Out of equilibrium

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- Boltzmann equation

$$E\left(\partial_t - Hp\partial_p\right) f_\chi = C_{\rm ann}[f_\chi] + C_{\rm el}[f_\chi]\,,$$

$$\begin{aligned} C_{\rm ann} &= \frac{1}{2g_\chi}\int \frac{d^3\tilde{p}}{(2\pi)^3 2\tilde{E}}\int \frac{d^3k}{(2\pi)^3 2\omega}\int \frac{d^3k}{(2\pi)^3 2\tilde{\omega}}(2\pi)^4\delta^{(4)}(\tilde{p}+p-\tilde{k}-k)\,|\mathcal{M}|^2_{\chi\chi\leftrightarrow\psi\psi} \\ &\quad\times\left[f_\psi(\omega)f_\psi(\tilde{\omega})\bar{f}_\chi(E)\bar{f}_\chi(\tilde{E}) - f_\chi(E)f_\chi(\tilde{E})\bar{f}_\psi(\omega)\bar{f}_\psi(\tilde{\omega})\right]\,, \qquad (\\ C_{\rm el} &= \frac{1}{2g_\chi}\int \frac{d^3\tilde{p}}{(2\pi)^3 2\tilde{E}}\int \frac{d^3k}{(2\pi)^3 2\omega}\int \frac{d^3\tilde{k}}{(2\pi)^3 2\tilde{\omega}}(2\pi)^4\delta^{(4)}(\tilde{p}+p-\tilde{k}-k)\,|\mathcal{M}|^2_{\chi\psi\leftrightarrow\chi\psi} \\ &\quad\times\left[f_\chi(E)f_\psi(\omega)\bar{f}_\chi(\tilde{E})\bar{f}_\psi(\tilde{\omega}) - f_\chi(\tilde{E})f_\psi(\tilde{\omega})\bar{f}_\chi(E)\bar{f}_\psi(\omega)\right] \qquad (\end{aligned}$$

- Full thermal equilibrium and kinetic equilibrium during freeze-out -> after integration (Gondolo&Gelmini)

$$\dot{n}_\chi + 3Hn_\chi = -\langle\sigma v\rangle\left(n_\chi^2 - n_{\chi,\rm MB}^2\right)$$

- Departure from kinetic equilibrium (velocity dependent cross-sections): narrow resonances, Sommerfeld enhancement, near degeneracy in mass
- Binder, Bringmann, Gustafsson, Hryczuk, PRD96 (2017) 115010

- Beyond kinetic equilibrium - define

$$T_\chi \equiv g_\chi/(3n_\chi)\int d^3p\,(2\pi)^{-3}(p^2/E)f_\chi$$

$$y(x) \equiv m_\chi T_\chi s^{-2/3}.$$

- Coupled Boltzmann equations

$$\frac{x}{Y}\frac{dY}{dx} = \frac{sY}{\tilde{H}}\left[\frac{Y_{\rm eq}^2}{Y^2}\langle\sigma v\rangle_T - \langle\sigma v\rangle_{T_\chi}\right], \tag{8}$$

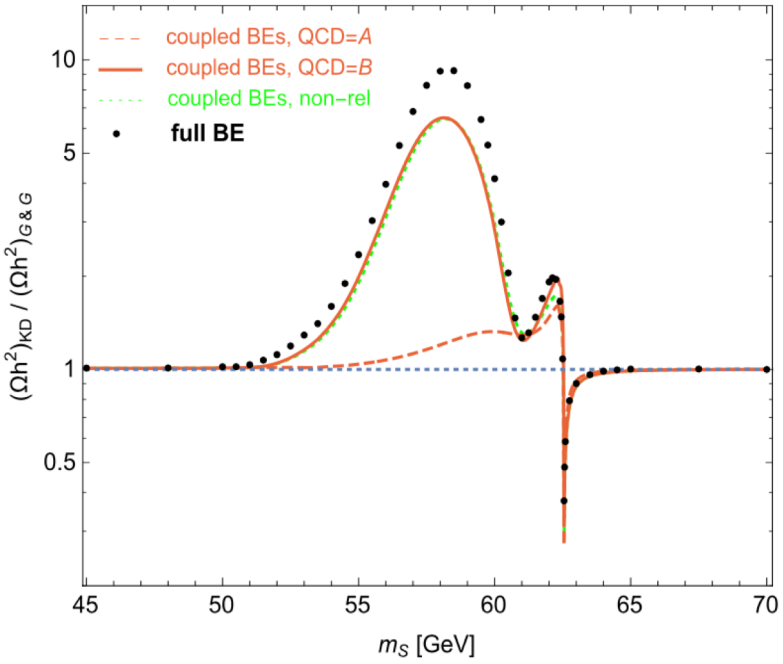
$$\frac{x}{y}\frac{dy}{dx} = \frac{\gamma w}{\tilde{H}}\left[\frac{y_{\rm eq}}{y}-1\right] + \frac{sY}{\tilde{H}}\left[\langle\sigma v\rangle_{T_\chi} - \langle\sigma v\rangle_{2,T_\chi}\right] + \frac{sY}{\tilde{H}}\frac{Y_{\rm eq}^2}{Y^2}\left[\frac{y_{\rm eq}}{y}\langle\sigma v\rangle_{2,T} - \langle\sigma v\rangle_T\right] + 2(1-w)\frac{H}{\tilde{H}}. \tag{9}$$

- where $w(T_\chi) \equiv 1 - \langle p^4/E^3\rangle_{T_\chi}/(6T_\chi)$

- Included in
  - DarkSUSY6 – Bringmann, Edsjo, Gondolo, Ullio, Bergstrom
  - DRAKE – Binder, Bringmann, Gustafsson, Hryczuk, 2103.01944
    - also solution to full Boltzmann equation

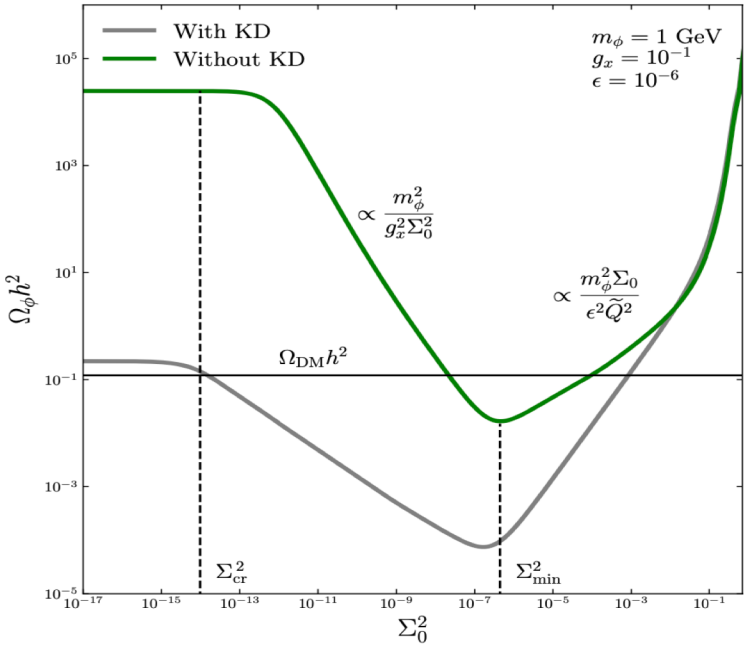
Examples

- Scalar singlet model



Binder et al, 1706.07433

Dark photon with scalar DM (p-wave)



$$\Sigma_0^2 \equiv 1 - 4m_\phi^2/m_x^2$$

GB, Chakraborti, Genolini, Salati, 2401.02513