# On-shell approach to spinning binaries in modified gravity

Panagiotis Marinellis

In collaboration with Adam Falkowski:

**[2407.16457], [2411.12909]**

IJCLab Irène Joliot-Curie Laboratoire de Physique des 2 Infinis

université PARIS-SACLAY

# Plan for this talk:

1. Motivation

2. Scattering Amplitudes and Observables

3. Scalar-tensor theories: Examples, compact objects, scalar hair and scattering waveforms

4. Outlook

# 1. Motivation

Image Credit: EGO*

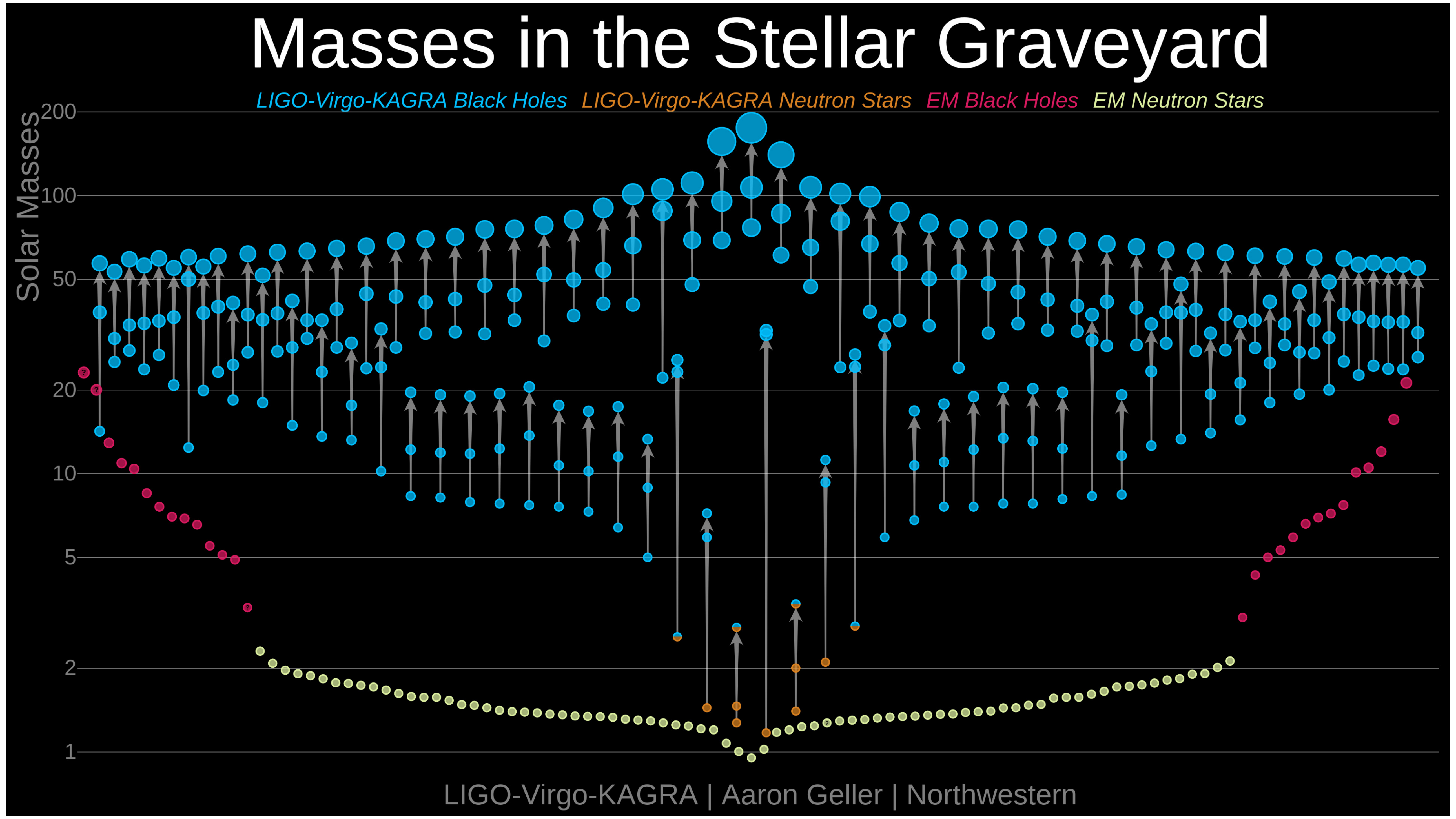
→ LIGO/VIRGO collaboration: First detection of **Gravitational Waves** (GWs) in **2015**

# 1. Motivation

Image Credit: EGO*

LIGO/VIRGO collaboration: First detection of **Gravitational Waves** (GWs) in **2015**

Has since then inspired an unprecedented interest in GW detection, especially with the upcoming **new generation of GW interferometers** (ET, Cosmic Explorer, LISA)



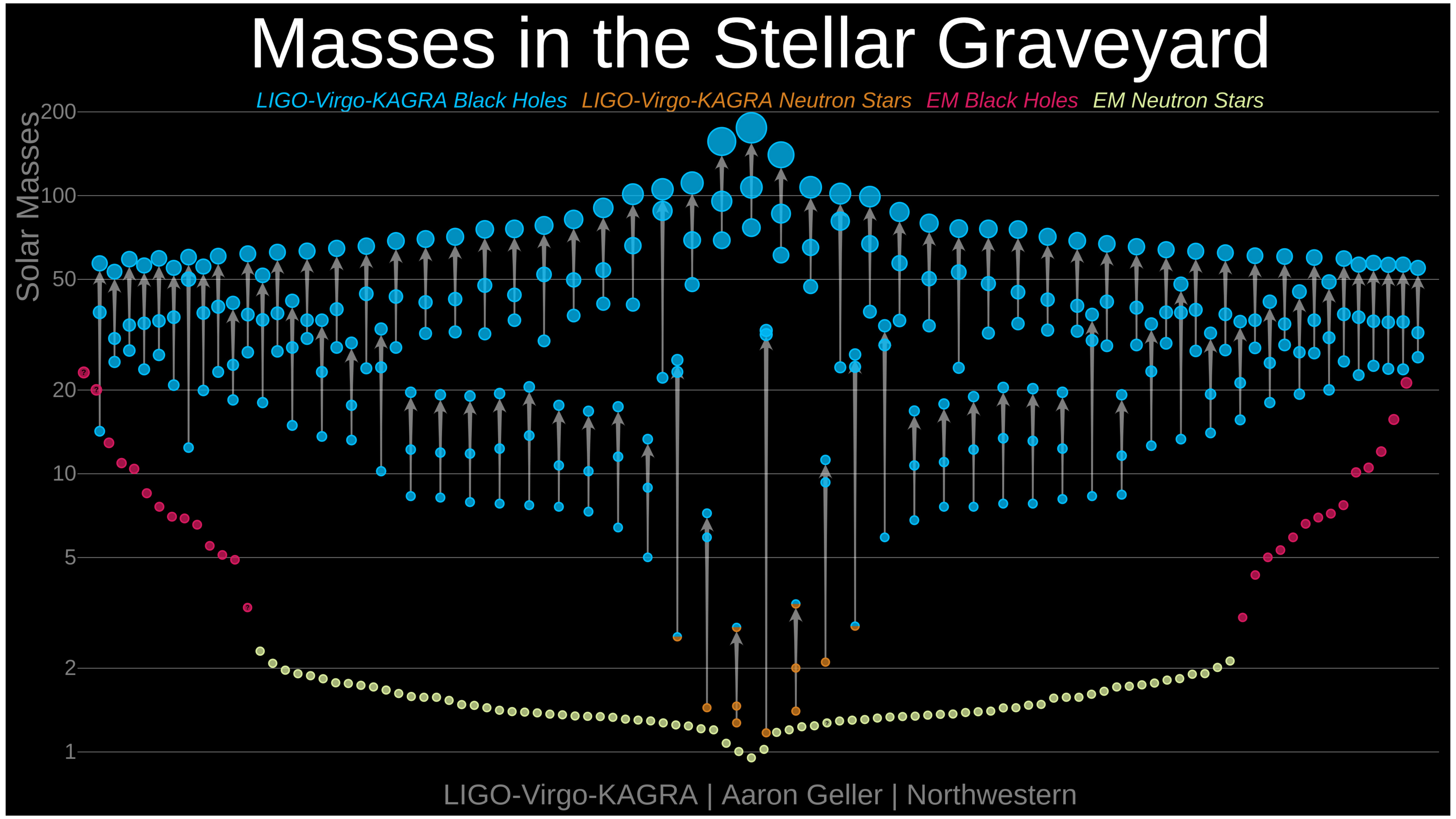

Credit:LIGO-Virgo/Aaron Geller/Northwestern University*

# 1. Motivation

Image Credit: EGO*

LIGO/VIRGO collaboration: First detection of **Gravitational Waves** (GWs) in **2015**

Has since then inspired an unprecedented interest in GW detection, especially with the upcoming **new generation of GW interferometers** (ET, Cosmic Explorer, LISA)




New era of **high precision** measurements of GWs:

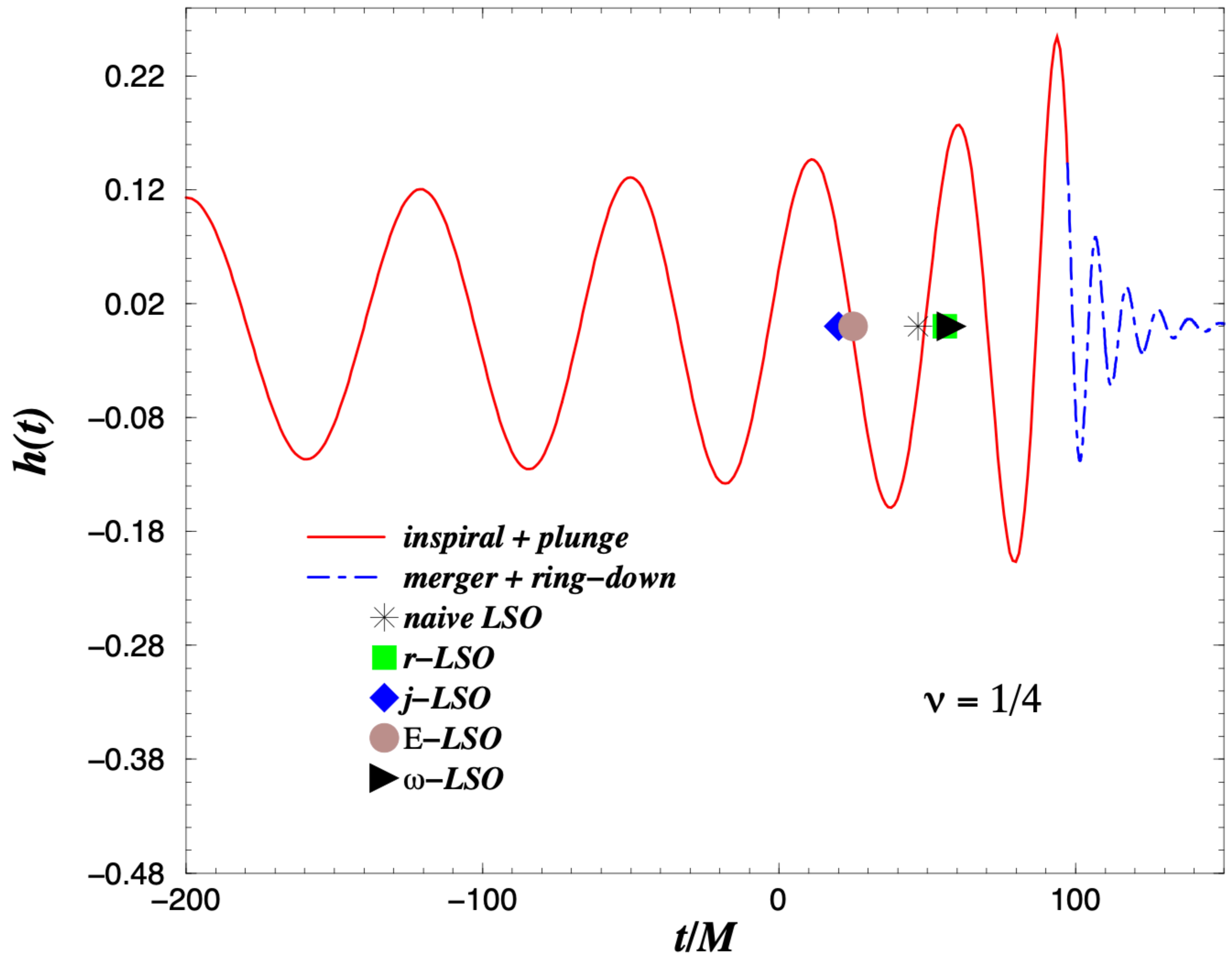
Highly accurate GW templates

**New window to test General Relativity (GR)**

Credit:LIGO-Virgo/Aaron Geller/Northwestern University*

# The phases of the binary problem:

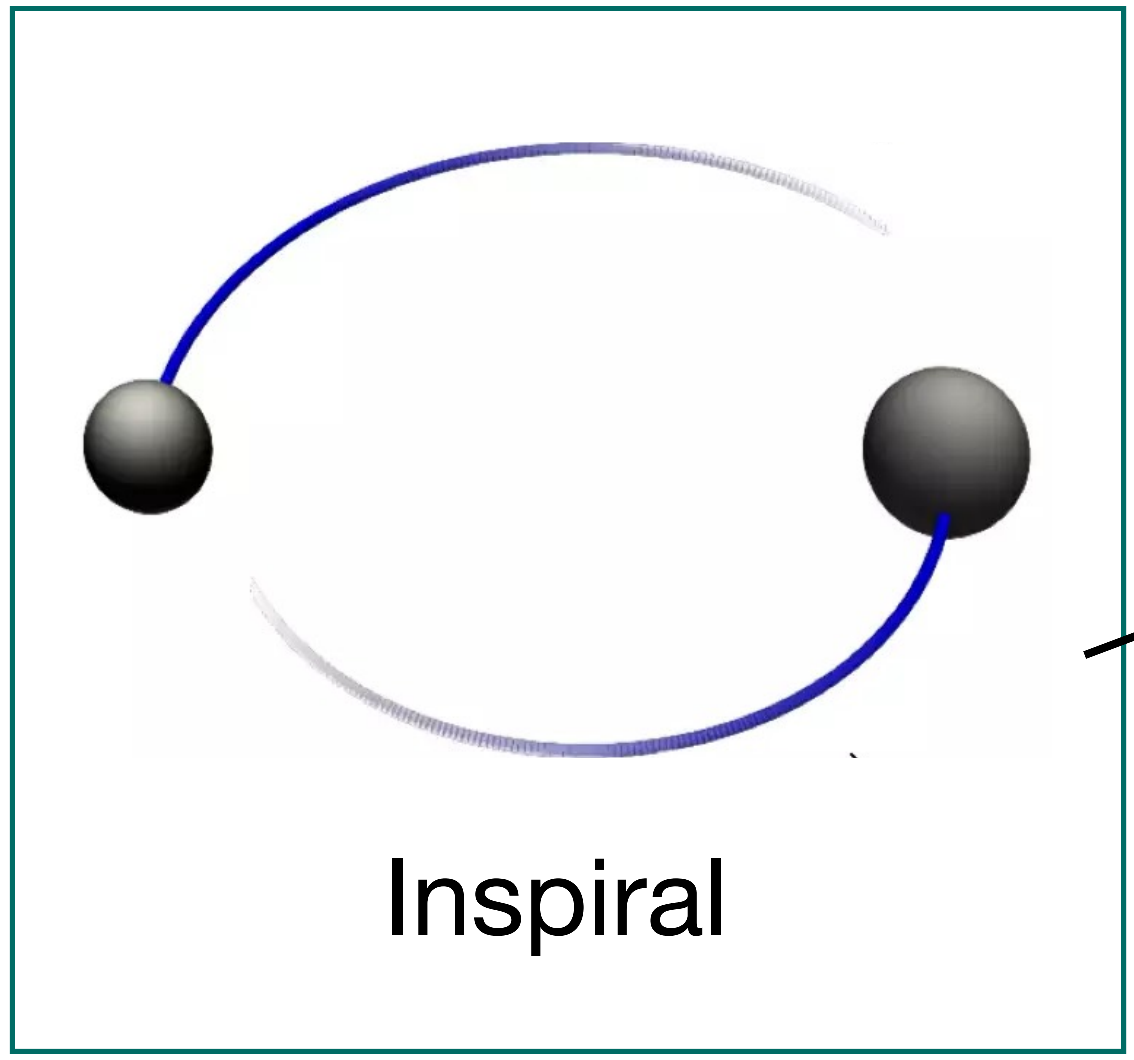
*Phys.Rev.D* 62 (2000) 064015 [Buonanno, Damour]*

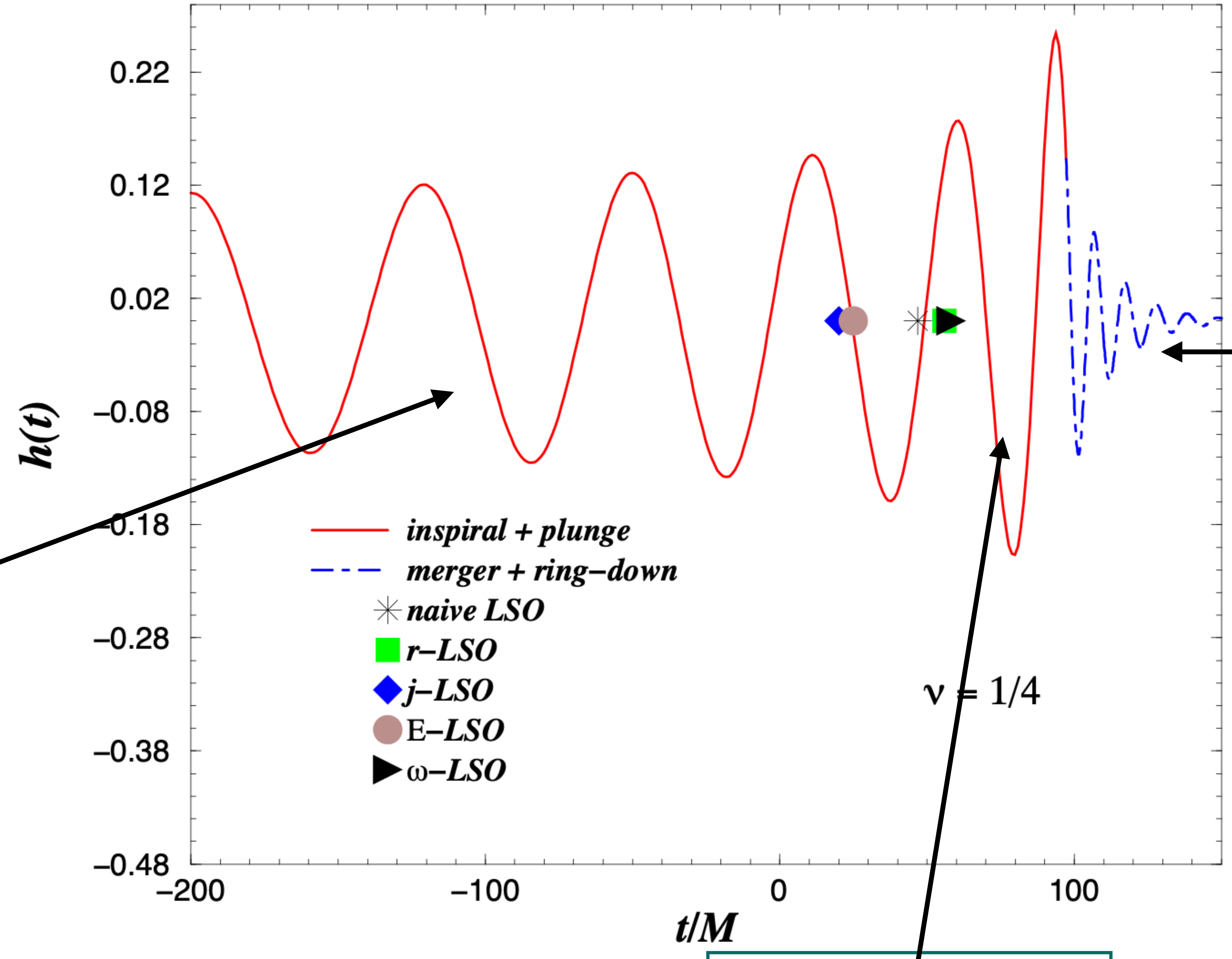


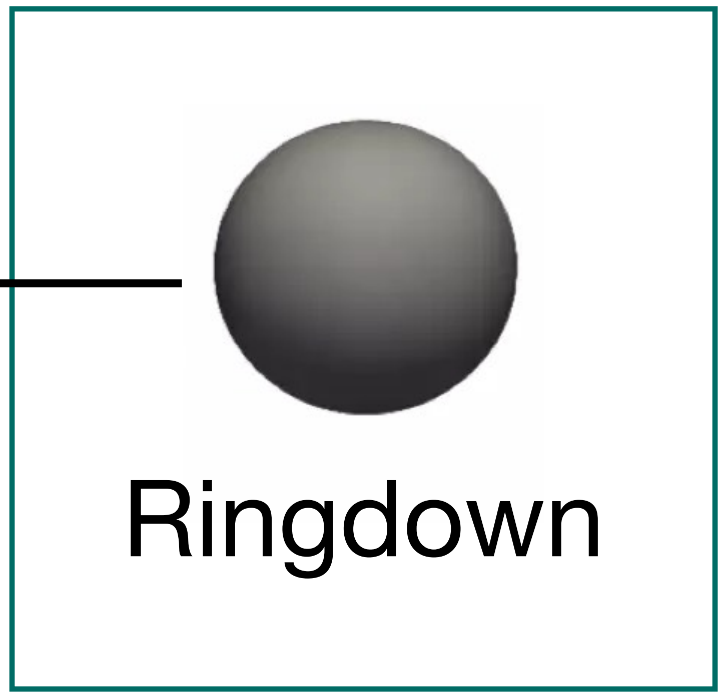
# The phases of the binary problem:

Phys.Rev.D 62 (2000) 064015 [Buonanno, Damour]*

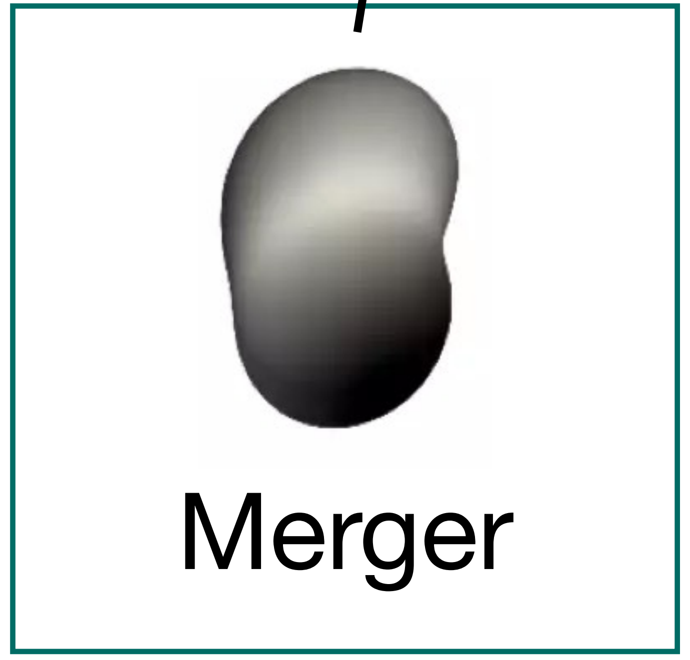
**Analytical approaches**




**Black Hole Perturbation Theory (BHPT)**

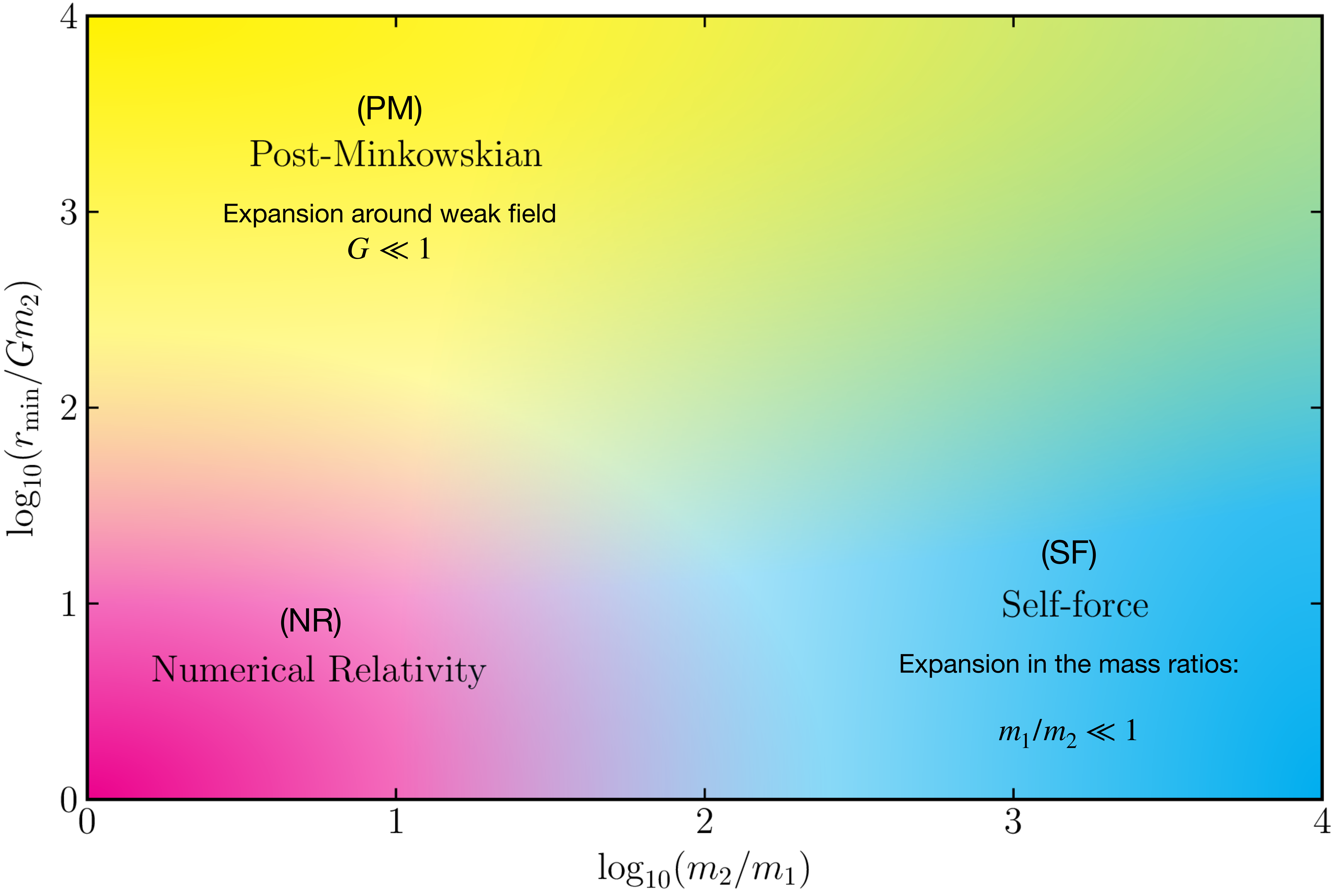



**Numerical Relativity**

Illustration Credit: Max-Planck-Gesellschaft*

# Analytical approaches:

Phys.Rev.D 108 (2023) 2, 024025 [Barack, Bern, Herrmann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng]



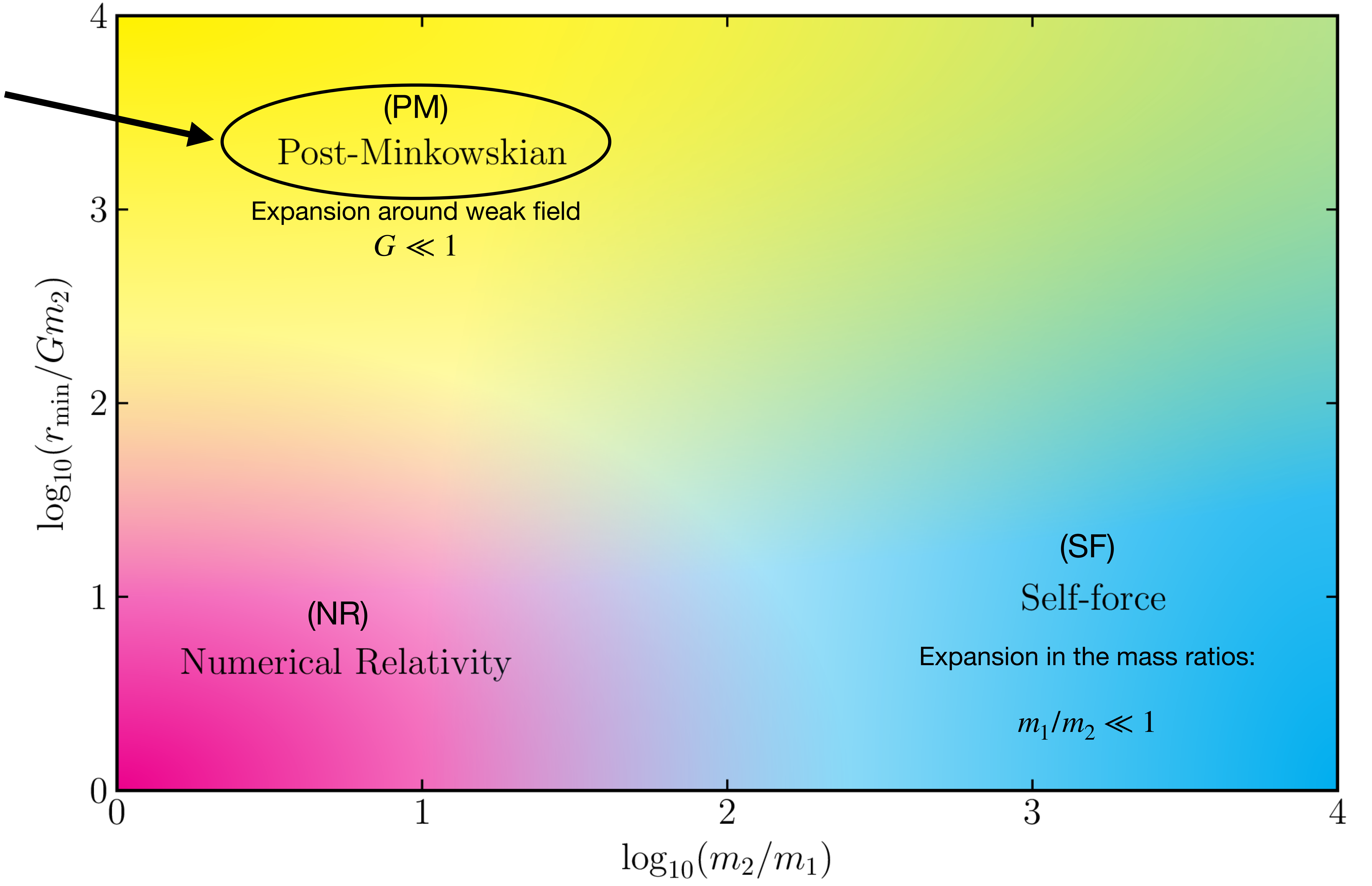
# Analytical approaches:

- Enormous program for accurately determining the binary dynamics and computing waveforms **at all orders in** $\frac{v}{c}$, in contrast with Post-Newtonian (PN) approaches.

- Inspiration from **techniques** used in **Scattering Amplitudes and Effective Field Theory**

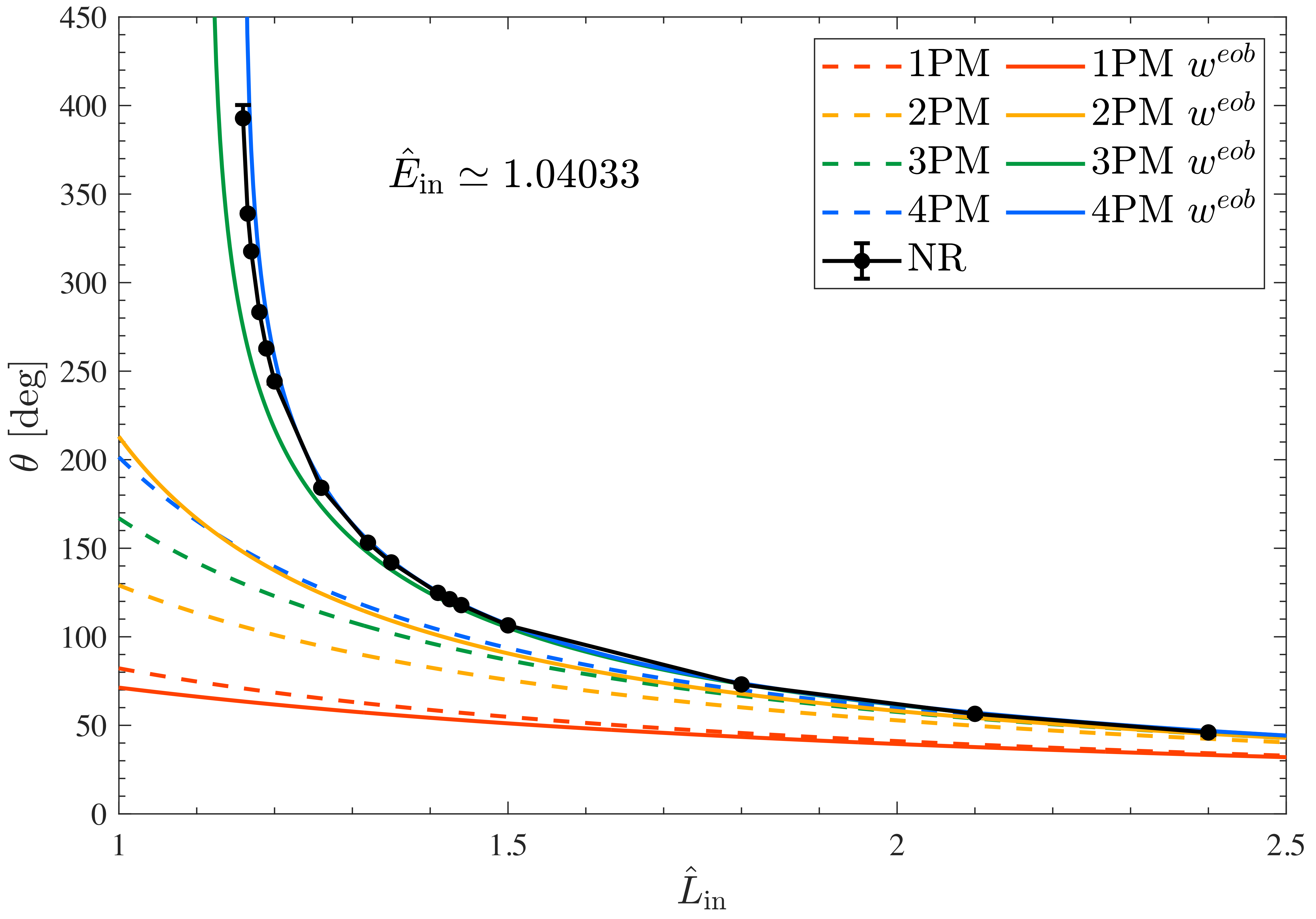
[Adamo, Alaverdian, Aoude, Bautista, Ben-Shahar, Bern, Bini, Brandhuber, Brown, Buonanno, Cachazo, Cangemi, Chiodaroli, Chen, Cordero, Cristofoli, de la Cruz, Damour, Damgaard, De Angelis, Driesse, Elkhidir, Gatica, Georgoudis, Goldberger, Gowdy, Gonzo, Guevara, Haddad, Heissenberg, Helset, Herrmann, Holstein, Huang, Huang, Ilderton, Jakobsen, Johansson, Kim, Kraus, Kosmopoulos, Kosower, Lee, Levi, Lin, Liu, Luna, Matasan, Maybee, Menezes, Mogull, Mougiakakos, Moynihan, Novichkov, O'Connell, Ochirov, Parra-Martinez, Pichini, Plefka, Porto, Riva, Roiban, Ross, Rothstein, Ruf, Russo, Saketh, Sauer, Scheopner, Sergola, Shen, Siemonsen, Smirnov, Smirnov, Steinhoff, Teng, Travaglini, Vanhove, Vazquez-Holm, di Vecchia, Veneziano, Vernizzi, Vines, Wong, Xu, Yang, Yin, Zeng, et al…]

Phys.Rev.D 108 (2023) 2, 024025 [Barack, Bern, Herrmann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng]



# PM expansion to the test and bound orbits:

*Phys.Rev.D* 108 (2023) 12, 124016 [Rettegno, Pratten, Thomas, Schmidt, Damour]

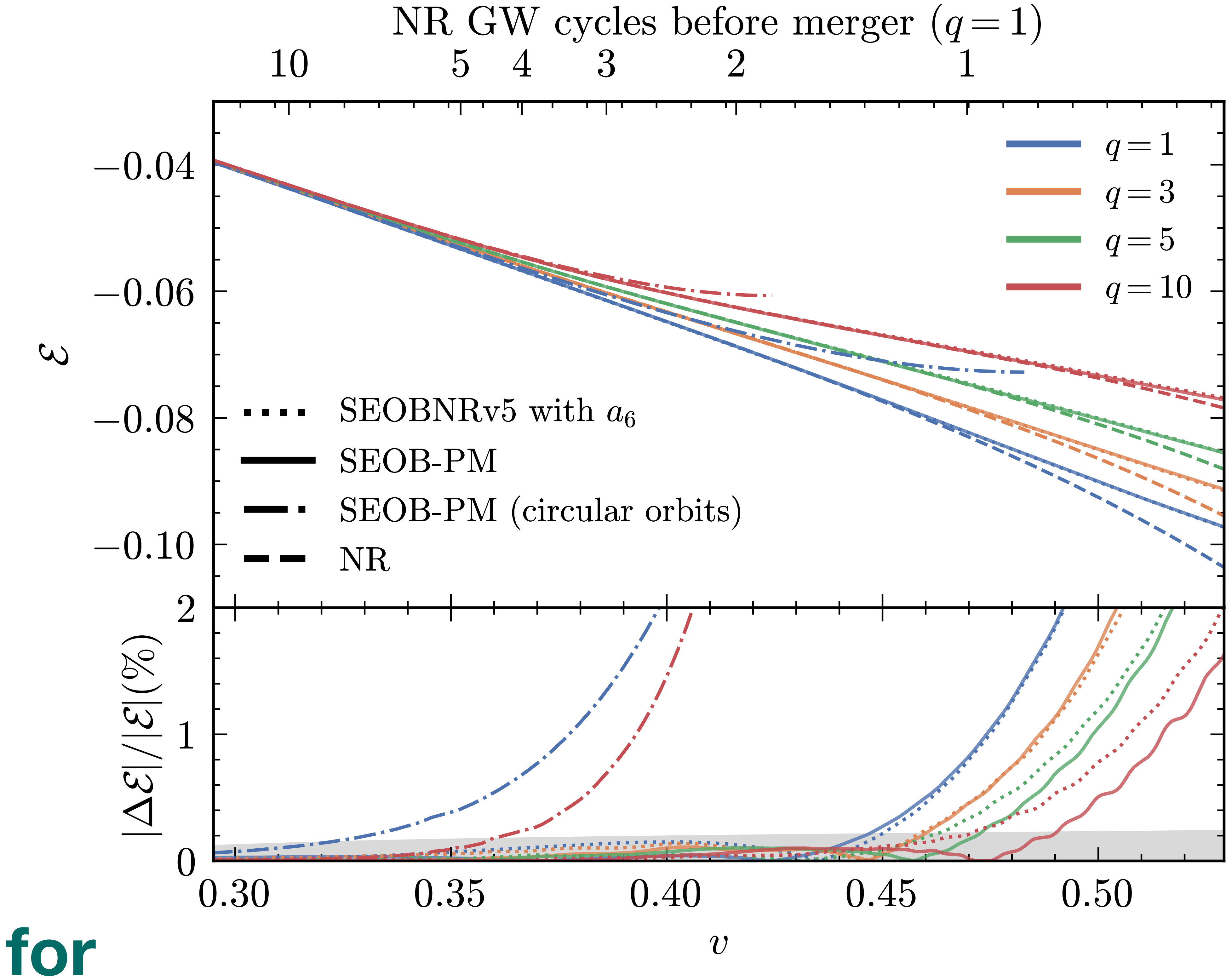



## Bound to Boundary map for binary dynamics:

JHEP 01 (2020) 072 [Kälin, Porto]
JHEP 02 (2020) 120 [Kälin, Porto]
JHEP 04 (2022) 154, JHEP 07 (2022) 002 (erratum) [Cho, Kälin, Porto]

## Recent work on a waveform map:

JHEP 05 (2024) 034 [Adamo, Gonzo, Ilderton]

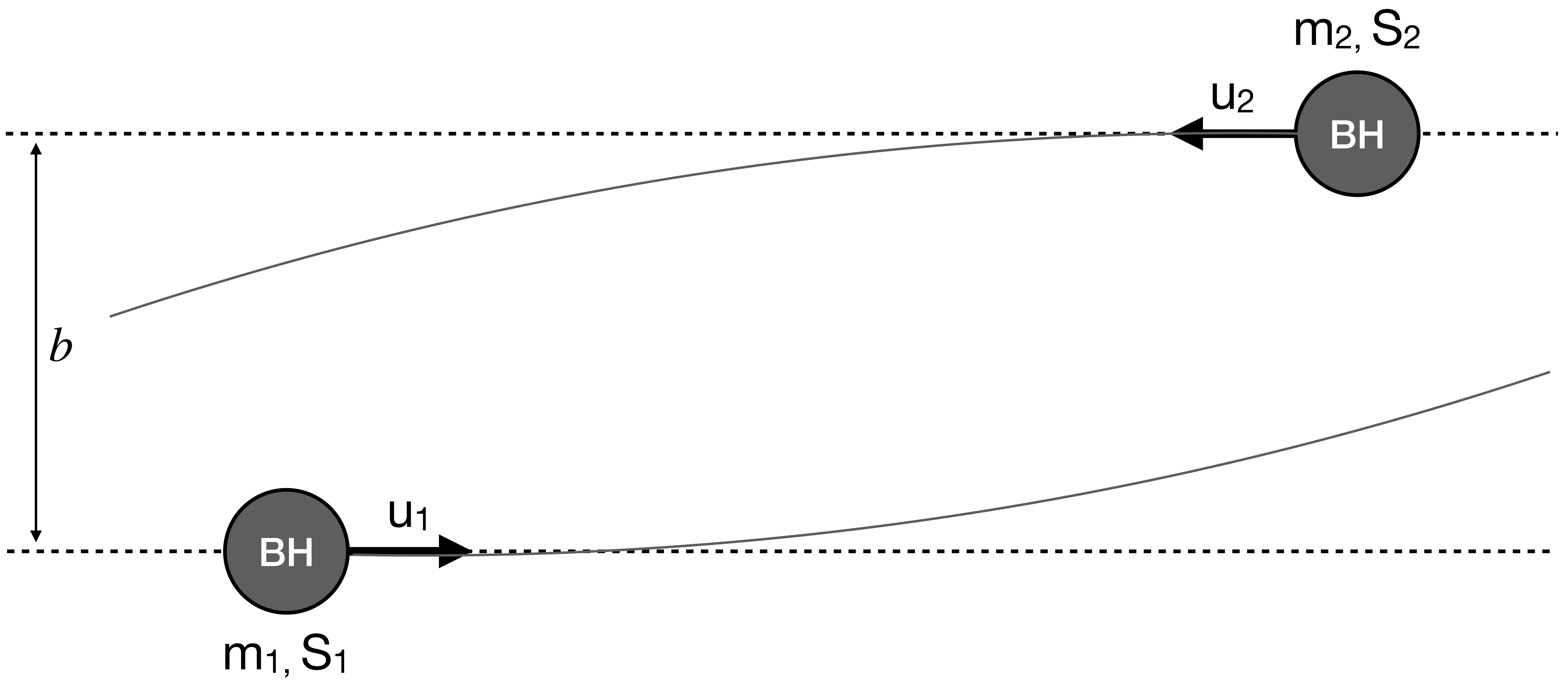



**Already PM is doing very well for black hole (BH) scattering!**

**Good agreement for the bound case as well!**

arxiv: 2405.19181 [Buonanno, Mogull, Patil, Pompili]

# 2. Scattering Amplitudes and Observables




**Weak field expansion:**

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_{Pl}} h_{\mu\nu}$$

**PM expansion:**

$$R_s \ll b$$

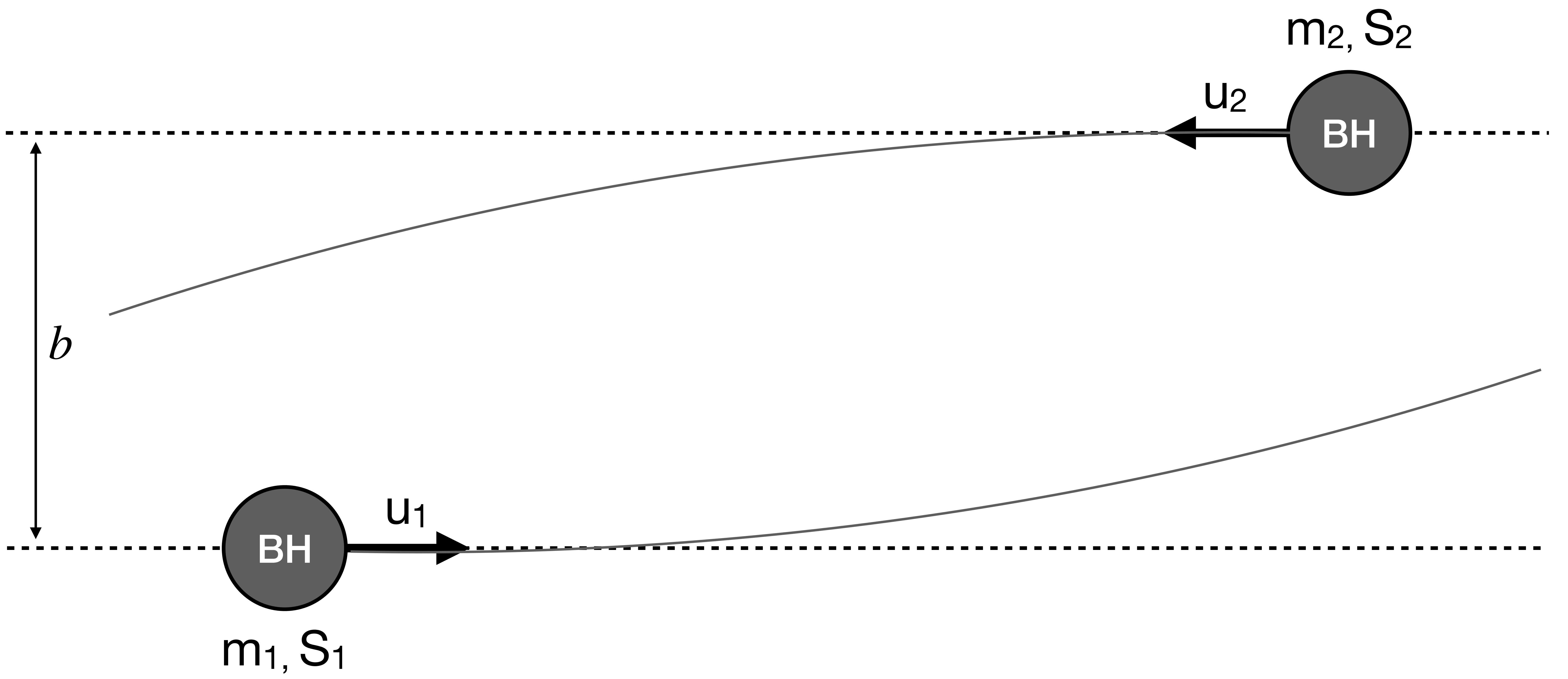
**Spin expansion:**

$$\frac{S}{m} \ll b$$

- **Focus: Classical scattering problem in GR**

⟶ Can we describe this problem using the Scattering Amplitudes used in QFT?

# 2. Scattering Amplitudes and Observables




**Weak field expansion:**

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_{Pl}} h_{\mu\nu}$$

**PM expansion:**

$$R_s \ll b$$

**Spin expansion:**

$$\frac{S}{m} \ll b$$

- **Focus: Classical scattering problem in GR**

⟶ Can we describe this problem using the Scattering Amplitudes used in QFT?

**Yes!** ⟶ Use of the **KMOC formalism**

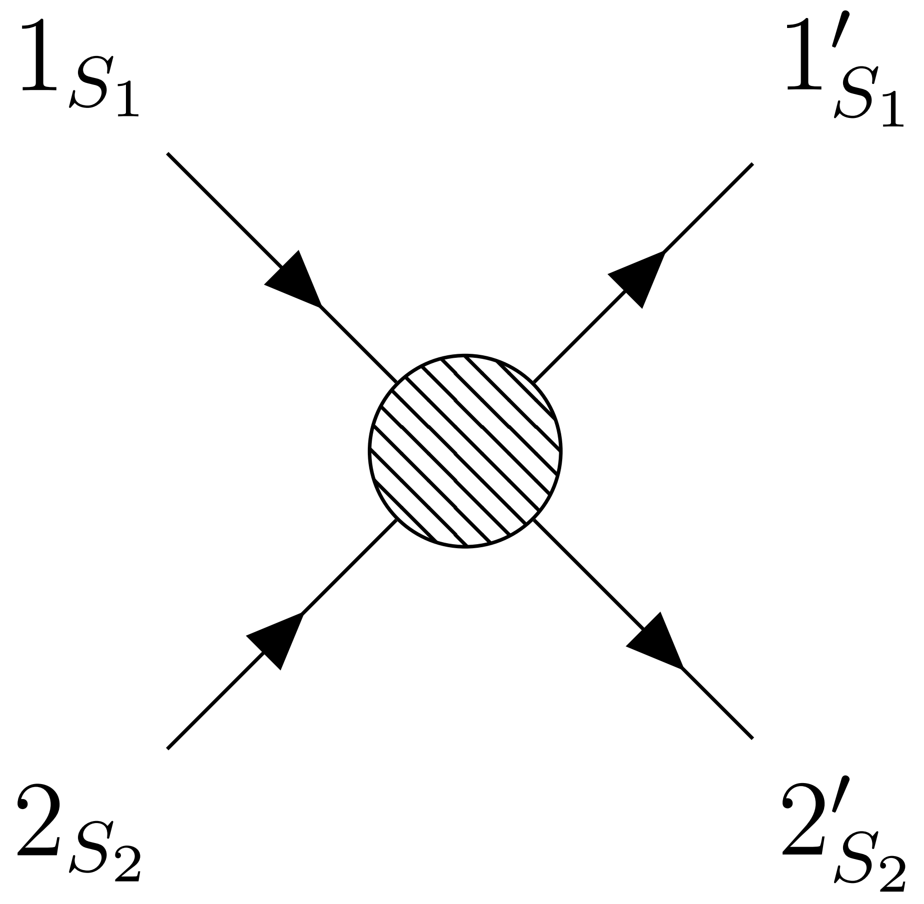
# KMOC (Kosower Maybee O'Connell) formalism

JHEP 02 (2019) 137 [Kosower, Maybee, O'Connell]
Phys.Rev.D 106 (2022) 5, 0567007 [Cristofoli, Gonzo, Kosower, O'Connell]

Idea: Relate **Scattering Amplitudes** directly to **classical observables**

⟶ Extract the classical piece of the amplitude through an "$\hbar$" counting prescription

e.g.:




⟶

Classical Impulse:
$$\Delta p_1^\mu = p^\mu_{1,fin.} - p^\mu_{1,in.}$$

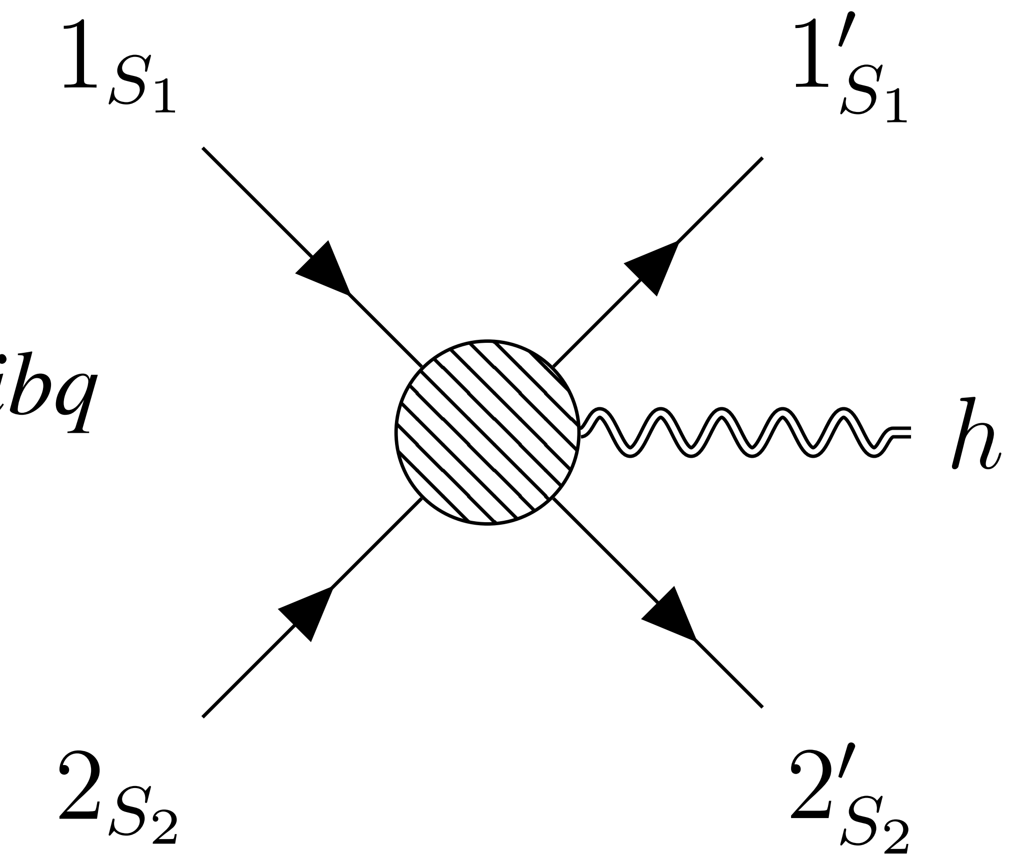
At leading order: $\Delta p_1^{\mu,LO} = \frac{i}{4}\left\langle\left\langle \hbar^2 \int \hat{d}^4 q \hat{\delta}(q \cdot p_1)\hat{\delta}(q \cdot p_2) e^{-ib\cdot q} q^\mu \mathcal{M}^{LO}(p_1, p_2 \to p_1 + \hbar q, p_2 - \hbar q)\right\rangle\right\rangle$

# Waveforms at leading order:

strain

on-shell measure

$$h_{GR}(t) \equiv h_+ \pm i h_\times \sim \int d\omega e^{-i\omega t} \int d\Phi(q) e^{-ibq}$$



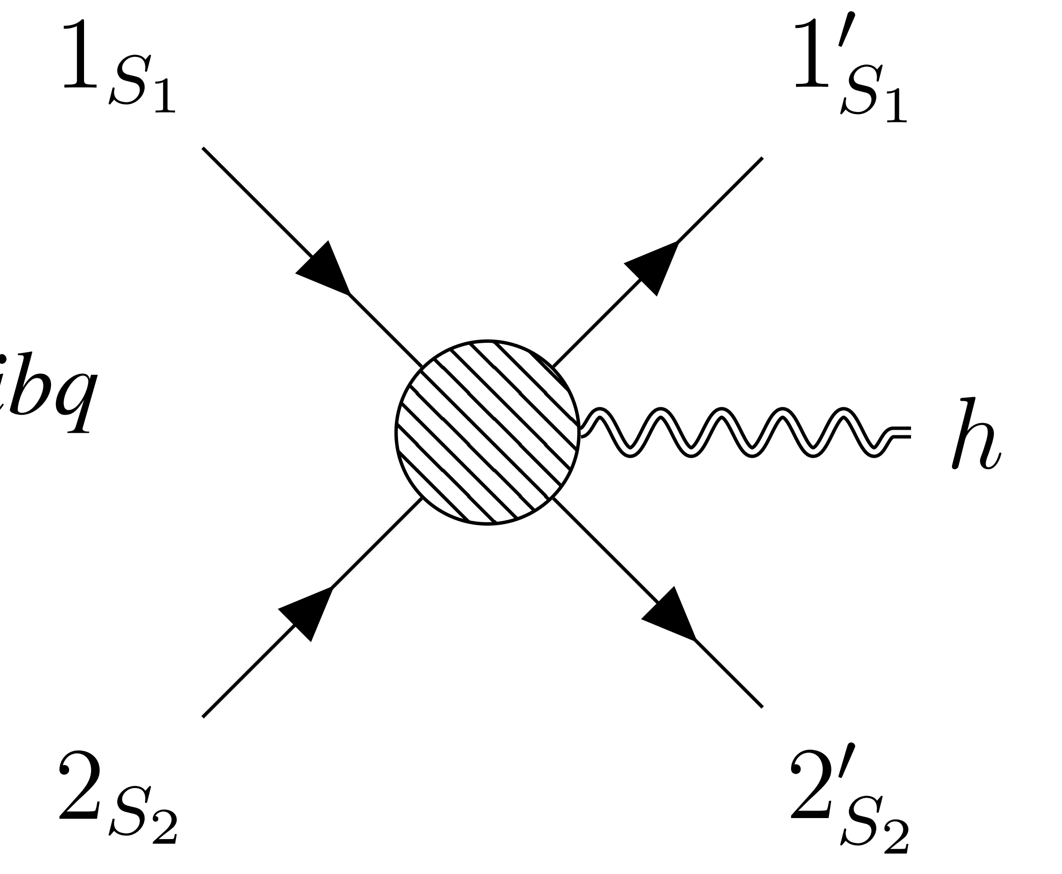

$\mathscr{M}_5^{\mathrm{cl}}(q,k)\,|_{k^\mu=\omega n^\mu}$

# Waveforms at leading order:

strain

on-shell measure

$$h_{GR}(t) \equiv h_+ \pm i h_\times \sim \int d\omega e^{-i\omega t} \int d\Phi(q) e^{-ibq}$$

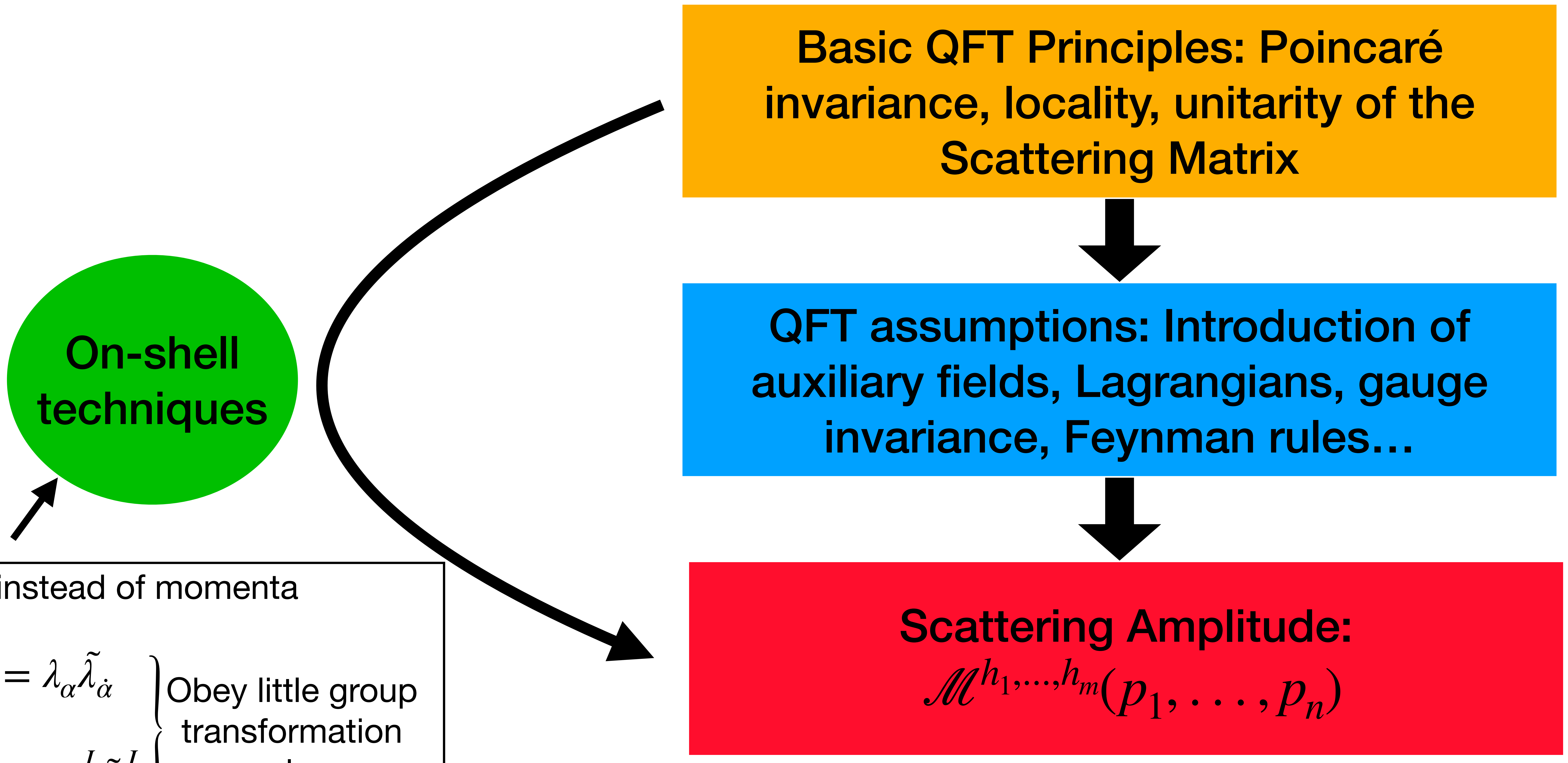



$$\mathscr{M}_5^{\mathrm{cl}}(q,k)\,|_{k^\mu=\omega n^\mu}$$

But, **why?**

- Computations organized in a **perturbative expansion** using a **simple algorithm**.
- **Analytic results**, in places where either PN approximations or NR was used before.
- Can exploit many **modern techniques used in particle physics** to simplify the problem.
- Can straightforwardly **extend to beyond GR predictions**

# On-shell techniques*:




Use spinors as variables instead of momenta

Massless spinors: $p_\mu \sigma^\mu_{\alpha\dot\alpha} = \lambda_\alpha \tilde\lambda_{\dot\alpha}$

Massive spinors: $p_\mu \sigma^\mu_{\alpha\dot\alpha} = \varepsilon_{IJ} \chi^I_\alpha \tilde\chi^J_{\dot\alpha}$

} Obey little group transformation rules

$\langle ij\rangle = \varepsilon_{\alpha\beta}\lambda^\alpha_i \lambda^\beta_j \quad , \quad [ij] = \varepsilon_{\dot\alpha\dot\beta}\tilde\lambda^{\dot\alpha}_i \tilde\lambda^{\dot\beta}_j$

$\langle \mathbf{ij}\rangle = \langle i^I j^J\rangle \quad , \quad [\mathbf{ij}] = [i^I j^J]$

*See yesterday's talk by Adam Falkowski

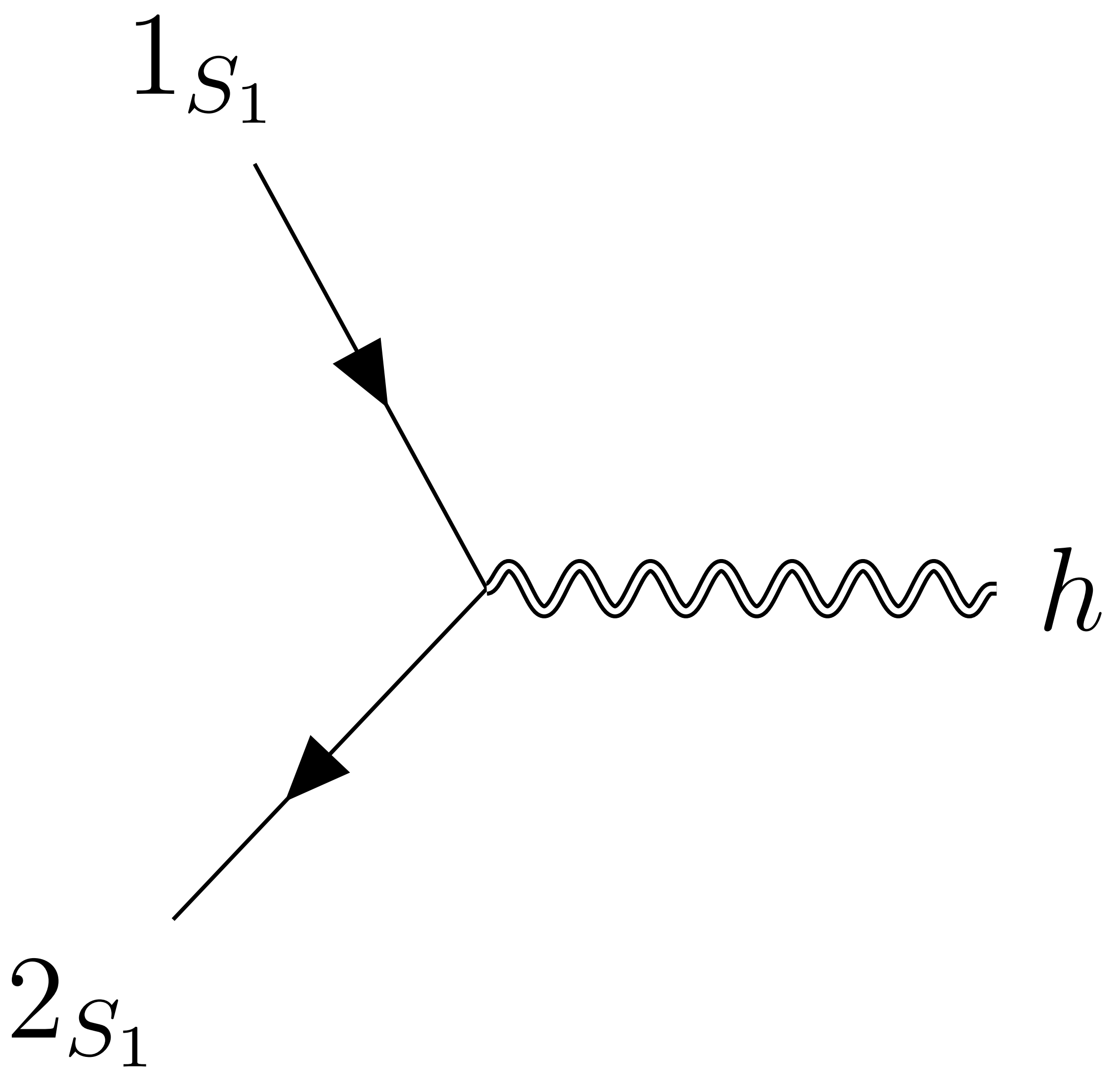
# On-shell techniques:


Idea: Build the **on-shell 3-point amplitudes** of the theory

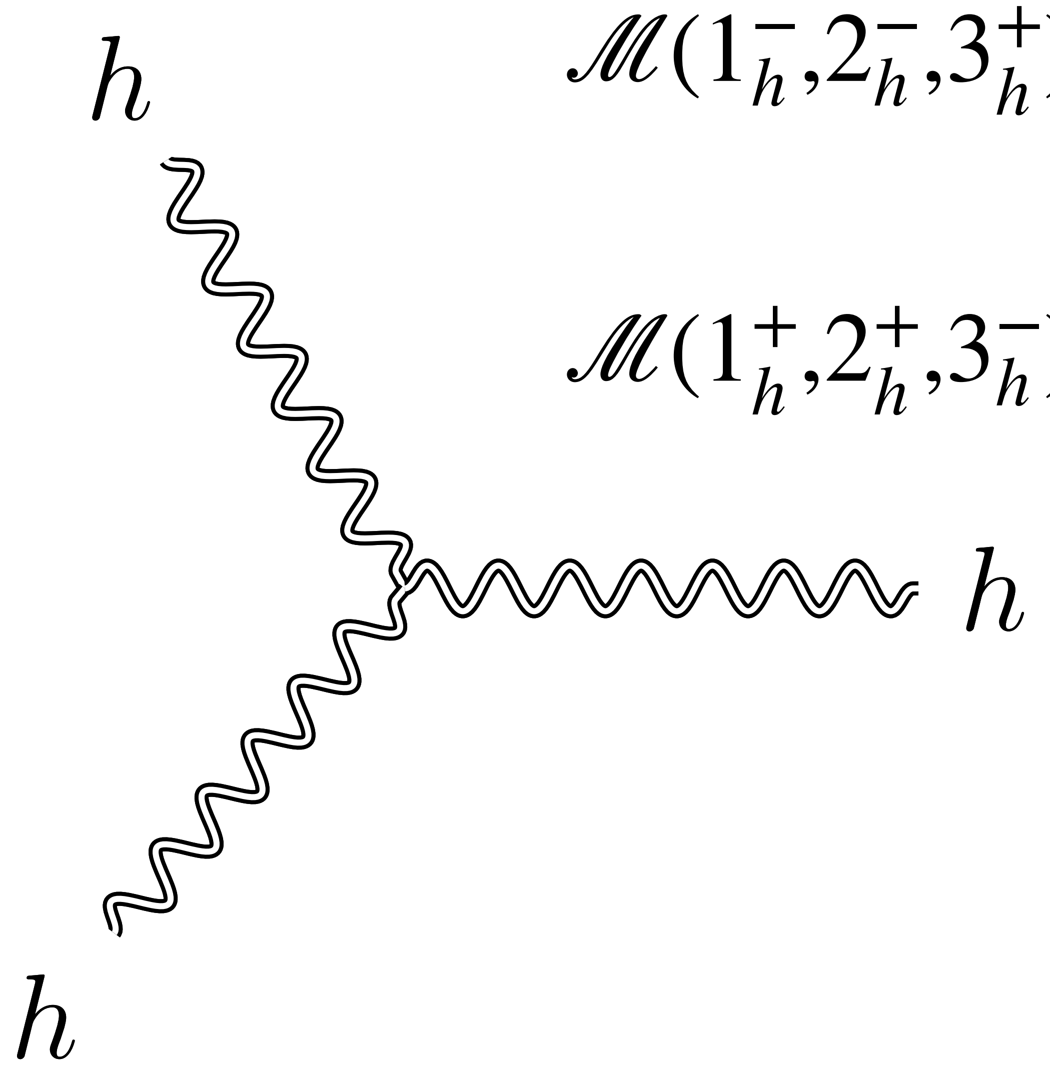
e.g.: **Spinning matter in GR**

$$\mathcal{M}[1_\Phi 2_{\bar{\Phi}} 3_h^-] = -\frac{\langle 3|p_1|\tilde{\zeta}]^2}{M_{Pl}[3\tilde{\zeta}]^2}\frac{[\mathbf{21}]^{2S}}{m^{2S}},$$

$$\mathcal{M}[1_\Phi 2_{\bar{\Phi}} 3_h^+] = -\frac{\langle \zeta|p_1|3]^2}{M_{Pl}\langle 3\zeta\rangle^2}\frac{\langle\mathbf{21}\rangle^{2S}}{m^{2S}},$$




,



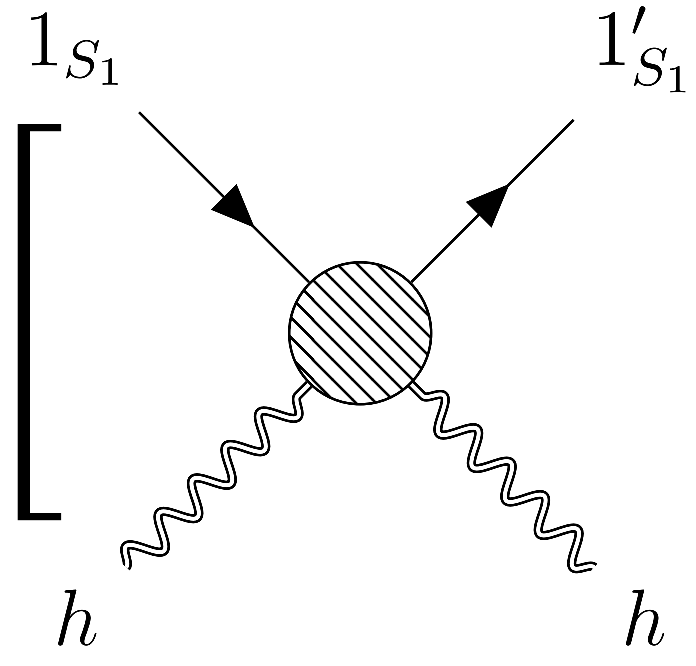

$$\mathcal{M}(1_h^-, 2_h^-, 3_h^+) = -\frac{1}{M_{Pl}}\frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 23\rangle^2}$$

$$\mathcal{M}(1_h^+, 2_h^+, 3_h^-) = -\frac{1}{M_{Pl}}\frac{[12]^6}{[13]^2[23]^2}.$$

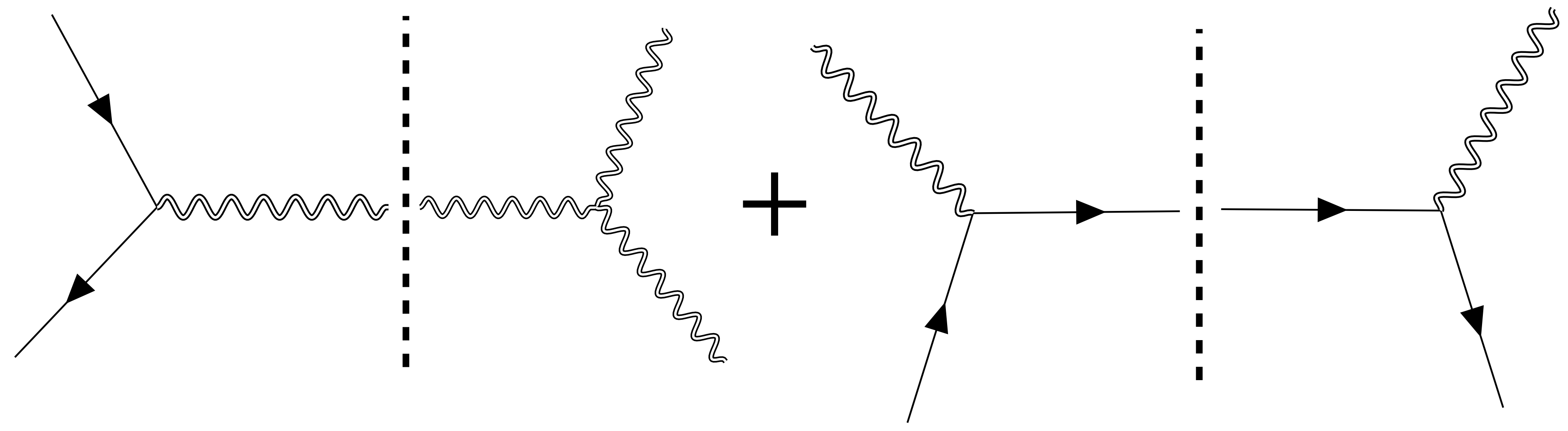
→ Build higher-point amplitudes from their **residues** at kinematic poles in the **complex** plane (up to contact terms)

e.g.:

$$\sum_{i=2,3,4} \text{Res}_{(p_1+p_i)^2\to 0}\Big[$$




$$\Big]\Big|_{tree} = -\Big[$$



$$+(t\leftrightarrow u)\Big]$$

# 3. Scalar-tensor theories: Examples, compact objects, scalar hair and scattering waveforms

- **Scalar-tensor theories** have long stood as a promising avenue to study **extensions of GR**
- They consist of gravity theories with the introduction of an additional **massless scalar** degree of freedom

$$S_{GR}[g_{\mu\nu}] \rightarrow S_{ST}[g_{\mu\nu}, \phi]$$

# 3. Scalar-tensor theories: Examples, compact objects, scalar hair and scattering waveforms

→ **Scalar-tensor theories** have long stood as a promising avenue to study **extensions of GR**

→ They consist of gravity theories with the introduction of an additional **massless scalar** degree of freedom

$$S_{GR}[g_{\mu\nu}] \rightarrow S_{ST}[g_{\mu\nu}, \phi]$$

**Example: Scalar Gauss-Bonnet and Dynamical Chern Simons gravity**

$$S = S_{EH}[R] + S_{SGB,DCS}[\phi, g_{\mu\nu}] + S_m[\Psi_m, \mathscr{A}(\phi) g_{\mu\nu}] \quad ,$$

- $S_{EH} = \int d^4x \frac{M_{Pl}^2}{2} \sqrt{-g} R$
- $S_{SGB,DCS} = \int d^4x \sqrt{-g} \left[ M_{Pl} \left( \frac{\alpha}{\Lambda^2} f(\phi) \mathscr{G} + \frac{\tilde{\alpha}}{\Lambda^2} \phi R \tilde{R} \right) + \frac{1}{2} (\partial^\mu \phi \partial_\mu \phi) \right]$

# 3. Scalar-tensor theories: Examples, compact objects, scalar hair and scattering waveforms

→ **Scalar-tensor theories** have long stood as a promising avenue to study **extensions of GR**

→ They consist of gravity theories with the introduction of an additional **massless scalar** degree of freedom

$$S_{GR}[g_{\mu\nu}] \rightarrow S_{ST}[g_{\mu\nu}, \phi]$$

**Example: Scalar Gauss-Bonnet and Dynamical Chern Simons gravity**

$$S = S_{EH}[R] + S_{SGB,DCS}[\phi, g_{\mu\nu}] + S_m[\Psi_m, \mathscr{A}(\phi) g_{\mu\nu}] \quad ,$$

$$R^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma} \quad , \quad \tilde{R}^{\mu}{}_{\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\rho\sigma}{}^{\alpha\beta}R^{\mu}{}_{\nu\alpha\beta}$$

$$R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$$

- $S_{EH} = \int d^4x \frac{M_{Pl}^2}{2}\sqrt{-g}R$
- $S_{SGB,DCS} = \int d^4x \sqrt{-g}\left[M_{Pl}\left(\frac{\alpha}{\Lambda^2}f(\phi)\mathscr{G} + \frac{\tilde{\alpha}}{\Lambda^2}\phi R\tilde{R}\right) + \frac{1}{2}(\partial^\mu\phi\partial_\mu\phi)\right]$

# 3. Scalar-tensor theories: Examples, compact objects, scalar hair and scattering waveforms

→ **Scalar-tensor theories** have long stood as a promising avenue to study **extensions of GR**

→ They consist of gravity theories with the introduction of an additional **massless scalar** degree of freedom

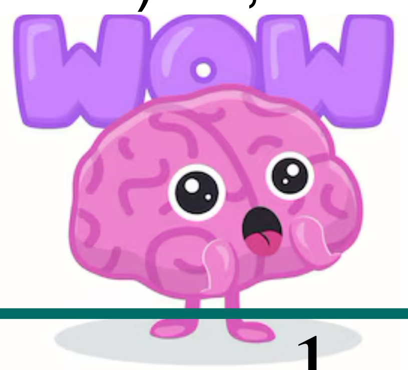
$$S_{GR}[g_{\mu\nu}] \to S_{ST}[g_{\mu\nu}, \phi]$$

**Broad window of resolvability!** arxiv: 2407.16457 [Falkowski, PM]

*Phys.Rev.D* 107 (2023) 4, 044030 [Silva, Ghosh, Buonanno]
arXiv: 2406.13654 [Julié, Pompili, Buonanno]
*Phys.Rev.Lett.* 126 (2021) 18, 181101 [Silva, Holgado, Cárdenas-Avendaño, Yunes]

**Example: Scalar Gauss-Bonnet and Dynamical Chern Simons gravity**

$$S = S_{EH}[R] + S_{SGB,DCS}[\phi, g_{\mu\nu}] + S_m[\Psi_m, \mathcal{A}(\phi) g_{\mu\nu}] \quad ,$$




**Already GW observations are used to constrain them!!!**

- $$S_{EH} = \int d^4x \frac{M_{Pl}^2}{2}\sqrt{-g}R$$
- $$S_{SGB,DCS} = \int d^4x \sqrt{-g}\left[M_{Pl}\left(\frac{\alpha}{\Lambda^2} f(\phi)\mathcal{G} + \frac{\tilde{\alpha}}{\Lambda^2}\phi R\tilde{R}\right) + \frac{1}{2}(\partial^\mu\phi\partial_\mu\phi)\right]$$

$\mathcal{G}$: $R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$

$R\tilde{R}$: $R^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma} \quad , \quad \tilde{R}^{\mu}{}_{\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\rho\sigma}{}^{\alpha\beta}R^{\mu}{}_{\nu\alpha\beta}$

Experimental window:

$$\frac{\sqrt{\alpha}}{\Lambda} \lesssim 0.22km \quad , \quad \frac{\sqrt{\tilde{\alpha}}}{\Lambda} \lesssim 9.5km$$

# Scalar hair in scalar-tensor theories:

Compact objects can acquire scalar hair in ST theories ⟶ Exactly the case for SGB and DCS!

Non-trivial configuration of the field $\phi \to$ Scalar hair

BH solution in ST theory

Far zone

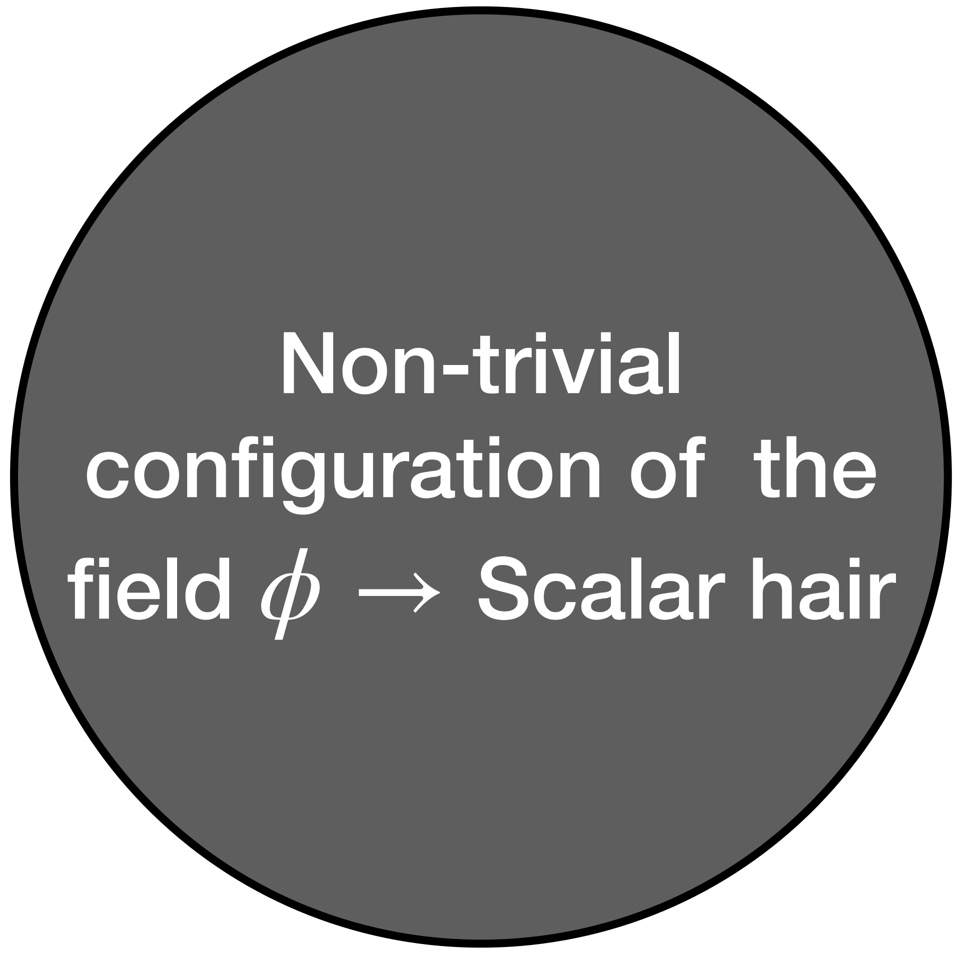
$x \to \infty$

"Monopole hair" "Dipole hair"

$$\phi = \frac{c_i}{r} + \frac{d_i}{r^2} + \dots$$

# Scalar hair in scalar-tensor theories:

Compact objects can acquire scalar hair in ST theories ⟶ Exactly the case for SGB and DCS!




BH solution in ST theory

Far zone

$x \to \infty$

$$\phi = \frac{c_i}{r} + \frac{d_i}{r^2} + \dots$$

"Monopole hair" ($c_i$) "Dipole hair" ($d_i$)

Neglect it, heavily suppressed

**We model the BH as a point-particle interacting with the scalar field in a ST fashion**

Most general effective metric that respects causality is:

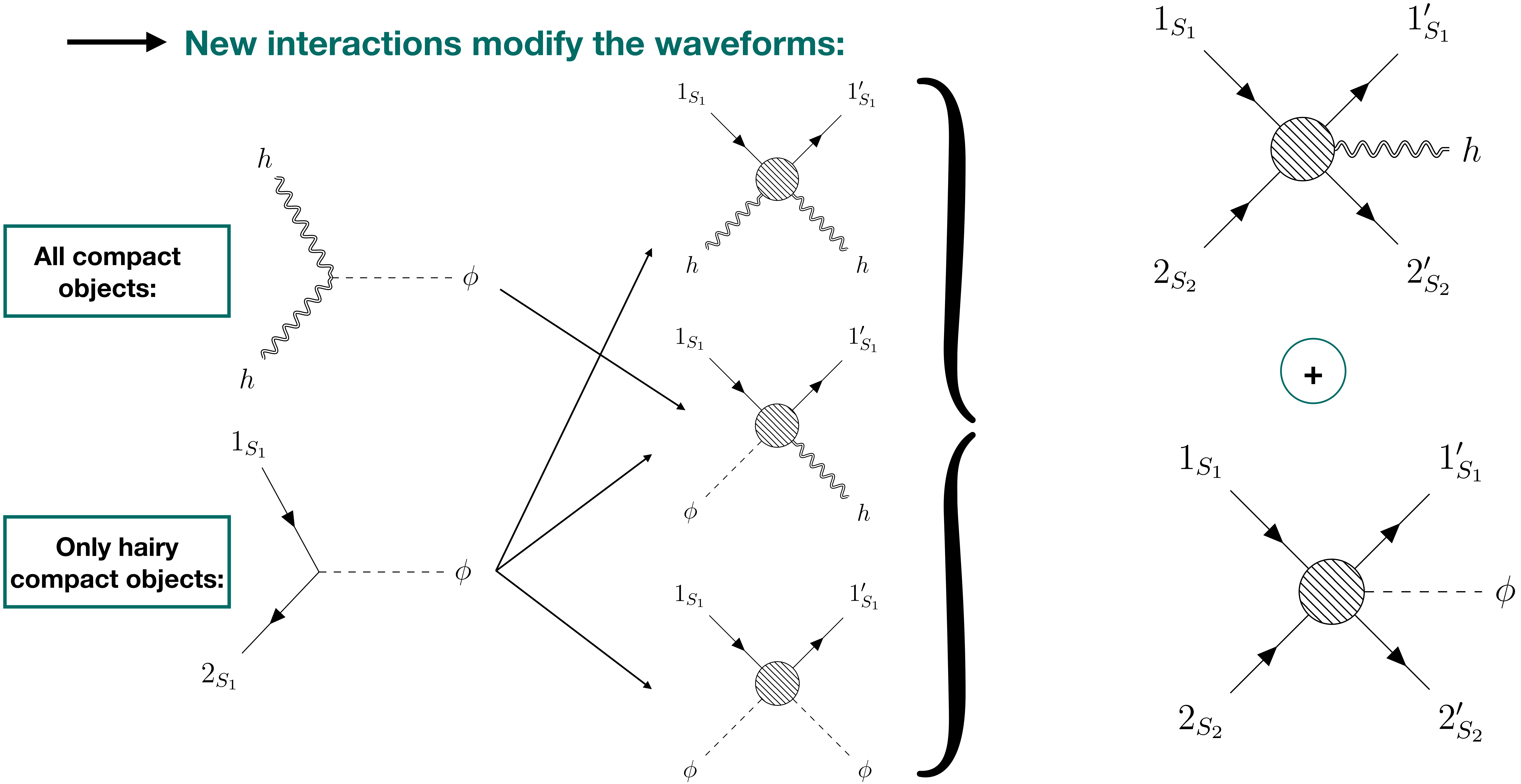
$$\tilde{g}_{\mu\nu} = \underbrace{\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] g_{\mu\nu}}_{\text{Conformal coupling}} + \underbrace{D\left(\frac{\phi}{M_{Pl}}\right)\frac{D_\mu\phi D_\nu\phi}{M_{Pl}^2\Lambda^2}}_{\text{Disformal coupling}},$$

**Generate 3-point amplitudes for arbitrary spinning BH!**

$$\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] \approx 1 + c\frac{\phi}{M_{Pl}}$$

# Corrections to Waveforms

→ **New interactions modify the waveforms:**



# Results for no-hair compact objects:

## LO scalar waveform for scalar Gauss-Bonnet:

$$W_\phi = \frac{m_1 m_2}{8\pi^2 b^3 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 \sqrt{\gamma^2-1}} \Bigg( \alpha \Bigg\{ -\frac{d^2}{dz^2}\left[\frac{1}{\sqrt{z^2+1}} \frac{2(\gamma^2-1)^2(\hat{u}_2 n)^2\left[\gamma(\hat{u}_2 n)-(\hat{u}_1 n)+(\tilde{b}n)z\right]}{\left[\gamma(\hat{u}_2 n)-(\hat{u}_1 n)+(\tilde{b}n)z\right]^2+(\tilde{v}n)^2(z^2+1)}\right] + \left[(\hat{u}_1 n)+\gamma\left(2\gamma^2-3\right)(\hat{u}_2 n)\right]\frac{2z^2-1}{(z^2+1)^{5/2}} + \left(2\gamma^2-1\right)(\tilde{b}n)\frac{3z}{(z^2+1)^{5/2}} \Bigg\}$$

$$+\frac{2\alpha}{b}\frac{d^3}{dz^3}\mathrm{Re}\left\{\frac{1}{\sqrt{z^2+1}}\frac{(\hat{u}_2 n)-\gamma(\hat{u}_1 n)+\gamma(\tilde{b}n)z+i\gamma(\tilde{v}n)\sqrt{z^2+1}}{\gamma(\hat{u}_2 n)-(\hat{u}_1 n)+(\tilde{b}n)z+i(\tilde{v}n)\sqrt{z^2+1}}\left[z(\tilde{v}n)-i\sqrt{z^2+1}(\tilde{b}n)\right]\times\left[\frac{(a_1 n)}{(\hat{u}_1 n)}-\gamma(a_1\hat{u}_2)-(a_2\hat{u}_1)-(a_1^A-a_2^A)(z\tilde{b}^A+i\sqrt{z^2+1}\tilde{v}^A)\right]\right\}$$

$$-\sqrt{\gamma^2-1}\frac{C_1}{b}\frac{d^3}{dz^3}\left\{\frac{1}{\sqrt{z^2+1}}\mathrm{Re}\left[\left(z\left[(\tilde{v}n)a_1^A+(a_1\tilde{v})n^A\right]-i\sqrt{z^2+1}\left[(\tilde{b}n)a_1^A+(a_1\tilde{b})n^A\right]\right)\times\left(\hat{u}_2^A-\gamma\hat{u}_1^A+\gamma\left[z\tilde{b}^A+i\sqrt{z^2+1}\tilde{v}^A\right]\right)\right]\right\}\Bigg)\Bigg|_{z=T_1} + (1\leftrightarrow 2)+\mathcal{O}(a^2)\,.$$

# Results for no-hair compact objects:

## LO scalar waveform for scalar Gauss-Bonnet:

$$
\begin{aligned}
W_\phi = \frac{m_1 m_2}{8\pi^2 b^3 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 \sqrt{\gamma^2-1}} \Bigg( \alpha \Bigg\{ -\frac{d^2}{dz^2}\Bigg[ \frac{1}{\sqrt{z^2+1}} \frac{2(\gamma^2-1)^2 (\hat{u}_2 n)^2 \big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]}{\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]^2 + (\tilde{v} n)^2 (z^2+1)} \Bigg] + \big[(\hat{u}_1 n) + \gamma\big(2\gamma^2-3\big)(\hat{u}_2 n)\big] \frac{2z^2-1}{(z^2+1)^{5/2}} + \big(2\gamma^2-1\big)(\tilde{b} n) \frac{3z}{(z^2+1)^{5/2}} \Bigg\} \\
+ \frac{2\alpha}{b} \frac{d^3}{dz^3} \mathrm{Re} \Bigg\{ \frac{1}{\sqrt{z^2+1}} \frac{(\hat{u}_2 n) - \gamma(\hat{u}_1 n) + \gamma(\tilde{b} n) z + i\gamma(\tilde{v} n)\sqrt{z^2+1}}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z + i(\tilde{v} n)\sqrt{z^2+1}} \Big[ z(\tilde{v} n) - i\sqrt{z^2+1}(\tilde{b} n) \Big] \times \Bigg[ \frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1 \hat{u}_2) - (a_2 \hat{u}_1) - (a_1^A - a_2^A)(z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A) \Bigg] \Bigg\} \\
-\sqrt{\gamma^2-1} \frac{C_1}{b} \frac{d^3}{dz^3} \Bigg\{ \frac{1}{\sqrt{z^2+1}} \mathrm{Re} \Bigg[ \Bigg( z\big[(\tilde{v} n) a_1^A + (a_1 \tilde{v}) n^A\big] - i\sqrt{z^2+1}\big[(\tilde{b} n) a_1^A + (a_1 \tilde{b}) n^A\big] \Bigg) \times \Bigg( \hat{u}_2^A - \gamma \hat{u}_1^A + \gamma\big[z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A\big] \Bigg) \Bigg] \Bigg\} \Bigg) \Bigg|_{z=T_1} + (1 \leftrightarrow 2) + \mathcal{O}(a^2) .
\end{aligned}
$$

**Connect to observables: Power emitted in scalar radiation**

$$\frac{dP_\phi}{d\Omega} = (\partial_t W_\phi)^2$$

$$\frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^0)} \sim \frac{\beta^6}{b^8} \quad \xrightarrow{\text{For closed orbits}} \quad \frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^0)} \sim \beta^{22}$$

$$\frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^1)} \sim \frac{\beta^4}{b^{10}} \quad \xrightarrow{\text{For closed orbits}} \quad \frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^1)} \sim \beta^{24}$$

$$\beta = \frac{v}{c}$$

# Results for no-hair compact objects:

## LO scalar waveform for scalar Gauss-Bonnet:

$$W_\phi = \frac{m_1 m_2}{8\pi^2 b^3 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 \sqrt{\gamma^2-1}} \Bigg(\alpha\Bigg\{ -\frac{d^2}{dz^2}\Bigg[\frac{1}{\sqrt{z^2+1}} \frac{2(\gamma^2-1)^2(\hat{u}_2 n)^2\big[\gamma(\hat{u}_2 n)-(\hat{u}_1 n)+(\tilde{b} n)z\big]}{\big[\gamma(\hat{u}_2 n)-(\hat{u}_1 n)+(\tilde{b} n)z\big]^2+(\tilde{v} n)^2(z^2+1)}\Bigg] + \big[(\hat{u}_1 n)+\gamma\big(2\gamma^2-3\big)(\hat{u}_2 n)\big]\frac{2z^2-1}{(z^2+1)^{5/2}} + \big(2\gamma^2-1\big)(\tilde{b} n)\frac{3z}{(z^2+1)^{5/2}}\Bigg\}$$

$$+\frac{2\alpha}{b}\frac{d^3}{dz^3}\mathrm{Re}\Bigg\{\frac{1}{\sqrt{z^2+1}}\frac{(\hat{u}_2 n)-\gamma(\hat{u}_1 n)+\gamma(\tilde{b} n)z+i\gamma(\tilde{v} n)\sqrt{z^2+1}}{\gamma(\hat{u}_2 n)-(\hat{u}_1 n)+(\tilde{b} n)z+i(\tilde{v} n)\sqrt{z^2+1}}\Big[z(\tilde{v} n)-i\sqrt{z^2+1}(\tilde{b} n)\Big]\times\Big[\frac{(a_1 n)}{(\hat{u}_1 n)}-\gamma(a_1\hat{u}_2)-(a_2\hat{u}_1)-(a_1^A-a_2^A)(z\tilde{b}^A+i\sqrt{z^2+1}\,\tilde{v}^A)\Big]\Bigg\}$$

$$-\sqrt{\gamma^2-1}\,\frac{C_1}{b}\frac{d^3}{dz^3}\Bigg\{\frac{1}{\sqrt{z^2+1}}\mathrm{Re}\Bigg[\Bigg(z\big[(\tilde{v} n)a_1^A+(a_1\tilde{v})n^A\big]-i\sqrt{z^2+1}\,\big[(\tilde{b} n)a_1^A+(a_1\tilde{b})n^A\big]\Bigg)\times\Bigg(\hat{u}_2^A-\gamma\hat{u}_1^A+\gamma\big[z\tilde{b}^A+i\sqrt{z^2+1}\,\tilde{v}^A\big]\Bigg)\Bigg]\Bigg\}\Bigg)\Bigg|_{z=T_1} + (1\leftrightarrow 2)+\mathcal{O}(a^2)\,.$$

**Connect to observables: Power emitted in scalar radiation**

$$\frac{dP_\phi}{d\Omega} = (\partial_t W_\phi)^2$$

$$\frac{dP_\phi}{d\Omega}\Big|_{\mathcal{O}(a^0)} \sim \frac{\beta^6}{b^8} \xrightarrow{\text{For closed orbits}} \frac{dP_\phi}{d\Omega}\Big|_{\mathcal{O}(a^0)} \sim \beta^{22}$$

**Bigger suppression** compared to $\beta^8$ previously computed with GR methods

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

$$\frac{dP_\phi}{d\Omega}\Big|_{\mathcal{O}(a^1)} \sim \frac{\beta^4}{b^{10}} \xrightarrow{\text{For closed orbits}} \frac{dP_\phi}{d\Omega}\Big|_{\mathcal{O}(a^1)} \sim \beta^{24}$$

$$\beta = \frac{v}{c}$$

# Results for hairy compact objects:

## LO Scalar Waveforms-Spinless part:

$$W_\phi^{(0)} = -\frac{m_1 m_2}{32\pi^2 M_{Pl}^3 \sqrt{\gamma^2-1}(\hat{u}_1 n)^2 b}\frac{1}{\sqrt{1+T_1^2}}\left\{ \frac{(\gamma^2-1)[c_1(\hat{u}_2 n)^2 + c_2(\hat{u}_1 n)^2][\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)T_1]}{[-(\hat{u}_1 n) + y(\hat{u}_2 n) + (\tilde{b}n)T_1]^2 + (\tilde{v}n)^2(1+T_1^2)} \right.$$

$$\left. -\frac{c_1}{2}\frac{(\hat{u}_1 n) + (2\gamma^2-3)y(\hat{u}_2 n) - (2\gamma^2-1)(\tilde{b}n)T_1}{y^2-1} + \frac{C_1^{(0)}}{2}c_2(\hat{u}_1 n) \right\} + (1 \leftrightarrow 2)\,.$$

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{1}{b^4}$$

**For closed orbits** →

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \beta^8$$

→ **Agreement with existing PN results for SGB!**

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

# Results for hairy compact objects:

scalar "monopole" charges $c_i = c_i(m_i, a_i, \frac{\alpha}{\Lambda^2})$

## LO Scalar Waveforms-Spinless part:

$$W_\phi^{(0)} = -\frac{m_1 m_2}{32\pi^2 M_{Pl}^3 \sqrt{\gamma^2-1}(\hat{u}_1 n)^2 b}\frac{1}{\sqrt{1+T_1^2}}\left\{\frac{(\gamma^2-1)[c_1(\hat{u}_2 n)^2 + c_2(\hat{u}_1 n)^2][\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)T_1]}{[-(\hat{u}_1 n) + y(\hat{u}_2 n) + (\tilde{b}n)T_1]^2 + (\tilde{v}n)^2(1+T_1^2)}\right.$$

$$\left. -\frac{c_1}{2}\frac{(\hat{u}_1 n) + (2\gamma^2-3)y(\hat{u}_2 n) - (2\gamma^2-1)(\tilde{b}n)T_1}{y^2-1} + \frac{C_1^{(0)}}{2}c_2(\hat{u}_1 n)\right\} + (1 \leftrightarrow 2)\,.$$

Zeroth spin order contact term

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{1}{b^4}$$

For closed orbits

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \beta^8$$

**Agreement with existing PN results for SGB!**

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

e.g.: $c_i \sim \alpha + \mathcal{O}(a^2)$ Phys.Rev.D 100 (2019) 10, 104061 [Julié, Berti]

# Results for hairy compact objects:

scalar “monopole” charges $c_i = c_i(m_i, a_i, \frac{\alpha}{\Lambda^2})$ → Can be matched to existing results if it’s secondary hair!

## LO Scalar Waveforms-Spinless part:

$$W_\phi^{(0)} = -\frac{m_1 m_2}{32\pi^2 M_{Pl}^3 \sqrt{\gamma^2-1}(\hat{u}_1 n)^2 b}\frac{1}{\sqrt{1+T_1^2}}\Bigg\{\frac{(\gamma^2-1)[c_1(\hat{u}_2 n)^2 + c_2(\hat{u}_1 n)^2][\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)T_1]}{[-(\hat{u}_1 n) + y(\hat{u}_2 n) + (\tilde{b}n)T_1]^2 + (\tilde{v}n)^2(1+T_1^2)}$$

$$-\frac{c_1}{2}\frac{(\hat{u}_1 n) + (2\gamma^2-3)y(\hat{u}_2 n) - (2\gamma^2-1)(\tilde{b}n)T_1}{y^2-1} + \frac{C_1^{(0)}}{2}c_2(\hat{u}_1 n)\Bigg\} + (1 \leftrightarrow 2)\,.$$

Zeroth spin order contact term

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{1}{b^4} \quad \xrightarrow{\text{For closed orbits}} \quad \left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \beta^8 \quad \longrightarrow$$

**Agreement with existing PN results for SGB!**

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

# Results for hairy compact objects:

## LO Gravitational Waveforms-Spinless part:

$$W_h^{(0)} = -\frac{c_1c_2m_1m_2}{512\pi^2M_{Pl}^3\sqrt{\gamma^2-1}(\hat{u}_1n)^2b}\frac{1}{\sqrt{1+T_1^2}}\text{Re}\left\{\frac{(\lambda_n[\gamma\hat{u}_2\sigma+T_1\tilde{b}\sigma+i\sqrt{T_1^2+1}\tilde{v}\sigma]\hat{u}_1\bar{\sigma}\lambda_n)^2}{y(\hat{u}_2n)-(\hat{u}_1n)+T_1(\tilde{b}n)+i\sqrt{T_1^2+1}(\tilde{v}n)}\right\}+(1\leftrightarrow 2)\,.$$

$$\left.\frac{dP_h}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{\beta^2}{b^4}$$

For closed orbits

$$\left.\frac{dP_h}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \beta^{10}$$

**Agreement and expected suppression compared to scalar radiation!**

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

# Results for hairy compact objects:

## LO Gravitational Waveforms-Spinless part:

$$W_h^{(0)} = -\frac{c_1c_2m_1m_2}{512\pi^2M_{Pl}^3\sqrt{\gamma^2-1}(\hat{u}_1n)^2b}\frac{1}{\sqrt{1+T_1^2}}\text{Re}\left\{\frac{(\lambda_n[\gamma\hat{u}_2\sigma + T_1\tilde{b}\sigma + i\sqrt{T_1^2+1}\tilde{v}\sigma]\hat{u}_1\bar{\sigma}\lambda_n)^2}{y(\hat{u}_2n) - (\hat{u}_1n) + T_1(\tilde{b}n) + i\sqrt{T_1^2+1}(\tilde{v}n)}\right\} + (1 \leftrightarrow 2)\,.$$

$$\left.\frac{dP_h}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{\beta^2}{b^4} \xrightarrow{\text{For closed orbits}} \left.\frac{dP_h}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \beta^{10} \longrightarrow$$

**Agreement and expected suppression compared to scalar radiation!**

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]
Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

**Comments:**

- Can show that there are no linear-in-spin corrections to $W_h^{(1)}$ at this order!
- Notice the **simplicity** of the waveform when expressed in the **spinor** language!
- In fact, we also derived that the NR **power** emitted is **exactly the same as in GR** up to setting $c_1c_2 \to 1$
- For reference, $\left.\frac{dP_{h,GR}}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \beta^{10} \longrightarrow$ Interactions impact severely the GW signal!

# Another perspective: The scalar-charge toy model

Recent work has been using a scalar-charge toy model in order to compare the PM and SF approaches.

$$\mathscr{L} \supset \sqrt{-g}\left[-2\sqrt{\pi}m_1 Q\psi\phi_1^2\right]$$

Phys.Rev.D 108 (2023) 2, 024025 [Barack, Bern, Hermann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng]



Comparison of post-Minkowskian and self-force expansions: Scattering in a scalar charge toy model

Leor Barack,[1] Zvi Bern,[2] Enrico Herrmann,[2] Oliver Long,[3,1] Julio Parra-Martinez,[4] Radu Roiban,[5] Michael S. Ruf,[2] Chia-Hsien Shen,[6] Mikhail P. Solon,[2] Fei Teng,[5] and Mao Zeng[7]

[1]*Mathematical Sciences, University of Southampton, Southampton SO17 1BJ, United Kingdom*
[2]*Mani L. Bhaumik Institute for Theoretical Physics, University of California at Los Angeles, Los Angeles, CA 90095, USA*
[3]*Max Planck Institute for Gravitational Physics (Albert Einstein Institute), Am Mühlenberg 1, Potsdam 14476, Germany*
[4]*Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, CA 91125, USA*
[5]*Institute for Gravitation and the Cosmos, Pennsylvania State University, University Park, PA 16802, USA*
[6]*Department of Physics, University of California at San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0319, USA*
[7]*Higgs Centre for Theoretical Physics, University of Edinburgh, James Clerk Maxwell Building, Peter Guthrie Tait Road, Edinburgh, EH9 3FD, United Kingdom*

**Abstract**

We compare numerical self-force results and analytical fourth-order post-Minkowskian (PM) calculations for hyperbolic-type scattering of a point-like particle carrying a scalar charge $Q$ off a Schwarzschild black hole, showing a remarkably good agreement. Specifically, we numerically compute the scattering angle including the full $\mathcal{O}(Q^2)$ scalar-field self-force term (but ignoring the gravitational self-force), and compare with analytical expressions obtained in a PM framework using scattering-amplitude methods. This example provides a nontrivial, high-precision test of both calculation methods, and illustrates the complementarity of the two approaches in the context of the program to provide high-precision models of gravitational two-body dynamics. Our PM calculation is carried out through 4PM order, i.e., including all terms through $\mathcal{O}(Q^2G^3)$. At the fourth post-Minkowskian order the point-particle description involves two a-priori undetermined coefficients, due to contributions from tidal effects in the model under consideration. These coefficients are chosen to align the post-Minkowskian results with the self-force ones.

# Another perspective: The scalar-charge toy model

Recent work has been using a scalar-charge toy model in order to compare the PM and SF approaches.

Phys.Rev.D 108 (2023) 2, 024025 [Barack, Bern, Hermann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng]

$$\mathscr{L} \supset \sqrt{-g}\left[\underbrace{-2\sqrt{\pi}m_1 Q}\psi\phi_1^2\right]$$

Related to our scalar charges $c_i$

scalar

Matter fields

→ The same type of coupling we have!

<u>Recall</u>: $m \to e^{C/2}m$ for any spin!

Our framework can be **mapped** to this model (in the limit of $c_2 \to 0$) and potentially **extend the results** in a consistent manner to incorporate effects that have not been compared before, e.g. **spin effects, gravitational waveform corrections, …**



Comparison of post-Minkowskian and self-force expansions: Scattering in a scalar charge toy model

Leor Barack,[1] Zvi Bern,[2] Enrico Herrmann,[2] Oliver Long,[3,1] Julio Parra-Martinez,[4] Radu Roiban,[5] Michael S. Ruf,[2] Chia-Hsien Shen,[6] Mikhail P. Solon,[2] Fei Teng,[5] and Mao Zeng[7]

[1] *Mathematical Sciences, University of Southampton, Southampton SO17 1BJ, United Kingdom*
[2] *Mani L. Bhaumik Institute for Theoretical Physics, University of California at Los Angeles, Los Angeles, CA 90095, USA*
[3] *Max Planck Institute for Gravitational Physics (Albert Einstein Institute), Am Mühlenberg 1, Potsdam 14476, Germany*
[4] *Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, CA 91125, USA*
[5] *Institute for Gravitation and the Cosmos, Pennsylvania State University, University Park, PA 16802, USA*
[6] *Department of Physics, University of California at San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0319, USA*
[7] *Higgs Centre for Theoretical Physics, University of Edinburgh, James Clerk Maxwell Building, Peter Guthrie Tait Road, Edinburgh, EH9 3FD, United Kingdom*

**Abstract**

We compare numerical self-force results and analytical fourth-order post-Minkowskian (PM) calculations for hyperbolic-type scattering of a point-like particle carrying a scalar charge $Q$ off a Schwarzschild black hole, showing a remarkably good agreement. Specifically, we numerically compute the scattering angle including the full $\mathcal{O}(Q^2)$ scalar-field self-force term (but ignoring the gravitational self-force), and compare with analytical expressions obtained in a PM framework using scattering-amplitude methods. This example provides a nontrivial, high-precision test of both calculation methods, and illustrates the complementarity of the two approaches in the context of the program to provide high-precision models of gravitational two-body dynamics. Our PM calculation is carried out through 4PM order, i.e., including all terms through $\mathcal{O}(Q^2G^3)$. At the fourth post-Minkowskian order the point-particle description involves two a-priori undetermined coefficients, due to contributions from tidal effects in the model under consideration. These coefficients are chosen to align the post-Minkowskian results with the self-force ones.

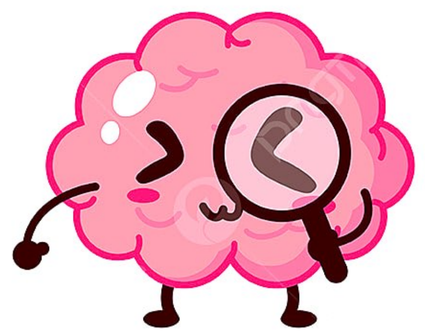
# 4. Outlook

- **Scattering Amplitudes techniques** can be proven to be extremely **useful** in the quest for precision measurements **in the GW era**, probing results to all orders in velocity.

- Computations for **spinning binaries** remarkably **simplify** in the **on-shell** language.

- Recasting already known problems in the amplitudes' language makes the **search for beyond GR effects easier to handle** and essentially the **usual QFT methods can be used**.

# 4. Outlook



- **Scattering Amplitudes techniques** can be proven to be extremely **useful** in the quest for precision measurements **in the GW era**, probing results to all orders in velocity.

- Computations for **spinning binaries** remarkably **simplify** in the **on-shell** language.

- Recasting already known problems in the amplitudes' language makes the **search for beyond GR effects easier to handle** and essentially the **usual QFT methods can be used**.

**What's next?**

- Employing similar methods to **study GR (and beyond) effects**, where current PN/NR results are poor, e.g. dipole hair in DCS.

- Could we employ our formalism to **effectively describe new Physics** related to **scalar particles** from GWs (boson clouds around BHs, soliton and boson stars, light axion-like particles, …) ?

- Dive more deeply into the synergy of QFT and GR and consider other applications as well, e.g. emergence of **couplings in ST theories from integrating out arbitrary spinning heavy particles** - use of **generalized unitarity and analytic properties of the amplitude** (work in progress).

# The universe seems to be extremely loud!

Artwork by Penelope Cowley*

## The universe seems to be extremely loud!

# Thank you for your attention!:)


Artwork by Penelope Cowley*

# Backup slides

# Integrals and variables in waveform parametrization:

Single-pole integrals

$$I_2^{\mu_1\dots\mu_n} = \int \hat{d}w_1 \hat{d}w_2 \hat{\delta}(u_1 w_1)\hat{\delta}(u_2 w_2)\hat{\delta}^{(4)}(w_1 + w_2 - k)\frac{e^{i(b_1 w_1 + b_2 w_2)}}{w_2^2} w_2^{\mu_1}\dots w_2^{\mu_n}$$
$$= e^{ib_1 k}\int \hat{d}^4 q \hat{\delta}(u_1\cdot q - u_1 \cdot k)\hat{\delta}(u_2 \cdot q)\frac{e^{-iq\cdot(b_1-b_2)}}{q^2} q^{\mu_1}\dots q^{\mu_n}$$

Double-pole integrals

$$J_2^{\mu_1\dots\mu_n} = \int \hat{d}w_1 \hat{d}w_2 \hat{\delta}(u_1 w_1)\hat{\delta}(u_2 w_2)\hat{\delta}^{(4)}(w_1 + w_2 - k)\frac{e^{i(b_1 w_1 + b_2 w_2)}}{w_2^2 w_1^2} w_2^{\mu_1}\dots w_2^{\mu_n}$$
$$= e^{ib_1 k}\int \hat{d}^4 q \hat{\delta}(u_1\cdot q - u_1 \cdot k)\hat{\delta}(u_2 \cdot q)\frac{e^{-iq\cdot(b_1-b_2)}}{q^2(k-q)^2} q^{\mu_1}\dots q^{\mu_n}$$

Parametrization: $q^\mu = z_1 u_1^\mu + z_2 u_2^\mu + z_v \tilde{v}^\mu + z_b \tilde{b}^\mu$

$$v^\mu \equiv \varepsilon^{\mu\alpha\beta\rho} u_{1\alpha} u_{2\beta} \tilde{b}_\rho \quad , \quad \tilde{v}^\mu = \frac{v^\mu}{\sqrt{-v^2}}$$

$$b \equiv \sqrt{-(b_1 - b_2)^2} \quad , \quad \tilde{b}^\mu = \frac{b_1^\mu - b_2^\mu}{b}$$

$$\hat{u}_i = \frac{u_i}{\sqrt{\gamma^2 - 1}}$$

Then follow a residue approach following the logic of

Phys.Rev.D 110 (2024) 4, L041502 [De Angelis, Novichkov, Gonzo]

# Integrals and variables in waveform parametrization:

$$I_2^{\mu_1\cdots\mu_n} = -\frac{\pi e^{ib_1 k}(\hat{u}_1 k)^n}{\sqrt{\gamma^2-1}}\int_{-\infty}^{\infty}\frac{dz e^{i(\hat{u}_1 k)bz}}{\sqrt{z^2+1}}\text{Re}\left\{\left[\gamma\hat{u}_2^{\mu_1} - \hat{u}_1^{\mu_1} + z\tilde{b}^{\mu_1} + i\sqrt{z^2+1}\,\tilde{v}^{\mu_1}\right]\left[\mu_1\to\mu_2\right]\ldots\left[\mu_1\to\mu_n\right]\right\}.$$

$$J_2^{\mu_1\cdots\mu_n} = \frac{\pi}{2\sqrt{\gamma^2-1}}\int_{-\infty}^{\infty}\frac{dz}{\sqrt{z^2+1}}\left\{(\hat{u}_1 k)^{n-1}e^{ib_1 k + i(\hat{u}_1 k)bz}\text{Re}\left[\frac{\left[\gamma\hat{u}_2^{\mu_1} - \hat{u}_1^{\mu_1} + z\tilde{b}^{\mu_1} + i\sqrt{z^2+1}\,\tilde{v}^{\mu_1}\right]\ldots\left[\mu_1\to\mu_n\right]}{\gamma(\hat{u}_2 k) - (\hat{u}_1 k) + (\tilde{b}k)z + i(\tilde{v}k)\sqrt{z^2+1}}\right]\right.$$
$$\left. + (\hat{u}_2 k)^{n-1}e^{ib_2 k + i(\hat{u}_2 k)bz}\text{Re}\left[\frac{\left[\frac{k^{\mu_1}}{\hat{u}_2 k} + \hat{u}_2^{\mu_1} - \gamma\hat{u}_1^{\mu_1} + z\tilde{b}^{\mu_1} - i\sqrt{z^2+1}\,\tilde{v}^{\mu_1}\right]\ldots\left[\mu_1\to\mu_n\right]}{\gamma(\hat{u}_1 k) - (\hat{u}_2 k) - (\tilde{b}k)z + i(\tilde{v}k)\sqrt{z^2+1}}\right]\right\}.$$

# Resolvability:

$$\alpha_g \sim \frac{m^2}{M_{Pl}^2} = mR_s$$

Scalar radiation for no-hair objects is resolvable if: $\alpha_g \left(\frac{R_s}{b}\right)^2 \frac{1}{\Lambda^4 b^4} \gg 1,$

For example, for $\Lambda \simeq 2 \times 10^{-18} GeV$ and $m \simeq M_\odot \simeq 10^{57} \sim GeV$ (corresponding to $R_s \simeq 1.5 \times 10^{19} GeV^{-1}$) we have $b_{\max} \sim 10^{11} R_s$

Linear in spin order effects for the same observable: $\alpha_g \left(\frac{R_s}{b}\right)^2 \frac{1}{\Lambda^4 b^4} \left(\frac{S}{bm}\right) \gg 1$

For our benchmark point, $m \sim M_\odot, \Lambda \sim 0.1 \mathrm{km}^{-1}$, this implies $S \gtrsim 10^6 (b/R_s)^7$, with $S_{\max} \sim 10^{76}$ for Kerr black holes in GR.

# More results:

## The scalar waveform for dynamical Chern-Simons:

$$
\begin{aligned}
W_{\tilde{\phi}} = \frac{m_1 m_2}{8\pi^2 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 b^3} \Bigg( & 2\tilde{\alpha}(\tilde{v}n) \frac{d^2}{dz^2} \left\{ \frac{1}{\sqrt{z^2+1}} \left[ \gamma z - (\gamma^2-1)(\hat{u}_2 n) \frac{z\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n)\big] - (\tilde{b}n)}{\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z\big]^2 + (\tilde{v}n)^2(z^2+1)} \right] \right\} \\
& + \frac{\tilde{\alpha}}{b\sqrt{\gamma^2-1}} \mathrm{Re} \frac{d^3}{dz^3} \left\{ \frac{1}{\sqrt{z^2+1}} \left( \frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1 \hat{u}_2) - (a_2 \hat{u}_1) + (a_2^A - a_1^A)\big[z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A\big] \right) \left( \frac{2(\gamma^2-1)^2(\hat{u}_2 n)^2}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z + i(\tilde{v}n)\sqrt{z^2+1}} - (\hat{u}_1 n) - \gamma\big(2\gamma^2-3\big)(\hat{u}_2 n) + \big(2\gamma^2-1\big)\big[z(\tilde{b}n) + i\sqrt{z^2+1}(\tilde{v}n)\big] \right) \right\} \\
& - \frac{1}{\sqrt{\gamma^2-1}} \frac{\tilde{C}_1 a_1^A}{b} \frac{d^3}{dz^3} \left\{ \frac{1}{\sqrt{z^2+1}} \left[ (2\gamma^2-1)\big[z^2(\tilde{b}n)\tilde{b}^A - (z^2+1)(\tilde{v}n)\tilde{v}^A\big] - (\gamma^2-1)n^A + \gamma(\gamma^2-2)(\hat{u}_1 n)\hat{u}_2^A - (\gamma^2-2)(\hat{u}_2 n)\hat{u}_2^A + z\gamma(\tilde{b}n)\hat{u}_2^A - z\gamma^2(\hat{u}_1 n)\tilde{b}^A + z\gamma(\hat{u}_2 n)\tilde{b}^A \right] \right\} \Bigg) \Bigg|_{z=T_1} + (1 \leftrightarrow 2) + \mathcal{O}(a^2) \,.
\end{aligned}
$$

## LO Scalar Waveforms for CC coupling-Spinning part:

$$
\begin{aligned}
W_{\phi}^{(1)} = \frac{m_1 m_2}{32\pi^2 M_{Pl}^3 (\hat{u}_1 n)^2 b^2} \frac{\partial}{\partial z} \Bigg[ \frac{1}{\sqrt{z^2+1}} \mathrm{Re} \Bigg\{ & c_1 \big[(\tilde{v}n)z - i s_1 (\tilde{b}n)\sqrt{z^2+1}\big] \big[-(\hat{u}_1 a_2) + z(\tilde{b}a_2) + i s_1 \sqrt{z^2+1}(\tilde{v}a_2)\big] \left( \frac{\gamma}{\gamma^2-1} - \frac{(\hat{u}_2 n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b}n) + i s_1 \sqrt{z^2+1}(\tilde{v}n)} \right) \\
& - \frac{c_2 (\hat{u}_1 n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b}n) + i s_1 \sqrt{z^2+1}(\tilde{v}n)} \left\{ [i s_1 (\tilde{b}n)\sqrt{z^2+1} - (\tilde{v}n)z](\hat{u}_2 a_1) + \big[\gamma(\tilde{v}n) + i s_1 [\gamma(\hat{u}_1 n) - (\hat{u}_2 n)]\sqrt{z^2+1}\big](\tilde{b}a_1) + \big[[(\hat{u}_2 n) - \gamma(\hat{u}_1 n)]z - \gamma(\tilde{b}n)\big](\tilde{v}a_1) \right\} \Bigg\} \Bigg] + (1 \leftrightarrow 2) \,.
\end{aligned}
$$

# Results obtained in standard GR:

LO order spinning waveform obtained from different approaches (consensus up to $\mathscr{O}(a^4)$):

*Phys.Rev.D* 110 (2024) 4, L041502 [De Angelis, Novichkov, Gonzo]
*Phys.Rev.D* 109 (2024) 3, 036007 [Aoude, Haddad, Heissenberg, Helset]
*JHEP* 02 (2024) 026 [Brandhuber, Brown, Chen, Gowdy, Travaglini]

e.g.: LO spinless waveform:

$$h_f(x)|_{\mathfrak{a}_i=0} = \sum_{i=1}^{2} \frac{\tilde{r}^{-,\mu\nu}_{(i),0} + \tilde{r}^{+,\mu\nu}_{(i),0}}{(p_i\cdot\rho)^2} \mathcal{I}_{(i),\mu\nu}(b_0)$$

$$\mathcal{I}^{\mu\nu}_{(1)}(b) = \frac{K^{\mu\nu}_{(1)}(v_1\cdot K_{(1)}\cdot\rho) - 2(v_1\cdot K_{(1)})^{(\mu}(\rho\cdot K_{(1)})^{\nu)}}{4\pi(\gamma^2-1)(\rho\cdot v_2)^2|b|^2|\boldsymbol{b}|_{(1)}|b|^2_{2\mathrm{d}}}$$

$$\begin{aligned}
r^{+,\mu\nu}_{(1),0} &= \langle k|p_1p_2\gamma^\mu p_1|k\rangle\langle k|p_1p_2\gamma^\nu p_1|k\rangle \\
r^{+,\mu\nu}_{(1),1} &= \langle k|p_1p_2\gamma^\mu p_1|k\rangle\langle k|p_1p_2\gamma^\nu \mathfrak{a}_1|k\rangle \\
r^{+,\mu\nu}_{(1),2} &= \langle k|p_1p_2\gamma^\mu \mathfrak{a}_1|k\rangle\langle k|p_1p_2\gamma^\nu \mathfrak{a}_1|k\rangle \\
r^{+,\mu\nu}_{(1),3} &= \langle k|p_1p_2\gamma^\mu \mathfrak{a}_1|k\rangle\langle k|\mathfrak{a}_1p_2\gamma^\nu \mathfrak{a}_1|k\rangle \\
r^{+,\mu\nu}_{(1),4} &= \langle k|\mathfrak{a}_1p_2\gamma^\mu \mathfrak{a}_1|k\rangle\langle k|\mathfrak{a}_1p_2\gamma^\nu \mathfrak{a}_1|k\rangle
\end{aligned}$$

$$\begin{aligned}
r^{-,\mu\nu}_{(1),0} &= m_1^4\langle k|p_2\gamma^\mu|k\rangle\langle k|p_2\gamma^\nu|k\rangle, \\
r^{-,\mu\nu}_{(1),1} &= m_1^2\langle k|p_2\gamma^\mu|k\rangle\langle k|\mathfrak{a}_1p_1p_2\gamma^\nu|k\rangle \\
r^{-,\mu\nu}_{(1),2} &= \langle k|\mathfrak{a}_1p_1p_2\gamma^\mu|k\rangle\langle k|\mathfrak{a}_1p_1p_2\gamma^\nu|k\rangle, \\
r^{-,\mu\nu}_{(1),3} &= \frac{1}{m_1^2}\langle k|\mathfrak{a}_1p_1p_2\gamma^\mu|k\rangle\langle k|\mathfrak{a}_1p_1p_2\gamma^\nu p_1\mathfrak{a}_1|k\rangle \\
r^{-,\mu\nu}_{(1),4} &= \frac{1}{m_1^4}\langle k|\mathfrak{a}_1p_1p_2\gamma^\mu p_1\mathfrak{a}_1|k\rangle\langle k|\mathfrak{a}_1p_1p_2\gamma^\nu p_1\mathfrak{a}_1|k\rangle
\end{aligned}$$

NLO order waveform also looks like a closed case:

JHEP 06 (2023) 048 [Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini]
JHEP 07 (2024) 272 [Elkhidir, O'Connell, Sergola, Vazquez-Holm]
JHEP 06 (2023) 004 [Herderschee, Roiban, Teng]
JHEP 2023 (2023) 06, 126 [Georgoudis, Heissenberg, Vazquez-Holm]
JHEP 01 (2024) 139 [Caron-Huot, Giroux, Hannesdottir, Mizera]

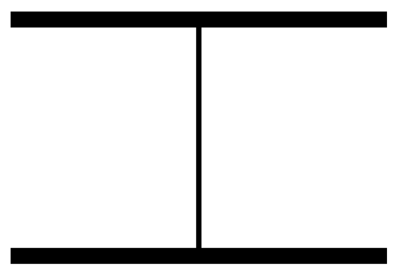
Recent result for NLO linear-in-spin effects:

arxiv: 2312.14859 [Bohnenblust, Ita, Kraus, Schlenk]

# Results obtained in standard GR:

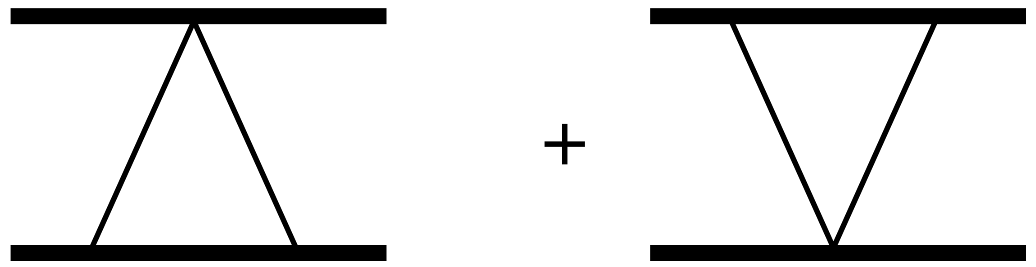
*JHEP* 10 (2021) 148 [Herrmann, Parra-Martinez, Ruf, Zeng]

tree:



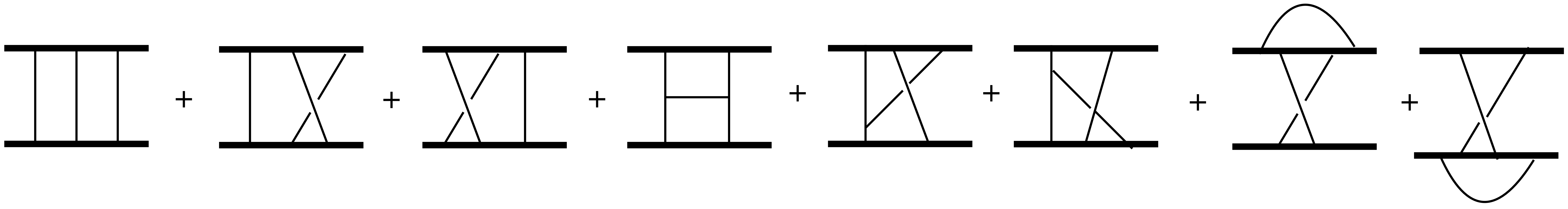
$$\Delta p^{\mu,(0)}_{1,\mathrm{GR}} = \frac{GM^2\nu}{|b|}\frac{2(2\sigma^2-1)}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}\,.$$

1-loop:



$$\Delta p^{\mu,(1)}_{1,\perp} = \frac{G^2M^3\nu}{|b|^2}\frac{3\pi}{4}\frac{(5\sigma^2-1)}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}$$

2-loop:



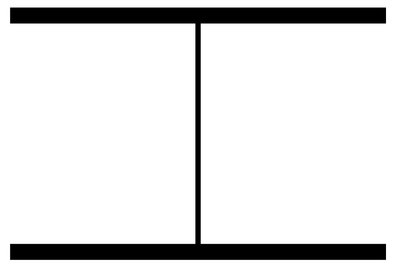
$$\Delta p^{\mu,(2)}_{1,\perp,\mathrm{cons}} = \frac{G^3M^4\nu}{|b|^3}\frac{2}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}\Bigg[h^2(\sigma,\nu)\left(16\sigma^2-\frac{1}{(\sigma^2-1)^2}\right) - \frac{4}{3}\nu\,\sigma\left(14\sigma^2+25\right) - 8\nu\left(4\sigma^4-12\sigma^2-3\right)\frac{\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\Bigg]$$

$$\Delta p^{\mu,(2)}_{1,\mathrm{rad}} = \frac{G^3M^4\nu^2}{|b|^3}\Bigg\{\frac{4}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}\left[f_1^{\mathrm{LS}}(\sigma)+f_3^{\mathrm{LS}}(\sigma)\frac{\sigma\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right] + \pi\,\check{u}_2^\mu\left[f_1(\sigma)+f_2(\sigma)\log\left(\frac{\sigma+1}{2}\right)+f_3(\sigma)\frac{\sigma\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right]\Bigg\}$$

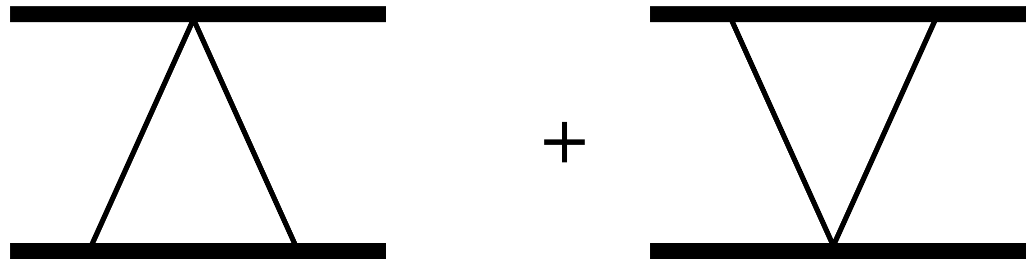
…

# Results obtained in standard GR:

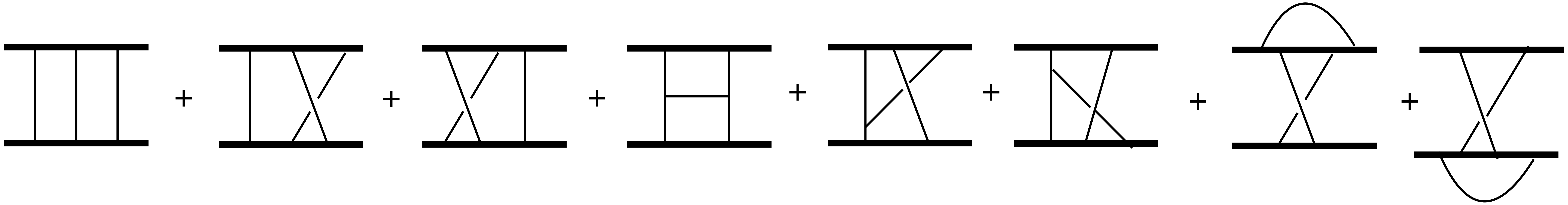
*JHEP* 10 (2021) 148 [Herrmann, Parra-Martinez, Ruf, Zeng]

tree: 

$$\Delta p^{\mu,(0)}_{1,\mathrm{GR}} = \frac{GM^2\nu}{|b|}\frac{2(2\sigma^2-1)}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}\,.$$

1-loop: 

$$\Delta p^{\mu,(1)}_{1,\perp} = \frac{G^2M^3\nu}{|b|^2}\frac{3\pi}{4}\frac{(5\sigma^2-1)}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}$$

2-loop: 

$$\Delta p^{\mu,(2)}_{1,\perp,\mathrm{cons}} = \frac{G^3M^4\nu}{|b|^3}\frac{2}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}\Bigg[h^2(\sigma,\nu)\left(16\sigma^2-\frac{1}{(\sigma^2-1)^2}\right) - \frac{4}{3}\nu\,\sigma\,(14\sigma^2+25) - 8\nu\,(4\sigma^4-12\sigma^2-3)\,\frac{\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\Bigg]$$

$$\Delta p^{\mu,(2)}_{1,\mathrm{rad}} = \frac{G^3M^4\nu^2}{|b|^3}\Bigg\{\frac{4}{\sqrt{\sigma^2-1}}\frac{b^\mu}{|b|}\Bigg[f_1^{\mathrm{LS}}(\sigma)+f_3^{\mathrm{LS}}(\sigma)\frac{\sigma\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\Bigg] + \pi\,\check{u}_2^\mu\Bigg[f_1(\sigma)+f_2(\sigma)\log\left(\frac{\sigma+1}{2}\right)+f_3(\sigma)\frac{\sigma\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\Bigg]\Bigg\}$$

… → **State of the art is pushing towards 4-loops (5PM) binary dynamics**

Phys.Rev.Lett. 132 (2024) 24, 241402 [Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch]
arxiv: 2406.01554 [Bern, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng]

# Results obtained in standard GR:

LO order spinning waveform obtained from different approaches (consensus up to $\mathcal{O}(a^4)$):

*Phys.Rev.D* 110 (2024) 4, L041502 [De Angelis, Novichkov, Gonzo]
*Phys.Rev.D* 109 (2024) 3, 036007 [Aoude, Haddad, Heissenberg, Helset]
*JHEP* 02 (2024) 026 [Brandhuber, Brown, Chen, Gowdy, Travaglini]

e.g.: LO spinless waveform:

$$h_f(x)|_{\mathfrak{a}_i=0} = \sum_{i=1}^{2} \frac{\tilde{r}^{-,\mu\nu}_{(i),0} + \tilde{r}^{+,\mu\nu}_{(i),0}}{(p_i \cdot \rho)^2} \mathcal{I}_{(i),\mu\nu}(b_0)$$

$$\mathcal{I}^{\mu\nu}_{(1)}(b) = \frac{K^{\mu\nu}_{(1)}(v_1 \cdot K_{(1)} \cdot \rho) - 2(v_1 \cdot K_{(1)})^{(\mu}(\rho \cdot K_{(1)})^{\nu)}}{4\pi(\gamma^2-1)(\rho \cdot v_2)^2 |b|^2 |\boldsymbol{b}|_{(1)} |b|^2_{2\mathrm{d}}}$$

$$\begin{aligned}
r^{+,\mu\nu}_{(1),0} &= \langle k|p_1 p_2 \gamma^\mu p_1|k\rangle \langle k|p_1 p_2 \gamma^\nu p_1|k\rangle \\
r^{+,\mu\nu}_{(1),1} &= \langle k|p_1 p_2 \gamma^\mu p_1|k\rangle \langle k|p_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle \\
r^{+,\mu\nu}_{(1),2} &= \langle k|p_1 p_2 \gamma^\mu \mathfrak{a}_1|k\rangle \langle k|p_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle \\
r^{+,\mu\nu}_{(1),3} &= \langle k|p_1 p_2 \gamma^\mu \mathfrak{a}_1|k\rangle \langle k|\mathfrak{a}_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle \\
r^{+,\mu\nu}_{(1),4} &= \langle k|\mathfrak{a}_1 p_2 \gamma^\mu \mathfrak{a}_1|k\rangle \langle k|\mathfrak{a}_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle
\end{aligned}$$

$$\begin{aligned}
r^{-,\mu\nu}_{(1),0} &= m_1^4 \langle k|p_2 \gamma^\mu|k\rangle \langle k|p_2 \gamma^\nu|k\rangle, \\
r^{-,\mu\nu}_{(1),1} &= m_1^2 \langle k|p_2 \gamma^\mu|k\rangle \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu|k\rangle \\
r^{-,\mu\nu}_{(1),2} &= \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\mu|k\rangle \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu|k\rangle, \\
r^{-,\mu\nu}_{(1),3} &= \frac{1}{m_1^2} \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\mu|k\rangle \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu p_1 \mathfrak{a}_1|k\rangle \\
r^{-,\mu\nu}_{(1),4} &= \frac{1}{m_1^4} \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\mu p_1 \mathfrak{a}_1|k\rangle \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu p_1 \mathfrak{a}_1|k\rangle
\end{aligned}$$

# Results obtained in standard GR:

LO order spinning waveform obtained from different approaches (consensus up to $\mathcal{O}(a^4)$):

*Phys.Rev.D* 110 (2024) 4, L041502 [De Angelis, Novichkov, Gonzo]
*Phys.Rev.D* 109 (2024) 3, 036007 [Aoude, Haddad, Heissenberg, Helset]
*JHEP* 02 (2024) 026 [Brandhuber, Brown, Chen, Gowdy, Travaglini]

e.g.: LO spinless waveform:

$$h_f(x)|_{\mathfrak{a}_i=0} = \sum_{i=1}^{2} \frac{\tilde{r}^{-,\mu\nu}_{(i),0} + \tilde{r}^{+,\mu\nu}_{(i),0}}{(p_i \cdot \rho)^2} \mathcal{I}_{(i),\mu\nu}(b_0)$$

$$\mathcal{I}^{\mu\nu}_{(1)}(b) = \frac{K^{\mu\nu}_{(1)}(v_1 \cdot K_{(1)} \cdot \rho) - 2(v_1 \cdot K_{(1)})^{(\mu}(\rho \cdot K_{(1)})^{\nu)}}{4\pi(\gamma^2-1)(\rho \cdot v_2)^2 |b|^2 |\boldsymbol{b}|_{(1)} |b|^2_{2\mathrm{d}}}$$

$$\begin{aligned}
r^{+,\mu\nu}_{(1),0} &= \langle k|p_1 p_2 \gamma^\mu p_1|k\rangle \langle k|p_1 p_2 \gamma^\nu p_1|k\rangle \\
r^{+,\mu\nu}_{(1),1} &= \langle k|p_1 p_2 \gamma^\mu p_1|k\rangle \langle k|p_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle \\
r^{+,\mu\nu}_{(1),2} &= \langle k|p_1 p_2 \gamma^\mu \mathfrak{a}_1|k\rangle \langle k|p_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle \\
r^{+,\mu\nu}_{(1),3} &= \langle k|p_1 p_2 \gamma^\mu \mathfrak{a}_1|k\rangle \langle k|\mathfrak{a}_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle \\
r^{+,\mu\nu}_{(1),4} &= \langle k|\mathfrak{a}_1 p_2 \gamma^\mu \mathfrak{a}_1|k\rangle \langle k|\mathfrak{a}_1 p_2 \gamma^\nu \mathfrak{a}_1|k\rangle
\end{aligned}$$

$$\begin{aligned}
r^{-,\mu\nu}_{(1),0} &= m_1^4 \langle k|p_2 \gamma^\mu|k\rangle \langle k|p_2 \gamma^\nu|k\rangle, \\
r^{-,\mu\nu}_{(1),1} &= m_1^2 \langle k|p_2 \gamma^\mu|k\rangle \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu|k\rangle \\
r^{-,\mu\nu}_{(1),2} &= \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\mu|k\rangle \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu|k\rangle, \\
r^{-,\mu\nu}_{(1),3} &= \frac{1}{m_1^2} \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\mu|k\rangle \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu p_1 \mathfrak{a}_1|k\rangle \\
r^{-,\mu\nu}_{(1),4} &= \frac{1}{m_1^4} \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\mu p_1 \mathfrak{a}_1|k\rangle \langle k|\mathfrak{a}_1 p_1 p_2 \gamma^\nu p_1 \mathfrak{a}_1|k\rangle
\end{aligned}$$

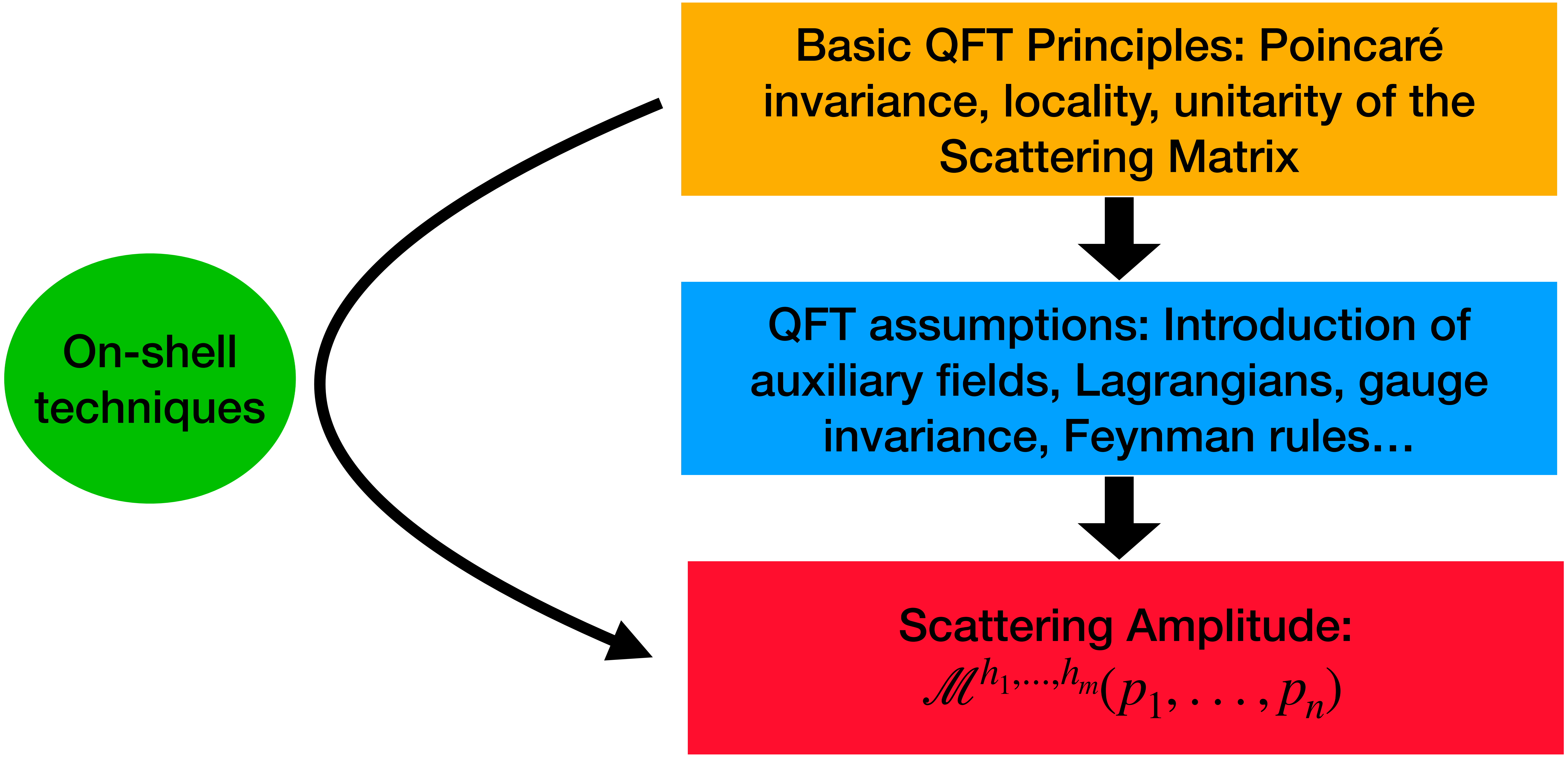
NLO order waveform also looks like a closed case:

JHEP 06 (2023) 048 [Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini]
JHEP 07 (2024) 272 [Elkhidir, O'Connell, Sergola, Vazquez-Holm]
JHEP 06 (2023) 004 [Herderschee, Roiban, Teng]
JHEP 2023 (2023) 06, 126 [Georgoudis, Heissenberg, Vazquez-Holm]
JHEP 01 (2024) 139 [Caron-Huot, Giroux, Hannesdottir, Mizera]

Recent result for NLO linear-in-spin effects:

arxiv: 2312.14859 [Bohnenblust, Ita, Kraus, Schlenk]

# On-shell techniques:



# On-shell techniques:

Basic QFT Principles: Poincaré invariance, locality, unitarity of the Scattering Matrix

↓

QFT assumptions: Introduction of auxiliary fields, Lagrangians, gauge invariance, Feynman rules…

↓

Scattering Amplitude:
$\mathscr{M}^{h_1,\ldots,h_m}(p_1,\ldots,p_n)$

# Contact terms' contributions: An inevitable problem

Huge list of authors who have contributed in solving this problem in QED and Gravity

→ The four -point amplitudes we construct are **healthy at all spin orders**, but **still have to include contact terms' deformations** which can contribute **classically** (very similar to what has already been observed in QED and GR): **Can be done order by order** in the spin expansion, but need a matching procedure to fix their coefficients

$$\mathscr{M}[1_{\Phi_i}2_{\bar{\Phi}_i}3_h^s4_\phi] = \frac{4\hat{\alpha}}{M_{Pl}^2\Lambda^2}t(p_1\epsilon_s)^2\left\{e^{q\cdot a} + P_{\xi,s}(p_3a_1, p_4a_1, wa_1, |a_1|')\right\} \quad ,$$

$$q^\mu = p_3^\mu + p_4^\mu \quad , \quad w^\mu = \frac{(t-m^2)}{2(p_1\epsilon_{3,-})}\epsilon_{3,-}^\mu \quad , \quad |a_1|' = \frac{(t-m^2)}{2m}|a_1|$$

Extending the treatment of
JHEP 05 (2023) 211[Bautista, Guevara, Kavanagh, Vines]

# Contact terms' contributions: An inevitable problem

Huge list of authors who have contributed in solving this problem in QED and Gravity

→ The four -point amplitudes we construct are **healthy at all spin orders**, but **still have to include contact terms' deformations** which can contribute **classically** (very similar to what has already been observed in QED and GR): **Can be done order by order** in the spin expansion, but need a matching procedure to fix their coefficients

$$\mathscr{M}[1_{\Phi_i} 2_{\bar{\Phi}_i} 3_h^s 4_\phi] = \frac{4\hat{\alpha}}{M_{Pl}^2 \Lambda^2} t (p_1 \epsilon_s)^2 \left\{ e^{q\cdot a} + \underbrace{P_{\xi,s}(p_3 a_1, p_4 a_1, w a_1, |a_1|')}_{\text{Contact terms' deformations}} \right\} \quad ,$$

"Unitary" piece
No spurious poles!

Contact terms' deformations

$$q^\mu = p_3^\mu + p_4^\mu \quad , \quad w^\mu = \frac{(t-m^2)}{2(p_1 \epsilon_{3,-})} \epsilon_{3,-}^\mu \quad , \quad |a_1|' = \frac{(t-m^2)}{2m} |a_1|$$

e.g. up to $\mathcal{O}(a_1^2)$ :

$$P_{\xi,-}\left((p_3 a_1), (p_4 a_1), (w a_1), |a_1|'\right)\Big|_{\mathcal{O}(a_1^2)} = \xi^{-1}\left(C_{000}^{-1}(w a_1)^2 + D_{000}^{-1}(w a_1)|a_1|'\right)$$

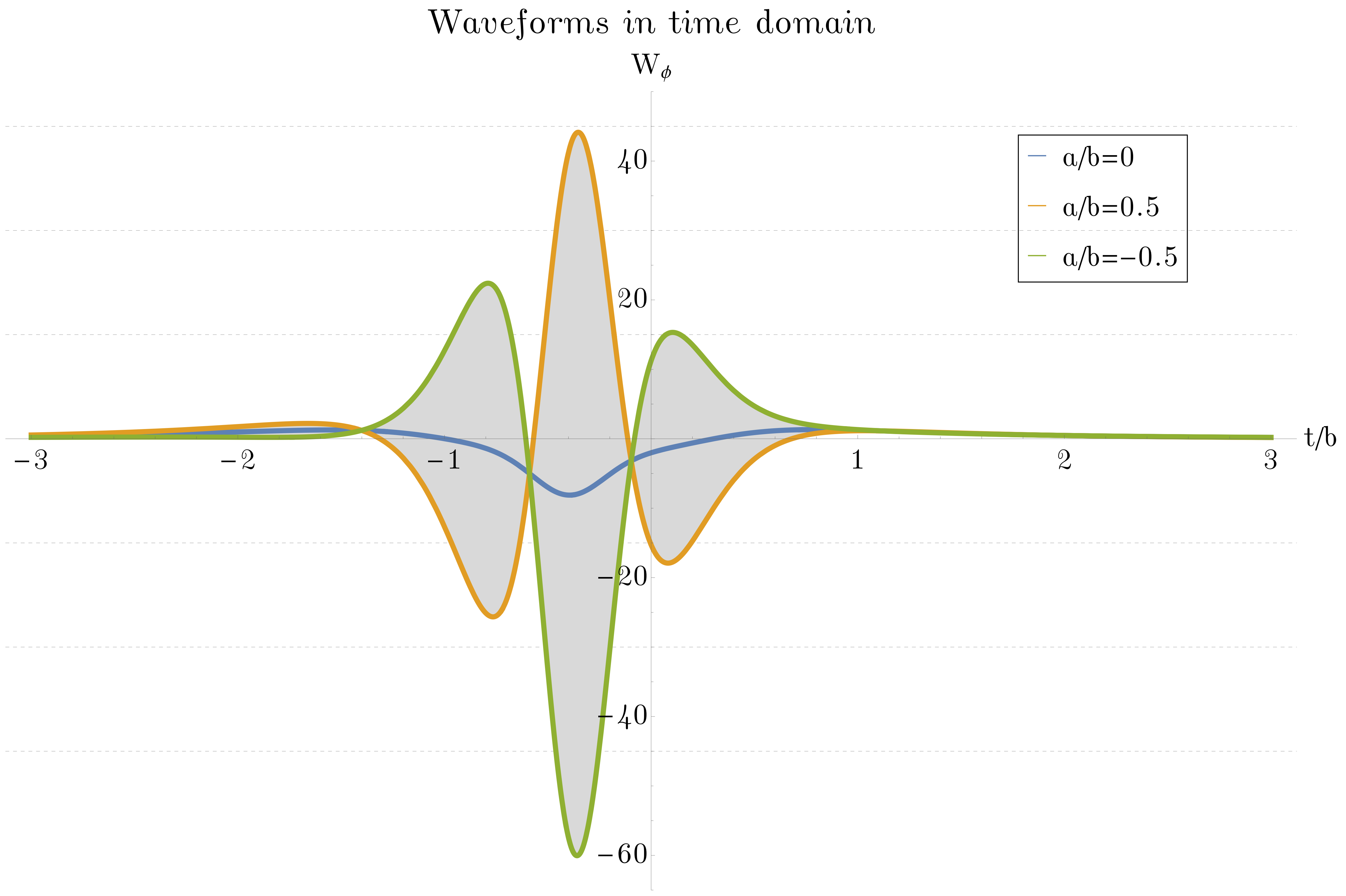
$$+\xi^0 \Big( C_{000}^0((w a_1) - (p_3 a_1)) + C_{100}^0((w a_1) - (p_3 a_1))(p_3 a_1) + C_{010}^0((w a_1) - (p_3 a_1))(p_4 a_1)$$

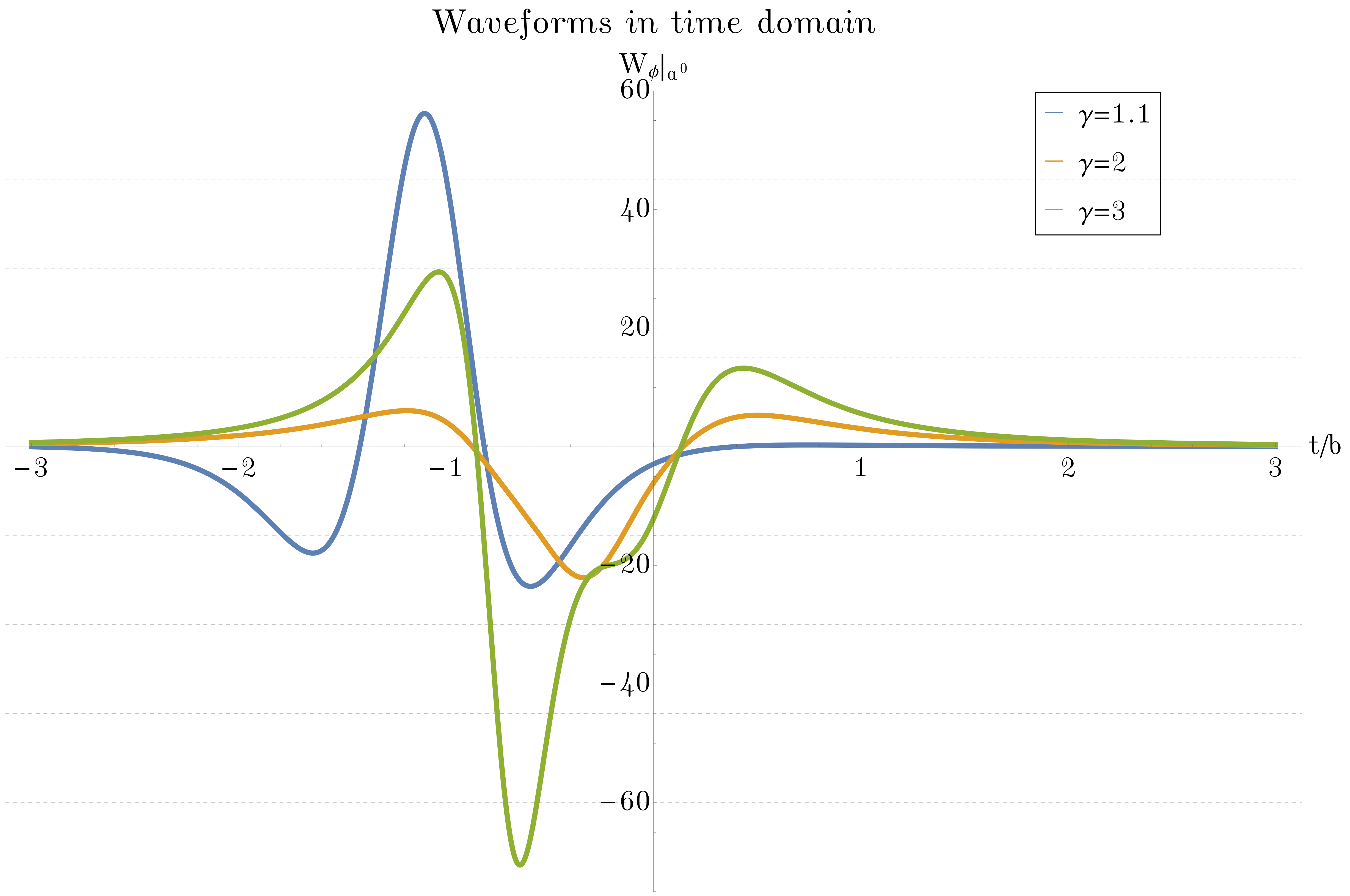
$$+ C_{001}^0((w a_1) - (p_3 a_1))(w a_1) + C_{000}^0 C^0((w a_1) - (p_3 a_1))|a_1|' \Big) .$$

Extending the treatment of
JHEP 05 (2023) 211[Bautista, Guevara, Kavanagh, Vines]

# Results:

## LO scalar waveform for scalar Gauss-Bonnet:



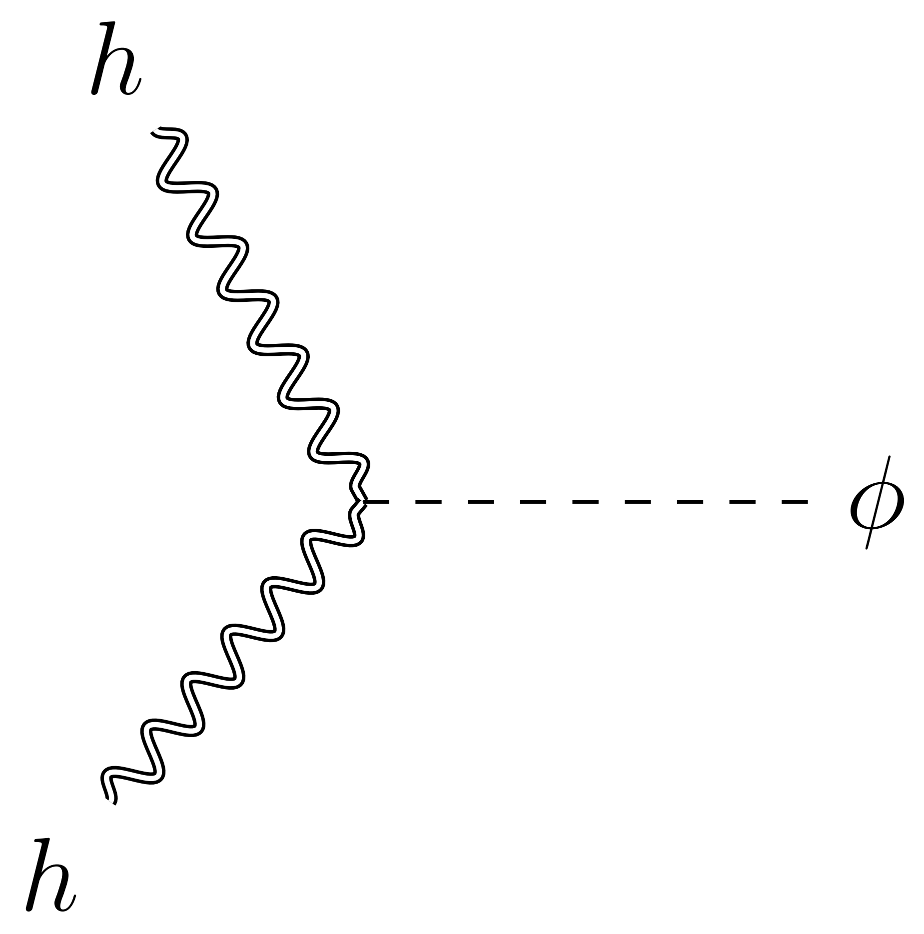

**1. Scalar waveform for the SGB case up to linear order in spin for different values of the spin magnitude $a \in [-b/2, b/2]$.**

**2. Scalar piece of the SGB waveform for different values of $\gamma$.**

# On-shell amplitudes:

Let's work by expanding $f(\phi) \approx c + \phi + \mathcal{O}(\phi^2)$.

Naively, this action produces an extra 3-point **shift-symmetric** on-shell amplitude which we should consider:

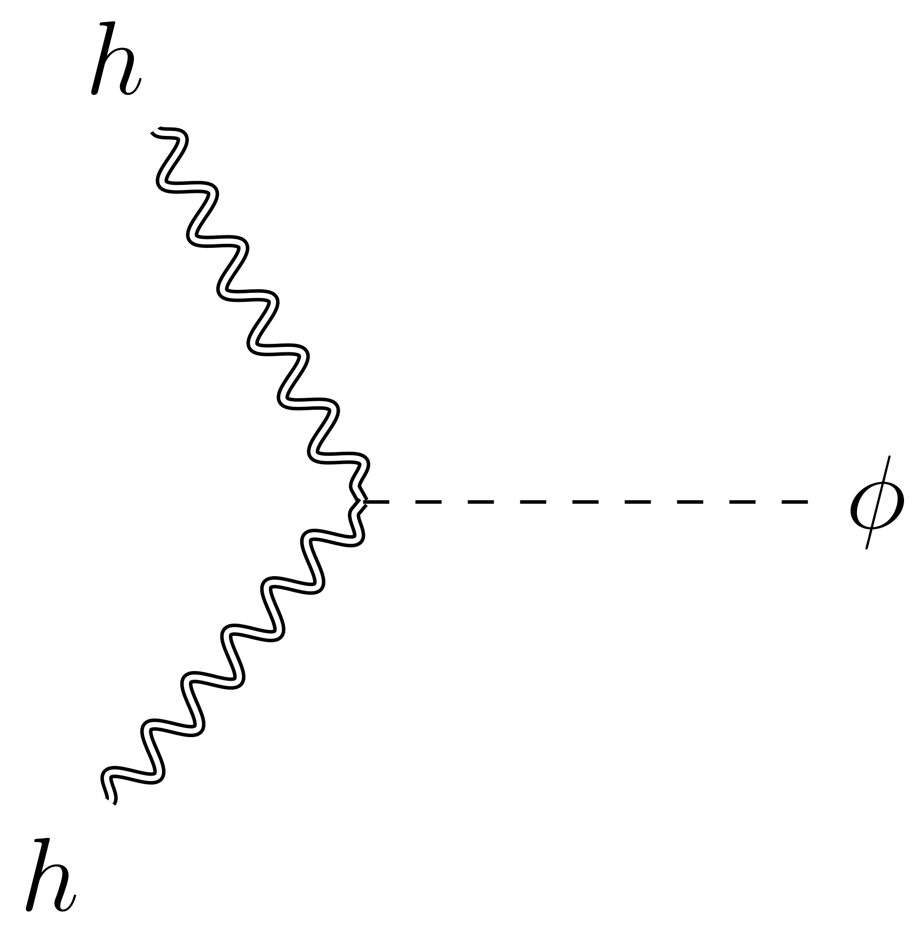



$$\sim \frac{\hat{\alpha}}{\Lambda^2} = \frac{\alpha + i\tilde{\alpha}}{\Lambda^2}$$

# On-shell amplitudes:

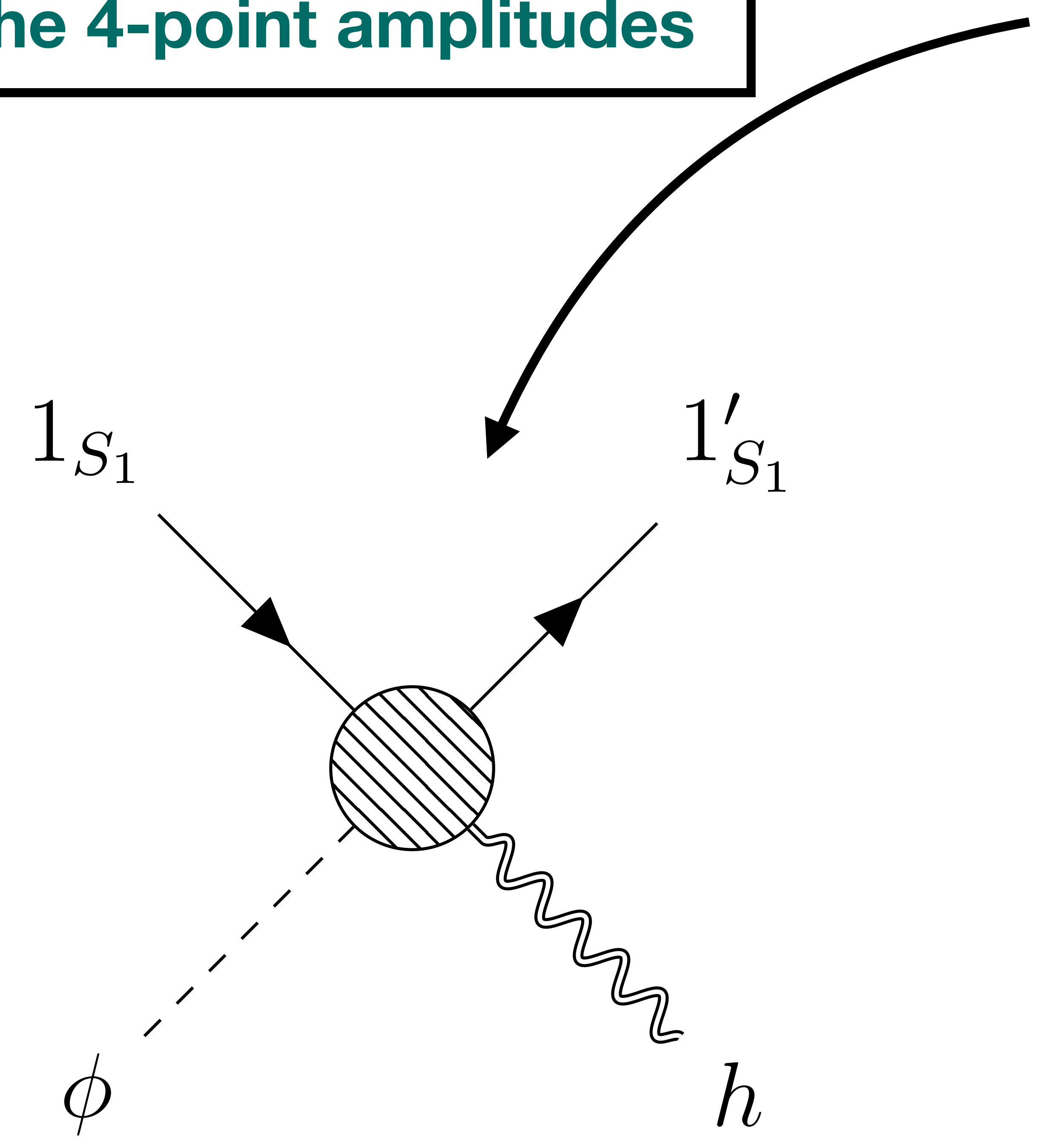
Let's work by expanding $f(\phi) \approx c + \phi + \mathcal{O}(\phi^2)$.

Naively, this action produces an extra 3-point **shift-symmetric** on-shell amplitude which we should consider:




$$\sim \frac{\hat{\alpha}}{\Lambda^2} = \frac{\alpha + i\tilde{\alpha}}{\Lambda^2}$$

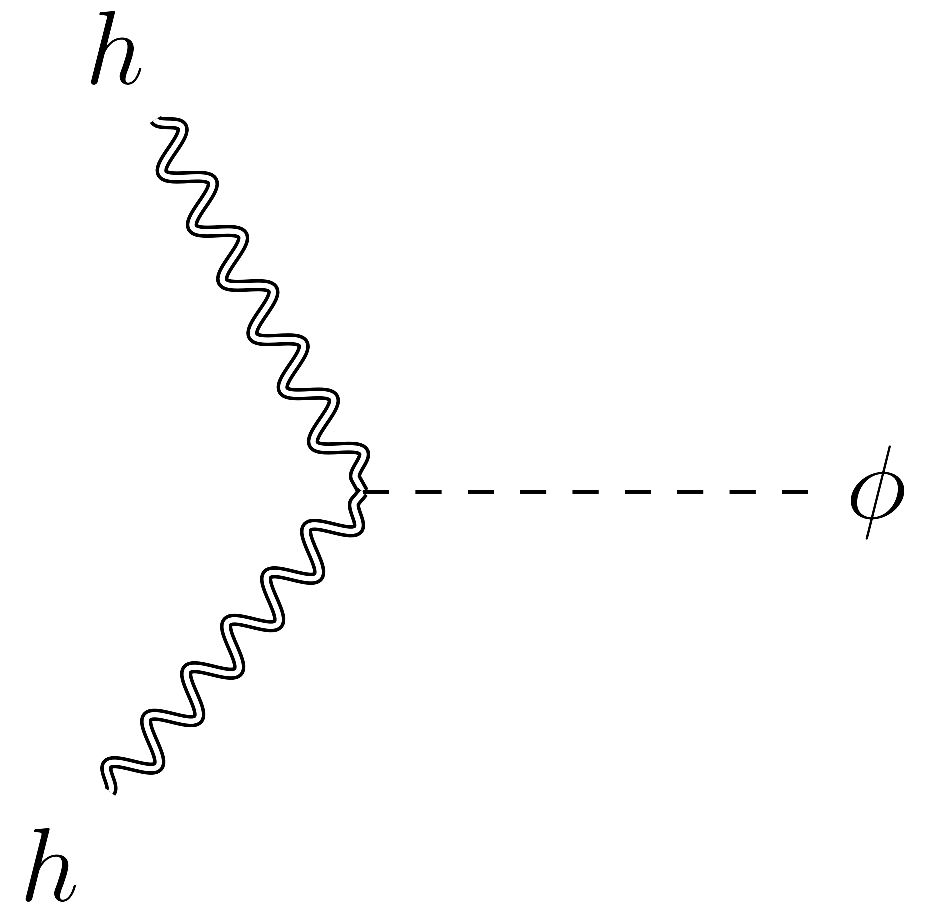
**Gives an extra contribution to the 4-point amplitudes**



# On-shell amplitudes:

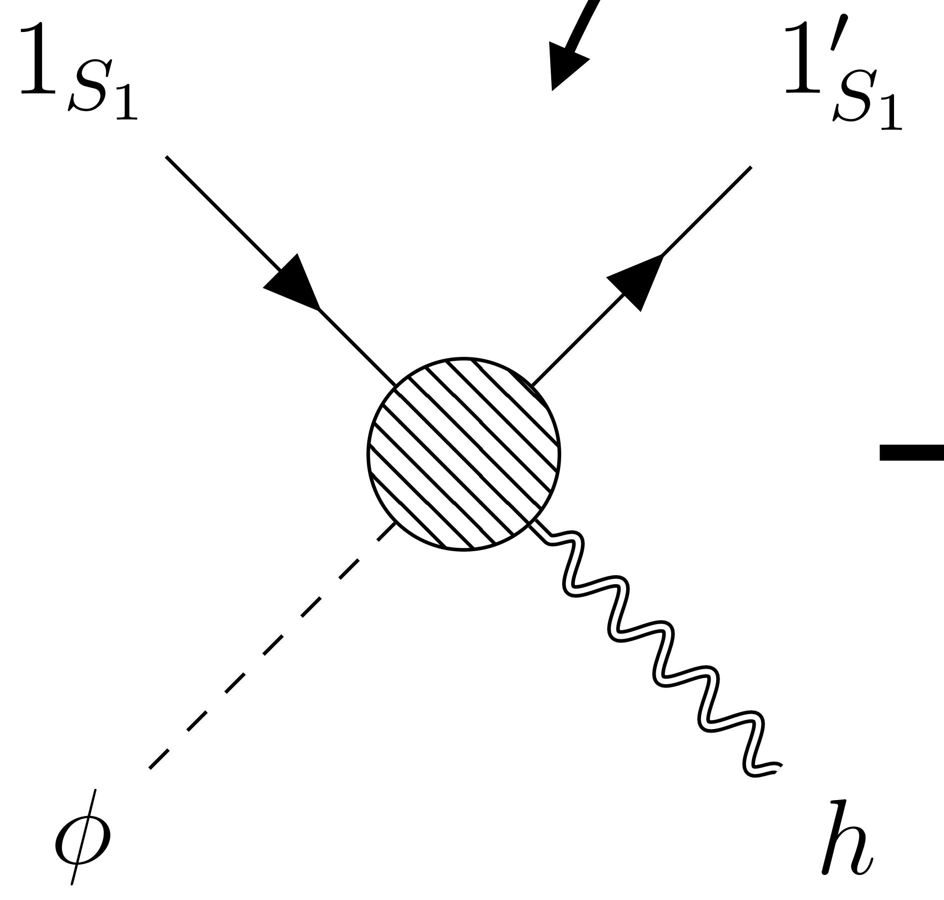
Let's work by expanding $f(\phi) \approx c + \phi + \mathcal{O}(\phi^2)$.

Naively, this action produces an extra 3-point **shift-symmetric** on-shell amplitude which we should consider:

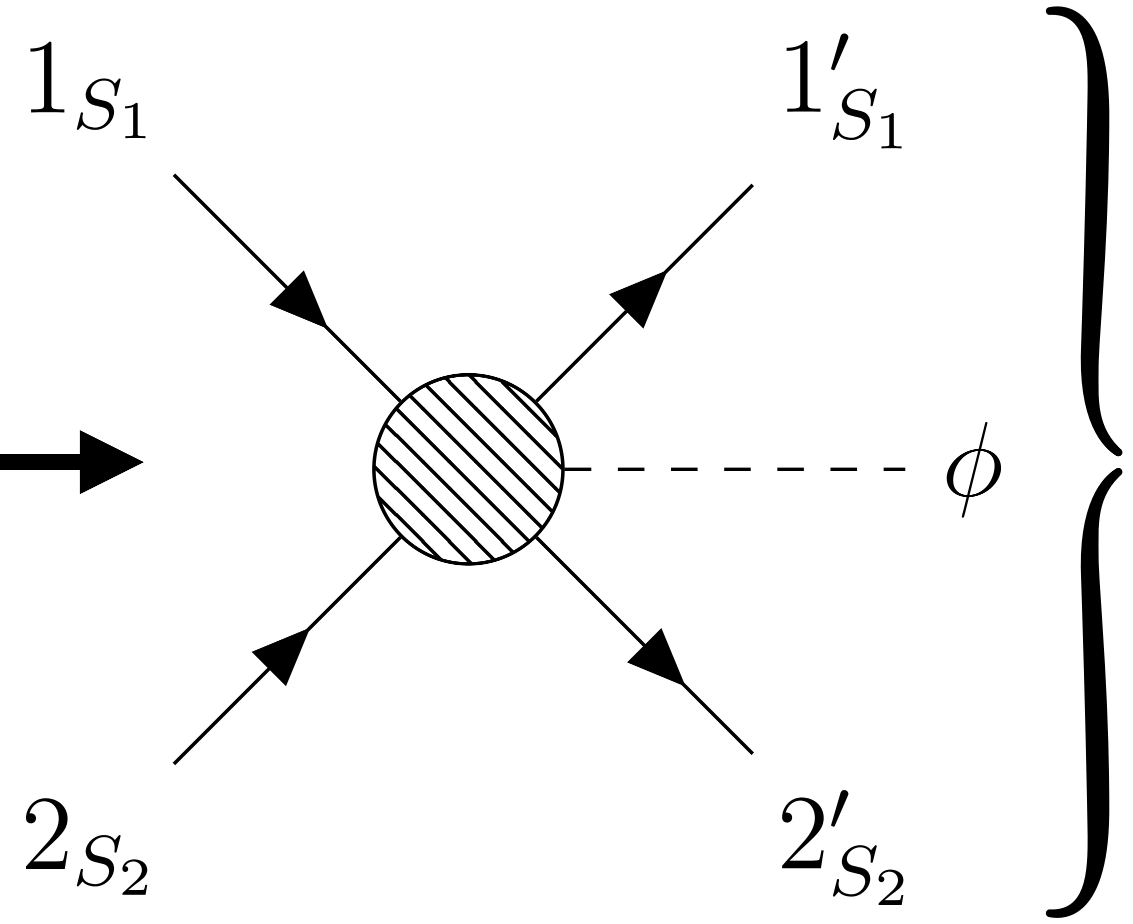



$$\sim \frac{\hat{\alpha}}{\Lambda^2} = \frac{\alpha + i\tilde{\alpha}}{\Lambda^2}$$

**Gives an extra contribution to the 4-point amplitudes**




**Contributes to the leading order 5-point amplitude**




**Can compute the Scalar Waveform in the KMOC formalism!**

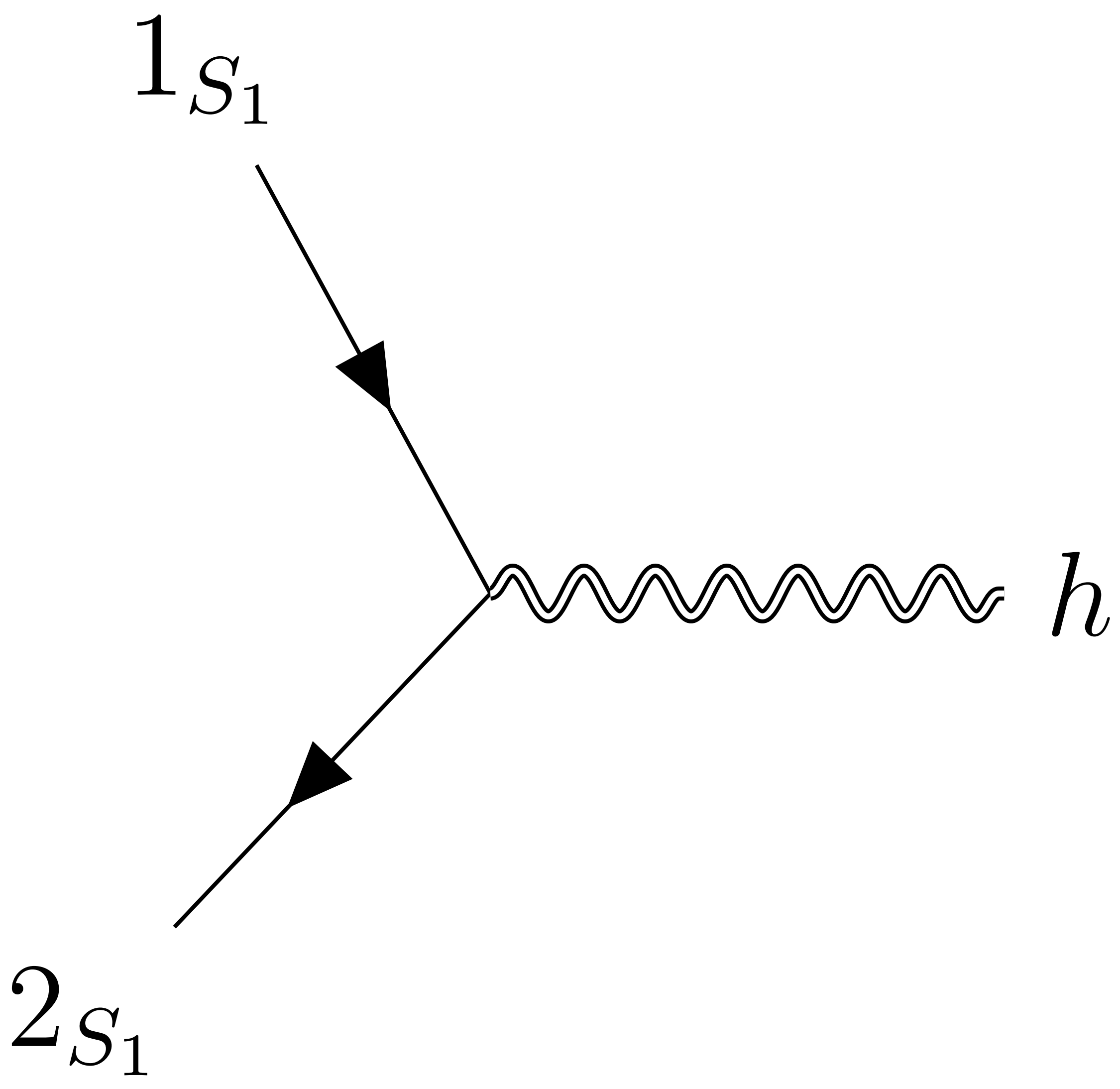
# On-shell techniques:


Idea: Build the **on-shell 3-point amplitudes** of the theory

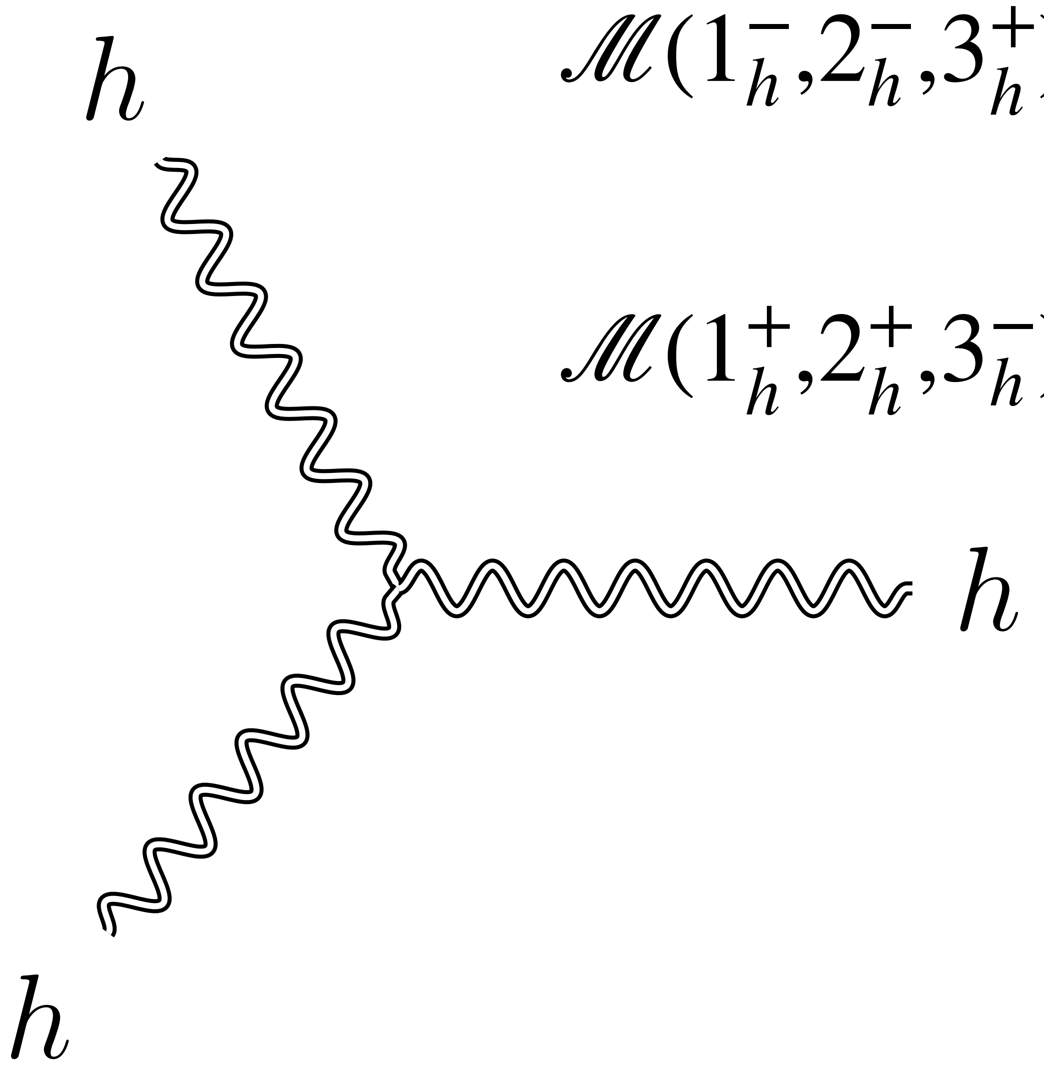
e.g.: **Spinning matter in GR**

$$\mathcal{M}\left[1_{\Phi}2_{\bar{\Phi}}3_h^-\right] = -\frac{\langle 3|p_1|\tilde{\zeta}]^2}{M_{Pl}[3\tilde{\zeta}]^2}\frac{[\mathbf{21}]^{2S}}{m^{2S}},$$

$$\mathcal{M}\left[1_{\Phi}2_{\bar{\Phi}}3_h^+\right] = -\frac{\langle\zeta|p_1|3]^2}{M_{Pl}\langle 3\zeta\rangle^2}\frac{\langle\mathbf{21}\rangle^{2S}}{m^{2S}},$$




,




$$\mathcal{M}(1_h^-,2_h^-,3_h^+) = -\frac{1}{M_{Pl}}\frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 23\rangle^2}$$

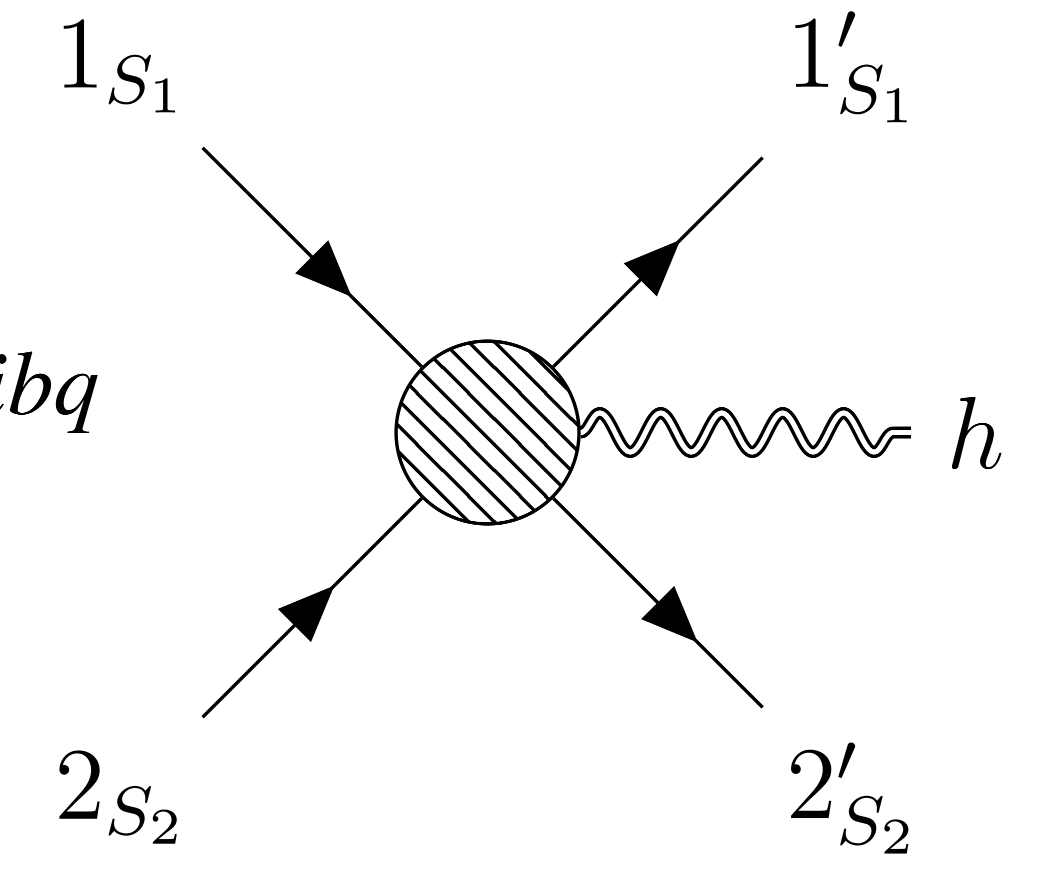
$$\mathcal{M}(1_h^+,2_h^+,3_h^-) = -\frac{1}{M_{Pl}}\frac{[12]^6}{[13]^2[23]^2}.$$

# Waveforms at leading order:

strain

on-shell measure

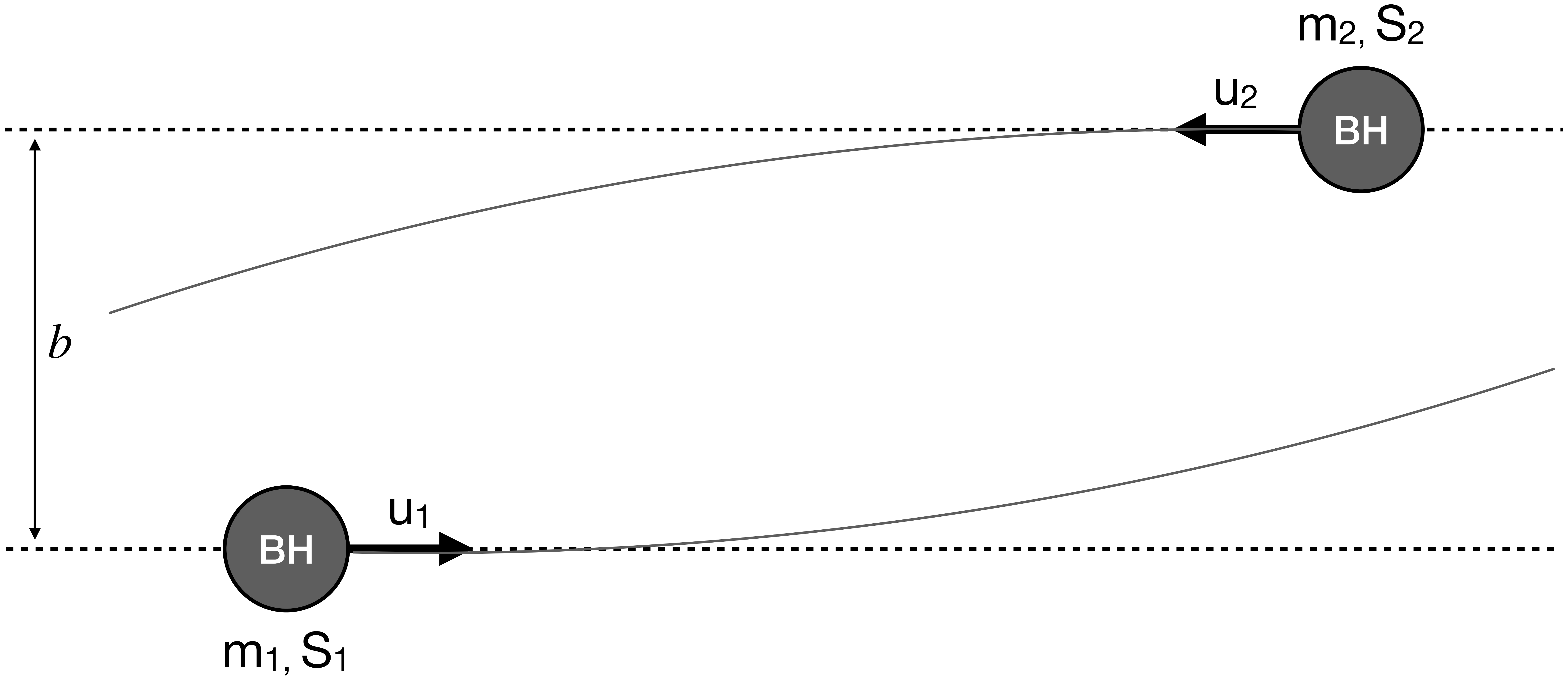
$$h_{GR}(t) \equiv h_+ \pm i h_\times \sim \int d\omega e^{-i\omega t} \int d\Phi(q) e^{-ibq}$$




$$\mathscr{M}_5^{\text{cl}}(q,k)\,|_{k^\mu = \omega n^\mu}$$

But, **why?**

# 2. Scattering Amplitudes and Observables




**Weak field expansion:**

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_{Pl}} h_{\mu\nu}$$

**PM expansion:**

$$R_s \ll b$$

**Spin expansion:**

$$\frac{S}{m} \ll b$$

- **Focus**: **Classical scattering problem in GR**

# The phases of the binary problem:

*Phys.Rev.D* 62 (2000) 064015 [Buonanno, Damour]*

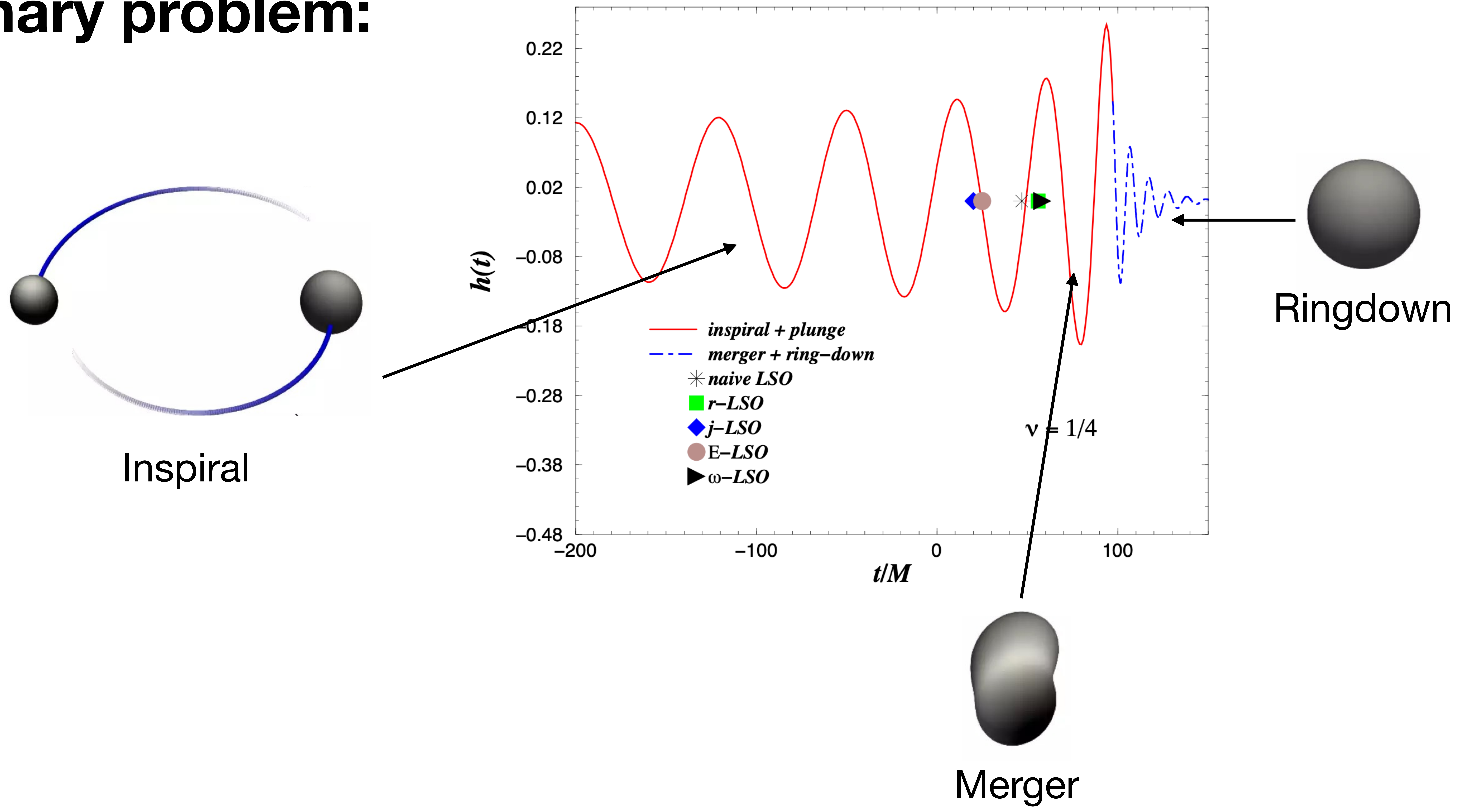
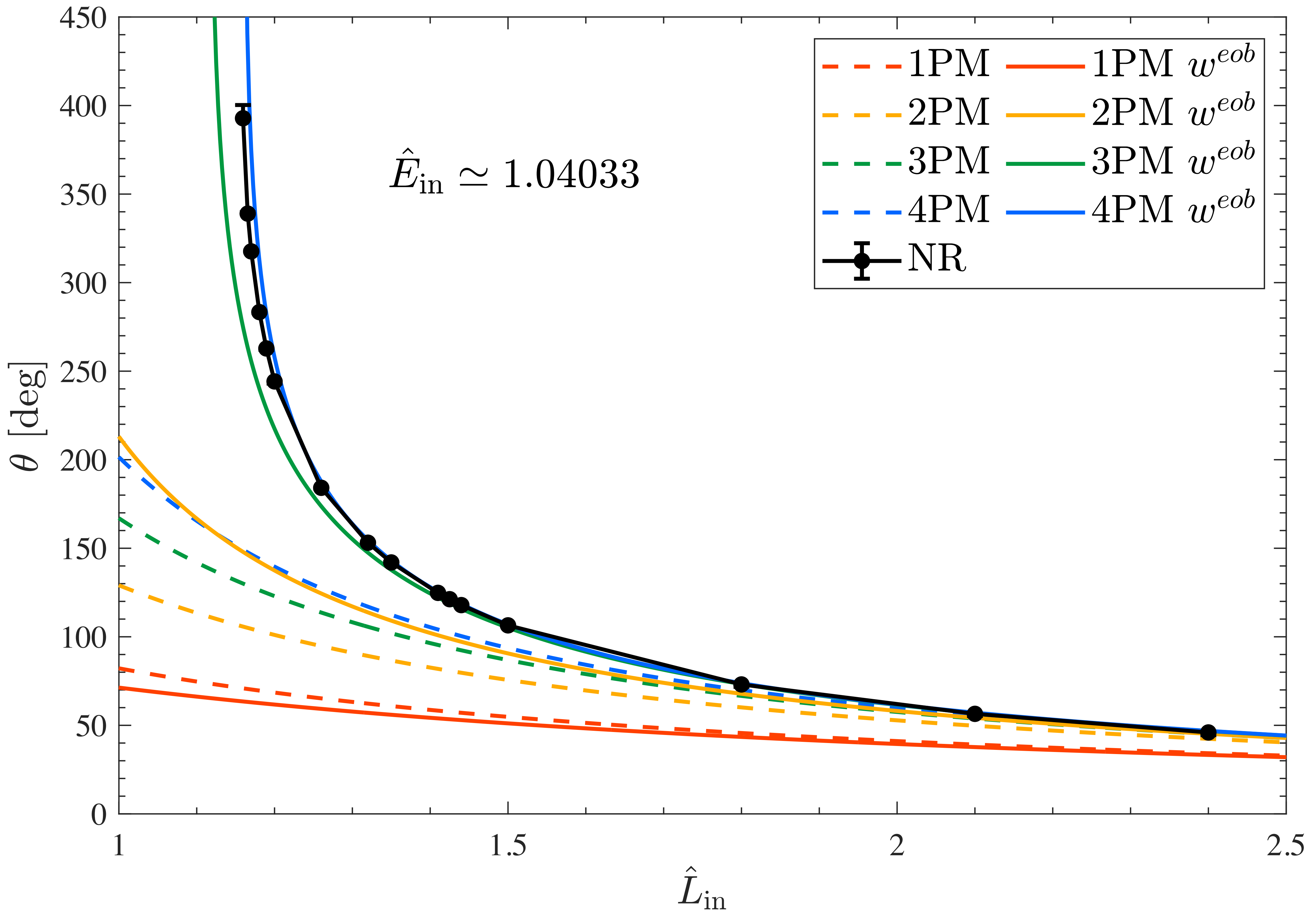



Illustration Credit: Max-Planck-Gesellschaft*

# PM expansion to the test and bound orbits:

*Phys.Rev.D* 108 (2023) 12, 124016 [Rettegno, Pratten, Thomas, Schmidt, Damour]




**Already PM is doing very well for black hole (BH) scattering!**

# Scalar hair in scalar-tensor theories:

Compact objects can acquire scalar hair in ST theories → Exactly the case for SGB and DCS!


BH solution in ST theory

$$\phi = \frac{c_i}{r} + \frac{d_i}{r^2} + \ldots$$

# Scalar hair in scalar-tensor theories:

Compact objects can acquire scalar hair in ST theories ⟶ Exactly the case for SGB and DCS!

Non-trivial configuration of the field $\phi \rightarrow$ Scalar hair

BH solution in ST theory

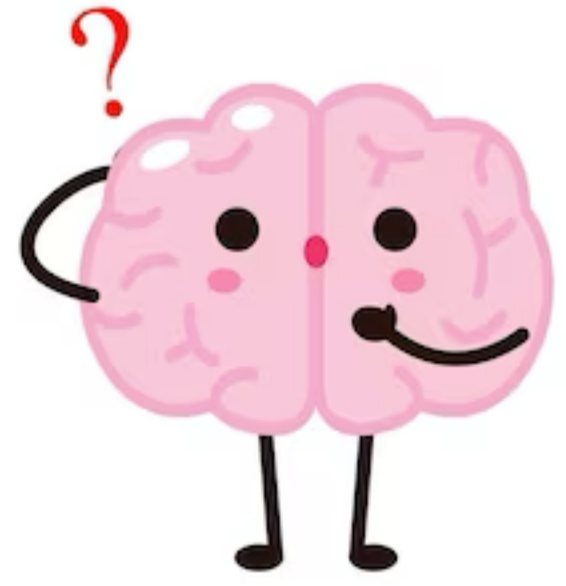
Far zone

$x \rightarrow \infty$

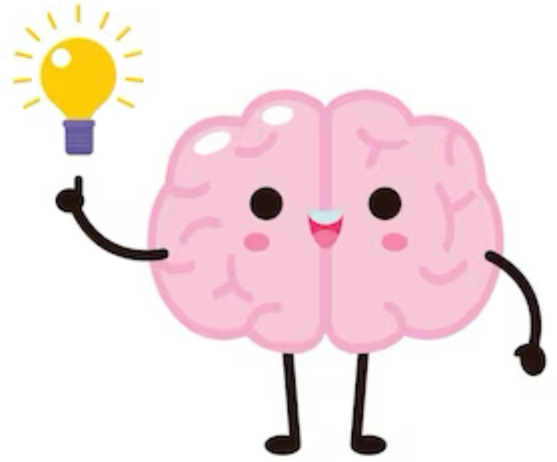
"Monopole hair" "Dipole hair"

$$\phi = \frac{c_i}{r} + \frac{d_i}{r^2} + \ldots$$

**How can we model this behaviour with amplitudes?**



# The on-shell way again:



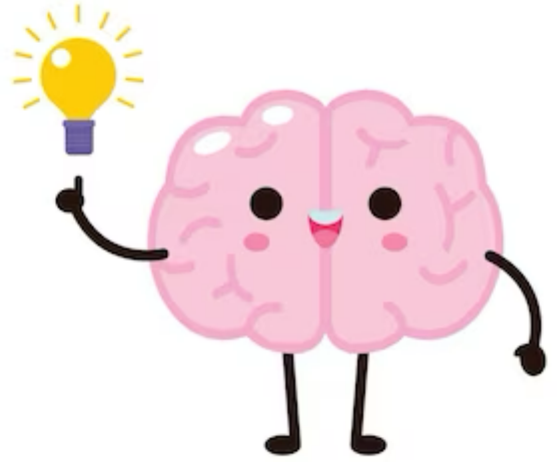
Phys.Rev.D 48 (1993) 3641-3647 [Bekenstein]

We model the BH as a point-particle interacting with the scalar field in a ST fashion

Most general effective metric that respects causality is:

$$\tilde{g}_{\mu\nu} = \underbrace{\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] g_{\mu\nu}}_{\text{Conformal coupling}} + \underbrace{D\left(\frac{\phi}{M_{Pl}}\right)\frac{D_\mu\phi D_\nu\phi}{M_{Pl}^2\Lambda^2}}_{\text{Disformal coupling}},$$

# The on-shell way again:



Phys.Rev.D 48 (1993) 3641-3647 [Bekenstein]

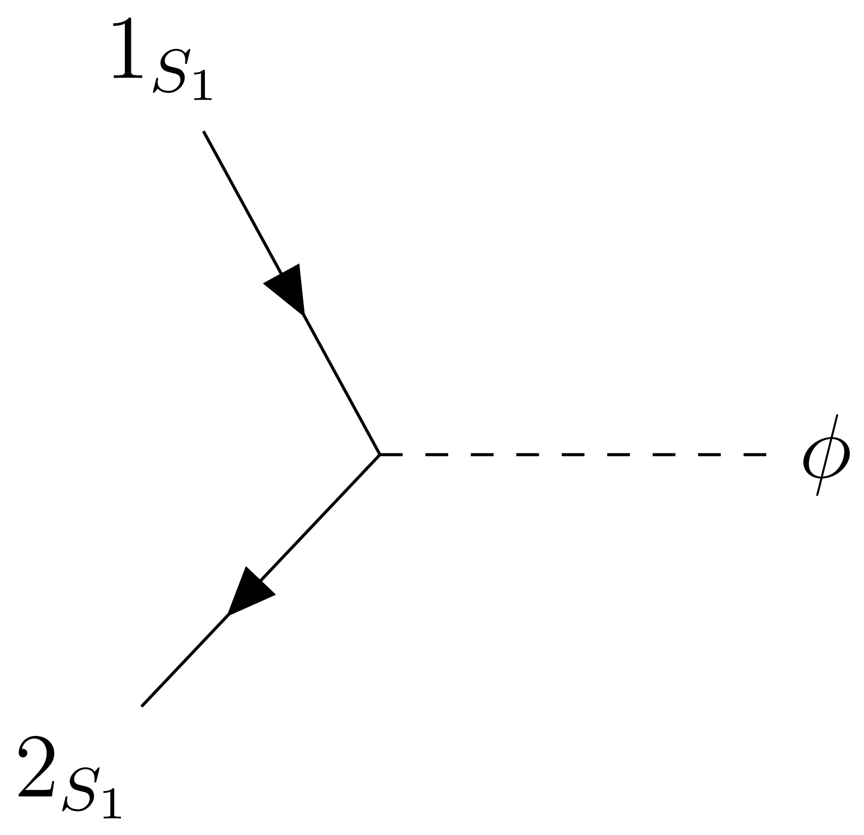
We model the BH as a point-particle interacting with the scalar field in a ST fashion

Neglect it, heavily suppressed

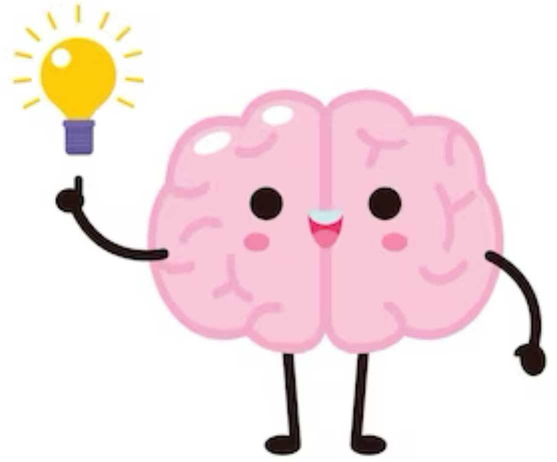
Most general effective metric that respects causality is: $$\tilde{g}_{\mu\nu} = \underbrace{\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] g_{\mu\nu}}_{\text{Conformal coupling}} + \underbrace{D\left(\frac{\phi}{M_{Pl}}\right)\frac{D_\mu\phi D_\nu\phi}{M_{Pl}^2\Lambda^2}}_{\text{Disformal coupling}},$$

Generate 3-point amplitudes for arbitrary spinning BH:

$$\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] \approx 1 + c\frac{\phi}{M_{Pl}}$$



# The on-shell way again:



Phys.Rev.D 48 (1993) 3641-3647 [Bekenstein]

We model the BH as a point-particle interacting with the scalar field in a ST fashion

Neglect it, heavily suppressed

Most general effective metric that respects causality is: $\tilde{g}_{\mu\nu} = \underbrace{\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] g_{\mu\nu}}_{\text{Conformal coupling}} + \underbrace{D\left(\frac{\phi}{M_{Pl}}\right)\frac{D_\mu\phi D_\nu\phi}{M_{Pl}^2\Lambda^2}}_{\text{Disformal coupling}},$

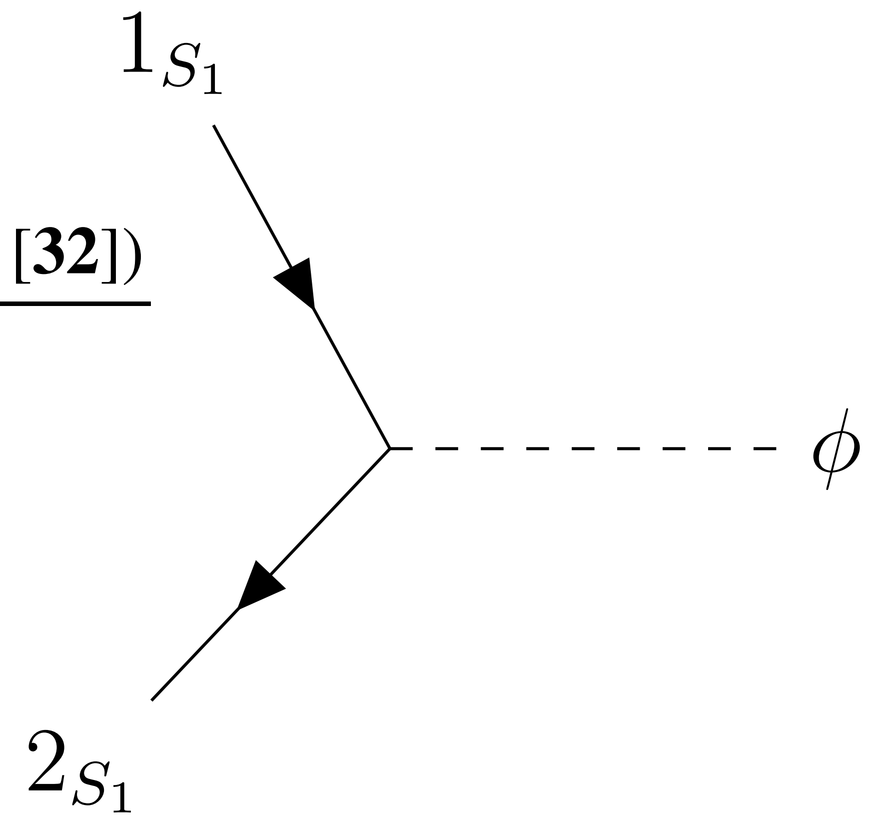
Conformal coupling Disformal coupling

Generate 3-point amplitudes for arbitrary spinning BH:

$$\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] \approx 1 + c\frac{\phi}{M_{Pl}}$$

$$\mathscr{M}_{3,bos.}(1_\phi, 2_A, 3_A) = \frac{c_1}{M_{Pl}} \frac{\langle \mathbf{32}\rangle^S [\mathbf{32}]^S}{m^{2S-2}}$$

$$\mathscr{M}_{3,ferm.}(1_\phi, 2_\Psi, 3_{\bar{\Psi}}) = \frac{c_1}{M_{Pl}} \frac{\langle \mathbf{32}\rangle^{S-1/2} [\mathbf{32}]^{S-1/2} (\langle \mathbf{32}\rangle + [\mathbf{32}])}{m^{2S-2}}$$




**Simple mass redefinition at all spin orders:**

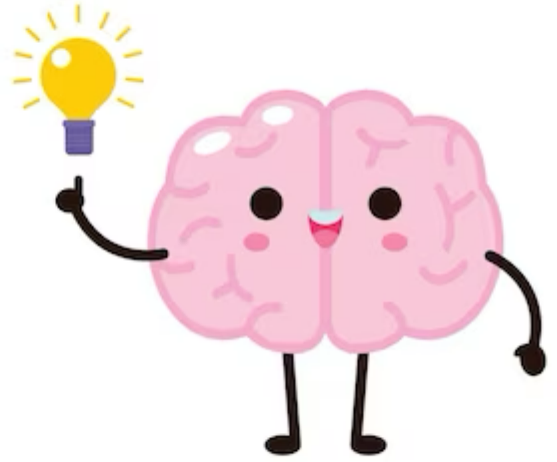
$$m \rightarrow e^{C/2} m$$

**Resemblance to <u>skeletonized action</u> used in GR literature:**

Astrophysical Journal, vol. 196, Mar. 1, 1975, pt. 2, p. L59-L62. [Eardley]

1992 Class. Quantum Grav. 9 2093 [Damour, Esposito-Farese]

# The on-shell way again:



Phys.Rev.D 48 (1993) 3641-3647 [Bekenstein]

We model the BH as a point-particle interacting with the scalar field in a ST fashion

Most general effective metric that respects causality is: $\tilde{g}_{\mu\nu} = \exp\left[C(\frac{\phi}{M_{Pl}})\right]g_{\mu\nu} + D(\frac{\phi}{M_{Pl}})\frac{D_\mu\phi D_\nu\phi}{M_{Pl}^2\Lambda^2},$

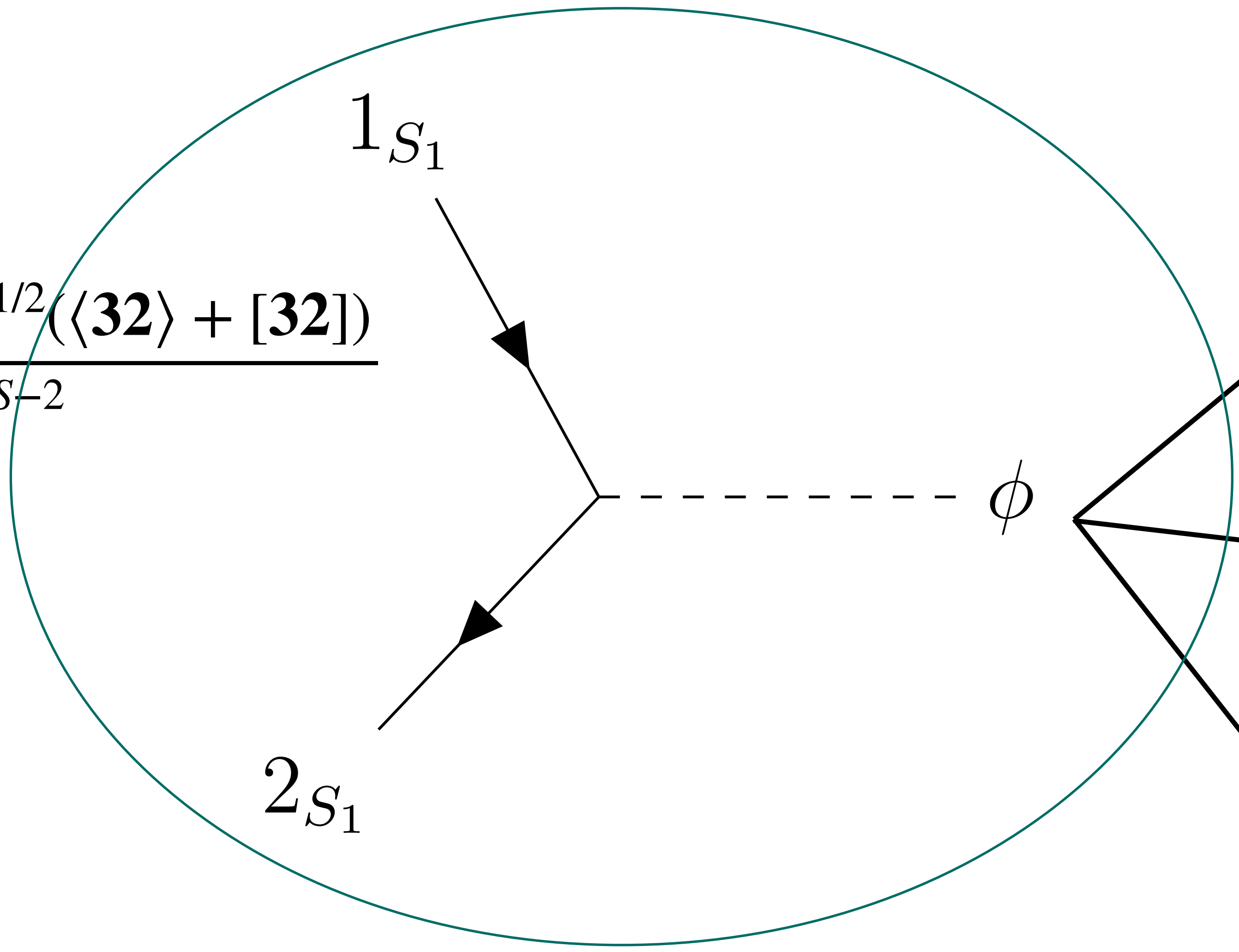
Conformal coupling    Disformal coupling

Neglect it, heavily suppressed

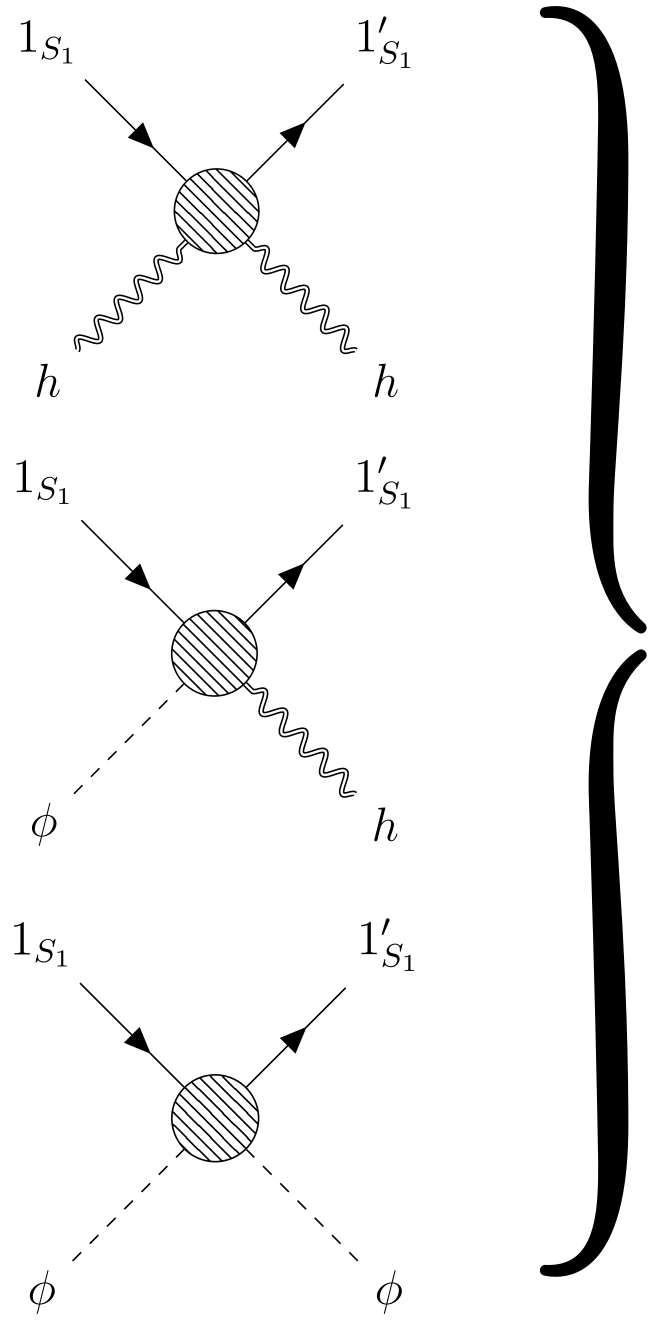
Generate 3-point amplitudes for arbitrary spinning BH:

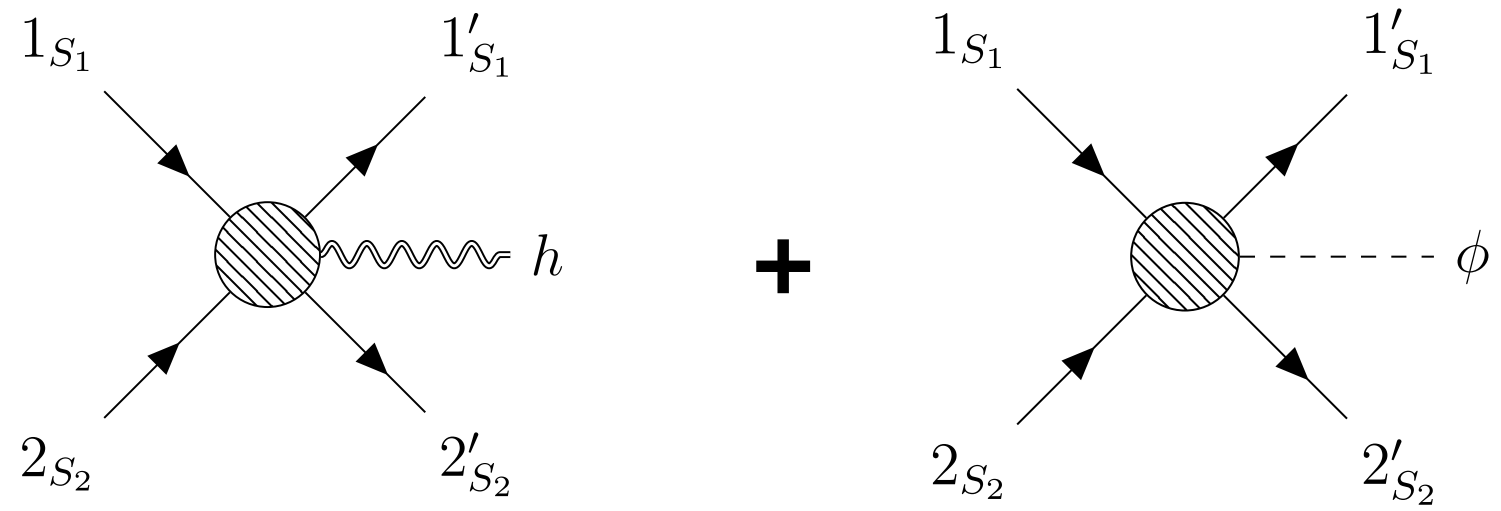
$$\mathcal{M}_{3,bos.}(1_\phi,2_A,3_A) = \frac{c_1}{M_{Pl}}\frac{\langle\mathbf{32}\rangle^S[\mathbf{32}]^S}{m^{2S-2}}$$

$$\mathcal{M}_{3,ferm.}(1_\phi,2_\Psi,3_{\bar{\Psi}}) = \frac{c_1}{M_{Pl}}\frac{\langle\mathbf{32}\rangle^{S-1/2}[\mathbf{32}]^{S-1/2}(\langle\mathbf{32}\rangle+[\mathbf{32}])}{m^{2S-2}}$$




$$\exp\left[C(\frac{\phi}{M_{Pl}})\right] \approx 1 + c\frac{\phi}{M_{Pl}}$$




**Simple mass redefinition at all spin orders:**

$$m \to e^{C/2}m$$

**Resemblance to <u>skeletonized action</u> used in GR literature:**

Astrophysical Journal, vol. 196, Mar. 1, 1975, pt. 2, p. L59-L62. [Eardley]

1992 Class. Quantum Grav. 9 2093 [Damour, Esposito-Farese]

# Results:

## LO scalar waveform for scalar Gauss-Bonnet:

$$
\begin{aligned}
W_\phi = \frac{m_1 m_2}{8\pi^2 b^3 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 \sqrt{\gamma^2-1}} \Bigg( \alpha \Bigg\{ -\frac{d^2}{dz^2}\Bigg[ \frac{1}{\sqrt{z^2+1}} \frac{2(\gamma^2-1)^2(\hat{u}_2 n)^2 \big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]}{\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]^2 + (\tilde{v} n)^2 (z^2+1)} \Bigg] + \big[(\hat{u}_1 n) + \gamma\big(2\gamma^2 - 3\big)(\hat{u}_2 n)\big] \frac{2z^2-1}{(z^2+1)^{5/2}} + \big(2\gamma^2-1\big)(\tilde{b} n) \frac{3z}{(z^2+1)^{5/2}} \Bigg\} \\
+ \frac{2\alpha}{b} \frac{d^3}{dz^3} \mathrm{Re} \Bigg\{ \frac{1}{\sqrt{z^2+1}} \frac{(\hat{u}_2 n) - \gamma(\hat{u}_1 n) + \gamma(\tilde{b} n) z + i\gamma(\tilde{v} n)\sqrt{z^2+1}}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z + i(\tilde{v} n)\sqrt{z^2+1}} \Big[ z(\tilde{v} n) - i\sqrt{z^2+1}(\tilde{b} n) \Big] \times \Bigg[ \frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1 \hat{u}_2) - (a_2 \hat{u}_1) - (a_1^A - a_2^A)(z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A) \Bigg] \Bigg\} \\
- \sqrt{\gamma^2-1} \frac{C_1}{b} \frac{d^3}{dz^3} \Bigg\{ \frac{1}{\sqrt{z^2+1}} \mathrm{Re} \Bigg[ \Bigg( z\big[(\tilde{v} n) a_1^A + (a_1 \tilde{v}) n^A\big] - i\sqrt{z^2+1}\big[(\tilde{b} n) a_1^A + (a_1 \tilde{b}) n^A\big] \Bigg) \times \Bigg( \hat{u}_2^A - \gamma \hat{u}_1^A + \gamma\big[z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A\big] \Bigg) \Bigg] \Bigg\} \Bigg) \Bigg|_{z=T_1} + (1 \leftrightarrow 2) + \mathcal{O}(a^2)\,.
\end{aligned}
$$

**Connect to observables: Power emitted in scalar radiation**

$$\frac{dP_\phi}{d\Omega} = (\partial_t W_\phi)^2$$

$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{\beta^6}{b^8} \quad \xrightarrow{\text{For closed orbits}} \quad \left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \beta^{22}$$

**Bigger suppression** compared to $\beta^8$ previously computed with GR methods

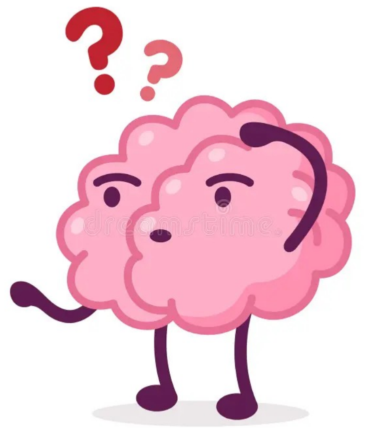
Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

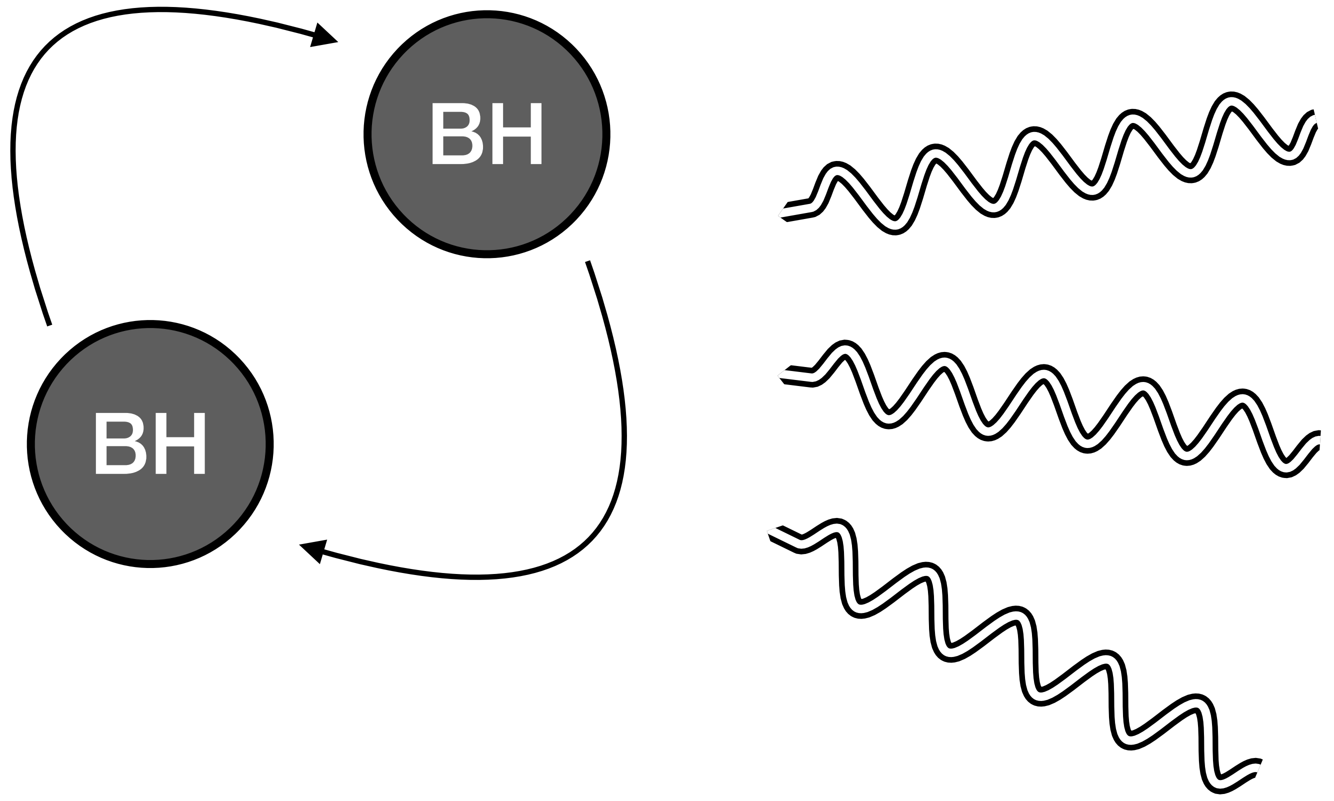
$$\left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^1)} \sim \frac{\beta^4}{b^{10}} \quad \xrightarrow{\text{For closed orbits}} \quad \left.\frac{dP_\phi}{d\Omega}\right|_{\mathcal{O}(a^1)} \sim \beta^{24}$$

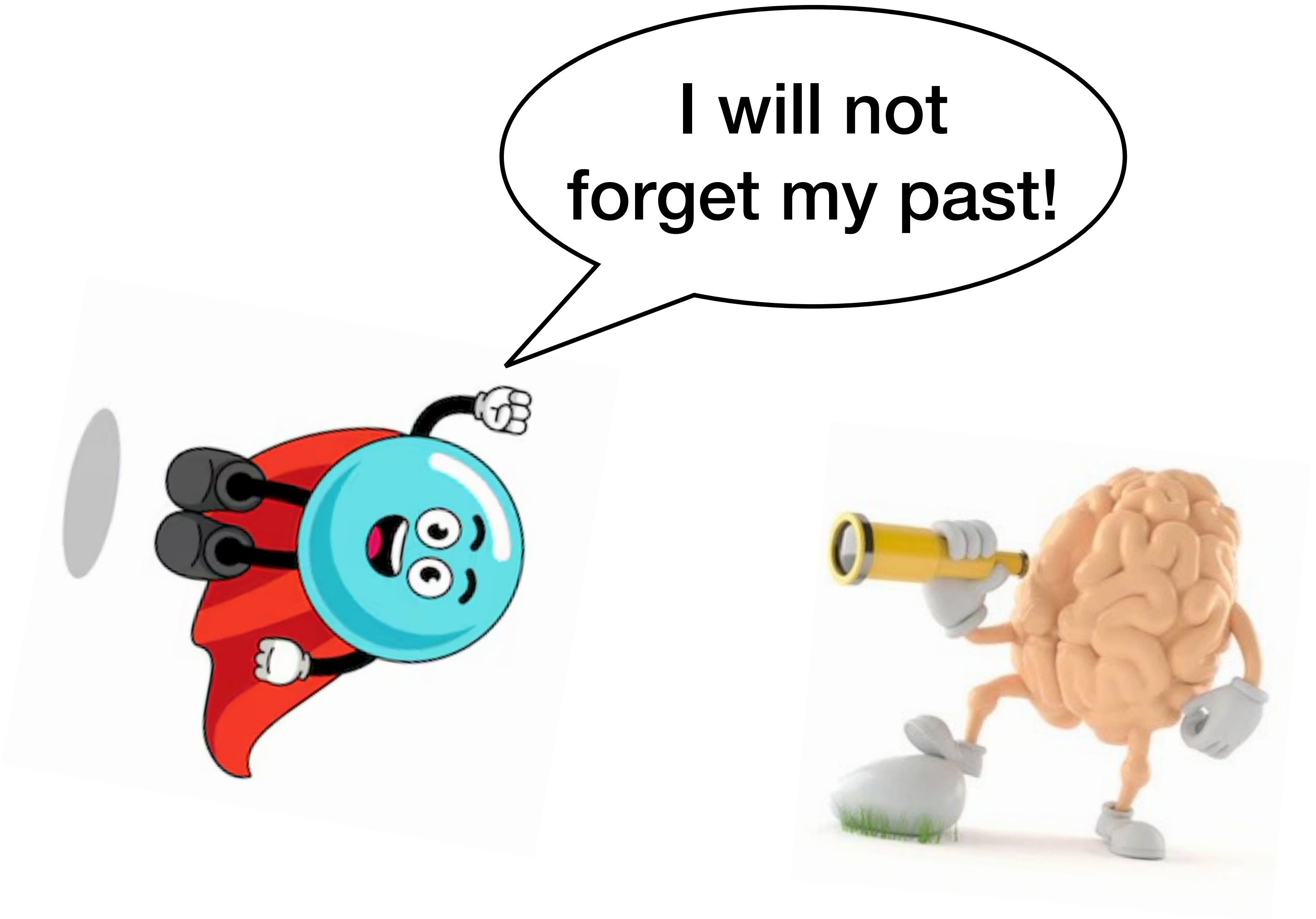
$$\beta = \frac{v}{c}$$

**So what's the difference?**



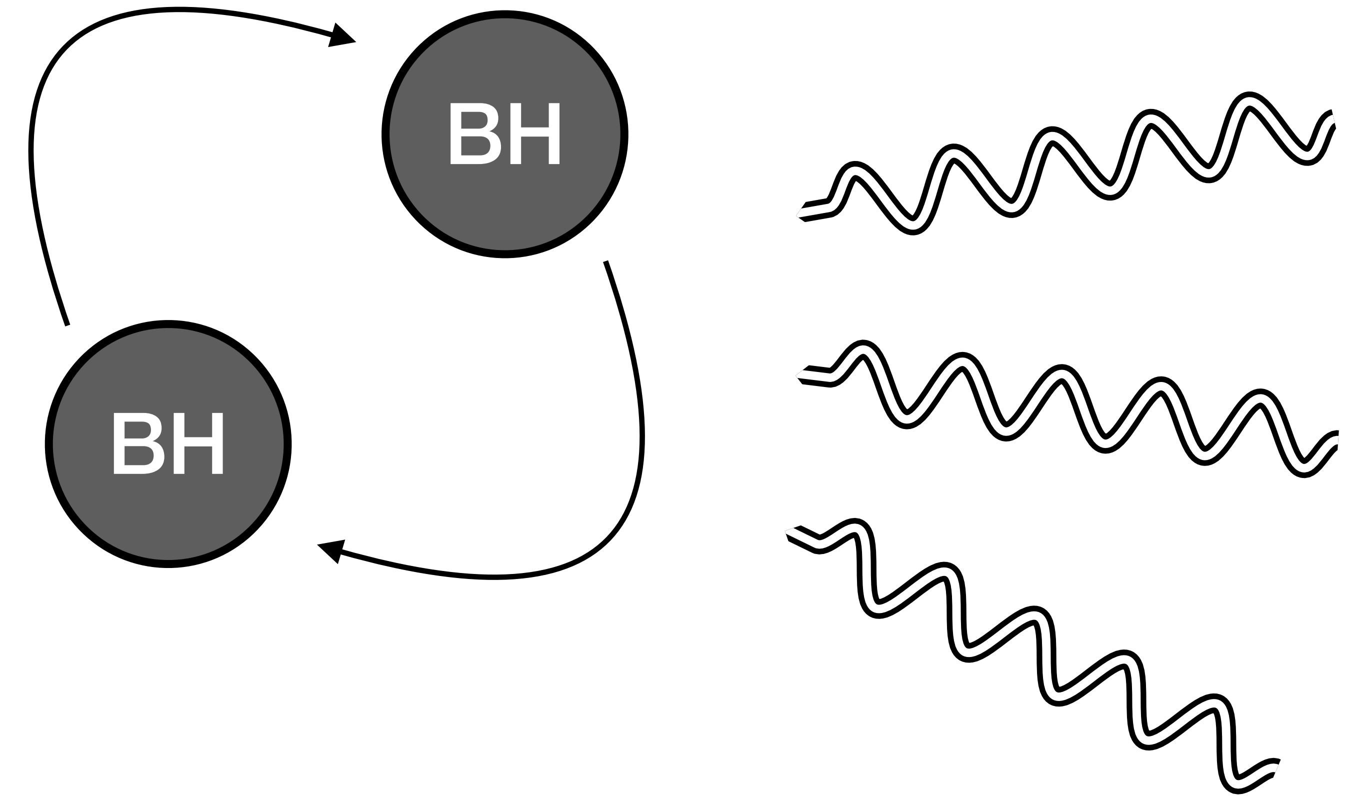
# Memory effect:

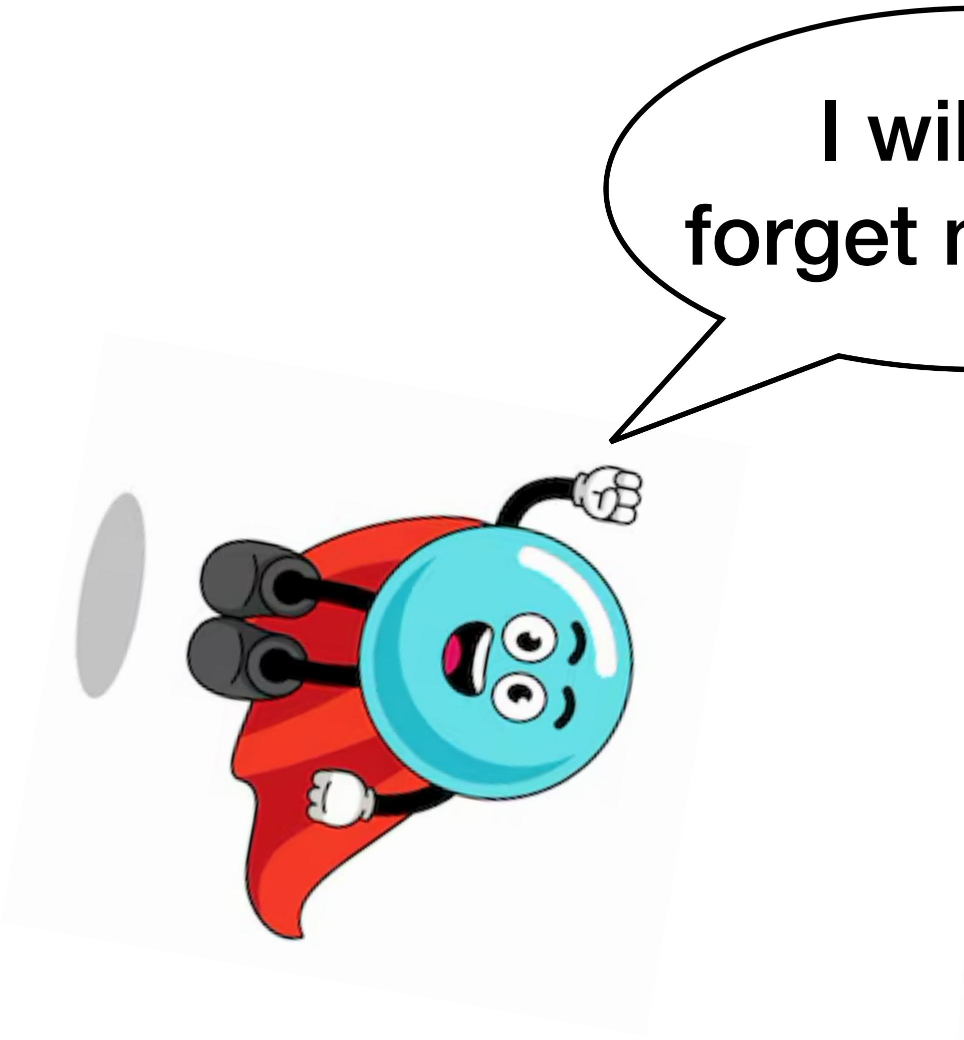



- **Scalar memory effect:** $\Delta\phi = W_\phi(t=\infty) - W_\phi(t=-\infty)$
- **Gravitational memory effect:** $\Delta h = h(t=\infty) - h(t=-\infty)$

# Memory effect:



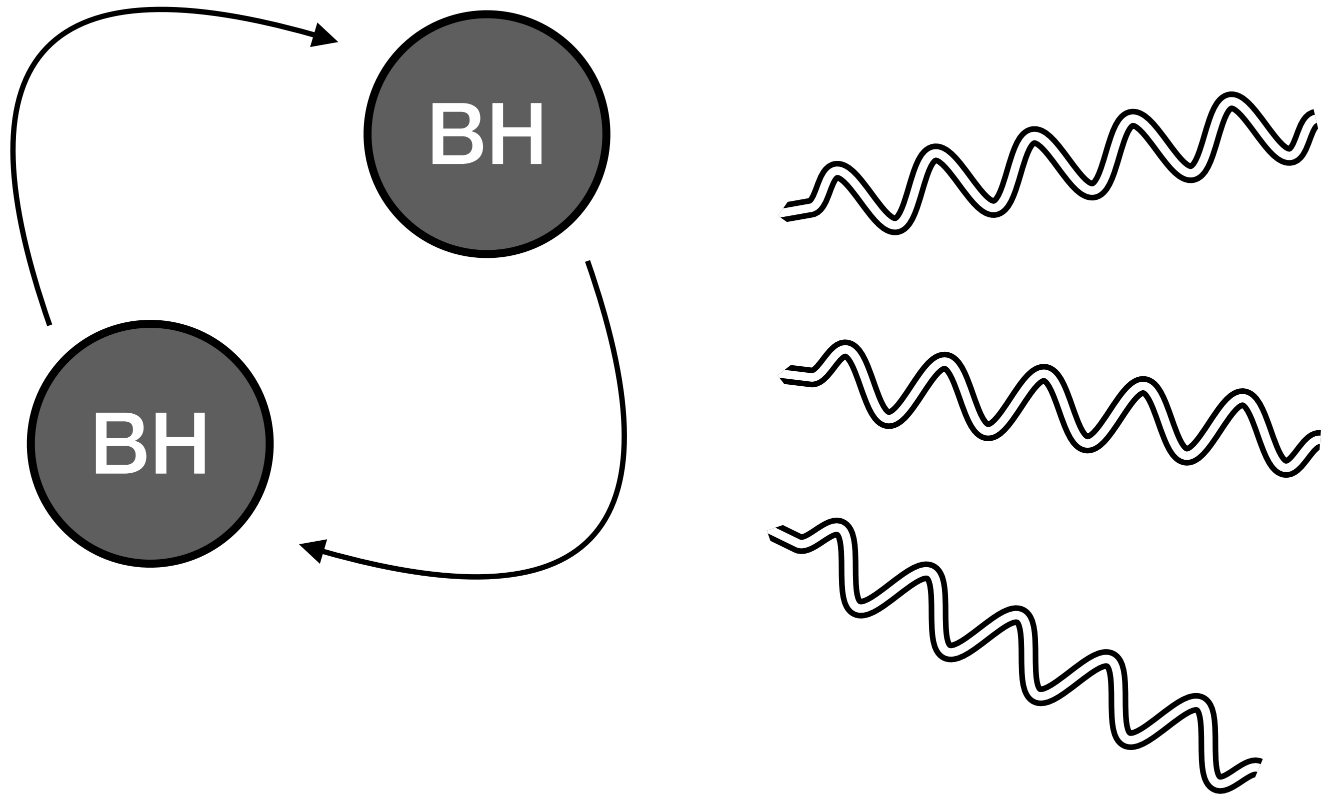



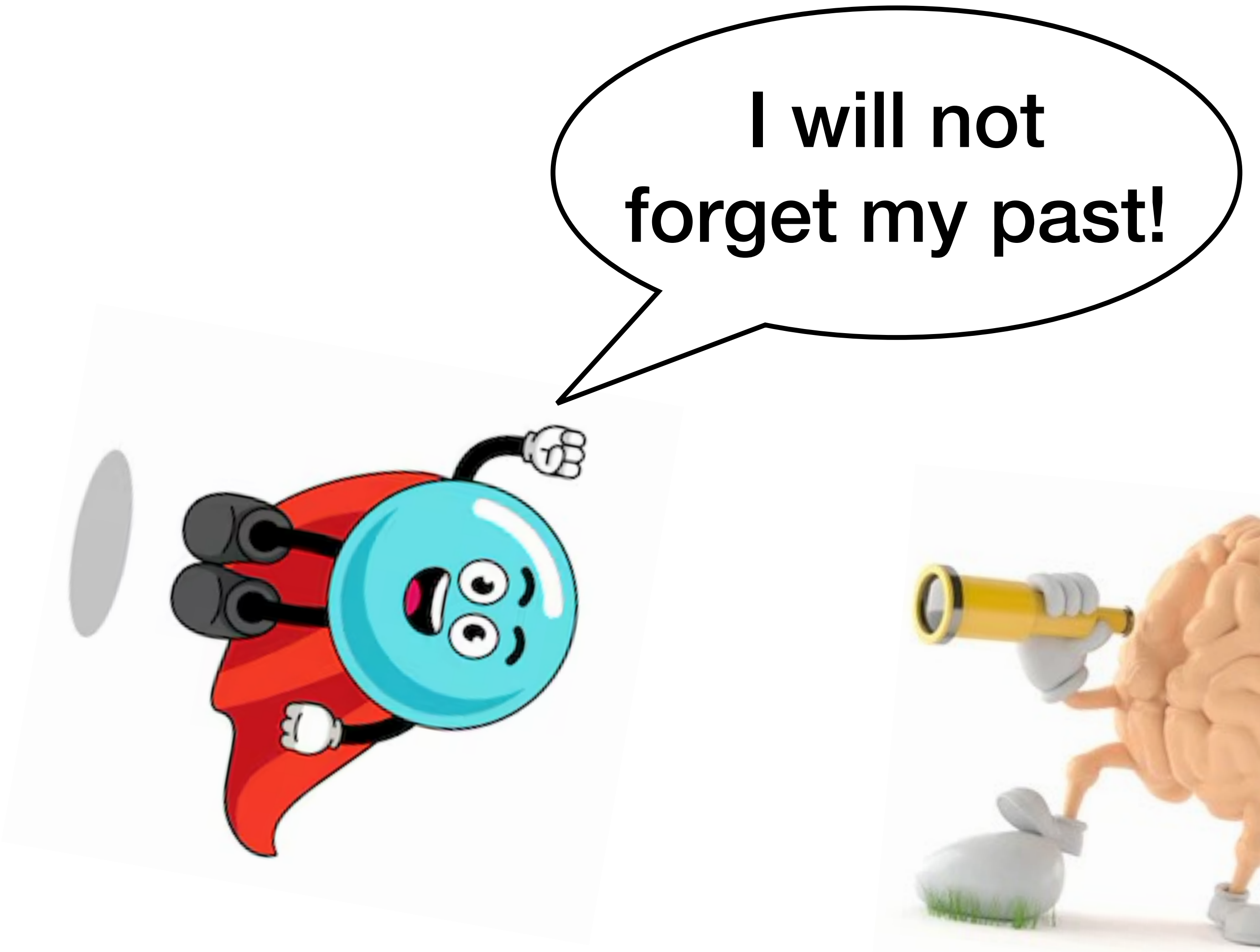

- **Scalar memory effect:** $\Delta\phi = W_\phi(t=\infty) - W_\phi(t=-\infty)$
- **Gravitational memory effect:** $\Delta h = h(t=\infty) - h(t=-\infty)$

At LO, can be related to the soft behaviour of the 5-point amplitude

$$\mathscr{M}_5 \to Soft \times \mathscr{M}_4$$

# Memory effect:




- **Scalar memory effect:** $\Delta\phi = W_\phi(t=\infty) - W_\phi(t=-\infty)$
- **Gravitational memory effect:** $\Delta h = h(t=\infty) - h(t=-\infty)$

At LO, can be related to the soft behaviour of the 5-point amplitude

$$\mathscr{M}_5 \to Soft \times \mathscr{M}_4$$

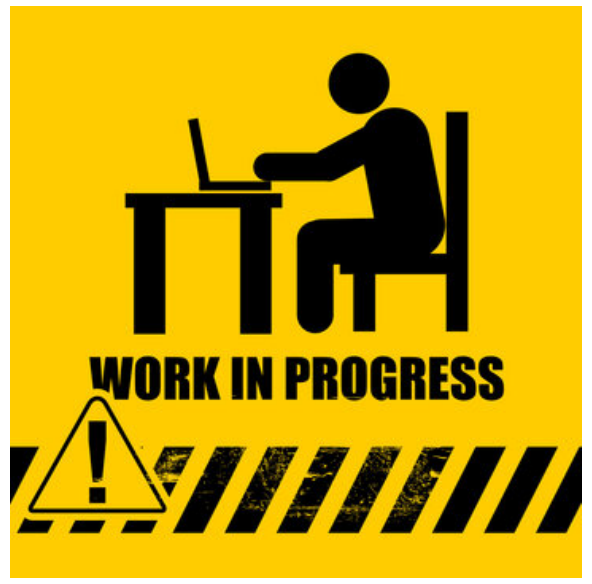
→ **No-hair objects** have **no leading order memory!**
**Reason**: There are no modifications to the 4-point amplitudes at tree-level.

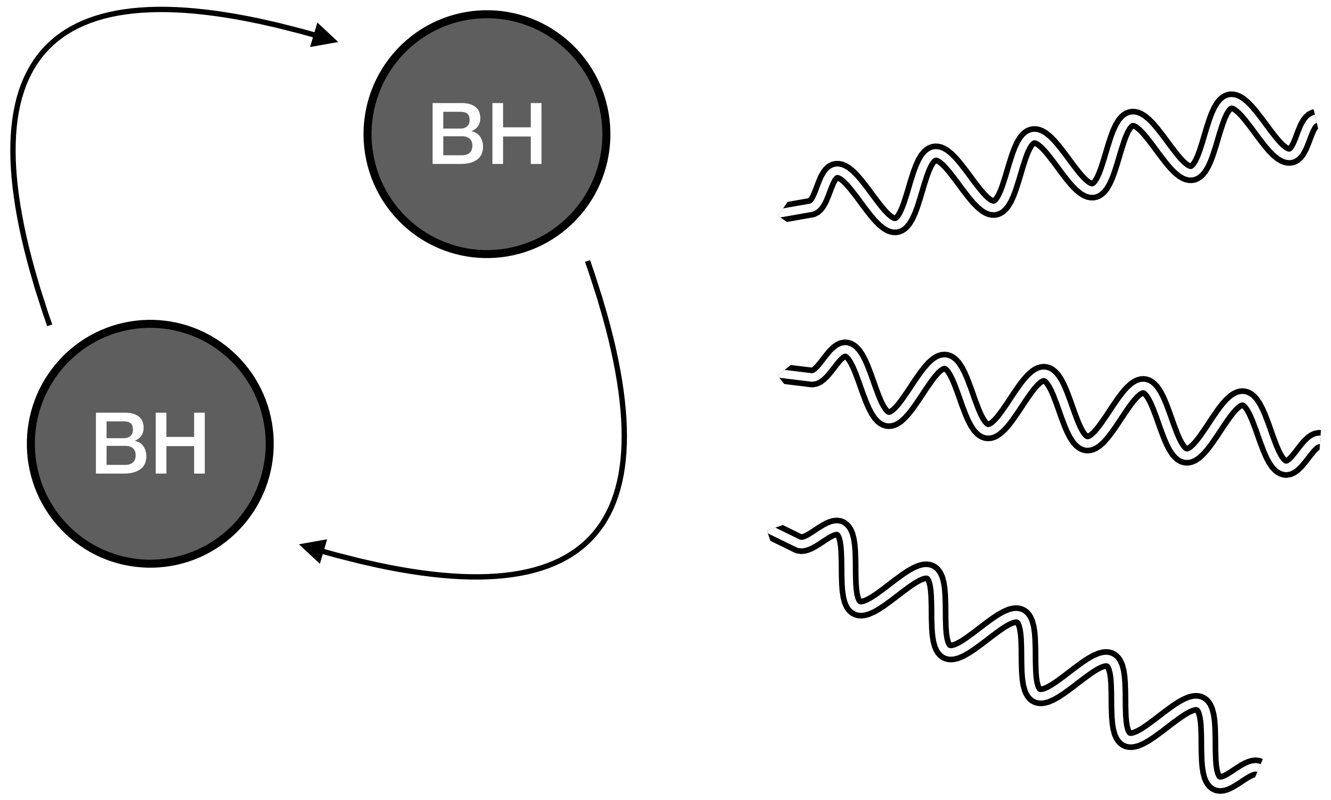
**A more general understanding**: Higher order interactions essentially contribute factors of $\frac{\partial^n}{\Lambda^n}$ in the action. At the level of the 5-point Amplitude, this translates to:

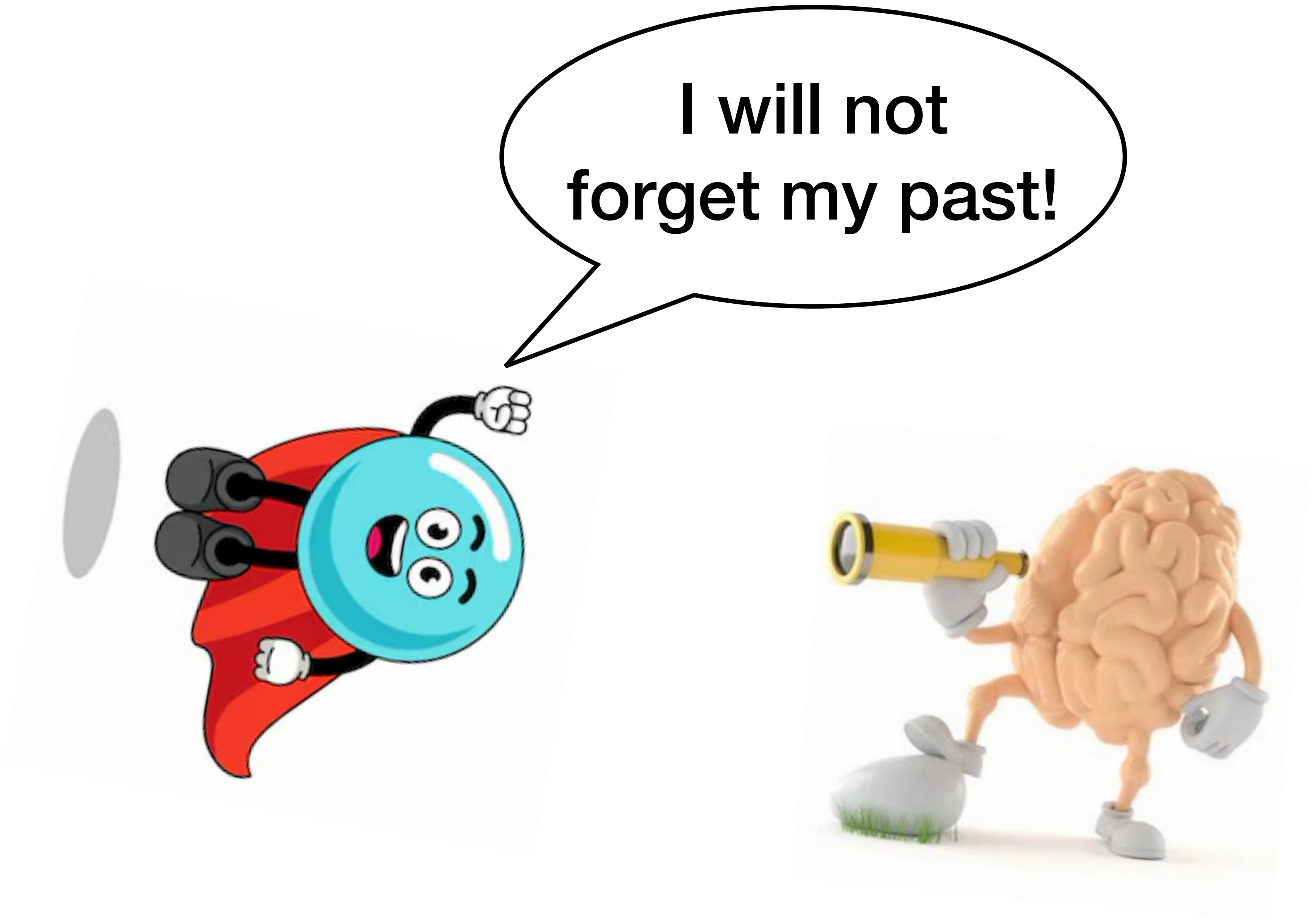
$$\frac{\partial^n}{\Lambda^n} \to \frac{k^n}{\Lambda^n} \sim \frac{\omega^n}{\Lambda^n}$$

, for $k^\mu = \omega n^\mu$ → kills the amplitude in the soft limit

# Memory effect:




→ **Hairy objects do have leading order memory!**

→ The gravitational memory effect can be elegantly expressed in terms of the classical soft factor:

$$\Delta h = -\frac{c_1 c_2 m_1 m_2}{4\pi M_{Pl}^3 \sqrt{\gamma^2 - 1}} \frac{1}{|b|} S^{\rm cl}_{W,s}(n, \tilde{b})$$

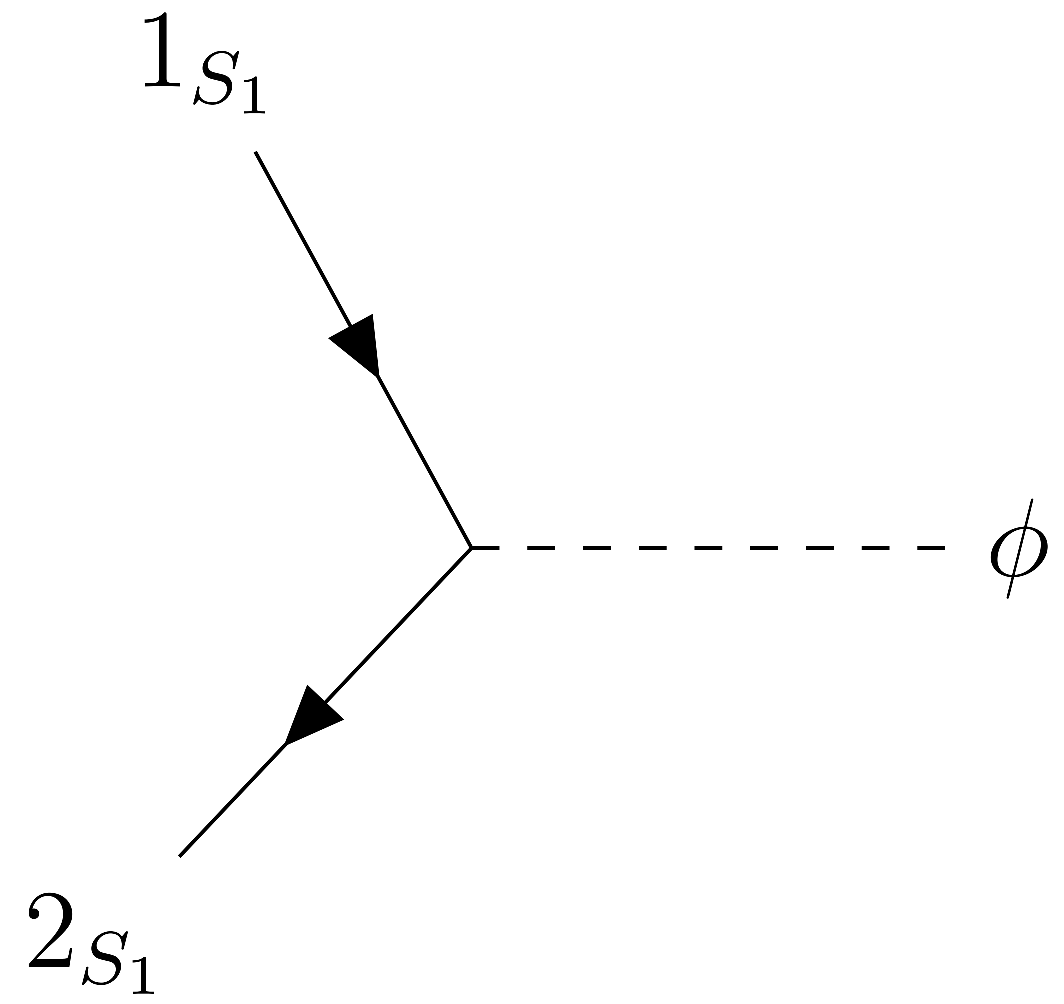
$$\Cap$$

$$\mathscr{M}^{(0)}_{ST}[1^{S_1}_{\Phi_1} 2^{S_2}_{\Phi_2} 3^{S_1}_{\bar{\Phi}_1} 4^{S_2}_{\bar{\Phi}_2}]$$

$$S^{\rm cl}_{W,s}(k,q) = \epsilon_{\mu\nu,s}\left[\frac{p_1^\mu q^\nu + p_1^\nu q^\mu}{p_1 k} - \frac{p_1^\mu p_1^\nu (qk)}{(p_1 k)^2}\right] + (1 \leftrightarrow 2)$$

→ Scalar memory trivially computed: $\Delta\phi = \dfrac{m_1 m_2 (2\gamma^2 - 1)(\tilde{b}n)\left[c_1(\hat{u}_2 n)^2 - c_2(\hat{u}_1 n)^2\right]}{32\pi^2 M_{Pl}^3 (\gamma^2 - 1)^{3/2} (\hat{u}_1 n)^2 (\hat{u}_2 n)^2 b}$

# EFT matching and heavy spinning particles:



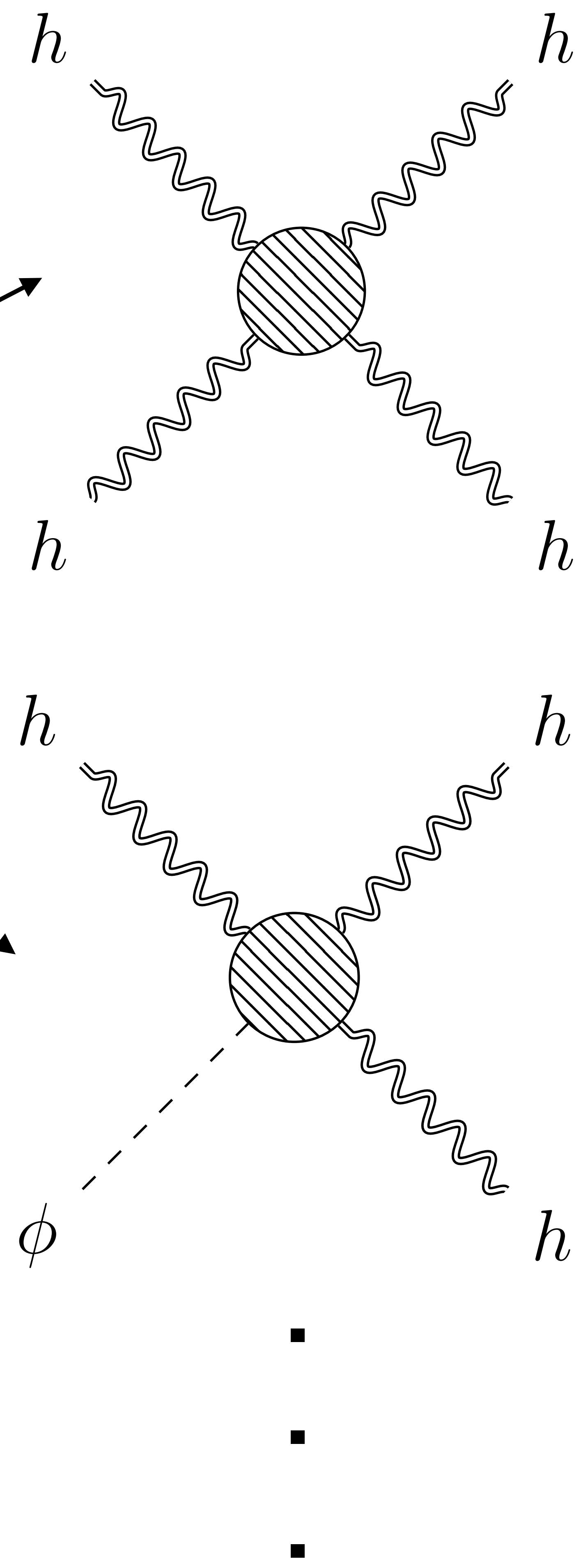

How should the matter couple in the UV to produce this interaction?

⟶ Interaction has to produce the **shift-symmetry** we see in the IR!

Simple in the **on-shell** language: $\mathscr{M}_3^{\mathrm{ss}}[1_\phi, 2_\Phi, 3_{\bar{\Phi}}] = \beta \left\{ \langle 32 \rangle + [32] \right\}^{2S-1} [\langle 32 \rangle - [32]]\,.$

# EFT matching and heavy spinning particles:

**IR**

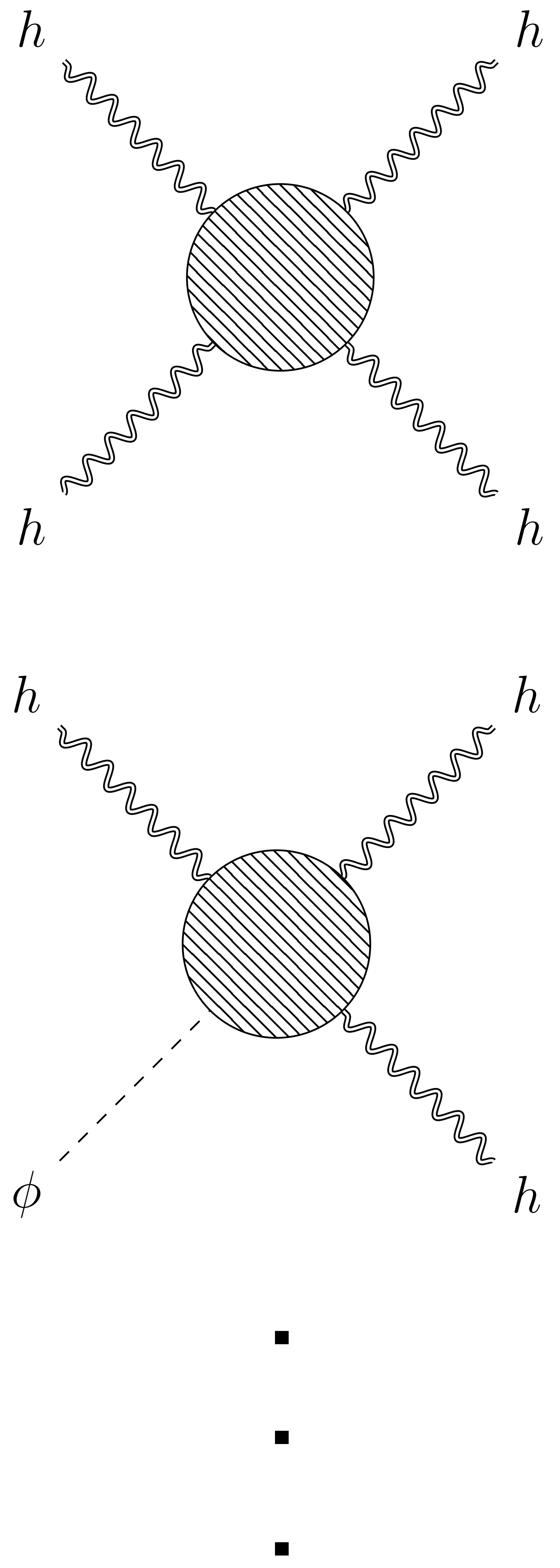



tree-level diagrams through the SGB/ DCS interactions

Match amplitudes' discontinuities in different channels!

Use of generalized unitarity makes the treatment systematic and simple

**UV**




matter running inside the loops