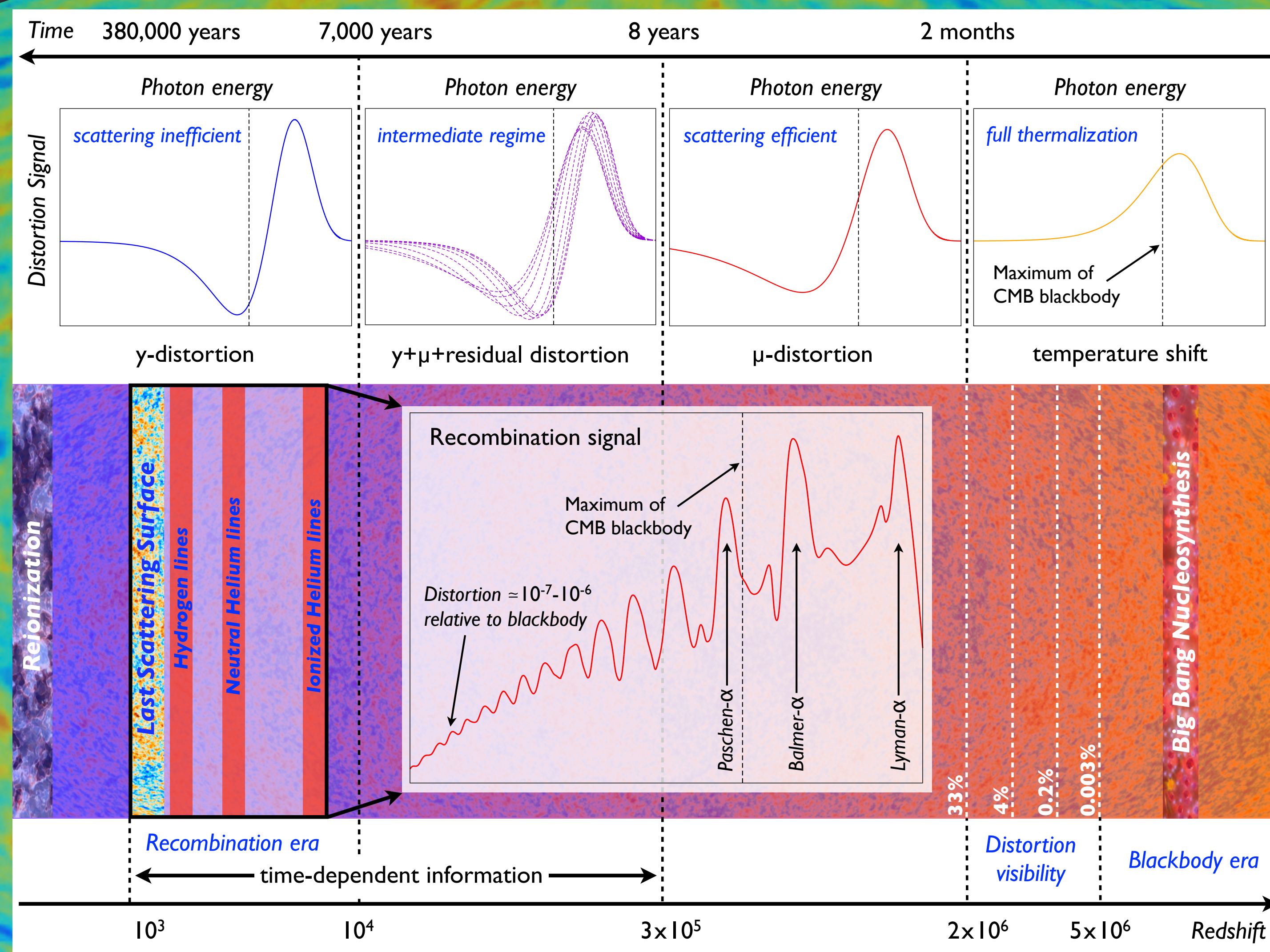
CMB spectral distortion science beyond the Monopole



MANCHESTER 1824
The University of Manchester

Jens Chluba

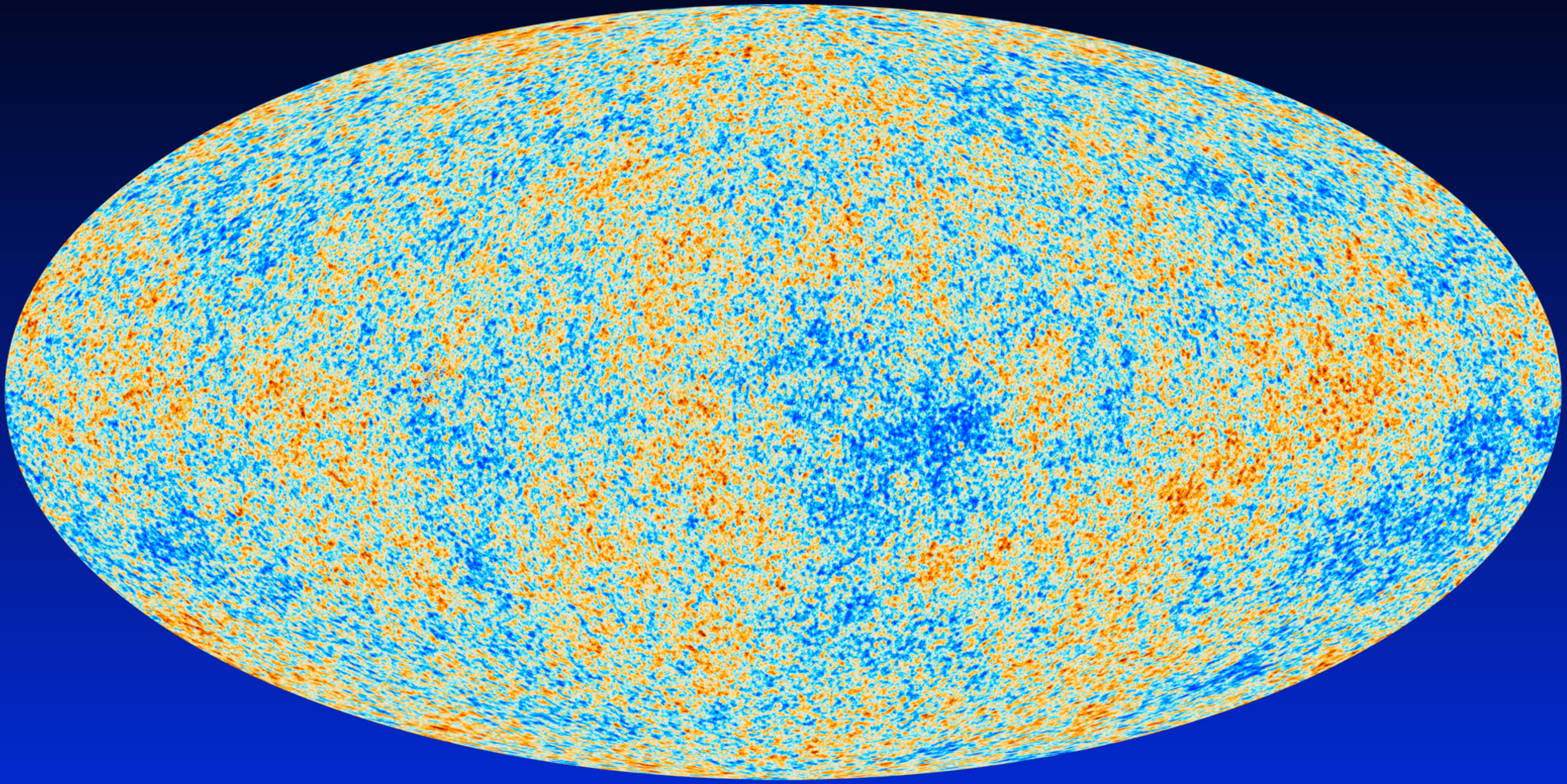
Paris-Saclay AstroParticle Workshop

November 21th 2024

erc THE ROYAL SOCIETY

* CMB ≙ Cosmic Microwave Background

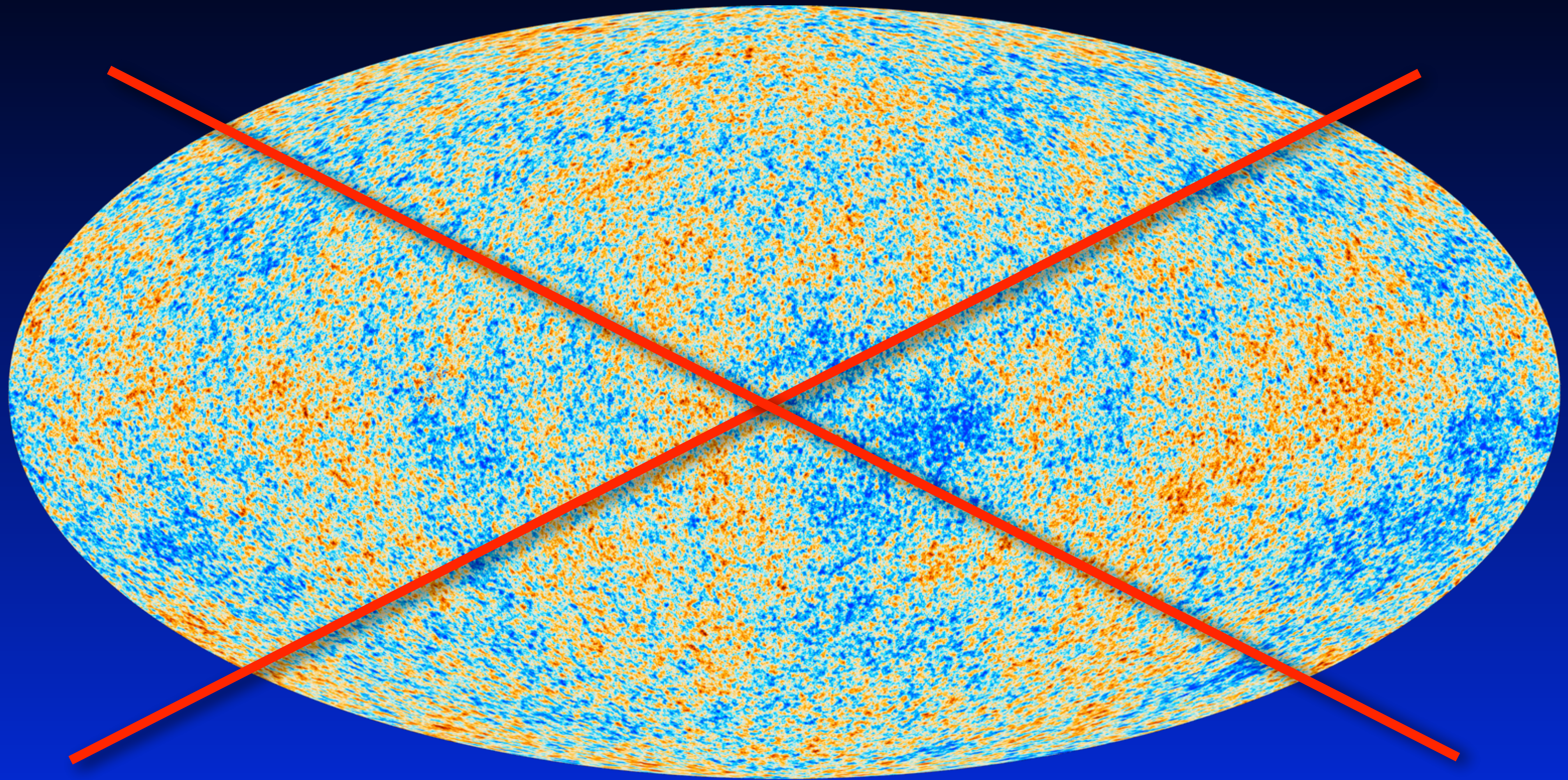
Cosmic Microwave Background Anisotropies



Planck all-sky temperature map

- CMB has a blackbody spectrum in every direction
- tiny variations of the CMB temperature $\Delta T/T \sim 10^{-5}$

Cosmic Microwave Background Anisotropies



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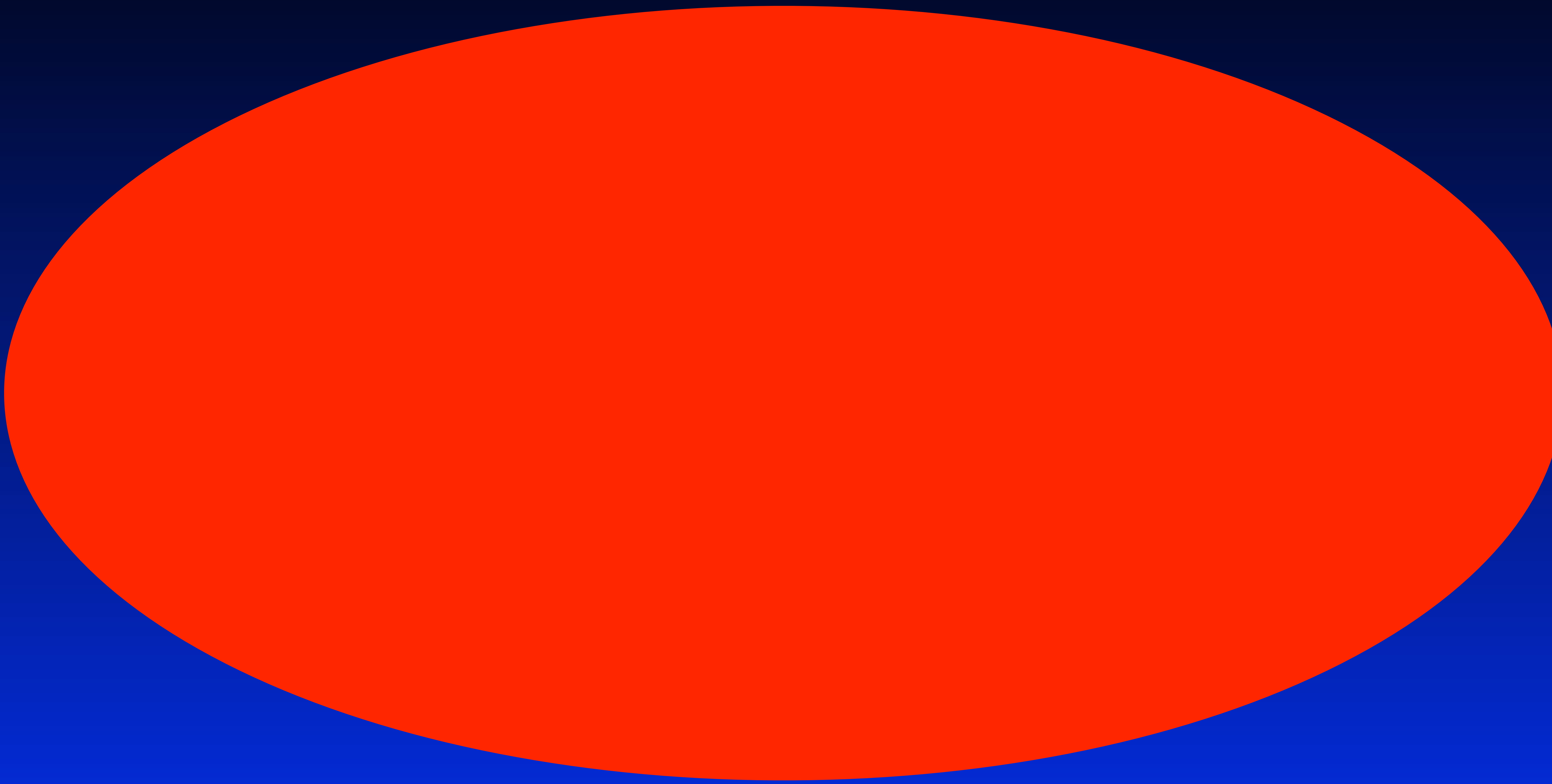
CMB spectral distortion science is all about the monopole!



Mather et al., 1994, ApJ, 420, 439
Fixsen et al., 1996, ApJ, 473, 576
Fixsen, 2003, ApJ, 594, 67
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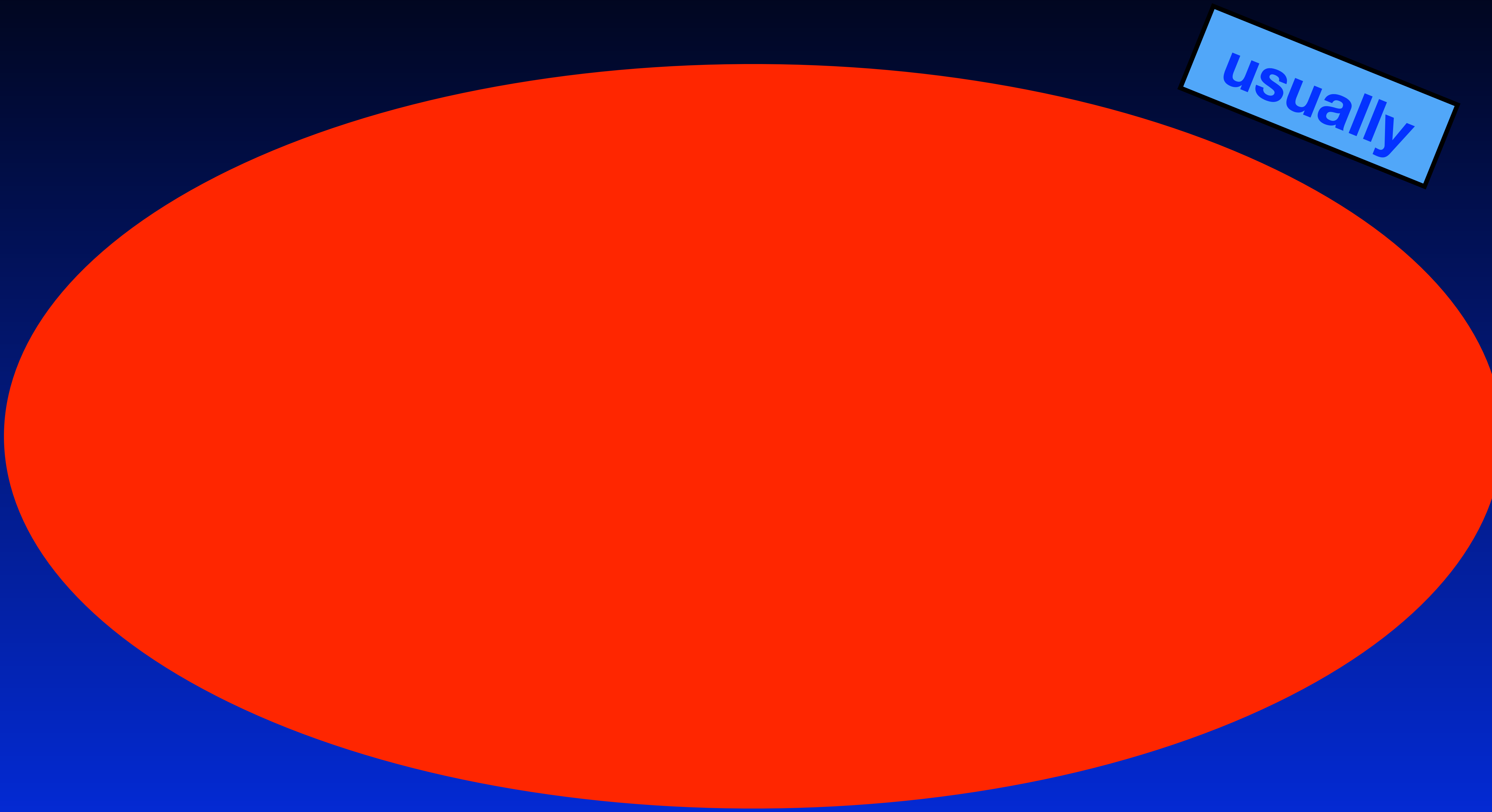

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COBE / FIRAS (Far InfraRed Absolute Spectrophotometer)

$$T_0 = 2.725 \pm 0.001\,\mathrm{K}$$

$$|y| \leq 1.5 \times 10^{-5}$$

$$|\mu| \leq 9 \times 10^{-5}$$

Mather et al., 1994, ApJ, 420, 439
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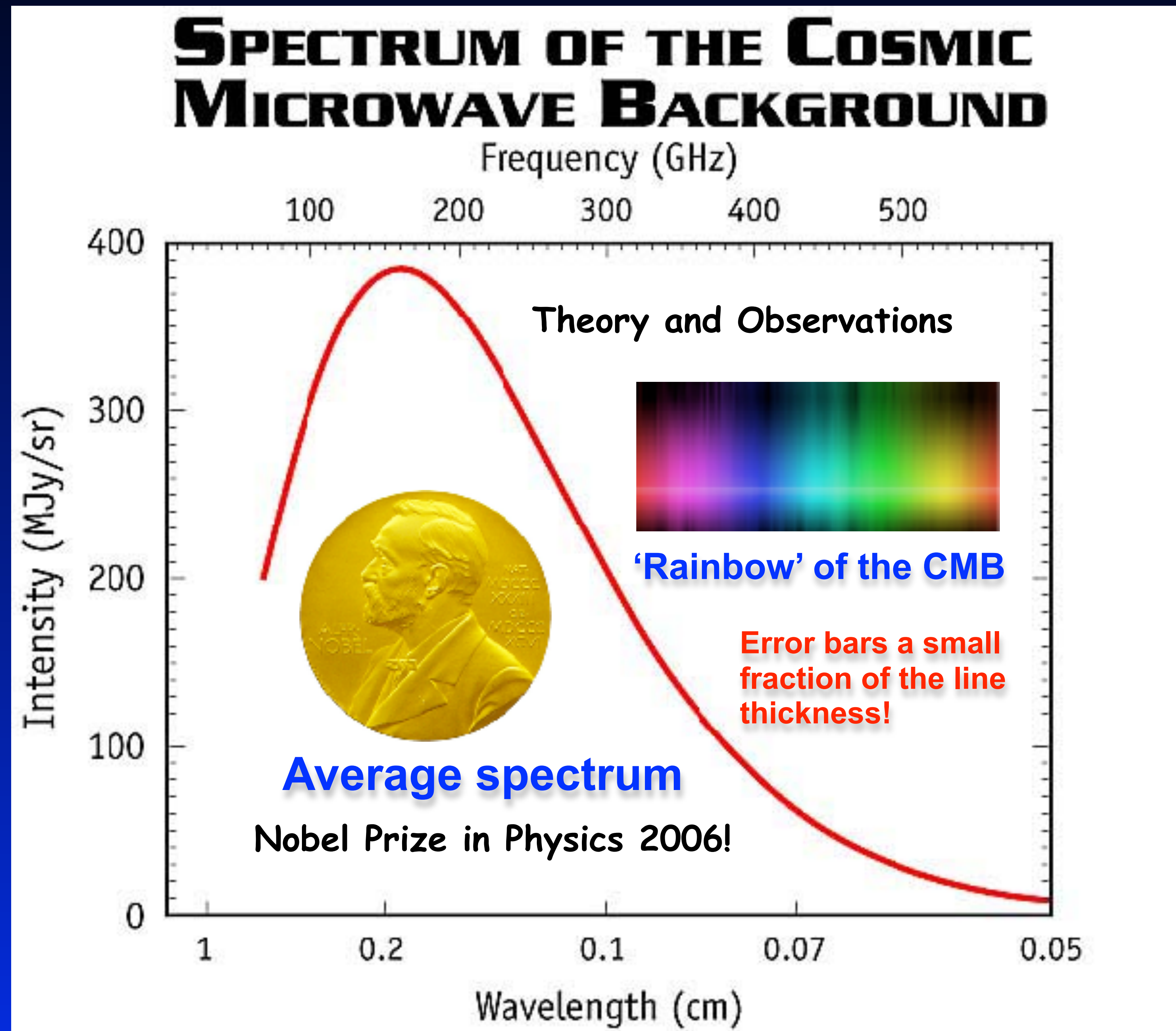
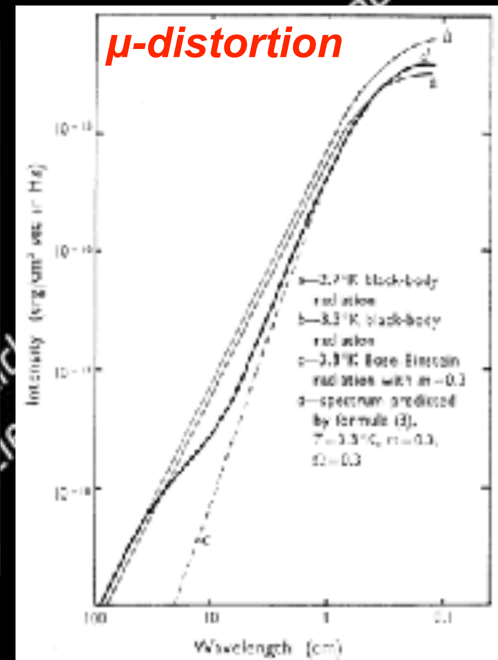
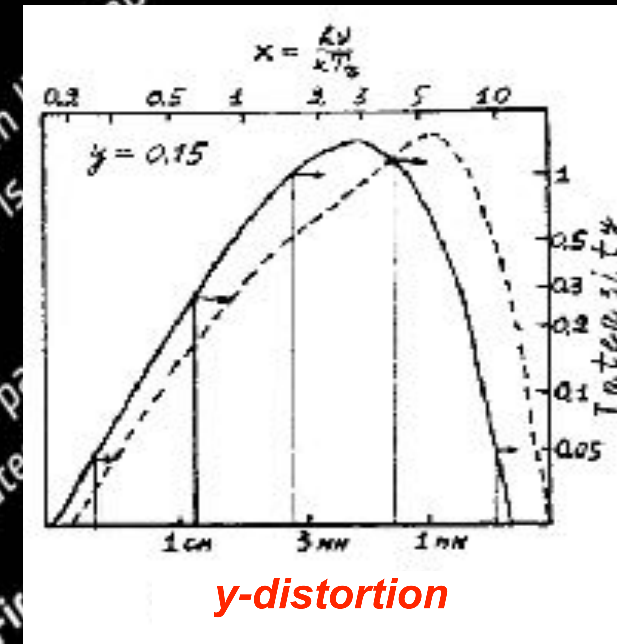
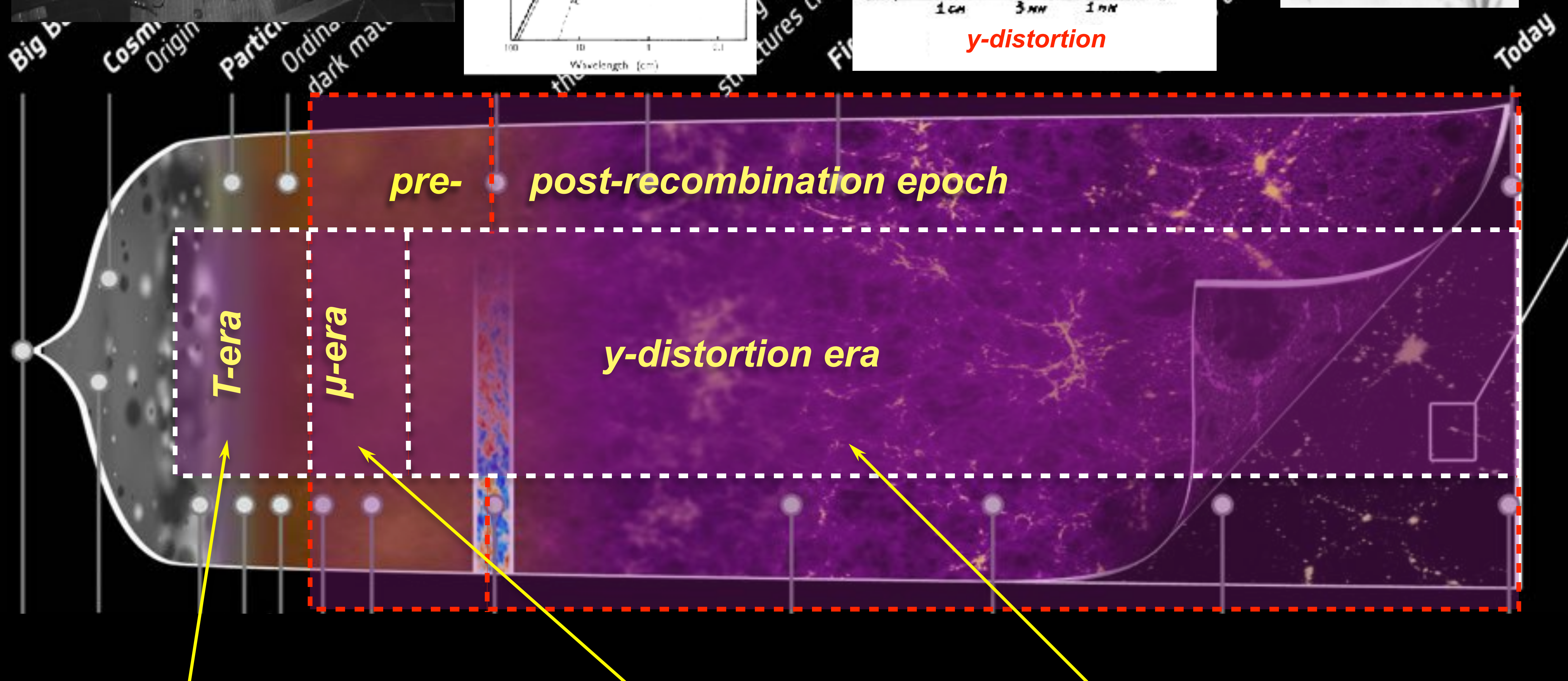


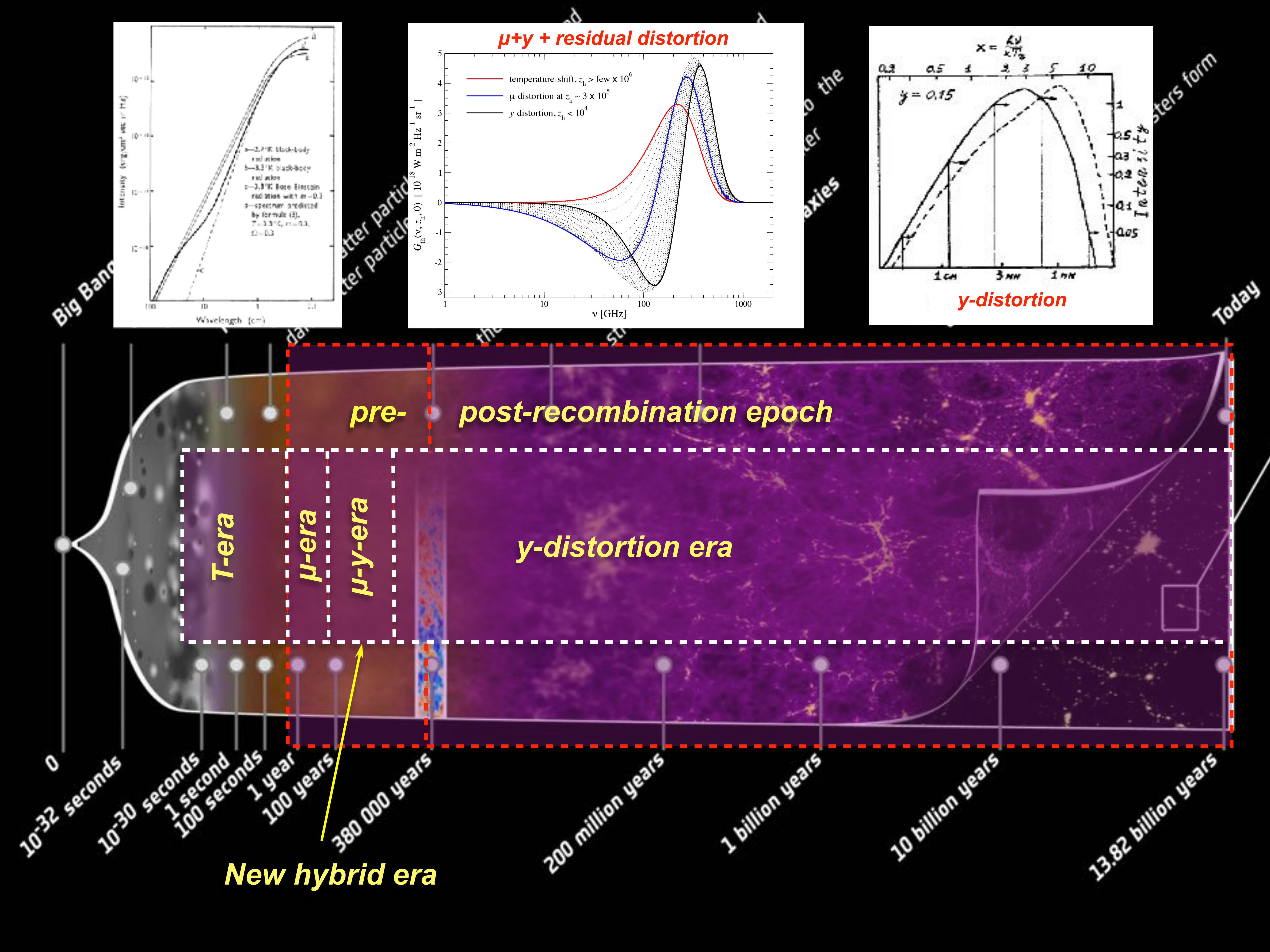
Blackbody spectrum to very high precision



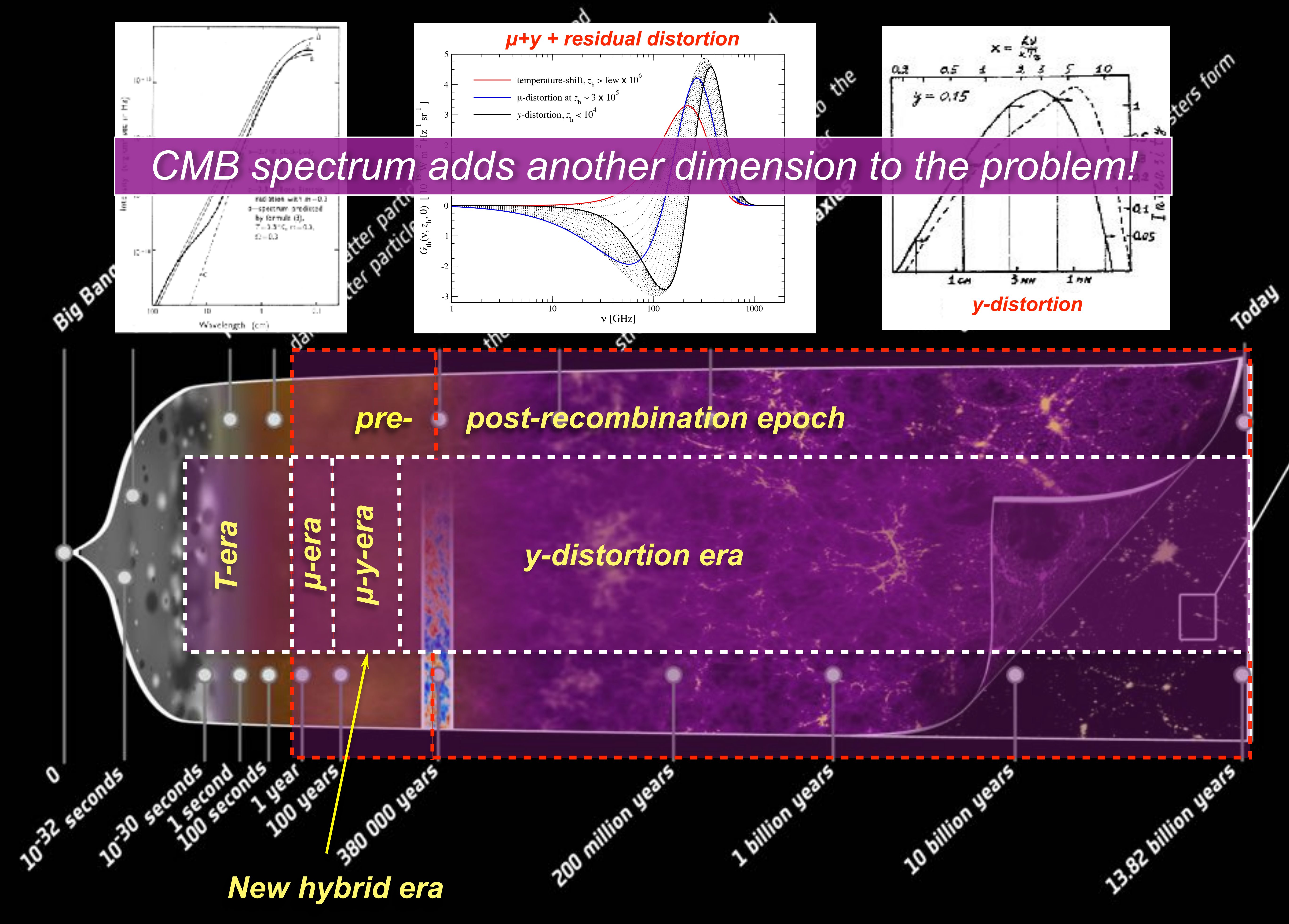




$$\frac{\Delta T}{T} \simeq \frac{1}{4} \left.\frac{\Delta \rho_\gamma}{\rho_\gamma}\right|_T \qquad \mu \simeq 1.4 \left.\frac{\Delta \rho_\gamma}{\rho_\gamma}\right|_\mu \qquad y \simeq \frac{1}{4} \left.\frac{\Delta \rho_\gamma}{\rho_\gamma}\right|_y$$



μ+y + residual distortion
temperature-shift, z_h > few x 10^6
μ-distortion at z_h ~ 3 x 10^5
y-distortion, z_h < 10^4
ν [GHz]
y-distortion
Big Bang
Today
pre-
post-recombination epoch
T-era
μ-era
μ-y-era
y-distortion era
0
10-32 seconds
10-30 seconds
1 second
100 seconds
1 year
100 years
380 000 years
200 million years
1 billion years
10 billion years
13.82 billion years
New hybrid era


CMB spectrum adds another dimension to the problem!
Wavelength (cm)
μ+y + residual distortion
temperature-shift, z_h > few x 10^6
μ-distortion at $z_h \sim 3 \times 10^5$
y-distortion, $z_h < 10^4$
ν [GHz]
y = 0.15
y-distortion
Big Bang
Today
pre-
post-recombination epoch
T-era
μ-era
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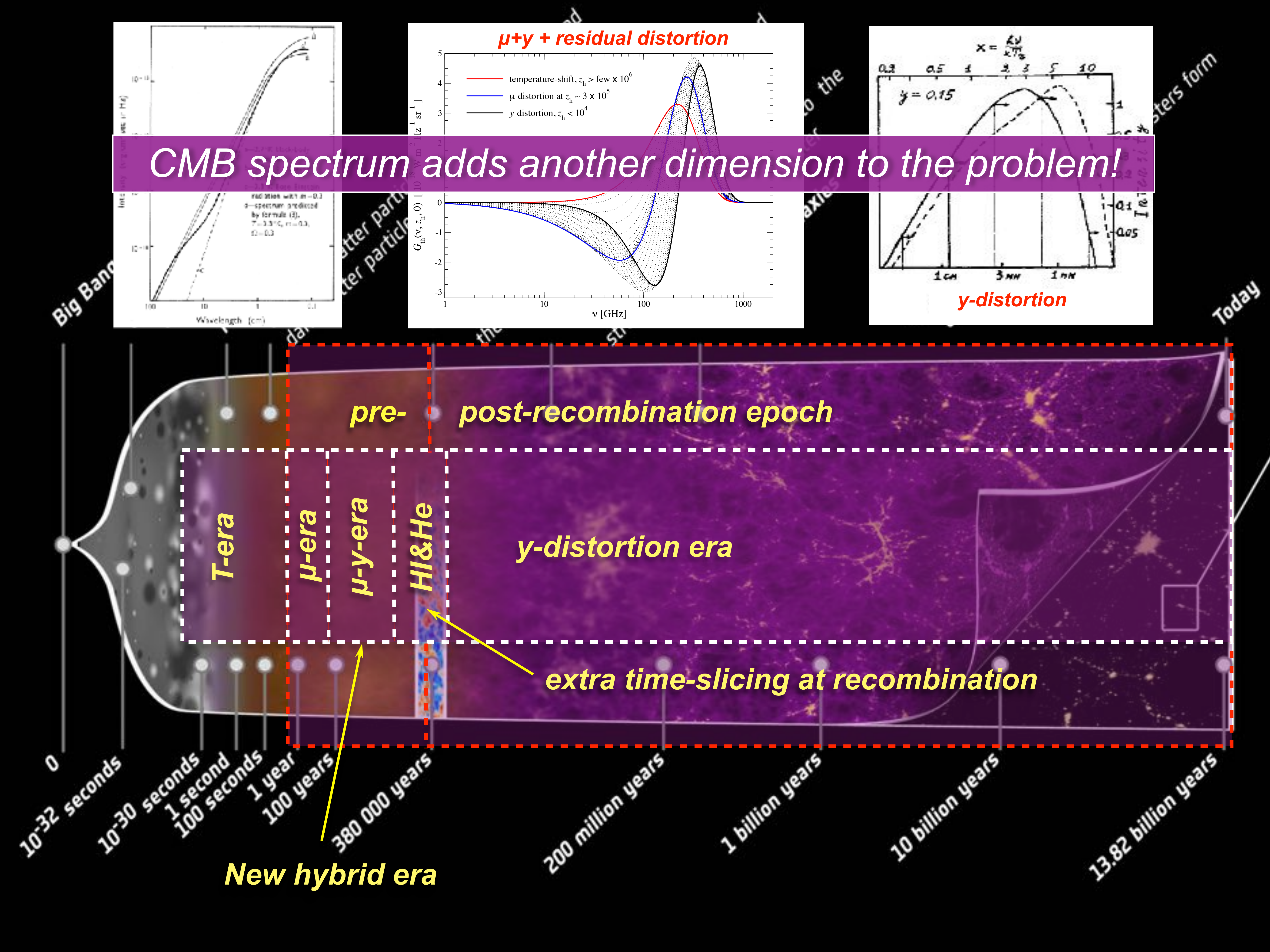
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pre-
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HI&He
y-distortion era
extra time-slicing at recombination
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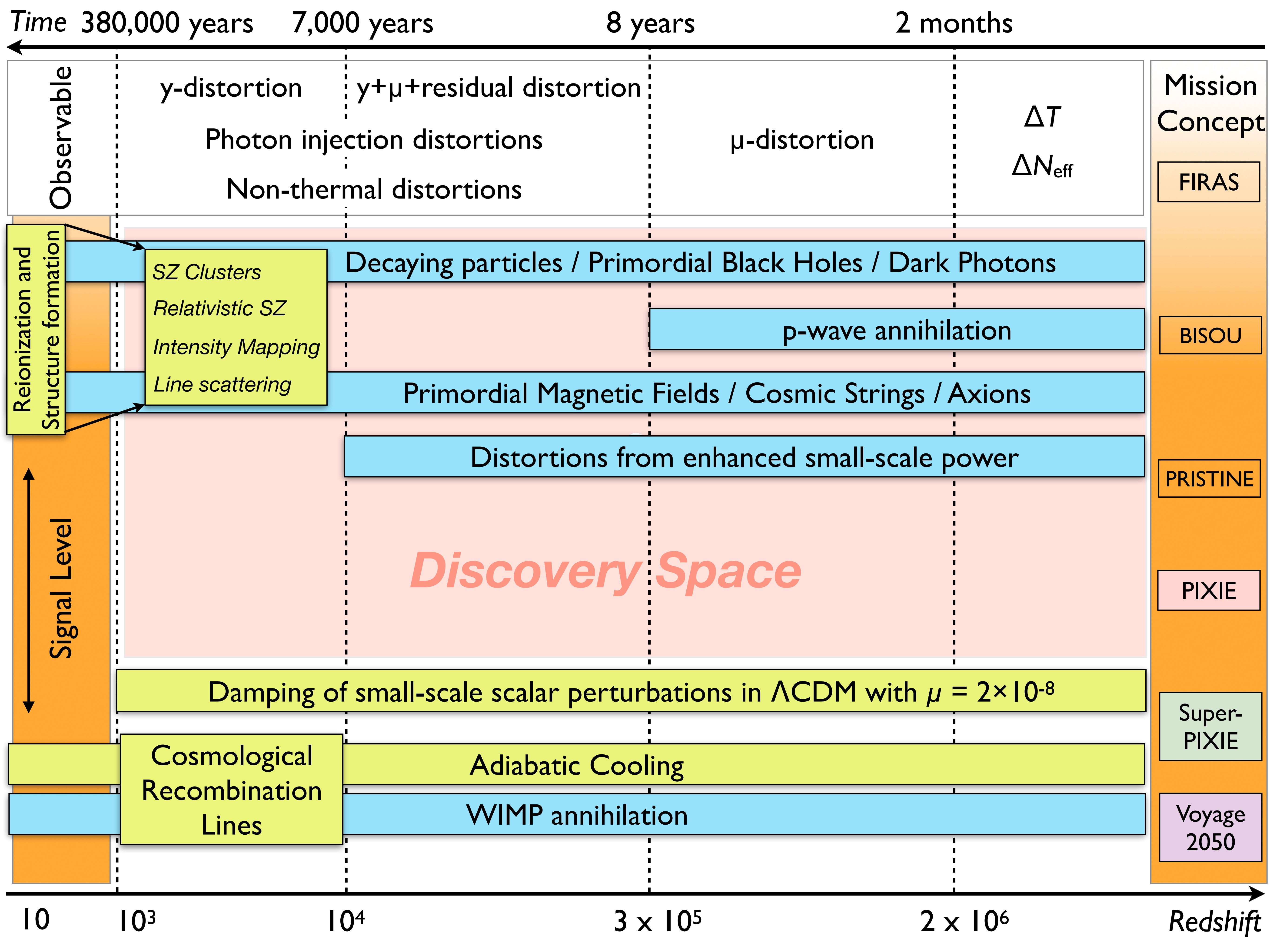
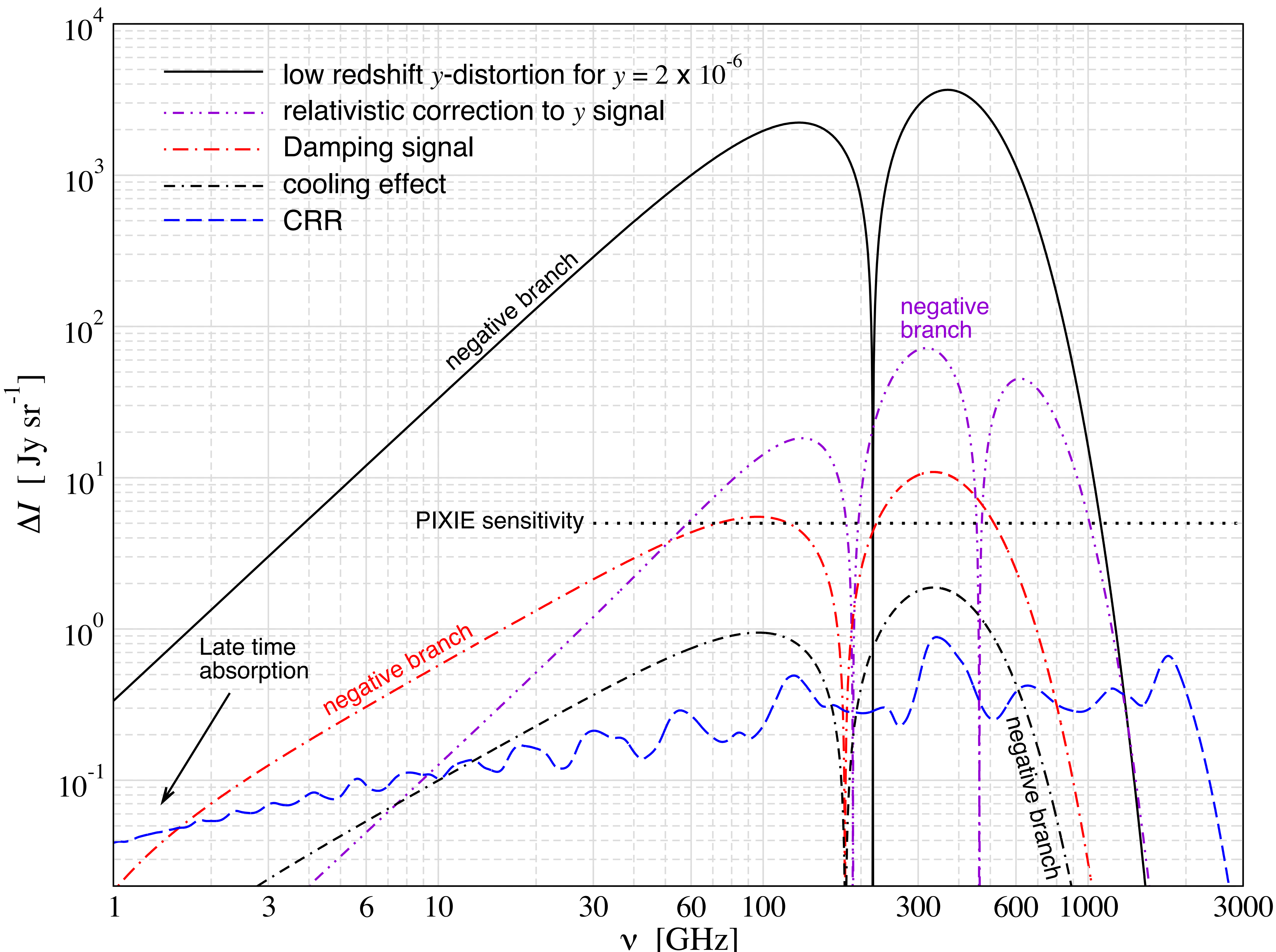


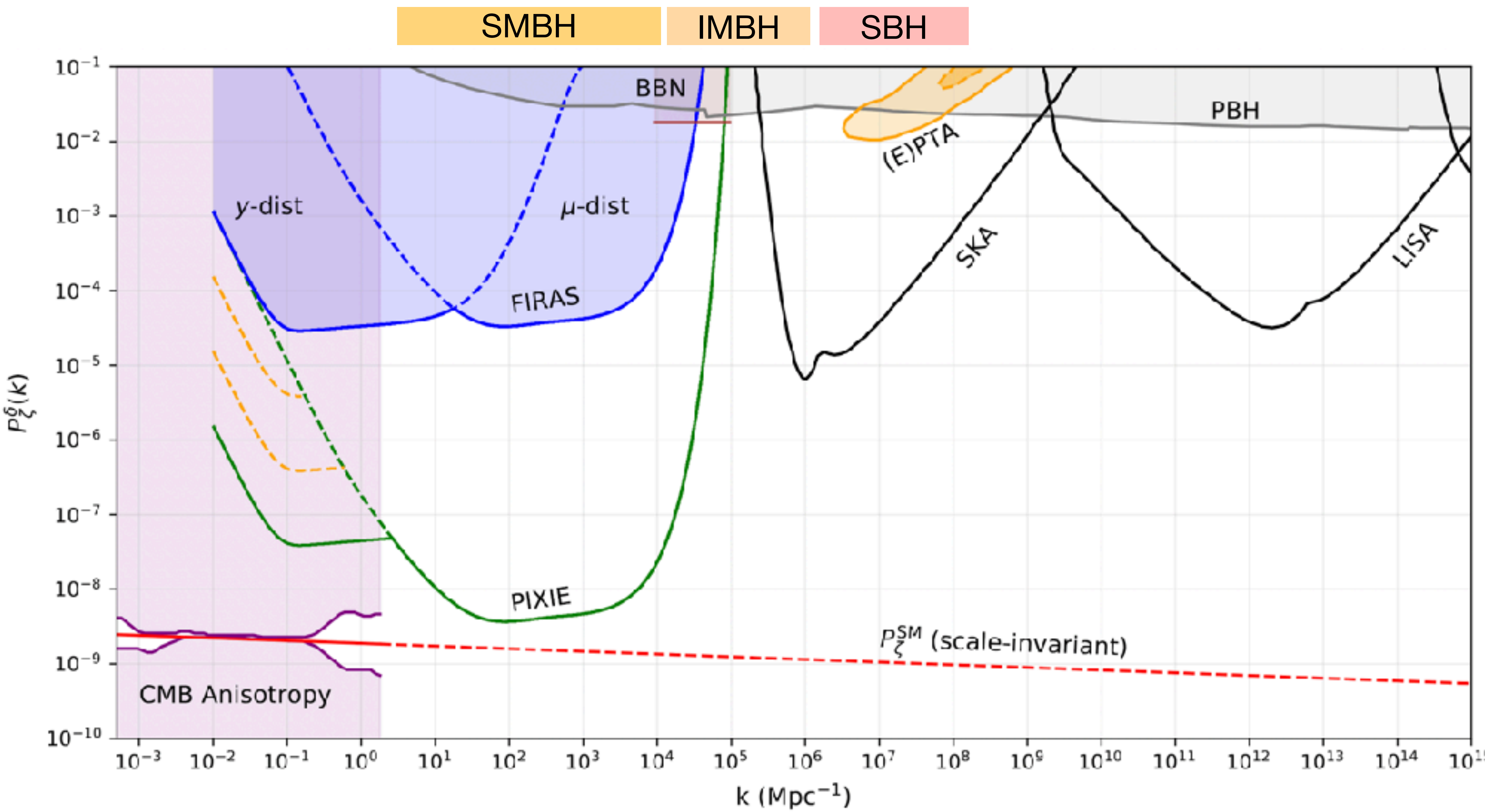
Time 380,000 years
7,000 years
8 years
2 months
Observable
y-distortion
y+μ+residual distortion
Photon injection distortions
Non-thermal distortions
μ-distortion
ΔT
ΔNeff
Mission Concept
FIRAS
BISOU
PRISTINE
PIXIE
Super-PIXIE
Voyage 2050
Reionization and Structure formation
SZ Clusters
Relativistic SZ
Intensity Mapping
Line scattering
Decaying particles / Primordial Black Holes / Dark Photons
p-wave annihilation
Primordial Magnetic Fields / Cosmic Strings / Axions
Distortions from enhanced small-scale power
Signal Level
Discovery Space
Damping of small-scale scalar perturbations in ΛCDM with μ = 2×10-8
Cosmological Recombination Lines
Adiabatic Cooling
WIMP annihilation
10
10³
10⁴
3 x 10⁵
2 x 10⁶
Redshift

Average ΛCDM spectral distortions



JC, 2016, MNRAS (ArXiv:1603.02496)

Small-scale power constraints and PBH formation



JC, Khatri & Sunyaev, 2012
JC, Erickcek & Ben-Dayan, 2012
Cyr et al., 2023, ArXiv:2309.02366

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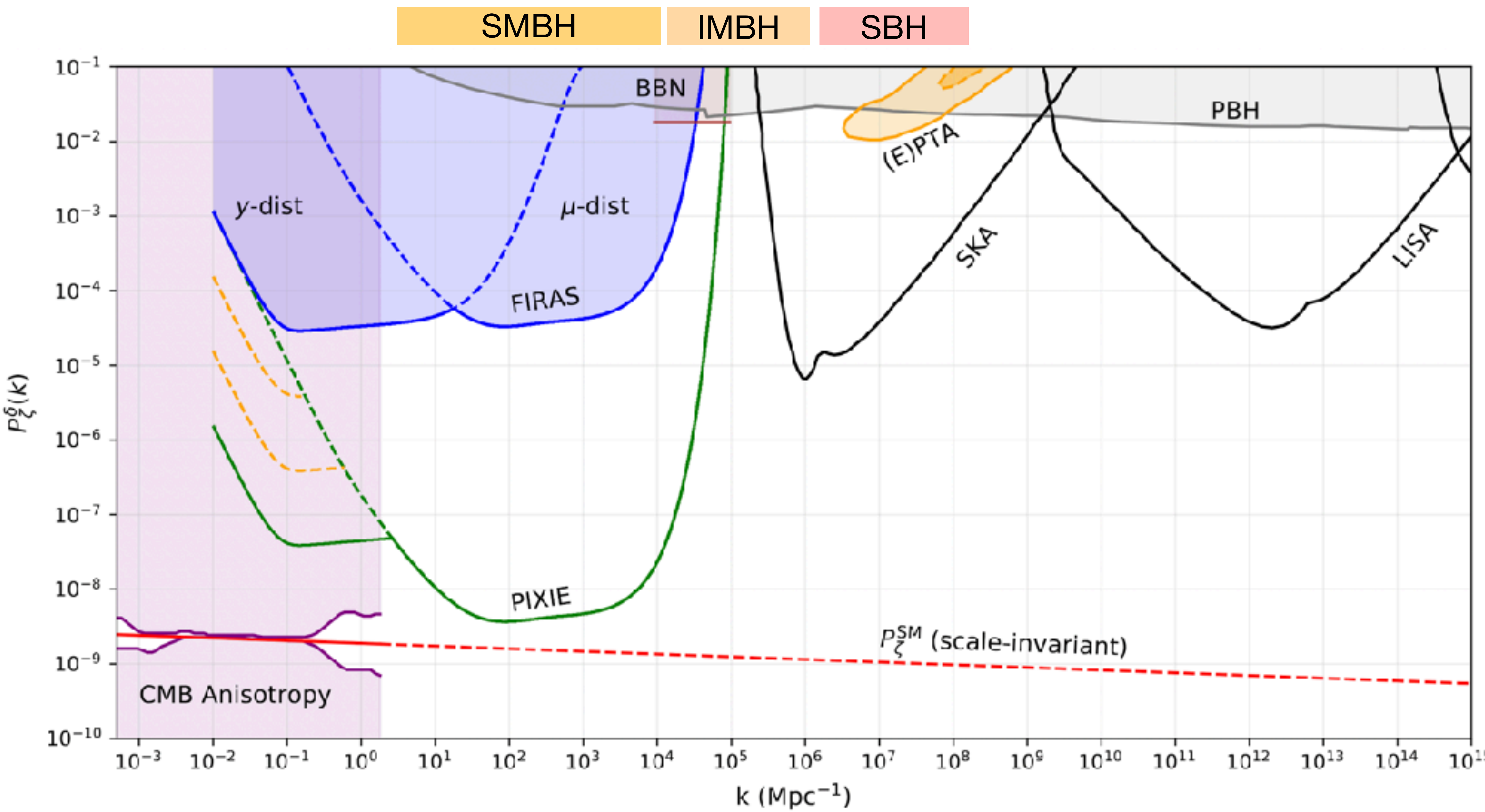

JC, Khatri & Sunyaev, 2012
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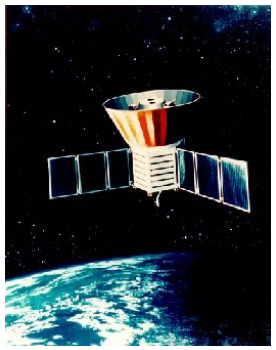
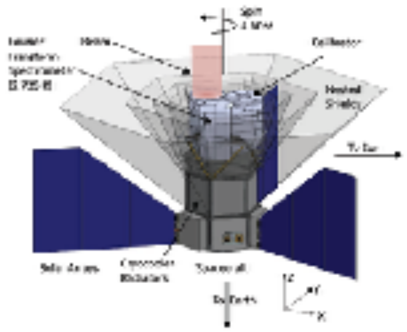
A CMB spectrometer could shed light on primordial black hole formation

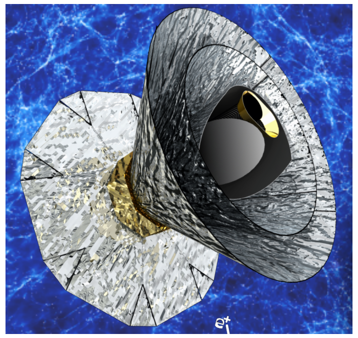
History of distortion experiments and proposals



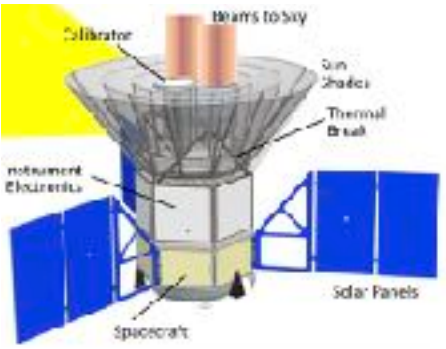
COBE/FIRAS
Mather & Fixsen



PIXIE
Kogut & Fixsen



PRISM
De Bernardis



PIXIE
Kogut & Fixsen

(PIXIE)

Super-PIXIE

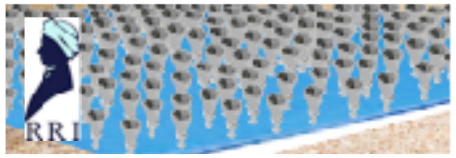
PRISTINE
Aghanim

FOSSIL
Aghanim

ARCADE 2
Kogut & Fixsen

Maffei
BISOU

TRIS
Gervasi, Zannoni & Tartari



APSERa
Subrahmanyan & Rao

COSMO
De Bernardis, Masi & Battistelli

TMS
Rubiño-Martin

1989 ≀≀ 2008 2011 2013 2015 2016 2018 2022

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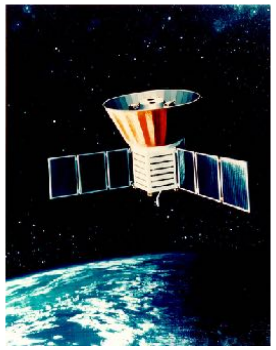

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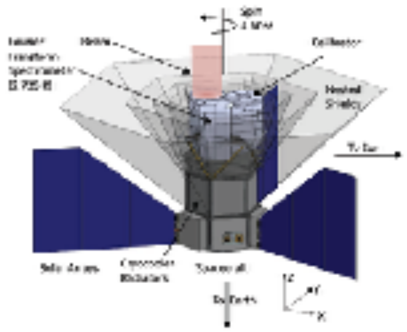

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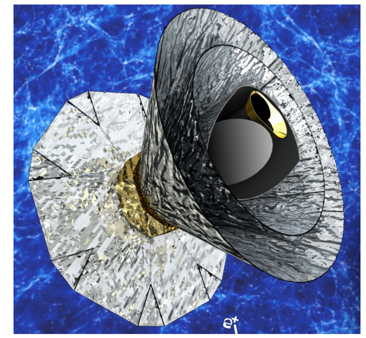

PRISM
De Bernardis

PIXIE
Kogut & Fixsen

Super-PIXIE

(PIXIE)

PRISTINE
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ARCADE 2
Kogut & Fixsen

Not just fiction anymore!

Maffei

BISOU

TRIS
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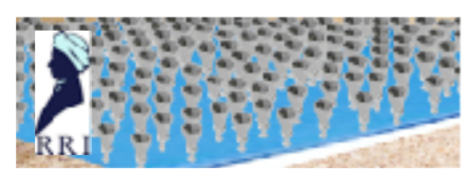

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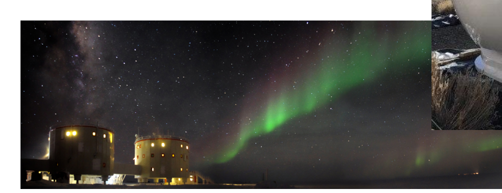

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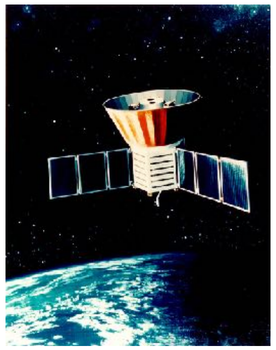

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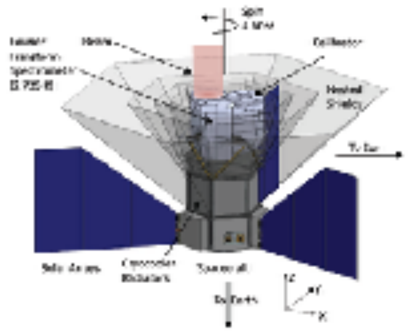

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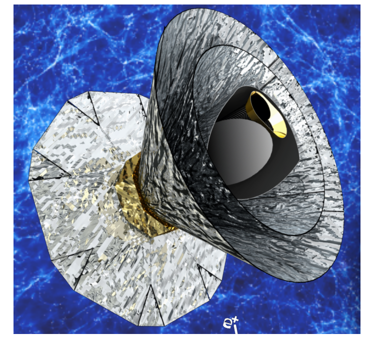
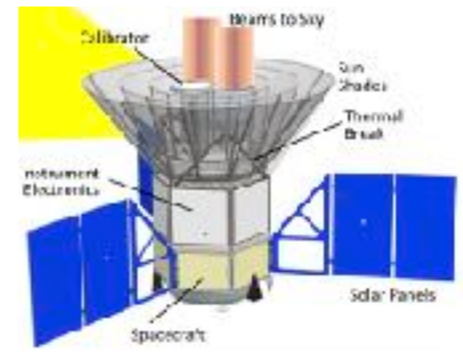


PRISM
De Bernardis



PIXIE
Kogut & Fixsen

Super-PIXIE

PRISTINE
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(PIXIE)

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BISOU officially in Phase A since May!

Maffei

BISOU

TRIS
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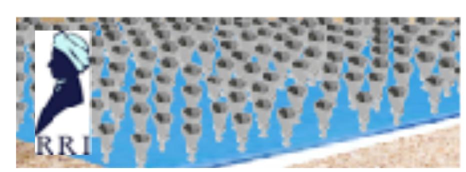

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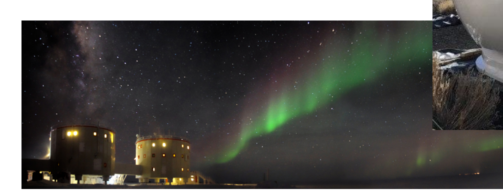

COSMO
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TMS
Rubiño-Martin

1989 ≀≀ 2008 2011 2013 2015 2016 2018 2022

- > 100 WP evaluated
- Identified three L-Class themes
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  - From temperate Exoplanets to the Milky Way
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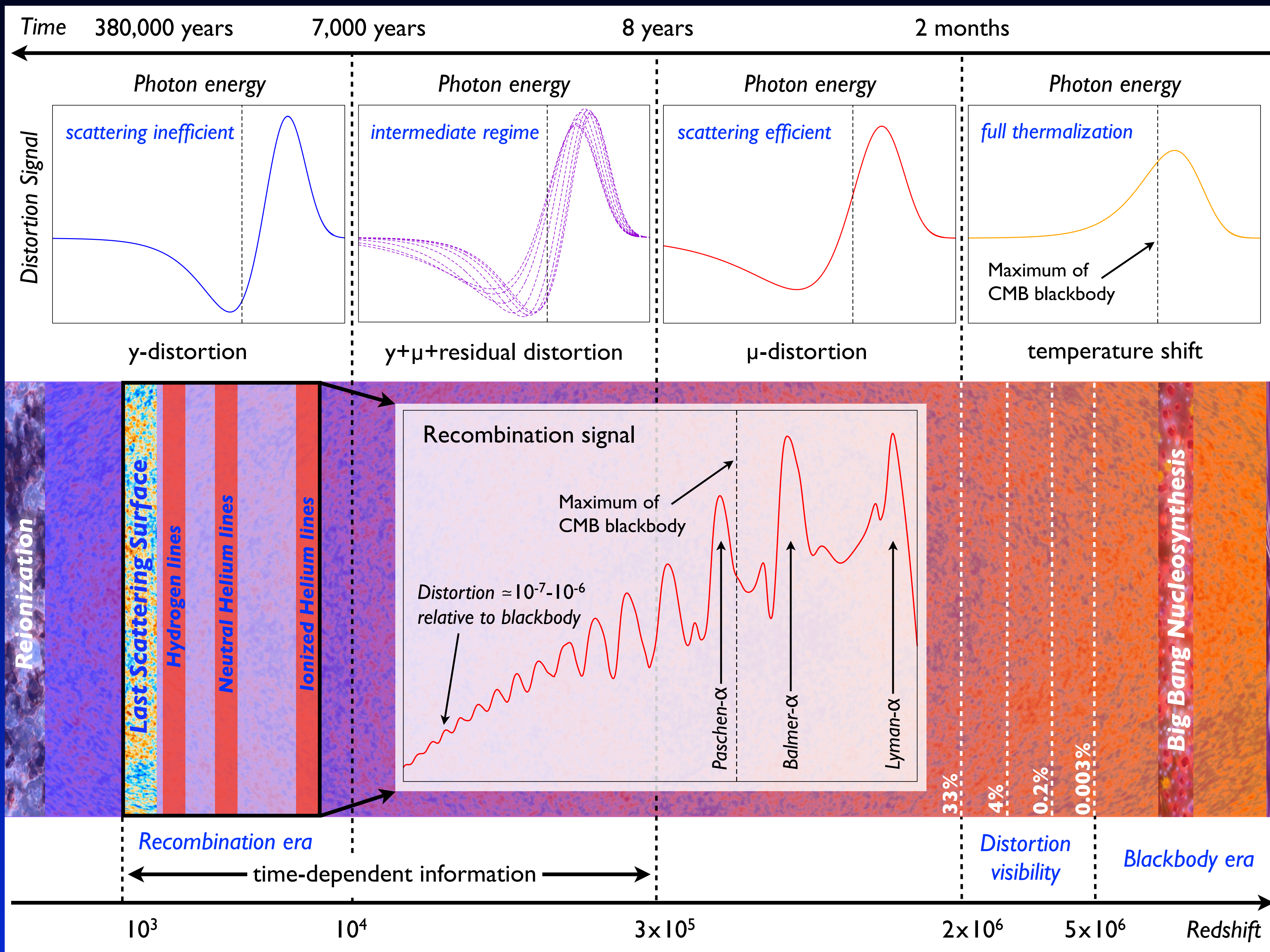
Uniqueness of CMB Spectral Distortion Science



Guaranteed distortion signals in ΛCDM

New tests of inflation and particle/dark matter physics

Signals from the reionization and recombination eras

Huge discovery potential

Complementarity and synergy with CMB anisotropy studies

JC & Sunyaev, MNRAS, 419, 2012
JC et al., MNRAS, 425, 2012
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COSMO
TMS
BISOU
Voyage 2050

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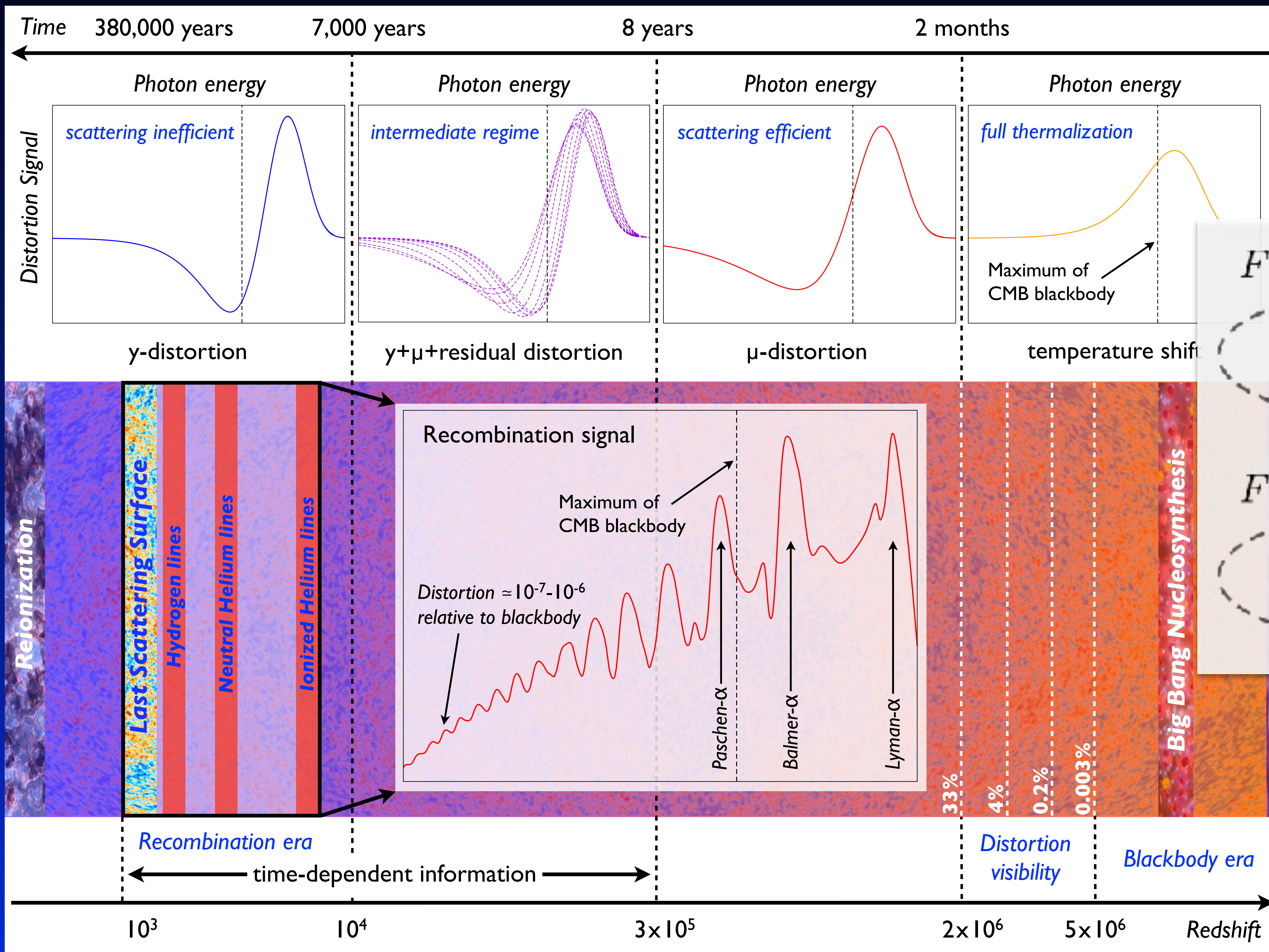

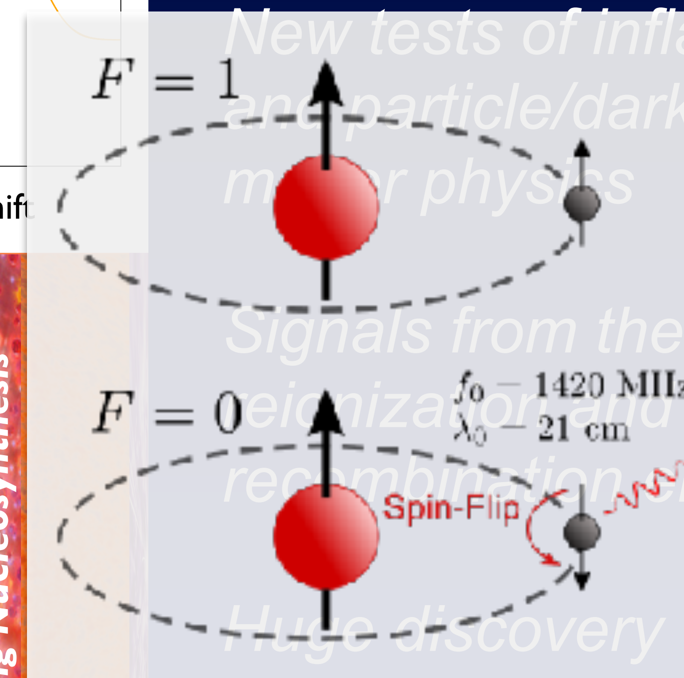

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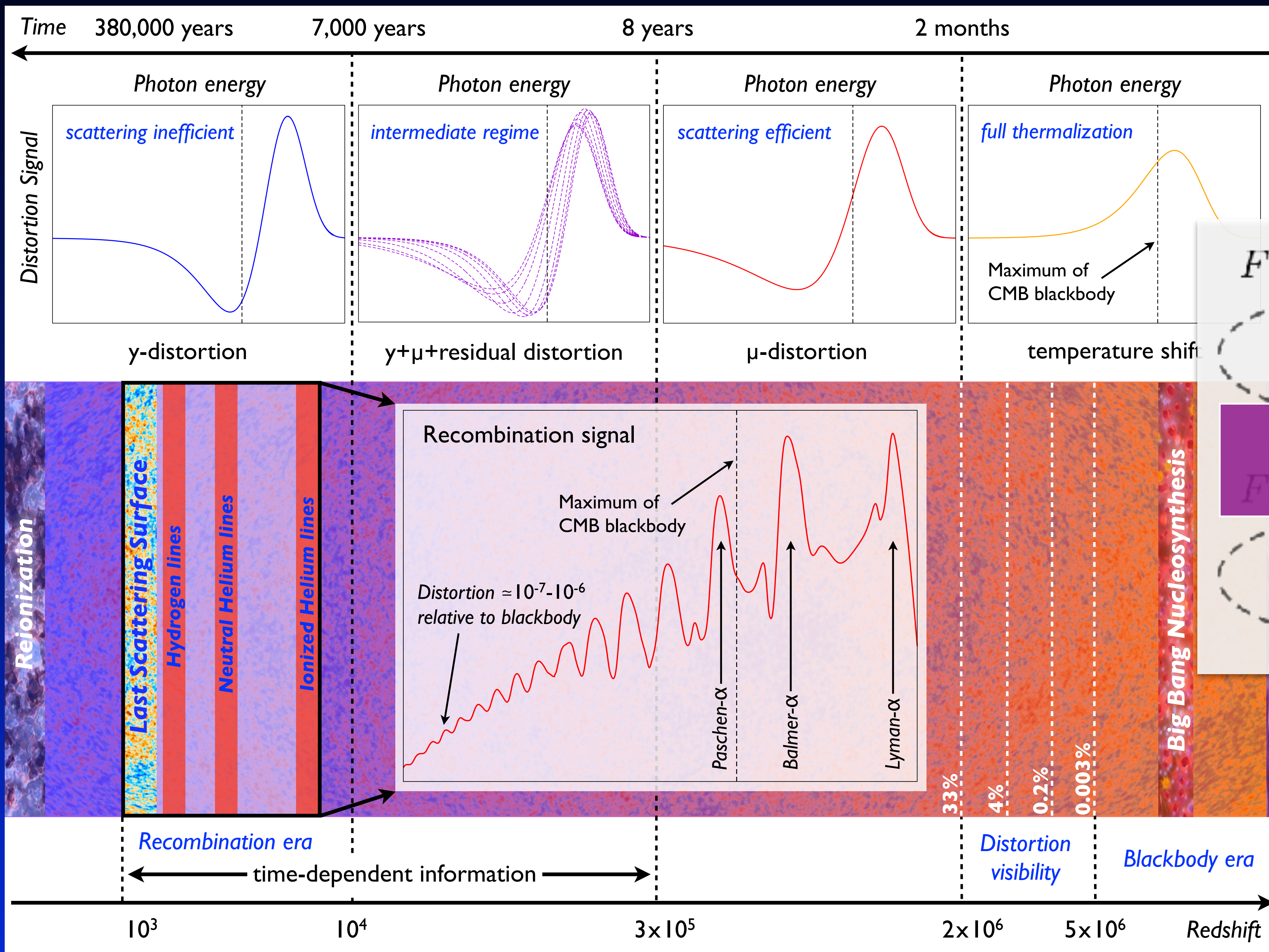

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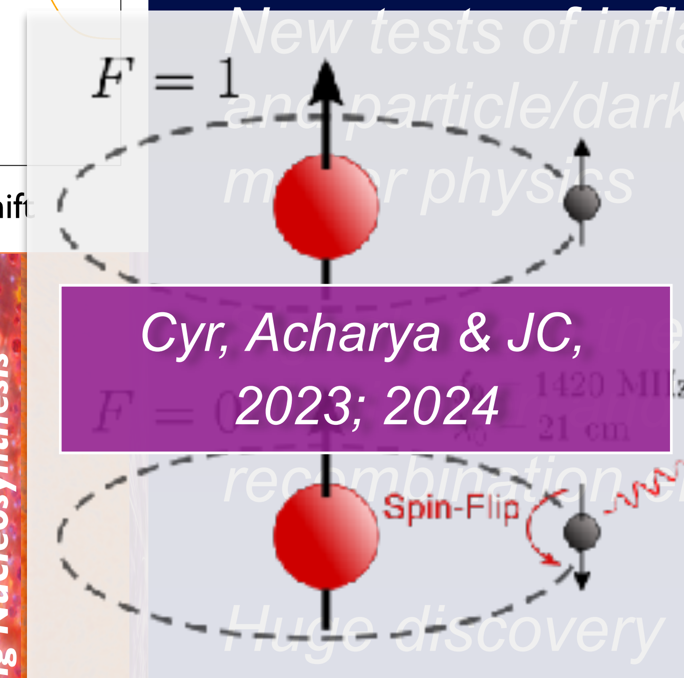

Cyr, Acharya & JC, 2023; 2024

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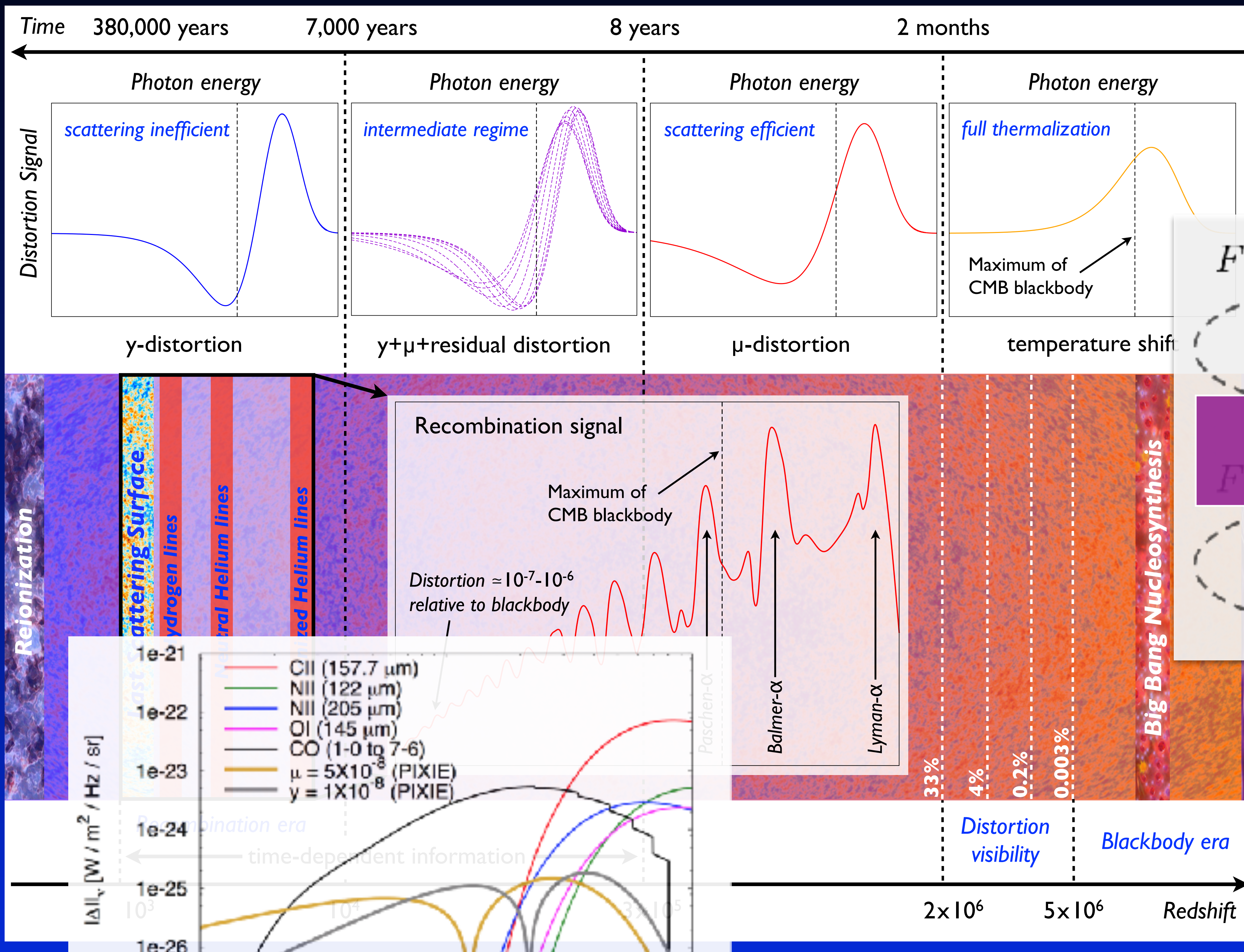

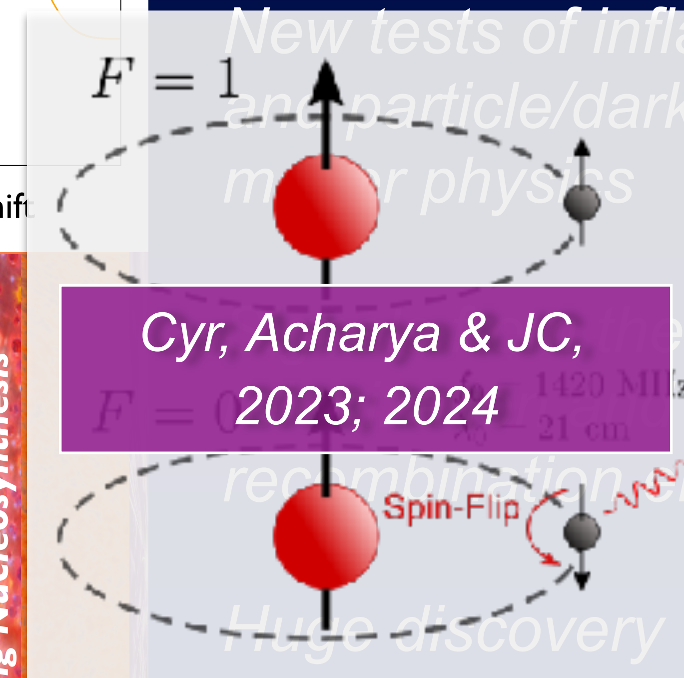

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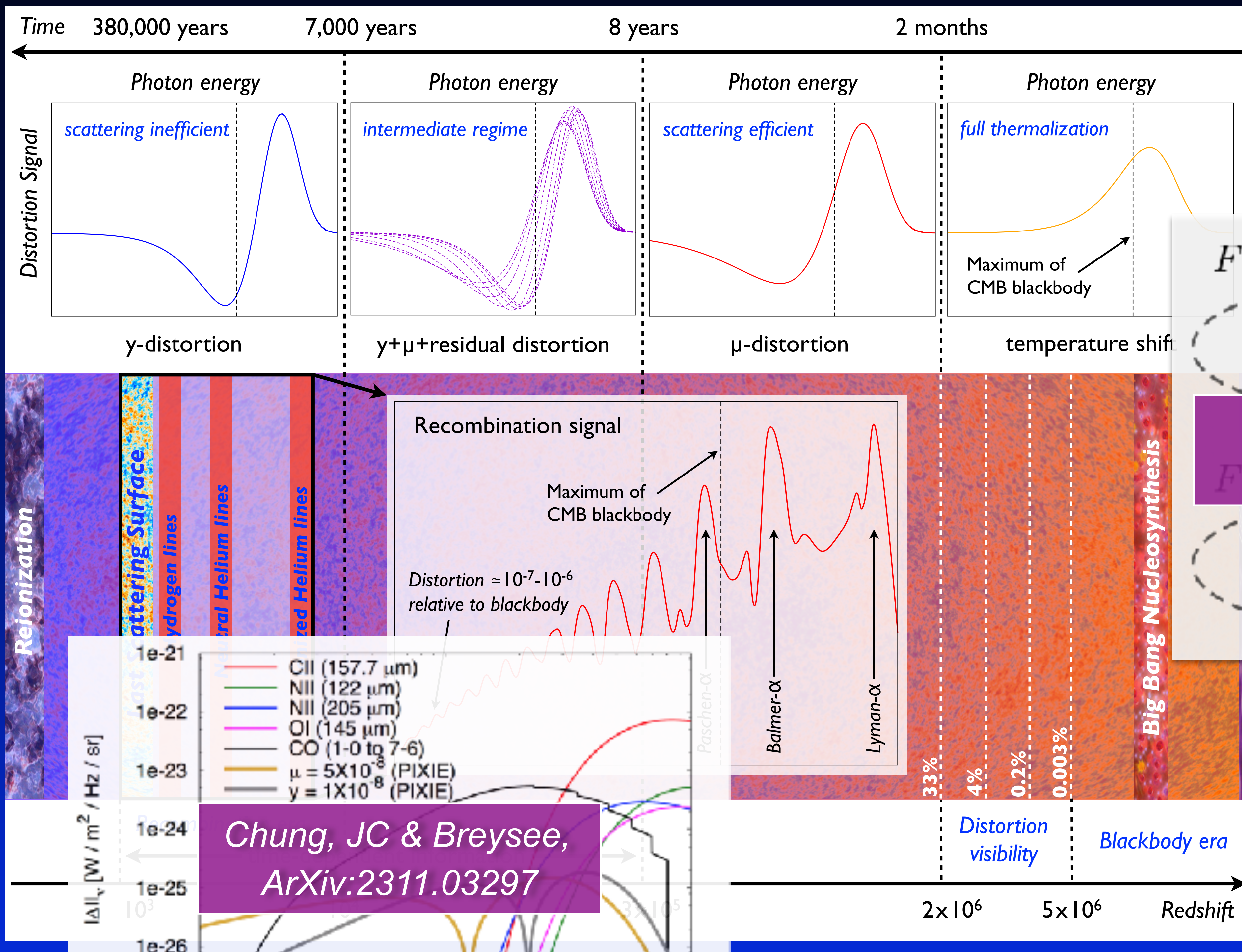

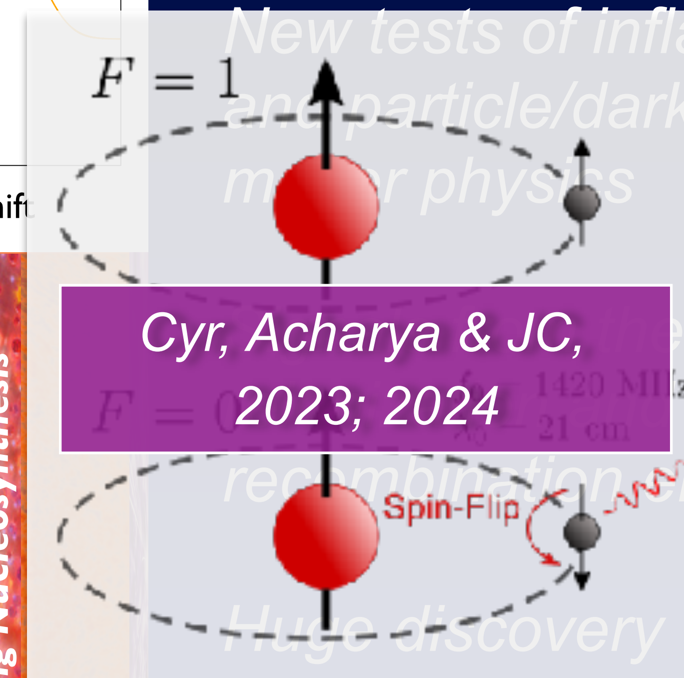

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COSMO
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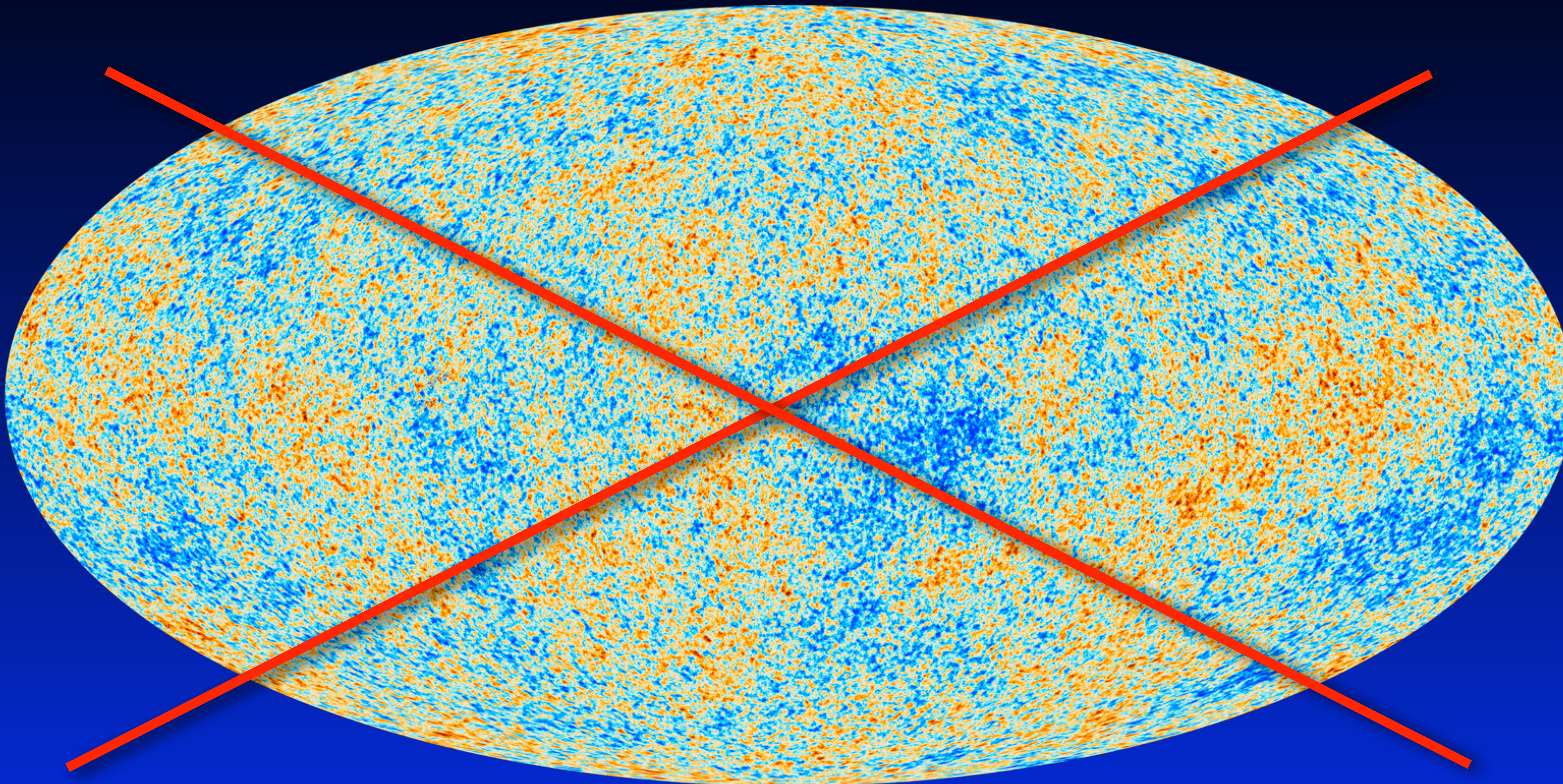
Cosmic Microwave Background Anisotropies



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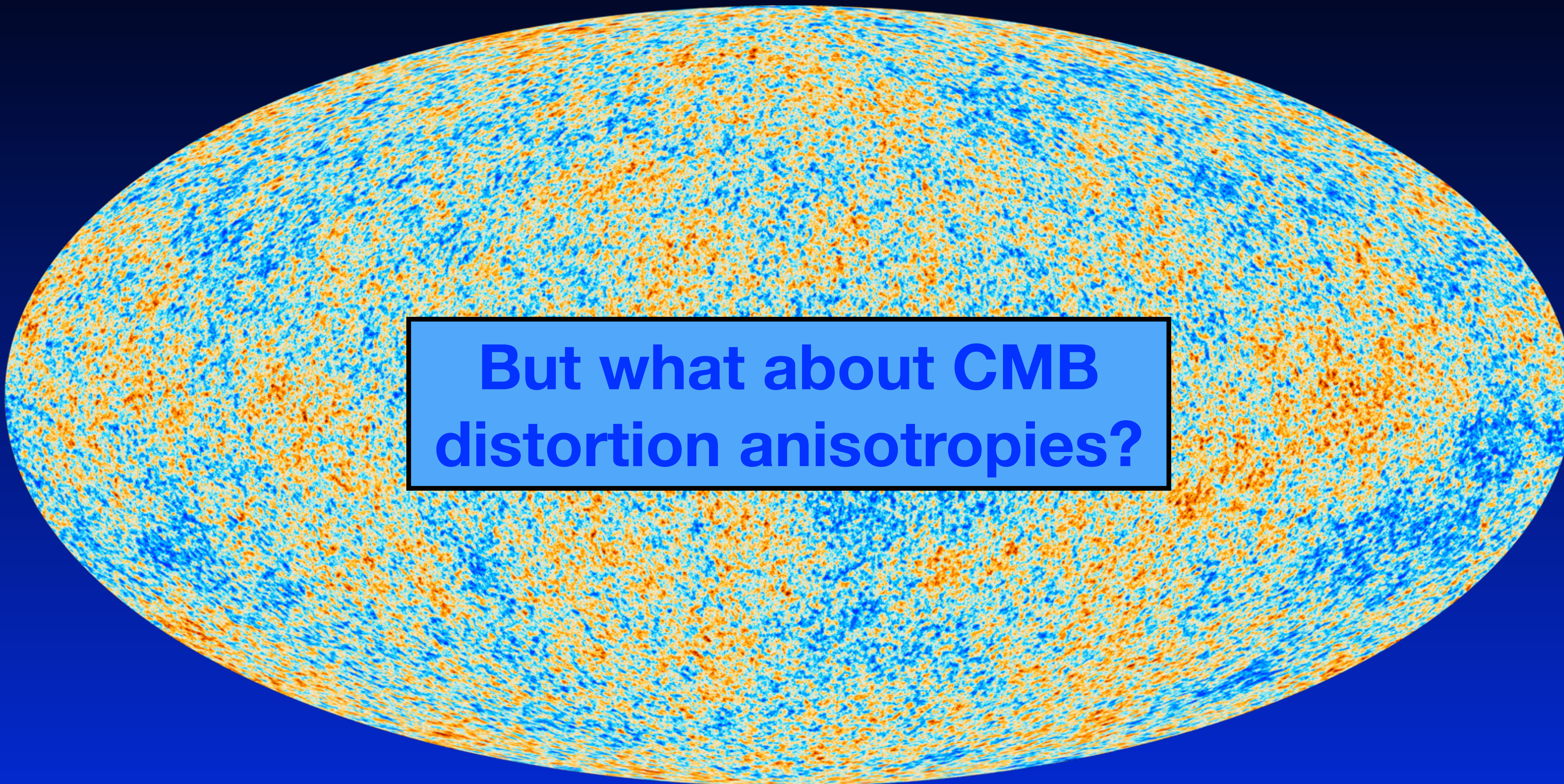

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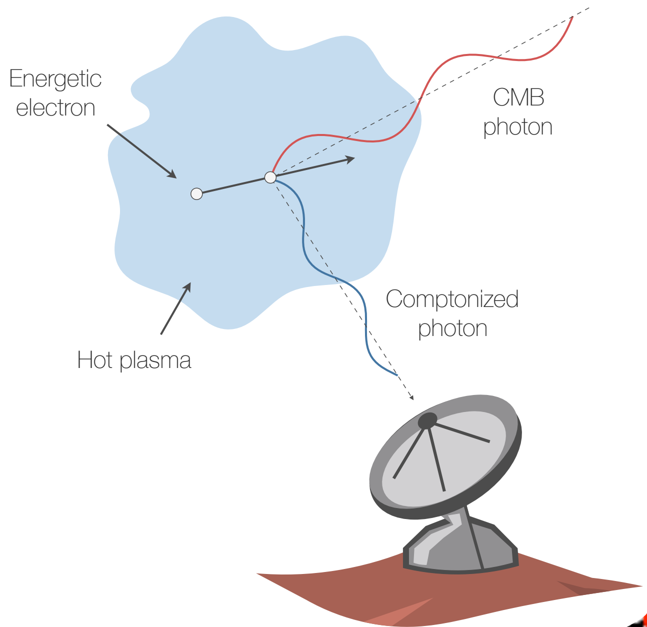
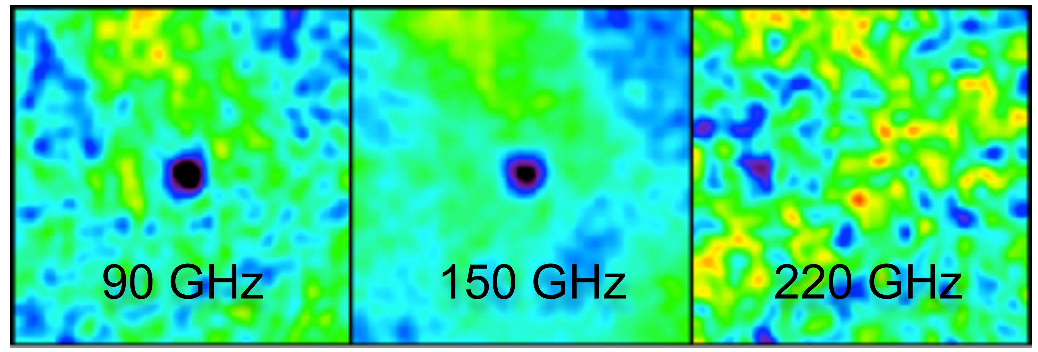
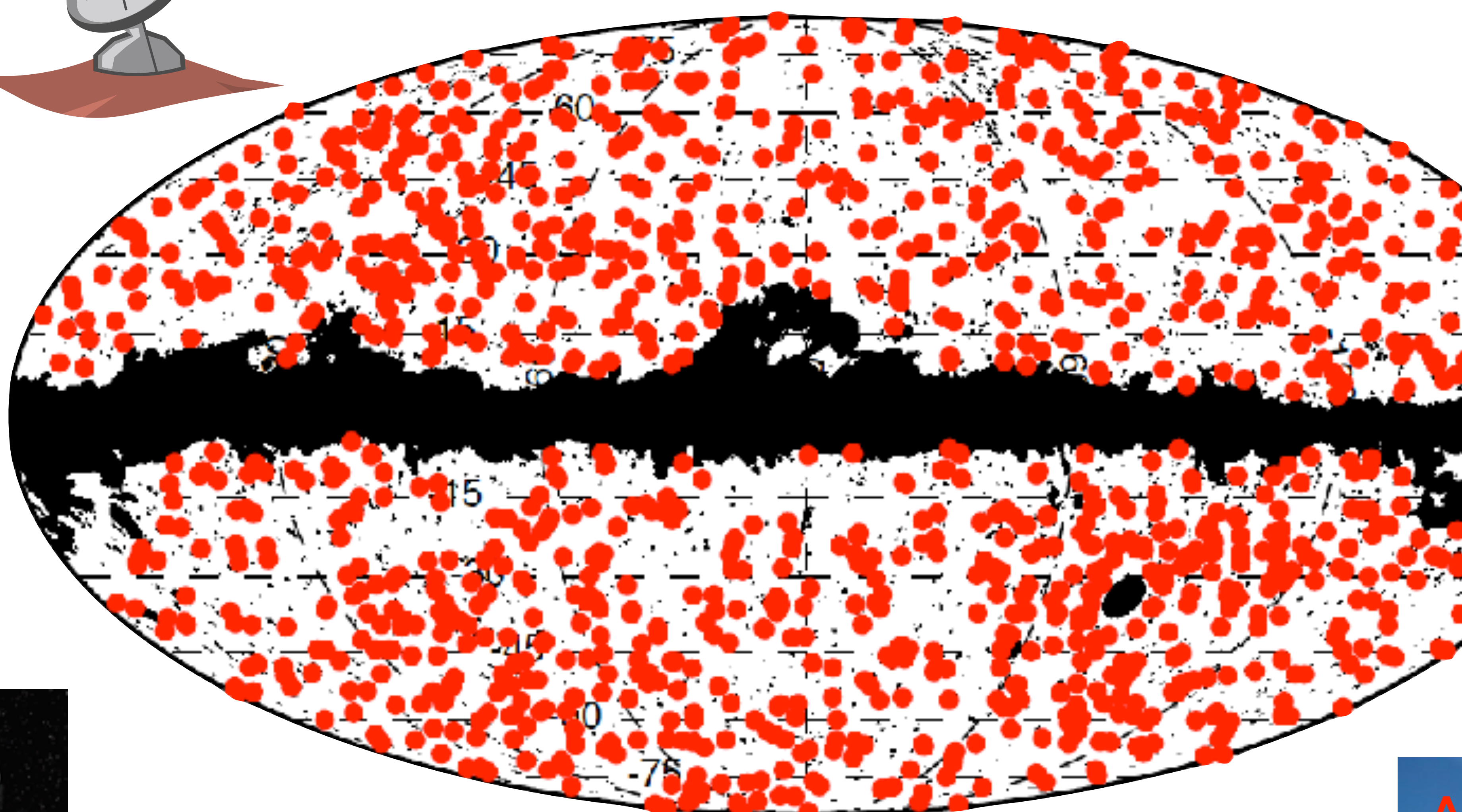
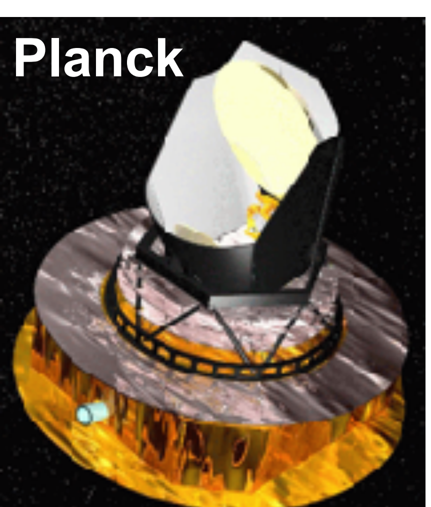
Thermal SZ effect is now routinely observed!





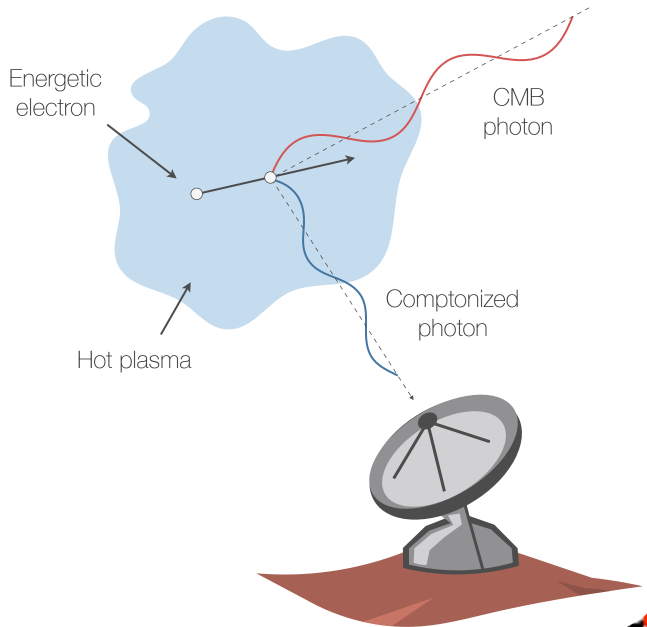
~ 1230 objects

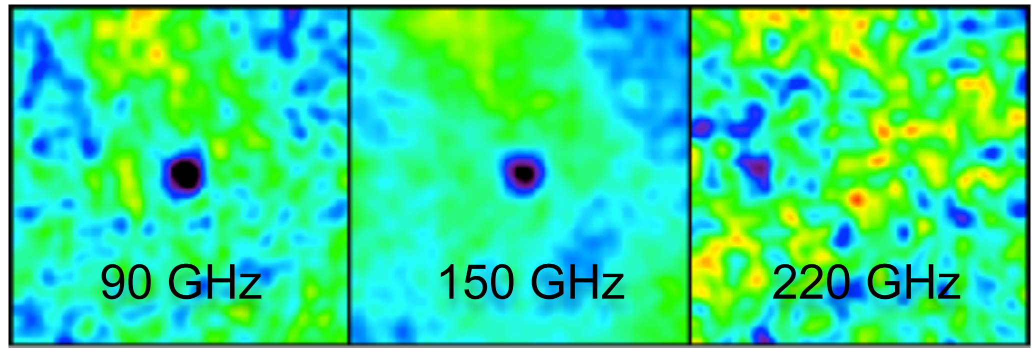




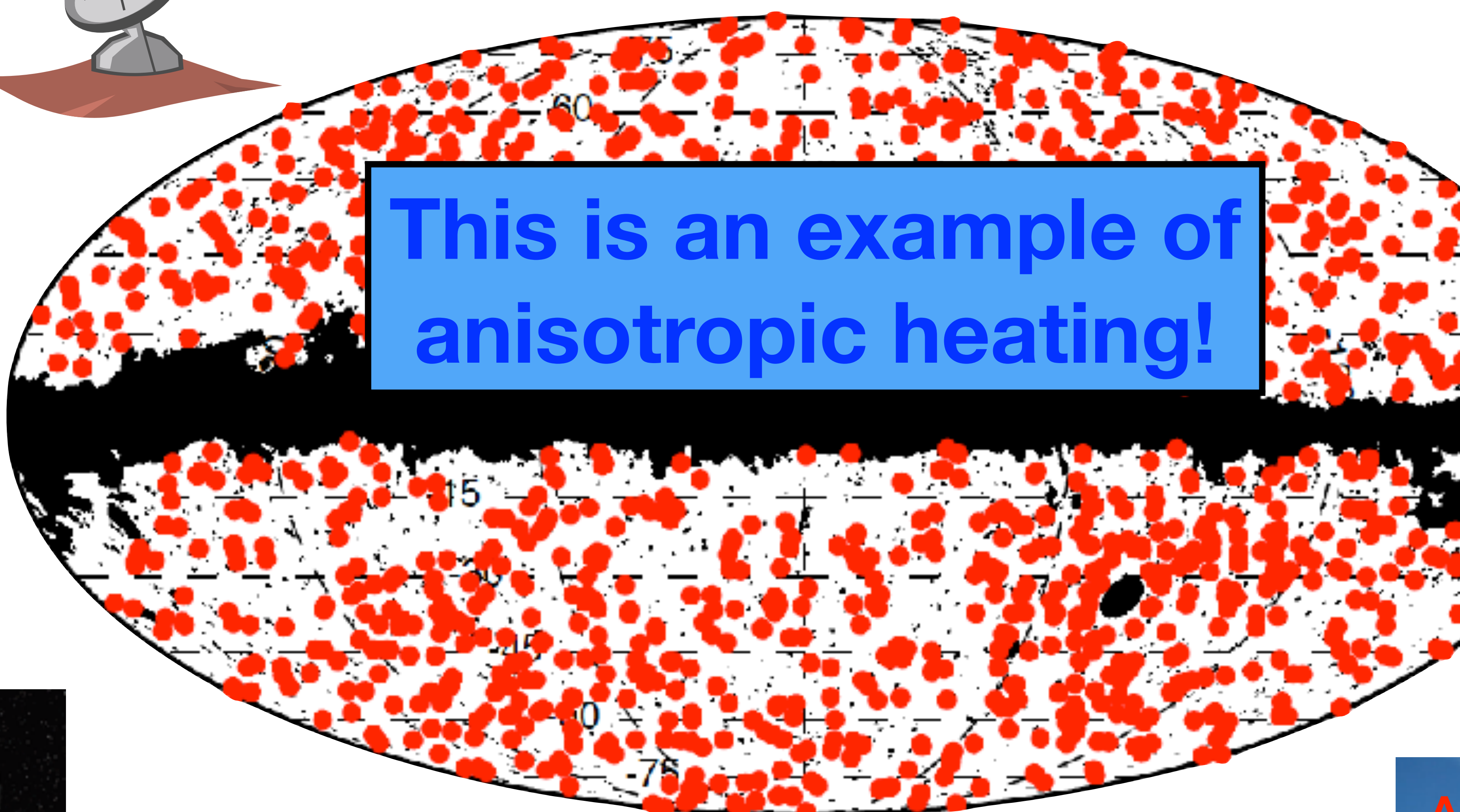
Planck Collaboration, 2013, paper XXIX

Thermal SZ effect is now routinely observed!

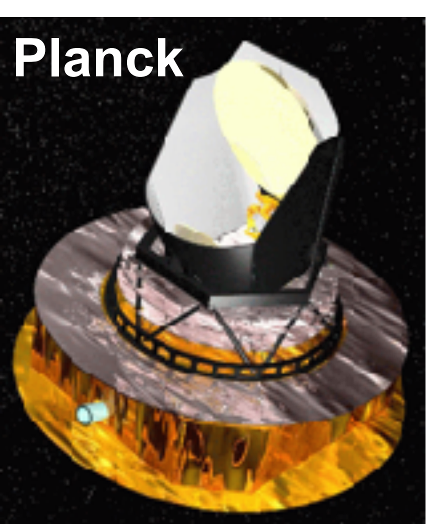




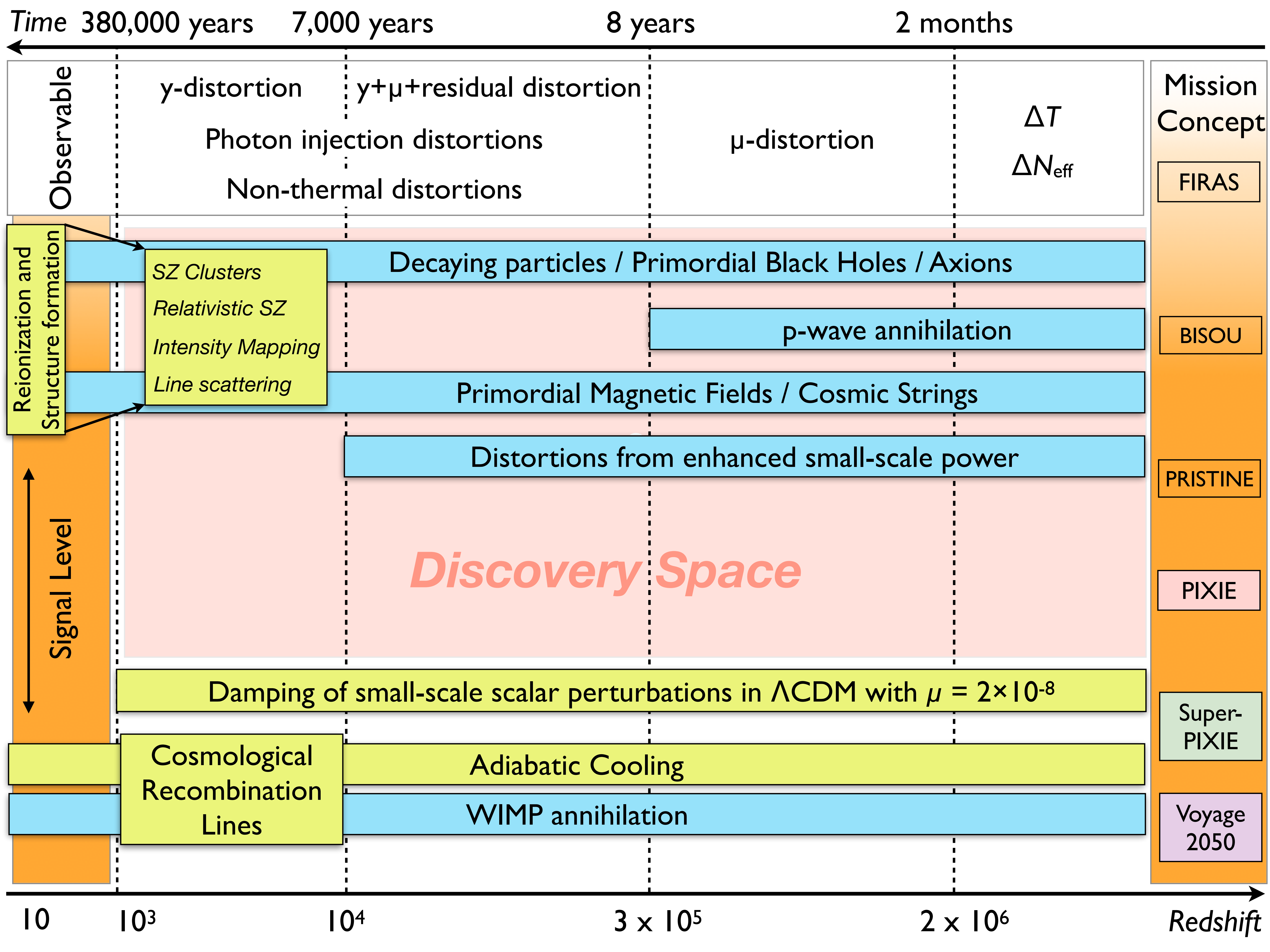
~ 1230 objects



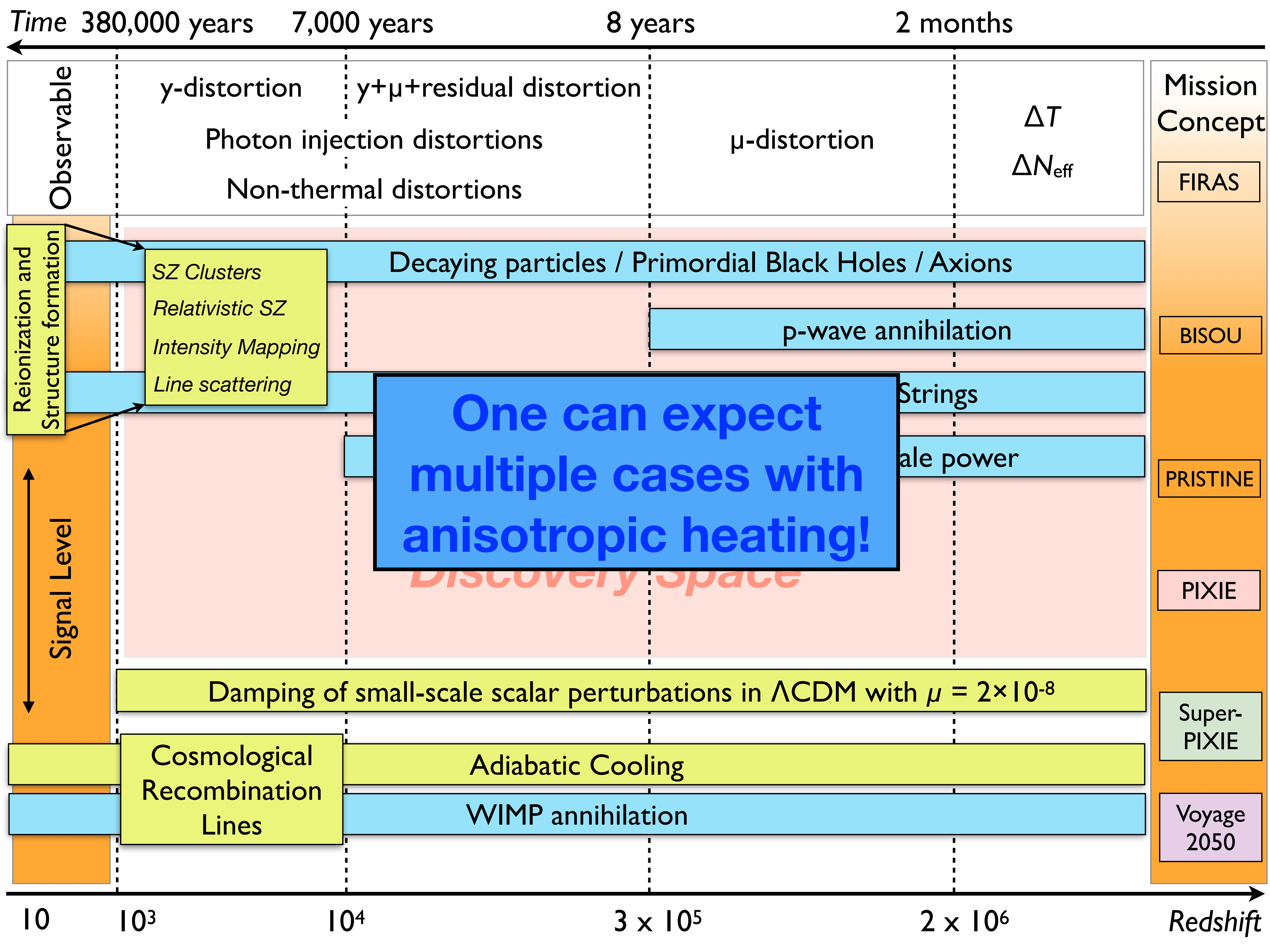
This is an example of anisotropic heating!



Planck Collaboration, 2013, paper XXIX



Time 380,000 years
7,000 years
8 years
2 months
Observable
y-distortion
y+μ+residual distortion
Photon injection distortions
Non-thermal distortions
μ-distortion
ΔT
ΔN_eff
Mission Concept
FIRAS
BISOU
PRISTINE
PIXIE
Super-PIXIE
Voyage 2050
Reionization and Structure formation
SZ Clusters
Relativistic SZ
Intensity Mapping
Line scattering
Decaying particles / Primordial Black Holes / Axions
p-wave annihilation
Primordial Magnetic Fields / Cosmic Strings
Distortions from enhanced small-scale power
Discovery Space
Signal Level
Damping of small-scale scalar perturbations in ΛCDM with μ = 2×10^-8
Cosmological Recombination Lines
Adiabatic Cooling
WIMP annihilation
10
10^3
10^4
3 x 10^5
2 x 10^6
Redshift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One can expect multiple cases with anisotropic heating!
Discovery Space
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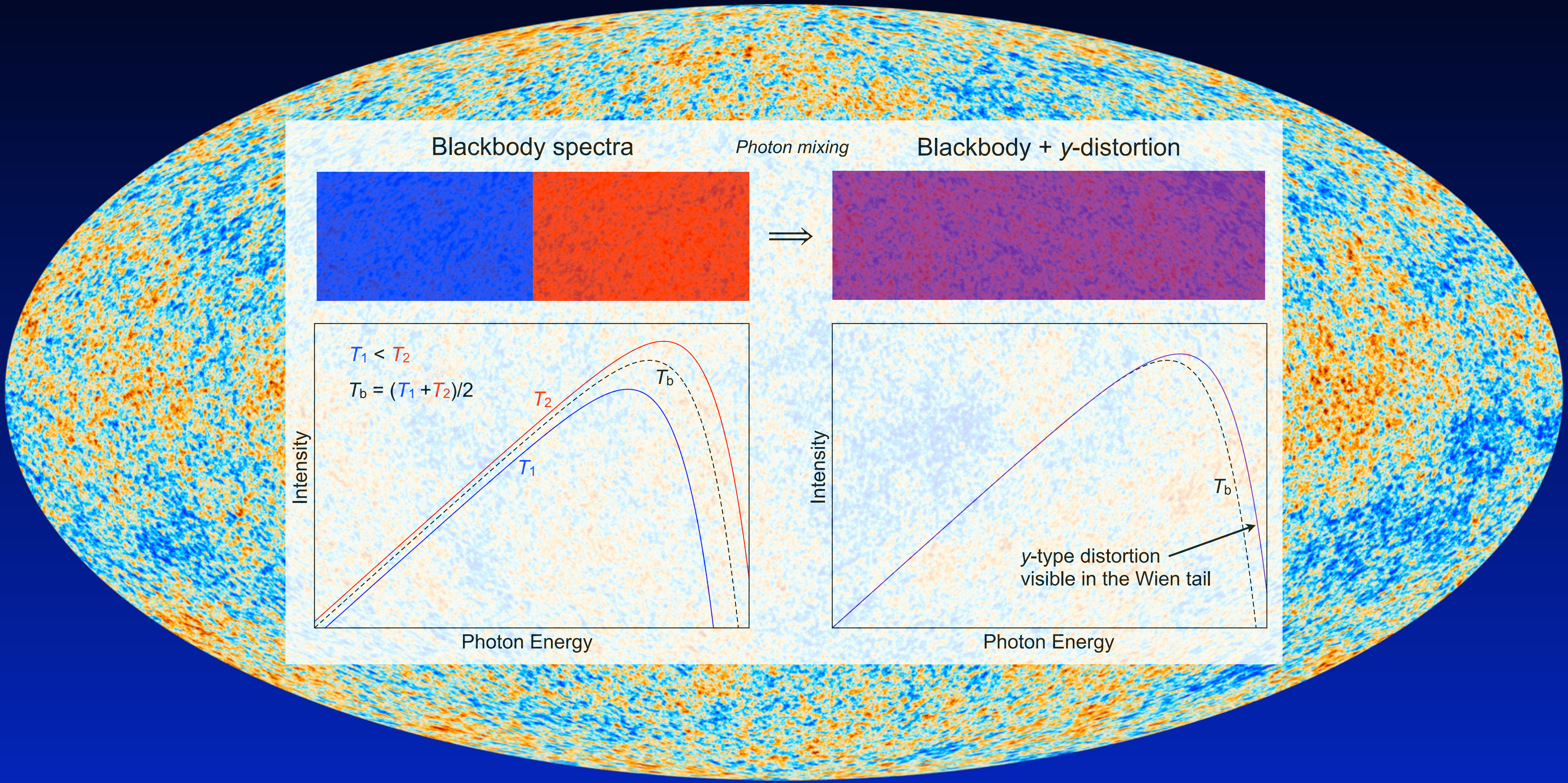
Spatially-varying heating from the dissipation of acoustic modes for non-Gaussian perturbations

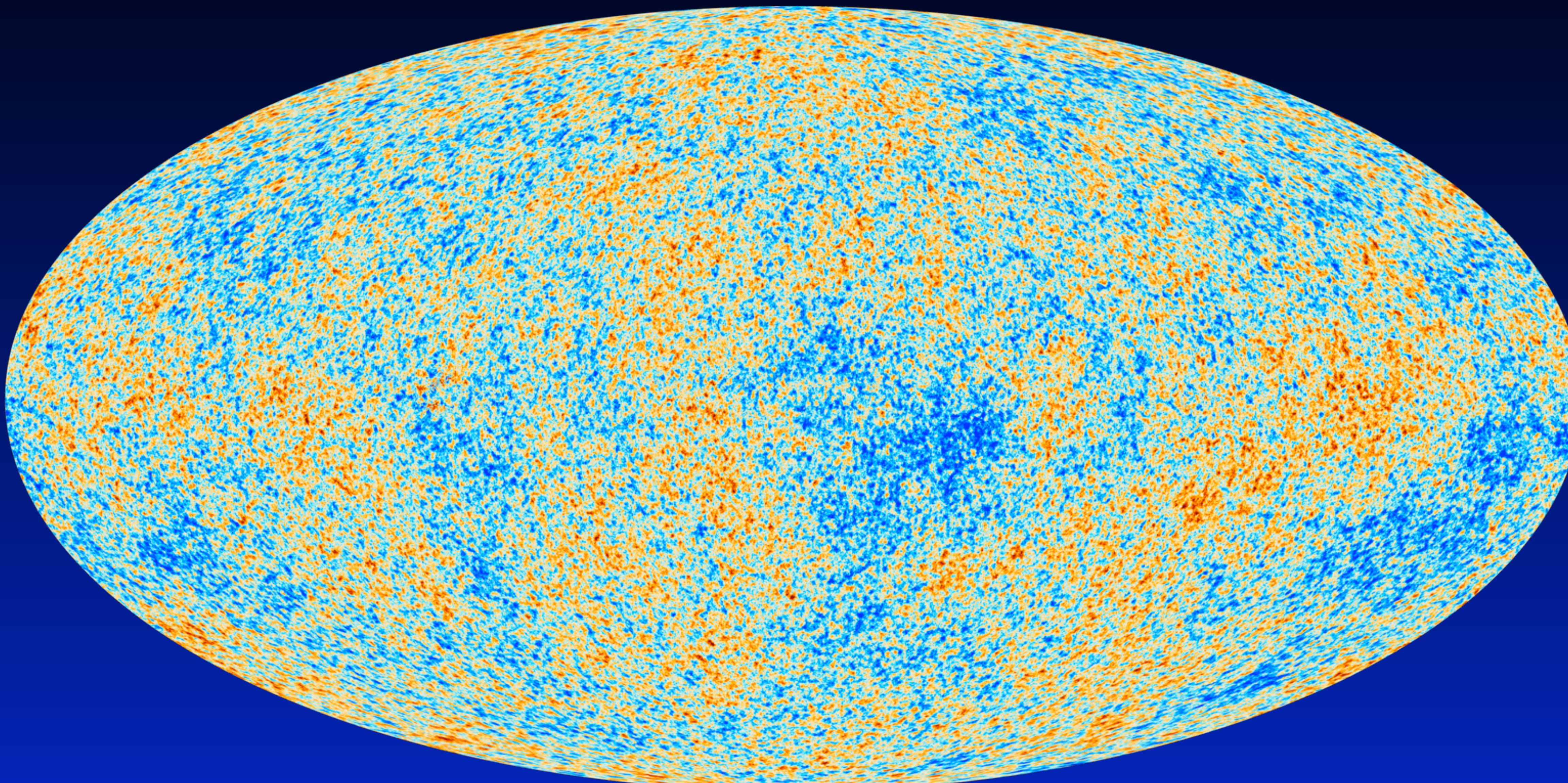
Spatially-varying heating and dissipation of acoustic modes for non-Gaussian perturbations



- Uniform heating (e.g., dissipation in Gaussian case or quasi-uniform energy release)
  → distortion practically the same in different directions

Pajer & Zaldarriaga, 2012; Ganc & Komatsu, 2012; Biagetti et al., 2013; JC et al., 2016

Spatially-varying heating and dissipation of acoustic modes for non-Gaussian perturbations



- Uniform heating (e.g., dissipation in Gaussian case or quasi-uniform energy release)
  → distortion practically the same in different directions
- Spatially varying heating rate (e.g., due to *ultra-squeezed limit non-Gaussianity* or *cosmic bubble collisions*)
  → distortion varies in different directions
  → probe of *scale-dependent* non-Gaussianity at k~10 Mpc^{-1} and ~740 Mpc^{-1}

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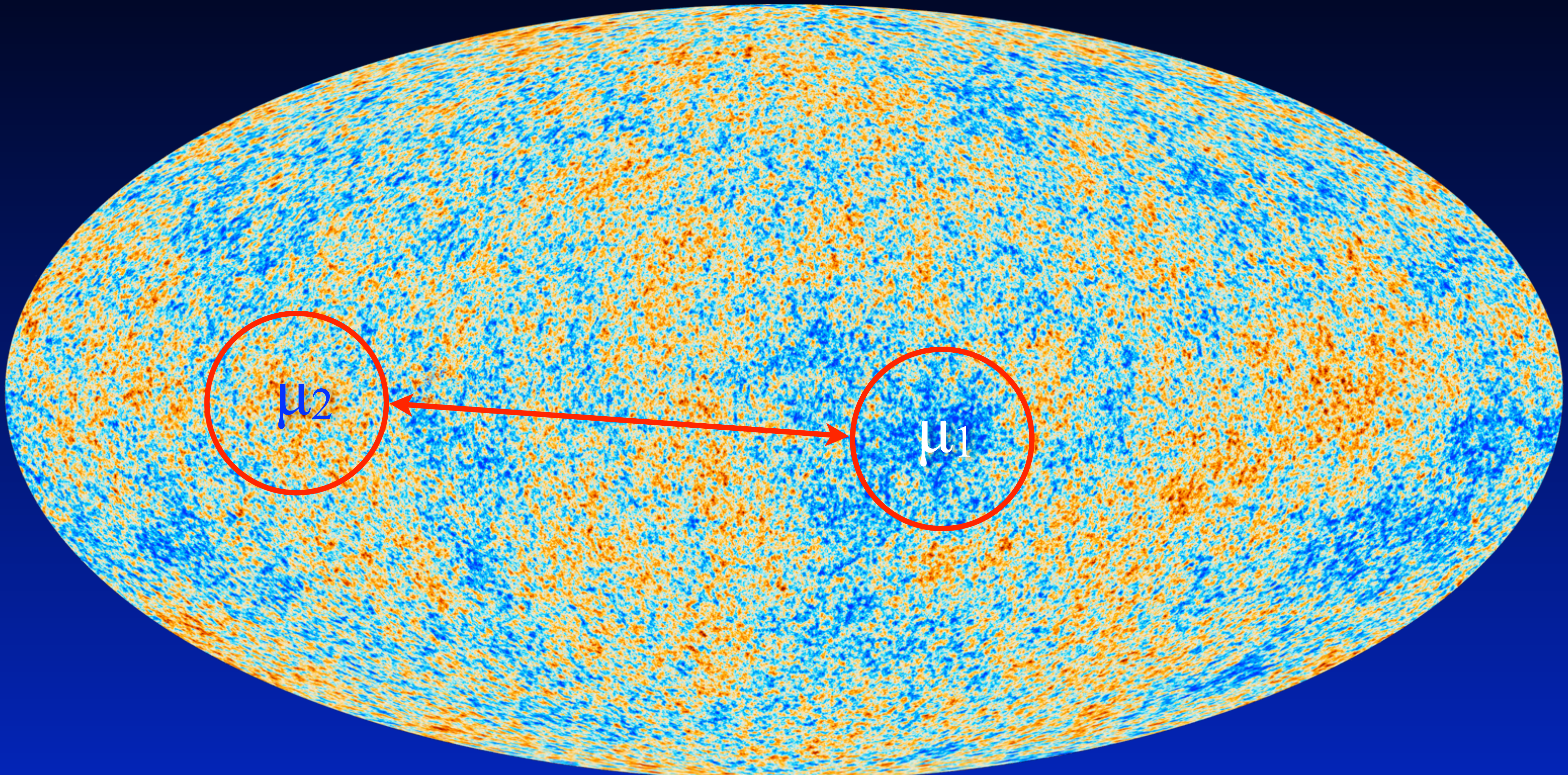

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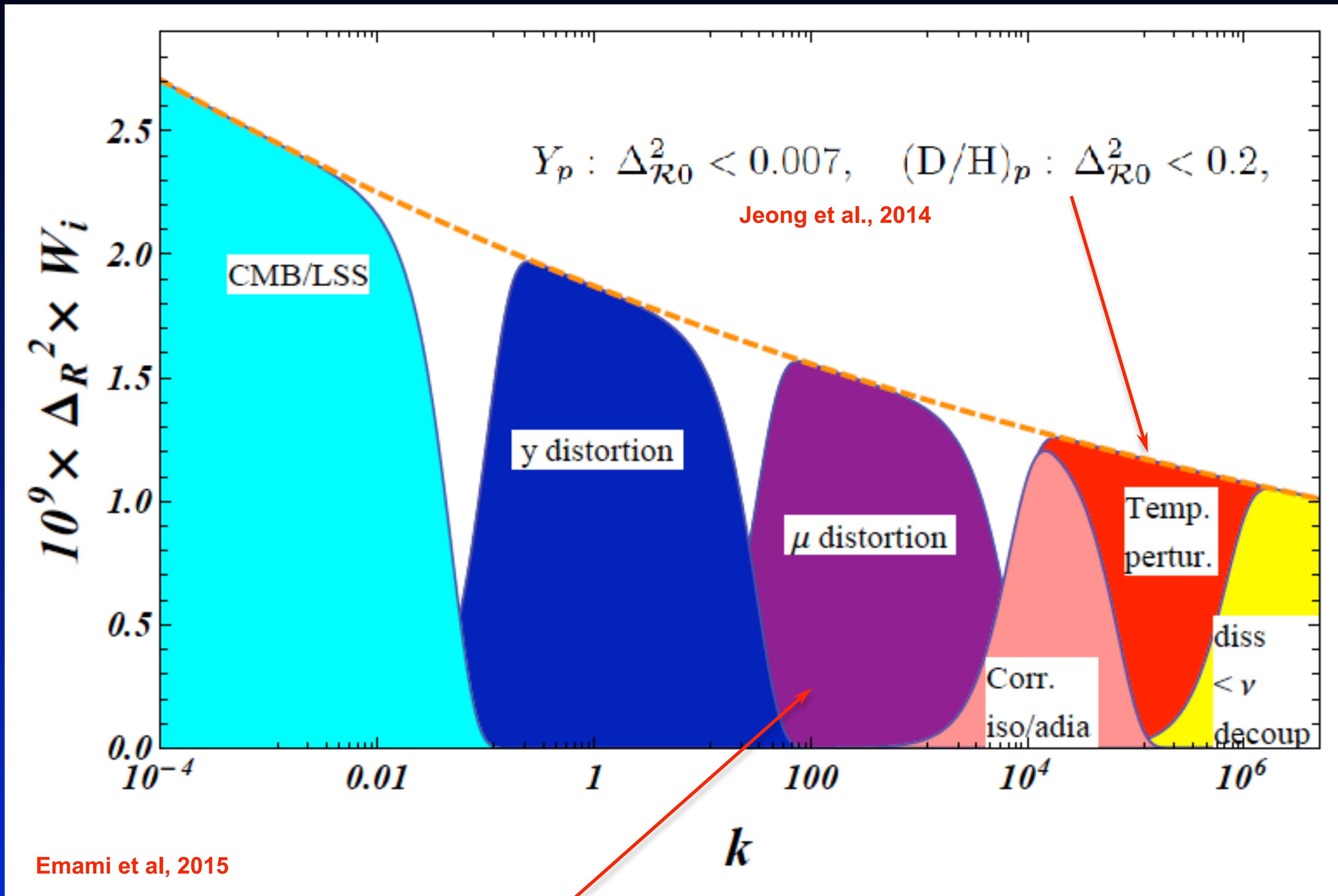
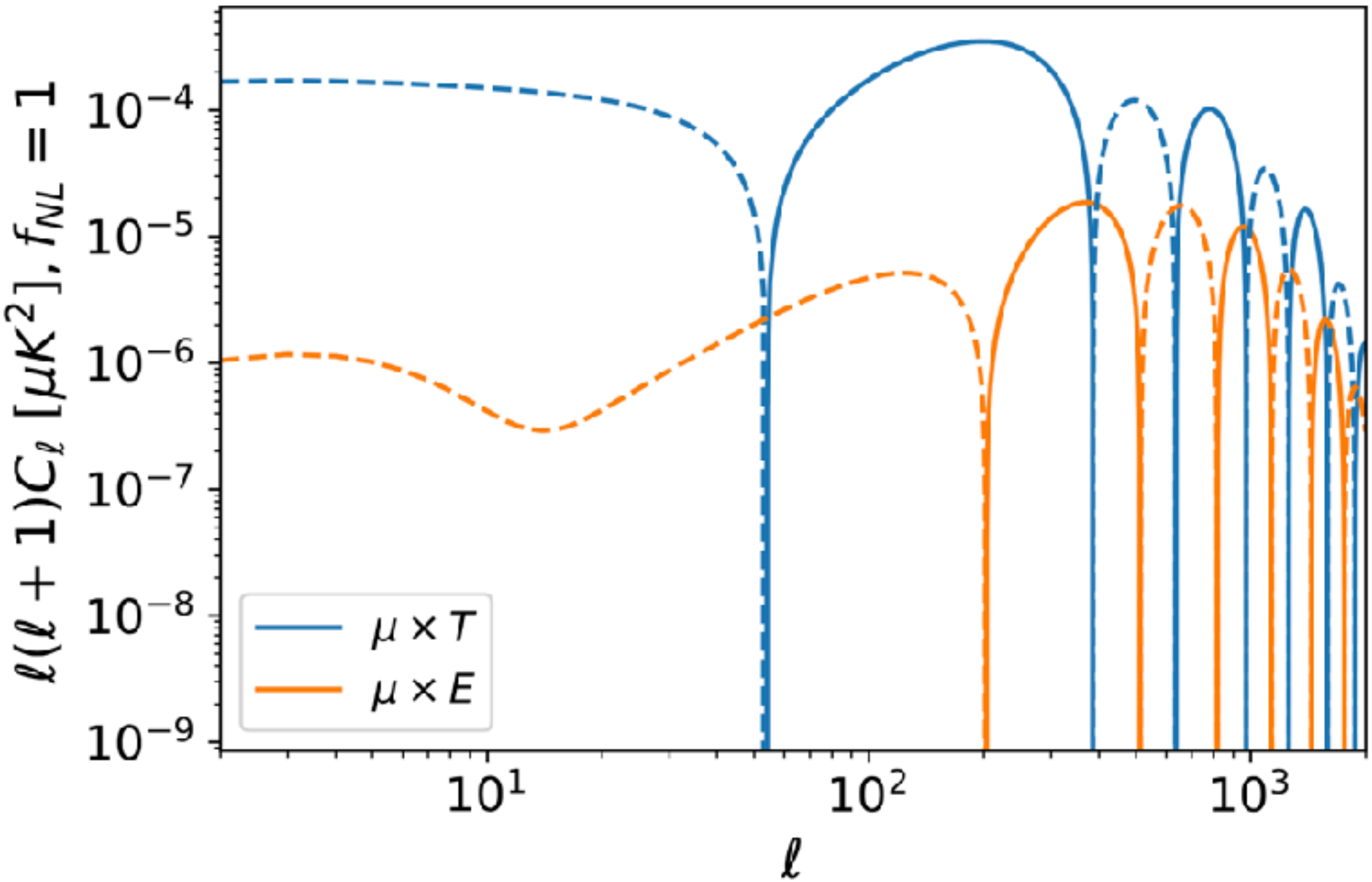


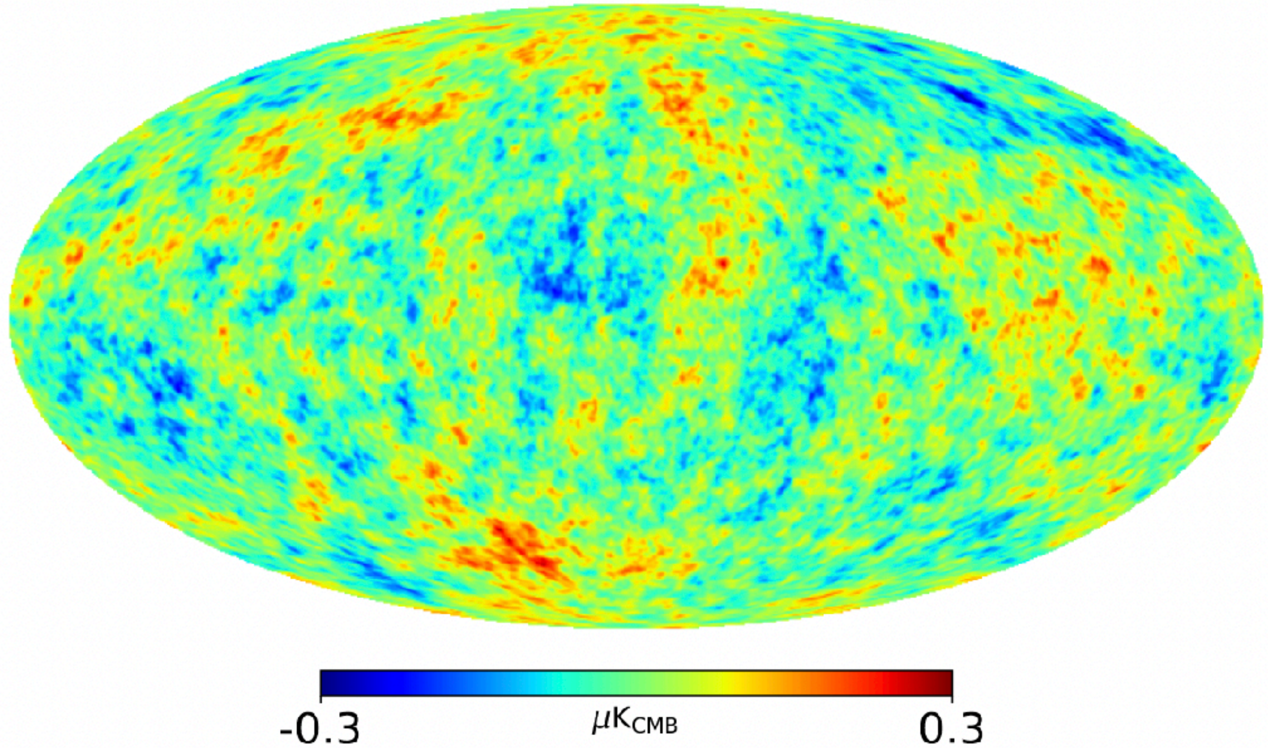
- *μ for ultra-squeezed limit non-Gaussianity*
  (Pajer & Zaldarriaga, 2012; Ganc & Komatsu, 2012; Biagetti et al., 2013, JC et al., 2016)

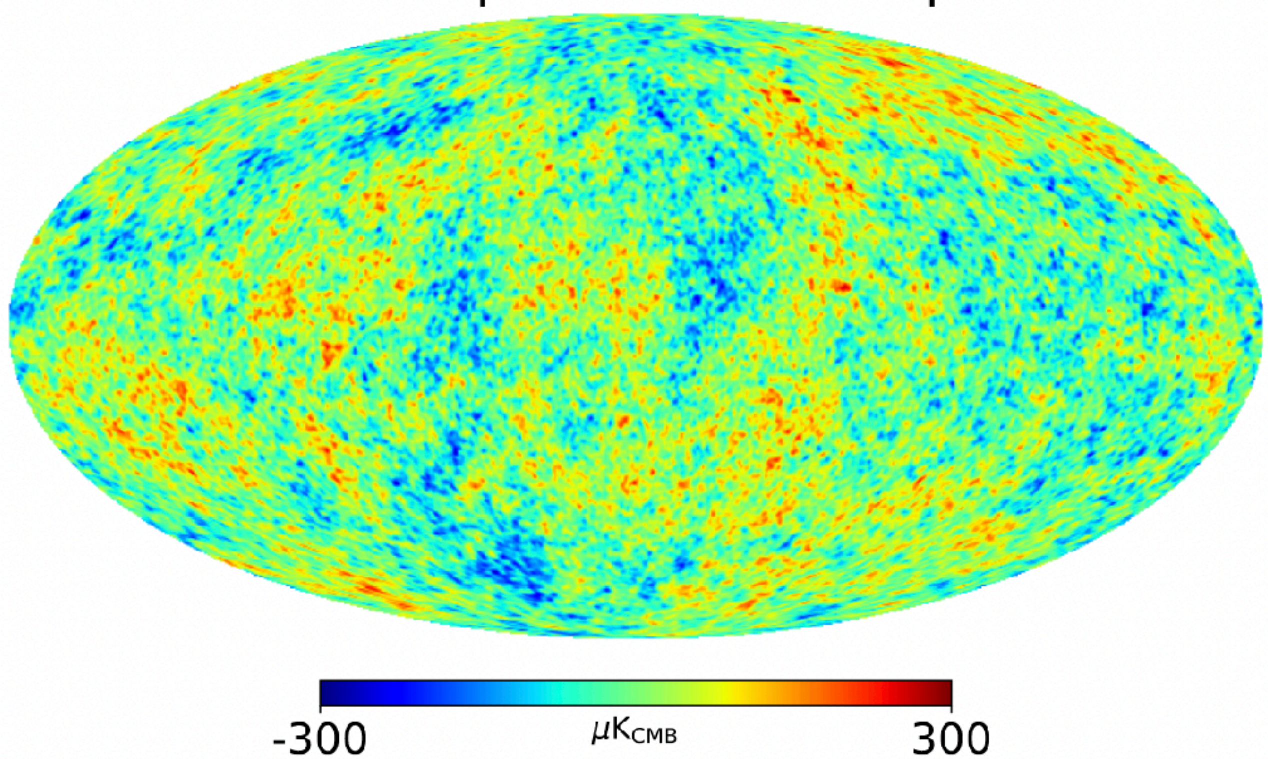
Primordial (local-type) non-Gaussianity causes *μ-T/E* correlations



Pajer & Zaldarriaga, 2012;
Ganc & Komatsu, 2012;
Biagetti et al., 2013;
Ota et al., 2014;
Zegeye et al., 2023, ArXiv:2303.00916

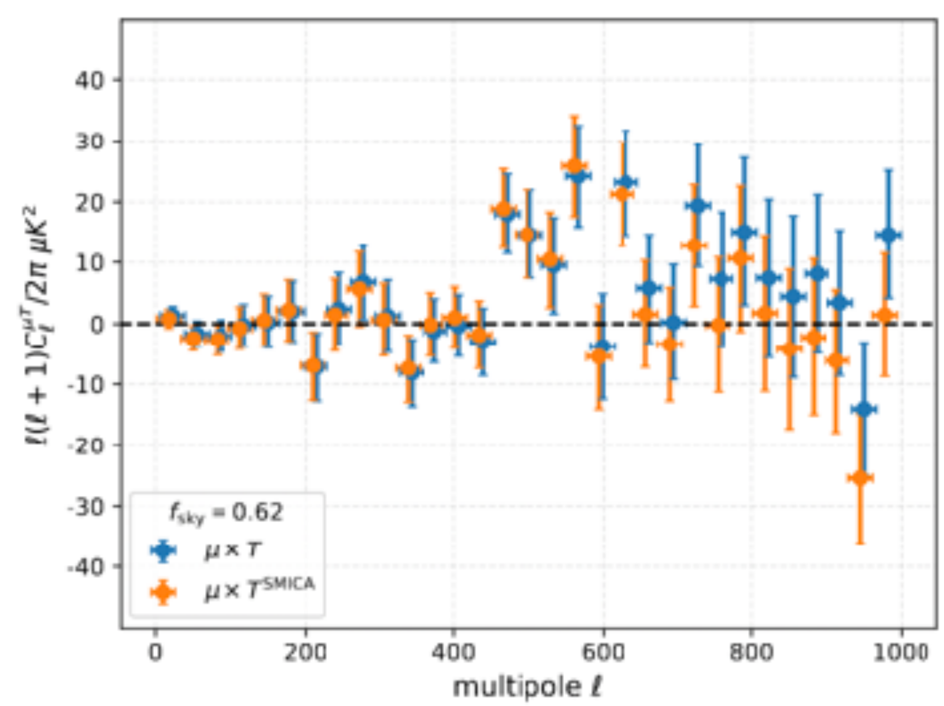
Figure from Zegeye et al., 2023





Distortion anisotropies

- Probe of primordial non-Gaussianity (local type)
- Sensitive to correlations at much *smaller scales* $k \sim 740\ \mathrm{Mpc}^{-1}$
- Complements other probes
- Science target for future CMB imagers (e.g., *Litebird,* CMB-S4 & *PICO*)
- Polarization correlations can also be used (reduced foreground challenge?)



Planck: $|f_{NL}| \lesssim 6800$ (95% c.l.)

Remazeilles & JC, 2018, ArXiv:1802.10101
Remazeilles, Ravenni & JC, 2021, ArXiv:2110.14664
Orlando, Meerburg & Patil, 2021, ArXiv:2109.01095
Zegeye et al., 2023, ArXiv:2303.00916

Rotti et al., MNRAS 2022

Distortions from effect of perturbations on the *average* spectrum

*using conformal Newtonian gauge

Distortions from effect of perturbations on the *average* spectrum

For temperature anisotropies

- *Direct* source of anisotropies from initial conditions (i.e., inflation)
- *New* sources/conversion from evolution of perturbations
  - Doppler peaks
  - Potential driving
  - Free streaming process

$$\frac{\partial n^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla n^{(1)} + \hat{O}_x n^{(0)} \left(\frac{\partial \Phi^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla \Psi^{(1)} \right) \approx \dot{\tau} \left[n_0^{(1)} + \frac{1}{10} n_2^{(1)} - n^{(1)} + \beta^{(1)} \chi \hat{O}_x n^{(0)} \right]$$

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➡ Doppler and potential driving due to presence of average CMB!

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➡ Doppler and potential driving due to presence of average CMB!

If the average spectrum is distorted this will lead to distortion anisotropies is a similar manner!

*using conformal Newtonian gauge

Challenge of the exact calculation

Challenge of the exact calculation

Standard evolution equations for photon temperature

$$\frac{\partial \Theta^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla \Theta^{(1)} + \frac{\partial \Phi^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla \Psi^{(1)} = \dot{\tau} \left[\Theta_0^{(1)} + \frac{1}{10} \Theta_2^{(1)} - \Theta^{(1)} + \beta^{(1)} \chi \right]$$

- Solve for multiple *k* modes (depending on how clever the sampling is…)
- Multipoles $\ell \sim$ 5-20 (depending on required precision and truncation scheme…)

➡ Need to solve $\mathcal{O}(30)$ coupled ODEs for ~10^2-10^3 values in *k*

Challenge of the exact calculation

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For spectrum also frequency evolution has to be followed!

- `CosmoTherm` runs usually require ~4000 bin in frequency (~30-60 seconds)
- Each ℓ mode in principle has a different spectrum

➡ Need to solve $\mathcal{O}(10^4)$ coupled ODEs for ~10^2-10^3 values in *k*

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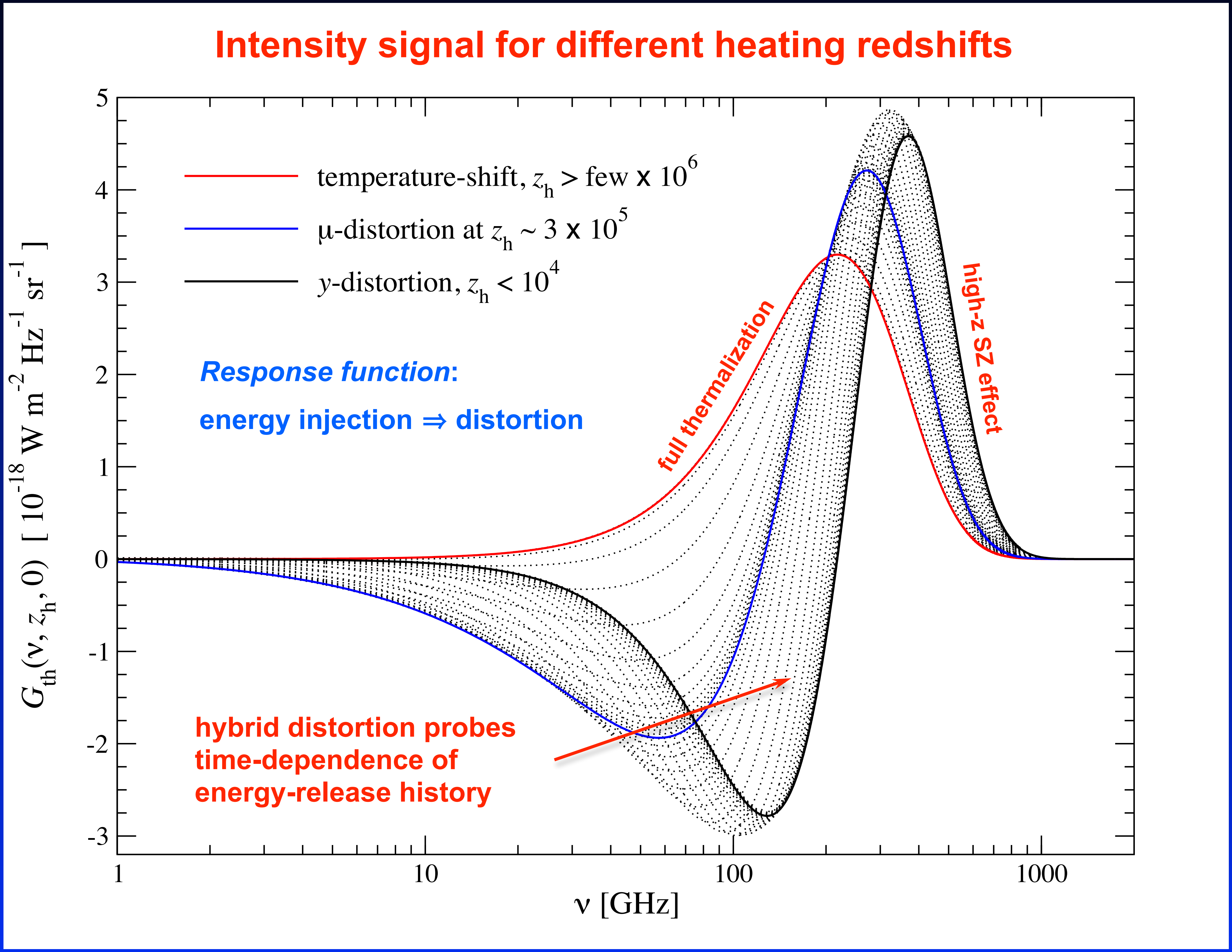
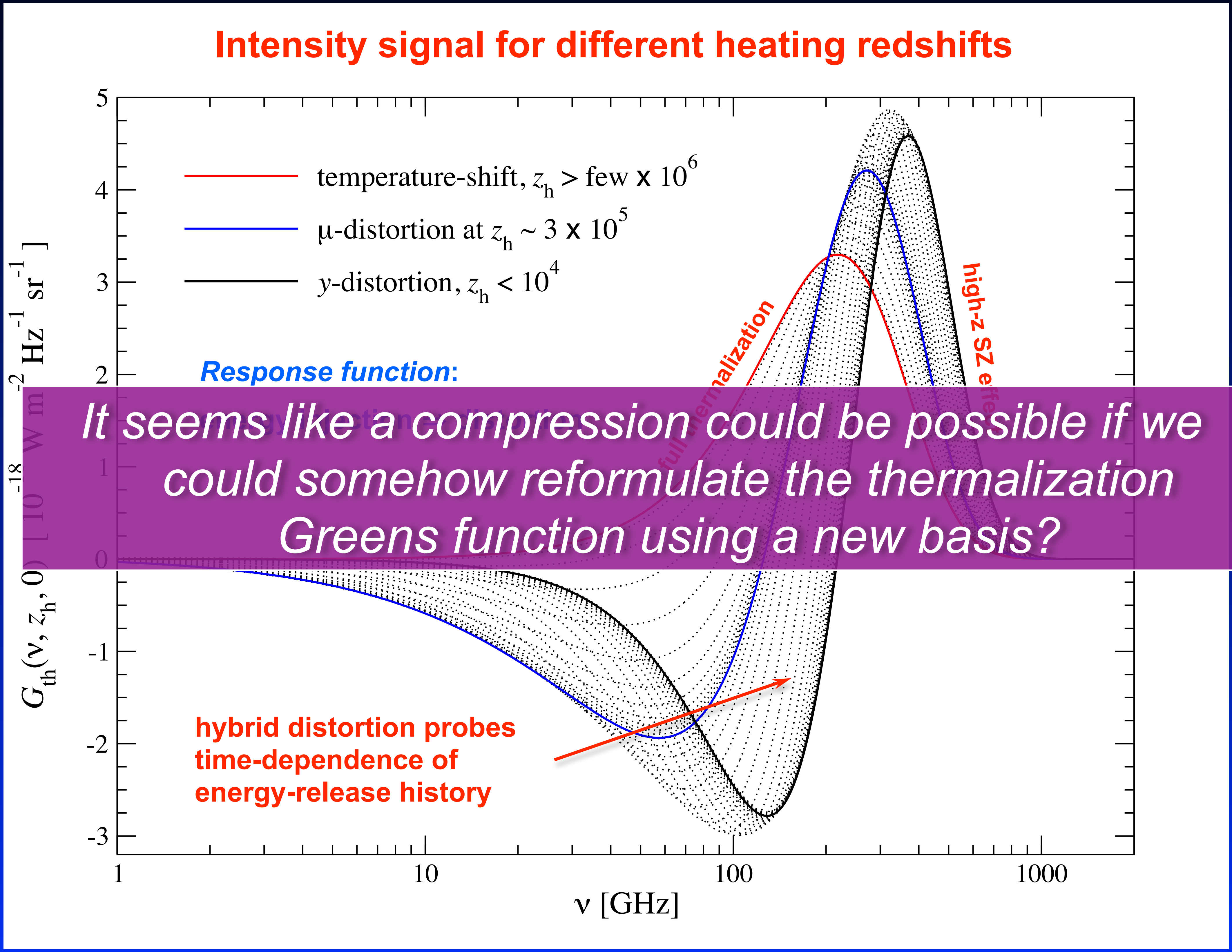
Problem becomes 'uncomfortable' even with additional approximations and HPC!

Greens function method for thermalization process



JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120

Greens function method for thermalization process



It seems like a compression could be possible if we could somehow reformulate the thermalization Greens function using a new basis?

JC & Sunyaev, 2012, ArXiv:1109.6552
JC, 2013, ArXiv:1304.6120

Simplest G, M and Y decomposition

Ansatz: $\Delta n = \Theta\, G(x) + \mu\, M(x) + y\, Y(x)$

$$G(x) = \frac{x\,\mathrm{e}^x}{(\mathrm{e}^x - 1)^2}, \qquad Y(x) = G(x)\left[x\frac{\mathrm{e}^x + 1}{\mathrm{e}^x - 1} - 4\right], \qquad M(x) = G(x)\left[\frac{1}{\beta_M} - \frac{1}{x}\right]$$

Temperature shift   y-distortion μ-distortion

$$x = \frac{h\nu}{kT}$$

Redshift-independent photon frequency

Simplest G, M and Y decomposition

Ansatz: $\Delta n = \Theta\, G(x) + \mu\, M(x) + y\, Y(x)$

$$\frac{\partial \Theta}{\partial t} \approx \gamma_T\, x_c\, \dot{\tau}\theta_z\, \mu, \qquad \frac{\partial y}{\partial t} \approx \frac{1}{4}\dot{Q} - 4\dot{\tau}\theta_z\, y \qquad \frac{\partial \mu}{\partial t} \approx \gamma_\rho\, 4\dot{\tau}\theta_z\,(4y) - \gamma_N\, x_c\, \dot{\tau}\theta_z\, \mu$$

Source of distortions

$$\theta_z = \frac{kT_z}{m_e c^2}$$

Dimensionless temperature

$$\gamma_T, \gamma_N, \gamma_\rho$$

Coefficients from energy integrals

$$x_c$$

Double Compton / Bremsstrahlung critical frequency

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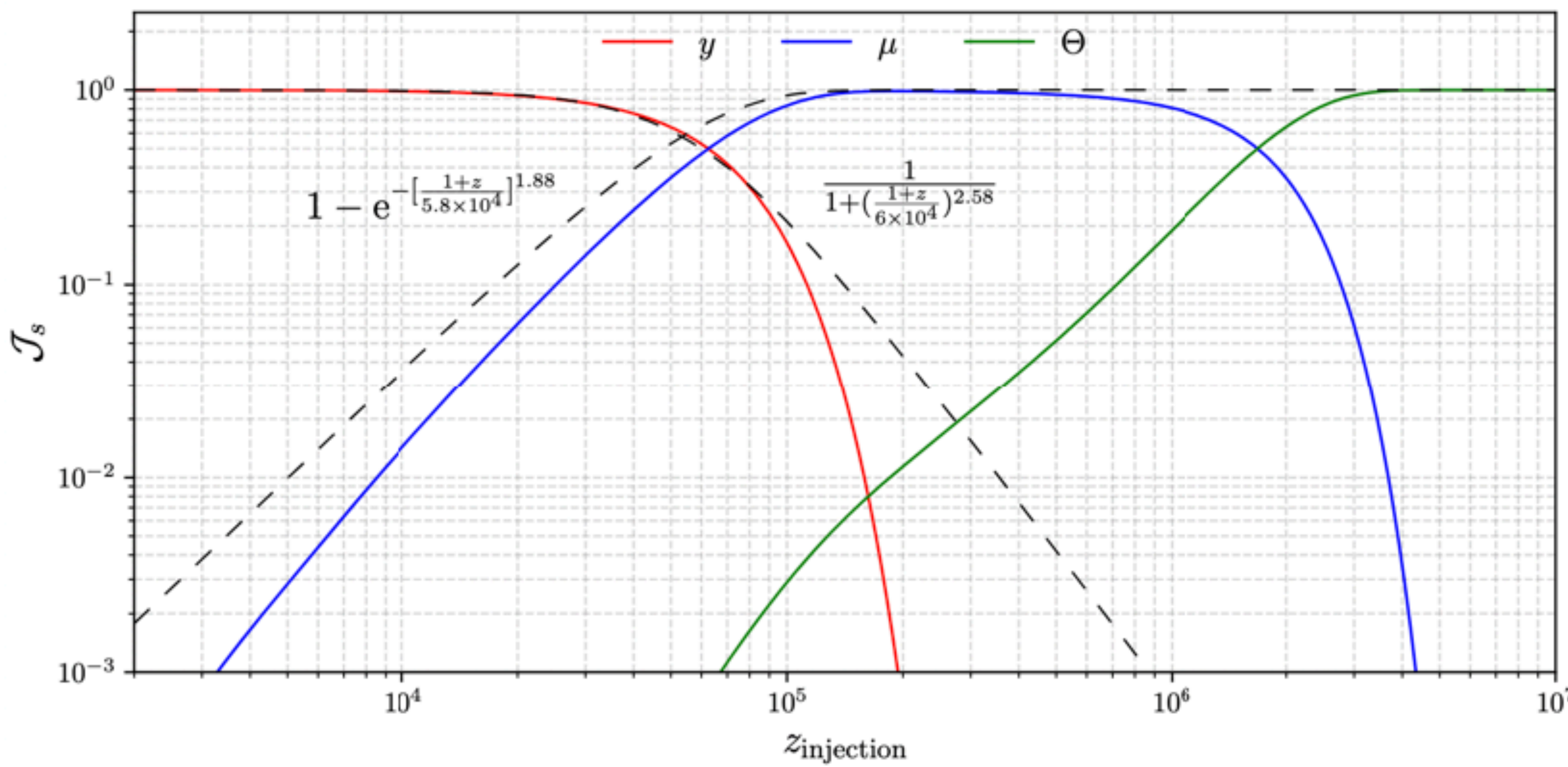

$$\mathcal{J}_y \approx e^{-4y_z}, \qquad \mathcal{J}_\mu \approx (1 - \mathcal{J}_y)\, \mathcal{J}_{\rm bb}, \qquad \text{and} \quad \mathcal{J}_T \approx 1 - \mathcal{J}_{\rm bb} \qquad \mathcal{J}_{\rm bb} \approx e^{-(z/z_\mu)^{2.5}}$$

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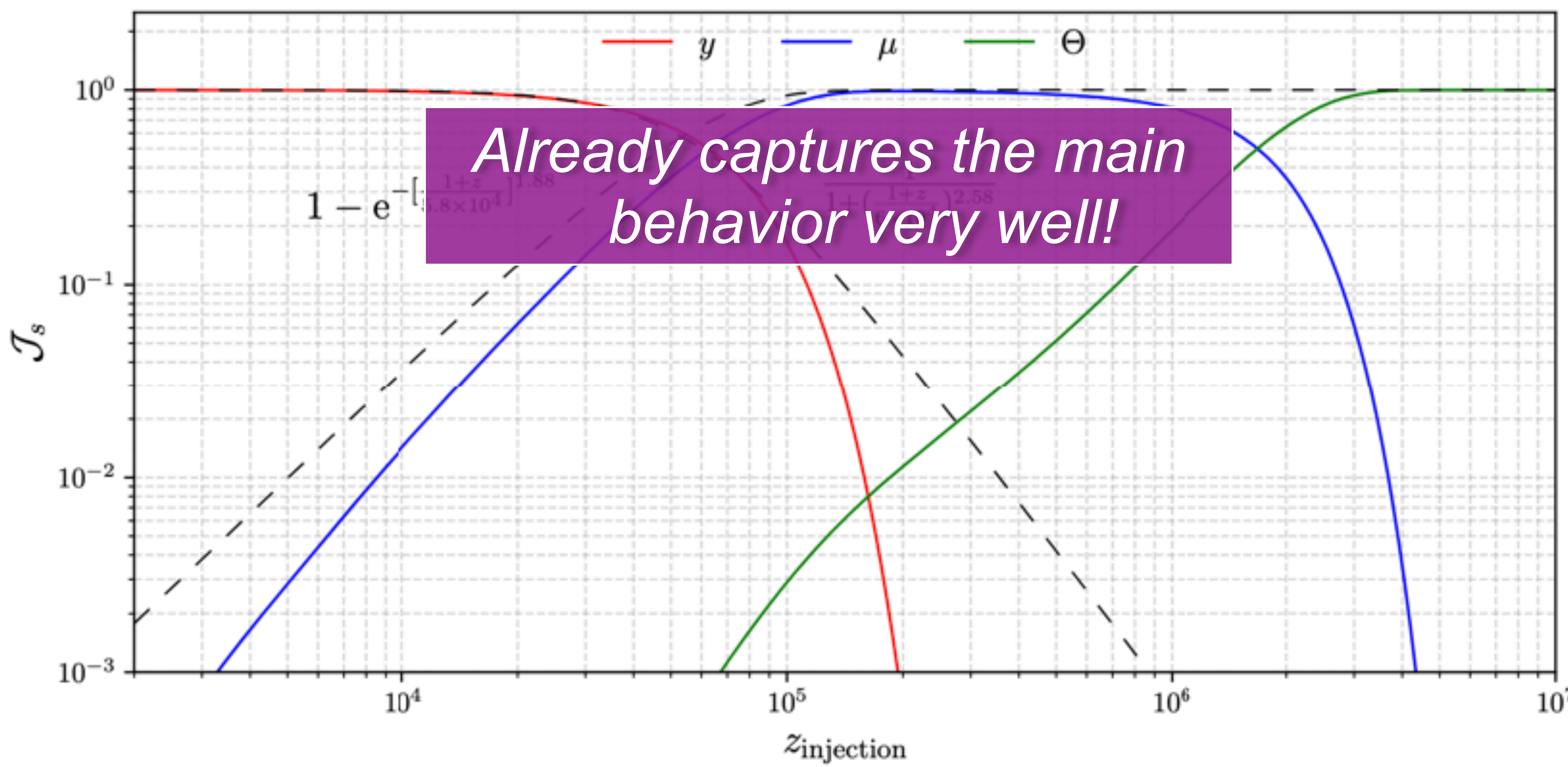

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Beyond simplest G, M and Y decomposition

Ansatz: $$\Delta n = \Theta\, G(x) + \mu\, M(x) + y\, Y(x) + \sum_{k=1}^{\infty} y_k\, Y_k(x)$$

$$Y_k(x) = (1/4)^k \hat{O}_x^k Y(x)$$

'Boosts' of Y

$$\hat{O}_x = -x\partial_x$$

Boost operator

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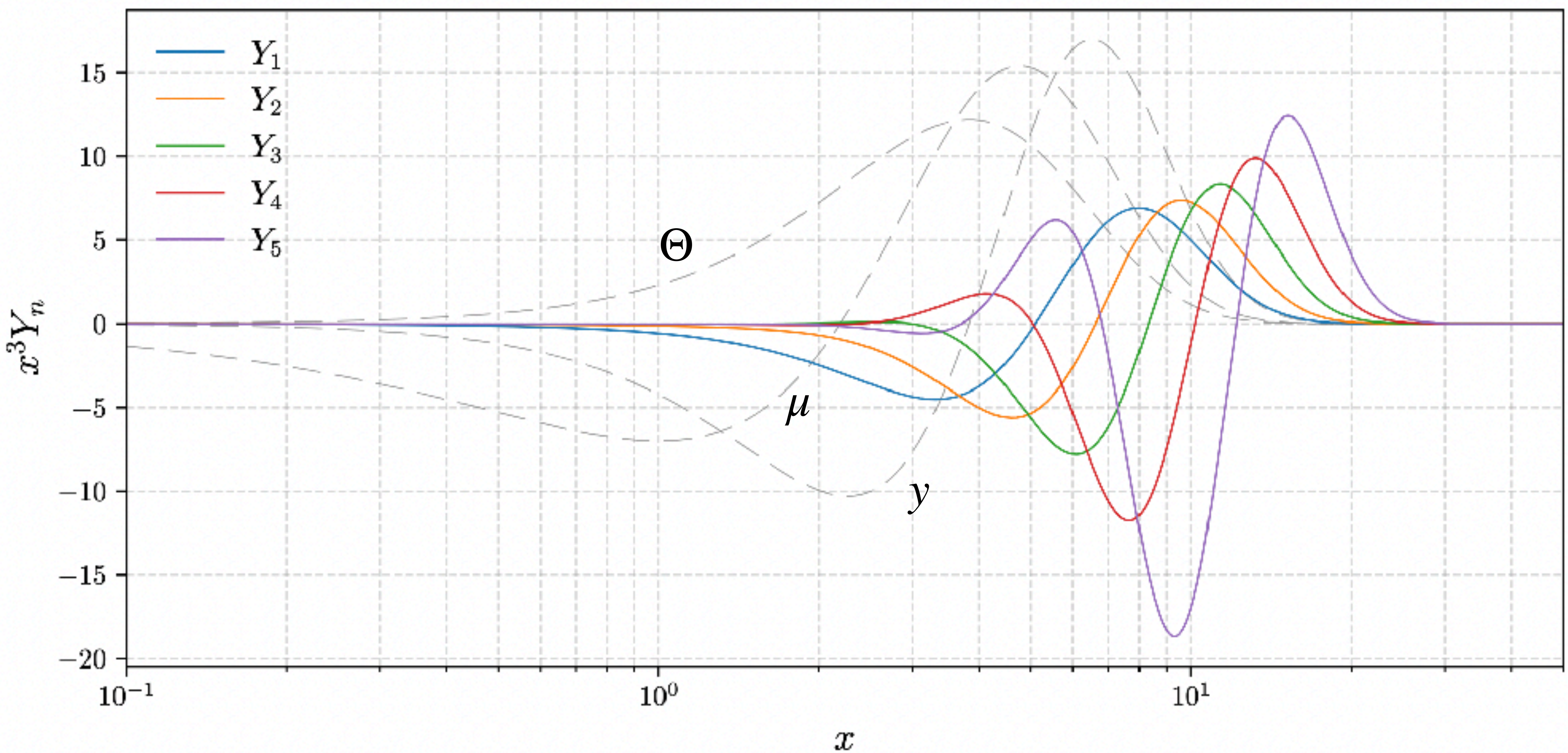
Illustration of the Y_k basis



- Basis extension focuses on high frequency part
- Imperfections of representation at intermediate frequencies expected

Beyond simplest G, M and Y decomposition

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$$\Delta n' = \Theta_{\rm e}\, Y + \hat{\mathcal{K}}_x\, \Delta n \qquad \longleftrightarrow \qquad \boldsymbol{y}' \approx M_{\rm K}\, \boldsymbol{y}$$

Kompaneets Equation

Kompaneets Matrix

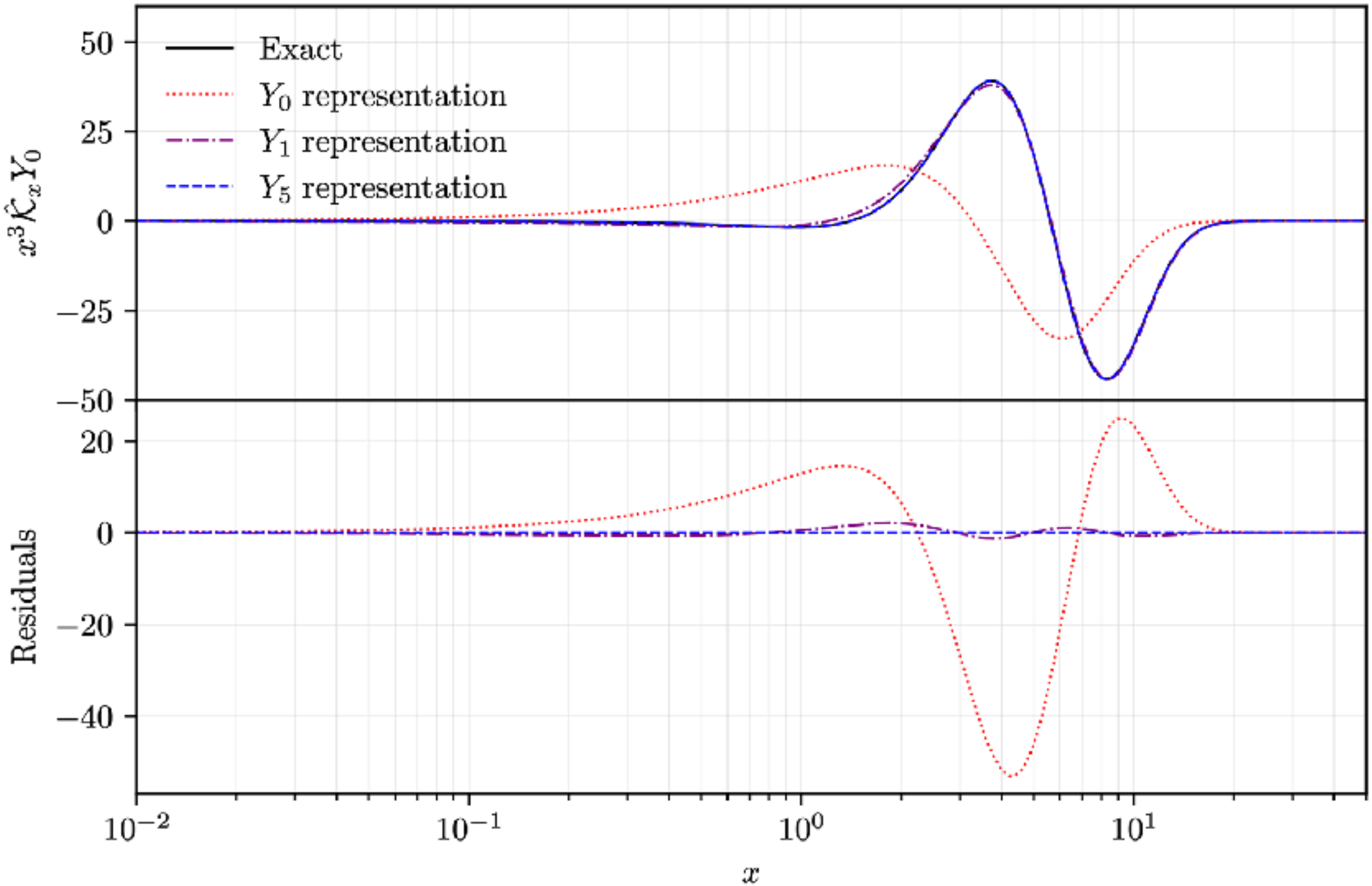
$$\boldsymbol{y} = (\Theta, y, y_1, \ldots, y_N, \mu)^T$$

Spectral parameter vector

$$\hat{\mathcal{K}}_x = x^{-2}\partial_x x^4 \partial_x + x^{-2}\partial_x x^4 [1 + 2n_{\rm bb}]$$

Kompaneets operator

Comptonization of y-spectrum



Representation improved with inclusion of more basis functions

Beyond simplest G, M and Y decomposition

Ansatz: $$\Delta n = \Theta\, G(x) + \mu\, M(x) + y\, Y(x) + \sum_{k=1}^{\infty} y_k\, Y_k(x)$$

Emission/Absorption terms

Heating term

$$\Delta n' = \Theta_e\, Y + \hat{\mathcal{K}}_x\, \Delta n + \Delta n'|_{\rm em/abs} + \Delta n'|_{\rm h} \quad \longleftrightarrow \quad \boldsymbol{y}' \approx M_{\rm K}\, \boldsymbol{y} + \boldsymbol{D} + \frac{\boldsymbol{Q}'}{4},$$

$$\boldsymbol{D} = (\gamma_T x_c\, \mu, 0, 0, \dots, 0, -\gamma_N x_c\, \mu)^T, \quad \boldsymbol{Q}' = (0, Q', 0, \dots, 0, 0)^T.$$

$$\boldsymbol{y} = (\Theta, y, y_1, \dots, y_N, \mu)^T$$

Beyond simplest G, M and Y decomposition

Ansatz: $\Delta n = \Theta\, G(x) + \mu\, M(x) + y\, Y(x) + \sum_{k=1}^{\infty} y_k\, Y_k(x)$

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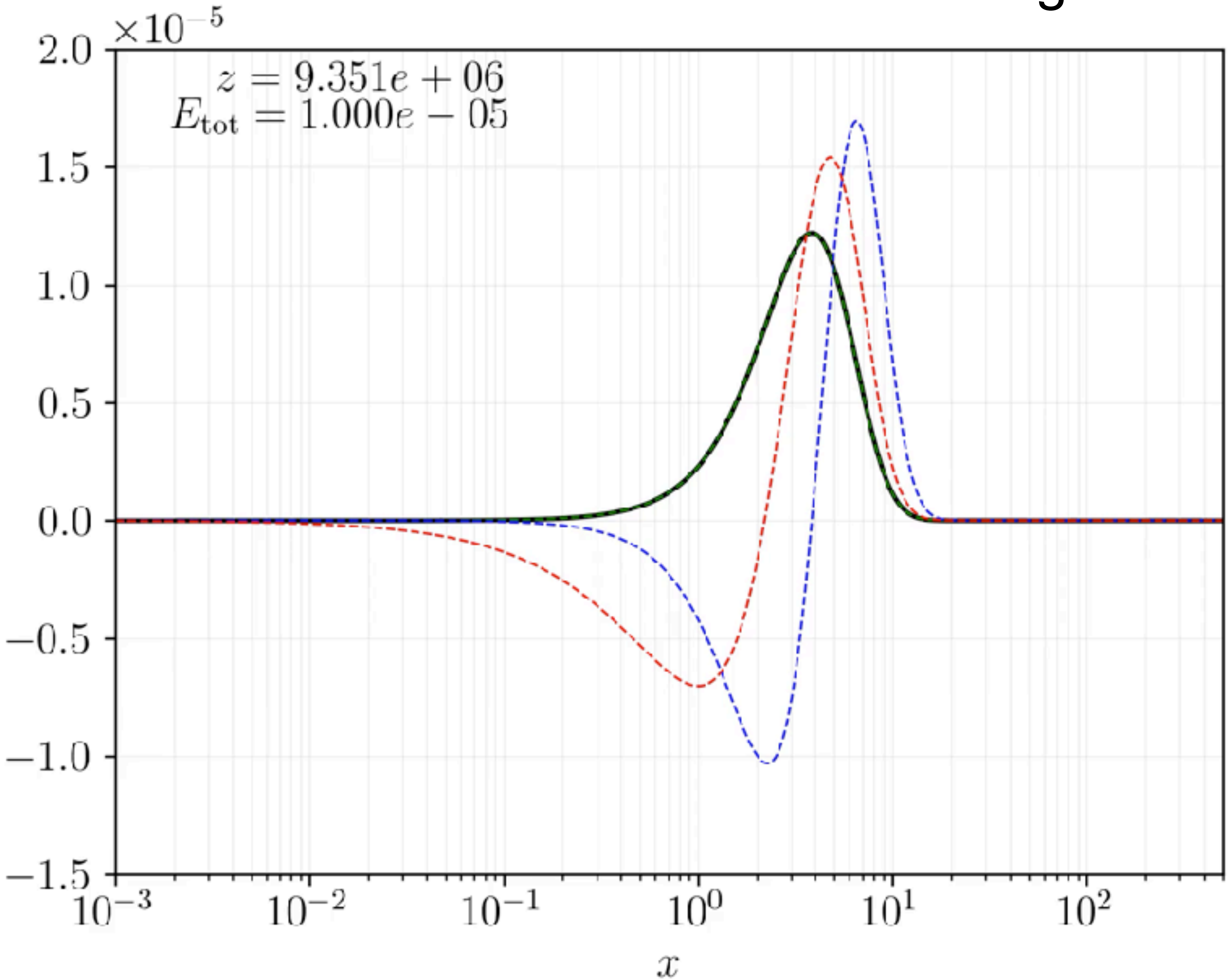
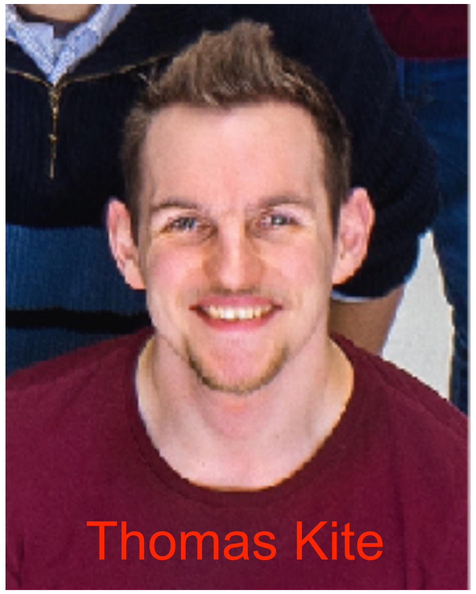
$$\boldsymbol{y} = (\Theta, y, y_1, \ldots, y_N, \mu)^T$$

Thermalization problem reduced to rotation of the spectral distortion parameters with y-source!

Main thermalization stages

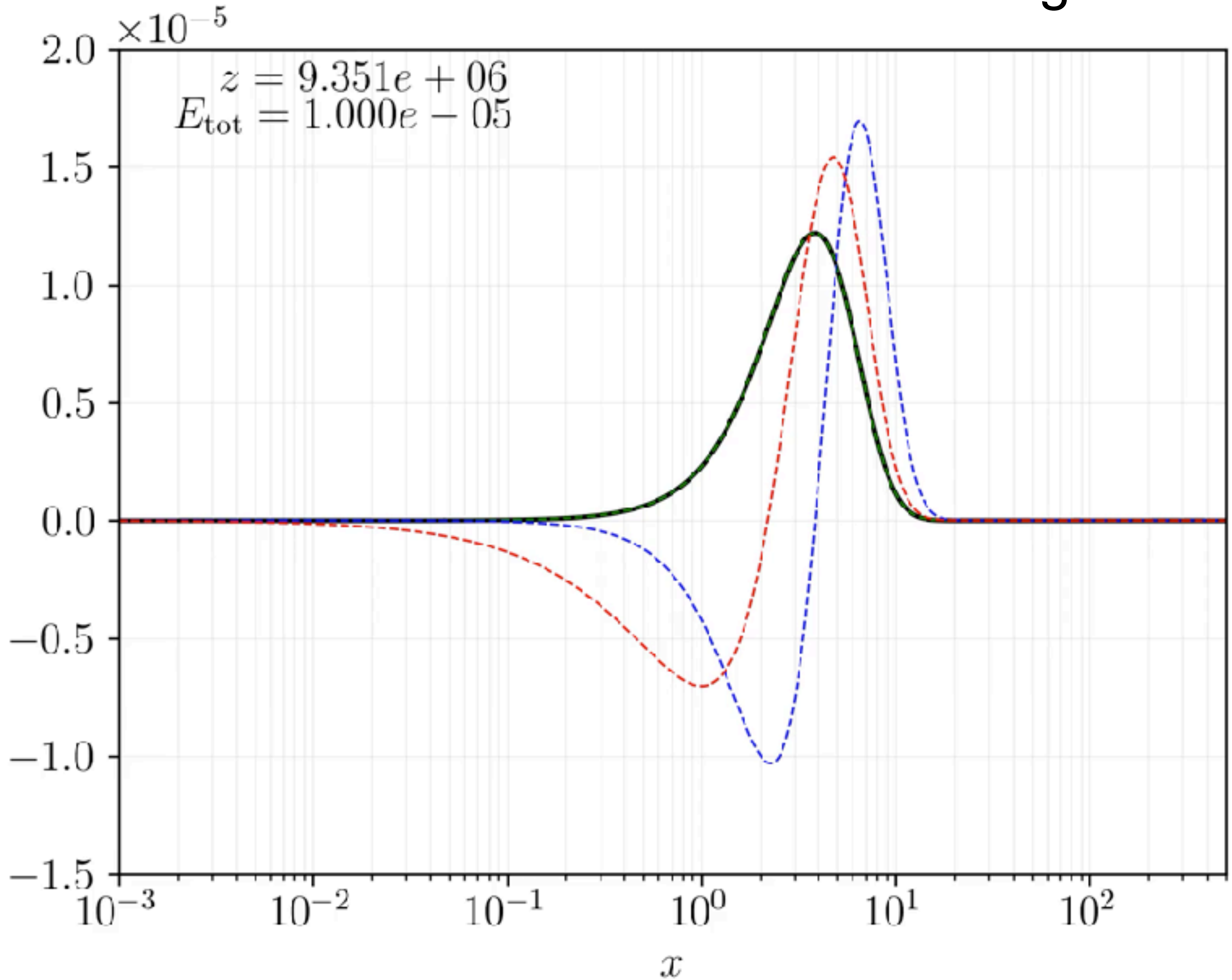


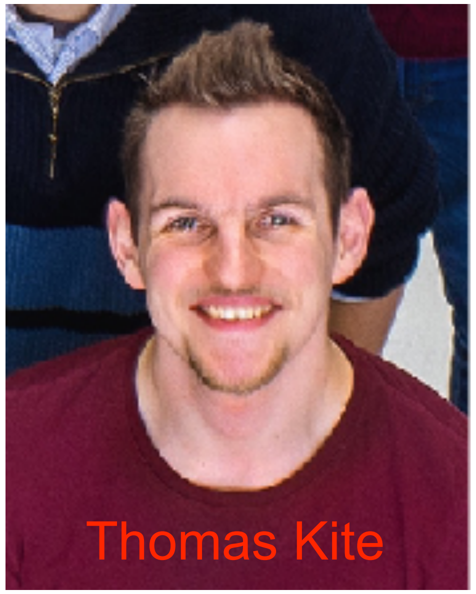


JC, ArXiv:1304.6120
JC, Kite & Ravenni, 2022, papers I
JC, Ravenni & Kite, 2022, papers II
Kite, Ravenni & JC, 2022, papers III

New discretization of the Greens function

Main thermalization stages





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New discretization of the Greens function

Generalizing the Boltzmann hierarchy

Generalizing the Boltzmann hierarchy

$$\frac{\partial n^{(1)}}{\partial t} + \frac{c\hat{\boldsymbol{\gamma}}}{a} \cdot \nabla n^{(1)} + \hat{O}_x n^{(0)} \left(\frac{\partial \Phi^{(1)}}{\partial t} + \frac{c\hat{\boldsymbol{\gamma}}}{a} \cdot \nabla \Psi^{(1)} \right) \approx \dot{\tau} \left[n_0^{(1)} + \frac{1}{10} n_2^{(1)} - n^{(1)} + \beta^{(1)} \chi \hat{O}_x n^{(0)} \right]$$

Generalizing the Boltzmann hierarchy

$$\frac{\partial n^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla n^{(1)} + \hat{O}_x n^{(0)} \left(\frac{\partial \Phi^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla \Psi^{(1)} \right) \approx \dot{\tau} \left[n_0^{(1)} + \frac{1}{10} n_2^{(1)} - n^{(1)} + \beta^{(1)} \chi \hat{O}_x n^{(0)} \right]$$

$$\hat{O}_x n^{(0)} \approx \hat{\bar{O}}_x n_{\rm bb} \approx G(x)$$

Generalizing the Boltzmann hierarchy

$$\frac{\partial n^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla n^{(1)} + \hat{O}_x n^{(0)} \left(\frac{\partial \Phi^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla \Psi^{(1)} \right) \approx \dot{\tau} \left[n_0^{(1)} + \frac{1}{10} n_2^{(1)} - n^{(1)} + \beta^{(1)} \chi \hat{O}_x n^{(0)} \right]$$

$$\hat{O}_x n^{(0)} \approx \hat{\bar{O}}_x n_{\rm bb} \approx G(x)$$

$$\frac{\partial \Theta^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla \Theta^{(1)} + \frac{\partial \Phi^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla \Psi^{(1)} = \dot{\tau} \left[\Theta_0^{(1)} + \frac{1}{10} \Theta_2^{(1)} - \Theta^{(1)} + \beta^{(1)} \chi \right]$$

Generalizing the Boltzmann hierarchy

$$\frac{\partial n^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla n^{(1)} + \hat{O}_x n^{(0)} \left(\frac{\partial \Phi^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla \Psi^{(1)} \right) \approx \dot{\tau} \left[n_0^{(1)} + \frac{1}{10} n_2^{(1)} - n^{(1)} + \beta^{(1)} \chi \hat{O}_x n^{(0)} \right]$$

$$\hat{O}_x n^{(0)} \approx \hat{\bar{O}}_x n_{\rm bb} \approx G(x)$$

$$\frac{\partial \Theta^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla \Theta^{(1)} + \frac{\partial \Phi^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla \Psi^{(1)} = \dot{\tau} \left[\Theta_0^{(1)} + \frac{1}{10} \Theta_2^{(1)} - \Theta^{(1)} + \beta^{(1)} \chi \right]$$

This gives the standard photon Boltzmann hierarchy

Generalizing the Boltzmann hierarchy

$$\frac{\partial n^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla n^{(1)} + \hat{O}_x n^{(0)} \left(\frac{\partial \Phi^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla \Psi^{(1)} \right) \approx \dot{\tau} \left[n_0^{(1)} + \frac{1}{10} n_2^{(1)} - n^{(1)} + \beta^{(1)} \chi \hat{O}_x n^{(0)} \right]$$

$$\hat{O}_x n^{(0)} \approx \hat{O}_x n_{\rm bb} \approx G(x)$$

Add a distortion of monopole

$$\hat{O}_x \Delta n^{(0)} = \Theta^{(0)} [3G(x) + Y(x)] + 4 \sum_{k=1}^{N} y_{k-1}^{(0)} Y_k(x) + y_N^{(0)} \hat{O}_x Y_N(x) + \mu^{(0)} \hat{O}_x M(x),$$

Generalizing the Boltzmann hierarchy

$$\frac{\partial n^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla n^{(1)} + \hat{O}_x n^{(0)} \left(\frac{\partial \Phi^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla \Psi^{(1)} \right) \approx \dot{\tau} \left[n_0^{(1)} + \frac{1}{10} n_2^{(1)} - n^{(1)} + \beta^{(1)} \chi \hat{O}_x n^{(0)} \right]$$

$$\hat{O}_x n^{(0)} \approx \hat{\bar{O}}_x n_{\mathrm{bb}} \approx G(x)$$

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$$\hat{O}_x \Delta n^{(0)} = \Theta^{(0)}[3G(x) + Y(x)] + 4 \sum_{k=1}^{N} y_{k-1}^{(0)} Y_k(x) + y_N^{(0)} \hat{O}_x Y_N(x) + \mu^{(0)} \hat{O}_x M(x),$$

$$\frac{\partial \boldsymbol{y}^{(1)}}{\partial \eta} + \hat{\gamma} \cdot \nabla \boldsymbol{y}^{(1)} \approx -\boldsymbol{b}_0^{(0)} \left(\frac{\partial \Phi^{(1)}}{\partial \eta} + \hat{\gamma} \cdot \nabla \Psi^{(1)} \right) + \tau' \left[\boldsymbol{y}_0^{(1)} + \frac{1}{10} \boldsymbol{y}_2^{(1)} - \boldsymbol{y}^{(1)} + \beta^{(1)} \chi \boldsymbol{b}_0^{(0)} \right]$$

Generalizing the Boltzmann hierarchy

$$\frac{\partial n^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla n^{(1)} + \hat{O}_x n^{(0)} \left(\frac{\partial \Phi^{(1)}}{\partial t} + \frac{c\hat{\gamma}}{a} \cdot \nabla \Psi^{(1)} \right) \approx \dot{\tau} \left[n_0^{(1)} + \frac{1}{10} n_2^{(1)} - n^{(1)} + \beta^{(1)} \chi \hat{O}_x n^{(0)} \right]$$

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Add a distortion of monopole

$$\hat{O}_x \Delta n^{(0)} = \Theta^{(0)}[3G(x) + Y(x)] + 4 \sum_{k=1}^{N} y_{k-1}^{(0)} \, Y_k(x) + y_N^{(0)} \, \hat{O}_x Y_N(x) + \mu^{(0)} \, \hat{O}_x M(x).$$

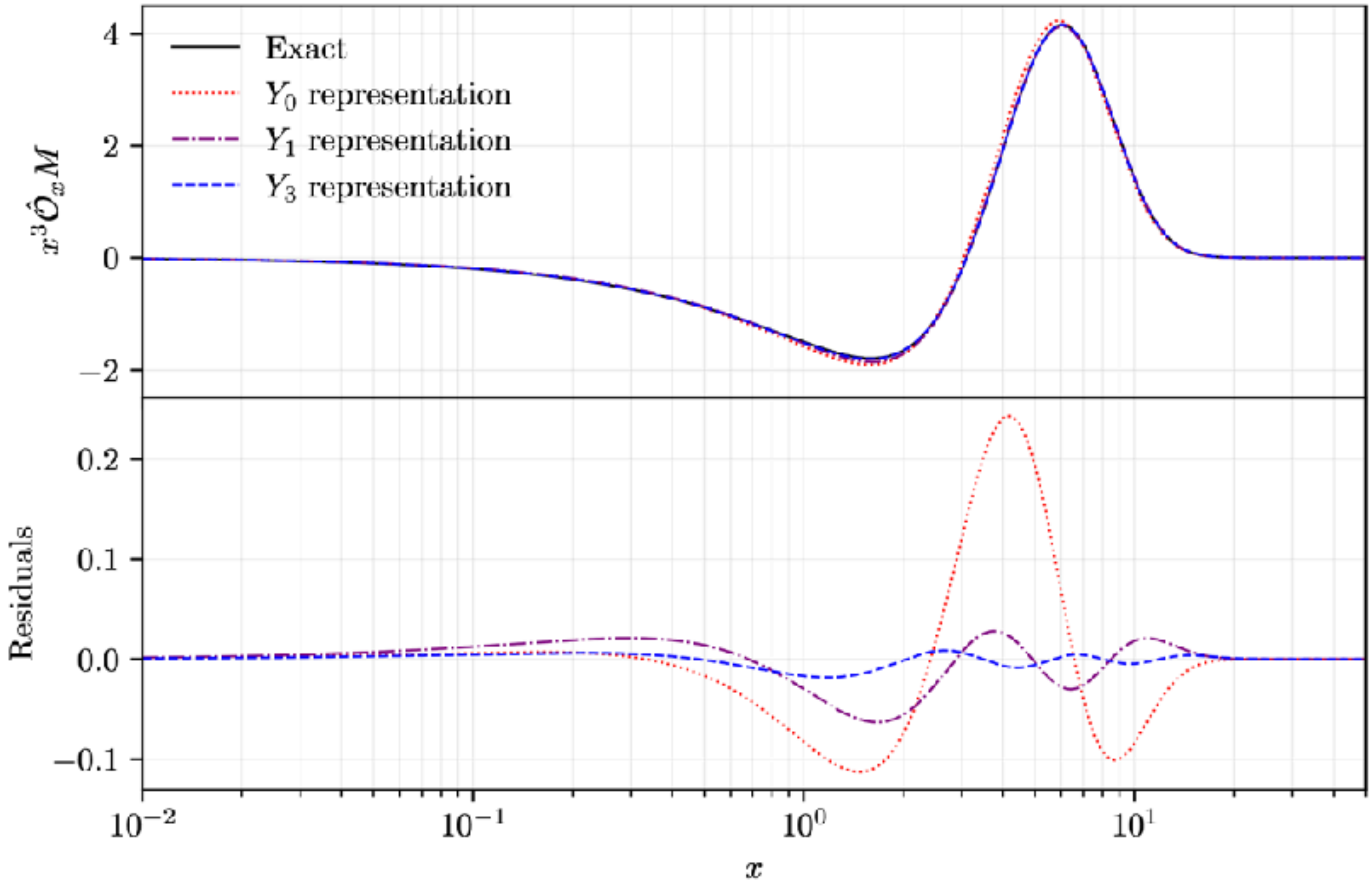
$$\frac{\partial \boldsymbol{y}^{(1)}}{\partial \eta} + \hat{\boldsymbol{\gamma}} \cdot \nabla \boldsymbol{y}^{(1)} \approx -\boldsymbol{b}_0^{(0)} \left(\frac{\partial \Phi^{(1)}}{\partial \eta} + \hat{\boldsymbol{\gamma}} \cdot \nabla \Psi^{(1)} \right) + \tau' \left[\boldsymbol{y}_0^{(1)} + \frac{1}{10} \boldsymbol{y}_2^{(1)} - \boldsymbol{y}^{(1)} + \beta^{(1)} \chi \, \boldsymbol{b}_0^{(0)} \right]$$

Time-dependent change of Doppler and potential driving

More general source vector due to coupling with average background

$$
\boldsymbol{b}^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + M_{\mathrm{B}}\,\boldsymbol{y}^{(0)} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix}
3 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & \cdots & 0 & 0 & O_{N,1} & O_{\mu,1} \\
0 & 4 & 0 & 0 & \cdots & 0 & 0 & O_{N,2} & O_{\mu,2} \\
0 & 0 & 4 & 0 & \cdots & 0 & 0 & O_{N,3} & O_{\mu,3} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & O_{N,N-2} & O_{\mu,N-2} \\
0 & 0 & 0 & 0 & \cdots & 4 & 0 & O_{N,N-1} & O_{\mu,N-1} \\
0 & 0 & 0 & 0 & \cdots & 0 & 4 & O_{N,N} & O_{\mu,N} \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & O_{N,N+1} & O_{\mu,N+1}
\end{pmatrix} \begin{pmatrix} \Theta^{(0)} \\ y_0^{(0)} \\ y_1^{(0)} \\ y_2^{(0)} \\ \vdots \\ y_{N-2}^{(0)} \\ y_{N-1}^{(0)} \\ y_N^{(0)} \\ \mu^{(0)} \end{pmatrix};
$$

Representation of Boosts of μ-spectrum



$$\hat{O}_x M(x) \approx 0.3736\, Y(x) + 1.9069\, M(x)$$

Even very lowest order projections work well in this case

Including thermalization effects

Greens function discretization

$$\Delta n' = \Theta_e \, Y + \hat{\mathcal{K}}_x \, \Delta n + \Delta n'|_{\rm em/abs} + \Delta n'|_{\rm h} \quad \longleftrightarrow \quad \boldsymbol{y}' \approx M_{\rm K} \, \boldsymbol{y} + \boldsymbol{D} + \frac{\boldsymbol{Q}'}{4},$$

$$\boldsymbol{D} = (\gamma_T x_c \, \mu, 0, 0, \ldots, 0, -\gamma_N x_c \, \mu)^T, \quad \boldsymbol{Q}' = (0, Q', 0, \ldots, 0, 0)^T.$$

$$\boldsymbol{y} = (\Theta, y, y_1, \ldots, y_N, \mu)^T$$

Including thermalization effects

Greens function discretization

$$\Delta n' = \Theta_{\rm e}\, Y + \hat{\mathcal{K}}_x\, \Delta n + \Delta n'|_{\rm em/abs} + \Delta n'|_{\rm h} \quad \longleftrightarrow \quad \boldsymbol{y}' \approx M_{\rm K}\, \boldsymbol{y} + \boldsymbol{D} + \frac{\boldsymbol{Q}'}{4},$$

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$$\boldsymbol{y} = (\Theta, y, y_1, \ldots, y_N, \mu)^T$$

$$\frac{\partial \boldsymbol{y}_0^{(0)}}{\partial \eta} = \tau' \theta_z \left[M_{\rm K}\, \boldsymbol{y}_0^{(0)} + \boldsymbol{D}_0^{(0)} \right] + \frac{\boldsymbol{Q}'^{(0)}}{4}$$

Obvious new terms

Including thermalization effects

Greens function discretization

$$\Delta n' = \Theta_e \, Y + \hat{\mathcal{K}}_x \, \Delta n + \Delta n'|_{\rm em/abs} + \Delta n'|_{\rm h} \quad \longleftrightarrow \quad \boldsymbol{y}' \approx M_{\rm K} \, \boldsymbol{y} + \boldsymbol{D} + \frac{\boldsymbol{Q}'}{4},$$

$$\boldsymbol{D} = (\gamma_T x_{\rm c} \, \mu, 0, 0, \dots, 0, -\gamma_N x_{\rm c} \, \mu)^T, \quad \boldsymbol{Q}' = (0, Q', 0, \dots, 0, 0)^T.$$

$$\boldsymbol{y} = (\Theta, y, y_1, \dots, y_N, \mu)^T$$

$$\frac{\partial \boldsymbol{y}_0^{(0)}}{\partial \eta} = \tau' \theta_z \left[M_{\rm K} \, \boldsymbol{y}_0^{(0)} + \boldsymbol{D}_0^{(0)} \right] + \frac{\boldsymbol{Q}'^{(0)}}{4}$$

$$\frac{\partial \boldsymbol{y}^{(1)}}{\partial \eta} + \hat{\boldsymbol{\gamma}} \cdot \nabla \boldsymbol{y}^{(1)} = -\boldsymbol{b}_0^{(0)} \left(\frac{\partial \Phi^{(1)}}{\partial \eta} + \hat{\boldsymbol{\gamma}} \cdot \nabla \Psi^{(1)} \right) + \tau' \left[\boldsymbol{y}_0^{(1)} + \frac{1}{10} \boldsymbol{y}_2^{(1)} - \boldsymbol{y}^{(1)} + \beta^{(1)} \chi \, \boldsymbol{b}_0^{(0)} \right]$$

Thomson terms as we just discussed

Including thermalization effects

Greens function discretization

$$\Delta n' = \Theta_{\rm e}\, Y + \hat{\mathcal{K}}_x\, \Delta n + \Delta n'|_{\rm em/abs} + \Delta n'|_{\rm h} \quad \longleftrightarrow \quad \boldsymbol{y}' \approx M_{\rm K}\, \boldsymbol{y} + \boldsymbol{D} + \frac{\boldsymbol{Q}'}{4},$$

$$\boldsymbol{D} = (\gamma_T x_{\rm c}\, \mu, 0, 0, \ldots, 0, -\gamma_N x_{\rm c}\, \mu)^T\,, \quad \boldsymbol{Q}' = (0, Q', 0, \ldots, 0, 0)^T\,.$$

$$\boldsymbol{y} = (\Theta, y, y_1, \ldots, y_N, \mu)^T$$

$$\downarrow$$

$$\frac{\partial \boldsymbol{y}_0^{(0)}}{\partial \eta} = \tau' \theta_z \left[M_{\rm K}\, \boldsymbol{y}_0^{(0)} + \boldsymbol{D}_0^{(0)}\right] + \frac{\boldsymbol{Q}'^{(0)}}{4}$$

$$\frac{\partial \boldsymbol{y}^{(1)}}{\partial \eta} + \hat{\boldsymbol{\gamma}} \cdot \nabla \boldsymbol{y}^{(1)} = -\boldsymbol{b}_0^{(0)} \left(\frac{\partial \Phi^{(1)}}{\partial \eta} + \hat{\boldsymbol{\gamma}} \cdot \nabla \Psi^{(1)}\right) + \tau' \left[\boldsymbol{y}_0^{(1)} + \frac{1}{10}\, \boldsymbol{y}_2^{(1)} - \boldsymbol{y}^{(1)} + \beta^{(1)} \chi\, \boldsymbol{b}_0^{(0)}\right] + \frac{\boldsymbol{Q}'^{(1)}}{4}$$
$$+ \tau' \theta_z \left\{M_{\rm K}\, \boldsymbol{y}_0^{(1)} + \boldsymbol{D}_0^{(1)} + \left[\delta_{\rm b}^{(1)} + \Psi^{(1)}\right]\left(M_{\rm K}\, \boldsymbol{y}_0^{(0)} + \boldsymbol{D}_0^{(0)}\right) + \Theta_0^{(1)} \left(\boldsymbol{D}_0^{(0)} + M_{\rm D}\, \boldsymbol{y}^{(0)} - \boldsymbol{S}^{(0)}\right)\right\}$$

New thermalization terms in perturbed monopole

Including thermalization effects

Greens function discretization

$$\Delta n' = \Theta_e Y + \hat{\mathcal{K}}_x \Delta n + \Delta n'|_{\rm em/abs} + \Delta n'|_{\rm h} \quad \longleftrightarrow \quad \boldsymbol{y}' \approx M_{\rm K}\,\boldsymbol{y} + \boldsymbol{D} + \frac{\boldsymbol{Q}'}{4},$$

$$\boldsymbol{D} = (\gamma_T x_c \mu, 0, 0, \dots, 0, -\gamma_N x_c \mu)^T, \quad \boldsymbol{Q}' = (0, Q', 0, \dots, 0, 0)^T.$$

$$\boldsymbol{y} = (\Theta, y, y_1, \dots, y_N, \mu)^T$$

$$\frac{\partial \boldsymbol{y}_0^{(0)}}{\partial \eta} = \tau'\theta_z \left[M_{\rm K}\,\boldsymbol{y}_0^{(0)} + \boldsymbol{D}_0^{(0)}\right] + \frac{\boldsymbol{Q}'^{(0)}}{4}$$

$$\frac{\partial \boldsymbol{y}^{(1)}}{\partial \eta} + \hat{\boldsymbol{\gamma}}\cdot\nabla \boldsymbol{y}^{(1)} = -\boldsymbol{b}_0^{(0)}\left(\frac{\partial \Phi^{(1)}}{\partial \eta} + \hat{\boldsymbol{\gamma}}\cdot\nabla\Psi^{(1)}\right) + \tau'\left[\boldsymbol{y}_0^{(1)} + \frac{1}{10}\boldsymbol{y}_2^{(1)} - \boldsymbol{y}^{(1)} + \beta^{(1)}\chi\,\boldsymbol{b}_0^{(0)}\right] + \frac{\boldsymbol{Q}'^{(1)}}{4}$$
$$+ \tau'\theta_z\left\{M_{\rm K}\,\boldsymbol{y}_0^{(1)} + \boldsymbol{D}_0^{(1)} + \left[\delta_{\rm b}^{(1)} + \Psi^{(1)}\right]\left(M_{\rm K}\,\boldsymbol{y}_0^{(0)} + \boldsymbol{D}_0^{(0)}\right) + \Theta_0^{(1)}\left(\boldsymbol{D}_0^{(0)} + M_{\rm D}\,\boldsymbol{y}^{(0)} - \boldsymbol{S}^{(0)}\right)\right\}$$

This gives a new photon Boltzmann hierarchy with thermalization

Final photon Boltzmann hierarchy with thermalization effects

$$\frac{\partial \boldsymbol{y}_0^{(0)}}{\partial \eta} = \tau' \theta_z \left[M_{\rm K}\, \boldsymbol{y}_0^{(0)} + \boldsymbol{D}_0^{(0)} \right] + \frac{\boldsymbol{Q}'^{(0)}}{4},$$

$$\frac{\partial \tilde{\boldsymbol{y}}_0^{(1)}}{\partial \eta} = -k\, \tilde{\boldsymbol{y}}_1^{(1)} - \frac{\partial \tilde{\Phi}^{(1)}}{\partial \eta} \boldsymbol{b}_0^{(0)} + \frac{\boldsymbol{Q}'^{(1)}}{4}$$
$$+ \tau' \theta_z \left\{ M_{\rm K}\, \tilde{\boldsymbol{y}}_0^{(1)} + \boldsymbol{D}_0^{(1)} + \left[\tilde{\delta}_{\rm b}^{(1)} + \tilde{\Psi}^{(1)} \right] \left(M_{\rm K}\, \boldsymbol{y}_0^{(0)} + \boldsymbol{D}_0^{(0)} \right) + \tilde{\Theta}_0^{(1)} \left(\boldsymbol{D}_0^{(0)} + M_{\rm D}\, \boldsymbol{y}^{(0)} - \boldsymbol{S}^{(0)} \right) \right\},$$

$$\frac{\partial \tilde{\boldsymbol{y}}_1^{(1)}}{\partial \eta} = k \left(\frac{1}{3} \tilde{\boldsymbol{y}}_0 - \frac{2}{3} \tilde{\boldsymbol{y}}_2 \right) + \frac{k}{3} \tilde{\Psi}^{(1)}\, \boldsymbol{b}_0^{(0)} - \tau' \left[\tilde{\boldsymbol{y}}_1^{(1)} - \frac{\tilde{\beta}^{(1)}}{3} \boldsymbol{b}_0^{(0)} \right],$$

$$\frac{\partial \tilde{\boldsymbol{y}}_2^{(1)}}{\partial \eta} = k \left(\frac{2}{5} \tilde{\boldsymbol{y}}_1^{(1)} - \frac{3}{5} \tilde{\boldsymbol{y}}_3^{(1)} \right) - \frac{9}{10}\, \tau'\, \tilde{\boldsymbol{y}}_2^{(1)},$$

$$\frac{\partial \tilde{\boldsymbol{y}}_{\ell \geq 3}^{(1)}}{\partial \eta} = k \left(\frac{\ell}{2\ell + 1} \tilde{\boldsymbol{y}}_{\ell - 1} - \frac{\ell + 1}{2\ell + 1} \tilde{\boldsymbol{y}}_{\ell + 1} \right) - \tau' \tilde{\boldsymbol{y}}_\ell^{(1)}.$$

JC, Kite & Ravenni, 2022, papers I, ArXiv:2210.09327
JC, Ravenni & Kite, 2022, papers II, ArXiv:2210.15308
Kite, Ravenni & JC, 2022, papers III, ArXiv:2212.02817

Final photon Boltzmann hierarchy with thermalization effects

$$\frac{\partial \boldsymbol{y}_0^{(0)}}{\partial \eta} = \tau' \theta_z \left[M_{\rm K}\, \boldsymbol{y}_0^{(0)} + \boldsymbol{D}_0^{(0)} \right] + \frac{\boldsymbol{Q}'^{(0)}}{4},$$

$$\frac{\partial \tilde{\boldsymbol{y}}_0^{(1)}}{\partial \eta} = -k\, \tilde{\boldsymbol{y}}_1^{(1)} - \frac{\partial \tilde{\Phi}^{(1)}}{\partial \eta} \boldsymbol{b}_0^{(0)} + \frac{\boldsymbol{Q}'^{(1)}}{4}$$
$$+ \tau' \theta_z \left\{ M_{\rm K}\, \tilde{\boldsymbol{y}}_0^{(1)} + \boldsymbol{D}_0^{(1)} + \left[\tilde{\delta}_{\rm b}^{(1)} + \tilde{\Psi}^{(1)} \right] \left(M_{\rm K}\, \boldsymbol{y}_0^{(0)} + \boldsymbol{D}_0^{(0)} \right) + \tilde{\Theta}_0^{(1)} \left(\boldsymbol{D}_0^{(0)} + M_{\rm D}\, \boldsymbol{y}^{(0)} - \boldsymbol{S}^{(0)} \right) \right\},$$

$$\frac{\partial \tilde{\boldsymbol{y}}_1^{(1)}}{\partial \eta} = k \left(\frac{1}{3} \tilde{\boldsymbol{y}}_0 - \frac{2}{3} \tilde{\boldsymbol{y}}_2 \right) + \frac{k}{3} \tilde{\Psi}^{(1)} \boldsymbol{b}_0^{(0)} - \tau' \left[\tilde{\boldsymbol{y}}_1^{(1)} - \frac{\tilde{\beta}^{(1)}}{3} \boldsymbol{b}_0^{(0)} \right],$$

$$\frac{\partial \tilde{\boldsymbol{y}}_2^{(1)}}{\partial \eta} = k \left(\frac{2}{5} \tilde{\boldsymbol{y}}_1^{(1)} - \frac{3}{5} \tilde{\boldsymbol{y}}_3^{(1)} \right) - \frac{9}{10} \tau' \, \tilde{\boldsymbol{y}}_2^{(1)},$$

$$\frac{\partial \tilde{\boldsymbol{y}}_{\ell \geq 3}^{(1)}}{\partial \eta} = k \left(\frac{\ell}{2\ell + 1} \tilde{\boldsymbol{y}}_{\ell - 1} - \frac{\ell + 1}{2\ell + 1} \tilde{\boldsymbol{y}}_{\ell + 1} \right) - \tau' \tilde{\boldsymbol{y}}_\ell^{(1)}.$$

These are the new thermalization and generalized source terms

JC, Kite & Ravenni, 2022, papers I, ArXiv:2210.09327
JC, Ravenni & Kite, 2022, papers II, ArXiv:2210.15308
Kite, Ravenni & JC, 2022, papers III, ArXiv:2212.02817

Final photon Boltzmann hierarchy with thermalization effects

$$\frac{\partial y_0^{(0)}}{\partial \eta} = \tau' \theta_z \left[M_{\rm K}\, y_0^{(0)} + D_0^{(0)} \right] + \frac{Q'^{(0)}}{4},$$

$$\frac{\partial \tilde{y}_0^{(1)}}{\partial \eta} = -k\, \tilde{y}_1^{(1)} - \frac{\partial \tilde{\Phi}^{(1)}}{\partial \eta} b_0^{(0)} + \frac{Q'^{(1)}}{4}$$
$$+ \tau' \theta_z \left\{ M_{\rm K}\, \tilde{y}_0^{(1)} + D_0^{(1)} + \left[\tilde{\delta}_{\rm b}^{(1)} + \tilde{\Psi}^{(1)} \right] \left(M_{\rm K}\, y_0^{(0)} + D_0^{(0)} \right) + \tilde{\Theta}_0^{(1)} \left(D_0^{(0)} + M_{\rm D}\, y^{(0)} - S^{(0)} \right) \right\},$$

$$\frac{\partial \tilde{y}_1^{(1)}}{\partial \eta} = k \left(\frac{1}{3} \tilde{y}_0 - \frac{2}{3} \tilde{y}_2 \right) + \frac{k}{3} \tilde{\Psi}^{(1)} b_0^{(0)} - \tau' \left[\tilde{y}_1^{(1)} - \frac{\tilde{\beta}^{(1)}}{3} b_0^{(0)} \right],$$

$$\frac{\partial \tilde{y}_2^{(1)}}{\partial \eta} = k \left(\frac{2}{5} \tilde{y}_1^{(1)} - \frac{3}{5} \tilde{y}_3^{(1)} \right) - \frac{9}{10} \tau' \tilde{y}_2^{(1)},$$

$$\frac{\partial \tilde{y}_{\ell \geq 3}^{(1)}}{\partial \eta} = k \left(\frac{\ell}{2\ell+1} \tilde{y}_{\ell-1} - \frac{\ell+1}{2\ell+1} \tilde{y}_{\ell+1} \right) - \tau' \tilde{y}_\ell^{(1)}.$$

These are the new thermalization and generalized source terms

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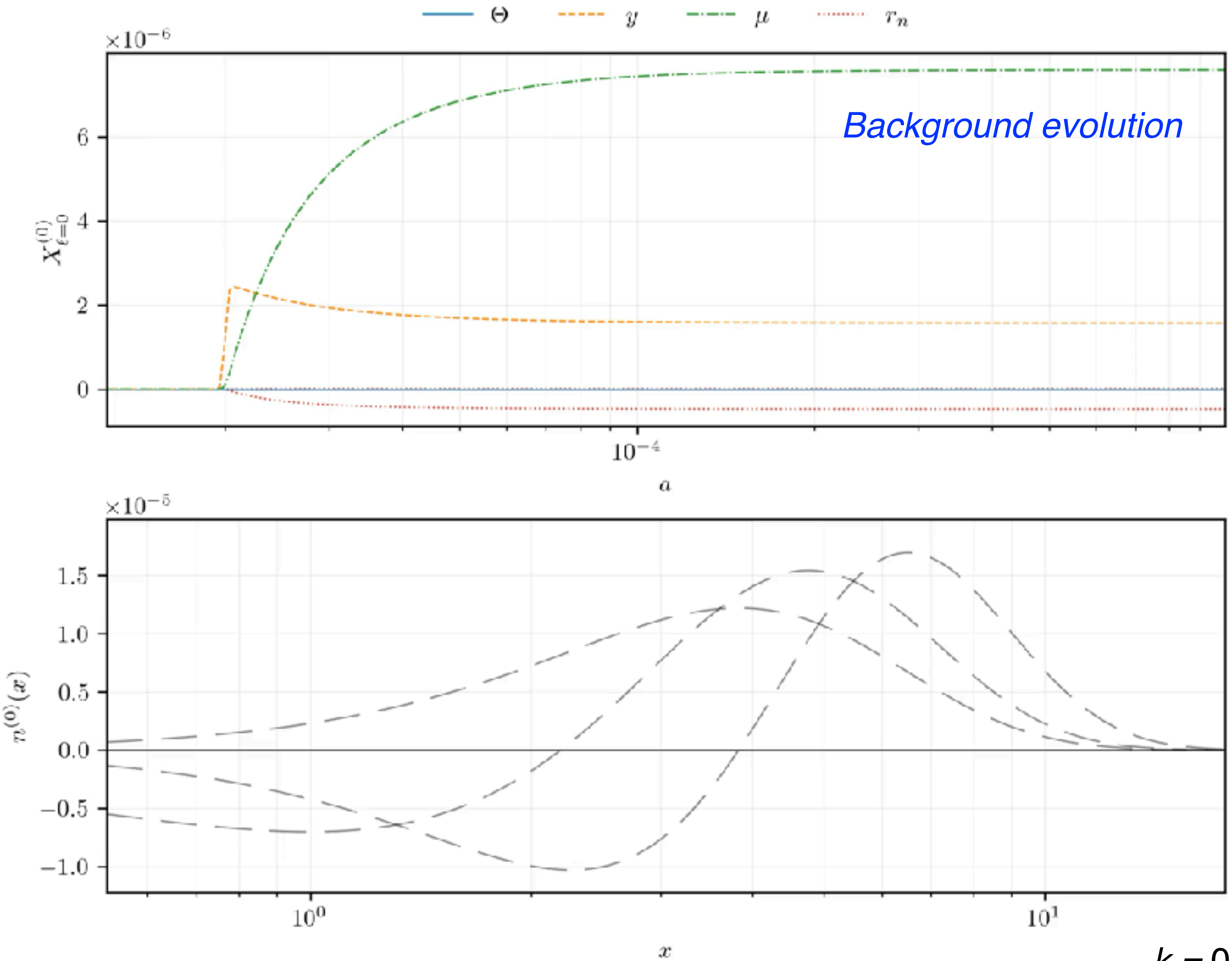
Kite, Ravenni & JC, 2022, papers III, ArXiv:2212.02817

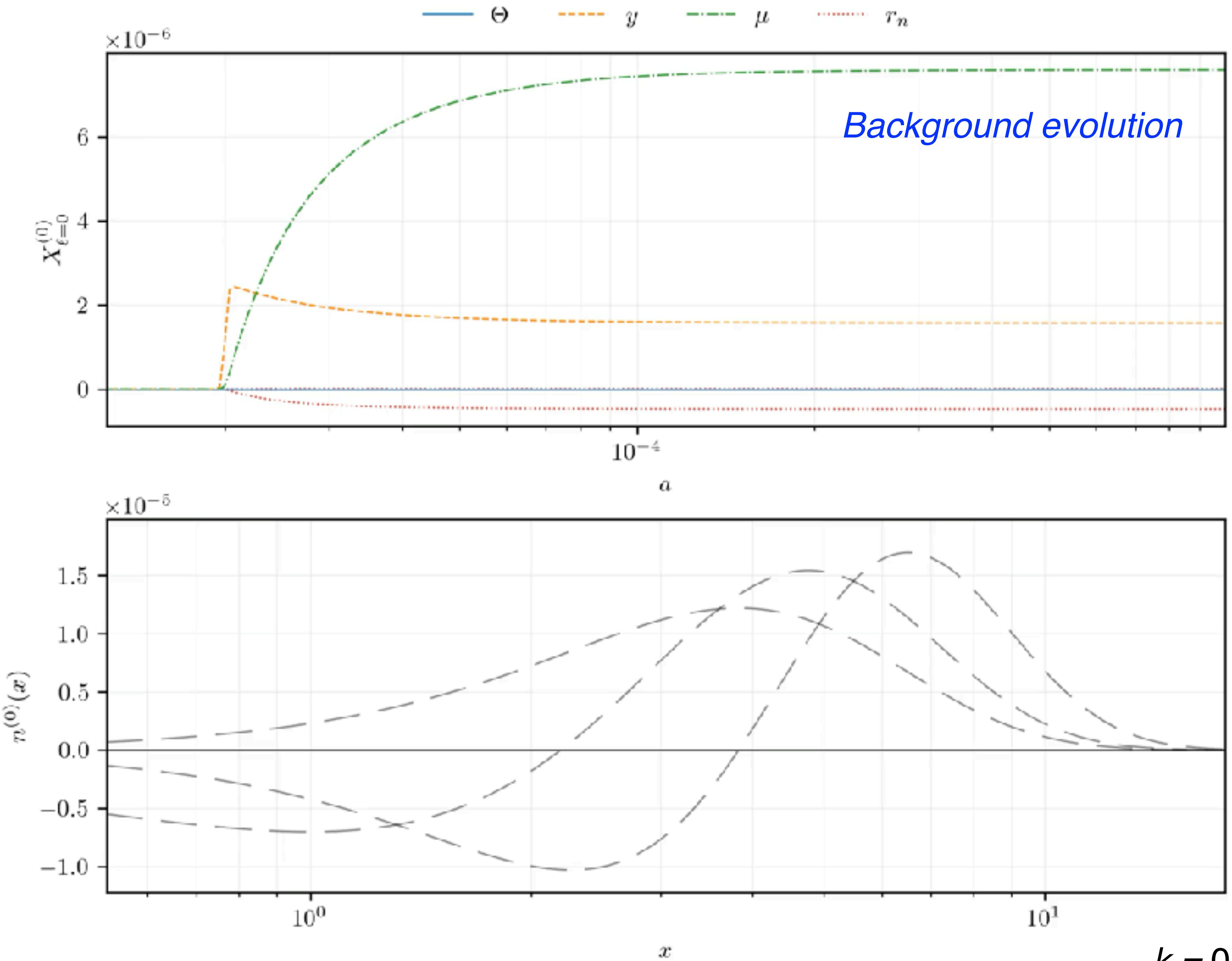
Now implemented in `CosmoTherm` but can be done with any Boltzmann code (e.g., `CAMB` or `CLASS`)

Evolution of modes for injection at z = 50,000



k = 0.01 Mpc^{-1}

Evolution of modes for injection at z = 50,000



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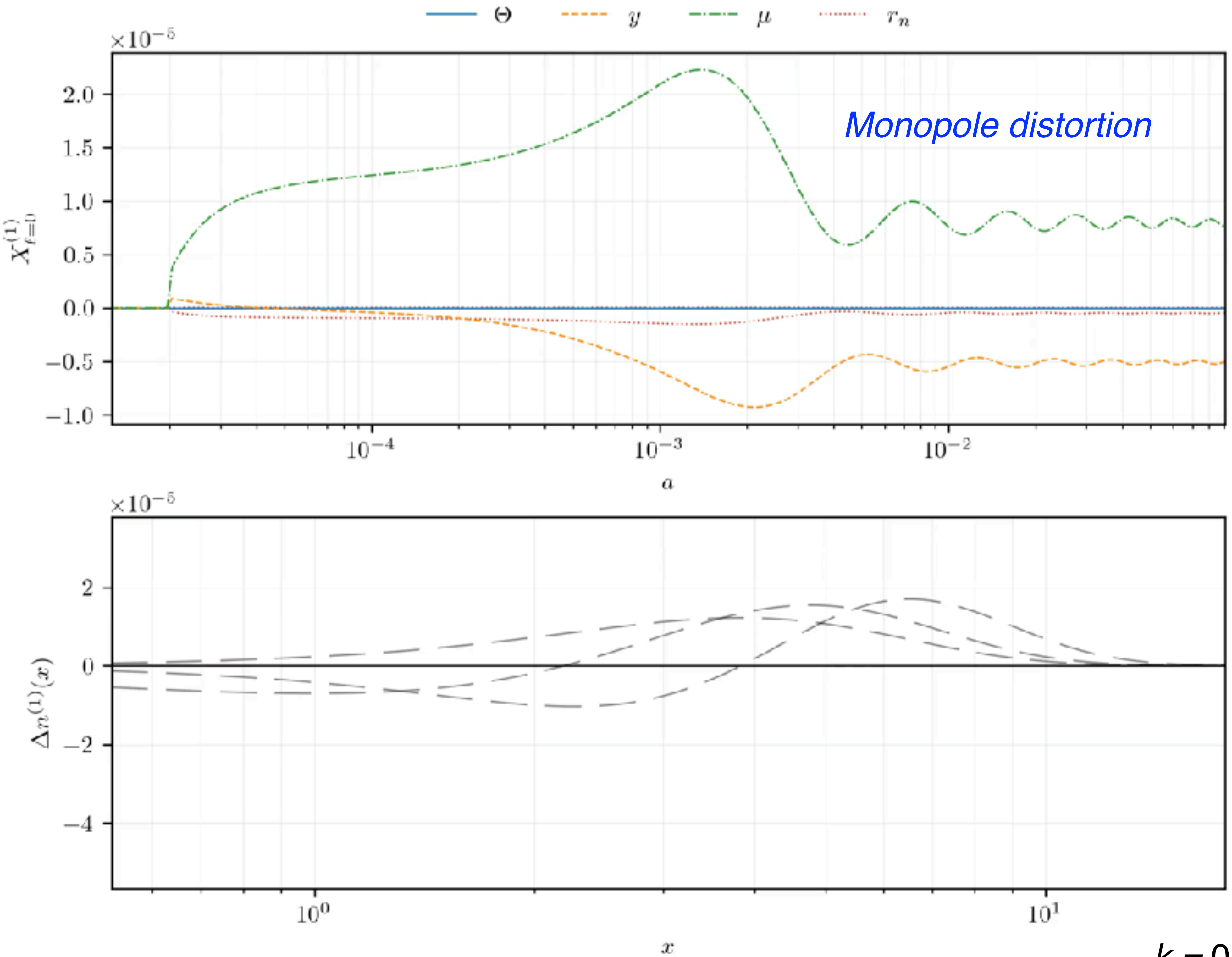
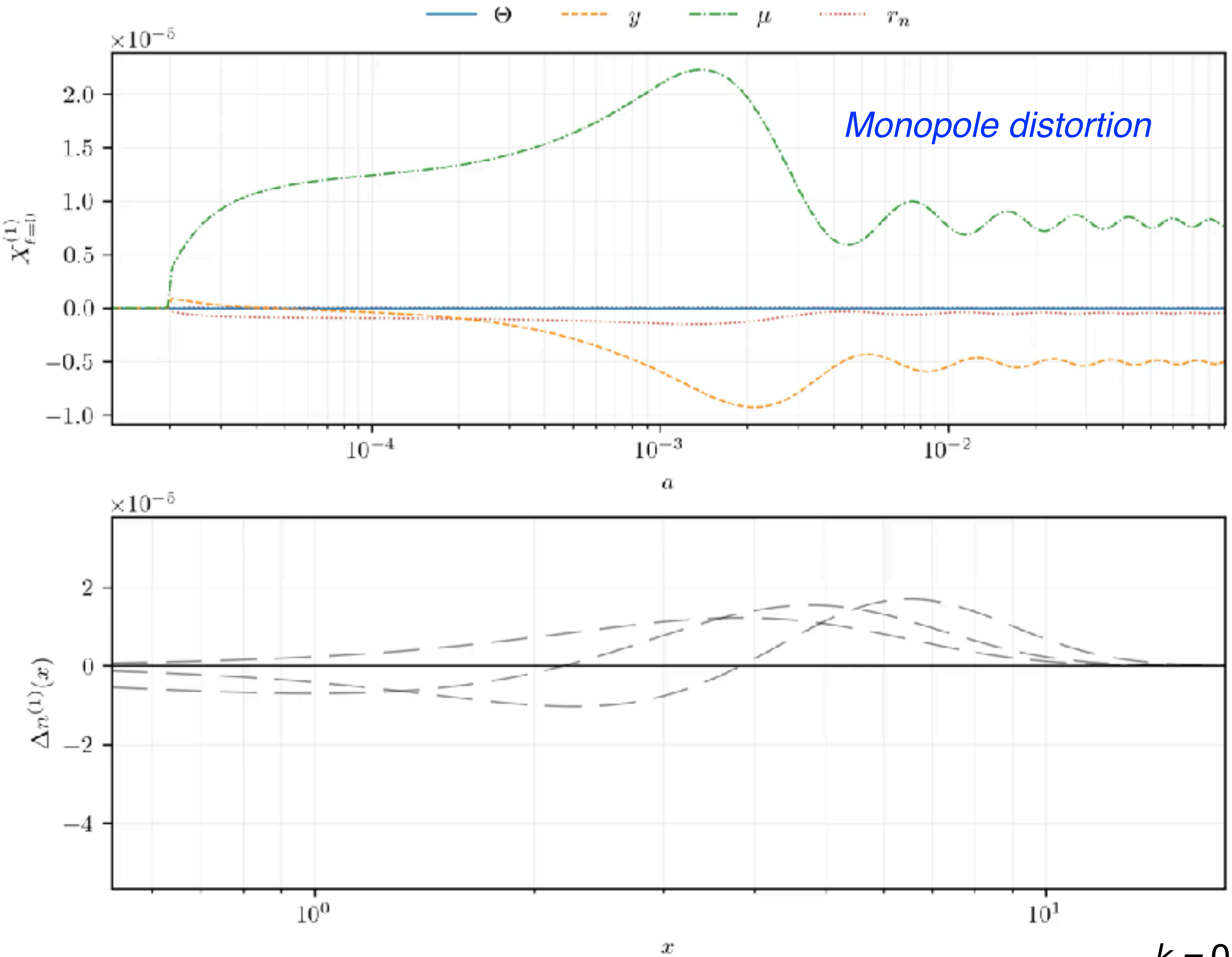


k = 0.01 Mpc^{-1}

Evolution of modes for injection at z = 50,000



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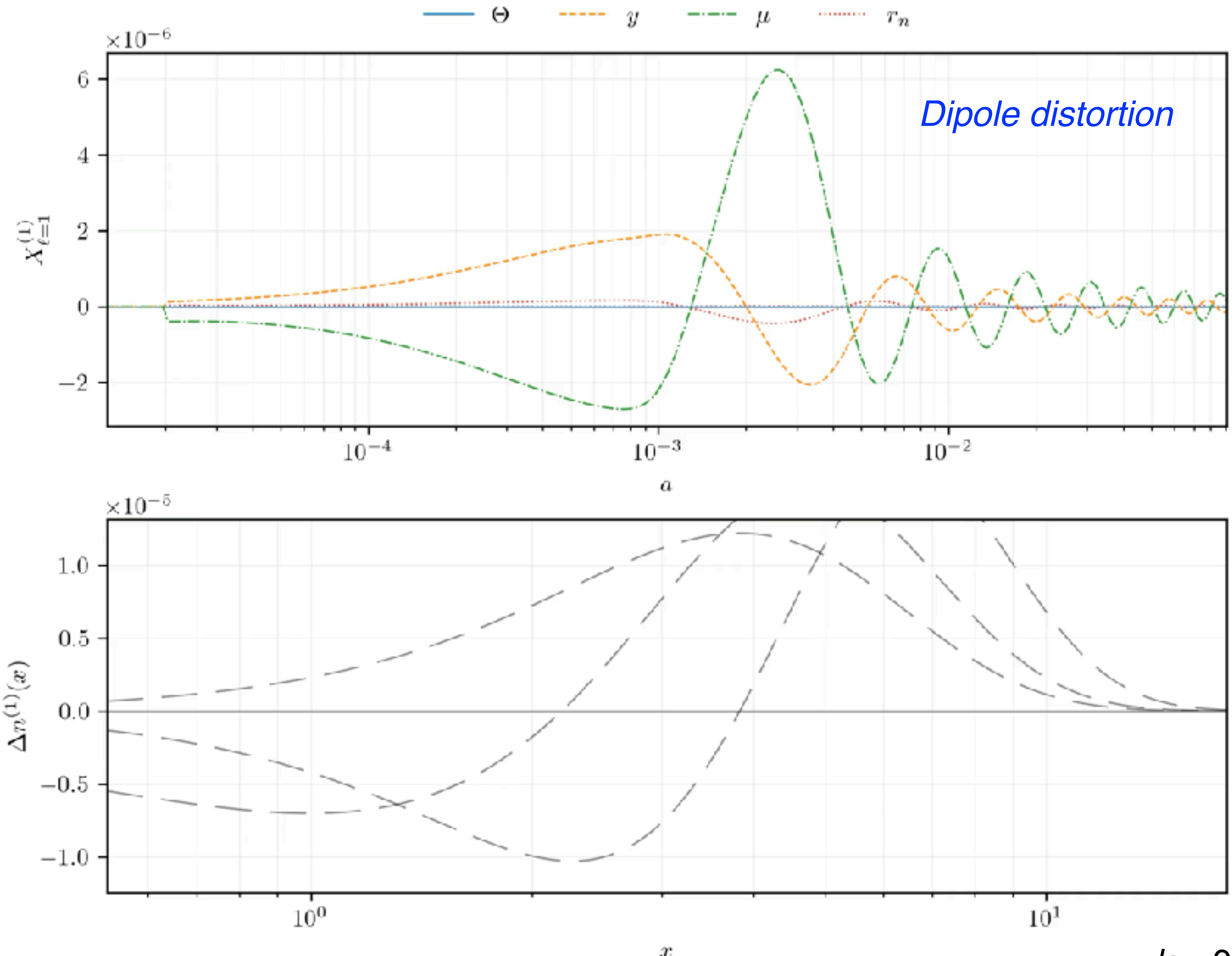
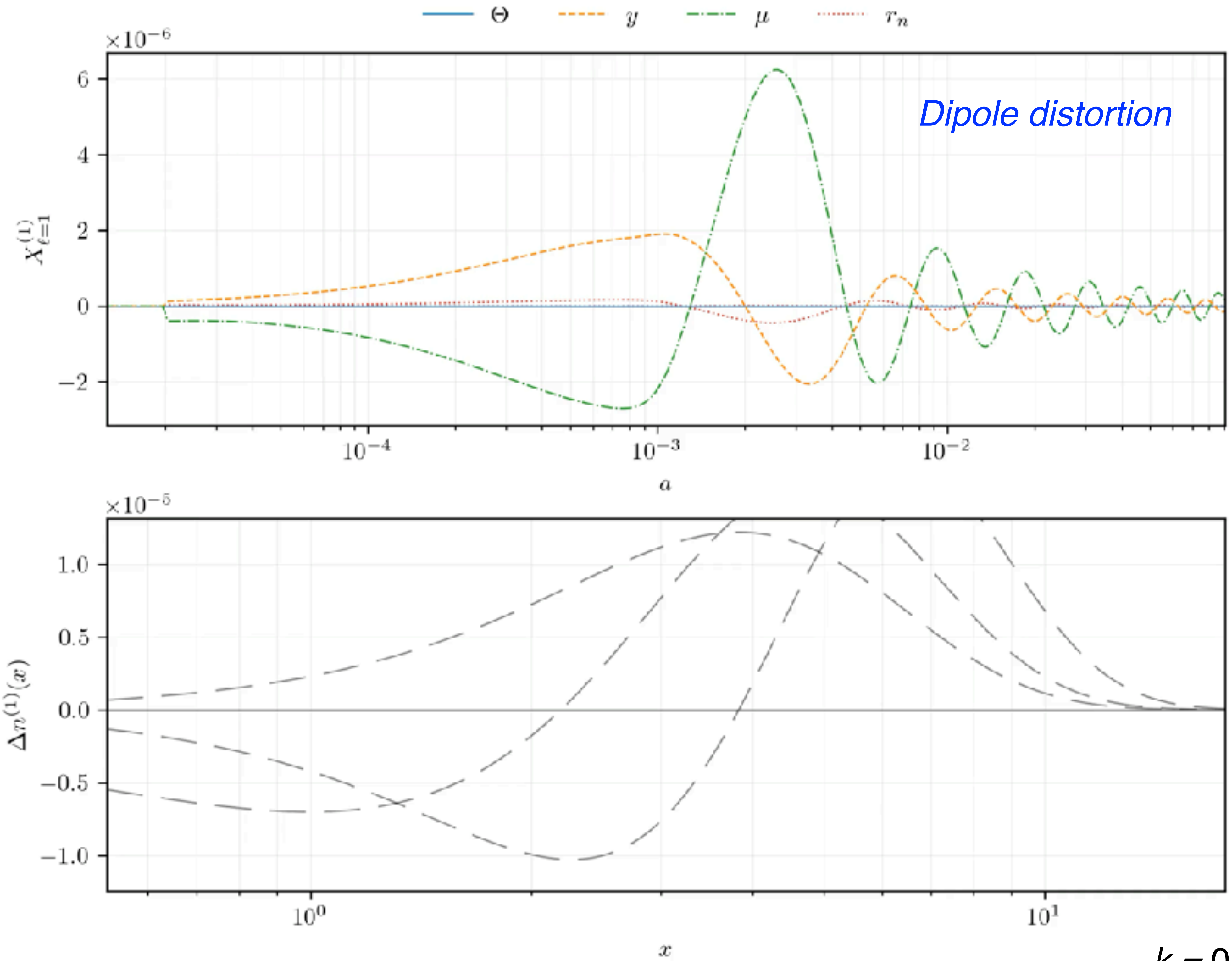


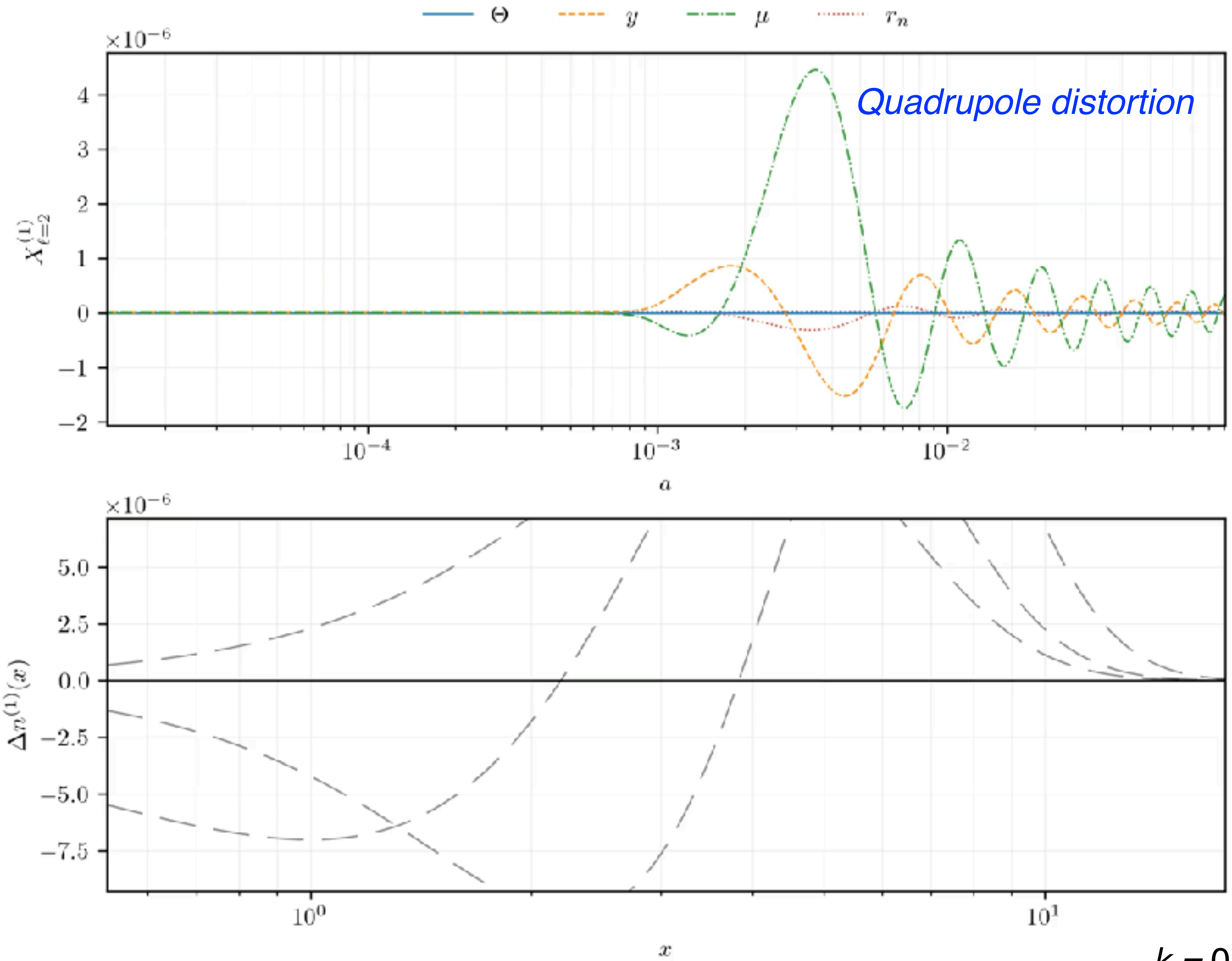
k = 0.01 Mpc^{-1}

Evolution of modes for injection at z = 50,000



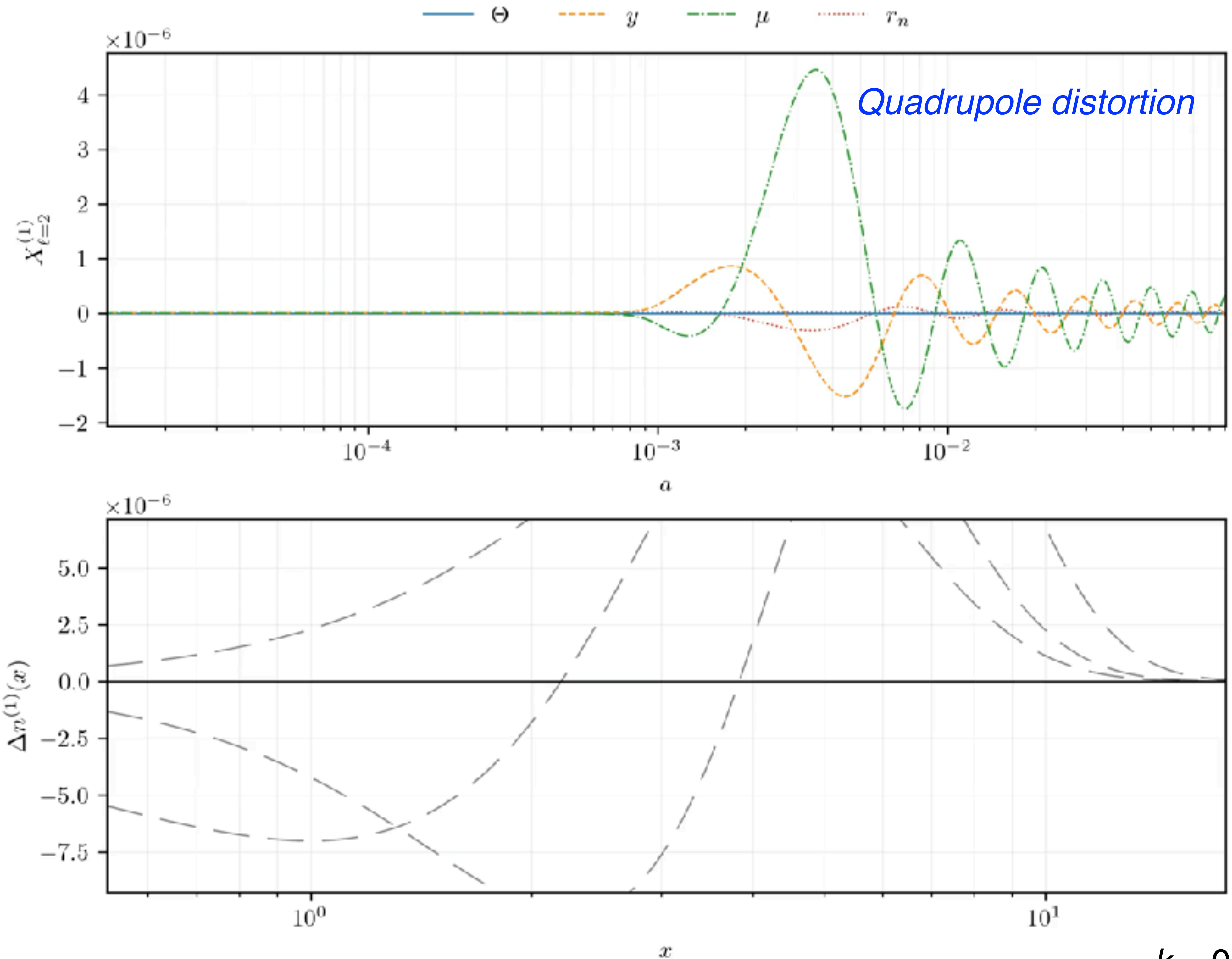
$k = 0.01$ Mpc^{-1}

Evolution of modes for injection at z = 50,000



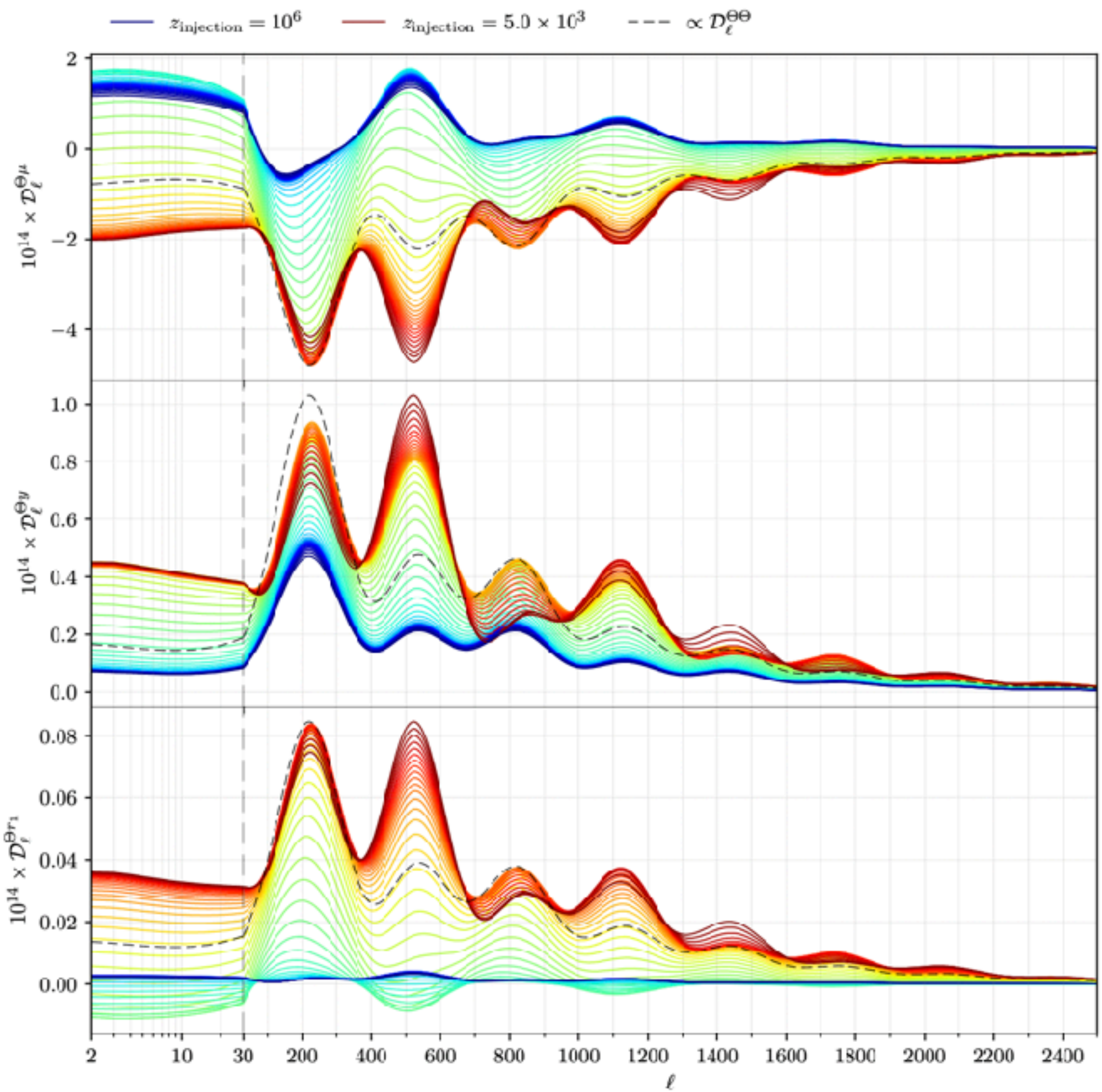
k = 0.01 Mpc^{-1}

Evolution of modes for injection at z = 50,000



$k = 0.01$ Mpc^{-1}

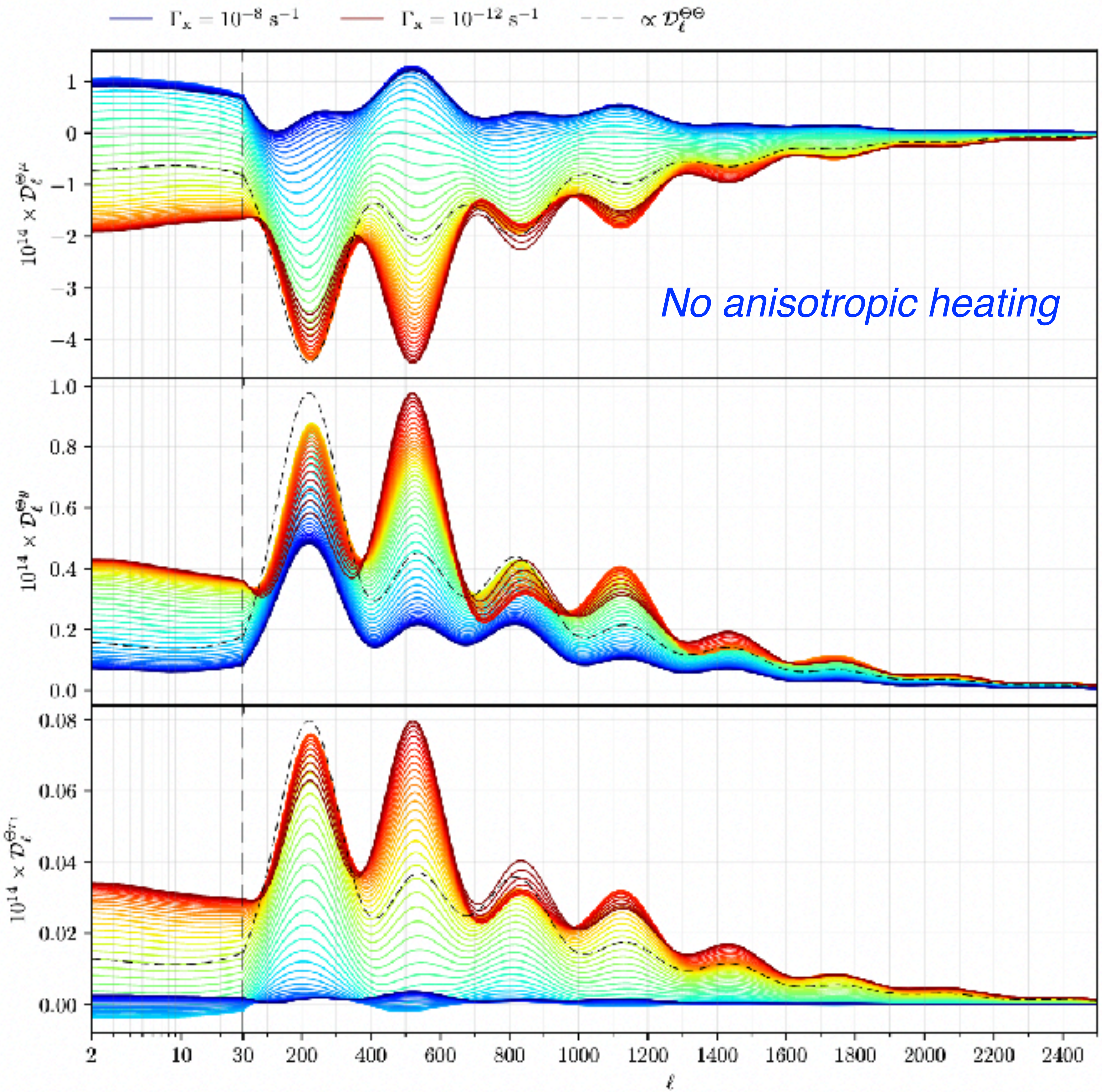
CMB power spectra for injection at various z



- `CosmoTherm` is able to reproduce standard CMB temperature signals
- Auto spectra indeed quite small…
- Cross-power spectra are within reach without violating COBE/FIRAS!
- Time-dependent information visible through spatial modes
- Extra information in residual distortion signals
- Polarization available

Kite, Ravenni & JC, 2022, papers III, ArXiv:2212.02817

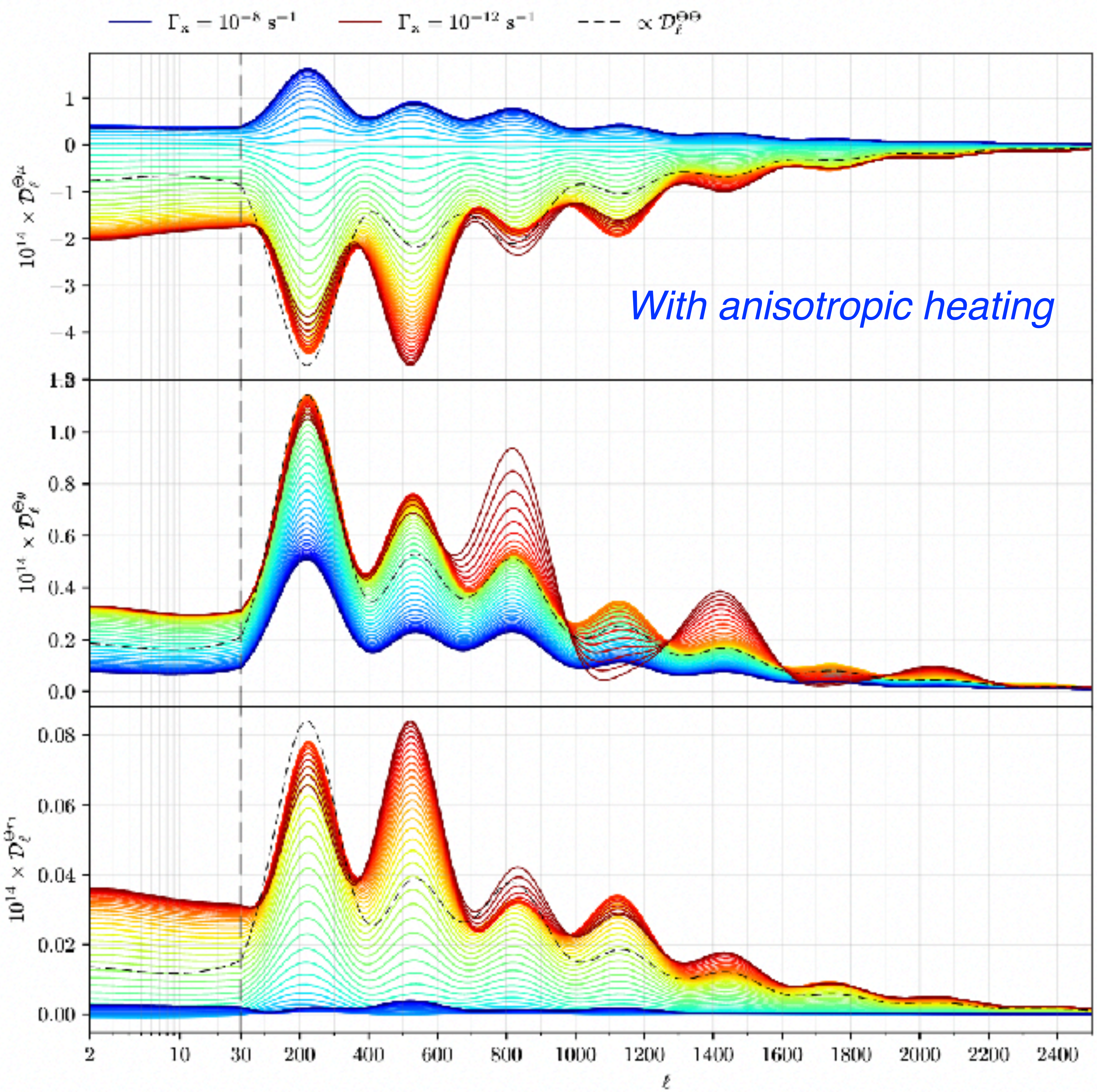
CMB power spectra for decaying particles



- New way to constrain these scenarios
- Anisotropic heating is important!
- Degeneracy between lifetime and abundance can in principle be broken by ℓ-dependence

Kite, Ravenni & JC, 2022, papers III, ArXiv:2212.02817

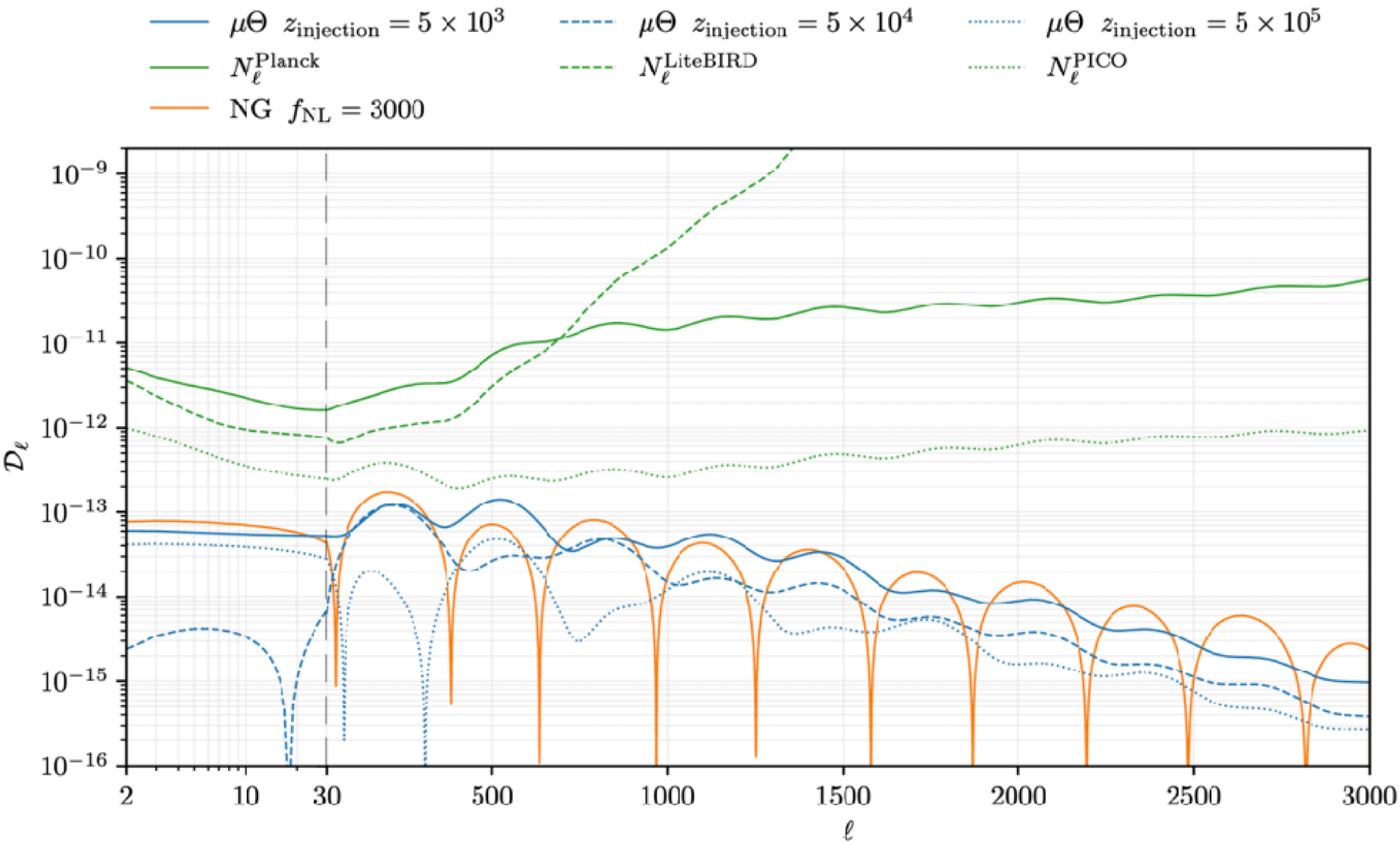
CMB power spectra for decaying particles



- New way to constrain these scenarios
- Anisotropic heating is important!
- Degeneracy between lifetime and abundance can in principle be broken by ℓ-dependence

Kite, Ravenni & JC, 2022, papers III, ArXiv:2212.02817

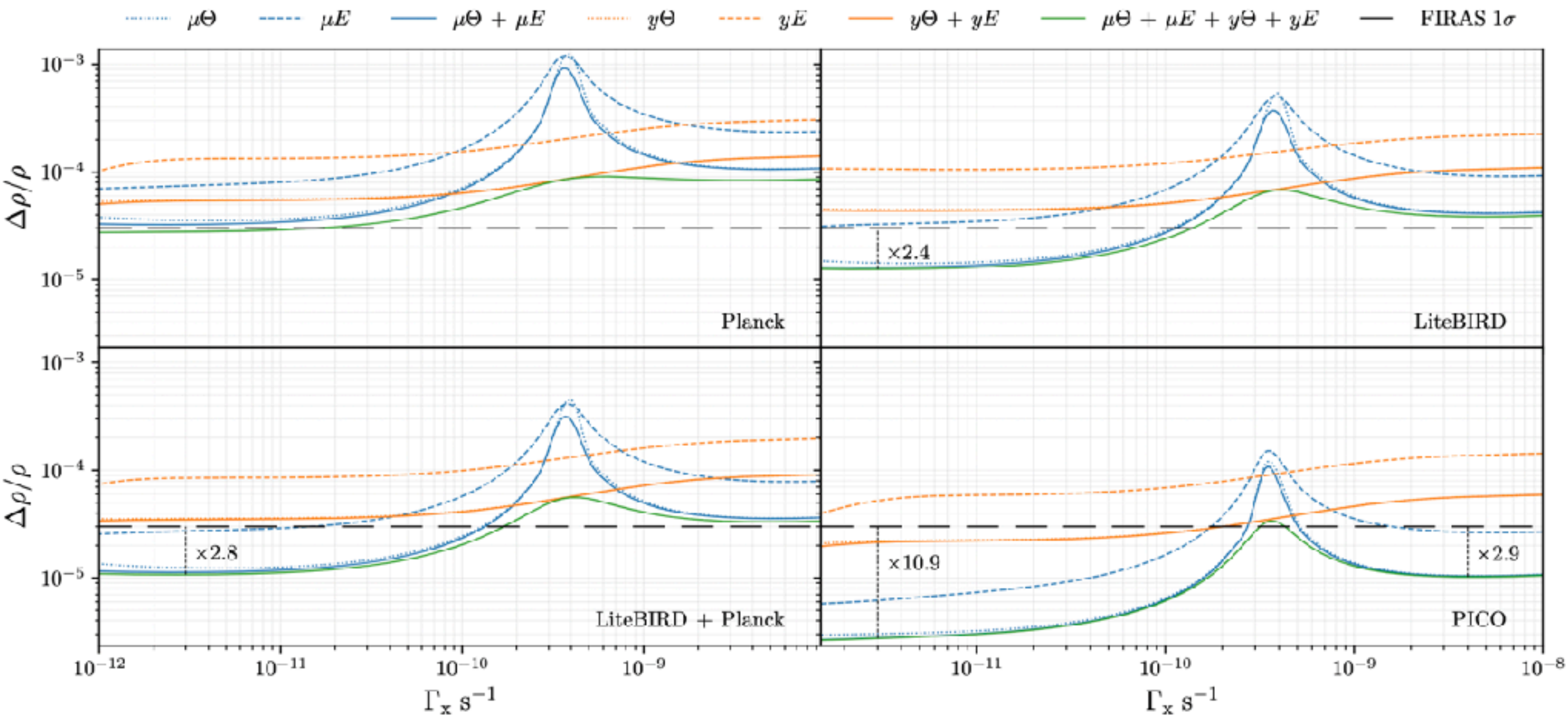
These new signals are *not* negligible!



Could be primordial ‘foreground’ for non-Gaussian signals

Kite, Ravenni & JC, 2022, papers III, ArXiv:2212.02817

Future CMB imagers can improve over FIRAS limit on *average* energy release



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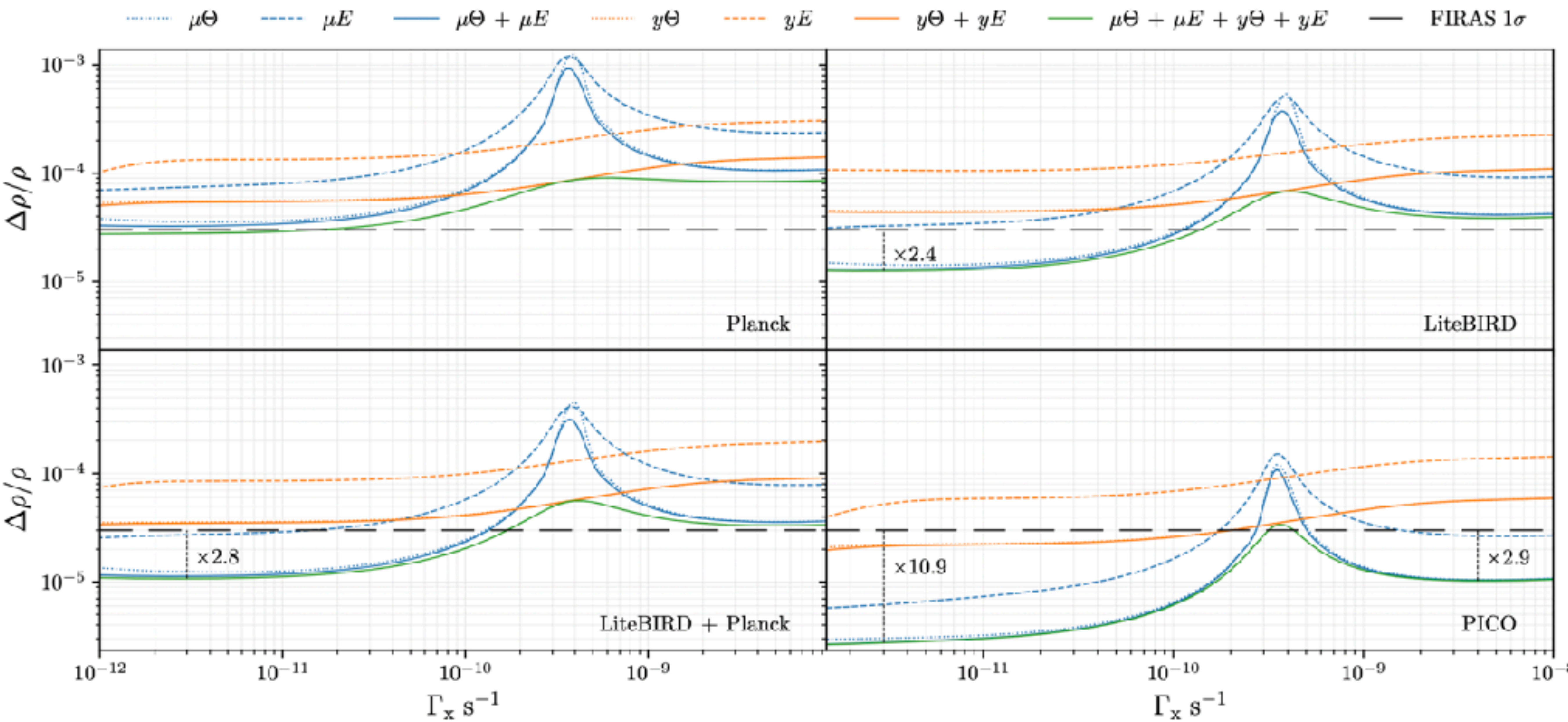


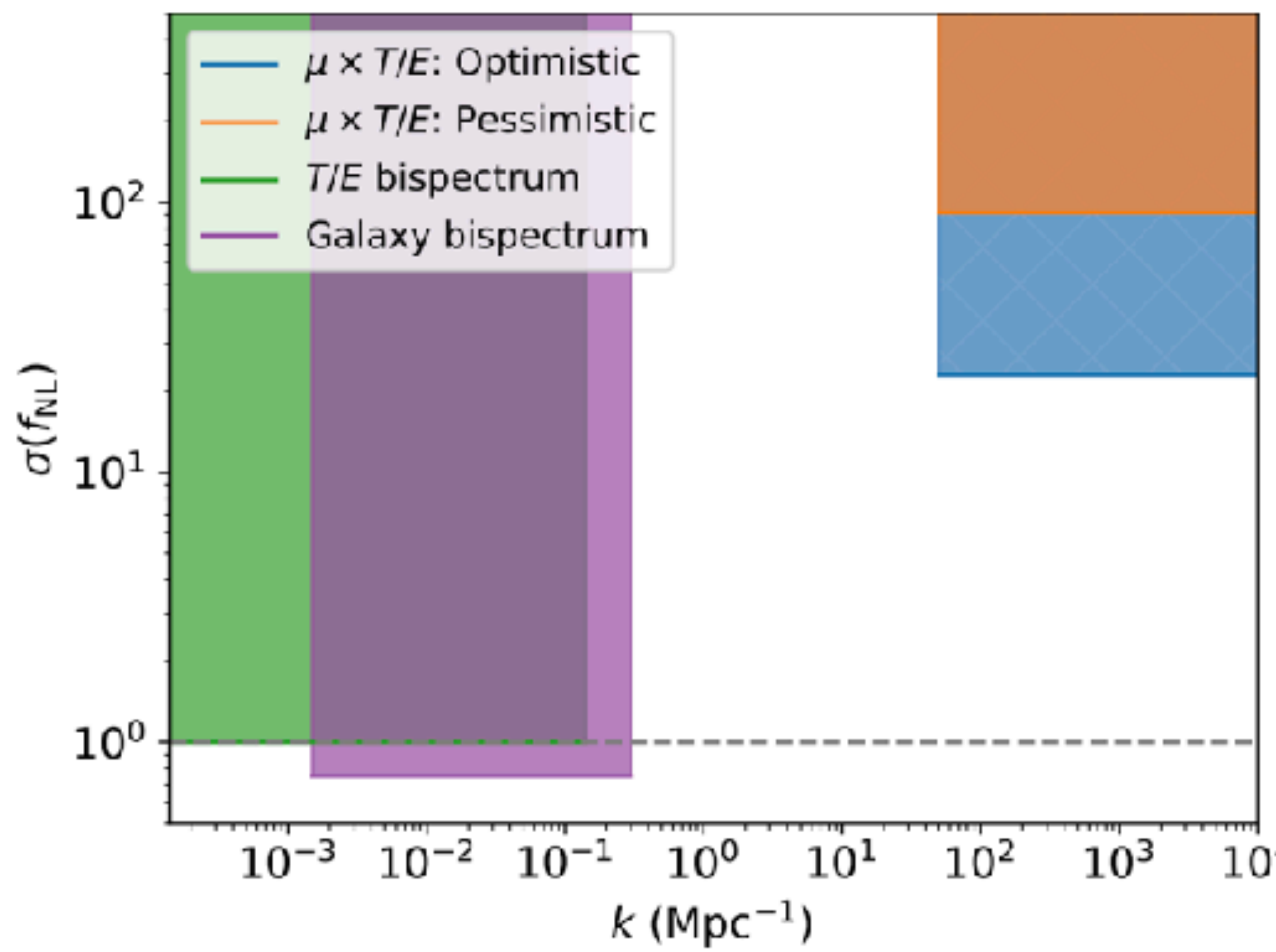
Novel synergistic targets for both CMB anisotropy and distortion enthusiasts!

Kite, Ravenni & JC, 2022, papers III, ArXiv:2212.02817

SKA as a CMB experiment

- Single dish mode is enough for μ-T constraints
- Low frequency foreground monitor

- Constraints on small scales
- SKA+Litebird equivalent to PICO in terms of μ-T
- *SKA could even do B-modes…*



Zegeye et al., ArXiv:2406.04326

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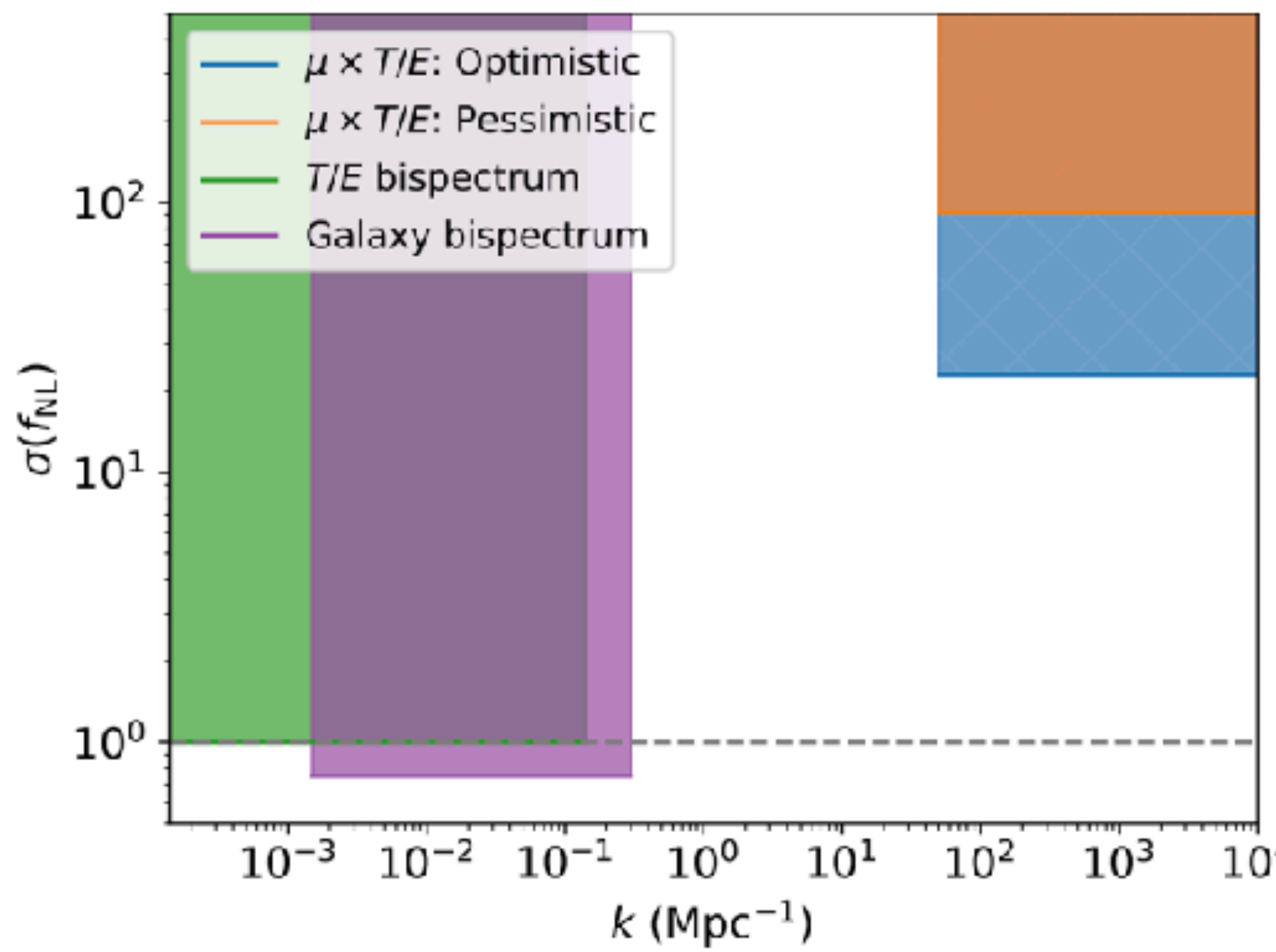
Detailed study with realistic foregrounds and systematics is required!



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Importance for LSSxCMB

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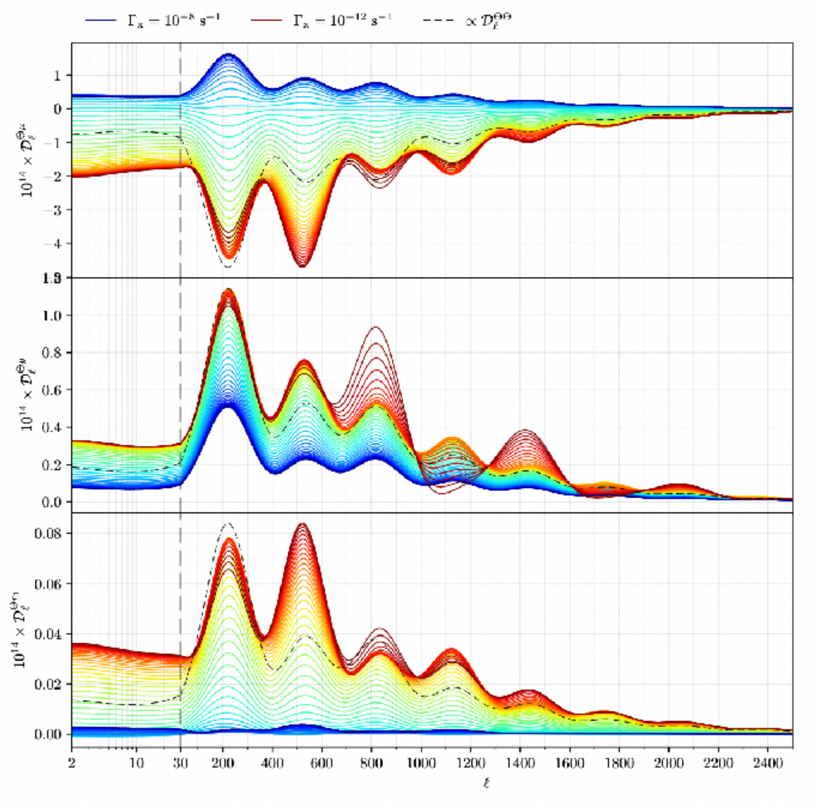
New types of observables

- Many explorations have now become possible
- New way of ‘slicing’ the last scattering surface
- Signals can be extracted using familiar analysis methods



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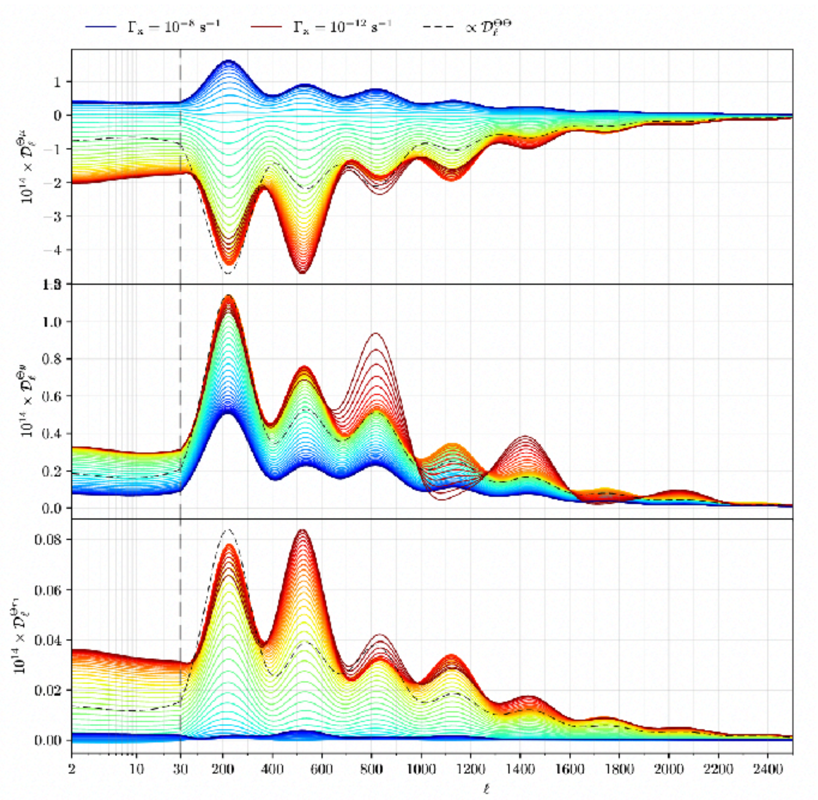

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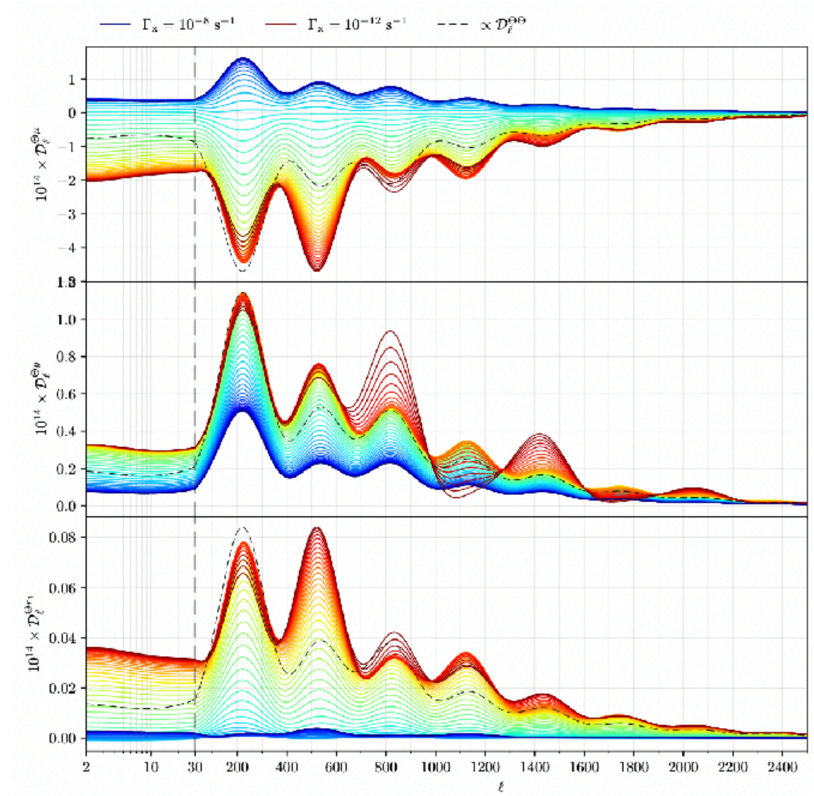

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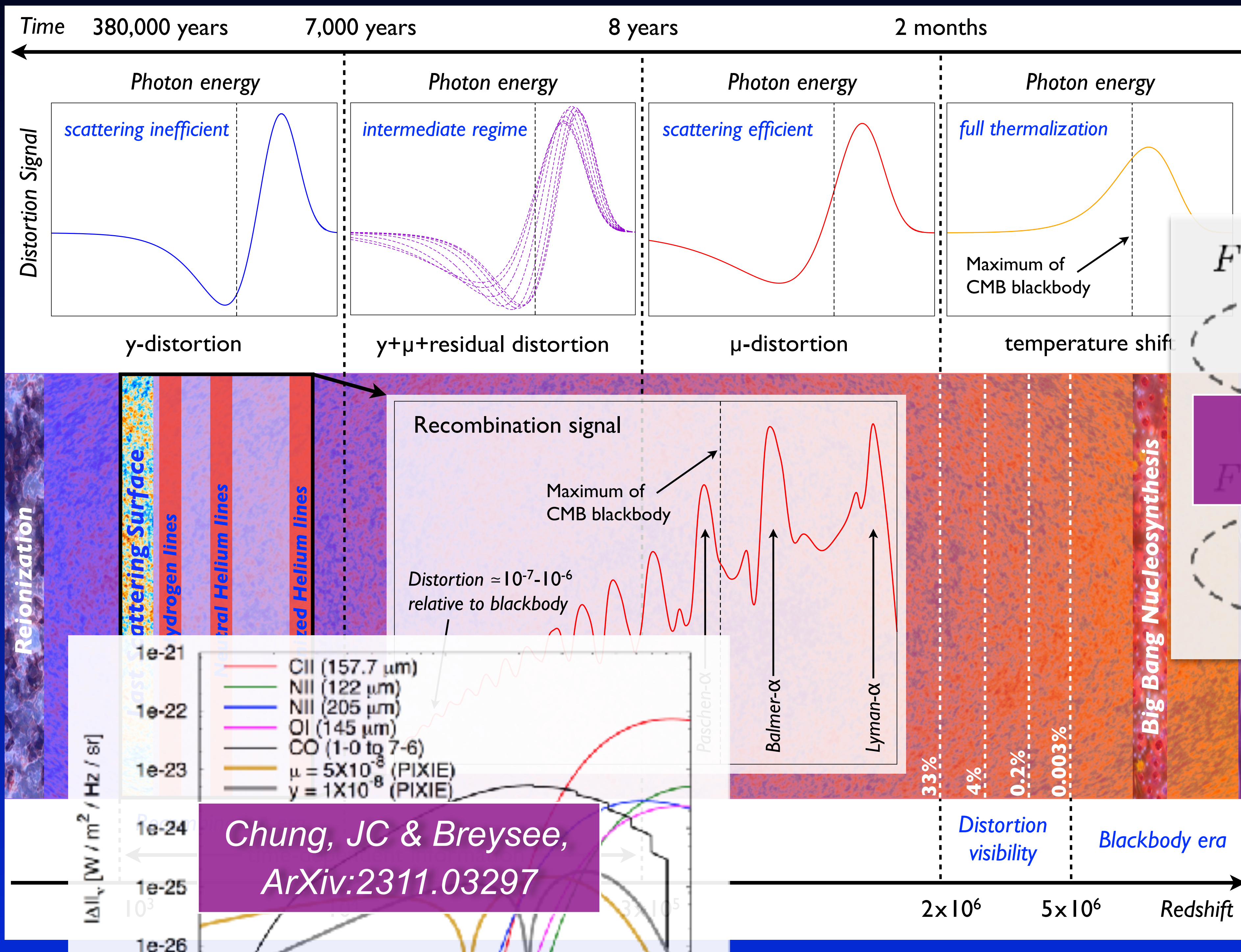
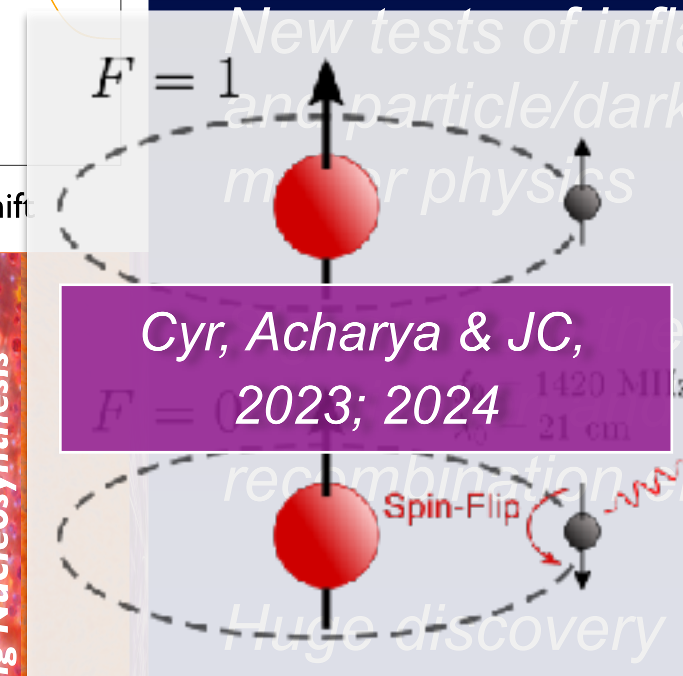
CMB distortion searches need modeling of low-*z* universe

- Foreground challenge for extraction of μ-distortion monopole
- Synergistic approach combining *all observables* will be required

Uniqueness of CMB Spectral Distortion Science





Guaranteed distortion signals in ΛCDM

New tests of inflation and particle/dark matter physics

recombination eras

Huge discovery potential

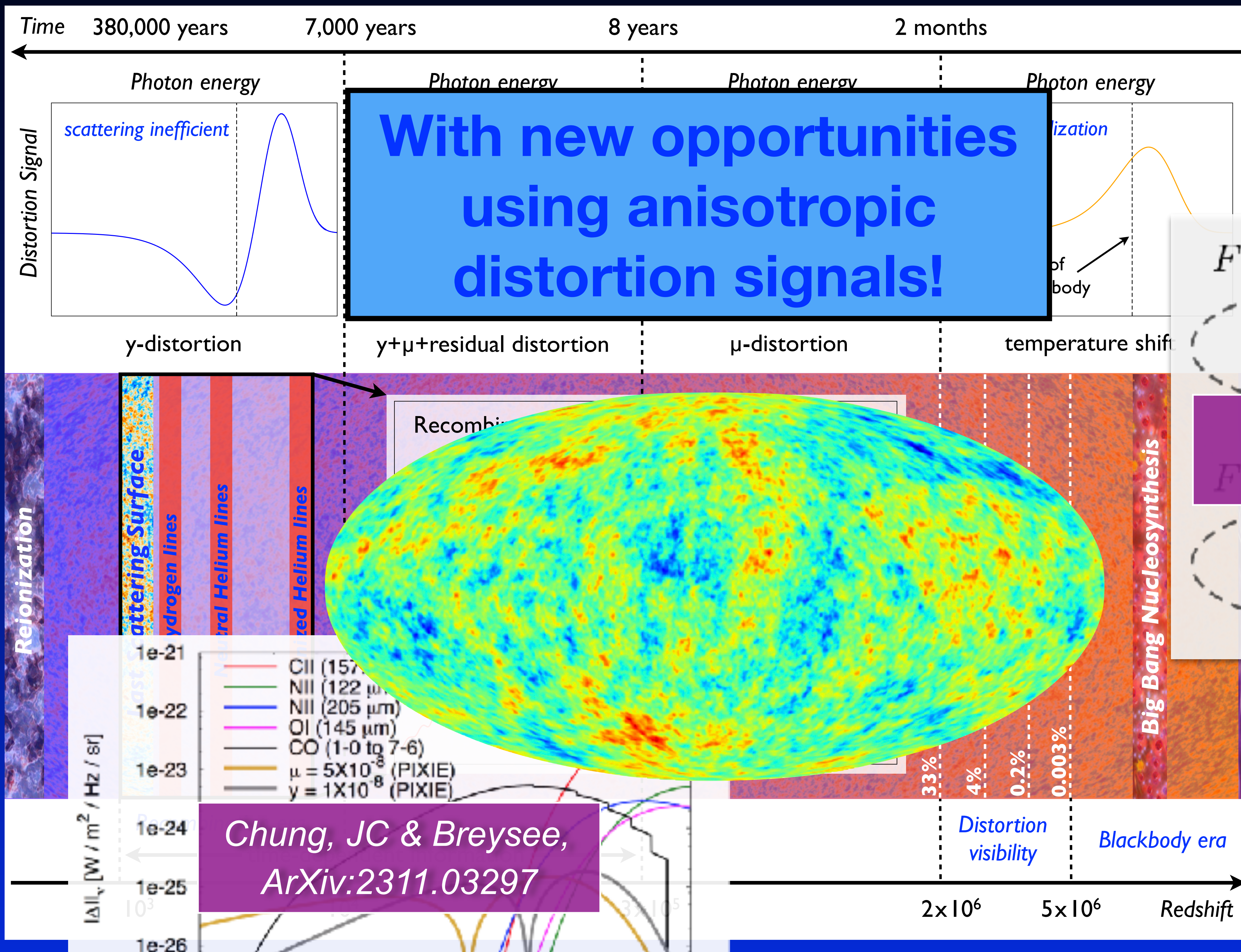
Complementarity and synergy with CMB anisotropy studies

COSMO
TMS
BISOU
Voyage 2050

JC & Sunyaev, MNRAS, 419, 2012
JC et al., MNRAS, 425, 2012
Silk & JC, Science, 2014
JC, MNRAS, 2016
JC et al., 2019, arXiv:1909.01593

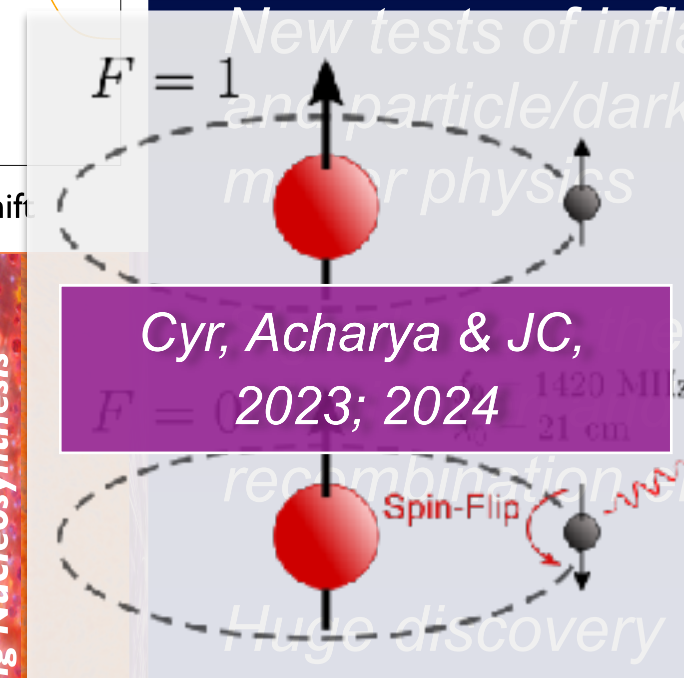
Uniqueness of CMB Spectral Distortion Science

Litebird, CMB-S4, PICO



With new opportunities using anisotropic distortion signals!

Chung, JC & Breysee, ArXiv:2311.03297



Cyr, Acharya & JC, 2023; 2024

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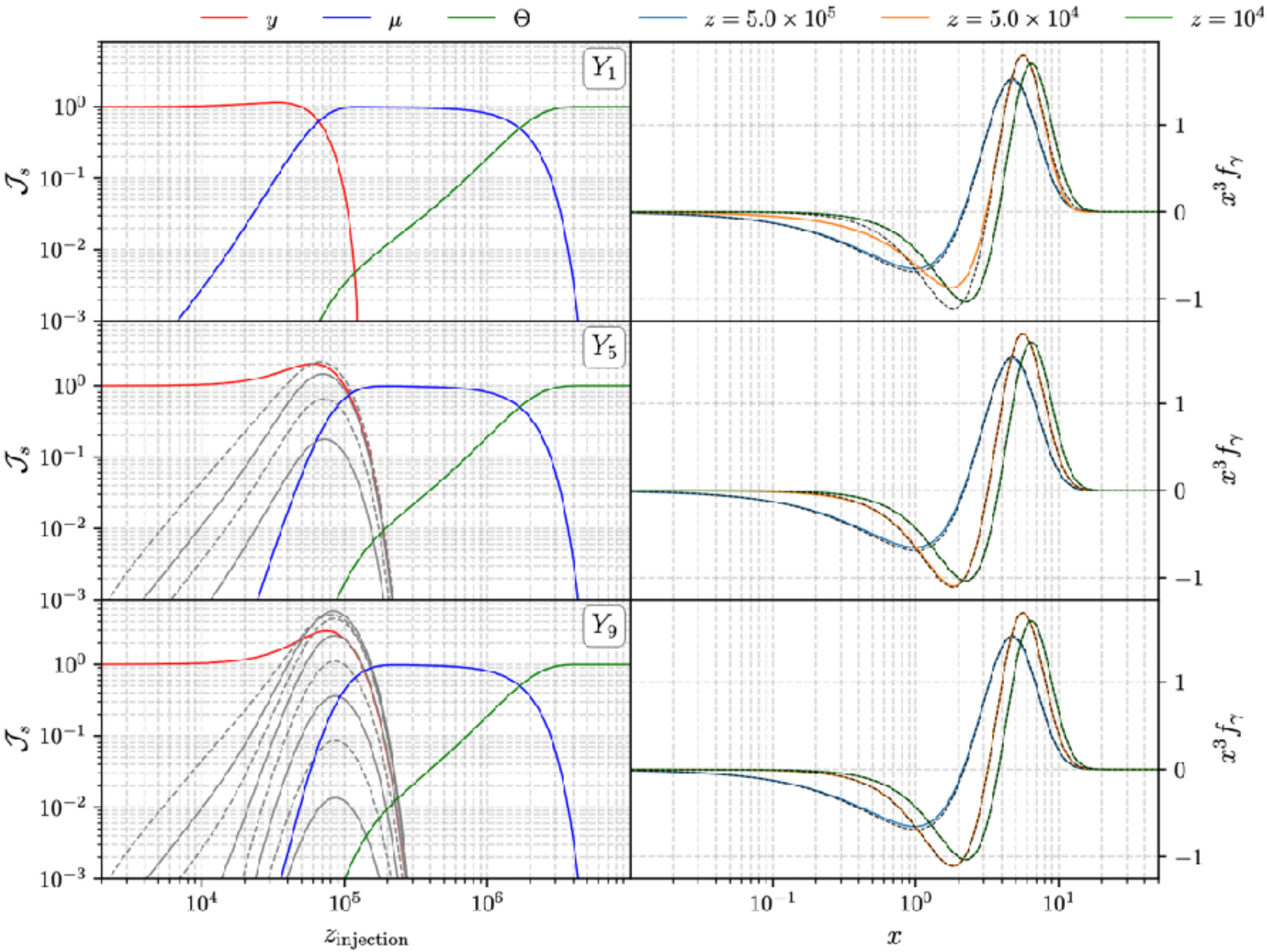
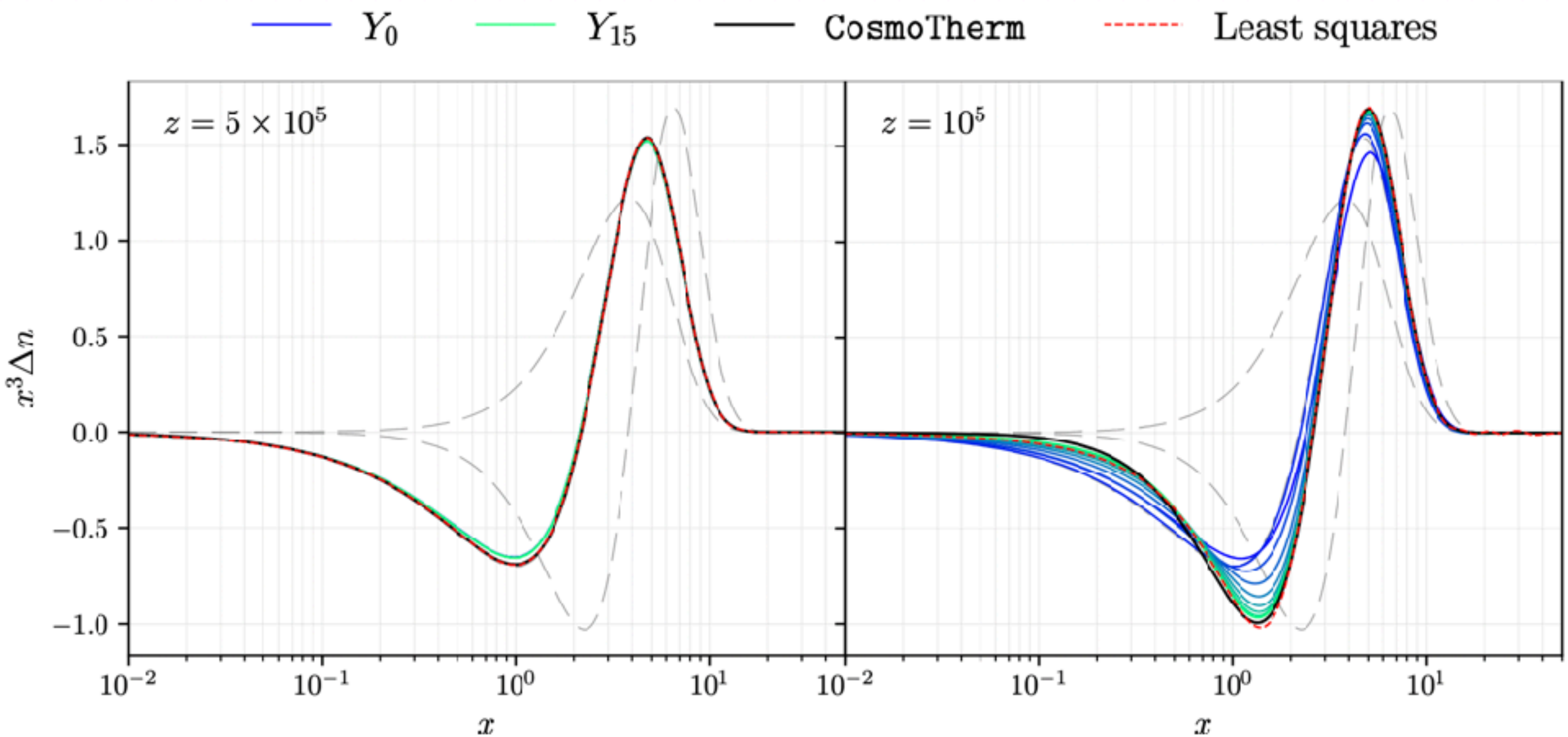
Beyond Simplest G, M and Y decomposition



Basis representation is not perfect



- `CosmoTherm` result not *exactly* captured at around z~10^5
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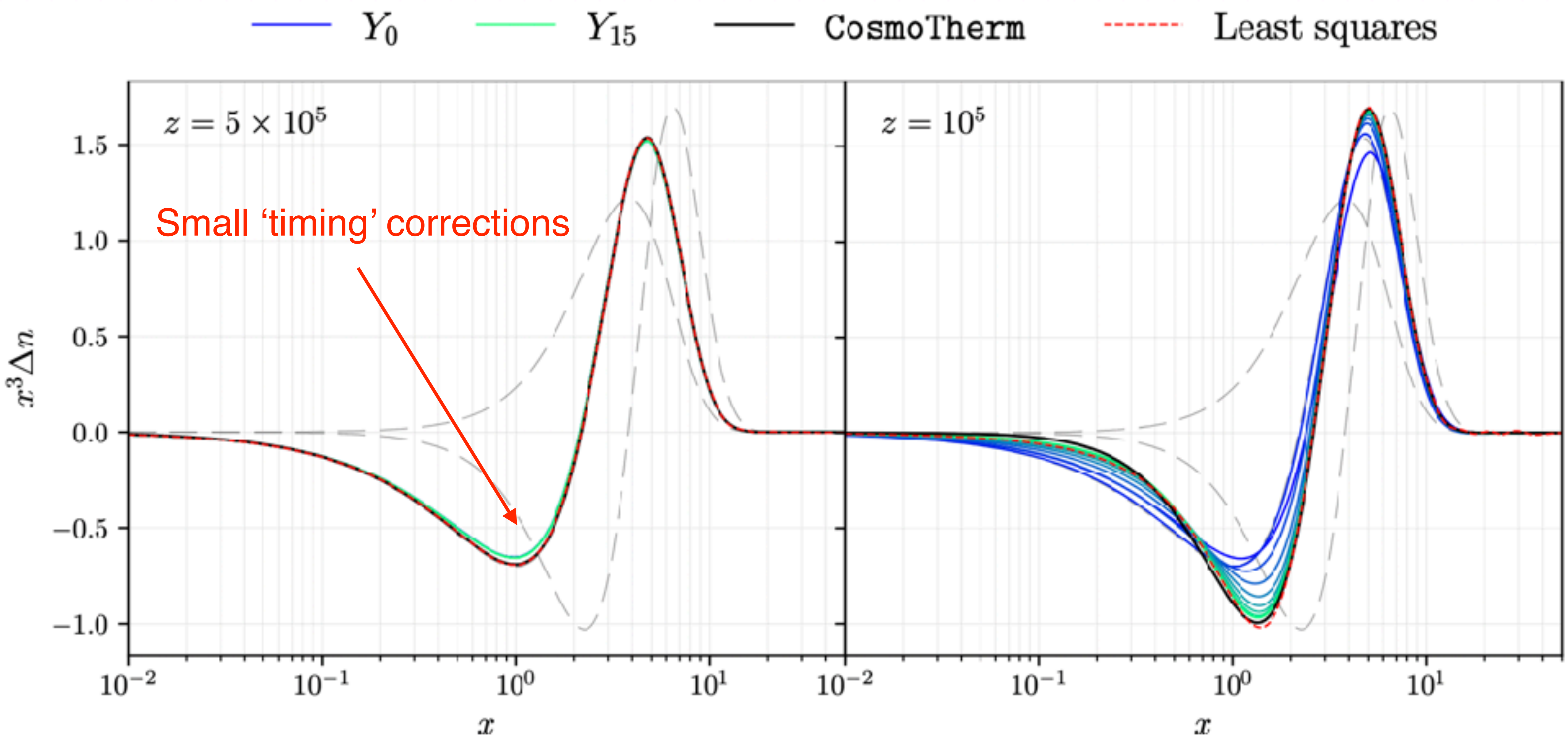

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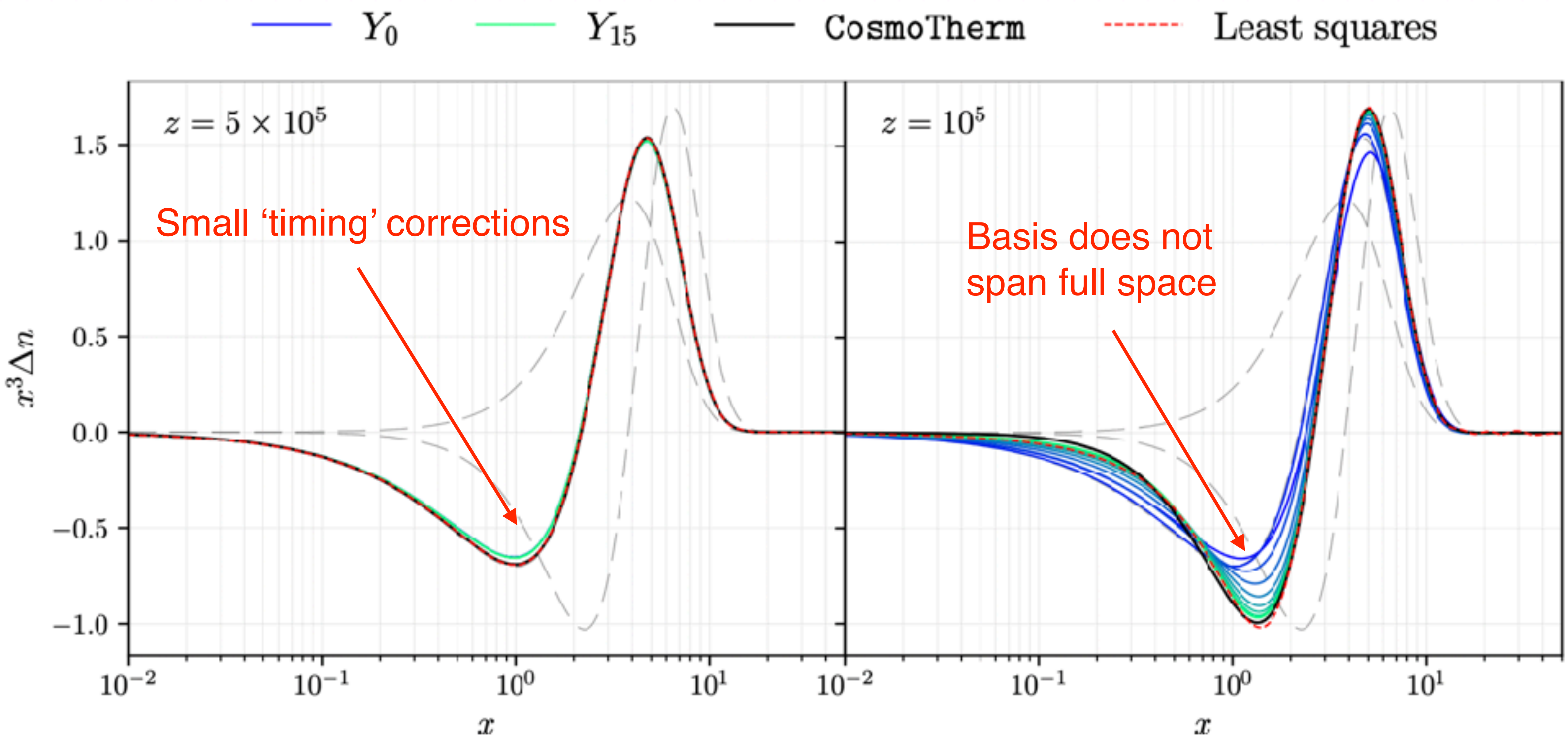

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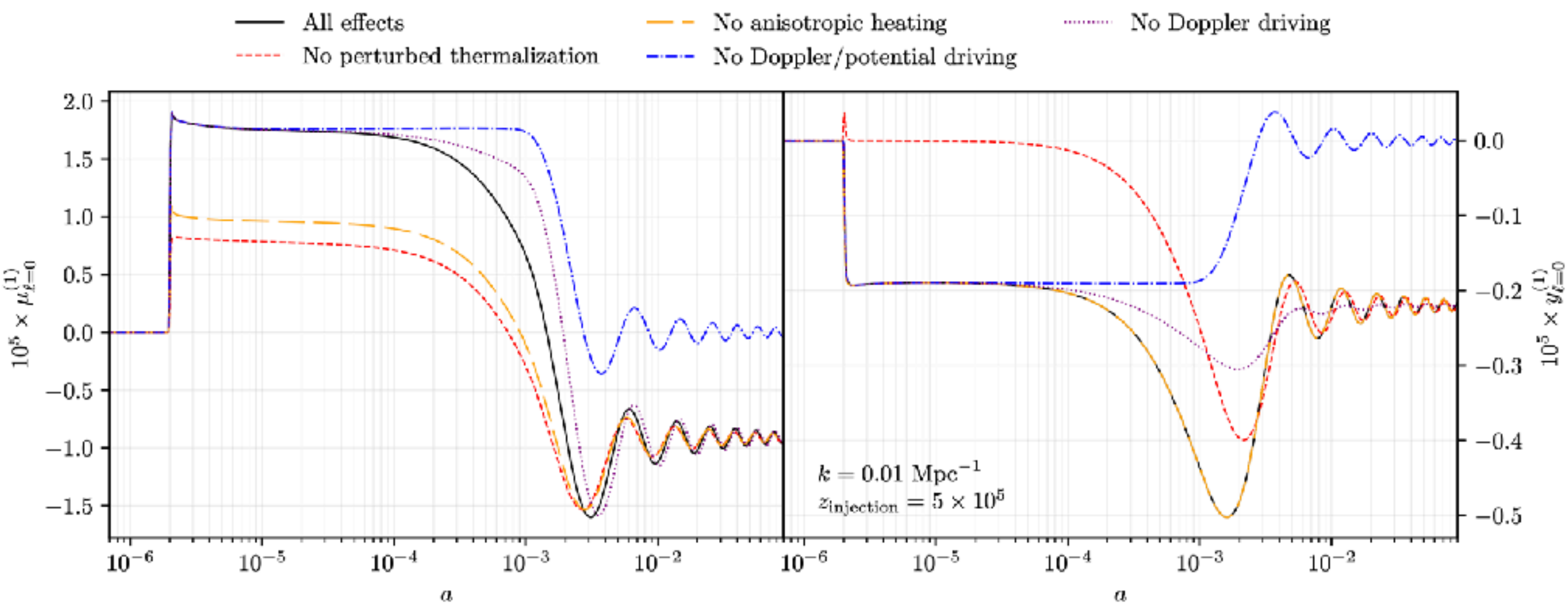
Evolution of modes for injection at z = 500,000

Effects of physical settings



$k = 0.01$ Mpc^{-1}

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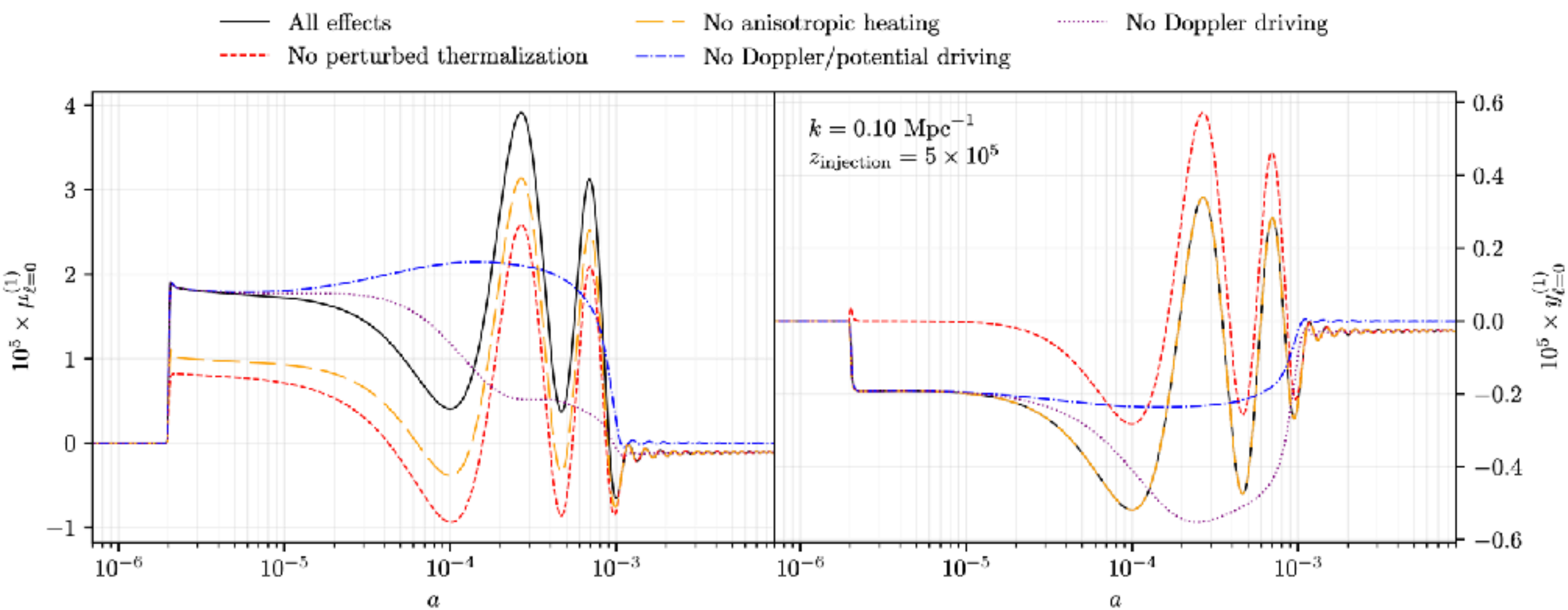

k = 0.1 Mpc^{-1}