Astroparticule symposium 2024

A Separate Universe approach to multifield inflation

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Outline

Introduction and Motivations

- Conceptual problems

→ No origin for the initial conditions

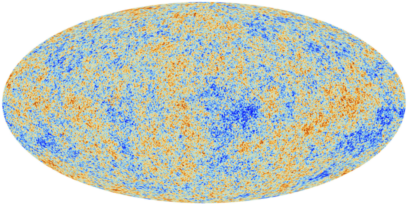
→ Horizon problem

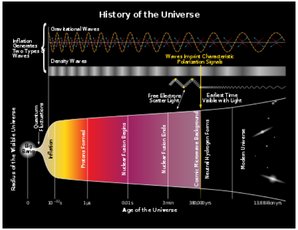
→ Flatness problem

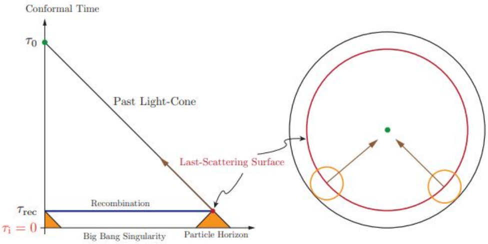
→ Scale invariance origin

→ origin of CMB and LSS

⇒ Accelerated expansion = **Inflation**







PBH = DM ?

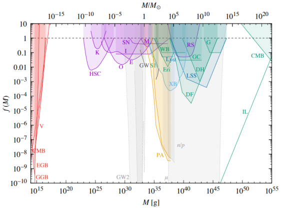
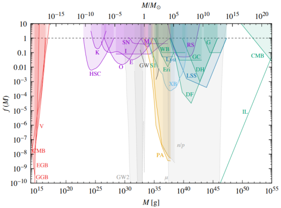


Figure: PBH as DM constraints, Carr+ 20

- Dark Matter = Primordial Black Holes ?
- Need for a boost in the power spectrum
- Need for a inflection point in the potential the inflaton
- Multifield inflation model

PBH = DM ?



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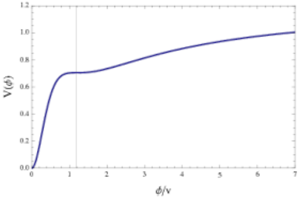


Figure: Inflection point for single field equation

Bauman TASI lectures

- Dark Matter = Primordial Black Holes ?
- Need for a boost in the power spectrum
- Need for a inflection point in the potential the inflaton
- Multifield inflation model

$\Rightarrow$ Produce even more large scale fluctuations

$\Rightarrow$ Large fluctuations have an effect on collapsed structures

- Best formalism for this : Stochastic inflation

  V.Vennin 22

Outline

Sub and Super horizon modes

- H : Hubble radius decreases during inflation
- Modes k start under the horizon : **subhorizon**
- They the cross the horizon : **superhorizon**
- Treat sub horizon modes and super horizon modes differently :

  When sub horizon modes cross the horizon they act as a stochastic noise in the dynamics of the super horizon modes

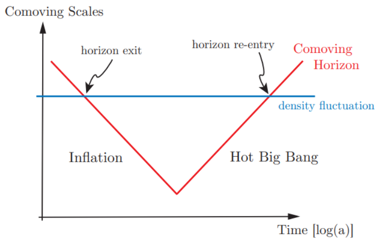


Figure: Solution to the horizon problem via inflation.
[Bauman TASI lectures]

Stochastic inflation

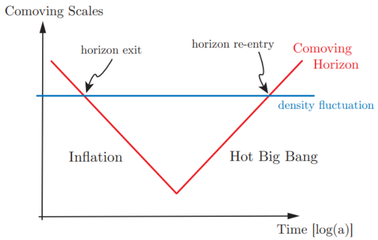


Figure: Solution to the horizon problem via inflation.Bauman TASI lectures

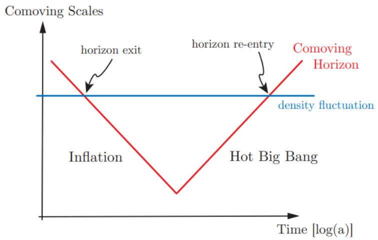
$\Rightarrow$ Langevin equation

$$\dot{\phi}_{IR} = -\frac{\partial V}{\partial \phi} + \xi_\phi$$

With ξ_ϕ a "noise" depending on the sub horizon scales $\rightarrow$ linear

Non linearity comes from $\dfrac{\partial V}{\partial \phi}$.

Stochastic inflation



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Non linearity comes from $\dfrac{\partial V}{\partial \phi}$.

- Note that there are no gradients here, so **no spatial interaction**
- Each Hubble patch evolves in time independently from other $\Rightarrow$ Seperate Universe Artigas+ 22 Pattison+ 19

The Separate Universe approach

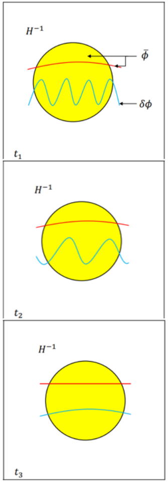
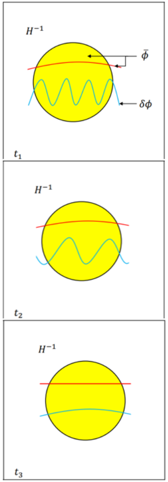


Figure: Quantum to classical transition of fluctuations Artigas Grav&Cosmo 2024

- In a Hubble patch the long wavelength modes eventually evolve as the background
- **GOAL :** evolve the superhorizon modes non linearly in homogeneous patches **VS** evolve the subhorizon modes linearly in non homogeneous patches
- When are these two operations compatible ?

The Separate Universe approach



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$\Rightarrow$ Proven to work at leading order in perturbation theory for single field inflation Artigas+ 22 Pattison+ 19

$\Rightarrow$ Proof doesn't tell **how** to execute this matching correctly $\longrightarrow$ if you gauge fix in SU you can't guaranty a good gauge choice in CPT

Outline

Stochastic inflation :

Stochastic inflation as I have presented it : suppose Slow Roll, de Sitter universe $\Rightarrow$ simple Langevin equation.

NOT always the case. No SR approximation $\Rightarrow$ no attractor solution $\Rightarrow$ need to keep the complete phase space, i.e. the associated momenta to our field(s) $\Rightarrow$ Hamiltonian theory to keep track of everything.

Hamiltonian approach & multifield inflation

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Multifield models :

Phenomenology

- PBH,
- Comological collider,
- Reheating : coupling inflaton to Standard model.

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Multifield models :

Phenomenology

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Theory

- EFT of inflation,
- non linear couplings,
- non minimally coupled to gravity,

$\Rightarrow$ coupling metric for a covariant theory.

Multifield inflation & ADM

Start from the most generic possible Lagrangian for multifield models of inflation, with $I, J \in [\![1, n]\!]$

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}M_{pl}^2\mathcal{R} - \frac{1}{2}g^{\mu\nu}G_{IJ}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi^I)\right]$$

Langlois+ 2008, Linde 1997, Kaiser+ 2012 ...

Make use of the ADM formalism:

$$ds^2 = -N^2(\tau,\vec{x})d\tau^2 + \gamma_{ij}(\tau,\vec{x})\left[dx^i + N^i(\tau,\vec{x})d\tau\right]\left[dx^j + N^j(\tau,\vec{x})d\tau\right]$$

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Compute the background equations, do some perturbation theory for full dynamics or for SU:

$$\phi^I = \bar{\phi}^I + \delta\phi^I, \quad N = \bar{N} + \delta N, \quad N^i = \delta N^i, \quad \gamma_{ij} = \bar{\gamma}_{ij} + \delta\gamma_{ij}$$

OR

$$\phi^I = \bar{\phi}^I + \delta\bar{\phi}^I, \quad N = \bar{N} + \delta\bar{N}, \quad N^i = \delta\bar{N}^i, \quad \gamma_{ij} = \bar{\gamma}_{ij} + \delta\bar{\gamma}_{ij}$$

Hamiltonian CPT

We write the hamiltonian action and separate it in background, first and second order.

$$S = \int \mathrm{d}\tau \int \mathrm{d}^3 x \left(\pi_I \dot{\phi}^I + \pi^{ij} \dot{\gamma}_{ij} - N\mathcal{C} - N^i \mathcal{D}_i \right) = S^{(0)} + S^{(1)} + S^{(2)}.$$

With $\mathcal{D}_i = \mathcal{D}_i^{(1)}$ the diffeormorphism constraint, $\mathcal{C} = \mathcal{C}^{(0)} + \mathcal{C}^{(1)} + \mathcal{C}^{(2)}$ the scalar constraint.

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$$\mathcal{C}^{(2)\,\phi} \supset \frac{1}{4v} G^{IJ}{}_{,K}\,\pi_J \left(\delta\phi^K \delta\pi_I^\star + \mathrm{c.c.} \right)$$

$\Rightarrow$ not explicitly covariant expression despite a covariant theory

$\Rightarrow$ Because $\left(\delta\phi^I, \delta\pi_I\right)$ themselves are not covariant in the phase space.

Adiabatic and entropic directions

- Isolate the adiabatic direction in order to (almost) recover single field dynamics

→ rotate these covariant variables to define the adiabatic and entropic directions. Pinol+ 2020

$$D_t e^I_a = \Omega_a{}^b e^I_a$$

$$\Omega = \begin{pmatrix} 0 & \omega_1 & 0 & \cdots & 0 \\ -\omega_1 & 0 & \omega_2 & \cdots & 0 \\ 0 & -\omega_2 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & -\omega_{n-1} & 0 \end{pmatrix}$$

$$\mathcal{Q}_a = e^a_I \mathcal{Q}^I, \quad \mathcal{P}_b = e^b_J \mathcal{P}_J$$

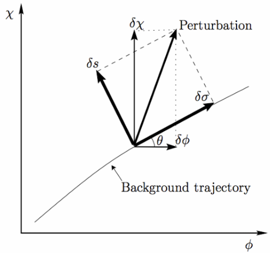

Figure: The adiabatic and entropic directions for two fields Gordon+ 2000

Remark n°1 : Note that we can do another complete canonical transform again but we don't need to since we have the equations of motion we can project on the right directions

Remark n°2 : We define the adiabatic and entropic directions after the first canonical transform simply for convenience.

Validity conditions for SU

Compute equations of motion for $\{\delta\phi^I/\mathcal{Q}^I/\mathcal{Q}_c, \delta\pi_J/\mathcal{P}_J/\mathcal{P}_b, \delta\gamma_1, \delta\pi_1, \delta\gamma_2, \delta\pi_2\}$ and $\{\bar{\delta\phi}^I/\bar{\mathcal{Q}}^I/\bar{\mathcal{Q}}_c, \bar{\delta\pi}_J/\bar{\mathcal{P}}_J/\bar{\mathcal{P}}_b, \bar{\delta\gamma}_1, \bar{\delta\pi}_1\}$

Compare them at large scales and check when they match. Next step : Verify whether we have $\delta N = \bar{\delta N}$

The separate universe approach is valid if :

$$\left(\frac{k}{aH}\right)^2 \ll 3\left(1-\epsilon_1\right), \quad \left(\frac{k}{aH}\right)^2 \ll 16\left(1-\epsilon_1\right), \quad \left|\left(\frac{k}{aH}\right)^2 G_{IJ}\right| \ll H^{-2}\left|\rho_{,IJ}\right|$$

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Matching the constraint → 1

Matching the dynamics → { 2, 3 }

1, 2 } → We are working at large scales

3 → The effective mass has to be big enough

⚠ Dynamics play a role here

Role of dynamics and adiabatic direction

The equations of motion of the adiabatic and entropic directions are :

$$
\begin{aligned}
\dot{\mathcal{Q}}_c =& \delta_{c\sigma}\dot{\sigma}\left(\frac{\delta N}{N} - \frac{\sqrt{3}}{2}\frac{\delta\gamma_1}{v^{2/3}}\right) + \frac{N}{v}P_c - \Omega_a{}^c\mathcal{Q}_a \\
\dot{\mathcal{P}}_c =& -\delta N v V_{,c} + N v^{1/3}\left(k^2\delta ac + V_{;ac} - \frac{v^2}{N^2}R_{a\sigma\sigma c}\dot{\sigma}^2\right)\mathcal{Q}_a + N\frac{\sqrt{3}}{2}v^{1/3}V_{,c}\,\delta\gamma_1 \\
& - \frac{v}{N}\delta_{c\sigma}\dot{\sigma}\delta N_1 - \Omega^c{}_a\mathcal{P}_a
\end{aligned}
$$

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$$-\frac{v}{N}\delta_{c\sigma}\dot{\sigma}\delta N_1 - \Omega^c{}_a\mathcal{P}_a$$

$\Rightarrow$ Almost single field dynamics for the adiabatic direction : extra source term that comes from the coupling to the first entropic direction. The effective mass condition is the only one that is affected

$$\left|\left(\frac{k}{aH}\right)^2 G_{IJ}\right| \ll H^{-2}\left|\rho_{,IJ}\right| \Leftrightarrow \left(\frac{k}{aH}\right)^2\delta_{ac} \ll H^{-2}\left|V_{,ac} - \frac{v^2}{N^2}R_{a\sigma\sigma c}\dot{\sigma}^2\right|$$

Outline

Mukhanov Sasaki variables

Next step : gauge fixing both approaches $\Rightarrow$ write gauge invariant equations of motion : The Mukhanov-Sasaki variables

$$Q_a = e_a^I \left(\delta\phi^I + \frac{M_{pl}^2 G^{IJ} \pi_J}{\sqrt{6} v^{5/3} \theta} \left(\sqrt{2} \delta\gamma_1 - \delta\gamma_2 \right) \right) , \qquad \bar{Q}_a = e_a^I \left(\delta\bar{\phi}^I + \frac{M_{pl}^2 G^{IJ} \pi_J}{\sqrt{3} \bar{v}^{5/3} \bar{\theta}} \delta\bar{\gamma}_1 \right)$$

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$$\ddot{Q}_\sigma + 3H\dot{Q}_\sigma + \left[k^2 + m_{\sigma\sigma} - \omega_1^2 - \frac{1}{M_{pl}^2 a^3}\partial_t\left(\frac{a^3\dot{\sigma}^2}{H}\right)\right]Q_\sigma = 2\partial_t(\omega_1 Q_1) - 2\left(\frac{V_{,\sigma}}{\dot{\sigma}} + \frac{\dot{H}}{H}\right)\omega_1 Q_1$$

$$\ddot{Q}_\alpha + (3H\delta_{\alpha\beta} + 2\Omega_{\alpha\beta})\dot{Q}_\beta + \left[k^2\delta_{\alpha\beta} + m^2_{\alpha\beta} + 3H\Omega_{\alpha\beta} + \dot{\Omega}_{\alpha\beta}\right]Q_\beta = \delta_{\alpha 1}2\left[\left(\frac{V_{,\sigma}}{\dot{\sigma}} + \frac{\dot{H}}{H}\right)\omega_1 Q_\sigma + \partial_t(\omega_1 Q_\sigma)\right]$$

$$m^2_{\alpha\beta} = V_{;\alpha\beta} - R_{\alpha\sigma\sigma\beta} + \Omega^2_{\alpha\beta}$$

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$$\ddot{\bar{Q}}_\sigma + 3H\dot{\bar{Q}}_\sigma + \left[m_{\sigma\sigma} - \omega_1^2 - \frac{1}{M_{pl}^2 a^3}\partial_t\left(\frac{a^3\dot{\sigma}^2}{H}\right)\right]\bar{Q}_\sigma = 2\partial_t(\omega_1\bar{Q}_1) - 2\left(\frac{V_{,\sigma}}{\dot{\sigma}} + \frac{\dot{H}}{H}\right)\omega_1\bar{Q}_1 + \boxed{\sqrt{\frac{\varepsilon}{2}}\eta M_{pl}H^2\bar{\mathcal{D}}^{(1)}}$$

$$\ddot{\bar{Q}}_\alpha + (3H\delta_{\alpha\beta} + 2\Omega_{\alpha\beta})\dot{\bar{Q}}_\beta + \left[m^2_{\alpha\beta} + 3H\Omega_{\alpha\beta} + \dot{\Omega}_{\alpha\beta}\right]\bar{Q}_\beta =$$

$$\delta_{\alpha 1}2\left[\left(\frac{V_{,\sigma}}{\dot{\sigma}} + \frac{\dot{H}}{H}\right)\omega_1\bar{Q}_\sigma + \partial_t(\omega_1\bar{Q}_\sigma)\right] + \boxed{2\sqrt{2\varepsilon}HM_{pl}\omega_1\delta_{1c}\bar{\mathcal{D}}^{(1)}}$$

$$m^2_{\alpha\beta} = V_{;\alpha\beta} - R_{\alpha\sigma\sigma\beta} + \Omega^2_{\alpha\beta}$$

$\Rightarrow$ Only works if we impose a "diffeomorphism constraint" in SU.

Conclusion and next steps

- The SU approach works as long as we are at a big enough scale **and** our fields are heavy enough
- Works in all gauges if we start from CPT and go to large scales
- Doesn't necessarily work if we try to gauge fix directly in SU
- **next steps :** go beyond the leading order, allow the perturbation to evolve non linearly as well $\Rightarrow$ order three Hamiltonian theory.
- **Large deviation principle** in inflation $\longrightarrow$ analytical asymptotical power spectrum in stochastic inflation
- **Long term goals :** find observables from multifield stochastic inflation models.

The end

Thank You

for listening

Spatially flat gauge

CPT

Spatially flat gauge :

- $\delta\gamma_1 = \delta\gamma_2 = 0$

$\Rightarrow \; = \dot{\delta\gamma_1} = \dot{\delta\gamma_2} = 0$

$$\Rightarrow \frac{\delta N}{N} = -\frac{\dot{\sigma}\delta\sigma}{N\theta}$$

SU

Spatially flat gauge :

- $\overline{\delta\gamma}_1 = 0$

$\Rightarrow \; = \dot{\overline{\delta\gamma}}_1 = 0$

$$\Rightarrow \frac{\overline{\delta N}}{N} = -\frac{2}{3}\frac{M_{pl}^2}{\theta^2}\left(\frac{\dot{\sigma}}{vN}\delta\overline{\pi}_\sigma + V_{,c}\,\overline{\delta}_c\right)$$

Both gauges are properly defined, **BUT** clear discrepancy because we don't have $\frac{\delta N}{N} = \frac{\overline{\delta N}}{N}$.

A proper spatially flat gauge fixing in the SU framework does not lead to a spatially flat gauge fixed CPT approach taken to large scales.

uniform expansion gauge

CPT

Uniform expansion gauge :

- $\delta\gamma_1 = 0, \quad \delta N_1 = 0$

$\Rightarrow$ = $\dot{\delta\gamma}_1 = 0$

- Still one unfixed gauge mode

$$\Rightarrow \frac{\delta N}{N} = -\frac{2}{3}\frac{M_{pl}^2}{\theta^2}\left(\frac{\dot{\sigma}}{vN}\delta\pi_\sigma + V_{,c}\,\delta_c\right) - \frac{2M_{pl}^4 k^2}{3\sqrt{6}v^{4/3}\theta^2}\delta\gamma_2$$

SU

Uniform expansion gauge :

- $\bar{\delta\gamma}_1 = 0$

$\Rightarrow$ = $\dot{\bar{\delta\gamma}}_1 = 0$

$$\Rightarrow \frac{\bar{\delta N}}{N} = -\frac{2}{3}\frac{M_{pl}^2}{\theta^2}\left(\frac{\dot{\sigma}}{vN}\bar{\delta\pi}_\sigma + V_{,c}\,\bar{\delta}_c\right)$$

The CPT framework is not completely gauge fixed, **BUT** we have matched two gauge fixing procedures at l large scales : $\boxed{\frac{\delta N}{N} = \frac{\bar{\delta N}}{N}}$.

Short term questions

Same effort but in Jordan Frame

We know that such models are classically equivalent to multifield models with $G^{IJ} = \delta^{IJ}$ but with non minimal couplings to gravity. Kaiser 2010 Geller+ 2022

Proceed with the same computations $\Rightarrow$ we get a fourth condition which can be seen as a derivative w.r.t $\delta\phi^I$ of the third condition.

Where should we coarse grain ?

We have coarse grained our theory at hubble scale $\Rightarrow$ not a problem as long as SR because of scale invariance. What if we do not have scale invariance ($\longrightarrow$ often happens in multifield models) ?

$\Rightarrow$ coarse grain at a different scale : at adiabatic effective mass scale ?

Third order Hamiltonian

In langevin equation : what if ξ_ϕ can evolve non lienarly as well ?

$\Longrightarrow$ need for third order Hamiltonian.

Questions and more applications of said hamiltonian :

- How/where to coarse grain if we want to do SU : sub horizon and superhorizon scales mix
- How will gauge fixing work in this approach, even without going to SU. Already very technical at second order Artigas+ 2023
- Recently : many debates on loop corrections in inflation with different results in different gauges $\longrightarrow$ no gauge fixing *a priori* so could contribute to this discussion
- Same story for the total derivatives in this debate : not an issue in Hamiltonian theory
- backreaction of IR modes on UV modes

Once all of this is done, we are only half way : still need to solve the Langevin equation and make observanble predictions.

Third order hamiltonian

We limited ourselves to linear evolution for the sub horizon modes, i.e. the second order hamiltonian $\longrightarrow$ lets go one step further : ongoing work with Matteo Braglia, Julien Grain and Lucas Pinol.

A few preliminary steps : Find gauge invariant variables at second order, compute constraints at third order.

$$
\begin{aligned}
\delta\phi^I =& \mathcal{Q}^I - \frac{1}{2}\Gamma^I_{JK}\mathcal{Q}^J\mathcal{Q}^K + \frac{1}{3}\left(\Gamma^I_{JK}\Gamma^J_{LM} - \frac{1}{2}\Gamma^I_{MK,L}\right)\mathcal{Q}^K\mathcal{Q}^L\mathcal{Q}^M \\
\delta\pi_I =& \mathcal{P}_I + \Gamma^K_{IJ}\bar{\pi}_K\mathcal{Q}^J + \Gamma^K_{IJ}\mathcal{P}_K\mathcal{Q}^J + \frac{1}{2}\left(\Gamma^S_{IJ,K} - \Gamma^S_{IR}\Gamma^R_{JK} + \Gamma^R_{IJ}\Gamma^S_{RK}\right)\bar{\pi}_S\mathcal{Q}^J\mathcal{Q}^K \\
&+ \left(3\Gamma^K_{IM,L} + \Gamma^K_{IR}\Gamma^R_{LM} + 3\Gamma^R_{IM}\Gamma^K_{RL}\right)\mathcal{P}_K\mathcal{Q}^L\mathcal{Q}^M \\
&+ \left(\Gamma^S_{IJ,ML} - 2\Gamma^S_{IK,J}\Gamma^K_{LM} + 3\Gamma^S_{KM,L}\Gamma^K_{IJ} - \Gamma^S_{IJ,K}\Gamma^K_{LM} + 2\Gamma^K_{IJ,L}\Gamma^S_{KM}\right. \\
&+ \Gamma^K_{JM,L}\Gamma^S_{IK} + \Gamma^K_{IR}\Gamma^R_{LM}\Gamma^S_{KJ} - 2\Gamma^K_{IJ}\Gamma^S_{KR}\Gamma^R_{ML} + \Gamma^K_{IJ}\Gamma^R_{KM}\Gamma^S_{RL} \\
&\left.+ 2\Gamma^K_{LM}\Gamma^S_{IR}\Gamma^R_{KJ} + \Gamma^K_{LM}\Gamma^R_{IK}\Gamma^S_{RJ} + \Gamma^K_{LM}\Gamma^R_{IJ}\Gamma^S_{RK}\right)\mathcal{Q}^L\mathcal{Q}^M\mathcal{Q}^J\bar{\pi}_S \\
F^{(3)}\left(\delta\phi^I\,\mathcal{P}_I\,t\right) =& \delta\phi^I\mathcal{P}_I + \frac{1}{2}\Gamma^K_{IJ}\pi_K\delta\phi^I\delta\phi^J + \frac{1}{2}\Gamma^K_{IJ}\mathcal{P}_K\delta\phi^I\delta\phi^J + \frac{1}{6}\left(\Gamma^K_{IJ\,L} + \Gamma^K_{LR}\Gamma^R_{IJ}\right)\pi_K\delta\phi^I\delta\phi^J\delta\phi^L
\end{aligned}
$$

Large deviation principle

The large deviation principle : often used in particle physics to quantify the probability of rare events in stochastic processes by describing the probability as a decaying exponential with a rate function. See review Touchette 2008

Used in cosmology before, for Large Scale Structure Bernardeau+ 2015, and in soft de Sitter for inflation Cohen+ 2022.

Our goal : solve stochastic inflation equations analytically, albeit asymptotically. **e.g:** do this in USR.

Other possibility : find the asymptotical PDF for the anisotropic stress. In Grain+ 2020, only second order moments were computed, we'd go one step further.