Probing non-minimal coupling through super-horizon instability and secondary gravitational waves

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Overview

1. Motivation
2. Spectrum of non-minimally coupled scalar fluctuations
3. Generation of secondary gravitational waves by the scalar field source
4. Constraining non-minimal coupling strength(ξ) based on observational bound
5. Important findings

- Scalar fluctuations, non-minimally coupled to gravity, can be treated as a potential source of secondary gravitational waves.

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- Significant post-inflationary long-wavelength(IR) instability of the source field beyond a certain coupling strength, leaves a visible imprint on secondary gravitational wave spectrum, which can be probed by various future GW observatories.

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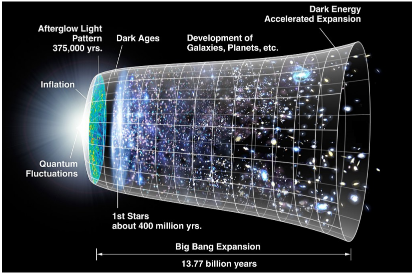
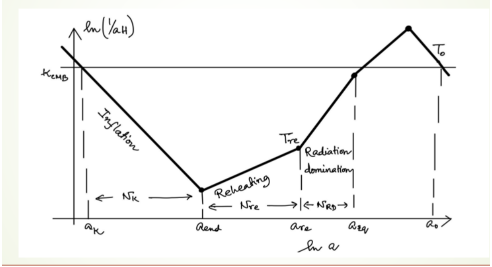
- Significant post-inflationary long-wavelength(IR) instability of the source field beyond a certain coupling strength, leaves a visible imprint on secondary gravitational wave spectrum, which can be probed by various future GW observatories.

- Constraining non-minimal coupling through PLANCK bound on tensor-to-scalar ratio and $\Delta N_{\rm eff}$.

Cosmic Evolution





- Cosmic evolution and dynamics of Hubble horizon through modified expansion history.

Why do we need reheating phase?

- At the end of early accelerated expansion(Inflation), universe was left in a super cold state of vanishing entropy, and particle no. density.
- To achieve successful nucleosynthesis, universe must transit to a hot, thermalized radiation-dominated phase.

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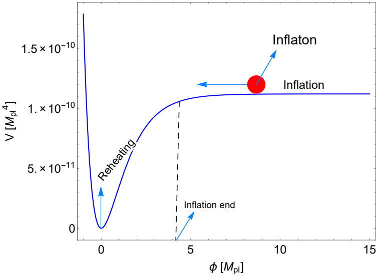
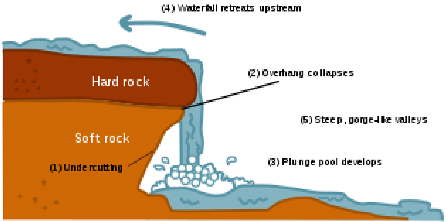
- Inflaton$\longrightarrow$ SM+BSM $\longrightarrow$ hot thermal bath $\longrightarrow$ reheating





General set up of non-minimally coupled scalar field(χ) system

- **Lagrangian of the system:**

$$\mathcal{L}_{[\phi,\chi]} = -\underbrace{\sqrt{-g}}_{a^4(\eta)}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\xi R\chi^2\right)$$

$a \rightarrow$ scale factor; $R \rightarrow$ Ricci scalar; $\xi \rightarrow$ non-minimal coupling

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- **Fourier decomposition:**

$\chi(\eta,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\ \chi_k(\eta)\ e^{i\vec{k}.\vec{x}}$

- **EoM of rescaled field mode($X_k = a(\eta)\chi_k(\eta)$):**

$$X_k'' + \left[k^2 + a^2 m_\chi^2 - \frac{a''}{a}(1-6\xi)\right]X_k = 0 \qquad (1)$$

$R = \left(6a''/a^3\right)$

Dynamical equation and appearance of IR instability(Tachyonic instability)

- **Form of scale factor:**

$$a(\eta) = a_{\rm end}\left(\frac{1+3w_\phi}{2|\eta_{\rm end}|}\right)^{\frac{2}{1+3w_\phi}}\left(\eta - \eta_{\rm end} + \frac{2|\eta_{\rm end}|}{1+3w_\phi}\right)^{\frac{2}{1+3w_\phi}};\ \eta_i < \eta \leq \eta$$

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- **Inflationary evolution:** $X_k'' + \underbrace{\left[k^2 - \frac{2(1-6\xi)}{\eta^2}\right]}_{\omega_k^2<0(\text{Instability})\rightarrow\ \text{for}\ \xi<1/6} X_k = 0$
- **Post-inflationary evolution:**
$$X_k'' + \underbrace{\left[k^2 - \frac{2(1-3w_\phi)(1-6\xi)}{(1+3w_\phi)^2\left(\eta + \frac{3(1+w_\phi)}{a_{\rm end}H_{\rm end}(1+3w_\phi)}\right)^2}\right]}_{\omega_k^2<0\rightarrow\ \text{for}\ w_\phi>1/3,\ \xi>1/6,\ \text{for}\ w_\phi<1/3,\ \xi<1/6} X_k = 0$$

Inflationary and post-inflationary vacuum solution

Adiabatic vacuum solution

Inflationary vacuum solution: $X_k^{(\mathrm{inf})} = \frac{\sqrt{\pi|\eta|}}{2} e^{i(\pi/4+\pi\nu_1/2)} H_{\nu_1}^{(1)}(k|\eta|)$

Post-inflationary vacuum solution:

$$X_k^{(\mathrm{reh})} = \sqrt{\frac{\bar{\eta}}{\pi}} \exp\left[\frac{3ik\mu}{a_{\mathrm{end}} H_{\mathrm{end}}} + \frac{i\pi}{4}\right] K_{\nu_2}(ik\bar{\eta})$$

- **EoS and ξ dependent indices:** $\nu_1 = \sqrt{9-48\xi}/2$; $\mu = \frac{(1+w_\phi)}{(1+3w_\phi)}$;

$$\nu_2 = \frac{\sqrt{3(1+w_\phi)\left(3(1-w_\phi)^2+16\xi(3w_\phi-1)\right)}}{2\sqrt{1+3w_\phi}\sqrt{1+4w_\phi+3w_\phi^2}}; \; \bar{\eta} = (\eta + 3\mu/a_{\mathrm{end}} H_{\mathrm{end}})$$

▸ Introduction to Bogoliubov coefficients

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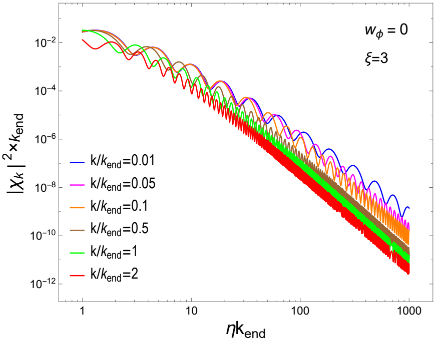
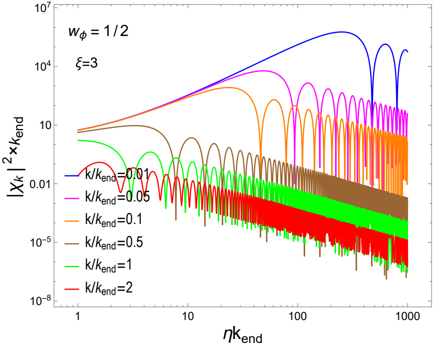
- **General reheating field solution:** $X_k(\eta) = \alpha_k X_k^{(\mathrm{reh})} + \beta_k X_k^{*(\mathrm{reh})}$
  $\alpha_k,\ \beta_k \longrightarrow$ Bogoliubov coefficients

▸ Introduction to Bogoliubov coefficients

Time-evolution of long-wavelength(IR) modes of scalar fluctuations





[1] A.chakraborty, S.Maiti, and D.Maity [arxiv: 2408.07767]

Defining field power spectrum and energy-density spectrum

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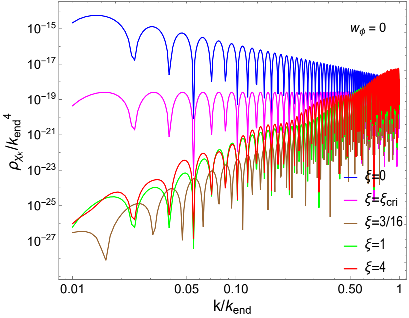
- **Field energy-density spectrum:**

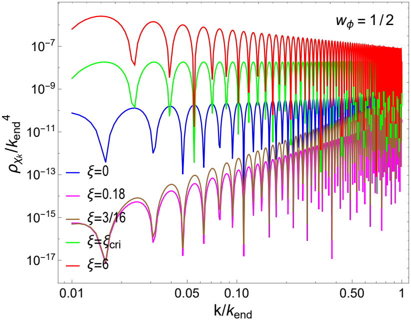
$$\rho_{\chi_k}(\eta) = \frac{k^3}{4\pi^2 a^4}(|X_k'|^2 + k^2|X_k|^2) = (k^2/a^2)\mathcal{P}_\chi(k,\eta)$$

Energy spectrum of IR modes for $1/3 < w_\phi \leq 1$

$$\rho_{\chi_k}(\eta > \eta_{\rm end}) \propto \begin{cases} (\sin^2(k\eta)/a(\eta)^4)(k/k_{\rm end})^{2(2-\nu_1-\nu_2)} & \text{for} \quad 0 \leq \xi < 3/16 \\ (\sin^2(k\eta)/a(\eta)^4)(k/k_{\rm end})^{2(2-\nu_2)} & \text{for} \quad \xi = 3/16 \\ (\cos^2(k\eta)/a(\eta)^4)(k/k_{\rm end})^{2(2-\nu_2)} & \text{for} \quad \xi > 3/16 \end{cases} \tag{2}$$

Behavior of energy-density spectrum





- $w_\phi = 0 \rightarrow \xi_{\rm cri} \approx 5/48$

  $w_\phi = 1/2 \rightarrow \xi_{\rm cri} \approx 4.073$

[1] A.chakraborty, S.Maiti, and D.Maity [arxiv: 2408.07767]

Model independent definition of reheating parameters($N_{\rm re}$, $T_{\rm re}$)

Reheating point: $\rho_{\rm R}(a_{\rm re}) = \rho_\phi(a_{\rm re})$

Reheating e-folding number: $N_{\rm re} = \frac{1}{3(1+w_\phi)} \ln\left(\frac{90 H_{\rm end}^2 M_{pl}^2}{\pi^2 g_{\rm re} T_{\rm re}^4}\right)$

Defining $k_{\rm end}$ and $k_{\rm re}$:
$(k_{\rm end}/a_0) = \left(\frac{43}{11 g_{\rm re}}\right)^{1/3} \left(\frac{\pi^2 g_{\rm re}}{90}\right)^{\alpha} \frac{H_{\rm end}^{1-2\alpha} T_{\rm re}^{4\alpha-1} T_0}{M_{pl}^{2\alpha}}$, $(k_{\rm end}/k_{\rm re}) = \exp\left(\frac{N_{\rm re}(1+3w_\phi)}{2}\right)$, $\alpha = 1/3(1+w_\phi)$, $a_0 \rightarrow$ present scale factor, and $T_0 = 2.725$ K is the present CMB temperature

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[1] L. Dai, M. Kamionkowski and J. Wang, PRL. 113, 041302 (2014)

[2] J. L. Cook, et al. JCAP 04 (2015) 047

Generation of secondary(induced) gravitational wave(SGW)

- **Perturbed FLRW metric:**
  $ds^2 = a^2(\eta)\left[-d\eta^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]$, transverse-traceless tensor $\rightarrow \partial^i h_{ij} = h^i_i = 0$

- **anisotropic stress tensor:** $\Pi_{ij} \sim (1-2\xi)\partial_i\chi\partial_j\chi - 2\xi\chi\partial_i\partial_j\chi + \xi\chi^2 G_{ij}$

- **Evolution equation:**
  $h^{\lambda''}_{\mathbf{k}} + 2\frac{a'}{a}h^{\lambda'}_{\mathbf{k}} + k^2 h^{\lambda}_{\mathbf{k}} = \frac{2}{M^2_{pl}} e^{ij}_{\lambda}(k) P^{lm}_{ij}(\hat{k}) T_{lm}(k,\eta)$, $P^{lm}_{ij}(\hat{k}) \rightarrow$ transverse-traceless projector

▸ outline of evolution Equation

Defining secondary tensor power spectrum in presence of scalar field source

- **Tensor power spectrum:** $\mathcal{P}_{\mathrm{T}}(k,\eta) = 4\frac{k^3}{2\pi^2}|h_{\mathbf{k}}(\eta)|^2, \;\; h_{\mathbf{k}}(\eta) = h_{\mathbf{k}}^{\mathrm{vac}} + \frac{2e^{ij}(\mathbf{k})}{M_{pl}^2}\int d\eta_1 \mathcal{G}_k(\eta,\eta_1)\Pi_{ij}^{\mathrm{TT}}(\mathbf{k},\eta_1)$

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- **Secondary tensor power spectrum:** $\mathcal{P}_{\rm T}^{\rm sec}(k,\eta_{\rm re}) \propto \frac{1}{M_{\rm pl}^4}\left(\int_{x_{\rm e}}^{x_{\rm re}} dx_1 \frac{\mathcal{G}_k^{\rm re}(x_{\rm re},x_1)}{a^2(x_1)}\right)^2 \times \int_{k_{\rm min}}^{k_{\rm end}} \frac{dq}{k}\int_{-1}^{1} d\gamma(1-\gamma^2)^2 \times \frac{(q/k)^3\mathcal{P}_X(q,\eta_1)\mathcal{P}_X(|\mathbf{k}-\mathbf{q}|\eta_1)}{|1-q/k|^3}$, $x = k\eta$, $\cos\gamma = \hat{k}.\hat{q}$

For $w_\phi > 1/3$, $\xi > 3/16$

$$\mathcal{P}_{\rm T}^{\rm sec}(k<k_{\rm re},\eta_{\rm re}) \propto \left(\frac{k_{\rm end}}{k_{\rm re}}\right)^{4-2\delta}\left(\frac{k}{k_{\rm end}}\right)^{4(2-\nu_2)} ; \delta = 4/(1+3w_\phi) \quad (3)$$

$$\mathcal{P}_{\rm T}^{\rm sec}(k>k_{\rm re},\eta_{\rm re}) \propto \left(\frac{k_{\rm re}}{k_{\rm end}}\right)^{\delta}\left(\frac{k}{k_{\rm end}}\right)^{4+\delta-4\nu_2} \quad (4)$$

Defining Gravitational wave(GW) energy spectrum

- **GW energy spectrum:**
  $\Omega_{\rm gw}(k,\eta) = (\Omega_{\rm gw}^{\rm pri} + \Omega_{\rm gw}^{\rm sec}) = \frac{(1+k^2/k_{\rm re}^2)}{24}(\mathcal{P}_{\rm T}^{\rm pri}(k,\eta_{\rm re}) + \mathcal{P}_{\rm T}^{\rm sec}(k,\eta_{\rm re}))$
- **Energy spectrum for today:**
  $\Omega_{\rm gw}(k)h^2 \approx \left(\frac{g_{r,0}}{g_{r,eq}}\right)^{1/3}\Omega_R h^2 \Omega_{\rm gw}(k,\eta),\;\; \Omega_R h^2 = 4.3\times 10^{-5}$

For $w_\phi > 1/3$, $\xi > 3/16$

$$\Omega_{\rm gw}^{\rm pri}(k<k_{\rm re})h^2 \propto (k/k_{\rm re})^0 \quad (5)$$

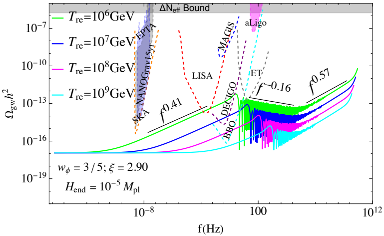
$$\Omega_{\rm gw}^{\rm pri}(k>k_{\rm re})h^2 \propto (k_{\rm end}/k_{\rm re})^{n_w}(k/k_{\rm end})^{n_w}\,;\, n_w = 2(3w_\phi - 1)/(1+3w_\phi) \quad (6)$$

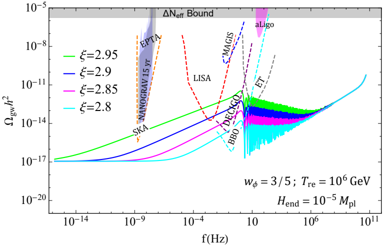
$$\Omega_{\rm gw}^{\rm sec}(k<k_{\rm re})h^2 \propto (k_{\rm end}/k_{\rm re})^{4-2\delta}(k/k_{\rm end})^{2(4-2\nu_2)} \quad (7)$$

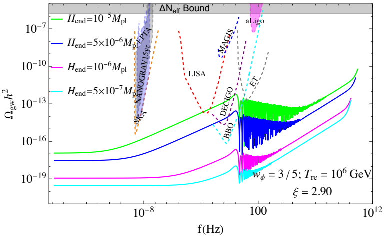
$$\Omega_{\rm gw}^{\rm sec}(k>k_{\rm re})h^2 \propto (k_{\rm end}/k_{\rm re})^{2-\delta}(k/k_{\rm end})^{6+\delta-4\nu_2} \quad (8)$$

[1] A.chakraborty, S.Maiti, and D.Maity [arxiv: 2408.07767]

GW spectrum for varying ξ, $T_{\rm re}$ and inflationary energy scale($H_{\rm end}$)







Constraining ξ through tensor-to-scalar ratio($r_{0.05}$) and $\Delta N_{\rm eff}$(for the scalar field)

- For $w_\phi > 1/3$, in the regime $k < k_{\rm re}$; $r_{0.05} \propto \left(\frac{90 H_{\rm end}^2 M_{\rm pl}^2}{\pi^2 g_{\rm re} T_{\rm re}^4}\right)^{\frac{2(3w_\phi - 1)}{3(1+w_\phi)}} \left(\frac{k_*}{k_{\rm end}}\right)^{4(2-\nu_2)} \leq 0.036$; $(k_*/a_0) = 0.05\ \mathrm{Mpc}^{-1}$

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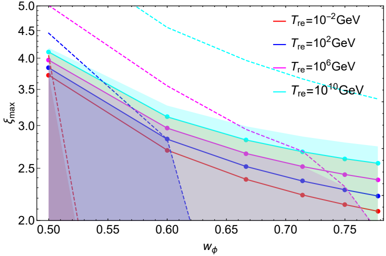
- This massless scalar, possible candidate for dark radiation, solely contributes to $\Delta N_{\rm eff} \rightarrow$ $\left(\frac{g_{r,0}}{g_{r,eq}}\right)^{1/3} \Omega_{\rm R} h^2\, \Omega_\chi(\eta_{\rm re}) \simeq 1.6 \times 10^{-6} \left(\frac{\Delta N_{\rm eff}}{0.284}\right)$



Contribution of GWs to $\Delta N_{\rm eff}$

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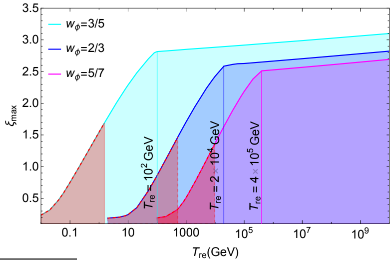
- **Minimum bound on $T_{\rm re}$(avoiding overproduction of extra degrees of freedom):**
  $T_{\rm re}^{\rm min} \geq \left(\frac{90 H_{\rm end}^2 M_{\rm pl}^2}{\pi^2 g_{\rm re}}\right)^{1/4} \beta^{\frac{3(1+w_\phi)}{4(3w_\phi-1)}} \left(\frac{0.284}{\Delta N_{\rm eff}}\right)^{\frac{3(1+w_\phi)}{4(3w_\phi-1)}}$, $\beta \rightarrow \left(1.43 \times 10^{-11}/n_w\right)\left(H_{\rm end}/10^{-5} M_{\rm pl}\right)^2$



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Important outcomes

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- Post-inflationary instability effect is dominant for higher EoS $w_\phi > 1/3$. The longer the wavelength, the more the enhancement owing to prolonged instability for a larger coupling strength ξ.
- For $w_\phi > 1/3$, significant IR instability beyond a certain large ξ leaves a visible imprint on the SGW spectrum overcoming the PGW strength at the low and intermediate frequency ranges.
- Combining two strong observational bounds, $r_{0.05}$ and $\Delta N_{\rm eff}$, to prevent the overproduction of tensor fluctuations at the CMB scale and the overproduction of extra relativistic degrees of freedom, we have found a tight constraint on coupling strength. We find that $\xi_{\rm max} \lesssim 4$ for any $w_\phi \geq 1/2$ for a wide range of reheating temperatures. Unlike $w_\phi > 1/3$, for $w_\phi < 1/3$, we put lower bound on ξ. For $w_\phi = 0$, we find the lower bound $\xi_{\rm min} \gtrsim 0.02$.

Thank you!

◂ Back

- **Bogoliubov coefficients** (α_k, β_k): Making the adiabatic vacuum solutions $X_k^{(\mathrm{inf})}$ and $X_k^{(\mathrm{reh})}$, and their first derivatives continuous at the junction $\eta = \eta_{\mathrm{end}}$, we compute the Bogoliubov coefficients as follows [1]:

$$\alpha_k = i\left(X_k^{(\mathrm{inf})'}(\eta_{\mathrm{end}})X_k^{(\mathrm{reh})^*}(\eta_{\mathrm{end}}) - X_k^{(\mathrm{inf})}(\eta_{\mathrm{end}})X_k^{(\mathrm{reh})^{*\prime}}(\eta_{\mathrm{end}})\right)$$
$$\beta_k = -i\left(X_k^{(\mathrm{inf})'}(\eta_{\mathrm{end}})X_k^{(\mathrm{reh})}(\eta_{\mathrm{end}}) - X_k^{(\mathrm{reh})'}(\eta_{\mathrm{end}})X_k^{(\mathrm{inf})}(\eta_{\mathrm{end}})\right) \quad (9)$$

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[1] M. R. de Garcia Maia Phys. Rev. D 48, 647 (1993)

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[2] M. R. de Garcia Maia Phys. Rev. D 48, 647 (1993)

◂ Back

- **Action with anisotropic stress:**
  $S_{GW} = \int dx^4 \sqrt{-g} \left[-\frac{g^{\mu\nu}}{64\pi G} \partial_\mu h_{ij} \partial_\nu h^{ij} + \frac{1}{2} \Pi^{ij} h_{ij} \right]$
- **Fourier decomposition:**
  $h_{ij}(\eta, \mathbf{x}) = \sum_{\lambda=(+,\times)} \int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} e^{\lambda}_{ij}(\mathbf{k}) h^{\lambda}_{\mathbf{k}}(\eta) \mathrm{e}^{i\mathbf{k}\cdot\mathbf{x}}, \quad e^{\lambda}_{ij}(\mathbf{k}) \rightarrow$ polarization tensor