Functional methods for Effective Field Theory calculations

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Don’t hesitate to write if you want to discuss about these topics!

Université Paris-Saclay, HEP seminar, 12/12/2024

Effective Theories

Introduction and state-of-the-art

Why Effective (Field) Theories?

The concept is very general: consider an apple falling from a tree. If you want to know its falling velocity, you will probably use

$$mgh = \frac{mv^2}{2} \quad \Longrightarrow \quad v = \sqrt{2gh}$$

But

… the gravitational potential is not linear in h

[Corrections of $\mathcal{O}(h/R) \sim 10^{-6}$]

… Newtonian gravity is itself an effective theory of General Relativity

Physics decouples!

No need to know all details to describe a system at a given precision

Why Effective (Field) Theories?

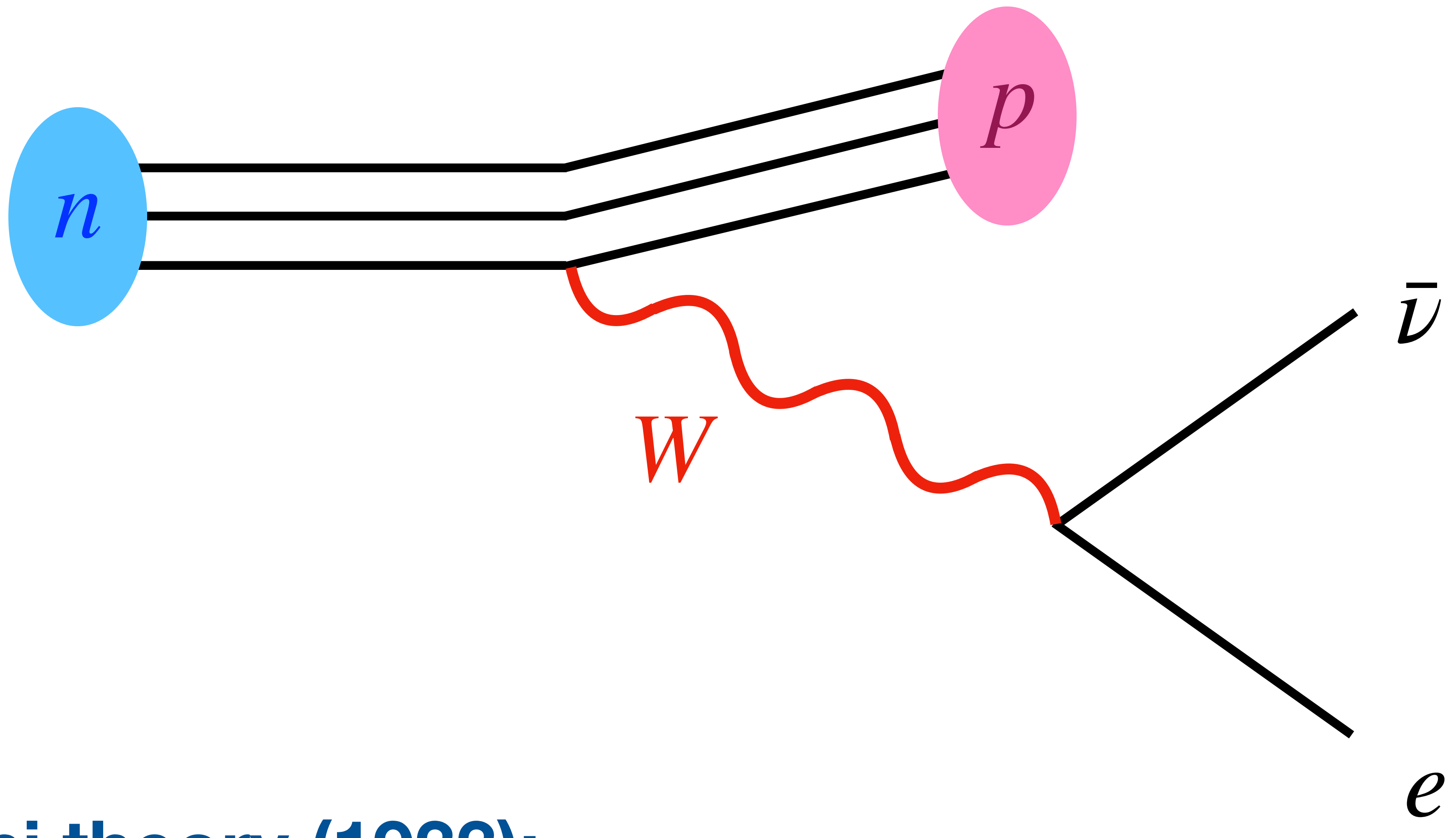
Effective Theories (ET) are ubiquitous in Physics:

- GR → Newtonian gravity
- Charge distribution → Multipolar expansion
- QED → Hidrogen atom
- QCD → Nuclear Physics
- …

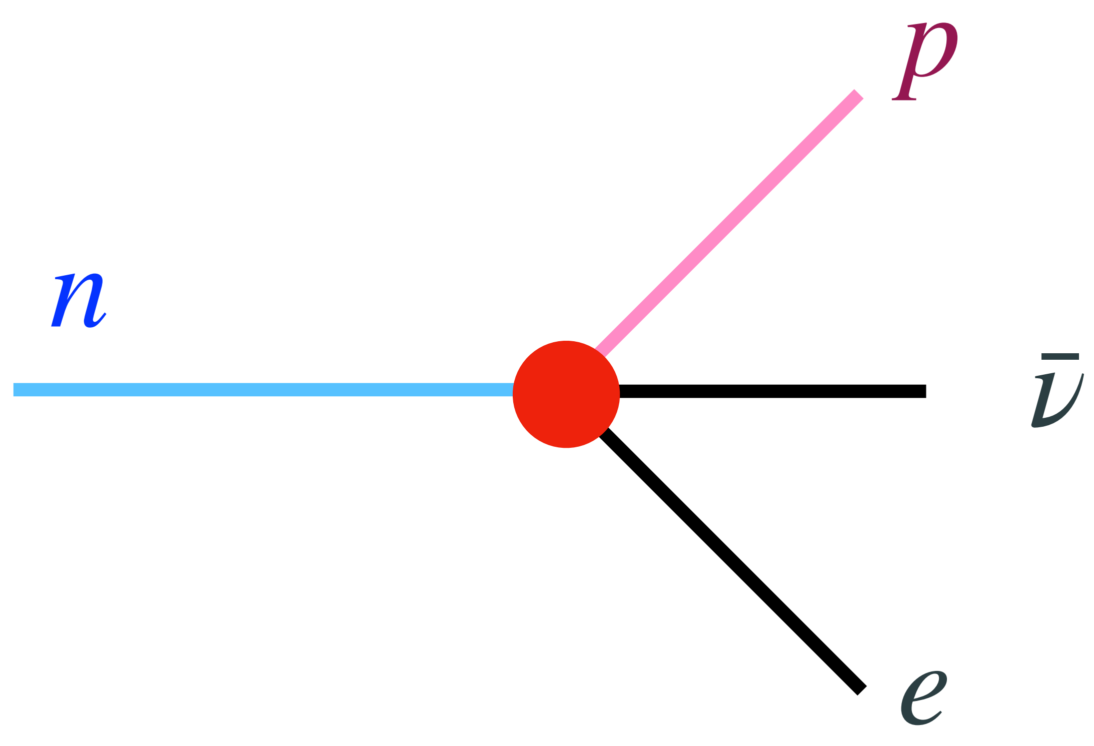
They efficiently separate energy scales:

- ETs are simpler (and more powerful)
- Can be formulated without knowing the full theory
- All theories break down eventually, so they can all be regarded as ETs

Electroweak theory + QCD (1983):



Fermi theory (1933):



What is experiment telling us?

No **direct evidence for NP** despite the many reasons for it

[**presence of a mass gap?**]

Indirect NP searches?

ATLAS Heavy Particle Searches* - 95% CL Upper Exclusion Limits

Status: March 2023

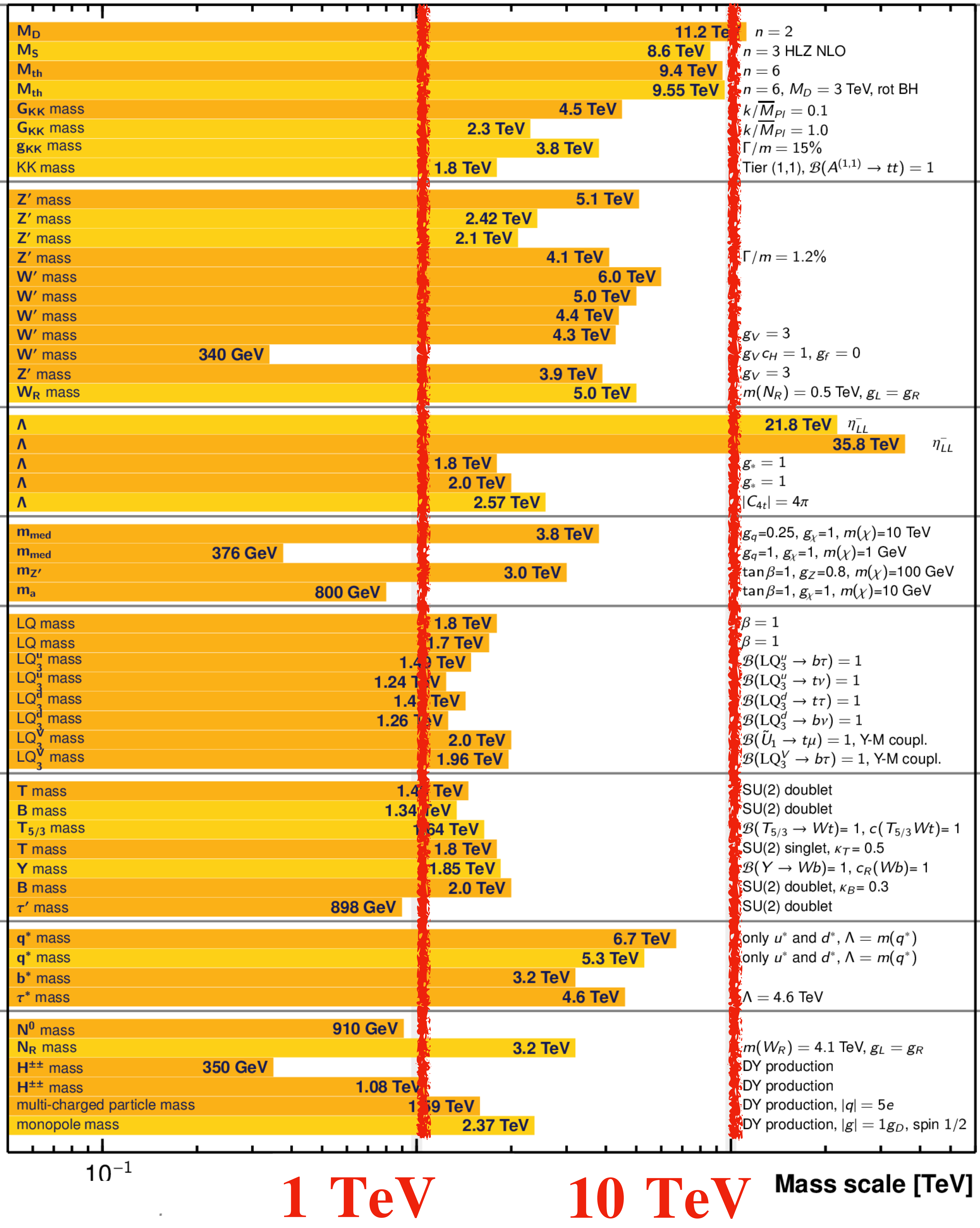
ATLAS Preliminary

$\int \mathcal{L}\, dt = (3.6 - 139)\ \mathrm{fb}^{-1}$ $\sqrt{s} = 13$ TeV

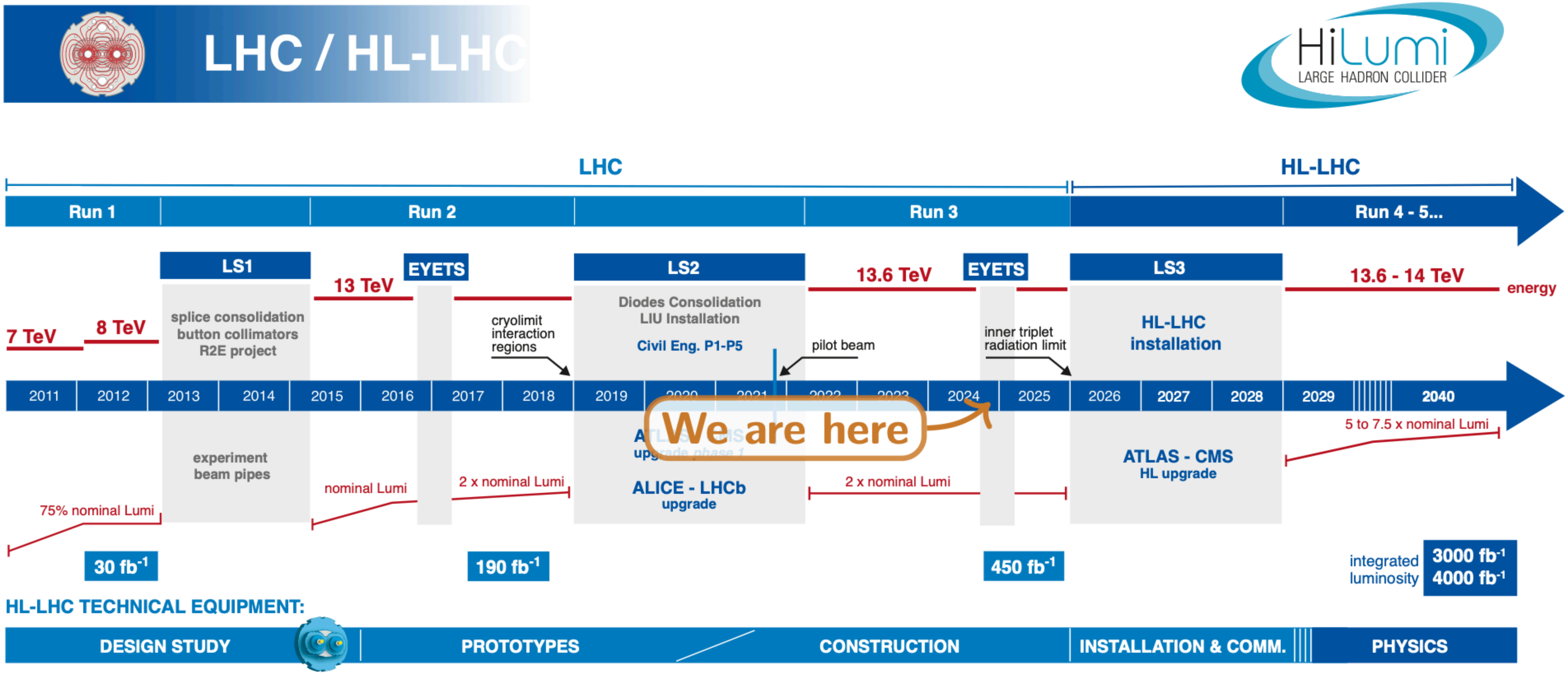
	Model	ℓ, γ	Jets†	E_T^{miss}	$\int \mathcal{L}\, dt[\mathrm{fb}^{-1}]$	Limit		Reference
Extra dimen.	ADD $G_{KK} + g/q$	$0\ e, \mu, \tau, \gamma$	$1-4$ j	Yes	139	M_D 11.2 TeV	$n = 2$	2102.10874
	ADD non-resonant $\gamma\gamma$	$2\ \gamma$	–	–	36.7	M_S 8.6 TeV	$n = 3$ HLZ NLO	1707.04147
	ADD QBH	–	2 j	–	139	M_{th} 9.4 TeV	$n = 6$	1910.08447
	ADD BH multijet	–	≥3 j	–	3.6	M_{th} 9.55 TeV	$n = 6$, $M_D = 3$ TeV, rot BH	1512.02586
	RS1 $G_{KK} \to \gamma\gamma$	$2\ \gamma$	–	–	139	G_{KK} mass 4.5 TeV	$k/\overline{M}_{Pl} = 0.1$	2102.13405
	Bulk RS $G_{KK} \to WW/ZZ$	multi-channel			36.1	G_{KK} mass 2.3 TeV	$k/\overline{M}_{Pl} = 1.0$	1808.02380
	Bulk RS $g_{KK} \to tt$	$1\ e, \mu$	≥1 b, ≥1J/2j	Yes	36.1	g_{KK} mass 3.8 TeV	$\Gamma/m = 15\%$	1804.10823
	2UED / RPP	$1\ e, \mu$	≥2 b, ≥3 j	Yes	36.1	KK mass 1.8 TeV	Tier (1,1), $\mathcal{B}(A^{(1,1)} \to tt) = 1$	1803.09678
Gauge bosons	SSM $Z' \to \ell\ell$	$2\ e, \mu$	–	–	139	Z' mass 5.1 TeV		1903.06248
	SSM $Z' \to \tau\tau$	$2\ \tau$	–	–	36.1	Z' mass 2.42 TeV		1709.07242
	Leptophobic $Z' \to bb$	–	2 b	–	36.1	Z' mass 2.1 TeV		1805.09299
	Leptophobic $Z' \to tt$	$0\ e, \mu$	≥1 b, ≥2 J	Yes	139	Z' mass 4.1 TeV	$\Gamma/m = 1.2\%$	2005.05138
	SSM $W' \to \ell\nu$	$1\ e, \mu$	–	Yes	139	W' mass 6.0 TeV		1906.05609
	SSM $W' \to \tau\nu$	$1\ \tau$	–	Yes	139	W' mass 5.0 TeV		ATLAS-CONF-2021-025
	SSM $W' \to tb$	–	≥1 b, ≥1 J	–	139	W' mass 4.4 TeV		ATLAS-CONF-2021-043
	HVT $W' \to WZ$ model B	0-2 e, μ	2 j / 1 J	Yes	139	W' mass 4.3 TeV	$g_V = 3$	2004.14636
	HVT $W' \to WZ \to \ell\nu\, \ell'\ell'$ model C	$3\ e, \mu$	2 j (VBF)	Yes	139	W' mass 340 GeV	$g_V c_H = 1$, $g_f = 0$	2207.03925
	HVT $Z' \to WW$ model B	$1\ e, \mu$	2 j / 1 J	Yes	139	Z' mass 3.9 TeV	$g_V = 3$	2004.14636
	LRSM $W_R \to \mu N_R$	$2\ \mu$	1 J	–	80	W_R mass 5.0 TeV	$m(N_R) = 0.5$ TeV, $g_L = g_R$	1904.12679
CI	CI $qqqq$	–	2 j	–	37.0	Λ 21.8 TeV	η_{LL}^-	1703.09127
	CI $\ell\ell qq$	$2\ e, \mu$	–	–	139	Λ 35.8 TeV	η_{LL}^-	2006.12946
	CI $eebs$	$2\ e$	1 b	–	139	Λ 1.8 TeV	$g_* = 1$	2105.13847
	CI $\mu\mu bs$	$2\ \mu$	1 b	–	139	Λ 2.0 TeV	$g_* = 1$	2105.13847
	CI $tttt$	≥1 e,μ	≥1 b, ≥1 j	Yes	36.1	Λ 2.57 TeV	$\lvert C_{4t}\rvert = 4\pi$	1811.02305
DM	Axial-vector med. (Dirac DM)	–	2 j	–	139	m_{med} 3.8 TeV	g_q=0.25, g_χ=1, $m(\chi)$=10 TeV	ATL-PHYS-PUB-2022-036
	Pseudo-scalar med. (Dirac DM)	$0\ e, \mu, \tau, \gamma$	$1-4$ j	Yes	139	m_{med} 376 GeV	g_q=1, g_χ=1, $m(\chi)$=1 GeV	2102.10874
	Vector med. Z'-2HDM (Dirac DM)	$0\ e, \mu$	2 b	Yes	139	$m_{Z'}$ 3.0 TeV	$\tan\beta$=1, g_Z=0.8, $m(\chi)$=100 GeV	2108.13391
	Pseudo-scalar med. 2HDM+a	multi-channel			139	m_a 800 GeV	$\tan\beta$=1, g_χ=1, $m(\chi)$=10 GeV	ATLAS-CONF-2021-036
LQ	Scalar LQ 1st gen	$2\ e$	≥2 j	Yes	139	LQ mass 1.8 TeV	$\beta = 1$	2006.05872
	Scalar LQ 2nd gen	$2\ \mu$	≥2 j	Yes	139	LQ mass 1.7 TeV	$\beta = 1$	2006.05872
	Scalar LQ 3rd gen	$1\ \tau$	2 b	Yes	139	LQ_3^u mass 1.4[illegible] TeV	$\mathcal{B}(LQ_3^u \to b\tau) = 1$	2303.01294
	Scalar LQ 3rd gen	$0\ e, \mu$	≥2 j, ≥2 b	Yes	139	LQ_3^u mass 1.24 T[illegible]V	$\mathcal{B}(LQ_3^u \to t\nu) = 1$	2004.14060
	Scalar LQ 3rd gen	≥2 e, μ, ≥1 τ	≥1 j, ≥1 b	–	139	LQ_3^d mass 1.4[illegible] TeV	$\mathcal{B}(LQ_3^d \to t\tau) = 1$	2101.11582
	Scalar LQ 3rd gen	$0\ e, \mu$, ≥1 τ	$0-2$ j, 2 b	Yes	139	LQ_3^d mass 1.26 T[illegible]V	$\mathcal{B}(LQ_3^d \to b\nu) = 1$	2101.12527
	Vector LQ mix gen	multi-channel	≥1 j, ≥1 b	Yes	139	LQ_3^V mass 2.0 TeV	$\mathcal{B}(\tilde{U}_1 \to t\mu) = 1$, Y-M coupl.	ATLAS-CONF-2022-052
	Vector LQ 3rd gen	$2\ e, \mu, \tau$	≥1 b	Yes	139	LQ_3^V mass 1.96 TeV	$\mathcal{B}(LQ_3^V \to b\tau) = 1$, Y-M coupl.	2303.01294
Vector-like fermions	VLQ $TT \to Zt + X$	$2e/2\mu/\geq 3e,\mu$	≥1 b, ≥1 j	–	139	T mass 1.4[illegible] TeV	SU(2) doublet	2210.15413
	VLQ $BB \to Wt/Zb + X$	multi-channel			36.1	B mass 1.34 TeV	SU(2) doublet	1808.02343
	VLQ $T_{5/3}T_{5/3}\lvert T_{5/3} \to Wt + X$	2(SS)/≥3 e,μ	≥1 b, ≥1 j	Yes	36.1	$T_{5/3}$ mass [illegible]64 TeV	$\mathcal{B}(T_{5/3} \to Wt)$= 1, $c(T_{5/3}Wt)$= 1	1807.11883
	VLQ $T \to Ht/Zt$	$1\ e, \mu$	≥1 b, ≥3 j	Yes	139	T mass 1.8 TeV	SU(2) singlet, κ_T= 0.5	ATLAS-CONF-2021-040
	VLQ $Y \to Wb$	$1\ e, \mu$	≥1 b, ≥1 j	Yes	36.1	Y mass 1.85 TeV	$\mathcal{B}(Y \to Wb)$= 1, $c_R(Wb)$= 1	1812.07343
	VLQ $B \to Hb$	$0\ e,\mu$	≥2b, ≥1j, ≥1J	–	139	B mass 2.0 TeV	SU(2) doublet, κ_B= 0.3	ATLAS-CONF-2021-018
	VLL $\tau' \to Z\tau/H\tau$	multi-channel	≥1 j	Yes	139	τ' mass 898 GeV	SU(2) doublet	2303.05441
Exctd ferm.	Excited quark $q^* \to qg$	–	2 j	–	139	q^* mass 6.7 TeV	only u^* and d^*, $\Lambda = m(q^*)$	1910.08447
	Excited quark $q^* \to q\gamma$	$1\ \gamma$	1 j	–	36.7	q^* mass 5.3 TeV	only u^* and d^*, $\Lambda = m(q^*)$	1709.10440
	Excited quark $b^* \to bg$	–	1 b, 1 j	–	139	b^* mass 3.2 TeV		1910.08447
	Excited lepton τ^*	$2\ \tau$	≥2 j	–	139	τ^* mass 4.6 TeV	$\Lambda = 4.6$ TeV	2303.09444
Other	Type III Seesaw	2,3,4 e, μ	≥2 j	Yes	139	N^0 mass 910 GeV		2202.02039
	LRSM Majorana ν	$2\ \mu$	2 j	–	36.1	N_R mass 3.2 TeV	$m(W_R) = 4.1$ TeV, $g_L = g_R$	1809.11105
	Higgs triplet $H^{\pm\pm} \to W^\pm W^\pm$	2,3,4 e, μ (SS)	various	Yes	139	$H^{\pm\pm}$ mass 350 GeV	DY production	2101.11961
	Higgs triplet $H^{\pm\pm} \to \ell\ell$	2,3,4 e, μ (SS)	–	–	139	$H^{\pm\pm}$ mass 1.08 TeV	DY production	2211.07505
	Multi-charged particles	–	–	–	139	multi-charged particle mass 1.[illegible]9 TeV	DY production, $\lvert q\rvert = 5e$	ATLAS-CONF-2022-034
	Magnetic monopoles	–	–	–	34.4	monopole mass 2.37 TeV	DY production, $\lvert g\rvert = 1 g_D$, spin 1/2	1905.10130

$\sqrt{s}$ = 13 TeV partial data

$\sqrt{s}$ = 13 TeV full data

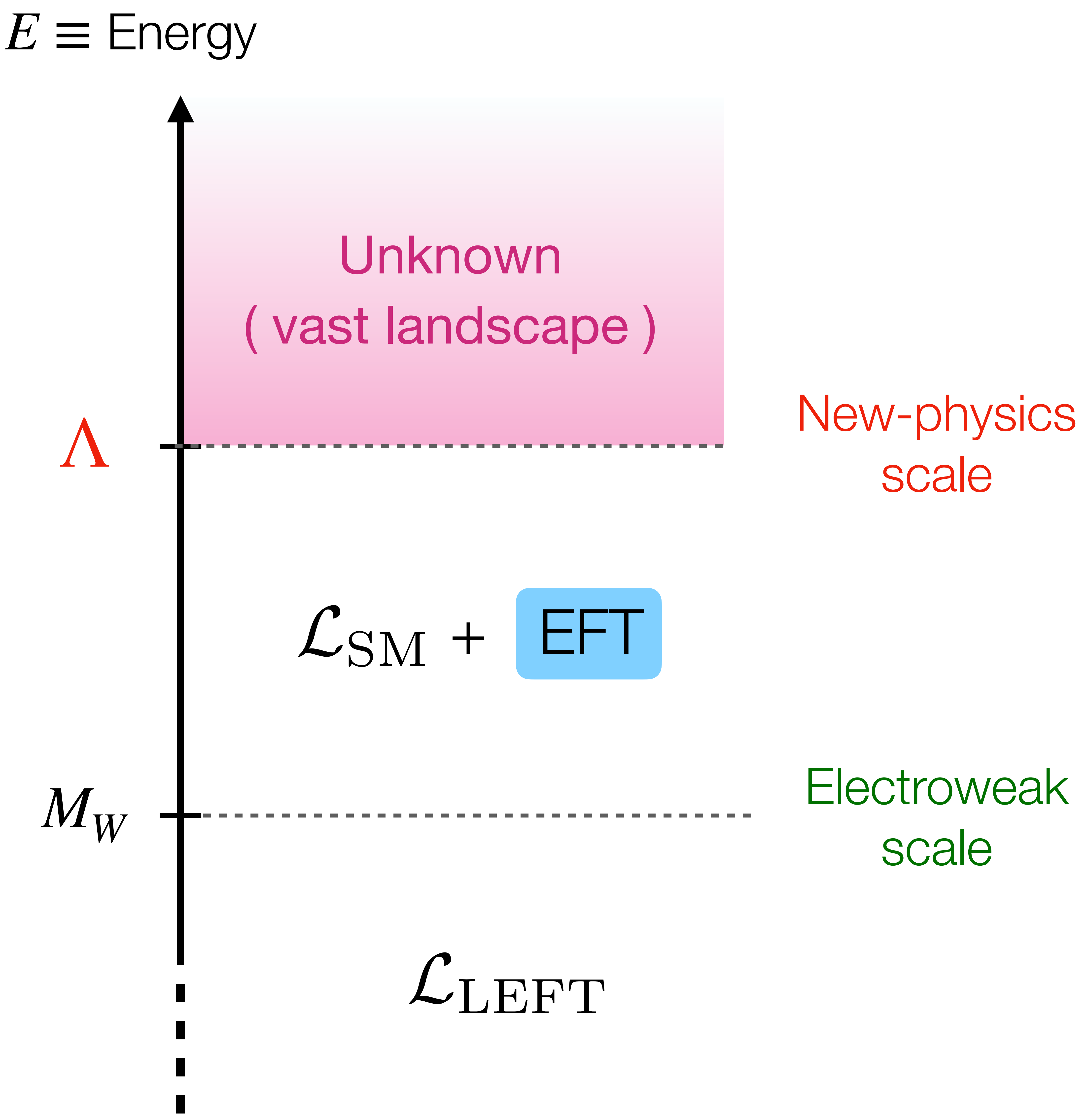


A new precision era



Marginal increase in energy, but $\sim 20\times$ more luminosity!

Effective Field Theories (EFT): bottom-up



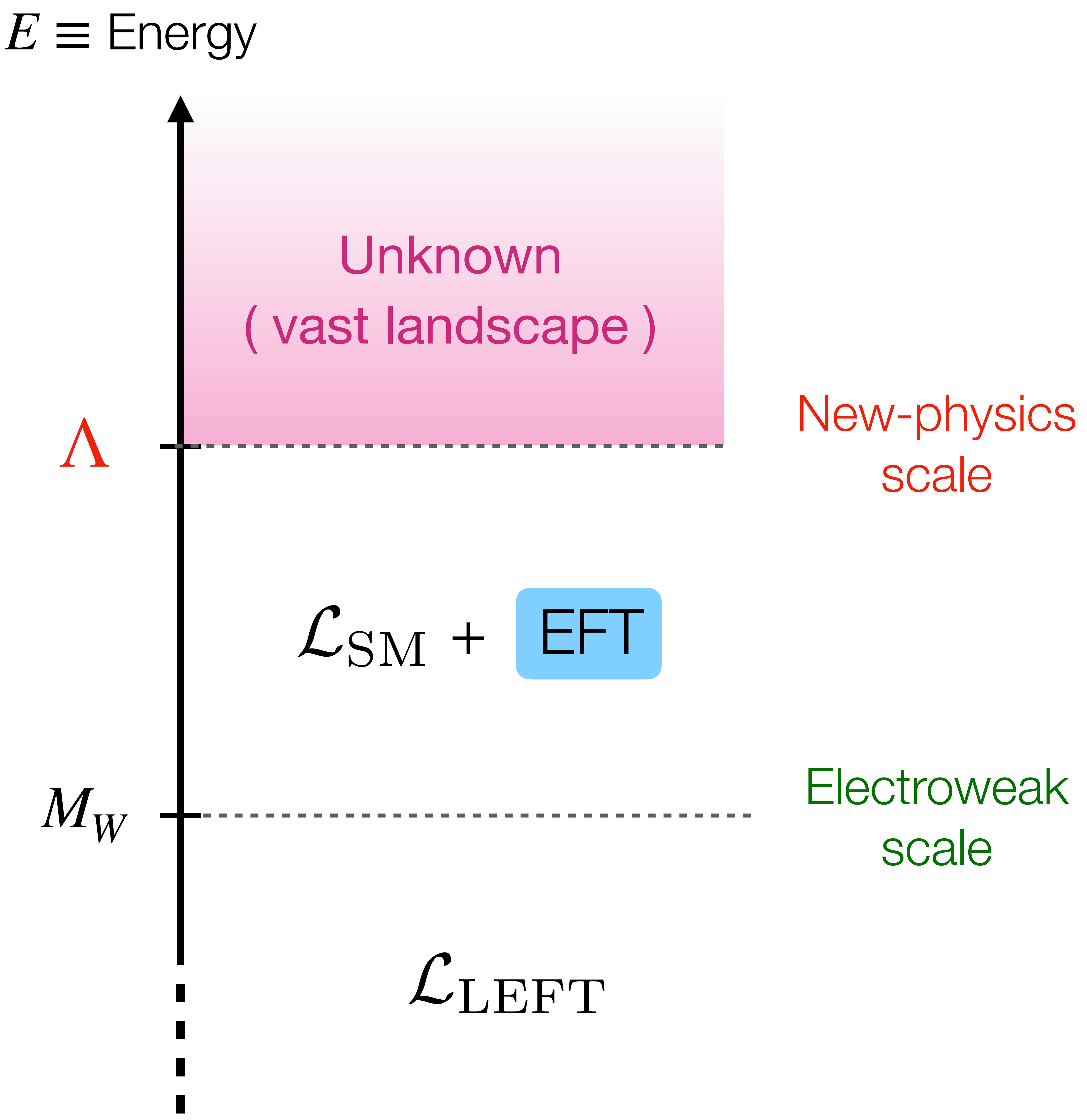
EFTs are great for parametrizing the unknown:

- Can be formulated without knowing the full theory
- Systematically improvable by adding extra terms in a double expansion in quantum corrections and E/Λ

UV physics

$$\mathscr{L}_{\rm EFT}(\eta_L) = \mathscr{L}_{d=4}(\eta_L) + \sum_{\ell=0}^{\infty} \sum_{n=5}^{\infty} \sum_{k} \frac{C^{(\ell)}_{n,k}}{(16\pi^2)^{\ell}\, \Lambda^{n-4}} O_{n,k}(\eta_L)$$

Effective Field Theories (EFT): bottom-up



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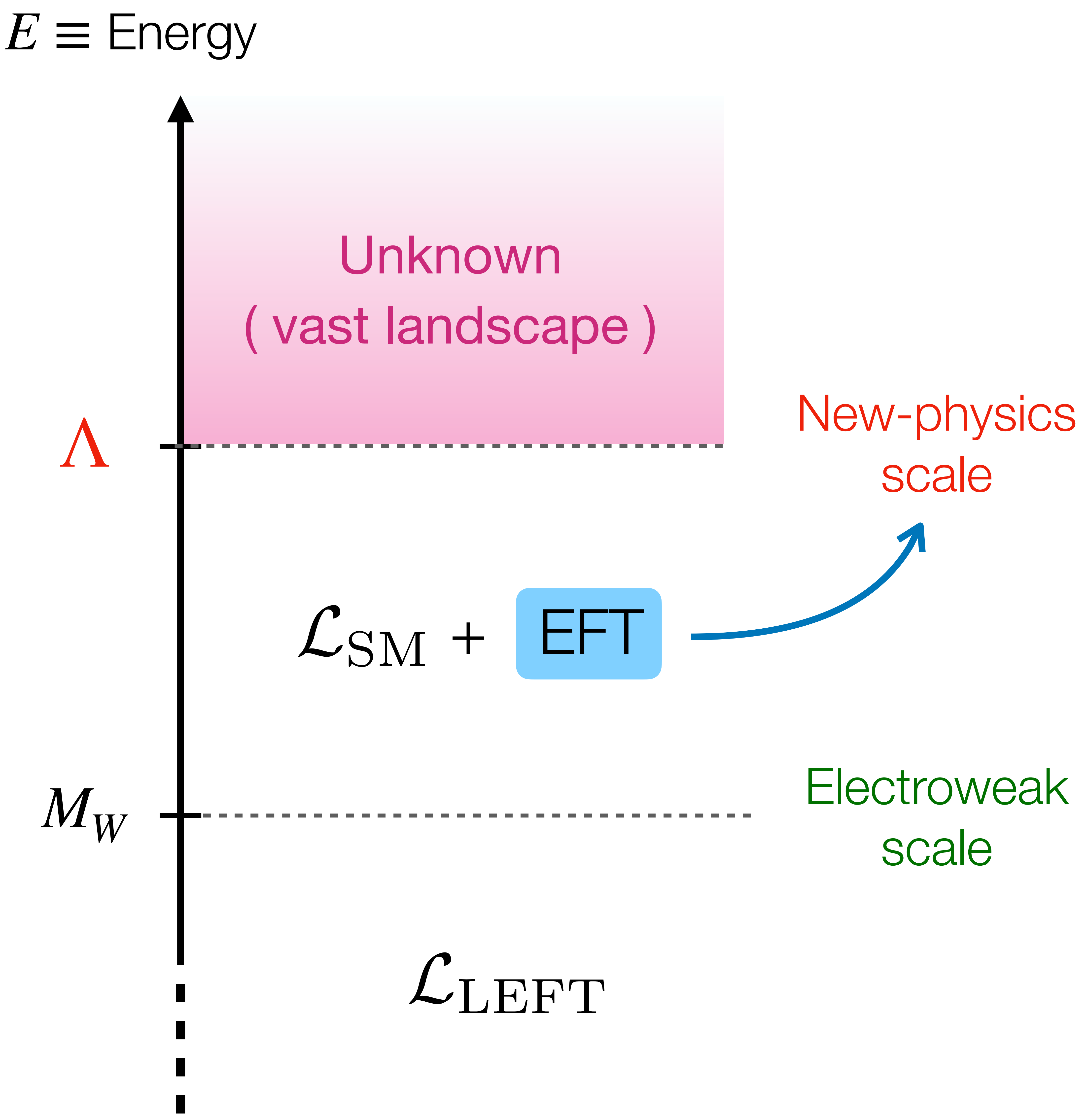
LEFT : $\dfrac{p}{\Lambda_{\rm EW}} \sim \dfrac{1}{100}$ SMEFT : $\dfrac{p}{\Lambda} \sim\, ?$

Effective Field Theories (EFT): bottom-up



EFTs are great for parametrizing the unknown:

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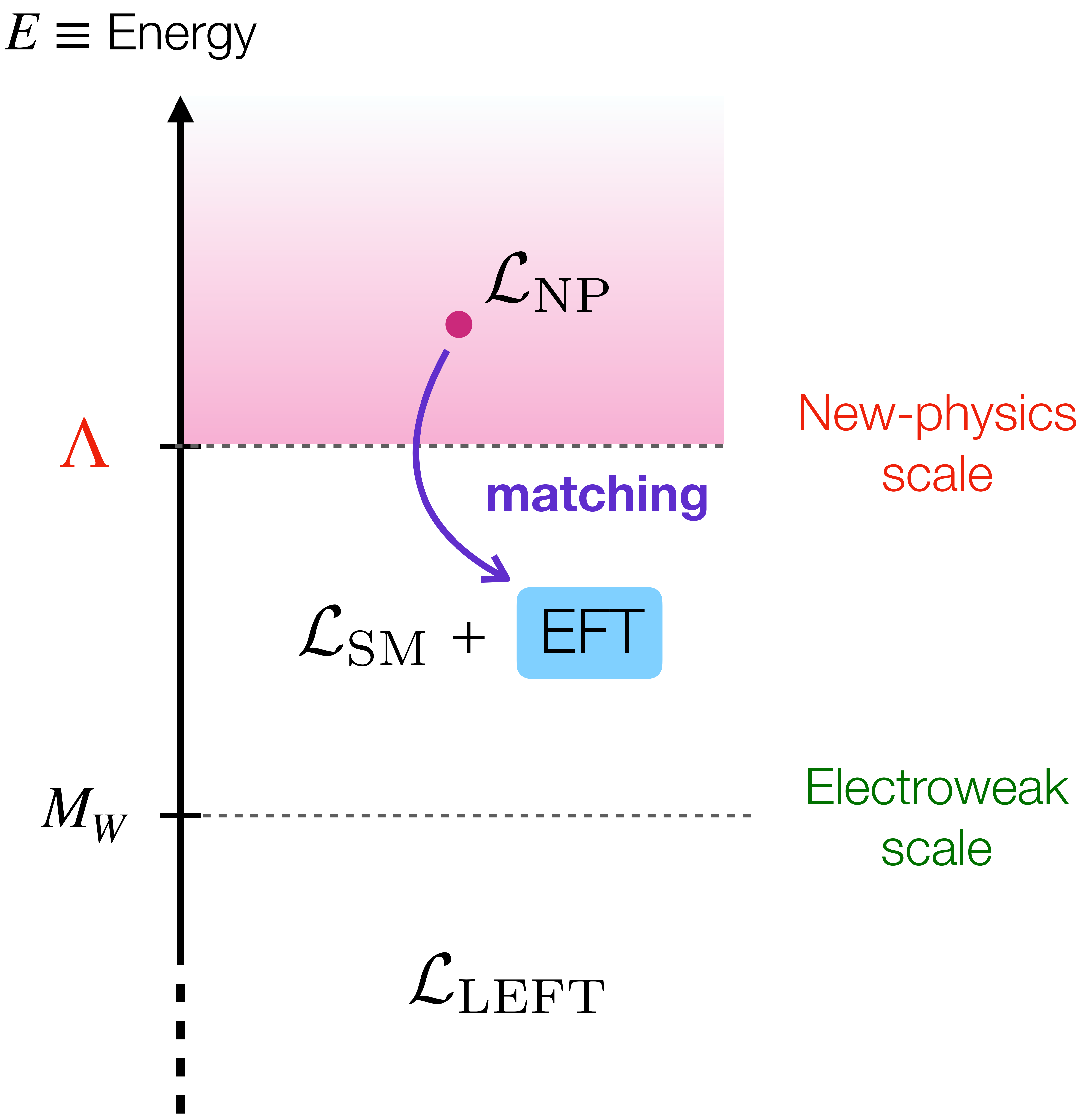
They give an indication on new-physics scales where a new fundamental theory has to be formulated

For example,

LEFT [Fermi Theory] $\rightarrow$ Electroweak scale $\rightarrow$ Standard Model (SM)

Effective Field Theories (EFT): top-down



Given a specific new-physics idea:

- Many models share the same EFT, providing a universal framework to connect models with data

- Precision necessitates EFTs: summation of (large) logarithms of E/Λ arising from the quantum corrections

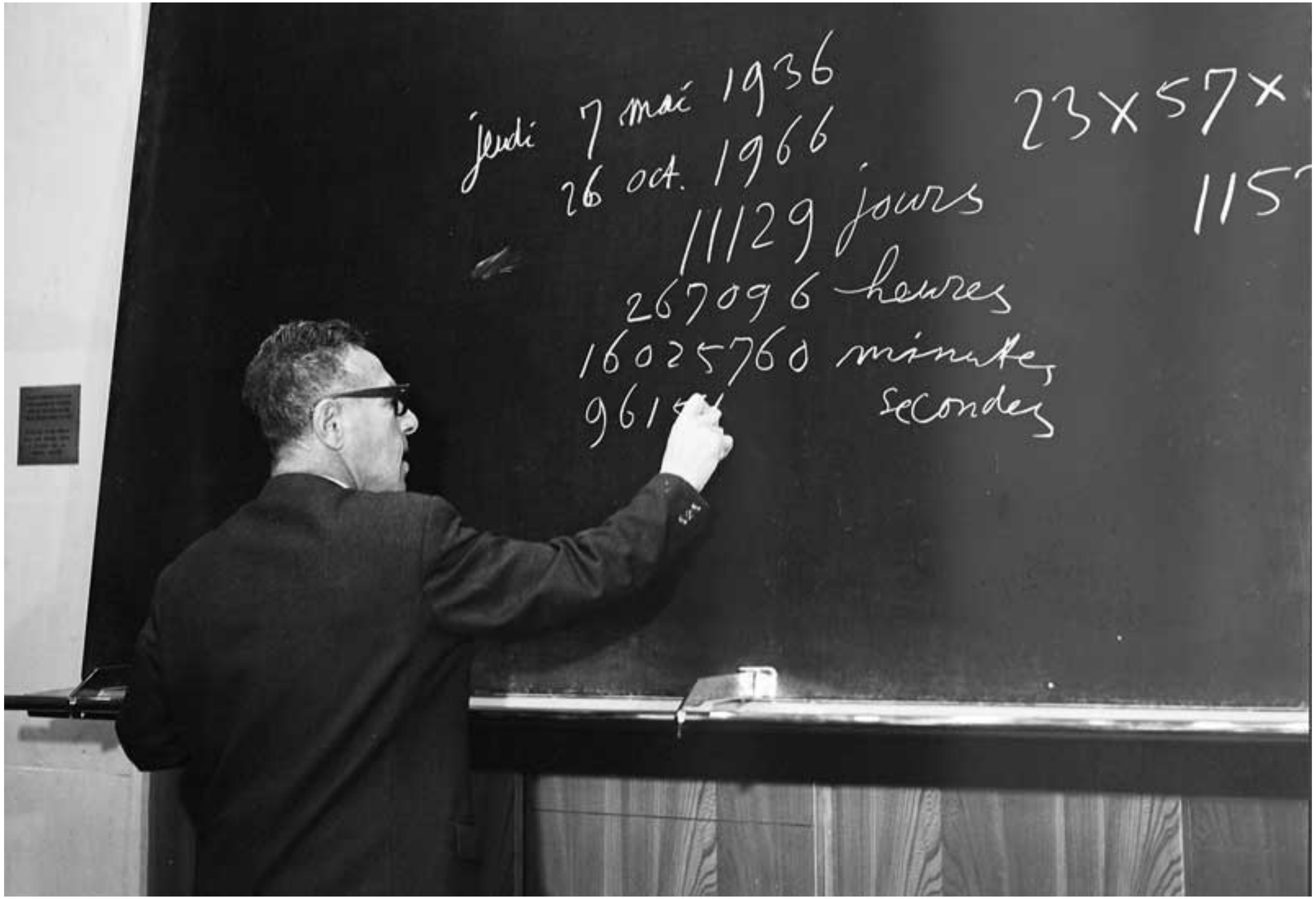
The step to build an EFT from a model is called **matching**

→ extremely repetitive and time-consuming task

Entire journal publications for the (one-loop) matching of *simple* NP models

The case for automation

Wim Klein, CERN “human computer”



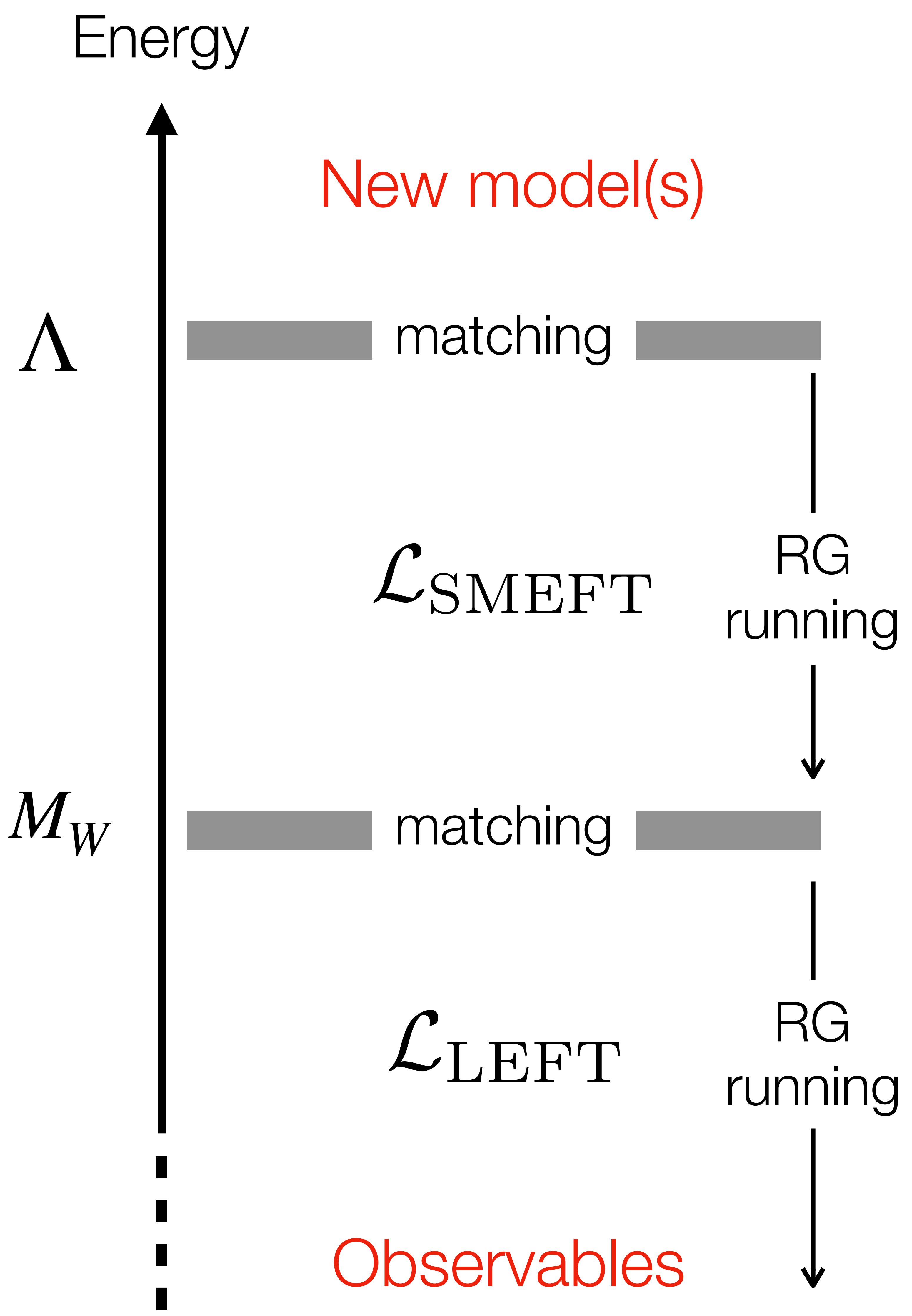
The case for automation

CERN first electronic computer

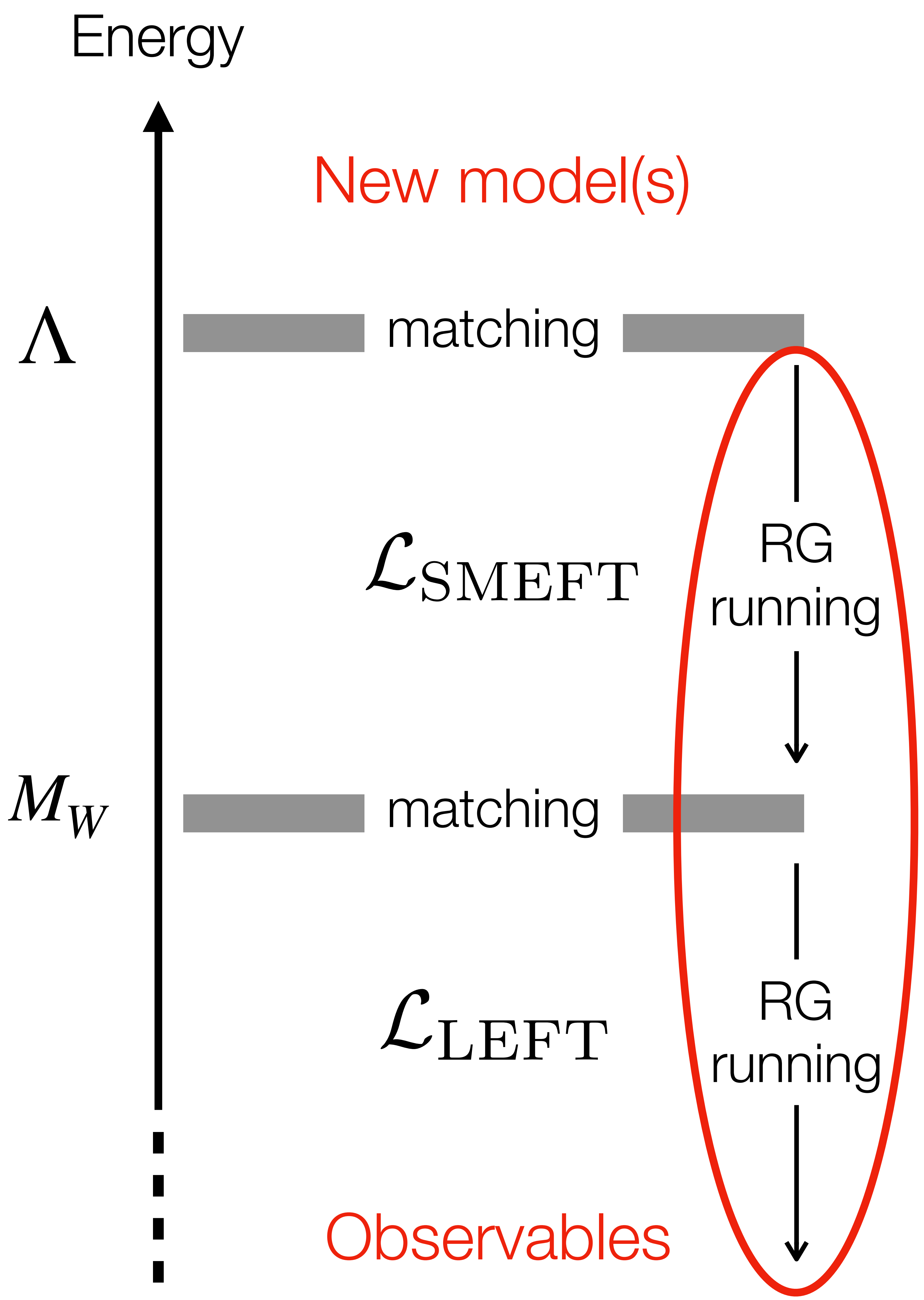
The (SM)EFT software project:

Upgrading from “human computers” to computers

The rise of automation



The rise of automation



DsixTools

JFM et al. '17 & '21

wilson

Aebischer et al. '18

RGEsolver

Di Noi, Silvestrini '22

“Hard-coded” one-loop results based on:

SMEFT running: Jenkins et al. '13, '14; Alonso et al. '14

LEFT basis: Jenkins et al. '18

SMEFT-LEFT matching: Jenkins et al. '17 Dekens, Stoffer '19

LEFT running: Jenkins et al. '18

The rise of automation

Energy

New model(s)

Λ matching

$\mathcal{L}_{\text{SMEFT}}$ RG running

M_W matching

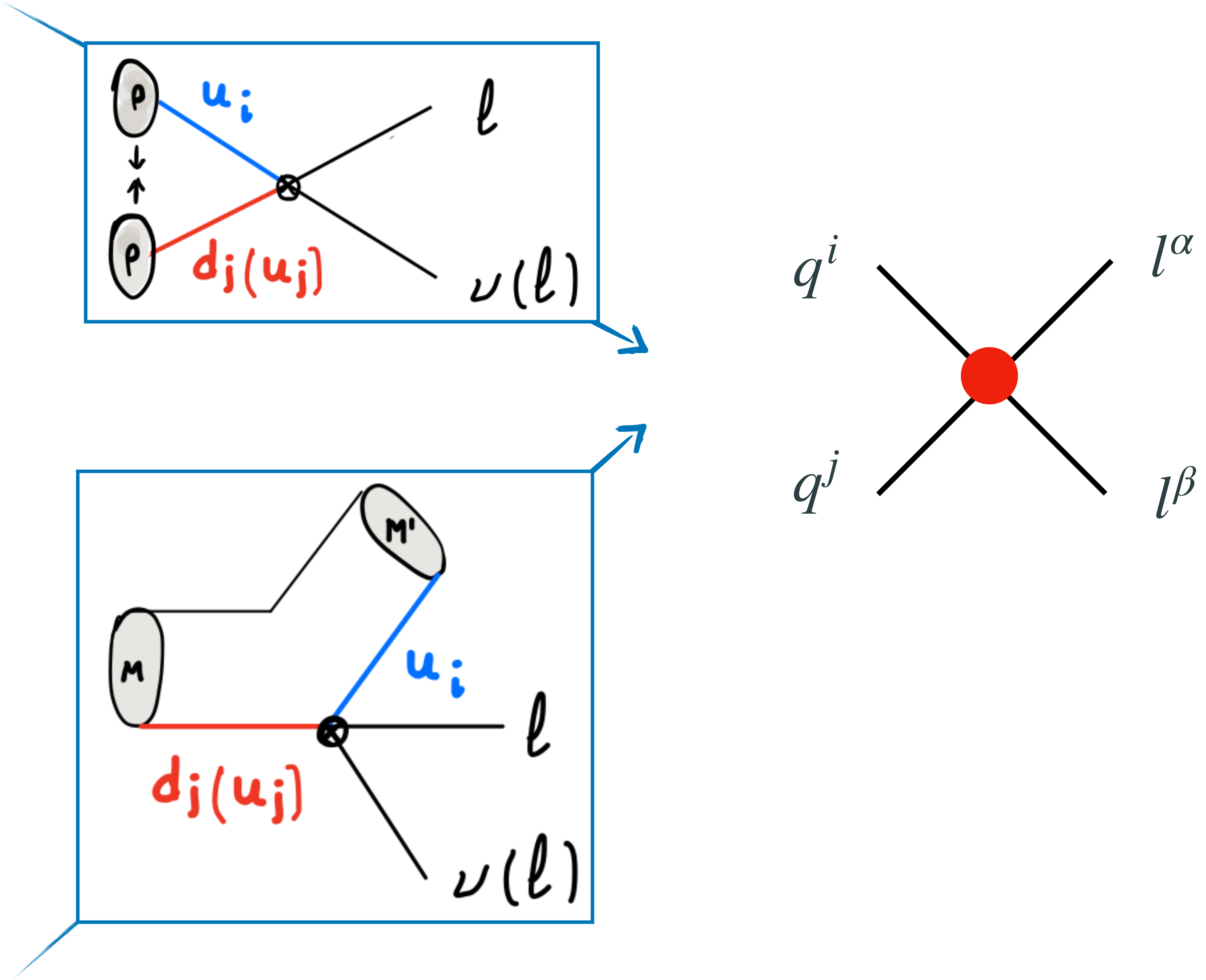
$\mathcal{L}_{\text{LEFT}}$ RG running

Observables

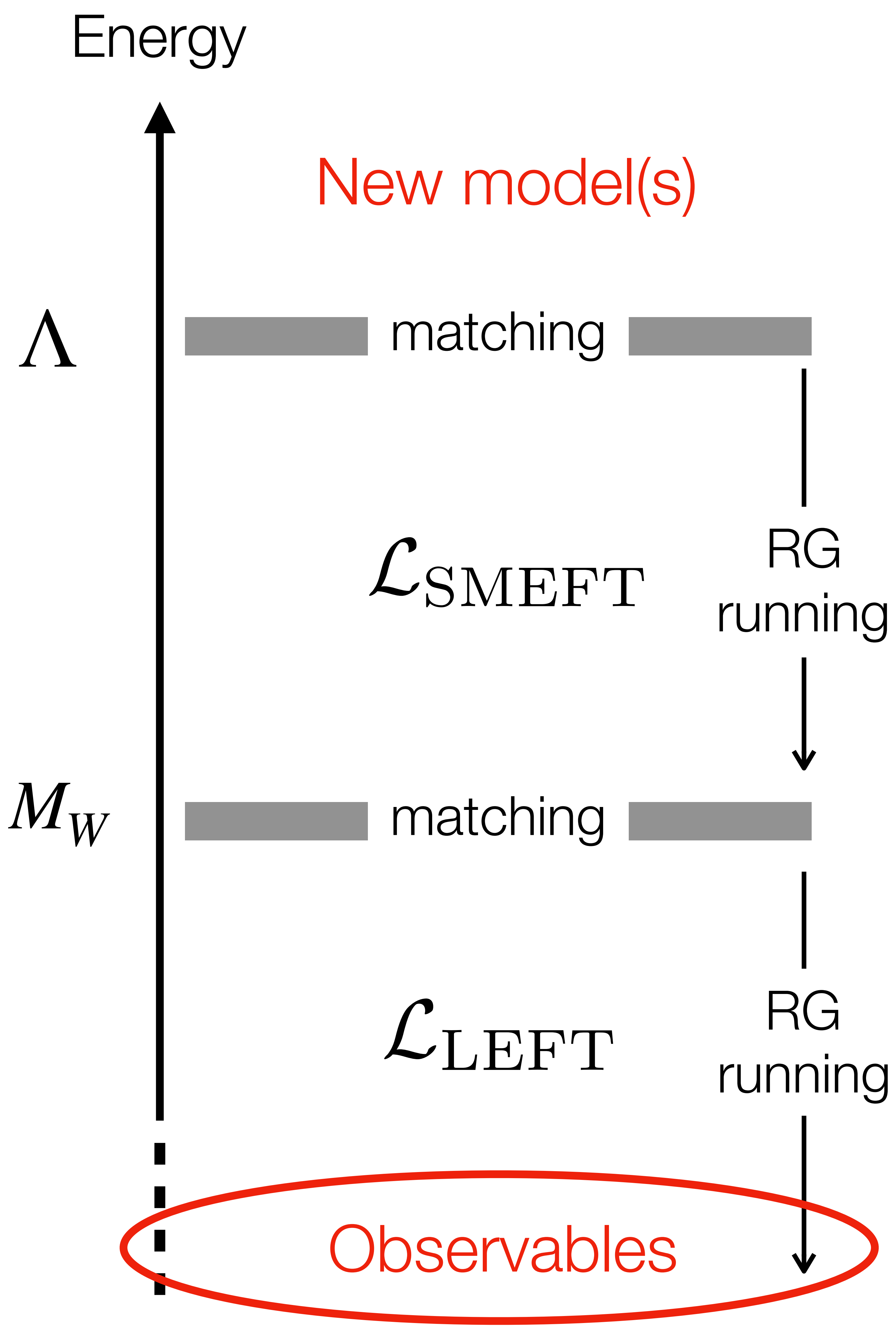
Exploit the complementary information in high- and low-energy probes:

→ For this, RG running effects are crucial !!

...



The rise of automation



SMEFT likelihood (smelli)

Aebischer et al. '18

flavio

Straub '16

HighPT

Allwicher et al. '22

HEPfit

De Blas et al. '19

Fitmaker

Ellis et al '20

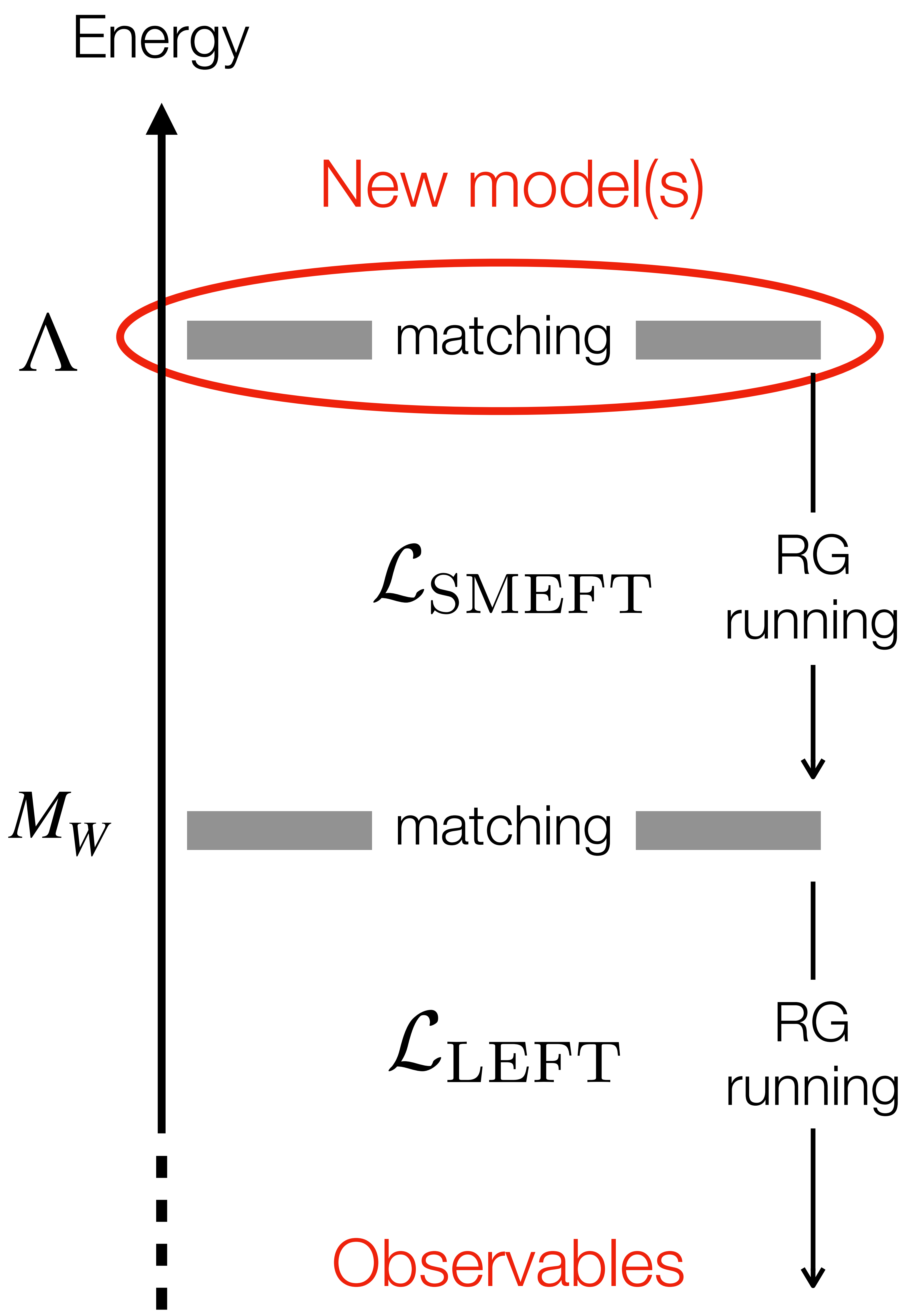
SMEFiT

Giani et al. '23

+ others

Growing involvement of experimental collaborations into this program!

The rise of automation

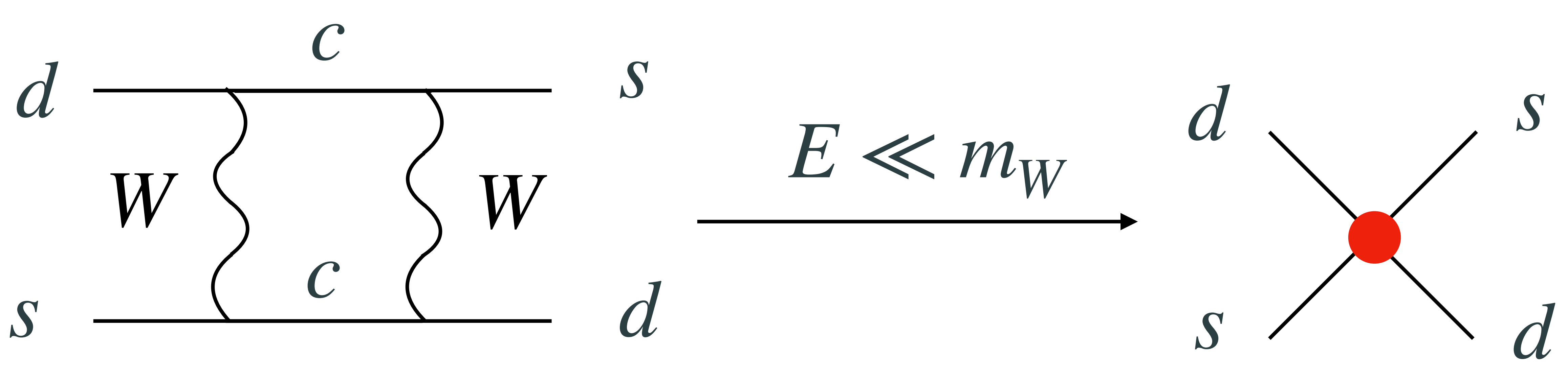


Much progress has been made:

- Tree-level matching to the SMEFT is a solved problem

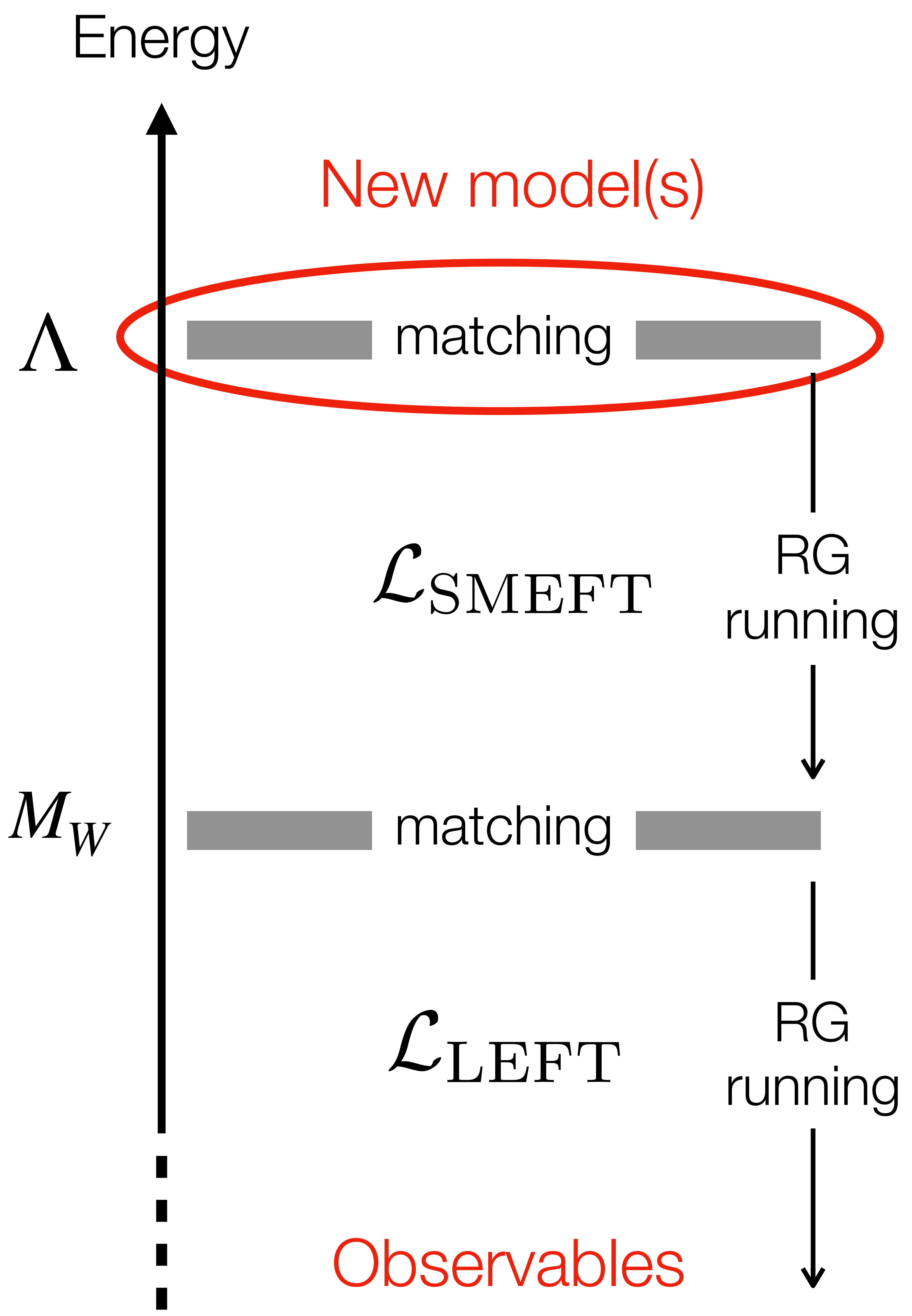
  [de Blas, Criado, Pérez-Victoria, Santiago, '17]   MatchingTools: [Criado '17]

- One-loop can be the leading effect in important processes. E.g., in the SM



Similarly, in BSM models: dipoles, FCNCs, EW precision…

The rise of automation



matchmakereft
Carmona et al. '22

MATCHETE
JFM et al. '23

Automated one-loop matching of *many* models

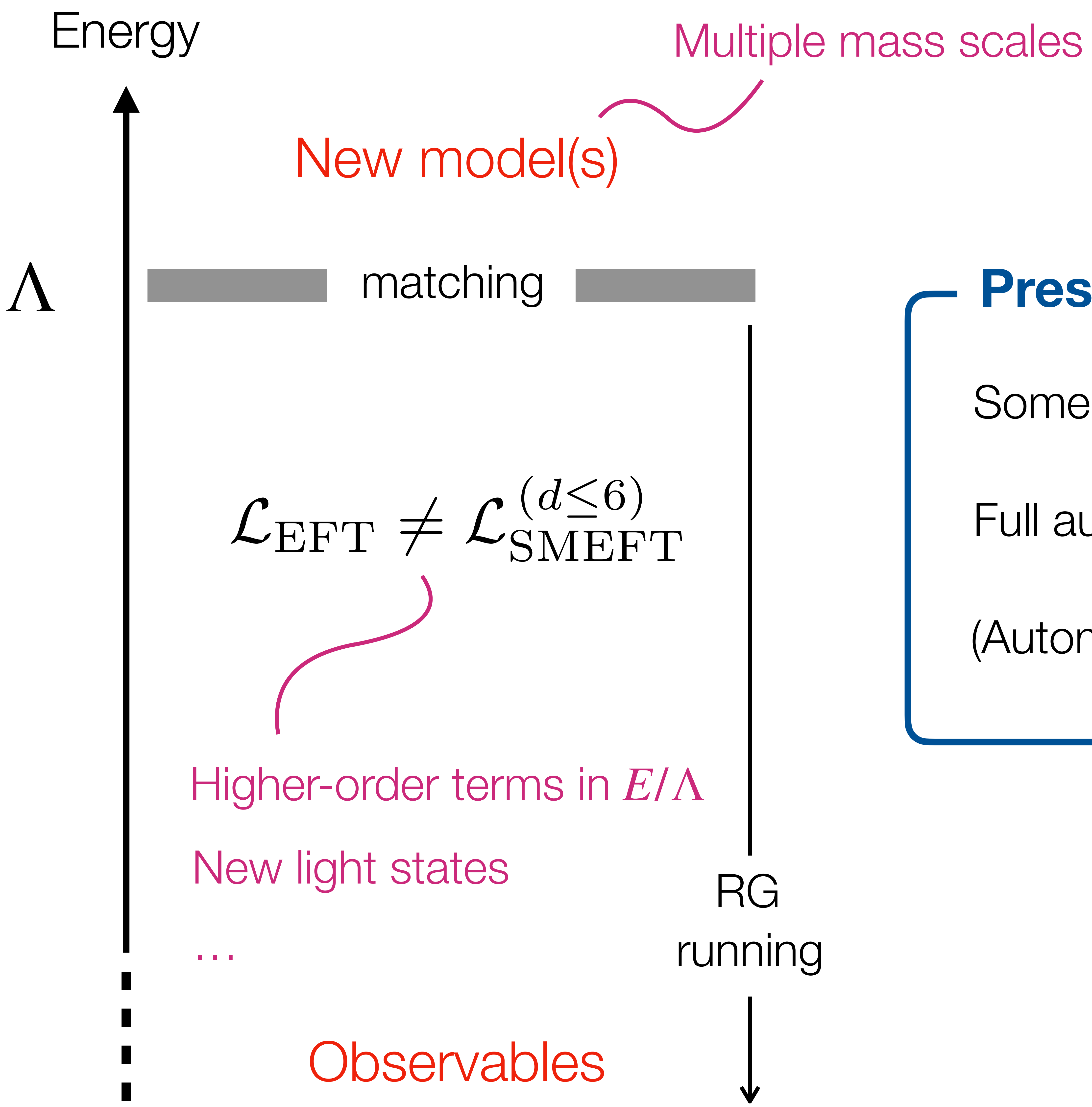
SOLD
Guedes et al. '23

UV-SMEFT dictionaries

"Breaking SMEFT operators"
UV-to-SMEFT mapping

Cepedello et al. '23

The rise of automation



Present limitations

Some steps/approaches require prior knowledge of the target EFT

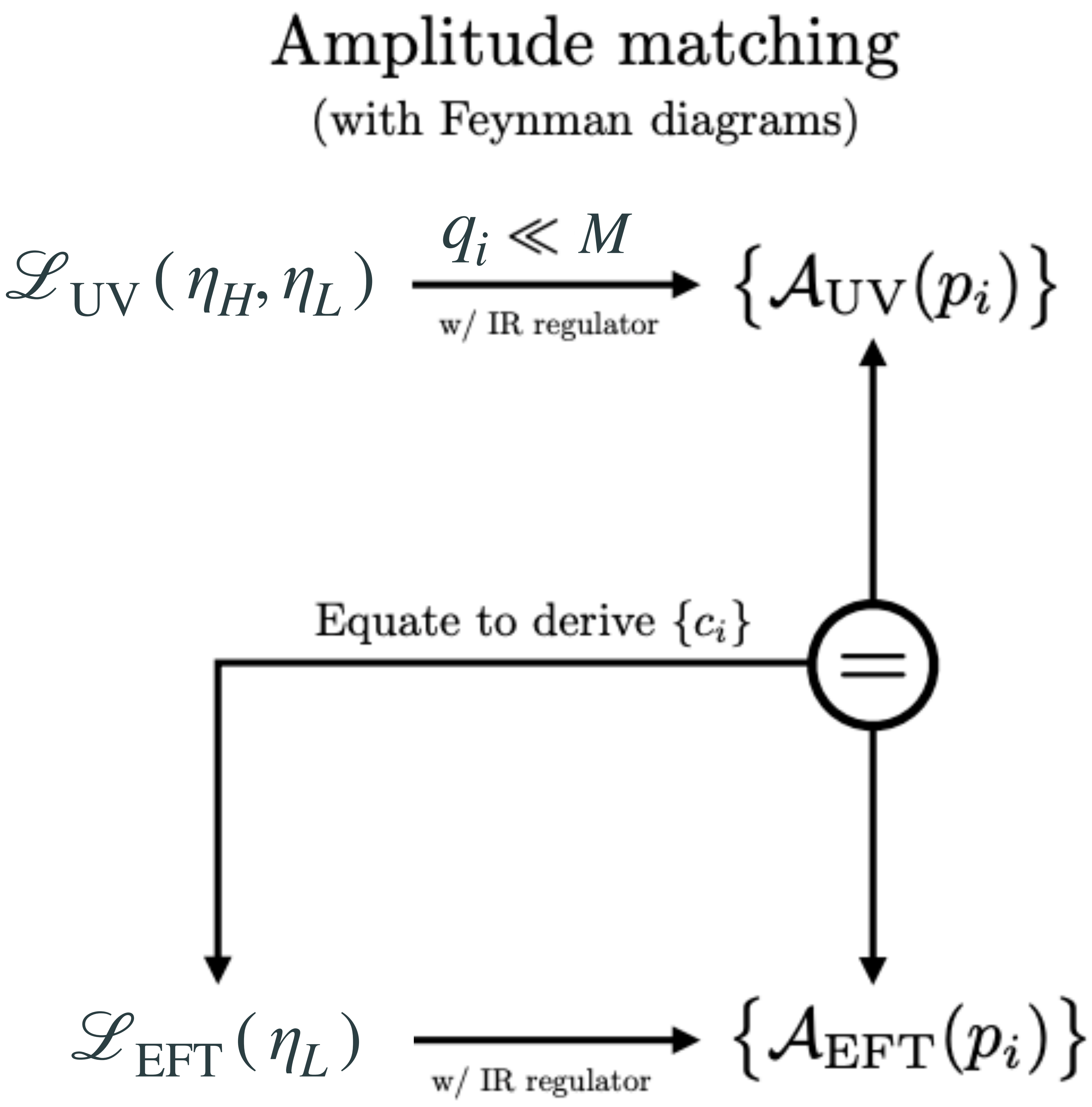
Full automation only for simpler scenarios (no heavy vectors yet!)

(Automated) inclusion of two-loop effects is (so far) non-trivial

EFT matching

The path-integral approach in a nutshell

Diagrammatic matching: off-shell and on-shell



[Figure from Cohen, Lu, Zhang, 2011.02484]

✔ Traditional, well-established procedure

✘ Requires a priori knowledge of the EFT Lagrangian

- **Off-shell amplitudes**: only 1PI diagrams but EFT Lagrangian contains many operators (off-shell d-dimensional Lagrangian)

- **On-shell matching**: only need to know on-shell EFT Lagrangian but, a priori, more complicated calculations. See Chala et al., 2411.12798 for a numerical approach using rational kinematics

✘ Breaks gauge invariance in intermediate steps

Simplified diagrammatic matching: method of regions

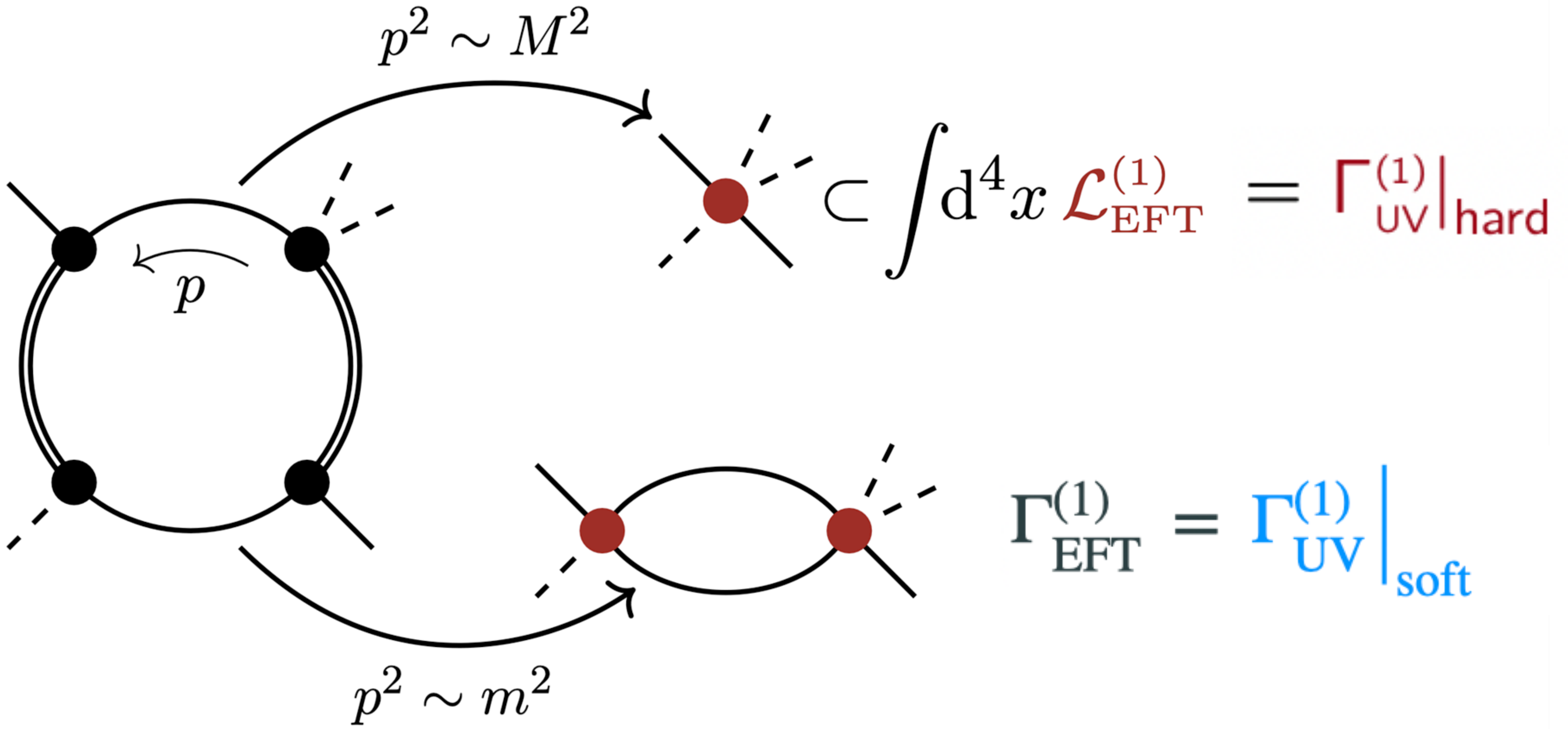
We can separate loop integrals in two regions (for $q^2, m^2 \ll M^2$): hard ($p^2 \sim M^2$) & soft ($p^2 \sim m^2$)

Method of regions: Beneke, Smirnov ’97, Jantzen ‘11

If only the <u>hard part of the loop</u> is considered, we get the EFT Lagrangian *directly*

JFM, Portolés, Ruiz-Femenía, '16

Zhang, '16



The functional approach to EFT matching

Functional matching
(path-integral methods)

$$\mathscr{L}_{\mathrm{UV}}(\eta_H, \eta_L)$$

$$\Big\downarrow \text{ Using Equations of Motion (EOMs)}$$

$$\Gamma_{\mathrm{UV}}[\,\hat{\eta}_H(\eta_L), \eta_L\,]$$

$$\Big\downarrow \text{ Method of regions}$$

$$\mathscr{L}_{\mathrm{EFT}}(\eta_L)$$

Not in a basis

Based on the Wilsonian approach : split vibrating fields into fast (η_H) and slow (η_L) vibrations and "integrate out" the fast ones

[Wilson, 1965]

- ✔ More systematic and efficient approach
- ✔ The EFT Lagrangian comes out directly
- ✔ Manifestly gauge invariant at every step
- ✘ Final results need to be reduced to a basis (off-shell matching)

Functional matching

- **Lagrangian:** $\mathscr{L}_{\mathrm{UV}}$ with fields $\eta = (\eta_H \; \eta_L)$ and hierarchy $m_H \gg m_L$

- **Background field method:** shift *all* fields $\eta \to \hat{\eta} + \eta$

  $\hat{\eta}$: background fields (satisfy the quantum EOM) [Tree lines in Feynman graphs]

  $\hat{\eta}$: quantum fluctuations [Loop lines in Feynman graphs]

- **Quantum effective action:**

$$e^{i\,\Gamma_{\mathrm{UV}}[\hat{\eta}]} = \int \mathscr{D}\eta \;\exp\left(i \int \mathrm{d}^d x \; \mathscr{L}_{\mathrm{UV}}(\eta + \hat{\eta}) \right)$$

Goal: Evaluate the path integral
(“integrate out” the quantum fluctuations)
and isolate the EFT contribution

General EFT matching formula

The EFT action is given by

$$S_{\rm EFT}[\phi] = \Gamma_{\rm UV}[\hat{\Phi}, \phi]\Big|_{\rm hard} \qquad \frac{\delta\Gamma_{\rm UV}\big|_{\rm hard}}{\delta\Phi}[\hat{\Phi}, \phi] = 0$$

Φ : Heavy

ϕ : Light

"hard" denotes the loop region where all loop momenta are $p \gtrsim \Lambda$ (incl. tree-level contributions) $^{(*)}$

- Explicit proof to two-loop order and (constructive) proof to any loop order in progress

  [JFM, Palavrić, Thomsen, 2311.13630] [JFM, Moreno, Palavrić, Thomsen, **coming soon!**]

- The hard region is by far the easiest to compute (only vacuum integrals at zero external momenta)
- The method can be trivially adapted to extract UV divergences needed for RG running
- Enables functional matching and RG running at any loop order

$^{(*)}$ Method of regions: Beneke, Smirnov, '97; Jantzen, '11

Functional matching

- **Expanding the Lagrangian in η:**

$$\mathscr{L}_{\text{UV}}(\hat{\eta}+\eta) = \mathscr{L}_{\text{UV}}(\hat{\eta}) + \left.\frac{\delta \mathscr{L}_{\text{UV}}}{\delta \eta_a}\right|_{\eta=\hat{\eta}} \eta_a + \frac{1}{2}\eta_a(x) \left.\frac{\delta^2 \mathscr{L}_{\text{UV}}}{\delta \eta_b(x')\,\delta \eta_a(x)}\right|_{\eta=\hat{\eta}} \eta_b(x') + \mathcal{O}(\eta^3)$$

Functional matching

- **Expanding the Lagrangian in η:**

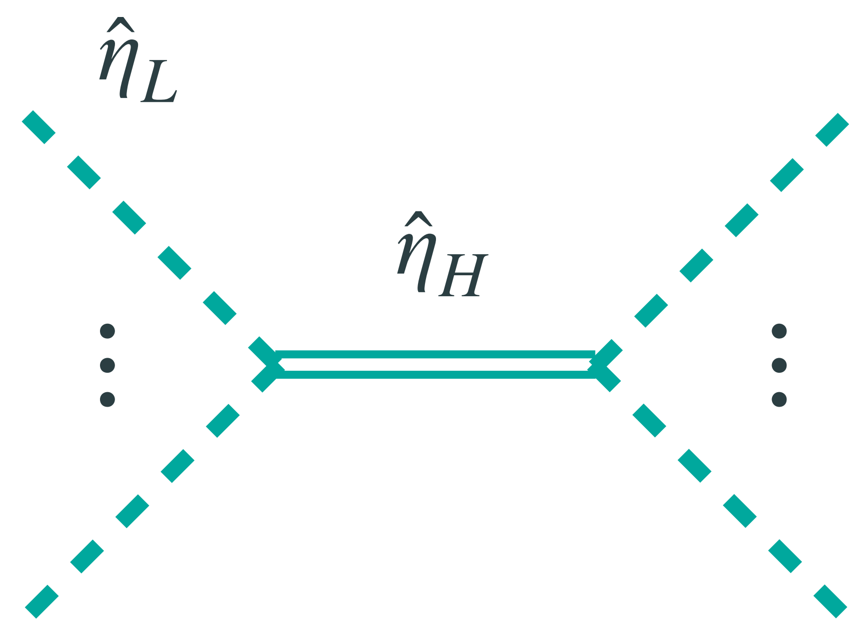
$$\mathscr{L}_{\text{UV}}(\hat{\eta}+\eta) = \boxed{\mathscr{L}_{\text{UV}}(\hat{\eta})} + \left.\frac{\delta \mathscr{L}_{\text{UV}}}{\delta \eta_a}\right|_{\eta=\hat{\eta}} \eta_a + \frac{1}{2}\,\eta_a(x) \left.\frac{\delta^2 \mathscr{L}_{\text{UV}}}{\delta \eta_b(x')\,\delta \eta_a(x)}\right|_{\eta=\hat{\eta}} \eta_b(x') + \mathscr{O}(\eta^3)$$

- **Tree-level:** $\mathscr{L}^{(0)}_{\text{EFT}} = \mathscr{L}_{\text{UV}}\big(\hat{\eta}_L, \hat{\eta}_H(\hat{\eta}_L)\big)$

— Substitute $\hat{\eta}_H$ by its EOM expanded in m_H^{-1}

[Simpler than computing Feynman graphs]

$$\left.\frac{\delta \mathscr{L}_{\text{UV}}}{\delta \eta_H}\right|_{\eta=\hat{\eta}} = 0$$



Functional matching

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$$\mathscr{L}_{\text{UV}}(\hat\eta+\eta) = \boxed{\mathscr{L}_{\text{UV}}(\hat\eta)} + \underbrace{\cancel{\frac{\delta \mathscr{L}_{\text{UV}}}{\delta \eta_a}\bigg|_{\eta=\hat\eta}}}_{0} \eta_a + \frac{1}{2}\eta_a(x)\frac{\delta^2\mathscr{L}_{\text{UV}}}{\delta\eta_b(x')\,\delta\eta_a(x)}\bigg|_{\eta=\hat\eta}\eta_b(x') + \mathcal{O}(\eta^3)$$

Functional matching

- **Expanding the Lagrangian in η:**

$$\mathcal{L}_{\text{UV}}(\hat{\eta}+\eta) = \boxed{\mathcal{L}_{\text{UV}}(\hat{\eta})} + \cancelto{0}{\frac{\delta \mathcal{L}_{\text{UV}}}{\delta \eta_a}\bigg|_{\eta=\hat{\eta}}} \eta_a + \frac{1}{2}\,\eta_a(x) \overbrace{\boxed{\frac{\delta^2 \mathcal{L}_{\text{UV}}}{\delta \eta_b(x')\,\delta \eta_a(x)}\bigg|_{\eta=\hat{\eta}}}}^{\int_{x'} \mathcal{Q}_{ab}(x,x')\ \equiv} \eta_b(x') + \mathcal{O}(\eta^3)$$

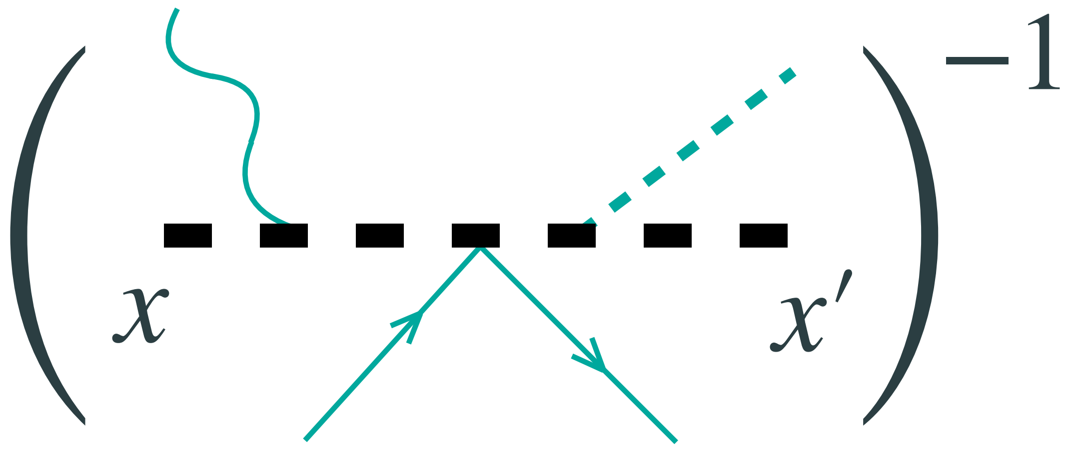
- **Inverse quantum-field propagator:**

Wilson line

[parallel transport $x \leftrightarrow x'$]

New!!

$$\mathcal{Q}_{ab}(x,x') = Q_{ac}\Big(\hat{\eta}(x), \hat{D}_x^{\mu_1}\cdots\hat{D}_x^{\mu_n}\hat{\eta}(x), \hat{D}_x^{\mu}\Big)\, \boldsymbol{U_{cb}(x,x')}\, \delta(x-x')$$



$$\hat{D}_x^{\mu_1}\cdots\hat{D}_x^{\mu_n}\, U(x,x')\Big|_{x=x'} = p_n\left(G^{\mu\nu}, D^{\mu}G^{\nu\rho}, \ldots\right)$$

[Kuzenko, McArthur, ’03]

[JFM, Moreno, Palavrić, Thomsen, **coming soon!**]

Functional matching

- **Expanding the Lagrangian in η:**

$$\mathscr{L}_{\mathrm{UV}}(\hat{\eta}+\eta) = \boxed{\mathscr{L}_{\mathrm{UV}}(\hat{\eta})} + \cancelto{0}{\left.\frac{\delta \mathscr{L}_{\mathrm{UV}}}{\delta \eta_a}\right|_{\eta=\hat{\eta}}} \eta_a + \frac{1}{2}\,\eta_a(x) \overbrace{\boxed{\left.\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta \eta_b(x')\,\delta \eta_a(x)}\right|_{\eta=\hat{\eta}}}}^{\int_{x'} \mathcal{Q}_{ab}(x,x') \;\equiv} \eta_b(x') + \boxed{\mathcal{O}(\eta^3)}$$

Higher-loop orders
(more later)

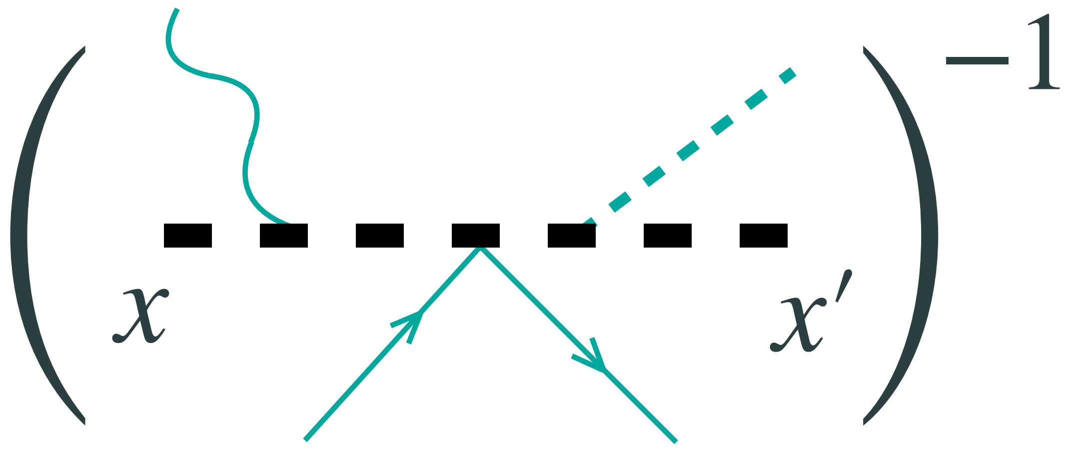
- **Inverse quantum-field propagator:**

Wilson line
[parallel transport $x \leftrightarrow x'$]

New!!

$$\mathcal{Q}_{ab}(x,x') = Q_{ac}\left(\hat{\eta}(x), \hat{D}_x^{\mu_1}\cdots\hat{D}_x^{\mu_n}\hat{\eta}(x), \hat{D}_x^{\mu}\right) \boldsymbol{U_{cb}(x,x')}\,\delta(x-x')$$

$$\left.\hat{D}_x^{\mu_1}\cdots\hat{D}_x^{\mu_n}\, U(x,x')\right|_{x=x'} = p_n\left(G^{\mu\nu}, D^{\mu}G^{\nu\rho}, \ldots\right)$$



[Kuzenko, McArthur, '03]
[JFM, Moreno, Palavrić, Thomsen, **coming soon!**]

Functional matching

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$$\mathscr{L}_{\mathrm{UV}}(\hat\eta+\eta) = \boxed{\mathscr{L}_{\mathrm{UV}}(\hat\eta)} + \cancelto{0}{\frac{\delta \mathscr{L}_{\mathrm{UV}}}{\delta \eta_a}}\bigg|_{\eta=\hat\eta} \eta_a + \frac{1}{2}\,\eta_a(x) \underbrace{\boxed{\frac{\delta^2 \mathscr{L}_{\mathrm{UV}}}{\delta \eta_b(x')\,\delta \eta_a(x)}\bigg|_{\eta=\hat\eta}}}_{\equiv\ \int_{x'} \mathcal{Q}_{ab}(x,x')} \eta_b(x') + \boxed{\mathcal{O}(\eta^3)}$$

Higher-loop orders
(more later)

- **1-loop effective action:**

$$e^{i\,\Gamma^{(1)}_{\mathrm{UV}}[\hat\eta]} = \int \mathscr{D}\eta\ \exp\!\left(i \int_{x,x'} \eta_a(x)\, \mathcal{Q}_{ab}(x,x')\, \eta_b(x') \right) \quad \Longrightarrow \quad \Gamma^{(1)}_{\mathrm{UV}}[\hat\eta] = -\,i \ln \mathrm{SDet}\, \mathcal{Q}^{-1/2} = \frac{i}{2}\, \mathrm{STr} \ln \mathcal{Q}$$

Gaussian integration

How to evaluate supertraces

$$\Gamma^{(1)}_{\mathrm{UV}}[\hat{\eta}] = \frac{i}{2}\,\mathrm{STr}\ln\mathcal{Q} = \pm\frac{i}{2}\int_{x,x'}\delta(x-x')\,\overbrace{[\,\ln Q(x,iD^{\mu}_{x})\,]_{ab}\,U_{ba}(x,x')\,\delta(x-x')}^{[\ln\mathcal{Q}(x,x')]_{aa}}$$

How to evaluate supertraces

$$[\ln \mathcal{Q}(x,x')]_{aa}$$

$$\int_k e^{ik(x-x')}$$

$$\Gamma^{(1)}_{\mathrm{UV}}[\hat{\eta}] = \frac{i}{2}\,\mathrm{STr}\ln\mathcal{Q} = \pm\frac{i}{2}\int_{x,x'} \delta(x-x')\,[\,\ln Q(x, iD^{\mu}_{x})\,]_{ab}\, U_{ba}(x,x')\,\delta(x-x')$$

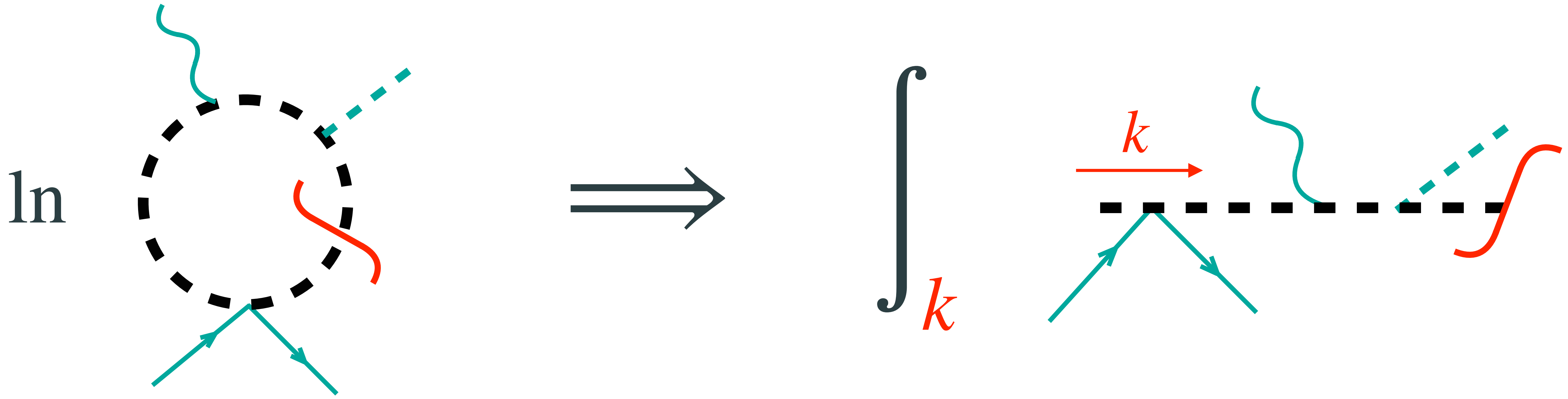
$$= \pm\frac{i}{2}\int_{x,k} [\ln Q\big(x, i\hat{D}^{\mu}_{x} - k\big)]_{ab}\, U_{ba}(x,x')\Big|_{x=x'}$$

How to evaluate supertraces

$$\Gamma^{(1)}_{\mathrm{UV}}[\hat{\eta}] = \frac{i}{2}\,\mathrm{STr}\ln\mathcal{Q} = \pm\frac{i}{2}\int_{x,x'} \overbrace{\delta(x-x')\,[\,\ln Q(x, iD^{\mu}_{x})\,]_{ab}\,U_{ba}(x,x')\,\delta(x-x')}^{[\ln \mathcal{Q}(x,x')]_{aa}}$$

$$\int_k e^{ik(x-x')}$$

$$= \pm\frac{i}{2}\int_{x,k} [\ln Q\big(x, i\hat{D}^{\mu}_{x} - k\big)]_{ab}\,U_{ba}(x,x')\Big|_{x=x'}$$



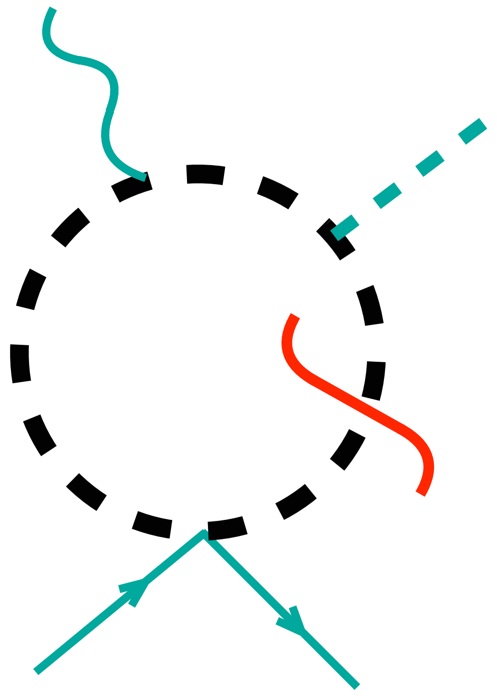
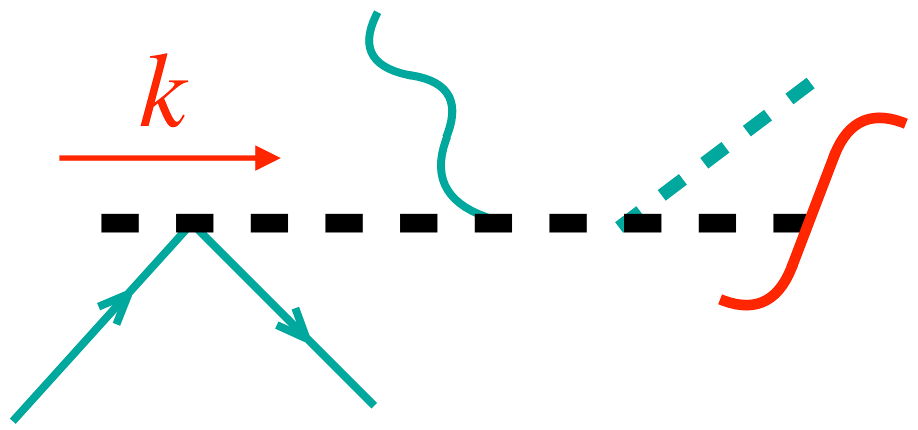
How to evaluate supertraces

$[\ln \mathcal{Q}(x,x')]_{aa}$

$\int_k e^{ik(x-x')}$

$$\Gamma^{(1)}_{\text{UV}}[\hat\eta] = \frac{i}{2}\,\text{STr}\ln\mathcal{Q} = \pm\frac{i}{2}\int_{x,x'}\delta(x-x')\,[\,\ln Q(x, iD^\mu_x)\,]_{ab}\,U_{ba}(x,x')\,\delta(x-x')$$

$$= \pm\frac{i}{2}\int_{x,k}[\ln Q\big(x, i\hat{D}^\mu_x - k\big)]_{ab}\,U_{ba}(x,x')\Big|_{x=x'}$$

ln  $\Longrightarrow$ $\Gamma^{(1)}_{\text{EFT}} = \int_k$ 

$\Big|_{\text{hard}}$

The EFT action (to arbitrary EFT order) is obtained *directly* from the hard-momentum expansion: $k \gtrsim m_H$

[JFM, Portolés, Ruiz-Femenía, 1607.02142; Z. Zhang 1610.00710]

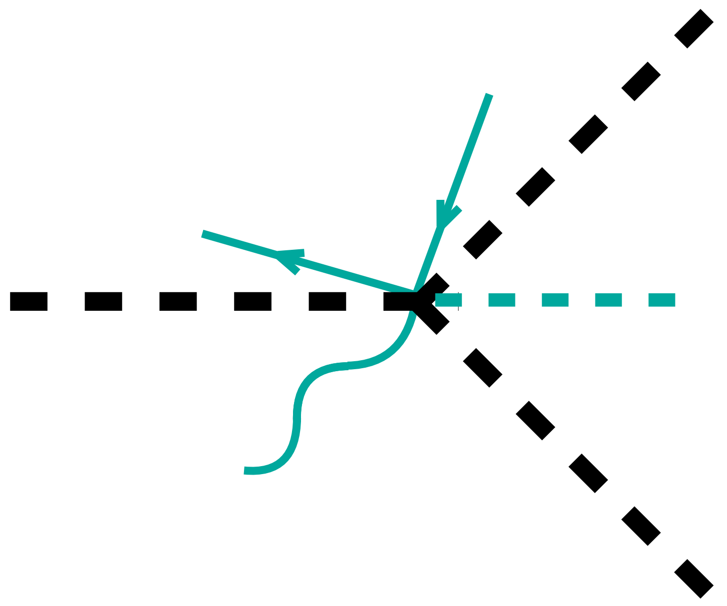
Going beyond one loop

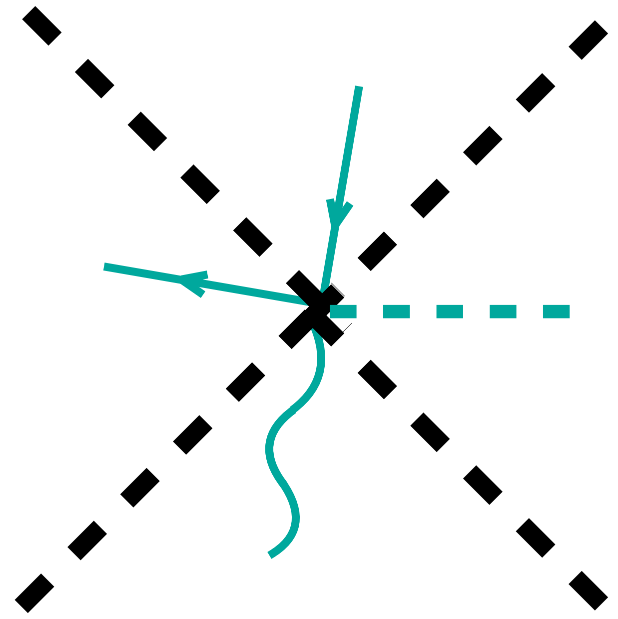
[JFM, Palavrić, Thomsen, 2311.13630]

[JFM, Moreno, Palavrić, Thomsen, **coming soon!**]

$$\Gamma_{\rm UV}[\hat{\eta}] = S_{\rm UV}[\hat{\eta}] - i\ln\int \mathscr{D}\eta\, \exp\left[i\left(\frac{1}{2}\,\eta_I \mathscr{Q}_{IJ}\eta_J + \frac{1}{3\,!}\,\eta_K\eta_J\eta_I \mathscr{V}_{KJI} + \frac{1}{4\,!}\,\eta_L\eta_K\eta_J\eta_I\mathscr{V}_{IJKL} + \dots\right)\right]$$

$$\mathscr{V}_{IJK} \equiv \left.\frac{\delta^3 \mathscr{L}_{\rm UV}}{\delta\eta_I\,\delta\eta_J\,\delta\eta_K}\right|_{\eta=\hat{\eta}} \qquad \mathscr{V}_{IJKL} \equiv \left.\frac{\delta^4 \mathscr{L}_{\rm UV}}{\delta\eta_I\,\delta\eta_J\,\delta\eta_K\,\delta\eta_L}\right|_{\eta=\hat{\eta}}$$





N.B.: $\mathscr{V}_{IJK} = \sum_{m,n} V^{(m,n)}_{ade}(x)\, P^m_y P^n_z\, U(x,y)_{db} U(x,z)_{ec}\, \delta(x-y)\delta(x-z)$

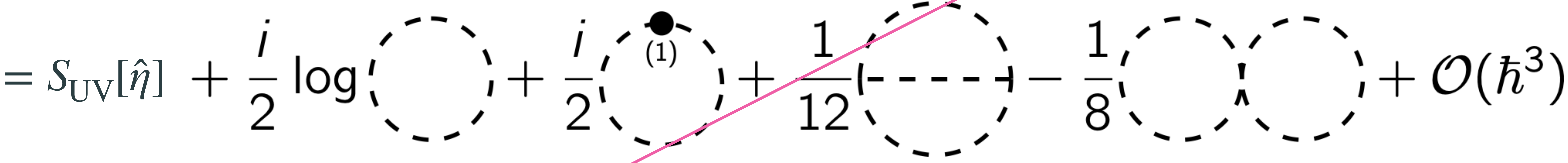
Going beyond one loop

[JFM, Palavrić, Thomsen, 2311.13630]

[JFM, Moreno, Palavrić, Thomsen, **coming soon!**]

$$\Gamma_{\mathrm{UV}}[\hat{\eta}] = S_{\mathrm{UV}}[\hat{\eta}] - i\ln\int \mathscr{D}\eta \exp\left[i\left(\frac{1}{2}\eta_I \mathscr{Q}_{IJ}\eta_J + \frac{1}{3!}\eta_I\eta_J\eta_K \mathscr{V}_{IJK} + \frac{1}{4!}\eta_I\eta_J\eta_K\eta_L \mathscr{V}_{IJKL} + \dots\right)\right]$$

$$= S_{\mathrm{UV}}[\hat{\eta}] + \frac{i\hbar}{2}\mathrm{STr}\ln\mathscr{Q} + \frac{i\hbar^2}{2}\mathscr{Q}^{-1}_{IJ}\mathscr{V}^{(1)}_{JI} + \frac{\hbar^2}{12}\mathscr{V}_{IJK}\,\mathscr{Q}^{-1}_{IL}\mathscr{Q}^{-1}_{JM}\mathscr{Q}^{-1}_{KN}\,\mathscr{V}_{LMN} - \frac{\hbar^2}{8}\mathscr{Q}^{-1}_{IJ}\,\mathscr{V}_{IJKL}\,\mathscr{Q}^{-1}_{KL} + \mathscr{O}(\hbar^3)$$



Tree level

One loop

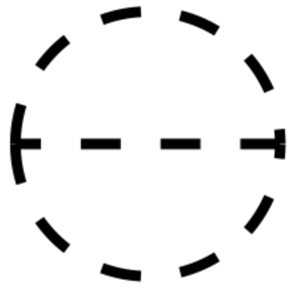
Two loops

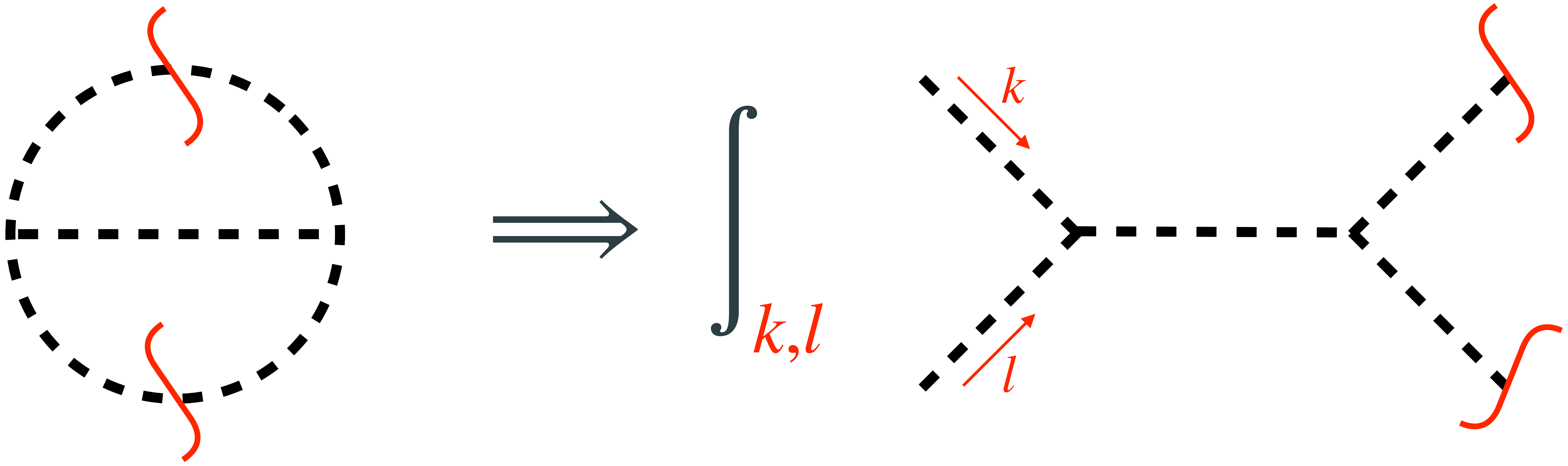
Every two-loop contribution is included here!

Two-loop functional evaluation

[JFM, Palavrić, Thomsen, 2311.13630]

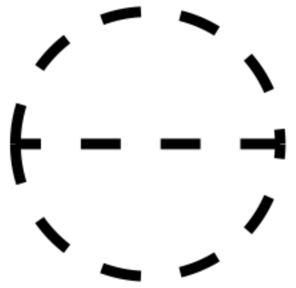
[JFM, Moreno, Palavrić, Thomsen, **coming soon!**]

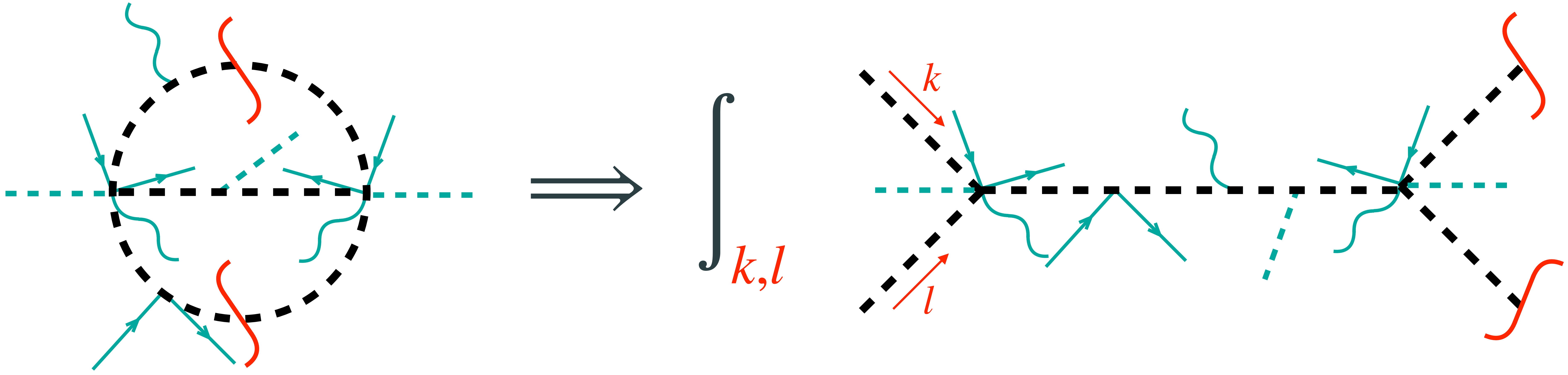
How to evaluate  ?



[JFM, Palavrić, Thomsen, 2311.13630]
[JFM, Moreno, Palavrić, Thomsen, **coming soon!**]

Two-loop functional evaluation

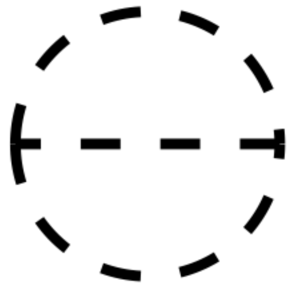
How to evaluate  ?

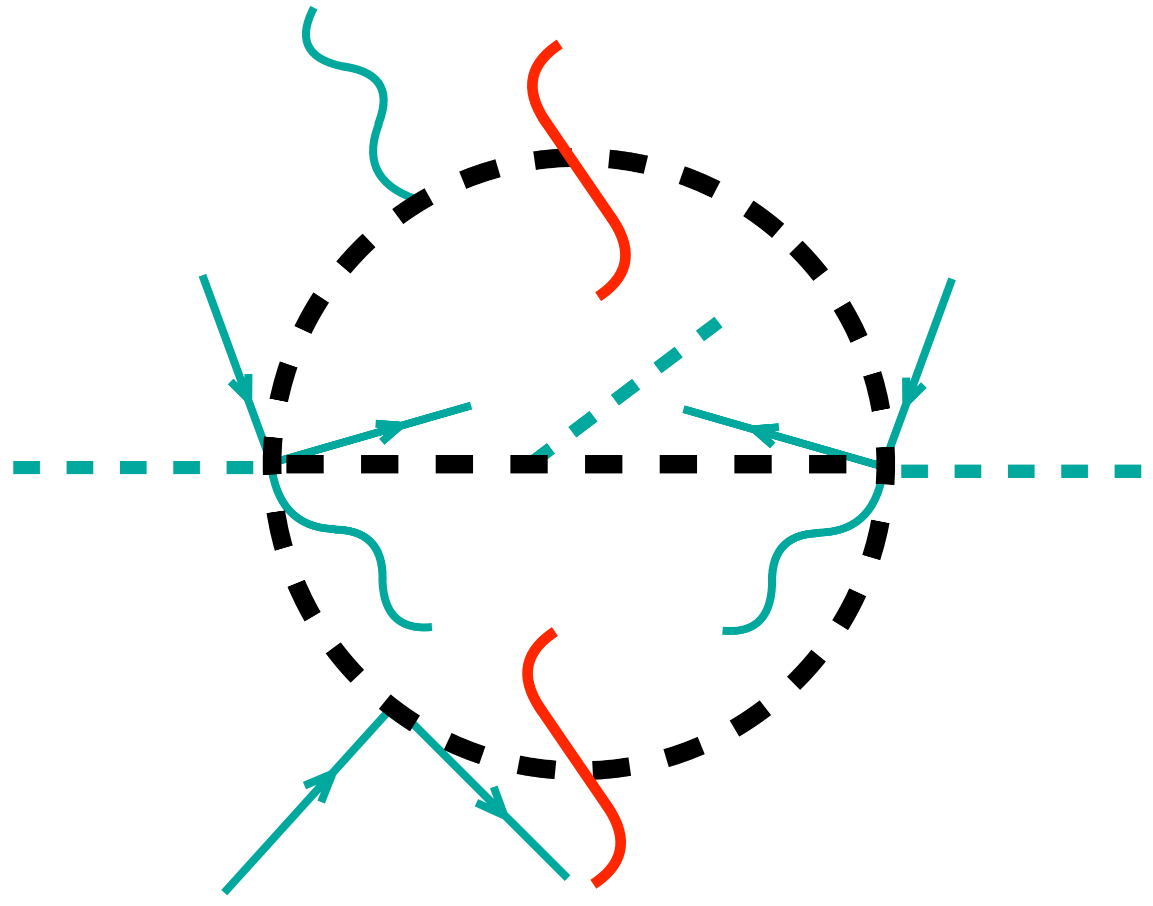


Every (non-factorizable) two-loop contribution is included here!

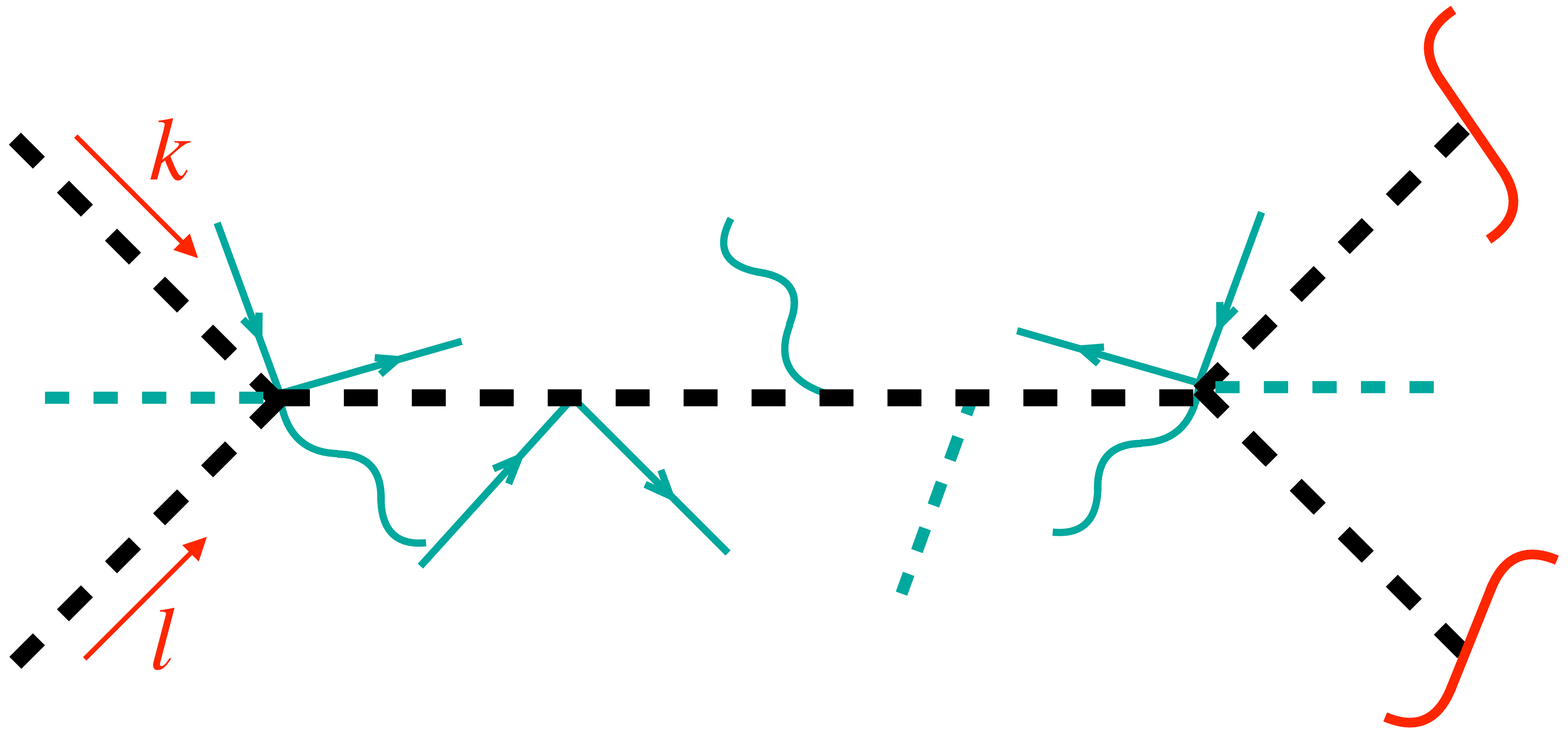
[JFM, Palavrić, Thomsen, 2311.13630]
[JFM, Moreno, Palavrić, Thomsen, **coming soon!**]

Two-loop functional evaluation

How to evaluate  ?



$$\Longrightarrow \int_{k,l}$$



$$G_{\ominus} = \int_x \int_{k,l} V_{abc}(x)\, Q^{-1}_{aa'}(x', iD_{x'} + k + l)\, V_{a'b'c'}(x') \left[Q^{-1}_{bd}(x, iD_x - k) U_{db'}(x, x')\right] \left[Q^{-1}_{ce}(x, iD_x - l) U_{ec'}(x, x')\right]$$

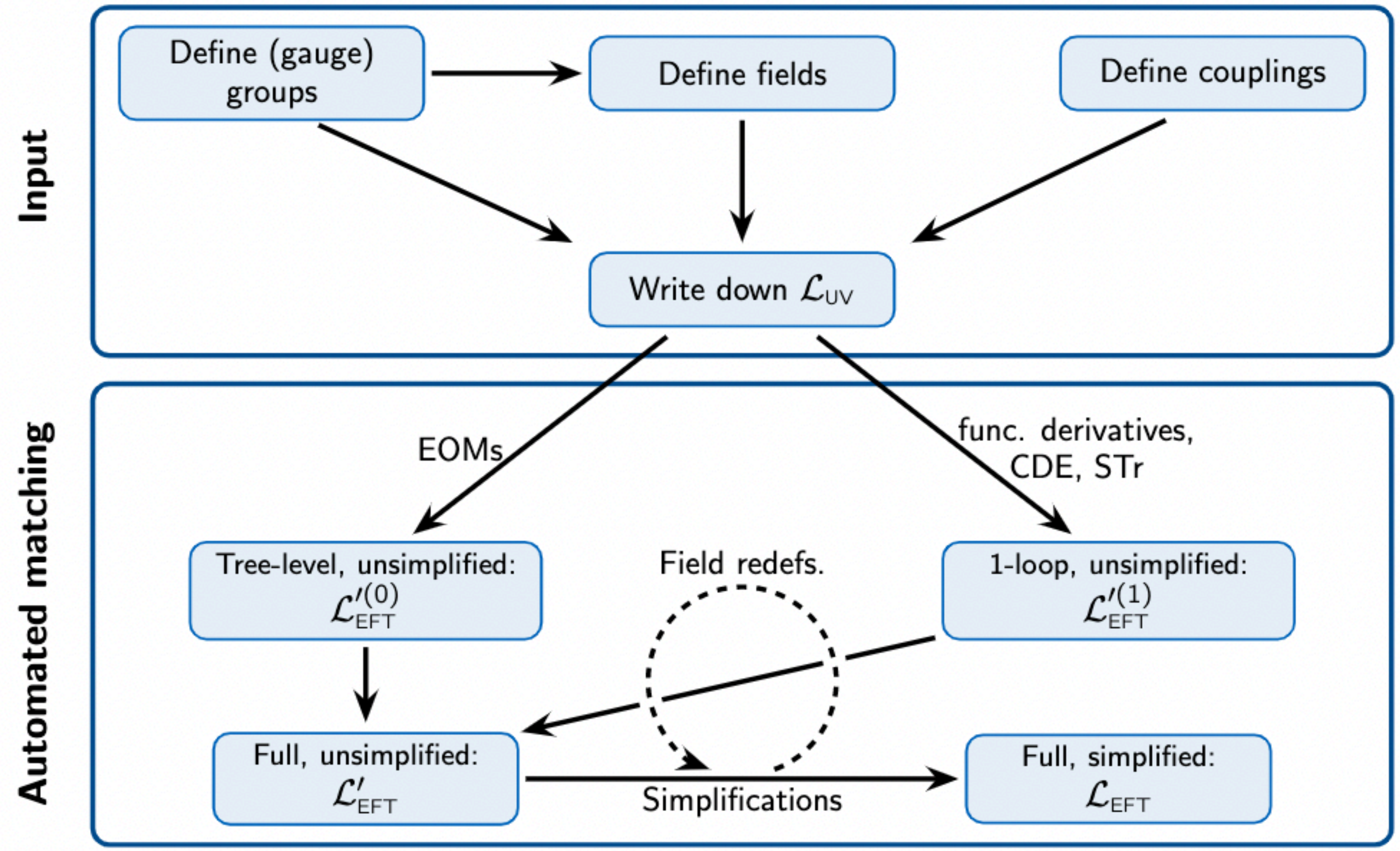
Valid to *all orders* in the EFT expansion!

MATCHETE

To make your way through the BSM jungle

The Matchete package

MATCHETE is a Mathematica package aimed at fully automating EFT matching and RG evolution of arbitrary weakly-coupled UV theories using functional methods

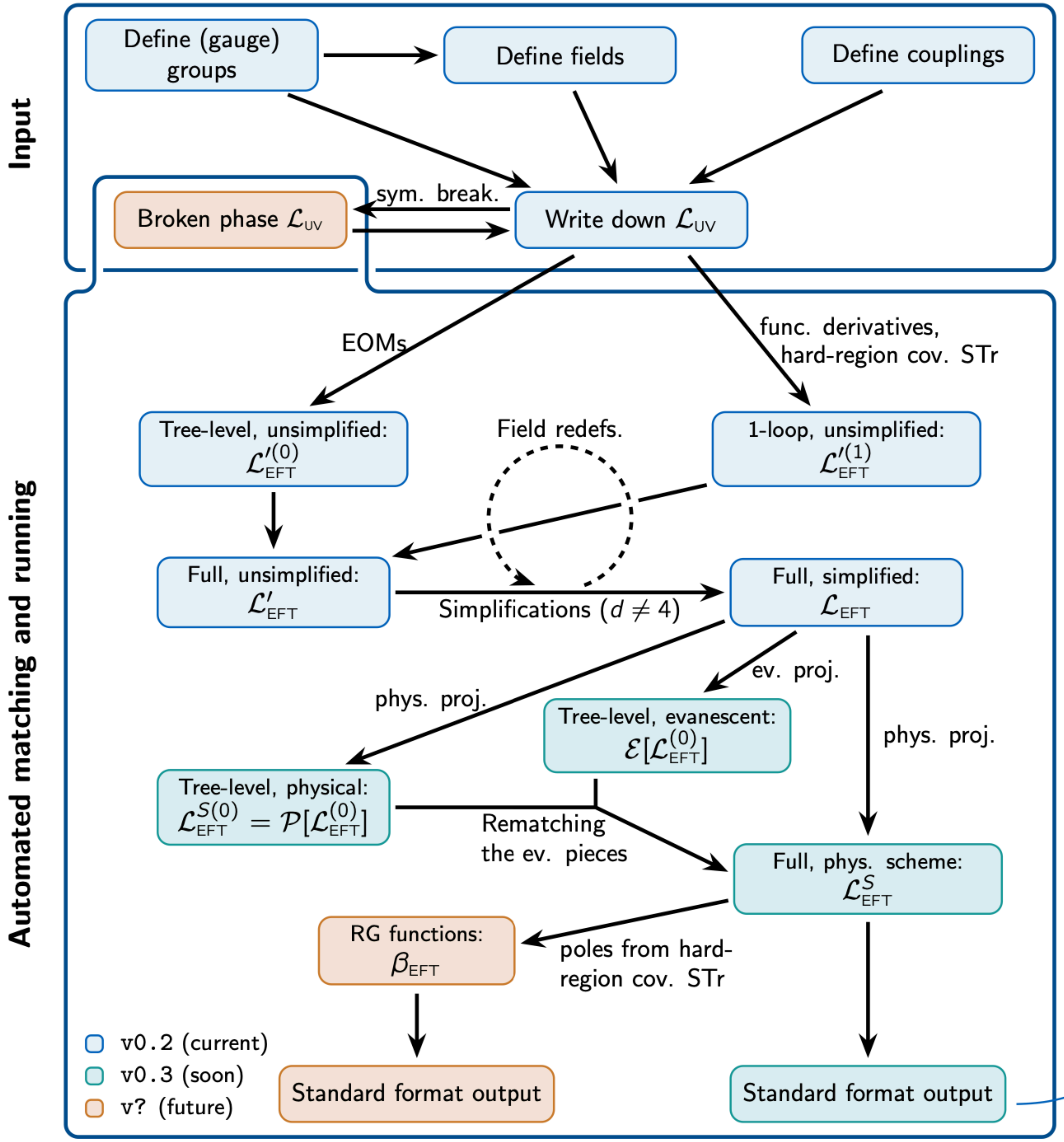


Matchete v0.2 now publicly available:

- One-loop matching of *any* model with heavy scalars and/or fermions
- Simple and intuitive input/output
- Handles *all* group theory (any group and reps)
- Fully automated simplifications to EFT basis (IBP, field redefinitions/EOMs,…)

[JFM, König, Pagès, Thomsen, Wilsch, 2212.04510]

Work in progress and future plans



Currently with

Upcoming!

- Handling of evanescent contributions
- Interface with other EFT tools

Coming “soon”

- One-loop RG computations

Longer term

- RG and matching beyond one loop
  — 2-loop RG in the bosonic SMEFT
  [Born, JFM, Kvedaraitė, Thomsen, 2410.07320]
- Heavy vectors and symmetry breaking

Example: SM + vector-like lepton

Setup

SM Lagrangian

In[3]:=
```
LSM = LoadModel["SM"];
```

Define new field

In[4]:=
```
DefineField[EE, Fermion, Charges → {U1Y[-1]}, Mass → {Heavy, ME}]
```

Define new coupling

In[5]:=
```
DefineCoupling[yE, EFTOrder → 0, Indices → {Flavor}]
```

Write interactions

In[6]:=
```
Lint = -yE[p] × Bar@l[i, p] ** PR ** EE[] × H[i] // PlusHc;
Lint // NiceForm
```

Out[7]//NiceForm=

$$-\overline{yE}^{p}\,\bar{H}_{i}\,\left(\overline{EE}\cdot P_{L}\cdot l^{ip}\right)-yE^{p}\,H^{i}\,\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot EE\right)$$

Define full UV Lagrangian

In[8]:=
```
LUV = LSM + FreeLag[EE] + Lint;
LUV // NiceForm
```

Out[9]//NiceForm=

$$-\frac{1}{4}B^{\mu\nu 2}-\frac{1}{4}G^{\mu\nu A 2}-\frac{1}{4}W^{\mu\nu I 2}+D_{\mu}\bar{H}_{i}\,D_{\mu}H^{i}+\mu^{2}\,\bar{H}_{i}\,H^{i}+i\,\left(\bar{d}^{p}_{a}\cdot\gamma_{\mu}\,P_{R}\cdot D_{\mu}d^{ap}\right)+i\,\left(\bar{e}^{p}\cdot\gamma_{\mu}\,P_{R}\cdot D_{\mu}e^{p}\right)+$$
$$i\,\left(\overline{EE}\cdot\gamma_{\mu}\cdot D_{\mu}EE\right)-ME\,\left(\overline{EE}\cdot EE\right)+i\,\left(\bar{l}^{p}_{i}\cdot\gamma_{\mu}\,P_{L}\cdot D_{\mu}l^{ip}\right)+i\,\left(\bar{q}^{p}_{ai}\cdot\gamma_{\mu}\,P_{L}\cdot D_{\mu}q^{aip}\right)+i\,\left(\bar{u}^{p}_{a}\cdot\gamma_{\mu}\,P_{R}\cdot D_{\mu}u^{ap}\right)-$$
$$\frac{1}{2}\lambda\,\bar{H}_{i}\,\bar{H}_{j}\,H^{i}\,H^{j}-\overline{Yd}^{pr}\,\bar{H}_{i}\,\left(\bar{d}^{r}_{a}\cdot P_{L}\cdot q^{aip}\right)-\overline{Ye}^{pr}\,\bar{H}_{i}\,\left(\bar{e}^{r}\cdot P_{L}\cdot l^{ip}\right)-Ye^{pr}\,H^{i}\,\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot e^{r}\right)-Yd^{pr}\,H^{i}\,\left(\bar{q}^{p}_{ai}\cdot P_{R}\cdot d^{ar}\right)-$$
$$Yu^{pr}\,\bar{H}_{i}\,\left(\bar{q}^{p}_{aj}\cdot P_{R}\cdot u^{ar}\right)\,\varepsilon^{ji}-\overline{Yu}^{pr}\,H^{j}\,\left(\bar{u}^{r}_{a}\cdot P_{L}\cdot q^{aip}\right)\,\bar{\varepsilon}_{ij}-\overline{yE}^{p}\,\bar{H}_{i}\,\left(\overline{EE}\cdot P_{L}\cdot l^{ip}\right)-yE^{p}\,H^{i}\,\left(\bar{l}^{p}_{i}\cdot P_{R}\cdot EE\right)$$

Example: SM + vector-like lepton

Main matching routine

In[9]:=
```
LEFT = Match[LUV, LoopOrder → 1, EFTOrder → 6] /. ϵ^-1 → 0;
```

Simplification to on-shell basis

In[10]:=
```
LEFTOnShell = LEFT // EOMSimplify;
Length@%
```

» The Lagrangian contains terms of lower power than dimension 4. Defining effective couplings and assuming these terms to be dimension 4. Use 'PrintEffectiveCouplings' and 'ReplaceEffectiveCouplings' to recover explicit expressions.

» Added new CG cg1 with indices {Bar[SU2L[fund]], SU2L[adj], Bar[SU2L[fund]]}

Out[11]= 66

Select Higgs-lepton current operator

In[12]:=
```
SelectOperatorClass[LEFTOnShell, {e, Bar@e, H, Bar@H}, 1] // GreensSimplify // NiceForm
```

Out[12]//NiceForm=

$$\frac{i}{360}\hbar\frac{1}{ME^2}\left(48\,gY^4\,\delta^{pr} + 5\,\overline{yE}^s\left(3\,yE^t\,\overline{Ye}^{tr}\,Ye^{sp}\left(1 + 6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\right) - 2\,yE^s\,gY^2\left(13 + 6\,\mathrm{Log}\left[\frac{\bar{\mu}^2}{ME^2}\right]\right)\delta^{pr}\right)\right)$$
$$\left(-D_\mu\bar{H}_i\,H^i\,\left(\bar{e}^r\cdot\gamma_\mu\,P_R\cdot e^p\right) + \bar{H}_i\,D_\mu\,H^i\,\left(\bar{e}^r\cdot\gamma_\mu\,P_R\cdot e^p\right)\right)$$

$$Q_{He}^{pr} = (H^\dagger i\overleftrightarrow{D}_\mu H)(\bar{e}_p\gamma^\mu e_r)$$

Example: SM + vector-like lepton

```
LEFTOnShell // NiceForm
```

iceForm=

$$-\frac{1}{4} G^{\mu\nu A2} - \frac{1}{4} W^{\mu\nu I2} + \left(-\frac{1}{4} - \frac{1}{3}\hbar\, gY^2 \text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) B^{\mu\nu 2} + D_\mu \bar{H}_i D_\mu H^i + \left(C_{H2} + \frac{1}{6}\hbar\, \overline{yE}^p yE^p C_{H2} \frac{1}{ME^2}\left(2 C_{H2} - 3 ME^2\left(1 + 2\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right)\bar{H}_i H^i + i\,(\bar{d}^r_a \cdot \gamma_\mu P_R \cdot D_\mu d^{ap})\,\delta^{pr} + i\,(\bar{e}^r \cdot \gamma_\mu P_R \cdot D_\mu e^p)\,\delta^{pr} +$$

$$i\,(\bar{l}^r_i \cdot \gamma_\mu P_L \cdot D_\mu l^{ip})\,\delta^{pr} + i\,(\bar{q}^r_{ai} \cdot \gamma_\mu P_L \cdot D_\mu q^{aip})\,\delta^{pr} + i\,(\bar{u}^r_a \cdot \gamma_\mu P_R \cdot D_\mu u^{ap})\,\delta^{pr} + \Bigg(-\frac{1}{2}\lambda + \hbar\,\Bigg(-\frac{1}{2}\overline{yE}^p\left(4\, yE^r\, \overline{Ye}^{rs}\, Ye^{ps}\left(1 + \text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) - yE^p\left(-2\,\overline{yE}^r yE^r \text{Log}\left[\frac{\mu^2}{ME^2}\right] + \lambda\left(1 + 2\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right) -$$

$$\frac{1}{180} C_{H2}\frac{1}{ME^2}\left(12\, gY^4 - 5\,\overline{yE}^p yE^p gY^2\left(13 + 6\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) + 5\,\overline{yE}^p\left(-12\,(\overline{yE}^r yE^p yE^r + 6\, yE^r\, \overline{Ye}^{rs}\, Ye^{ps} - 2\, yE^p \lambda) + yE^p gL^2\left(5 + 6\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right)\Bigg)\Bigg)$$

$$\bar{H}_i \bar{H}_j H^i H^j + \left(-\overline{Yd}^{pr} + \frac{1}{12}\hbar\,\overline{yE}^s yE^s\, \overline{Yd}^{pr}\frac{1}{ME^2}\left(-4\, C_{H2} + 3\, ME^2\left(1 + 2\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right)\bar{H}_i\,(\bar{d}^r_a \cdot P_L \cdot q^{aip}) +$$

$$\left(-\overline{Ye}^{pr} + \frac{1}{24}\hbar\, yE^s\frac{1}{ME^2}\left(-3\,\overline{yE}^p\,\overline{Ye}^{sr}\,(2\, C_{H2} - ME^2)\left(3 + 2\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) + 2\,\overline{yE}^s\,\overline{Ye}^{pr}\left(-4\, C_{H2} + 3\, ME^2\left(1 + 2\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right)\right)\bar{H}_i\,(\bar{e}^r \cdot P_L \cdot l^{ip}) +$$

$$\left(-Ye^{rp} + \frac{1}{24}\hbar\,\overline{yE}^s\frac{1}{ME^2}\left(3\, ME^2\left(2\, yE^s\, Ye^{rp}\left(1 + 2\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) + yE^r\, Ye^{sp}\left(3 + 2\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right) - 2\, C_{H2}\left(4\, yE^s\, Ye^{rp} + 3\, yE^r\, Ye^{sp}\left(3 + 2\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right)\right) H^i\,(\bar{l}^r_i \cdot P_R \cdot e^p) +$$

$$\left(-Yd^{rp} + \frac{1}{12}\hbar\,\overline{yE}^s yE^s\, Yd^{rp}\frac{1}{ME^2}\left(-4\, C_{H2} + 3\, ME^2\left(1 + 2\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right) H^i\,(\bar{q}^r_{ai} \cdot P_R \cdot d^{ap}) +$$

$$\left(-Yu^{rp} + \frac{1}{12}\hbar\,\overline{yE}^s yE^s\, Yu^{rp}\frac{1}{ME^2}\left(-4\, C_{H2} + 3\, ME^2\left(1 + 2\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right)\bar{H}_i\,(\bar{q}^r_{aj} \cdot P_R \cdot u^{ap})\,\varepsilon^{ji} +$$

$$\left(-\overline{Yu}^{pr} + \frac{1}{12}\hbar\,\overline{yE}^s yE^s\, \overline{Yu}^{pr}\frac{1}{ME^2}\left(-4\, C_{H2} + 3\, ME^2\left(1 + 2\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right) H^j\,(\bar{u}^r_a \cdot P_L \cdot q^{aip})\,\bar{\varepsilon}_{ij} +$$

$$\frac{1}{180}\hbar\frac{1}{ME^2}\left(12\,\lambda\, gY^4 + 5\,\overline{yE}^p\left(12\,\overline{yE}^r yE^p\,(\overline{yE}^s yE^r yE^s + 6\, yE^s\, \overline{Ye}^{st}\, Ye^{rt} - yE^r\lambda) - 72\, yE^r\,\overline{Ye}^{rs}\left(Ye^{ps}\lambda + \overline{Ye}^{tu}\, Ye^{pu}\, Ye^{ts}\left(1 + \text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right) + yE^p\lambda\left(12\,\lambda + gL^2\left(5 + 6\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) - gY^2\left(13 + 6\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right)\right)$$

$$\bar{H}_i \bar{H}_j \bar{H}_k H^i H^j H^k + \frac{1}{90}\hbar\frac{1}{ME^2}\left(-12\, gY^4 + 5\,\overline{yE}^p yE^p gY^2\left(13 + 6\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) + 45\,\overline{yE}^p yE^r\left(-\overline{yE}^r yE^p + \overline{Ye}^{rs}\, Ye^{ps}\left(1 + 2\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right)\bar{H}_i D_\mu \bar{H}_j D_\mu H^i H^j +$$

$$\frac{1}{180}\hbar\frac{1}{ME^2}\left(-12\, gY^4 + 5\,\overline{yE}^p yE^p gY^2\left(13 + 6\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) - 15\,\overline{yE}^p\left(yE^p gL^2\left(5 + 6\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) + 4\, yE^r\left(2\,\overline{yE}^r yE^p - 3\,\overline{Ye}^{rs}\, Ye^{ps}\left(3 + 2\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right)\right)\bar{H}_i D_\mu \bar{H}_j H^i D_\mu H^j +$$

$$\frac{1}{8}\hbar\,\overline{yE}^p yE^p gY^2\frac{1}{ME^2}\bar{H}_i H^i B^{\mu\nu 2} - \frac{1}{3}\hbar\, gL\, gY\,\overline{yE}^p yE^p\frac{1}{ME^2}\bar{H}_i H^j B^{\mu\nu} W^{\mu\nu I} T^{Ii}_j + \frac{1}{24}\hbar\,\overline{yE}^p yE^p gL^2\frac{1}{ME^2}\bar{H}_i H^i W^{\mu\nu I2} +$$

$$\frac{1}{360}\hbar\,\overline{Yd}^{pr}\frac{1}{ME^2}\left(12\, gY^4 - 5\,\overline{yE}^s yE^s gY^2\left(13 + 6\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) + 5\,\overline{yE}^s\left(-12\,(\overline{yE}^t yE^s yE^t + 6\, yE^t\,\overline{Ye}^{tu}\, Ye^{su} - 2\, yE^s\lambda) + yE^s gL^2\left(5 + 6\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right)\bar{H}_i \bar{H}_j H^j\,(\bar{d}^r_a \cdot P_L \cdot q^{aip}) +$$

$$\Bigg(\frac{1}{2}\overline{yE}^p yE^s\,\overline{Ye}^{sr}\frac{1}{ME^2} + \frac{1}{720}\hbar\frac{1}{ME^2}\Bigg(30\,\overline{yE}^s yE^u\,\overline{Ye}^{pt}\,\overline{Ye}^{ur}\, Ye^{st}\left(37 + 18\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) - 45\,\overline{yE}^p\left(\overline{yE}^s yE^s yE^t\,\overline{Ye}^{tr}\left(19 + 18\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) + 4\,\overline{Ye}^{sr}\left(yE^t\,\overline{Ye}^{tu}\, Ye^{su} - 2\, yE^s\lambda\left(5 + 4\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right) +$$

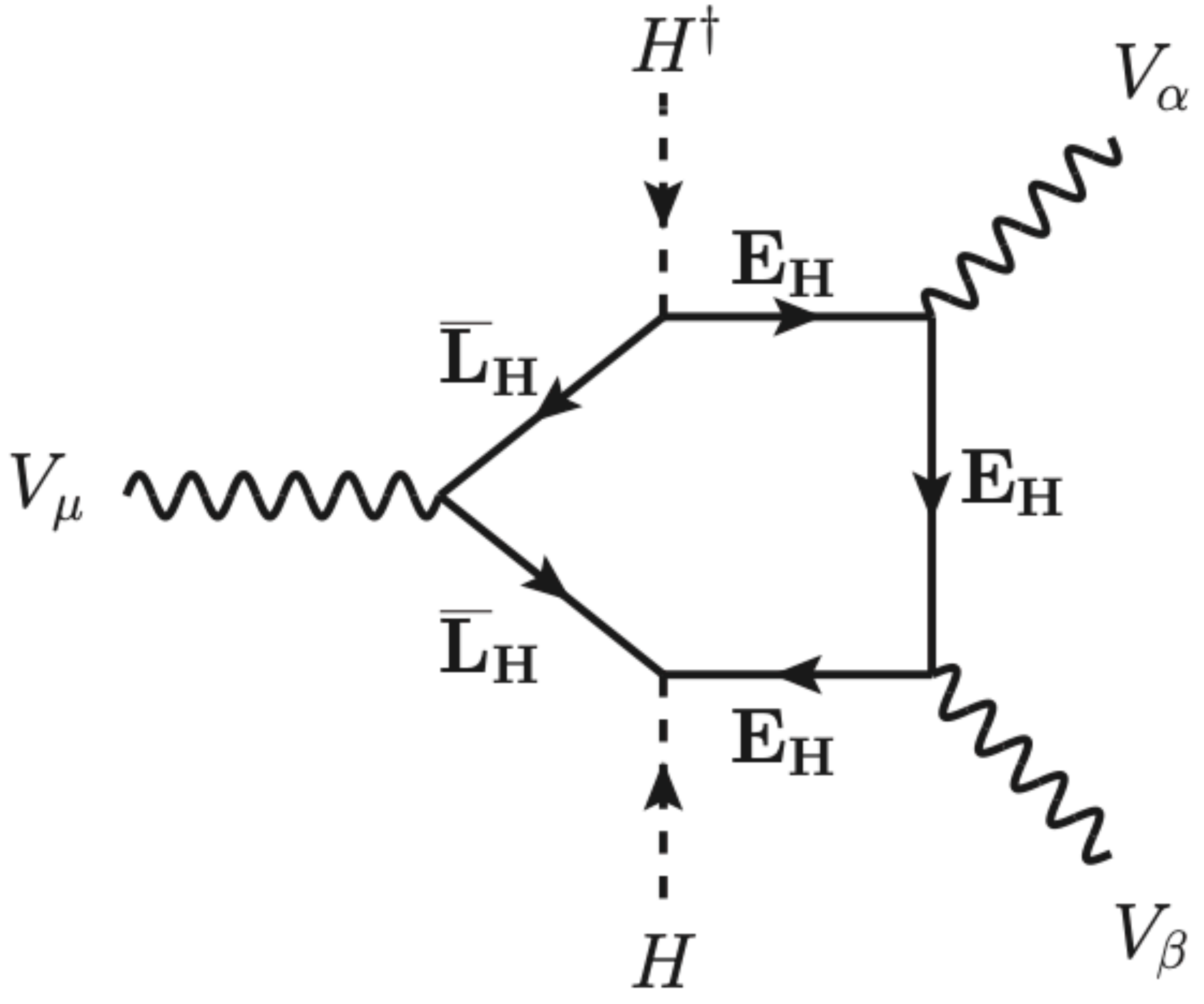
$$2\,\overline{Ye}^{pr}\left(12\, gY^4 - 5\,\overline{yE}^s yE^s gY^2\left(13 + 6\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) + 5\,\overline{yE}^s\left(-12\,(\overline{yE}^t yE^s yE^t + 6\, yE^t\,\overline{Ye}^{tu}\, Ye^{su} - 2\, yE^s\lambda) + yE^s gL^2\left(5 + 6\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right)\Bigg)\Bigg)\bar{H}_i \bar{H}_j H^i\,(\bar{e}^r \cdot P_L \cdot l^{jp}) +$$

$$\Bigg(\frac{1}{2}\overline{yE}^s yE^r\, Ye^{sp}\frac{1}{ME^2} + \frac{1}{720}\hbar\frac{1}{ME^2}\Bigg(24\, Ye^{rp} gY^4 - 10\,\overline{yE}^s yE^s\, Ye^{rp} gY^2\left(13 + 6\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) + 5\,\overline{yE}^s\left(2\, yE^s\, Ye^{rp} gL^2\left(5 + 6\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right) - 3\,\overline{yE}^t yE^t\left(8\, yE^s\, Ye^{rp} + 3\, yE^r\, Ye^{sp}\left(19 + 18\text{Log}\left[\frac{\mu^2}{ME^2}\right]\right)\right)\right) +$$

$$6\left(4\,\lambda\left(2\, yE^s\, Ye^{rp} + 3\, yE^r\, Ye^{sp}\left(5 + 4\text{Log}\left[\frac{\mu^2}{\ldots}\right]\right)\right) + \overline{Ye}^{tu}\left(-6\, yE^r\, Ye^{su}\, Ye^{tp} + yE^t\left(-24\, Ye^{rp}\, Ye^{su} + Ye^{ru}\, Ye^{sp}\left(37 + 18\text{Log}\left[\frac{\mu^2}{\ldots}\right]\right)\right)\right)\right)\Bigg)\Bigg)\bar{H}_i H^i H^j\,(\bar{l}^r_i \cdot P_R \cdot e^p) +$$

Example: neutral triple-gauge interactions

New physics in $Z(\gamma, Z)(\gamma^*, Z^*)$?



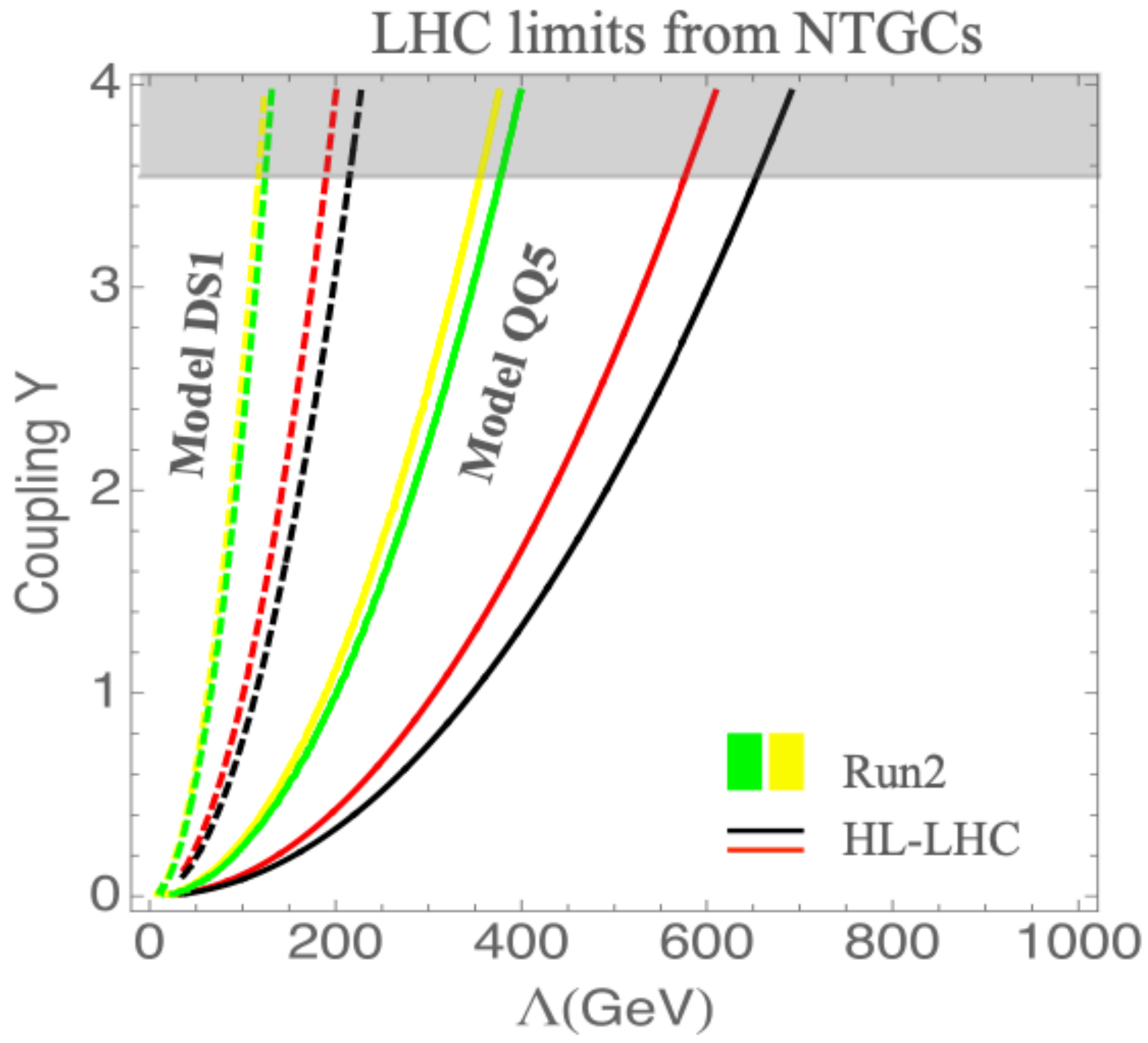


*Diagram and plot from [2402.04306]

22 BSM models with dimension-8 SMEFT contributions to NTG analyzed using **Matchete** by *Cepedello, Esser, Hirsch, and Sanz [2402.04306]*

Summary and conclusions

- (Automated) EFT matching and RG evolution is crucial to BSM phenomenology
- Functional matching is ideal for automation (also useful for pen-and-paper computations!)
- Huge progress towards complete (one-loop) automation: Lagrangian in, fully simplified EFT Lagrangian out
- The ultimate goal is a tool (or chain of tools) that fully automates
  - Matching
  - RG evolution

  Multi-step matching

  - Connection to observables / fit to data **Interface with other EFT pheno codes**

  streamlining future BSM analyses

https://gitlab.com/matchete/matchete

MATCHETE

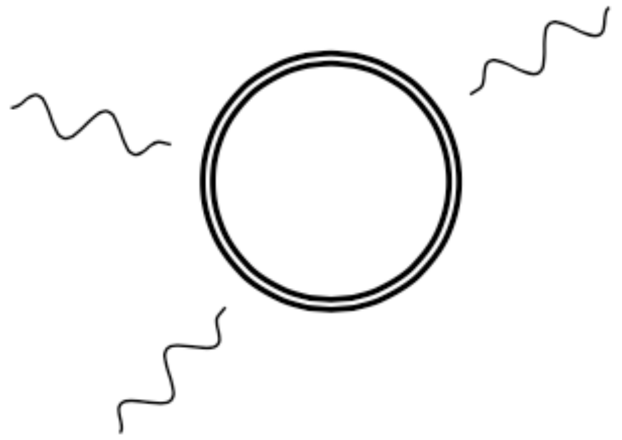
Thank you

BSM phenomenology is about to become easy!

Backup

Linear simplifications

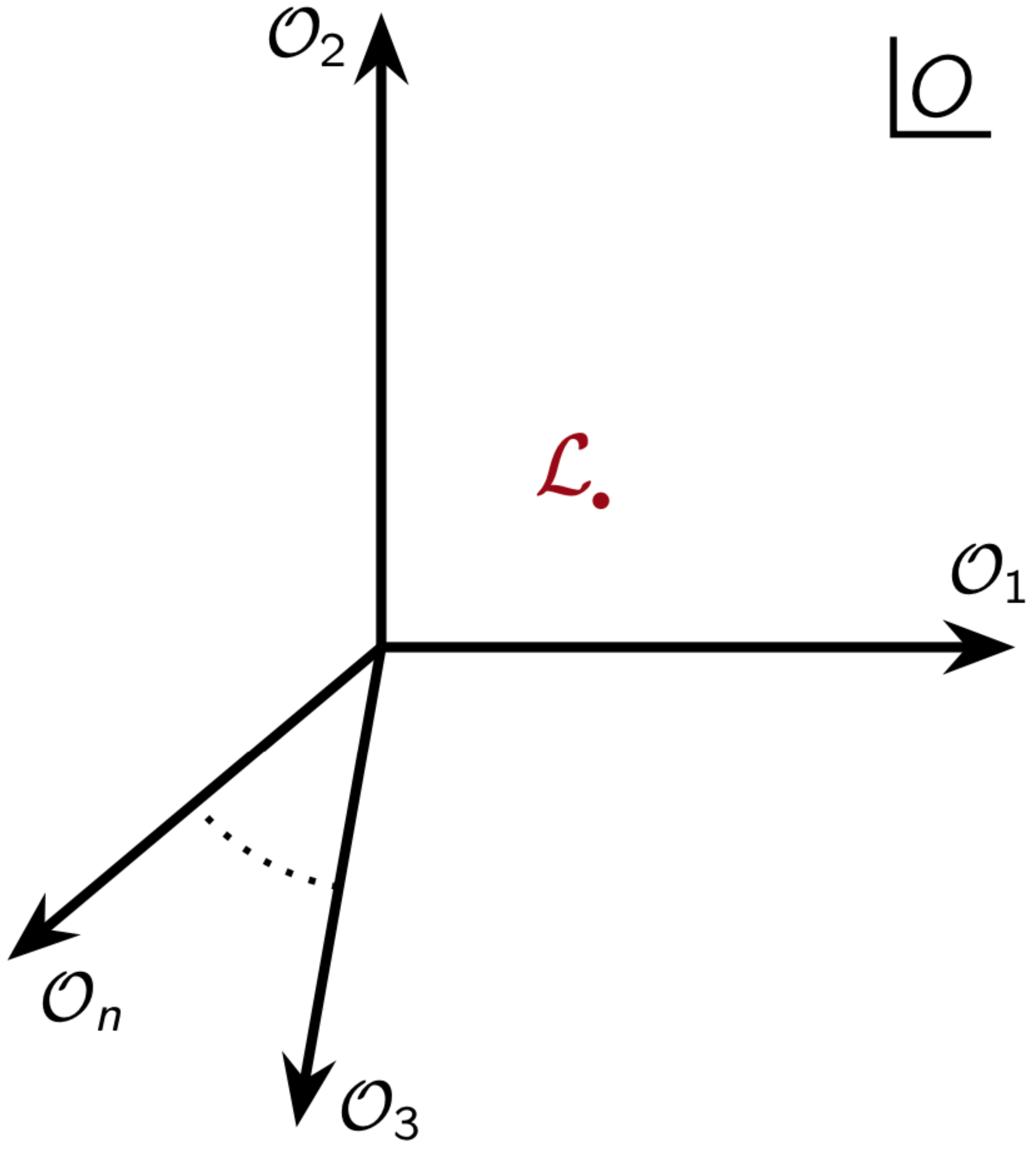
Example: Integrating out a heavy fermion in the fundamental representation of SU(3)



In[12]:=
```
LEFT // NiceForm
```

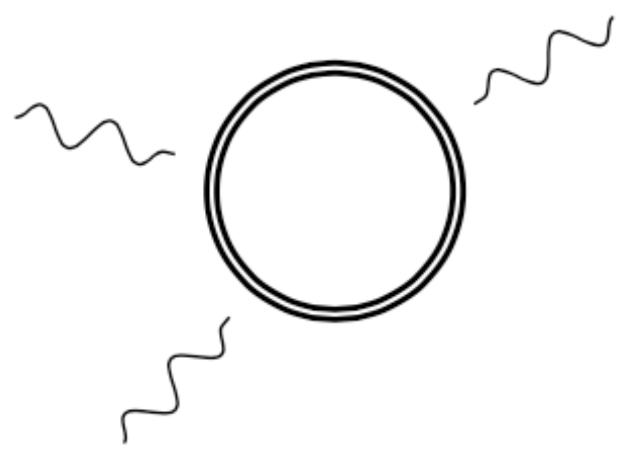
Out[12]//NiceForm=

$$\frac{7}{540}\hbar g^2 \frac{1}{M\Psi^2}\left(D_\rho G^{\mu\nu A}\right)^2 + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D^2 G^{\mu\nu A} + \frac{7}{540}\hbar g^2 \frac{1}{M\Psi^2} D_\rho G^{\mu\nu A} D_\nu G^{\mu\rho A} - \frac{1}{180}\hbar g^2 \frac{1}{M\Psi^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\nu D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24}\hbar g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$



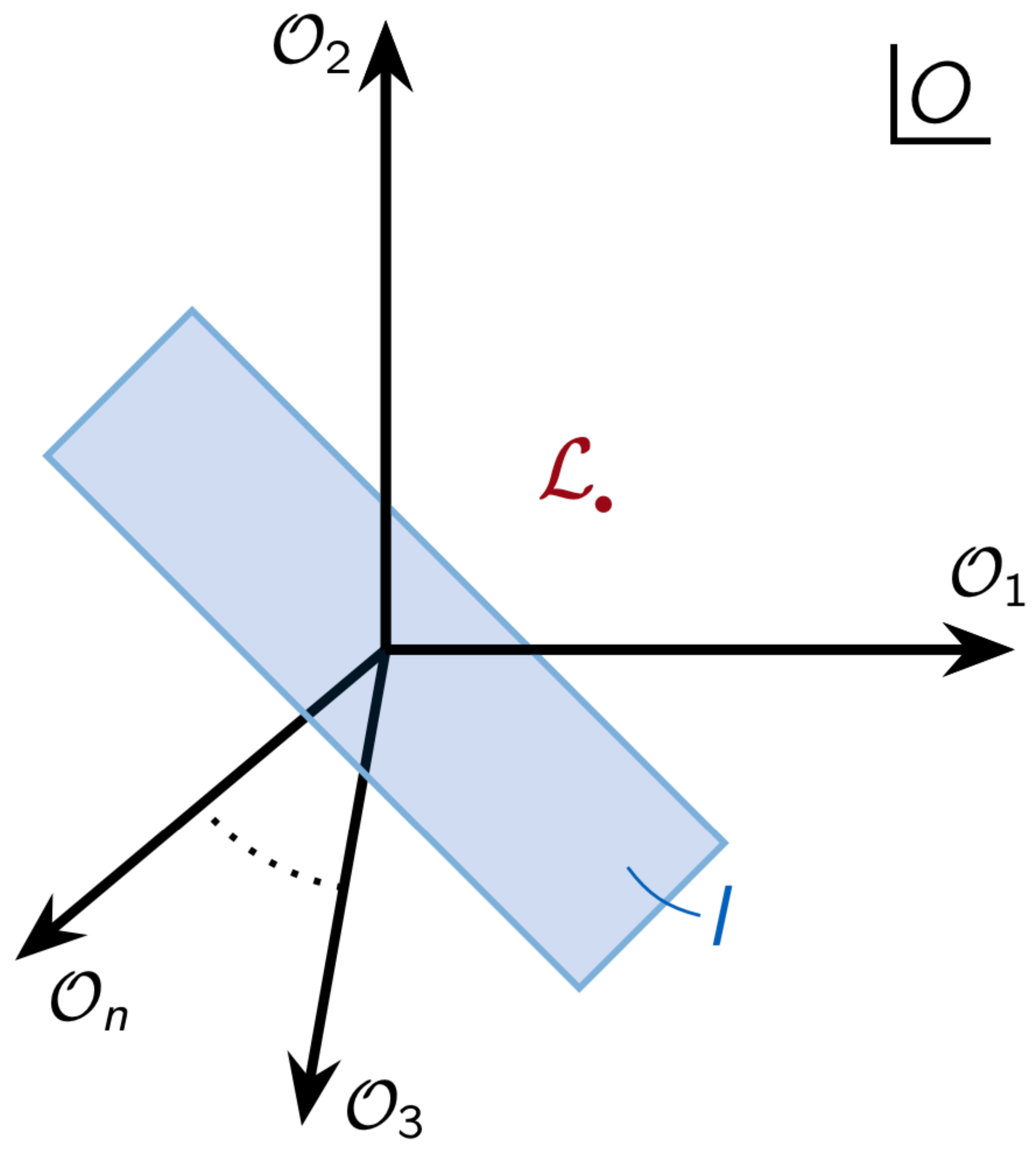
Linear simplifications

Example: Integrating out a heavy fermion in the fundamental representation of SU(3)



```
In[12]:= LEFT // NiceForm
Out[12]//NiceForm=
```

$$\frac{7}{540}\,\hbar\, g^2\, \frac{1}{M\Psi^2}\left(D_\rho G^{\mu\nu A}\right)^2 + \frac{1}{40}\,\hbar\, g^2\, \frac{1}{M\Psi^2}\, G^{\mu\nu A}\, D^2 G^{\mu\nu A} + \frac{7}{540}\,\hbar\, g^2\, \frac{1}{M\Psi^2}\, D_\rho G^{\mu\nu A}\, D_\nu G^{\mu\rho A} - \frac{1}{180}\,\hbar\, g^2\, \frac{1}{M\Psi^2}\, D_\nu G^{\mu\nu A}\, D_\rho G^{\mu\rho A} + \frac{1}{40}\,\hbar\, g^2\, \frac{1}{M\Psi^2}\, G^{\mu\nu A}\, D_\nu D_\rho G^{\mu\rho A} + \frac{1}{40}\,\hbar\, g^2\, \frac{1}{M\Psi^2}\, G^{\mu\nu A}\, D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24}\,\hbar\, g^3\, \frac{1}{M\Psi^2}\, G^{\mu\nu A}\, G^{\mu\rho B}\, G^{\nu\rho C}\, f^{ABC}$$



$I \subseteq O$ is the space of all operators identities, such as IBP relations, yielding e.g.

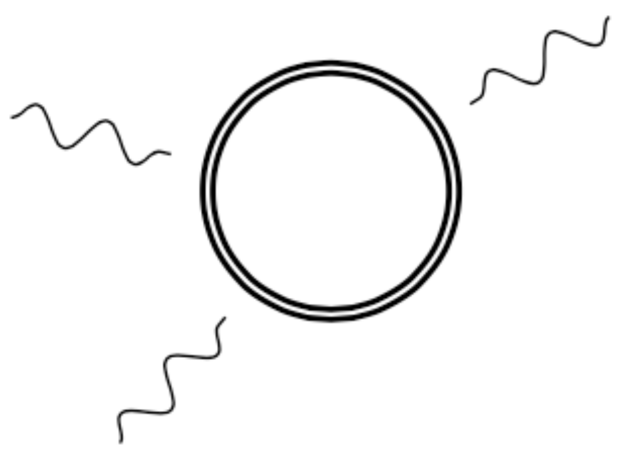
$$\mathcal{O}_1 + 2\,\mathcal{O}_3 = 0$$

interpreted as

$$\mathcal{O}_1 + 2\,\mathcal{O}_3 \in I$$

Linear simplifications

Example: Integrating out a heavy fermion in the fundamental representation of SU(3)



```
In[12]:= LEFT // NiceForm
```

Out[12]//NiceForm=

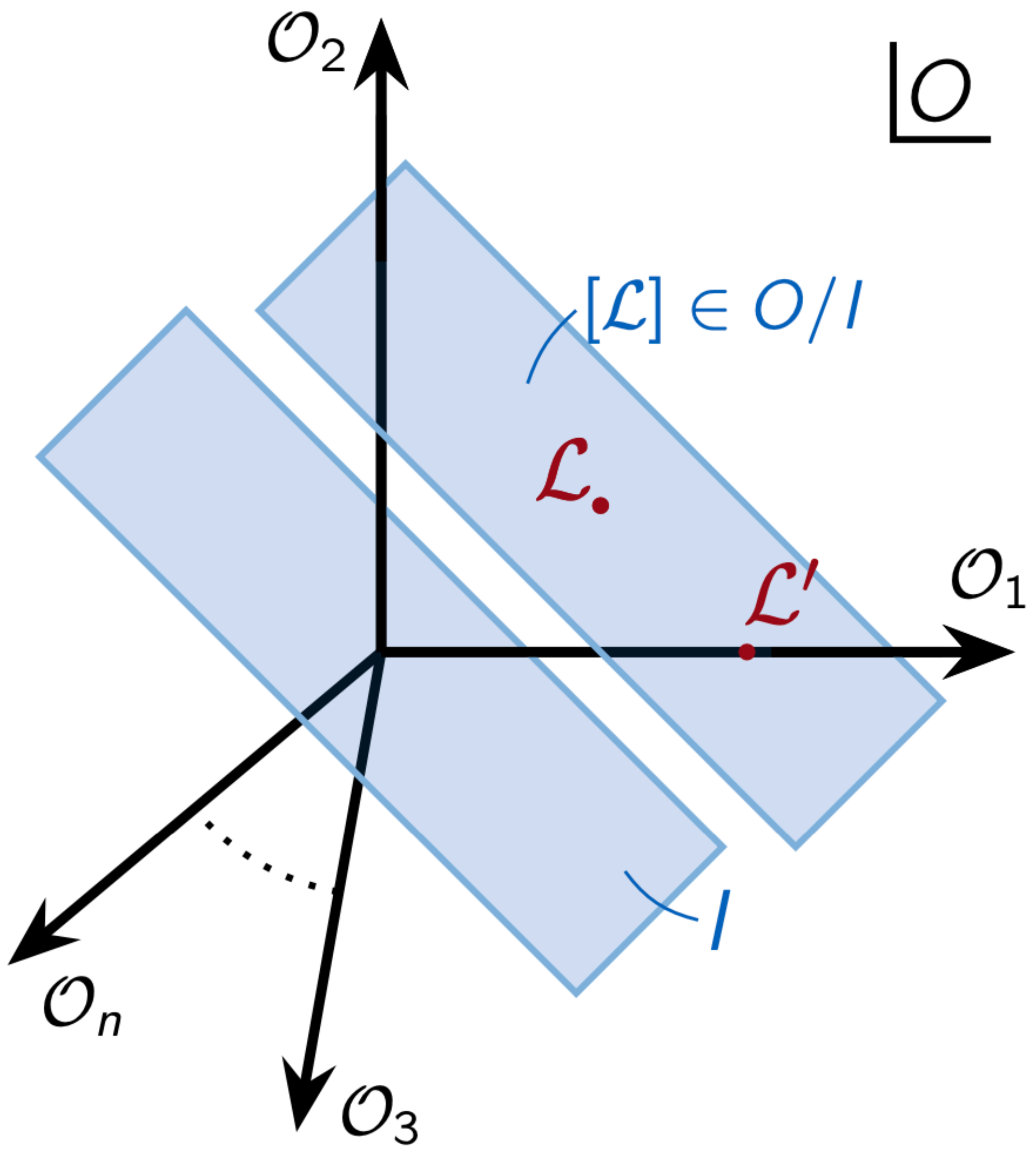
$$\frac{7}{540}\hbar\, g^2 \frac{1}{M\Psi^2}\left(D_\rho G^{\mu\nu A}\right)^2 + \frac{1}{40}\hbar\, g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D^2 G^{\mu\nu A} + \frac{7}{540}\hbar\, g^2 \frac{1}{M\Psi^2} D_\rho G^{\mu\nu A} D_\nu G^{\mu\rho A} - \frac{1}{180}\hbar\, g^2 \frac{1}{M\Psi^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar\, g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\nu D_\rho G^{\mu\rho A} + \frac{1}{40}\hbar\, g^2 \frac{1}{M\Psi^2} G^{\mu\nu A} D_\rho D_\nu G^{\mu\rho A} - \frac{1}{24}\hbar\, g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$

By gaussian elimination, we can *choose* a representative element for $[\mathscr{L}_{\mathrm{EFT}}] \in O/I$ to get an EFT basis

```
In[13]:= LEFT // GreensSimplify // NiceForm
```

Out[13]//NiceForm=

$$-\frac{1}{15}\hbar\, g^2 \frac{1}{M\Psi^2} D_\nu G^{\mu\nu A} D_\rho G^{\mu\rho A} - \frac{1}{180}\hbar\, g^3 \frac{1}{M\Psi^2} G^{\mu\nu A} G^{\mu\rho B} G^{\nu\rho C} f^{ABC}$$



$I \subseteq O$ is the space of all operators identities, such as IBP relations, yielding e.g.

$$\mathcal{O}_1 + 2\,\mathcal{O}_3 = 0$$

interpreted as

$$\mathcal{O}_1 + 2\,\mathcal{O}_3 \in I$$

Evanescent operators

Evanescent operators appear from a special type of linear simplification (valid only for $d = 4$)

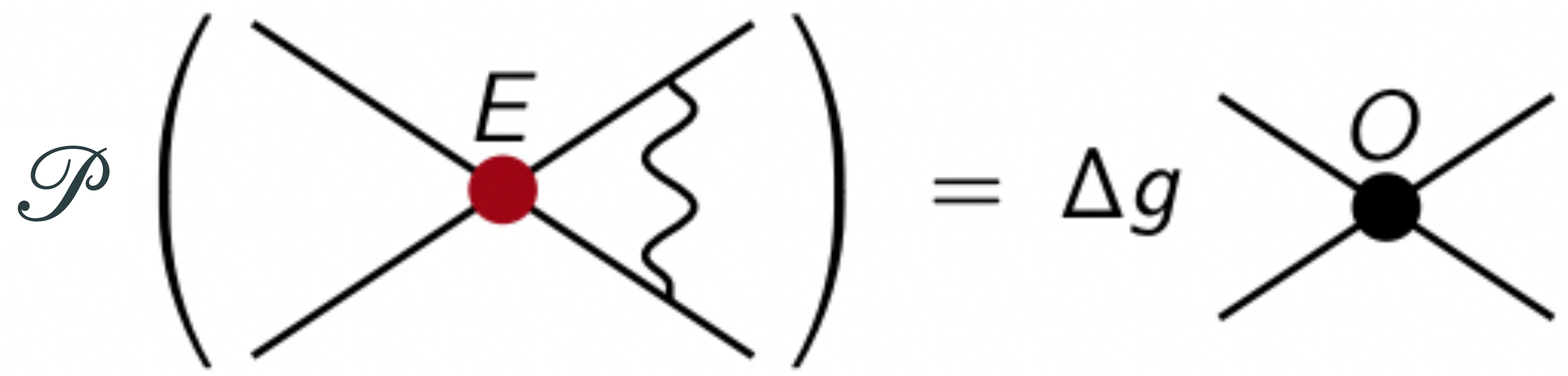
$$O_d = \underbrace{\mathscr{P}\, O_d}_{\text{Physical part}} + \underbrace{\overset{Id - \mathscr{P}}{\mathscr{E}}\, O_d}_{\text{Evanescent part}}$$

$\mathscr{P} \equiv$ Projection to the physical ($d = 4$) basis

E.g. Fierz identities

$$(\bar{\ell}_p e_r)(\bar{e}_s \ell_t) = -\frac{1}{2}(\bar{\ell}_p \gamma_\mu \ell_t)(\bar{e}_s \gamma^\mu e_r) + \overset{\text{rank}\,(\,d-4\,)}{E_{\ell e}^{prst}} \longrightarrow (\bar{\ell}_p e_r)(\bar{e}_s \ell_t) + \frac{1}{2}(\bar{\ell}_p \gamma_\mu \ell_t)(\bar{e}_s \gamma^\mu e_r) - E_{\ell e}^{prst} \in I$$

Representative elements are chosen so evanescent operators are retained. Afterwards, they are removed by shifting the coefficients of physical operators



e.g. $E_{\ell e}^{prst} \to -\dfrac{g_L\, y_e^{ts}}{128\pi^2}\, Q_{eW}^{pr} +$ [other contributions]