

A visualization of a gravitational well, showing two black holes in the center with concentric blue and purple rings representing the curvature of spacetime. The background is a dark blue grid with small white stars.

On-shell approach to spinning binaries in modified gravity

Panagiotis Marinellis

In collaboration with Adam Falkowski:
[2407.16457], [2411.12909]

Plan for this talk:

1. Motivation

2. Scattering Amplitudes and Observables

3. Scalar-tensor theories: Examples, compact objects, scalar hair and scattering waveforms

4. Outlook

1. Motivation



Image Credit: EGO*

→ LIGO/VIRGO collaboration: First detection of **Gravitational Waves** (GWs) in **2015**

1. Motivation

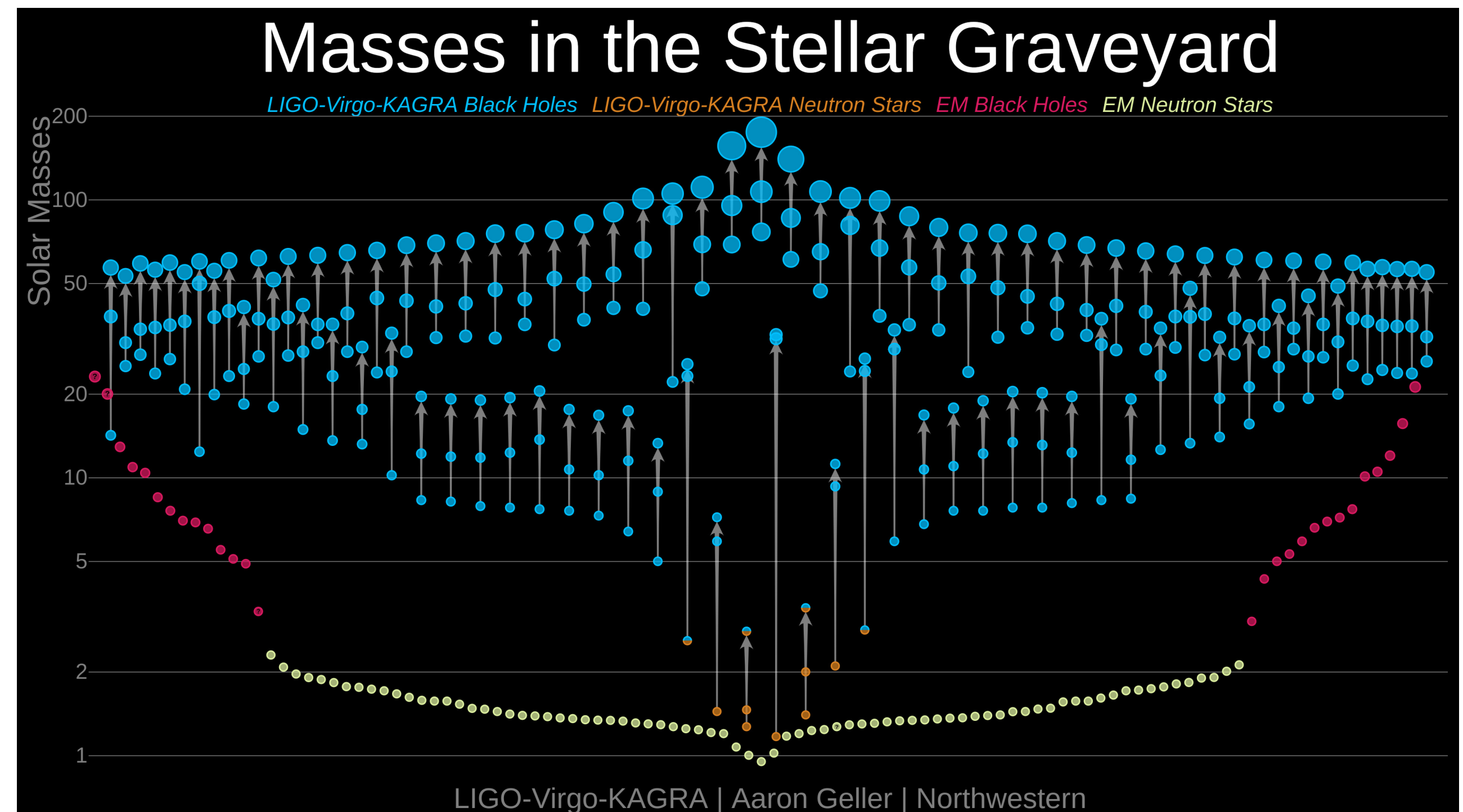


Image Credit: EGO*

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Has since then inspired an unprecedented interest in GW detection, especially with the upcoming **new generation of GW interferometers** (ET, Cosmic Explorer, LISA)



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Image Credit: EGO*

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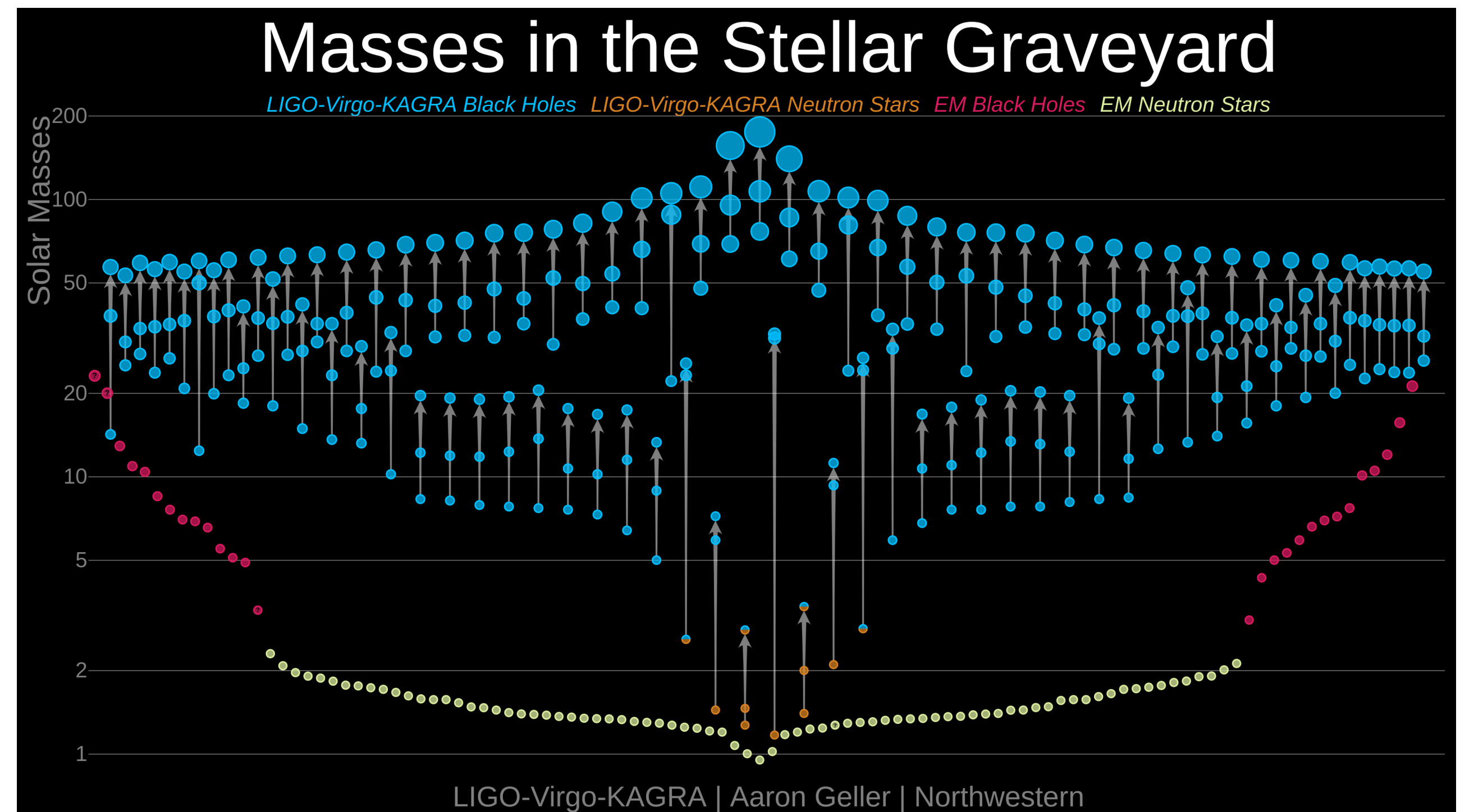
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New era of **high precision** measurements of GWs:

Highly accurate GW templates

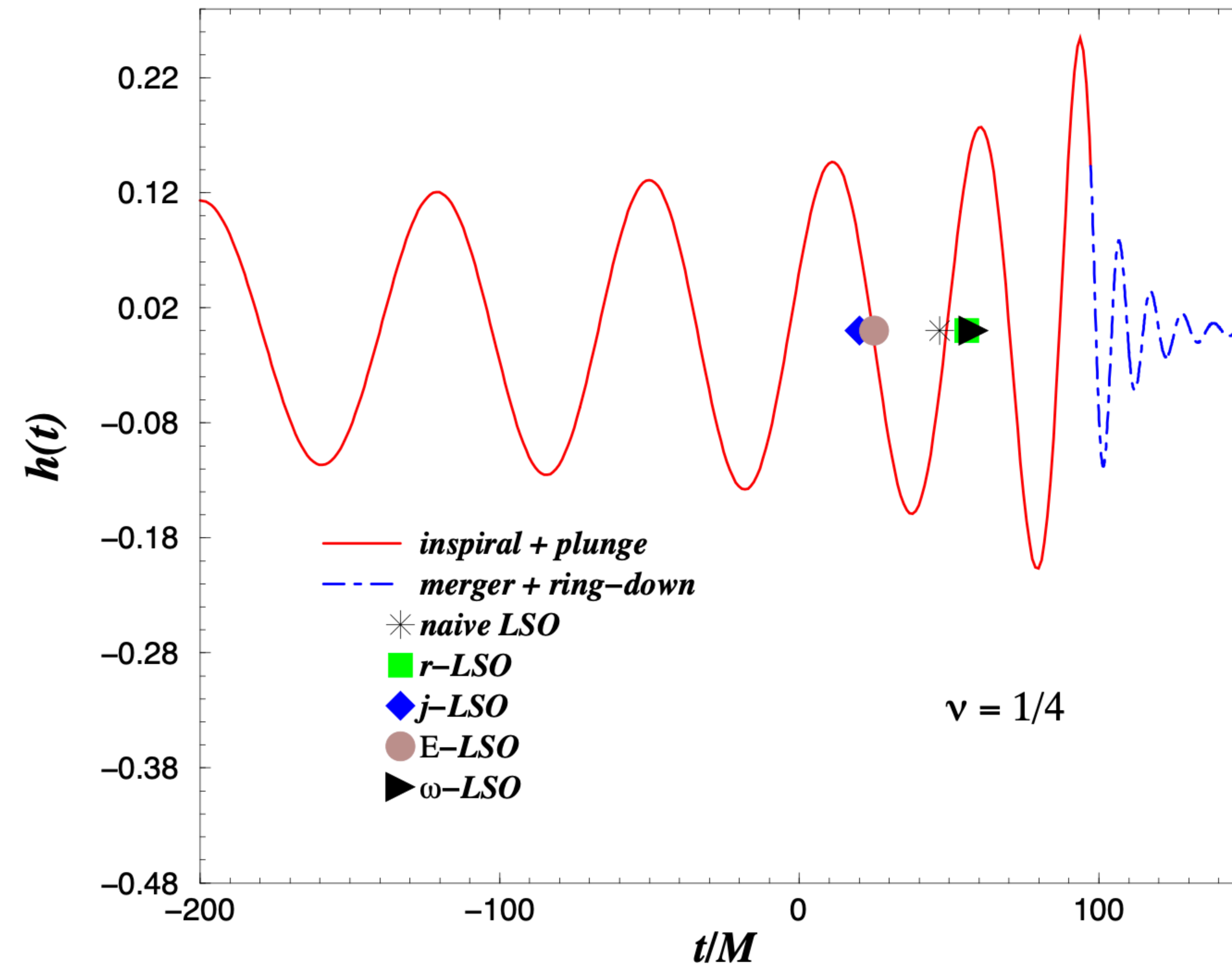


New window to test General Relativity (GR)



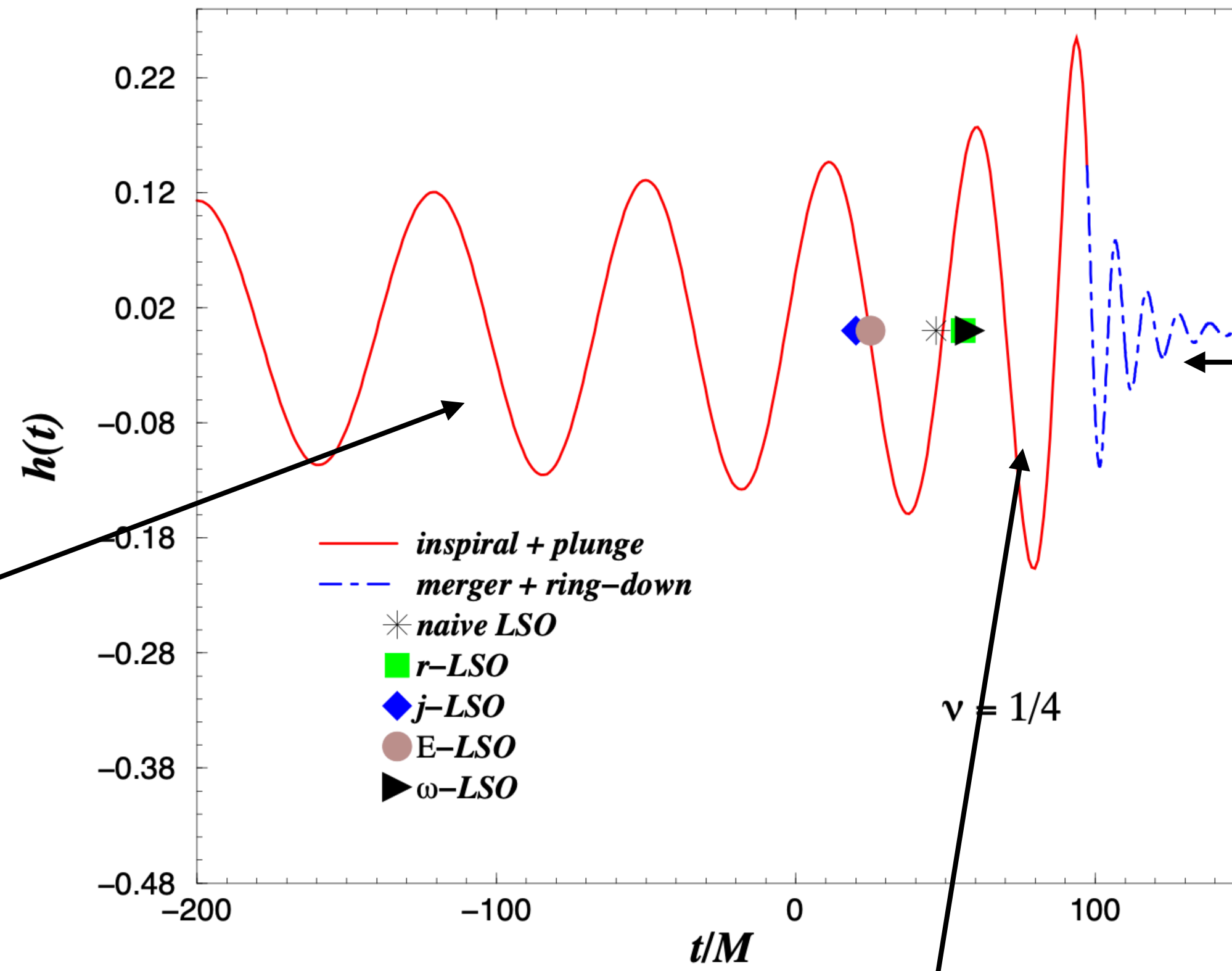
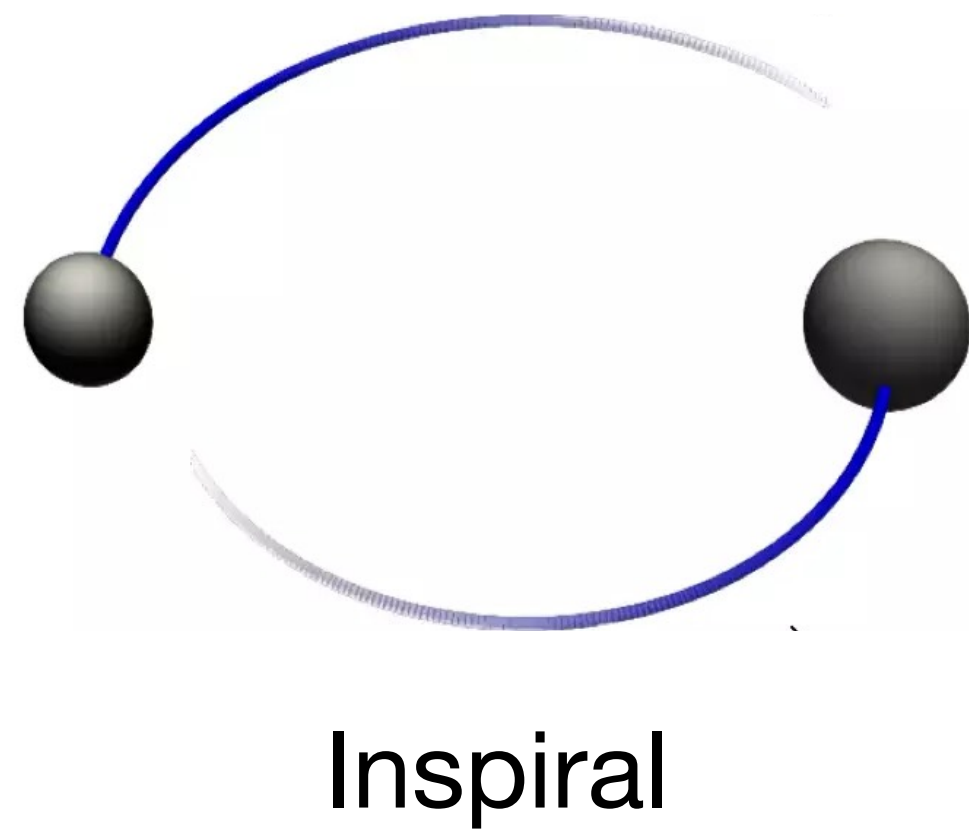
The phases of the binary problem:

Phys.Rev.D 62 (2000) 064015 [Buonanno, Damour]*



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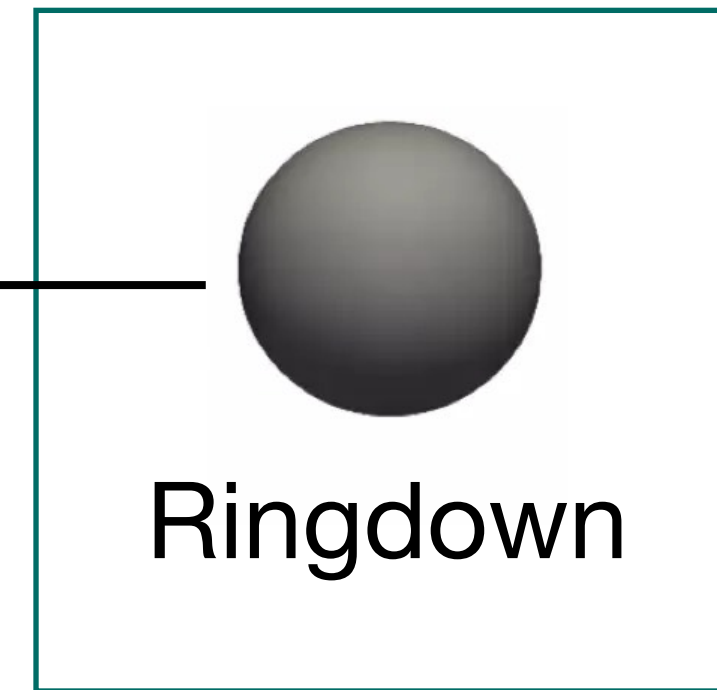
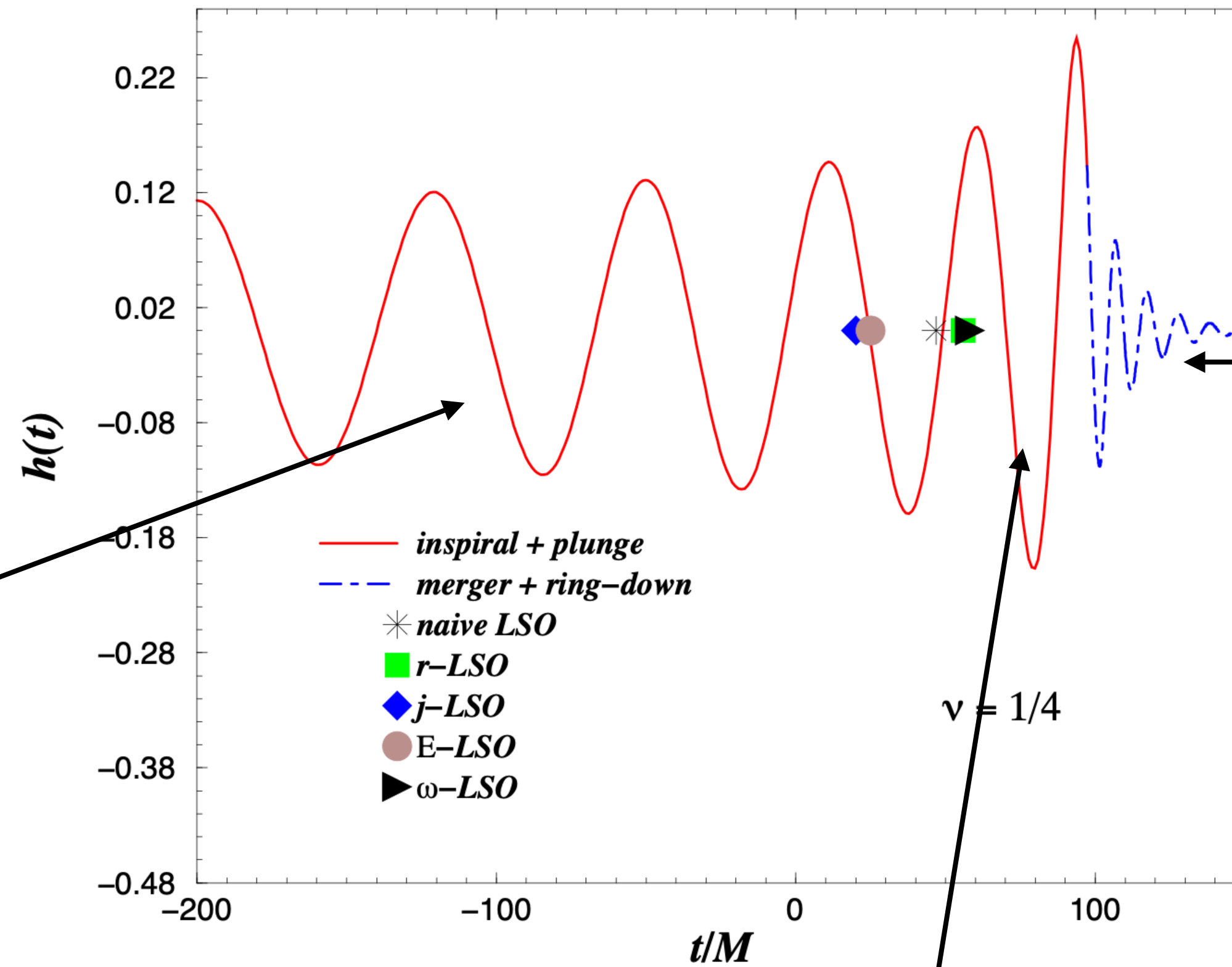
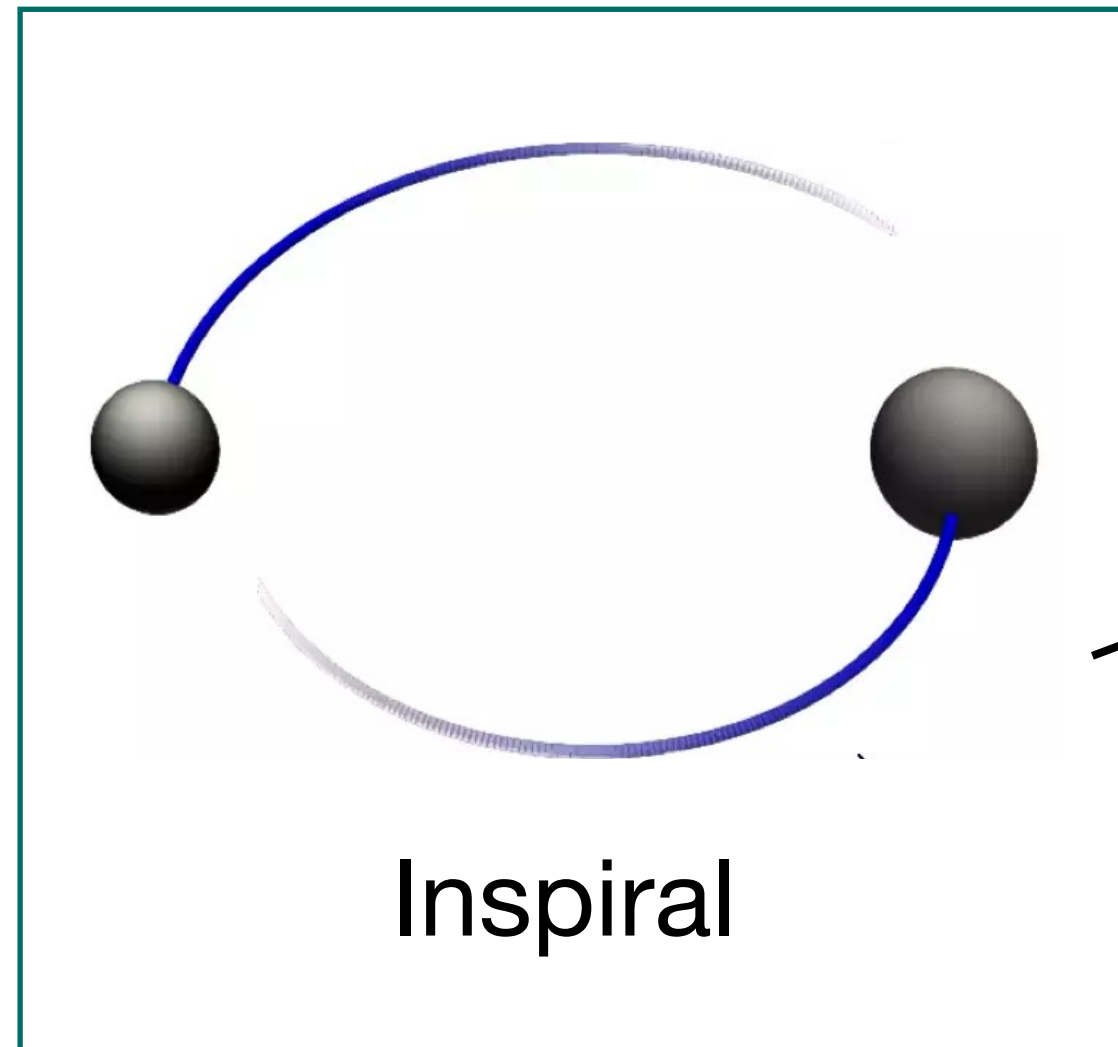
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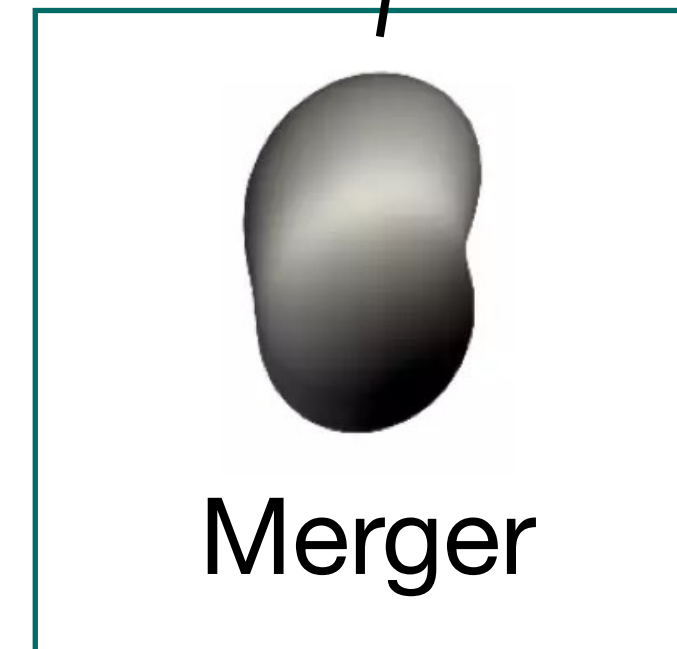
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Analytical approaches



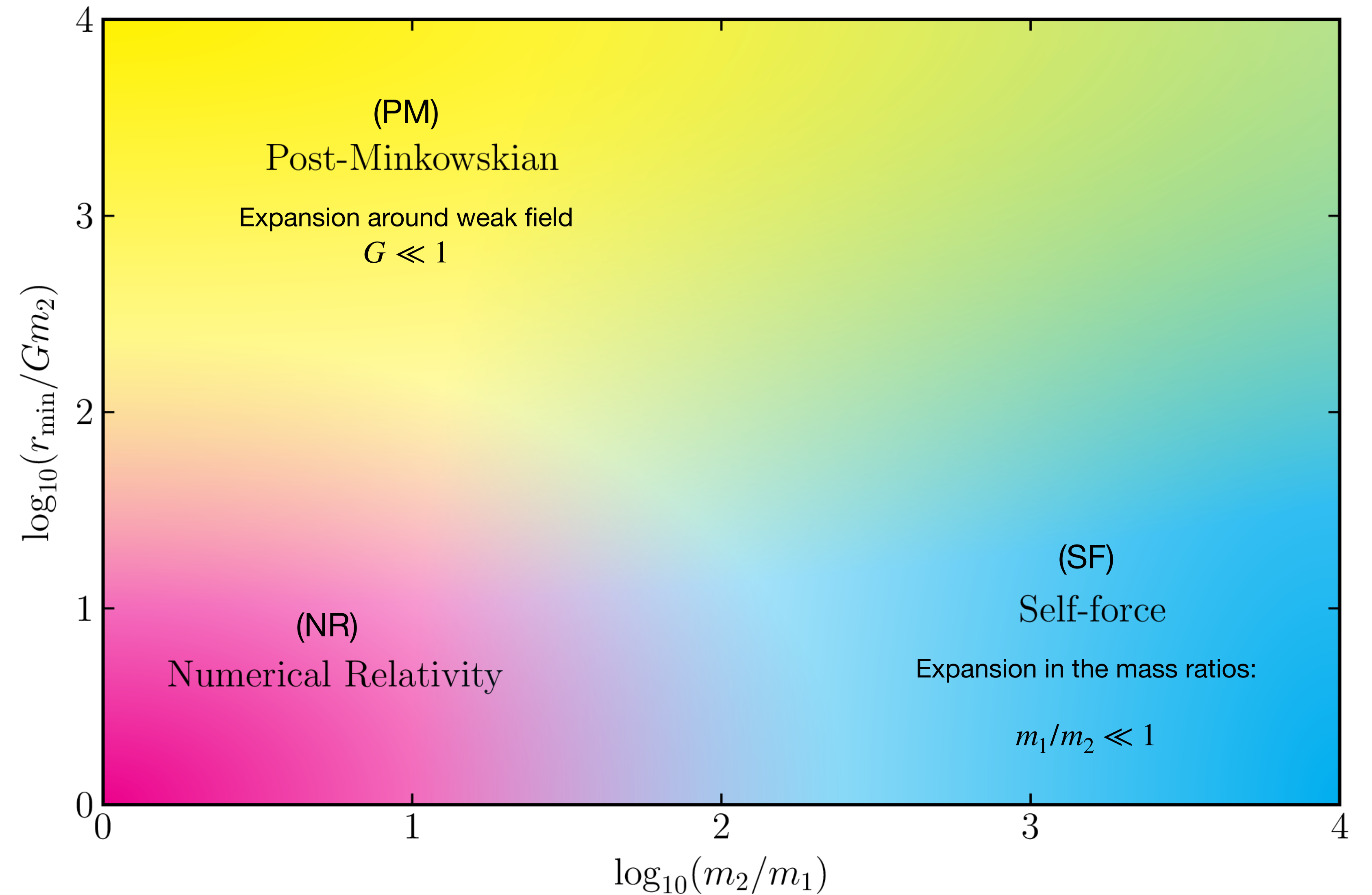
**Black Hole
Perturbation
Theory (BHPT)**



Numerical Relativity

Analytical approaches:

Phys.Rev.D 108 (2023) 2, 024025 [Barack, Bern, Herrmann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng]



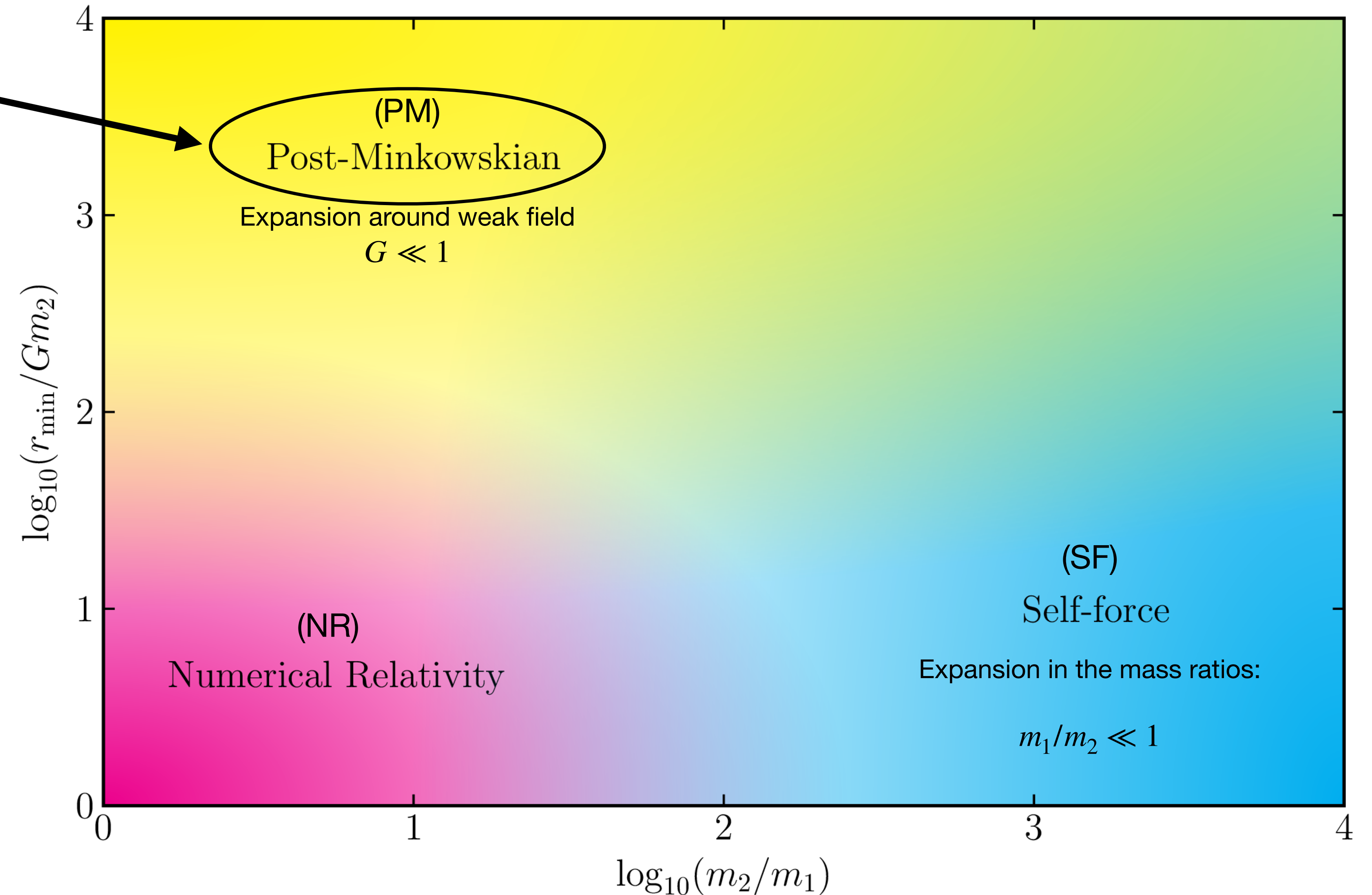
Analytical approaches:

- Enormous program for accurately determining the binary dynamics and computing waveforms **at all orders in** $\frac{v}{c}$, in contrast with Post-Newtonian (PN) approaches.

- Inspiration from **techniques** used in **Scattering Amplitudes and Effective Field Theory**

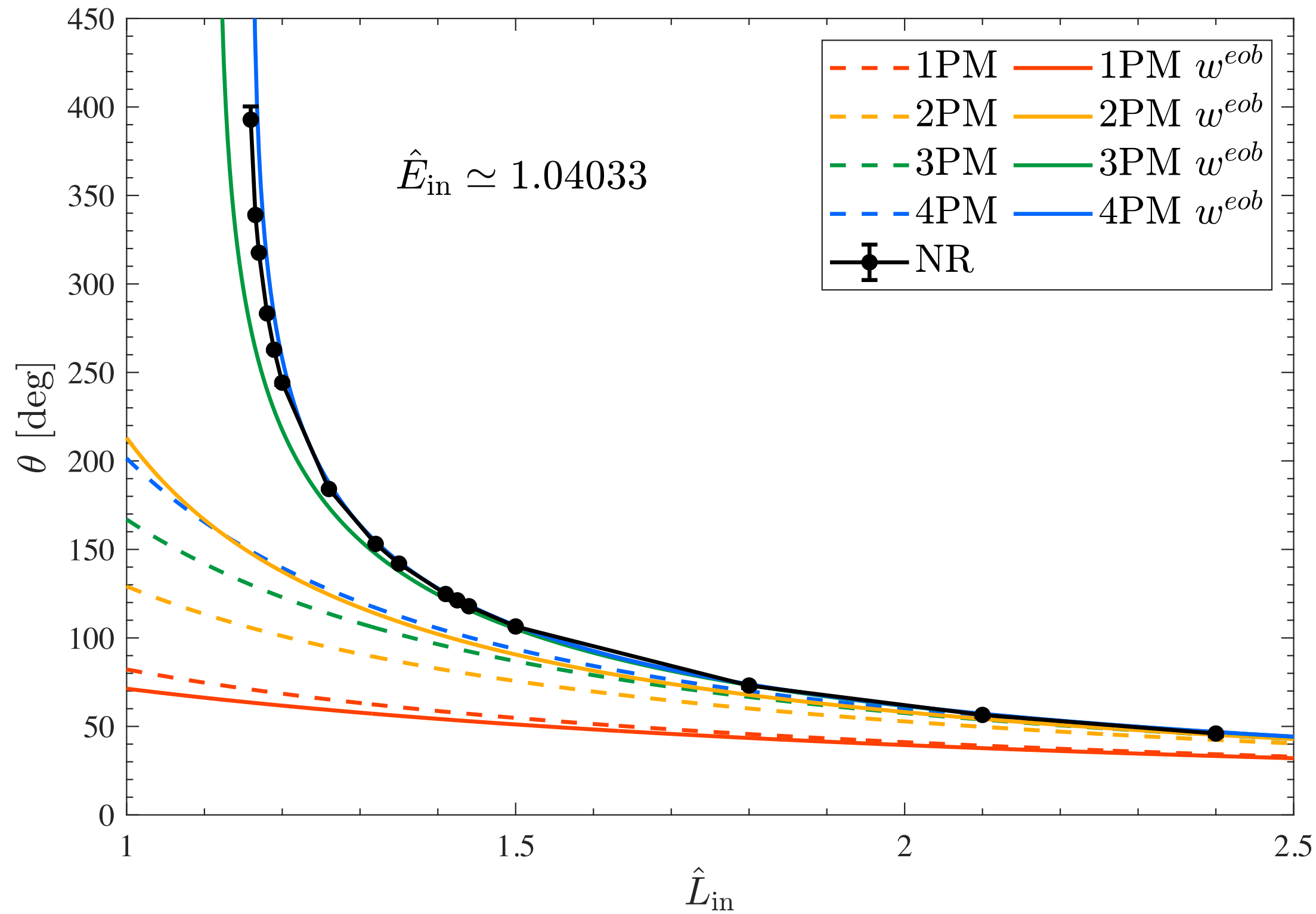
[Adamo, Alaverdian, Aoude, Bautista, Ben-Shahar, Bern, Bini, Brandhuber, Brown, Buonanno, Cachazo, Cangemi, Chiodaroli, Chen, Cordero, Cristofoli, de la Cruz, Damour, Damgaard, De Angelis, Driesse, Elkhidir, Gatica, Georgoudis, Goldberger, Gowdy, Gonzo, Guevara, Haddad, Heissenberg, Helset, Herrmann, Holstein, Huang, Huang, Ilderton, Jakobsen, Johansson, Kim, Kraus, Kosmopoulos, Kosower, Lee, Levi, Lin, Liu, Luna, Matasan, Maybee, Menezes, Mogull, Mougikakos, Moynihan, Novichkov, O'Connell, Ochirov, Parra-Martinez, Pichini, Plefka, Porto, Riva, Roiban, Ross, Rothstein, Ruf, Russo, Saketh, Sauer, Scheopner, Sergola, Shen, Siemonsen, Smirnov, Smirnov, Steinhoff, Teng, Travaglini, Vanhove, Vazquez-Holm, di Vecchia, Veneziano, Vernizzi, Vines, Wong, Xu, Yang, Yin, Zeng, et al...]

Phys.Rev.D 108 (2023) 2, 024025 [Barack, Bern, Herrmann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng]



PM expansion to the test and bound orbits:

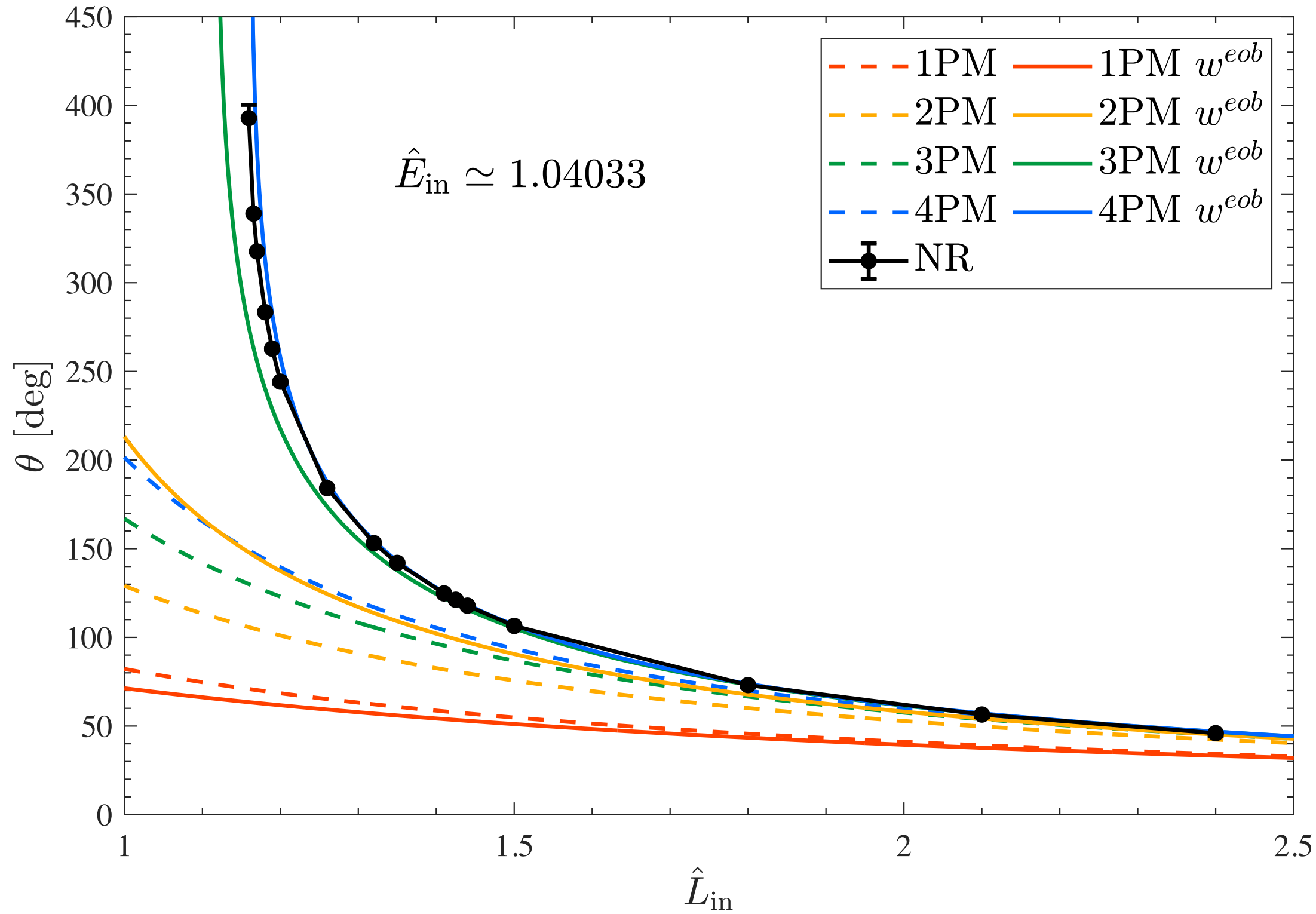
Phys.Rev.D 108 (2023) 12, 124016 [Rettegno, Pratten, Thomas, Schmidt, Damour]



**Already PM is doing very well for
black hole (BH) scattering!**

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Good agreement for the bound case as well!

Bound to Boundary map for binary dynamics:

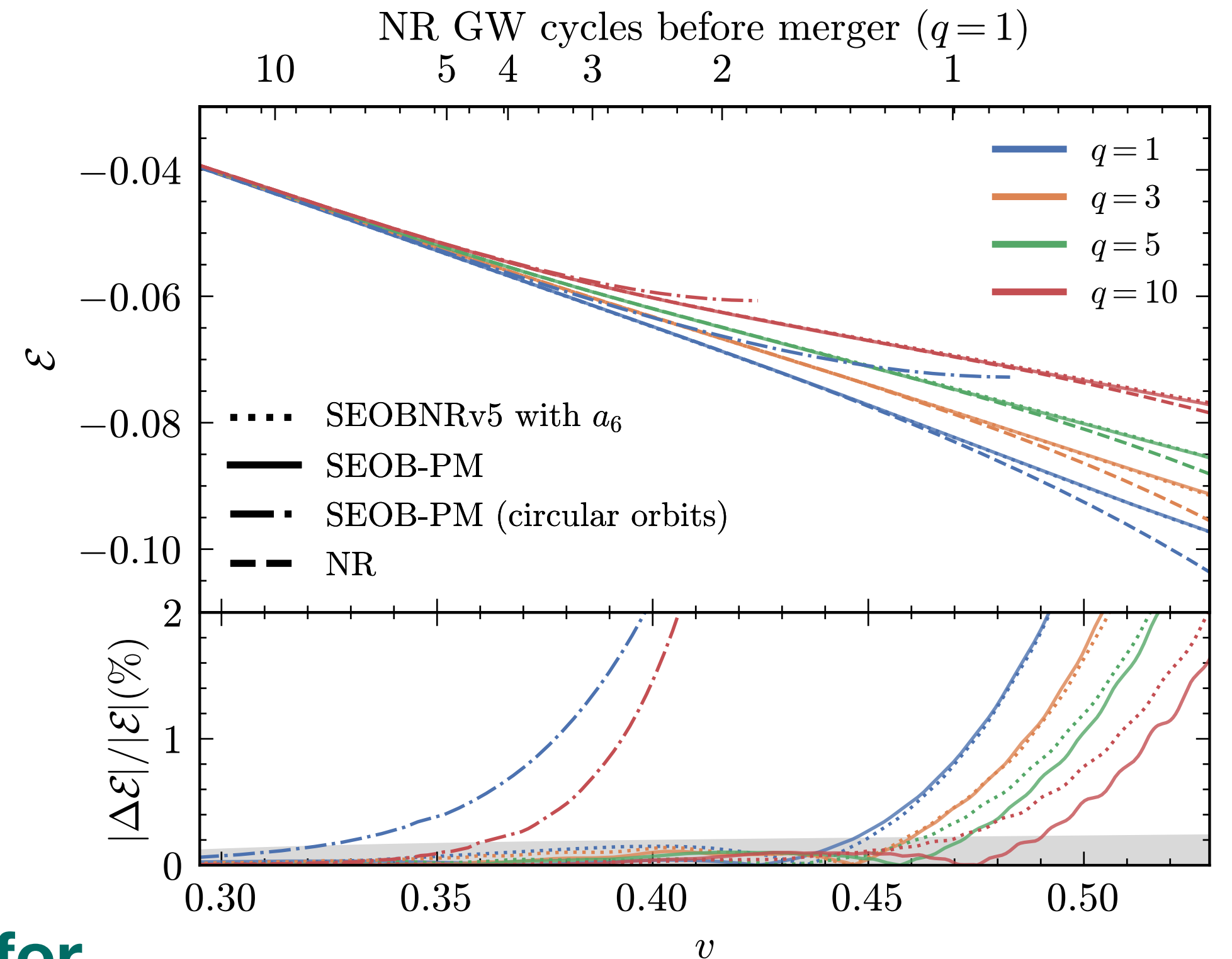
JHEP 01 (2020) 072 [Kälin, Porto]

JHEP 02 (2020) 120 [Kälin, Porto]

JHEP 04 (2022) 154, JHEP 07 (2022) 002 (erratum) [Cho, Kälin, Porto]

Recent work on a waveform map:

JHEP 05 (2024) 034 [Adamo, Gonzo, Ilderton]



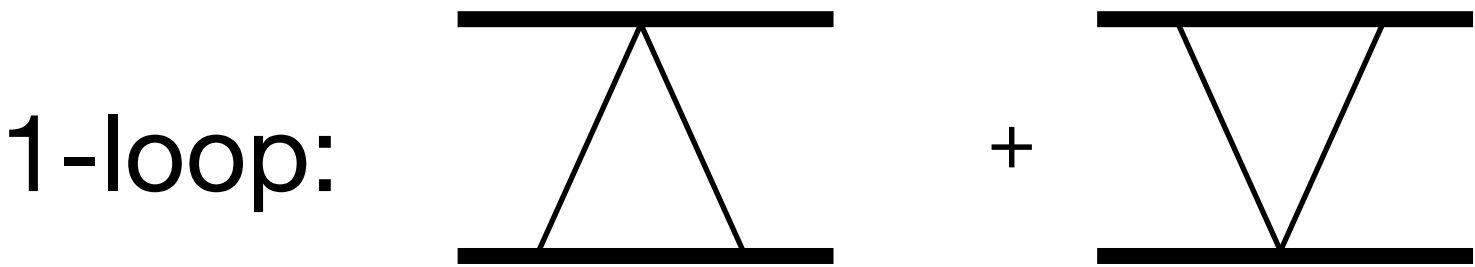
arxiv: 2405.19181 [Buonanno, Mogull, Patil, Pompili]

Results obtained in standard GR:

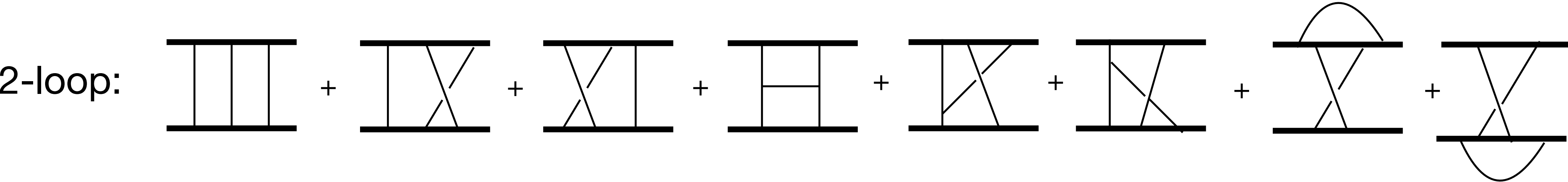
JHEP 10 (2021) 148 [Herrmann, Parra-Martinez, Ruf, Zeng]



$$\Delta p_{1,\text{GR}}^{\mu,(0)} = \frac{GM^2\nu}{|b|} \frac{2(2\sigma^2 - 1)}{\sqrt{\sigma^2 - 1}} \frac{b^\mu}{|b|}.$$



$$\Delta p_{1,\perp}^{\mu,(1)} = \frac{G^2 M^3 \nu}{|b|^2} \frac{3\pi}{4} \frac{(5\sigma^2 - 1)}{\sqrt{\sigma^2 - 1}} \frac{b^\mu}{|b|}$$



$$\Delta p_{1,\perp,\text{cons}}^{\mu,(2)} = \frac{G^3 M^4 \nu}{|b|^3} \frac{2}{\sqrt{\sigma^2 - 1}} \frac{b^\mu}{|b|} \left[h^2(\sigma, \nu) \left(16\sigma^2 - \frac{1}{(\sigma^2 - 1)^2} \right) - \frac{4}{3} \nu \sigma (14\sigma^2 + 25) - 8\nu (4\sigma^4 - 12\sigma^2 - 3) \frac{\text{arcsinh} \sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}} \right]$$

$$\Delta p_{1,\text{rad}}^{\mu,(2)} = \frac{G^3 M^4 \nu^2}{|b|^3} \left\{ \frac{4}{\sqrt{\sigma^2 - 1}} \frac{b^\mu}{|b|} \left[f_1^{\text{LS}}(\sigma) + f_3^{\text{LS}}(\sigma) \frac{\sigma \text{arcsinh} \sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}} \right] + \pi \ddot{u}_2^\mu \left[f_1(\sigma) + f_2(\sigma) \log \left(\frac{\sigma + 1}{2} \right) + f_3(\sigma) \frac{\sigma \text{arcsinh} \sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}} \right] \right\}$$

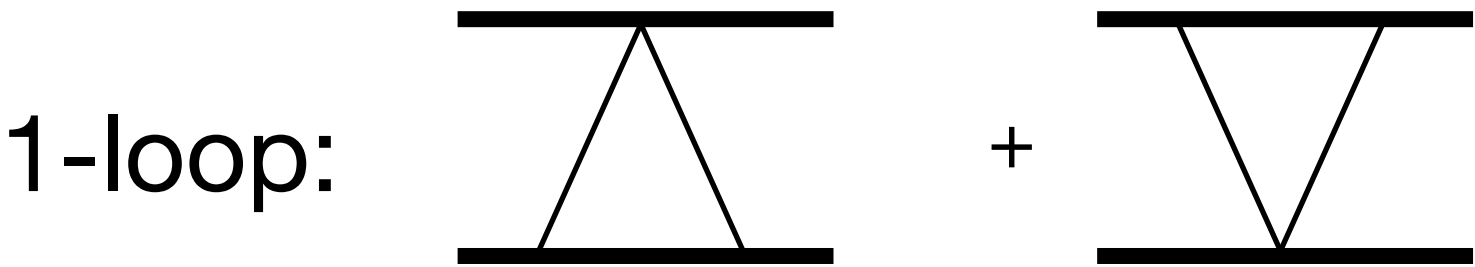
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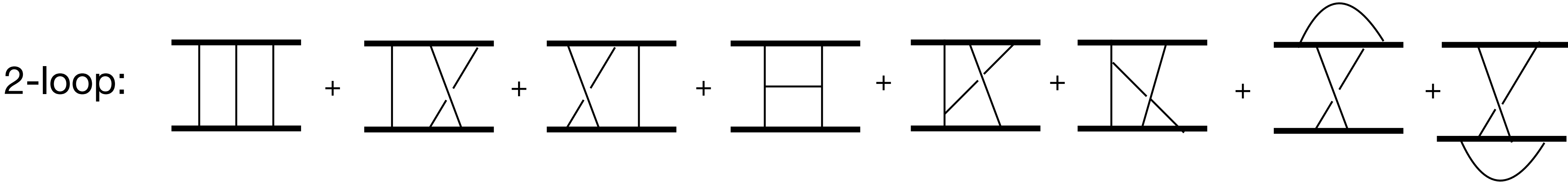
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...



State of the art is pushing towards 4-loops (5PM) binary dynamics

Phys.Rev.Lett. 132 (2024) 24, 241402 [Driesse, Jakobsen, Mogull, Plefka, Sauer, Usovitsch]
 arxiv: 2406.01554 [Bern, Herrmann, Roiban, Ruf, Smirnov, Smirnov, Zeng]

Results obtained in standard GR:

LO order spinning waveform obtained from different approaches (consensus up to $\mathcal{O}(a^4)$):

Phys.Rev.D 110 (2024) 4, L041502 [De Angelis, Novichkov, Gonzo]

Phys.Rev.D 109 (2024) 3, 036007 [Aoude, Haddad, Heissenberg, Helset]

JHEP 02 (2024) 026 [Brandhuber, Brown, Chen, Gowdy, Travaglini]

e.g.: LO spinless waveform:

$$h_f(x)|_{\mathbf{a}_i=0} = \sum_{i=1}^2 \frac{\tilde{r}_{(i),0}^{-,\mu\nu} + \tilde{r}_{(i),0}^{+,\mu\nu}}{(p_i \cdot \rho)^2} \mathcal{I}_{(i),\mu\nu}(b_0)$$

$$\mathcal{I}_{(1)}^{\mu\nu}(b) = \frac{K_{(1)}^{\mu\nu}(v_1 \cdot K_{(1)} \cdot \rho) - 2(v_1 \cdot K_{(1)})^{(\mu}(\rho \cdot K_{(1)})^{\nu)}}{4\pi(\gamma^2 - 1)(\rho \cdot v_2)^2 |b|^2 |\mathbf{b}_{(1)}| |b|_{2d}^2}$$

$$\begin{aligned} r_{(1),0}^{+,\mu\nu} &= \langle k | p_1 p_2 \gamma^\mu p_1 | k \rangle \langle k | p_1 p_2 \gamma^\nu p_1 | k \rangle & r_{(1),0}^{-,\mu\nu} &= m_1^4 \langle k | p_2 \gamma^\mu | k \rangle \langle k | p_2 \gamma^\nu | k \rangle, \\ r_{(1),1}^{+,\mu\nu} &= \langle k | p_1 p_2 \gamma^\mu p_1 | k \rangle \langle k | p_1 p_2 \gamma^\nu \mathbf{a}_1 | k \rangle & r_{(1),1}^{-,\mu\nu} &= m_1^2 \langle k | p_2 \gamma^\mu | k \rangle \langle k | \mathbf{a}_1 p_1 p_2 \gamma^\nu | k \rangle, \\ r_{(1),2}^{+,\mu\nu} &= \langle k | p_1 p_2 \gamma^\mu \mathbf{a}_1 | k \rangle \langle k | p_1 p_2 \gamma^\nu \mathbf{a}_1 | k \rangle & r_{(1),2}^{-,\mu\nu} &= \langle k | \mathbf{a}_1 p_1 p_2 \gamma^\mu | k \rangle \langle k | \mathbf{a}_1 p_1 p_2 \gamma^\nu | k \rangle, \\ r_{(1),3}^{+,\mu\nu} &= \langle k | p_1 p_2 \gamma^\mu \mathbf{a}_1 | k \rangle \langle k | \mathbf{a}_1 p_2 \gamma^\nu \mathbf{a}_1 | k \rangle & r_{(1),3}^{-,\mu\nu} &= \frac{1}{m_1^2} \langle k | \mathbf{a}_1 p_1 p_2 \gamma^\mu | k \rangle \langle k | \mathbf{a}_1 p_1 p_2 \gamma^\nu p_1 \mathbf{a}_1 | k \rangle, \\ r_{(1),4}^{+,\mu\nu} &= \langle k | \mathbf{a}_1 p_2 \gamma^\mu \mathbf{a}_1 | k \rangle \langle k | \mathbf{a}_1 p_2 \gamma^\nu \mathbf{a}_1 | k \rangle & r_{(1),4}^{-,\mu\nu} &= \frac{1}{m_1^4} \langle k | \mathbf{a}_1 p_1 p_2 \gamma^\mu p_1 \mathbf{a}_1 | k \rangle \langle k | \mathbf{a}_1 p_1 p_2 \gamma^\nu p_1 \mathbf{a}_1 | k \rangle \end{aligned}$$

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NLO order waveform also looks like a closed case:

JHEP 06 (2023) 048 [Brandhuber, Brown, Chen, De Angelis, Gowdy, Travaglini]

JHEP 07 (2024) 272 [Elkhidir, O'Connell, Sergola, Vazquez-Holm]

JHEP 06 (2023) 004 [Herderschee, Roiban, Teng]

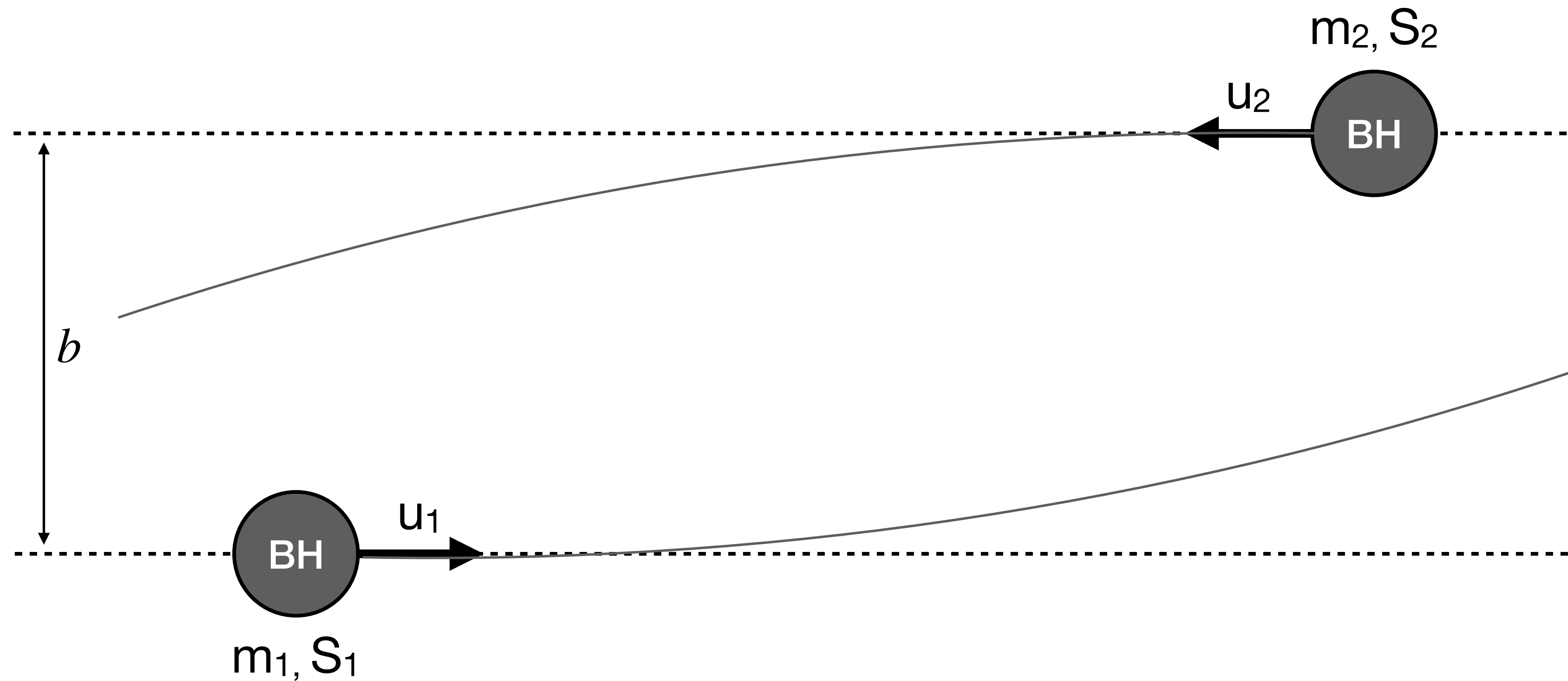
JHEP 2023 (2023) 06, 126 [Georgoudis, Heissenberg, Vazquez-Holm]

JHEP 01 (2024) 139 [Caron-Huot, Giroux, Hannesdottir, Mizera]

Recent result for NLO linear-in-spin effects:

arxiv: 2312.14859 [Bohnenblust, Ita, Kraus, Schlenk]

2. Scattering Amplitudes and Observables



Weak field expansion:

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_{Pl}} h_{\mu\nu}$$

PM expansion:

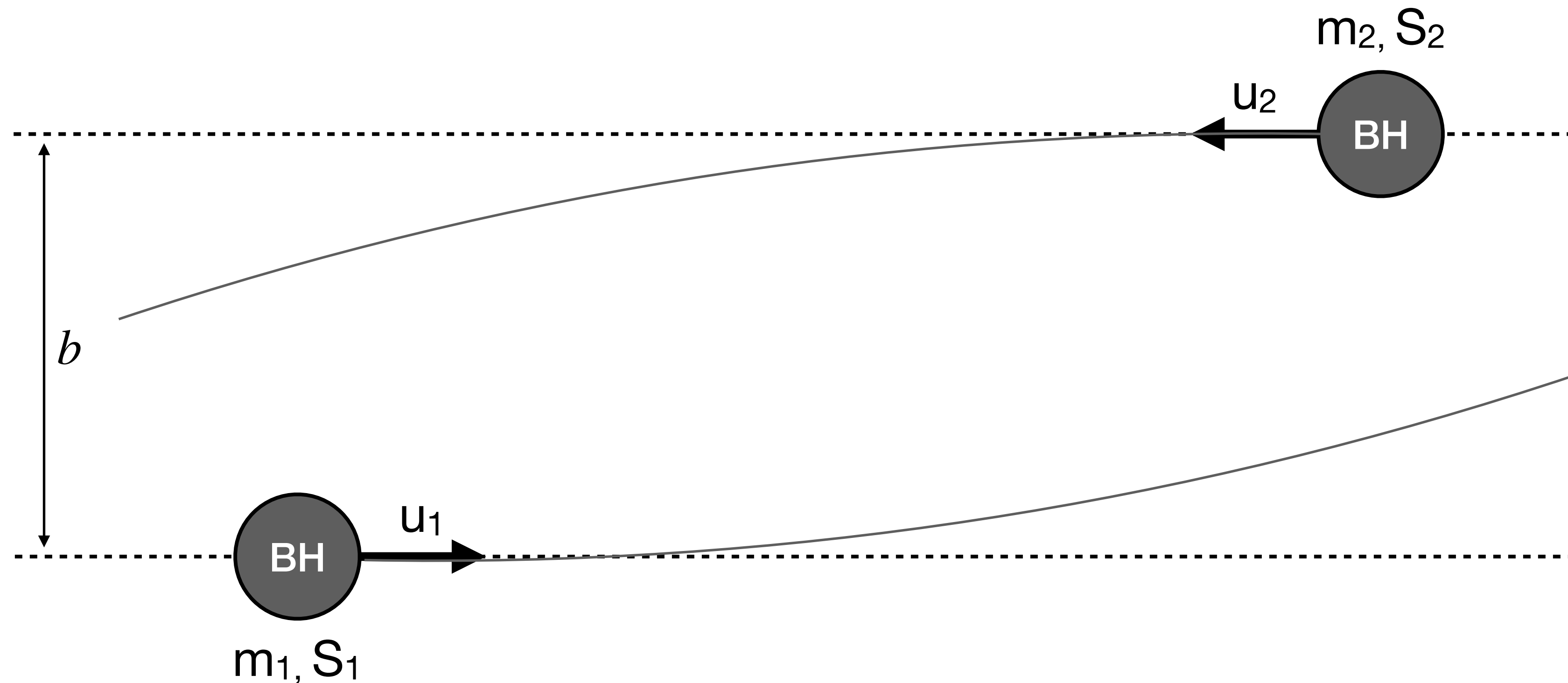
$$R_s \ll b$$

Spin expansion:

$$\frac{S}{m} \ll b$$

- **Focus: Classical scattering problem in GR**

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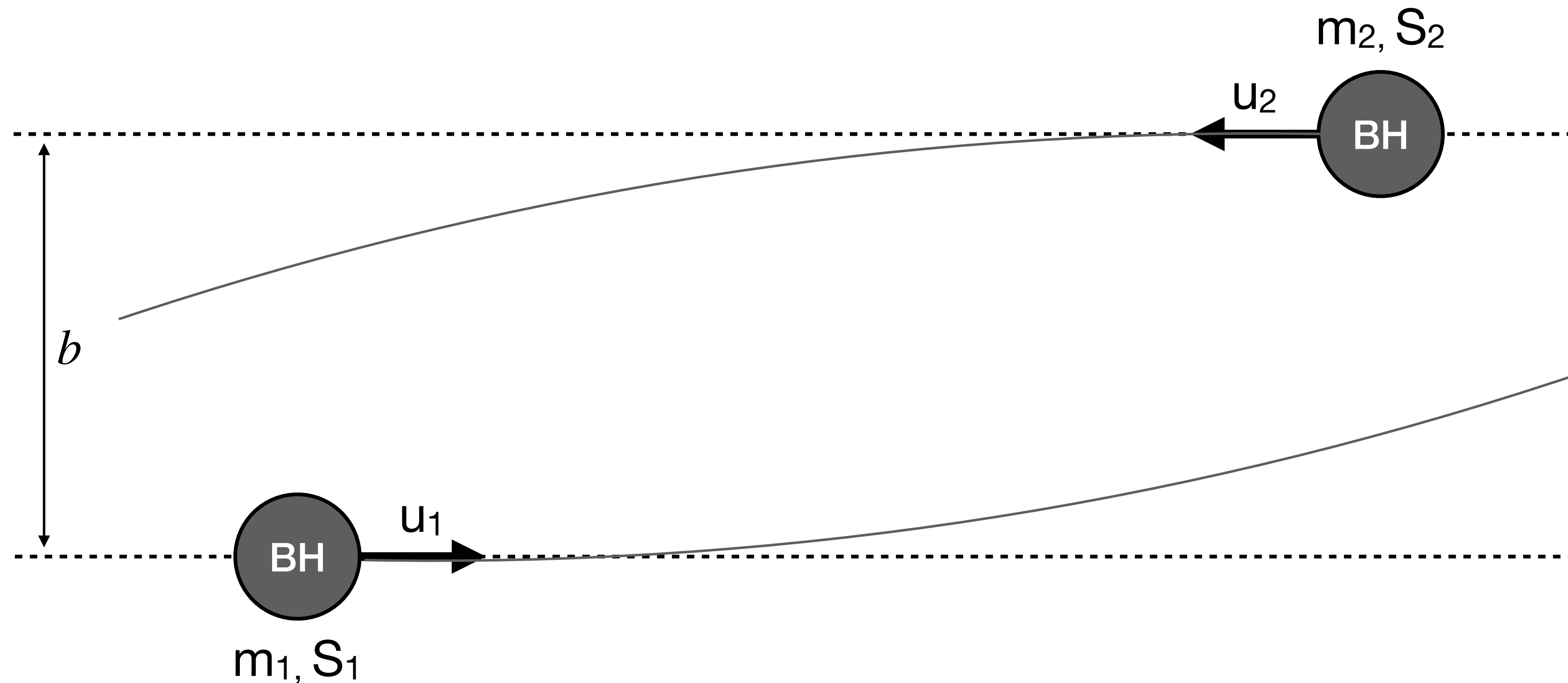
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- **Focus: Classical scattering problem in GR**

→ Can we describe this problem using the Scattering Amplitudes used in QFT?

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- **Focus: Classical scattering problem in GR**

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Yes! → Use of the **KMOC formalism**

KMOC (Kosower Maybee O'Connell) formalism

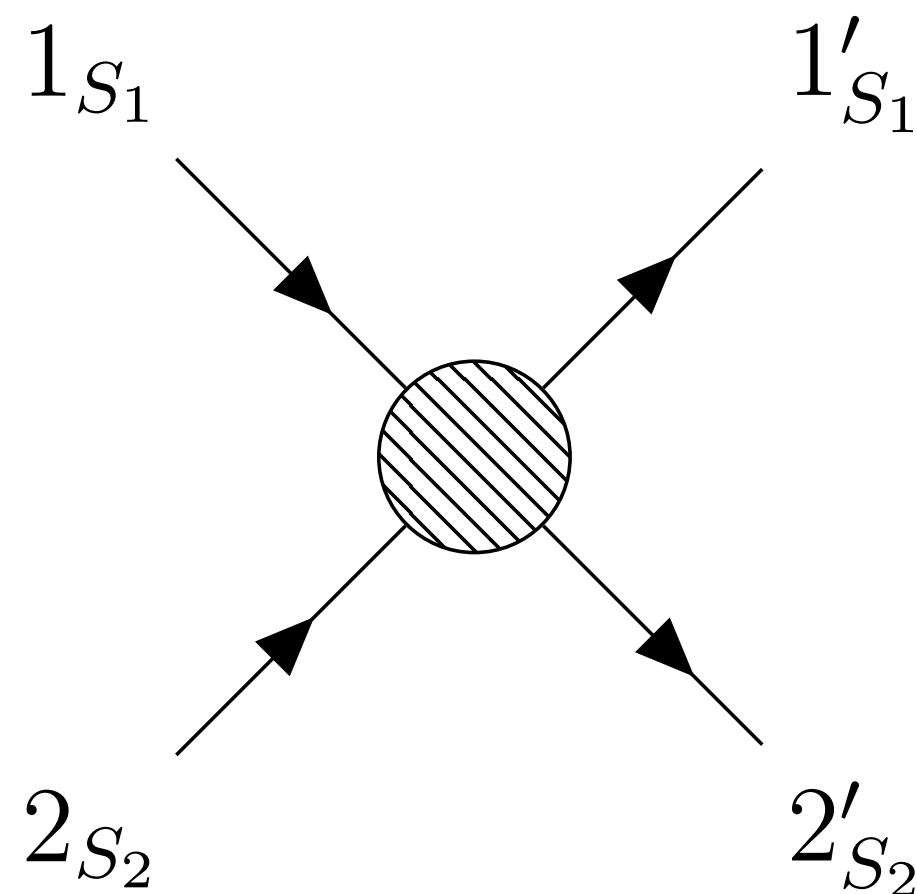
JHEP 02 (2019) 137 [Kosower, Maybee, O'Connell]

Phys.Rev.D 106 (2022) 5, 0567007 [Cristofoli, Gonzo, Kosower, O'Connell]

Idea: Relate **Scattering Amplitudes** directly to **classical observables**

→ Extract the classical piece of the amplitude through an “ \hbar ” counting prescription

e.g.:



Classical Impulse:

$$\Delta p_1^\mu = p_{1,fin.}^\mu - p_{1,in.}^\mu$$

KMOC (Kosower Maybee O'Connell) formalism

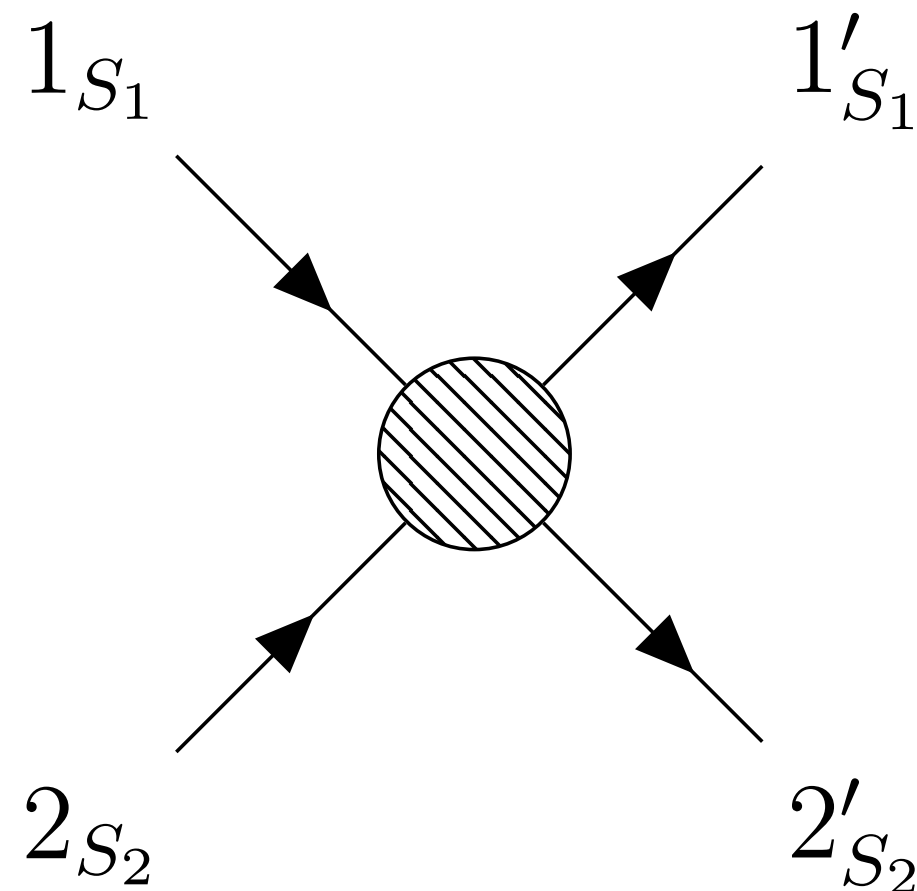
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At leading order:
$$\Delta p_1^{\mu, LO} = \frac{i}{4} \left\langle \left\langle \hbar^2 \int \hat{d}^4 q \hat{\delta}(q \cdot p_1) \hat{\delta}(q \cdot p_2) e^{-ib \cdot q} q^\mu \mathcal{M}^{LO}(p_1, p_2 \rightarrow p_1 + \hbar q, p_2 - \hbar q) \right\rangle \right\rangle$$

Waveforms at leading order:

strain

on-shell measure

$$h_{GR}(t) \equiv h_+ \pm ih_\times \sim \int d\omega e^{-i\omega t} \int d\Phi(q) e^{-ibq}$$

1_{S_1} $1'_{S_1}$
 2_{S_2} $2'_{S_2}$
 h
 $\mathcal{M}_5^{cl}(q, k) |_{k^\mu = \omega n^\mu}$

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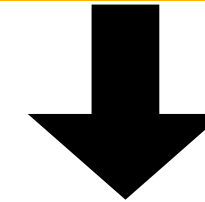
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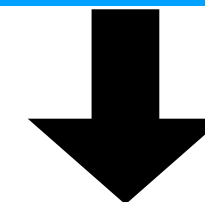
- Computations organized in a **perturbative expansion** using a **simple algorithm**.
- **Analytic results**, in places where either PN approximations or NR was used before.
- Can exploit many **modern techniques used in particle physics** to simplify the problem.
- Can straightforwardly **extend to beyond GR predictions**

On-shell techniques:

Basic QFT Principles: Poincaré invariance, locality, unitarity of the Scattering Matrix

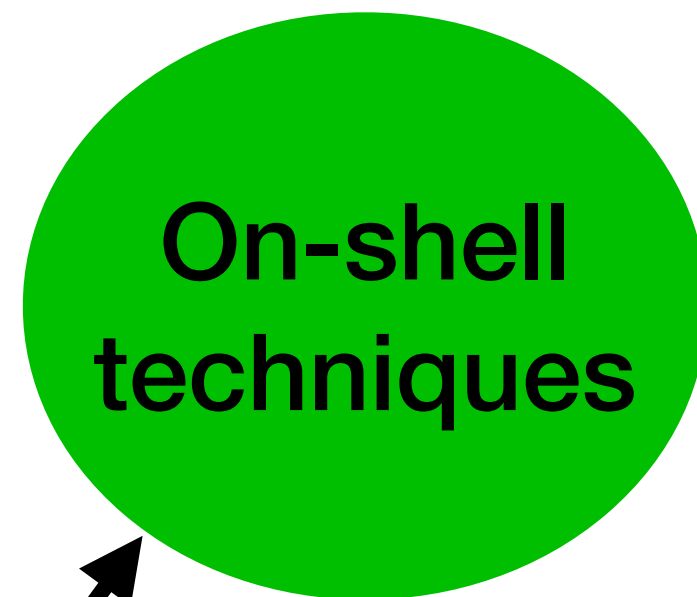


QFT assumptions: Introduction of auxiliary fields, Lagrangians, gauge invariance, Feynman rules...



Scattering Amplitude:
 $\mathcal{M}^{h_1, \dots, h_m}(p_1, \dots, p_n)$

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Scattering Amplitude:
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Use spinors as variables instead of momenta

Massless spinors: $p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$
 Massive spinors: $p_\mu \sigma^\mu_{\alpha\dot{\alpha}} = \epsilon_{IJ} \chi_\alpha^I \tilde{\chi}_{\dot{\alpha}}^J$ } Obey little group transformation rules

$$\langle ij \rangle = \epsilon_{\alpha\beta} \lambda_i^\alpha \lambda_j^\beta, \quad [ij] = \epsilon_{\dot{\alpha}\dot{\beta}} \tilde{\lambda}_i^{\dot{\alpha}} \tilde{\lambda}_j^{\dot{\beta}}$$

$$\langle \mathbf{ij} \rangle = \langle i^I j^J \rangle, \quad [\mathbf{ij}] = [i^I j^J]$$

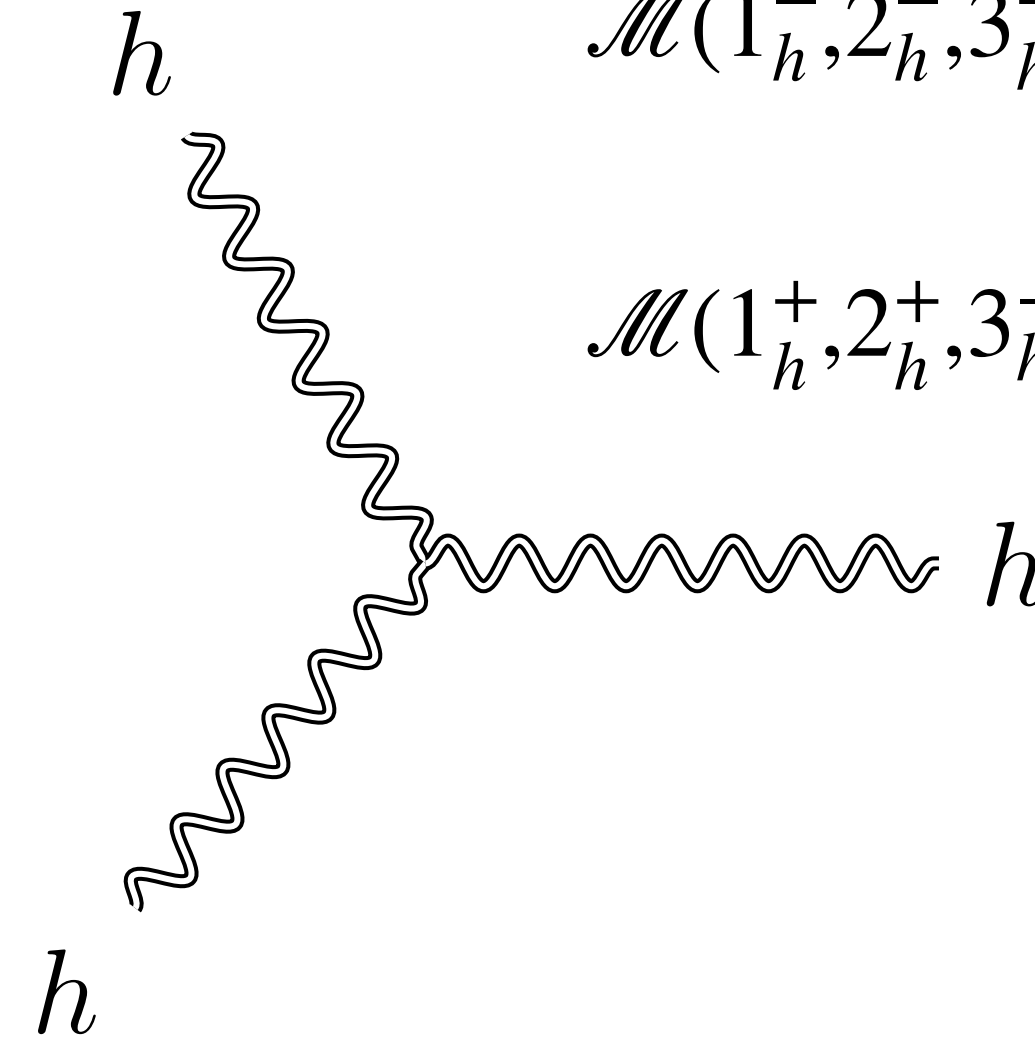
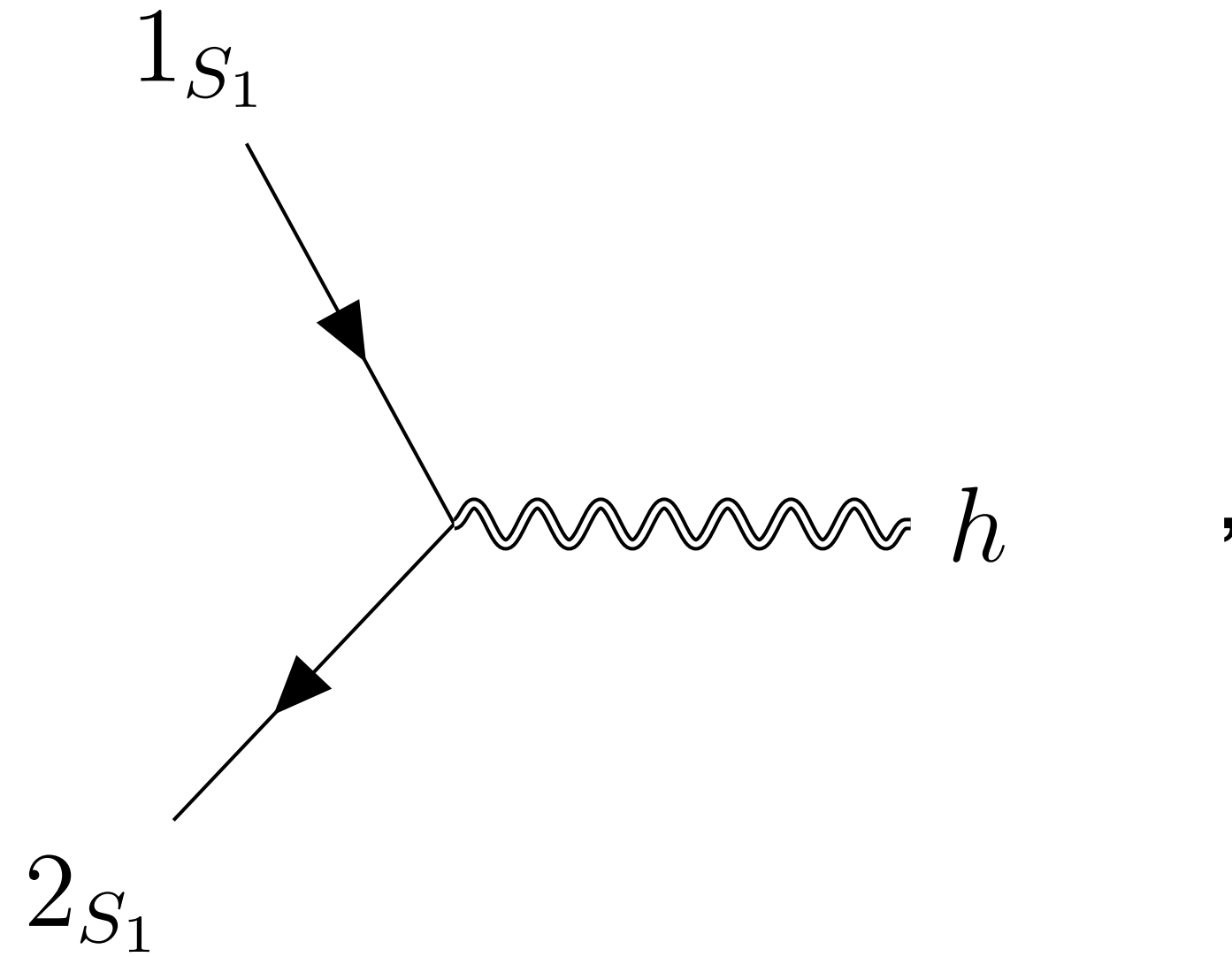
On-shell techniques:

Idea: Build the **on-shell 3-point amplitudes** of the theory

e.g.: **Spinning matter in GR**

$$\mathcal{M}[1_\Phi 2_{\bar{\Phi}} 3_h^-] = -\frac{\langle 3 | p_1 | \tilde{\zeta} \rangle^2 [21]^{2S}}{M_{Pl} [3\tilde{\zeta}]^2 m^{2S}},$$

$$\mathcal{M}[1_\Phi 2_{\bar{\Phi}} 3_h^+] = -\frac{\langle \zeta | p_1 | 3 \rangle^2 \langle 21 \rangle^{2S}}{M_{Pl} \langle 3\zeta \rangle^2 m^{2S}},$$



$$\mathcal{M}(1_h^-, 2_h^-, 3_h^+) = -\frac{1}{M_{Pl}} \frac{\langle 12 \rangle^6}{\langle 13 \rangle^2 \langle 23 \rangle^2}$$

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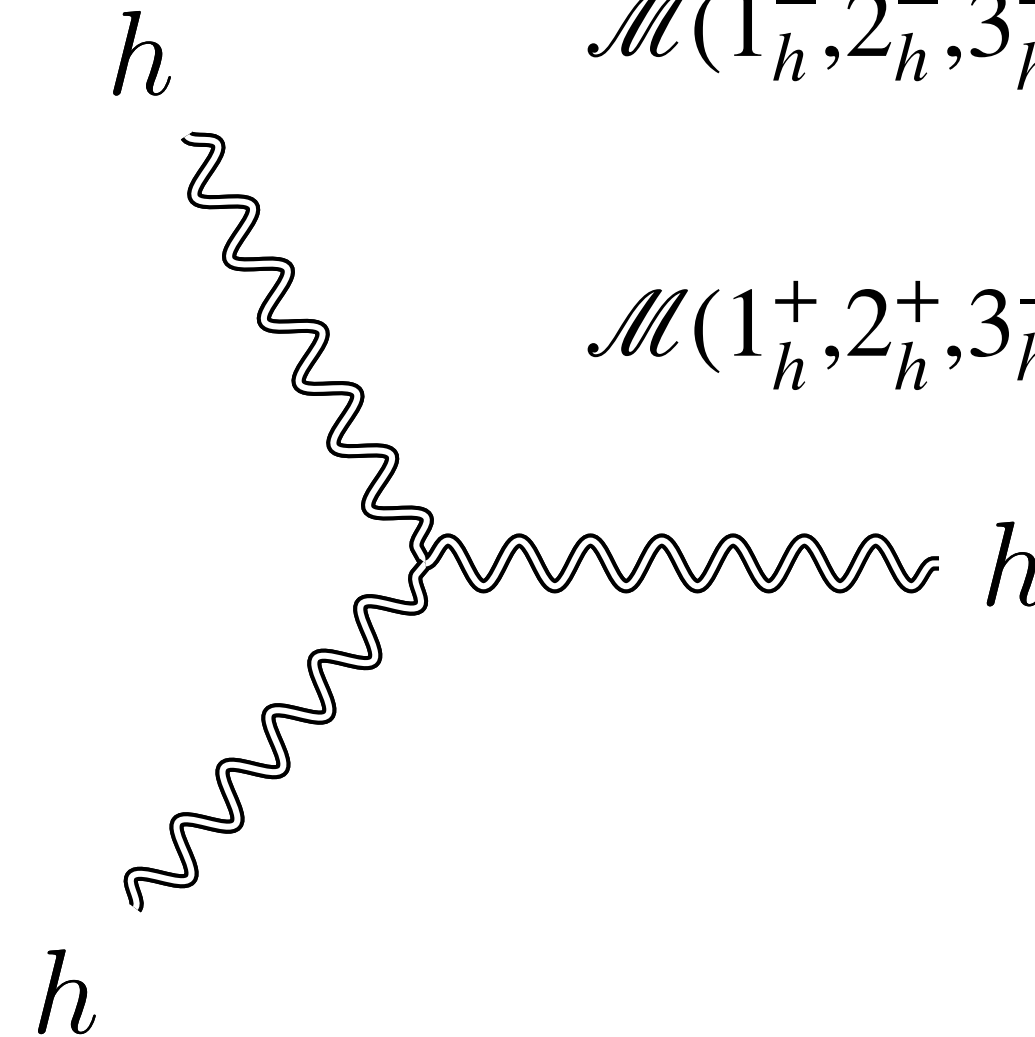
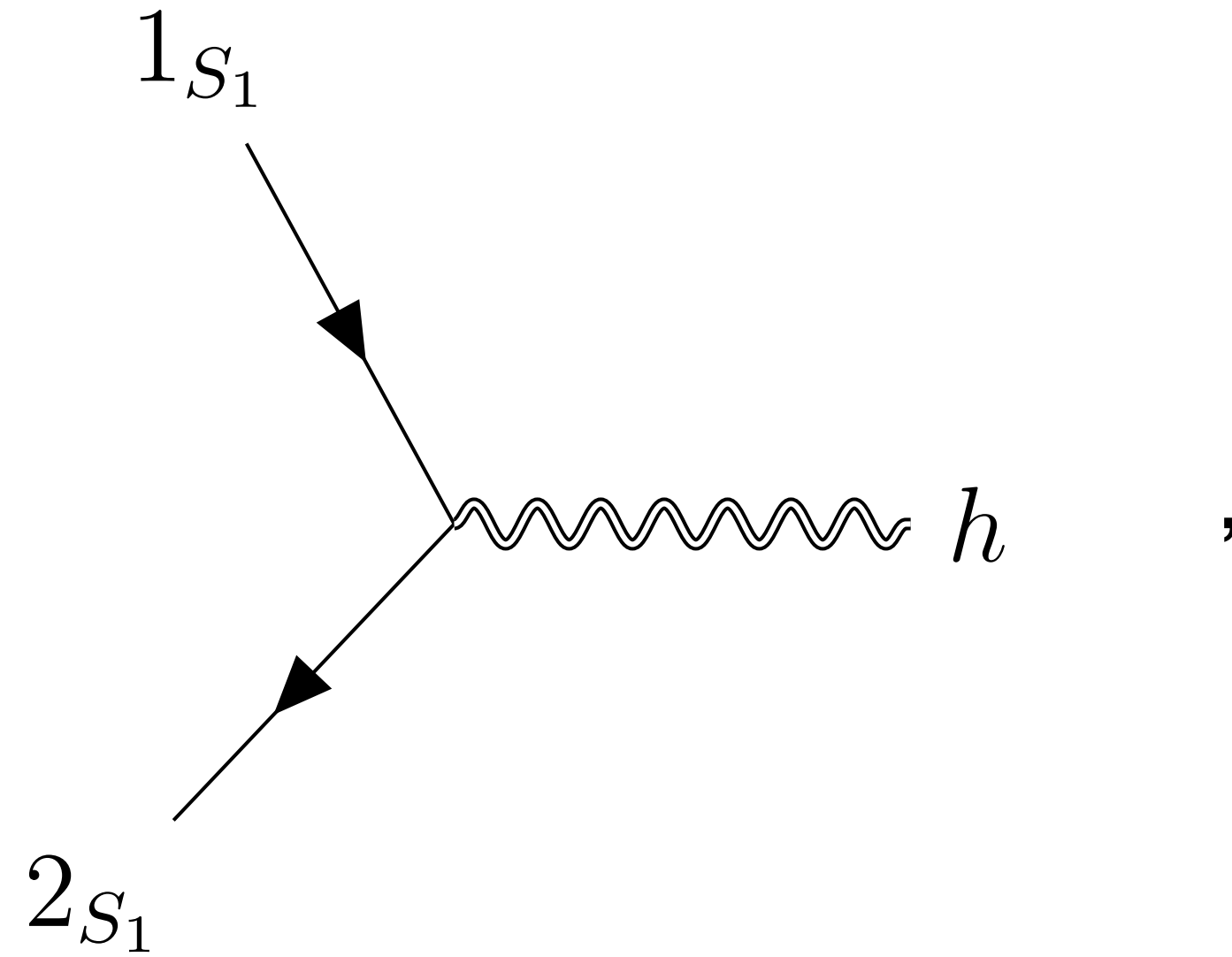


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→ Build higher-point amplitudes from their **residues** at kinematic poles in the **complex** plane (up to contact terms)

e.g.:

$$\sum_{i=2,3,4} \text{Res}_{(p_1+p_i)^2 \rightarrow 0} \left[\text{Diagram with blob} \right] \Big|_{tree} = - \left[\text{Diagram 1} + \text{Diagram 2} + (t \leftrightarrow u) \right]$$

3. Scalar-tensor theories: Examples, compact objects, scalar hair and scattering waveforms

- **Scalar-tensor theories** have long stood as a promising avenue to study **extensions of GR**
- They consist of gravity theories with the introduction of an additional **massless scalar** degree of freedom

$$S_{GR}[g_{\mu\nu}] \rightarrow S_{ST}[g_{\mu\nu}, \phi]$$

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Example: Scalar Gauss-Bonnet and Dynamical Chern Simons gravity

$$S = S_{EH}[R] + S_{SGB,DCS}[\phi, g_{\mu\nu}] + S_m[\Psi_m, \mathcal{A}(\phi)g_{\mu\nu}] \quad ,$$

$$\bullet S_{EH} = \int d^4x \frac{M_{Pl}^2}{2} \sqrt{-g} R$$

$$\bullet S_{SGB,DCS} = \int d^4x \sqrt{-g} \left[M_{Pl} \left(\frac{\alpha}{\Lambda^2} f(\phi) \mathcal{G} + \frac{\tilde{\alpha}}{\Lambda^2} \phi R \tilde{R} \right) + \frac{1}{2} (\partial^\mu \phi \partial_\mu \phi) \right]$$

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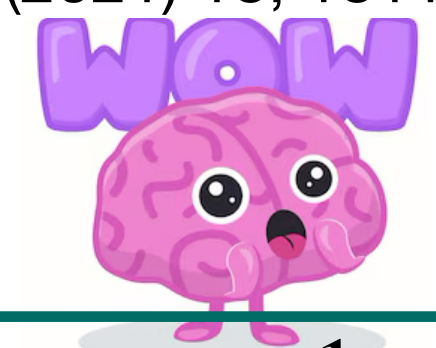
Broad window of resolvability! arxiv: 2407.16457 [Falkowski, PM]

Phys.Rev.D 107 (2023) 4, 044030 [Silva, Ghosh, Buonanno]

Example: Scalar Gauss-Bonnet and Dynamical Chern Simons gravity arXiv: 2406.13654 [Julié, Pompili, Buonanno]

Phys.Rev.Lett. 126 (2021) 18, 181101 [Silva, Holgado, Cárdenas-Avendaño, Yunes]

$$S = S_{EH}[R] + S_{SGB,DCS}[\phi, g_{\mu\nu}] + S_m[\Psi_m, \mathcal{A}(\phi)g_{\mu\nu}] \quad ,$$



Already GW observations are used to constrain them!!!

$$S_{EH} = \int d^4x \frac{M_{Pl}^2}{2} \sqrt{-g} R \quad \boxed{R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2}$$

$$R^{\mu\nu\rho\sigma} \tilde{R}_{\mu\nu\rho\sigma} \quad , \quad \tilde{R}^{\mu}_{\nu\rho\sigma} = \frac{1}{2} \epsilon_{\rho\sigma}^{\alpha\beta} R^{\mu}_{\nu\alpha\beta}$$

$$S_{SGB,DCS} = \int d^4x \sqrt{-g} \left[M_{Pl} \left(\frac{\alpha}{\Lambda^2} f(\phi) \mathcal{G} + \frac{\tilde{\alpha}}{\Lambda^2} \phi R \tilde{R} \right) + \frac{1}{2} (\partial^\mu \phi \partial_\mu \phi) \right]$$

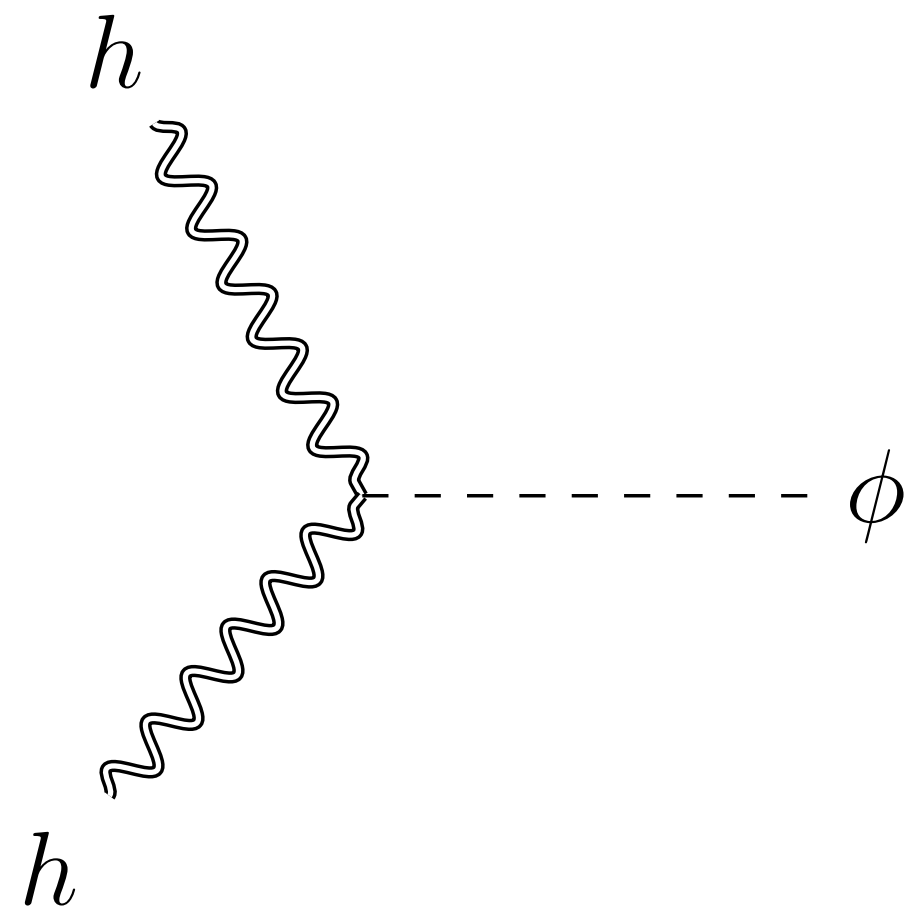
Experimental window:

$$\frac{\sqrt{\alpha}}{\Lambda} \lesssim 0.22 km \quad , \quad \frac{\sqrt{\tilde{\alpha}}}{\Lambda} \lesssim 9.5 km$$

On-shell amplitudes:

Let's work by expanding $f(\phi) \approx c + \phi + \mathcal{O}(\phi^2)$.

Naively, this action produces an extra 3-point **shift-symmetric** on-shell amplitude which we should consider:

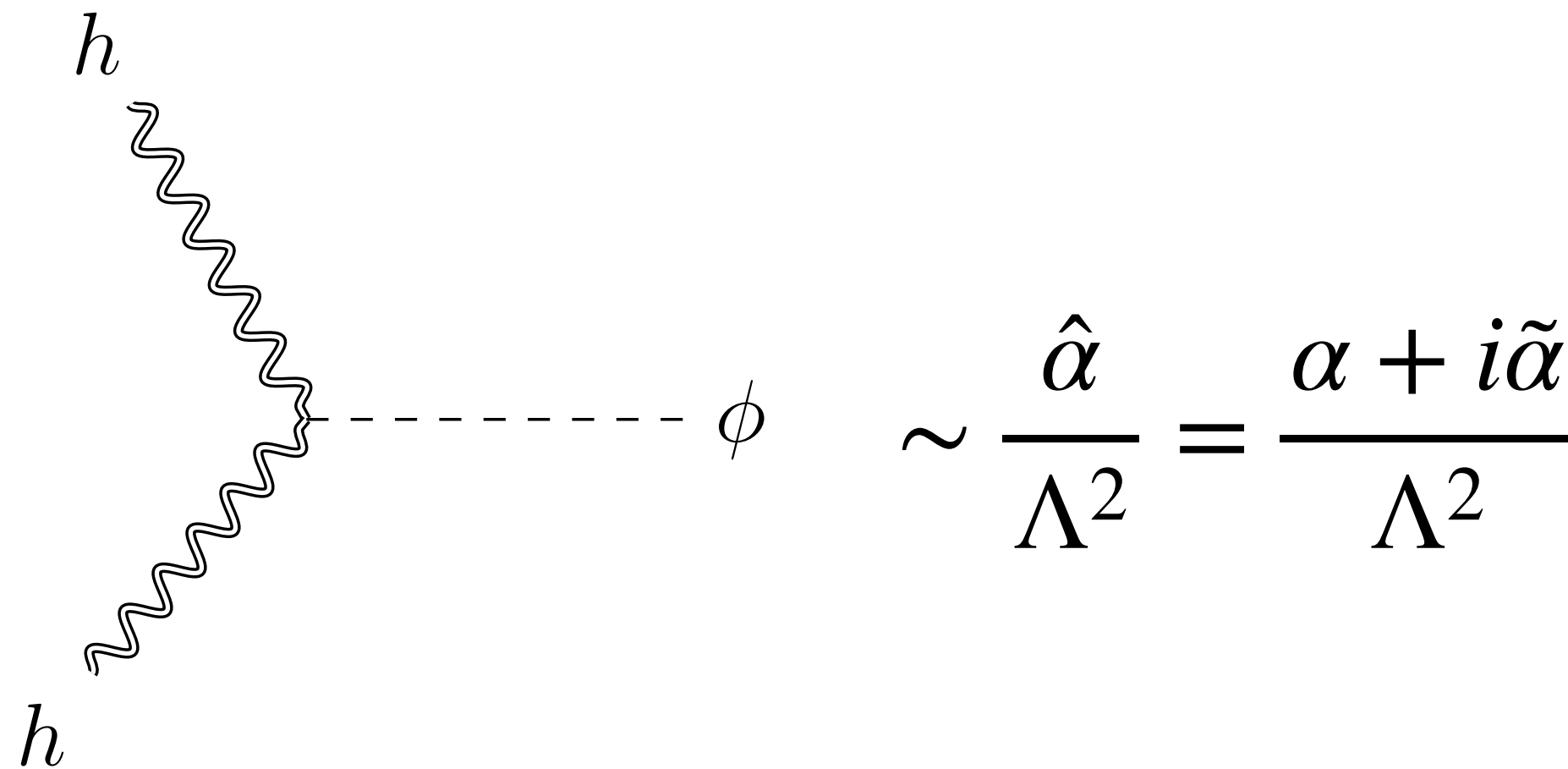
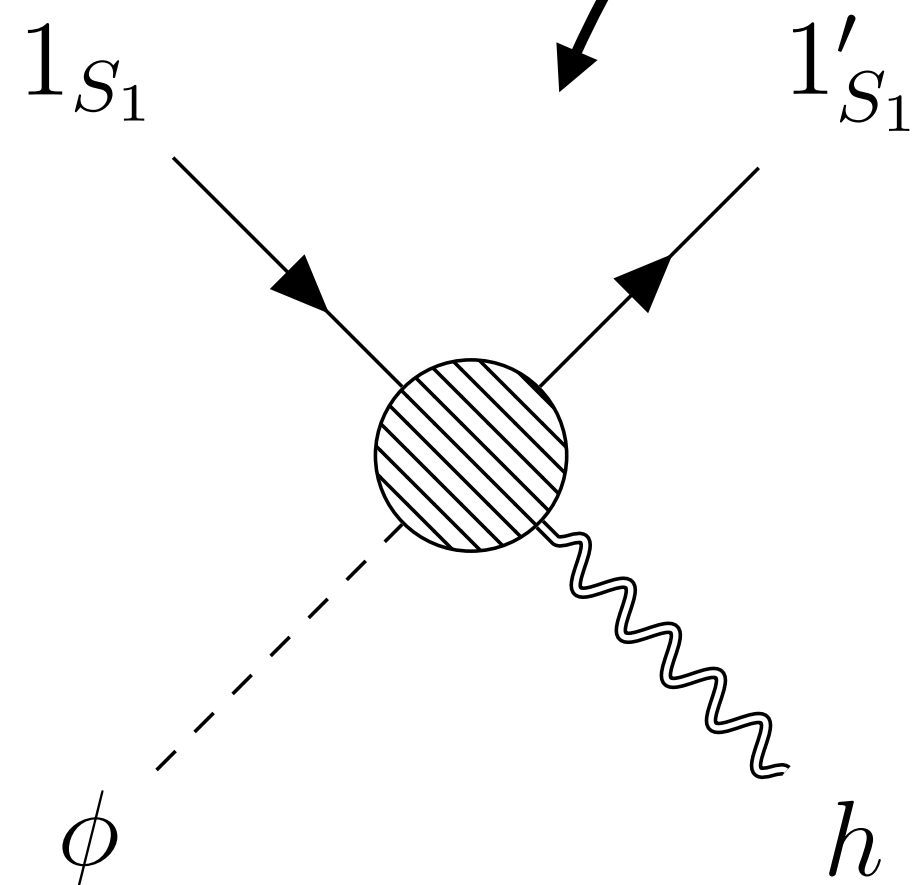

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Gives an extra contribution to the 4-point amplitudes

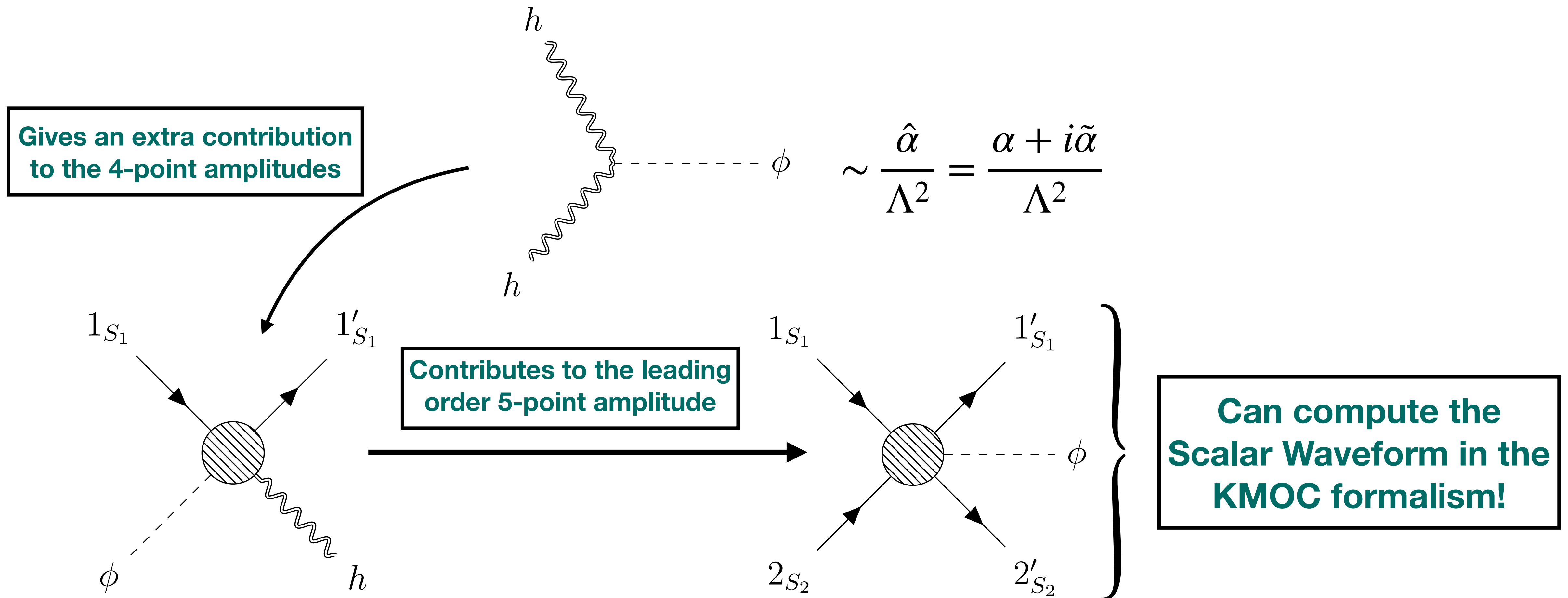


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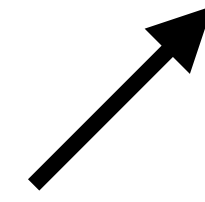
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Contact terms' contributions: An inevitable problem

Huge list of authors who have contributed in solving this problem in QED and Gravity



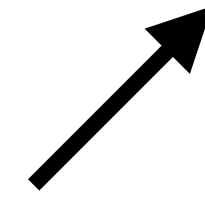
→ The four -point amplitudes we construct are **healthy at all spin orders**, but **still have to include contact terms' deformations** which can contribute **classically** (very similar to what has already been observed in QED and GR): **Can be done order by order** in the spin expansion, but need a matching procedure to fix their coefficients

$$\mathcal{M}[1_{\Phi_i} 2_{\bar{\Phi}_i} 3_h^s 4_\phi] = \frac{4\hat{\alpha}}{M_{Pl}^2 \Lambda^2} t(p_1 \epsilon_s)^2 \left\{ e^{q \cdot a} + P_{\xi,s}(p_3 a_1, p_4 a_1, w a_1, |a_1|') \right\} ,$$

$$q^\mu = p_3^\mu + p_4^\mu , \quad w^\mu = \frac{(t - m^2)}{2(p_1 \epsilon_{3,-})} \epsilon_{3,-}^\mu , \quad |a_1|' = \frac{(t - m^2)}{2m} |a_1|$$

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“Unitary” piece
No spurious poles!

Contact terms' deformations

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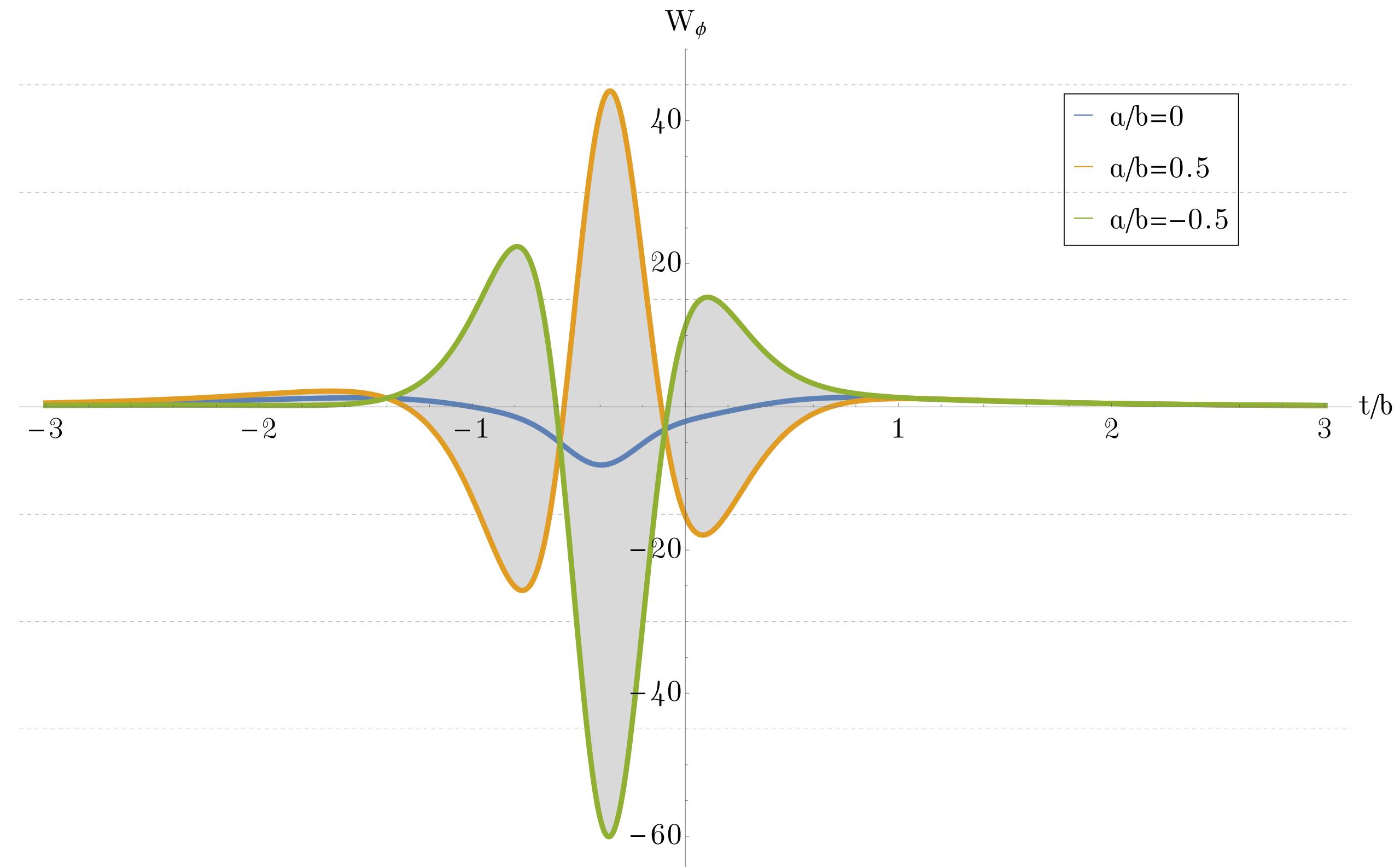
e.g. up to $\mathcal{O}(a_1^2)$:

$$P_{\xi,-} \left((p_3 a_1), (p_4 a_1), (w a_1), |a_1|' \right) \Big|_{\mathcal{O}(a_1^2)} = \xi^{-1} \left(C_{000}^{-1} (w a_1)^2 + D_{000}^{-1} (w a_1) |a_1|' \right) + \xi^0 \left(C_{000}^0 ((w a_1) - (p_3 a_1)) + C_{100}^0 ((w a_1) - (p_3 a_1))(p_3 a_1) + C_{010}^0 ((w a_1) - (p_3 a_1))(p_4 a_1) + C_{001}^0 ((w a_1) - (p_3 a_1))(w a_1) + C_{000}^0 C^0 ((w a_1) - (p_3 a_1)) |a_1|' \right).$$

Results:

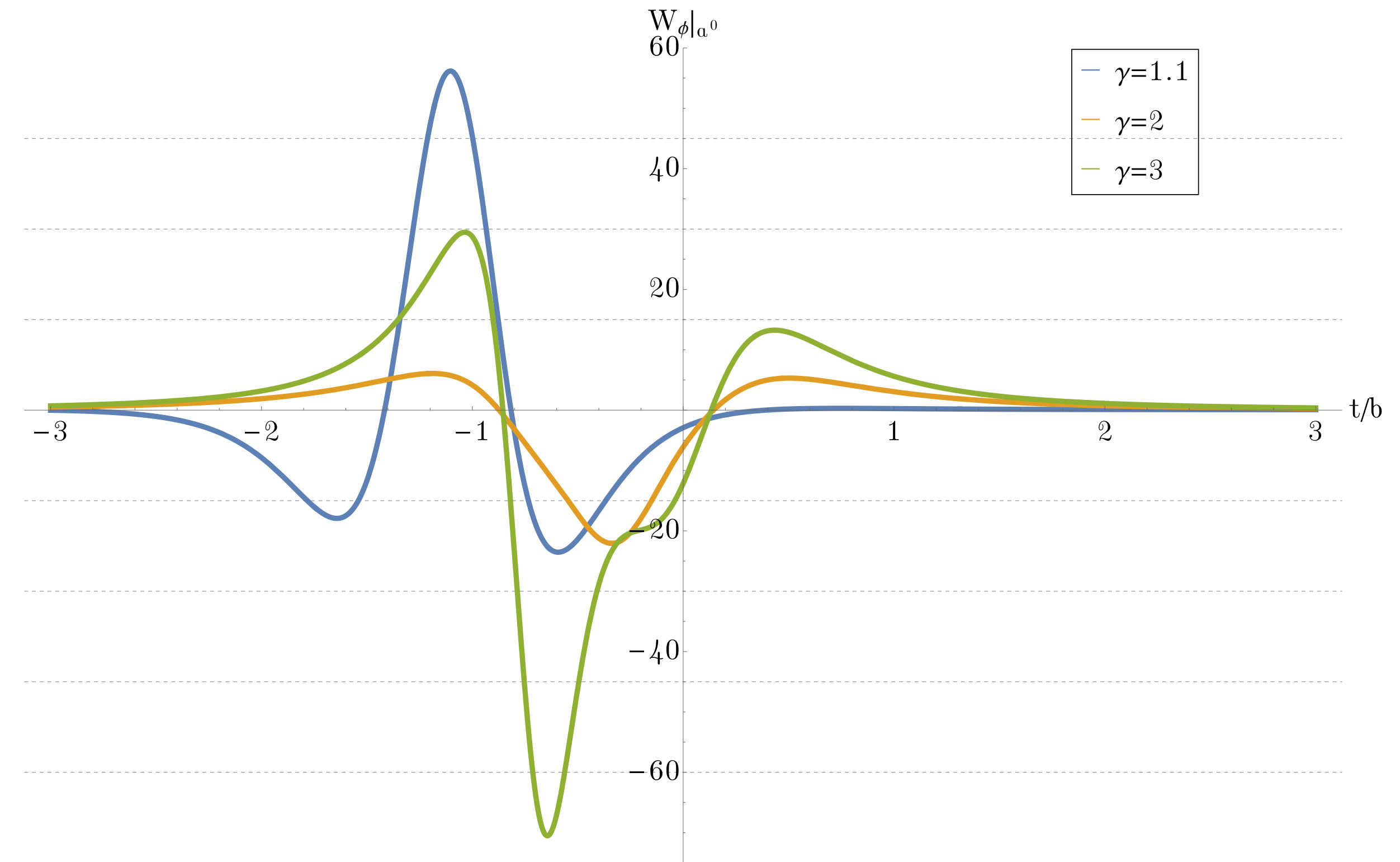
LO scalar waveform for scalar Gauss-Bonnet:

Waveforms in time domain



1. Scalar waveform for the SGB case up to linear order in spin for different values of the spin magnitude $a \in [-b/2, b/2]$.

Waveforms in time domain



2. Scalar piece of the SGB waveform for different values of γ .

Results:

LO scalar waveform for scalar Gauss-Bonnet:

$$\begin{aligned}
 W_\phi = & \frac{m_1 m_2}{8\pi^2 b^3 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 \sqrt{\gamma^2 - 1}} \left(\alpha \left\{ -\frac{d^2}{dz^2} \left[\frac{1}{\sqrt{z^2 + 1}} \frac{2(\gamma^2 - 1)^2 (\hat{u}_2 n)^2 [\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z]}{[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z]^2 + (\tilde{v}n)^2 (z^2 + 1)} \right] + [(\hat{u}_1 n) + \gamma(2\gamma^2 - 3)(\hat{u}_2 n)] \frac{2z^2 - 1}{(z^2 + 1)^{5/2}} + (2\gamma^2 - 1)(\tilde{b}n) \frac{3z}{(z^2 + 1)^{5/2}} \right\} \right. \\
 & + \frac{2\alpha}{b} \frac{d^3}{dz^3} \text{Re} \left\{ \frac{1}{\sqrt{z^2 + 1}} \frac{(\hat{u}_2 n) - \gamma(\hat{u}_1 n) + \gamma(\tilde{b}n)z + i\gamma(\tilde{v}n)\sqrt{z^2 + 1}}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z + i(\tilde{v}n)\sqrt{z^2 + 1}} \left[z(\tilde{v}n) - i\sqrt{z^2 + 1}(\tilde{b}n) \right] \times \left[\frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1 \hat{u}_2) - (a_2 \hat{u}_1) - (a_1^A - a_2^A)(z\tilde{b}^A + i\sqrt{z^2 + 1}\tilde{v}^A) \right] \right\} \\
 & \left. - \sqrt{\gamma^2 - 1} \frac{C_1}{b} \frac{d^3}{dz^3} \left\{ \frac{1}{\sqrt{z^2 + 1}} \text{Re} \left[\left(z[(\tilde{v}n)a_1^A + (a_1 \tilde{v})n^A] - i\sqrt{z^2 + 1}[(\tilde{b}n)a_1^A + (a_1 \tilde{b})n^A] \right) \times \left(\hat{u}_2^A - \gamma\hat{u}_1^A + \gamma[z\tilde{b}^A + i\sqrt{z^2 + 1}\tilde{v}^A] \right) \right] \right\} \right) \Big|_{z=T_1} + (1 \leftrightarrow 2) + \mathcal{O}(a^2).
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 \end{aligned}$$

Connect to
observables: Power
emitted in scalar
radiation

$$\frac{dP_\phi}{d\Omega} = (\partial_t W_\phi)^2$$

$$\frac{dP_\phi}{d\Omega} \Big|_{\mathcal{O}(a^0)} \sim \frac{\beta^6}{b^8}$$

For closed orbits

$$\frac{dP_\phi}{d\Omega} \Big|_{\mathcal{O}(a^0)} \sim \beta^{22}$$

$$\frac{dP_\phi}{d\Omega} \Big|_{\mathcal{O}(a^1)} \sim \frac{\beta^4}{b^{10}}$$

For closed orbits

$$\frac{dP_\phi}{d\Omega} \Big|_{\mathcal{O}(a^1)} \sim \beta^{24}$$

$$\beta = \frac{v}{c}$$

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For closed orbits

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Bigger suppression
compared to β^8 previously
computed with GR methods

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016)
2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou,
Hinderer, Nissanke, Ortiz, Witek]

$$\frac{dP_\phi}{d\Omega} \Big|_{\mathcal{O}(a^1)} \sim \frac{\beta^4}{b^{10}}$$

For closed orbits

$$\frac{dP_\phi}{d\Omega} \Big|_{\mathcal{O}(a^1)} \sim \beta^{24}$$

$$\beta = \frac{v}{c}$$

Results:

LO scalar waveform for scalar Gauss-Bonnet:

$$W_\phi = \frac{m_1 m_2}{8\pi^2 b^3 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 \sqrt{\gamma^2 - 1}} \left(\alpha \left\{ -\frac{d^2}{dz^2} \left[\frac{1}{\sqrt{z^2 + 1}} \frac{2(\gamma^2 - 1)^2 (\hat{u}_2 n)^2 [\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z]}{[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z]^2 + (\tilde{v}n)^2 (z^2 + 1)} \right] + [(\hat{u}_1 n) + \gamma(2\gamma^2 - 3)(\hat{u}_2 n)] \frac{2z^2 - 1}{(z^2 + 1)^{5/2}} + (2\gamma^2 - 1)(\tilde{b}n) \frac{3z}{(z^2 + 1)^{5/2}} \right\} \right. \\ \left. + \frac{2\alpha}{b} \frac{d^3}{dz^3} \operatorname{Re} \left\{ \frac{1}{\sqrt{z^2 + 1}} \frac{(\hat{u}_2 n) - \gamma(\hat{u}_1 n) + \gamma(\tilde{b}n)z + i\gamma(\tilde{v}n)\sqrt{z^2 + 1}}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z + i(\tilde{v}n)\sqrt{z^2 + 1}} \left[z(\tilde{v}n) - i\sqrt{z^2 + 1}(\tilde{b}n) \right] \times \left[\frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1 \hat{u}_2) - (a_2 \hat{u}_1) - (a_1^A - a_2^A)(z\tilde{b}^A + i\sqrt{z^2 + 1}\tilde{v}^A) \right] \right\} \right. \\ \left. - \sqrt{\gamma^2 - 1} \frac{C_1}{b} \frac{d^3}{dz^3} \left\{ \frac{1}{\sqrt{z^2 + 1}} \operatorname{Re} \left[\left(z[(\tilde{v}n)a_1^A + (a_1 \tilde{v})n^A] - i\sqrt{z^2 + 1}[(\tilde{b}n)a_1^A + (a_1 \tilde{b})n^A] \right) \times \left(\hat{u}_2^A - \gamma\hat{u}_1^A + \gamma[z\tilde{b}^A + i\sqrt{z^2 + 1}\tilde{v}^A] \right) \right] \right\} \right) \Big|_{z=T_1} + (1 \leftrightarrow 2) + \mathcal{O}(a^2).$$

Connect to observables: Power emitted in scalar radiation

$$\frac{dP_\phi}{d\Omega} = (\partial_t W_\phi)^2$$

$$\frac{dP_\phi}{d\Omega} \Big|_{\mathcal{O}(a^0)} \sim \frac{\beta^6}{b^8}$$

For closed orbits

$$\frac{dP_\phi}{d\Omega} \Big|_{\mathcal{O}(a^0)} \sim \beta^{22}$$

Bigger suppression
compared to β^8 previously computed with GR methods

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016)

2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

$$\frac{dP_\phi}{d\Omega} \Big|_{\mathcal{O}(a^1)} \sim \frac{\beta^4}{b^{10}}$$

For closed orbits

$$\frac{dP_\phi}{d\Omega} \Big|_{\mathcal{O}(a^1)} \sim \beta^{24}$$

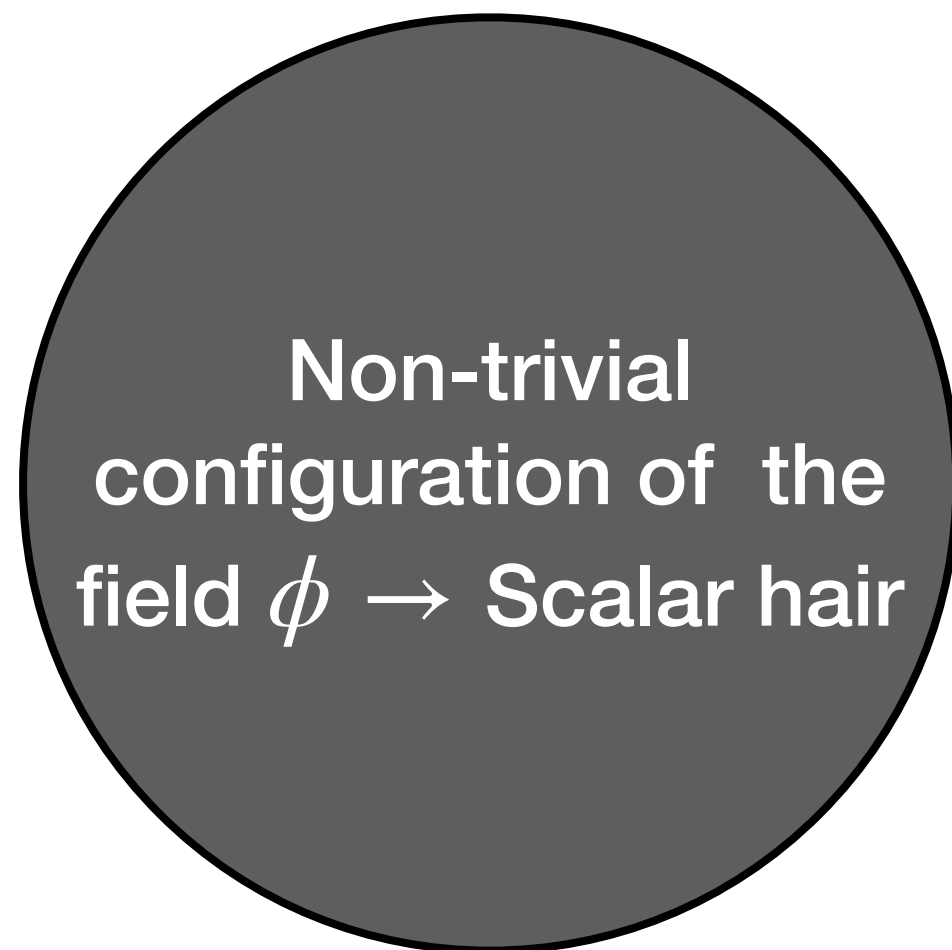
$$\beta = \frac{v}{c}$$

So what's the difference?



Scalar hair in scalar-tensor theories:

Compact objects can acquire scalar hair in ST theories \longrightarrow Exactly the case for SGB and DCS!



BH solution in ST theory

Far zone
 $x \rightarrow \infty$

$$\phi = \frac{c_i}{r} + \frac{d_i}{r^2} + \dots$$

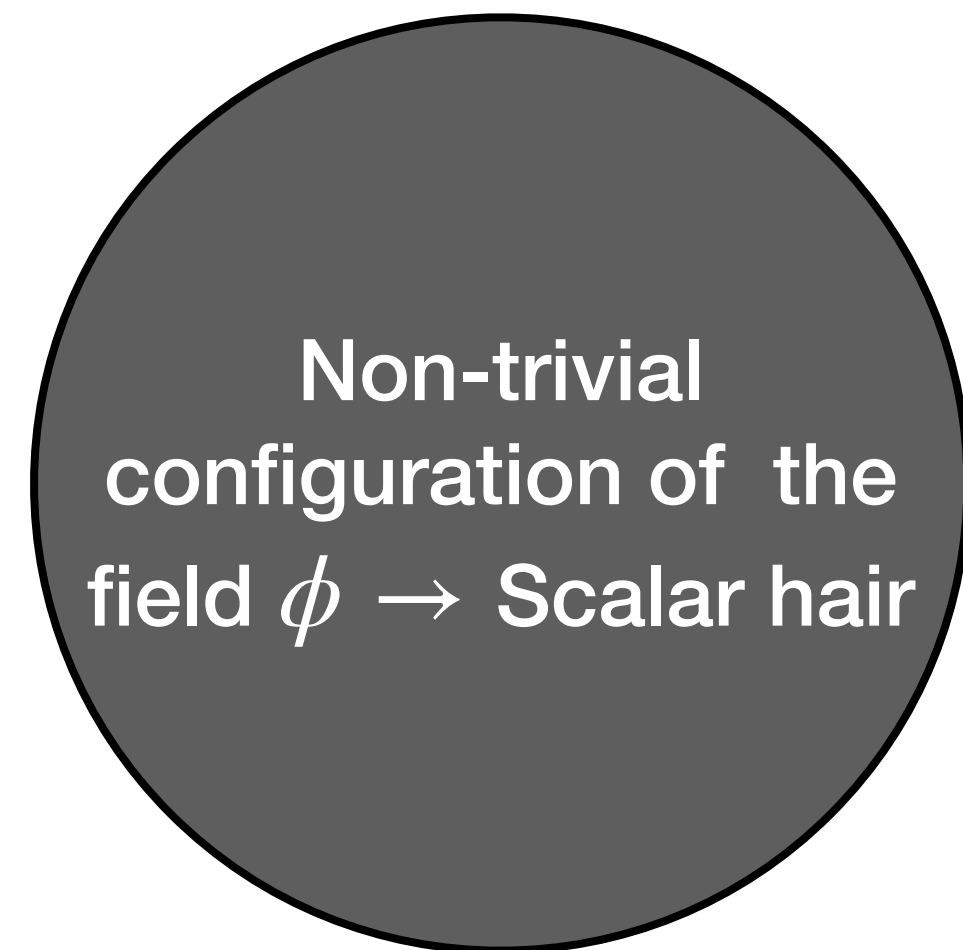
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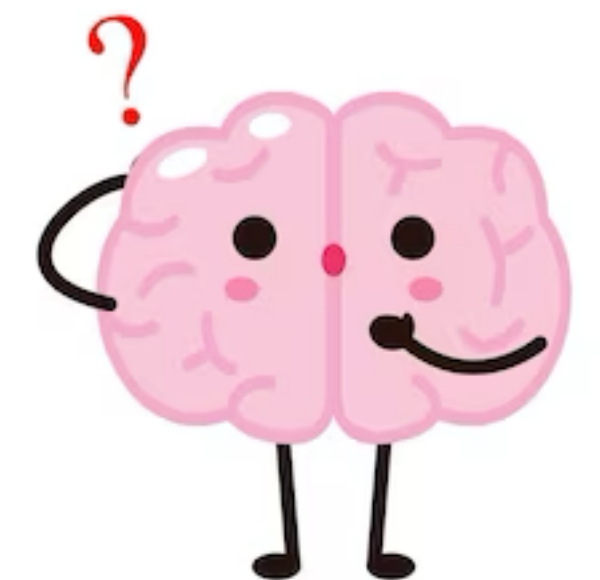
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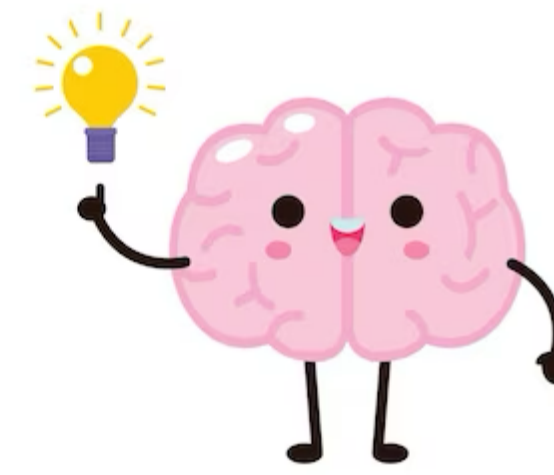
“Monopole hair” “Dipole hair”

$$\phi = \frac{c_i}{r} + \frac{d_i}{r^2} + \dots$$

How can we model this behaviour with amplitudes?



The on-shell way again:



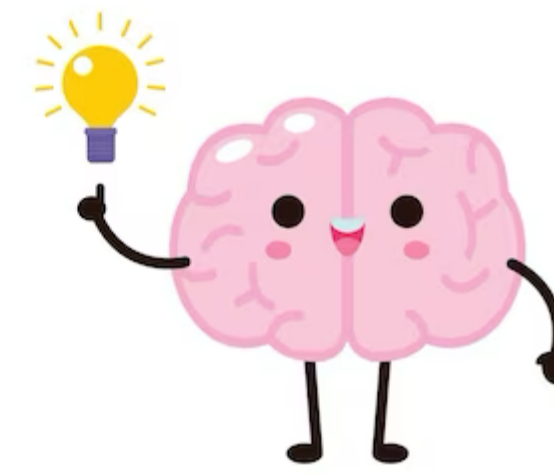
Phys.Rev.D 48 (1993) 3641-3647 [Bekenstein]

We model the BH as a point-particle interacting with the scalar field in a ST fashion

Most general effective metric that respects causality is:

$$\tilde{g}_{\mu\nu} = \underbrace{\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right]}_{\text{Conformal coupling}} g_{\mu\nu} + \underbrace{D\left(\frac{\phi}{M_{Pl}}\right) \frac{D_\mu\phi D_\nu\phi}{M_{Pl}^2 \Lambda^2}}_{\text{Disformal coupling}},$$

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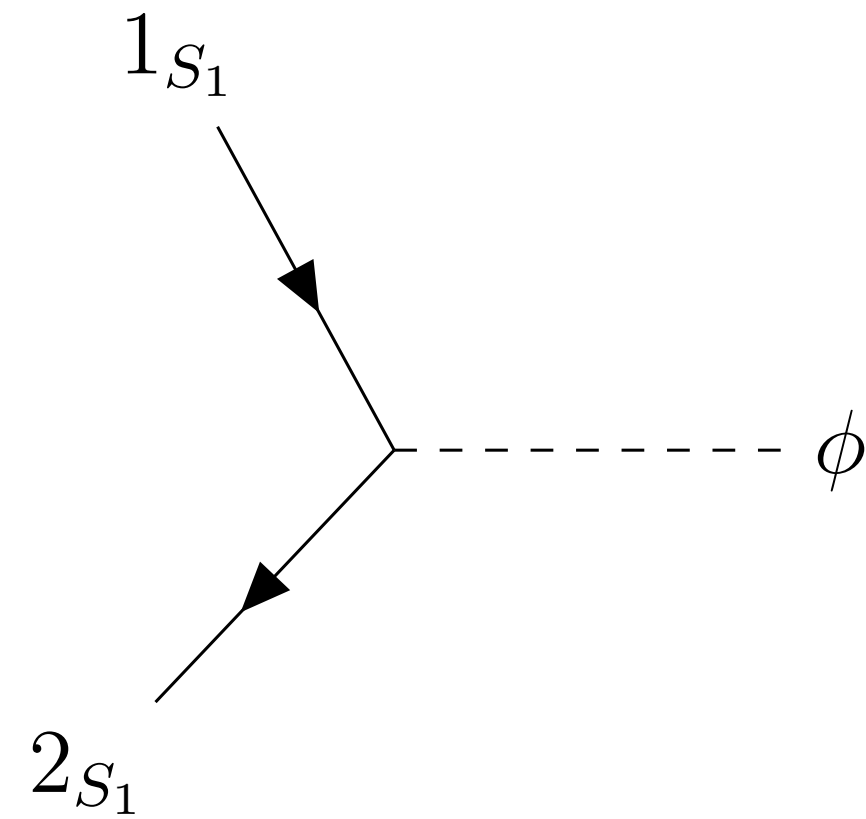
We model the BH as a point-particle interacting with the scalar field in a ST fashion

Neglect it, heavily suppressed

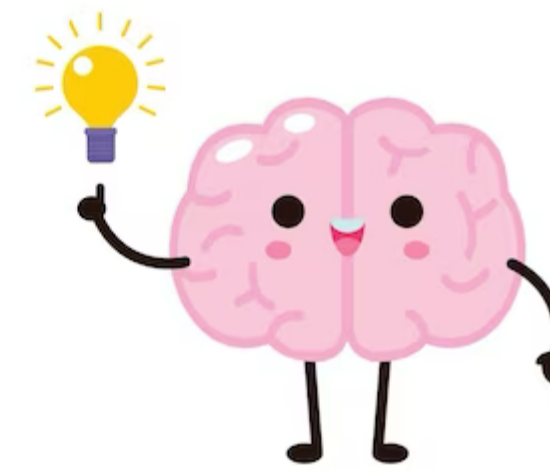
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Generate 3-point amplitudes for arbitrary spinning BH:

$$\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] \approx 1 + c \frac{\phi}{M_{Pl}}$$



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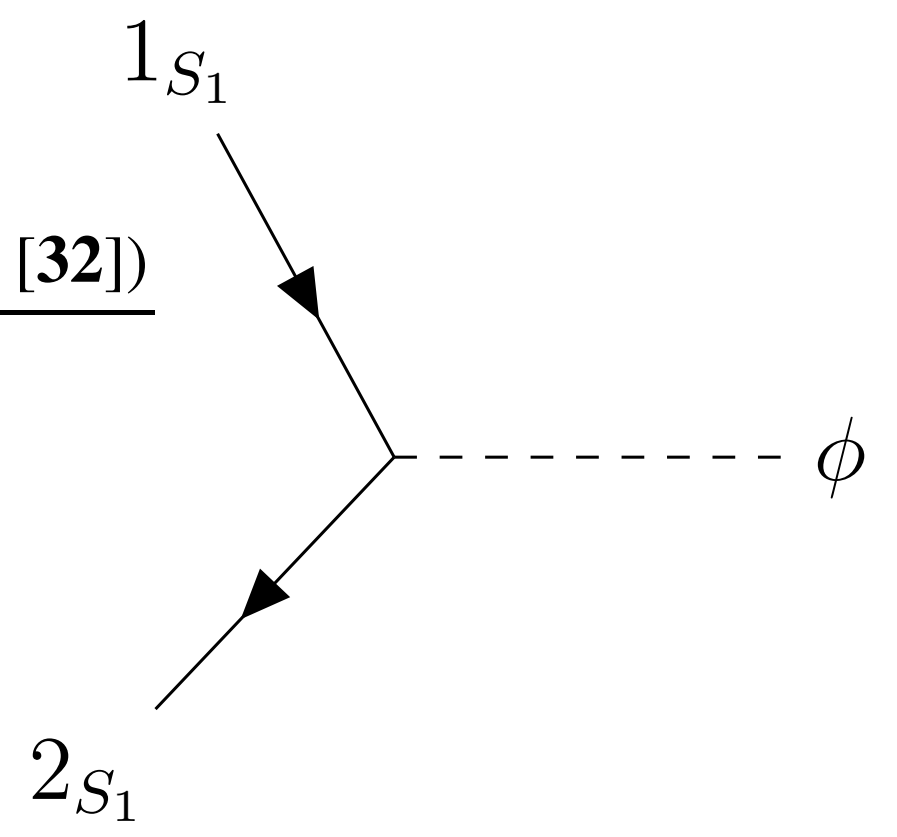
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Generate 3-point amplitudes for arbitrary spinning BH:

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$$\mathcal{M}_{3,ferm.}(1_\phi, 2_\Psi, 3_{\bar{\Psi}}) = \frac{c_1}{M_{Pl}} \frac{\langle 32 \rangle^{S-1/2} [32]^{S-1/2} (\langle 32 \rangle + [32])}{m^{2S-2}}$$



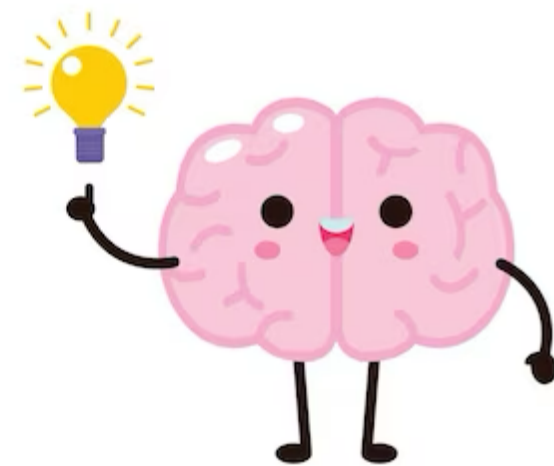
Simple mass redefinition at all spin orders:

$$m \rightarrow e^{C/2} m$$

Resemblance to skeletonized action used in GR literature:

Astrophysical Journal, vol. 196, Mar. 1, 1975, pt. 2, p. L59-L62. [Eardley]
 1992 Class. Quantum Grav. 9 2093 [Damour, Esposito-Farese]

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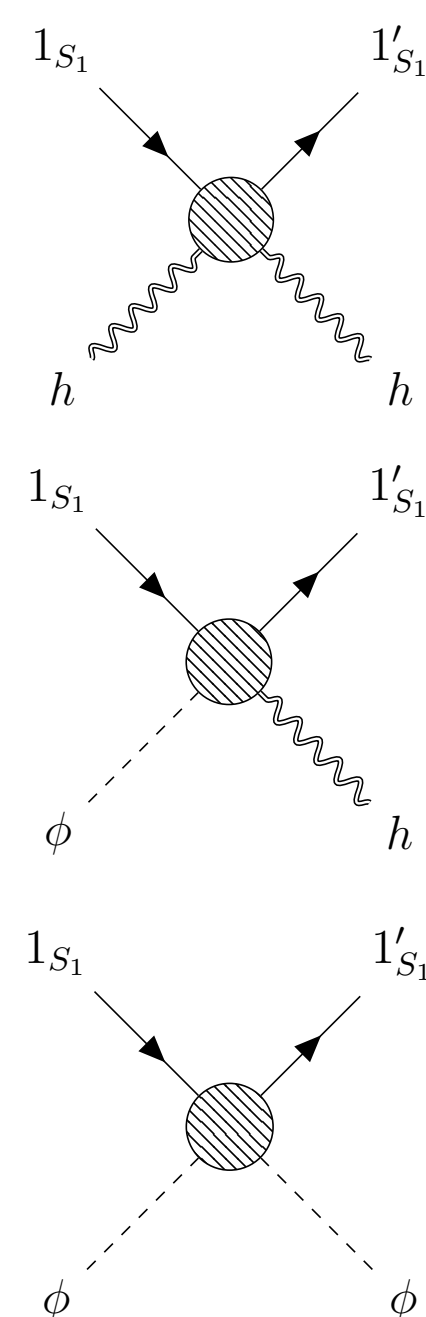
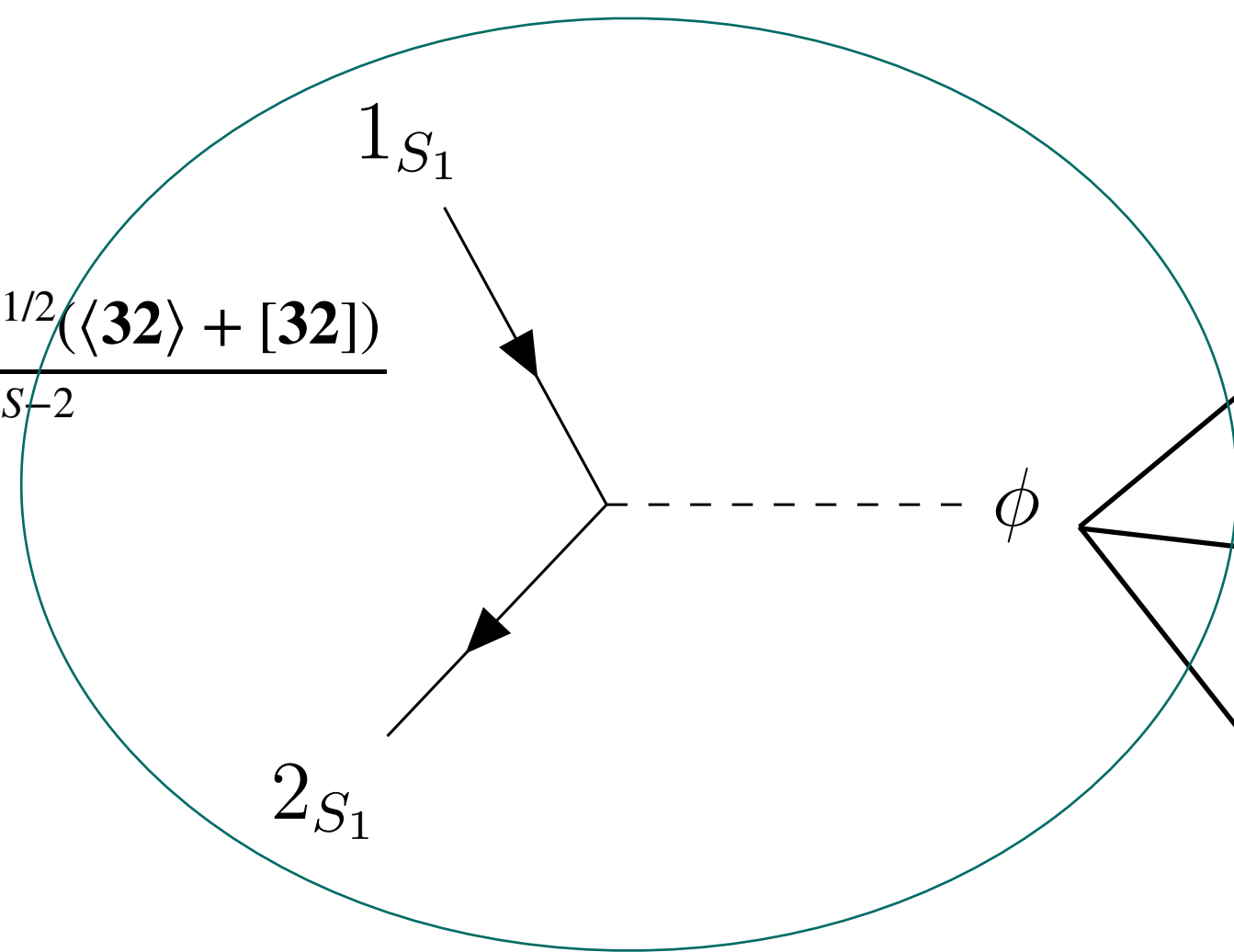
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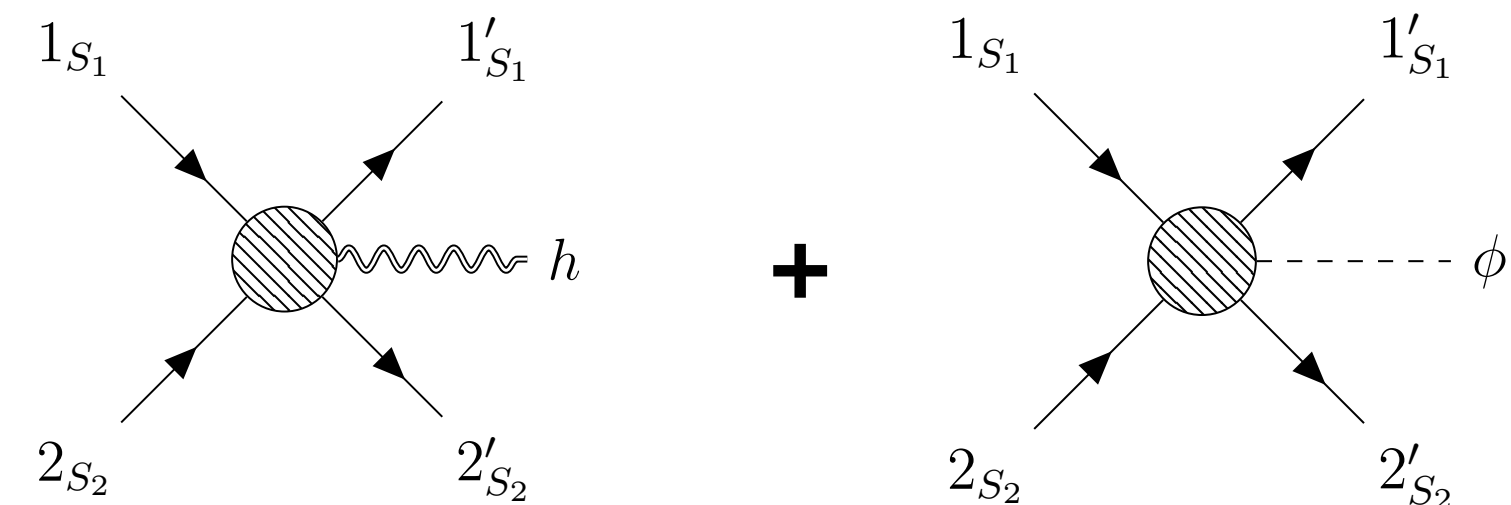
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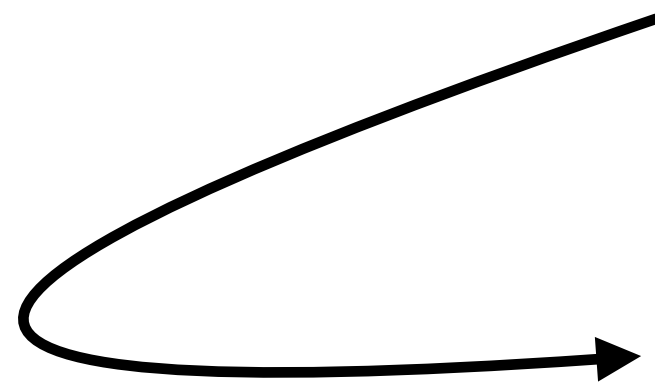
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Results:

LO Scalar Waveforms-Spinless part:

$$W_{\phi}^{(0)} = -\frac{m_1 m_2}{32\pi^2 M_{Pl}^3 \sqrt{\gamma^2 - 1} (\hat{u}_1 n)^2 b} \frac{1}{\sqrt{1 + T_1^2}} \left\{ \frac{(\gamma^2 - 1)[c_1(\hat{u}_2 n)^2 + c_2(\hat{u}_1 n)^2][\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)T_1]}{[-(\hat{u}_1 n) + y(\hat{u}_2 n) + (\tilde{b}n)T_1]^2 + (\tilde{v}n)^2(1 + T_1^2)} - \frac{c_1(\hat{u}_1 n) + (2\gamma^2 - 3)y(\hat{u}_2 n) - (2\gamma^2 - 1)(\tilde{b}n)T_1}{y^2 - 1} + \frac{C_1^{(0)}}{2} c_2(\hat{u}_1 n) \right\} + (1 \leftrightarrow 2).$$



$$\left. \frac{dP_{\phi}}{d\Omega} \right|_{\mathcal{O}(a^0)} \sim \frac{1}{b^4}$$

For closed orbits

$$\left. \frac{dP_{\phi}}{d\Omega} \right|_{\mathcal{O}(a^0)} \sim \beta^8$$

Agreement with existing PN results for SGB!

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]
 Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]



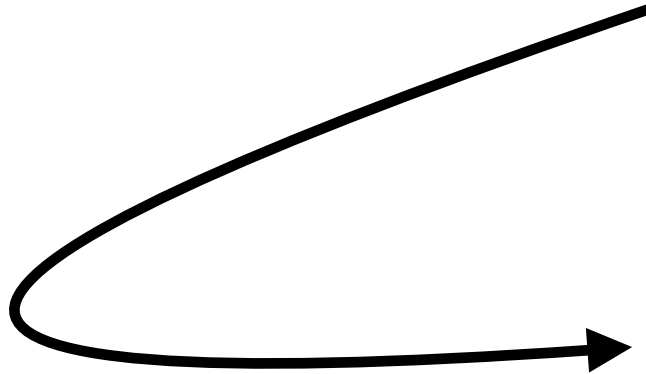
Results:

scalar “monopole” charges $c_i = c_i(m_i, a_i, \frac{\alpha}{\Lambda^2})$

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Zeroth spin order contact term



$$\frac{dP_\phi}{d\Omega} \Big|_{\mathcal{O}(a^0)} \sim \frac{1}{b^4}$$

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Results:

e.g.: $c_i \sim \alpha + \mathcal{O}(a^2)$ Phys.Rev.D 100 (2019) 10, 104061 [Julié, Berti]

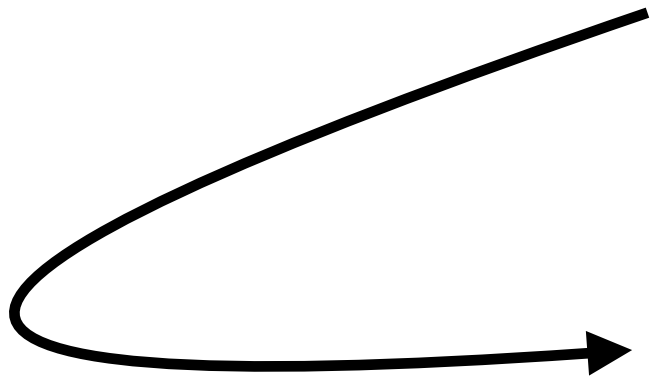
scalar "monopole" charges $c_i = c_i(m_i, a_i, \frac{\alpha}{\Lambda^2})$

Can be matched to existing results if it's secondary hair!

LO Scalar Waveforms-Spinless part:

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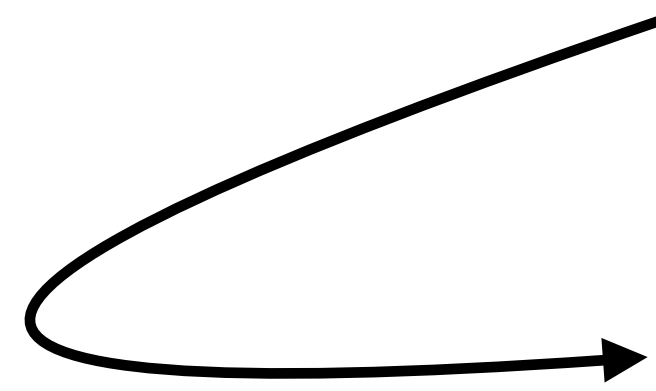
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Results:

LO Gravitational Waveforms-Spinless part:

$$W_h^{(0)} = - \frac{c_1 c_2 m_1 m_2}{512 \pi^2 M_{Pl}^3 \sqrt{\gamma^2 - 1} (\hat{u}_1 n)^2 b} \frac{1}{\sqrt{1 + T_1^2}} \operatorname{Re} \left\{ \frac{(\lambda_n [\gamma \hat{u}_2 \sigma + T_1 \tilde{b} \sigma + i \sqrt{T_1^2 + 1} \tilde{v} \sigma] \hat{u}_1 \bar{\sigma} \lambda_n)^2}{y(\hat{u}_2 n) - (\hat{u}_1 n) + T_1(\tilde{b} n) + i \sqrt{T_1^2 + 1}(\tilde{v} n)} \right\} + (1 \leftrightarrow 2).$$



$$\left. \frac{dP_h}{d\Omega} \right|_{\mathcal{O}(a^0)} \sim \frac{\beta^2}{b^4}$$

For closed orbits

$$\left. \frac{dP_h}{d\Omega} \right|_{\mathcal{O}(a^0)} \sim \beta^{10}$$

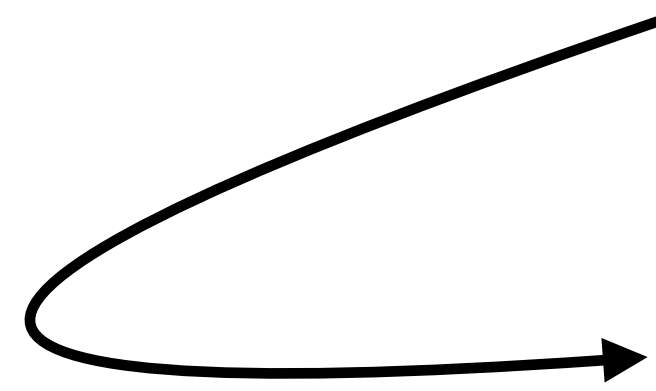
**Agreement and
expected suppression
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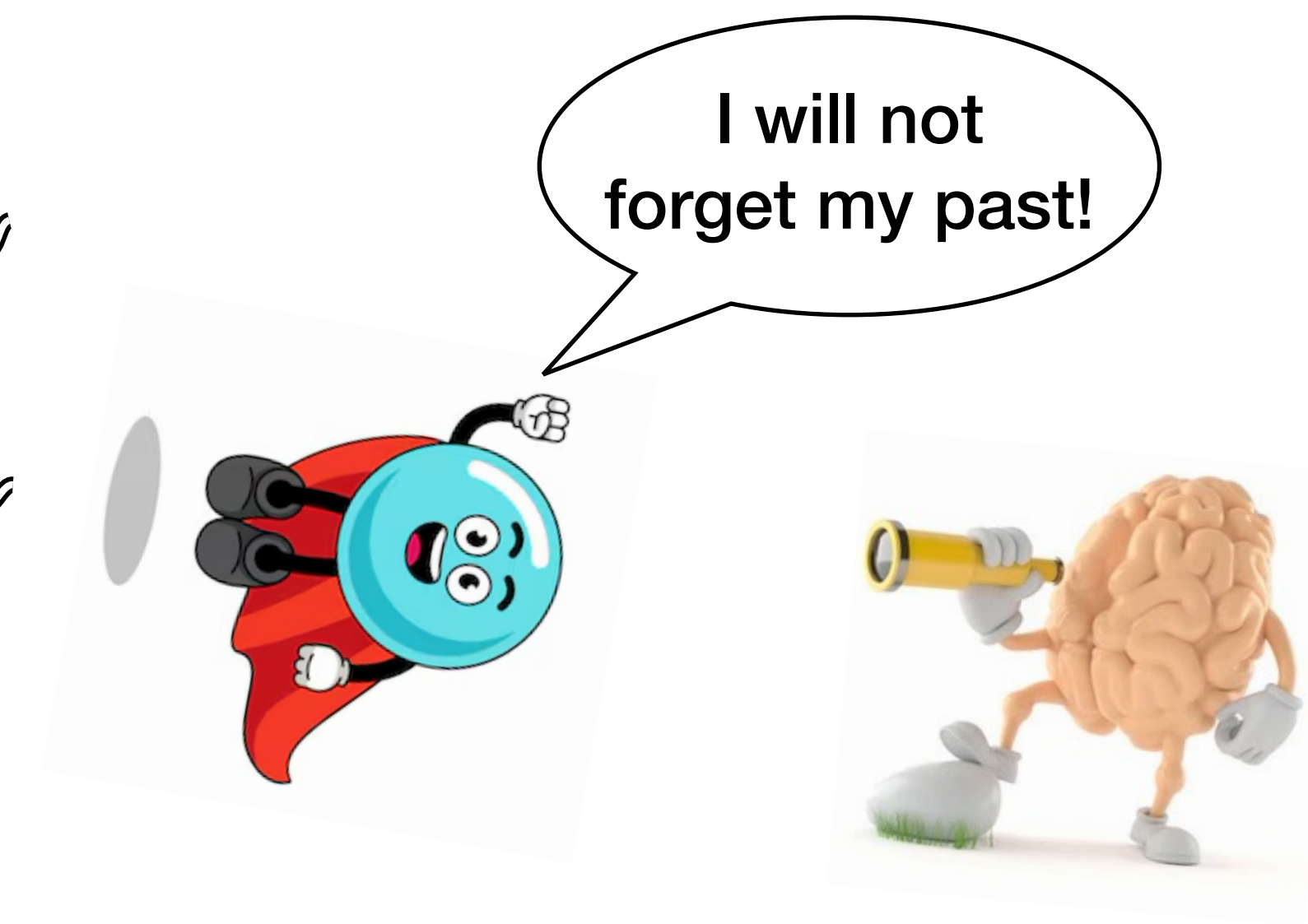
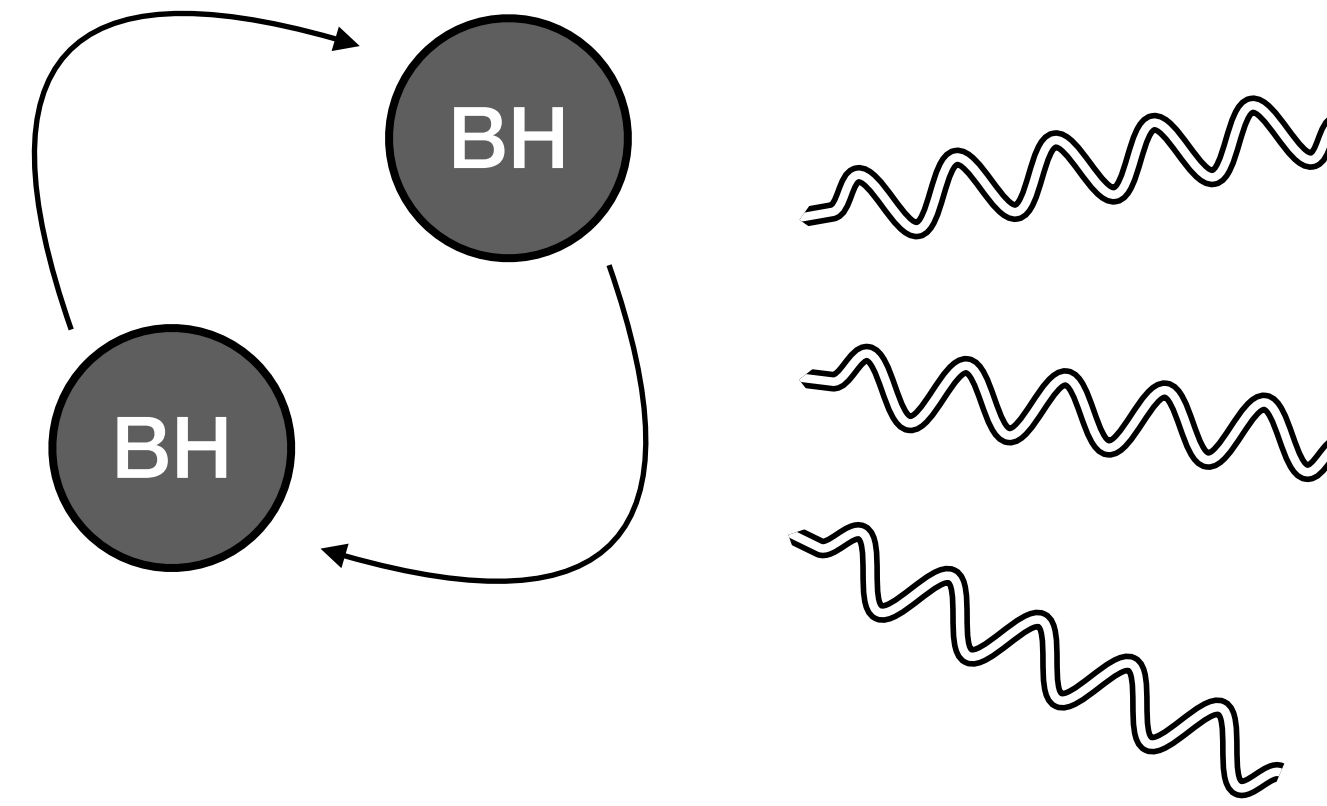
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Comments:

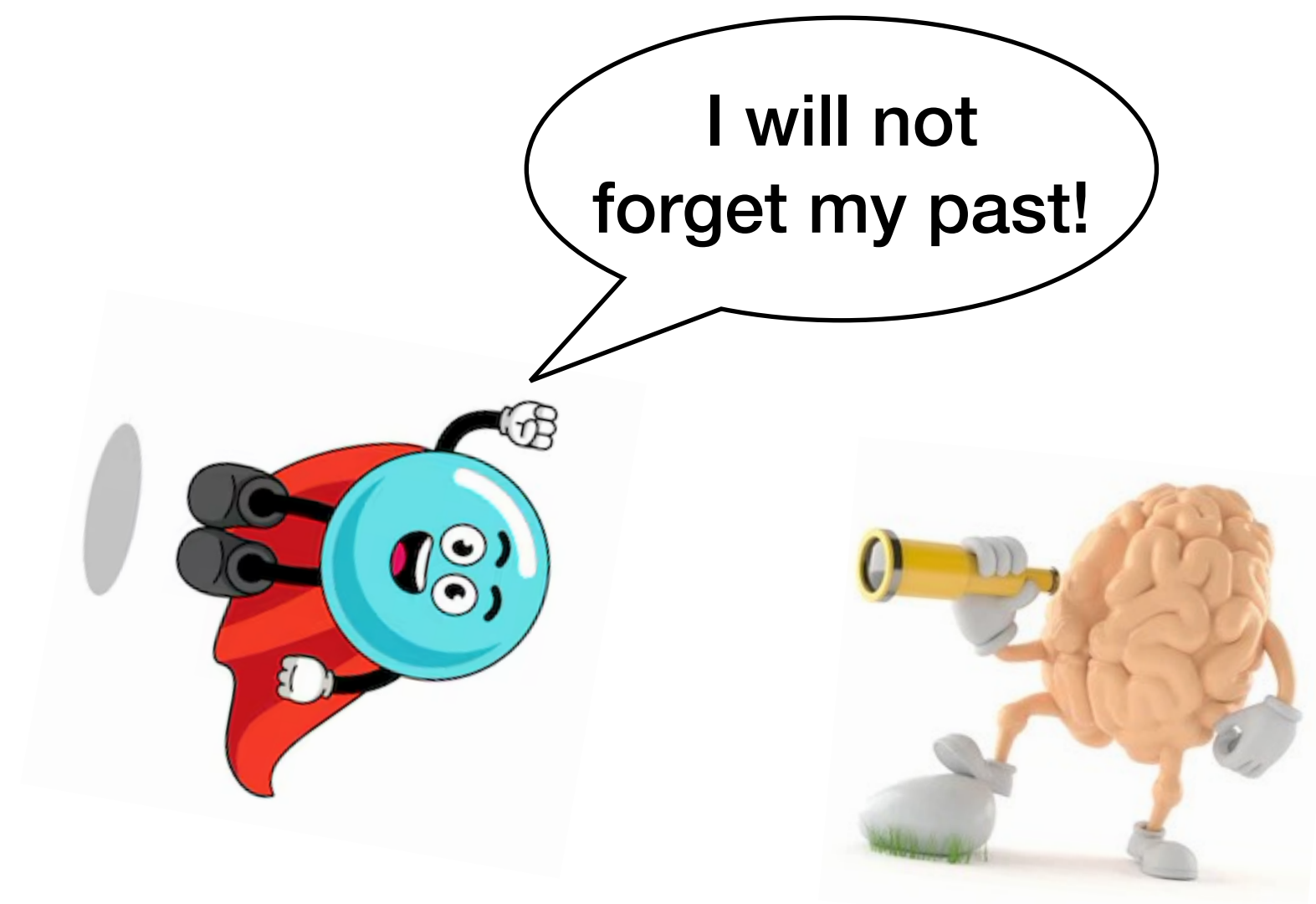
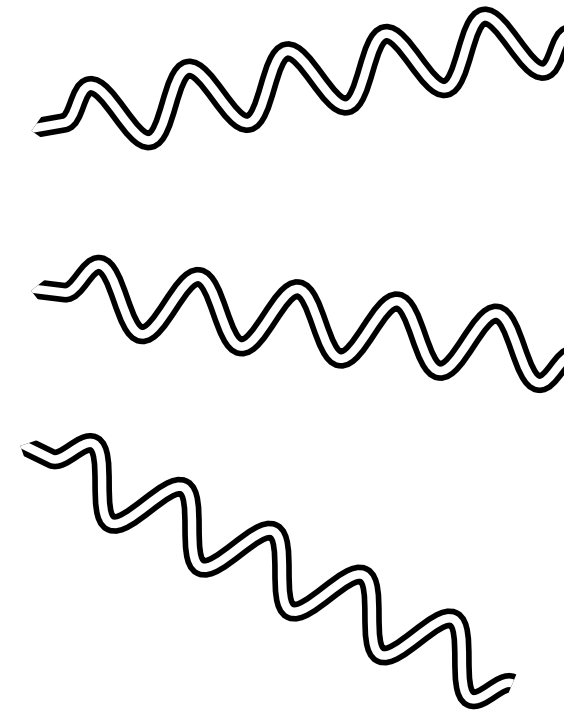
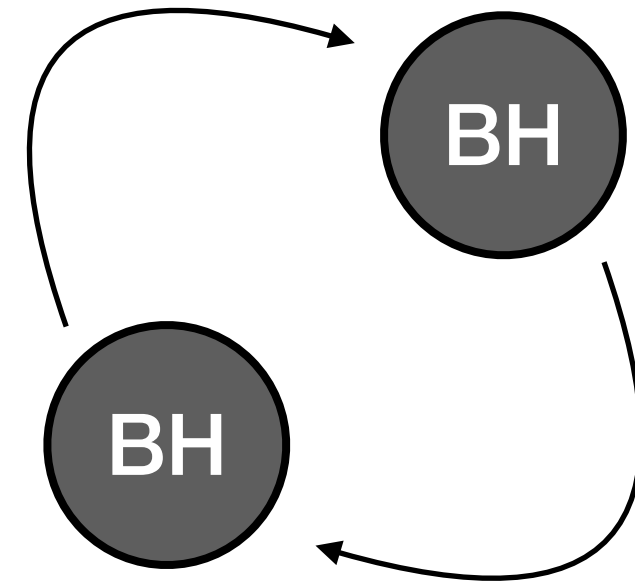
- Can show that there are no linear-in-spin corrections to $W_h^{(1)}$ at this order!
- Notice the **simplicity** of the waveform when expressed in the **spinor** language!
- In fact, we also derived that the NR **power** emitted is **exactly the same as in GR** up to setting $c_1 c_2 \rightarrow 1$
- For reference, $\left. \frac{dP_{h,GR}}{d\Omega} \right|_{\mathcal{O}(a^0)} \sim \beta^{10} \longrightarrow$ Interactions impact severely the GW signal!

Memory effect:



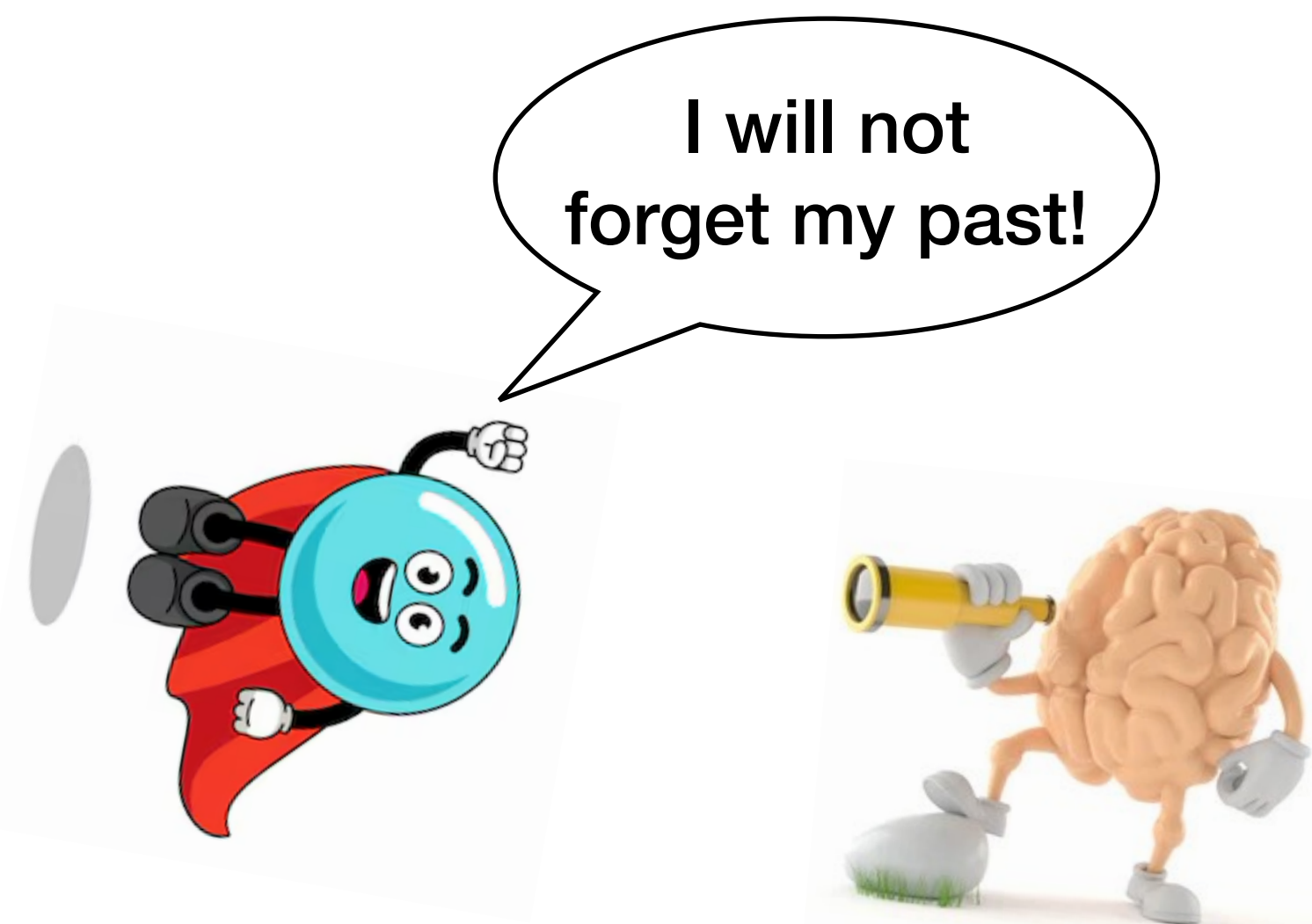
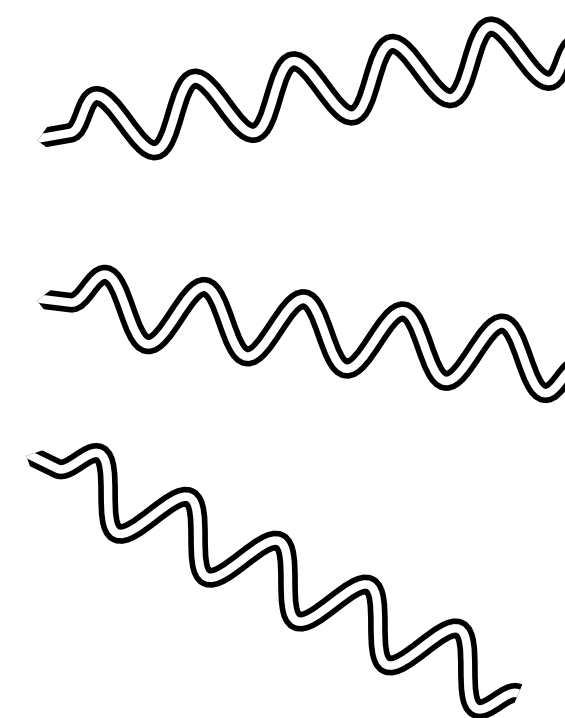
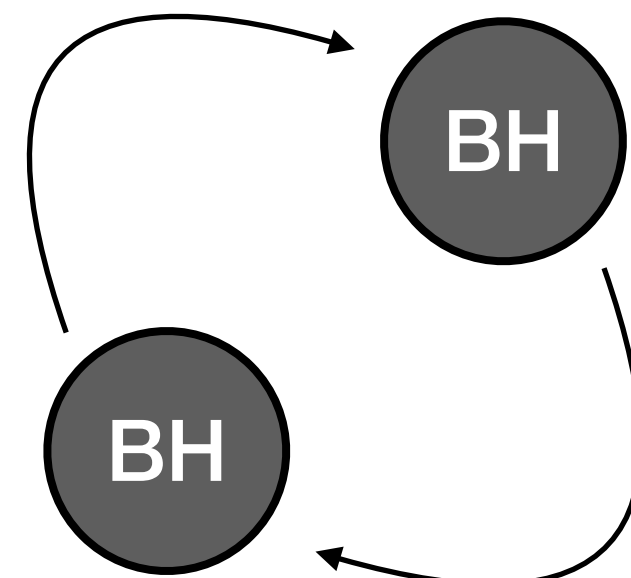
- **Scalar memory effect:** $\Delta\phi = W_\phi(t = \infty) - W_\phi(t = -\infty)$
- **Gravitational memory effect:** $\Delta h = h(t = \infty) - h(t = -\infty)$

Memory effect:



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- } At LO, can be related to the soft behaviour of the 5-point amplitude
- $\mathcal{M}_5 \rightarrow \text{Soft} \times \mathcal{M}_4$

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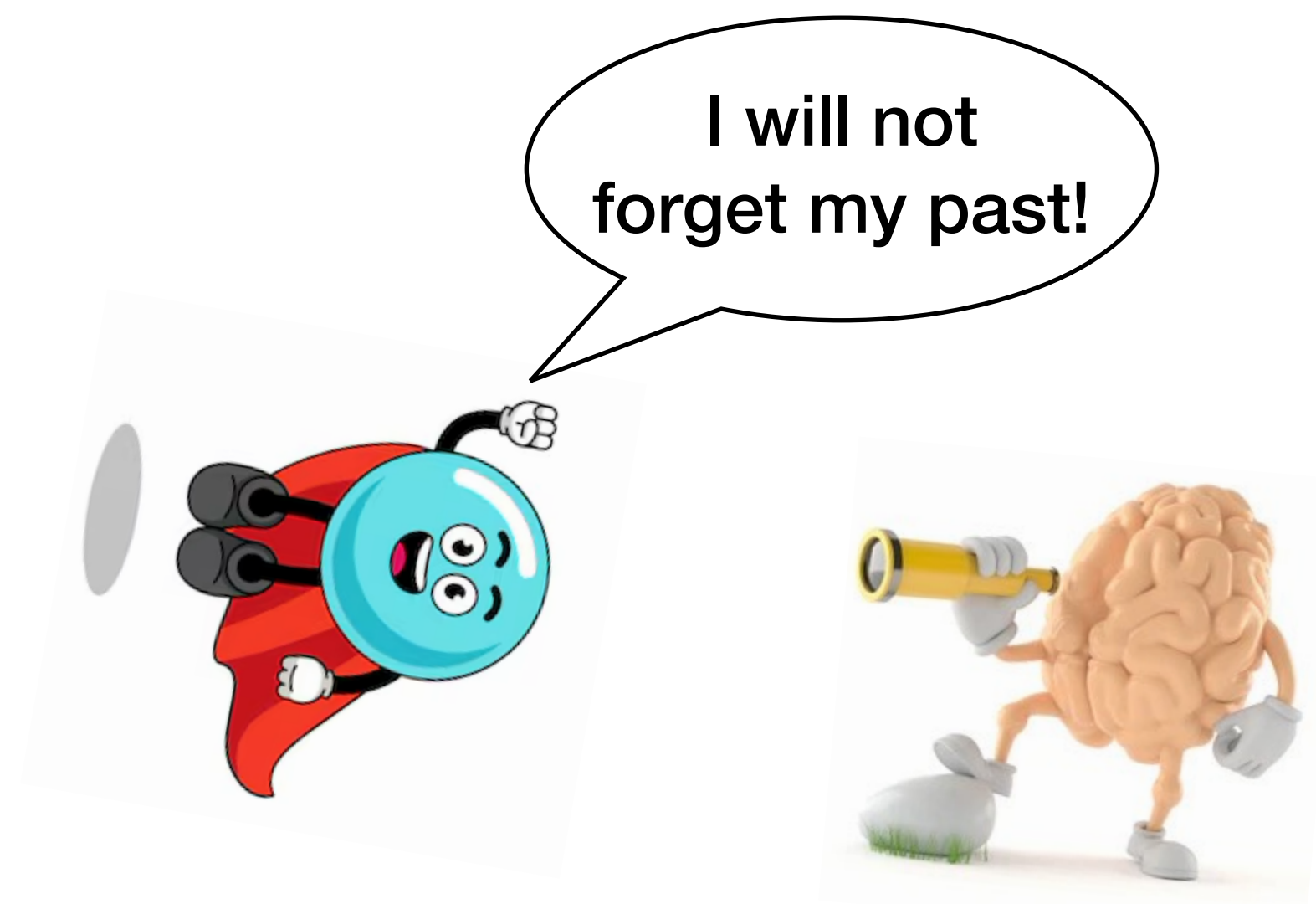
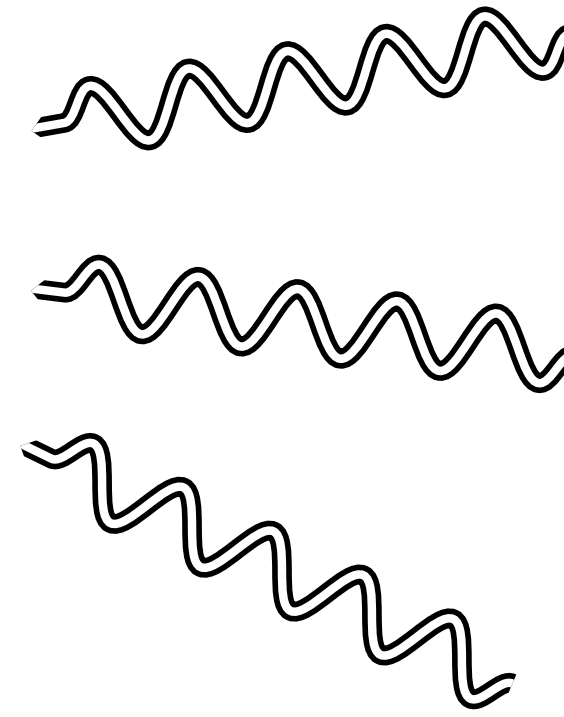
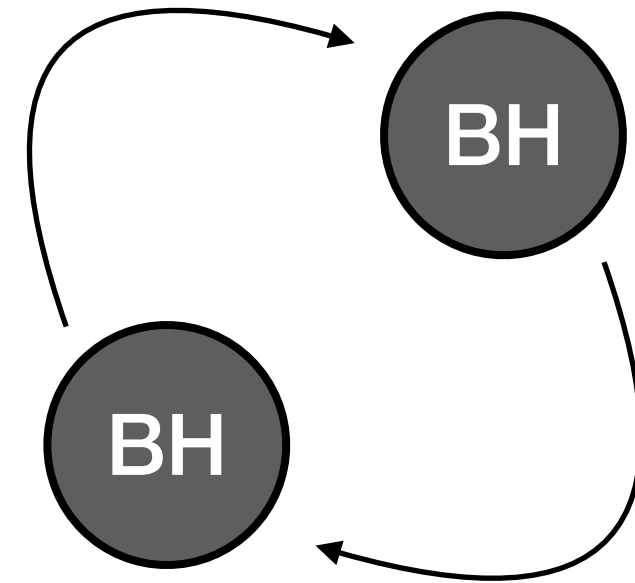
→ **No-hair objects** have **no leading order memory!**

Reason: There are no modifications to the 4-point amplitudes at tree-level.

A more general understanding: Higher order interactions essentially contribute factors of $\frac{\partial^n}{\Lambda^n}$ in the action. At the level of the 5-point Amplitude, this translates to:

$$\frac{\partial^n}{\Lambda^n} \rightarrow \frac{k^n}{\Lambda^n} \sim \frac{\omega^n}{\Lambda^n}, \text{ for } k^\mu = \omega n^\mu \rightarrow \text{kills the amplitude in the soft limit}$$

Memory effect:



→ Hairy objects do have leading order memory!

→ The gravitational memory effect can be elegantly expressed in terms of the classical soft factor:

$$\Delta h = - \frac{c_1 c_2 m_1 m_2}{4\pi M_{Pl}^3 \sqrt{\gamma^2 - 1}} \frac{1}{|b|} S_{W,s}^{cl}(n, \tilde{b})$$

$$\mathcal{M}_{ST}^{(0)} [1^{S_1} 2^{S_2} 3^{S_1} 4^{S_2}]$$

$\Phi_1 \quad \Phi_2 \quad \bar{\Phi}_1 \quad \bar{\Phi}_2$

$$S_{W,s}^{cl}(k, q) = \epsilon_{\mu\nu,s} \left[\frac{p_1^\mu q^\nu + p_1^\nu q^\mu}{p_1 k} - \frac{p_1^\mu p_1^\nu (qk)}{(p_1 k)^2} \right] + (1 \leftrightarrow 2)$$

→ Scalar memory trivially computed: $\Delta\phi = \frac{m_1 m_2 (2\gamma^2 - 1) (\tilde{b}n) \left[c_1 (\hat{u}_2 n)^2 - c_2 (\hat{u}_1 n)^2 \right]}{32\pi^2 M_{Pl}^3 (\gamma^2 - 1)^{3/2} (\hat{u}_1 n)^2 (\hat{u}_2 n)^2 b}$

Another perspective: The scalar-charge toy model

Recent work has been using a scalar-charge toy model in order to compare the PM and SF approaches.

$$\mathcal{L} \supset \sqrt{-g} \left[-2\sqrt{\pi} m_1 Q \psi \phi_1^2 \right]$$

Phys.Rev.D 108 (2023) 2, 024025 [Barack, Bern, Herrmann, Long, Parra-Martinez, Roiban, Ruf, Shen, Solon, Teng, Zeng]

Comparison of post-Minkowskian and self-force expansions: Scattering in a scalar charge toy model

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Abstract

We compare numerical self-force results and analytical fourth-order post-Minkowskian (PM) calculations for hyperbolic-type scattering of a point-like particle carrying a scalar charge Q off a Schwarzschild black hole, showing a remarkably good agreement. Specifically, we numerically compute the scattering angle including the full $\mathcal{O}(Q^2)$ scalar-field self-force term (but ignoring the gravitational self-force), and compare with analytical expressions obtained in a PM framework using scattering-amplitude methods. This example provides a nontrivial, high-precision test of both calculation methods, and illustrates the complementarity of the two approaches in the context of the program to provide high-precision models of gravitational two-body dynamics. Our PM calculation is carried out through 4PM order, i.e., including all terms through $\mathcal{O}(Q^2 G^3)$. At the fourth post-Minkowskian order the point-particle description involves two a-priori undetermined coefficients, due to contributions from tidal effects in the model under consideration. These coefficients are chosen to align the post-Minkowskian results with the self-force ones.

arXiv:2304.09200v2 [hep-th] 12 Jul 2023

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$$\mathcal{L} \supset \sqrt{-g} \left[\underbrace{-2\sqrt{\pi} m_1 Q}_{\text{Related to our scalar charges } c_i} \psi \phi_1^2 \right]$$

scalar Matter fields

→ The same type of coupling we have!

Recall: $m \rightarrow e^{C/2} m$ for any spin!

Our framework can be **mapped** to this model (in the limit of $c_2 \rightarrow 0$) and potentially **extend the results** in a consistent manner to incorporate effects that have not been compared before, e.g. **spin effects, gravitational waveform corrections, ...**

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4. Outlook



- **Scattering Amplitudes techniques** can be proven to be extremely **useful** in the quest for precision measurements **in the GW era**, probing results to all orders in velocity.
- Computations for **spinning binaries** remarkably **simplify** in the **on-shell** language.
- Recasting already known problems in the amplitudes' language makes the **search for beyond GR effects easier to handle** and essentially the **usual QFT methods can be used**.

4. Outlook



- **Scattering Amplitudes techniques** can be proven to be extremely **useful** in the quest for precision measurements **in the GW era**, probing results to all orders in velocity.
- Computations for **spinning binaries** remarkably **simplify** in the **on-shell** language.
- Recasting already known problems in the amplitudes' language makes the **search for beyond GR effects easier to handle** and essentially the **usual QFT methods can be used**.

What's next?

- Employing similar methods to **study GR (and beyond) effects**, where current PN/NR results are poor, e.g. dipole hair in DCS.
- Could we employ our formalism to **effectively describe new Physics** related to **scalar particles** from GWs (boson clouds around BHs, soliton and boson stars, light axion-like particles, ...) ?
- Dive more deeply into the synergy of QFT and GR and consider other applications as well, e.g. emergence of **couplings in ST theories from integrating out arbitrary spinning heavy particles** - use of **generalized unitarity and analytic properties of the amplitude** (work in progress).

The universe seems to be extremely loud!



The universe seems to be extremely loud!



Thank you for your attention!:)



Artwork by Penelope Cowley*

Backup slides

Integrals and variables in waveform parametrization:

Single-pole integrals

$$I_2^{\mu_1 \dots \mu_n} = \int \hat{d}w_1 \hat{d}w_2 \hat{\delta}(u_1 w_1) \hat{\delta}(u_2 w_2) \hat{\delta}^{(4)}(w_1 + w_2 - k) \frac{e^{i(b_1 w_1 + b_2 w_2)}}{w_2^2} w_2^{\mu_1} \dots w_2^{\mu_n}$$

$$= e^{ib_1 k} \int \hat{d}^4 q \hat{\delta}(u_1 \cdot q - u_1 \cdot k) \hat{\delta}(u_2 \cdot q) \frac{e^{-iq \cdot (b_1 - b_2)}}{q^2} q^{\mu_1} \dots q^{\mu_n}$$

Double-pole integrals

$$J_2^{\mu_1 \dots \mu_n} = \int \hat{d}w_1 \hat{d}w_2 \hat{\delta}(u_1 w_1) \hat{\delta}(u_2 w_2) \hat{\delta}^{(4)}(w_1 + w_2 - k) \frac{e^{i(b_1 w_1 + b_2 w_2)}}{w_2^2 w_1^2} w_2^{\mu_1} \dots w_2^{\mu_n}$$

$$= e^{ib_1 k} \int \hat{d}^4 q \hat{\delta}(u_1 \cdot q - u_1 \cdot k) \hat{\delta}(u_2 \cdot q) \frac{e^{-iq \cdot (b_1 - b_2)}}{q^2 (k - q)^2} q^{\mu_1} \dots q^{\mu_n}$$

Parametrization: $q^\mu = z_1 u_1^\mu + z_2 u_2^\mu + z_v \tilde{v}^\mu + z_b \tilde{b}^\mu$

$$v^\mu \equiv \varepsilon^{\mu\alpha\beta\rho} u_{1\alpha} u_{2\beta} \tilde{b}_\rho$$

$$\tilde{v}^\mu = \frac{v^\mu}{\sqrt{-v^2}}$$

$$b \equiv \sqrt{-(b_1 - b_2)^2}$$

$$\tilde{b}^\mu = \frac{b_1^\mu - b_2^\mu}{b}$$

$$\hat{u}_i = \frac{u_i}{\sqrt{\gamma^2 - 1}}$$

Then follow a residue approach following the logic of

Phys.Rev.D 110 (2024) 4, L041502 [De Angelis, Novichkov, Gonzo]

Integrals and variables in waveform parametrization:

$$I_2^{\mu_1 \dots \mu_n} = -\frac{\pi e^{ib_1 k} (\hat{u}_1 k)^n}{\sqrt{\gamma^2 - 1}} \int_{-\infty}^{\infty} \frac{dz e^{i(\hat{u}_1 k)bz}}{\sqrt{z^2 + 1}} \operatorname{Re} \left\{ [\gamma \hat{u}_2^{\mu_1} - \hat{u}_1^{\mu_1} + z \tilde{b}^{\mu_1} + i\sqrt{z^2 + 1} \tilde{v}^{\mu_1}] [\mu_1 \rightarrow \mu_2] \dots [\mu_1 \rightarrow \mu_n] \right\}.$$

$$J_2^{\mu_1 \dots \mu_n} = \frac{\pi}{2\sqrt{\gamma^2 - 1}} \int_{-\infty}^{\infty} \frac{dz}{\sqrt{z^2 + 1}} \left\{ (\hat{u}_1 k)^{n-1} e^{ib_1 k + i(\hat{u}_1 k)bz} \operatorname{Re} \left[\frac{[\gamma \hat{u}_2^{\mu_1} - \hat{u}_1^{\mu_1} + z \tilde{b}^{\mu_1} + i\sqrt{z^2 + 1} \tilde{v}^{\mu_1}] \dots [\mu_1 \rightarrow \mu_n]}{\gamma(\hat{u}_2 k) - (\hat{u}_1 k) + (\tilde{b}k)z + i(\tilde{v}k)\sqrt{z^2 + 1}} \right] \right. \\ \left. + (\hat{u}_2 k)^{n-1} e^{ib_2 k + i(\hat{u}_2 k)bz} \operatorname{Re} \left[\frac{\left[\frac{k^{\mu_1}}{\hat{u}_2 k} + \hat{u}_2^{\mu_1} - \gamma \hat{u}_1^{\mu_1} + z \tilde{b}^{\mu_1} - i\sqrt{z^2 + 1} \tilde{v}^{\mu_1} \right] \dots [\mu_1 \rightarrow \mu_n]}{\gamma(\hat{u}_1 k) - (\hat{u}_2 k) - (\tilde{b}k)z + i(\tilde{v}k)\sqrt{z^2 + 1}} \right] \right\}.$$

Resolvability:

$$\alpha_g \sim \frac{m^2}{M_{Pl}^2} = mR_s$$

Scalar radiation for no-hair objects is resolvable if: $\alpha_g \left(\frac{R_s}{b}\right)^2 \frac{1}{\Lambda^4 b^4} \gg 1$,

For example, for $\Lambda \simeq 2 \times 10^{-18} GeV$ and $m \simeq M_\odot \simeq 10^{57} \sim GeV$ (corresponding to $R_s \simeq 1.5 \times 10^{19} GeV^{-1}$) we have $b_{\max} \sim 10^{11} R_s$

Linear in spin order effects for the same observable: $\alpha_g \left(\frac{R_s}{b}\right)^2 \frac{1}{\Lambda^4 b^4} \left(\frac{S}{bm}\right) \gg 1$

For our benchmark point, $m \sim M_\odot, \Lambda \sim 0.1 km^{-1}$, this implies $S \gtrsim 10^6 (b/R_s)^7$, with $S_{\max} \sim 10^{76}$ for Kerr black holes in GR.

More results:

The scalar waveform for dynamical Chern-Simons:

$$\begin{aligned}
 W_{\tilde{\phi}} = & \frac{m_1 m_2}{8\pi^2 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 b^3} \left(2\tilde{\alpha}(\tilde{v}n) \frac{d^2}{dz^2} \left\{ \frac{1}{\sqrt{z^2+1}} \left[\gamma z - (\gamma^2 - 1)(\hat{u}_2 n) \frac{z[\gamma(\hat{u}_2 n) - (\hat{u}_1 n)] - (\tilde{b}n)}{[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z]^2 + (\tilde{v}n)^2(z^2+1)} \right] \right\} \right. \\
 & + \frac{\tilde{\alpha}}{b\sqrt{\gamma^2-1}} \text{Re} \frac{d^3}{dz^3} \left\{ \frac{1}{\sqrt{z^2+1}} \left(\frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1 \hat{u}_2) - (a_2 \hat{u}_1) + (a_2^A - a_1^A) [z\tilde{b}^A + i\sqrt{z^2+1}\tilde{v}^A] \right) \left(\frac{2(\gamma^2-1)^2(\hat{u}_2 n)^2}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)z + i(\tilde{v}n)\sqrt{z^2+1}} - (\hat{u}_1 n) - \gamma(2\gamma^2-3)(\hat{u}_2 n) + (2\gamma^2-1)[z(\tilde{b}n) + i\sqrt{z^2+1}(\tilde{v}n)] \right) \right\} \\
 & \left. - \frac{1}{\sqrt{\gamma^2-1}} \frac{\tilde{C}_1 a_1^A}{b} \frac{d^3}{dz^3} \left\{ \frac{1}{\sqrt{z^2+1}} \left[(2\gamma^2-1)[z^2(\tilde{b}n)\tilde{b}^A - (z^2+1)(\tilde{v}n)\tilde{v}^A] - (\gamma^2-1)n^A + \gamma(\gamma^2-2)(\hat{u}_1 n)\hat{u}_2^A - (\gamma^2-2)(\hat{u}_2 n)\hat{u}_1^A + z\gamma(\tilde{b}n)\hat{u}_2^A - z\gamma^2(\hat{u}_1 n)\tilde{b}^A + z\gamma(\hat{u}_2 n)\tilde{b}^A \right] \right\} \right) \Big|_{z=T_1} + (1 \leftrightarrow 2) + \mathcal{O}(a^2).
 \end{aligned}$$

LO Scalar Waveforms for CC coupling-Spinning part:

$$\begin{aligned}
 W_{\phi}^{(1)} = & \frac{m_1 m_2}{32\pi^2 M_{Pl}^3 (\hat{u}_1 n)^2 b^2} \frac{\partial}{\partial z} \left[\frac{1}{\sqrt{z^2+1}} \text{Re} \left\{ c_1 [(\tilde{v}n)z - is_1(\tilde{b}n)\sqrt{z^2+1}] [- (\hat{u}_1 a_2) + z(\tilde{b}a_2) + is_1\sqrt{z^2+1}(\tilde{v}a_2)] \left(\frac{\gamma}{\gamma^2-1} - \frac{(\hat{u}_2 n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b}n) + is_1\sqrt{z^2+1}(\tilde{v}n)} \right) \right. \right. \\
 & \left. \left. - \frac{c_2(\hat{u}_1 n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b}n) + is_1\sqrt{z^2+1}(\tilde{v}n)} \left\{ [is_1(\tilde{b}n)\sqrt{z^2+1} - (\tilde{v}n)z](\hat{u}_2 a_1) + [\gamma(\tilde{v}n) + is_1[\gamma(\hat{u}_1 n) - (\hat{u}_2 n)]\sqrt{z^2+1}](\tilde{b}a_1) + [[(\hat{u}_2 n) - \gamma(\hat{u}_1 n)]z - \gamma(\tilde{b}n)](\tilde{v}a_1) \right\} \right\} \right] + (1 \leftrightarrow 2).
 \end{aligned}$$

More results:

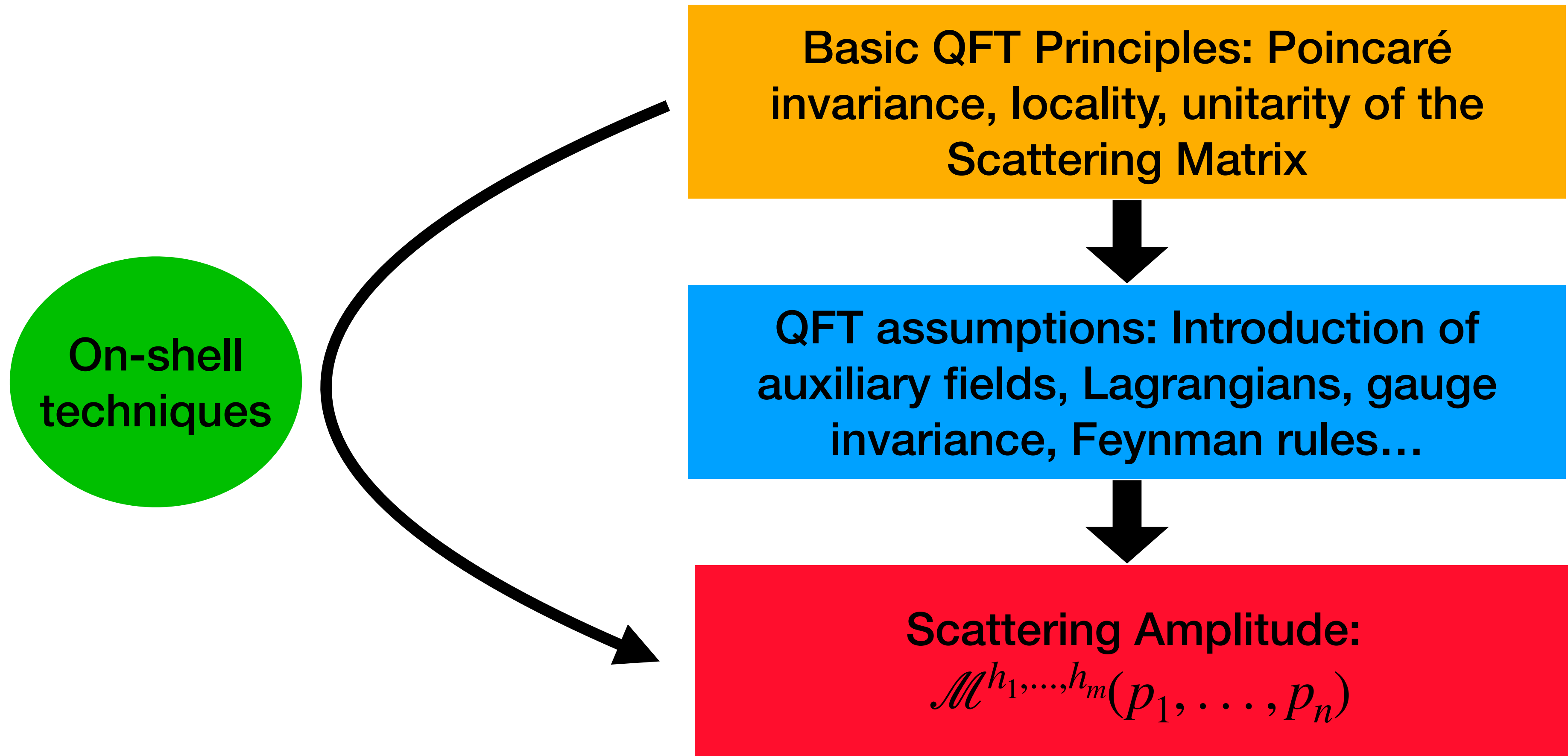
LO Scalar Waveforms from non-conformal couplings:

$$W_{non-conf,\phi}^{(1)} = \frac{m_1 m_2}{32\pi^2 M_{Pl}^3 (\hat{u}_1 n)^2 b^2} \frac{\partial}{\partial z} \left[\frac{1}{\sqrt{z^2 + 1}} \text{Re} \left\{ c_1 [(\tilde{v}n)z - is_1(\tilde{b}n)\sqrt{z^2 + 1}] [-(\hat{u}_1 a_2) + z(\tilde{b}a_2) + is_1\sqrt{z^2 + 1}(\tilde{v}a_2)] \left(\frac{\gamma}{\gamma^2 - 1} - \frac{(\hat{u}_2 n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b}n) + is_1\sqrt{z^2 + 1}(\tilde{v}n)} \right) \right. \right. \\ \left. \left. - \frac{c_2(\hat{u}_1 n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b}n) + is_1\sqrt{z^2 + 1}(\tilde{v}n)} \left\{ [is_1(\tilde{b}n)\sqrt{z^2 + 1} - (\tilde{v}n)z](\hat{u}_2 a_1) + [\gamma(\tilde{v}n) + is_1[\gamma(\hat{u}_1 n) - (\hat{u}_2 n)]\sqrt{z^2 + 1}](\tilde{b}a_1) + [(\hat{u}_2 n) - \gamma(\hat{u}_1 n)]z - \gamma(\tilde{b}n)](\tilde{v}a_1) \right\} \right\} \right] + (1 \leftrightarrow 2)$$

LO Gravitational Waveforms from non-conformal couplings:

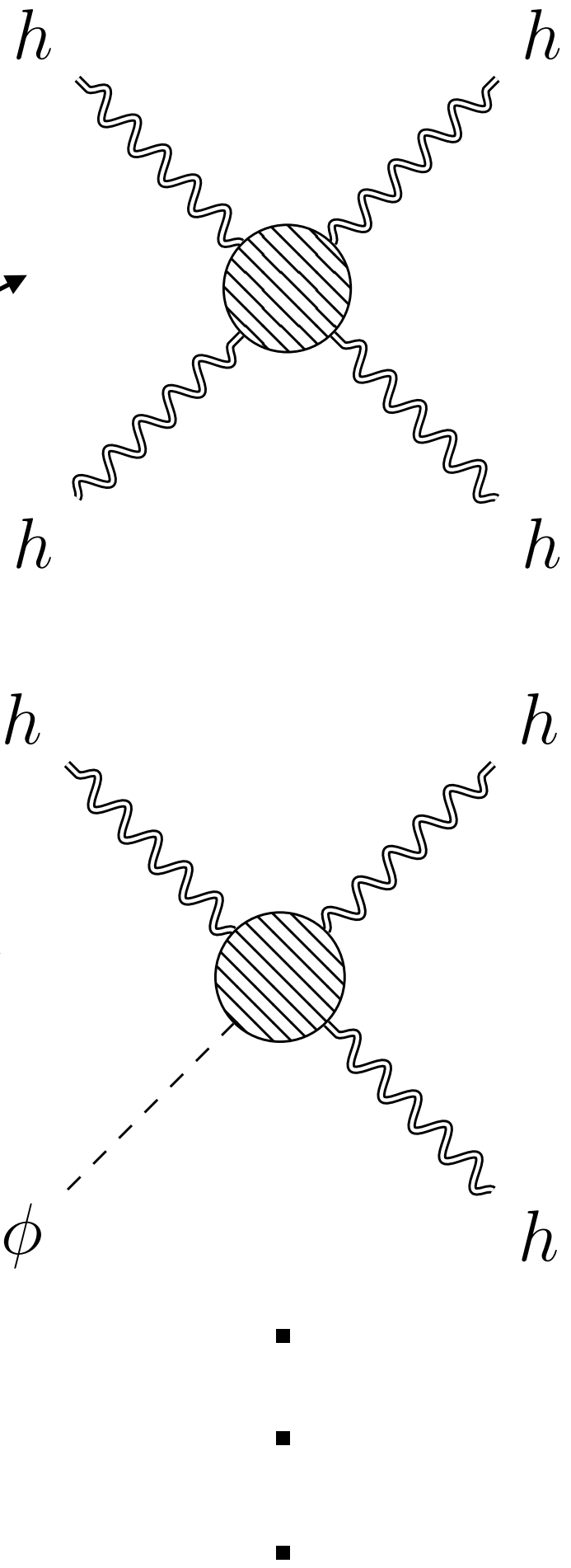
$$W_{non-conf,h}^{(1)} = - \frac{C_d c_2 m_1 m_2 \varepsilon^{\mu\nu\rho\alpha}}{1024\pi^2 M_{Pl}^3 \sqrt{\gamma^2 - 1}} \frac{\partial}{\partial z} \left\{ \frac{1}{\sqrt{z^2 + 1}} \text{"Re"} \left\{ \frac{(\lambda_n [\gamma \hat{u}_{2\mu} + z \tilde{b}_\mu + i\sqrt{z^2 + 1} \tilde{v}_\mu] \hat{u}_{1\nu} \sigma_\alpha \bar{\sigma}_\beta \lambda_n)}{-\hat{u}_1 n + \gamma(\hat{u}_2 n) + z(\tilde{b}n) + i\sqrt{z^2 + 1}(\tilde{v}n)} \right. \right. \\ \left. \left. \times (\lambda_n [\gamma \hat{u}_2 \sigma + z \tilde{b} \sigma + i\sqrt{z^2 + 1} \tilde{v} \sigma] (\hat{u}_1 \bar{\sigma}) \lambda_n) \left[\frac{(n a_1)}{(\hat{u}_1 n)} - \gamma(\hat{u}_2 a_1) - z(\tilde{b} a_1) - i\sqrt{z^2 + 1}(\tilde{v} a_1) \right] \right\} \right\} \Big|_{z=T_1} + + (1 \leftrightarrow 2).$$

On-shell techniques:



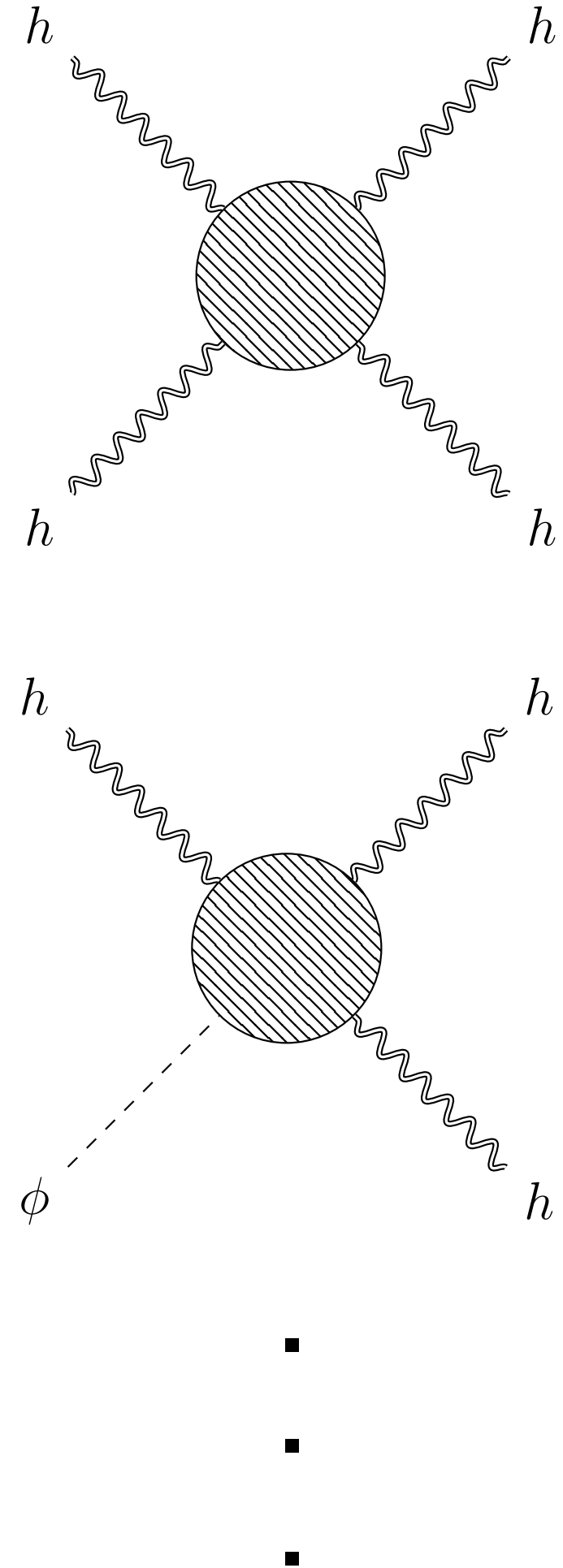
EFT matching and heavy spinning particles:

IR



tree-level diagrams
through the SGB/
DCS interactions

UV



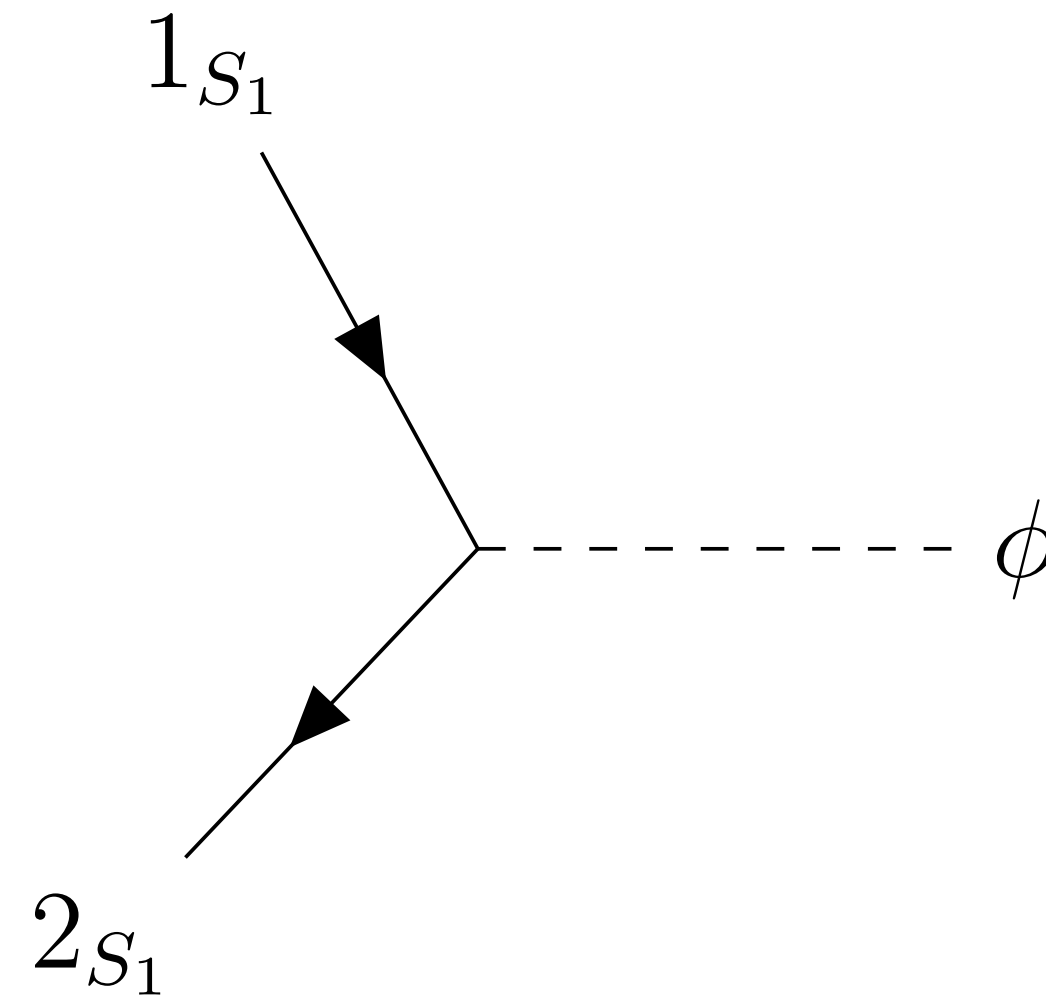
matter running
inside the loops

Match amplitudes' discontinuities
in different channels!



Use of generalized unitarity
makes the treatment systematic
and simple

EFT matching and heavy spinning particles:



How should the matter couple in the UV to produce this interaction?

→ Interaction has to produce the **shift-symmetry** we see in the IR!

Simple in the **on-shell** language: $\mathcal{M}_3^{\text{SS}}[1_\phi, 2_\Phi, 3_{\bar{\Phi}}] = \beta \left\{ \langle 32 \rangle + [32] \right\}^{2S-1} [\langle 32 \rangle - [32]]$.