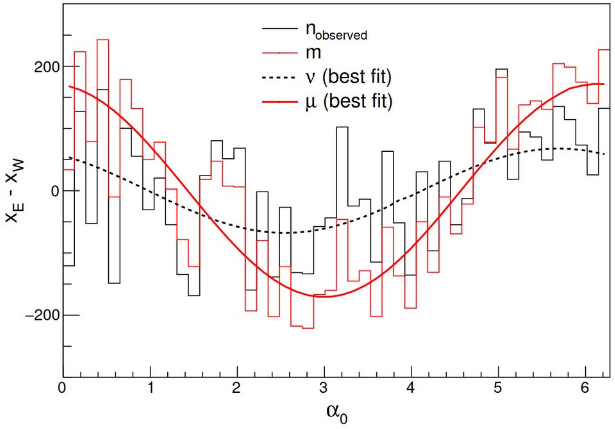
EW with Forward Folding

Martin Schimassek, Olivier Deligny,
Rogerio Menezes de Almeida

Introduction

- main question: can we account for resolution effects, i.e. bin-to-bin migrations, in LSA analyses?

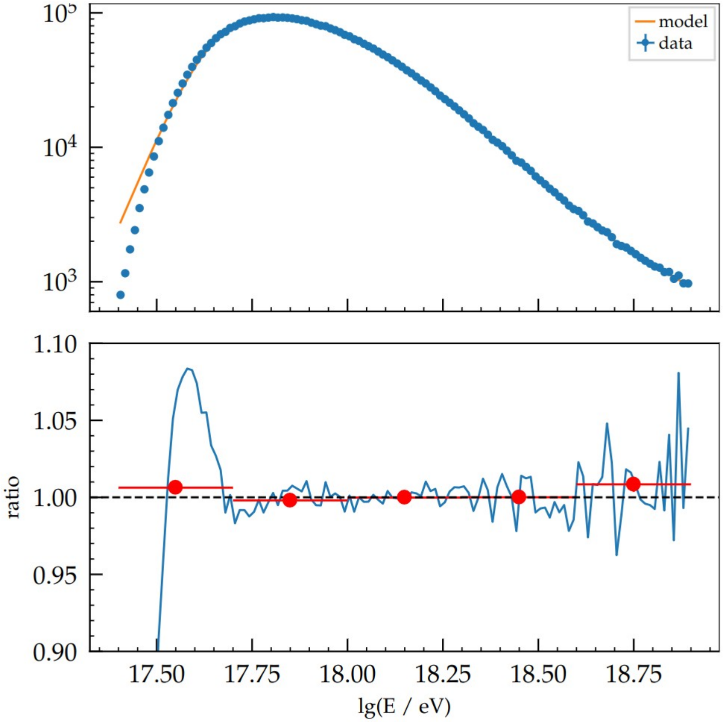


- necessary steps:
    - obtain a valid description of the detector response in the range of interest

    - re-formulate the analysis

Detector Response

- new response function derived from data: GAP-2024-022



GAP-2024-022

An updated and improved Model of the SD-Response Function

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April 2024

Forward-Folding

- now written up in GAP-note:
  - outline of the method
  - derivation of the formulas used
  - tests on simple MC-set up

→ we have OK for application to data (…)

GAP-2024-031

Accounting for the Detector Response in the East-West Method

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The math

- In theory: predicted count rate of events in given RA/energy/EW-bin: (under pure dipole assumption)

$$\nu^{(k)}_{\mathrm{E/W}}(\alpha_i) = \frac{\mathcal{E}}{2\pi} \int_{E'_k}^{E'_{k+1}} \mathrm{d}E' \int_0^{\infty} \mathrm{d}E \int \mathrm{d}\theta \sin\theta \cos\theta \varepsilon(E,\theta) r(E'|E,\theta) J(E) \int_{\mathrm{E/W}} \mathrm{d}\varphi (1 + \vec{d}\cdot\vec{n}_i)$$

- requires 5-dimensional integration, 4D if RA-is taken out

→ assume r and dn induce 'small' corrections:

$$r(1+\vec{d}\vec{n}) \approx (1+\Delta)(1+\vec{d}\vec{n}) = 1 + \Delta + \vec{d}\vec{n} + \mathcal{O}(\epsilon^2)$$

→ integral factorizes into transfer matrix like part and a dipole part

$$R^{(k)}_{\mathrm{E},ij} = R^{(k)}_{\mathrm{W},ij} = R_{ij} = \frac{\int_{\Delta\hat{E}_i} \mathrm{d}\hat{E} \int_{\Delta E_j} \mathrm{d}E \int_{\Delta\theta} \mathrm{d}\theta \, \cos\theta \, r(\hat{E}|E;\theta) \, J_0(E)}{\int_{\Delta E_j} \mathrm{d}E \int_{\Delta\theta} \mathrm{d}\theta \, \cos\theta \, J_0(E)}$$

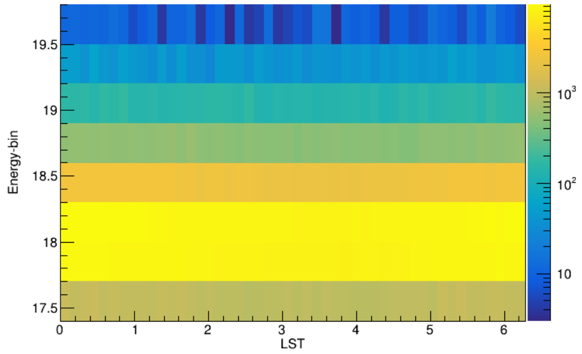
$$\nu^{(k)}_{\mathrm{E},i}(\mathbf{p}) - \nu^{(k)}_{\mathrm{W},i}(\mathbf{p}) = \sum_j R_{ij} \left(\mu^{(k)}_{\mathrm{E},i}(\mathbf{p}) - \mu^{(k)}_{\mathrm{W},i}(\mathbf{p}) \right)$$

How to fit?

- formulate a model d(E), phi(E) for dipole

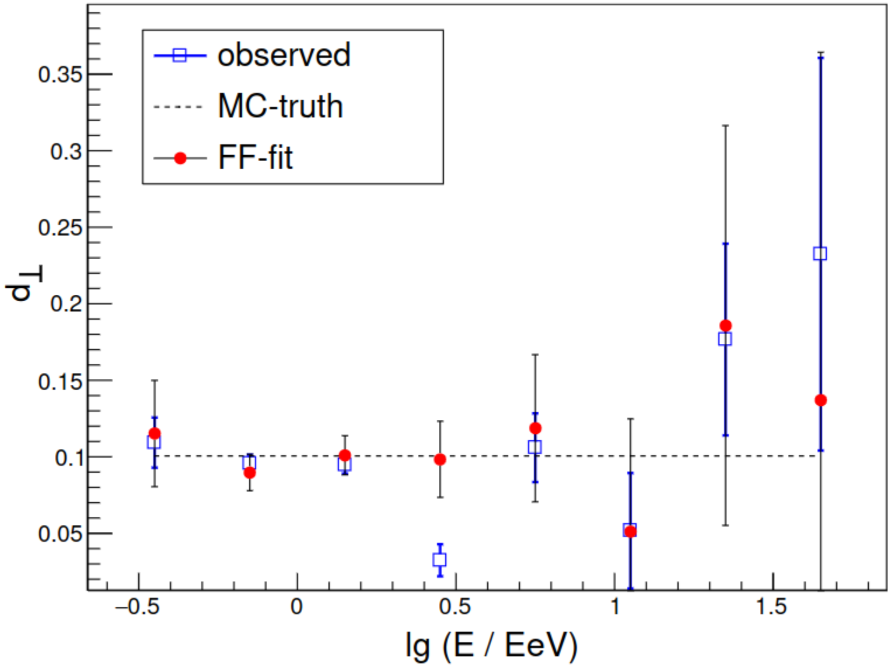
- forward fold to fit the 2D-data: n(E, RA)[East/West]

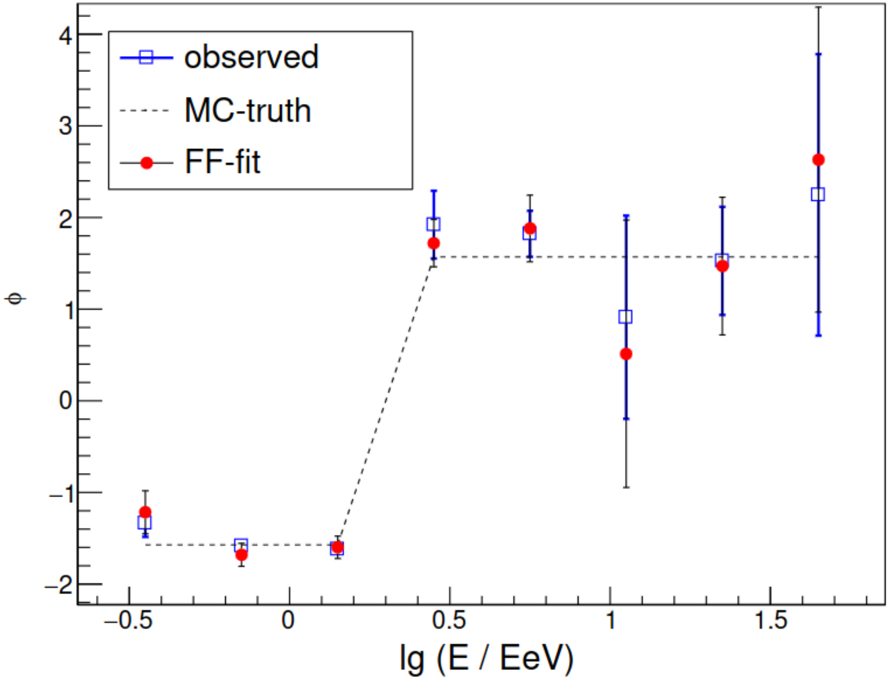
- likelihood: Skellam uses only differences → insensitive to spurious modulations



$$-2\log\mathcal{L} = \sum_{i,k}\left[\nu_{\mathrm{E}}^{(k)}(\alpha_i) + \nu_{\mathrm{W}}^{(k)}(\alpha_i) - \frac{n_{\mathrm{E}}^{(k)}(\alpha_i) - n_{\mathrm{W}}^{(k)}(\alpha_i)}{2}\log\left(\frac{\nu_{\mathrm{E}}^{(k)}(\alpha_i)}{\nu_{\mathrm{W}}^{(k)}(\alpha_i)}\right) - \log\left(\mathrm{I}_{\left|n_{\mathrm{E}}^{(k)}(i) - n_{\mathrm{W}}^{(k)}(i)\right|}\left(2\sqrt{\nu_{\mathrm{E}}^{(k)}(\alpha_i)\nu_{\mathrm{W}}^{(k)}(\alpha_i)}\right)\right)\right]$$

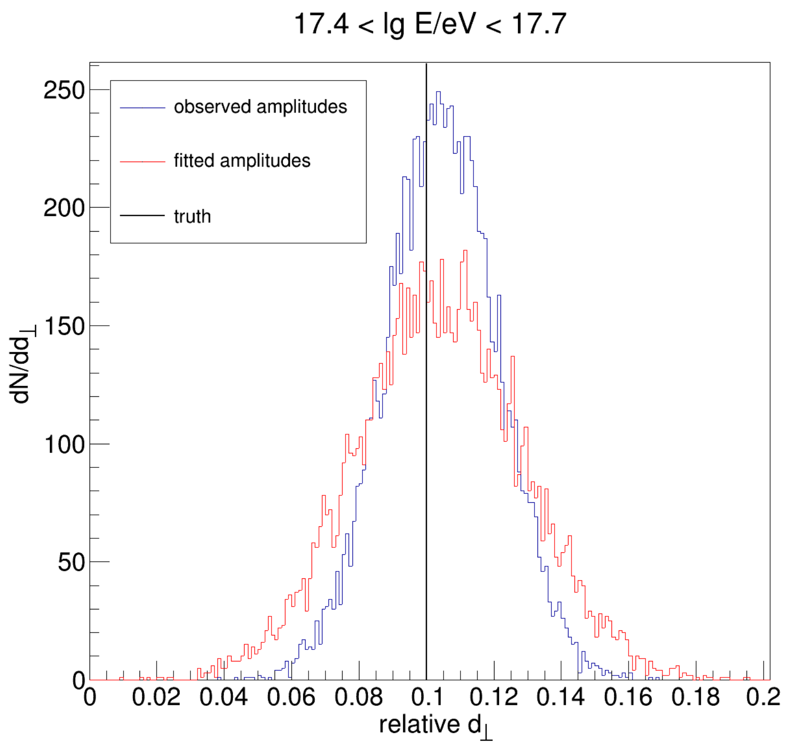
Example: Constant 10% Dipole

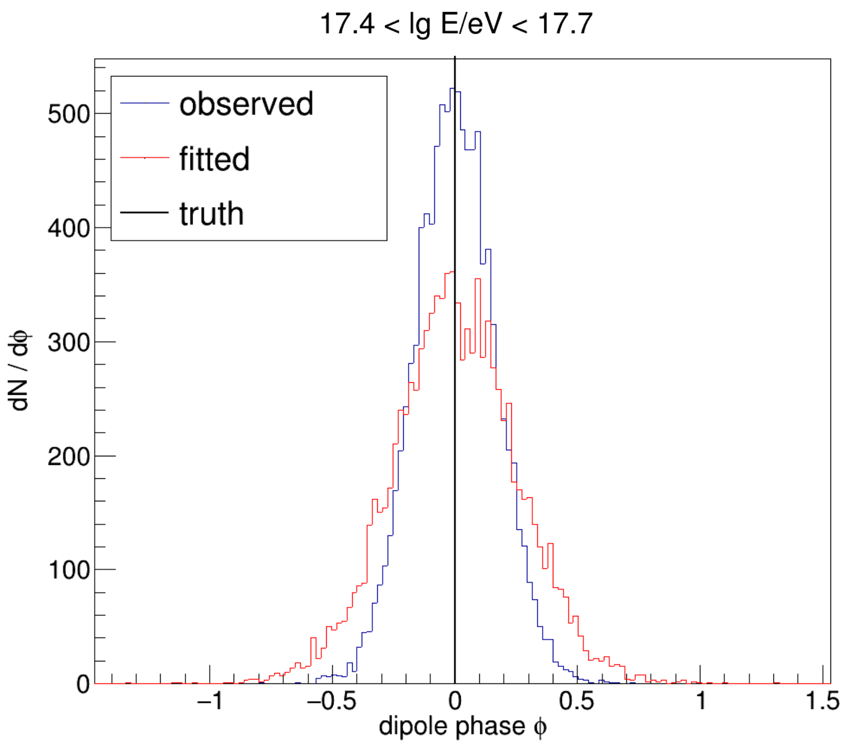




Bias and Resolution

- evaluate effects of resolution: compare distributions for 10 000 MC-samples



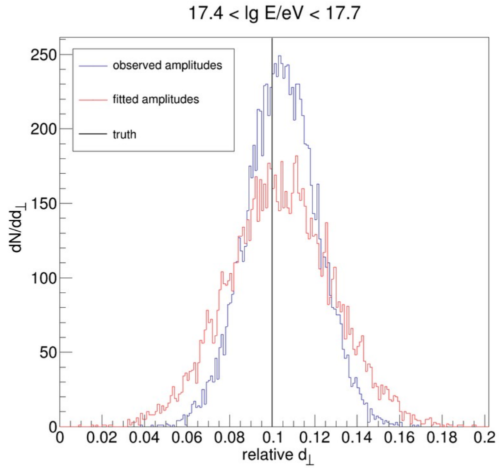


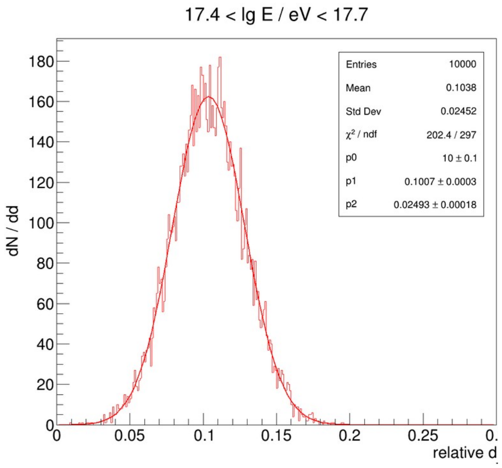
Bias and Resolution

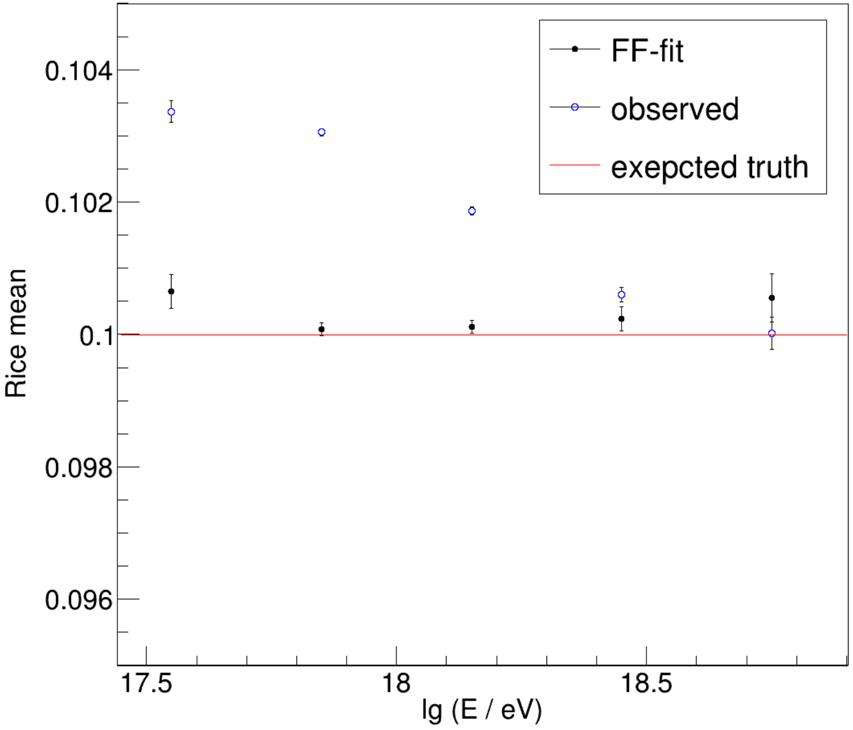
- fit Rice distribution to evaluate 'mean' bias

- small biases for this scenario

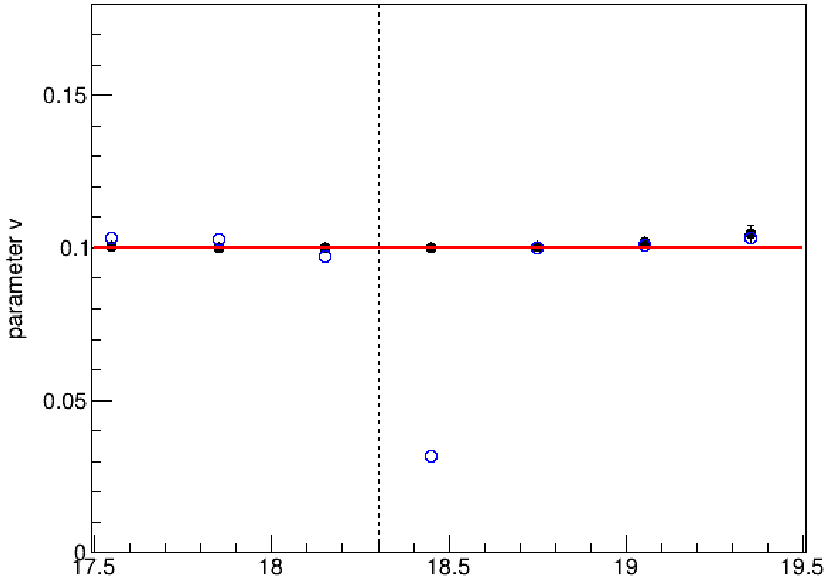
- worse resolution with FF-fit

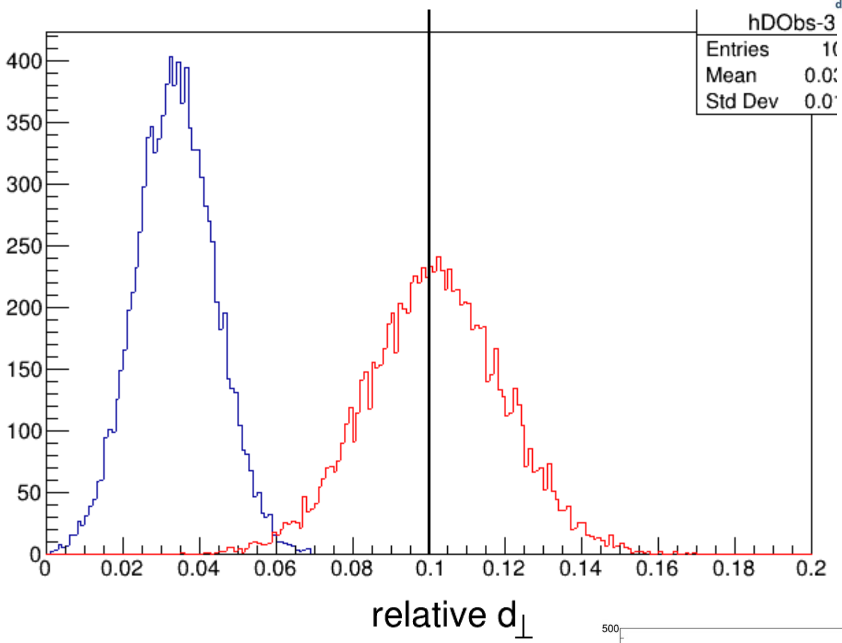


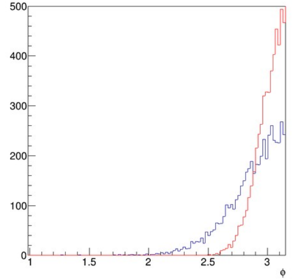




Scenario 2: sudden change of phase



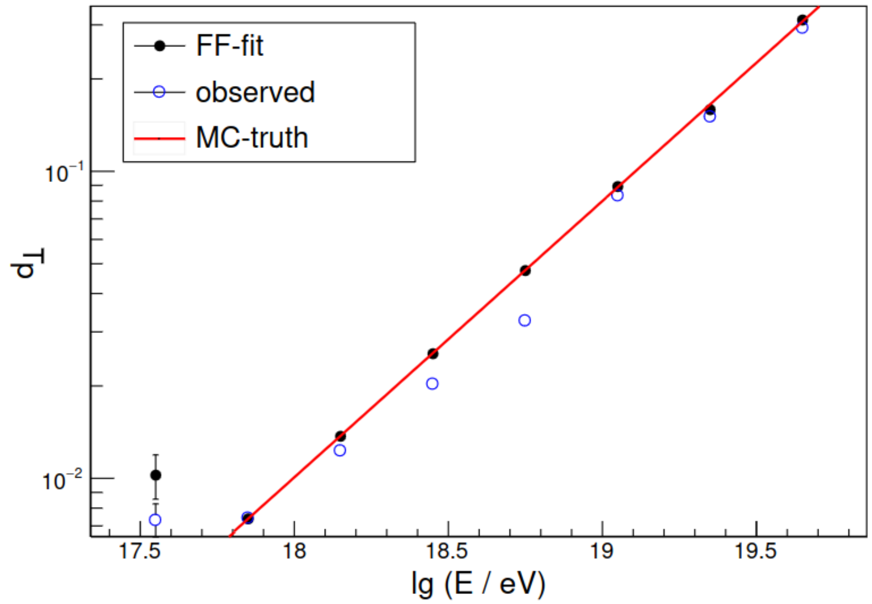


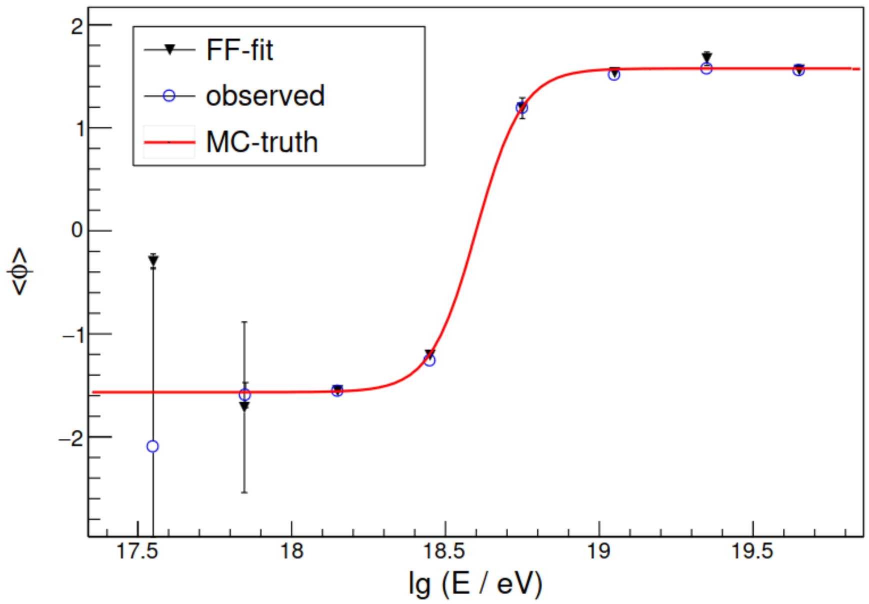


- insert a jump in phase at bin edge (2 EeV) from 0 to π

- starting values still from observed amplitudes

Scenario 3: ‘realistic’





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In Data?

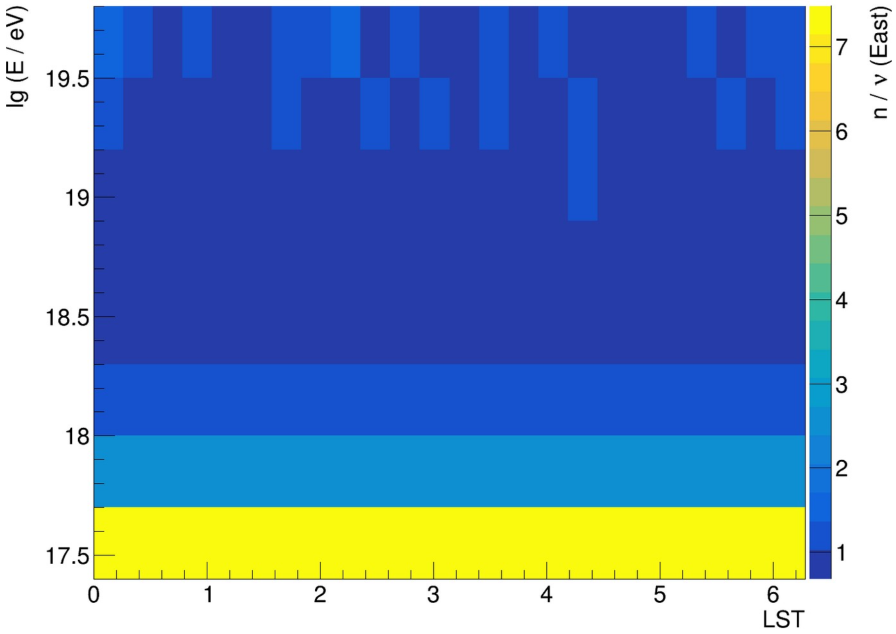
- application to data allowed by AD-task leadership

- first applied to scrambled data to be conservative

→ bug found: double counting of efficiency

$$\mu_0 = \frac{\mathcal{E}\Delta\alpha_0}{2} \int_{\Delta E_i} \mathrm{d}E\, J_0(E) \int \mathrm{d}\theta \sin\theta \cos\theta\, \varepsilon(E,\theta),$$

$$R^{(k)}_{\mathrm{E},ij} = R^{(k)}_{\mathrm{W},ij} = R_{ij} = \frac{\int_{\Delta \hat{E}_i} \mathrm{d}\hat{E} \int_{\Delta E_j} \mathrm{d}E \int_{\Delta\theta} \mathrm{d}\theta\ \cos\theta\ r(\hat{E}|E;\theta)\, J_0(E)}{\int_{\Delta E_j} \mathrm{d}E \int_{\Delta\theta} \mathrm{d}\theta\ \cos\theta\ J_0(E)}$$

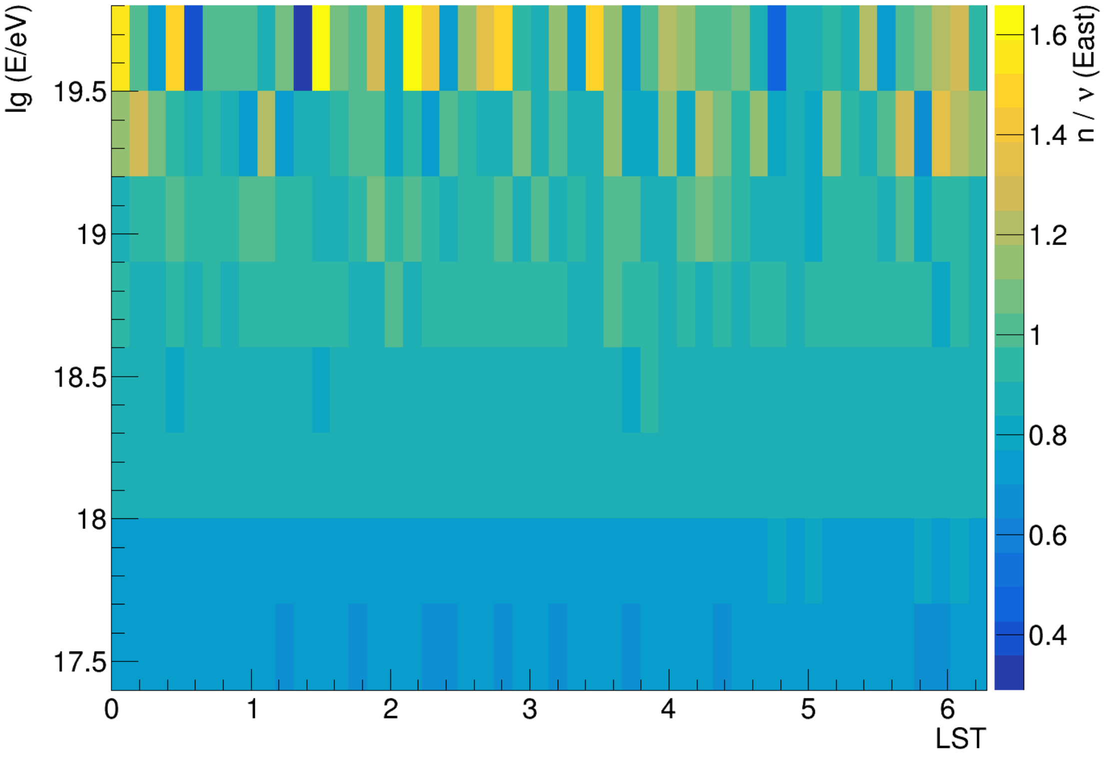


In Data: Bug-fix?

- remove efficiency from transfer matrix

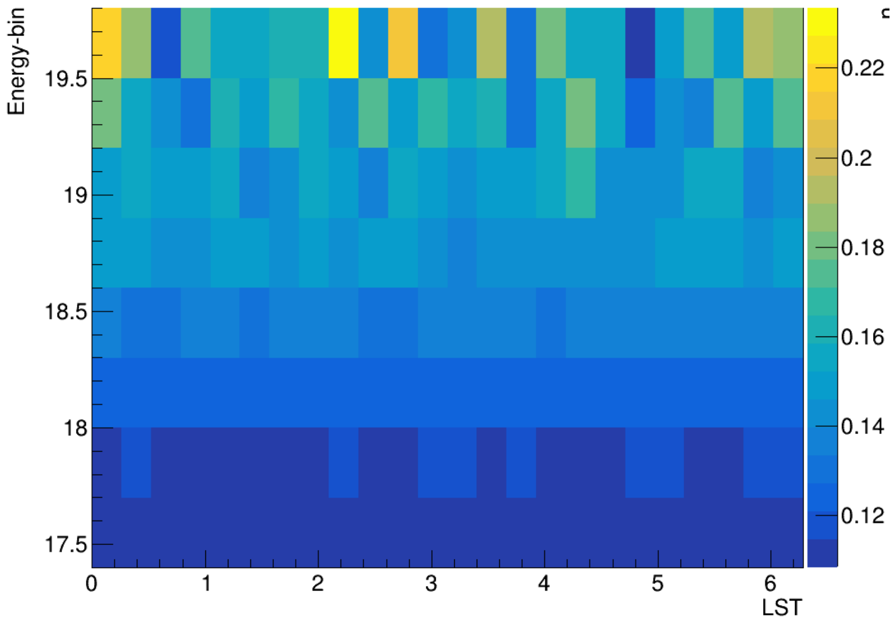
- alternatives: remove from μ, renormalise R with efficiency yield (almost) identical results

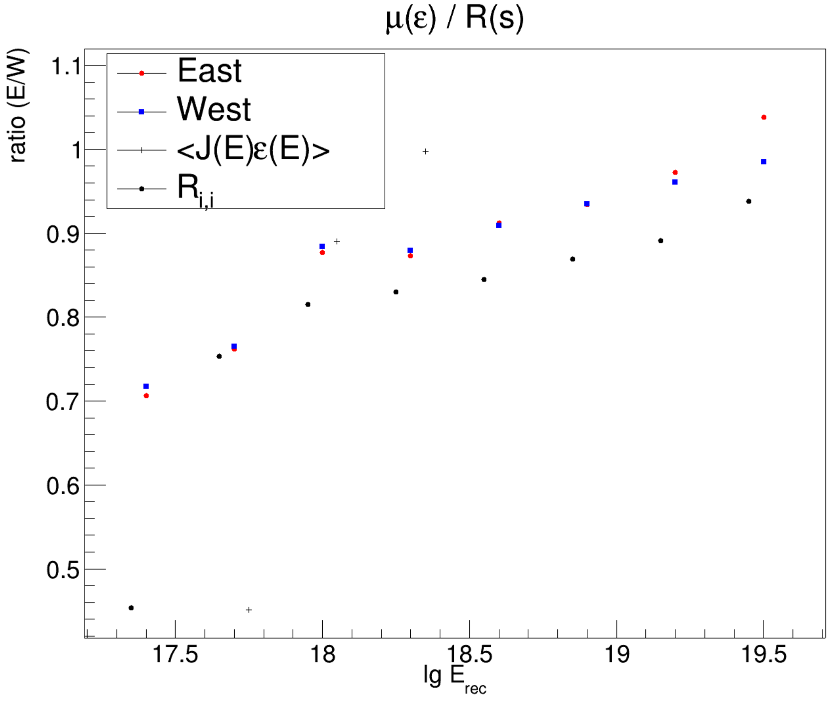
→ still some ~20% energy dependent problem



In Data: Bug-fix?

- re-implement full numerical integration to check approximation as origin (?)
- resembles R(i, I) elements…





Summary

- GAPs on EW-forward folding publish

- application to data pending due to bugs…

- unrelated: neutron paper finally submitted to PRD via arXiv:2406.11702

arXiv > hep-ph > arXiv:2406.11702

High Energy Physics - Phenomenology

[Submitted on 17 Jun 2024]

Neutron Production in Simulations of Extensive Air Showers

Martin Schimassek, Ralph Engel, Alfredo Ferrari, Markus Roth, David Schmidt, Darko Veberič

More Maths

- assuming $d \ll 1$, treat $\vec{d} \cdot \vec{n}$ as second order correction in (1)

→ we can describe $\nu^{(k)}_{\mathrm{E/W}}(\alpha_i)$ with a (constant) transfer matrix R, such that

$$\nu^{\mathrm{E,W}}_{ik}(\mathbf{s}) = \sum_j R_{ij}\,\mu^{\mathrm{E,W}}_{jk}(\mathbf{s})$$

and the expected counts $\mu^{(k)}_{\mathrm{E/W}}(\alpha_i)$ for an ideal detector (r → delta function)

- fit likelihood $\mathcal{L}$ for model given observed counts, as Skellam distribution*

$$-2\log\mathcal{L} = \sum_{i,k}\left[\nu^{(k)}_{\mathrm{E}}(\alpha_i) + \nu^{(k)}_{\mathrm{W}}(\alpha_i) - \frac{n^{(k)}_{\mathrm{E}}(\alpha_i) - n^{(k)}_{\mathrm{W}}(\alpha_i)}{2}\log\left(\frac{\nu^{(k)}_{\mathrm{E}}(\alpha_i)}{\nu^{(k)}_{\mathrm{W}}(\alpha_i)}\right) - \log\left(\mathrm{I}_{\left|n^{(k)}_{\mathrm{E}}(i) - n^{(k)}_{\mathrm{W}}(i)\right|}\left(2\sqrt{\nu^{(k)}_{\mathrm{E}}(\alpha_i)\nu^{(k)}_{\mathrm{W}}(\alpha_i)}\right)\right)\right]$$

*pdf of the difference of two independent Poisson variables

Code and Reproduction

- LSA-MC-study: we are preparing a note

- reproduction: in same repository (in auger sandbox)

Running the Codes

There are two executables related to the anisotropy part.

For test purposes, there is one that fits a powerlaw d(E) ~ d0 x E^b, with the executable

```
./create_and_fit_simple_model -o powerlaw_d010_b00_c400_n1000 -d .1 --beta 0.0 -c 400 -n 10000
```

where the parameters -- apart from `-o` for the output name -- steer the MC-setup. `-d` indicates the dipole strength at 1 EeV, `--beta` the powerlaw index for generation, `-c` the possibility of a phase shift at given energy in EeV, setting it to 400 effectively removes it from the MC. `-n` is the number of MC-samples to generate and fit.

The 'standard' analysis, using free parameters of amplitude and phase per bin, can be run with very similar parameters:

```
./create_and_fit_mc_data -o realistic_d0006_b09_c2 -d .006 --beta 0.9 -c 2 -n 10000
```

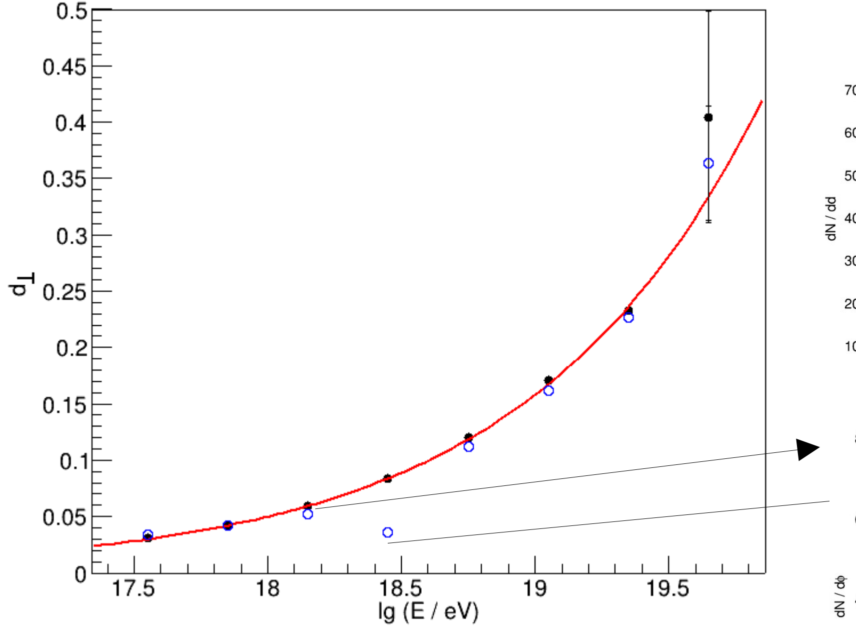
there are more command line options than in the simple model case. They are fairly self-explanatory if `-h` is used to get the help output.

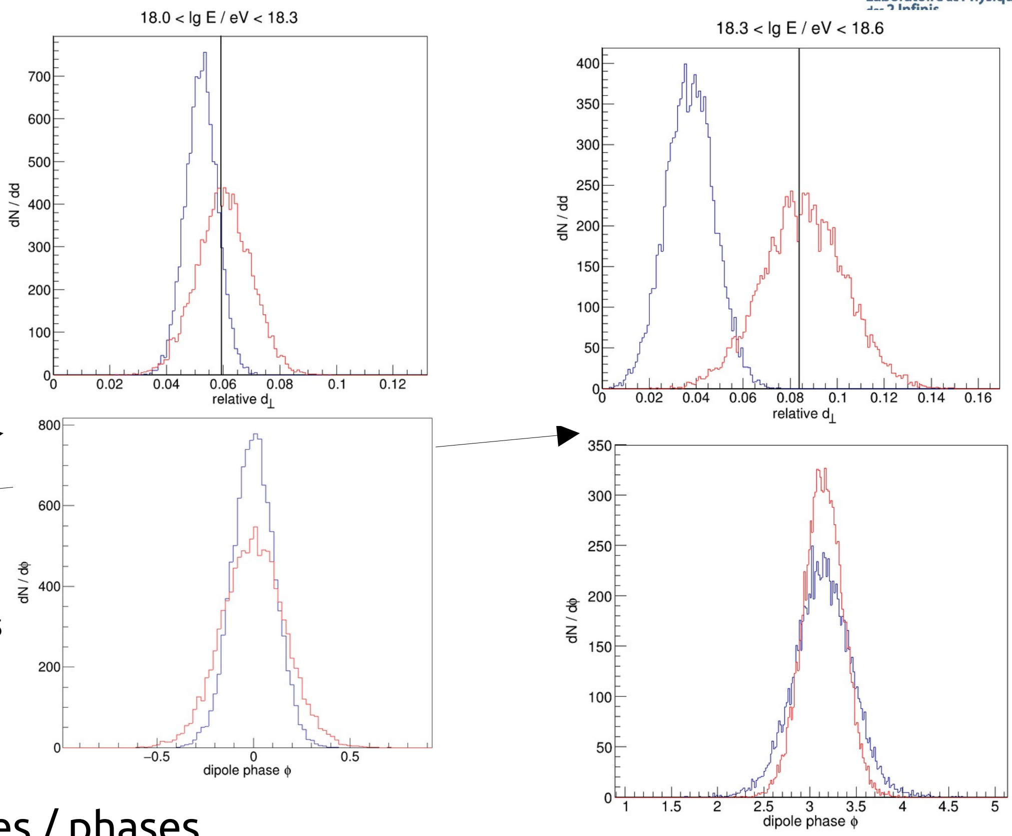
To fit the output distributions with a Rice distribution, you can use the utilities in the plotting directory. simply source root and run e.g.

```
root FitAmplitudeDistributions.C\(\"../realistic_d0006_b09_c2.root\"\)
```

It will produce a plot of the mean and width of the fitted Rice distribution as function of energy bins.

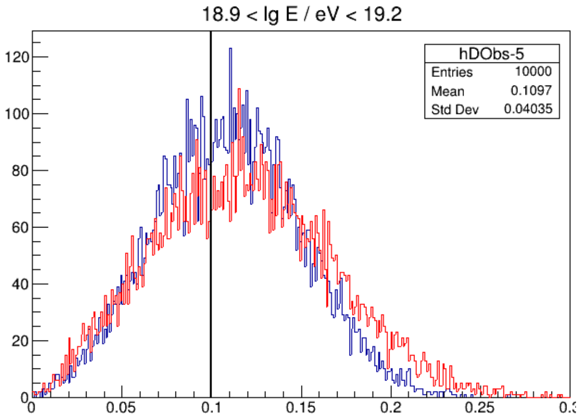
Scenario 3: d ~ $E^{0.5}$, sudden phase change



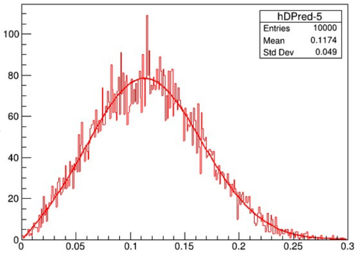


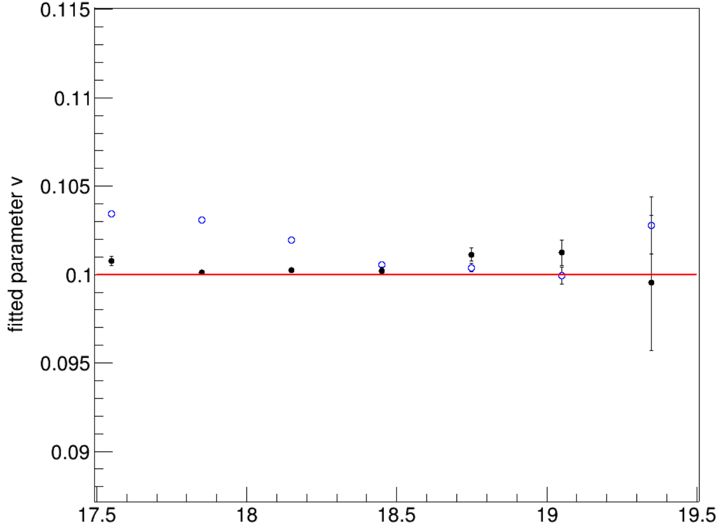
- change scenario to: d = 0.05 x (E / EeV)$^{0.5}$
- jump of phase from 0 to Pi at 2 EeV
- starting values from observed amplitudes / phases

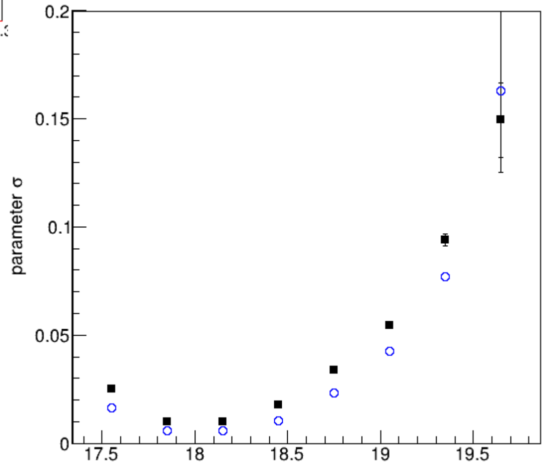
No Systematics: Constant 10% Dipole

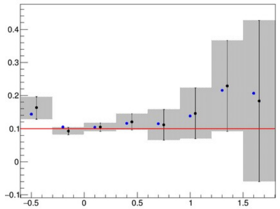


fit Rice

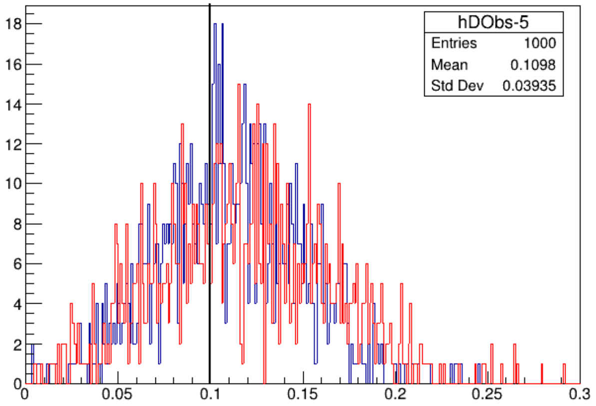




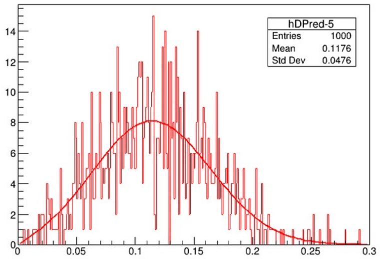


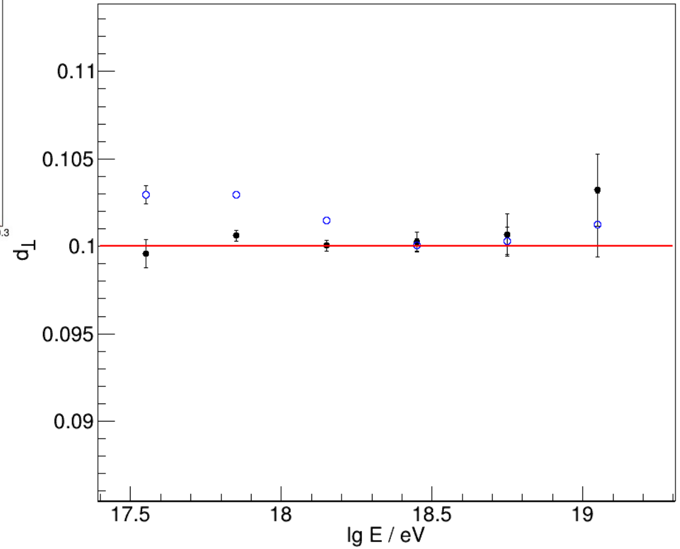


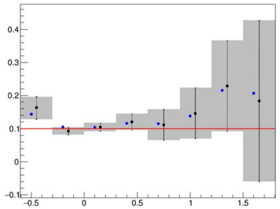
With Systematics: Constant 10% Dipole



fit Rice

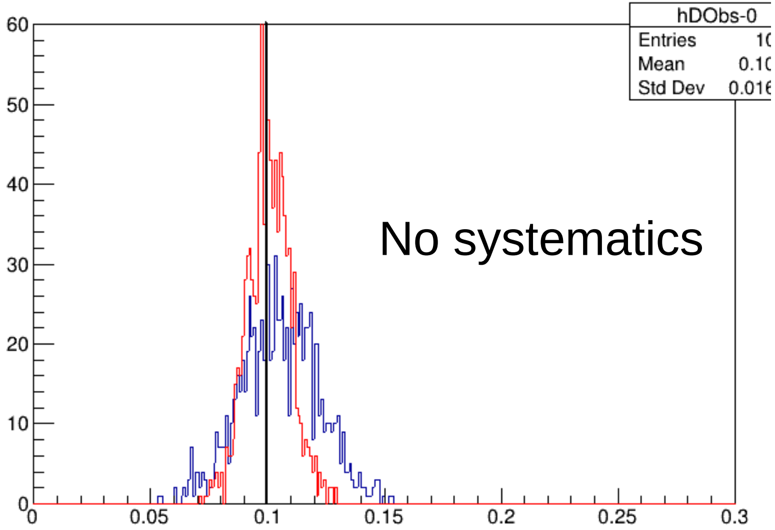


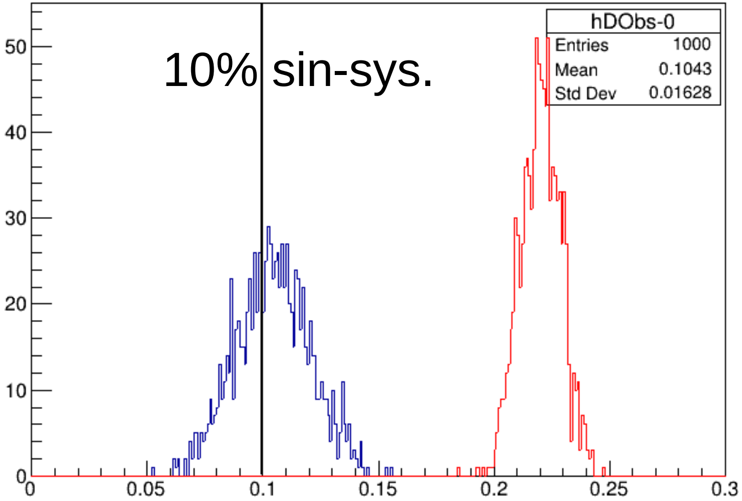




No bias visible
(might need more MC statistics though)

With Systematics: Poisson fit fails



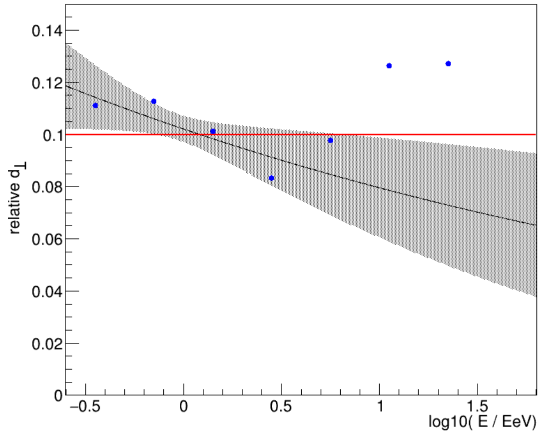


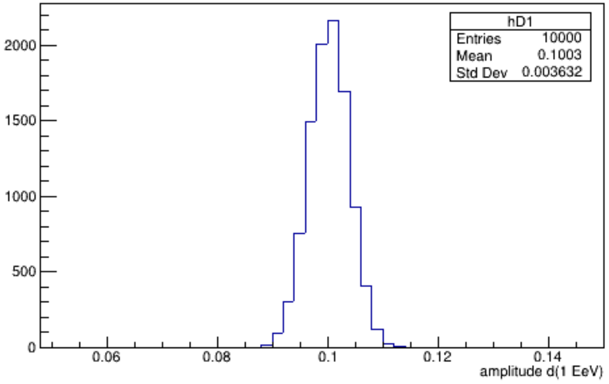
Constant 10% Dipole

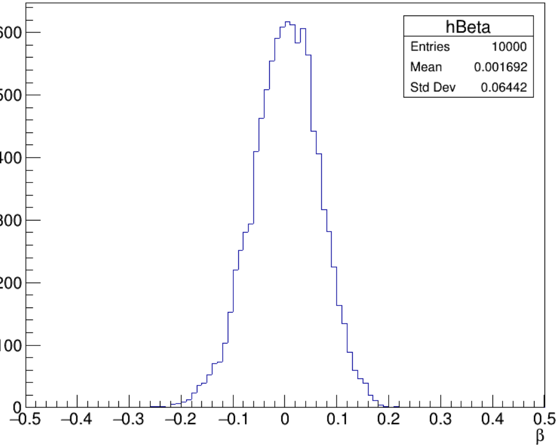
- $d_\perp(E) = d_0 \times (E/1\mathrm{EeV})^\beta$ with beta and d0 as free parameters

- starting values from observed amplitudes

- on average correct reconstruction of parameters, but inflexible



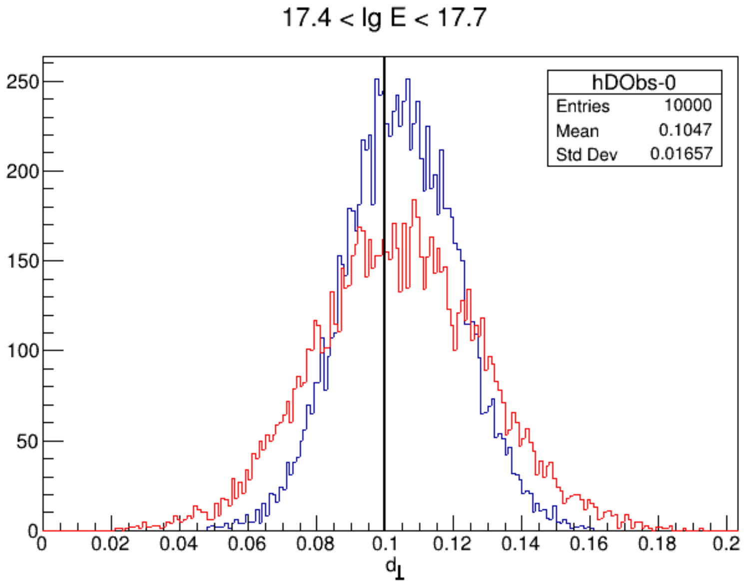


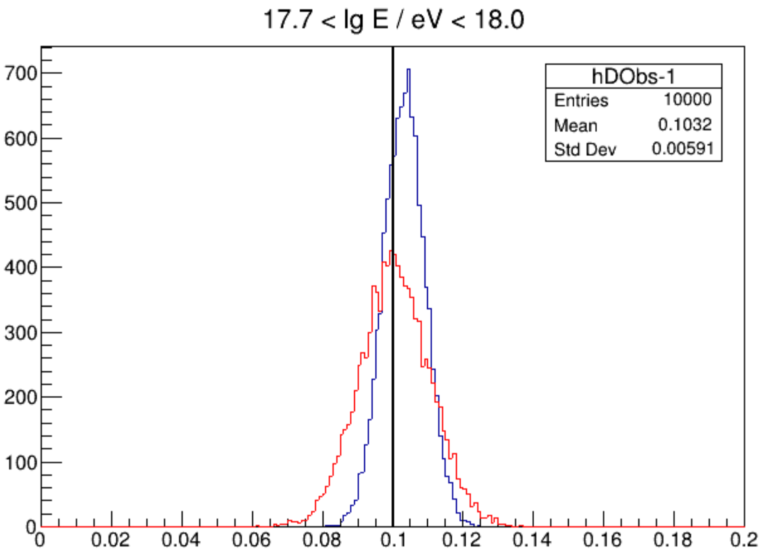


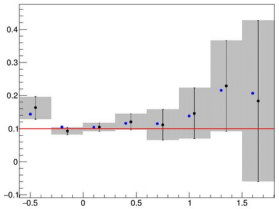
Constant 10% Dipole

- d(E), phi(E) pairs per bin → 16 parameters

- starting values from observed amplitudes







Tests: Implementation

- use 'perturbation' assumption: transfer matrix pre-calculated

- binning 0.3 in lg E

- 8 energy bins
  highest one 'unbound' [..., 400 EeV)

17.55	3.4%	1.4%	.05%	0	0	0	0	0
17.85	6.3%	31.4%	6.9%	0.03%	0	0	0	0
18.15	0.01%	12.2%	72.1%	10.7%	0	0	0	0
18.45	0	0	10.0%	82.8%	10.4%	0	0	0
18.75	0	0	0	6.2%	84.4%	9.1%	0	0
19.05	0	0	0	0	5.2%	86.9%	8.1%	0
19.35	0	0	0	0	0	4.0%	89.1%	6.2%
19.65	0	0	0	0	0	0	2.7%	93.8%

In Detail

- assuming $d \ll 1$, treat $\vec{d} \cdot \vec{n}$ as second order correction in (1)

→ we can describe $\nu^{(k)}_{\mathrm{E/W}}(\alpha_i)$ with a (constant) transfer matrix R $\nu^{\mathrm{E,W}}_{ik}(\mathbf{s}) = \sum_j R_{ij}\mu^{\mathrm{E,W}}_{jk}(\mathbf{s})$
and the expected counts $\mu^{(k)}_{\mathrm{E/W}}(\alpha_i)$ for an ideal detector (r → delta function)

$$R_{ij} = \frac{\int_{E'_i}^{E'_i+\Delta E'_i} \mathrm{d}E' \int_{E_j}^{E_j+\Delta E_j} \mathrm{d}E \int \mathrm{d}\theta \sin\theta \cos\theta \epsilon(E,\theta) J(E) r(E'|E;\theta)}{\int_{E_j}^{E_j+\Delta E_j} \mathrm{d}E \int \mathrm{d}\theta \sin\theta \cos\theta J(E)}$$

- fit likelihood $\mathcal{L}$ for model given observed counts, as Skellam distribution*

$$-2\log\mathcal{L} = \sum_{i,k}\left[\nu^{(k)}_{\mathrm{E}}(\alpha_i) + \nu^{(k)}_{\mathrm{W}}(\alpha_i) - \frac{n^{(k)}_{\mathrm{E}}(\alpha_i) - n^{(k)}_{\mathrm{W}}(\alpha_i)}{2}\log\left(\frac{\nu^{(k)}_{\mathrm{E}}(\alpha_i)}{\nu^{(k)}_{\mathrm{W}}(\alpha_i)}\right) - \log\left(\mathrm{I}_{\left|n^{(k)}_{\mathrm{E}}(i) - n^{(k)}_{\mathrm{W}}(i)\right|}\left(2\sqrt{\nu^{(k)}_{\mathrm{E}}(\alpha_i)\nu^{(k)}_{\mathrm{W}}(\alpha_i)}\right)\right)\right]$$