Gravitational waves from the primordial universe.

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CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Universität Hamburg

This talk:

Within standard Einsten gravity (no modified gravity!)

Non-standard physics comes from particle physics, not from the gravity side

Plan

— Generalities about primordial gravitational wave (GW) backgrounds
Review of the best-motivated sources: short versus long-lived sources

- First-order phase transitions
- Inflation
- Cosmic strings (local and global/axionic strings)

— Effect of new particles beyond the Standard Model on primordial GW backgrounds

- from new heavy particles coupling to the Standard Model
- from new particles completely decoupled from the Standard Model

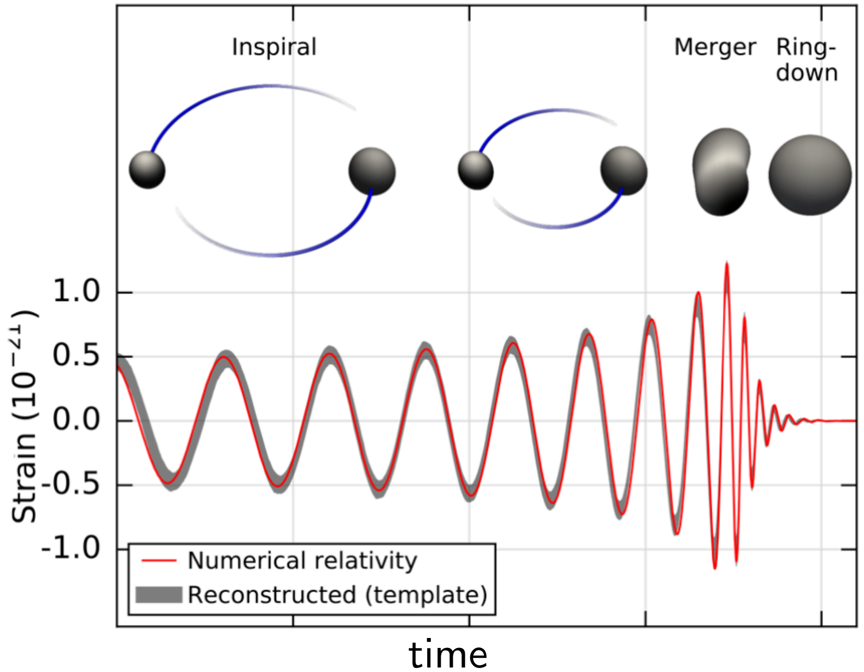
— GW backgrounds from axion early-universe dynamics:
3 distinct sources (beyond the ones above from axionic strings):

- GWs from the Peccei-Quinn phase transition
- GW signatures from kination induced by rotating axions
- GWs from axion fragmentation

Two types of gravitational-wave (GW) signals

- **Astrophysical signals (in the late universe)**

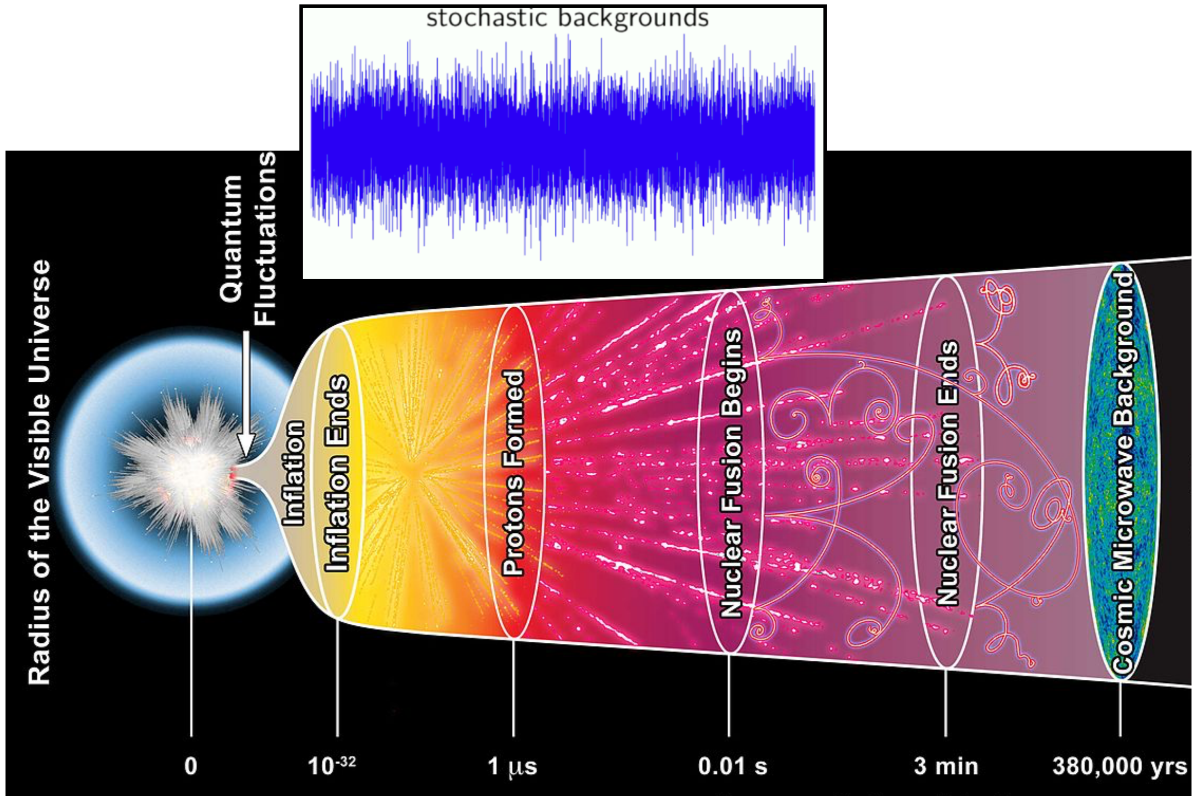
✔ detected



LIGO&Virgo, arXiv:1602.03841

- **Cosmological background filling the whole universe (a relic from the early universe)**

✘ not yet detected, some hint (PTAs?)

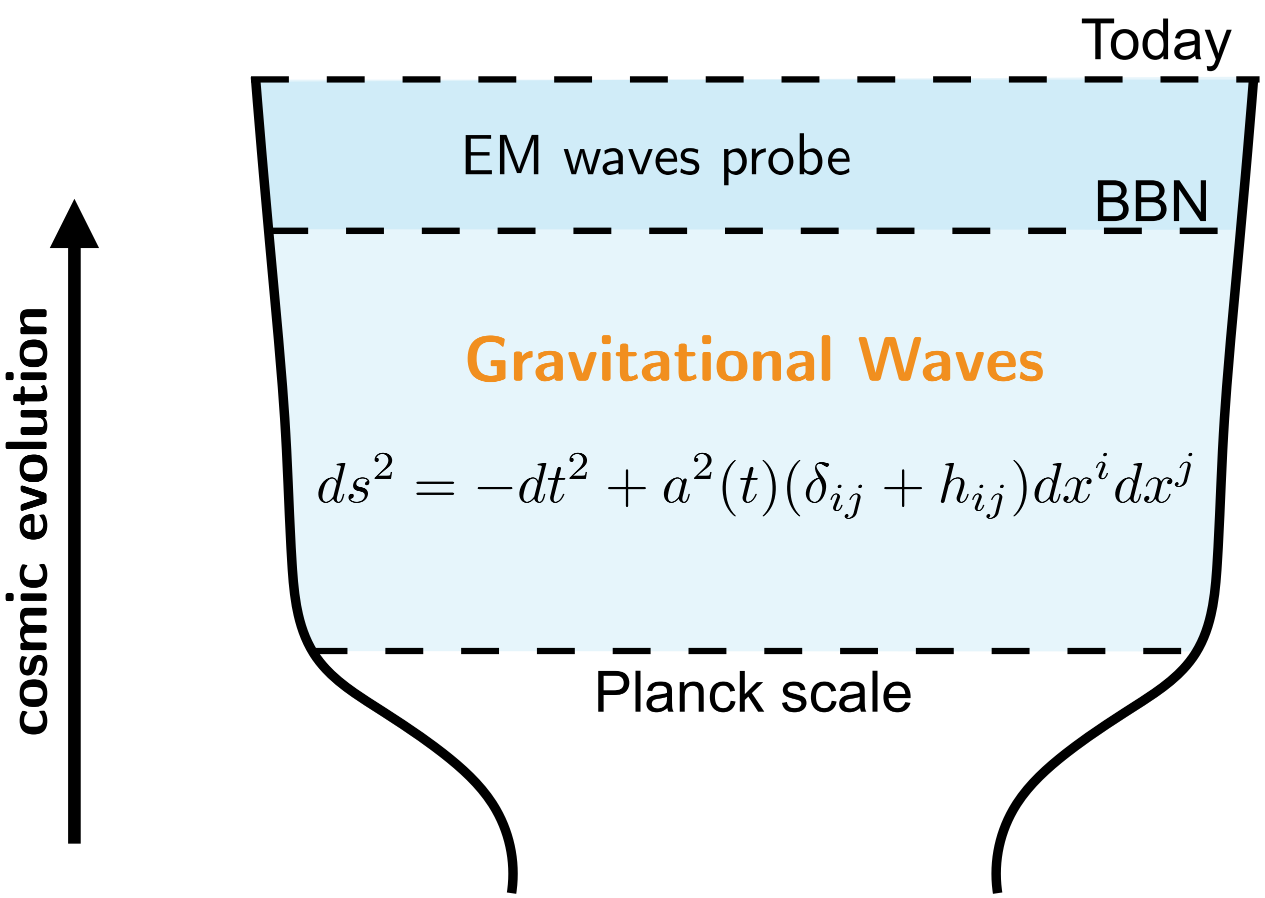


Note: Astrophysical signals can lead to a stochastic background if they cannot be resolved.

Primordial gravitational waves: Fossil radiation .

superposition of GW generated by an enormous number of causally independent sources, arriving at random times and from random directions.

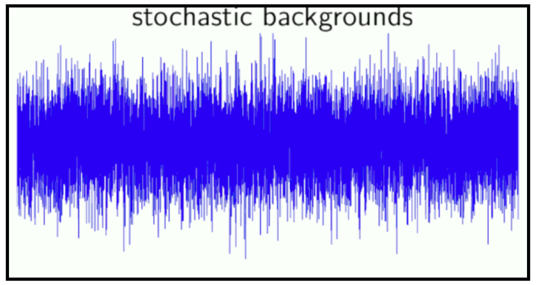
Individual waves are not detectable, sources can not be resolved but instead we can only observe a Stochastic GW Background. For most of the cosmological sources, it is homogeneous, isotropic, gaussian and unpolarized and appears as a noise in the detector.



Random h_{ij}

$\Downarrow$

Stochastic GW background (SGWB) characterized by energy density



$$\rho_{\rm GW} = \frac{\langle \dot{h}_{ij}\dot{h}^{ij} \rangle}{32\pi G}$$

Ensemble average

$\Updownarrow$

time/space average

Probing high-energy physics with gravitational waves .

Interaction rate of GW

Expansion rate

$$\frac{\Gamma_{\rm GW}(T)}{H(T)} \sim \frac{G^2T^5}{T^2/M_{\rm pl}} = \left(\frac{T}{M_{\rm pl}}\right)^3 \ll 1$$

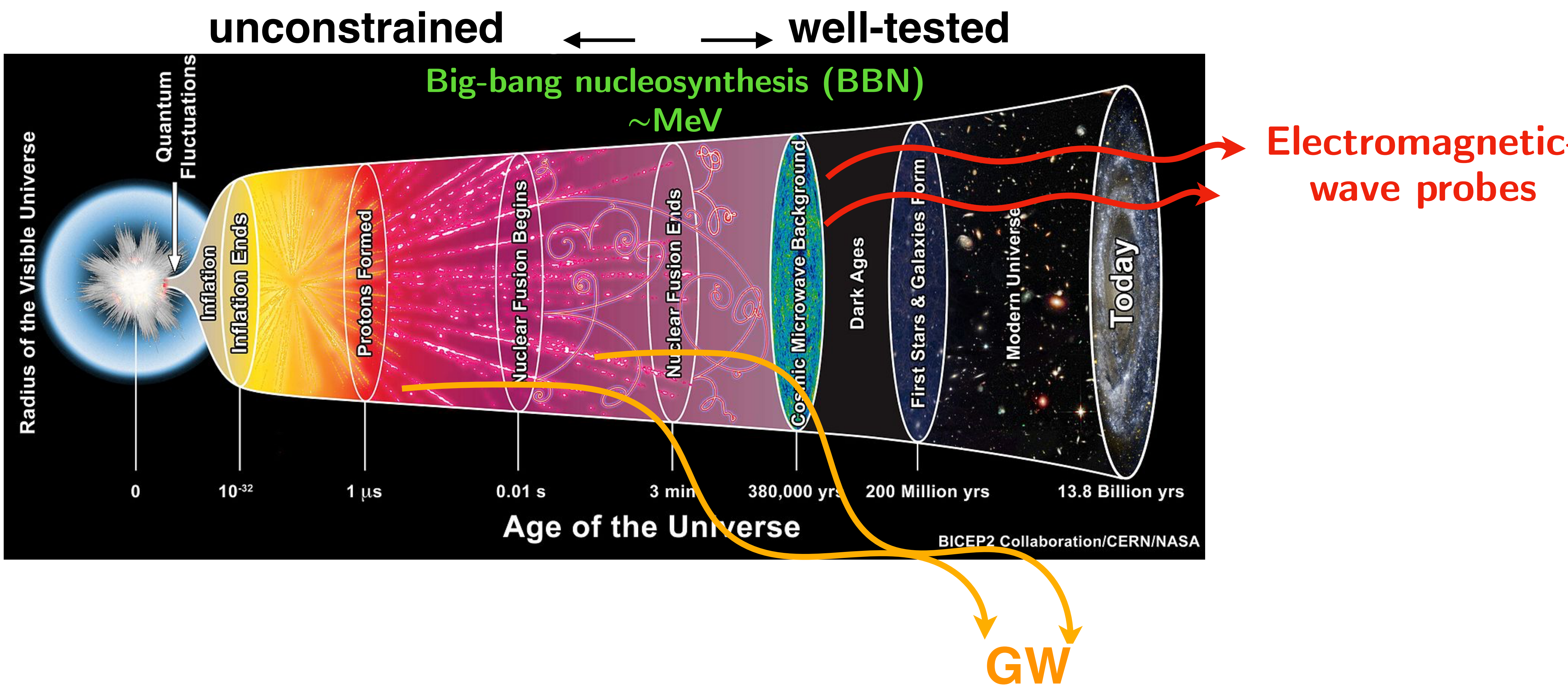
GW produced below the Planck scale are decoupled: They propagate freely in the universe until today.

They do not loose memory of conditions when produced.

They retain spectral shape, typical frequency and intensity characteristic of production mechanism, encoding information about particle physics at high-energy scales that cannot be probed by colliders.

Probing high-energy physics with gravitational waves .

High energies **Low energies**



Energy density of GW background:

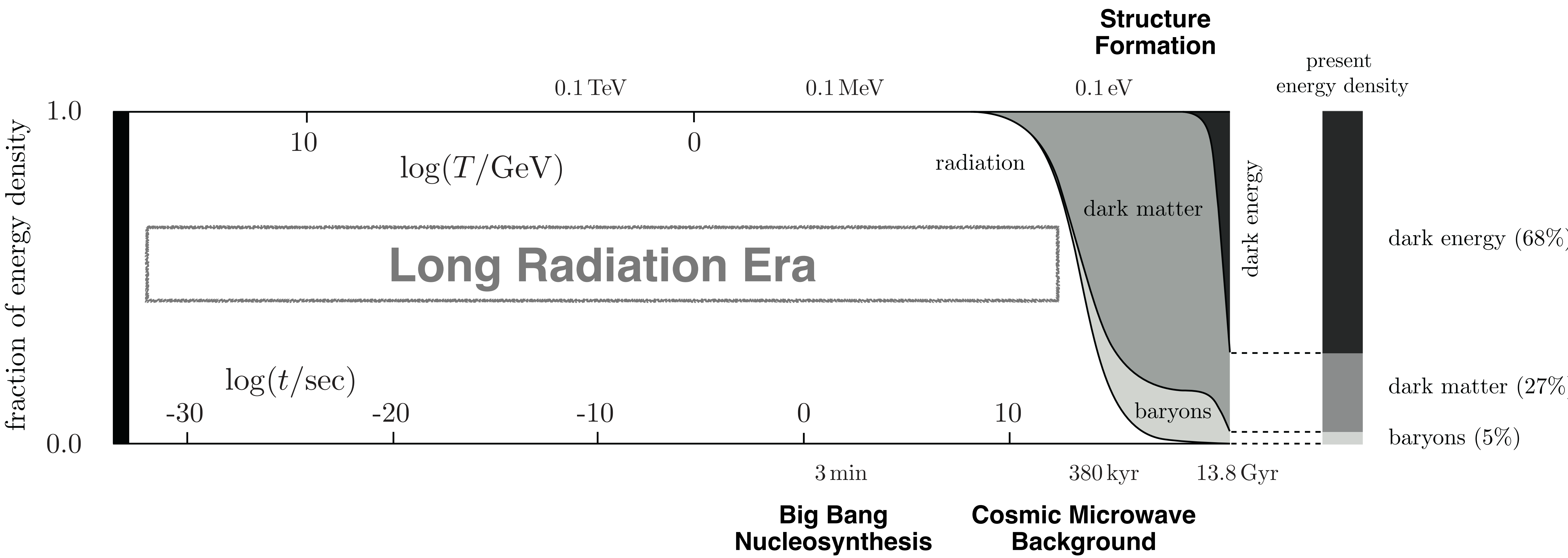
$$\rho^{\rm GW}_{\rm today} = \rho^{\rm GW}_{\rm prod} \left(\frac{a_{\rm prod}}{a_{\rm today}} \right)^4$$

The universe is expanding. ⇨ **BSM of cosmology**

its production mechanism ⇨ **particle physics beyond the Standard Model (BSM)**

What can we learn on particle physics and cosmological history from primordial gravitational waves?

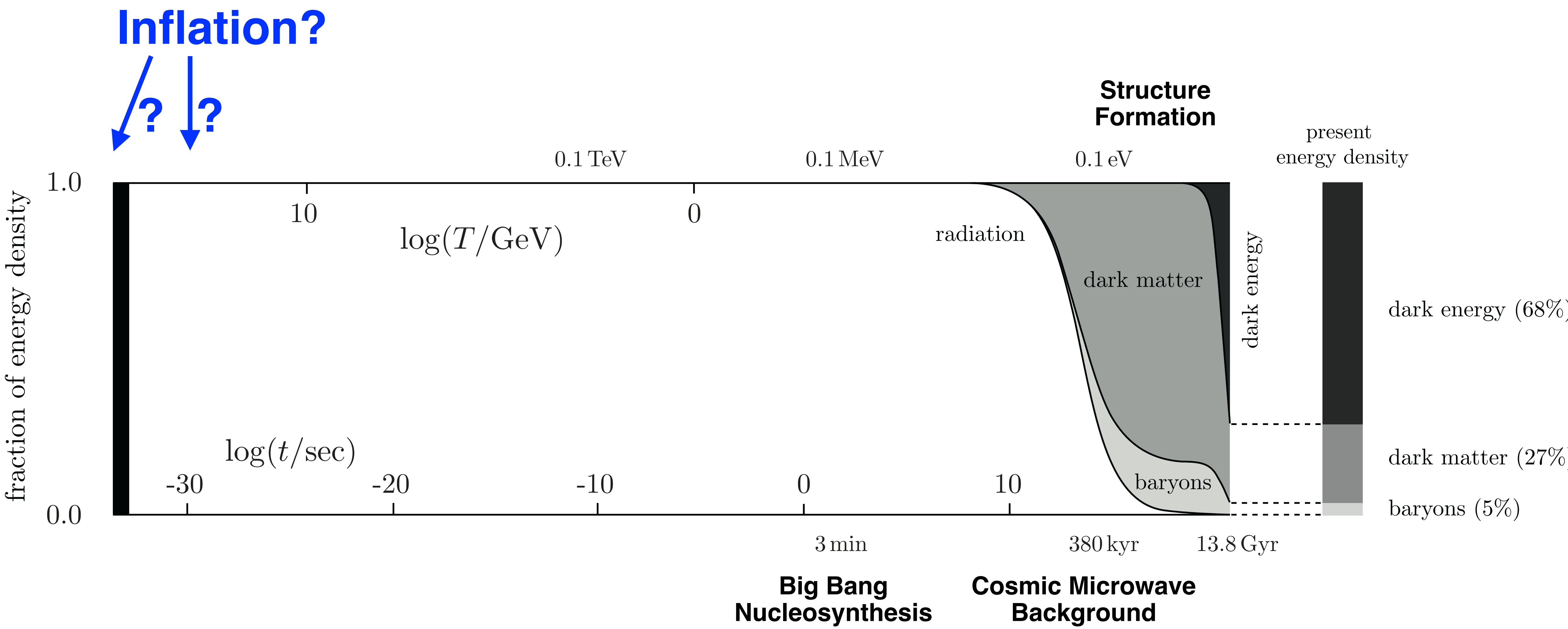
Standard Cosmological History



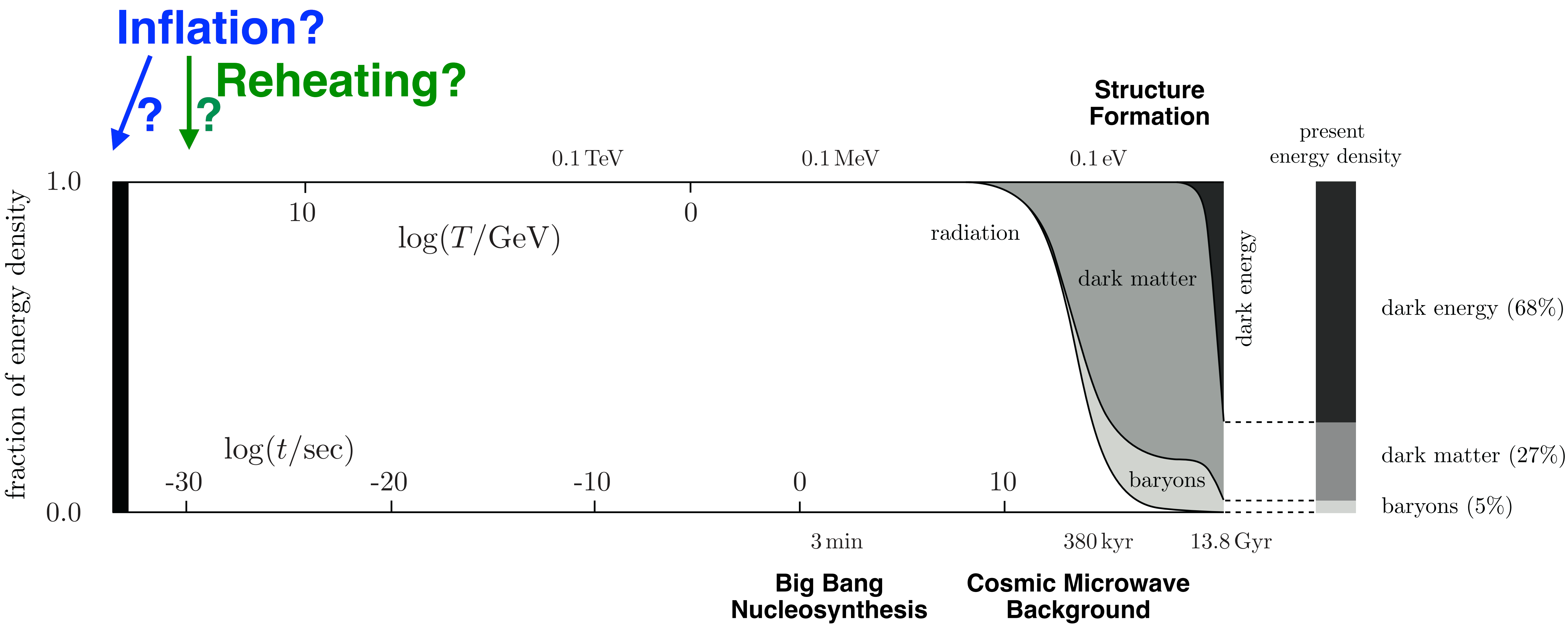
Well-tested

Almost unconstrained

Cosmological History

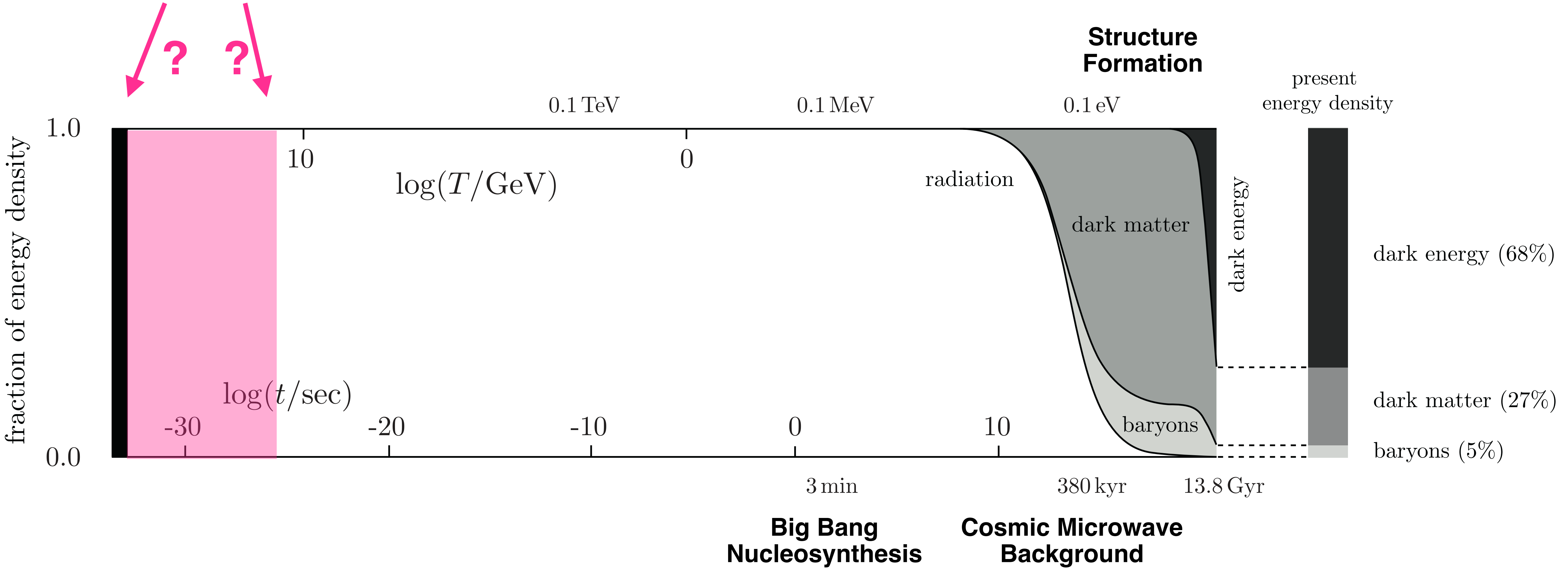


Cosmological History



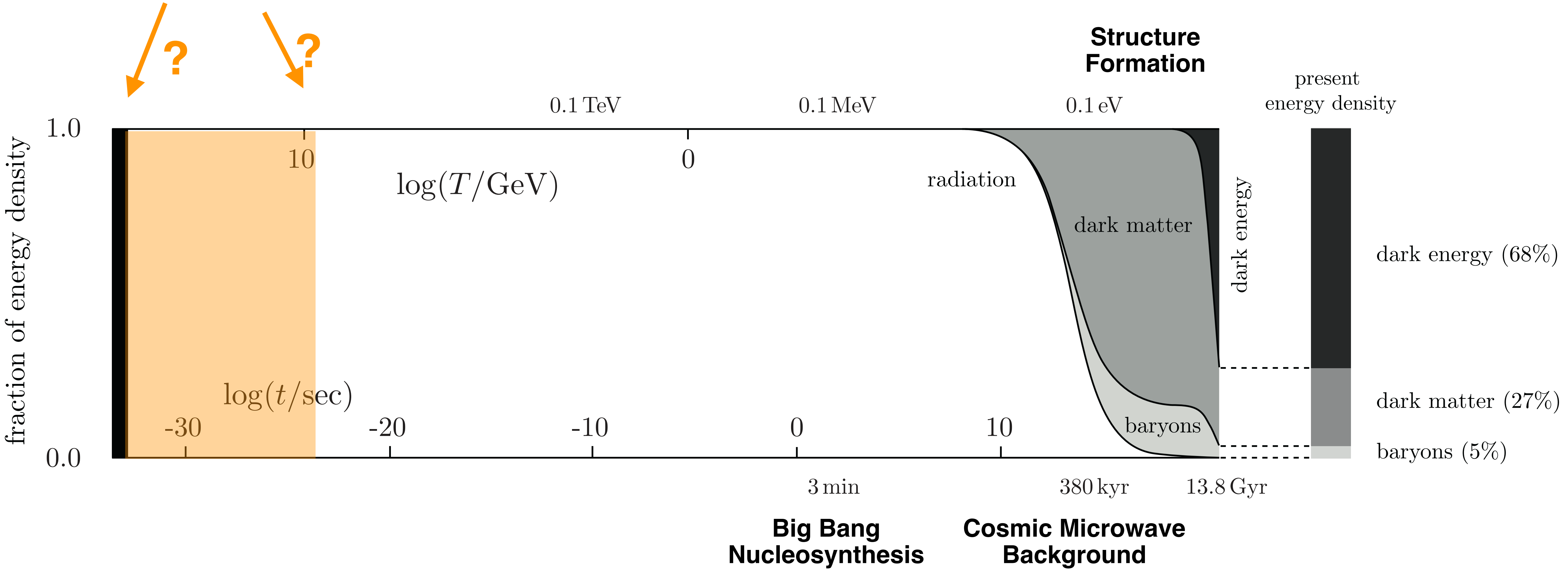
Cosmological History

Kination after inflation?

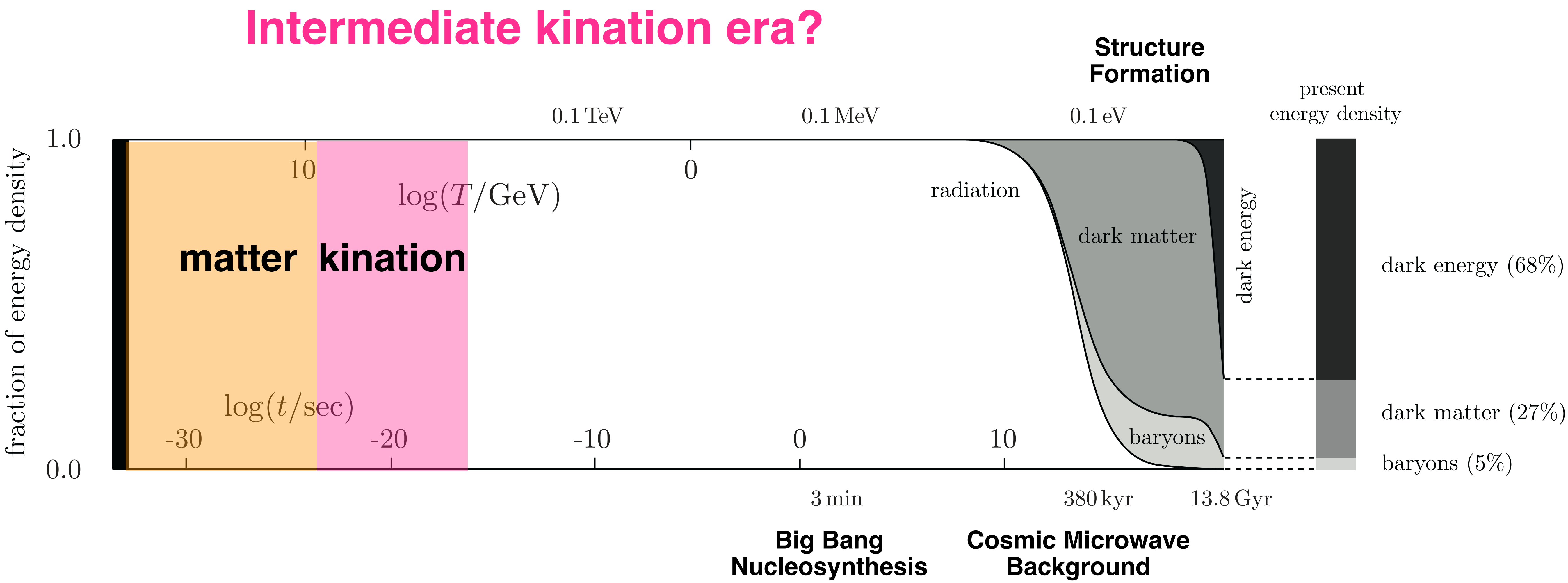


Cosmological History

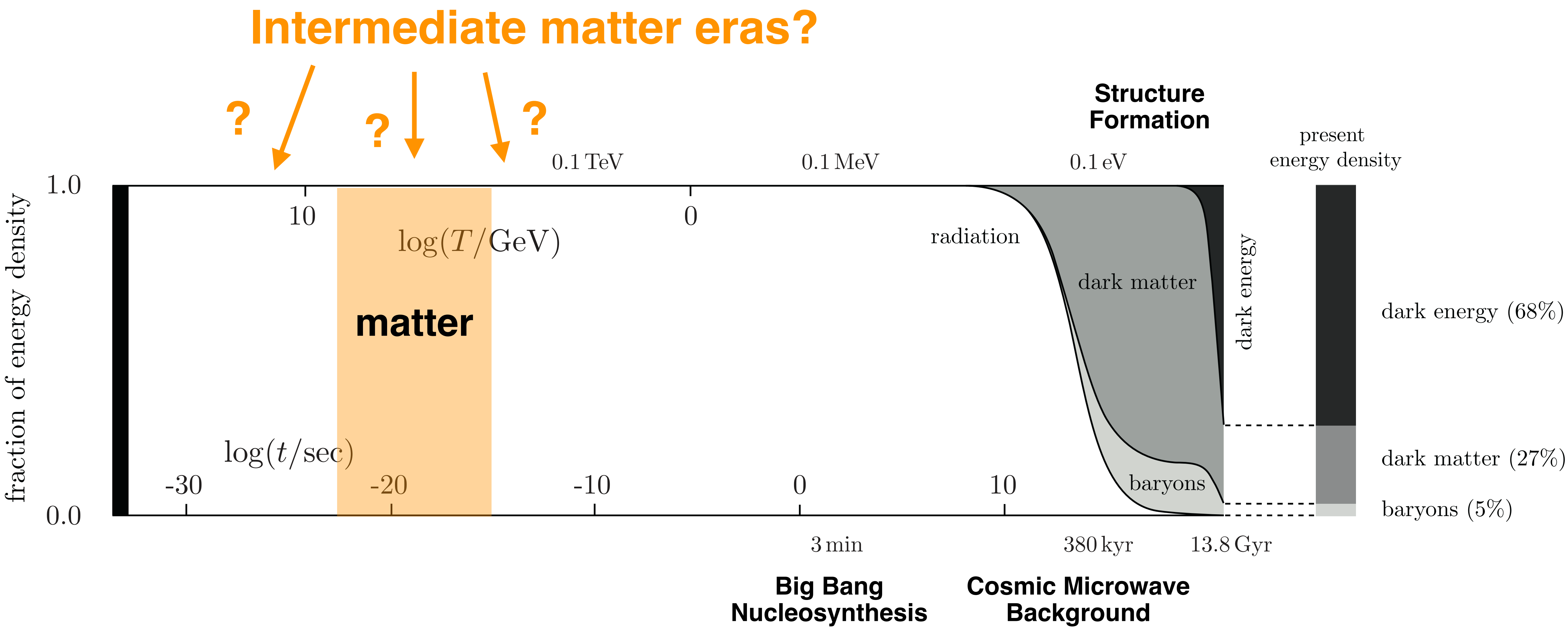
Early Matter era after inflation?



Cosmological History

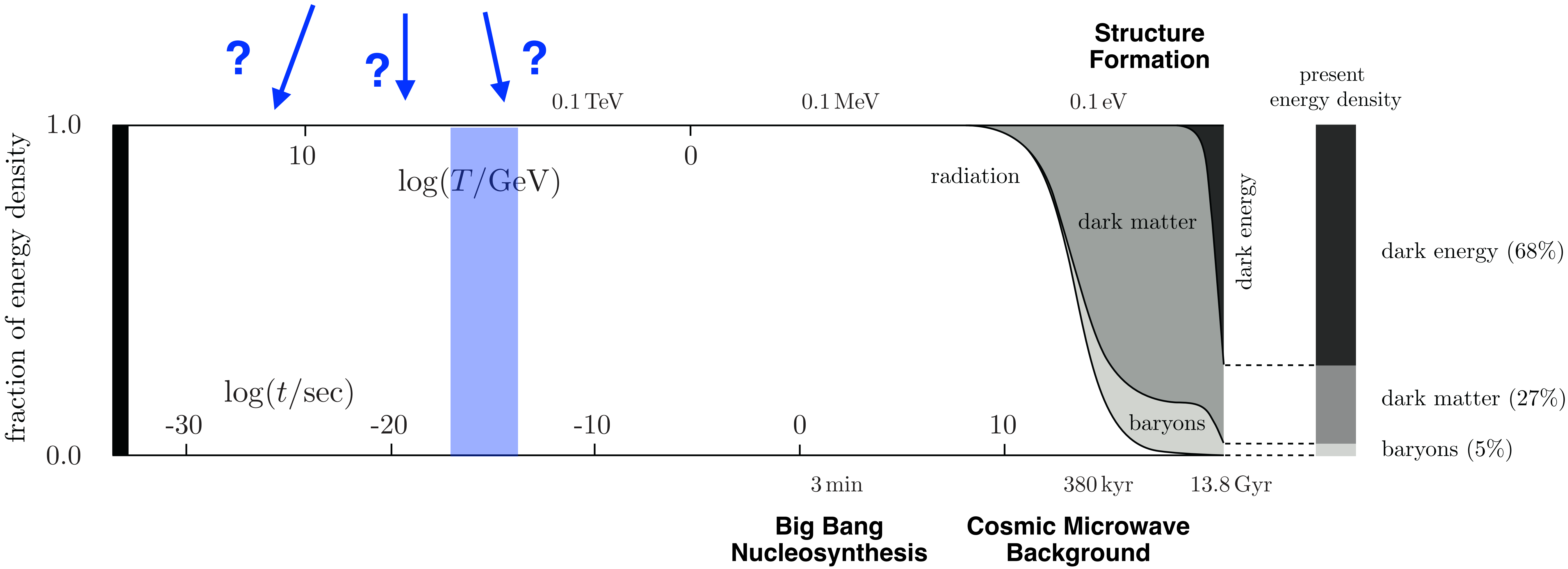


Cosmological History

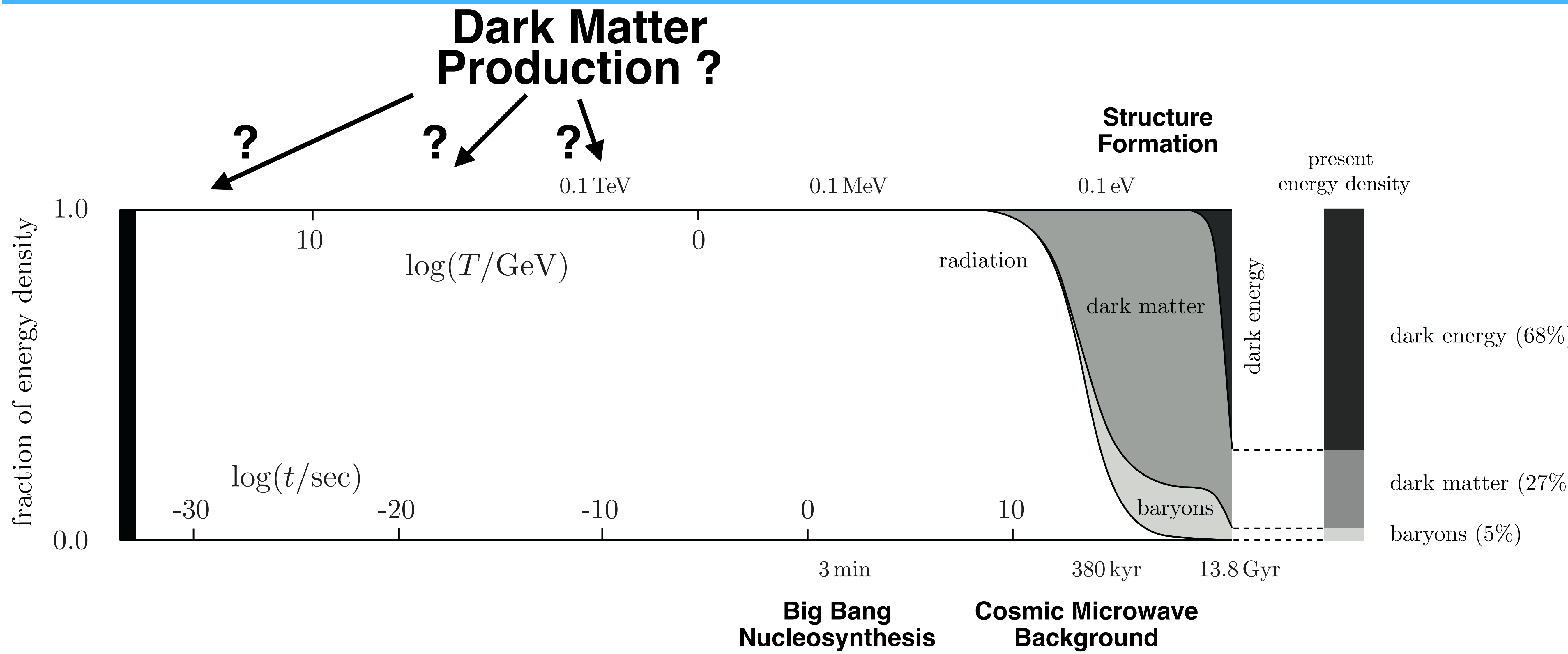


Cosmological History

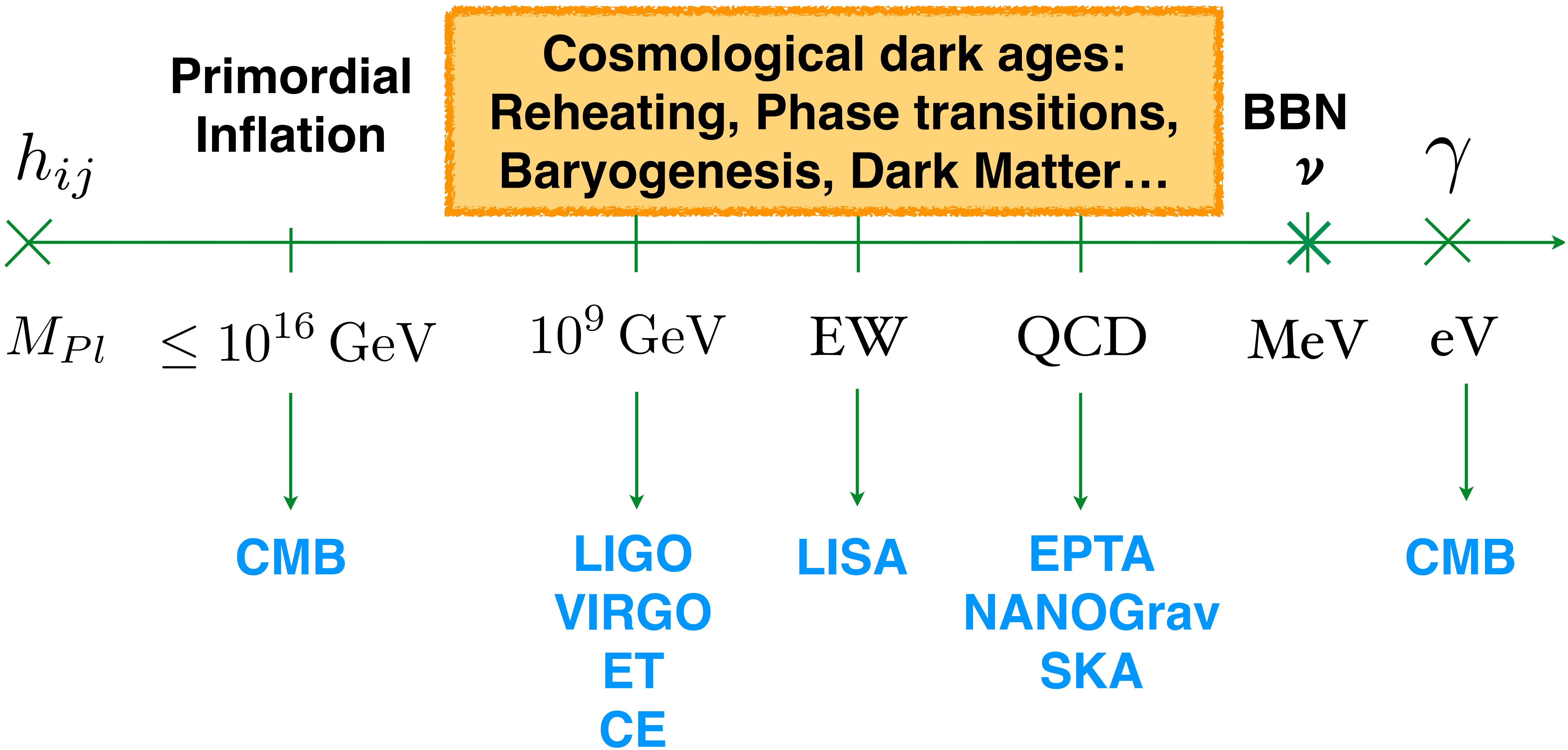
Secondary intermediate late inflation eras?



Cosmological History

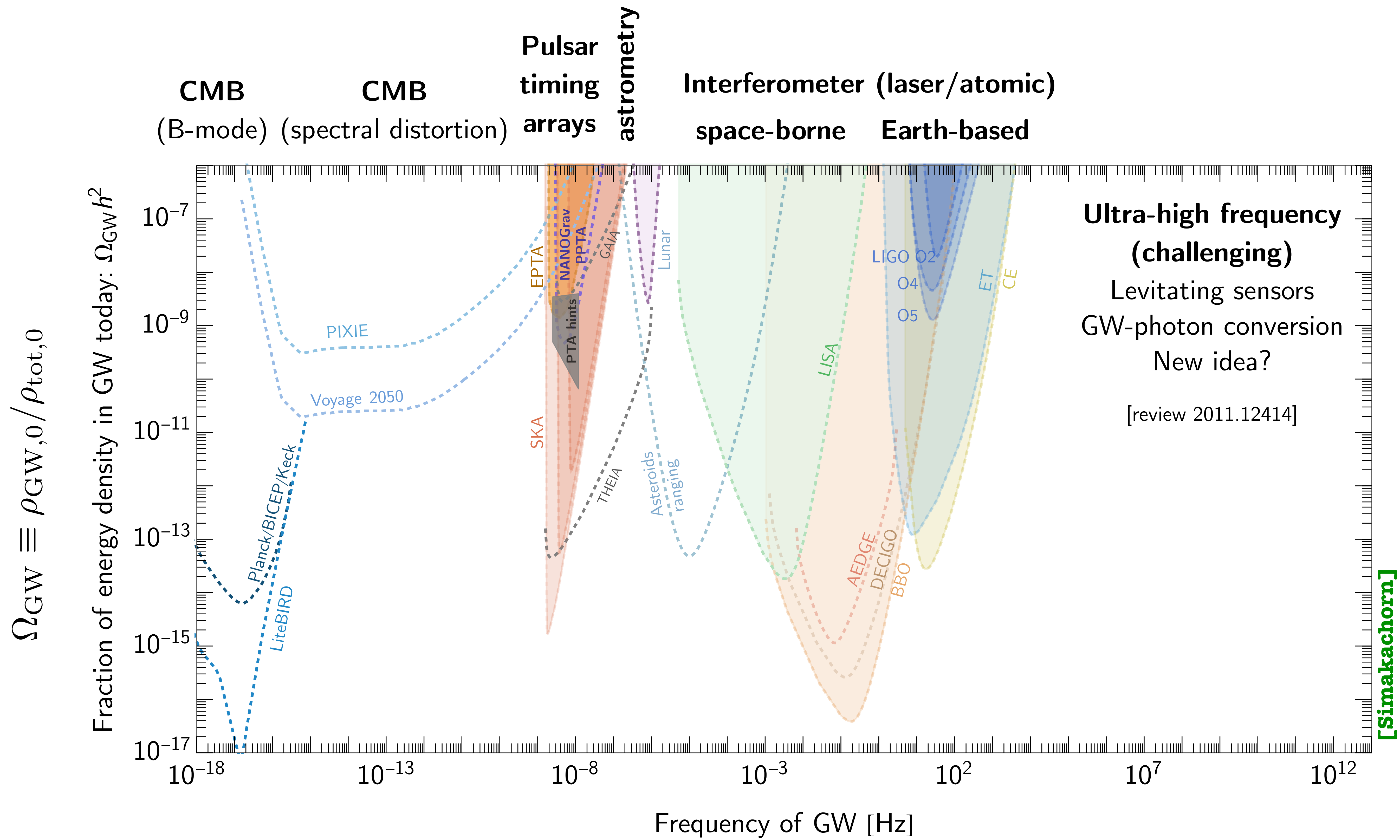


Probing the cosmological history with Gravitational Waves:



Current and future GW experiments constitute a new avenue of investigation in particle physics and cosmology.

The landscape of current & future GW experiments.



Primordial GW

Tensor perturbations of Friedmann-Robertson-Walker metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

Wave equation:

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$

Source:

Tensor anisotropic stress

=Transverse Traceless component of the energy-momentum tensor of the source $= (P_{il}P_{jm} - \frac{1}{2}P_{ij}P_{lm})T_{lm}$

$P_{ij} = \delta_{ij} - \hat{k}_i\hat{k}_j$

Well-known cosmological sources .

-> Cosmological Phase Transitions

-> Cosmic Strings

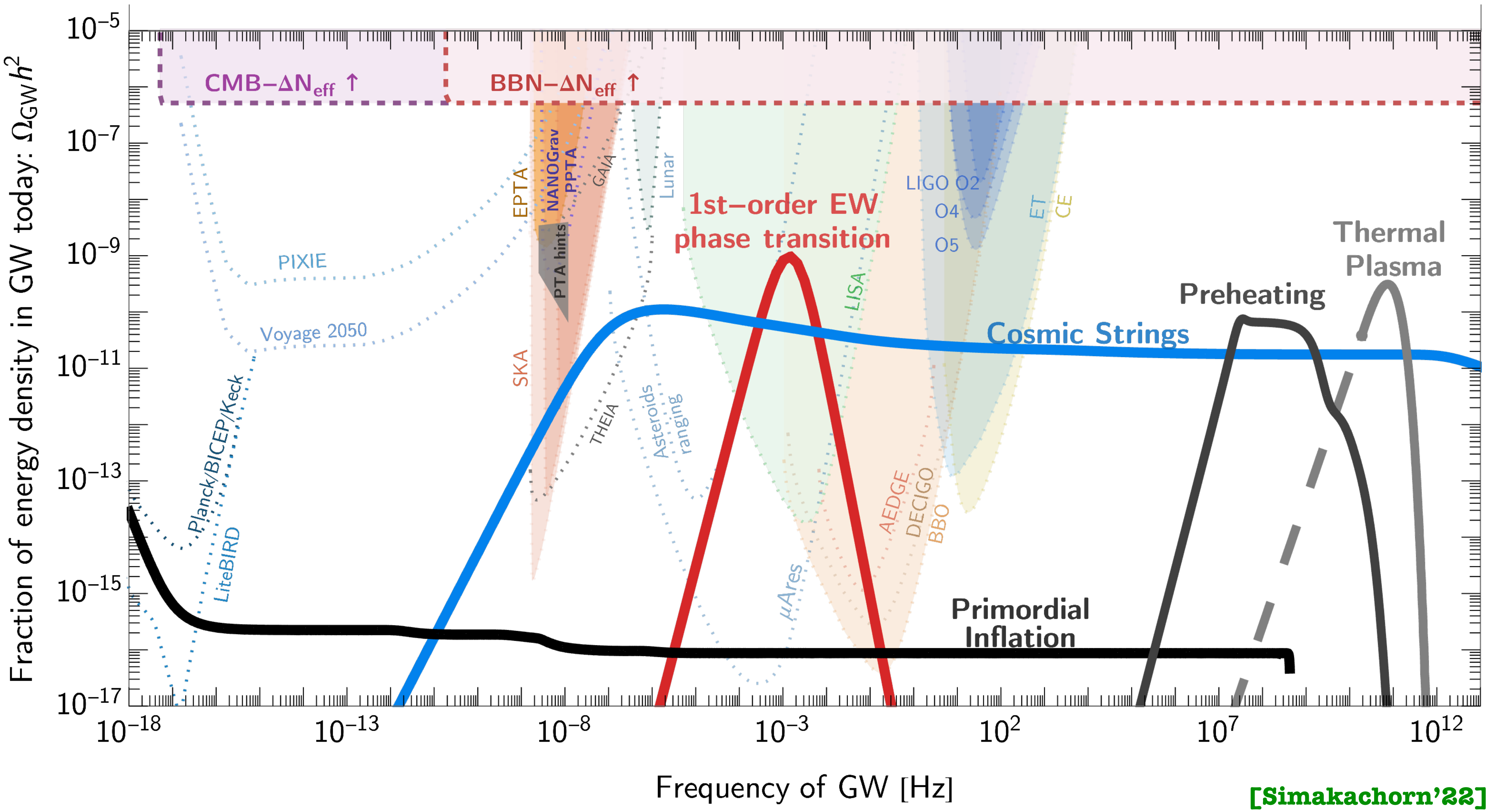
-> Inflation

-> Reheating of the universe

see
-review 1801.04268
-1912.02569 (cosmic strings)
-PhD thesis P. Simakachorn

Beyond-the-Standard Model sources.

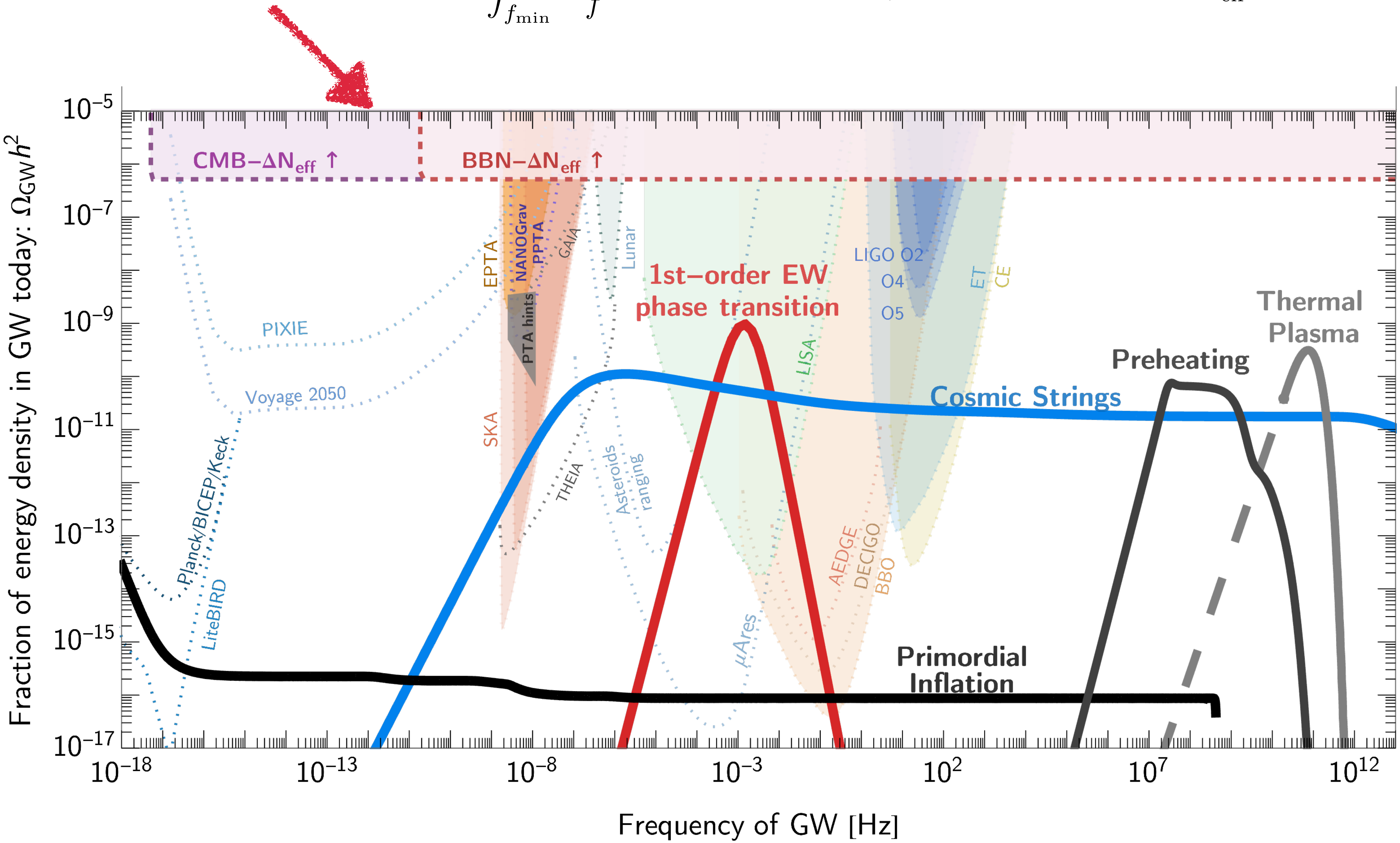
Preheating, first-order phase transitions, cosmic strings



[Simakachorn'22]

Upper theoretical bound.

GW as extra radiation: $$\int_{f_{\min}}^{f_{\max}} \frac{df}{f}\, \Omega_{\rm GW}(f) \lesssim 0.23\, \Omega_{\rm rad,0} \Delta N_{\rm eff} \qquad \text{where} \quad \Delta N_{\rm eff}^{\rm BBN,CMB} \lesssim 0.2$$



[Simakachorn'22]

Characteristic Frequencies for causal (and short-lasting) sources .

T_* **temperature of the universe at time of emission**

f_* **frequency at time of emission**

observed frequency: $f \sim f_* \frac{T_0}{T_*} \sim \mathcal{O}(H_*) \frac{T_0}{T_*} \sim \frac{T_*}{M_{Pl}} T_0 \sim T_* \times 10^{-13}$ 10^{-19} **GeV**

H_* = Hubble rate at T_*

If T_* **~ 100 GeV:** (Electroweak scale)

$$f \sim 10^{-30}\,\mathrm{GeV} \sim 10^{-30} \times 10^{25}\ \mathrm{Hz} \sim 10^{-5}\ \mathbf{Hz}$$

LISA !

If T_* **~** $\mathbf{10^{10}}$ **GeV:** (Peccei-Quinn scale)

f ~ $\mathbf{10^{3}}$ **Hz**

Frequencies limits

The lowest frequency of GW is that of those GW produced today with the largest possible source size, i.e., the Hubble horizon today:

$$f_{\mathrm{GW,lowest}} = H_0 \simeq 10^{-18}\ \mathrm{Hz}.$$

The highest frequency of primordial GW arises from the highest energy scale $H_{prod} \simeq M_P$

$$f_{\mathrm{GW,highest}} \simeq 10^{13}\ \mathrm{Hz}.$$

Largest possible amplitude

Early produced GW act as an effective number of neutrino relics which is strongly constrained by CMB measurement and by BBN predictions

$$\int_{f_{\rm BBN,CMB}}^{f_{\rm max}} \frac{df}{f}\Omega_{\rm GW}(f) \leq 5.6\times 10^{-6}\Delta N_\nu$$

$$\Delta N_\nu \leq 0.2$$

$$\Omega_{\rm GW,*} \leq 5.6\times 10^{-6}\Delta N_\nu \cdot \begin{cases} \log^{-1}\left[\frac{f_{\rm max}}{\max(f_{\rm ref},f_{\rm min})}\right] & \text{for flat spectrum,} \\ \beta\left[1-\left(\frac{f_{\rm ref}}{f_{\rm peak}}\right)^{\beta}\right]^{-1} & \text{for peak,} \end{cases}$$

$\Omega_{\rm GW} = \Omega_{\rm GW,*}$ for $f_{\rm min} < f < f_{\rm max}$,

$\Omega_{\rm GW}(f) = \Omega_{\rm GW,*}(f/f_{\rm peak})^{\beta}$

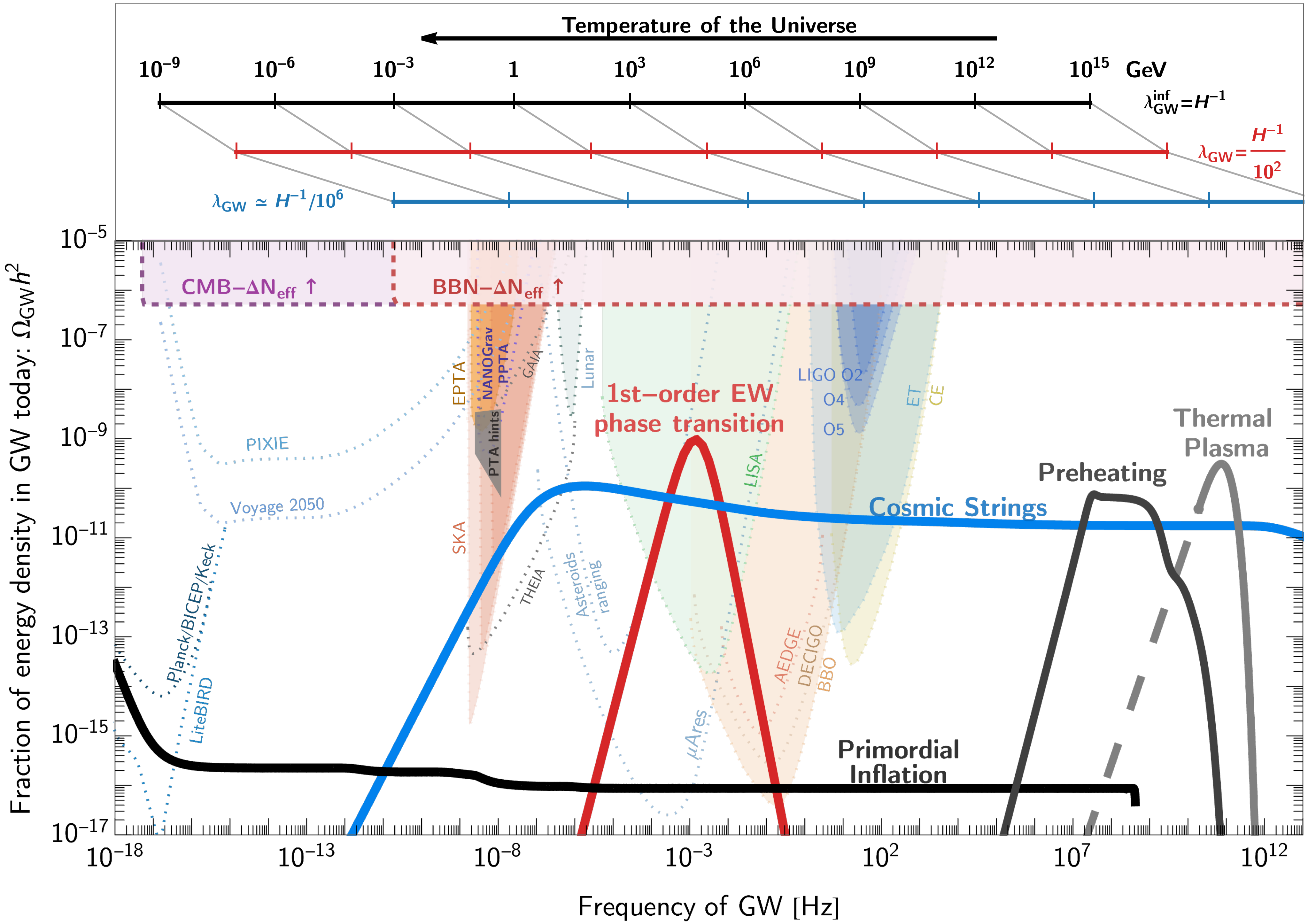
Reading the history of the universe.

GW frequency $f_{\mathrm{GW},0} \simeq \lambda_{\mathrm{GW}}^{-1}\left(\frac{a_{\mathrm{prod}}}{a_0}\right)$

Low-freq. lim

$f_{\mathrm{GW}}^{\mathrm{min}} \simeq H_0^{-1}$

$\simeq 10^{-18}$ Hz



High-freq. limit

$f_{\mathrm{GW}}^{\mathrm{max}} \simeq 10^{13}$ Hz

$(\lambda_{\mathrm{GW}} \sim H^{-1} \sim M_{\mathrm{pl}}^{-1})$

[Simakachorn]

Reading the history of the universe.

GW frequency $f_{\mathrm{GW},0} \simeq \lambda_{\mathrm{GW}}^{-1}\left(\frac{a_{\mathrm{prod}}}{a_0}\right)$

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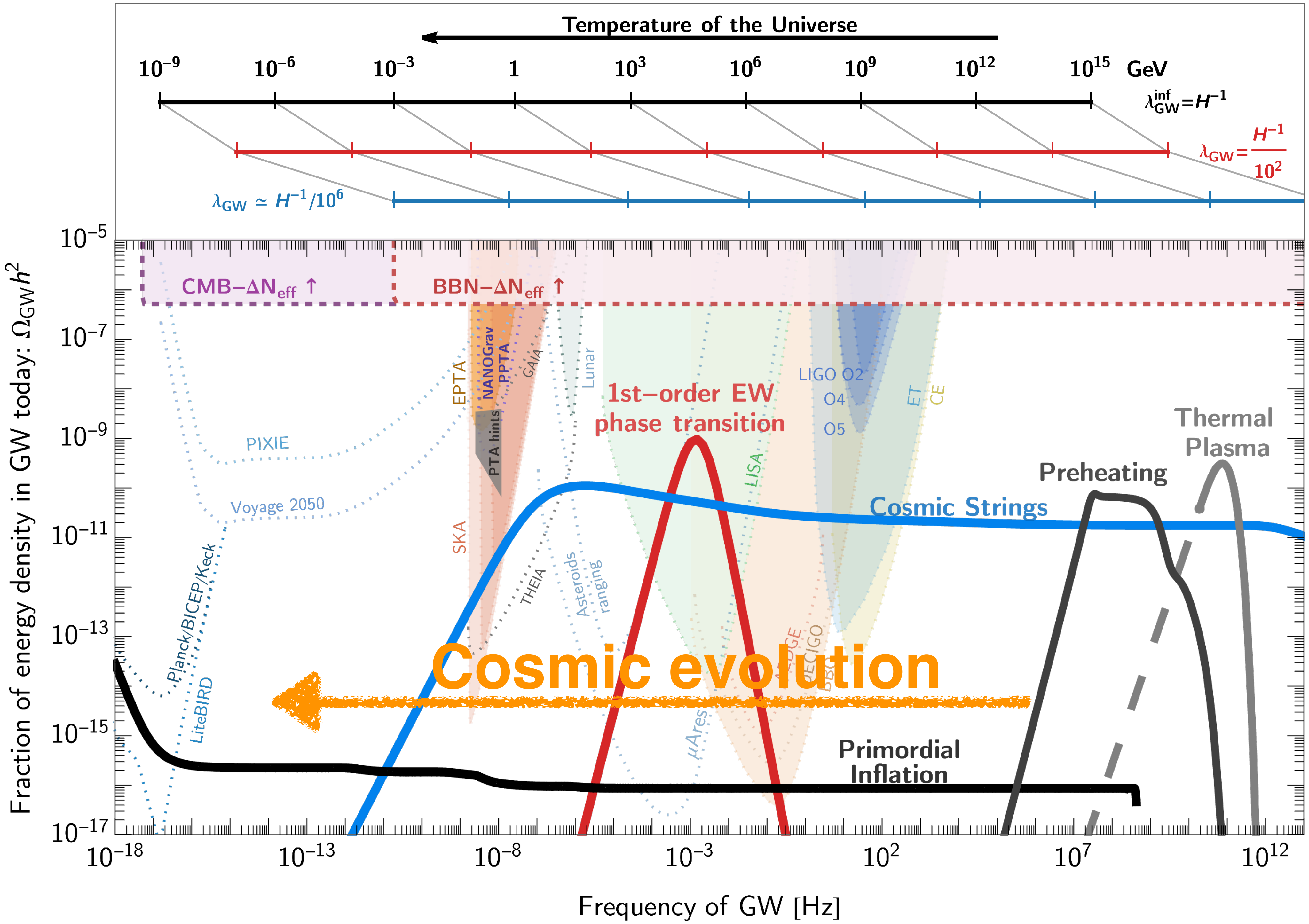
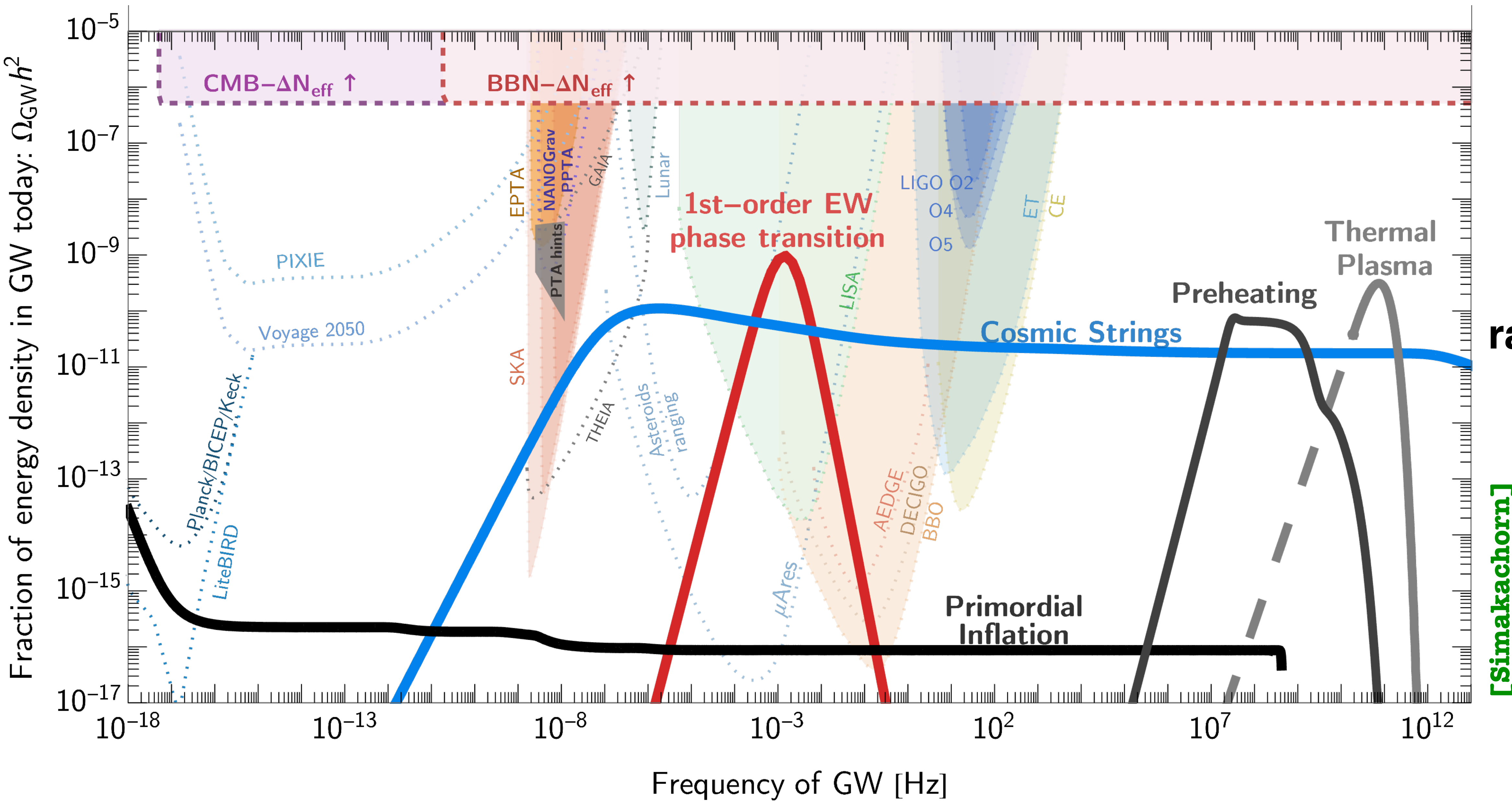


[Simakachorn]

GW spectra are sensitive to the cosmological history.

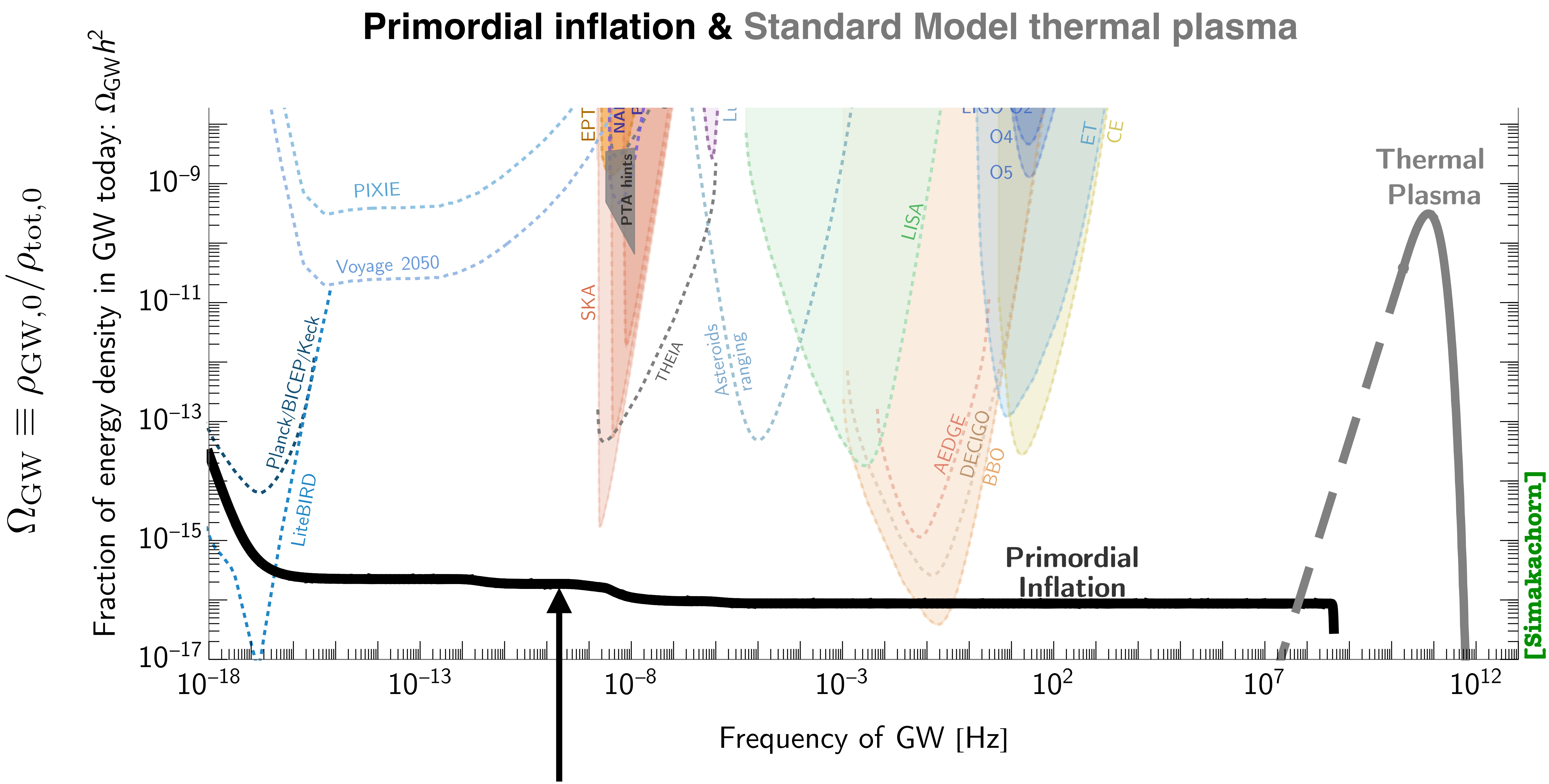
frequency $f_{\mathrm{GW},0} \simeq \lambda_{\mathrm{GW}}^{-1} \left(\frac{a_{\mathrm{prod}}}{a_0}\right)$ **energy density** $\rho_{\mathrm{GW},0} \simeq \rho_{\mathrm{GW}}^{\mathrm{prod}} \left(\frac{a_{\mathrm{prod}}}{a_0}\right)^4$



Standard Model radiation era at high energies

What if the universe is not radiation-dominated at high energies?

Standard Model sources of primordial GW.



Irreducible GW background from amplification of initial quantum fluctuations of the gravitational field during inflation

GWs from inflation.

Quantum fluctuations of some comoving scale k during inflation classicalize upon horizon exit (k > aH) and stay frozen afterwards.

After the end of the inflation phase, the increasing comoving horizon catches up with these modes and they re-enter (k < aH) the horizon.

Tensor perturbations of comoving wave number k = $a_k H_k$ lead to the

the stochastic GW today

$$\Omega_{\rm GW}^{\rm inf}(k) = \frac{k^2 a_k^2}{24 H_0^2}\Omega_T^{\rm inf},$$

$$\Omega_T^{\rm inf} \simeq \frac{2}{\pi^2}\left(\frac{H_{\rm inf}}{M_{\rm pl}}\right)^2 \left(\frac{k}{k_p}\right)^{n_t}$$

~ constant

Pivot scale $k_p/a_0 \simeq 0.002\ {\rm Mpc}^{-1}$

$f_p = 3.1 \times 10^{-18}\ {\rm Hz}$

$n_t \simeq -2\epsilon$

Taking for simplicity $n_t = 0$

GWs from inflation.

$$\Omega \sim (ka)^2 \sim (a^2 H)^2 \sim a^4 \rho \sim a^4 a^{-3(1+\omega)} \sim a^{1-3\omega}$$

$$f \sim aH \sim a\rho^{1/2} a^{-(1+3\omega)/2} \longrightarrow a \sim f^{-2/(1+3\omega)}$$

w: equation of state of the universe

$$\longrightarrow \Omega \sim a^{1-3\omega} \sim f^{-2\frac{(1-3\omega)}{(1+3\omega)}}$$

w=1/3, radiation era —> scale-invariant spectrum

w=0, matter era —> $\Omega \sim f^{-2}$

w=1, kination —> $\Omega \sim f$

The GW spectrum from inflation encodes the full cosmological evolution of the equation of state of the universe!

GWs from the SM plasma.

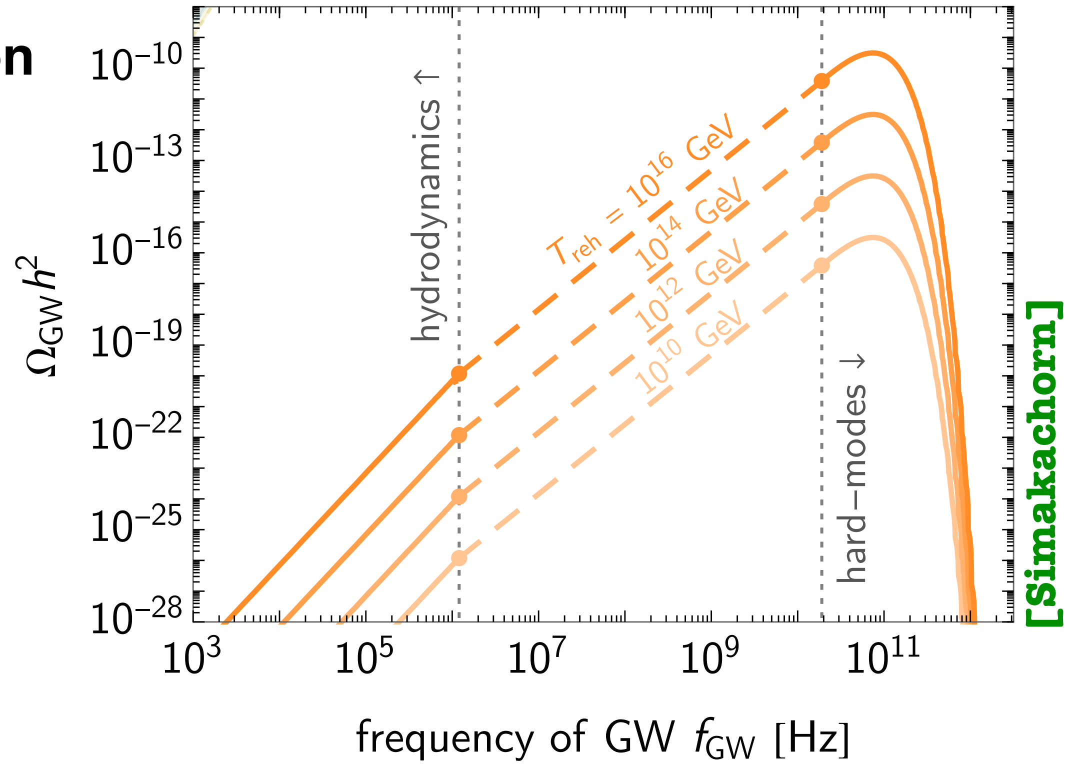
T_* **temperature of the universe at time of emission**

f_* **frequency at time of emission**

observed frequency: $f \sim f_* \frac{T_0}{T_*} \sim \mathcal{O}(H_*) \frac{T_0}{T_*}$

Instead f$_*$ ~ T$_*$ —-> cancellation

f$_*$ ~ T$_0$/2π ~ 5.10^{10} Hz



- J. Ghiglieri and M. Laine, 1504.02569
- A.Ringwald, J.Schütte-Engel and C.Tamarit, 2011.04731
- A.Ringwald and C.Tamarit, 2203.00621

Characteristic Frequencies for causal (and short-lasting) sources .

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LISA !

If T_* **~ 10^{10} GeV:** (Peccei-Quinn scale)

$f \sim 10^{3}$ Hz

Gravitational Waves from a 1st-order phase transition .

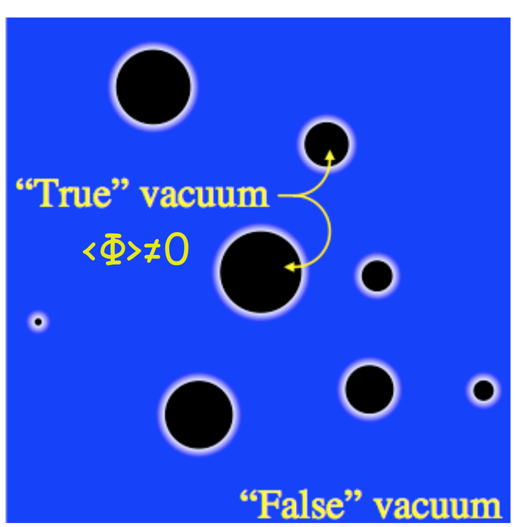
$$\ddot{h}_{ij} + 2\mathcal{H}\dot{h}_{ij} + k^2 h_{ij} = 8\pi G a^2 T^{(TT)}_{ij}(k,t)$$

$$T_{ab}(\mathbf{x}) = (\rho + p)\frac{v_a(\mathbf{x})v_b(\mathbf{x})}{1 - v^2(\mathbf{x})}$$

Source of GW: anisotropic stress

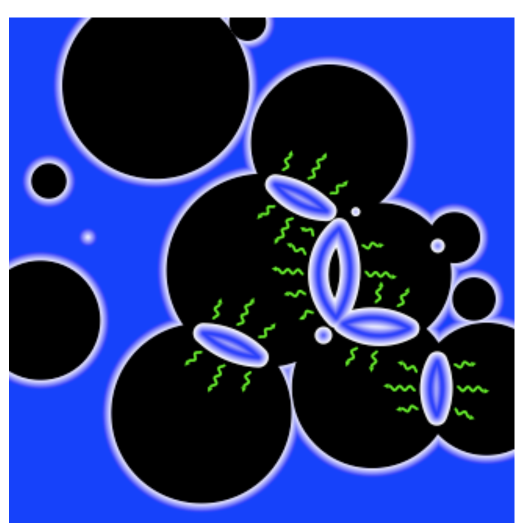
Generated mainly from fluid velocities in the vicinity of colliding bubble walls

Bubble nucleation



$\langle\Phi\rangle=0$

Bubble percolation



violent process if $v_b \sim O(1)$

Fluid flows

Magnetic fields

Turbulence

Stochastic bgd of gravitational radiation

Sources of GWs during a 1st-order phase transition

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$

Fluid (sound waves and turbulence)

$$\Pi_{ij} \sim \gamma^2(\rho + p)\,v_i v_j$$

Electromagnetic field (primordial magnetic fields MHD turbulence)

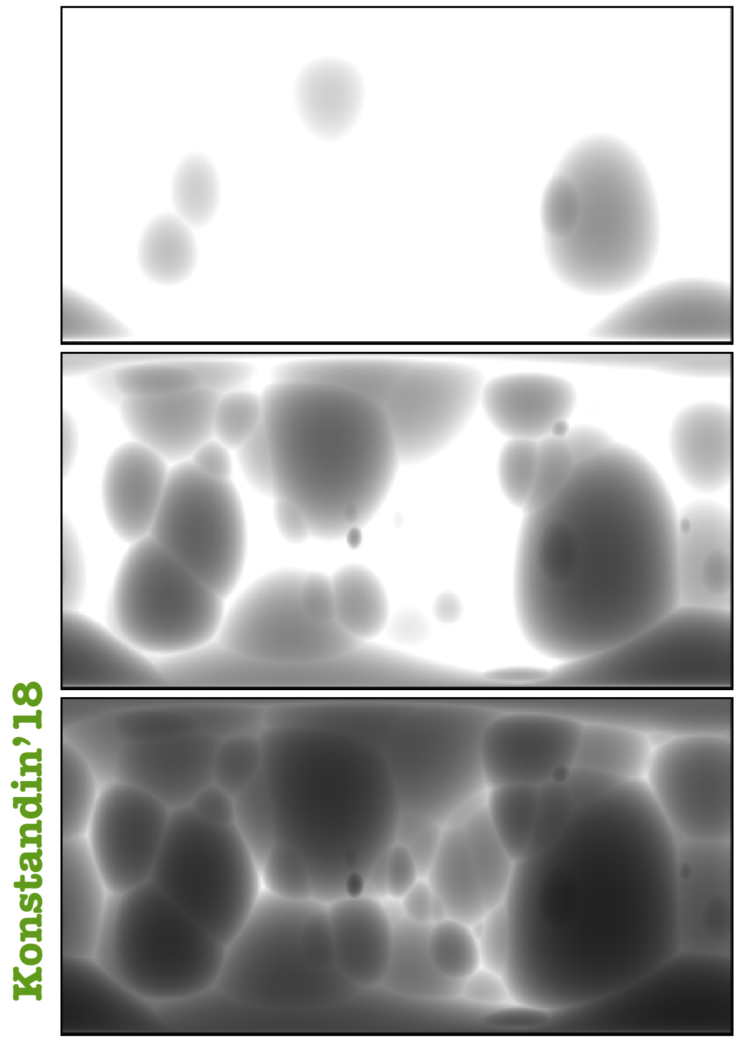
$$\Pi_{ij} \sim (E^2 + B^2)\frac{\delta_{ij}}{3} - E_i E_j - B_i B_j$$

Scalar field (collision of bubble walls)

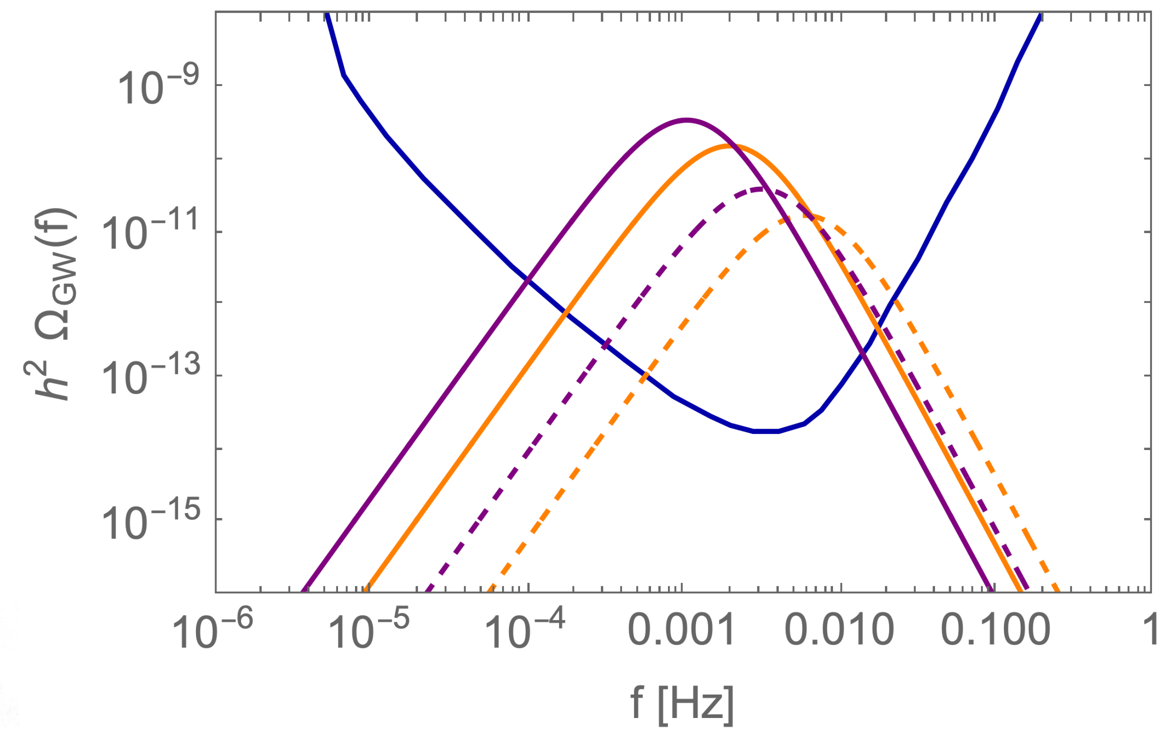
$$\Pi_{ij} \sim \partial_i\phi\,\partial_j\phi$$

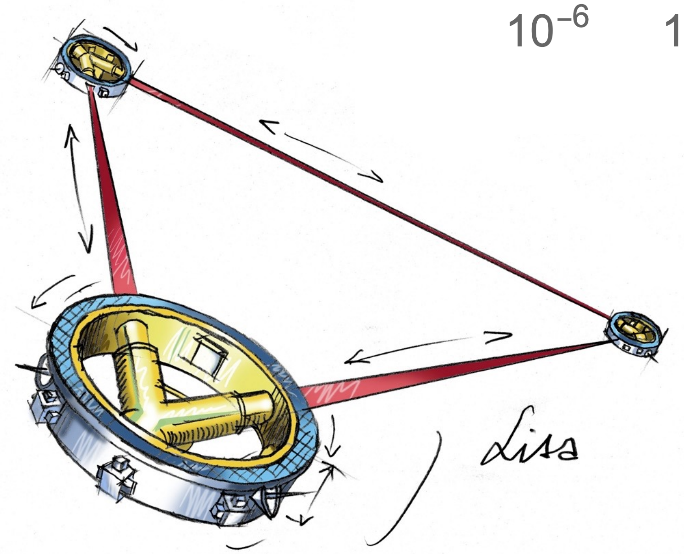
In principle entirely determined by the Higgs effective potential.

GWs from a 1st-order phase transition .



Electroweak Phase Transition
-> milli-Hertz —> LISA !





[LISA Cosmology Working group, 1512.06239]
+ update 1910.13125

LISA: a new window on the Weak Scale

hep-ph/0607107

complementary to collider informations

Two Characteristic quantities

- β : **inverse duration of the phase transition**

set by the tunneling probability $P \propto e^{\beta t} \propto \frac{T^4}{H^4} e^{-S_3/T} \sim 1$ ➜ $\frac{S_3}{T}$ ~ 140

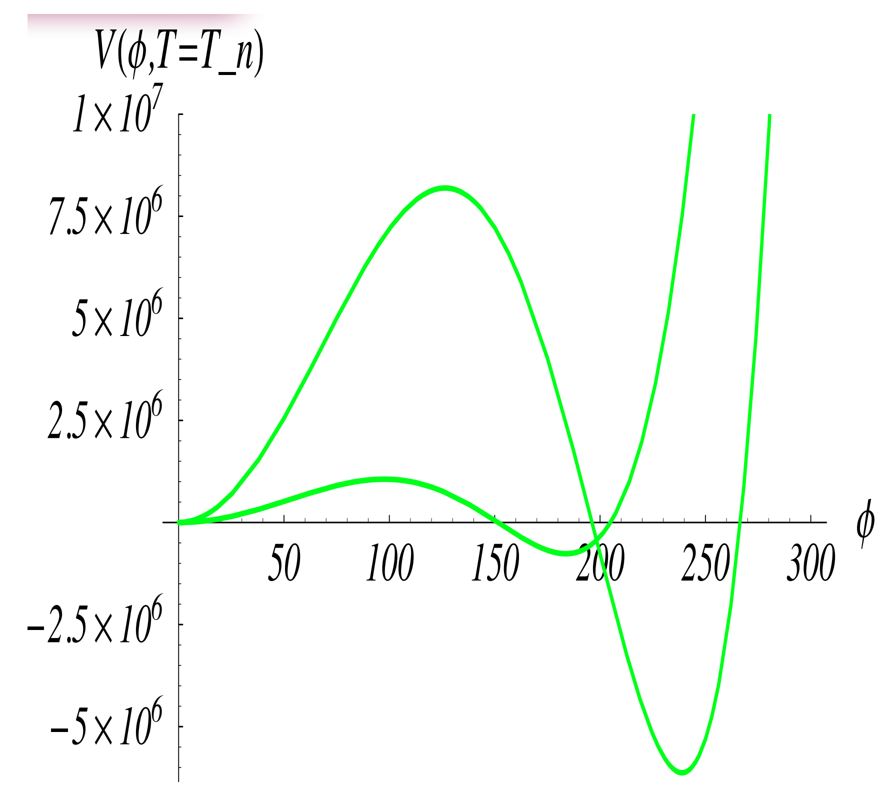
$$\beta \equiv \frac{dS}{dt}\Big|_* = -H_* T_* \frac{dS}{dT}\Big|_*$$

typically $\frac{\beta}{H} \sim \mathcal{O}(10^2 - 10^3)$

corresponds to the characteristic inverse size of bubbles at time of collisions $R_* = v_w/\beta$

sets the characteristic frequency $f_* \propto \frac{\beta}{v_w}$

$$f_0 = \frac{a_*}{a_0} f_* \approx 20 \frac{1}{v_w} \frac{\beta}{H_*} \frac{T_*}{100GeV} \left(\frac{g_*}{100}\right)^{1/6} \mu\text{Hz}$$



- α : **vacuum energy/radiation energy density**

α and β : entirely determined by the effective scalar potential at high temperature

Estimate of the GW energy density at the emission time .

$$\Omega_{GW,*} = \frac{\rho_{GW,*}}{\rho_{tot,*}} \sim \frac{G}{\beta^2}\Pi^2_{source} \times \frac{1}{\rho_{tot,*}} \sim \left(\frac{H_*}{\beta}\right)^2 \left(\frac{\Pi_{source}}{\rho_{tot,*}}\right)^2$$

$$\Pi_{source} \sim \kappa\rho_{vac}$$

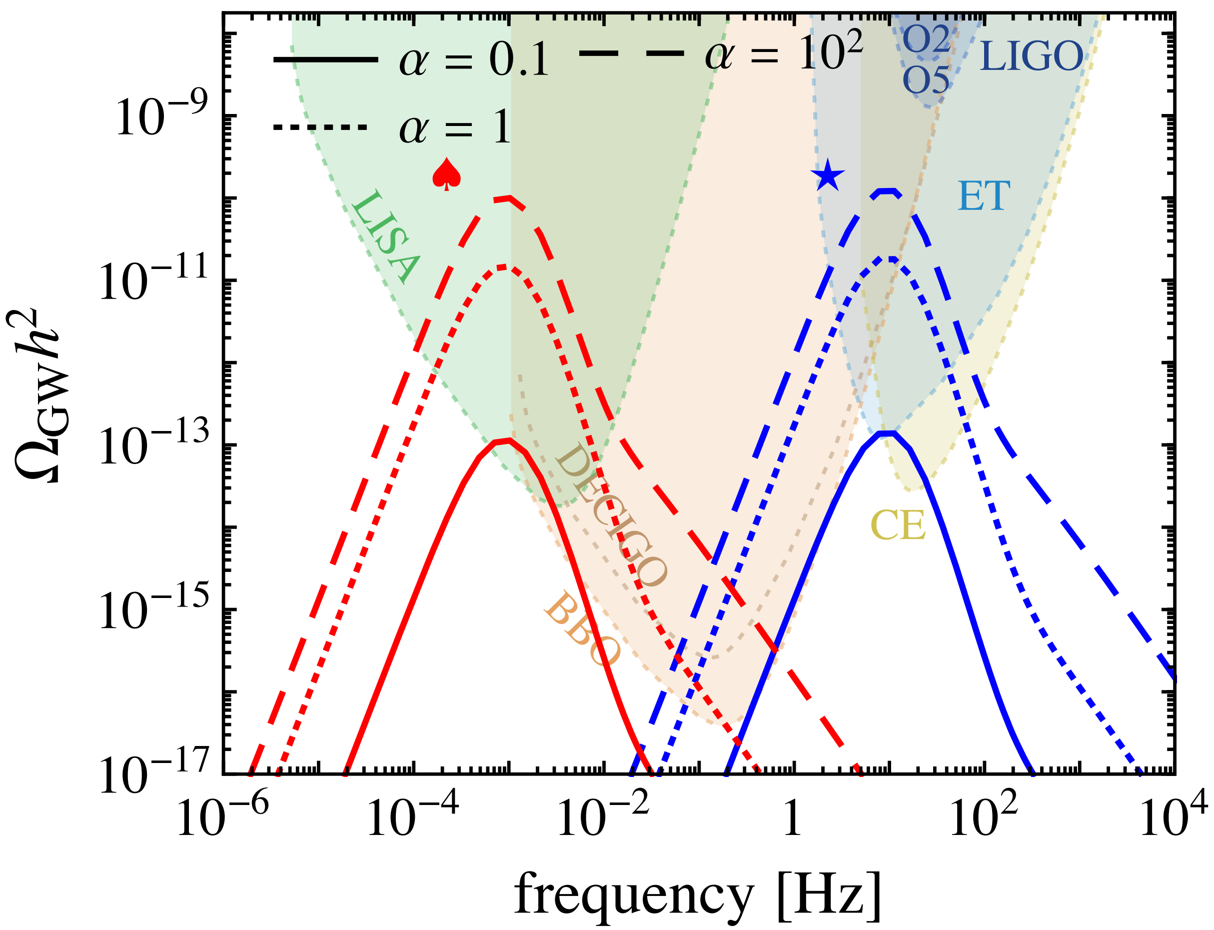
$$\kappa_\phi = \frac{\rho_\phi}{\rho_{\rm vac}} \qquad \kappa_v = \frac{\rho_v}{\rho_{\rm vac}} \qquad \kappa_t = \epsilon\,\kappa_v$$

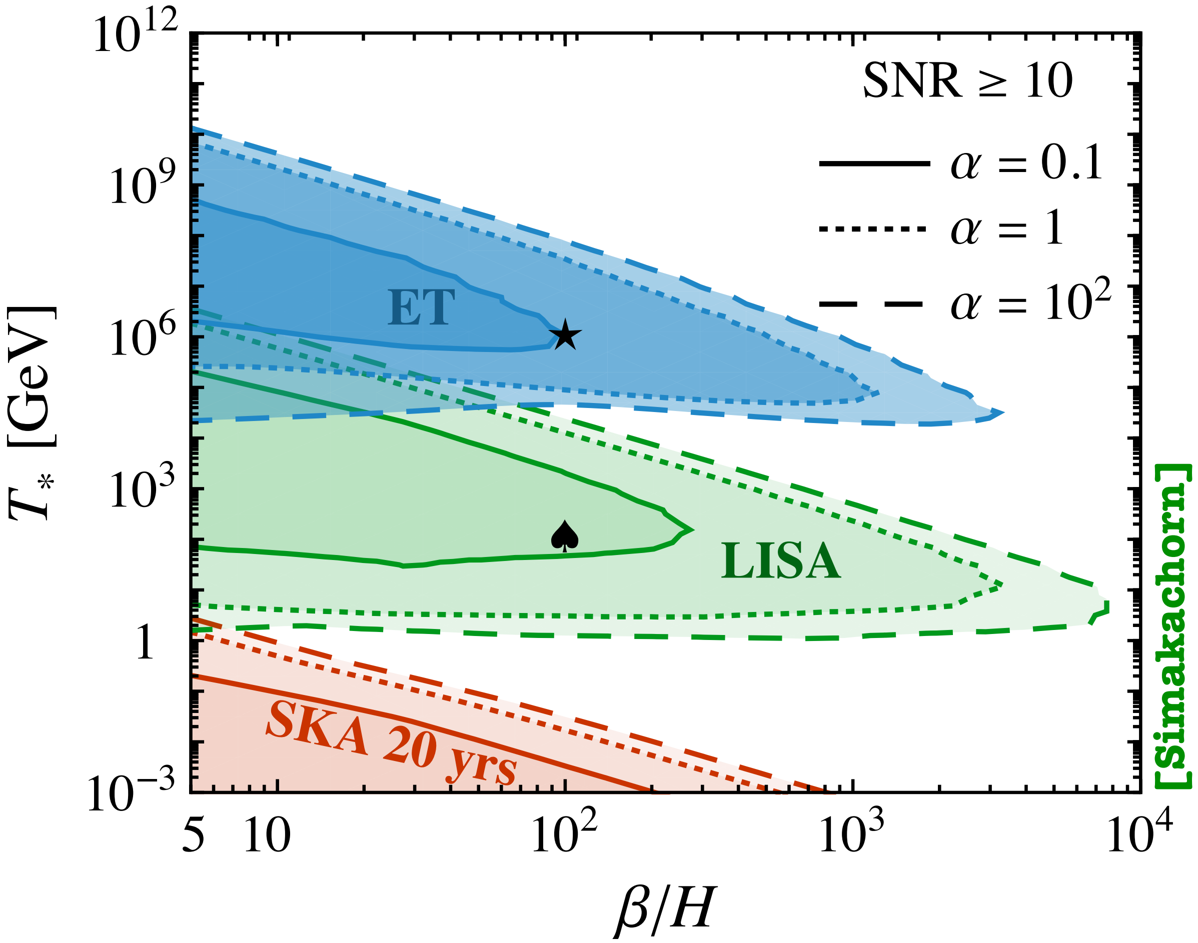
fractions of vacuum energy that goes into either gradient energy in bubble kinetic energy in the fluid o into turbulent motion.

$$\left(\frac{\Pi_{source}}{\rho_{tot,*}}\right)^2 \sim \frac{\kappa^2\alpha^2}{(1+\alpha)^2}$$

$$\Omega_{GW,*} \sim \left(\frac{H_*}{\beta}\right)^2 \times \frac{\kappa^2\alpha^2}{(1+\alpha)^2}$$

GWs from a 1st-order phase transition: Reach



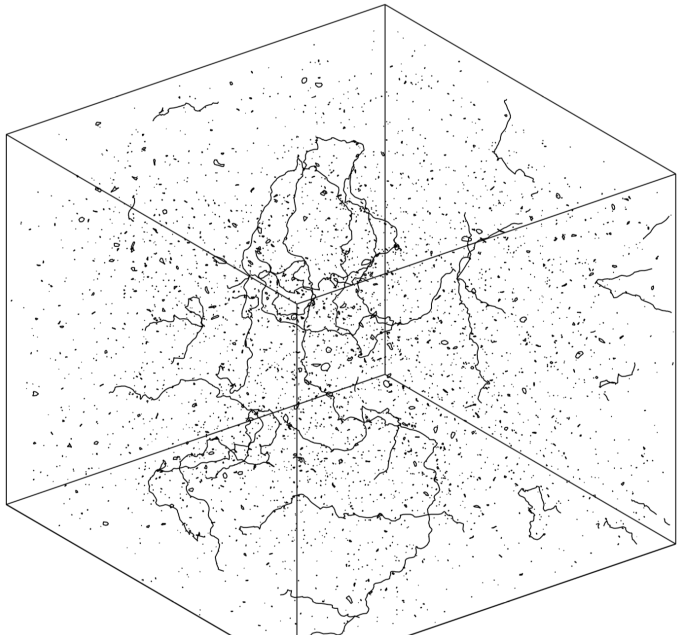


For reviews see e.g.

1512.06239, 1910.13125, 2204.05434

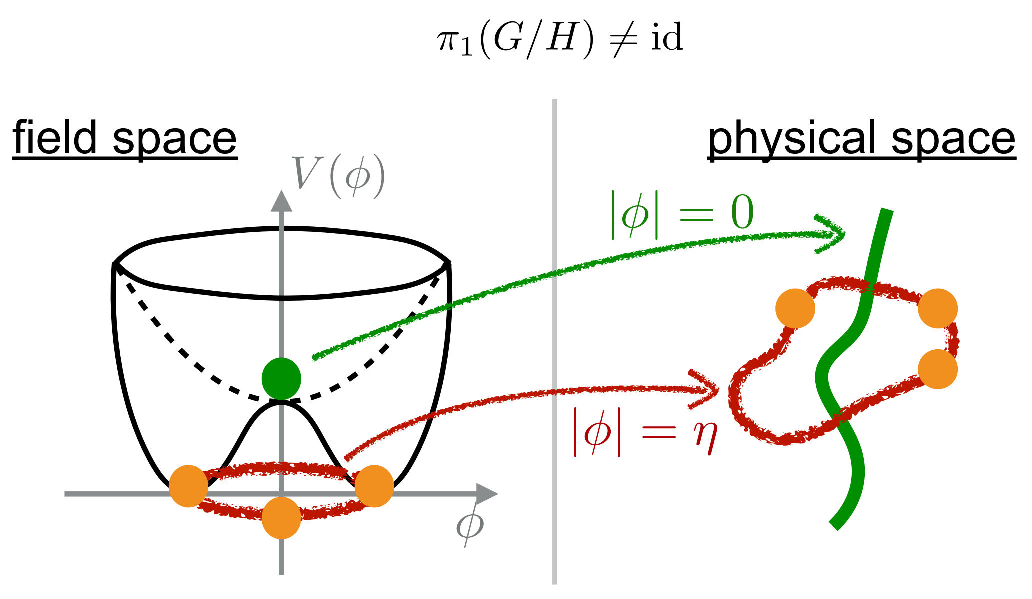
Gravitational Waves from cosmic strings.

recent reviews: [1909.00819, 1912.02569]



Network of cosmic strings
[Allen & Shellard, 1990]

Gravitational Waves from cosmic strings.



Cosmic string: Line-like topological defect arising after spontaneous U(1) symmetry breaking at some energy scale η.
The broken symmetry can be either local or global; —> local or global (axionic) cosmic strings.

string tension: $\mu \sim \eta^2$

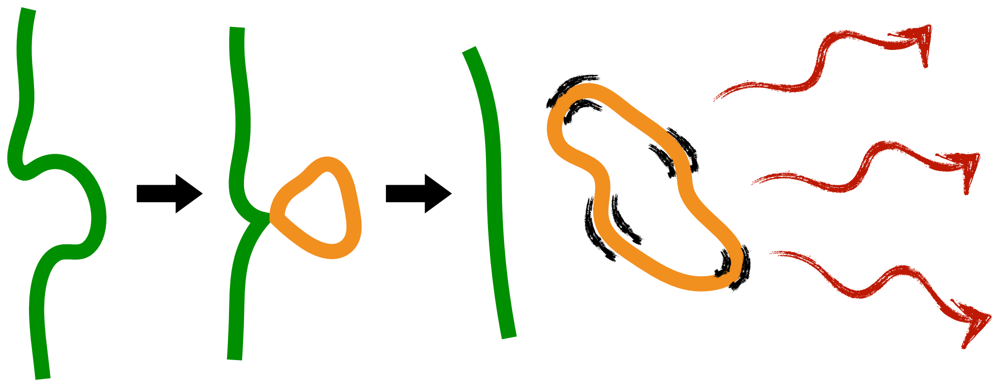
$$\mu \sim \eta^2 \times \begin{cases} 1, & \textbf{local} \\ \ln(m_\phi/H), & \textbf{global} \end{cases}$$

Axionic strings: $\eta = f_a$

Loop formation & scaling regime.

After the network formation, the string network keeps producing loops.

String intercommutation: loop formation depletes energy from the network.



Cosmic strings do not overclose the universe.

The energy density of the network tracks the total energy density of the Universe

Scaling regime: $\rho_{\rm net}(t) \simeq \mu/t^2 \simeq \; G\mu\rho_{\rm tot}(t)$

$$\rho_\infty \propto t^{-2} \propto \begin{cases} a^{-3} & \text{for matter} \\ a^{-4} & \text{for radiation} \\ a^{-6} & \text{for kination} \end{cases}$$

GW from cosmic strings.

Cosmic strings: Long-lasting source of GW

The produced loops decay into particles and GW.

Local-string loops decay dominantly into GW while global-string loops decay dominantly into Goldstone radiation.

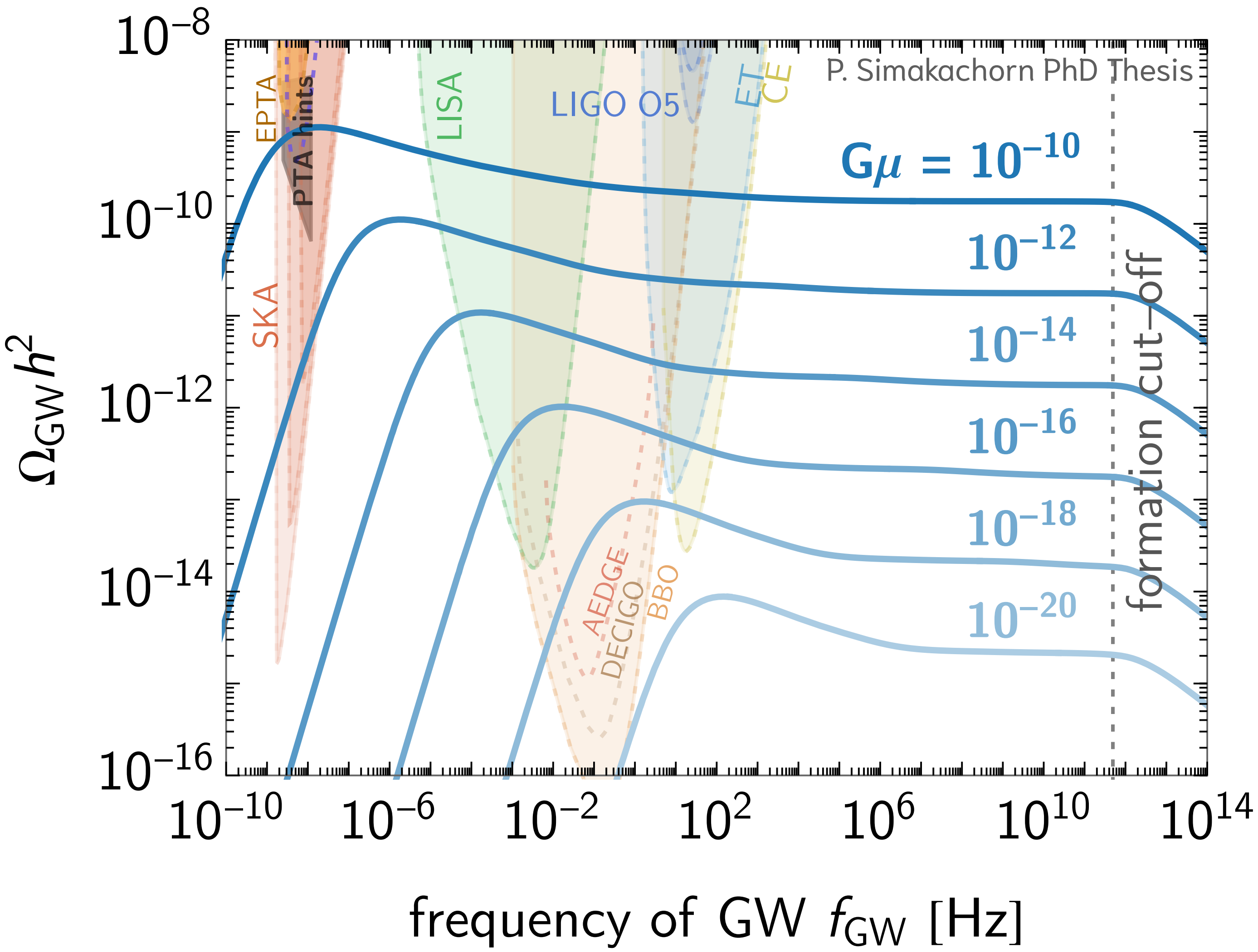
GW from local cosmic strings.

Superposition of many loop populations producing GW at time $\tilde{t}$ and of many oscillation k^{th}-modes,

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{1}{\rho_{c,0}} \sum_{k=1}^{k_{\max}} \frac{2k}{f_{\rm GW}} \cdot \Gamma^{(k)} G\mu^2 \int_{t_{\rm form}}^{t_0} n_{\rm loop}(\tilde{t}) \left[\frac{a(\tilde{t})}{a(t_0)}\right]^5 d\tilde{t},$$

GW from a loop

# of loops produced along cosmic history
(from production time until today)



GW from cosmic strings.

Cosmic strings: Long-lasting source of GW

GW spectrum ⇨ GW emission from a loop × loop-number density

$$\Omega_{\rm GW}^{(k)}(f) = \frac{1}{\rho_c}\cdot\frac{2k}{f}\cdot\frac{(0.1)\Gamma^{(k)}G\mu^2}{\alpha(\alpha+\Gamma G\mu)}\int_{t_F}^{t_0} d\tilde{t}\,\frac{C_{\rm eff}(t_i)}{t_i^4}\left[\frac{a(\tilde{t})}{a(t_0)}\right]^5\left[\frac{a(t_i)}{a(\tilde{t})}\right]^3\Theta(t_i - t_F)$$

string's nature — loop number — red-shift

a^{-3} (loop-number density)

GW: a^{-4}, loop size: a^{-1}

$t_i \equiv$ loop production, $\tilde{t} \equiv$ loop emission

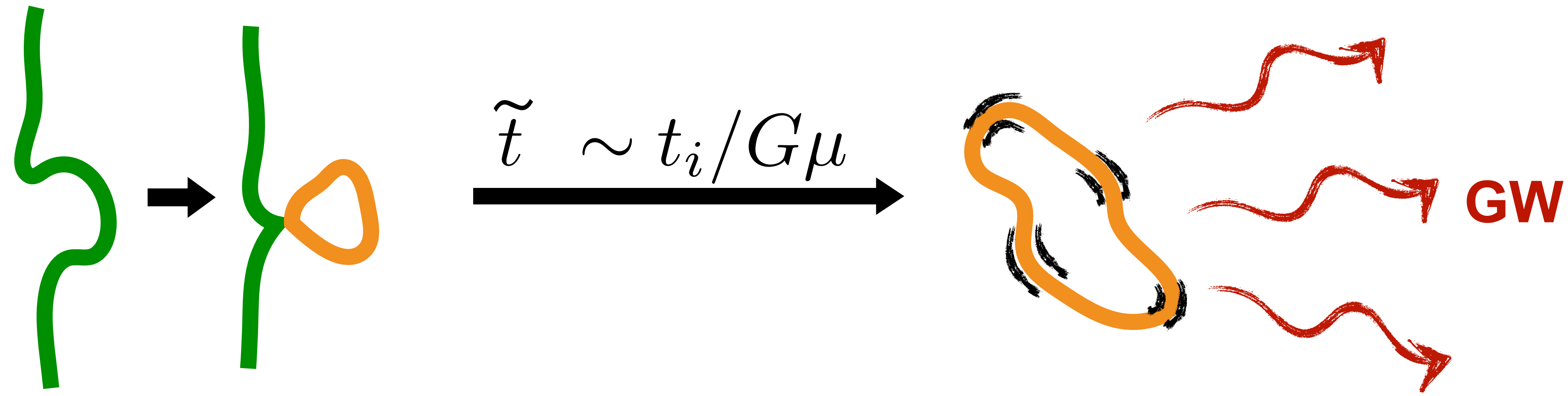
α = initial loop size as a fraction of Hubble horizon

Γ = GW radiation efficiency

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{1}{\rho_{c,0}}\sum_{k=1}^{k_{\rm max}}\frac{2k}{f_{\rm GW}}\cdot\Gamma^{(k)}G\mu^2 \times \int_{t_{\rm form}}^{t_0} n_{\rm loop}(\tilde{t})\left[\frac{a(\tilde{t})}{a(t_0)}\right]^5 d\tilde{t},$$

cosmic string (local)

loop production @ t_i loop emission @ $\tilde{t}$



Relation between observed frequency & Hubble radius at emission

$$f \approx H_i \left(\frac{a_i}{a_0}\right) \left(\frac{1}{G\mu}\right)^{1/2}$$

(the delayed emission happens at $a/a_i = (t/t_i)^{1/2} = 1/\sqrt{(G\mu)}$)

Relation between observed frequency & Hubble radius at emission.

The broad band GW spectrum is the result of the superposition of GW generated by many populations of loops produced at different temperatures.

Each emits GW at frequency $f_{\rm GW}^{\rm emit} \simeq 2k/l$

k : GW mode number of loop oscillation

l: loop's size

The GW contribution at higher frequencies comes from smaller loops produced at higher energy scales.

Time of GW emission for local strings: $\tilde{t} \simeq \alpha/(2\Gamma G\mu)t_i$

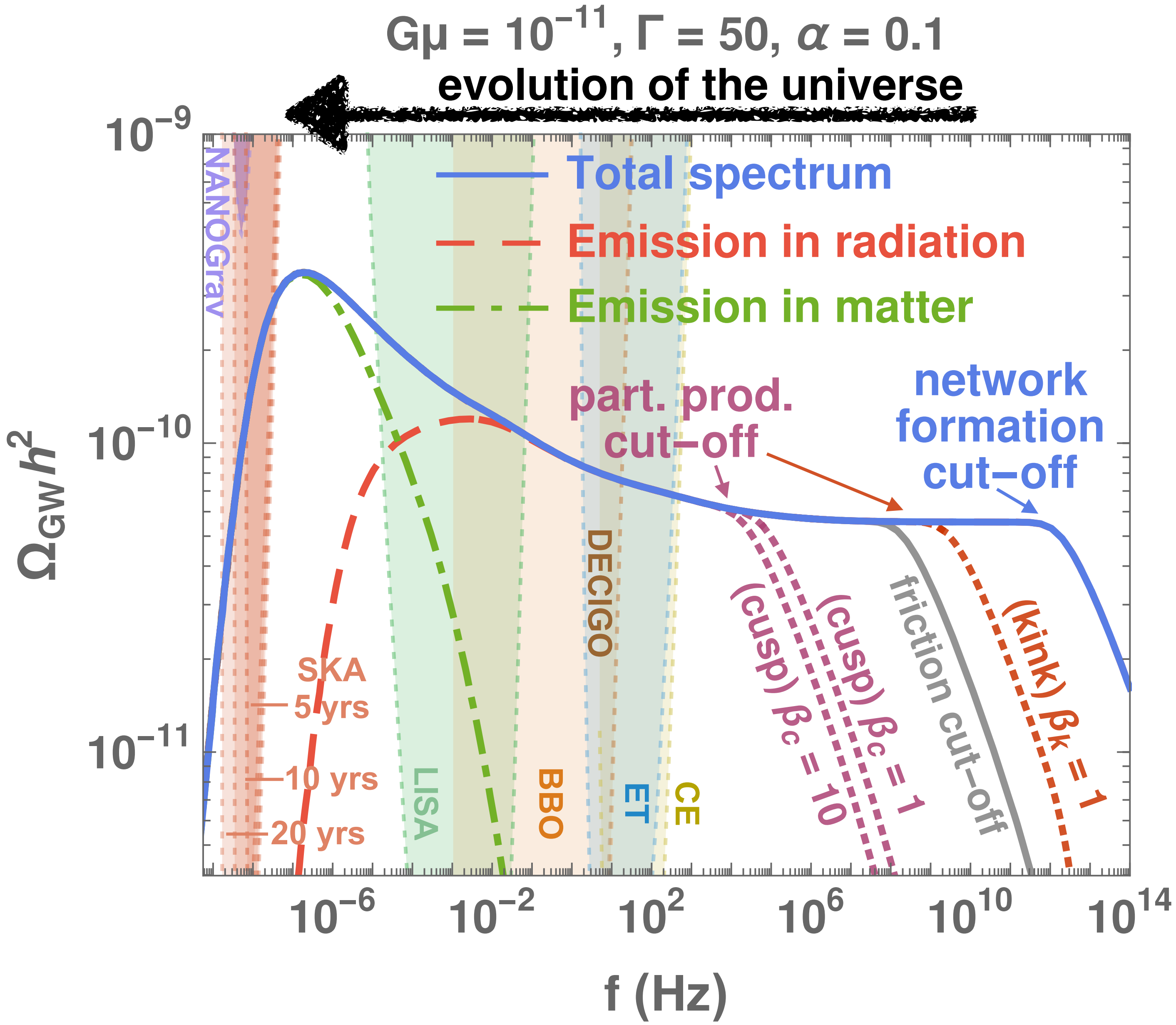
In contrast, global strings quickly emit GW after loop production: $\tilde{t} \simeq t_i$

t_i : time of loop formation

For a loop population created at temperature T, the GW spectrum is sourced maximally at a GW frequency today that is higher for local strings compared to global strings.

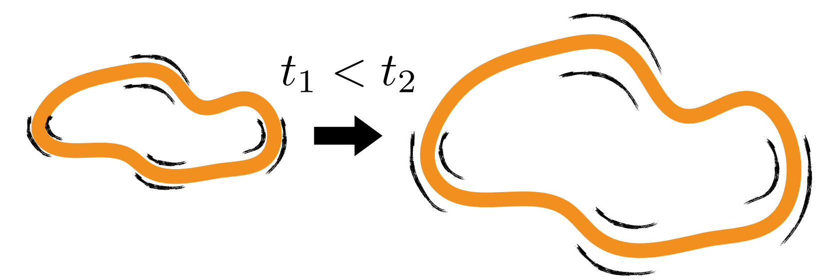
Gravitational Waves from Cosmic strings.

(long-lasting sources).



[1912.02569]

Higher f ⇔ Earlier emission



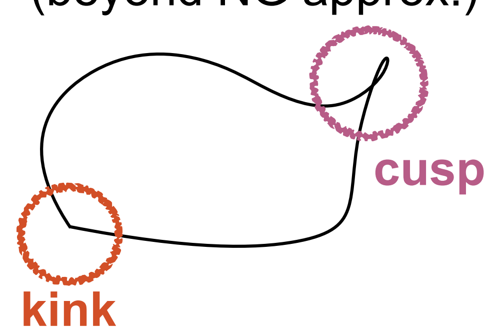
smaller loop ⇔ higher oscillation f

@ earlier t_i

more GW from more loops
but more red-shift

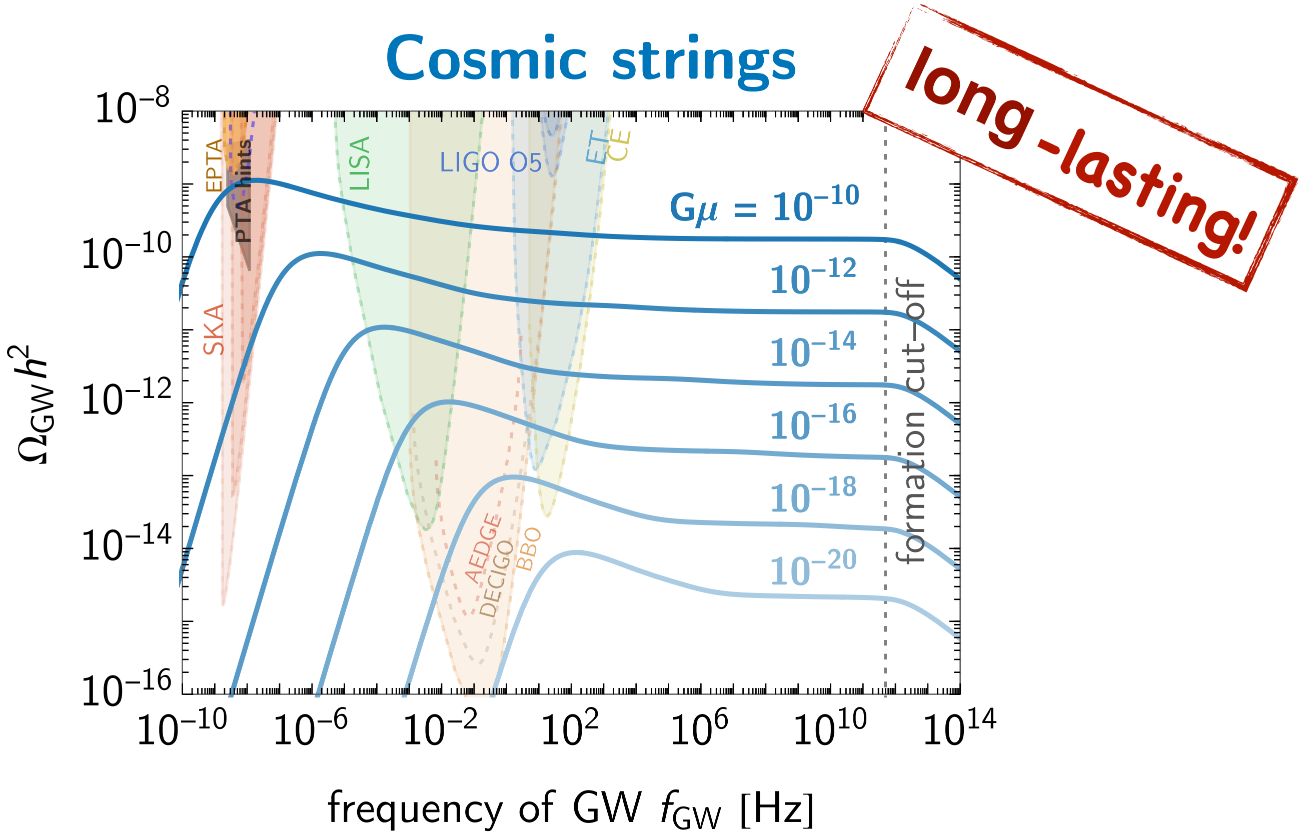
⇨ Flat during radiation

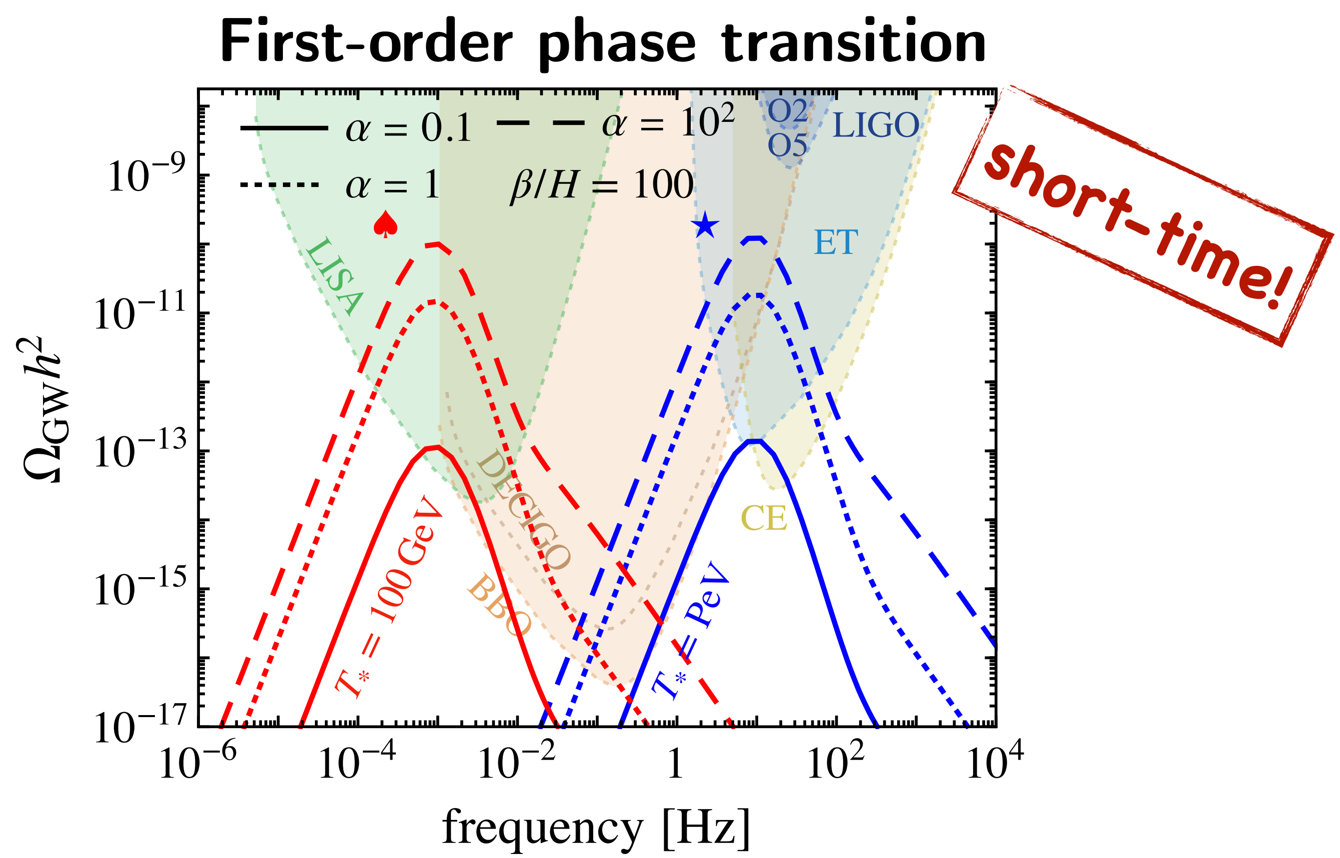
singular structures on loop
(beyond NG approx.)



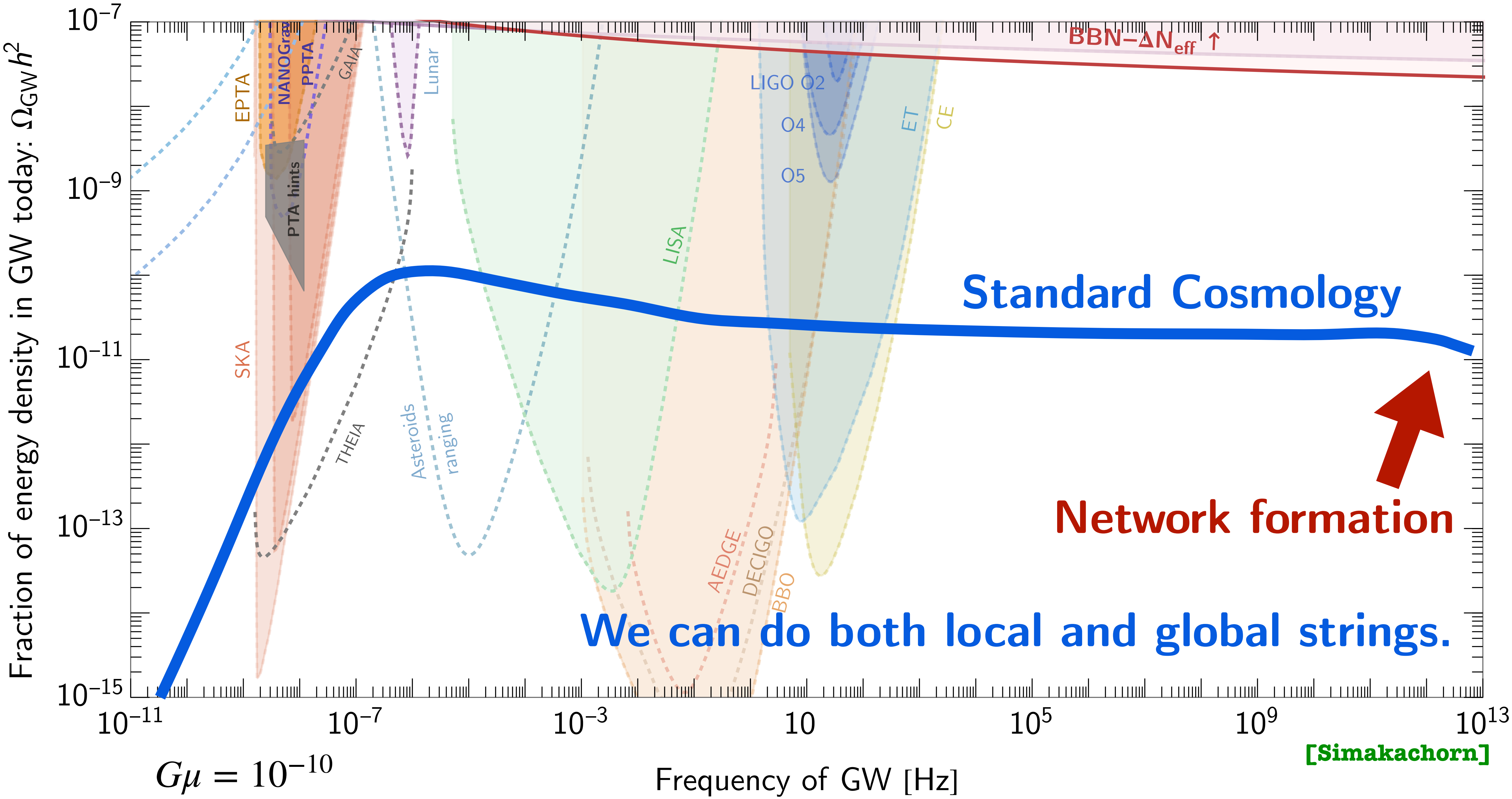
lead to particle emission

Short-lasting vs long-lasting primordial sources.





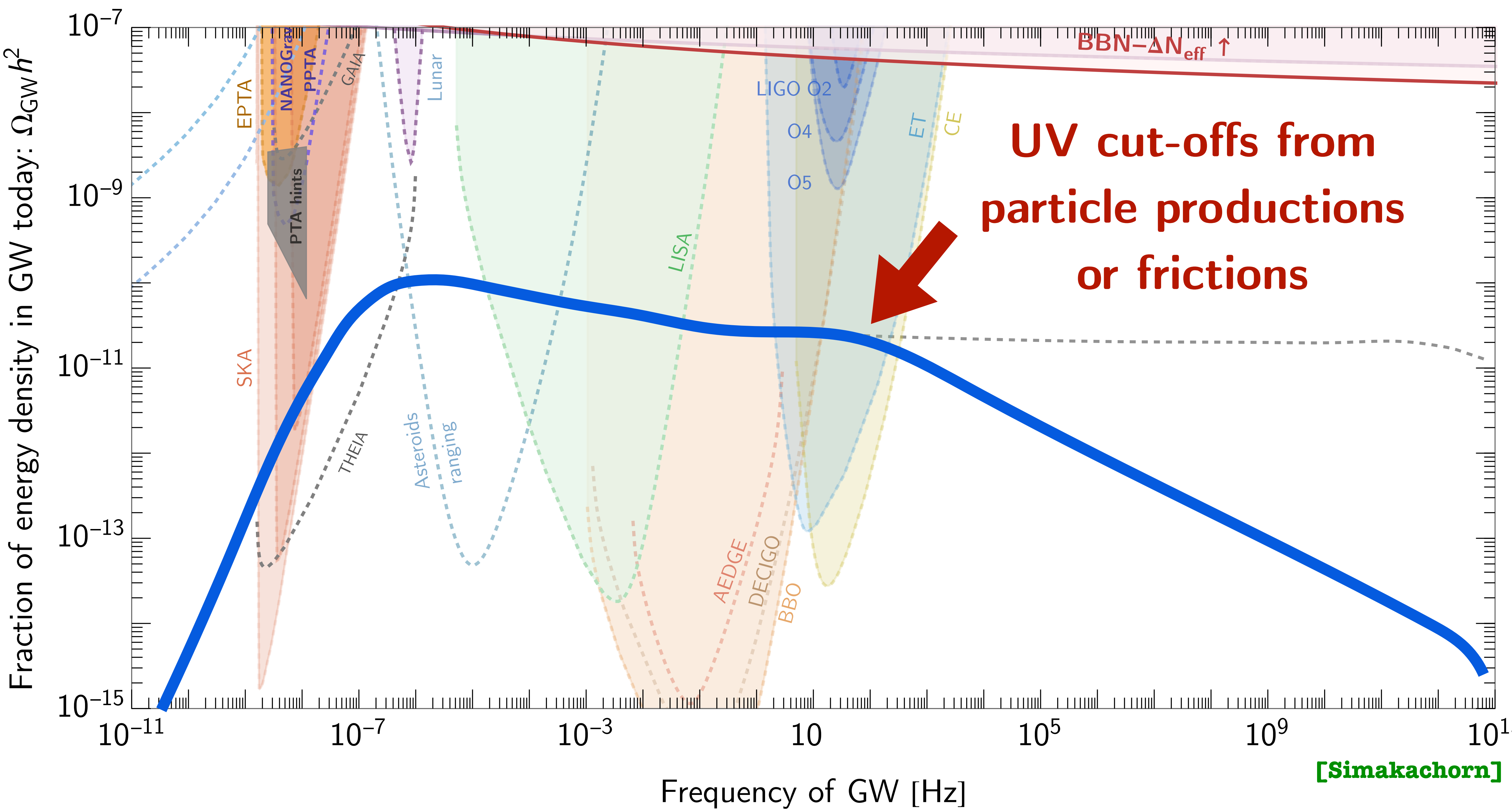
Gravitational Waves from cosmic strings.



$G\mu = 10^{-10}$

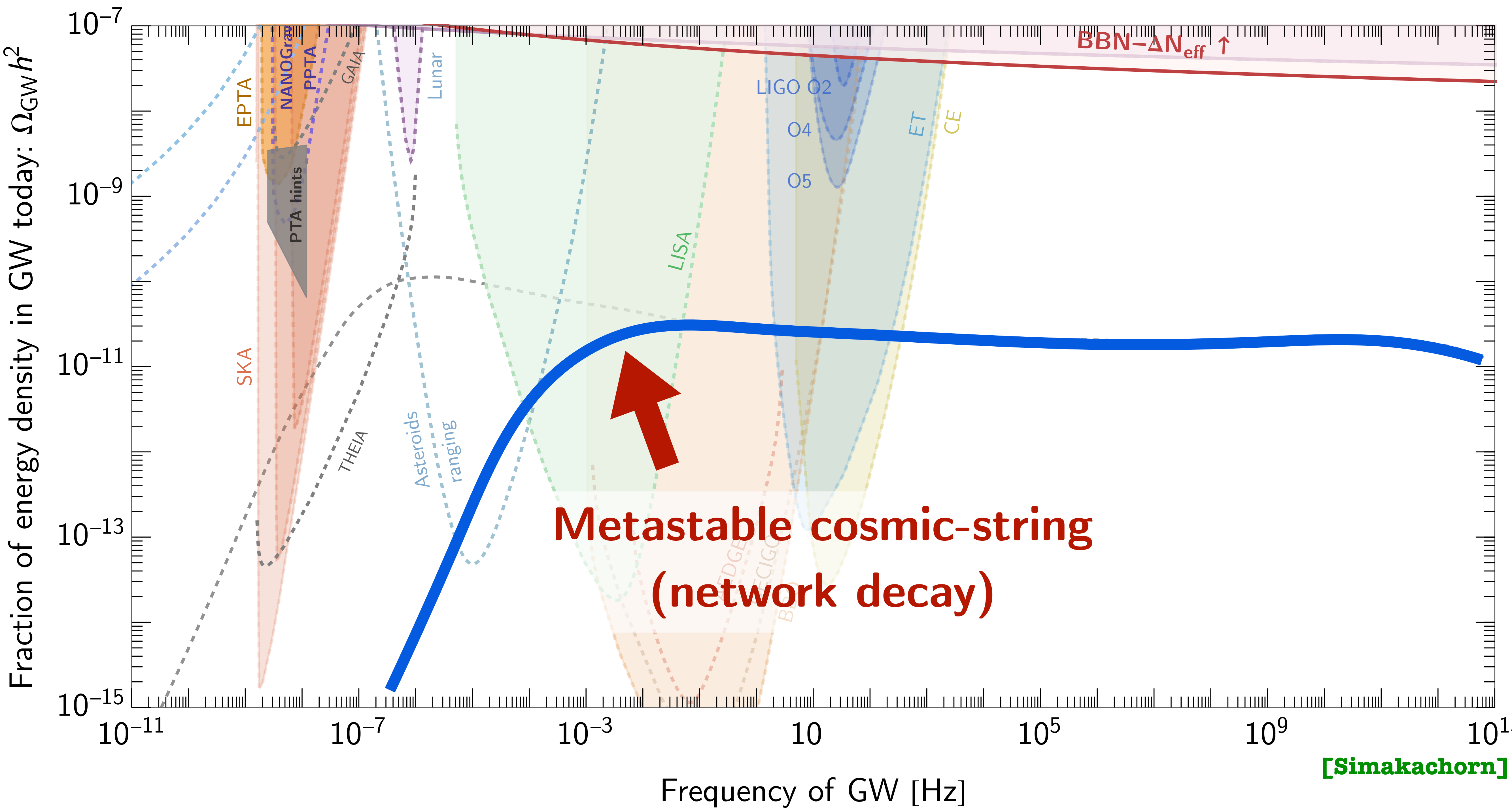
[Simakachorn]

Gravitational Waves from cosmic strings.



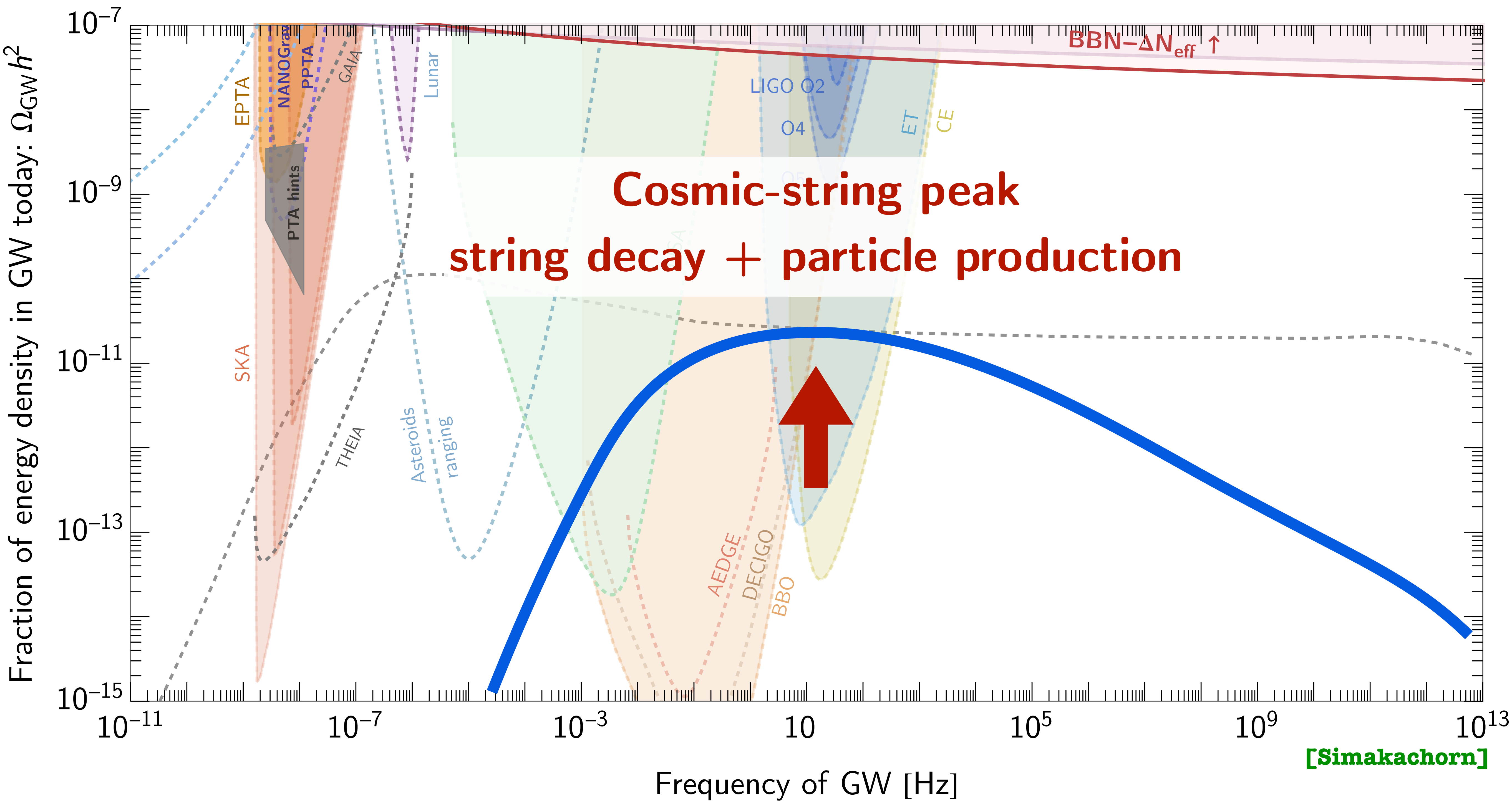
[Simakachorn]

Gravitational Waves from cosmic strings.



[Simakachorn]

Gravitational Waves from cosmic strings.



[Simakachorn]

LOCAL STRINGS
vs
GLOBAL STRINGS.

See comparison in Appendix F of [1912.02569] .

Loops from global strings : short-lived

Loops from local strings : long-lived.

—> different GW spectra in both frequency and amplitude.

LOCAL STRINGS vs GLOBAL STRINGS.

Global strings: no gauge field, instead massless Goldstone mode, with logarithmically-divergent gradient energy.

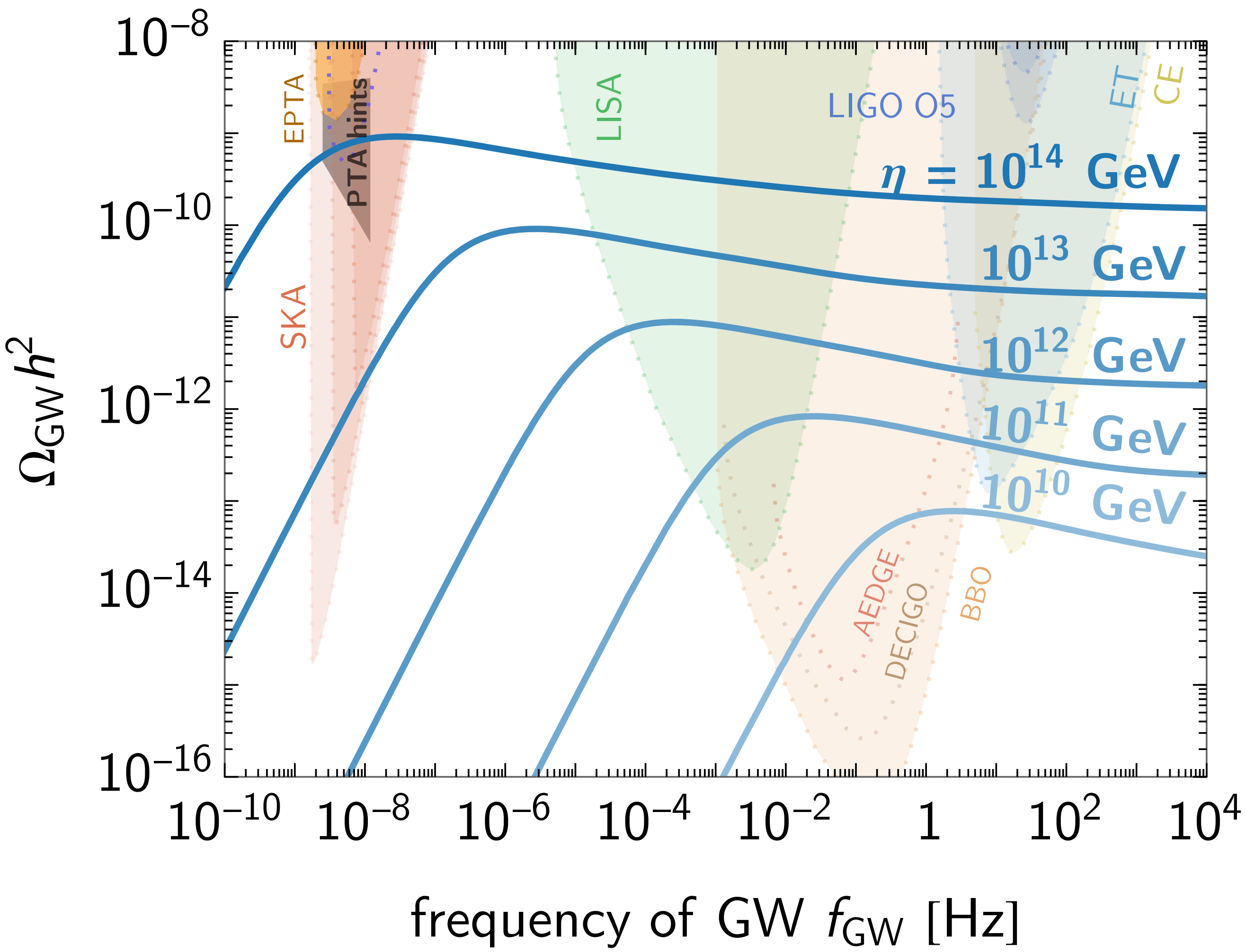
Loops quickly decay into axion particles.

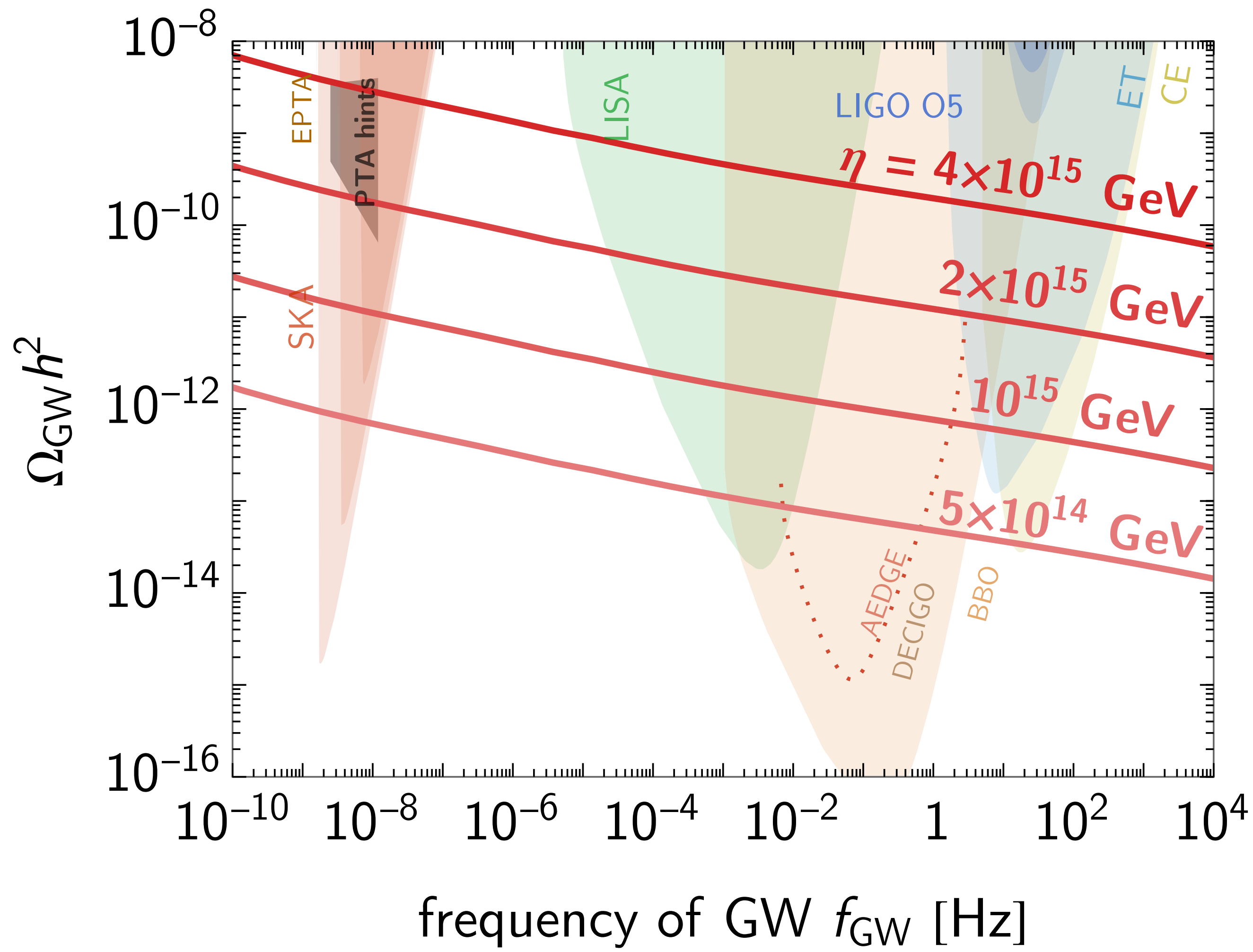
GW are mainly produced at the time of the loop production.

$$\Omega_{\rm GW}^{\rm local} \simeq \Omega_r \frac{\eta}{M_{\rm pl}}, \qquad \Omega_{\rm GW}^{\rm global} \simeq \Omega_r \left(\frac{\eta}{M_{\rm pl}}\right)^4 \log^3(\eta t_i).$$

$\eta = f_a$

LOCAL STRINGS **versus** GLOBAL STRINGS (m_a=0)





spectral shape changes with η

local loops live longer before decaying (& lifetime depends on η)

global loops decay fast.

To reach the same amplitude as the local strings, global strings need a larger η since GW production is not the leading energy loss.

Part 2:
GW signatures of BSM degrees of freedom.

[Servant+Simakachorn to appear]

Energy density in gravitational waves.

$$\rho_{\mathrm{GW,0}} = \rho_{\mathrm{GW,e}} \left(\frac{a}{a_0} \right)^4$$

$\rho_{\mathrm{GW,0}}$: **today**

$\rho_{\mathrm{GW,e}}$: **at the time of GW emission**

$\left(\frac{a}{a_0} \right)^4$: **massless —> redshifts as radiation**

GWs from cosmic strings and from inflation track the total energy density of the universe.

$$\boxed{\rho_{\mathrm{GW}}(a_{\mathrm{emit}}) \propto \rho_{\mathrm{tot}}(a_{\mathrm{emit}})}$$

$\rho_{\mathrm{tot}}(a_{\mathrm{emit}})$: **affected by BSM physics**

Effect of new BSM particles on primordial GW backgrounds.

Let us assume GWs are emitted in a radiation-dominated era

$$\rho_{\mathrm{GW},0} = \rho_{\mathrm{GW,e}} \left(\frac{a}{a_0}\right)^4$$

$$= \left(\frac{\rho_{\mathrm{GW,e}}}{\rho_{\mathrm{tot,e}}}\right) \rho_{\mathrm{rad,e}} \left(\frac{a}{a_0}\right)^4$$

Independent of the particle content for inflation & cosmic strings sources (refers to $\left(\frac{\rho_{\mathrm{GW,e}}}{\rho_{\mathrm{tot,e}}}\right)$)

Encodes information on the particle content (refers to $\rho_{\mathrm{rad,e}}$)

Effect of BSM physics on primordial GW backgrounds.

The total radiation energy density is modified by the existence of additional degrees of freedom

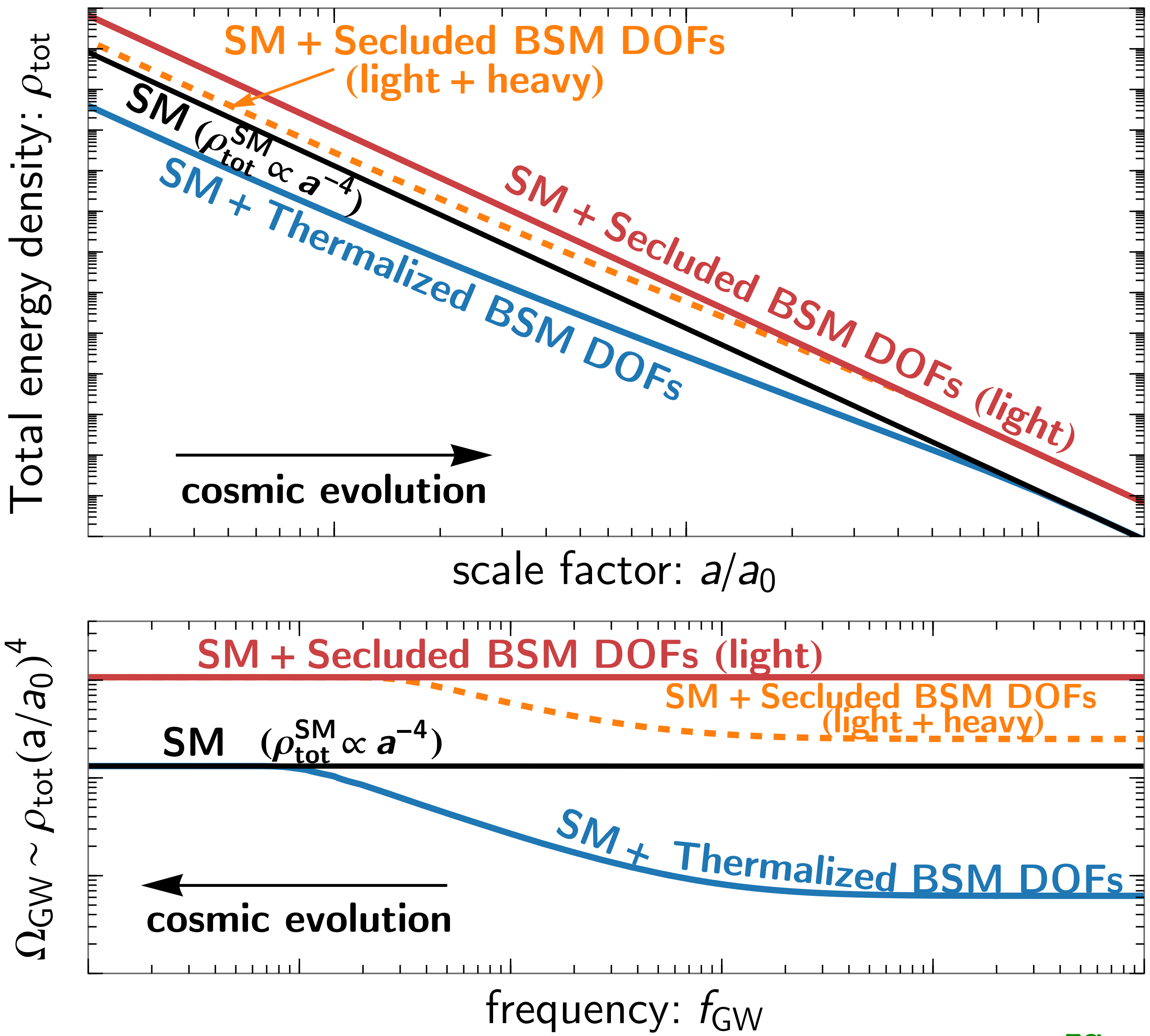
$$\rho_{\rm rad} = \frac{\pi^2}{30} T^4 \times \begin{cases} \left[g_*^{\rm SM}(T) + g_*^{\rm BSM}(T)\right] & \text{Thermalised BSM} \\ \left[g_*^{\rm SM}(T) + g_*^{\rm BSM}(T)\frac{T_D^4}{T^4}\right] & \text{Non-thermalised BSM} \end{cases}$$

T_D : Temperature of secluded sector

Using entropy conservation $T \sim \frac{1}{g_{*s}^{1/3}} \frac{1}{a}$ **and assuming** $g_* = g_{*s}$

$$\rho_{\rm rad} \sim \rho_{\rm rad,0} \left(\frac{a_0}{a}\right)^4 \times \begin{cases} \left[g_*^{\rm SM}(T) + g_*^{\rm BSM}(T)\right]^{-1/3} & \text{decreases wrt SM if } g_*^{\rm BSM} \neq 0 \\ \frac{1}{g_*^{\rm SM}(T)^{4/3}} \left[g_*^{\rm SM}(T) + g_*^{\rm BSM}(T_D)\frac{T_D^4}{T^4}\right] & \text{increases wrt SM if } g_*^{\rm BSM} \neq 0 \end{cases}$$

Effect of new particles on primordial inflationary GW background.

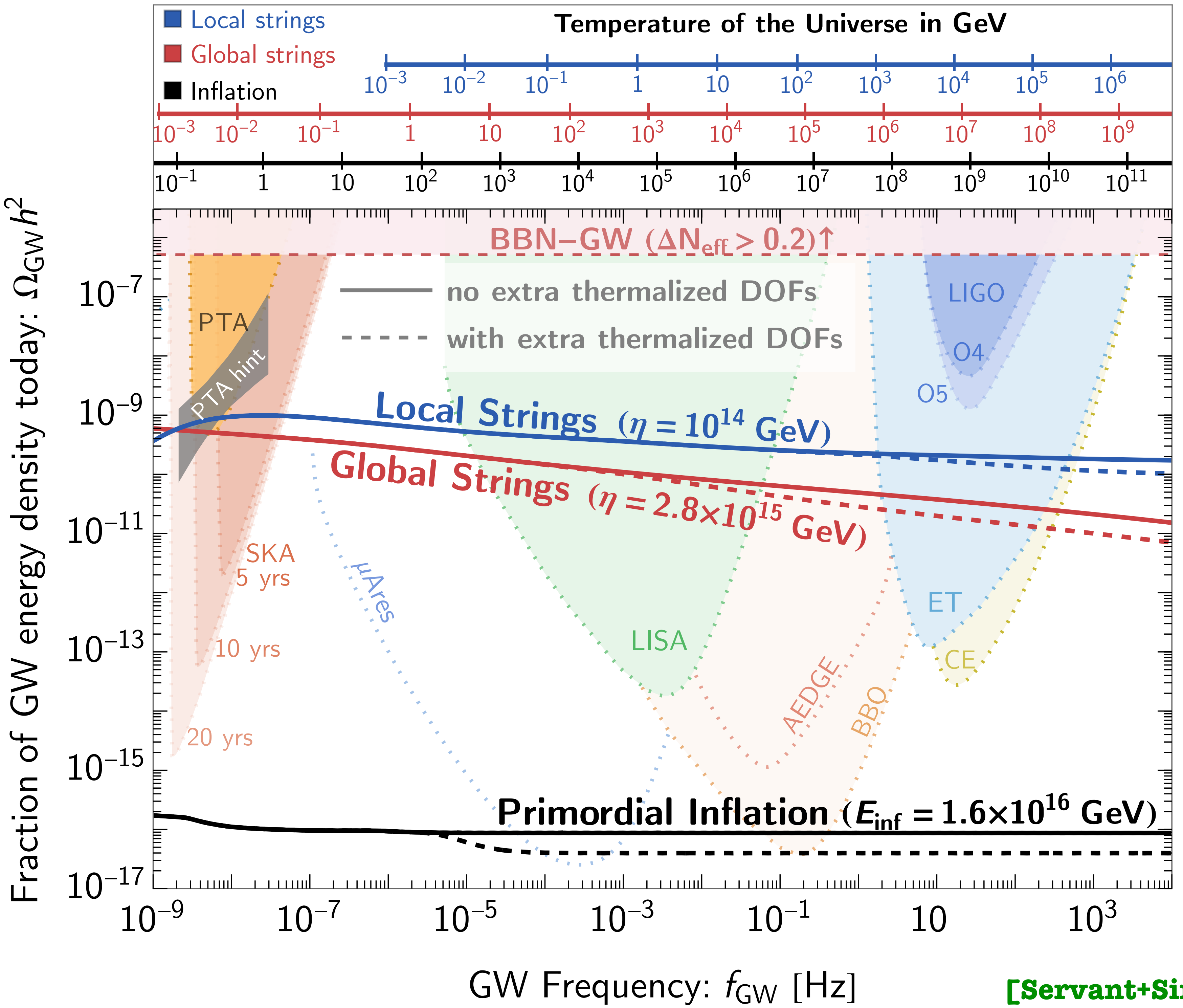


(Schematic, not to scale)

[Servant+Simakachorn to appear]

Effect of new particles coupled to the SM on primordial GW backgrounds.

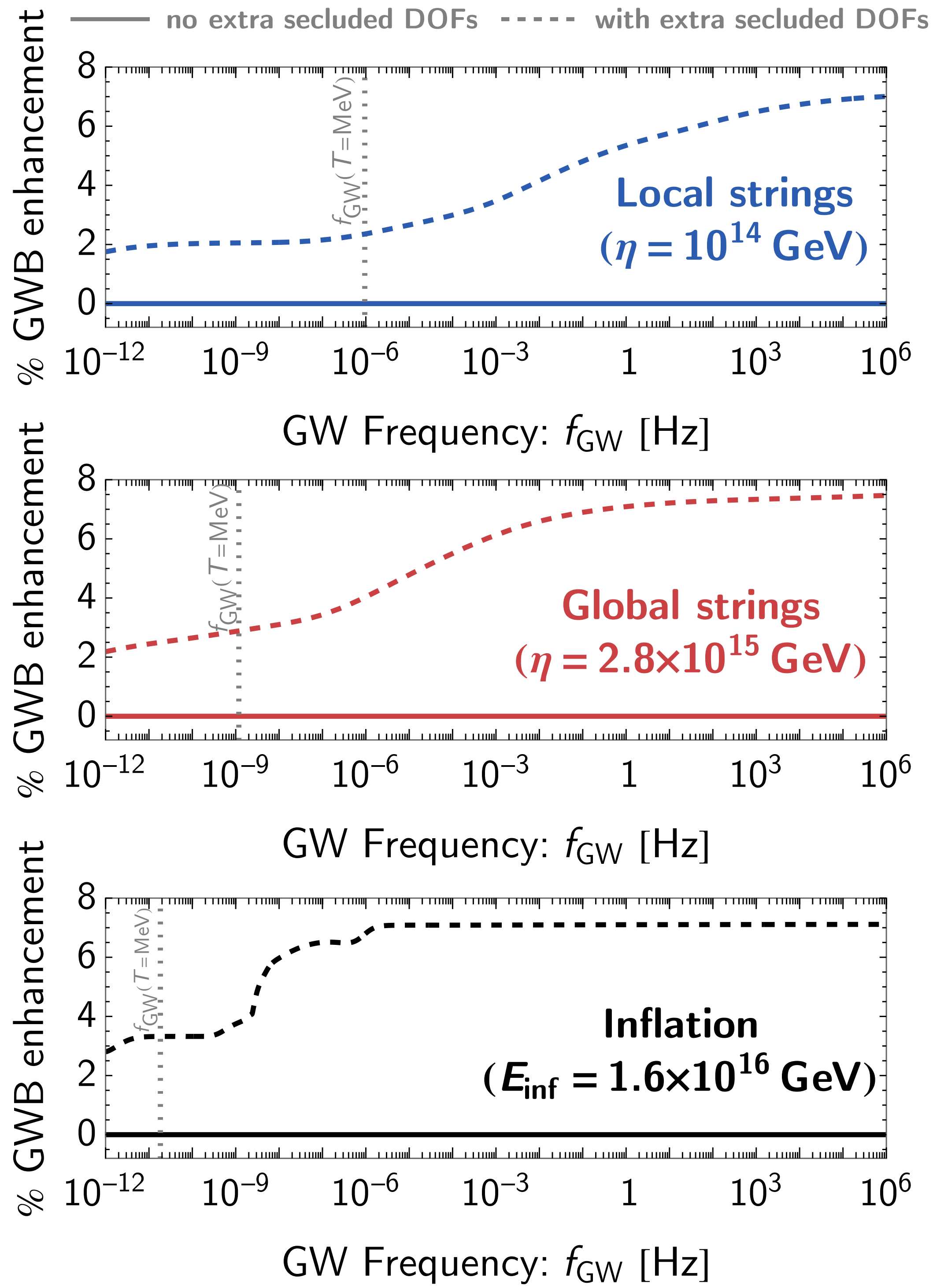
Effect of extra thermalized DOFs $m = 1 \text{ TeV and } \Delta g_*^{\text{th}} = 10^3$



[Servant+Simakachorn to appear]

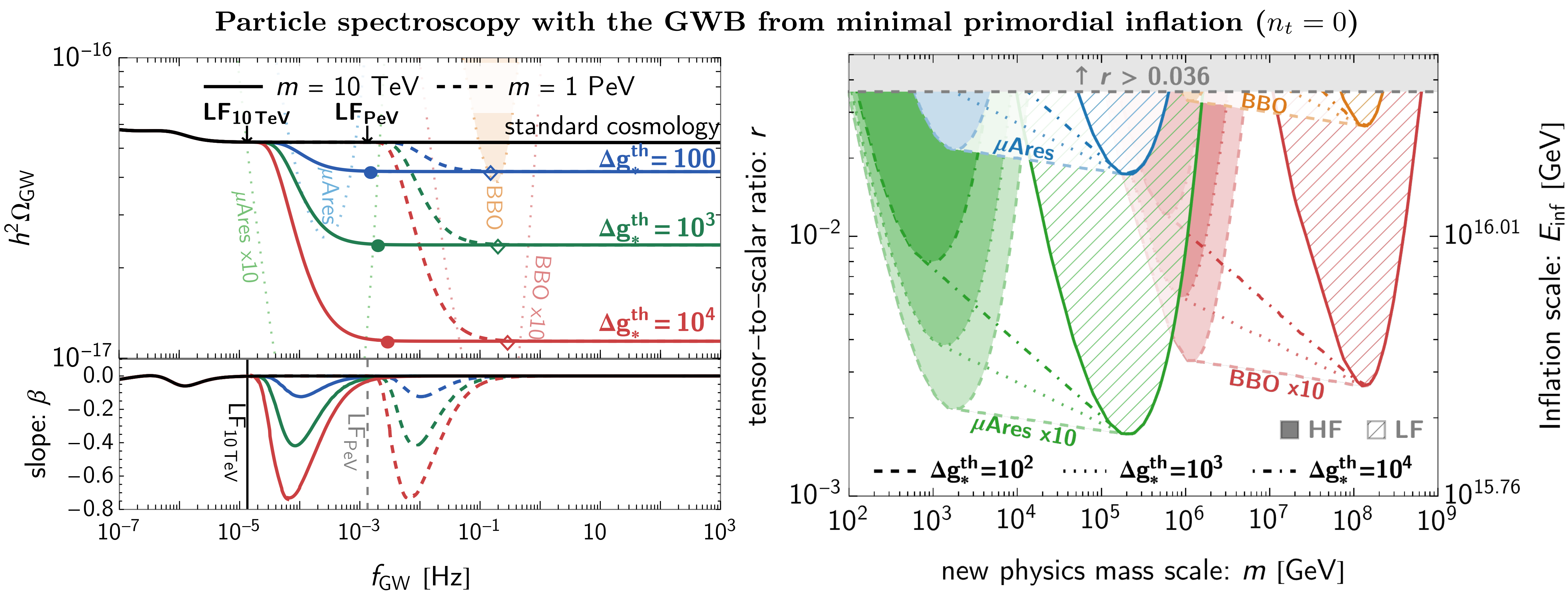
Effect of new particles secluded from the SM on primordial GW backgrounds.

(Maximal) Effect of extra secluded DOFs (saturating the ΔN_{eff} bound)



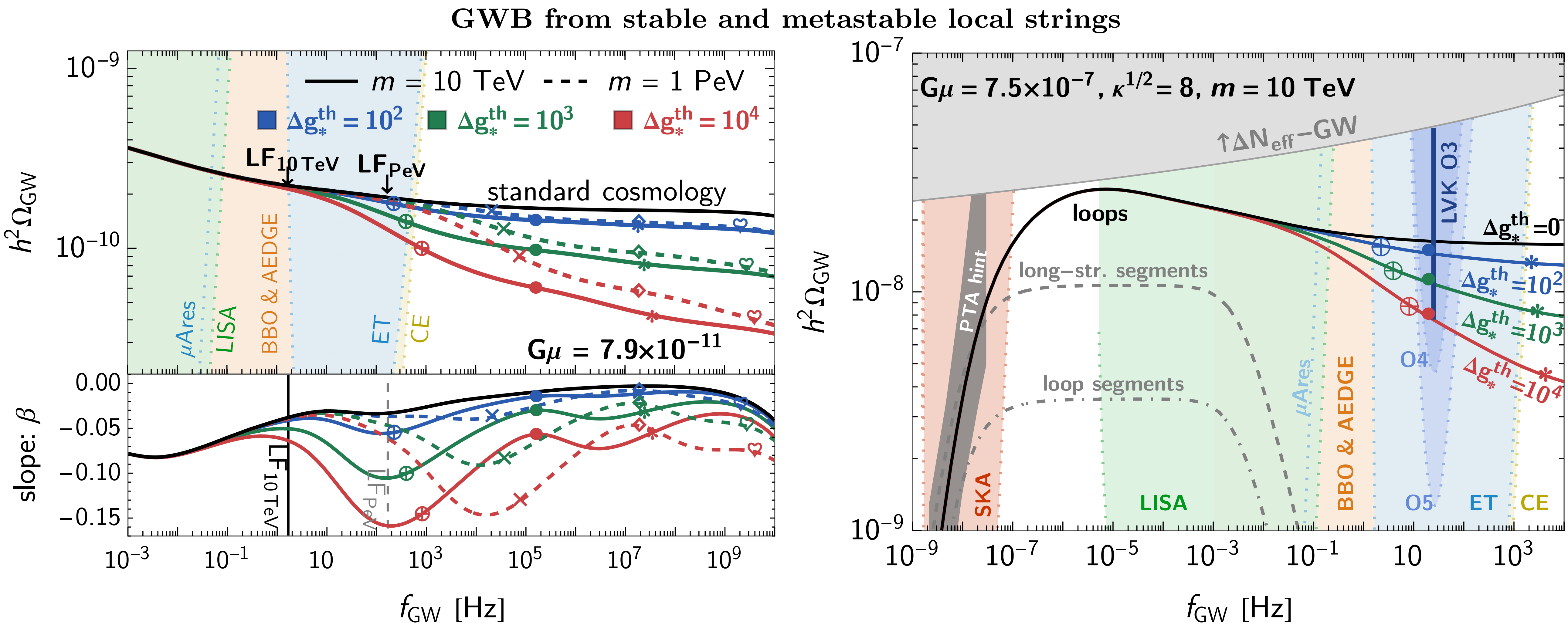
[Servant+Simakachorn to appear]

Effect of new particles coupled to the SM on the primordial inflationary GW background.



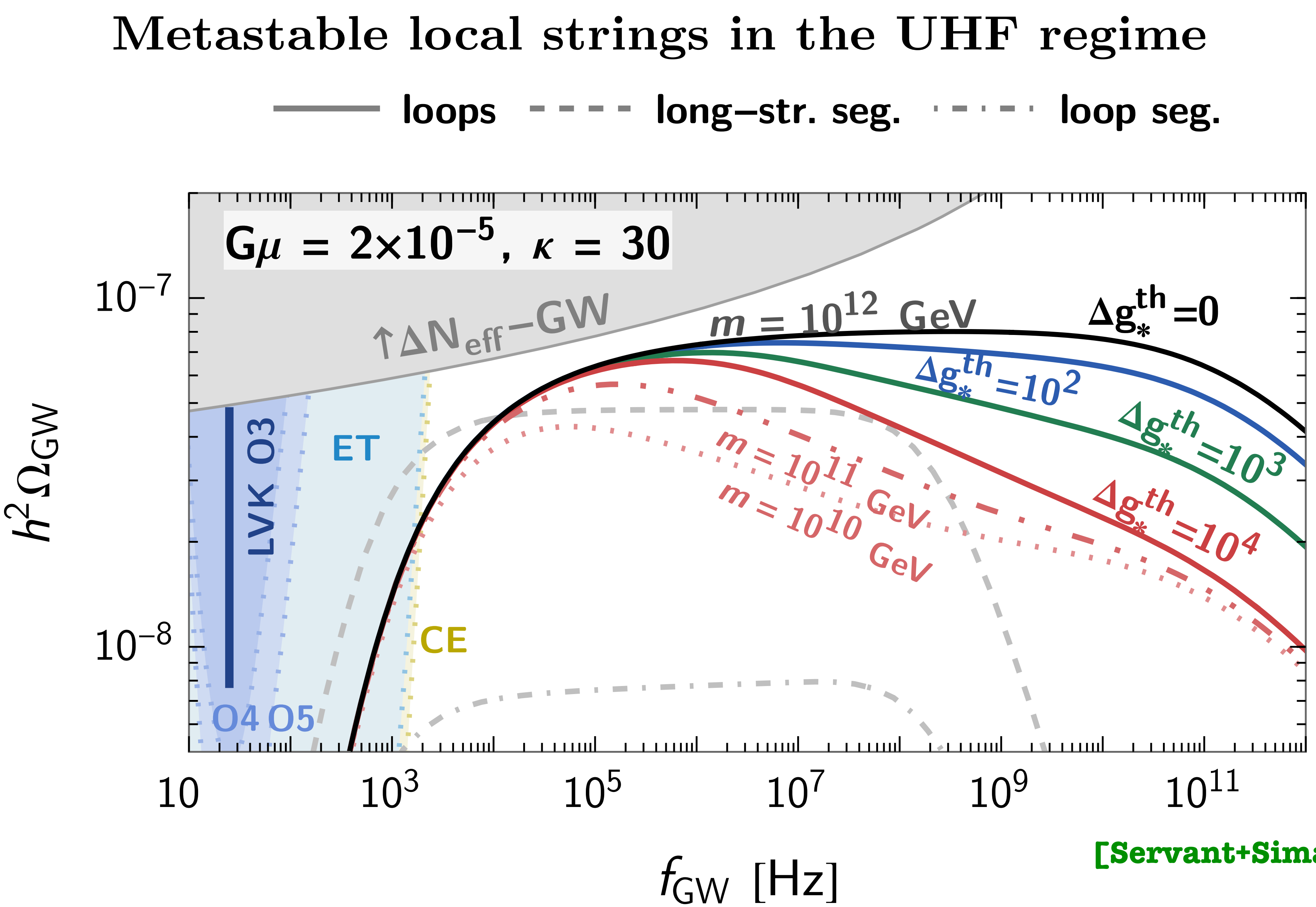
[Servant+Simakachorn to appear]

Effect of new particles coupled to the SM on the primordial GW background from local cosmic strings.



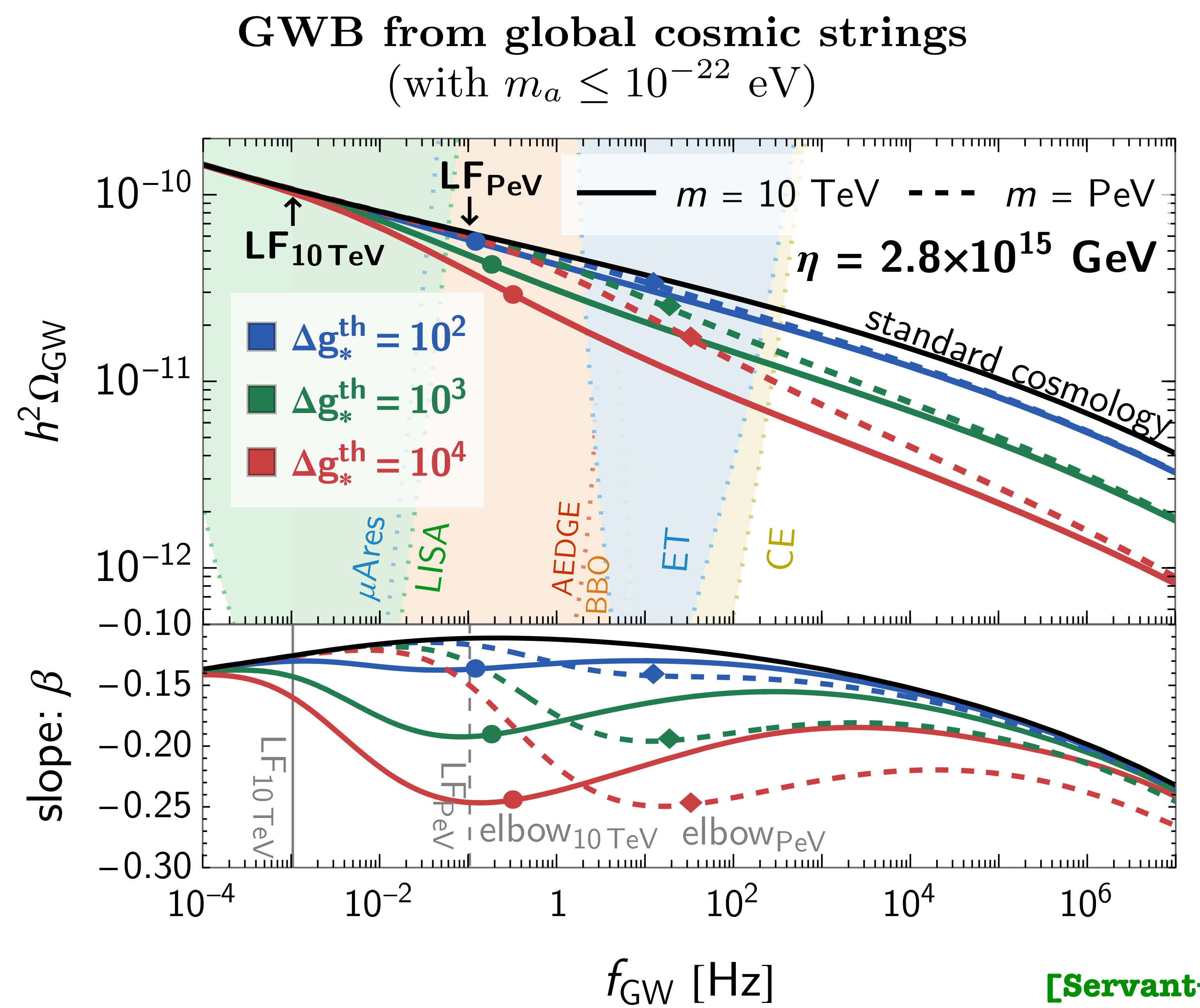
[Servant+Simakachorn to appear]

Effect of new particles coupled to the SM on the HIGH-FREQUENCY primordial GW background from local cosmic strings.



[Servant+Simakachorn to appear]

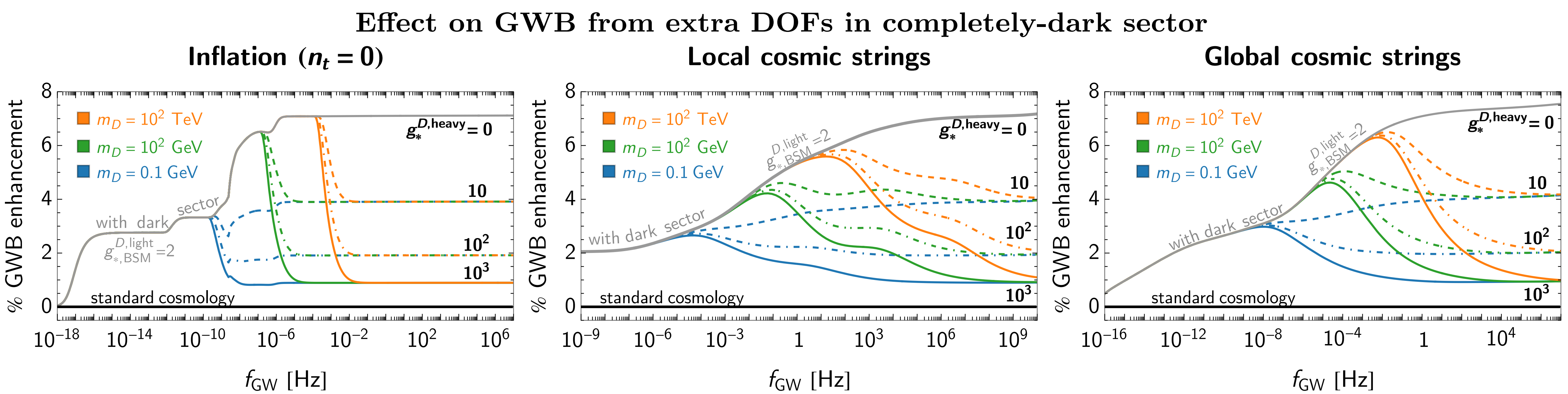
Effect of new particles coupled to the SM on the primordial GW background from global cosmic strings.



[Servant+Simakachorn to appear]

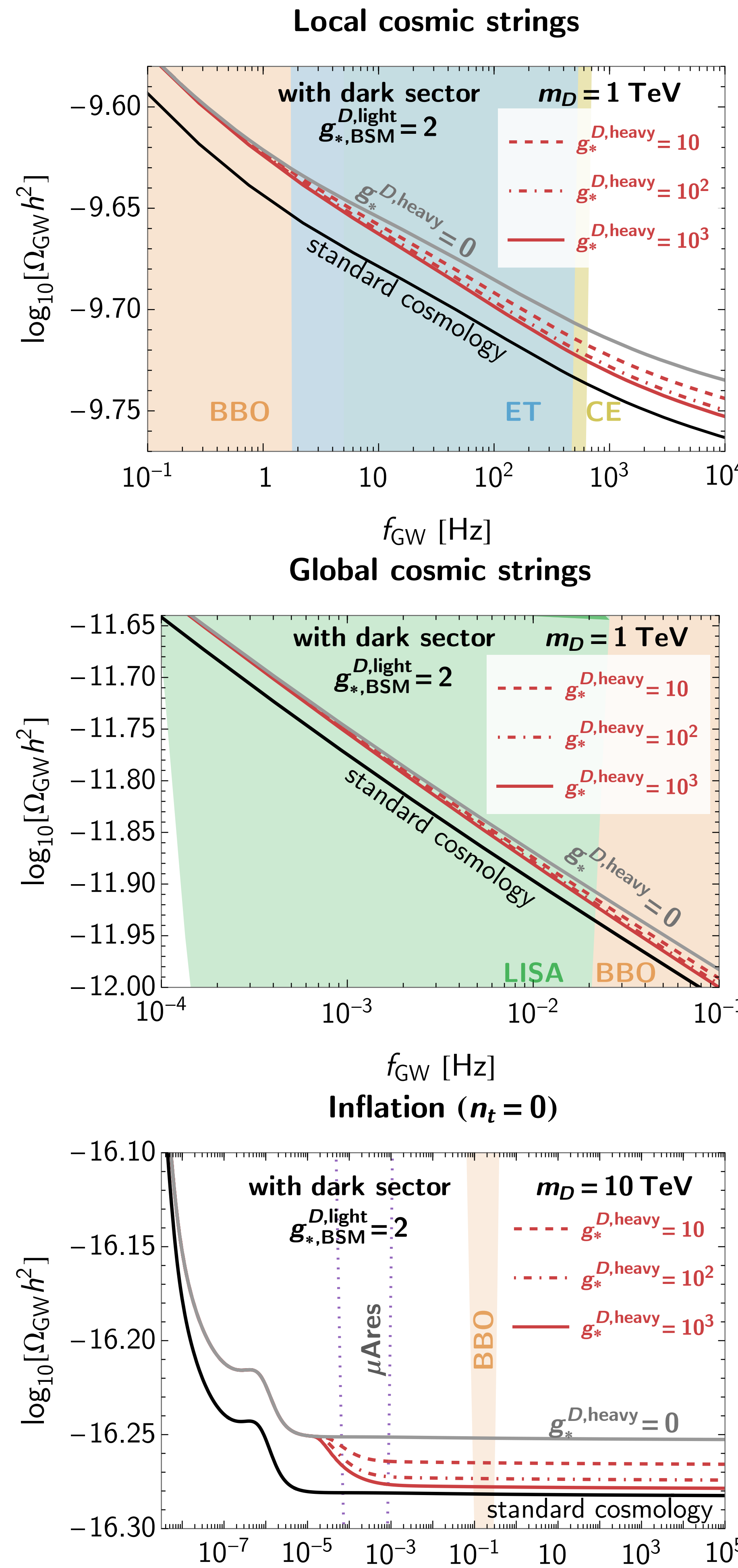
Effect of new particles secluded from the SM on primordial GW backgrounds.

Including the possibility that particles from the secluded sector are in thermal equilibrium within the dark sector and decay into the light secluded particles



[Servant+Simakachorn to appear]

Effect of new particles secluded from the SM on primordial GW backgrounds.

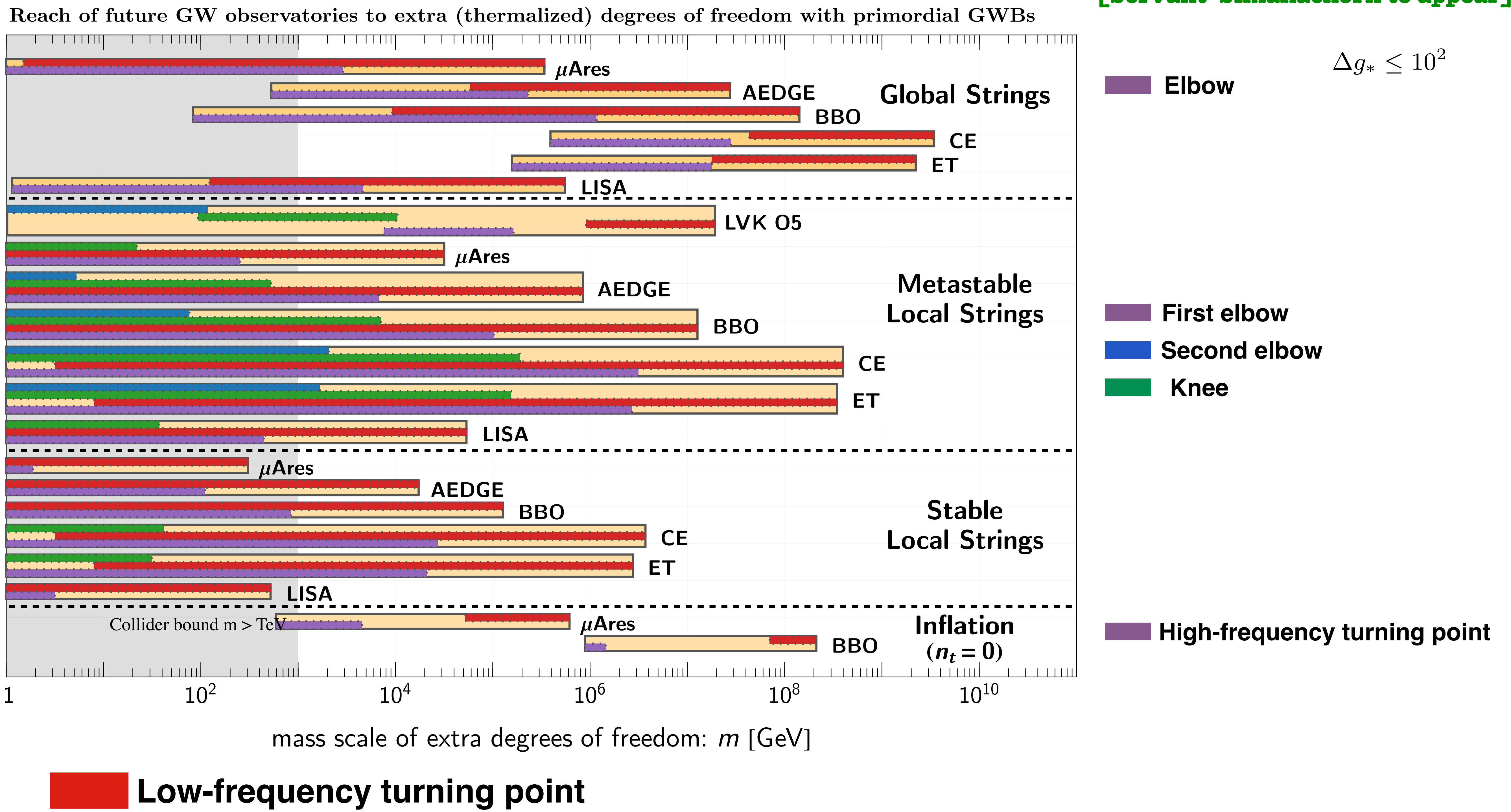


f_{GW} [Hz]

[Servant+Simakachorn to appear]

Reaches of future GW observatories.

[Servant+Simakachorn to appear]



For the completely dark DOFs of mass m, one can replace m by εm ($\epsilon \equiv T_D/T$) on the horizontal axis and remove the collider bound.

Part 3:
GW signatures of axion cosmology.

Axions

Among the most hunted particles.

Ubiquitous in many extensions of the Standard Model

Axion could arise either as a higher dimensional gauge field, or as a Pseudo- Nambu Goldstone boson (PNGB) from spontaneous breaking of global symmetry which is not exact but broken weakly.

I will assume the second possibility as a simple benchmark. Important for cosmology: Axion is accompanied by its partner, the radial mode of a complex scalar field.

Axion mass is proportional to this breaking.

Very general context.

Historically: QCD axion. Strong dynamics from QCD provides breaking of symmetry.

Axion-like-particles (ALPs): other axions whose mass is not affected by QCD. They get their mass from other sources.

Axion-Like-Particles (ALPs).

Consider complex scalar field

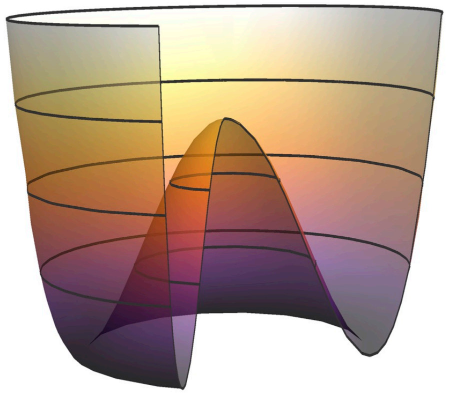
$$\Phi = \phi e^{i\theta}$$

charged under anomalous U(1) global symmmetry (Peccei-Quinn symmetry)

Spontaneously broken at scale f_a

$$V(\varphi) = \lambda \left(|\varphi|^2 - \frac{f_a^2}{2} \right)^2$$

$$\langle \boldsymbol{\varphi} \rangle = f_a/\sqrt{2}$$



Axion as Goldstone boson

$$\theta \rightarrow \theta + \text{const.}$$

$$\boxed{\theta = a / f_a}$$

ALPs.

Non-perturbative effects at energy Λ_b << f_a break the shift symmetry and generate a potential/mass for the axion

$$V = m_a^2(T) f_a^2 \left[1 - \cos(\theta)\right]$$

$$m_a = \Lambda_b^2 / f_a$$

QCD axion

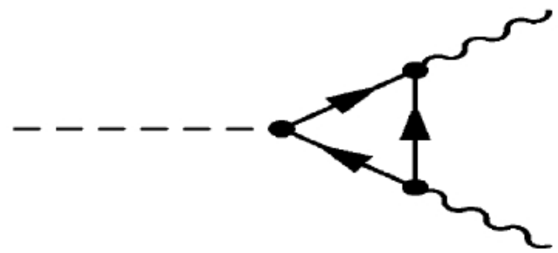
$$m_a^2 f_a^2 \approx (76\ \mathrm{MeV})^4$$

Generic ALP

m_a and f_a : free parameters

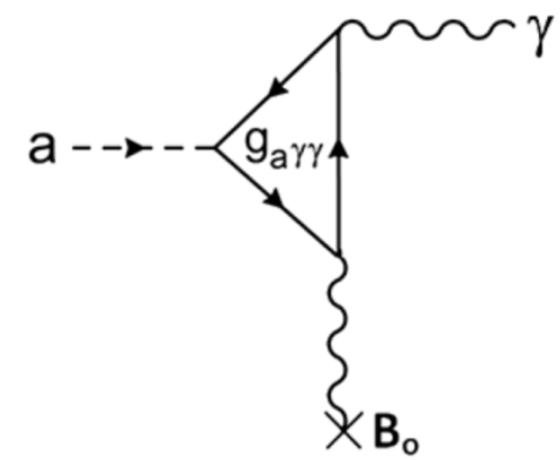
The hunt for axions.

Mainly through Axion-photon coupling



$$\frac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

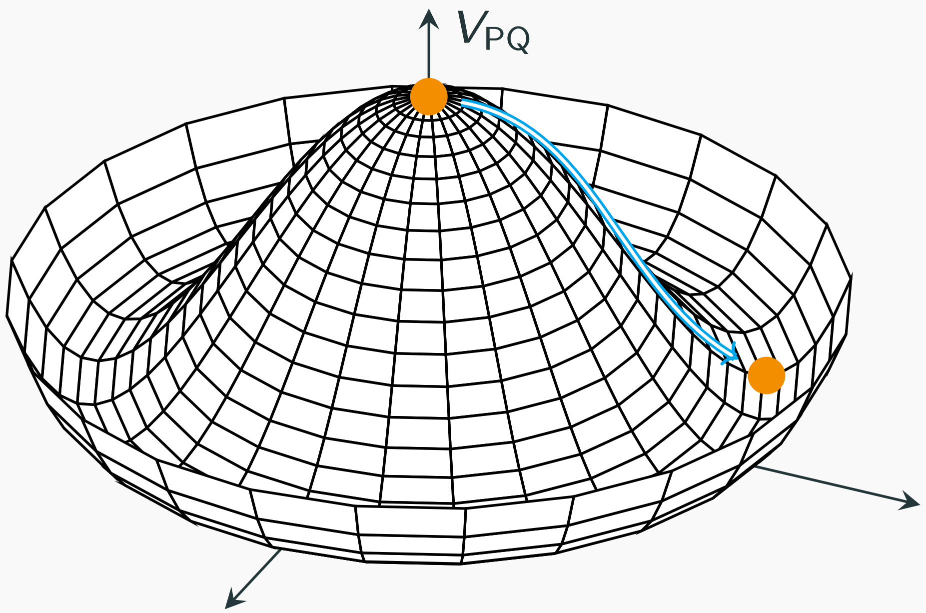
In a background magnetic field:
axion<->photon conversion



If long-lived: Dark Matter candidate

Lifetime depends on axion-photon coupling.
However, relic abundance only depends on f_a

Pre- and post-inflationary scenarios.



Post-inflationary scenario

- Different initial angle in each Hubble patch.
- Inhomogeneous including topological defects.

Pre-inflationary scenario

- Random initial angle in the observable universe.
- Initially homogeneous w/o topological defects.

↓

GLOBAL (axionic) COSMIC STRINGS

↓

primordial GW bgd

Gravitational-wave constraints on axion parameter space from axionic strings.

GW from axion-related defects.

Some references

GW from GLOBAL Cosmic Strings:

Chang & Cui, [1910.04781], [2106.09746].

Gouttenoire et al,[1912.02569].

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GW from Domain Walls:

T. Hiramatsu, M. Kawasaki and K. Saikawa, On the estimation of gravitational wave spectrum from cosmic domain walls, JCAP 02 (2014) 031 [1309.5001].

R. Zambujal Ferreira, A. Notari, O. Pujolas & F. Rompineve, High Quality QCD Axion at Gravitational Wave Observatories, Phys. Rev. Lett. 128 (2022) 141101 [2107.07542].

K. Saikawa, Gravitational waves from cosmic domain walls: a mini-review, J. Phys. Conf. Ser. 1586 (2020) 012039.

. R. Z. Ferreira, A. Notari, O. Pujolas and F. Rompineve, Gravitational waves from domain walls in Pulsar Timing Array datasets, JCAP 02 (2023) 001 [2204.04228].

. E. Madge, E. Morgante, C. P. Iba´n˜ez, N. Ramberg and S. Schenk, Primordial gravitational waves in the nano-Hertz regime and PTA data – towards solving the GW inverse problem, 2306.14856.

Temperature-frequency relation.

A loop population produced at temperature T quickly decays into GW of frequency

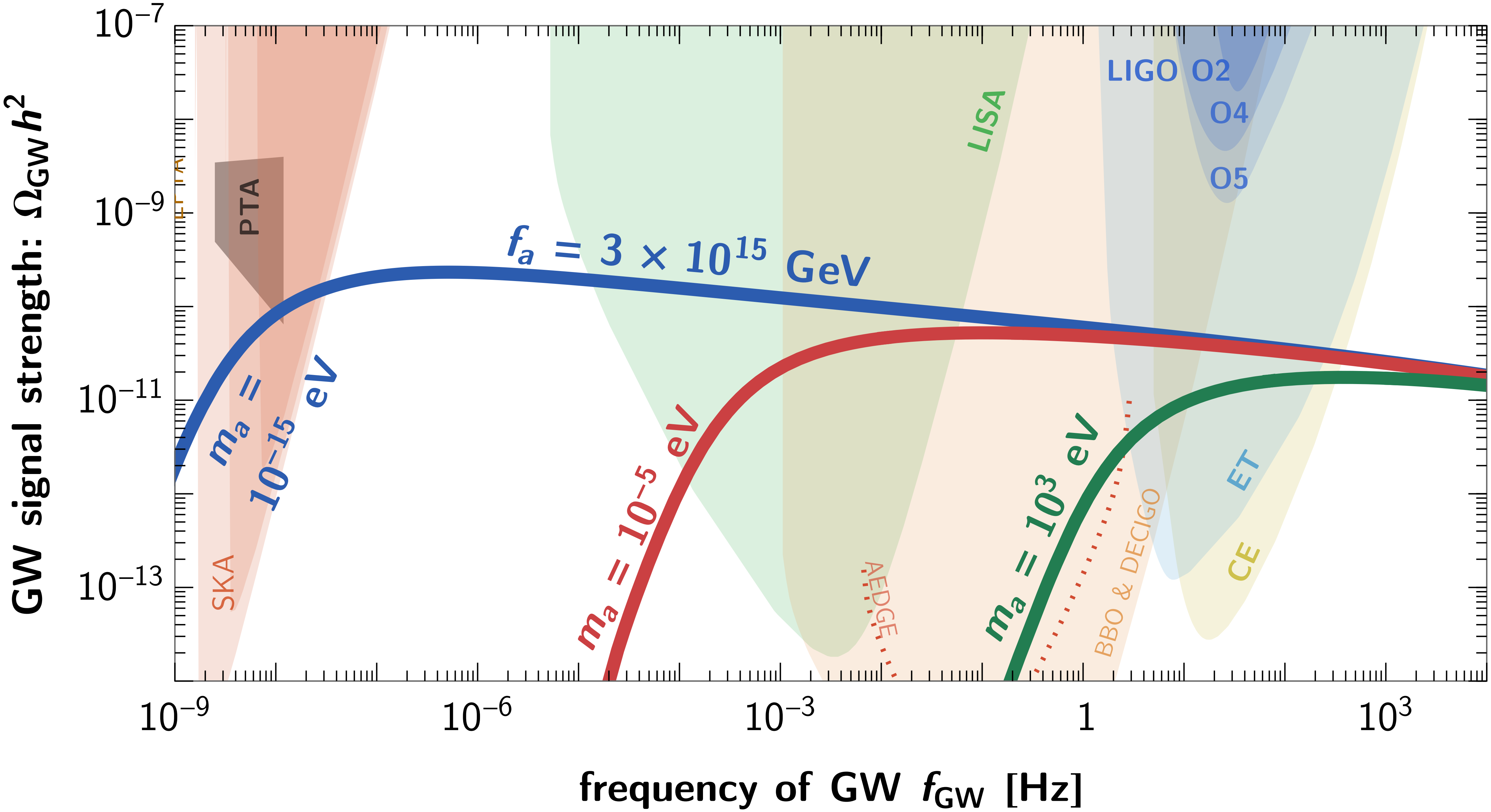
$$f_{\rm GW}^{\rm cs}(T) \simeq 63\ {\rm nHz} \left(\frac{\alpha}{0.1}\right) \left(\frac{T}{10{\rm MeV}}\right) \left[\frac{g_*(T)}{10.75}\right]^{\frac{1}{4}},$$

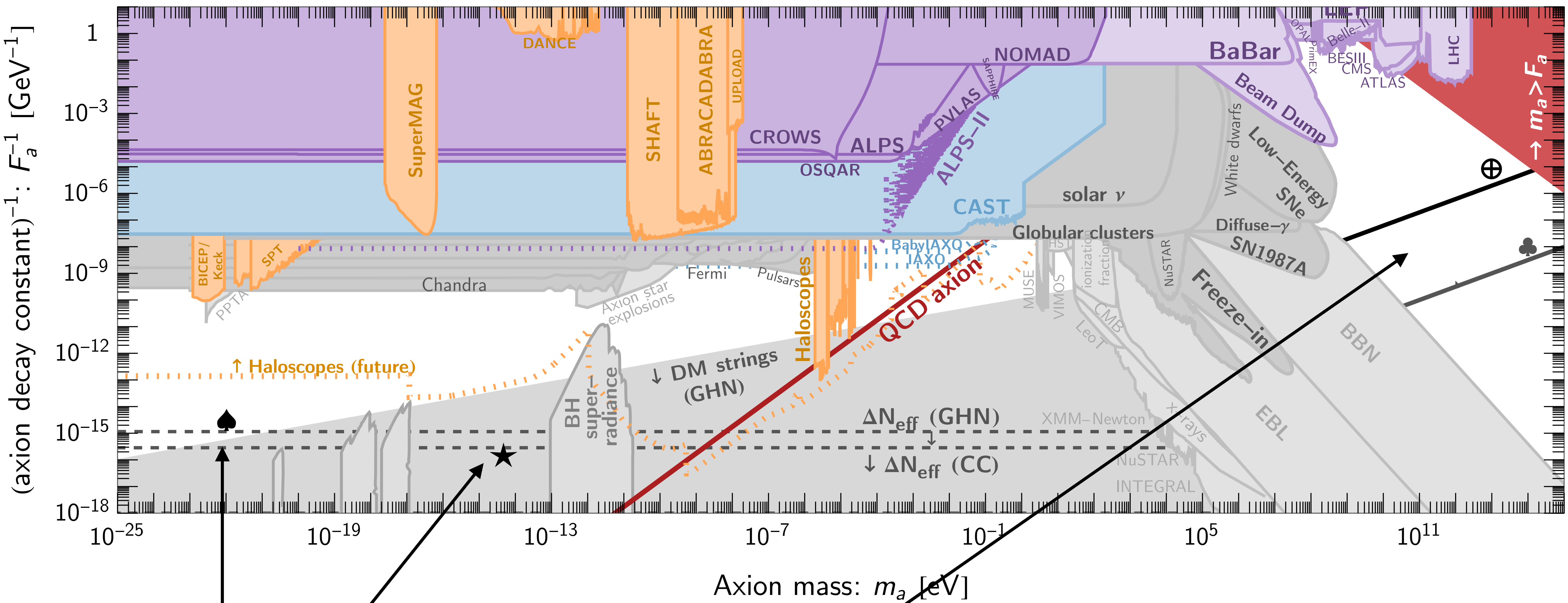
α: typical loop size in units of Hubble horizon

IR cutoff of GW spectrum fixed by axion mass .

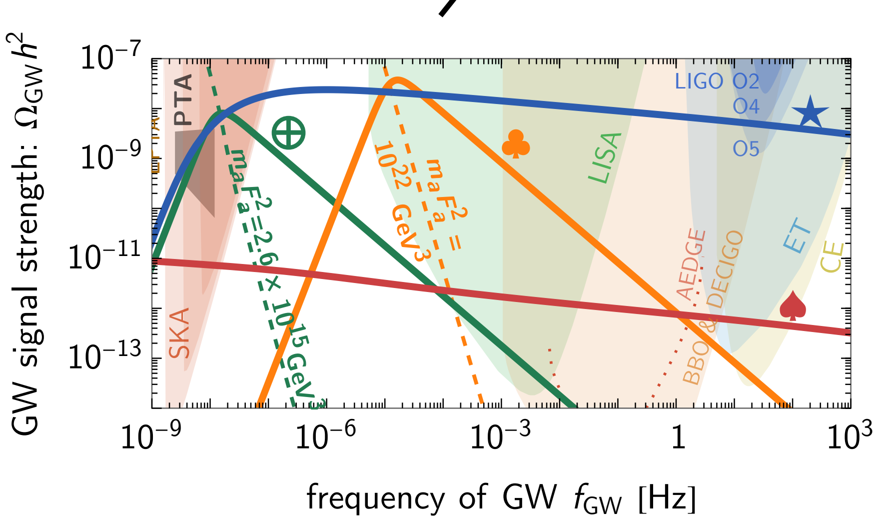
Network decays when H ~ m_a
Corresponding IR cutoff frequency:

$$f_{\rm GW}^{\rm cs}(m_a) \simeq 9.4\ {\rm nHz}\left(\frac{\alpha}{0.1}\right)\left(\frac{m_a}{10^{-15}{\rm eV}}\right)^{\frac{1}{2}}.$$



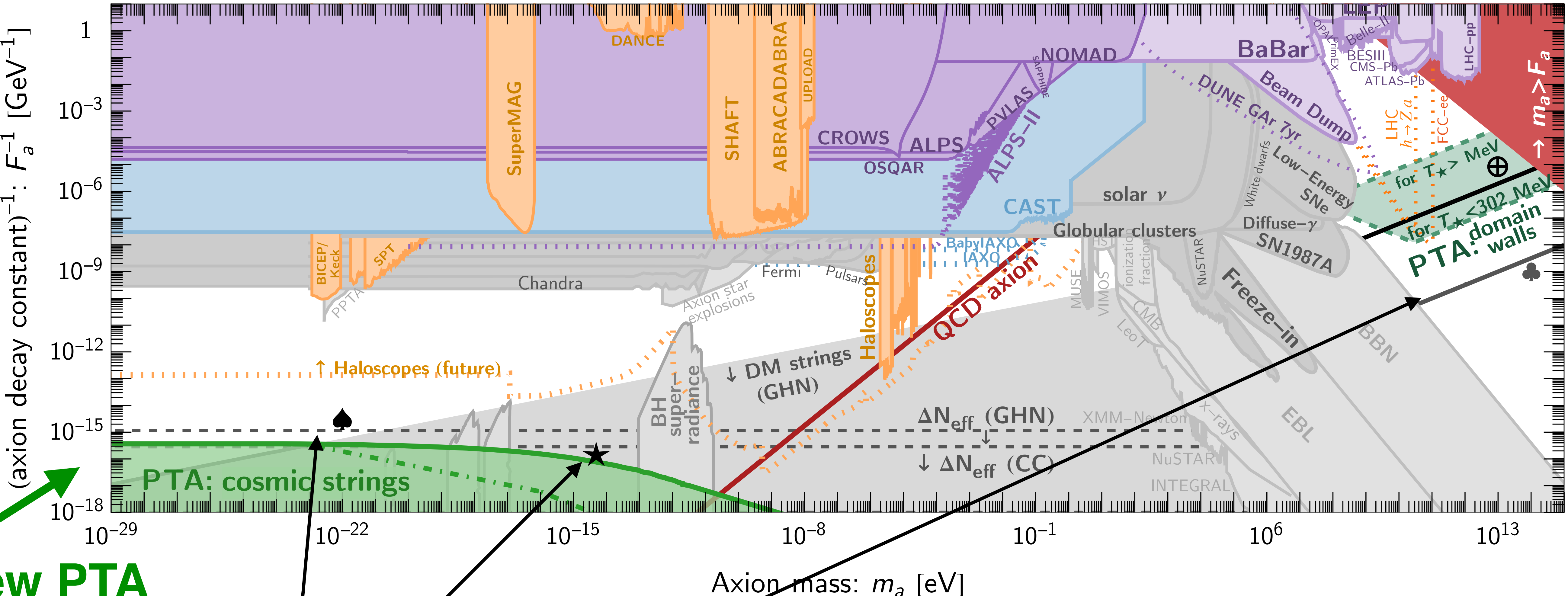


Benchmark spectra



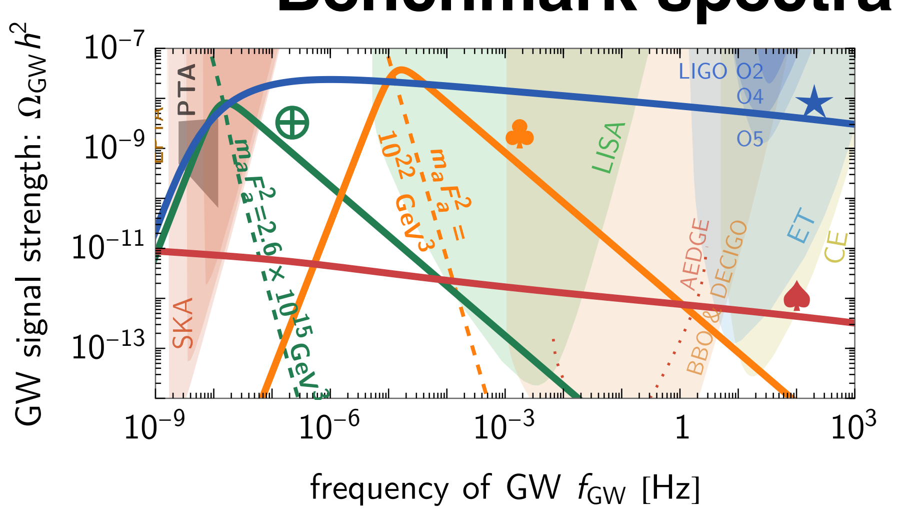
Constraining post-inflationary axions with Pulsar Timing Arrays

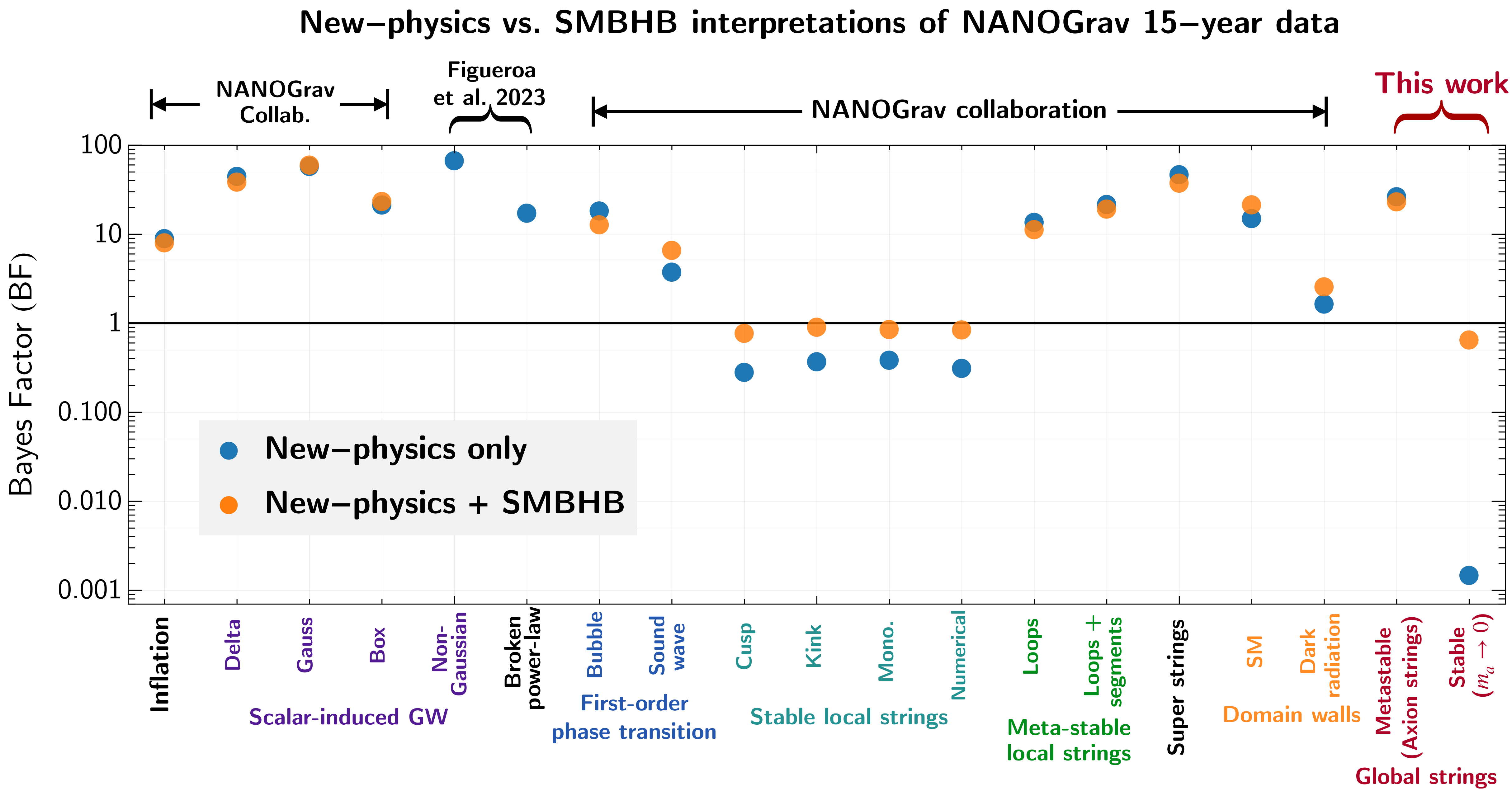
[Servant, Simakachorn, 2307.03121]



New PTA constraint

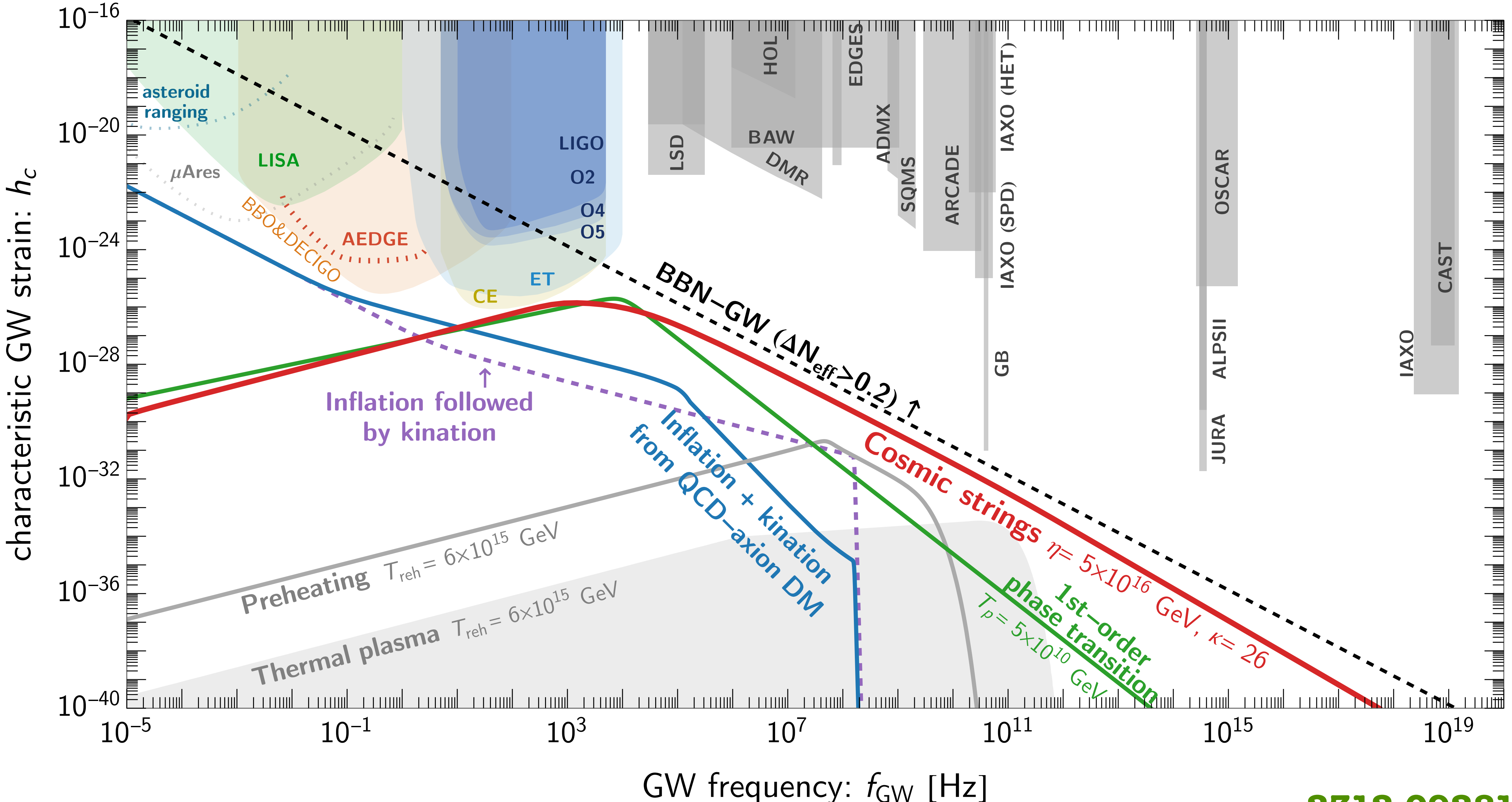
Benchmark spectra





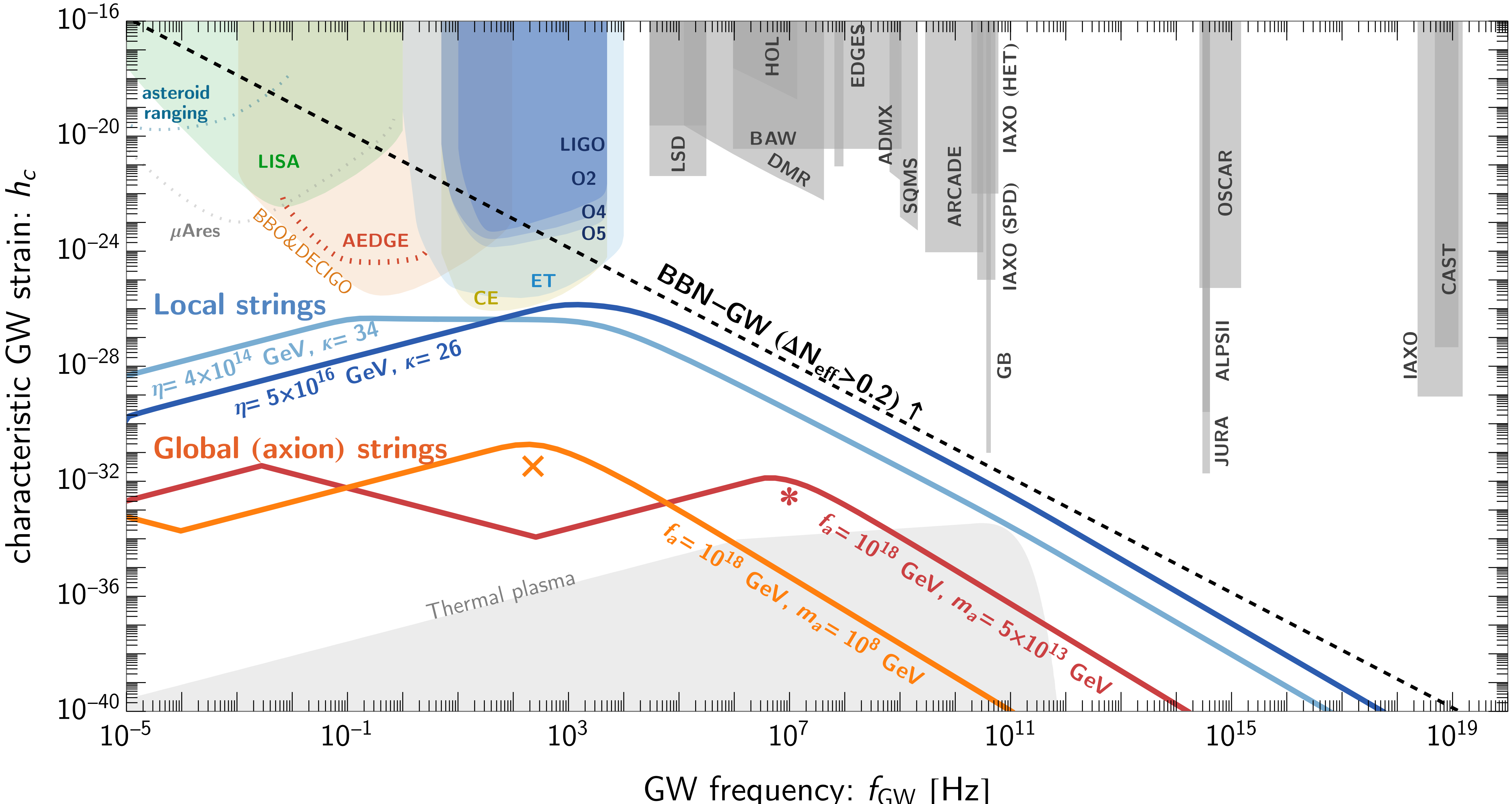
[Servant, Simakachorn, 2307.03121]

Ultra-high frequency primordial GWs .



2312.09281

Ultra-high frequency GWs from local versus global cosmic strings



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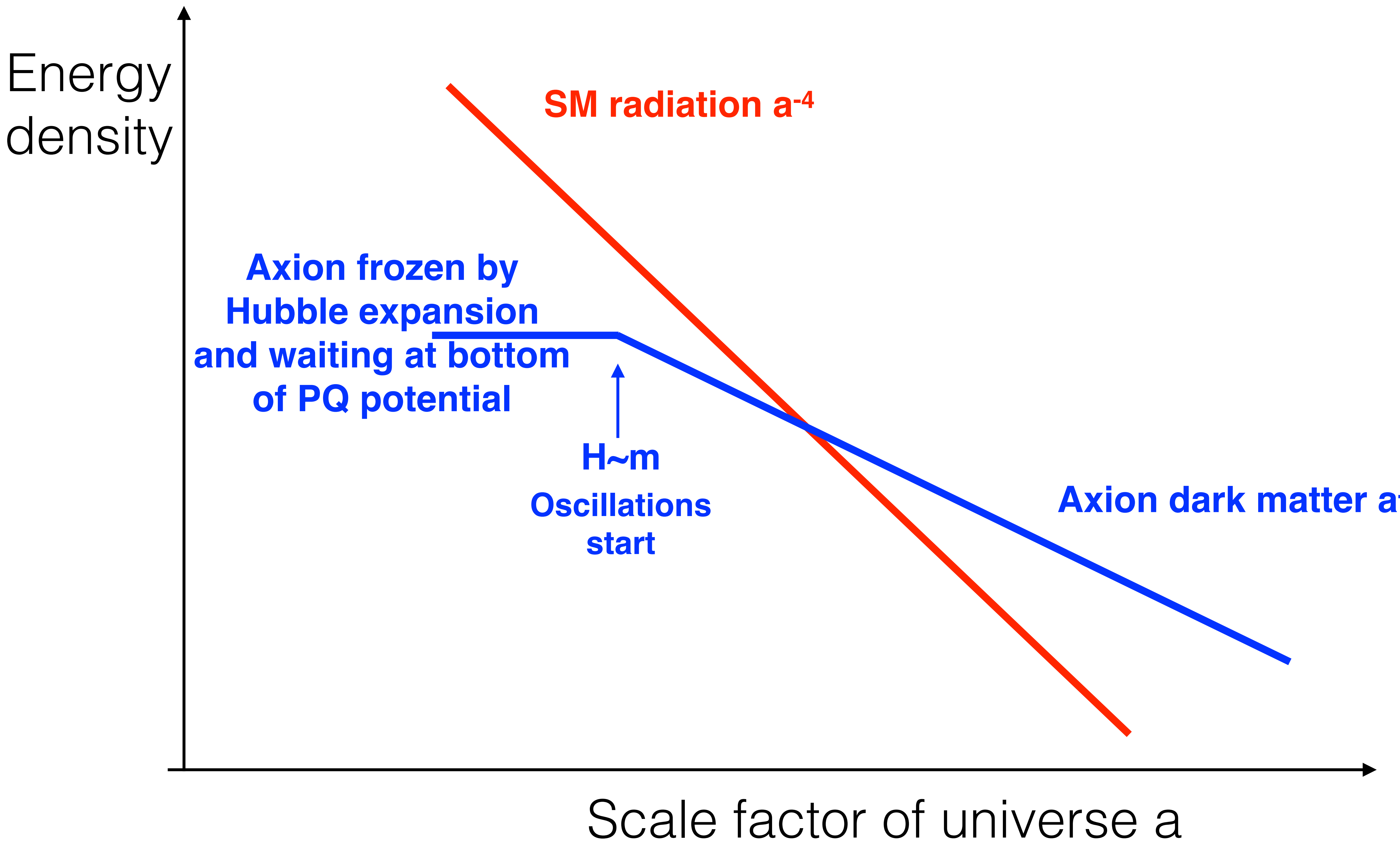
Axion cosmology & its GW signatures.

Usual story.

(Most axion cosmology literature is about the rather late cosmology from moment axion gets a mass)



Axions from the misalignment mechanism.

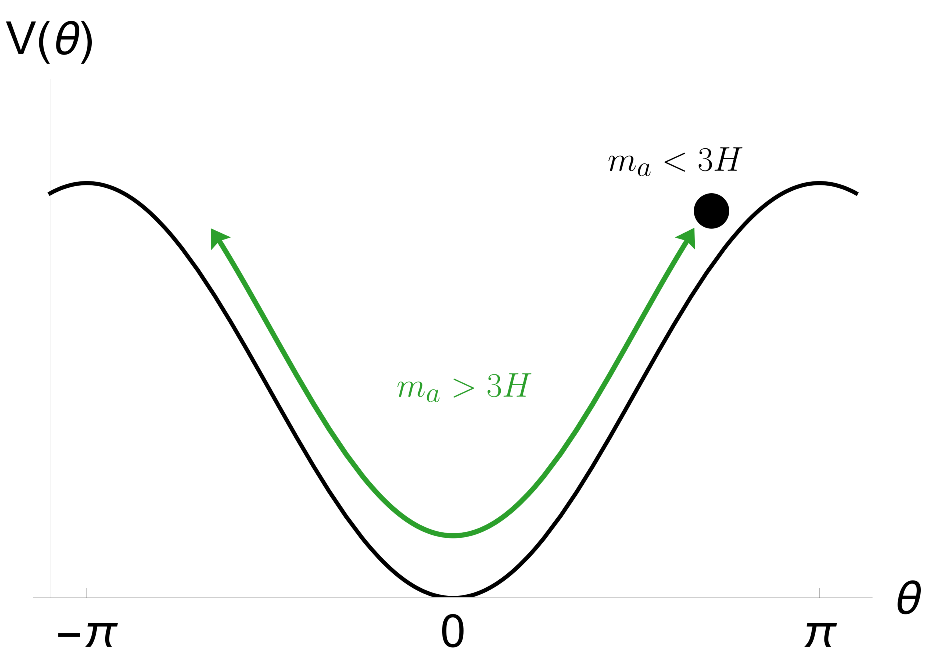
Axion late cosmology

Neglecting fluctuations, the homogeneous zero-mode satisfies

$$\ddot{\Theta} + 3H\dot{\Theta} + m_{\rm a}^2(T)\sin(\Theta) = 0,$$

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

With initial conditions: $\Theta(t_i) = \Theta_i, \quad \dot{\Theta}(t_i) = 0.$ ***standard assumption***



- $m_a \ll 3H \iff \rho_a \propto a^0$ (Frozen)
- $m_a \gg 3H \iff \rho_a \propto a^{-3}$ (Oscillating)

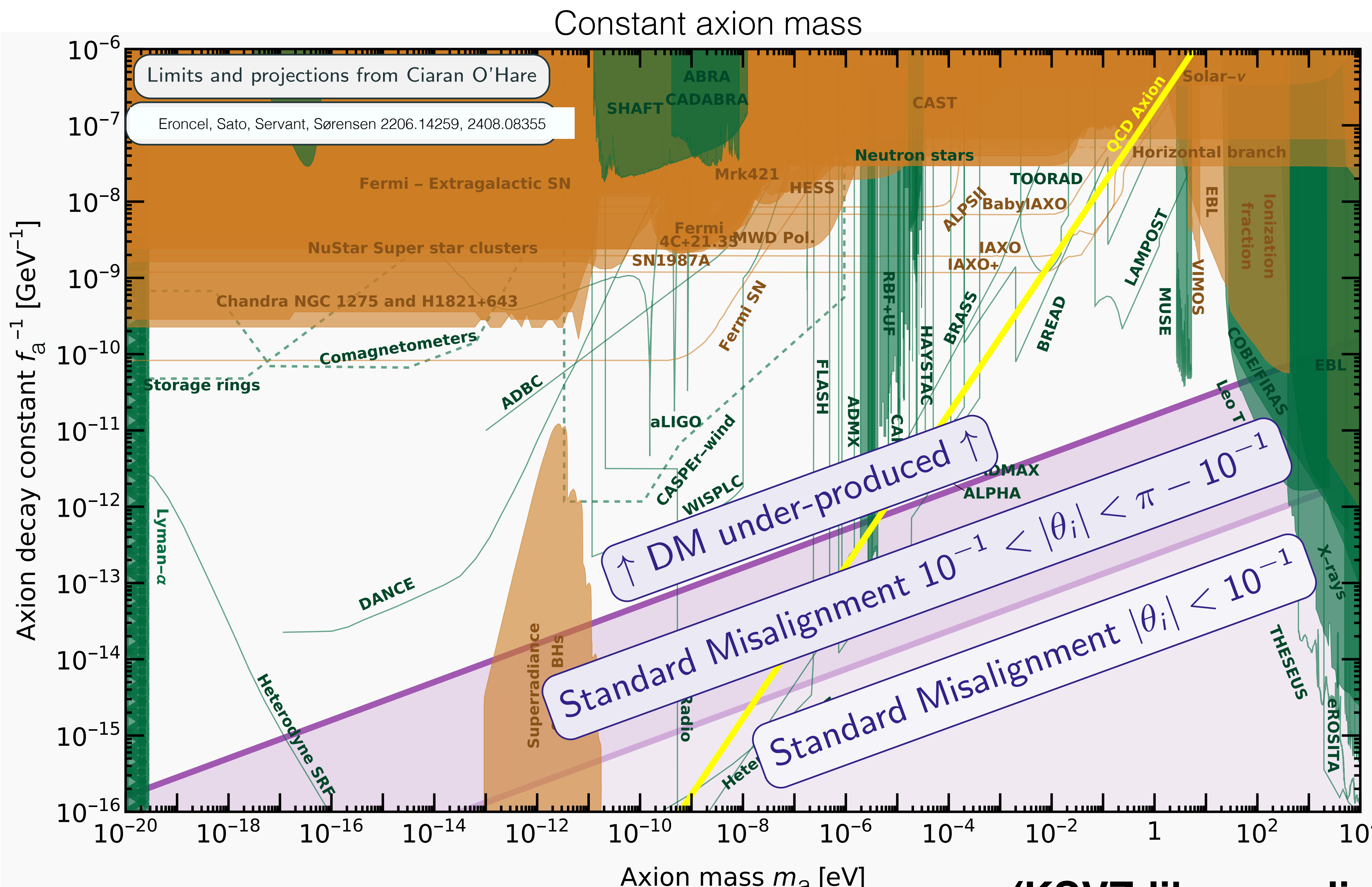
—> standard misalignment mechanism

For $\Theta_i \sim 1$ $\quad \rho_{\rm DM} \sim \rho_{osc}\left(\frac{a_{osc}}{a_0}\right)^3 \sim m_a^2 f_a^2 \left(\frac{T_0}{T_{osc}}\right)^3$

$$T_{osc} \sim \sqrt{m_a M_{Pl}}$$

$\rho_{\rm DM}$ grows with f_a —> Axion Dark Matter overabundance for too large f_a

Conventional misalignment makes too little DM for low f_a .



(KSVZ-like coupling)

$$g_{\theta\gamma} = (\alpha_{\mathbf{em}}/2\pi)(1.92/f)$$

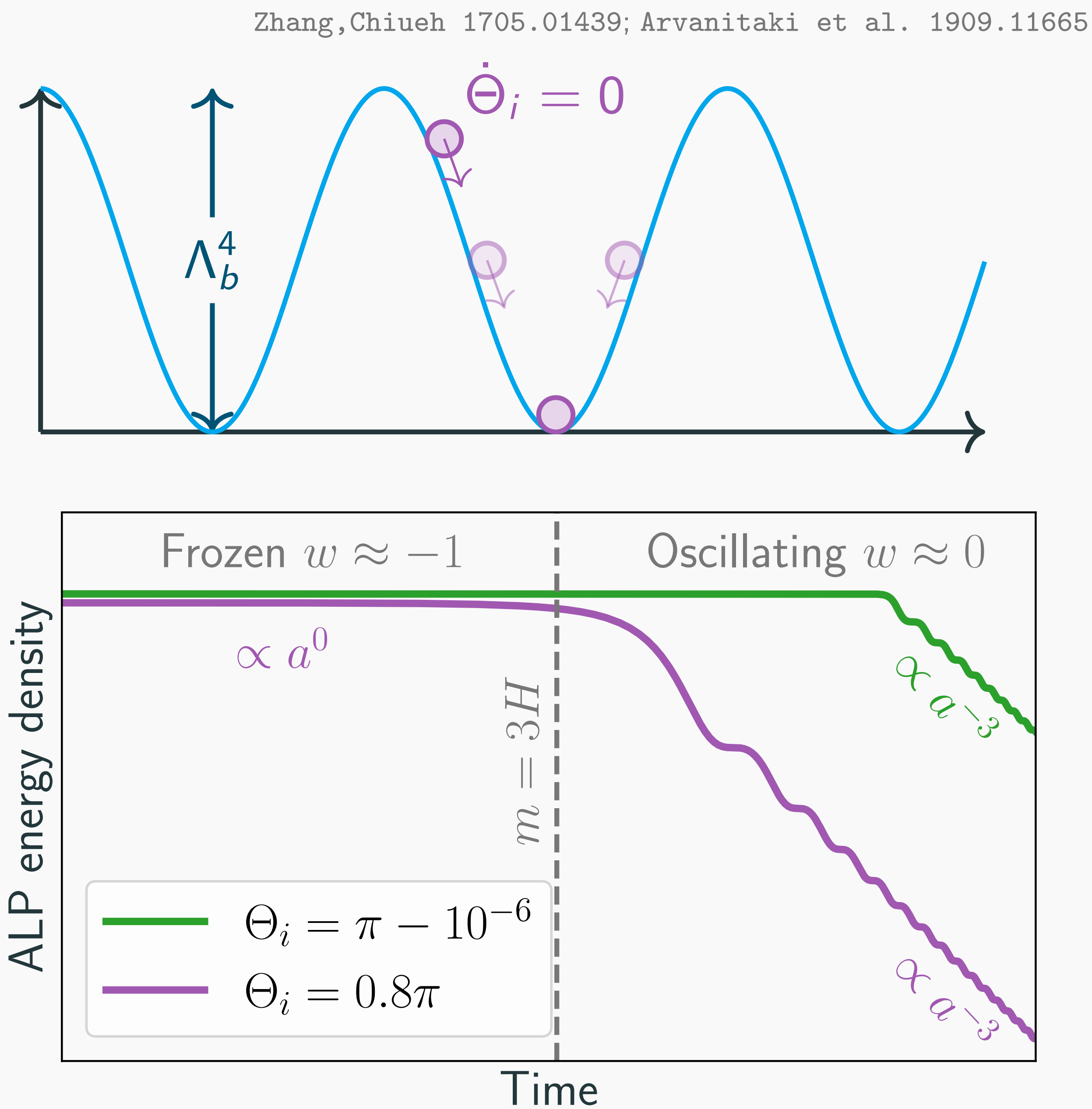
A way out: switch on initial velocity for the axion

Standard versus kinetic Misalignment.

Two ways to delay the onset of oscillations

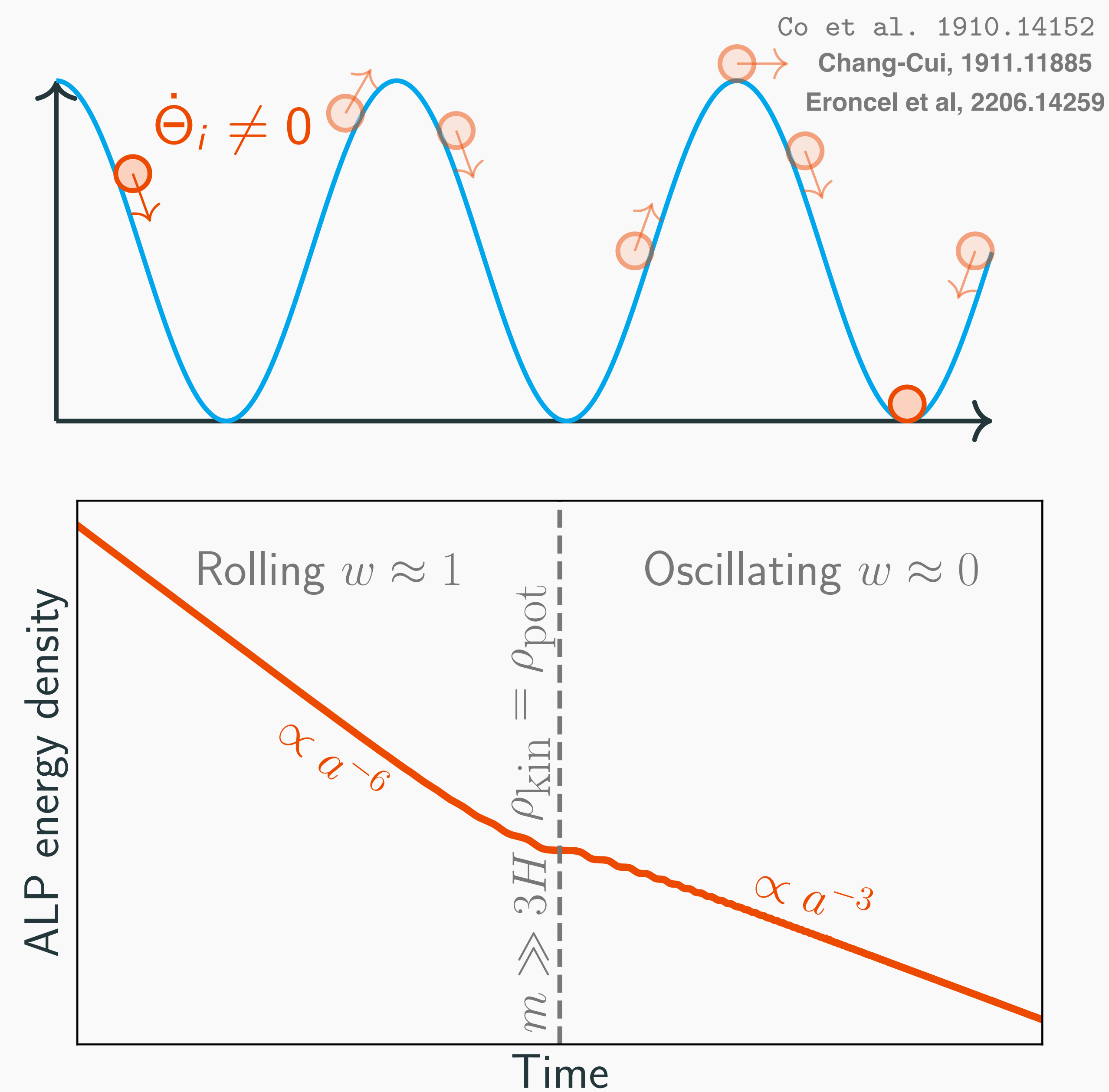
Initial field value tuned to top of potential:

Standard (Large) misalignment



Large initial velocity

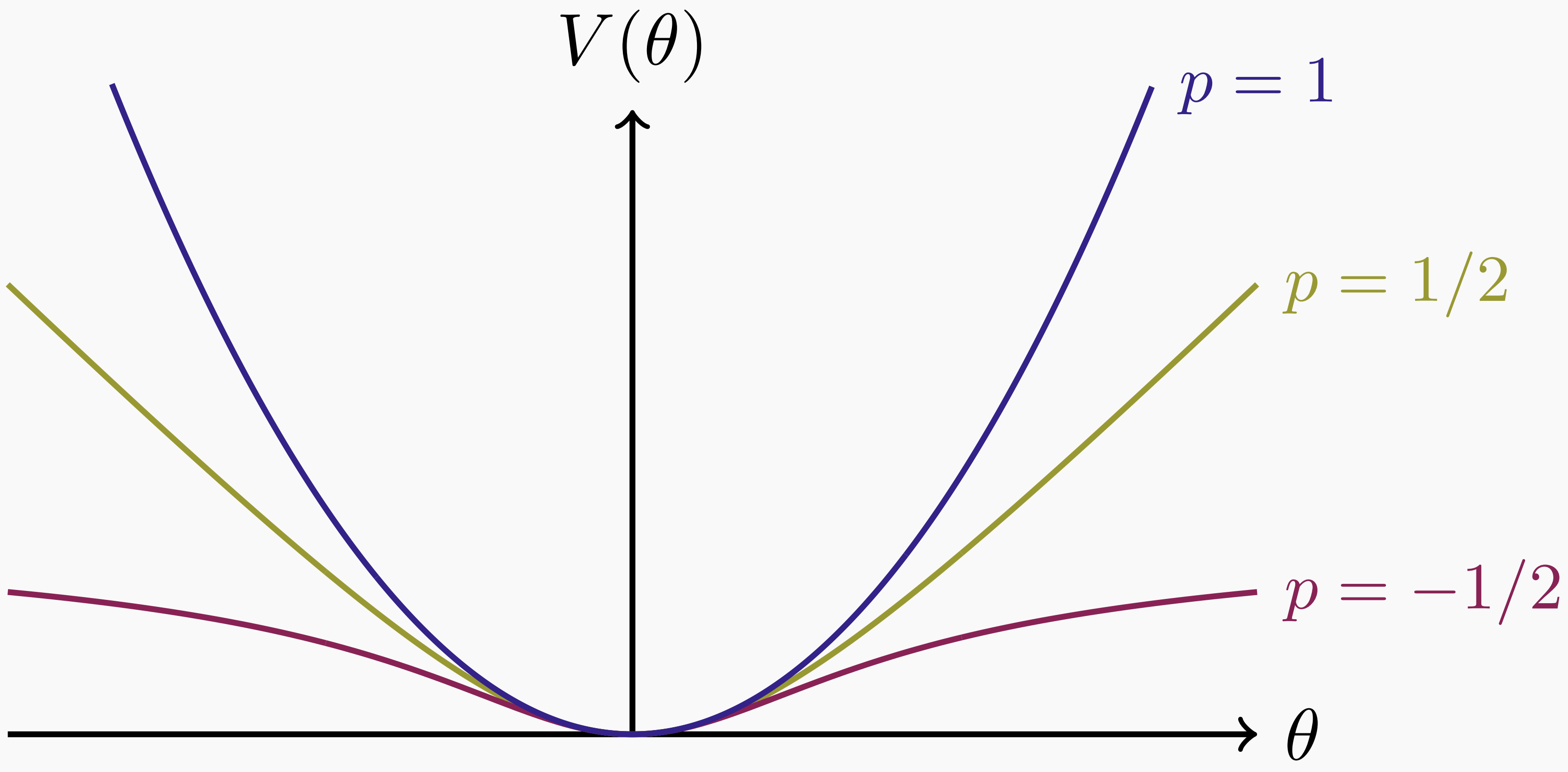
Kinetic misalignment



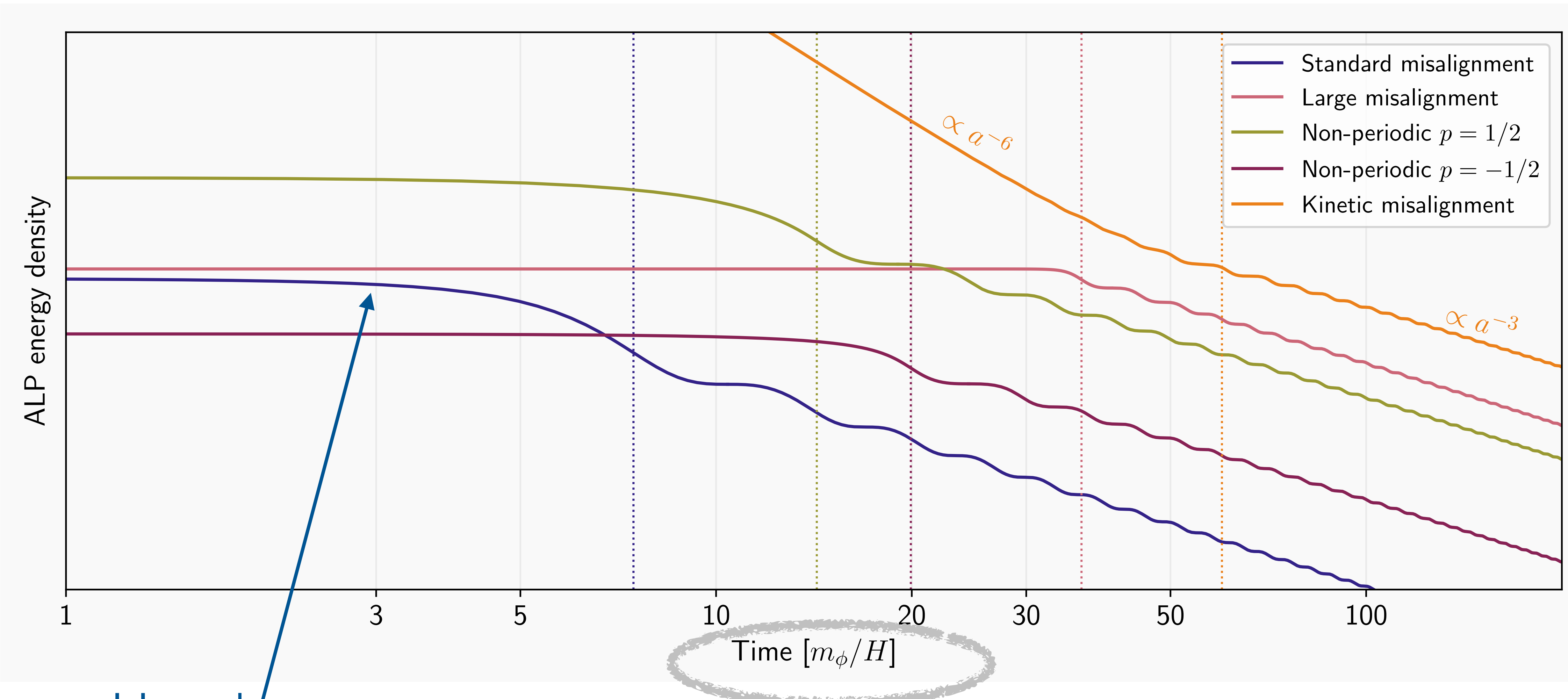
A third way to delay the onset of oscillations: a non-periodic potential.

1906.06352, 2305.03756

$$V(\theta) = \frac{m_\phi^2 f_\phi^2}{2p}\left[\left(1+\theta^2\right)^p - 1\right], \quad p < 1.$$

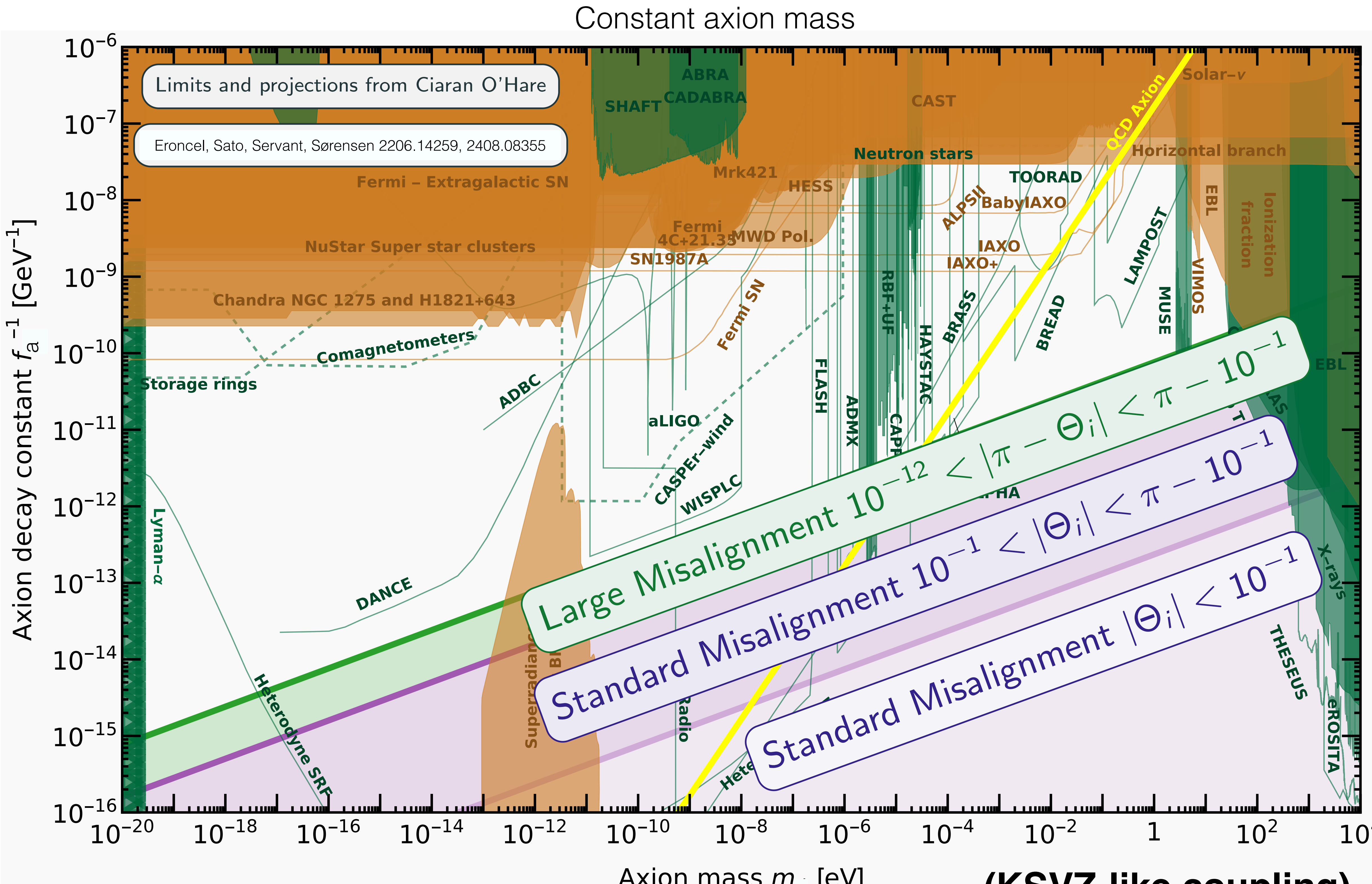


Common property of all these cases: onset of oscillations is delayed which boosts the dark matter abundance, and extends the ALP dark matter parameter space to lower decay constants.



Usual story

ALP DM parameter space.

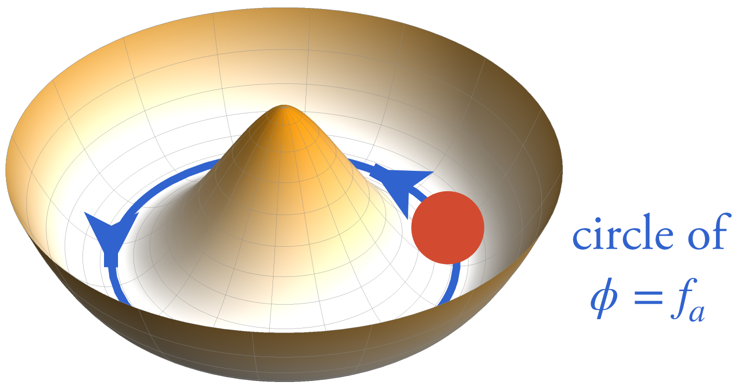


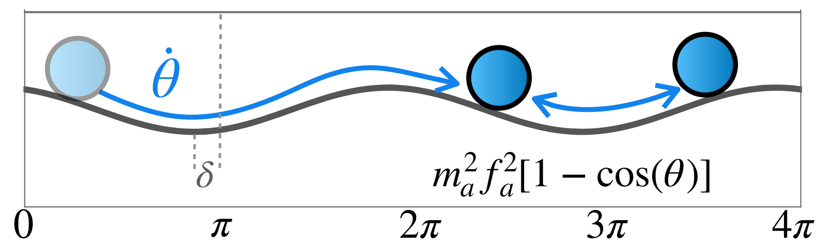
(KSVZ-like coupling)

$$g_{\theta\gamma} = (\alpha_{\mathbf{em}}/2\pi)(1.92/f)$$

Kinetic misalignment.

Add kinetic energy to delay onset of oscillations





> Delay oscillations
$\Rightarrow$ less redshift
$\Rightarrow$ more DM
$\Rightarrow$ lower f_a

-> ALP can be DM for low f_a

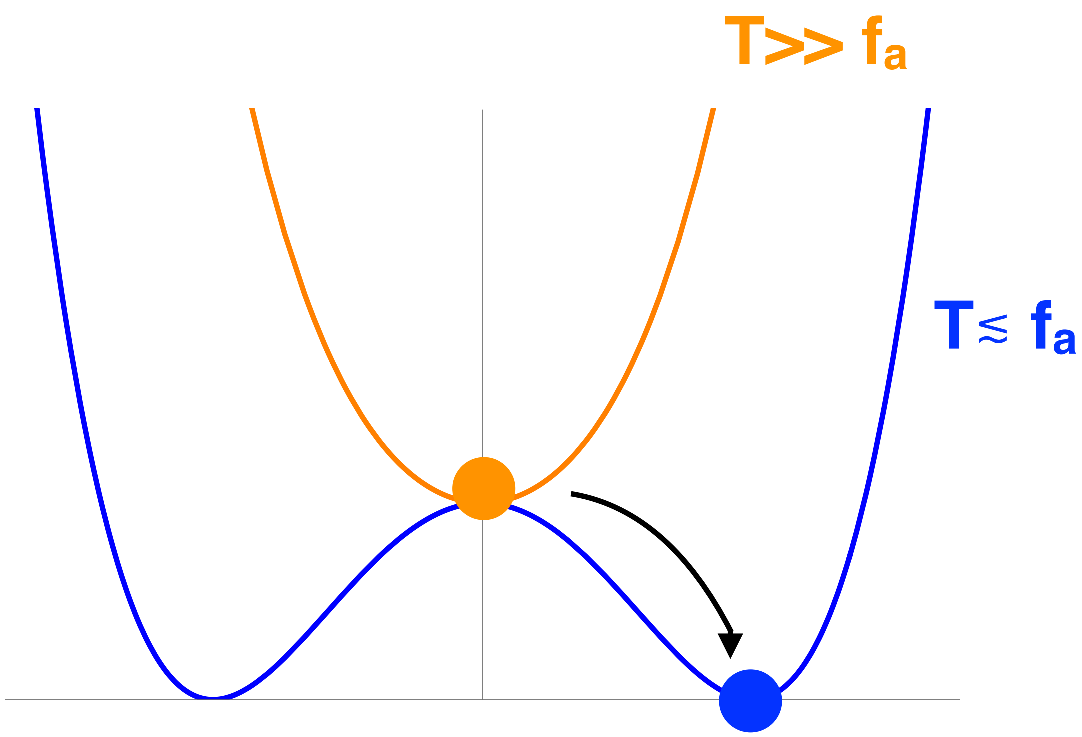
Co, Hall, Harigaya et al '19'20
Chang, Cui'19
Eröncel et al, '22, '24

Axion cosmology.

"Common" story:

Starts at $\langle\phi\rangle=0$

Studies axion cosmology ignoring the radial mode

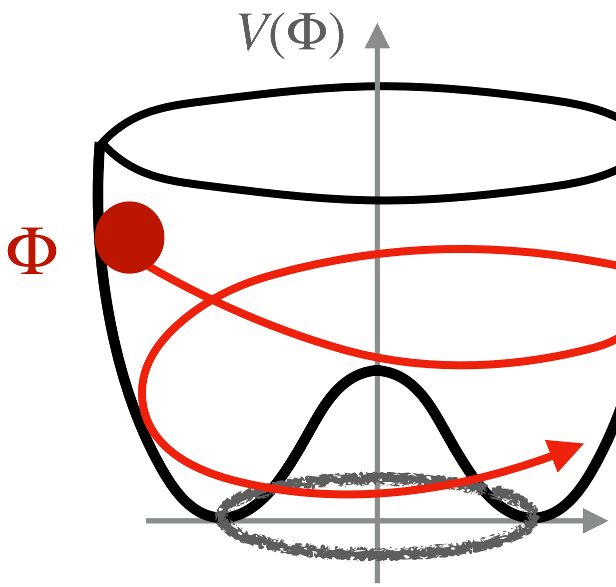


Alternative:

Starts at $\langle\phi\rangle \gg f_a$

(field can be driven naturally to these large field values during inflation due to a negative Hubble-induced mass term)

Radial mode /axion interplay



How did the axion acquire a kick?

If PQ symmetry is broken explicitly at high energies
—> mexican hat potential is tilted

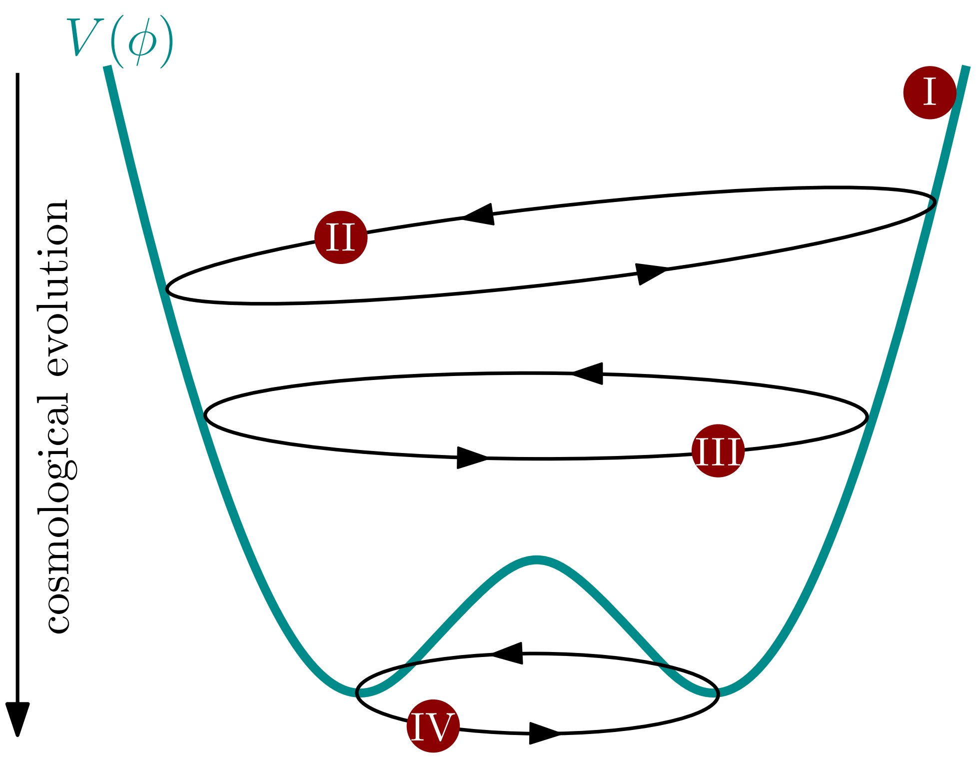


Figure by P. Simakachorn

If radial mode of PQ field starts at large VEV, the angular mode gets a large kick in the early universe

With initial conditions: $$\frac{1}{2}\dot{\Theta}_i^2 \gg 2m^2(T_i)$$

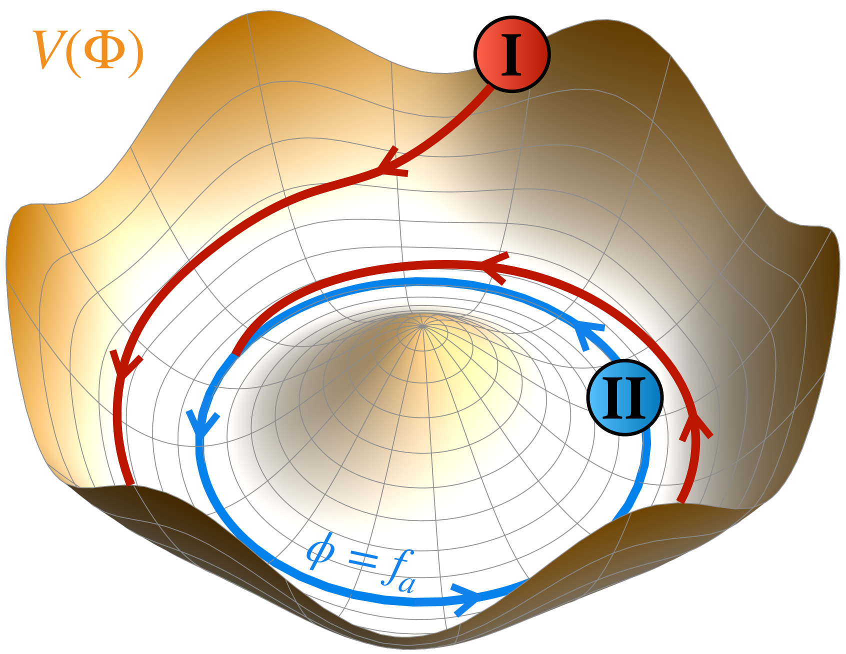
Delayed axion oscillations !

-> kinetic misalignment mechanism

[Co, Harigaya, Hall'19] 1910.14152

2004.00629

Initial conditions.



Similar to Affleck-Dine ’85 scenario

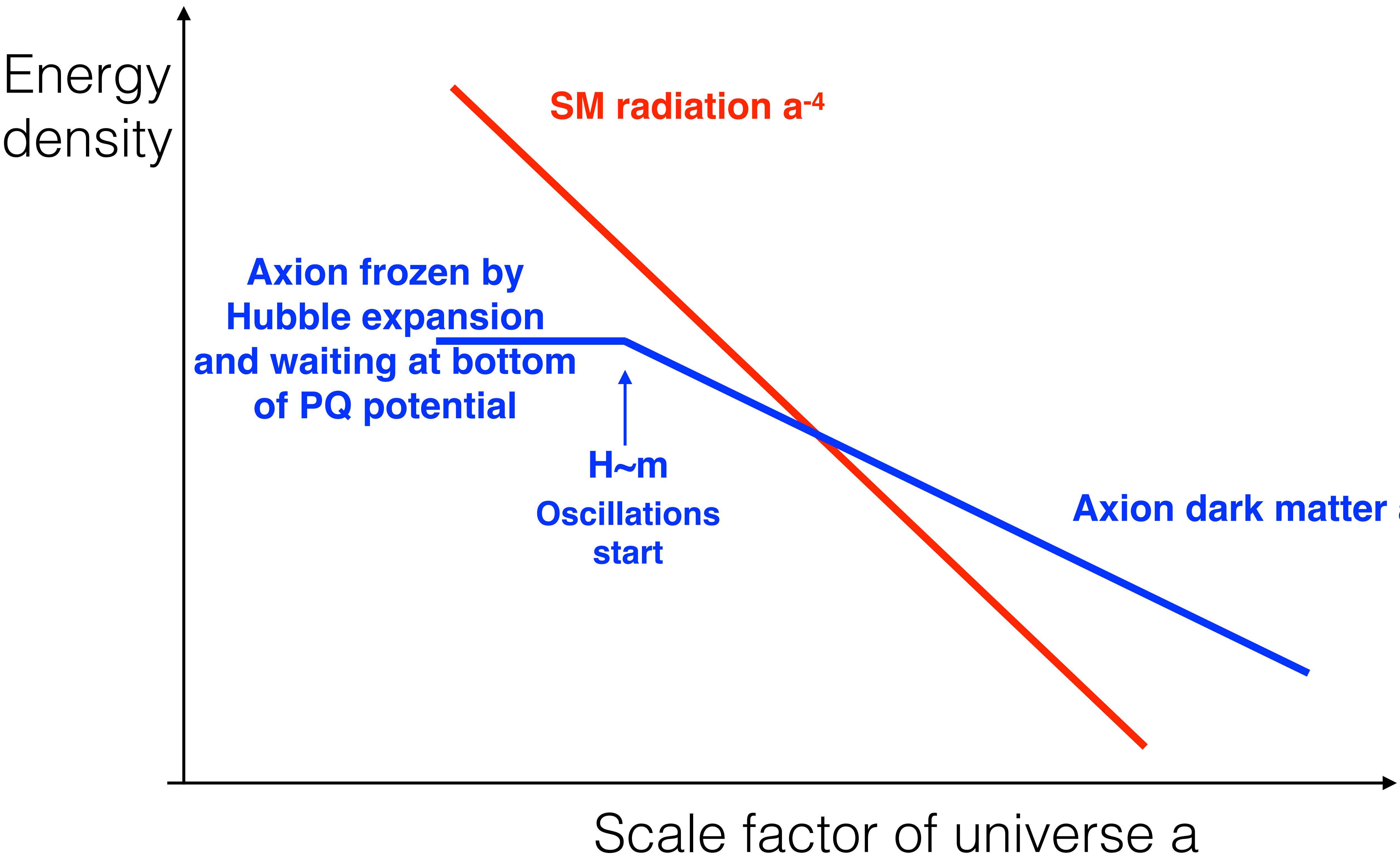
At early times, ϕ is driven away from ϕ =0,
towards $\langle\phi\rangle \gg f_a$
by negative Hubble-induced mass term $H \gg m_\phi$

$$V_H(\Phi, H) \supset -cH^2|\Phi|^2$$

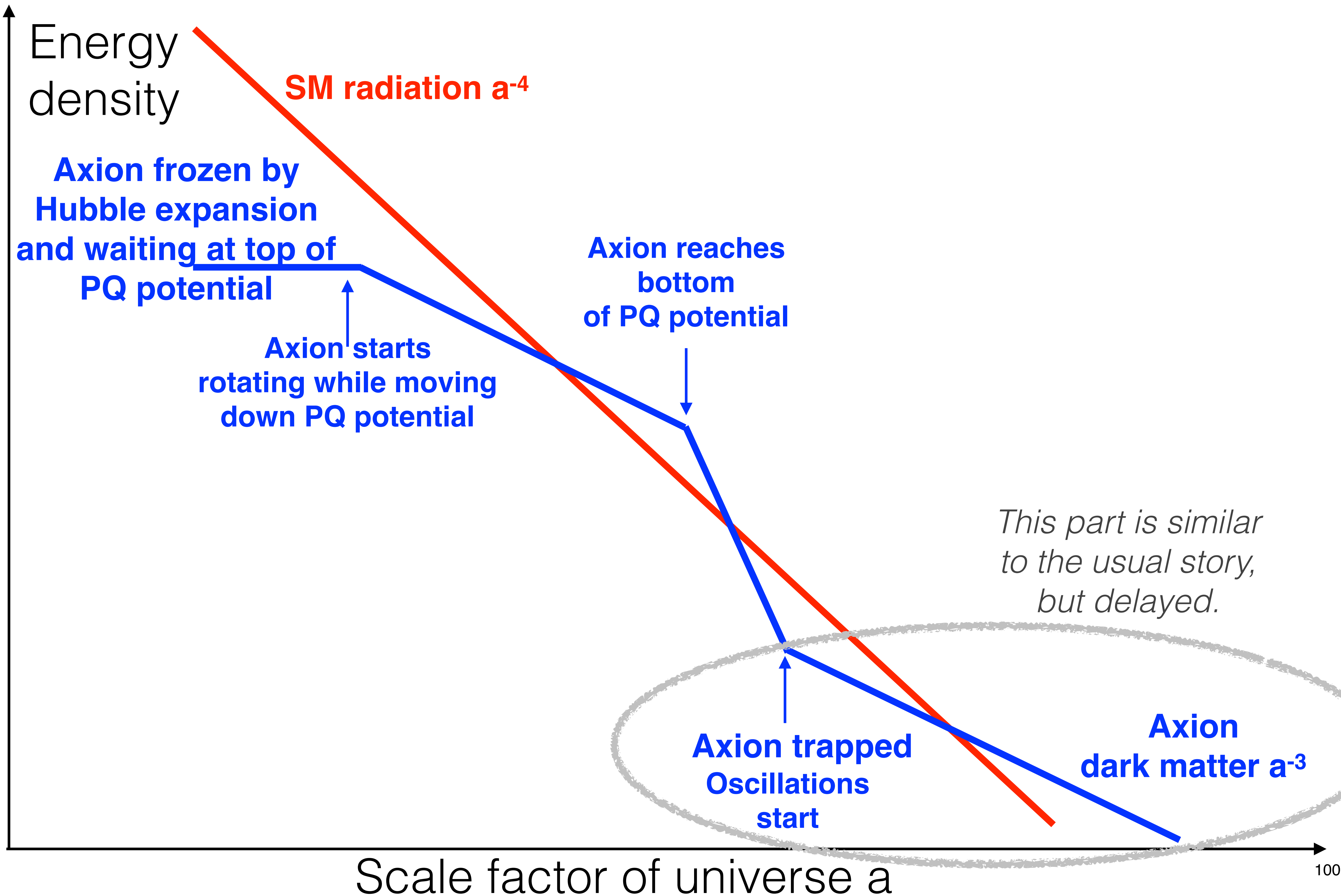
Dine, Randall, Thomas ‘95

+ explicit U(1) breaking term transfers radial mode motion into kick for the axion

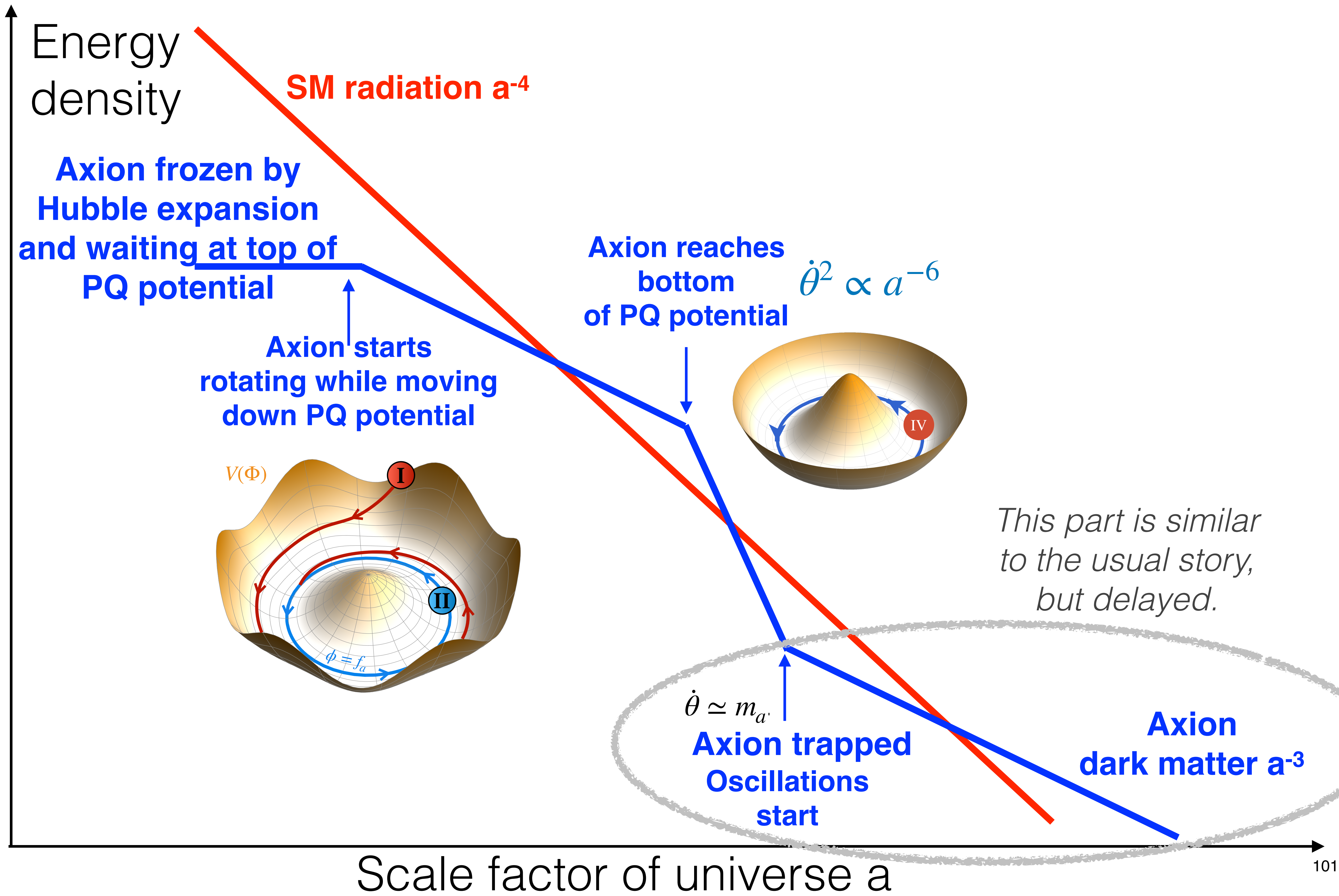
Usual story.


Energy density
SM radiation a-4
Axion frozen by Hubble expansion and waiting at bottom of PQ potential
H~m
Oscillations start
Axion dark matter a-3
Scale factor of universe a

New story.

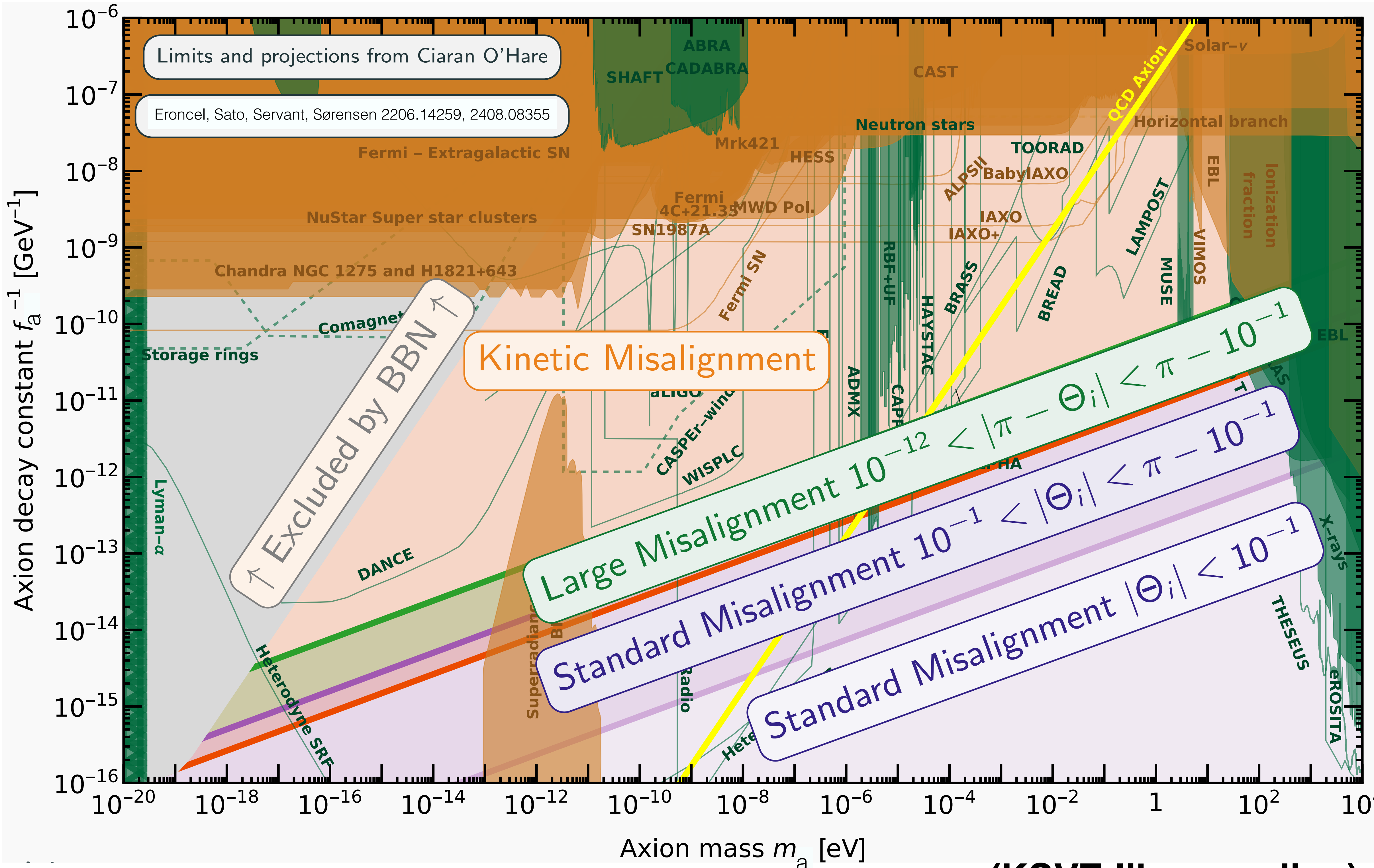

Energy density
SM radiation a-4
Axion frozen by Hubble expansion and waiting at top of PQ potential
Axion starts rotating while moving down PQ potential
Axion reaches bottom of PQ potential
This part is similar to the usual story, but delayed.
Axion trapped Oscillations start
Axion dark matter a-3
Scale factor of universe a

New story.



Energy density
SM radiation a-4
Axion frozen by Hubble expansion and waiting at top of PQ potential
Axion starts rotating while moving down PQ potential
V(Φ)
I
II
$\phi = f_a$
Axion reaches bottom of PQ potential
$\dot{\theta}^2 \propto a^{-6}$
IV
This part is similar to the usual story, but delayed.
$\dot{\theta} \simeq m_{a'}$
Axion trapped Oscillations start
Axion dark matter a-3
Scale factor of universe a

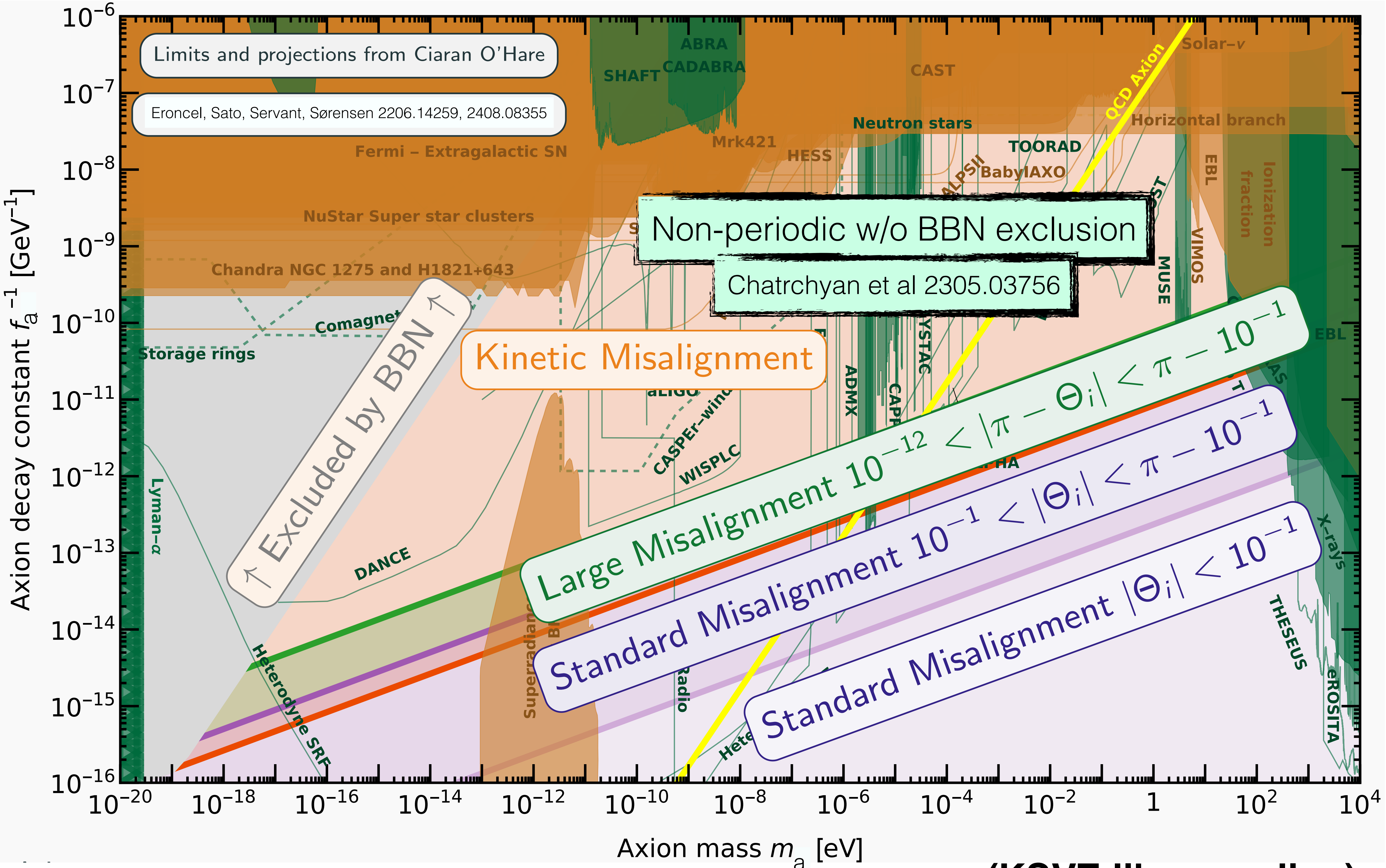
ALP DM parameter space.



(KSVZ-like coupling)

$$g_{\theta\gamma} = (\alpha_{\mathbf{em}}/2\pi)(1.92/f)$$

ALP DM parameter space.



(KSVZ-like coupling)

$$g_{\theta\gamma} = (\alpha_{\mathbf{em}}/2\pi)(1.92/f)$$

Model implementations of a rotating axion

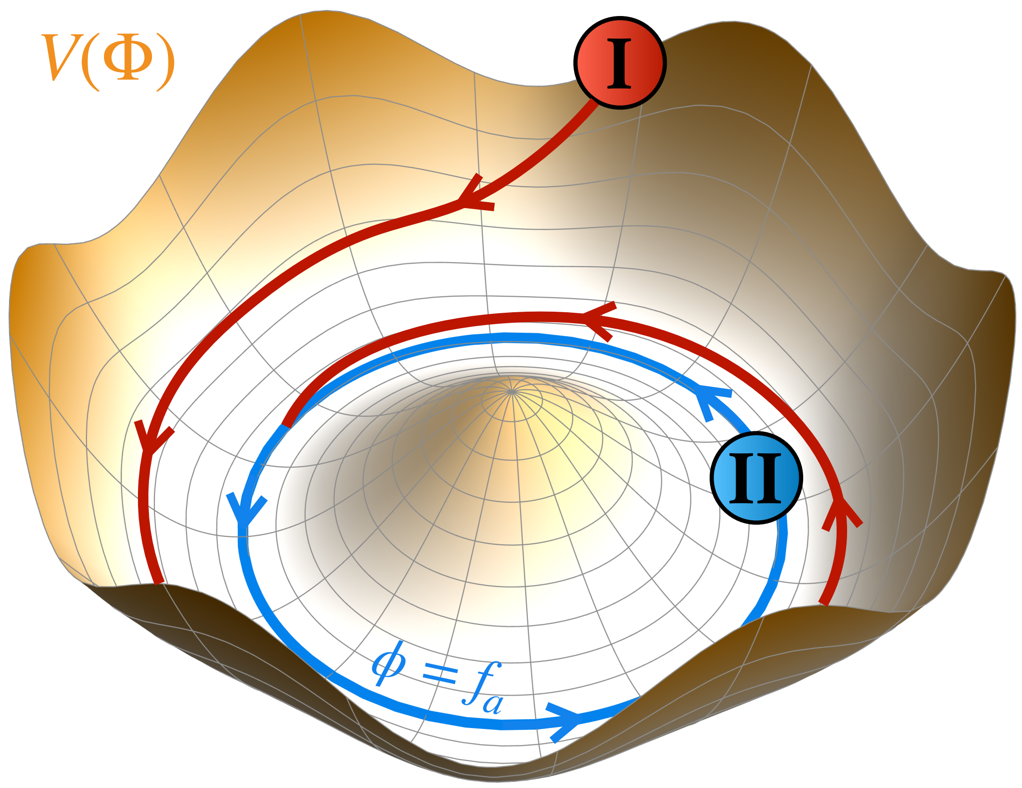
Complex scalar field

"Affleck-Dine Baryogenesis" (Affleck, Dine, 1985)

"Axiogenesis" (Co, Hall, Harigaya, et. al., '19)

"Kination cosmology" (Gouttenoire et al, '21)

"Axion DM from kinetic misalignment" Eroncel, Sato, Servant, Sørensen ,2408.08355



$\Phi \sim \phi e^{i\theta}$ with $U(1)$-symmetry

Radial mode ϕ oscillates in potential with mass $\sqrt{V''(\Phi)}$.

Angular mode θ "axion" spins, with large kinetic energy.

Requirements

1. $U(1)$-symmetric (quadratic) potential with spontaneous symmetry-breaking minimum

2. Large initial scalar VEV

3. Explicit $U(1)$-breaking term (wiggle for angular velocity)

4. Damping of radial motion

Ingredients 1 & 2 : scalar potential

$$V(\Phi) = m_r^2 |\Phi|^2 \left[\log\left(\frac{|\Phi|^2}{f_a^2}\right) - 1\right] + \left[A\frac{\Phi^n}{M_{Pl}^{n-3}}. + h.c\right] + \frac{|\Phi|^{2n-2}}{M_{Pl}^{2n-6}}$$

I. $U(1)$-conserving potential (quadratic) with a minimum f_a

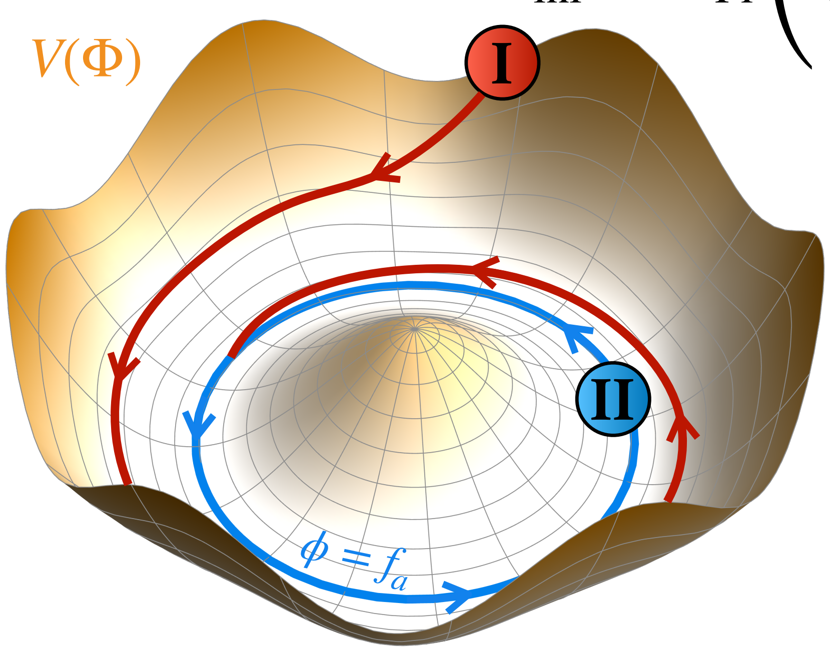
(motivated by supersymmetric setups)

$\propto \cos(l\theta)$

II. explicit breaking term (e.g. $U(1)$ is not exact at high scales.)

stabilization i.e., at large $|\Phi|$

$$\phi_{\rm ini} \simeq M_{\rm Pl}\left(\sqrt{\frac{c}{\lambda^2}}\cdot\frac{m_r}{M_{\rm Pl}}\right)^{1/(n-2)}$$



Ingredient 3 : large initial VEV $\phi_{\rm ini}$

By adding a negative Hubble mass

$$V_H(\Phi, H) \supset -cH^2|\Phi|^2$$

ϕ is driven away from $\phi = 0$ at early times $(H \gg m_r)$

(e.g. Dine, Randall, Thomas, 1995, Fujita & Harigaya 1607.07058)

Take-home message

ALPs can be the DM everywhere in the [m_a ,f_a] plane.

Kinetic Misalignment Mechanism:

A well-motivated alternative production mechanism for ALP Dark Matter

Moves the ALP Dark Matter window into testable territory.
->All axion experiments are in principle sensitive to axion dark matter (even helioscopes and light-shining-through-the-wall experiments)

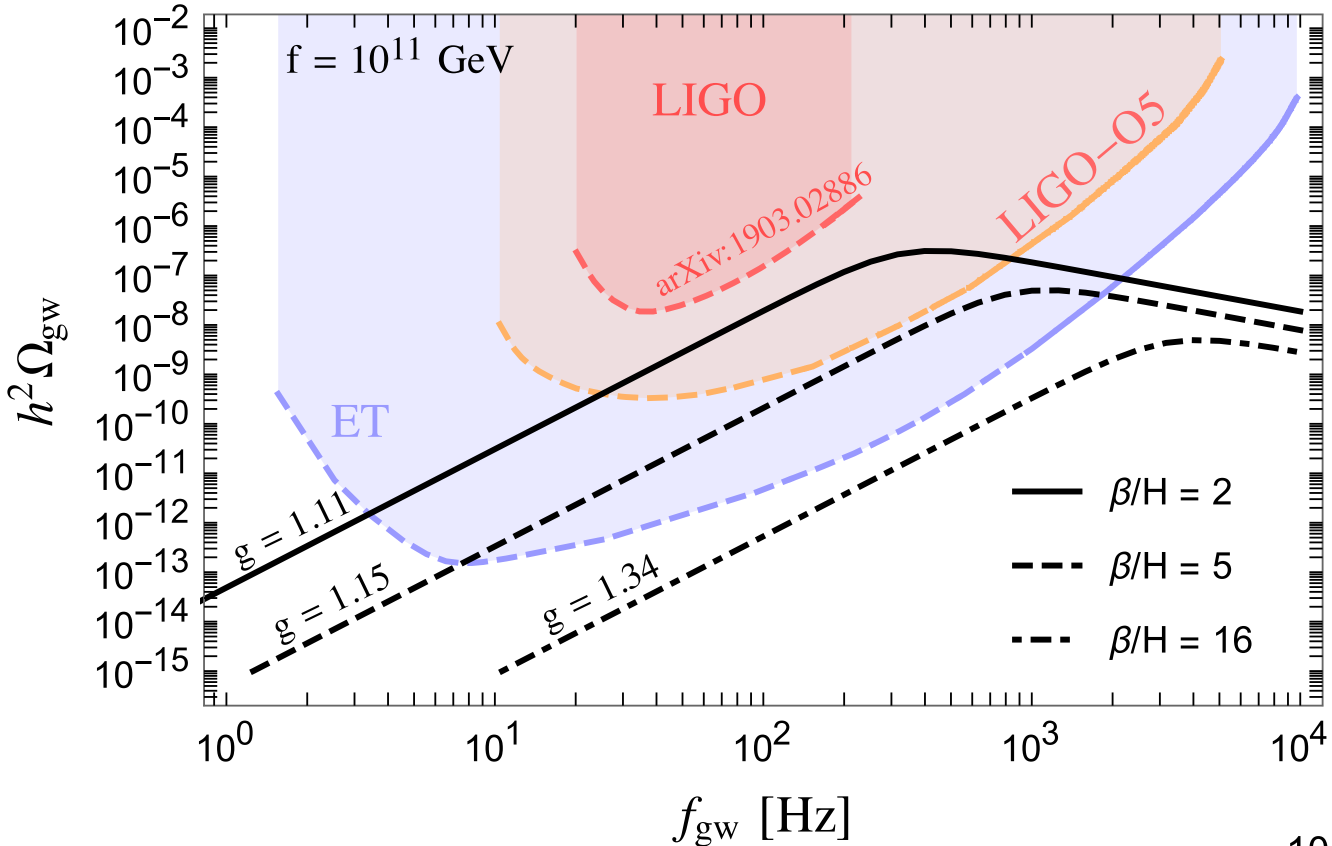
QCD axion Dark Matter inside MADMAX and Iaxo sensitivities

GW backgrounds from axion early-universe dynamics

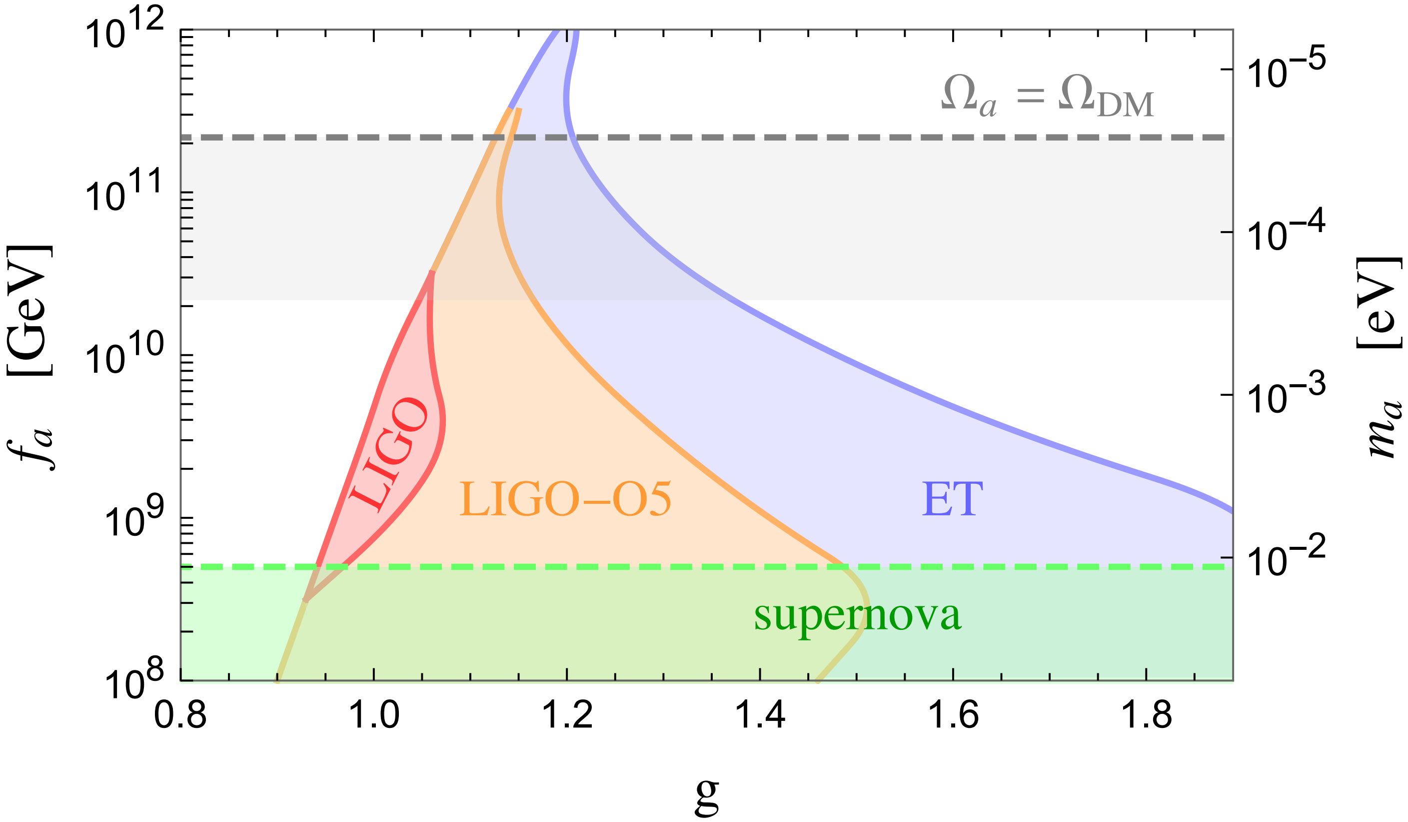
4 distinct sources:

- GWs from axionic (global) cosmic strings
- GWs from the Peccei-Quinn phase transition (if first-order)
- GW signatures from kination induced by rotating axions
- [GWs from axion fragmentation: not treated in this talk, see Sec.9 of 2206.14259]

GW from a first-order Peccei-Quinn phase transition.

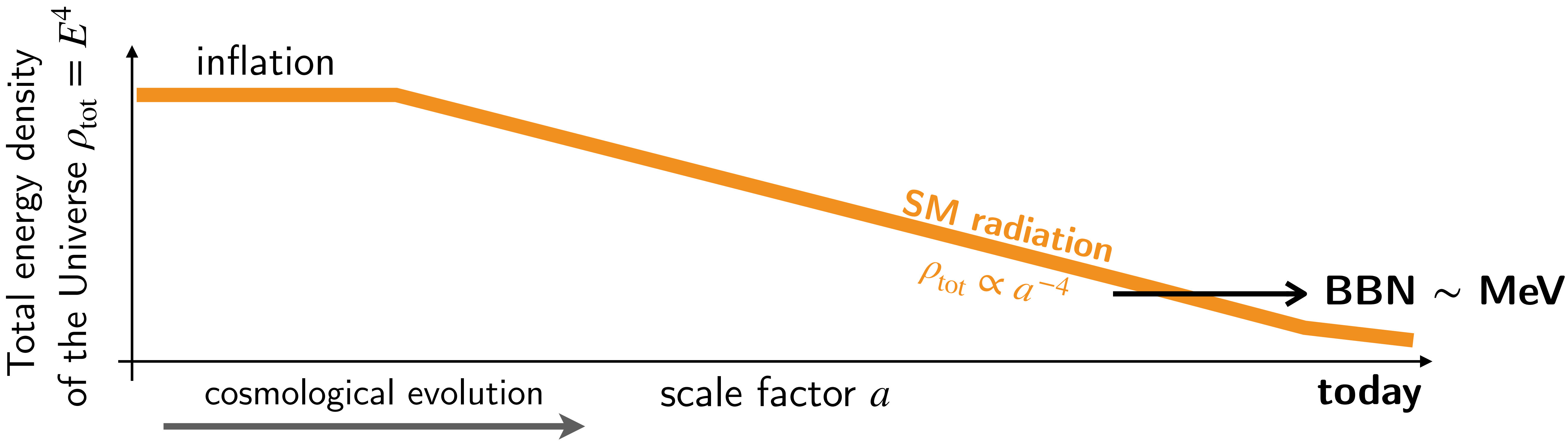


Delle Rose et al, 1912.06139
Von Harling et al, 1912.07587

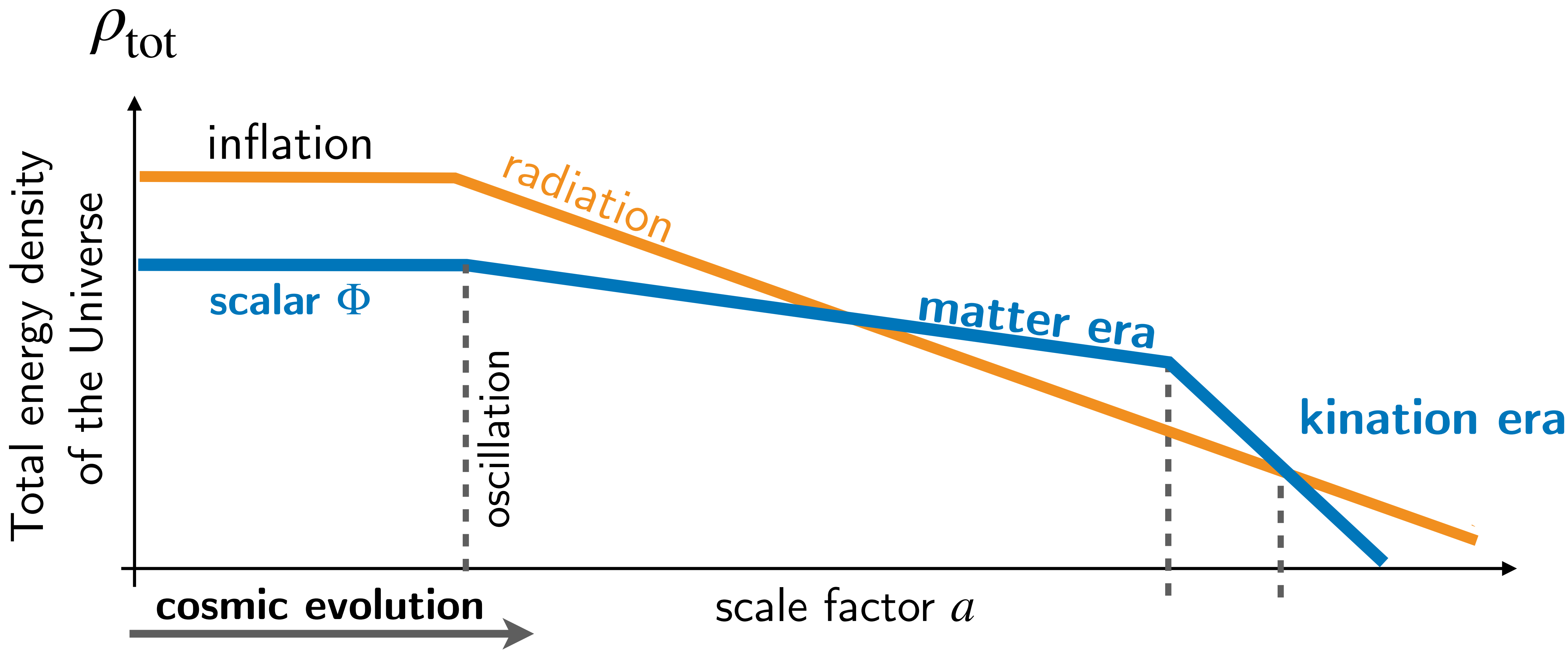


Effect of non-standard cosmology on primordial GW spectra.

Standard cosmological history



Early matter+kination era

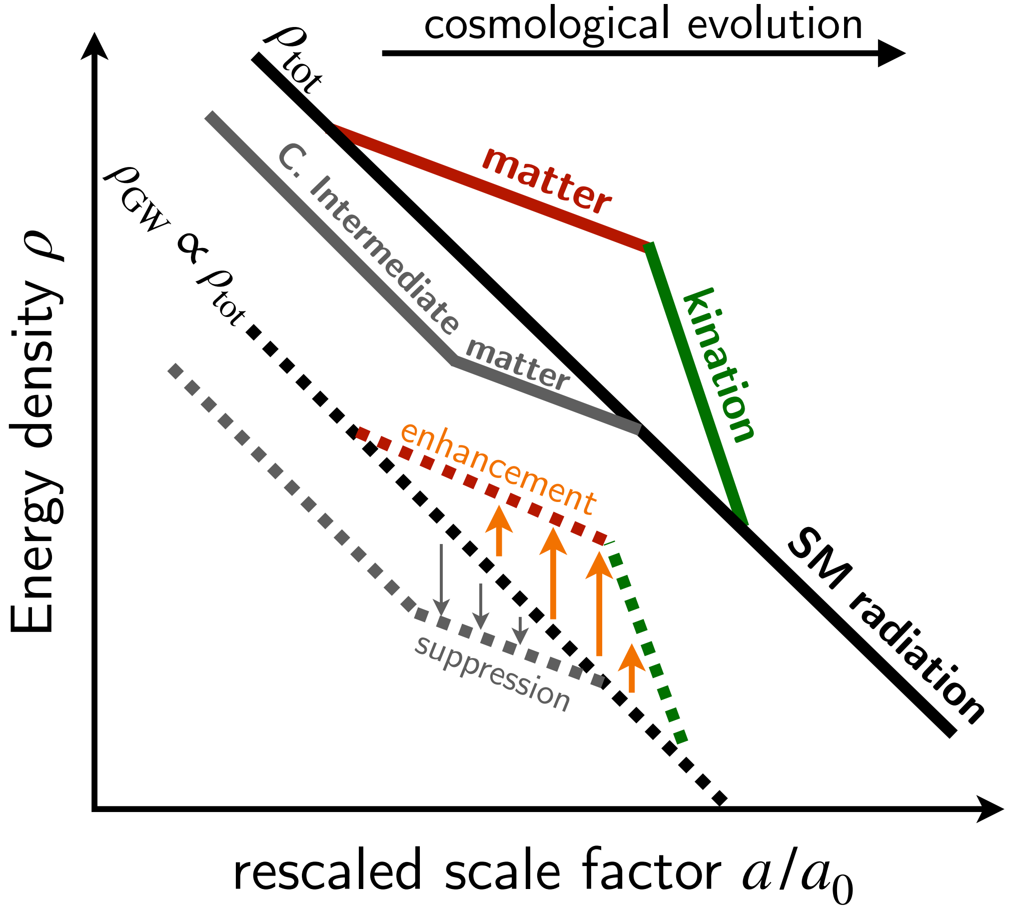


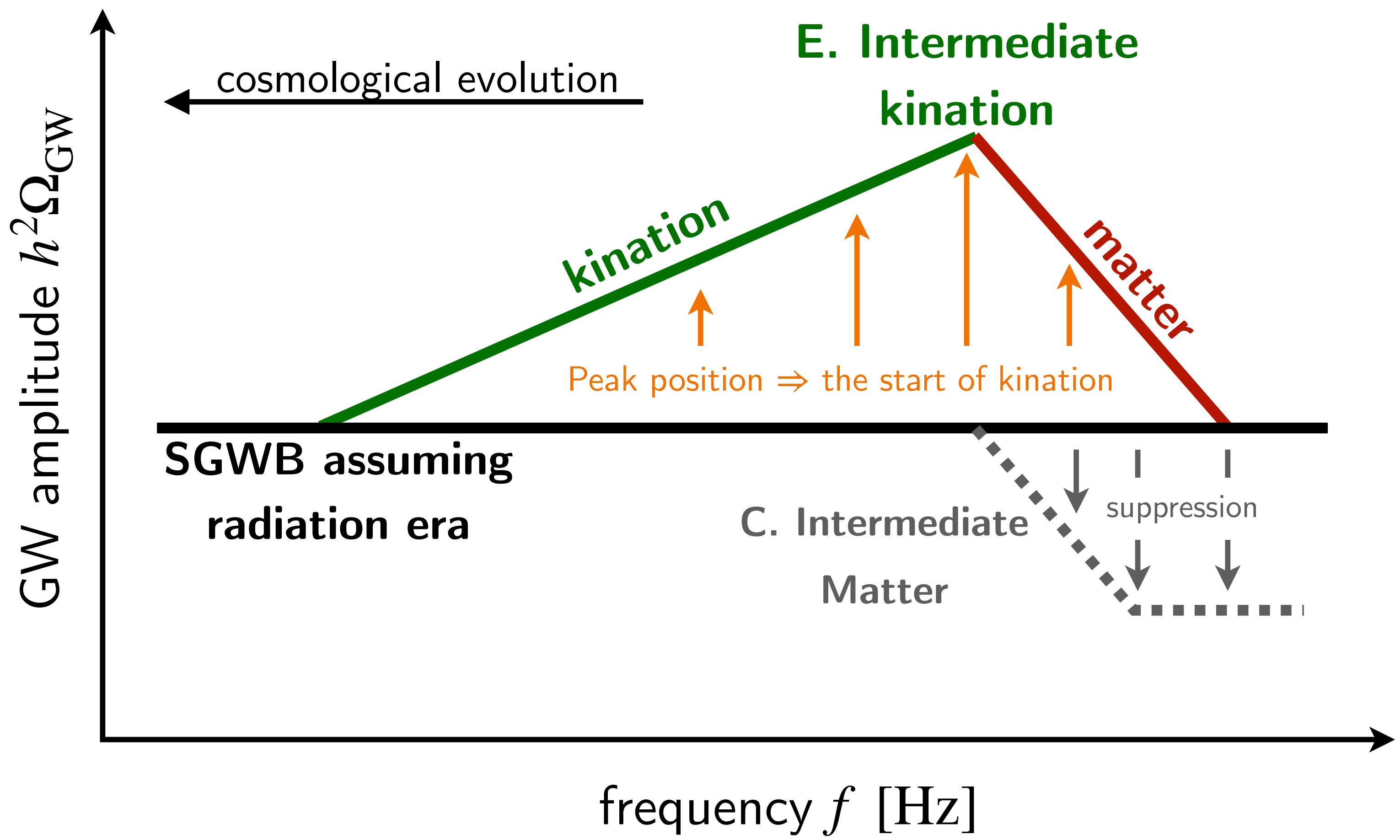
GW from cosmic strings and from inflation track the total energy density of the universe.

—> Significantly enhanced by a matter + kination era

Impact of the cosmological history on Gravitational Waves:

[1912.02569] [2111.01150]

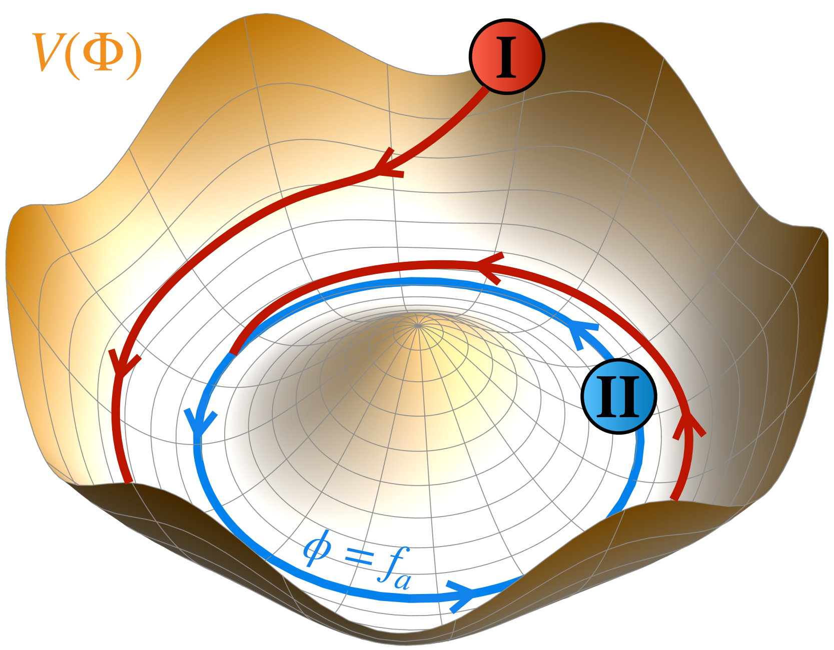


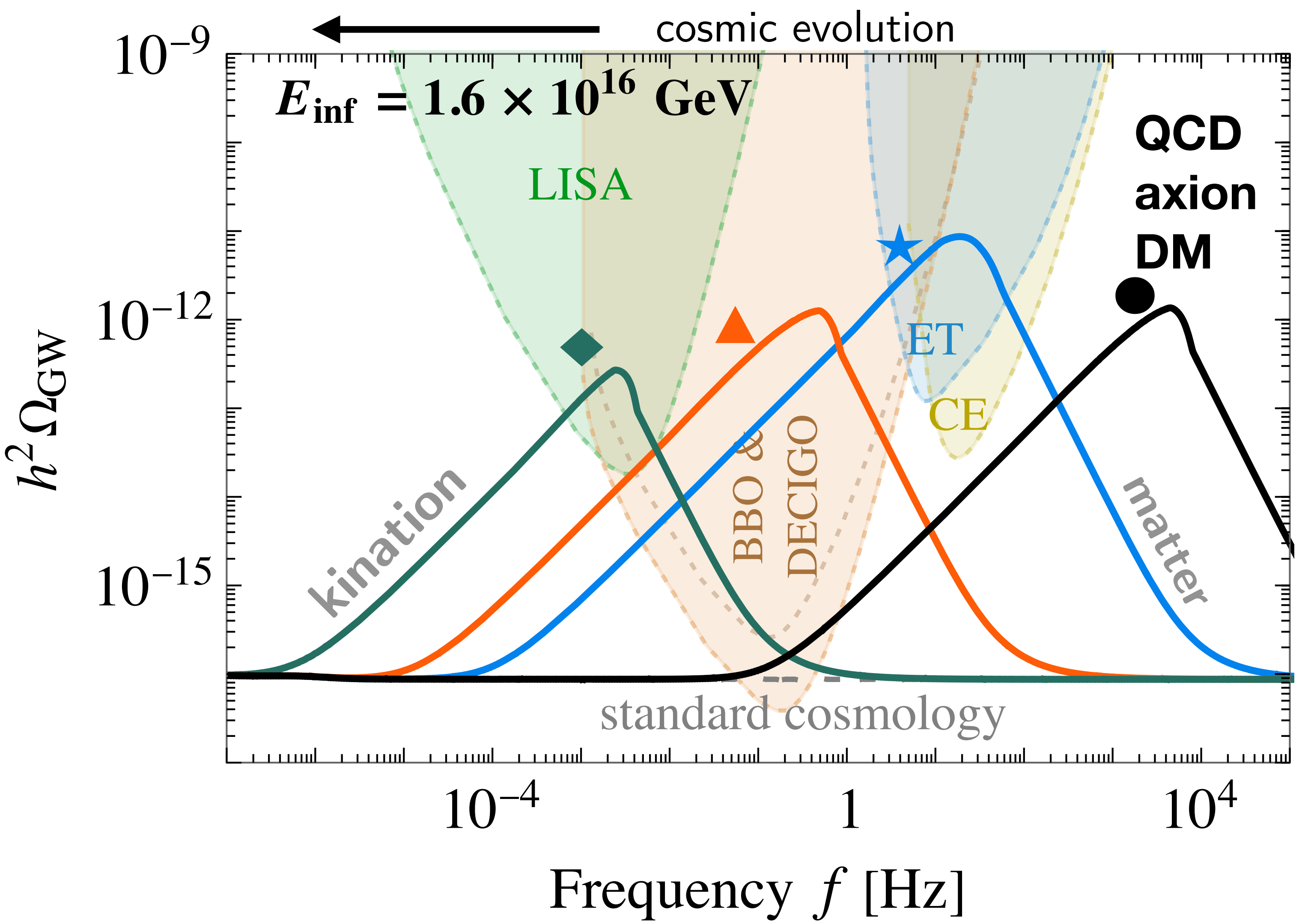


Fraction of energy density in GW today

$$\Omega_{GW,0} = \left(\frac{\rho_{GW,prod}}{\rho_{tot,0}}\right)\left(\frac{a_{prod}}{a_0}\right)^4 = \left(\frac{\rho_{GW,prod}}{\rho_{tot,prod}}\right)\left(\frac{\rho_{tot,prod}}{\rho_{tot,0}}\right)\left(\frac{a_{prod}}{a_0}\right)^4$$

Amplification of inflationary GW from axion-induced kination era.

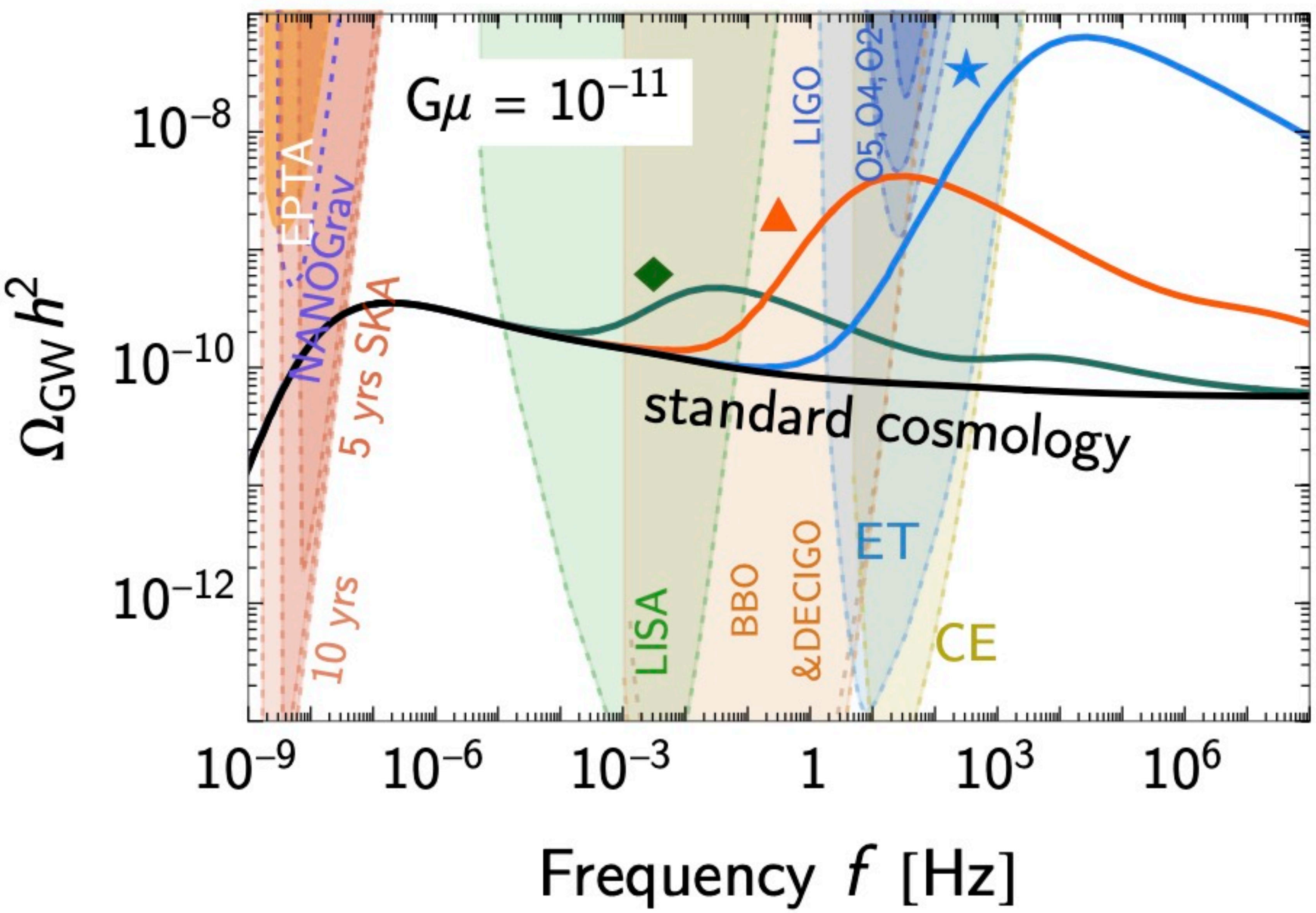




$f_a \sim 10^{10}$GeV
N_{KD} =5

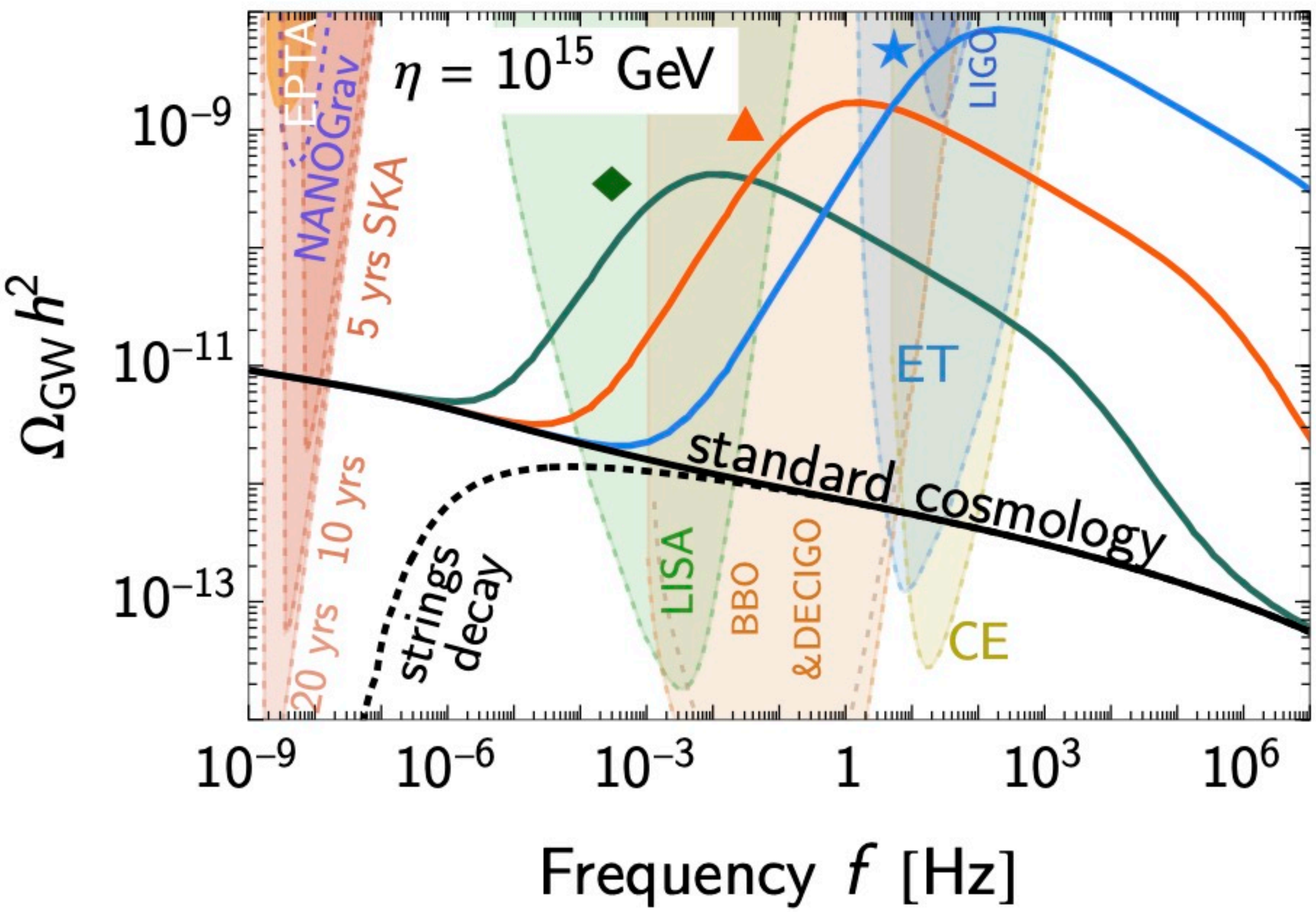
[Gouttenoire et al 2108.10328 & 2111.01150]

Amplification of GW from local cosmic strings due to an axion-induced kination era.



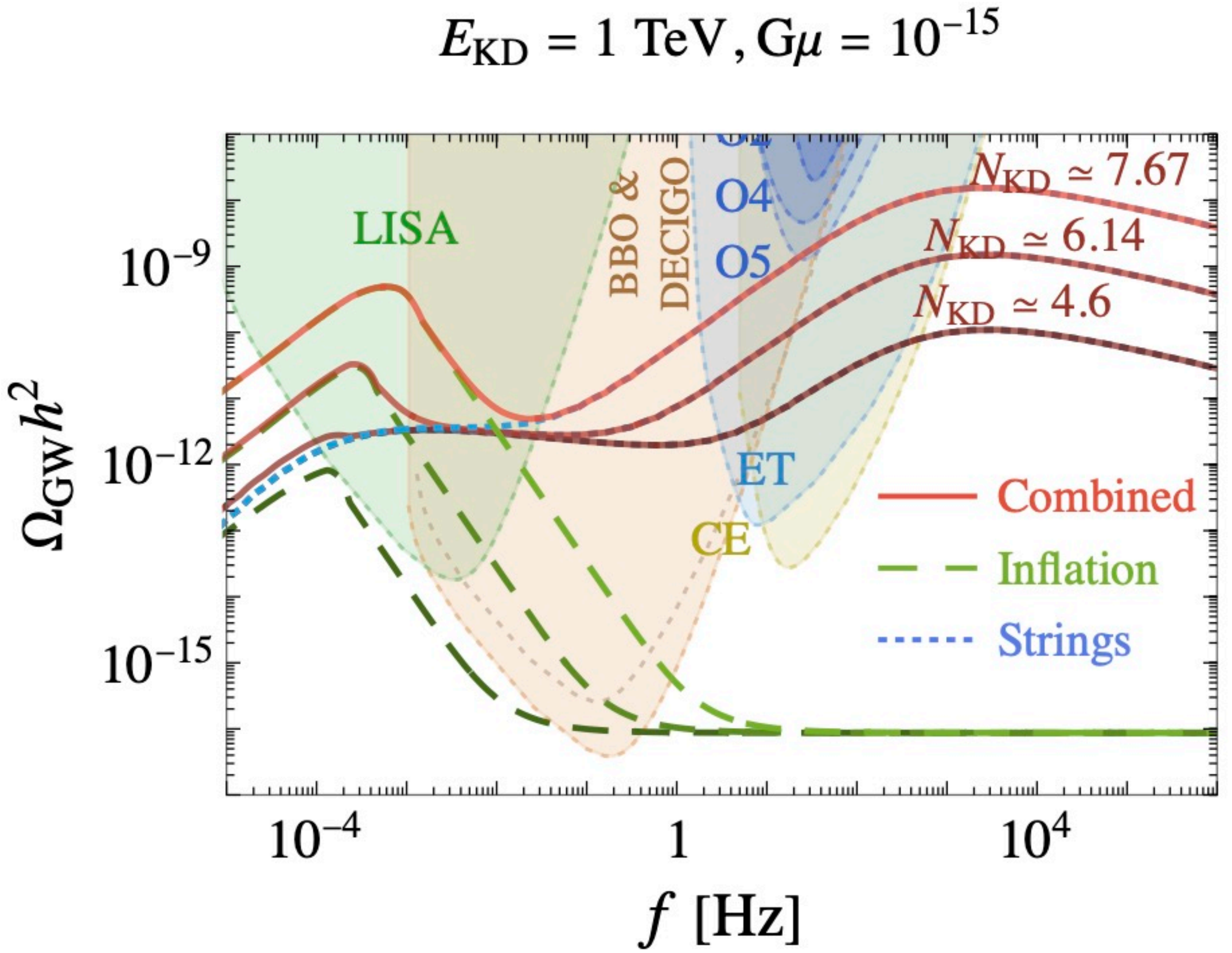
[2111.01150]

Amplification of GW from global cosmic strings due to an axion-induced kination era.



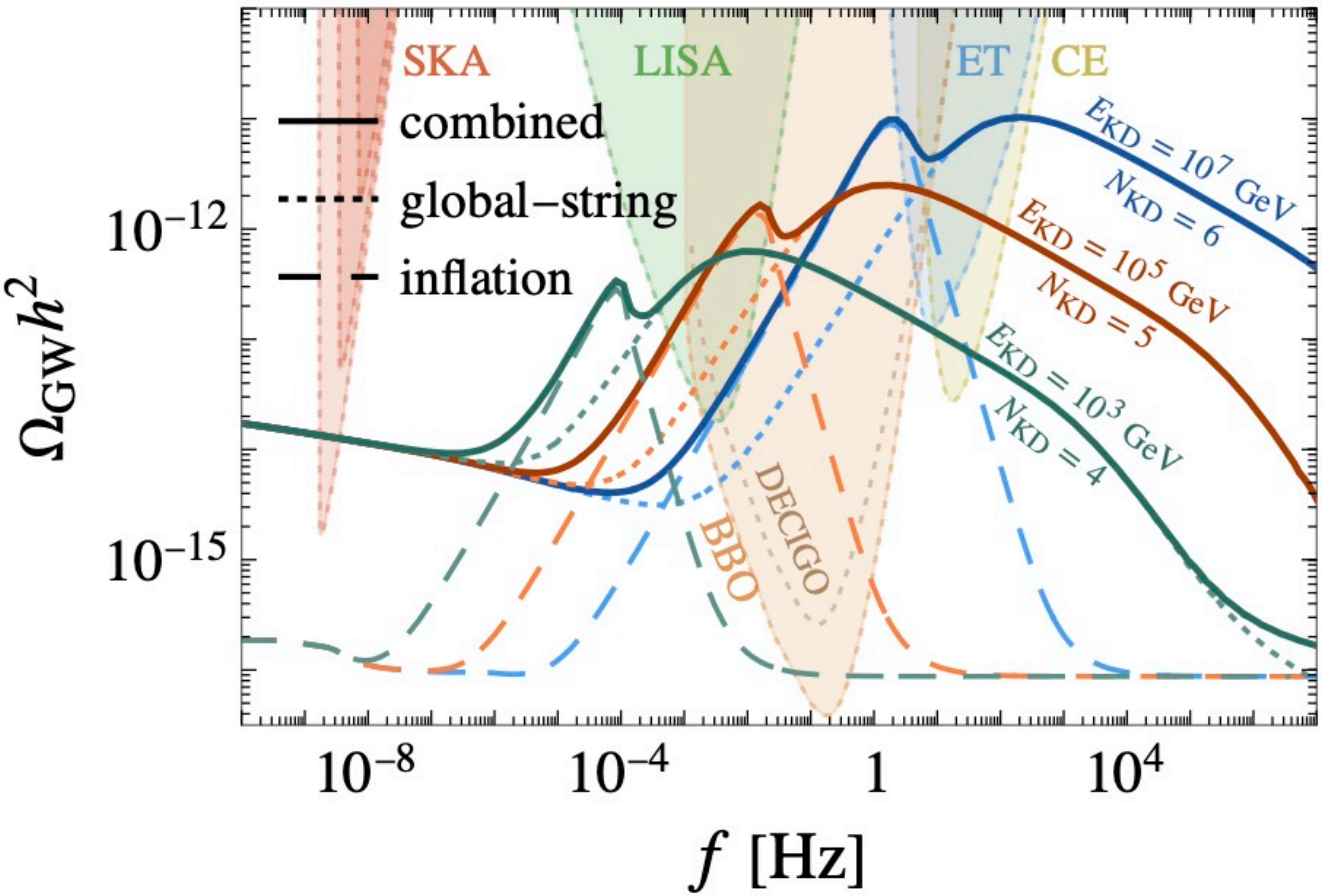
[2111.01150]

Gravitational Waves from inflation & local cosmic strings in non-standard cosmology induced by rotating axions.



[2111.01150]

Gravitational Waves from inflation & global cosmic strings in non-standard cosmology induced by rotating axions.



[2111.01150]

GWs
from axion fragmentation.

GWs from axion fragmentation.

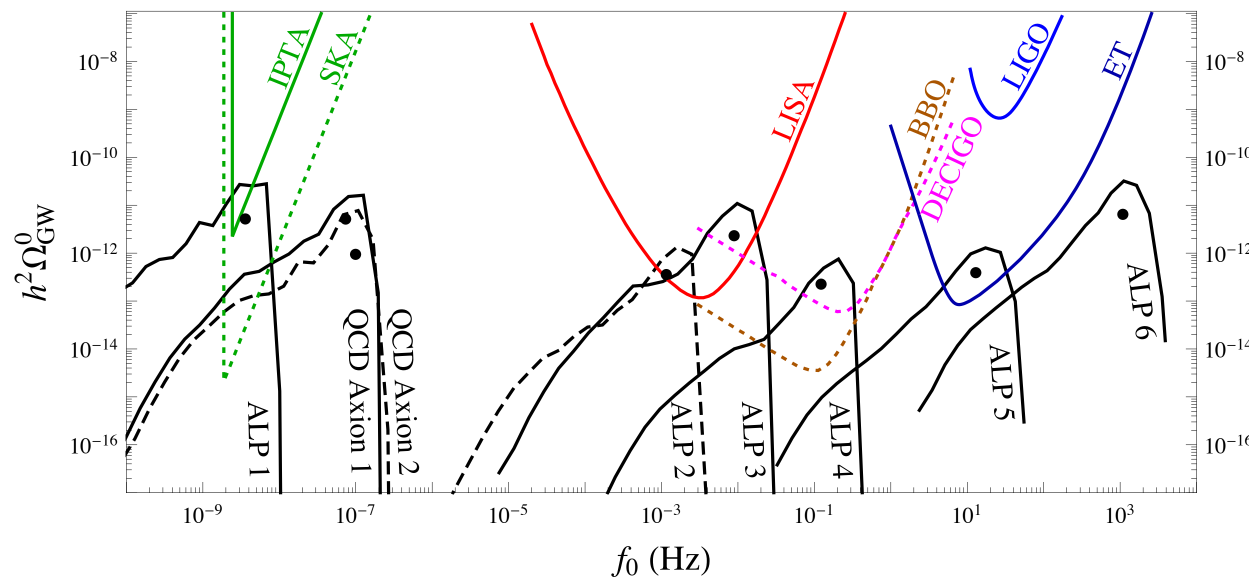
The transfer of energy in the early universe from the homogeneous axion field into axion quantum fluctuations, inevitably produces a stochastic background of gravitational waves of primordial origin with a peak frequency controlled by the axion mass.

$$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\Delta h_{ij}}{a^2} = \frac{16\pi}{M_{\rm pl}^2}\Pi_{ij}^{\rm TT},$$

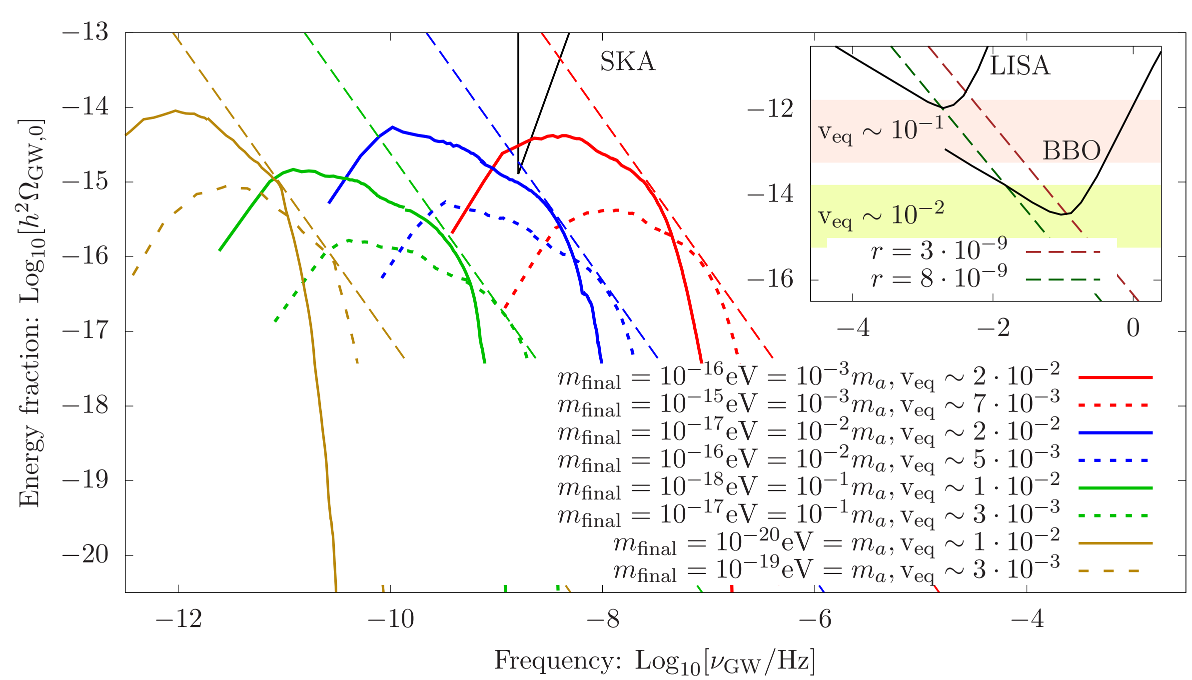
$$\Pi_{ij}^{\rm TT}(t,\vec{x}) = \frac{1}{a^2}\left[\partial_i\phi(t,\vec{x})\partial_j\phi(t,\vec{x}) - \frac{1}{3}\delta_{ij}(\partial_k\phi(t,\vec{x})\partial_k\phi(t,\vec{x}))\right]$$

Examples.

Machado et al,
1811.01950
'Audible axions'
(Excite dark photon)



Chatrchyan, Jaeckel
2004.07844



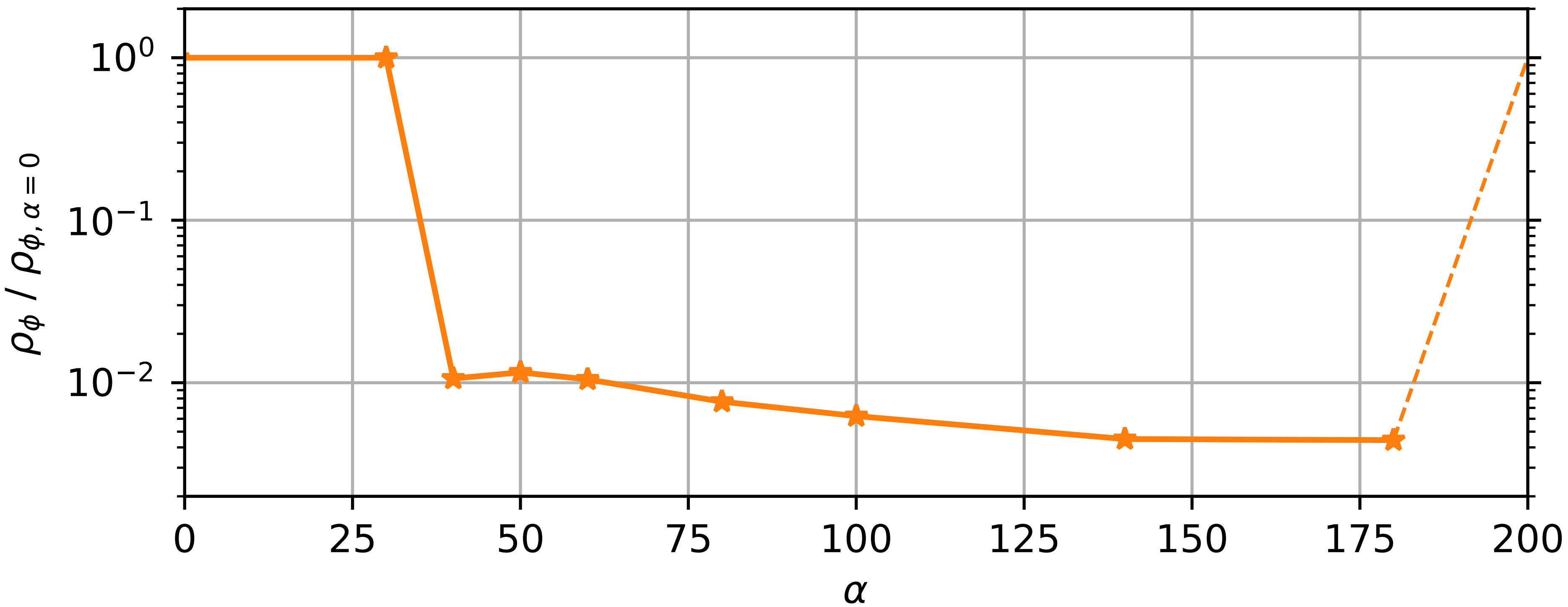
However.

The signal is generally suppressed when imposing the upper bounds from either the axion dark matter abundance or the axion dark radiation.

Schwaller et al, 2012.11584 (from coupling to dark photon)
Eroncel et al, 2206.14259
Geller et al, 2307.03724

—>Dilution of ALP energy density needed, not easy

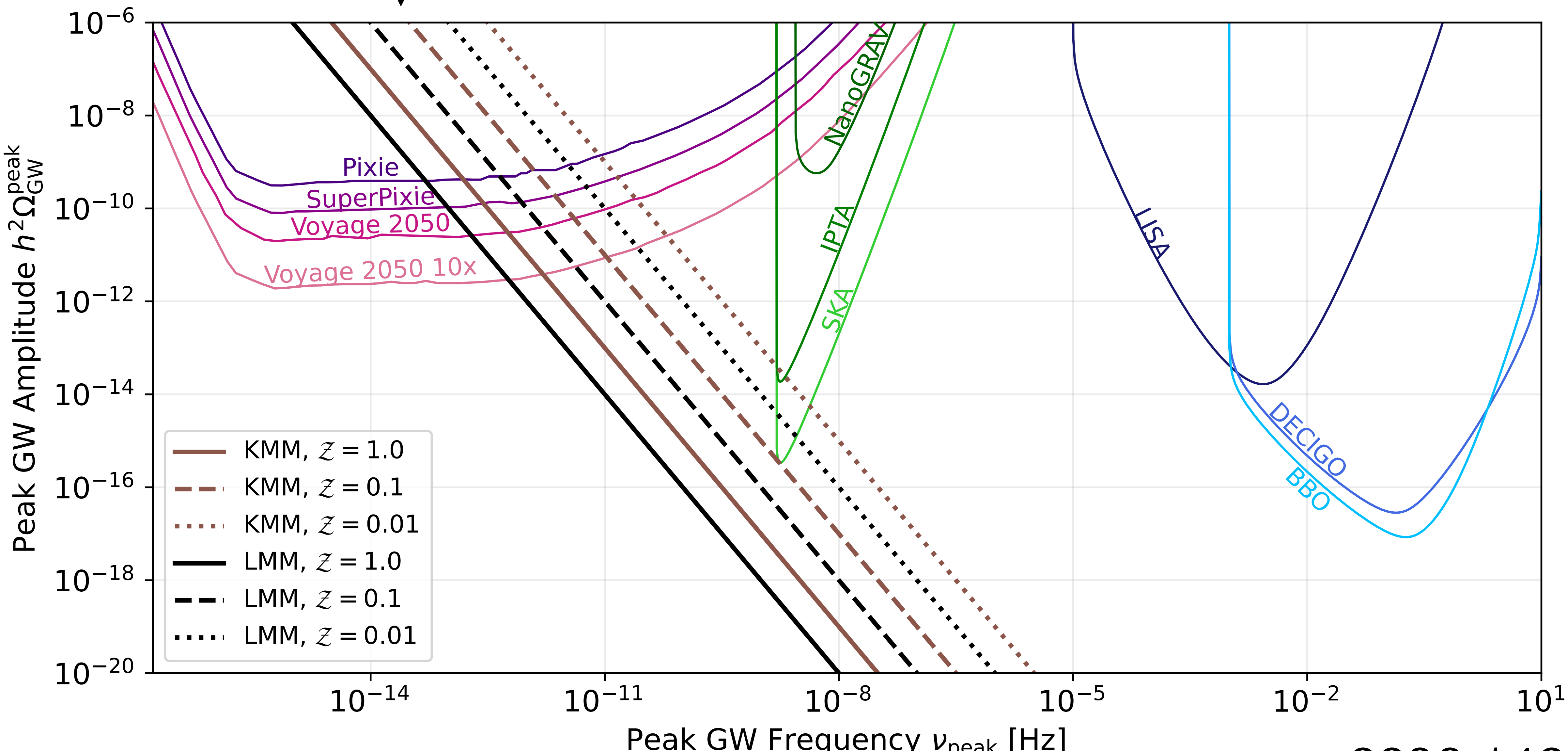
Achieved dilution factor of ALP energy density



Ratzinger, Schwaller, Stefanek, 2012.11584

Gravitational waves from ALP DM fragmentation.

position of the peak of the GW spectrum



2206.14259

$\mathcal{Z}$ = needed dilution factor of ALP energy density

(Can arise from non-linear effects after the fragmentation.)

Conclusion.

Very rich spectrum of GW signatures as tests of the early cosmological history

Gravitational waves: complementary probes of

- Cosmological phase transitions

- Early equation of state of the universe

- Scalar field dynamics (before/during/after inflation)

- Scalar Dark Matter production mechanism (from misalignment or from decay of defects)

Extra material.

Stochastic GW background of primordial origin.

Consider the cosmological perturbation theory on the isotropic-homogeneous expanding Universe, described by the Friedmann-Robertson-Walker metric,

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j,$$

GW: tensor perturbation satisfying the transverse traceless condition

The equation-of-motion follows from the linearized Einstein equation,

$$\ddot{h}_{ij}(\mathbf{x},t) + 3H\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}h_{ij}(\mathbf{x},t) = 16\pi G\Pi^{\mathrm{TT}}_{ij}(\mathbf{x},t),$$

transverse-traceless part of the

anisotropic stress tensor defined by

$$a^2\Pi_{ij} = T_{ij} - pa^2(\delta_{ij} + h_{ij})$$

Fourier decomposition:

$$\ddot{h}_{ij}(\mathbf{k},t) + 3H\dot{h}_{ij}(\mathbf{k},t) + \frac{k^2}{a^2}h_{ij}(\mathbf{k},t) = 16\pi G\Pi^{\mathrm{TT}}_{ij}(\mathbf{k},t),$$

2 limits:

$$h_\lambda(\mathbf{k},\tau) = \begin{cases} \frac{A_\lambda(\mathbf{k})}{a(\tau)}e^{ik\tau} + \frac{B_\lambda(\mathbf{k})}{a(\tau)}e^{-ik\tau}, & \text{for } k \gg aH \ \text{(sub-horizon)}, \\ A_\lambda(\mathbf{k}) + B_\lambda(\mathbf{k})\int^\tau \frac{d\tau'}{a^2(\tau')}, & \text{for } k \ll aH \ \text{(super-horizon)}, \end{cases}$$

oscillatory behavior redshifted by expansion

stays frozen and later re-enters the horizon and starts oscillating

$d\tau \equiv dt/a$ is the conformal time

Stochastic GW background of primordial origin.

Early-universe production process operates within a causal patch ($\lambda_{GW} \leq H^{-1}$), much smaller than the horizon size today,

$$\frac{\lambda_{\rm GW,0}}{H_0^{-1}} \leq \frac{H_{\rm prod}^{-1}}{H_0^{-1}} \left[\frac{a_0}{a_p}\right] \simeq \Omega_{r,0}^{-1/2} \left[\frac{T_0}{T_p}\right] \simeq 2 \cdot 10^{-13} \left[\frac{100 \text{ GeV}}{T_p}\right],$$

Primordial GW sources from many uncorrelated patches randomize the amplitude of h_{ij} (x, t) observed today and contribute to the *stochastic GW background*.

For an isotropic, homogeneous, unpolarized, stationary, and gaussian background, the correlation function reads $\langle h_{ij}(\mathbf{x},\tau)\, h_{ij}(\mathbf{x},\tau)\rangle = 2 \int d(\log k)\, h_c^2(k,\tau)$

dimensionless characteristic strain

$$\rho_{\rm GW} = \frac{\left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle}{32\pi G} = \frac{\left\langle h'_{ij}(\mathbf{x},\tau) h'_{ij}(\mathbf{x},\tau)\right\rangle}{32\pi G a^2}.$$

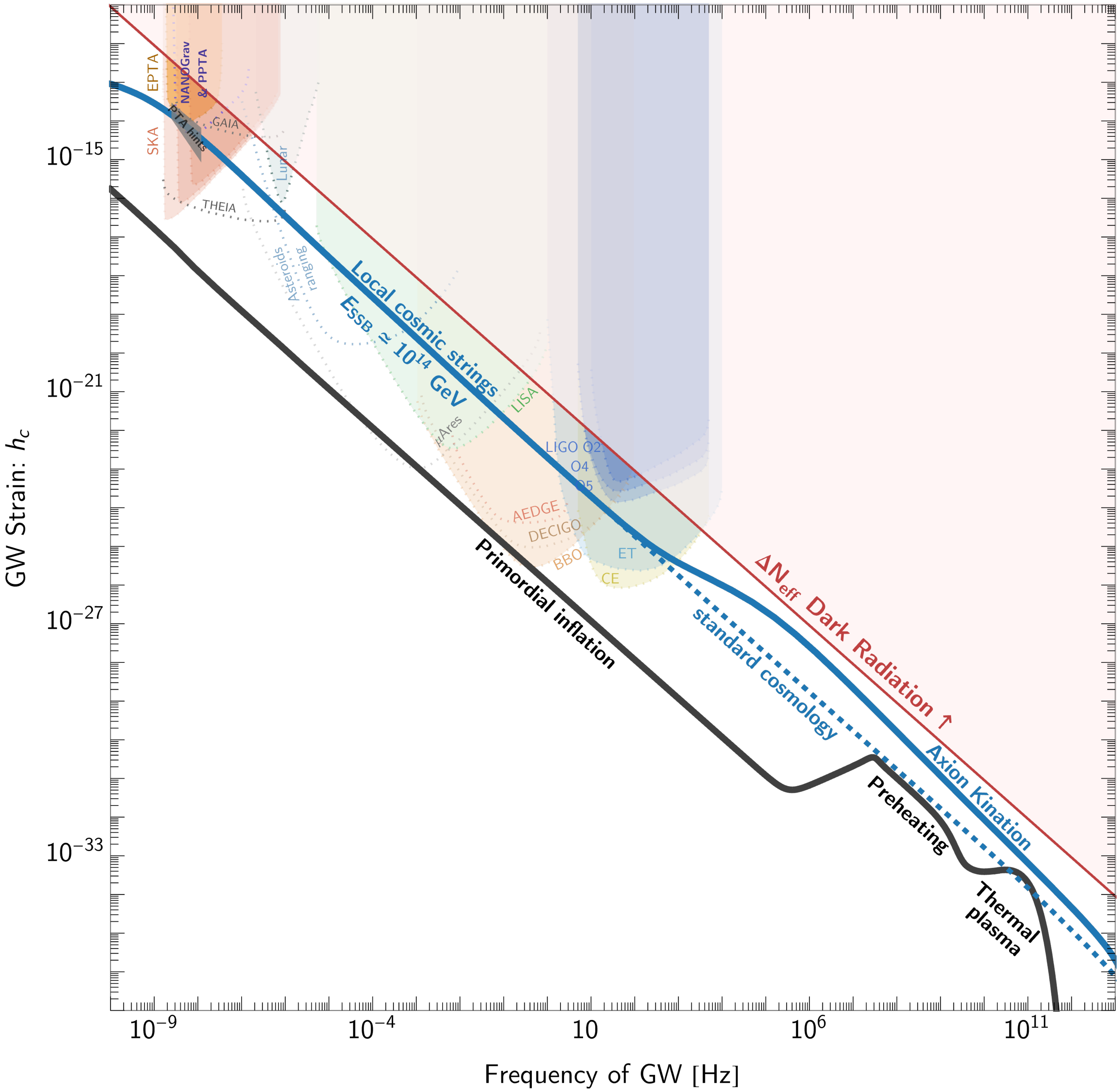
$$\rho_{\rm GW} = \int d(\log k)\frac{k^2 h_c^2(k,\tau)}{16\pi G a^2(\tau)} \equiv \int d(\log k)\frac{d\rho_{\rm GW}}{d\log k},$$

$$\Omega_{\rm GW,0}(f) = \frac{k^2 h_c^2(k,\tau_0)}{16\pi G a_0^2} = \frac{\rho_{\rm GW}^{\rm prod}(f)}{\rho_{\rm tot,0}}\left(\frac{a_{\rm prod}}{a_0}\right)^4,$$

$$h_c \simeq 1.26 \times 10^{-18} ({\rm Hz}/f_{\rm GW})\sqrt{\Omega_{\rm GW} h^2}.$$

Due to $h^2_c \sim a^{-2}$ for sub-horizon mode, *the GW energy density of some mode* k *red-shifts as radiation* a^{-4}.

$$h_c \simeq 1.26 \times 10^{-18}(\mathrm{Hz}/f_{\mathrm{GW}})\sqrt{\Omega_{\mathrm{GW}} h^2}.$$



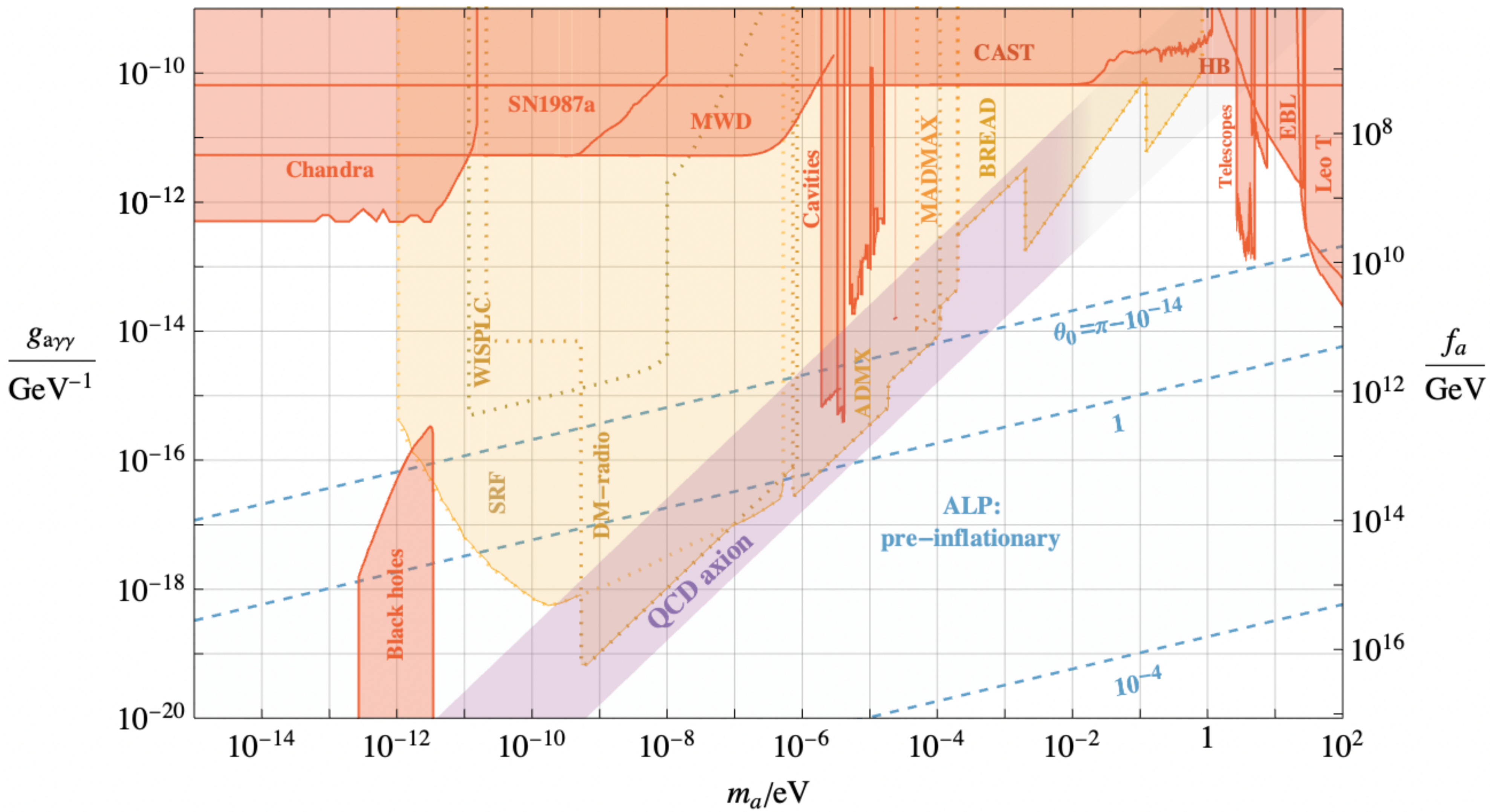
LOCAL STRINGS vs GLOBAL STRINGS.

With respect to local strings, the GW spectrum from global strings in standard radiation cosmology is:

- suppressed by the shorter Hubble time $\tilde{t}_M$ at the time of GW emission: factor $\tilde{t}_{\rm M}^{\rm global}/\tilde{t}_{\rm M}^{\rm local} \propto G\mu_{\rm local} \propto \left(\eta/M_{\rm pl}\right)^2$,
- suppressed by the larger GW redshift factor since emission occurs earlier: factor $\left[\frac{a\left(\tilde{t}_{\rm M}^{\rm global}\right)}{a\left(\tilde{t}_{\rm M}^{\rm local}\right)}\right]^4 \propto \left(\eta/M_{\rm pl}\right)^4$,
- enhanced by the lower loop redshift factor since GW emission occurs right after loop production: factor $\left(a\left(\tilde{t}_{\rm M}^{\rm local}\right)/a\left(\tilde{t}_{\rm M}^{\rm global}\right)\right)^3 \propto \left(\eta/M_{\rm pl}\right)^{-3}$,
- increased by the logarithmically-enhanced GW power emission rate: factor $\log^2\left(\eta t_i\right)$,
- increased by the logarithmically-enhanced loop lifetime: factor $\log\left(\eta t_i\right)$.

Slide by Marco Gorghetto

ALPs: Targets for haloscopes



$$\frac{\Omega_a^{\rm mis}}{\Omega_{\rm DM}} \simeq 2.2 \cdot 10^{-3} \left(\frac{f_a}{10^{12}\ \text{GeV}}\right)^2 \left(\frac{m_a}{10^{-6}\ \text{eV}}\right)^{1/2} h(\theta_0)\,\theta_0^2$$

$$g_{a\gamma\gamma} \simeq \frac{\alpha_{em}}{2\pi f_a}$$

The case $N_{DW} > 1$.

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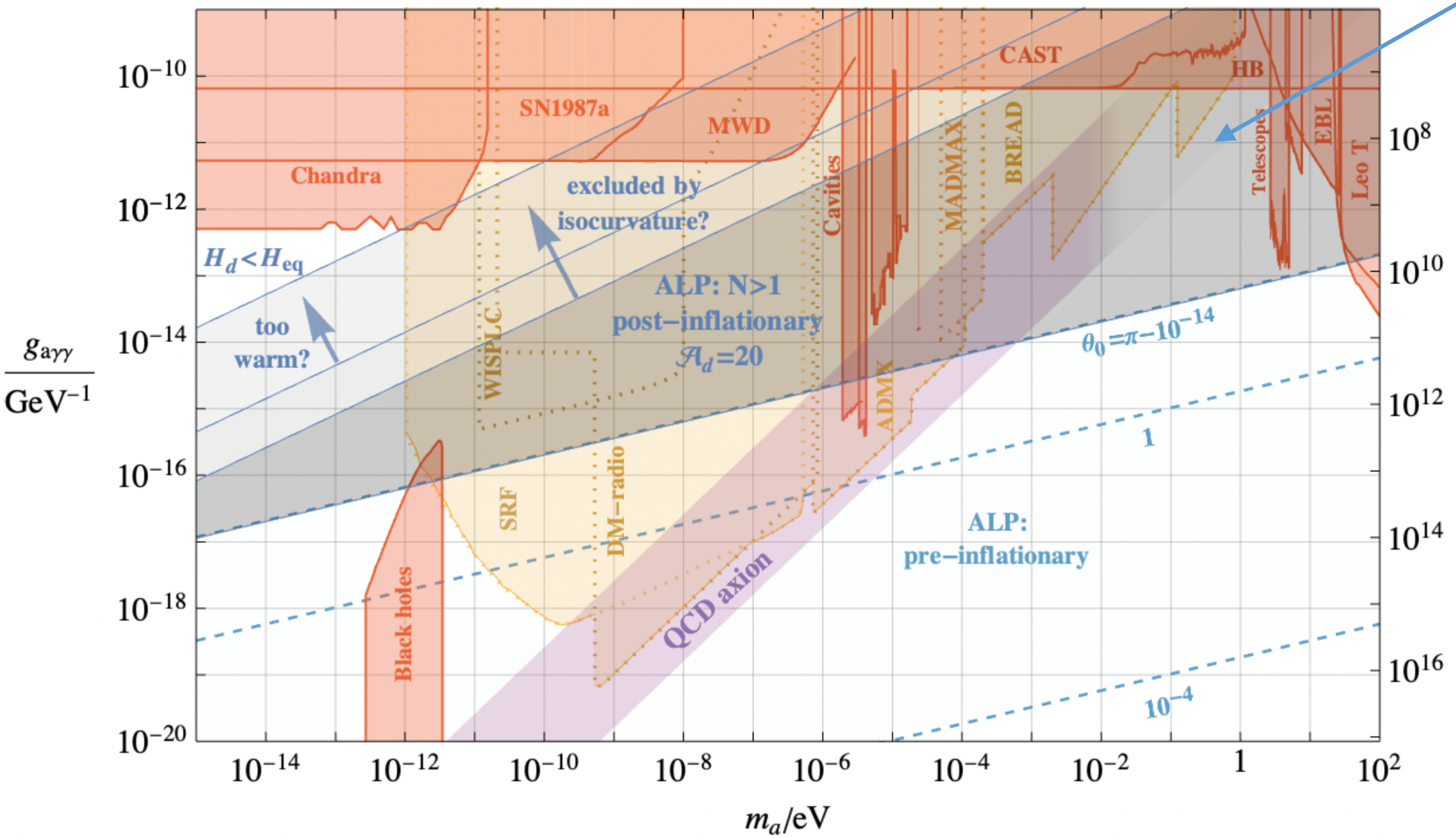
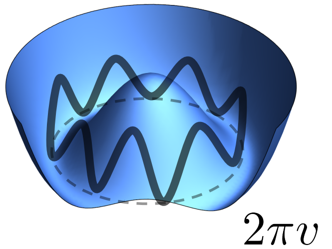
ALPs: Targets for haloscopes



Domain wall number



$v = N f_a$

$$\frac{\Omega_a}{\Omega_{DM}} \simeq 2\left(\frac{\mathcal{A}_d}{20}\right)\left(\frac{m_a}{H_d}\right)^{1/2}\left(\frac{f_a}{10^{12}\ \text{GeV}}\right)^2\left(\frac{m_a}{10^{-6}\ \text{eV}}\right)^{1/2}$$

[2212.13263]

M. Gorghetto, E.Hardy

Equation of motion of complex scalar field in expanding universe .

$$\ddot{\Phi} - a^{-2}\nabla^2\Phi + 3H\dot{\Phi} + \frac{\partial V}{\partial \Phi^\dagger} = 0$$

with $\Phi = \phi e^{i\theta}$

$$\ddot{\phi} - a^{-2}\nabla^2\phi + 3H\dot{\phi} + V'(\phi) = \phi\dot{\theta}^2 - a^{-2}\phi(\nabla\theta)^2,$$
$$\phi\ddot{\theta} - a^{-2}\phi\nabla^2\theta + 3H\phi\dot{\theta} = -2\dot{\phi}\dot{\theta} + 2a^{-2}\nabla\phi\nabla\theta.$$

For homogeneous field, these are Kepler problem:

centrifugal force

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = \phi\dot{\theta}^2$$

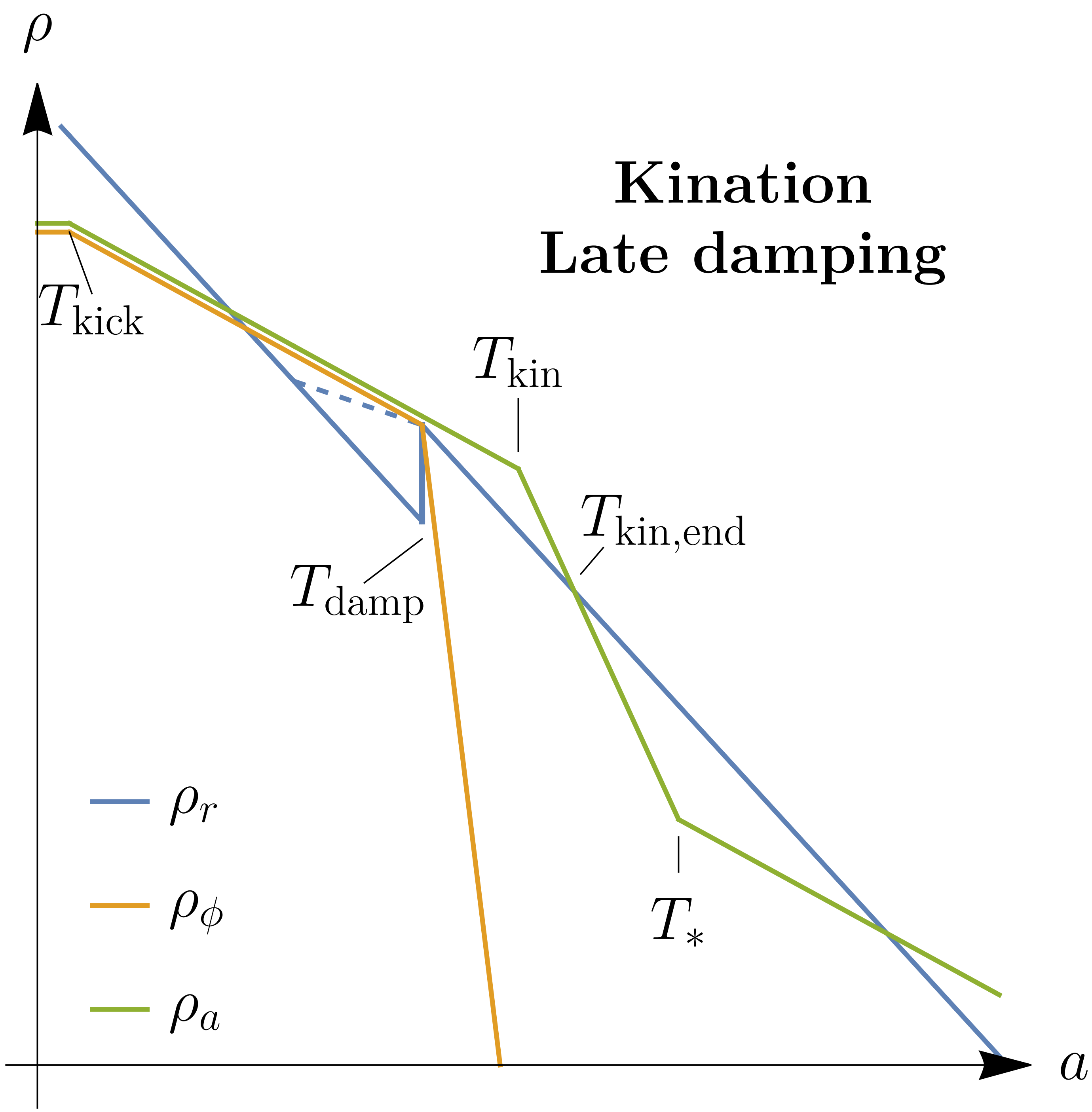
coriolis force

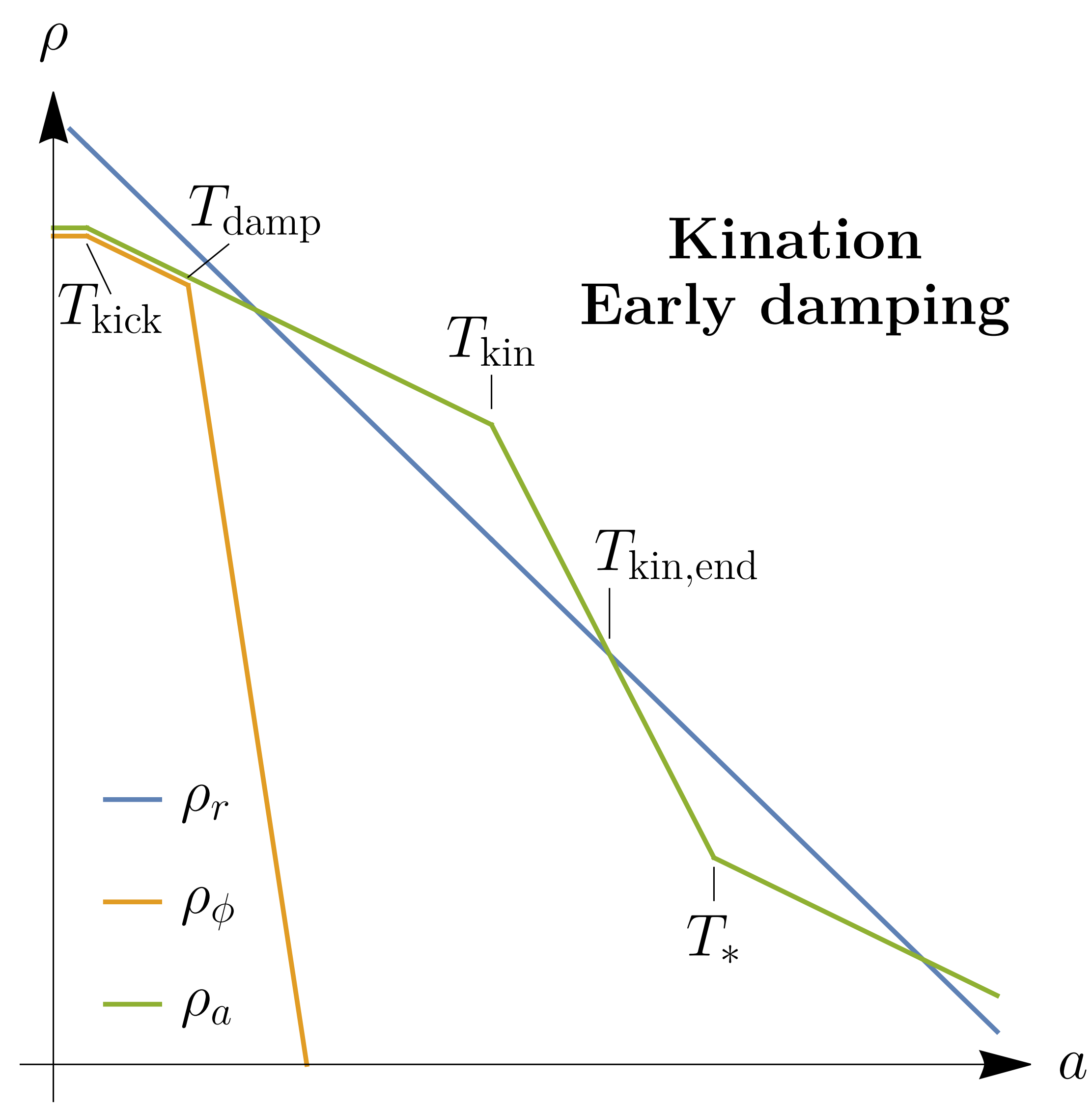
$$\ddot{\theta} + 3H\dot{\theta} = -2\frac{\dot{\phi}}{\phi}\dot{\theta}$$

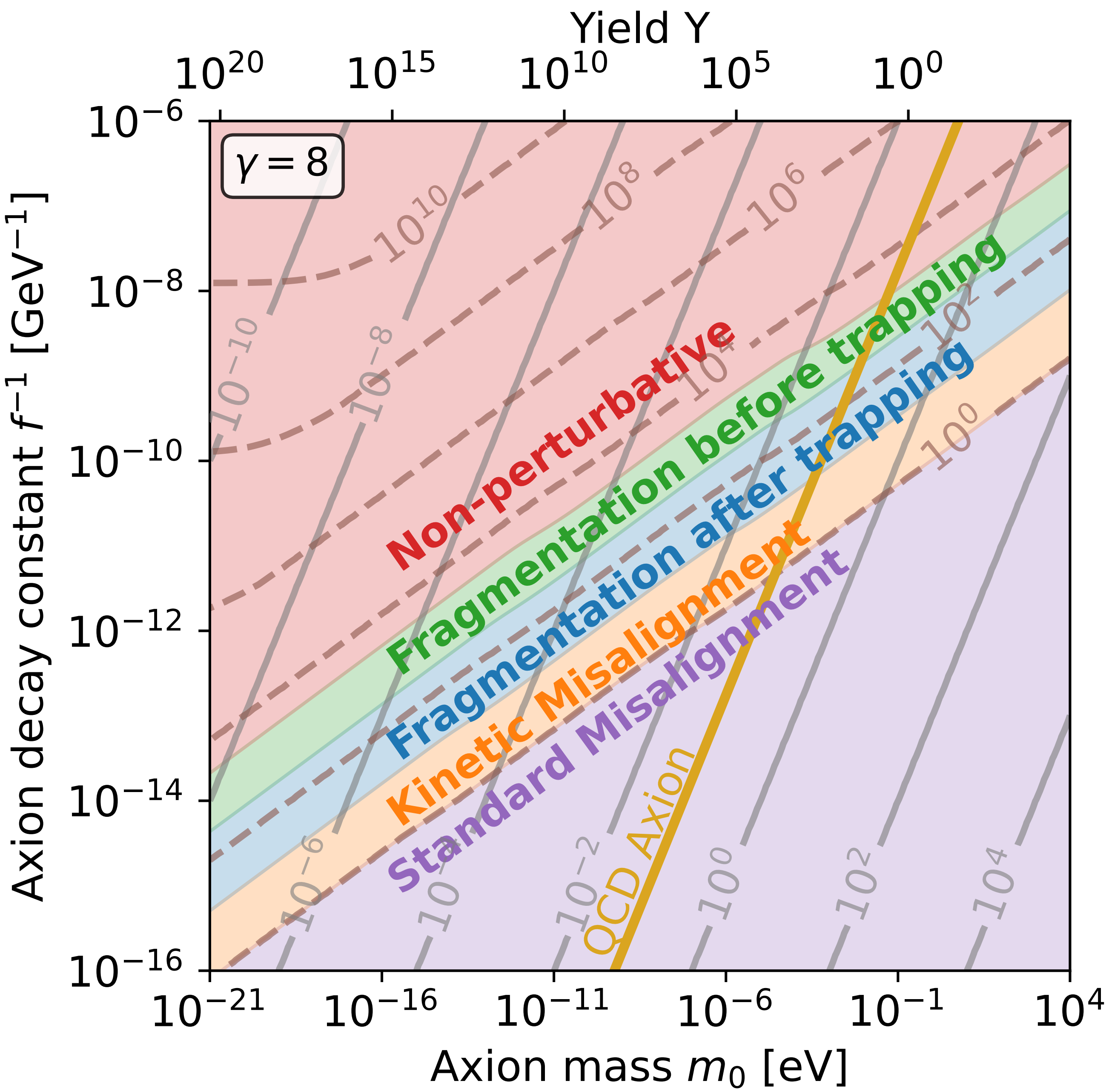
conservation of charge (angular momentum):

$$\frac{d}{dt}(a^3\phi^2\dot{\theta}) = 0$$

Ingredient 4 for kination: Damping of radial mode energy





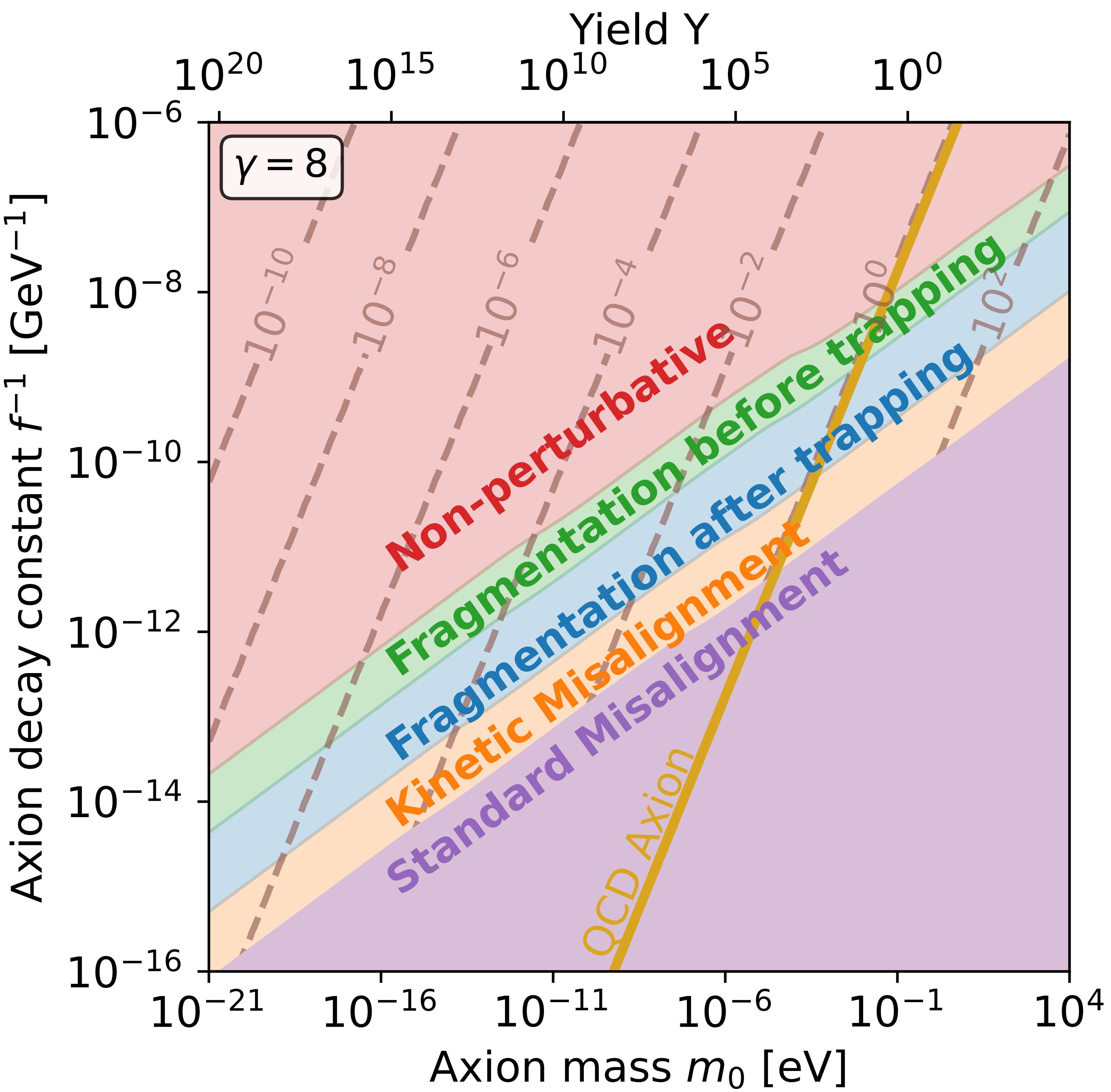


Solid lines: contours of zero-temperature barrier heights,

Dashed lines: (m/3H)_* contours

2206.14259

Contours of trapping temperature in GeV.



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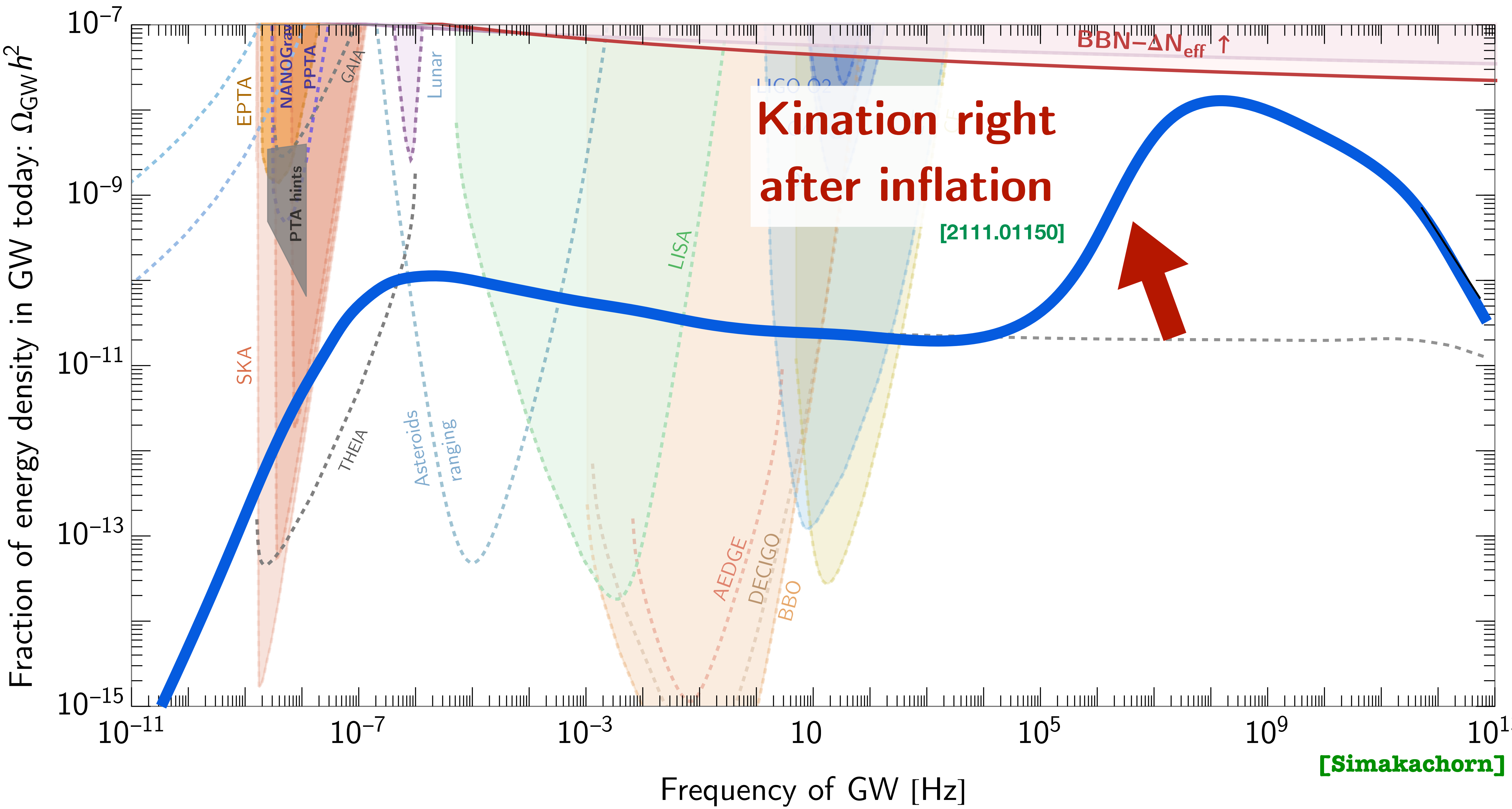
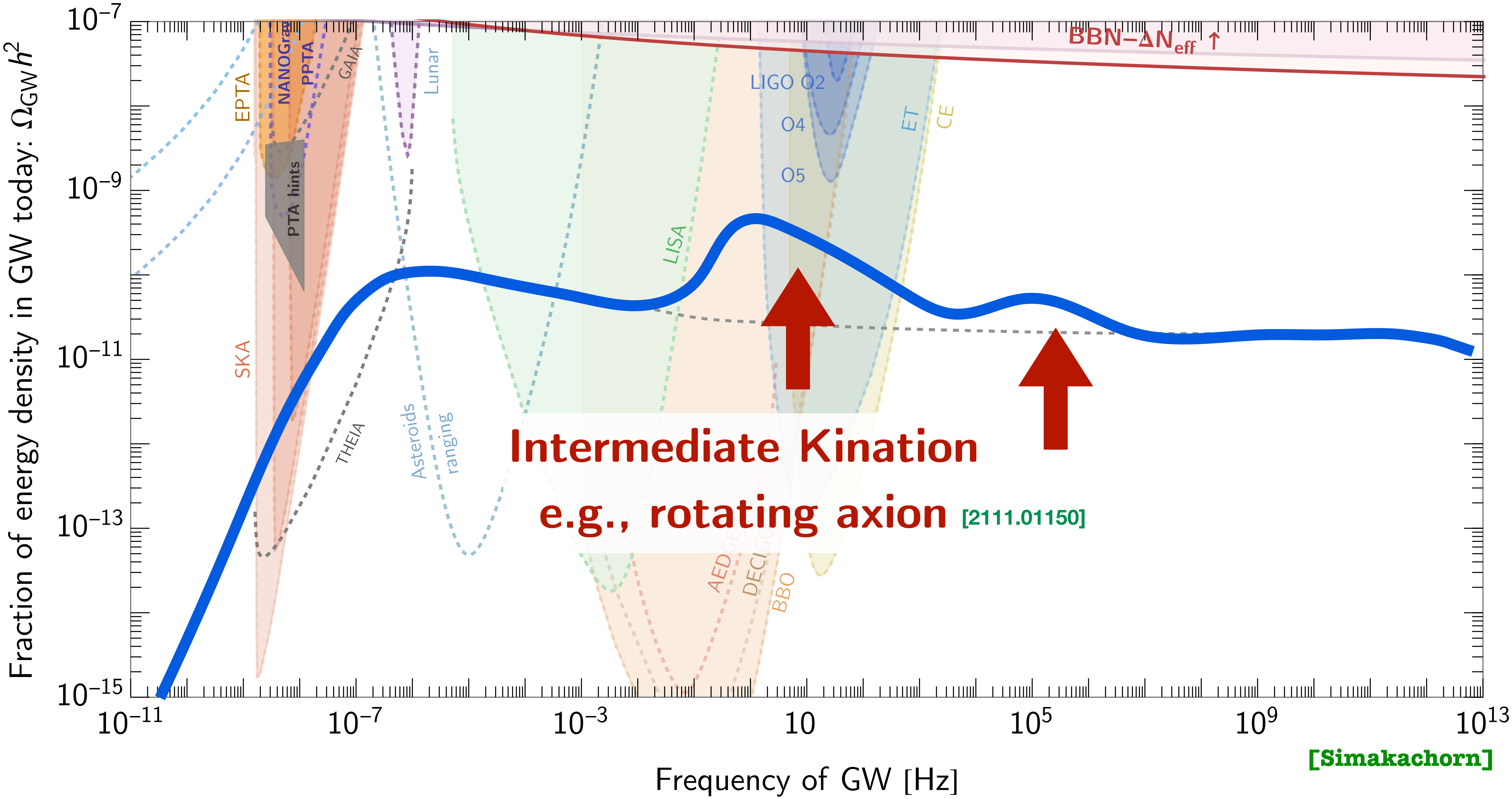
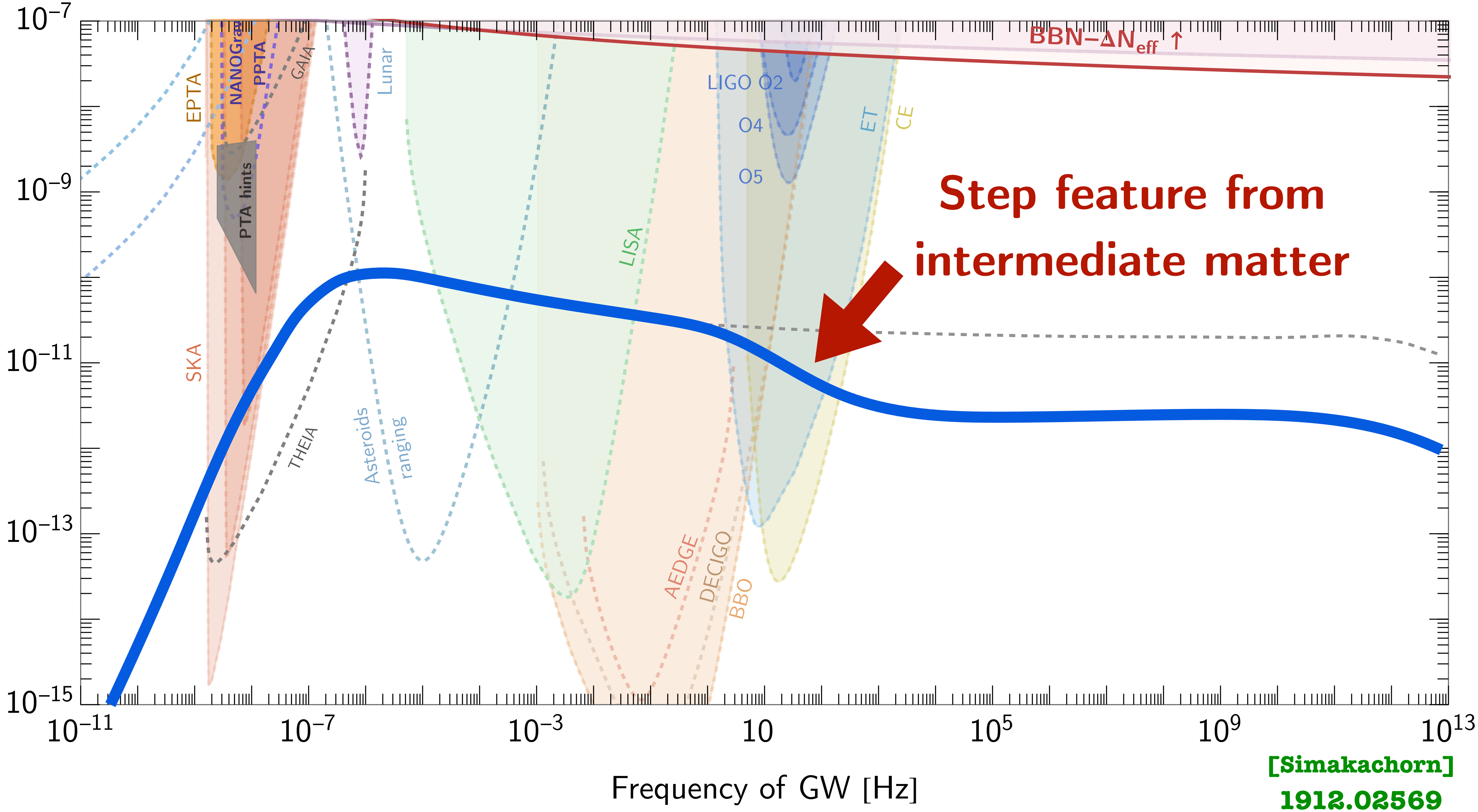
Gravitational Waves from cosmic strings in non-standard cosmology (kination after inflation).



Gravitational Waves from cosmic strings in non-standard cosmology induced by rotating axions.

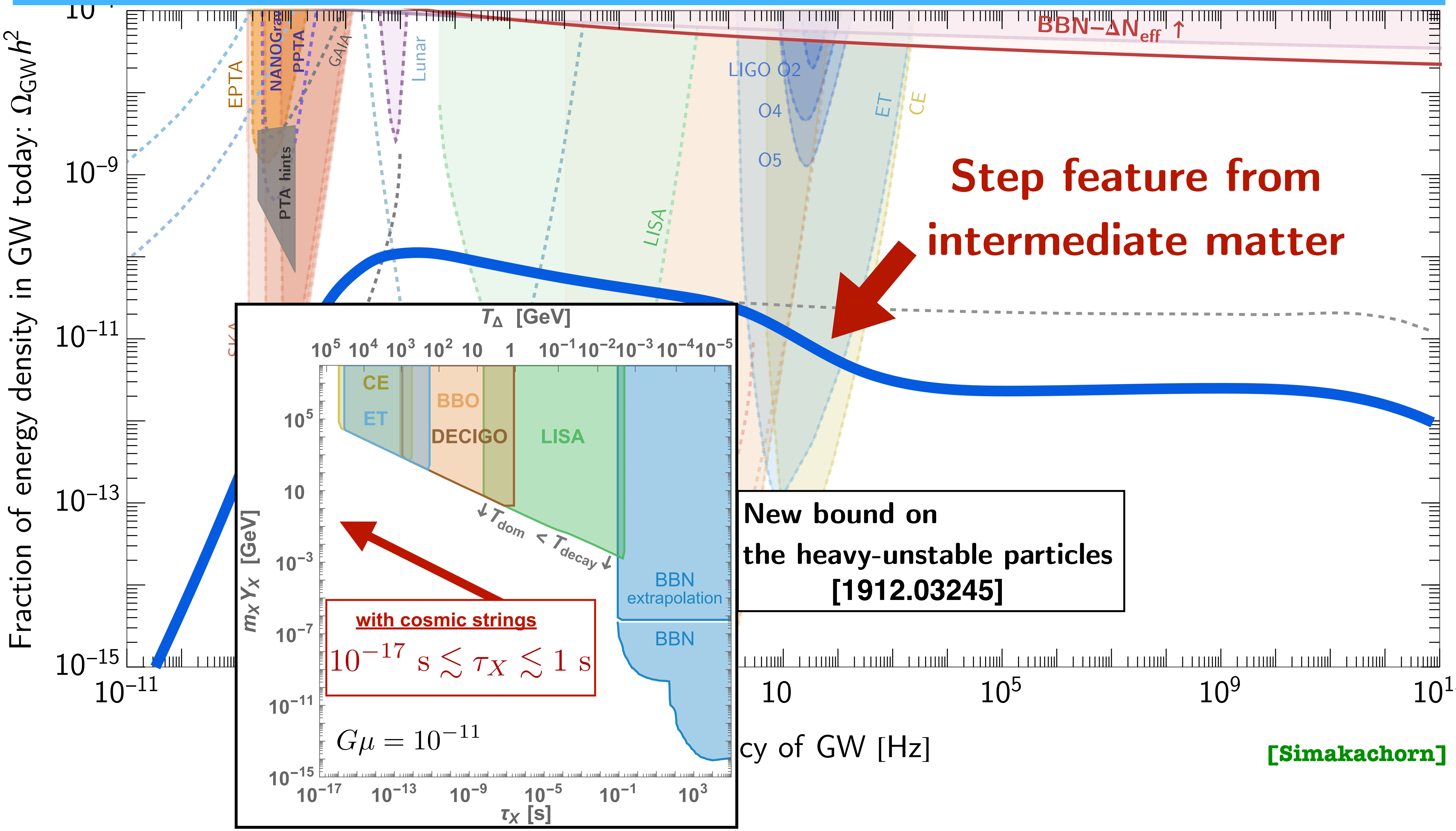


Gravitational Waves from cosmic strings in non-standard cosmology.



[Simakachorn] 1912.02569

Gravitational Waves from cosmic strings in non-standard cosmology.



Axion kinetic misalignment:

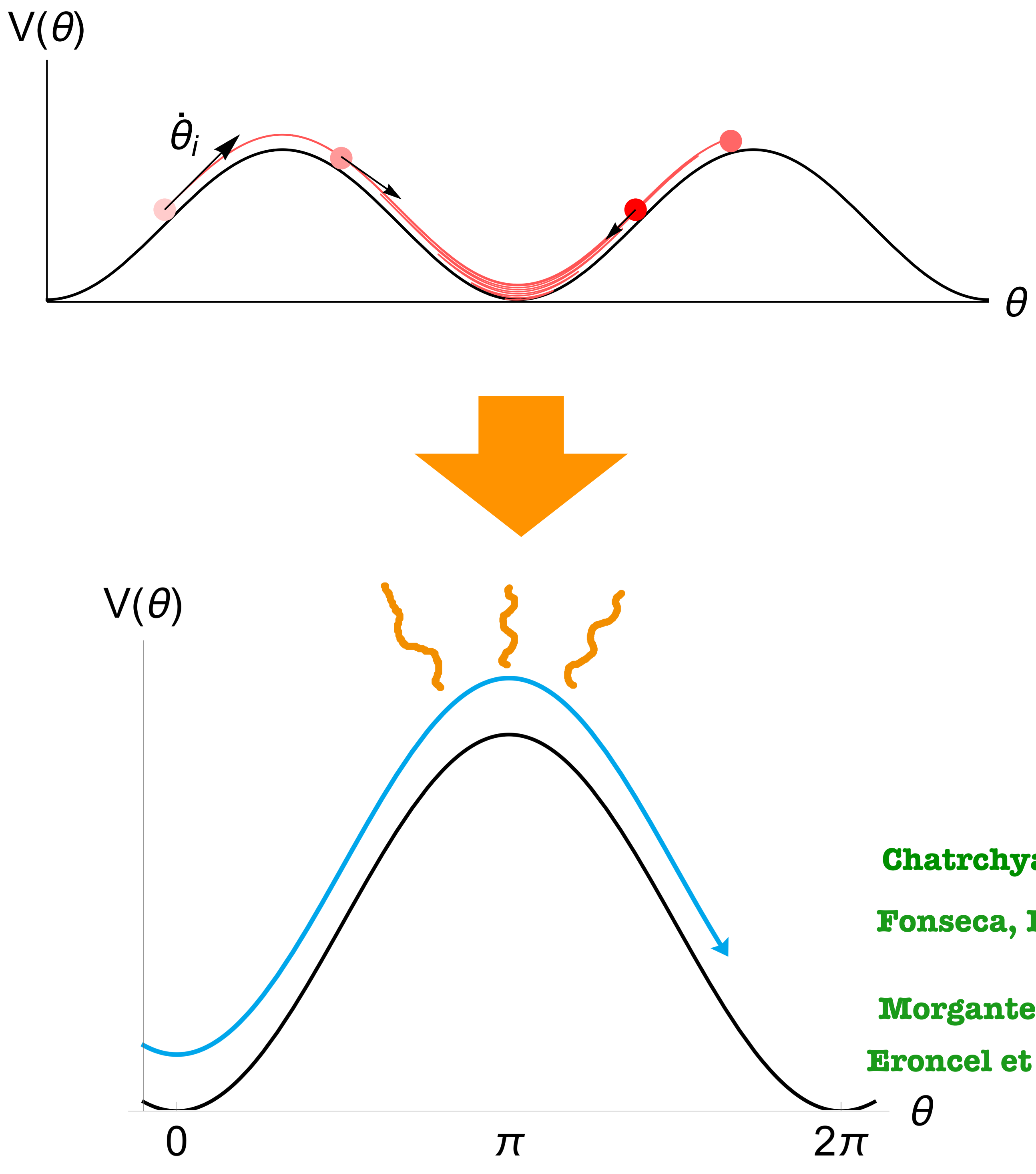
↓

Axion fragmentation.

↓

Compact axion halos.

Axion fragmentation .



Chatrchyan et al, 1903.03116, 2004.07844

Fonseca, Morgante, Sato, Servant, 1911.08472, 1911.08473

Morgante et al, 2109.13823

Eroncel et al'22, 2206.14259

Axion Fragmentation.

Not considered in usual axion phenomenology with oscillations around one minimum: Fragmentation suppressed unless the field starts very close to the top of the potential ("large misalignment mechanism") or for specific potentials with more than one cosine -> parametric resonance.

Greene, Kofman, Starobinsky, hep-ph/9808477
Chatrchyan et al, 1903.03116, 2004.07844
Arvanitaki et al, 1909.11665

However, becomes very relevant when field crosses many wiggles, with interesting implications, e.g. for the relaxion mechanism, but also as a new axion Dark Matter production mechanism.

Chatrchyan et al, 1903.03116, 2004.07844
Fonseca,Morgante,Sato, Servant'19
Morgante et al, 2109.13823

Generalization Eroncel et al, 2206.14259
(fragmentation before and after trapping + detailed application to DM)

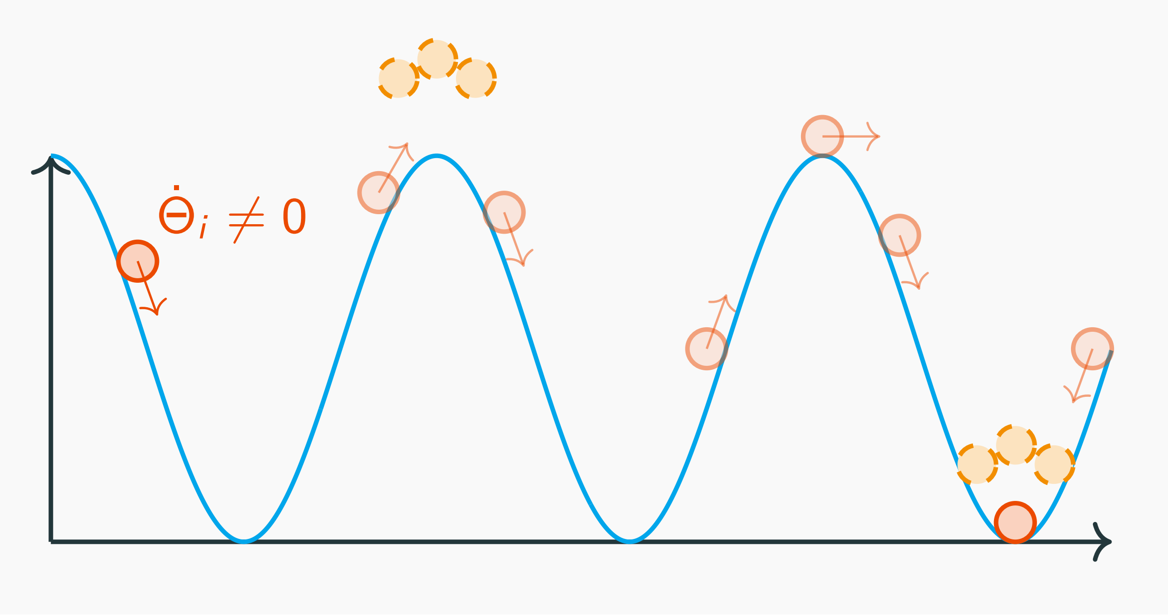
ALP fluctuations.

- Even in pre-inflationary scenario, ALP field has some fluctuations on top of the homogeneous background, which can be described by the mode functions in the Fourier space.

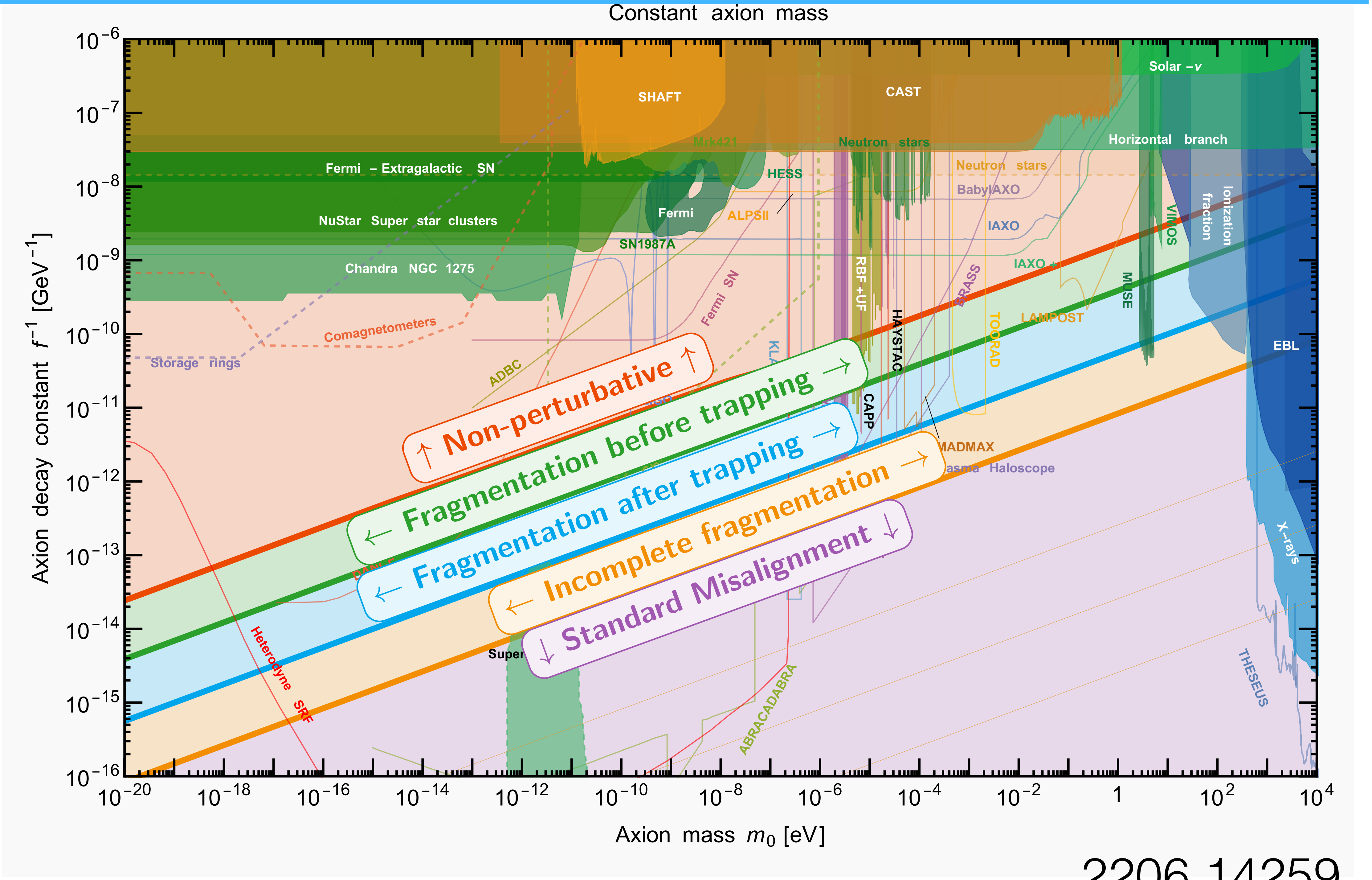
$$\theta(t, \mathrm{x}) = \Theta(t) + \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \theta_k e^{i\vec{k}\cdot\vec{x}} + \mathrm{h.c.}$$

- Even though the fluctuations are small initially, they can be enhanced exponentially later via parametric resonance yielding to fragmentation.

- In the case of efficient fragmentation, all the energy of the homogeneous mode can be transferred to the fluctuations. [Fonseca et al. 1911.08472; Morgante et al. 2109.13823]



Fragmentation regions in ALP parameter space.



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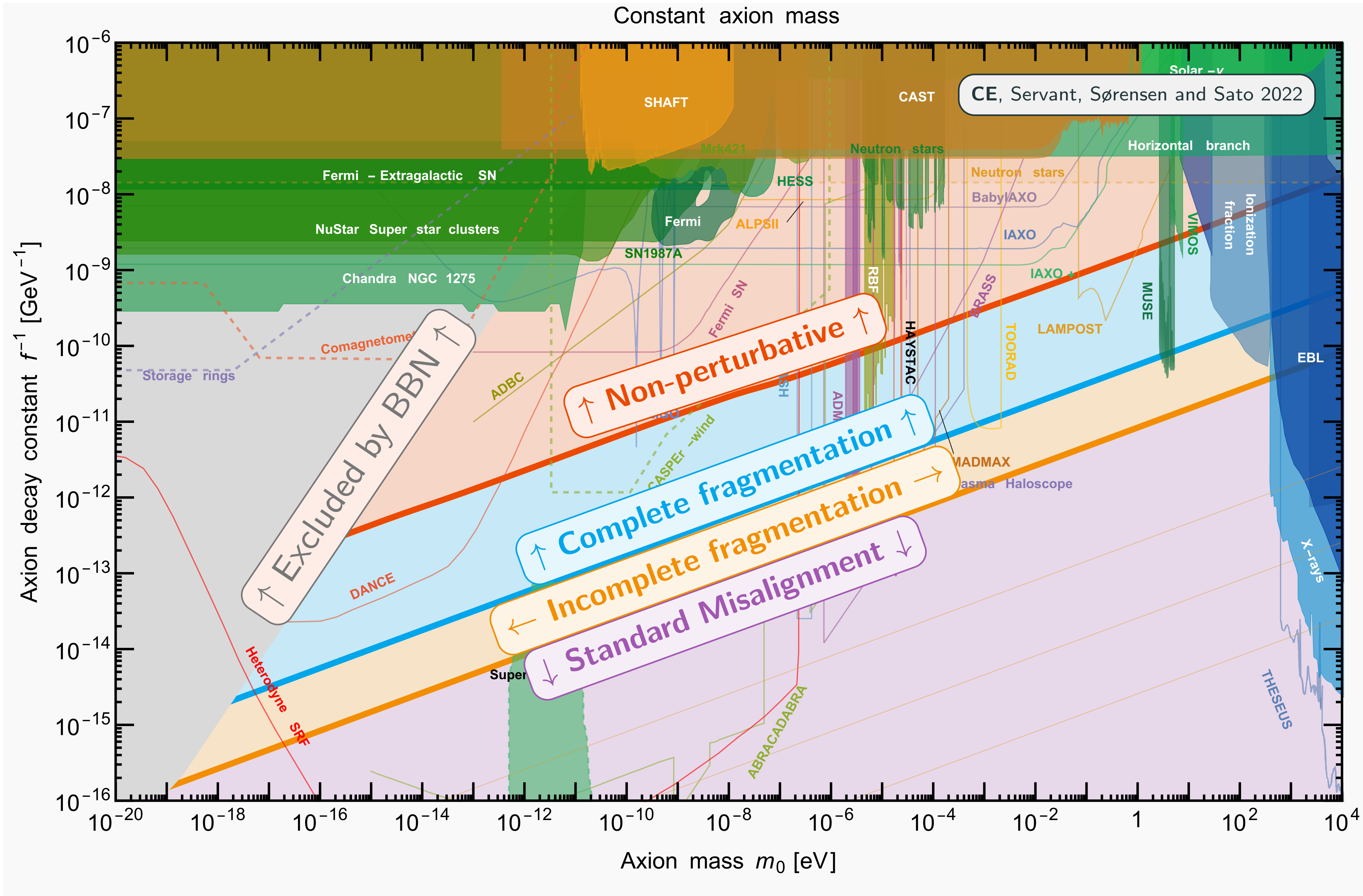
Fragmentation regions in ALP parameter space.



ALP fluctuations.

$$\phi(t,\mathbf{x}) = \bar{\phi}(t) + \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \phi_k e^{i\vec{k}\cdot\vec{x}} + \text{h.c.}$$

EoM for the unavoidable adiabatic perturbations :

$$\ddot{\phi}_k + 3H\dot{\phi}_k + \underbrace{\left[\frac{k^2}{a^2} + V''(\phi)\Big|_{\bar{\phi}}\right]}_{\text{eff. frequency}} \phi_k = \underbrace{2\,\Phi_k\; V'(\phi)\Big|_{\bar{\phi}} - 4\dot{\Phi}_k\dot{\bar{\phi}}}_{\text{source term}}$$

unstable when the effective frequency

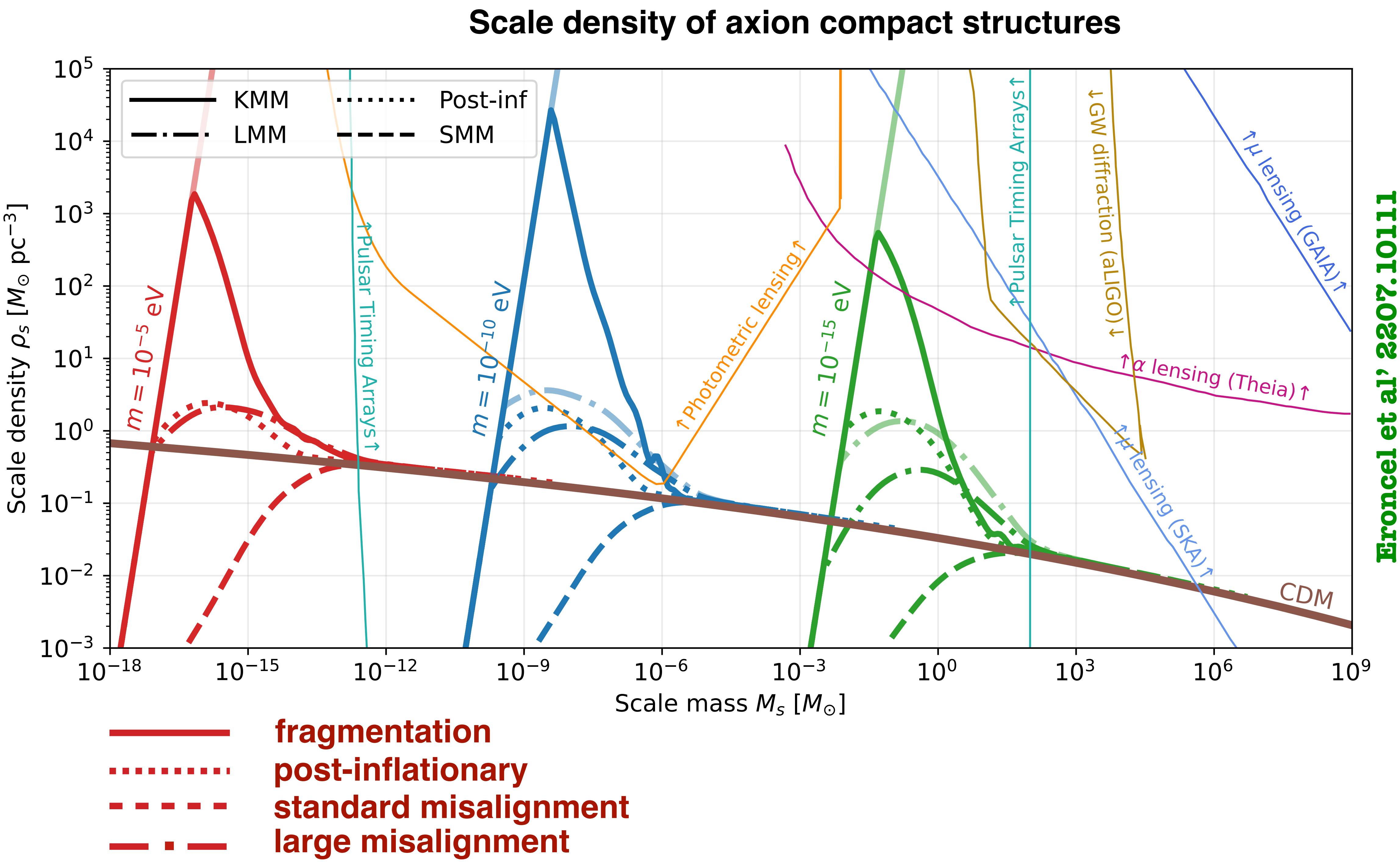
- **becomes negative ⇒ tachyonic instability**
- **is oscillating ⇒ parametric resonance**

Growth rate of the perturbations depend exponentially on $\left.\dfrac{m_\phi}{H}\right|_{\text{osc}}$

Dense and compact ALP mini-clusters can also be formed in the pre-inflationary scenario!

Observational tests: compact axion halos.

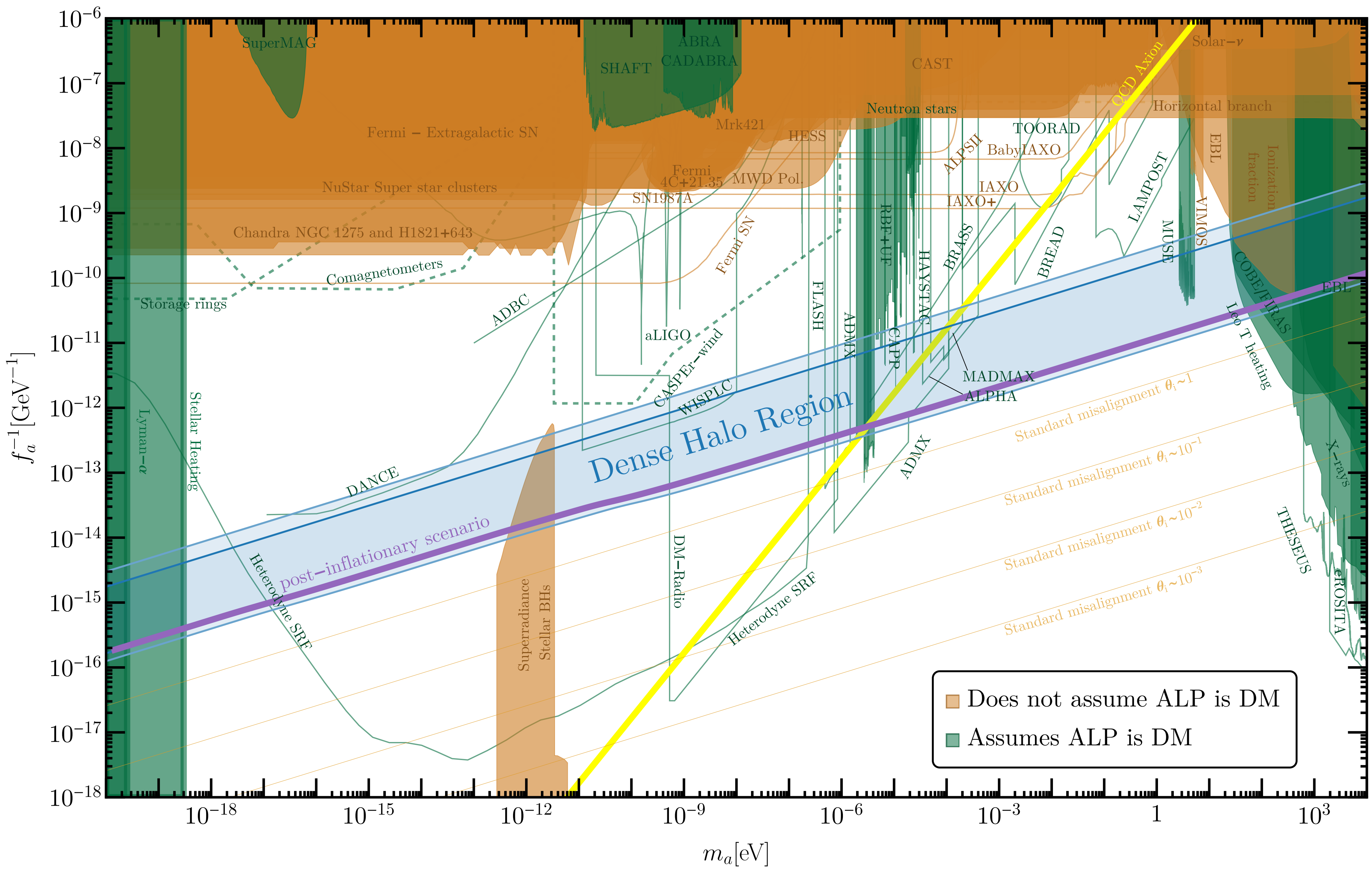
kinetic misalignment—>axion fragmentation-> structure formation enhancement



was studied in the context of large misalignment scenario in [Arvanitaki et al'19]

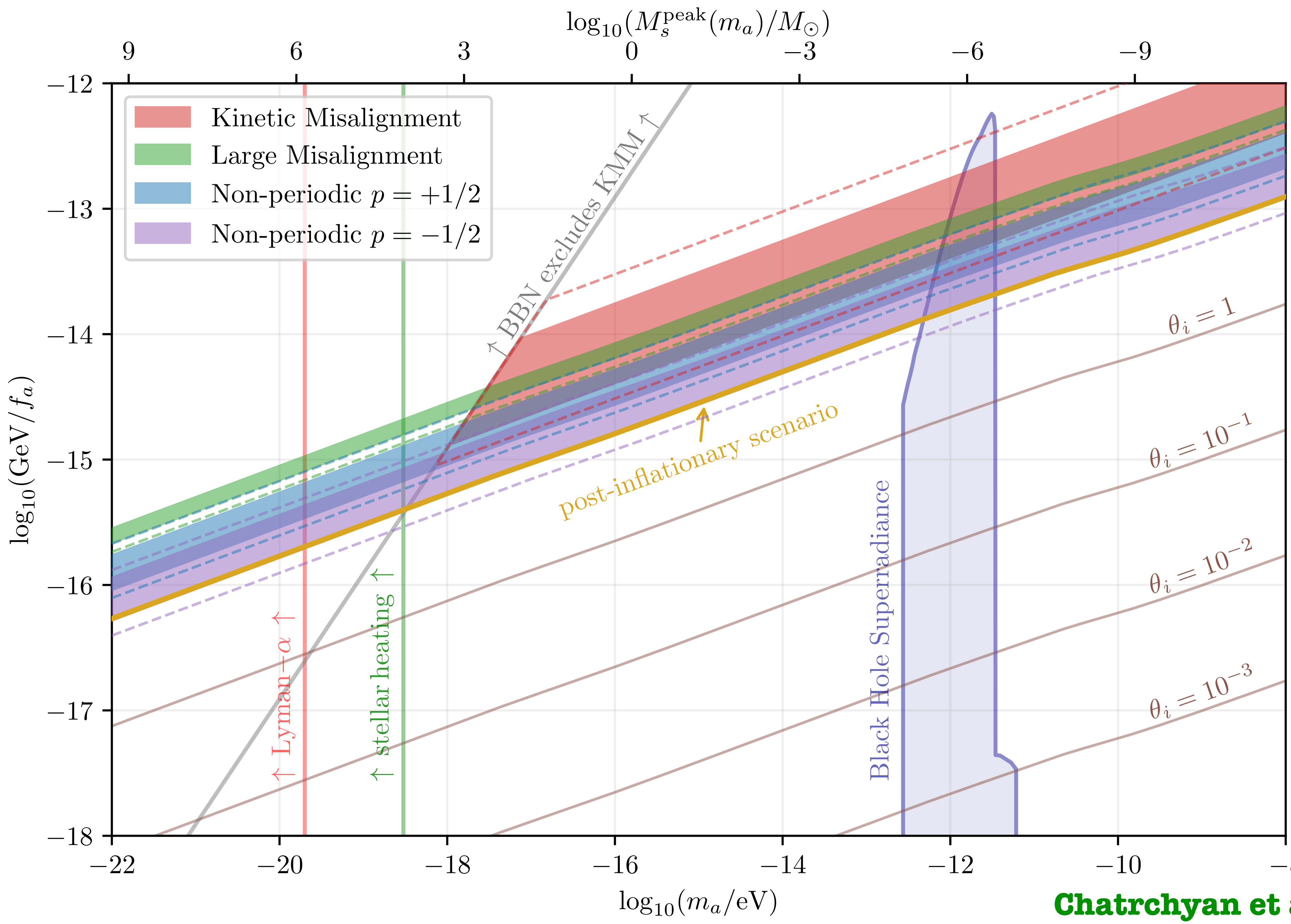
Different in the context of axion kinetic fragmentation: Eroncel et al , 2207.10111

Parameter space where parametric resonance can create compact halos.



Chatrchyan et al, 2305.03756

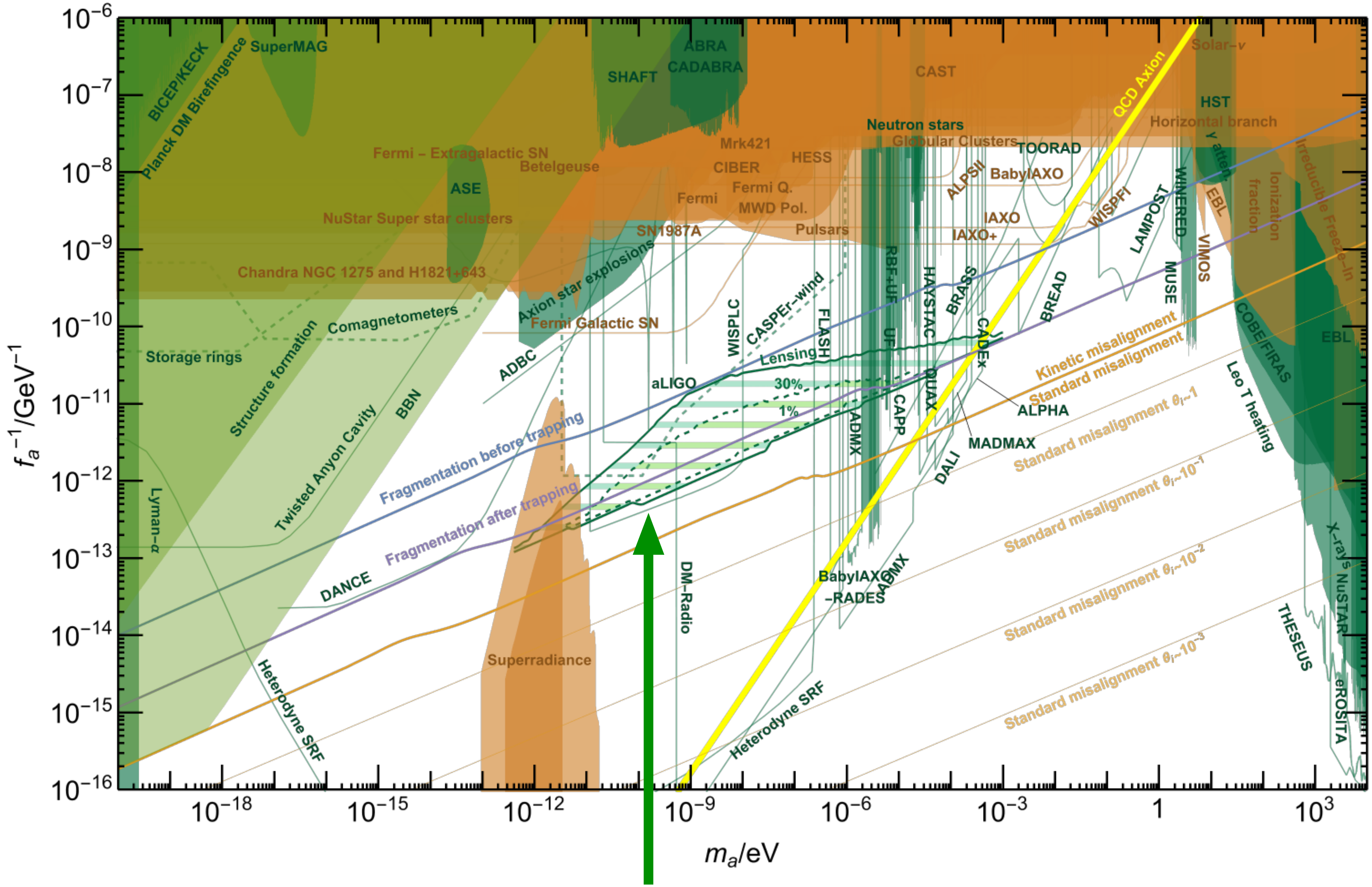
Parameter space where parametric resonance can create compact halos (with $\rho_s \gtrsim 10\, M_\odot\, \mathrm{pc}^{-3}$).



Chatrchyan et al 2305.03756

The dense halo regions from ≠ production mechanisms mostly overlap.
Difficult to infer the producion mechanism from observations.
However, observations of dense structure gives information about f_a even when ALP does not couple to the SM!

Observability of compact halos from kinetic misalignment.



Region that can be probed by photometric lensing