Gravitational Waves from Gravitational Particle Production

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2206.08940, 2303.07359, 2305.14446, **2412.XXXXX**

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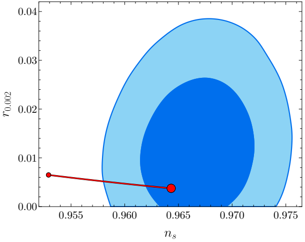
Scalar spectator

Consider a scalar χ (spin 0) which only interacts with gravity and/or the inflaton ϕ

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[-\frac{1}{2}M_P^2 R + \frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 \right.$$
$$\left. - \underbrace{6\lambda M_P^4 \tanh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)}_{\text{T-model inflation}} - \frac{1}{2}\underbrace{\sigma\phi^2\chi^2}_{\phi\text{-coupling}} - \underbrace{y\phi\bar{\psi}\psi}_{\text{reheating}} + \mathcal{L}_{\rm SM}\right]$$

Inflationary couplings normalized by

$$\lambda \simeq \frac{3\pi^2 A_{S*}}{N_*^2}, \qquad T_{\rm reh} \simeq \left(\frac{9\lambda}{20\pi^4 g_{\rm reh}}\right)^{1/4} y\,M_P$$



J. Ellis et al., PRD 105 (2022) 043504; Planck + BICEP/Keck PRL 127 (2021) 151301

Gravitational particle production

Introducing *conformal time*, $dt = a\, d\tau$, and the re-scaled field $X = a\chi$,

$$\left(\partial_\tau^2 - \nabla^2 + a^2 m_{\rm eff}^2\right) X \;=\; 0\,, \qquad m_{\rm eff}^2 \;=\; m_\chi^2 + \sigma\phi^2 + \frac{1}{6} R$$

Quantize as a superposition of oscillators

$$\hat{X}(\tau, \mathbf{x}) \;=\; \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}}\, e^{-i\boldsymbol{k}\cdot\mathbf{x}} \left[X_k(\tau)\hat{a}_{\boldsymbol{k}} + X_k^*(\tau)\hat{a}^\dagger_{-\boldsymbol{k}}\right]\,, \qquad [\hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{k}'}] = \delta(\boldsymbol{k} - \boldsymbol{k}')\,, \quad \hat{a}_{\boldsymbol{k}}|0\rangle \;=\; 0$$

obtaining

$$X_k'' + \omega_k^2 X_k \;=\; 0\,, \quad \text{with} \quad \omega_k^2 \;=\; k^2 + a^2 m_{\rm eff}^2$$

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For a mode inside the horizon,

$$\omega_k^2 \;=\; k^2 + \mathcal{O}\left(\frac{a^2H^2}{k^2}\right) \;>\; 0$$

free particle

interactions

Gravitational particle production

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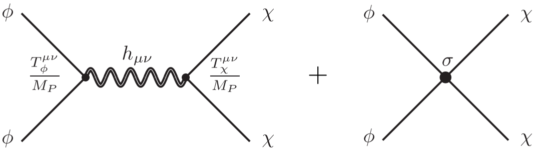


$$|\mathcal{M}|^2 \;=\; \frac{1}{8}\,\frac{\rho_\phi^2}{m_\phi^4}\,(\sigma-\lambda)^2$$

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{K}|\frac{\partial f_\chi}{\partial|\boldsymbol{K}|} \;=\; \mathcal{C}[f_\chi] \quad \Rightarrow \quad f_\chi \propto |\boldsymbol{K}|^{-9/2}$$

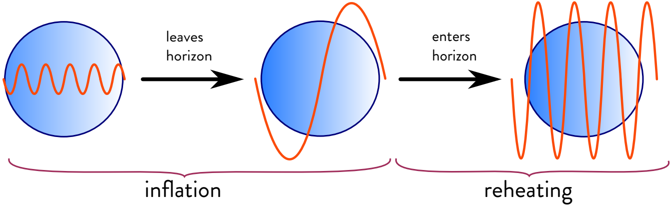
Gravitational particle production

Light scalar fields are unstable during inflation

$$X_k'' + \omega_k^2 X_k = 0\,, \quad \text{with} \quad \omega_k^2 = k^2 + 2(aH)^2\left[\frac{m_\chi^2}{2H^2} + \frac{\sigma\phi^2}{2H^2} - 1\right]$$

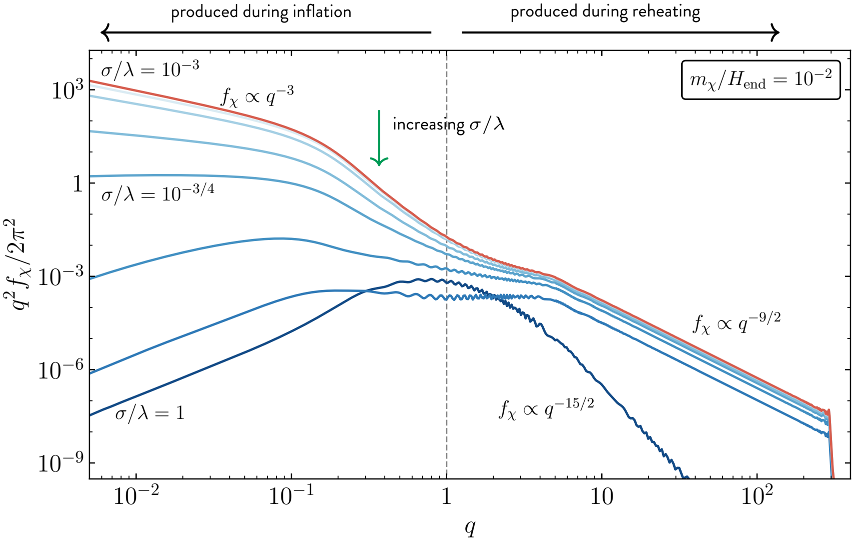
For a mode that is outside the horizon ($k/aH \ll 1$),

$$\omega_k^2 < 0 \qquad \text{if} \qquad m_\chi^2 < 2H^2\,,\ \sigma/\lambda \ll 1 \qquad \text{(tachyonic instability)}$$



No free particle state during inflation $\Rightarrow$ no perturbative picture

Weak inflaton coupling



Spectator as Dark Matter

Relic abundance

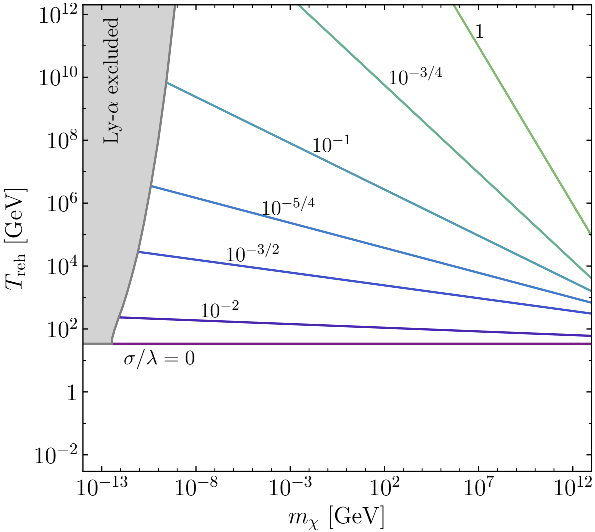
$$\Omega_{\rm DM} \simeq \frac{\rho_\chi}{\rho_c} \propto \frac{m_\chi T_{\rm reh}}{M_P^2} \int dq\, q^2 f_\chi(q)$$

Structure formation constraint

$$m_{\rm WDM} > (1.9 - 5.3)\ {\rm keV} \quad ({\rm Ly}\text{-}\alpha)$$

$$m_{\rm DM} = m_{\rm WDM} \left(\frac{T_\star}{T_{\rm WDM}}\right) \sqrt{\frac{\langle q^2 \rangle}{\langle q^2 \rangle_{\rm WDM}}}$$

G. Ballesteros, MG and M. Pierre, JCAP 03 (2021), 101



Isocurvature in the CMB

CDI: cold dark matter density isocurvature

NDI: neutrino density isocurvature

NVI: neutrino velocity isocurvature

However, they have not been detected,

$$\beta_{\mathrm{iso}} = \frac{\mathcal{P}_{\mathcal{S}}}{\mathcal{P}_{\mathcal{R}} + \mathcal{P}_{\mathcal{S}}} < \begin{cases} 2.5\% \text{ (CDI)} \\ 7.4\% \text{ (NDI)} \\ 6.8\% \text{ (NVI)} \end{cases}$$

This constraint applies only at

large scales ($k_* = 0.002\,\mathrm{Mpc}^{-1}$)

At smaller scales, ¯_(ツ)_/¯

Y. Akrami et al. [Planck], Astron. Astrophys. 641, A10 (2020)

Isocurvature spectrum of spectator field

The **full** isocurvature spectrum is given by

D. Chung, E. Kolb, A. Riotto, L. Senatore, PRD 72, 023511 (2005)

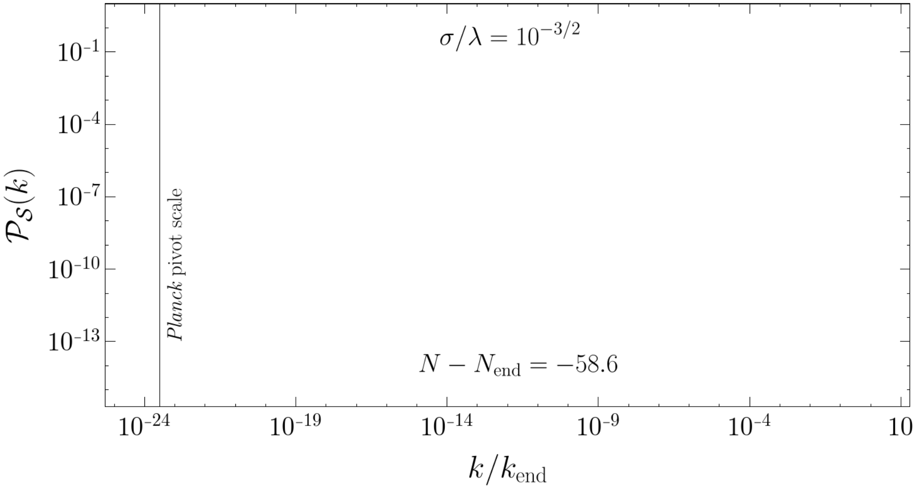
$$\mathcal{P}_{\mathcal{S}}(k) \;=\; \frac{k^3}{2\pi^2\rho_\chi^2}\int d^3\mathbf{x}\,\langle :\delta\rho_\chi(\mathbf{x})::\delta\rho_\chi(0):\rangle\, e^{-i\boldsymbol{k}\cdot\mathbf{x}} \;=\; \frac{k^3}{(2\pi)^5\rho_\chi^2}\int d^3\boldsymbol{p}\;P_\chi(p,|\boldsymbol{p}-\boldsymbol{k}|)$$

where

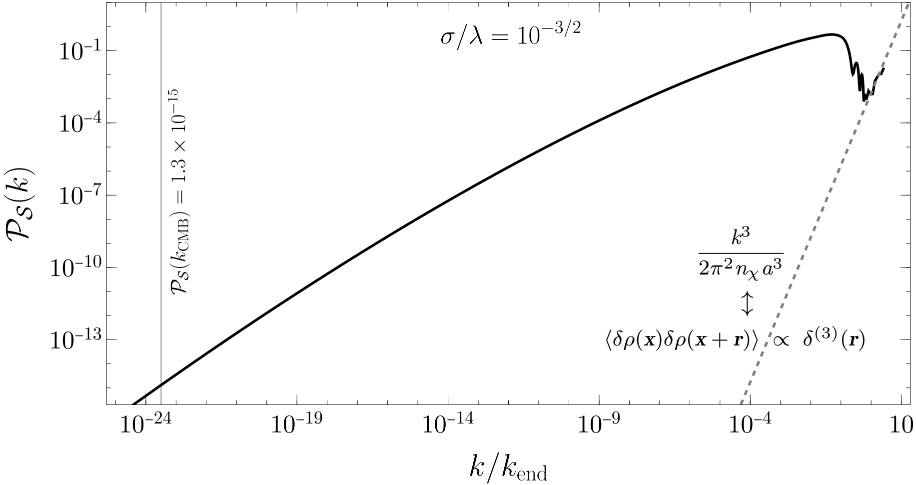
$$\begin{aligned}
P_\chi(p,q) \;=&\; |\chi_p'|^2|\chi_q'|^2 - H\Big[\chi_p\chi_p'^{\,*}|\chi_q'|^2 + \chi_q\chi_q'^{\,*}|\chi_p'|^2 - H(\chi_p\chi_p'^{*})(\chi_q'\chi_q^{*}) + \mathrm{h.c.}\Big] \\
&+ \left(\frac{p^2+q^2-k^2}{2a^2} + m_{\mathrm{eff}}^2 + H^2\right)\Big[(\chi_p\chi_p'^{*})(\chi_q\chi_q'^{*}) - H\left(\chi_p\chi_p'^{\,*}|\chi_q|^2 + \chi_q\chi_q'^{\,*}|\chi_p|^2\right) + \mathrm{h.c.}\Big] \\
&+ H^2\left(|\chi_p'|^2|\chi_q|^2 + |\chi_p|^2|\chi_q'|^2\right) + \left(\frac{p^2+q^2-k^2}{2a^2} + m_{\mathrm{eff}}^2 + H^2\right)^2|\chi_p|^2|\chi_q|^2
\end{aligned}$$

A. Long, unpublished

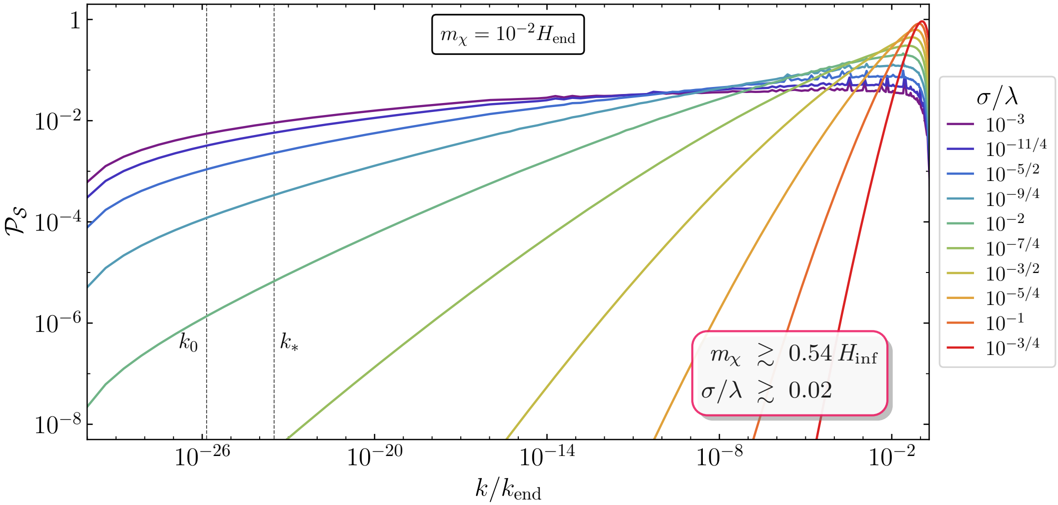
Isocurvature spectrum of spectator field



Isocurvature spectrum of spectator field



Isocurvature in gravitational production



$$m_\chi \gtrsim 0.54\, H_{\rm inf}$$
$$\sigma/\lambda \gtrsim 0.02$$

Induced gravitational waves

Gravitational waves sourced by the particle production

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} \;=\; P^{ab}_{ij}\left\{4\partial_a\Phi\partial_b\Phi + 2\partial_a\chi\partial_b\chi\right\}$$

In the uniform density gauge

$$\nabla^2\Phi \;=\; \frac{a^2}{2M_P^2}\left(\delta\rho_\chi - \frac{3\mathcal{H}}{a^2}\chi'\chi\right)$$

with

$$h_{ij}(\eta,\mathbf{x}) = \sum_\lambda \int \frac{d^3k}{(2\pi)^{3/2}} e^{i\mathbf{k}\cdot\mathbf{x}}\epsilon^\lambda_{ij}(\mathbf{k})h_{\mathbf{k},\lambda}(\eta)$$

one gets

$$h_{\mathbf{k},\lambda}(N) \;=\; -\frac{1}{M_P^4}\int^N dN'\mathcal{G}_{\mathbf{k}}(N,N')\left(\frac{a(N')}{H(N')}\right)^2\int\frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}}\frac{\epsilon^\lambda_{ij}\boldsymbol{p}^i(\boldsymbol{k}-\boldsymbol{p})^j}{|\boldsymbol{p}|^2|\boldsymbol{k}-\boldsymbol{p}|^2}\,\delta\rho_{\chi,\boldsymbol{p}}(N')\delta\rho_{\chi,\boldsymbol{k}-\boldsymbol{p}}(N') + \cdots$$

Define

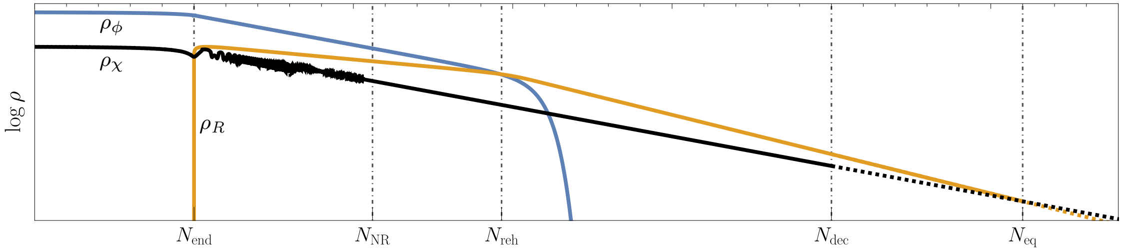
$$\langle h_{\mathbf{k}}(N)h_{\mathbf{k}'}(N)\rangle \;=\; \frac{2\pi^2}{k^3}\mathcal{P}_h(k)\,\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$$

Induced gravitational waves

$$\Omega_{\rm GW} = \frac{1}{48}\left(\frac{k}{aH}\right)^2 \mathcal{P}_h(k)$$

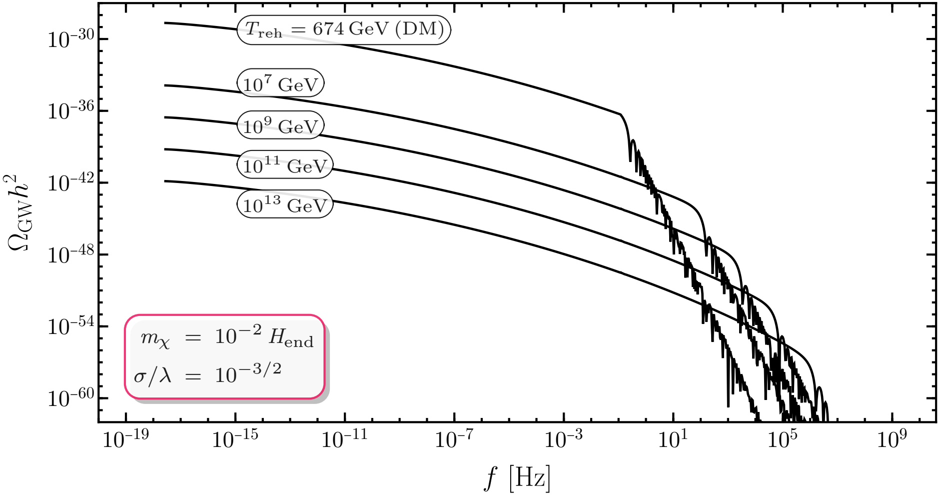
$$\mathcal{P}_h(k) = \left[\frac{1}{8}\int^N dN'\, \mathcal{G}_{\boldsymbol{k}}(N,N')\; T_{\mathcal{S}}(N') \left(\frac{a(N')H(N')}{k}\right)^2 \left(\frac{\rho_\chi(N')}{H^2(N')M_P^2}\right)^2\right]^2 \Delta(k/k_{\rm end}) + \cdots$$

$$\Delta(k/k_{\rm end}) = k^2 \int_0^\infty dp\, p \int_{|k-p|}^{k+p} dq\, q\, \frac{(k^4 - 2k^2(p^2+q^2) + (p^2-q^2)^2)^2}{p^7\, q^7}\, \mathcal{P}_{\mathcal{S}}(p)\mathcal{P}_{\mathcal{S}}(q)$$



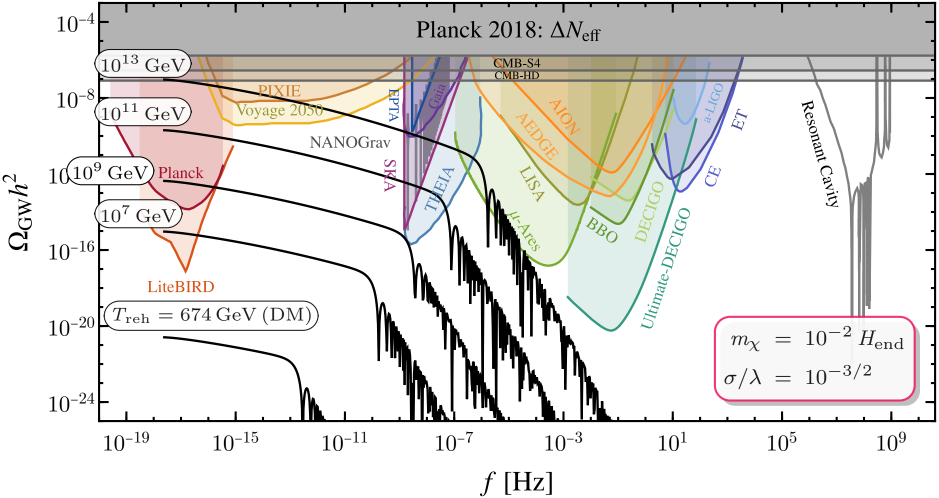
Peak contribution

The peak contribution freezes at N_{reh}, **but** $\rho_\chi \ll \rho_{\phi,R}$



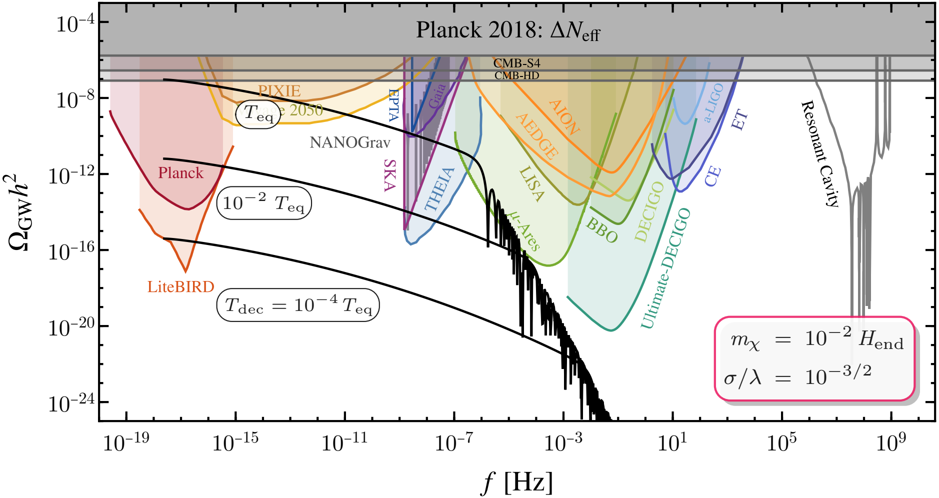
Tail contribution

The tail contribution freezes at $N_{\rm dec} = N_{\rm eq}$



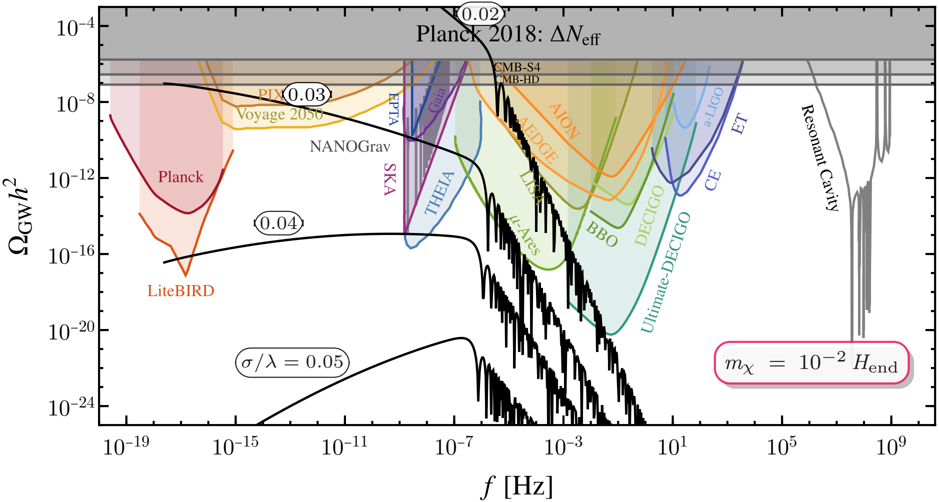
Tail contribution

The tail contribution freezes at $N_{\rm dec} \leq N_{\rm eq}$, $T_{\rm reh} = 10^{13}$ GeV



Tail contribution

The tail contribution freezes at $N_{\rm dec} = N_{\rm eq}$, $T_{\rm reh} = 10^{13}\ {\rm GeV}$



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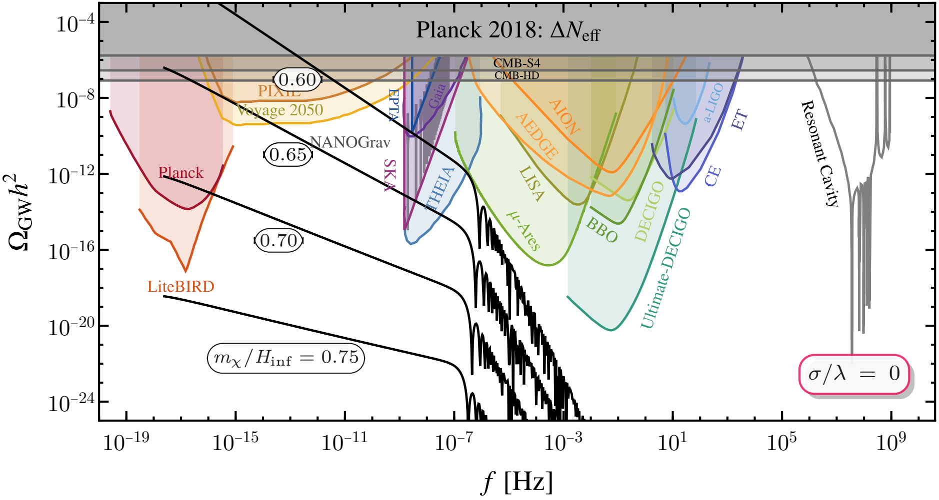
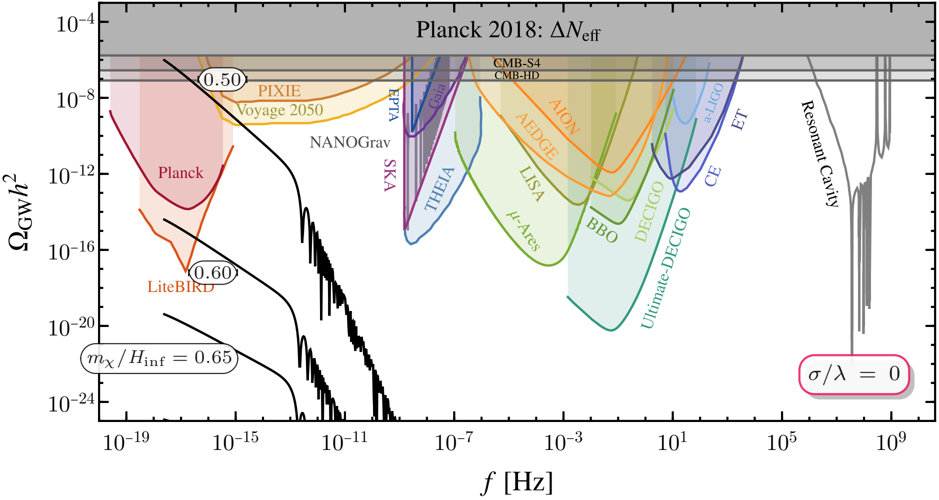
The tail contribution freezes at $N_{\mathrm{dec}} = N_{\mathrm{eq}}$, $T_{\mathrm{reh}} = 10^{13}\ \mathrm{GeV}$

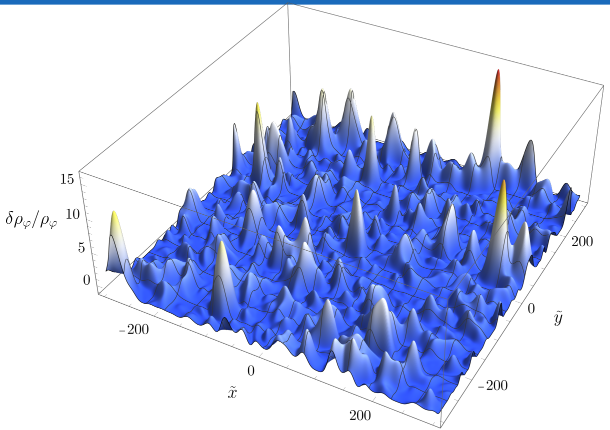


Tail contribution

Super-heavy DM scenario ($\Omega_\chi h^2 = 0.12$)



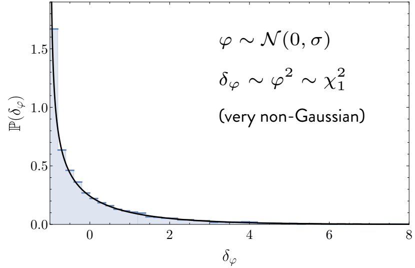
What about PBHs?



Configuration space approach

$$\langle\varphi(\mathbf{x})\varphi(\mathbf{x})\rangle = \int_0^\infty \frac{dk}{k}\,\mathcal{P}_\varphi(k)\,\frac{\sin(kr)}{kr}$$

$$\delta_\varphi(R,\mathbf{x}) = \int d^3\mathbf{x}'\;W(R,|\mathbf{x}-\mathbf{x}'|)\,\delta_\varphi(\mathbf{x}')$$



Stay tuned!