PQ inflation at the Pole

Adriana Menkara (DESY.), Hyun Min Lee, Myeong-Jung Seong, Jun-Ho Song (CAU)

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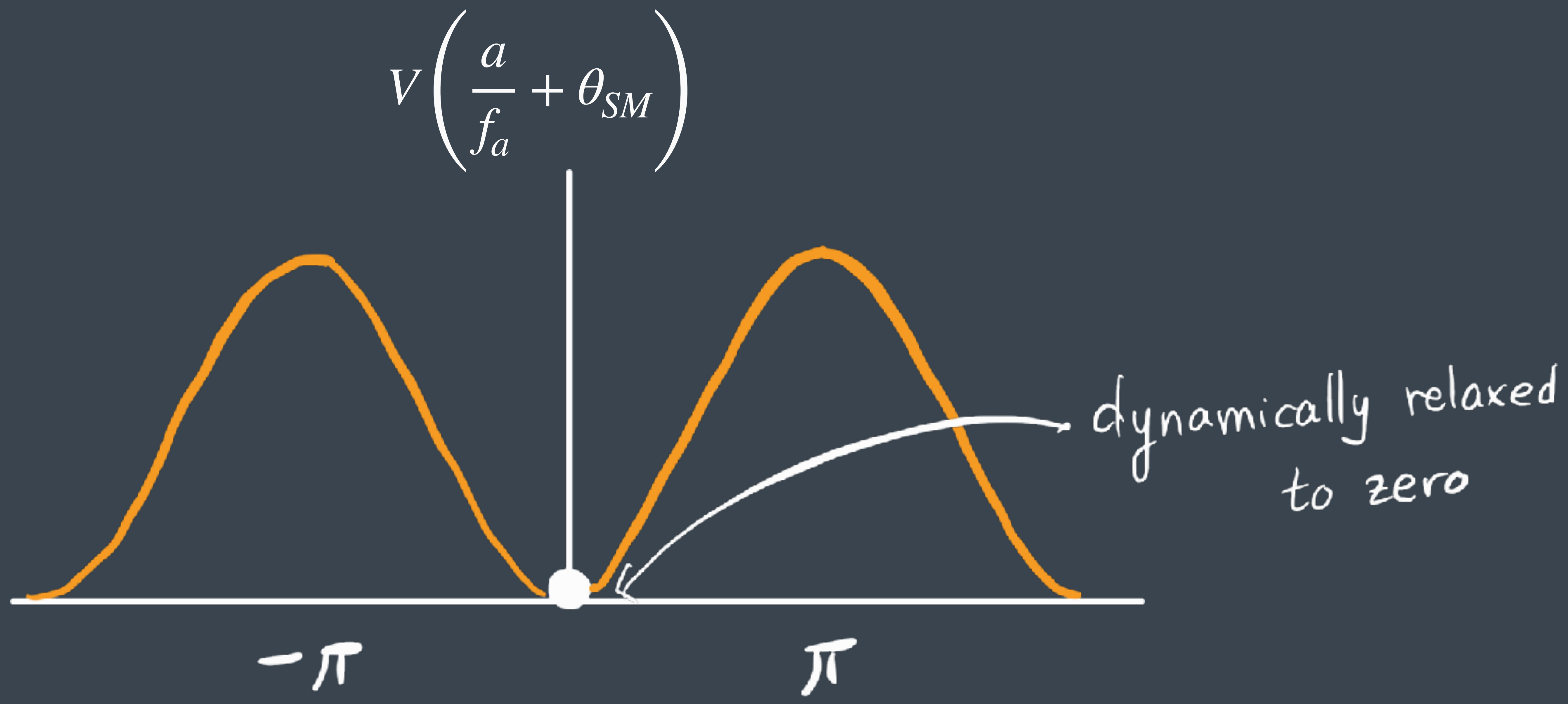
QCD axion and Peccei-Quinn

Strong CP Problem $\theta_{SM} = \theta_{QCD} + \arg[\det Y_u Y_d] \leq 10^{-10}$ **??**

Coupling to QCD $G\tilde{G}$

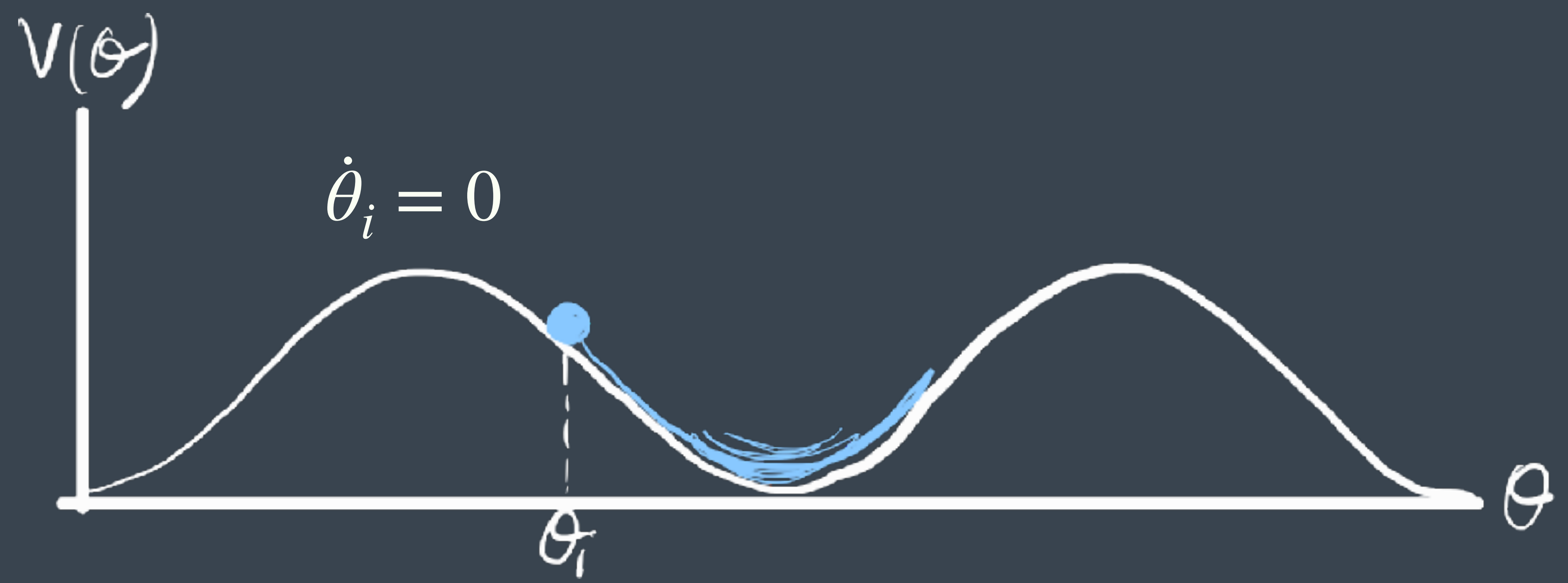
$$\Delta \mathscr{L}_{QCD} = \frac{a}{f_a} \frac{g_s^2}{32\pi^2} G_{\mu\nu} \tilde{G}^{\mu\nu}$$



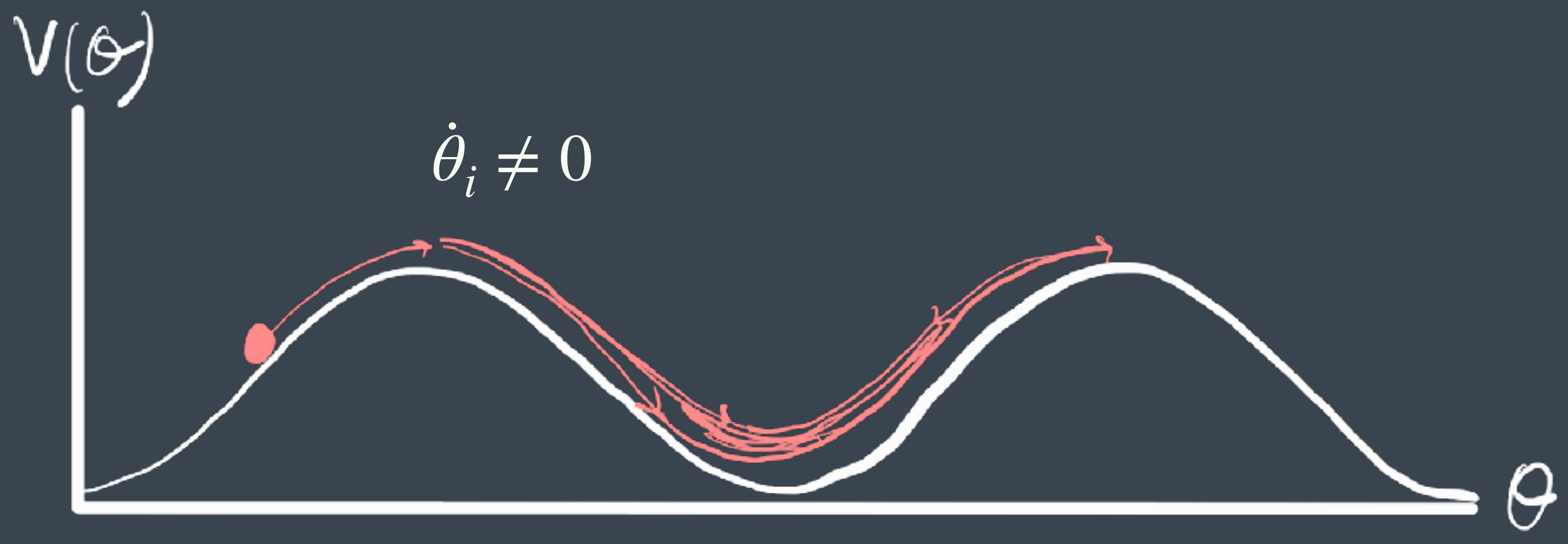
Shift symmetry $a = a + 2\pi f_a$

$$\mathscr{L}_{\text{int}} = c_1 \frac{\partial_\mu a}{f_a} \left(\bar{f} \gamma^\mu \gamma^5 f\right) + c_2 \frac{\alpha a}{8\pi f_a} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

Axions and Dark Matter



Misalignment mechanism



Kinetic misalignment

Axions bounds

$10^8\,\mathrm{GeV} < f_a$	**From supernovae**
$f_a < 1.5 \times 10^{11}\,\mathrm{GeV}$	**Model dependent !!**

Pole inflation

α-attractor properties arise from a non-minimal coupling to gravity,

$$\frac{\mathscr{L}_J}{\sqrt{-g_J}} = \frac{1}{2}M_P^2\left(1 - \frac{1}{6M_p^2}\phi^2\right)R_J + \frac{1}{2}\left(\partial_\mu\phi\right)^2 - V_J\left(\phi\right) \qquad V_J(\phi) = F(\phi)\left(1 - \frac{1}{6}\phi^2\right)^2$$

which in the Einstein frame translates as a **pole** in the kinetic term

$$\frac{\mathscr{L}_E}{\sqrt{-g_E}} = -\frac{1}{2}M_P^2R_E + \frac{1}{2}\frac{\left(\partial_\mu\phi\right)^2}{\left(1 - \frac{1}{6M_P^2}\phi^2\right)^2} - V_E(\phi)$$

Pole inflation

$$V_J(\phi) = c_m \Lambda^{4-2m} \phi^{2m} \left(1 - \frac{1}{6M_P^2} \phi^2 \right)^2$$

Inflation happens for a **vanishing Jordan frame potential**

$$\Updownarrow \qquad \phi = \sqrt{6} M_P \tanh\left(\frac{\chi}{\sqrt{6} M_P} \right)$$

$$V_E(\chi) = 3^m c_m \Lambda^{4-2m} M_P^{2m} \left[\tanh\left(\frac{\chi}{\sqrt{6} M_P} \right) \right]^{2m}$$

Inflation happens at the **pole of the kinetic Einstein frame**

1. KSVZ

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}M_P^2\,\Omega(\Phi)R(g_J) + |\partial_\mu\Phi|^2 - \Omega^2(\Phi)V_E(\Phi)$$

Conformal couplings

$$\Omega(\Phi) = 1 - \frac{1}{3M_P^2}|\Phi|^2$$

$$V_E(\Phi) = V_0' + \frac{\beta_m}{M_P^{2m-4}}|\Phi|^{2m} - m_\Phi^2|\Phi|^2 + \left(\sum_{k=0}^{[n/2]}\frac{c_k}{2M_P^{n-4}}|\Phi|^{2k}\Phi^{n-2k} + \mathrm{h.c.}\right)$$

PQ violating terms

The PQ violating terms are crucial for the axion non-zero velocity, but are constrained by the **axion quality problem**.

1. KSVZ

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PQ conserving terms

The PQ terms **drive inflation** and are responsible for the **SSB of the** $U(1)_{PQ}$

1.KSVZ

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+ Extra heavy quark **(QCD anomalies)** $\mathscr{L}_{Q,\mathrm{int}} = -y_Q\Phi\bar{Q}_R Q_L + \mathrm{h.c.}$

+ Higgs portal **(Reheating)** $\Delta V_E = \lambda_{H\Phi}|\Phi|^2|H|^2$

2. DFSZ Pole

SM + PQ + **extra Higgs doublet**

$$\frac{\mathcal{L}_J}{\sqrt{-g_J}} = -\frac{1}{2}M_P^2\Omega R(g_J) + |D_\mu H_1|^2 + |D_\mu H_2|^2 + |\partial_\mu \Phi|^2 - \Omega^2 V_E$$

We take conformal couplings: $\Omega = 1 - \frac{1}{3M_P^2}|H_1|^2 - \frac{1}{3M_P^2}|H_2|^2 - \frac{1}{3M_P^2}|\Phi|^2$

+ general Yukawa interactions $\mathcal{L}_Y = y_{ij}\bar{f}_L H_1 f_R + y'_{ij}\bar{f}_L H_2 f_R$

DFSZ Scalar potential

Inflation and SSB of the U(1)

$$V_{\rm PQ} = \lambda_\Phi |\Phi|^4 + \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 \left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right) + \lambda_{1\Phi} |H_1|^2 |\Phi|^2$$
$$+\lambda_{2\Phi} |H_2|^2 |\Phi|^2 + \left(2^{p/2-1} \kappa_p H_1^\dagger H_2 \Phi^p + \textbf{h.c.}\right) + \mu_\Phi^2 |\Phi|^2 + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + V_0$$

Axion rotation

$$V_{\rm PQV} = \sum_{n,l} \sum_{k=0}^{[l/2]} \frac{c_{n,l,k}}{2M^{2n+l-4}} (H_1^\dagger H_2)^n |\Phi|^{2k} \Phi^{l-2k} + {\rm h.c.}$$

DFSZ Scalar potential

Inflation and SSB of the U(1)

$$V_{\rm PQ} = \lambda_\Phi |\Phi|^4 + \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 \left(H_1^\dagger H_2\right)\left(H_2^\dagger H_1\right) + \lambda_{1\Phi} |H_1|^2 |\Phi|^2$$
$$+\lambda_{2\Phi} |H_2|^2 |\Phi|^2 + \left(2^{p/2-1}\kappa_p H_1^\dagger H_2 \Phi^p + \textbf{h.c.}\right) + \mu_\Phi^2 |\Phi|^2 + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + V_0$$

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Axion quality problem

Axion quality problem

After the QCD phase transition, we get the contribution $\Delta V_E = -\Lambda_{\rm QCD}^4 \cos\left(\bar\theta + \xi \frac{a}{f_a}\right)$

$$V_{\rm eff}(a) = -\Lambda_{\rm QCD}^4 \cos\left(\bar\theta + \xi \frac{a}{f_a}\right) + M_P^4 \left(\frac{f_a}{\sqrt{2} q_\Phi M_P}\right)^l \sum_{k=0}^{[l/2]} |c_{0,l,k}| \cos\left((l-2k)\frac{q_\Phi a}{f_a} + A_{0,l,k}\right)$$

In order to solve the strong CP problem we need

$$\left(\frac{f_a}{M_P}\right)^l \lesssim \frac{2^{l/2} \xi\, q_\Phi^{l-1}}{(l-2k)\,|c_{0,l,k}|} \left(\frac{\Lambda_{\rm QCD}}{M_P}\right)^4 \times 10^{-10}$$

$f_a = 10^{12}(10^8)\ {\rm GeV}$ requires $l \gtrsim 13(8)$

Domain Wall Number

	k_G	k_F	E/N
Type I	0	$3(pq_\Phi - q_1)$	$-$
Type II	$3pq_\Phi$	$4pq_\Phi$	$\frac{8}{3}$
Type X	0	$3pq_\Phi$	$-$
Type Y	$3pq_\Phi$	pq_Φ	$\frac{2}{3}$

$N_{DW} = K_G = 3pq_\phi$, so if PQ is **broken** after inflation we have a DW problem.

For comparison, for KSVZ models $N_{DW} = K_G = 1 \implies$ **no DW problem**.

Mixed Higgs-PQ inflation

- Inflation happens around the pole $\chi^2 \equiv \rho^2 + h_1^2 + h_2^2 \to 6$.
- The heavy angular direction $\tilde{A} \equiv -(1 + \langle \tau_1^2 \rangle / \langle \tau_2^2 \rangle)\eta_1 + p\theta$, is heavier that the Hubble rate and can be decoupled
- We need $\langle h_1/\rho \rangle \neq 0$ and $\langle h_2/\rho \rangle \neq 0$ due to tadpoles.

$$\Downarrow$$

Non-trivial relations between the quartic couplings or vanishing β functions.

Pure PQ inflation bounds

- Effective Lagrangian $\dfrac{\mathscr{L}_{\mathrm{bkg},E}}{\sqrt{-g_E}} = -\dfrac{1}{2}R + \dfrac{1}{2}(\partial_\mu \phi)^2 + 3\sinh^2\left(\dfrac{\phi}{\sqrt{6}}\right)\left(\partial_\mu \theta\right)^2 - V_E(\phi, \theta)$

- CMB normalization $\Longrightarrow \lambda_\Phi = 1.1 \times 10^{-11}$ during inflation.

- PQ invariant mass term is bounded by $|\mu_\Phi| < 1.4 \times 10^{13}\,\mathrm{GeV}$,

$$V_{\mathrm{PQV}} = 3^{l/2} \sum_{k=0}^{[l/2]} |c_{0,l,k}| \cos\Big((l-2k)\theta_i + A_{0,lk}\Big) < 1.0 \times 10^{-10}.$$

Reheating

$$\mathcal{L}_{\rm int} \supset -\sum_{i=1,2} \lambda_{i\Phi} |H_i|^2 |\Phi|^2 - 2^{p/2-1}\kappa_p H_1^\dagger H_2 \Phi^p - \frac{1}{2} y_N \overline{N_R^c} \Phi N_R + {\rm h.c.}$$

- Bounds due to $\lambda_\Phi = 1.1 \times 10^{-11}$

$\Gamma_{\phi\phi \to H_i^\dagger H_i}$: $|\lambda_{H_i\Phi}| \lesssim 1.2\sqrt{\lambda_\Phi} \simeq 4 \times 10^{-6}$

$\Gamma_{\phi\phi \to H_1^\dagger H_2}$: $|\kappa_2| \lesssim 10^{-5}$

$\Gamma_{\phi \to H_1^\dagger H_2}$: suppressed by Planck powers

$\Gamma_{\phi \to N_R N_R}$: $y_N \lesssim 10^{-3}$

Reheating temperature

$$\mathcal{L}_{\rm int} \supset -\sum_{i=1,2} \lambda_{i\Phi} |H_i|^2 |\Phi|^2 - 2^{p/2-1}\kappa_p H_1^\dagger H_2 \Phi^p - \frac{1}{2} y_N \overline{N_R^c} \Phi N_R + \mathrm{h.c.}$$

Low!!

- We need $|\kappa_1| \lesssim 10^3\,\mathrm{GeV}(f_a/10^8\,\mathrm{GeV})$ for EWSB $\Longrightarrow T_{\rm RH} \sim 10^5\,\mathrm{GeV}$

- A higher reheating temperature can be achieved through $\kappa_2 \lesssim \lambda_{H_i\Phi} \lesssim 10^{-7}$

PQ symmetry restoration after reheating

$$V_T(\Phi) \supset \frac{1}{24}T^2\left(\sum_b n_b m_{b,\mathrm{eff}}^2 + \sum_f n_f m_{f,\mathrm{eff}}^2\right) = \beta T^2 |\Phi|^2$$

$$\beta \equiv \frac{1}{24}(4\lambda_{H_1\Phi} + 4\lambda_{H_2\Phi} + 6y_N^2)$$

Symmetry restored for $\beta T^2 + \mu_\Phi^2 > 0$ **with** $\mu_\Phi^2 \simeq -\lambda_\Phi v_\Phi^2$

$$T_{\mathrm{reh}} < \sqrt{\frac{\lambda_\Phi}{\beta}} f_a \equiv T_{\mathrm{restore}}$$

For $\lambda_{H_1\Phi}, \lambda_{H_2\Phi} \sim 10^{-10}$ and $y_N \sim 10^{-6}$, we get the upper bound $T_{\mathrm{restore}} \simeq 0.57 f_a$

Post-inflationary Noether Charge

- If $\phi(a_{\rm RH}) > 3f_a$, we never have **early matter domination**

$$n_\theta(T_{\rm RH}) = n_{\theta,\rm end}\left(\frac{\pi^2 g_*(T_{\rm RH})T_{\rm RH}^4}{45V_E(\phi_{\rm end})}\right)^{3/4}$$

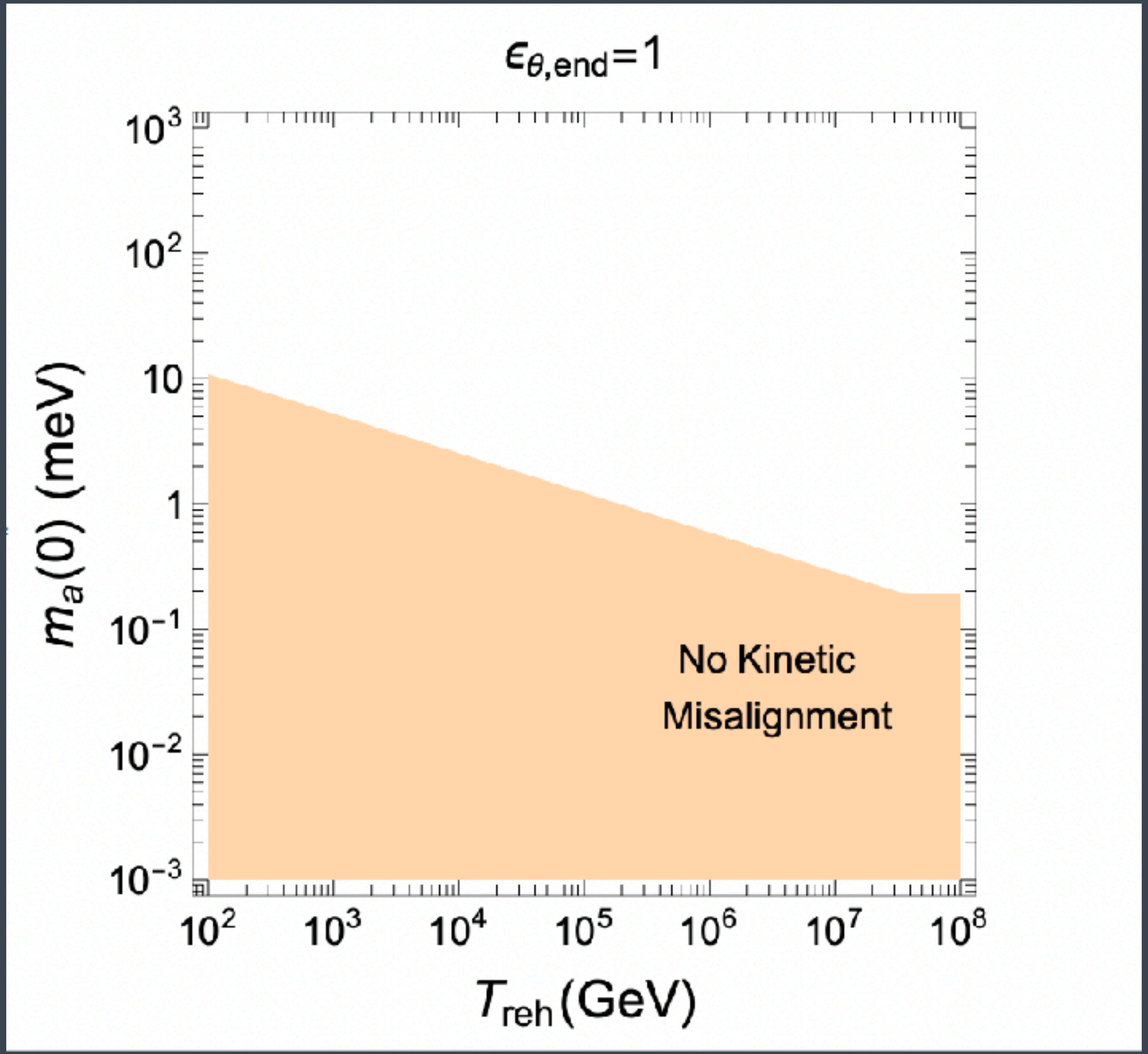
- If reheating is delayed

$$n_\theta(T_{\rm RH}) = n_{\theta,\rm end}\left(\frac{a_{\rm end}}{a_c}\right)^3\left(\frac{a_c}{a_{\rm RH}}\right)^3$$

$$= n_{\theta,\rm end}\left(\frac{\pi^2 g_*(T_{\rm RH})T_{\rm RH}^4}{45V_E(\phi_{\rm end})}\right)^{3/4}\left(\frac{T_{\rm RH}}{T_{\rm RH}^c}\right)$$

Suppression due to MD

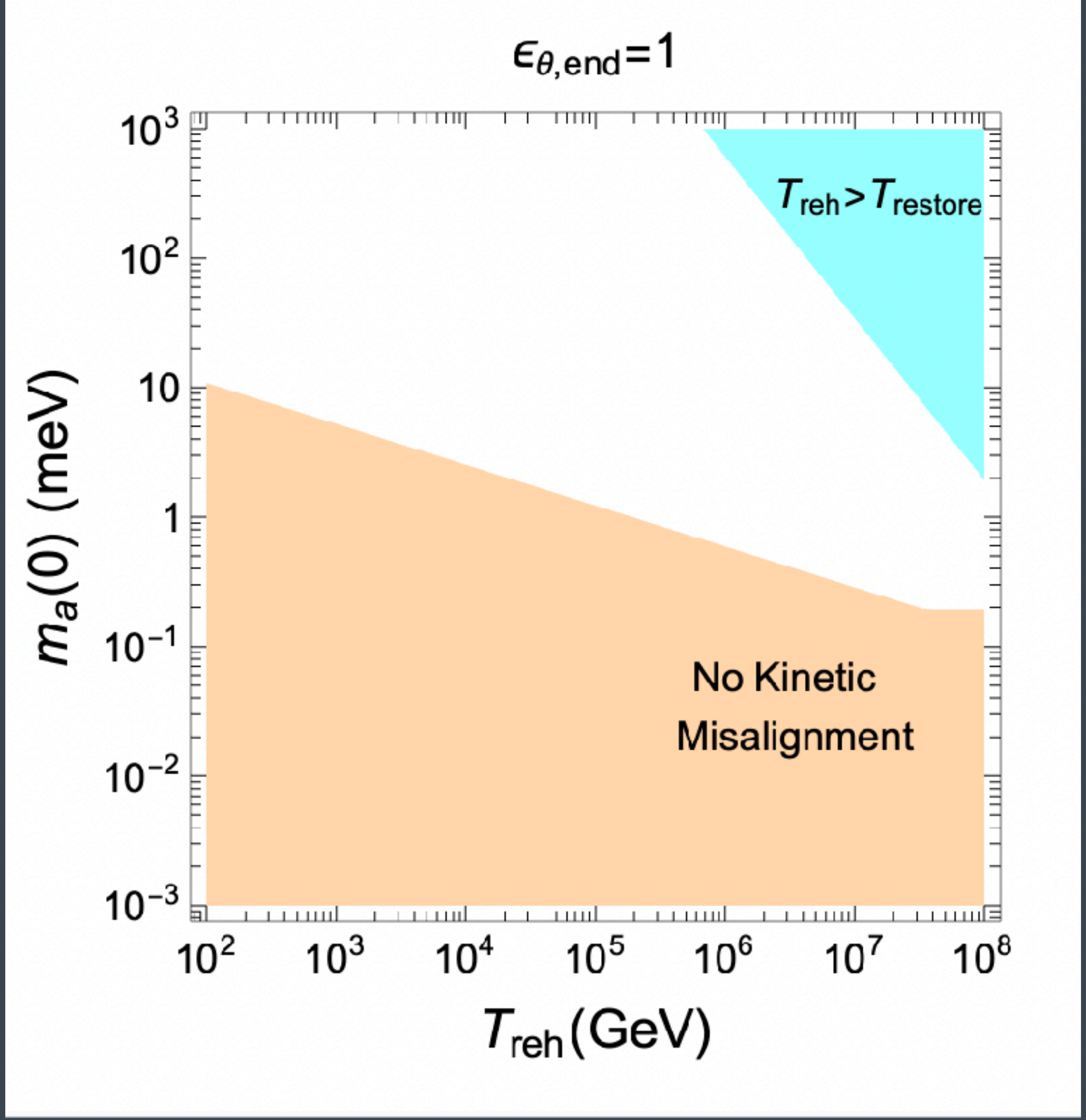
Results.



Condition for kinetic misalignment

$$\dot{\theta}(T_*) \geq 6H(T_{\mathrm{osc}})\dot{\theta}(T_*) \geq 6H(T_{\mathrm{osc}})$$

Results.



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Restoration of the PQ symmetry for large T_{reh}

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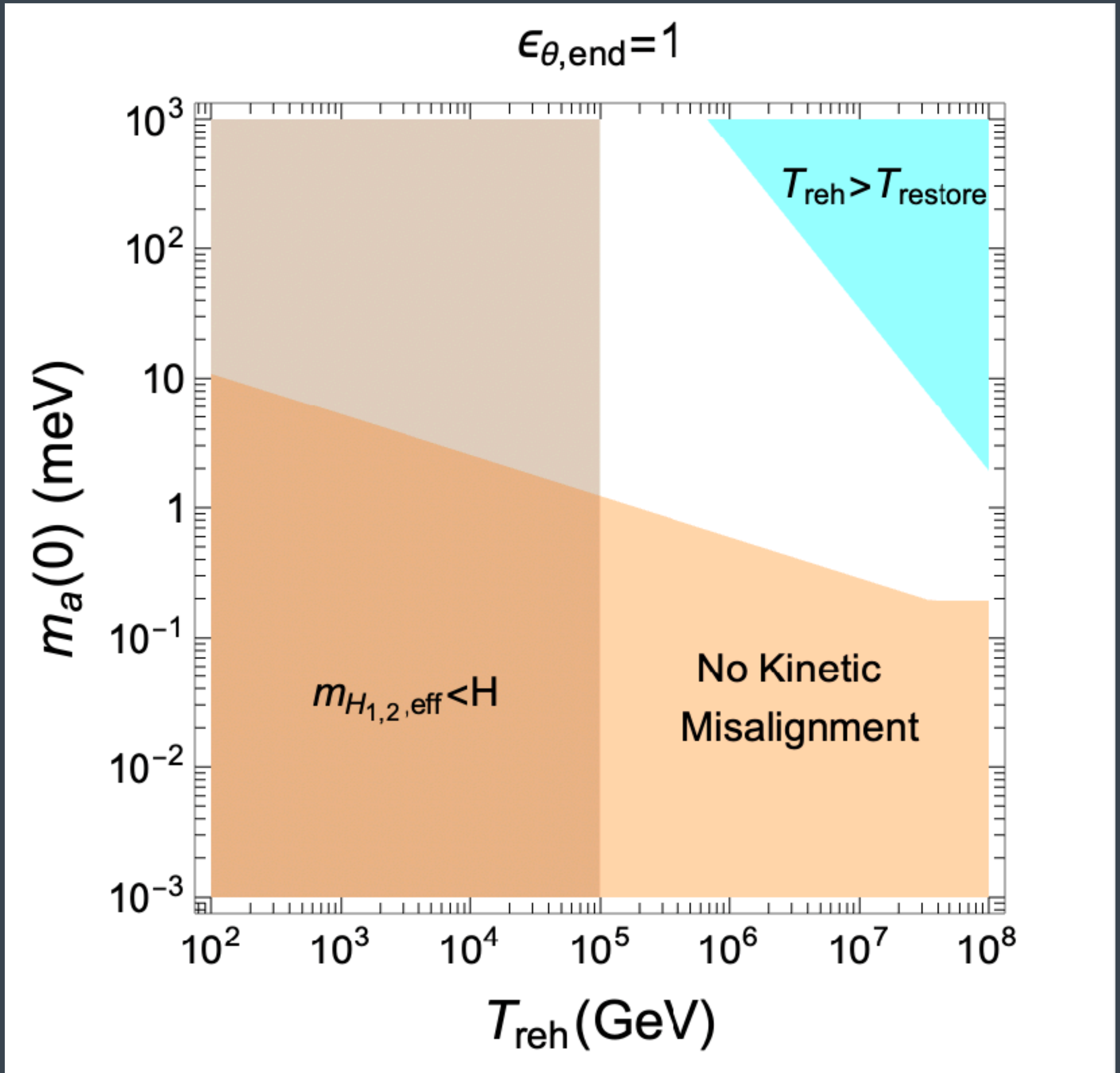

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For too low T_{reh}, extra fields NOT decoupled

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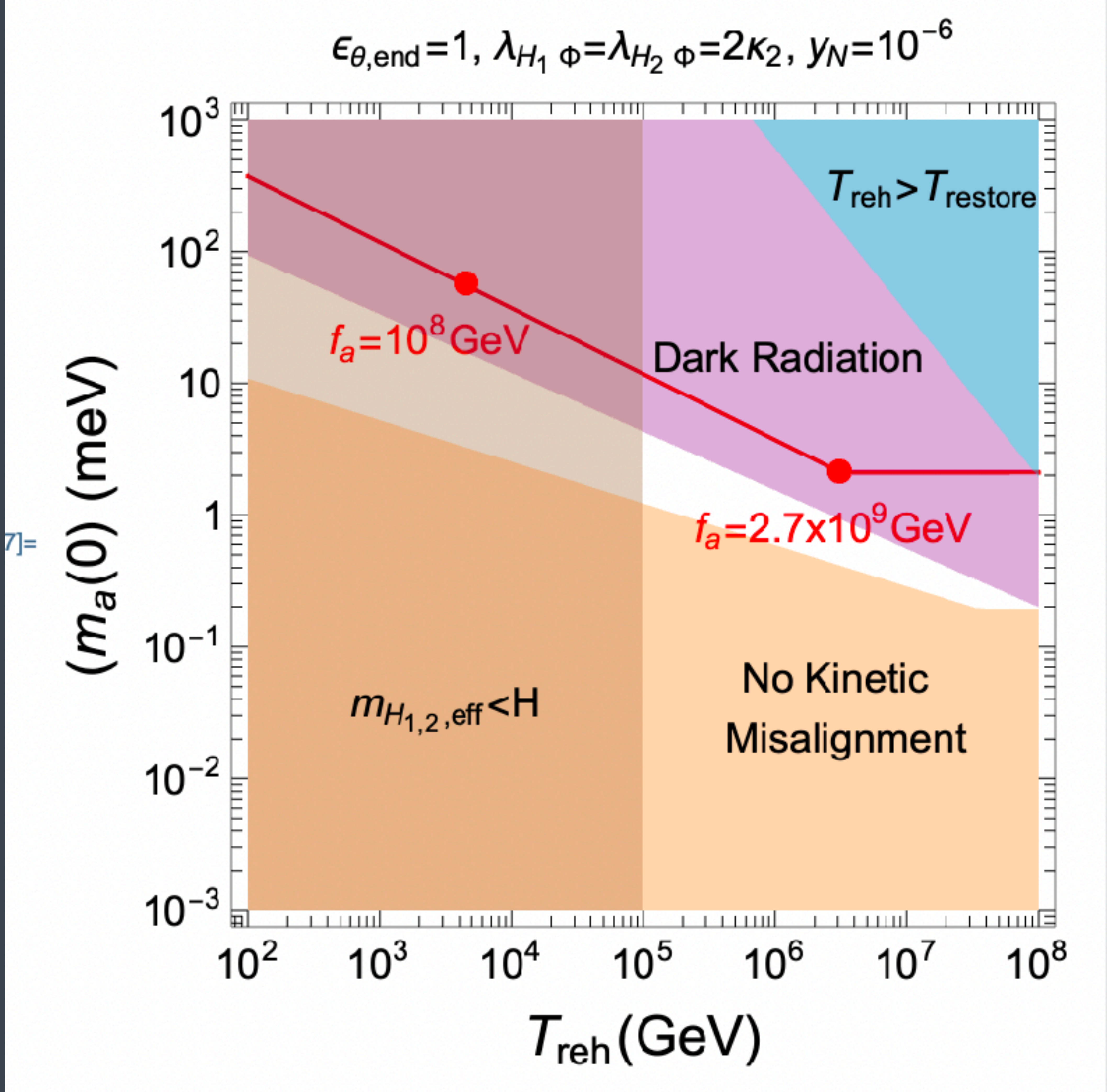

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Detectable Dark Radiation

Domain walls

$$\Delta V \gtrsim \rho_{\rm walls} \sim \frac{\sigma}{t} \gtrsim \frac{\sigma}{0.1 \rm sec}$$

- The PQ violating potential gives rise to a nonzero pressure $\Delta V = c\,\Lambda_{\rm QCD}^4 \times 10^{-10}$
- Domain walls never become dominant if $\Delta V \gtrsim \frac{\sigma^2}{M_P^2}(t_*/0.1\,{\rm s})$

$$c = |\,c_{0,l,k}|\left(\frac{f_a}{\sqrt{2}q_\Phi M_P}\right)^l\left(\frac{M_P}{\Lambda_{\rm QCD}}\right)^4 \times 10^{10}$$

- for $\sigma \sim \Lambda_{\rm QCD}^3$, there is no domain wall problem as long as $c \gtrsim 10^{-13}$,

A word on iscocurvature

- the power spectrum of the isocurvature perturbation depends only on $Y_{a,\mathrm{mis}}$
- $P_{\mathrm{iso}}(k_*) = \left(\frac{1}{Y_a}\frac{\partial Y_a}{\partial \theta_*}\right)^2 \left\langle \delta\theta_*^2 \right\rangle = \left[\frac{4}{\theta_*}\left(\frac{Y_{a,mis}}{Y_a}\right)^2 + \frac{1}{4}\left(\frac{1}{\epsilon_{\theta,*}}\frac{\partial \epsilon_{\theta,*}}{\partial \theta_*}\right)^2\left(\frac{Y_kin}{Y_a}\right)^2\right]\left\langle \partial\theta_*^2 \right\rangle$
- $\left\langle \delta\theta_*^2 \right\rangle = \frac{1}{f_{a,eff}^2}\left(\frac{H_I}{2\pi}\right)^2$ with $f_{a,eff} \equiv \sqrt{6}\left|\sinh\left(\frac{\phi_*}{\sqrt{6M_P}}\right)\right|$
- The large effective decay constant suppresses isogurvature perturbations.

Summary

- We built a consistent framework for the axion kinetic misalignment in the DFSZ axion set up.
- Inflation is driven by the radial direction.
- The PQ violating terms induce the non-zero velocity.
- We can reproduce the observed amount of Dark Matter.
- Domain walls never dominate due to the pressure of the PQ violating terms.
- Isocurvature perturbations are negligible due to a large $f_{a,\mathrm{eff}}$

Back up slides

2HDM

PQ	Φ	H_1	H_2	q_L	u_R	d_R	l_L	e_R
Type I	q_Φ	q_1	$q_1 - pq_\Phi$	0	$q_1 - pq_\Phi$	$-q_1 + pq_\Phi$	0	0
Type II	q_Φ	q_1	$q_1 - pq_\Phi$	0	$q_1 - pq_\Phi$	$-q_1$	0	$-q_1$
Type X	q_Φ	q_1	$q_1 - pq_\Phi$	0	$q_1 - pq_\Phi$	$-q_1 + pq_\Phi$	0	$-q_1$
Type Y	q_Φ	q_1	$q_1 - pq_\Phi$	0	$q_1 - pq_\Phi$	$-q_1$	0	$-q_1 + pq_\Phi$

$q_\phi \neq 0$ **for PQ symmetry breaking**

PQ conserving: If the PQ charges satisfy $pq_\phi - q_1 + q_2$, $H_1^\dagger H_2 \Phi^p$ is invariant.

PQ violating terms:

$$V_{\rm PQV} = \sum_{n,l} \sum_{k=0}^{[l/2]} \frac{c_{n,l,k}}{2M^{2n+l-4}} (H_1^\dagger H_2)^n |\Phi|^{2k} \Phi^{l-2k} + \mathrm{h.c.}$$

$$n(q_2 - q_1) + q_\Phi(l - 2k) \neq 0$$

Yukawa interactions

Z_2	Φ	H_1	H_2	q_L	u_R	d_R	l_L	e_R
Type I	+	−	+	+	+	+	+	+
Type II	+	−	+	+	+	−	+	−
Type X	+	−	+	+	+	+	+	−
Type Y	+	−	+	+	+	−	+	+

PQ Type II: $\mathscr{L}_Y = -Y_u \bar{q}_L \tilde{H}_2 u_R - Y_d \bar{q}_L H_1 d_R - Y_e \bar{l}_L H_1 e_R \,.$

Z even neutrinos: $\mathscr{L}_\nu^{(1)} = -Y_\nu \bar{l}_L \tilde{H}_2 N_R - \frac{1}{2} y_N \overline{N_R^c} \Phi N_R \,.$

Z odd neutrinos: $\mathscr{L}_\nu^{(2)} = -Y_\nu \bar{l}_L \tilde{H}_1 N_R - \frac{1}{2} y_N \overline{N_R^c} \Phi N_R$

Pure PQ inflation: decoupling

$$m_\pm^2 = \frac{1}{4}\left[\lambda_{1\Phi} + \lambda_{2\Phi} - 4\lambda_\Phi \pm \sqrt{(\lambda_{1\Phi} - \lambda_{2\Phi})^2 + 4\rho^{2p-4}\kappa_p^2}\right]\rho^4,$$

$$(\lambda_{1\Phi} - 2\lambda_\Phi)(\lambda_{2\Phi} - 2\lambda_\Phi) > \rho^{2p-4}\kappa_p^2 \simeq 6^{p-2}\kappa_p^2\,.$$

$$m_{\pm,\mathrm{eff}}^2 = \frac{m_\pm^2}{6\sinh^2\left(\frac{\phi}{\sqrt{6}}\right)} \simeq \frac{3}{2}\left[\lambda_{1\Phi} + \lambda_{2\Phi} - 4\lambda_\Phi \pm \sqrt{(\lambda_{1\Phi} - \lambda_{2\Phi})^2 + 4\kappa_2^2}\right]\cdot\frac{\sinh^2\left(\frac{\phi}{\sqrt{6}}\right)}{\cosh^4\left(\frac{\phi}{\sqrt{6}}\right)}$$

Inflationary dynamics

$$\dot{\phi} \simeq -\frac{1}{3H}\frac{\partial V_E}{\partial \phi} = -\sqrt{2\epsilon_\phi}\, M_P H$$

Angular velocity suppressed

$$\dot{\theta} \simeq -\frac{1}{3H}\frac{\frac{\partial V_E}{\partial \theta}}{6M_P^2 \sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)} = -\frac{\sqrt{2\epsilon_\theta}\, H}{6\sinh^2\left(\frac{\phi}{\sqrt{6}M_P}\right)}$$

$\phi \gg \sqrt{6}M_P$

Predictions insensitive to m

$$n_s = 1 - \frac{4N+3}{2\left(N^2 - \frac{9}{16m^2}\right)} \qquad r = \frac{12}{N^2 - \frac{9}{16m^2}}$$

For N=60, $n_s = 0.966$ and $r = 0.0033$

CMB bounds

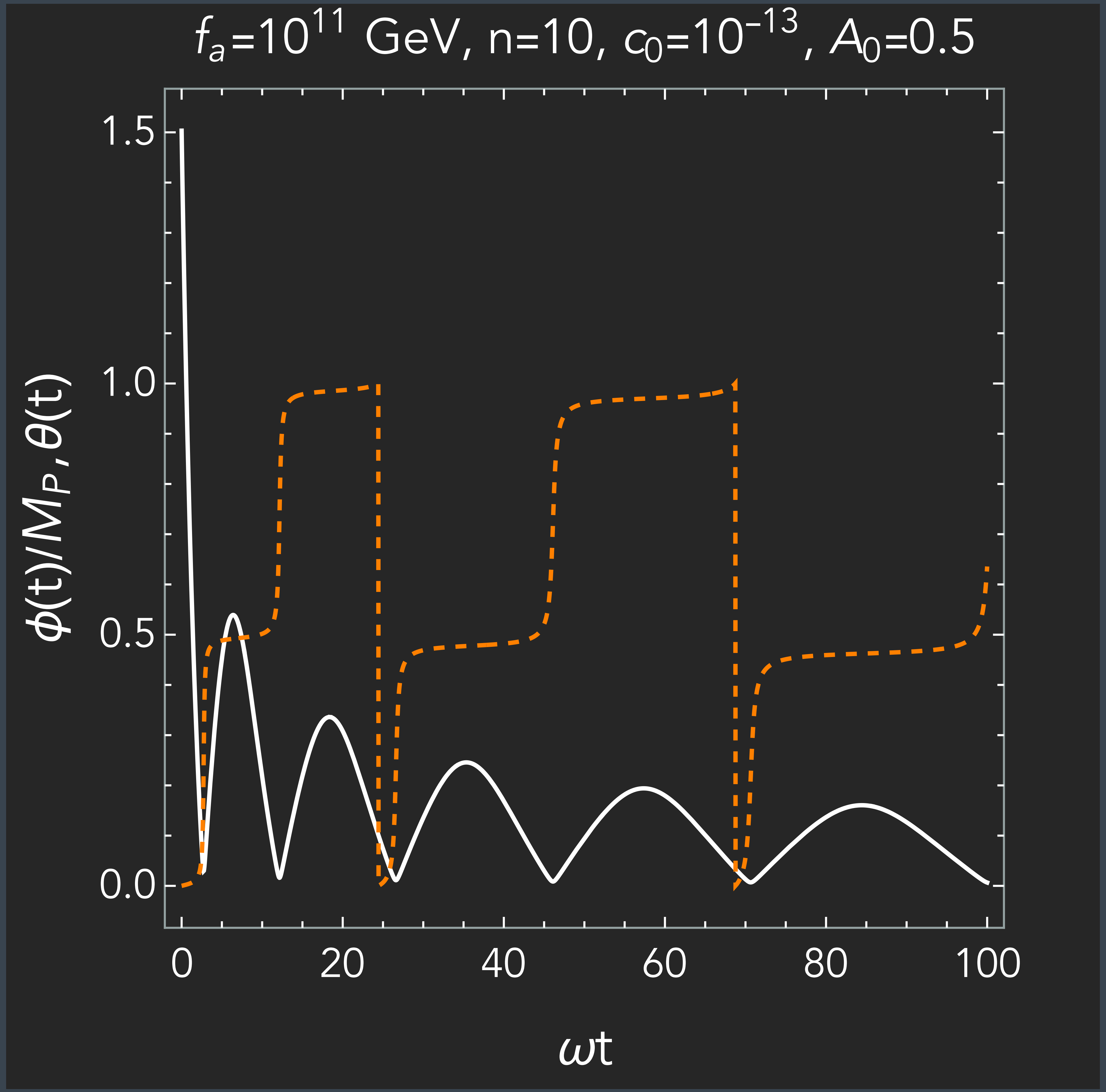
$$3^m \beta_m = (3.1 \times 10^{-8})\, r = 1.0 \times 10^{-10}$$

For $m = 2,\ \beta_m = \lambda_\Phi \implies \lambda_\Phi = 1.1 \times 10^{-11}$ during inflation

From PQ conservation

$$V_n(\theta_i)/M_P^4 = 3^{n/2} \sum_{k=0}^{[n/2]} |c_k| \cos\Big((n-2k)\theta_i + A_k\Big) < 1.0 \times 10^{-10}$$

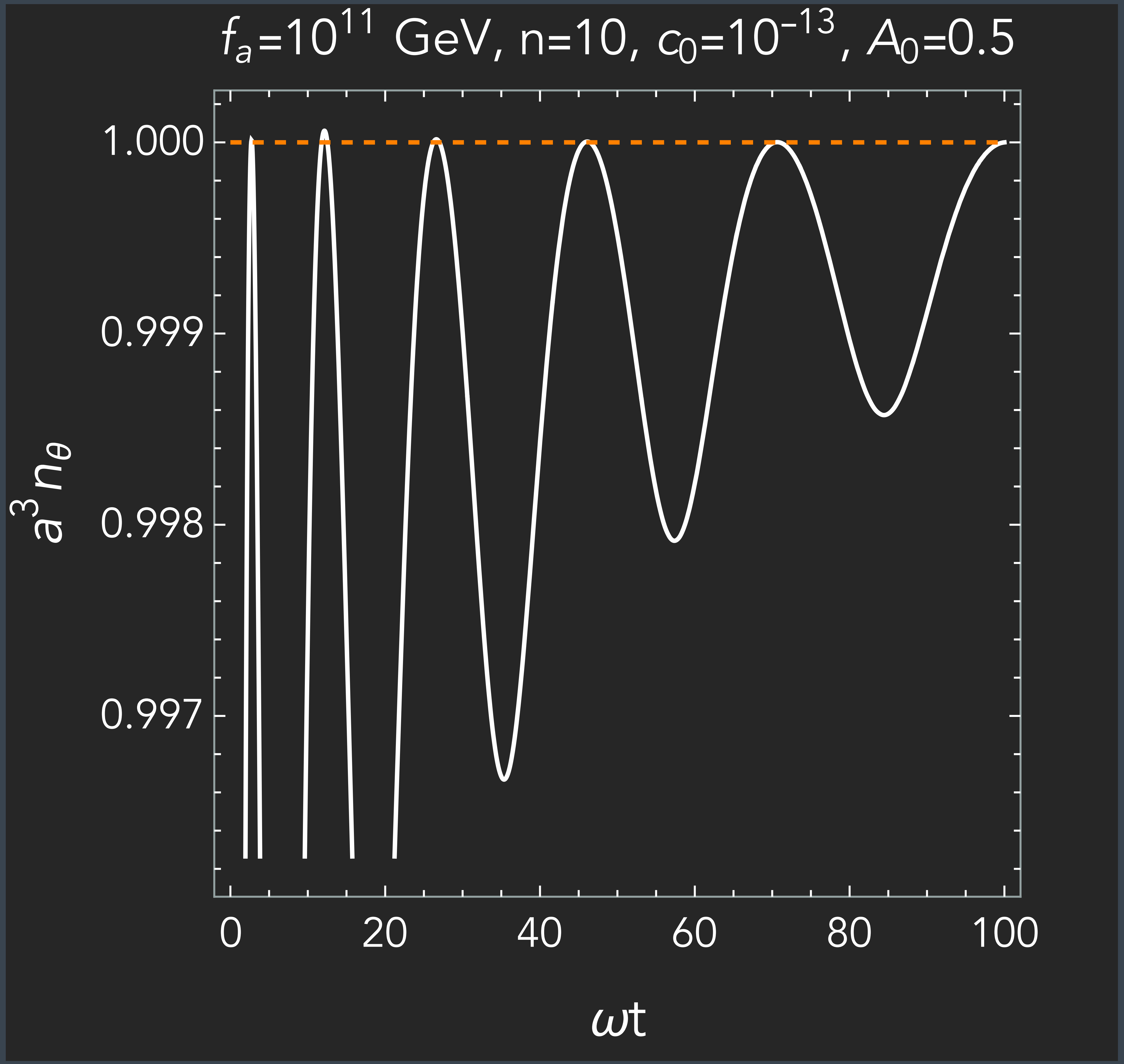
Post inflationary dynamics



$$\ddot{\phi} + 3H\dot{\phi} - \phi\,\dot{\theta}^2 \simeq -\frac{\partial V_E}{\partial \phi}$$

$$\phi^2(\ddot{\theta} + 3H\dot{\theta}) + 2\phi\dot{\phi}\,\dot{\theta} \simeq -\frac{\partial V_E}{\partial \theta}$$

Post inflationary dynamics



The PQ terms are small

↓

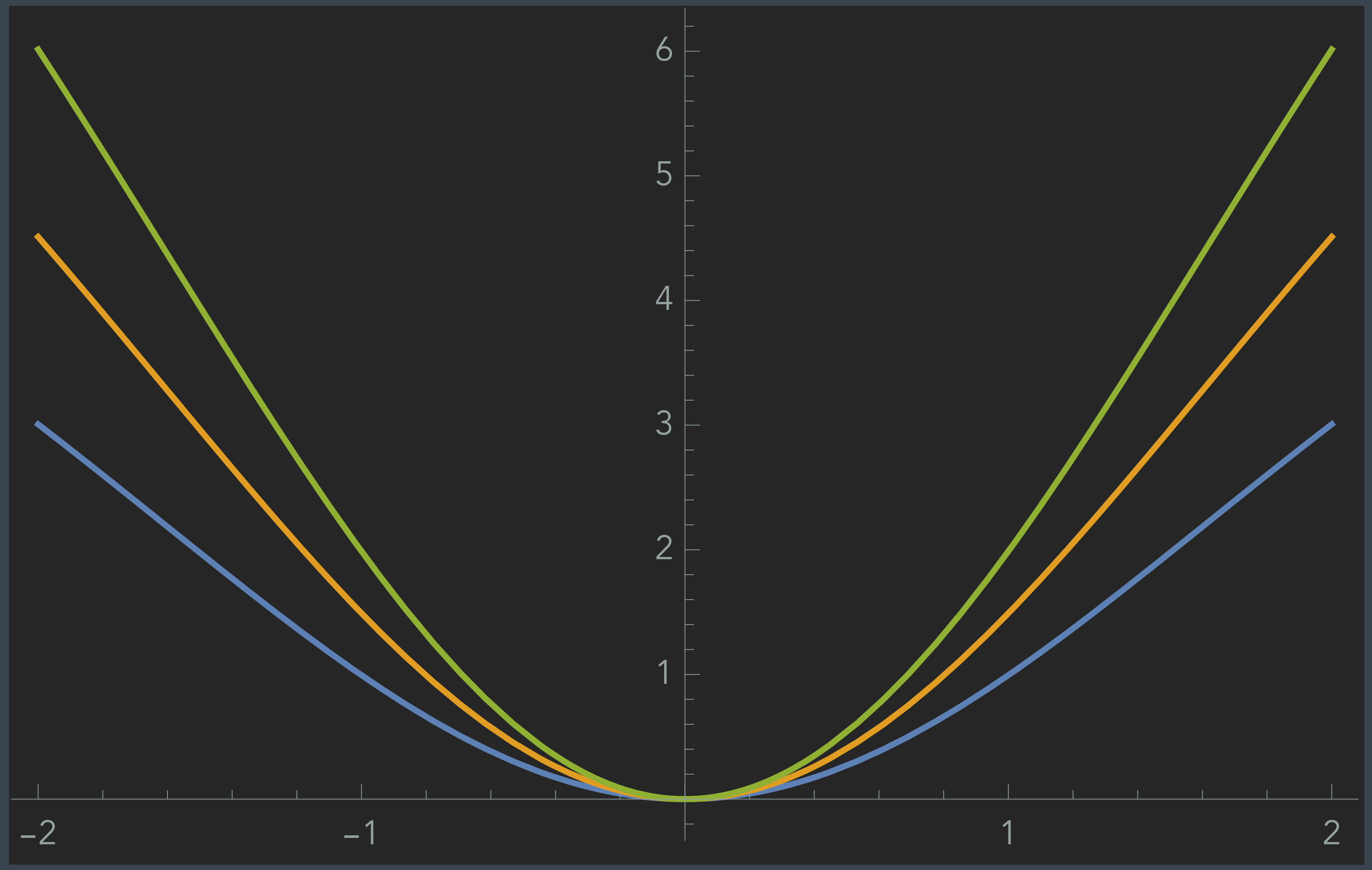
Noether charge approximately conserved

$$\ddot{\phi} + 3H\dot{\phi} \simeq \frac{C^2}{a^6\phi^3} - \lambda_\Phi \phi^3$$

Reheating

After inflation, the inflation condensate oscillates around the minima

$V_E(\phi) \simeq \alpha_m \phi^{2m}$ $\quad |H| \ll \sqrt{6} M_P$



General equation of state

$$\rho_\phi = \left\langle \frac{1}{2}\dot{\phi}^2 \right\rangle + \langle V_E(\phi) \rangle = (m+1)\langle V_E(\phi) \rangle$$

$$p_\phi = \left\langle \frac{1}{2}\dot{\phi}^2 \right\rangle - \langle V_E(\phi) \rangle = (m-1)\langle V_E(\phi) \rangle$$

$$\langle w_\phi \rangle = \frac{p_\phi}{\rho_\phi} = \frac{m-1}{m+1}$$

For $m \neq 1, \omega_\phi \neq 0$

The particular dynamics depends on the value of m