pomarol, B. Shakya, 2403.03252 [hep-ph]

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# Outline

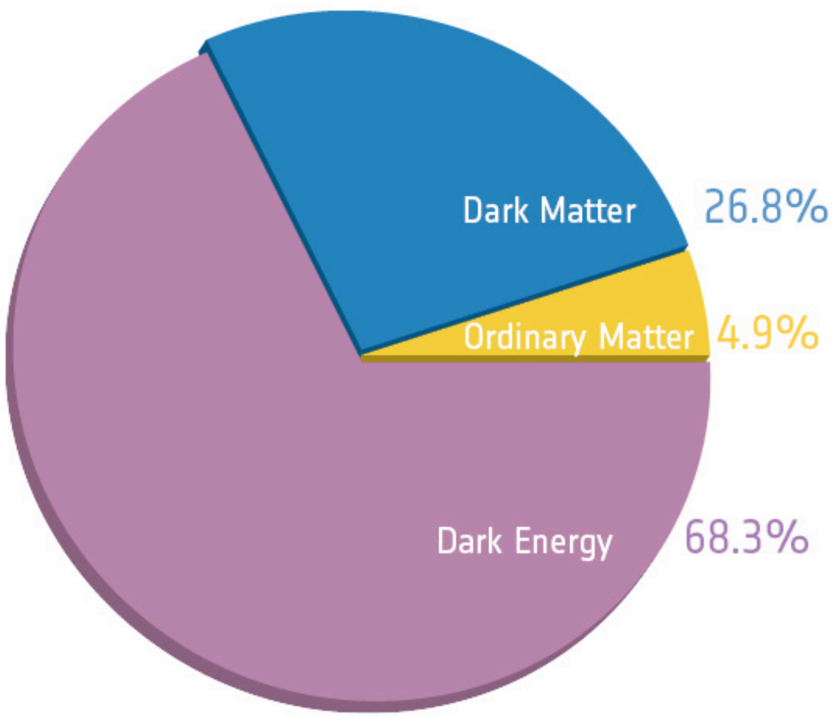
- Phase transition & bubble dynamics
- Dark matter from bubble collisions
- Conclusions

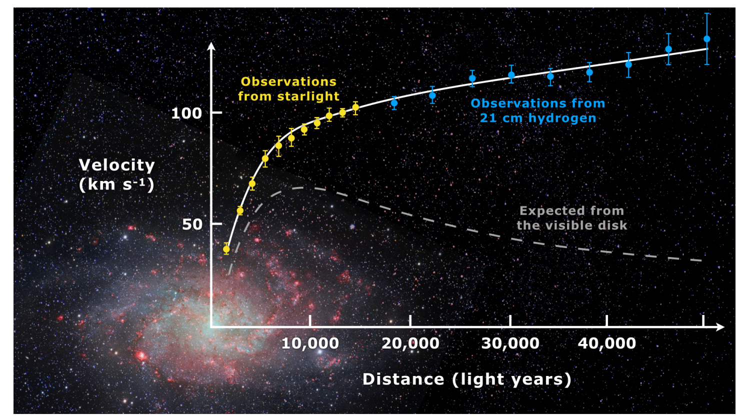
# Phase transitions and bubble dynamics

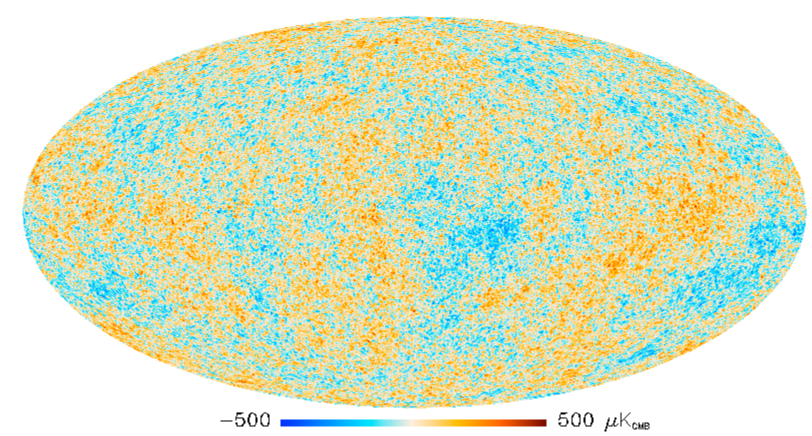
# Origin of dark matter

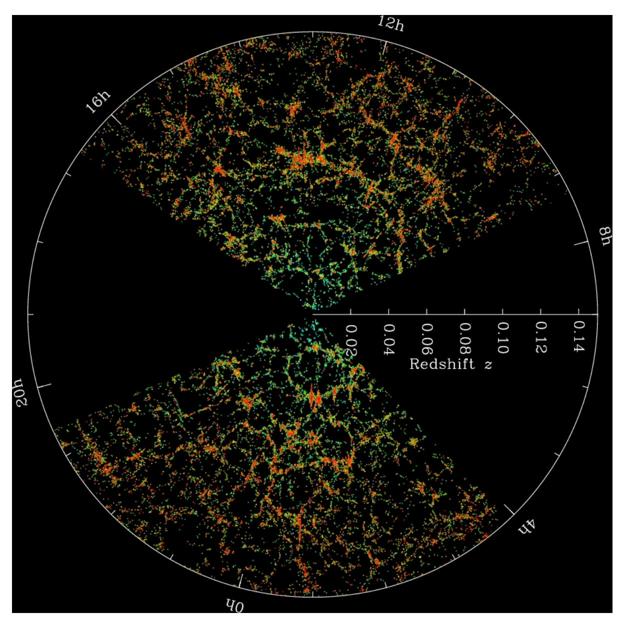


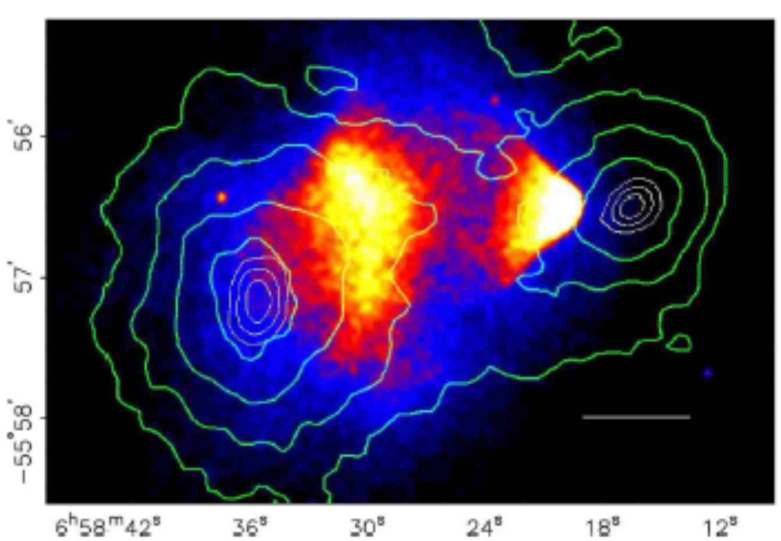
## We don’t have a clue to the nature of 27% dark matter and 68% dark energy.



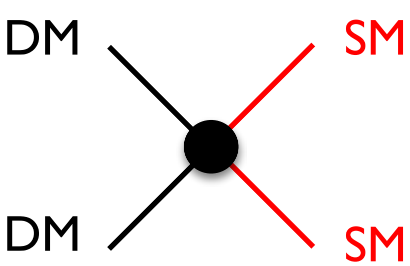




# Dark matter production 

**DM in thermal equilibrium**

- Freeze-out mechanism

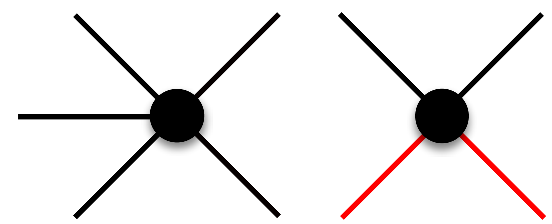
DM annihilation to thermal bath




$$\Omega_{\rm DM} \sim \frac{1\,{\rm pb}}{\langle \sigma v \rangle}$$

- DM self-interactions

DM self-annihilation;
small DM-SM interaction

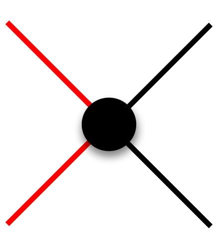


$$\Omega_{\rm DM} \sim \frac{1\,{\rm pb}}{(\alpha_{\rm self}\sigma_{\rm self}/(m_{\rm DM}M_P))^{1/2}}$$

**DM out of equilibrium**

- Freeze-in mechanism

Tiny DM-SM interaction



$$\langle \sigma v \rangle = \sigma_0 (T/m_{\rm DM})^n,\ T > m_{\rm DM}$$

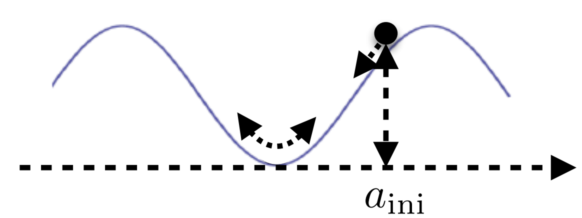
$$\Omega_{\rm DM} \propto \sigma_0 (T_{\rm RH})^{n+1},\ n > -1$$

$n < -1$ : $T_{\rm RH}$-independent

- Misalignment (boson DM)

DM initial potential density

Ultra light DM $\quad m_a \sim H \sim \frac{T_{\rm osc}^2}{M_P}$

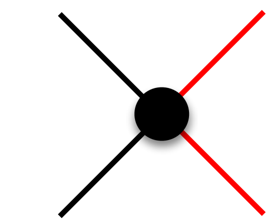



$$\Omega_{\rm DM} \sim \rho_a(a_{\rm ini})$$

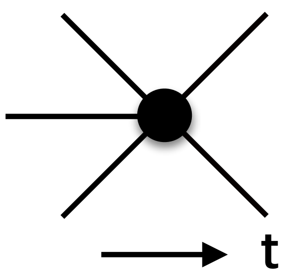
# Bounds on DM masses



- WIMP/SIMP: Unitarity bounds

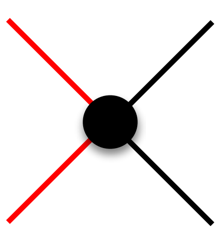


$$(\sigma v)_{\rm th} \simeq 3\times 10^{-26}\,{\rm cm}^3/s < \frac{16\pi}{m_{\rm DM}^2 v} \quad \longrightarrow \quad m_{\rm DM} \lesssim 100\,{\rm TeV}$$




$$\frac{\alpha_{\rm self}\sigma_{\rm self}}{m_{\rm DM}} \simeq 10^{-2}\,{\rm cm}^2/{\rm g} < \frac{16\pi\alpha_{\rm self}}{m_{\rm DM}^3 v^2} \quad \longrightarrow \quad m_{\rm DM} \lesssim 1\,{\rm GeV}$$

- FIMP: Temperature bound



$$\langle\sigma v\rangle \propto (\sigma v)\Big|_{T=m_{\rm DM}} \times e^{-m_{\rm DM}/T} \quad \longrightarrow \quad m_{\rm DM} \lesssim T_{\rm RH}$$

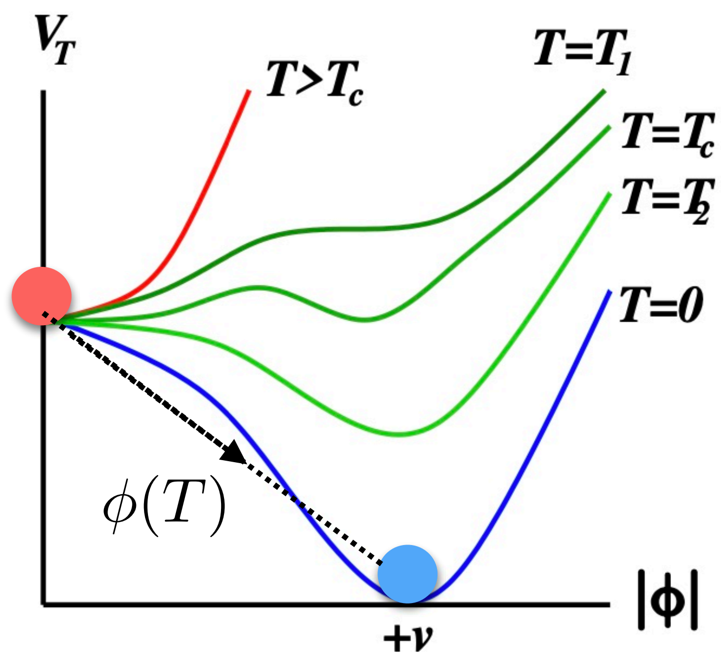
- Heavy dark matter beyond bounds on WIMP/SIMP or FIMP?

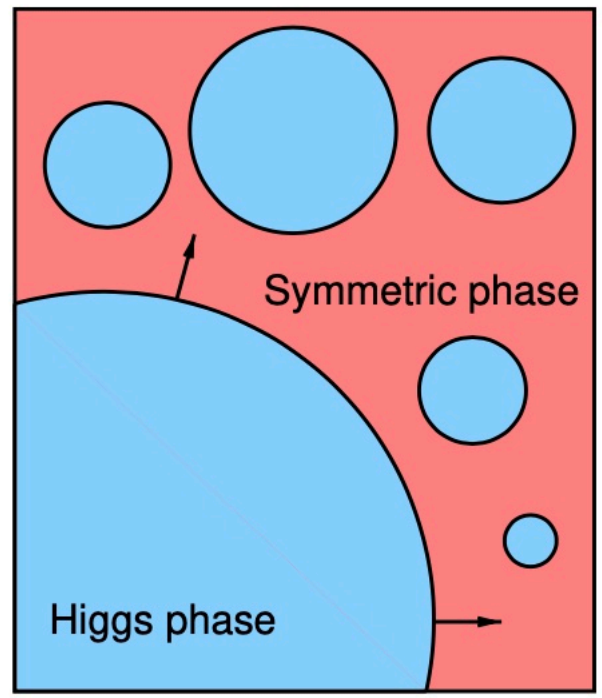
➡ New production mechanisms?

# Filtering dark matter



- DM can receive mass via 1st-order phase transitions.

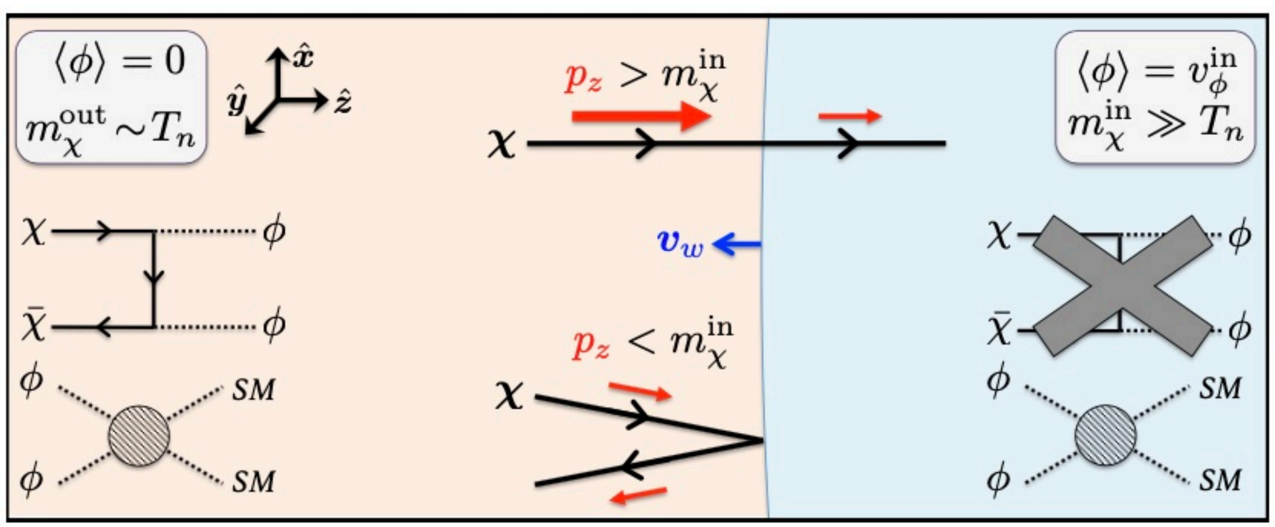



Inside bubbles: true vacuum

Outside bubbles: false vacuum

[M. Hindmarsh et al, 2020]




Outside bubbles Inside bubbles

[M. Baker et al ; D. Chway et al, 2019]

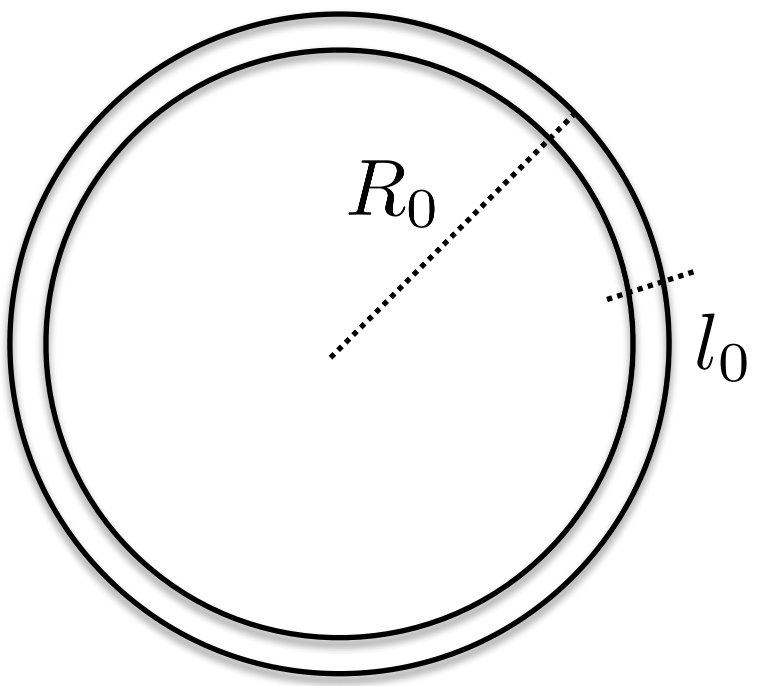
DM abundance is filtered for a non-relativistic bubble wall, $v_w<1$.

$$\Omega_{\rm DM}h^2 \sim 0.1\left(\frac{T_n}{1\,{\rm TeV}}\right)\left(\frac{x}{30}\right)^{5/2} e^{-(x-30)},\quad x = \frac{m_{\rm DM}}{T_n}$$

DM mass goes beyond temperature.

# Phase transitions and bubbles






$T \gg R_0^{-1}$ : O(3) symmetric bubble

Bounce action: $S_4 = \int d\tau d^3 x \mathcal{L}_E = \frac{S_3}{T}$,

$$S_3 = 4\pi \int_0^\infty s^2 ds \left[ \frac{1}{2} \left( \frac{d\phi}{ds} \right)^2 + V_T(\phi) \right], \; s = |\vec{x}|$$

Bubble profile: $\frac{d^2\phi}{ds^2} + \frac{2}{s}\frac{d\phi}{ds} - V_T'(\phi) = 0,$ w/ b.c. $\frac{d\phi}{dr} = 0, \;\text{at}\; r = 0,$

$\phi = 0, \;\text{at}\; r = \infty.$

Thin-wall approximation: $l_0 \ll R_0$

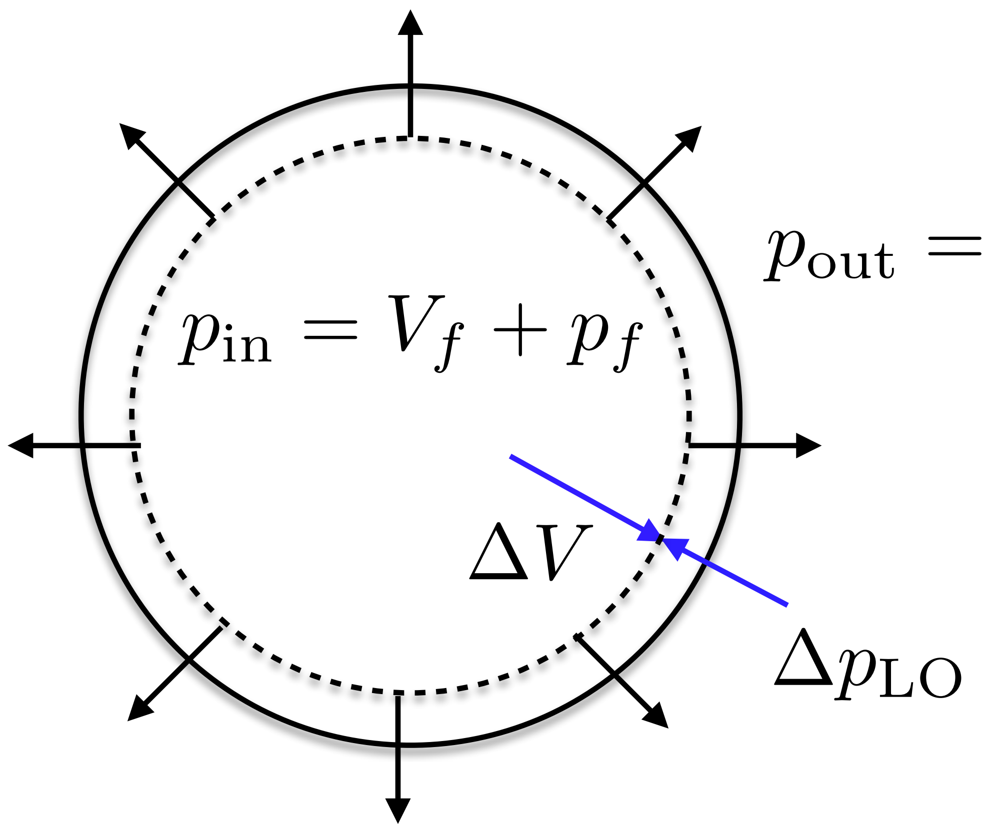
$$S_3 \simeq 4\pi\sigma R^2 - \frac{4\pi}{3}\Delta V\, R^3 = \frac{16\pi}{3}\frac{\sigma^3}{(\Delta V)^2} \longrightarrow R_0 = \frac{2\sigma}{\Delta V}$$ "critical radius"

Bubble nucleation rate & inverse duration time:

$$\Gamma_n \sim T_n^4 e^{-S_4(T_n)} \sim H(T_n), \qquad \frac{\beta}{H_*} = T\frac{dS_4}{dT}\bigg|_{T_*} - 4 \;\gtrsim 1$$

# Running bubbles






Lagrangian for bubbles:

$$\mathcal{L} = -2\pi\sigma R^2\sqrt{1-\dot{R}^2} + \frac{4\pi}{3}R^3 p,$$

$$p = \Delta V - \Delta p_{\rm LO} - \gamma\Delta p_{\rm NLO}$$

$\longrightarrow$ e.o.m: $\frac{d\gamma}{dR} + \frac{2\gamma}{R} = \frac{p}{\sigma}$

1) Difference in FT potential: $\Delta p_{\rm LO} = \Delta V_{\rm FT} = \frac{1}{24}(\Delta m^2)T^2$

maximum velocity: $\gamma_{\rm max} = \frac{2R_{\rm max}}{3R_0}$ , $R_{\rm max} \sim v_w \beta^{-1}$

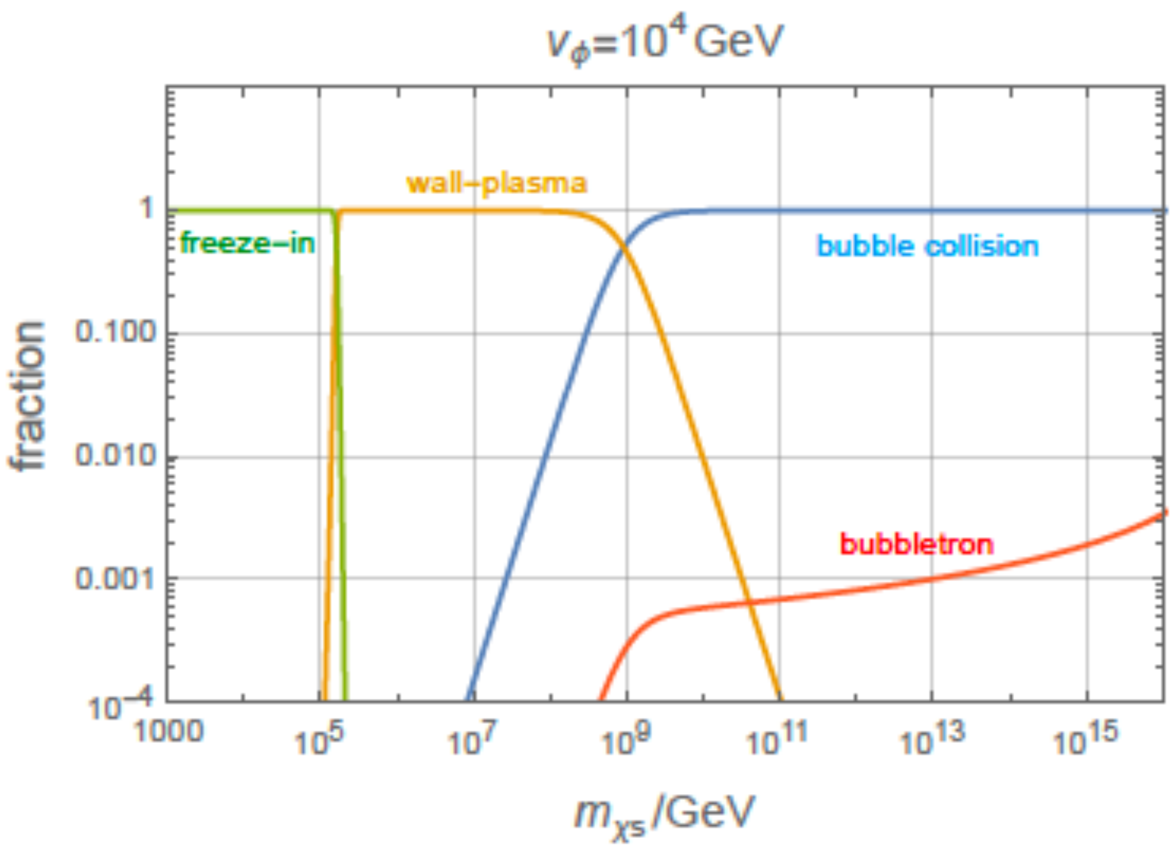
$\Delta V = c_V v_\phi^4,\ T \sim v_\phi$ $\longrightarrow$ $\gamma_{\rm max} \sim \frac{1}{(\beta/H_*)}\frac{M_P}{v_\phi} \gg 1$

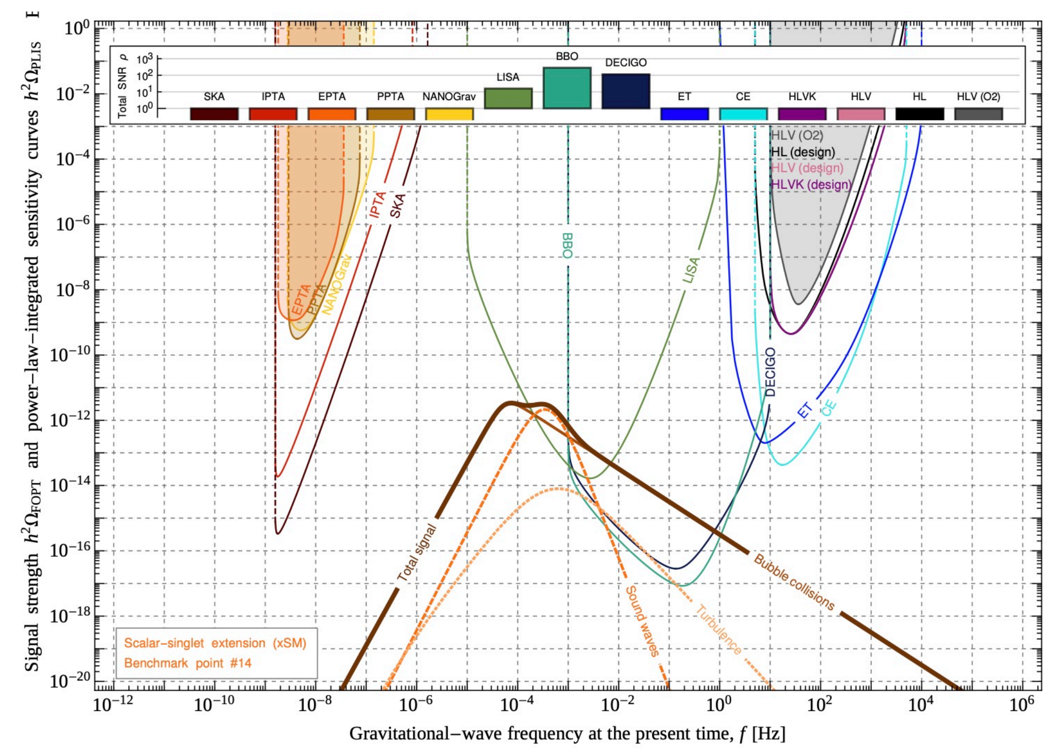
2) Pressure from radiation of light gauge boson: $\gamma\Delta p_{\rm NLO} = \gamma g^2 \Delta m_V\, T^3$

$p = 0$ $\longrightarrow$ $\gamma_{\rm eq} = \frac{\Delta V - \Delta p_{\rm LO}}{\Delta p_{\rm NLO}} \simeq \frac{\Delta V}{g^3 v_\phi T^3} \sim \frac{c_V}{g^3} \gg 1, \quad g \ll 1$

# Dark matter and GWs






[K. Schmitz, 2020]

DM density: $$\Omega_\chi h^2 \approx 0.1 \left(\frac{\beta/H}{10}\right)\left(\frac{\alpha}{(1+\alpha)g_* c_V}\right)^{1/4} \frac{m_\chi\, v_\phi}{(2.5\ \text{TeV})^2} \int_{p^2_{\min}}^{p^2_{\max}} \frac{dp^2}{p^4}\, \text{Im}[\tilde{\Gamma}^{(2)}(p^2)]$$

GW spectrum: $$\Omega_{\rm coll}(f)\, h^2 = 1.66 \times 10^{-5} \left(\frac{H_*}{\beta}\right)^2 \frac{\kappa^2_{\rm coll}\alpha^2}{(1+\alpha)^2} \left(\frac{100}{g_*(T_*)}\right)^{1/3} \frac{v_w^3}{1+2.4v_w^2} \cdot \frac{(f/f_{\rm coll})^{2.8}}{1+2.8(f/f_{\rm coll})^{3.8}}$$

| **Experiment** | $f_{optimal}$/**Hz** | $v_\phi$/**GeV** | $m_{\rm DM}$/GeV |
|---|---|---|---|
| Pulsar Timing Arrays (PTAs) [107] | $10^{-8}$ | 0.1 | $10^{13} - 10^{16}$ |
| LISA [108] | 0.001 | $10^4$ | $10^6 - 10^{15}$ |
| BBO [109], DECIGO [110] | 0.1 | $10^6$ | $10^5 - 10^{13}$ |
| Einstein Telescope (ET) [111], Cosmic Explorer (CE) [112] | 10 | $10^8$ | $10^6 - 10^{10}$ |

# Dark matter from bubble collisions

# Green function formalism



Probability for particle production: $\mathcal{P} = 2\,\mathrm{Im}\,(\,\Gamma[\phi]\,)$

$\Gamma[\phi]$, Effective action or generating function of 1PI Green functions

Quadratic in ϕ: $\Gamma[\phi] = \sum_{n=2}^{\infty} \frac{1}{n!} \int d^4x_1 ... d^4x_n \Gamma^{(n)}(x_1, ..., x_n) \phi(x_1) ... \phi(x_n)$

$$\longrightarrow \quad \mathcal{P} = \int \frac{d^4p}{(2\pi)^4} |\tilde{\phi}(p)|^2 \,\mathrm{Im}\Big(\widetilde{\Gamma}^{(2)}(p^2)\Big), \qquad \tilde{\phi}(p) = \int d^4x\, \phi(x)\, e^{ip\cdot x}, \text{ etc.}$$

Bubble profile in momentum space: $\tilde{\phi}(p) = (2\pi)^2 \delta(p_x) \delta(p_y) \tilde{\phi}(p_z, \omega)$

Number density of particles per area: [R. Watkins, L.M. Widrow, 1992]

$$\frac{N}{A} = 2 \int \frac{dp_z\, d\omega}{(2\pi)^2} \underbrace{|\tilde{\phi}(p_z, \omega)|^2}_{\text{Background dynamics}} \underbrace{\mathrm{Im}[\tilde{\Gamma}^{(2)}(\omega^2 - p_z^2)]}_{\text{Particle physics info.}}$$

Background dynamics

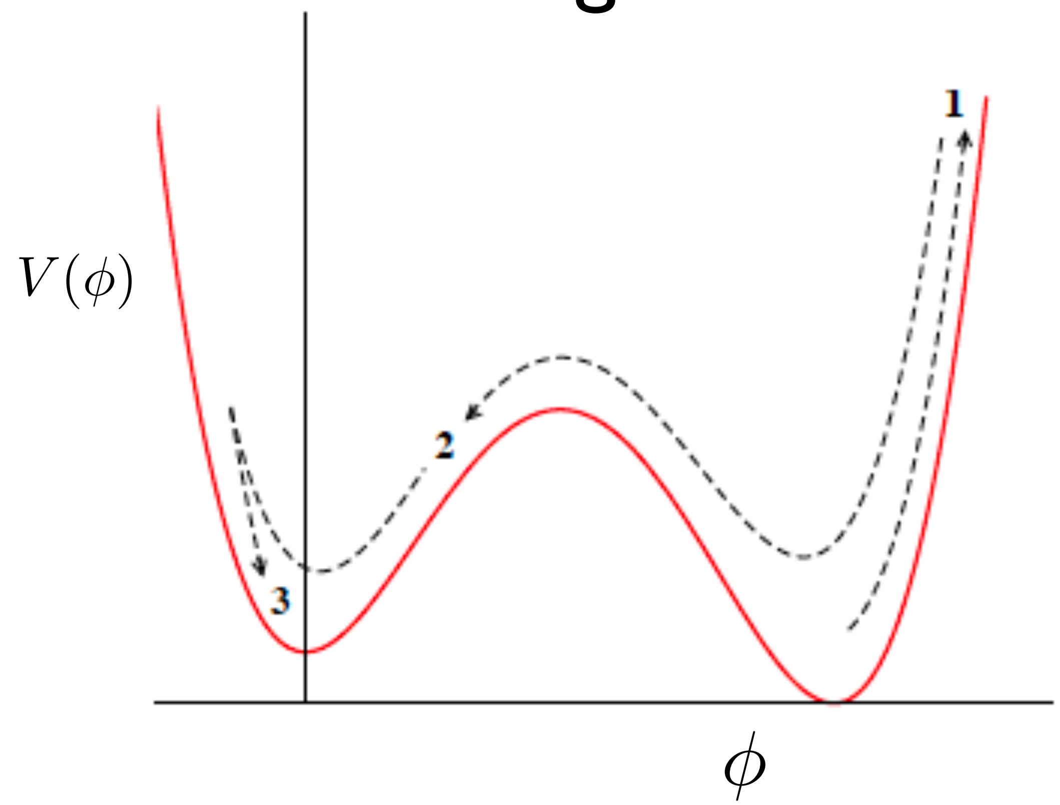
Particle physics info.

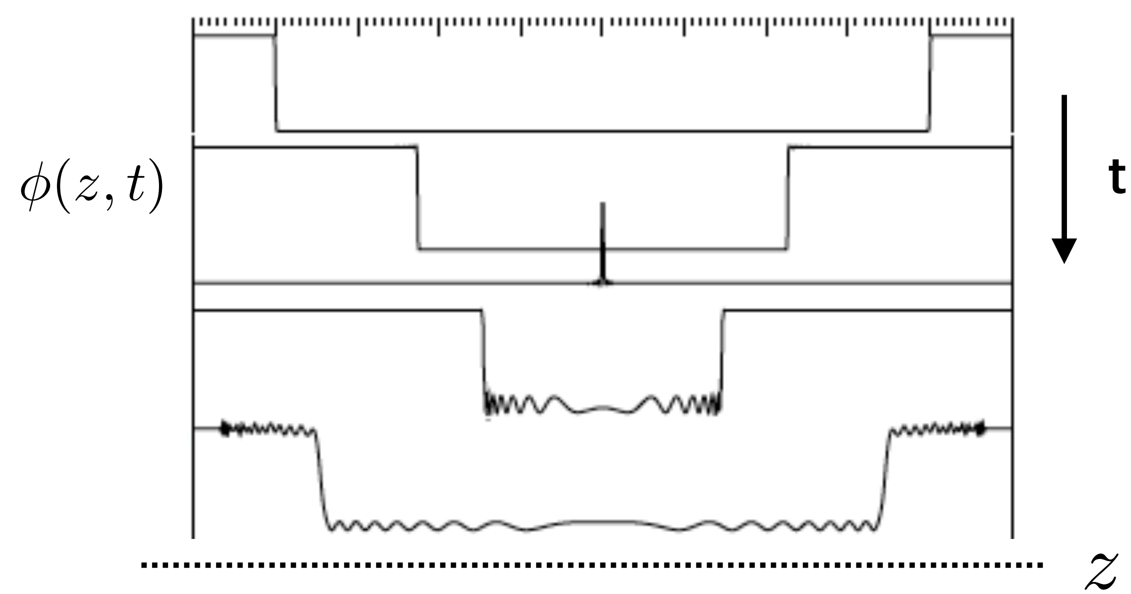
# Bubble collisions: elastic



Almost degenerate minima: elastic bubble collision

=> scalar waves, particle production




[A. Falkowski, J.M. No, 2012]

Thin wall: $$\phi(z,t) = \begin{cases} 0 \quad , & z^2 < v_w^2 t^2, \\ v_\phi \quad , & z^2 > v_w^2 t^2. \end{cases} \longrightarrow \tilde{\phi}(p_z,\omega) = \frac{4 v_w v_\phi}{\omega^2 - v_w^2 p_z^2}$$

$$v_w \simeq 1 \longrightarrow \tilde{\phi}(p_z,\omega) \simeq \frac{4 v_\phi}{p^2}$$ , IR dominant

Thick wall: $$\phi(z,t) = \frac{v_\phi}{2}\left(2 + \tanh\left(\frac{z-|t|}{l_w}\right) - \tanh\left(\frac{z+|t|}{l_w}\right)\right), \quad \underline{l_w = \frac{l_0}{\gamma_w}}.$$

$$\longrightarrow \tilde{\phi}(p,\omega) \simeq \frac{2\pi l_w v_\phi \omega}{\sinh(\frac{1}{2}\pi l_w \omega)} \frac{1}{p^2} \propto e^{-\pi l_w \omega/2}, \qquad \omega \gg 1/l_w.$$

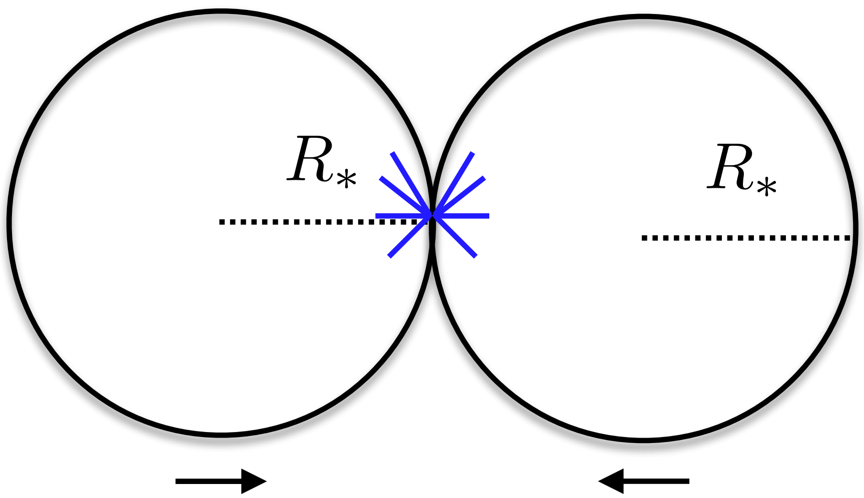
# Bubble collisions: elastic



Produced particles on surface diffuse over the bubble volume.

$$n = \frac{N}{V} = \frac{3}{4\pi^2 R_*} \int_{p^2_{\rm min}}^{p^2_{\rm max}} dp^2\, f(p^2) \mathrm{Im}\left[\widetilde{\Gamma}^{(2)}(p^2)\right],$$

$$p^2_{\rm max} = 2/l_w,\ p^2_{\rm min} = \min(2m, (2R_*)^{-1})$$




"Efficiency" factor for particle production

$$f(p^2) = \frac{1}{4} \int_{p^2}^{\infty} \frac{d\Psi}{\sqrt{\Psi^2 - p^4}} |\tilde{\phi}(p_z, \omega)|^2, \quad p^2 = \omega^2 - p_z^2$$

$$\tilde{\phi}(p, \omega) \simeq \frac{2\pi l_w v_\phi \omega}{\sinh(\frac{1}{2}\pi l_w \omega)} \frac{1}{p^2} \propto e^{-\pi l_w \omega/2}, \qquad \omega \gg 1/l_w.$$

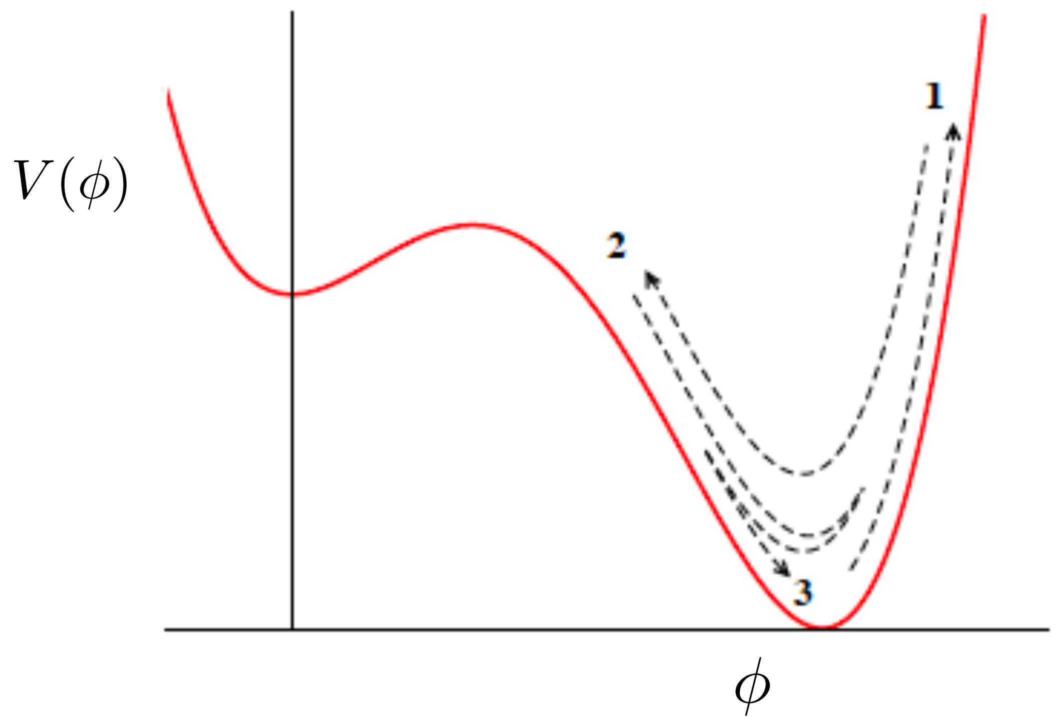
$m \sim 1/l_w \gg 1/l_0 \sim v_\phi \sim T$ => production of heavy dark matter.

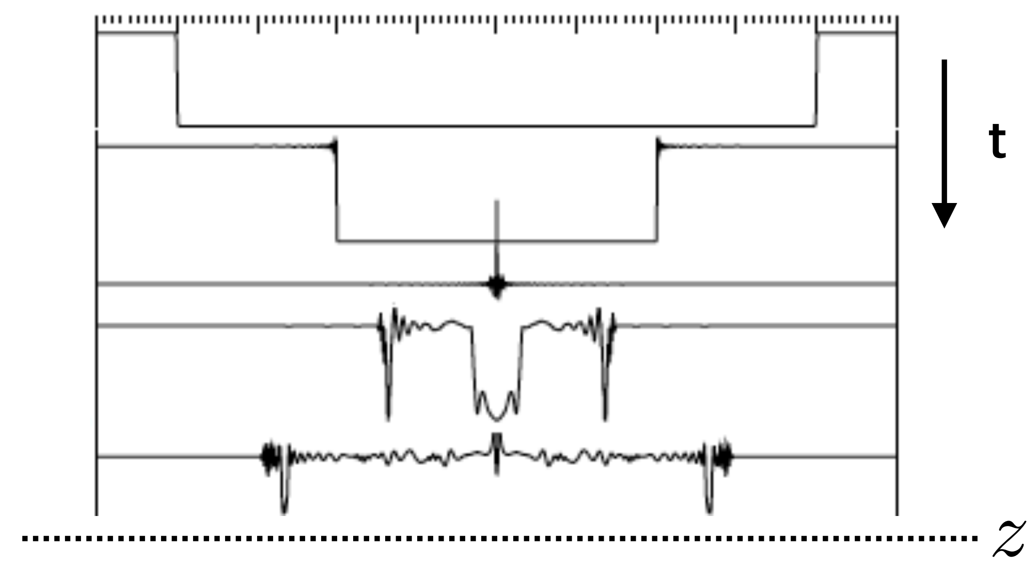
# Bubble collisions: inelastic



Non-degenerate minima: inelastic bubble collision

=> scalar waves, mainly




[A. Falkowski, J.M. No, 2012]

Solve for bubble profile: $(\partial_t^2 - \partial_z^2)\phi = -\dfrac{\partial V}{\partial \phi}\,,\quad t>0:\; V(\phi) \simeq \dfrac{1}{2} m_\phi^2 (\phi - v_\phi)^2,$

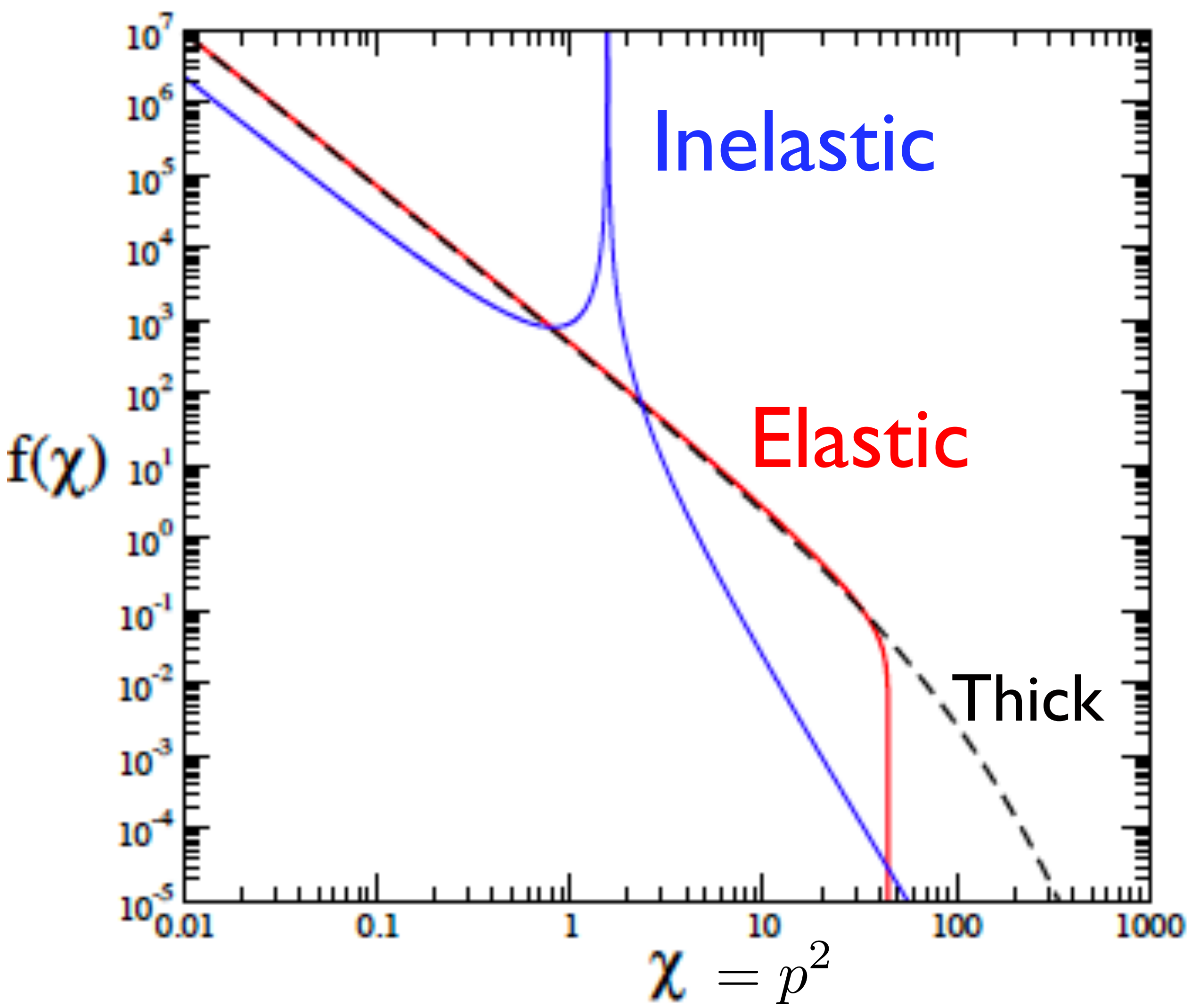
$$\longrightarrow \quad \phi(z,t) = v_\phi + v_\phi l_w \int_0^\infty dp_z \, \frac{p_z}{\sqrt{p_z^2 + m_\phi^2}} \frac{\cos(p_z z)}{\sinh(\frac{1}{2}\pi l_w p_z)} \sin(\sqrt{p_z^2 + m_\phi^2}\, t), \; t > 0.$$

$$\longrightarrow \quad \tilde{\phi}(p_z, \omega) = \frac{\pi l_w v_\phi p_z}{\sinh(\frac{1}{2}\pi l_w p_z)} \left( \frac{1}{\omega^2 - p_z^2} - \frac{1}{\omega^2 - p_z^2 - m_h^2} \right),$$

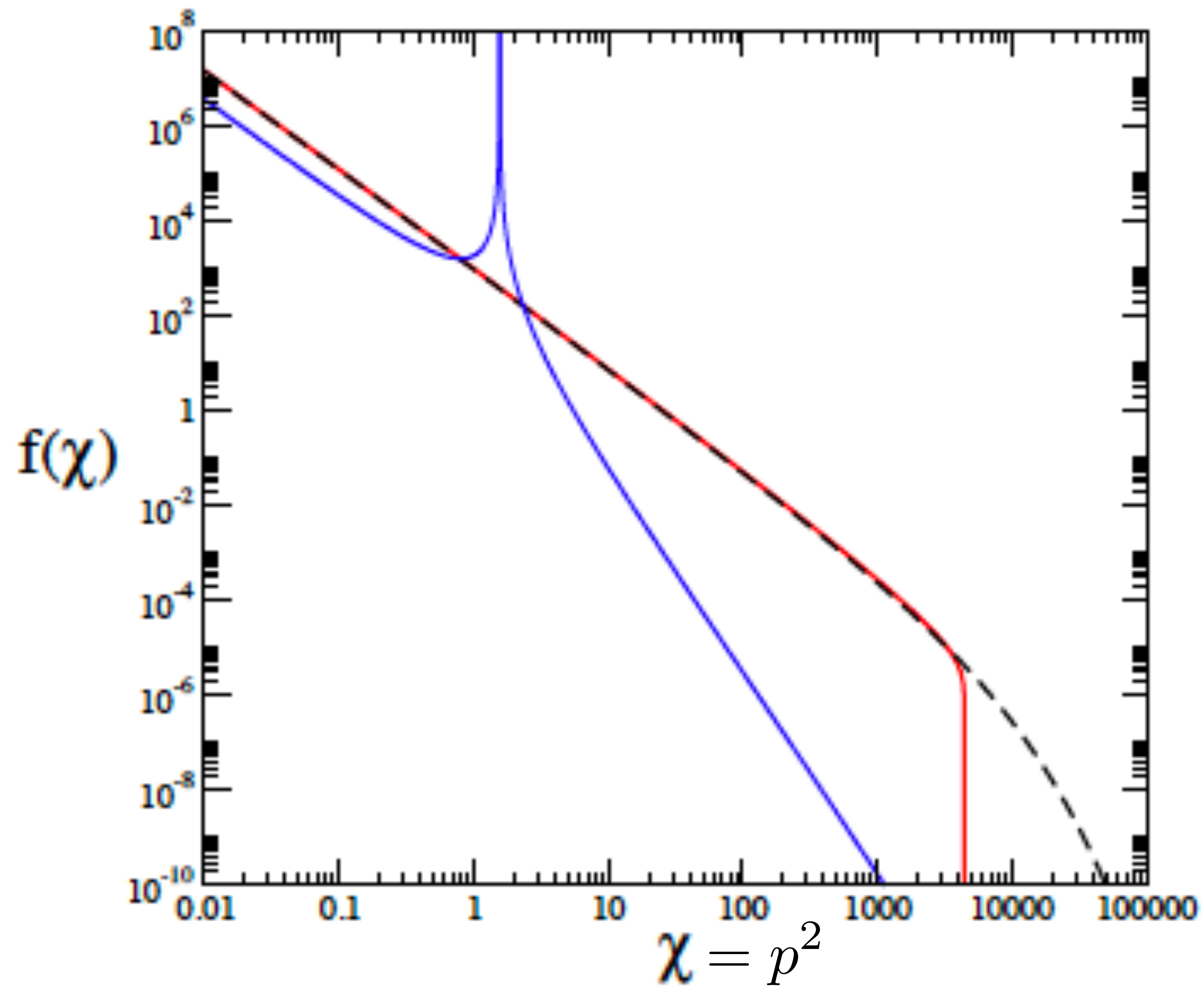
$$m_{\rm DM} \gg m_h : \qquad f(p^2) \simeq f(p^2)\Big|_{\rm elastic}$$

# Elastic vs inelastic collisions






$\gamma_w = 10^2$




$\gamma_w = 10^3$

[A. Falkowski, J.M. No, 2012]

# 2-body vs 3-body



$\widetilde{\Gamma}^{(2)}(p^2)$ : 2-point 1PI Green function

Optical theorem ⟶ $\mathrm{Im}[\tilde{\Gamma}^{(2)}(p^2)] = \frac{1}{2}\sum_k \int d\Pi_k |\bar{\mathcal{M}}(\phi_p^* \to \underline{k})|^2$

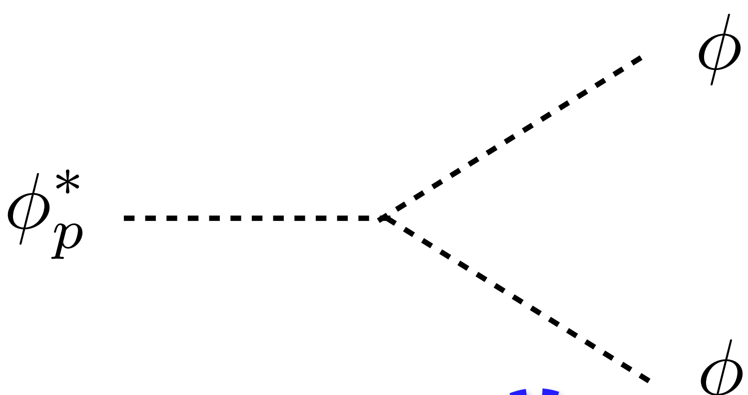
Decay of off-shell scalar

Scalar self-interaction:

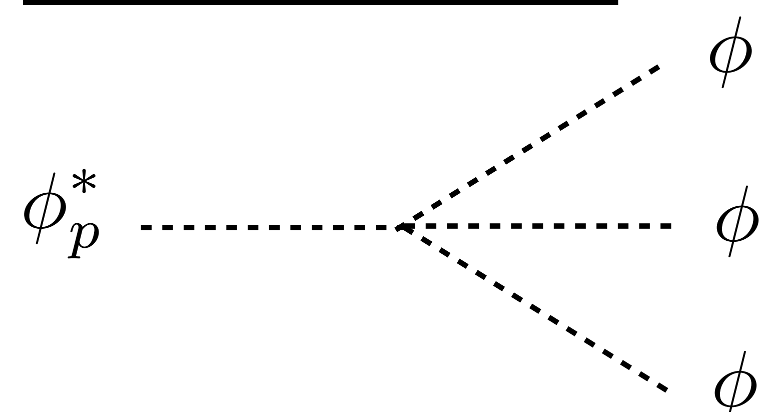
$$\frac{\lambda_\phi}{4!}\phi^4 \longrightarrow \frac{\lambda_\phi}{4!}(v_\phi + \bar{\phi})^4 \to \frac{\lambda_\phi}{3!} v_\phi \bar{\phi}^3$$

**2-body decay**

**3-body decay** [G. Giudice et al, 2024]




$\sim \lambda_\phi v_\phi$




$\sim \lambda_\phi$

$$\mathrm{Im}[\tilde{\Gamma}^{(2)}(p^2)]_{\phi_p^* \to \phi\phi} = \frac{\lambda_\phi^2 v_\phi^2}{8\pi}(1 - 4m_\phi^2/p^2)\,\Theta(p - 2m_\phi)$$

$$\mathrm{Im}[\tilde{\Gamma}^{(2)}(p^2)]_{\phi_p^* \to 3\phi} = \frac{\lambda_\phi^2 p^2}{3072\,\pi^3}(1 - 9m_\phi^2/p^2)\,\Theta(p - 3m_\phi)$$

$p^2 \gg v_\phi^2$ : 3-body process becomes more important!

# Scalar vs fermion DM



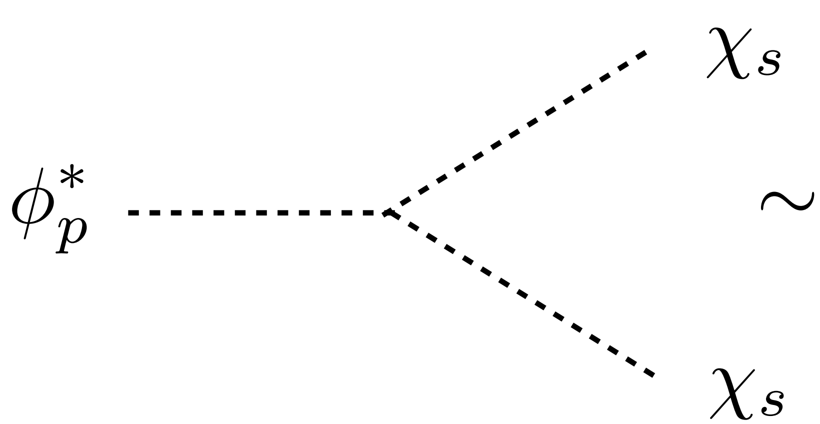
Applications to scalar DM and fermion DM

**Scalar DM** $\chi_s$

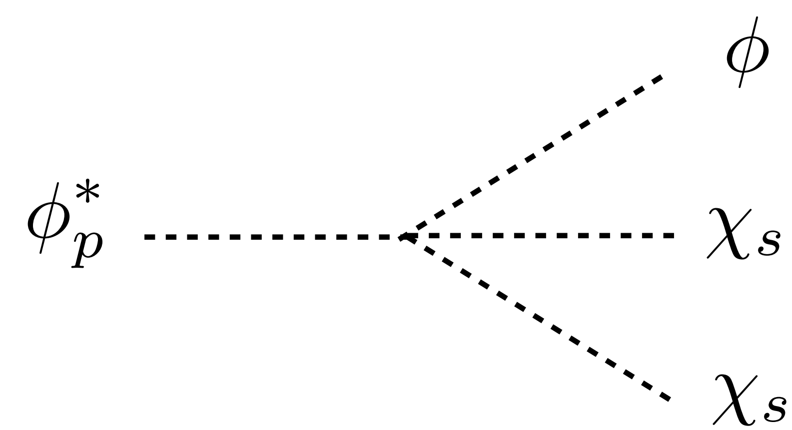
3-body process is similarly important.

[G. Giudice et al, 2024]

$$\frac{\lambda_s}{4}\phi^2\chi_s^2 \longrightarrow \frac{\lambda_s}{4}(v_\phi+\bar{\phi})^2\chi_s^2 \to \frac{\lambda_s}{2}v_\phi\bar{\phi}\chi_s^2$$




$\sim \lambda_s v_\phi$




$\sim \lambda_s$

**Fermion DM** $\chi_f$

2-body process is dominant.

$y_f\phi\bar{\chi}_f\chi_f\,,\quad \Delta m_{\chi_f}=y_f v_\phi \lesssim m_{\chi_f}$ DM mass not related to SSB.

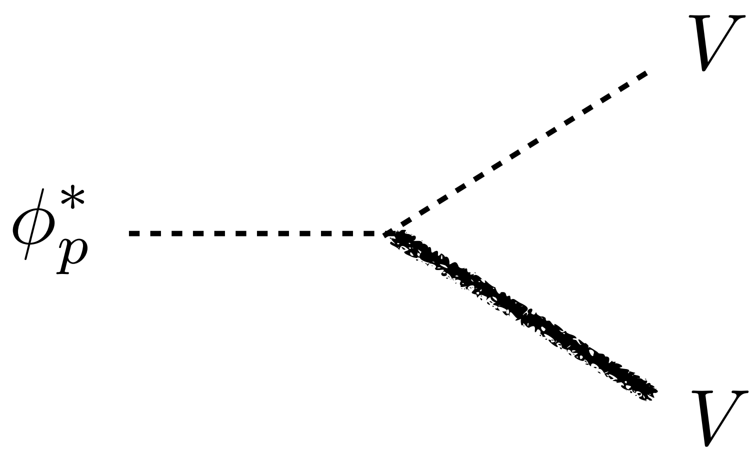
$$\mathrm{Im}[\tilde{\Gamma}^{(2)}(p^2)]_{\phi_p^*\to\chi_f\bar{\chi}_f} = \frac{y_f^2}{8\pi}p^2(1-4m_{\chi_f}^2/p^2)^{3/2}\,\Theta(p^2-4m_{\chi_f}^2)$$

# Vector DM production

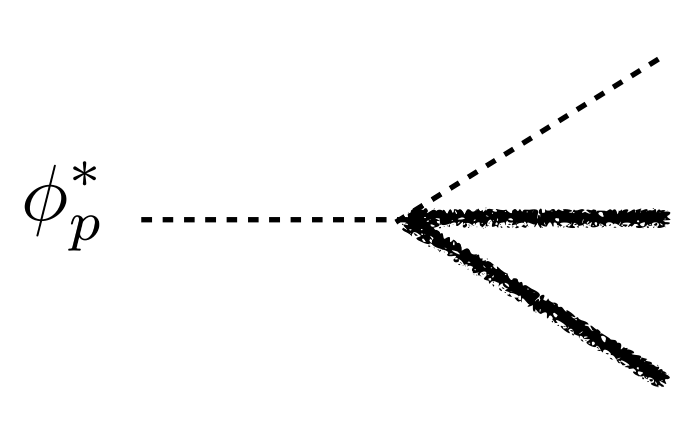


**Vector DM** $g^2\phi^2 V_\mu V^\mu \longrightarrow g^2(v_\phi + \bar{\phi})^2 V_\mu V^\mu$

e.g. in unitary gauge



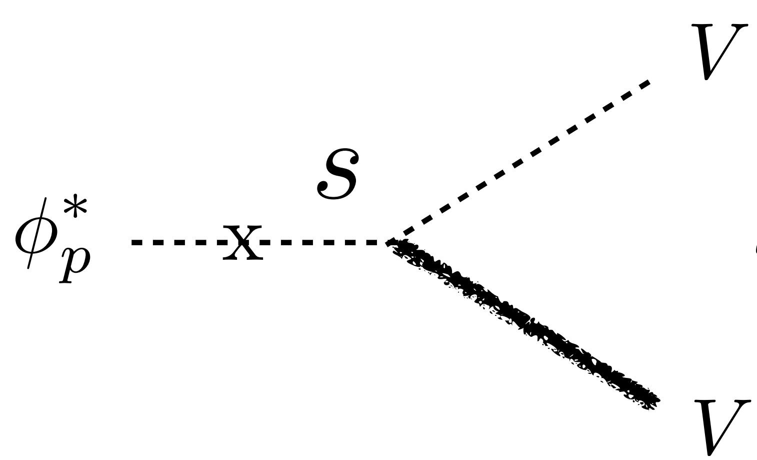

$\sim g^2 v_\phi$



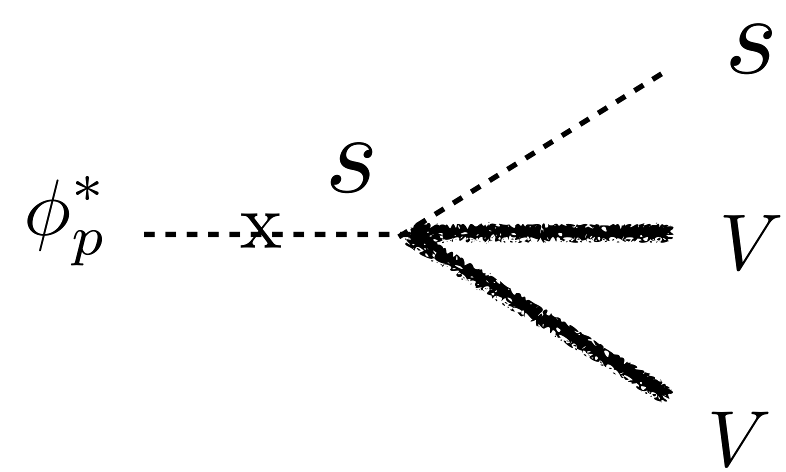

$\sim g^2$

$m_V > g v_\phi \longrightarrow$ Another mechanism for vector DM mass

e.g. mixing with extra Higgs $s$ induces effective interactions.




$\sim g_s^2 v_s \theta$




$\sim g_s^2 \theta$

# Gauge dependence



Production of on-shell gauge bosons:

Polarization sum, $\sum \epsilon^\mu \epsilon^\nu \to -g^{\mu\nu} + (1-\xi)\frac{p^\mu p^\nu}{p^2 - \xi m_V^2}$

+ ghosts, Goldstone fields with $m^2 = \xi m_V^2$

1) unitary gauge: $\xi \to \infty,$ $\sum \epsilon^\mu \epsilon^\nu \to -g^{\mu\nu} + \frac{p^\mu p^\nu}{m_V^2}$

$$|\bar{\mathcal{M}}(\phi_p^* \to VV)|^2 = g^2 m_V^2 \left(3 - \frac{p^2}{m_V^2} + \frac{p^4}{4m_V^4}\right)$$

[A. Falkowski, J.M. No, 2012]

$$p^2 \gg m_\phi^2, m_V^2, \quad |\bar{\mathcal{M}}(\phi_p^* \to VV)|^2 \sim \frac{g^2}{4m_V^2}\, p^4$$

2) Feynman-'t Hooft gauge: $\xi = 1,$ $\sum \epsilon^\mu \epsilon^\nu \to -g^{\mu\nu}$

+ ghosts, Goldstone fields with $m_c^2 = m_G^2 = m_V^2$

$$|\bar{\mathcal{M}}(\phi_p^* \to VV)|^2 = g^2 m_V^2 \left(3 - \frac{p^2}{m_V^2} + \frac{\lambda_\phi^2}{g^4}\right)$$ ⟶ still negative $p^2$ ?

# $R_\xi$ gauge



3) $R_\xi$ gauge: $m_c^2 = m_G^2 = \xi m_V^2 \neq m_V^2$

$$|\bar{\mathcal{M}}(\phi_p^* \to VV)|^2 = \underline{(g m_V)^2 \left(2 + \frac{1}{4m_V^4}(p^2 - 2m_V^2)^2\right)\sqrt{1 - \frac{4m_V^2}{p^2}}}$$

"gauge-independent"

$$+ \underline{\frac{g^2}{4m_V^2}(-p^2 + m_\phi^2)(p^2 + m_\phi^2)\sqrt{1 - \frac{4\xi m_V^2}{p^2}}}$$

[G. Giudice et al, 2024; J. Papavasilliou, A. Pilfatsis et al, 1997 ]

"gauge-dependent": vanishing for on-shell $\phi$

$$|\bar{\mathcal{M}}(\phi_p^* \to VV)|^2 = g^2 m_V^2 \times \begin{cases} \frac{(\xi-3)p^2}{2m_V^2} + \frac{\lambda_\phi^2}{g^4} + 3 & \text{for } \frac{p^2}{m_V^2} \gg \xi, 1 \\ \frac{p^4}{4m_V^4} - \frac{p^2}{m_V^2} + 3 & \text{for } \xi \gg \frac{p^2}{m_V^2}, 1 \end{cases}$$

Fried-Yennie gauge: $\xi = 3$ ⟶ no $p^2$ or $p^4$ terms!

[G. Giudice et al, 2024]

# Goldstone equivalence



**Goldstone Equivalence Theorem:** $\epsilon_L^\mu \frac{p_\mu}{m_V} = G$ (Goldstone)

Physical processes with longitudinal modes at high energy:

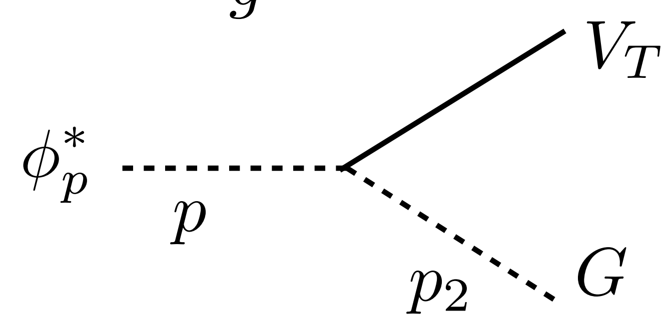
$$\bar{\mathcal{M}}(\phi; V_L) = \bar{\mathcal{M}}(\phi; G) + \mathcal{O}\Big(\frac{m_V^2}{p^2}\Big)$$

[B. W. Lee et al, 1977]

(i) TT: $|\bar{\mathcal{M}}|^2 \sim 2m_V^2,$

(ii) LL: $\phi_p^* \to GG, \qquad |\bar{\mathcal{M}}|^2 \sim \lambda_\phi^2 v_\phi^2 = \frac{\lambda_\phi^2}{g^2}\, m_V^2$

(iii) TL: $\phi_p^* \to V_T(p_1) G(p_2),$




$\sim ig(p^\mu + p_2^\mu)\epsilon_{\mu,T}$

In rest frame for $\phi_p^*, \quad p = (E, 0, 0, 0)$

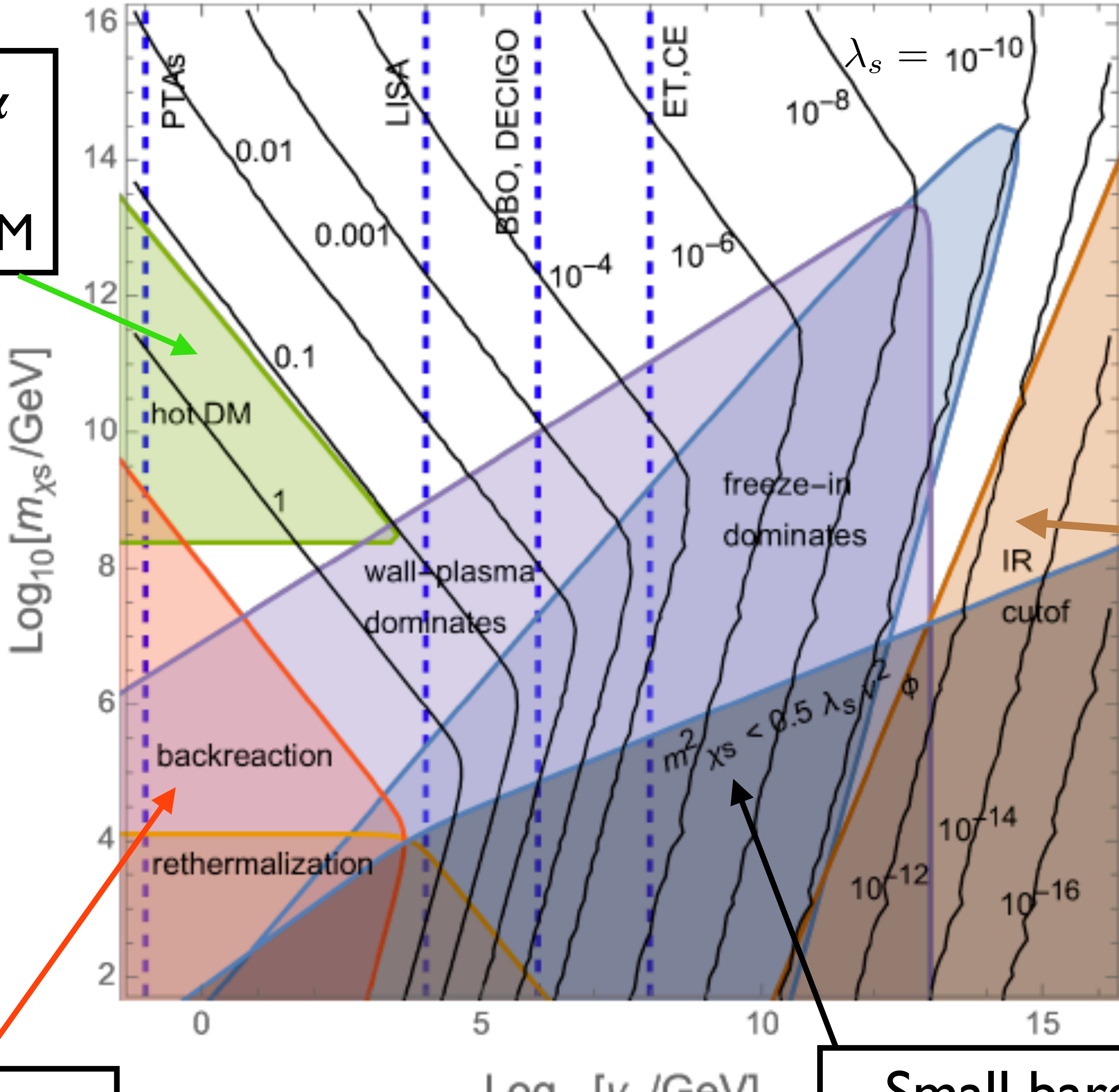
$V_T, G$ back-to-back $\longrightarrow$ $|\bar{\mathcal{M}}|^2 \sim 0$

$$|\bar{\mathcal{M}}|^2 \sim m_V^2 \Big(2g^2 + \frac{\lambda_\phi^2}{g^2}\Big)\Big(1 + \mathcal{O}\Big(\frac{m_V^2}{p^2}\Big)\Big), \quad p^2 \gg m_V^2$$

: finite at large p²

# Scalar dark matter

Lyman-$\alpha$ bounds on hot DM




Portal for scalar DM:

$$\mathcal{L}_{\chi_s} = \frac{\lambda_s}{4}\phi^2\chi_s^2$$

$m_{\chi_s} < R_*^{-1}$

Multiple bubbles in the Fourier analysis needed.

$\rho_{\chi_f} > \Delta V$

Small bare DM mass: higher order corrections

# Conclusions



- Bubble dynamics in1st-order phase transitions is an interesting testing ground for dark matter and GWs.

- Non-relativistic bubbles: DM receives mass from SSB.

  → "Filtering mechanism" DM with masses parametrically larger than T.

- Relativistic bubbles: DM mass is not related to SSB.

  → "Bubble collisions" produce DM with masses much larger that T & $v_\phi$ .

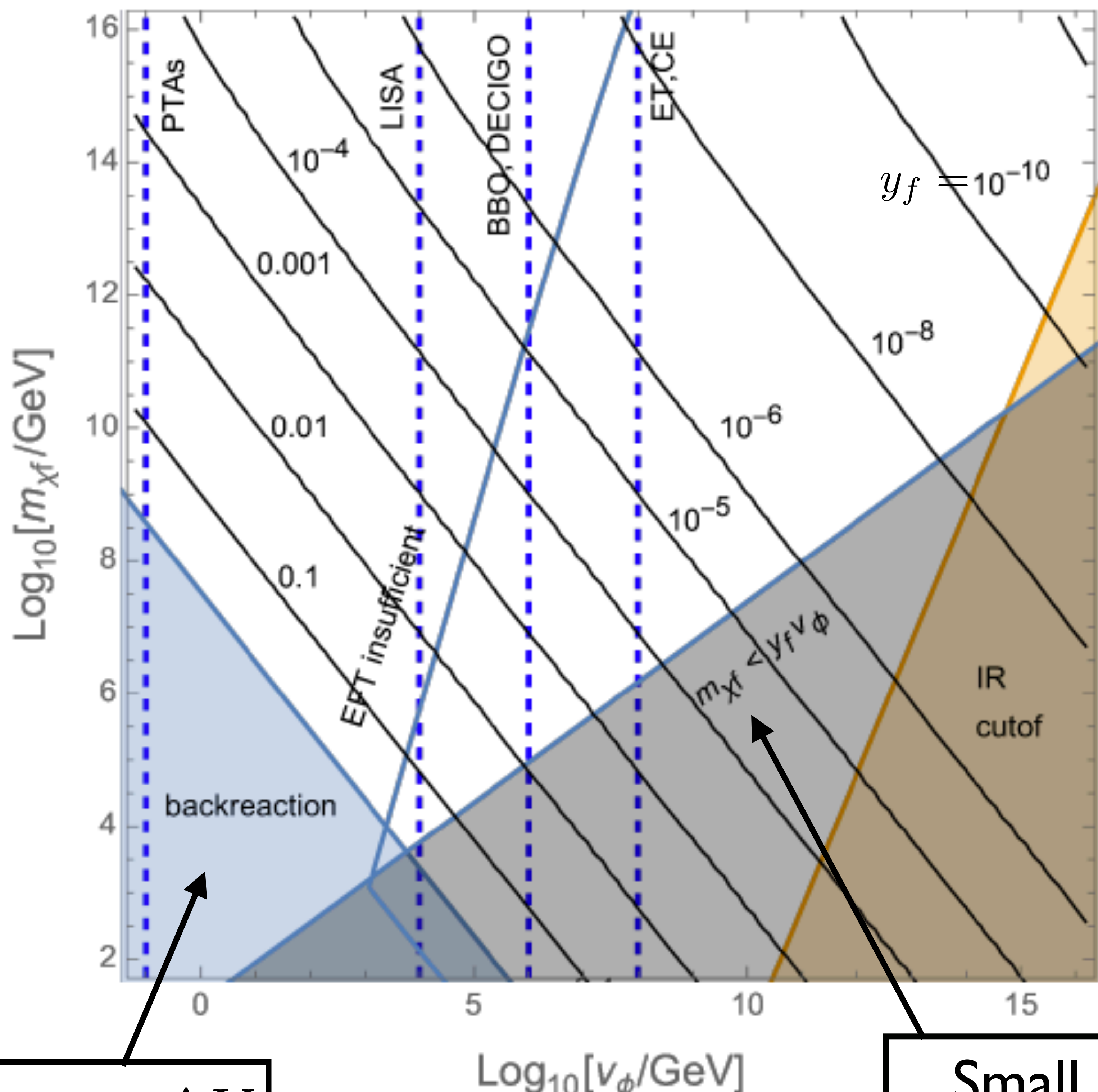
$$m_{\rm DM} \lesssim \frac{1}{l_w} \sim \gamma_w v_\phi \sim \gamma_w T, \quad \gamma_w \gg 1$$

- Practical solution provided for gauge-dependence issue for vector dark matter through GET.

# Backup slides

# Fermion dark matter






EFT for fermion DM:

$$\mathcal{L}_{\chi_f} = \frac{1}{\Lambda_f}\phi^2 \bar{\chi}_f \chi_f$$

$$\longrightarrow \mathcal{L}_{\rm eff} = y_f \phi\, \bar{\chi}_f \chi_f\,,$$

$$y_f = \frac{v_\phi}{\Lambda_f},\ \ \Lambda_f < m_{\chi_f}, v_\phi$$

Hot DM, rethermalization, freeze-in bounds are not shown: UV-dependent.

$\rho_{\chi_f} > \Delta V$

Small bare DM mass: higher order corrections

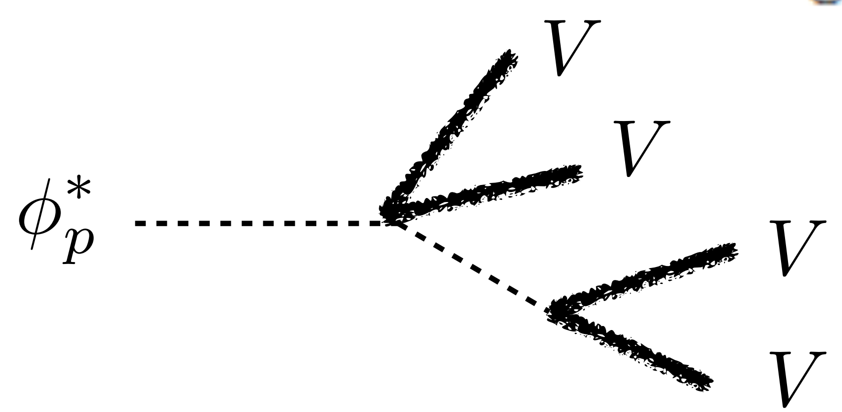
# Problem in unitary gauge

-2-

$p^2 \gg m_\phi^2, m_V^2, \quad |\bar{\mathcal{M}}(\phi_p^* \to VV)|^2 \sim \frac{g^2}{4m_V^2}\, p^4$ in unitary gauge

$\phi_p^* \to 2V$ : $\quad \mathrm{Im}[\tilde{\Gamma}^{(2)}(p^2)]_{\phi_p^* \to 2V} \sim \frac{g^2 m_V^2}{8\pi}\left(\frac{p^4}{4m_V^4}\right)$

$\phi_p^* \to 4V$ :




$$\mathrm{Im}[\tilde{\Gamma}^{(2)}(p^2)]_{\phi_p^* \to 4V} \sim \frac{g^6 m_V^2}{8\pi(4\pi^2)^2}\left(\frac{p^4}{4m_V^4}\right)^2$$

4-body process "grows faster" with large momentum.

Processes with higher vector boson multiplicity grows even faster with large momentum.

→ **Breakdown of perturbative expansion!**

# Other processes



3-body process for vector DM: $\phi_p^* \to \phi V V$

$$|\bar{\mathcal{M}}|^2_{p^2<v_\phi^2} \sim g^4\left(3 - \frac{p^2}{m_V^2} + \frac{p^4}{4m_V^4}\right), \qquad |\bar{\mathcal{M}}|^2_{p^2>v_\phi^2} \sim \lambda_\phi^2 + 2g^4$$

$$\longrightarrow \quad \mathrm{Im}\,\widetilde{\Gamma}^{(2)}(p^2)\Big|_{\phi^* \to \phi V V} \sim p^2 |\bar{\mathcal{M}}|^2\Big|_{\phi^* \to \phi V V}$$

cf. 2-body process: $\mathrm{Im}\,\widetilde{\Gamma}^{(2)}(p^2)\Big|_{\phi^* \to V V} \sim |\bar{\mathcal{M}}|^2\Big|_{\phi^* \to V V}$

$p^2 > v_\phi^2$ : 3-body process dominant for vector DM!

Higher order processes for DM: $\phi_{p_1}^* \phi_{p_2}^* \to \chi_s \chi_s, \bar{\chi}_f \chi_f, VV$

$\phi_{p_1}^* \sim \left(\frac{v_\phi}{p^2}\right)^2 \longrightarrow$ Suppressed for $p^2 \sim m_{\mathrm{DM}}^2 \gg v_\phi^2$

# Back-reaction issue



Energy density of particles can be comparable to latent heat:

$$\rho_{\rm DM} \sim \Delta V \ , \quad \rho_{\rm DM} \sim \int dp^2\, p f(p^2) {\rm Im}\, \widetilde{\Gamma}^{(2)}(p^2)$$

⟶ Bubble dynamics, DM production affected.

$$f(p^2) \sim \frac{1}{p^4}$$ (Elastic collisions)

$|\bar{\mathcal{M}}|^2 \sim$

- $(p^2)^0$, (scalar, 2-body) ; $p^2$, (scalar, 3-body)
- $p^2$, (fermion, 2-body)
- ~~$p^4$, (Vector, 2-body)~~ ~~Back-reaction issue!~~
- $(p^2)^0$, (vector, 2-body) ; $p^2$, (vector, 3-body)

Important to treat the growth of vector DM properly for back-reaction issue.