Probing the Cosmological Principle with weak lensing shear

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[arXiv:2411.08560], JCAP02(2025)016

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Introduction

Motivation

- Homogeneity and isotropy on large scales is foundational to modern cosmology
- Some dark energy, modified gravity models lead to large-scale anisotropies
- Fundamentally, this assumption must be tested
- Renewed interest in anisotropic cosmologies (e.g. SN Ia measurements, CMB dipole)
  $\Longrightarrow$ Can use weak lensing to probe anisotropies
- Formalism developed by Pitrou, Pereira, & Uzan to estimate B-mode shear generated by anisotropies:
  [arXiv:1503.01125], [arXiv:1503.01127]
- Incorporate non-linear corrections and tomography into results of Pitrou et al. and quantify possible SNR for Euclid, Euclid$\times$Simons, and Euclid$\times$Planck

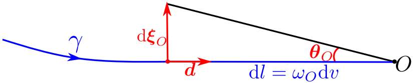
Lensing Formalism

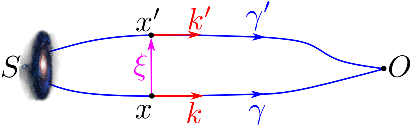
Lensing distortions

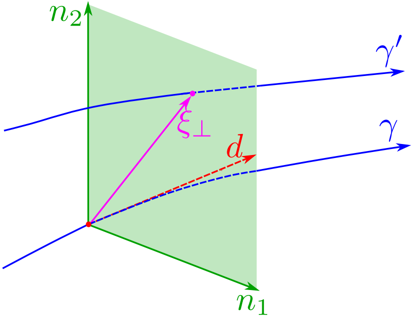
Jacobi matrix

- Observed angular size $\longmapsto$ physical separation

$$\xi^A\big|_S \propto \mathcal{D}^A{}_B\,\theta^B\big|_O$$







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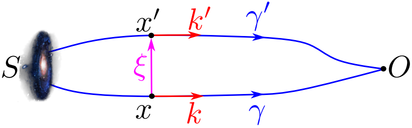
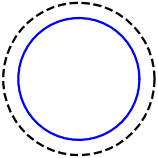


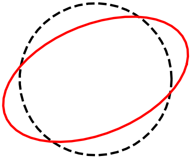
$$\boldsymbol{\mathcal{D}} \approx \bar{D}_A\left[\overbrace{\begin{pmatrix} 1-\kappa & 0 \\ 0 & 1-\kappa \end{pmatrix}}^{\text{Convergence}} + \underbrace{\begin{pmatrix} 0 & -\psi \\ \psi & 0 \end{pmatrix}}_{\text{Rotation}} + \overbrace{\begin{pmatrix} -\gamma_1 & \gamma_2 \\ \gamma_2 & \gamma_1 \end{pmatrix}}^{\text{Shear}}\right]$$

- Weak lensing: κ, $\gamma \ll 1$ and $\dot{\psi} = \mathcal{O}(\gamma^2)$
  $\implies$ Ignore rotation (usually)

Convergence:



Shear:



E-modes, B-modes, and multipoles

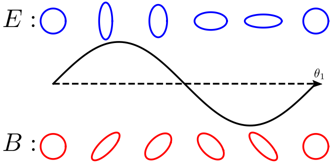
- Expand κ in spherical harmonics

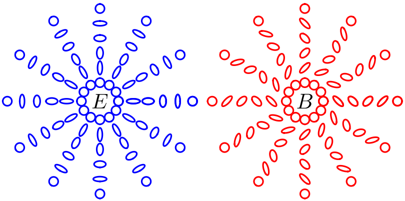
$$\kappa = \sum_{\ell,m} \kappa_{\ell m} Y_{\ell m}$$

- Expand γ in spin-weighted spherical harmonics

$$\gamma^{\pm} = \gamma_1 \pm i\gamma_2 = \sum_{\ell,m} (E_{\ell m} \pm i B_{\ell m}) Y^{\pm 2}_{\ell m}$$

- E =even parity, B =odd parity
- No B-modes on large scales (FLRW)*





Anisotropic Spacetime

Bianchi-I universes

Metric

$\mathrm{d}s^2 = a^2(-\mathrm{d}\eta^2 + \gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j)$

- $a =$ scale factor, $\gamma_{ij} =$ spatial metric
- Hubble rate: $\mathcal{H} \equiv \frac{a'}{a}$
- Spatial shear: $\sigma_{ij} \equiv \frac{1}{2}\gamma'_{ij}$

Dark Energy

$$T^{de}_{\mu\nu} = (\rho_{de}+P_{de})u_\mu u_\nu + P_{de}g_{\mu\nu} + \Pi_{\mu\nu}$$

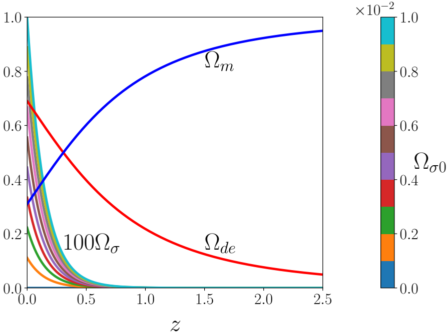
Anisotropic stress

- EoS: $P_{de} = -\rho_{de}$
- Anisotropic stress model: $\Pi^i{}_j \propto \Omega_{de} W^i_j$

Evolution

$$\mathcal{H}^2 = \frac{1}{3}\kappa a^2\rho + \frac{1}{6}\sigma^2$$

$$(\sigma^i{}_j)' = -2\mathcal{H}\sigma^i{}_j + \kappa\Pi^i{}_j$$



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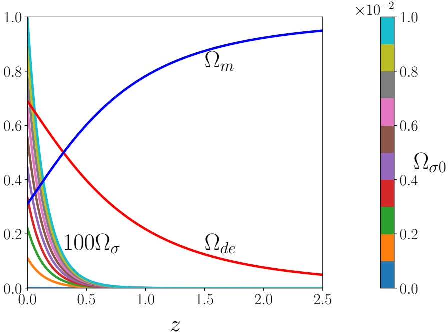


Weak shear limit: $\boxed{\left(\frac{\sigma}{\mathcal{H}}\right) \ll 1}$ $\Longrightarrow$ leading order in $\frac{\sigma}{\mathcal{H}}$

Results

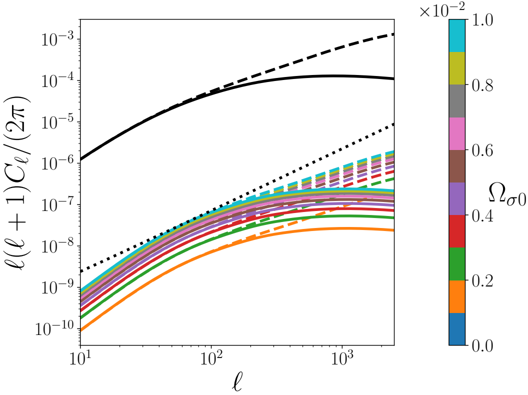
Angular power spectra

E-modes and convergence

- Standard textbook result
- First order in φ
- $C_\ell^{EE} = \langle E_{\ell m} E_{\ell m}^* \rangle \approx C_\ell^{\kappa\kappa} \sim \varphi^2$

B-modes

- Post-Born couples σ and φ $\implies$ Non-zero B-modes!
- $C_\ell^{BB} = \frac{1}{2\ell+1} \sum_m \langle B_{\ell m} B_{\ell m}^* \rangle \sim \left(\frac{\sigma}{\mathcal{H}}\right)^2 \varphi^2$



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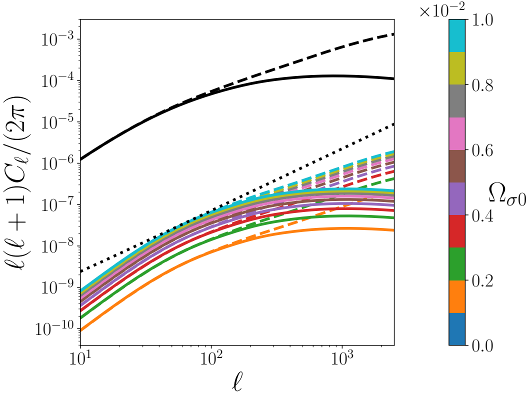

C_ℓ^{BB} likely too small to detect :(

$\implies$ Use cross-correlation!

E-B, κ-B cross-correlation

E-modes$\times$**B-modes**

- Parity $\implies$ off-diagonal correlations
- Isolate signal with BipoSH:

$$^{EB}\mathcal{A}^{\cdot\cdot}_{\cdot\cdot} = \sum \left\langle E_{\ell m} B^*_{\ell\pm1 m'} \right\rangle \sim \ell^{4.5}\,\mathcal{P}_{\ell M} \sim \left(\frac{\sigma}{\mathcal{H}}\right)\varphi^2$$

- Tomography for E and B from Euclid

Convergence$\times$**B-modes**

- Obtain $\mathcal{P}_{\ell M}$ again
- Use reconstructed CMB κ from Planck and Simons Obs. $\implies$ non-tomographic!
- Main topic of this talk

E-B, κ-B cross-correlation

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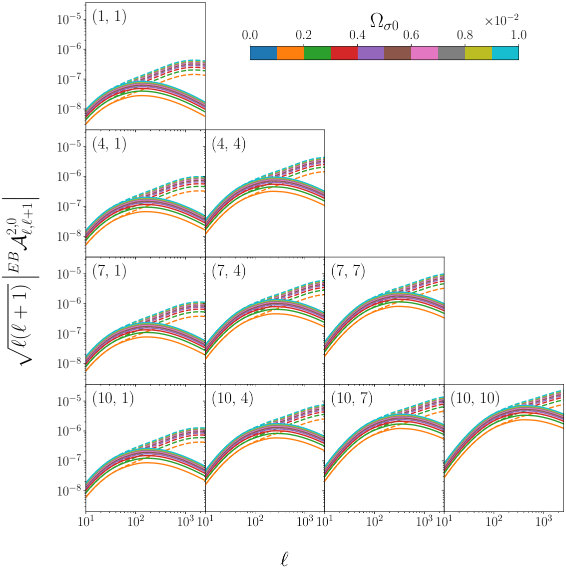

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What about $\langle EE \rangle$ and $\langle \kappa\kappa \rangle$?

E-B, κ-B cross-correlation

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Convergence$\times$B-modes

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What about $\langle EE\rangle$ and $\langle \kappa\kappa\rangle$?
$\implies$ Lower SNR!

E-B, κ-B cross-correlation

E-modes×B-modes

- Parity $\Longrightarrow$ off-diagonal correlations
- Isolate signal with BipoSH:

$$^{EB}\mathcal{A}^{\cdot\cdot}_{\cdot\cdot} = \sum \left\langle E_{\ell m} B^{*}_{\ell\pm 1 m'} \right\rangle \sim \ell^{4.5}\,\mathcal{P}_{\ell M} \sim \left(\frac{\sigma}{\mathcal{H}}\right)\varphi^2$$

- Tomography for E and B from Euclid

Convergence×B-modes

- Obtain $\mathcal{P}_{\ell M}$ again
- Use reconstructed CMB κ from Planck and Simons Obs. $\Longrightarrow$ <u>non-tomographic!</u>
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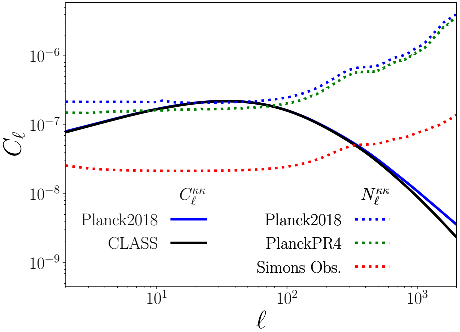
Signal-to-noise

Estimator and error

- Simple estimator $\hat{\mathcal{P}}_{\ell M}$
  $\implies$ Compute SNR:

$$\left(\frac{S}{N}\right)^2 \sim \sum_{\ell,\,\text{bins}} \left(\frac{\mathcal{P}_{\ell M}}{\Delta \mathcal{P}_{\ell M}}\right)^2$$

- SI variance:

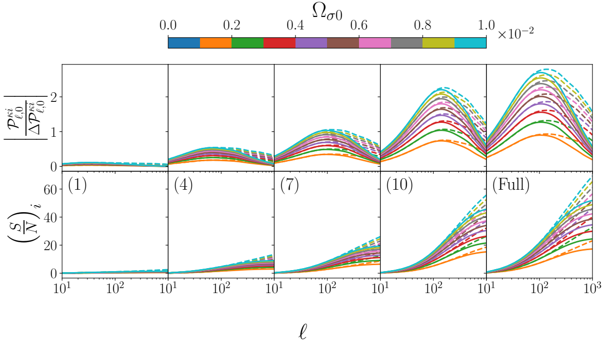


Spectrum + noise/bias   Shape noise

$$(\Delta \mathcal{P}_{\ell M})^2 = \text{Var}\left(\hat{\mathcal{P}}_{\ell M}\right)_{\text{SI}} \sim \frac{1}{f_{\text{sky}}} \frac{\left(C_\ell^{\kappa\kappa}\right)_{\text{SI}} \left(C_\ell^{BB}\right)_{\text{SI}}}{\ell^9}$$

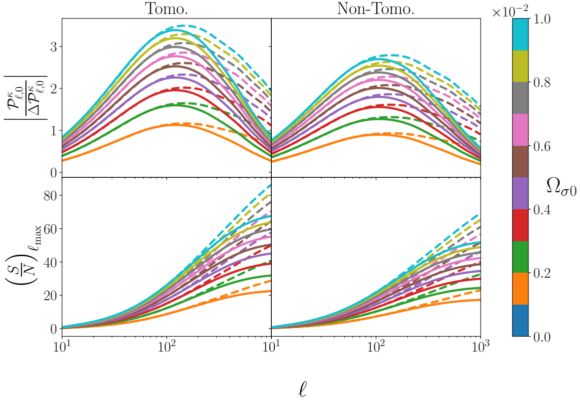
Survey overlap fraction

Cumulative SNR



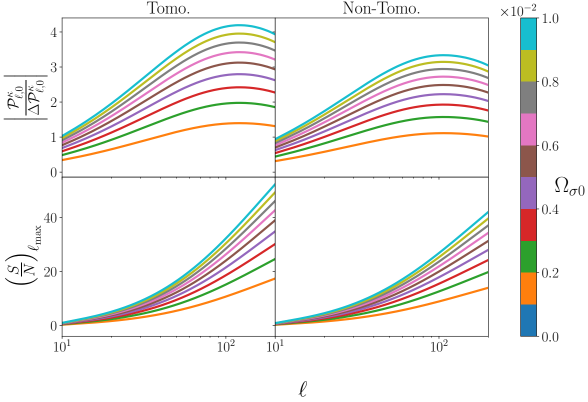
- Calculate SNR in each bin

Cumulative SNR



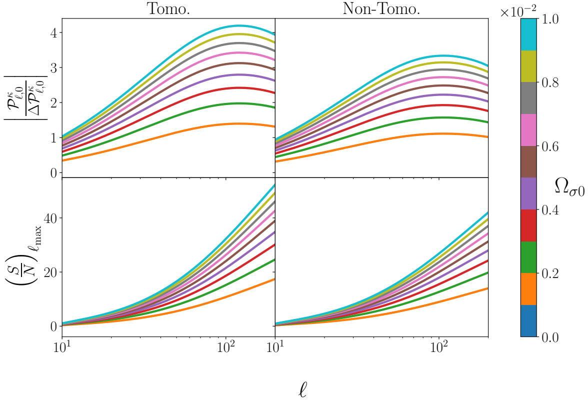
- Calculate SNR in each bin
- Sum up tomographic bins

Cumulative SNR



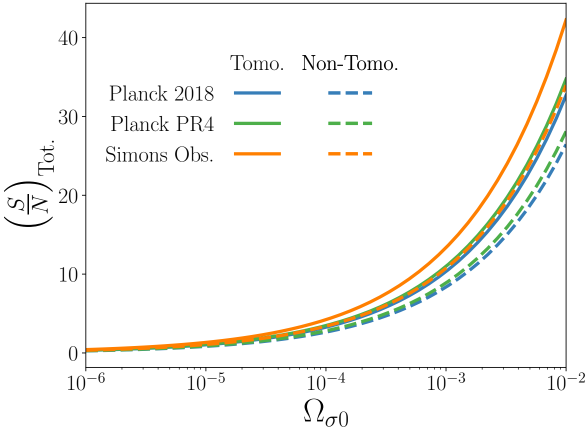
- Calculate SNR in each bin
- Sum up tomographic bins
- Check nonlinear behaviour using HaloFit

Cumulative SNR



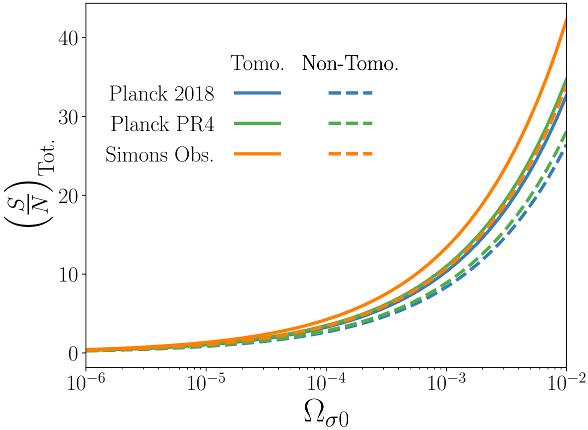
- Calculate SNR in each bin
- Sum up tomographic bins
- Check nonlinear behaviour using HaloFit
- Set cutoff scale $\ell_{\max} = 200$

Constraining power



SNR=10 cutoff

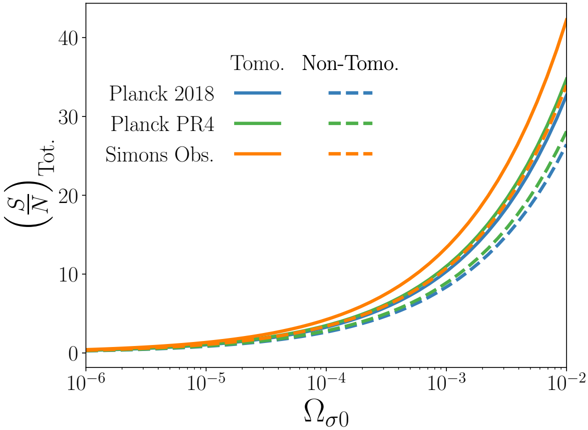
Constraining power



SNR=10 cutoff

- PL2018×Euclid: $\frac{\sigma}{\mathcal{H}} \lesssim 3.1\%$

Constraining power



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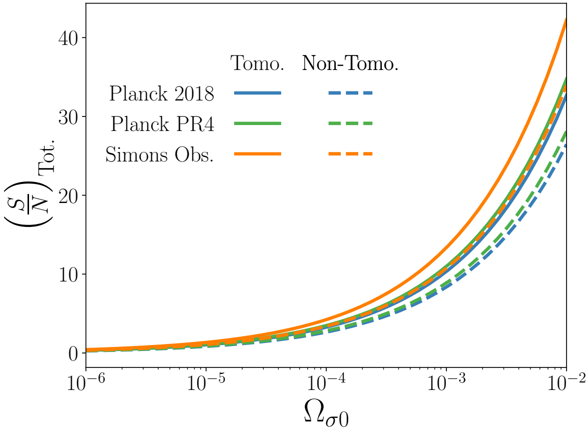
- PLPR4×Euclid: $\frac{\sigma}{\mathcal{H}} \lesssim 2.9\%$

Constraining power



SNR=10 cutoff

- PL2018×Euclid: $\frac{\sigma}{\mathcal{H}} \lesssim 3.1\%$
- PLPR4×Euclid: $\frac{\sigma}{\mathcal{H}} \lesssim 2.9\%$
- Simons×Euclid: $\frac{\sigma}{\mathcal{H}} \lesssim 2.4\%$

Conclusion

Concluding remarks

Summary

- Lightning review of lensing formalism
- E-B + κ-B cross-correlations can constrain late-time anisotropic expansion
  $\implies$ make use of large scales and tomography
- Should construct appropriate estimators for cross-correlations

Outlook

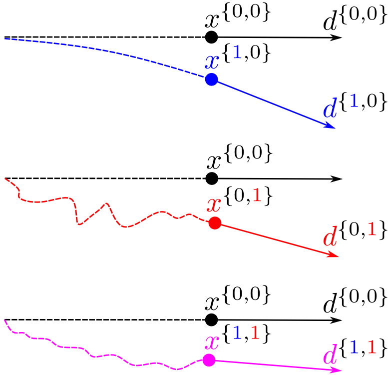
- Much to improve...
- Should take into account more systematics (e.g. masking, IA, ...)
- Weak lensing is of immense importance to upcoming surveys
- Hope to place constraints on $\sigma/\mathcal{H}$ at the percent level

Thank you :)

Extra slides

Perturbed metric

$$\mathrm{d}s^2 = a^2\big[-(1+2\Phi)\mathrm{d}\eta^2 + 2\bar{B}_i \mathrm{d}x^i \mathrm{d}\eta + (\gamma_{ij} + h_{ij})\mathrm{d}x^i \mathrm{d}x^j \big]$$



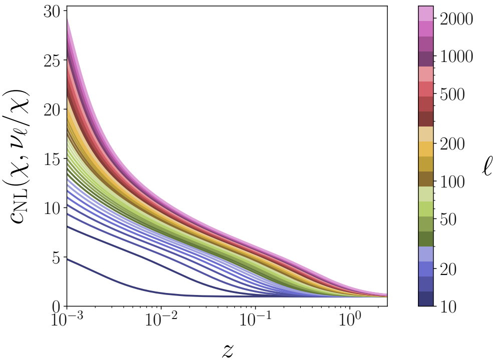
Perturbation scheme

- Treat $\sigma_{ij}/\mathcal{H} \ll 1$ as perturbation along with scalar-vector-tensor (SVT) perturbations
- SVT: Φ, $\bar{B}^i$, $h_{ij} = -2\Psi\left(\gamma_{ij} + \frac{\sigma_{ij}}{\mathcal{H}}\right) + 2E_{ij}$
- Two-fold perturbation $\{n, m\}$ for shear and SVT*:

$$X^{\{n,m\}} = \underbrace{X^{\{0,0\}}}_{\text{FLRW}} + \overbrace{\delta X^{\{0,1\}}}^{\text{SVT}} + \underbrace{\delta X^{\{1,0\}}}_{\text{Shear}} + \overbrace{\delta X^{\{1,1\}} + \ldots + \delta X^{\{n,m\}}}^{\text{Shear+SVT}}$$

Non-linear corrections

- Angular power spectra of the form $C_\ell \sim \ell^4 \int \frac{\mathrm{d}\chi}{\chi^2}\, P(k) T_\varphi^2(\chi, k)\big|_{k=\ell/\chi}$
- Calculate non-linear matter power spectrum P_m^{NL} with HaloFit
- Rescale transfer function $T_\varphi \longmapsto c_{NL} T_\varphi$
- Non-linear factor $c_{\mathsf{NL}} = \sqrt{P_m^{\mathsf{NL}}/P_m^{\mathsf{L}}}$
- CLASS: $P(k)$, $T_\varphi(\eta, k)$, HaloFit
- Smooth interpolation from linear to non-linear regimes
- Better understand limits of validity for predictions



Euclid tomography

- Lensing projects/flattens observables
- Tomography regains some projected info.
- Reduce noise and error
- Euclid:
    - 10 equi-populated bins $10^{-3} \leq z \leq 2.5$
    - Weight underlying distribution $n(z)$ with photometric error $p_{\mathrm{ph}}(z_{\mathrm{p}}|z)$