Weak lensing higher-order statistics in f(R) gravity

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ΛCDM

Phenomenological model

Exotic components

Cosmological tensions

ΛCDM

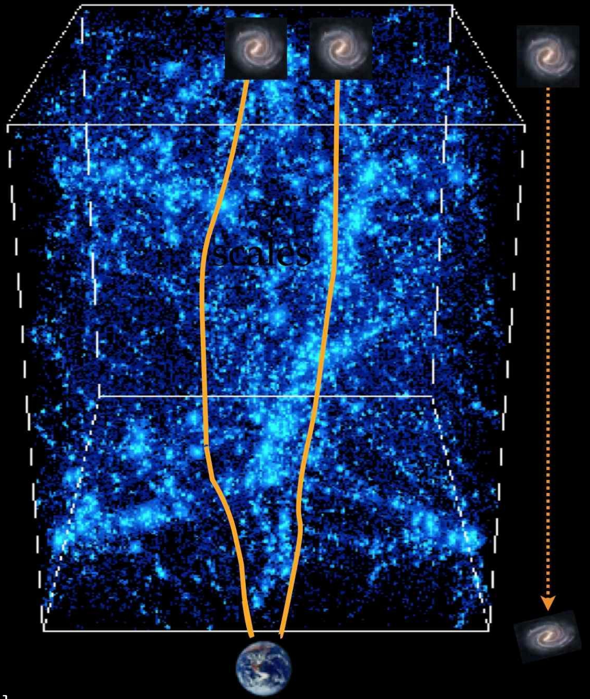
Phenomenological model

Exotic components

Cosmological tensions

Discriminate between models with higher-order statistics?

Weak lensing



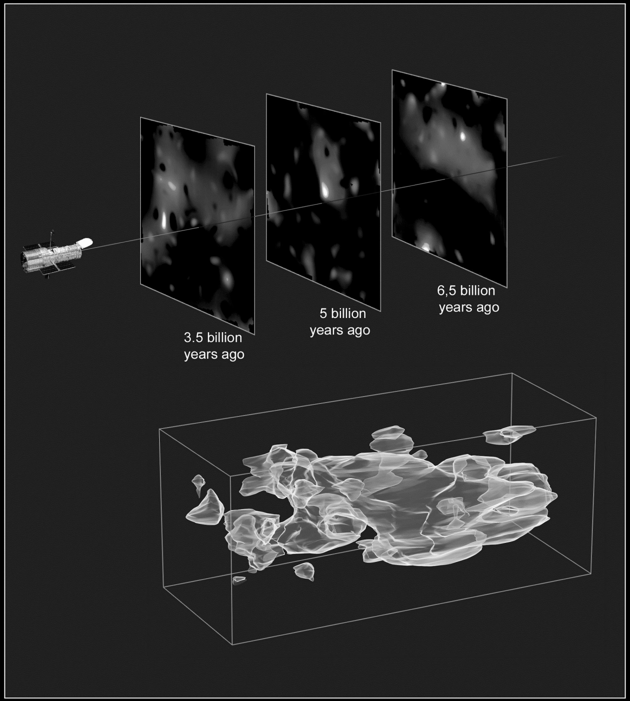
[CEA]

$$\gamma_1 = \frac{1}{2}(\partial_1^2 - \partial_2^2)\psi$$

$$\gamma_2 = \partial_1 \partial_2 \psi$$

Mass-mapping

$$\kappa = \frac{1}{2}(\partial_1^2 + \partial_2^2)\psi$$

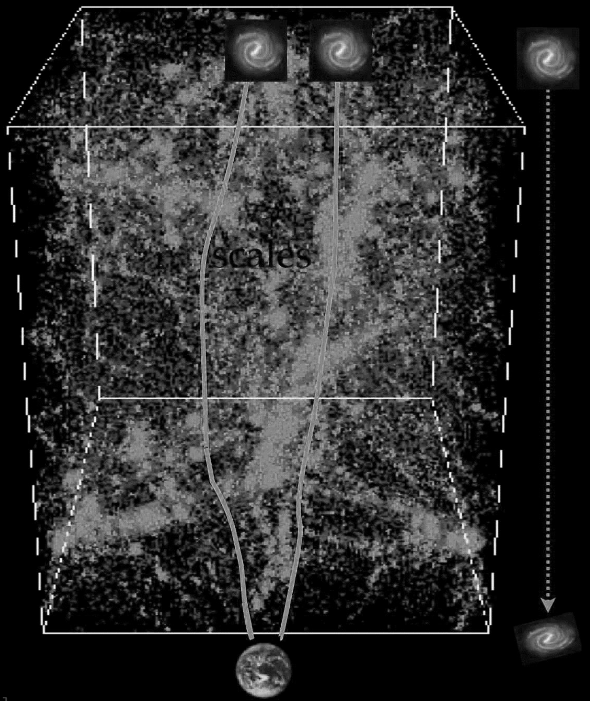


[Euclid/Weak lensing]

Cosmic shear
(coherent deformation)

Convergence maps
(projected mass)

Weak lensing



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Mass-mapping

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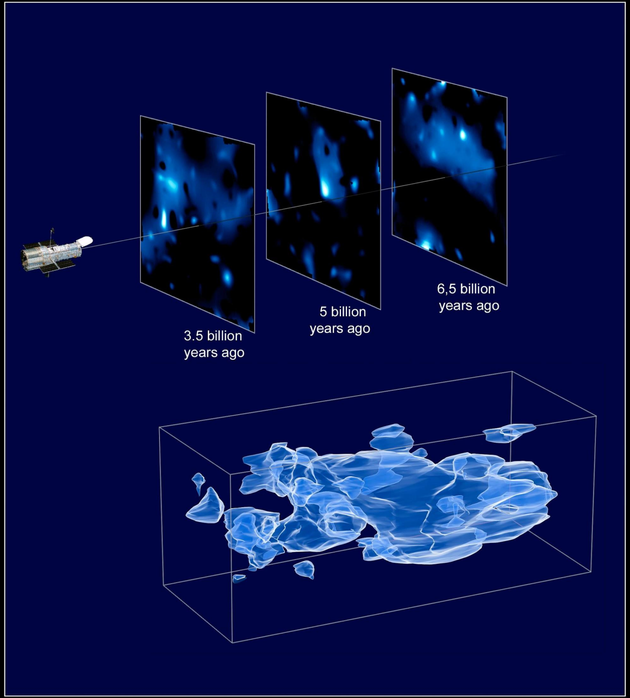


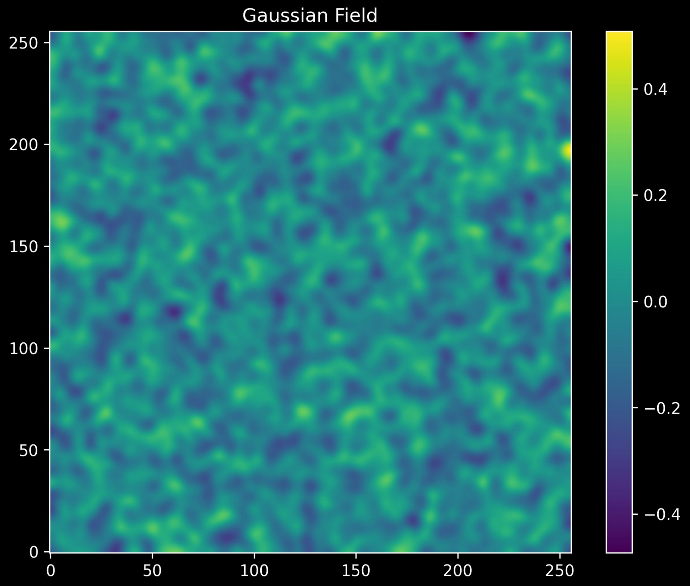
[Euclid/Weak lensing]

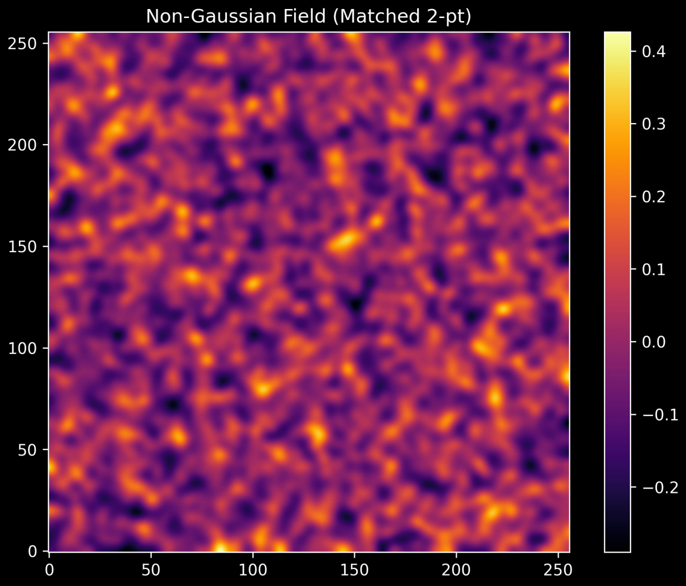
Cosmic shear (coherent deformation)

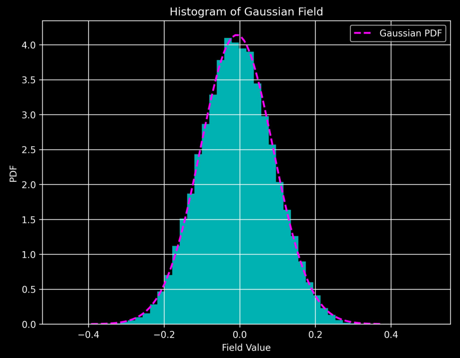
Convergence maps (projected mass)

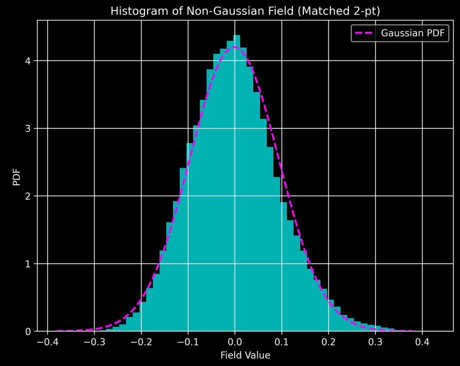
Why?

Why?

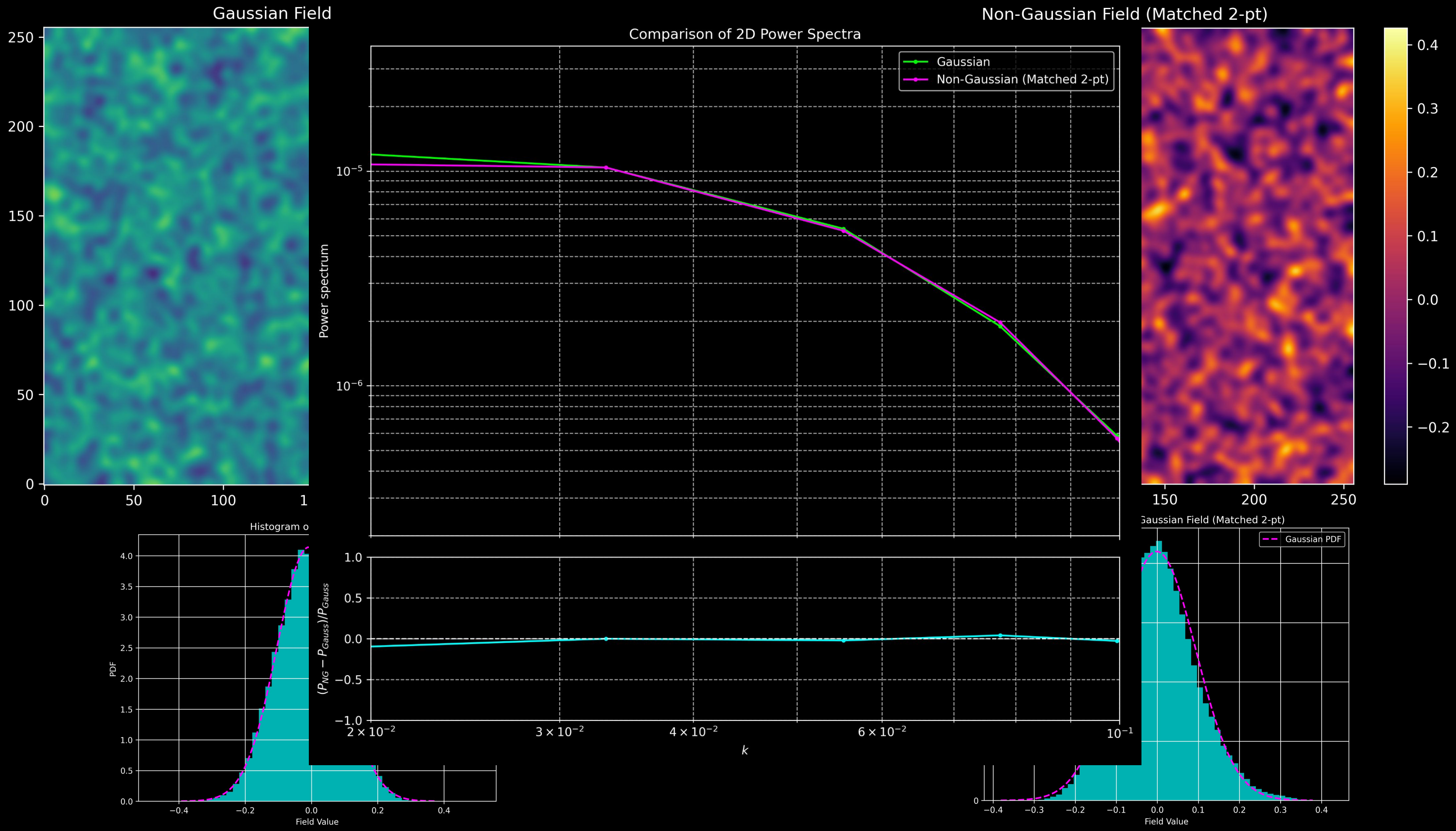

Gaussian Field
0
50
100
150
200
250
0.4
0.2
0.0
−0.2
−0.4


Non-Gaussian Field (Matched 2-pt)
0
50
100
150
200
250
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2


Histogram of Gaussian Field
Gaussian PDF
PDF
Field Value


Histogram of Non-Gaussian Field (Matched 2-pt)
Gaussian PDF
PDF
Field Value

Why?



Gaussian Field
Non-Gaussian Field (Matched 2-pt)
Comparison of 2D Power Spectra
Gaussian
Non-Gaussian (Matched 2-pt)
Power spectrum
$(P_{NG} - P_{Gauss})/P_{Gauss}$
k
Histogram o
PDF
Field Value
aussian Field (Matched 2-pt)
Gaussian PDF

HOSs

1pt PDF

Higher-order moments

Higher-order statistics

Peak counts

Topological statistics

HOSs

1pt PDF

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Higher-order moments

Higher-order statistics

Peak counts

Topological statistics

κ - peaks

M_{ap} - peaks

HOSs

1pt PDF

Higher-order moments

Higher-order statistics

Peak counts

κ - peaks

M_{ap} - peaks

Topological statistics

Betti nrs.

Minkowski fnc.

$f(R)$ gravity

$$S = \frac{1}{16\pi G_N} \int d^4x \sqrt{-g} \left[R + \boxed{f(R)}\right]$$

[arXiv:0705.1158]

$f(R)$ gravity

$$S = \frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,[R + \boxed{f(R)}]$$

Hu & Sawicki

$$f(R) = -m^2\frac{c_1\left(\frac{R}{m^2}\right)^n}{c_2\left(\frac{R}{m^2}\right)^n + 1}$$

~ ΛCDM at background

different at perturbations

$$f_R = \mathrm{d}f/\mathrm{d}R$$

$$f_{R0}$$

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$$f_R = \mathrm{d}f/\mathrm{d}R$$

$$f_{R0}$$

$$f_{R0} \longleftrightarrow m_\nu = \sum_i m_{\nu,i}$$

enhanced

suppressed

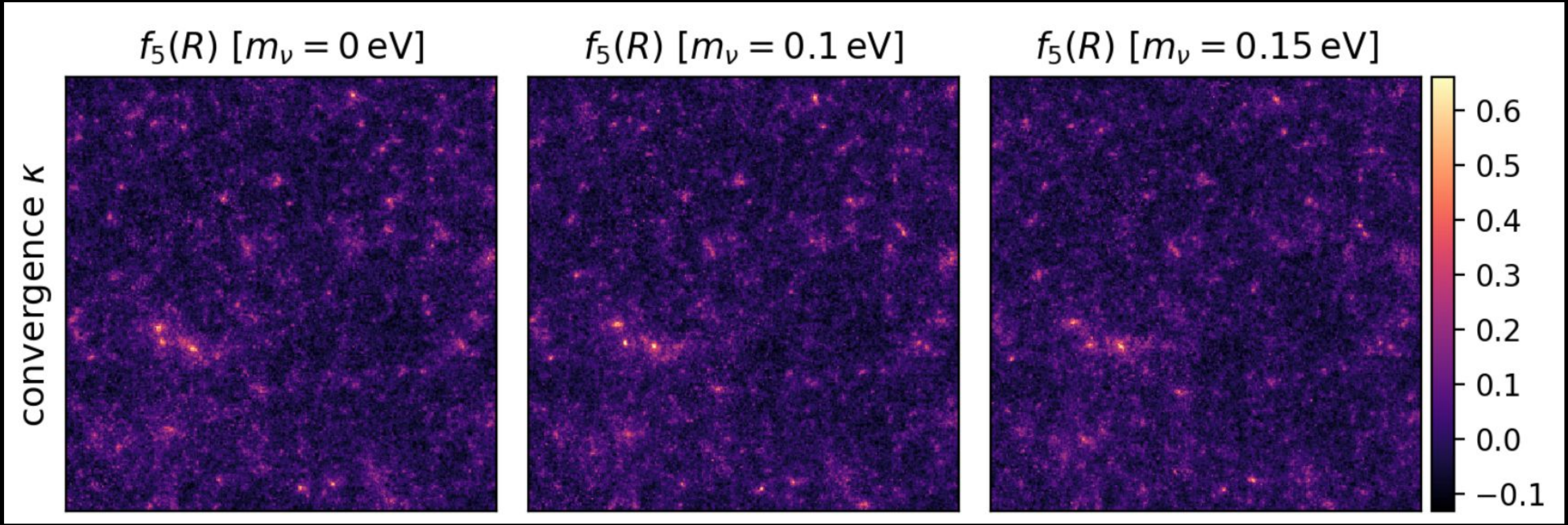
[arXiv:0705.1158]

DUSTGRAIN-pathfinder

$$-10^{-4} \leq f_{R0} \leq -10^{-6}$$

$$0 \text{ eV} \leq m_\nu \leq 0.3 \text{ eV}$$

8 + 1
cosmologies



256 maps - 5 x 5 deg^2

[arXiv:1806.04681]

Metrics

Metrics

$$\mathrm{SMAPE} = \frac{100}{n} \sum_{i=2}^{n} \frac{1}{2} \frac{|E_i - M_i|}{(|E_i| + |M_i|)}$$

Hellinger

$$d_H^2 = 1 - \sqrt{\frac{2\sigma_1\sigma_2}{\sigma_1^2 + \sigma_2^2}} \exp\left[-\frac{1}{4}\frac{(\mu_1 - \mu_2)^2}{\sigma_1^2 + \sigma_2^2}\right]$$

Mahalanobis

$$d_M = \sqrt{(\mathbf{E} - \mathbf{M})^{\mathrm{T}}\mathbf{C}^{-1}(\mathbf{E} - \mathbf{M})}$$

Wasserstein

$$d_W^2 = \|e_1 - e_2\|_2^2 + \mathrm{Tr}\left[\mathbf{C}_1 + \mathbf{C}_2 - 2\left(\mathbf{C}_1^{\frac{1}{2}}\mathbf{C}_1\mathbf{C}_2^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]$$

Discriminate GR vs. MG

PARENTAL
ADVISORY
EXPLICIT CONTENT

Discriminate GR vs. MG

PARENTAL ADVISORY EXPLICIT CONTENT

$\mathcal{M}$(*mod*; *hos*, *config*) value of a metric for

- model *mod*
- high order statistical probe *hos*
- configuration *config* (bins + range + Gaussianity cuts)

Discriminate GR vs. MG

PARENTAL ADVISORY EXPLICIT CONTENT

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IF $\mathfrak{M}(GR; hos, config) \neq \mathfrak{M}(MG; hos, config)$

- evidence for MG or statistical fluctuation? → error bars!

Discriminate GR vs. MG

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each $\mathcal{M}$(*mod*; *hos*, *config*) an associated error as

- mock dataset: 256 measurements of HOS sampled from multi Gaussian w/ mean and covariance as the GR one
- compute $\mathcal{M}$(*mod*; *hos*, *config*) for the mock
- repeat over Nrnd samples (removing unphysical realizations)
- estimate the final $\mathcal{M}$(*mod*; *hos*, *config*) from the distribution of Nrnd samples

Discriminate GR vs. MG

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Plot $\mathfrak{M}(GR;\, hos,\, config)$ vs $\mathfrak{M}(MG;\, hos,\, config)$ for different *config*

- larger departure from 1 to 1 line, stronger evidence for MG

Preliminary Results

For each *hos* and MG model, we vary

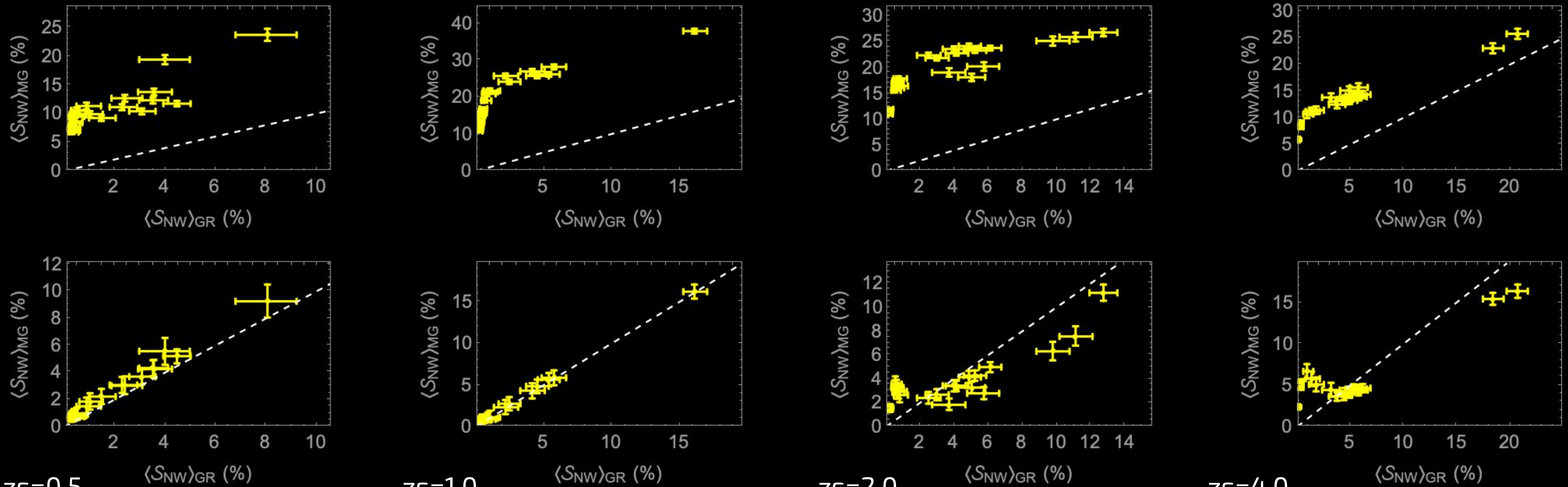
- redshift of the sources
- configuration *config* (subset, larger S/N)
- mass of the neutrino

Results will depend on

- metric
- HOS probe

1PDF - SMAPE – fR4

PRELIMINARY



zs=0.5 zs=1.0 zs=2.0 zs=4.0

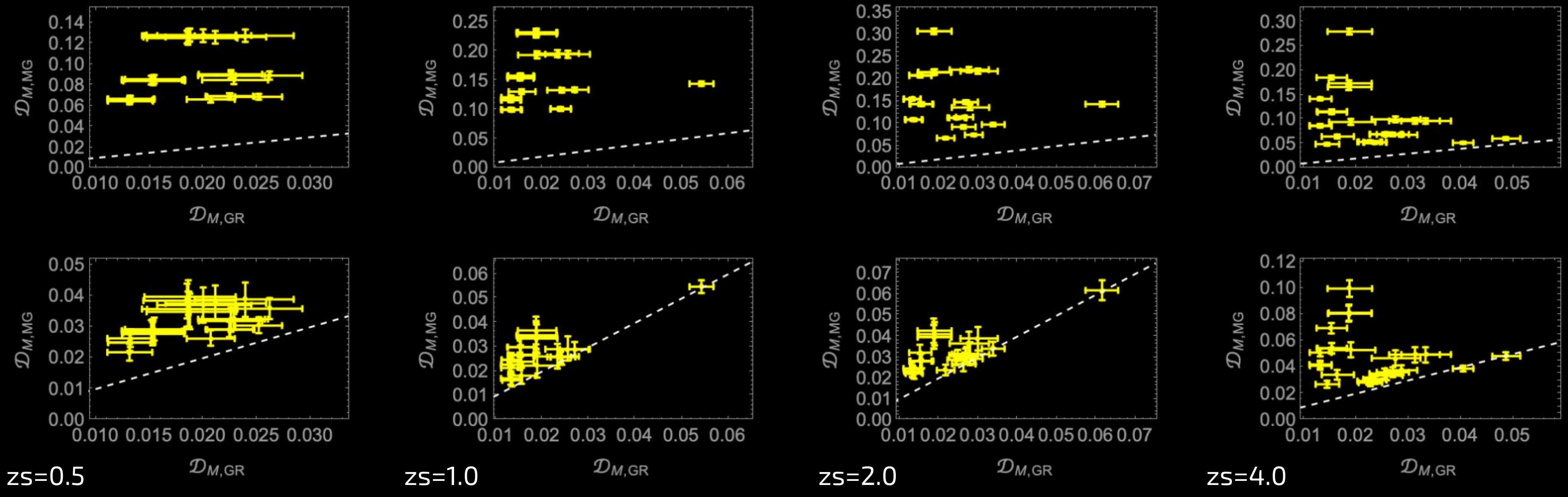
top panels: fR4 vs ΛCDM; *bottom panels*: fR4 + m_ν = 0.3eV

massive neutrinos strongly suppress MG signature

SMAPE does not account for errors on individual measurements

1PDF - Mahalanobis – fR4

PRELIMINARY



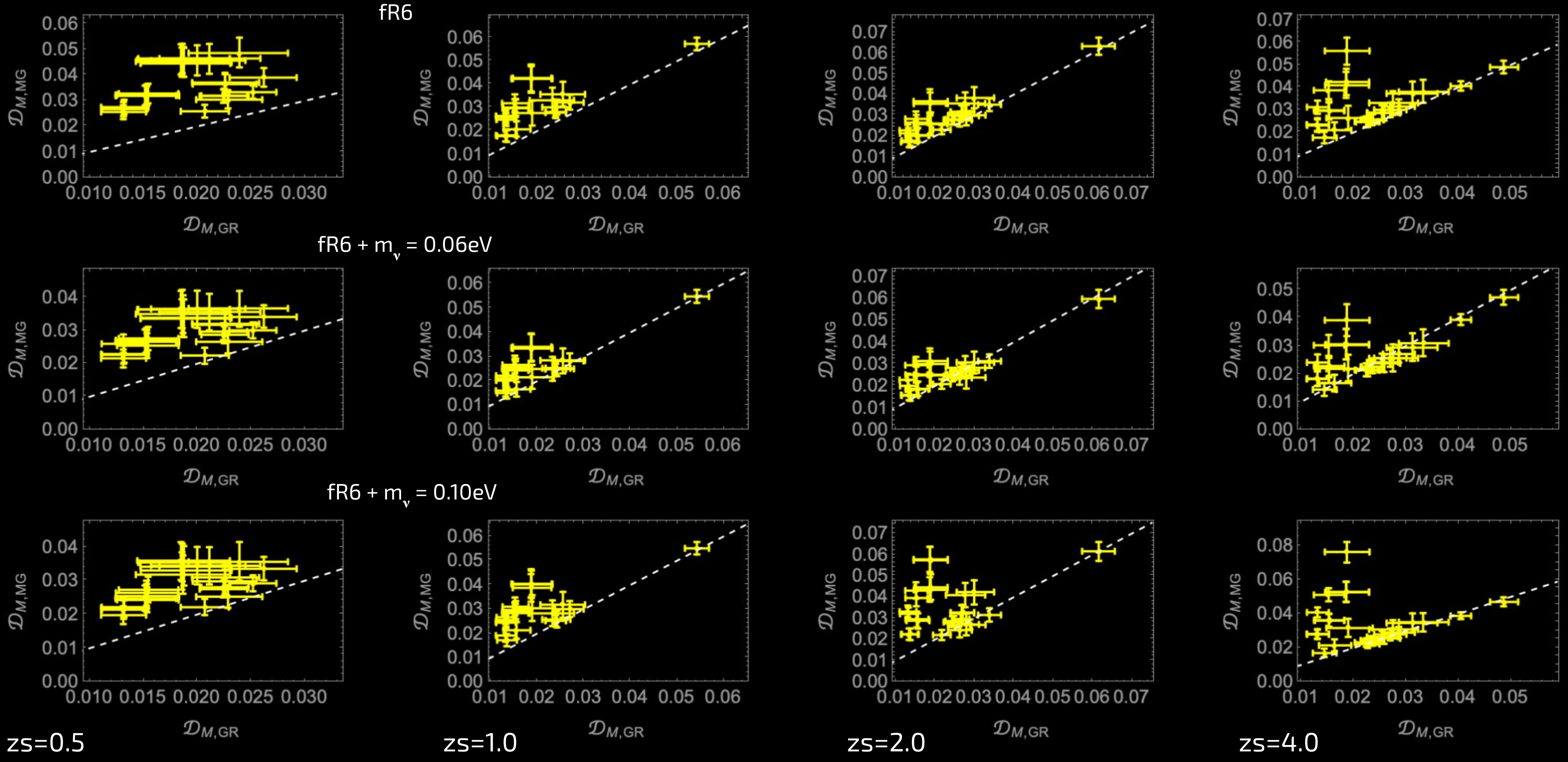
top panels: fR4 vs ΛCDM; *bottom panels*: fR4 + m_ν = 0.3eV

massive neutrinos strongly suppress MG signature

possible to discriminate fR4 vs GR with expected accuracy of data

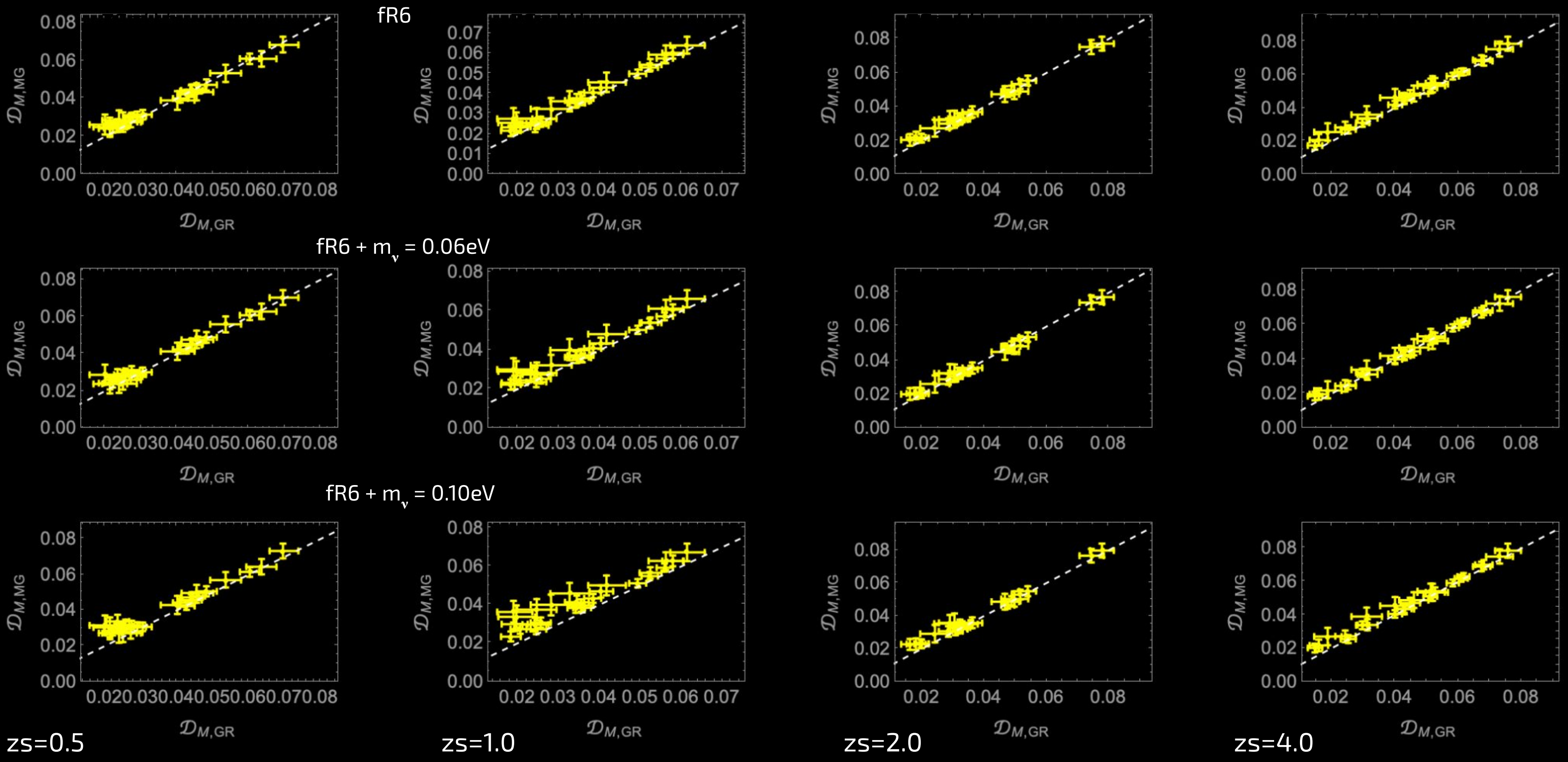
1PDF - Mahalanobis – fR6

PRELIMINARY



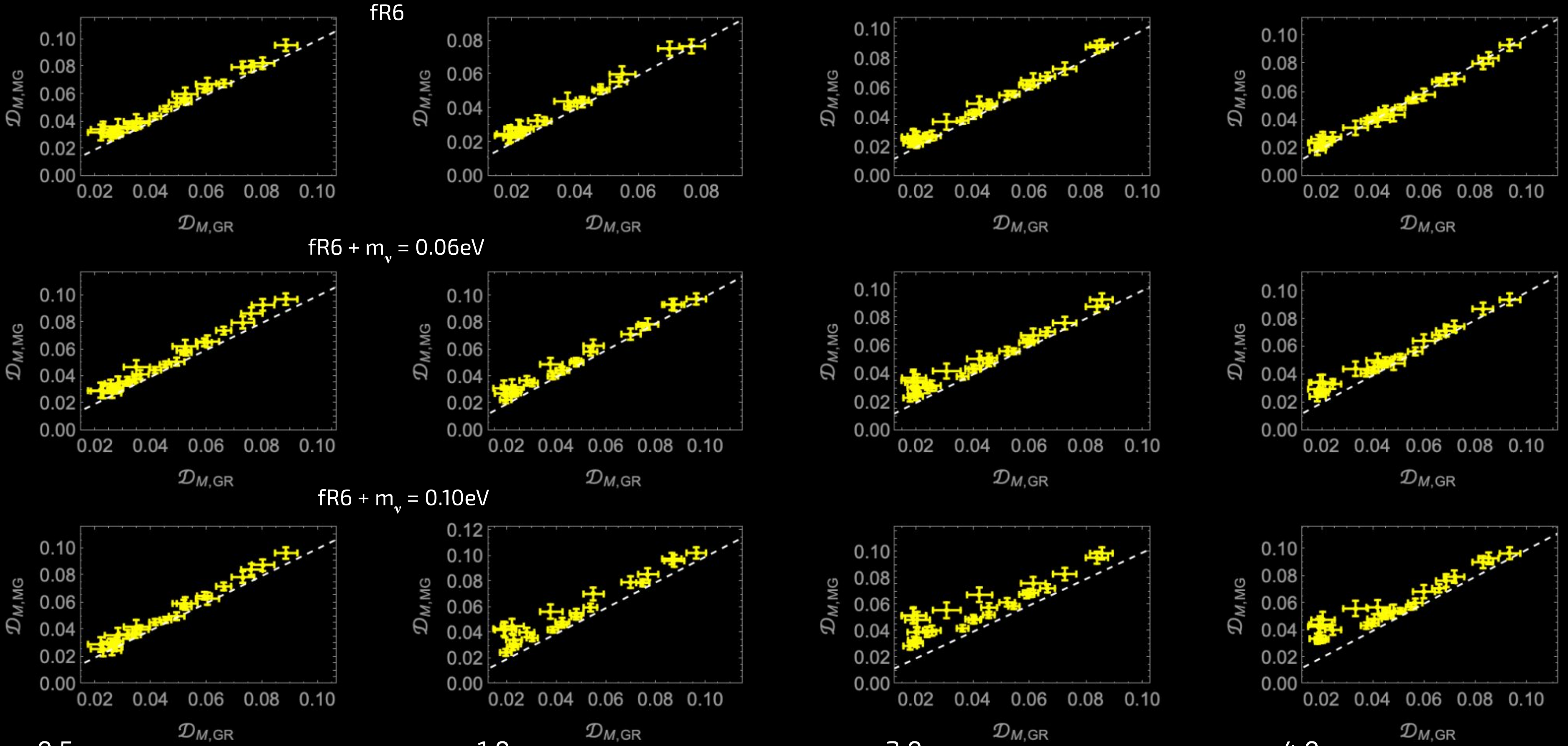
Peaks - Mahalanobis – fR6

PRELIMINARY



M_{ap} - Mahalanobis – fR6

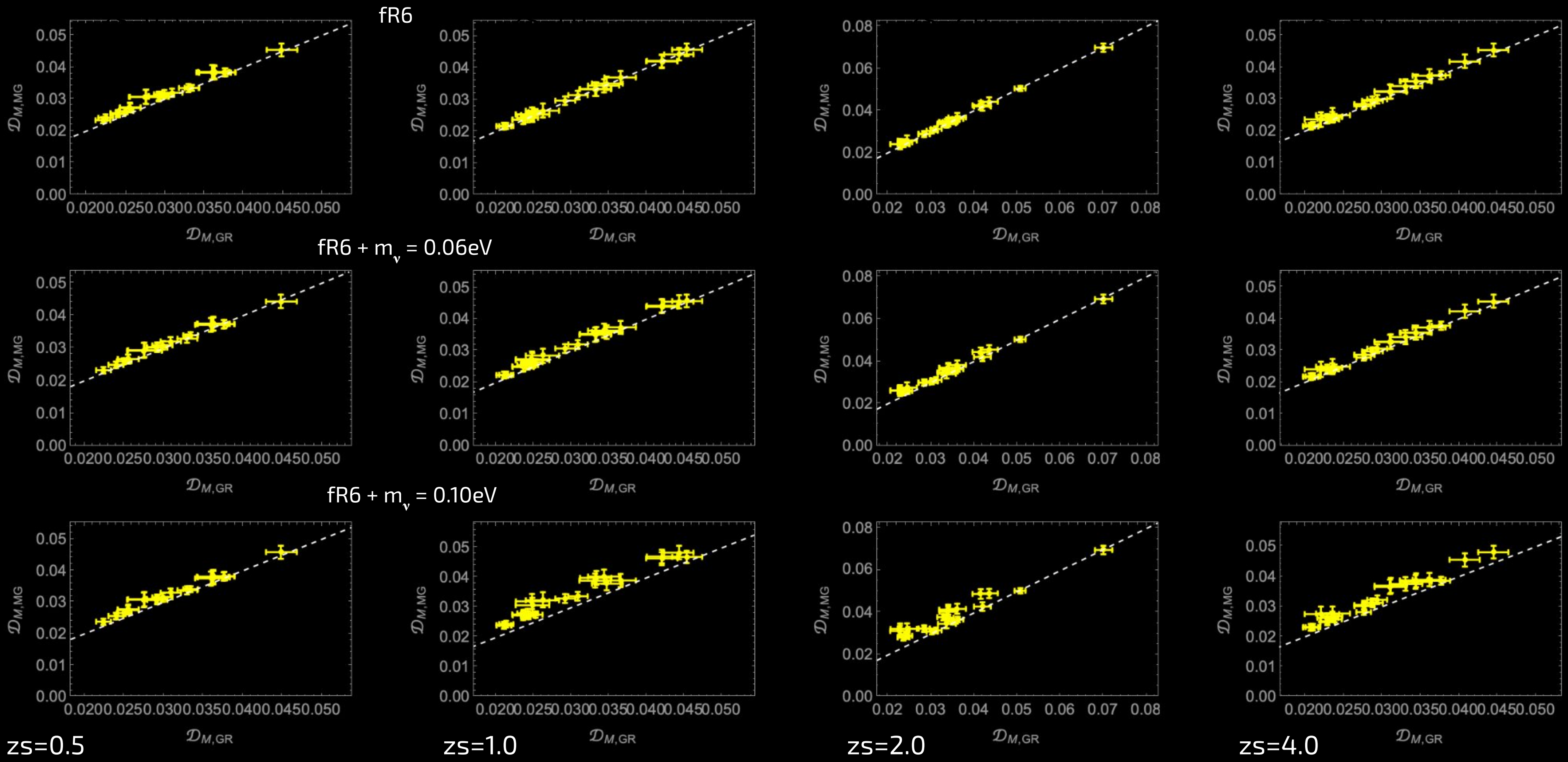
PRELIMINARY



zs=0.5 zs=1.0 zs=2.0 zs=4.0

BNs - Mahalanobis – fR6

PRELIMINARY



Conclusions

HOS can help discriminating among f(R) and ΛCDM provided one chooses

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Limitations

- massive neutrinos reducing the signature for f(R)
- preliminary results based on fixed source redshift

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- the most suitable probe and the correct metric
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Limitations

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Outlooks

- consider Euclid - like source redshift distribution and perform tomography and
- add systematics (e.g, IA and baryons) and observational effects (e.g., mask)

EXTRA SLIDES

Euclid project
(DR1 data)

Theory codes in MG

Outlooks

Theoretical characterization

$f(R)$ and nDGP

Weak lensing

$$\kappa = \frac{1}{2}(\partial_1\partial_1 + \partial_2\partial_2)\psi = \frac{1}{2}\nabla_2^2\psi$$

Convergence

$$\gamma_1 = \frac{1}{1}(\partial_1\partial_1 - \partial_2\partial_2)\psi$$

$$\gamma_2 = \partial_1\partial_2\psi.$$

Shear

$$\kappa(\boldsymbol{\theta},\chi) = \frac{3\Omega_m}{2}\left(\frac{H_0}{c}\right)^2 \int_0^{\chi} \frac{d\chi'}{a(\chi')} \frac{f_K(\chi-\chi')}{f_K(\chi)} f_K(\chi')\delta(f_K(\chi')\boldsymbol{\theta},\chi')$$

2PCF

$$C_\kappa(\ell) = \frac{c}{H_0}\int_0^{z_s} dz \frac{\mathcal{W}^2(z)}{\chi^2(z)E(z)}\Sigma^2(k_\ell, z)\mu^2(k_\ell, z)P_{mm}(k_\ell, z).$$

$$\mathcal{W}(z) = \frac{3}{2}\Omega_m\left(\frac{H_0}{c}\right)^2(1+z)\chi(z)\left[1-\frac{\chi(z)}{\chi(z_s)}\right].$$

$$\xi_\kappa(\theta) = \frac{1}{2\pi}\int d\ell\,\ell\,J_0(\ell\theta)\,C_\kappa(\ell).$$

1PDF

Cumulant generating function

$$\phi_{\kappa,\Theta}(y) = \int_0^{z_s} dz\, \chi'(z)\, \phi_{\delta,cyl}\left[\mathcal{W}(z)y, d_A(z), z\right]$$

Inverse Laplace transform

$$\mathcal{P}(\kappa) = \int_{-i\infty}^{+i\infty} \frac{dy}{2\pi i} \exp\left[-y\kappa + \phi_{\kappa,\Theta}(y)\right]$$

Peaks

$$M_{ap}(\boldsymbol{\theta};\vartheta) = \int d^2\theta' \, \underline{U_\vartheta(|\boldsymbol{\theta} - \boldsymbol{\theta}'|)} \, \kappa(\boldsymbol{\theta}')$$

$$\int d^2\theta \, U_\vartheta(\boldsymbol{\theta}) = 0$$

Compensated filter function

HOM – Hierarchical Ansatz

$$\xi_p(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_p) = \sum_\alpha \mathcal{Q}_p^\alpha \sum_{t_\alpha} \prod_{p-1} \xi(\boldsymbol{r}_i, \boldsymbol{r}_j)$$

p-point correlation function

$$\langle \kappa^t(\vartheta_s) \rangle = \mathcal{Q}_t \, \mathcal{C}_t \left[\kappa_{sm}(z, \vartheta_s) \right]$$

$$\kappa_{sm}(\vartheta_s) = 2\pi \int \mu(k_\ell, z) P_{mm}(k_\ell, z) W_f^2(\ell \vartheta_s) \, \ell \, d\ell$$

$$\mathcal{C}_t \left[\kappa_{sm}(z, \vartheta_s) \right] = \int dz \, \frac{\mathcal{W}^t(z)}{H(z) \left[\chi(z) \right]^{2(t-1)}} \kappa_{sm}^{t-1}(z, \vartheta_s).$$

Higher-order moments

$$\langle \kappa^3(\theta) \rangle = \int_0^{\chi_s} d\chi \, \frac{q^3(\chi)}{f_K^6(\chi)} \int \frac{d^2 \boldsymbol{\ell}_1}{(2\pi)^3} \Sigma(k_{\ell_1}, \chi) W_f(\ell_1 \vartheta_s) \ldots$$

$$\times \int \frac{d^2 \boldsymbol{\ell}_3}{(2\pi)^3} \Sigma(k_{\ell_3}, \chi) W_f(\ell_3 \vartheta_s) \underline{\tilde{B}_2(k_{\ell_1}, k_{\ell_2}, k_{\ell_3})}_{\sum_i \ell_i = 0}$$

$$\langle \kappa^4(\theta) \rangle = \int_0^{\chi_s} d\chi \, \frac{q^4(\chi)}{f_K^8(\chi)} \int \frac{d^2 \boldsymbol{\ell}_1}{(2\pi)^3} \Sigma(k_{\ell_1}, \chi) W_f(\ell_1 \vartheta_s) \times \cdots$$

$$\int \frac{d^2 \boldsymbol{\ell}_4}{(2\pi)^3} \Sigma(k_{\ell_4}, \chi) W_f(\ell_4 \vartheta_s) \times \underline{\tilde{B}_3(k_{\ell_1}, k_{\ell_2}, k_{\ell_3}, k_{\ell_4})}_{\sum_i \ell_i = 0}$$

Betti numbers

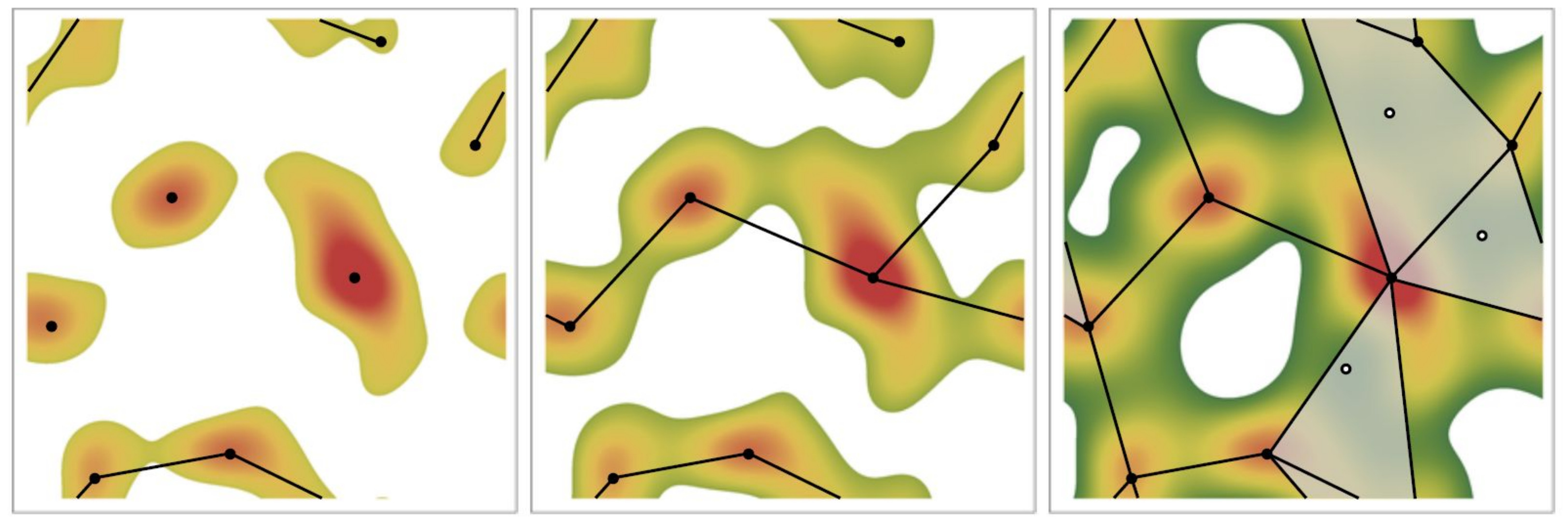
$$\beta_0(\nu) = \int_\nu^\infty d\nu' \, \{\mathcal{N}_2(\nu') - [1 - g(\nu')]\mathcal{N}_1(\nu')\}$$

$$\beta_1(\nu) = \int_\nu^\infty d\nu' \, [g(\nu')\mathcal{N}_1(\nu') - \mathcal{N}_0(\nu')]$$

$$\beta_2(\nu) = 0$$

$$\mathcal{N}_0, \mathcal{N}_1, \mathcal{N}_2$$

Density of minima, saddle points and maxima



[arXiv:1908.01619]

Minkowski functionals

$$V_0(\nu) = \frac{1}{A}\int_{Q_\nu} da$$

$$V_1(\nu) = \frac{1}{4A}\int_{\partial Q_\nu} dl$$

$$V_2(\nu) = \frac{1}{2\pi A}\int_{\partial Q_\nu}^{\partial} dl\, \mathcal{K}$$

$$Q_\nu = \left\{ (x_1, x_2) \middle| \frac{k(x_1, x_2)}{\sigma_0} \geq \nu \right\}$$

A area on which measured MFs

da, dl surface/line element boundary

K local geodesic curvature of boundary

$f(R)$ gravity

$$f(R) = -m^2 \frac{c_1 \left(\frac{R}{m^2}\right)^n}{c_2 \left(\frac{R}{m^2}\right)^n + 1}$$

Scalar field

$$f_R = \mathrm{d}f/\mathrm{d}R$$

$$f_R \simeq -n \frac{c_1}{c_2^2} \left(\frac{m^2}{R}\right)^{n+1}$$

1PDF

1 - fR4

2 - fR4 m_ν = 0.3eV

3 - fR5

4 - fR5 m_ν = 0.1eV

5 - fR5 m_ν = 0.15eV

6 - fR6

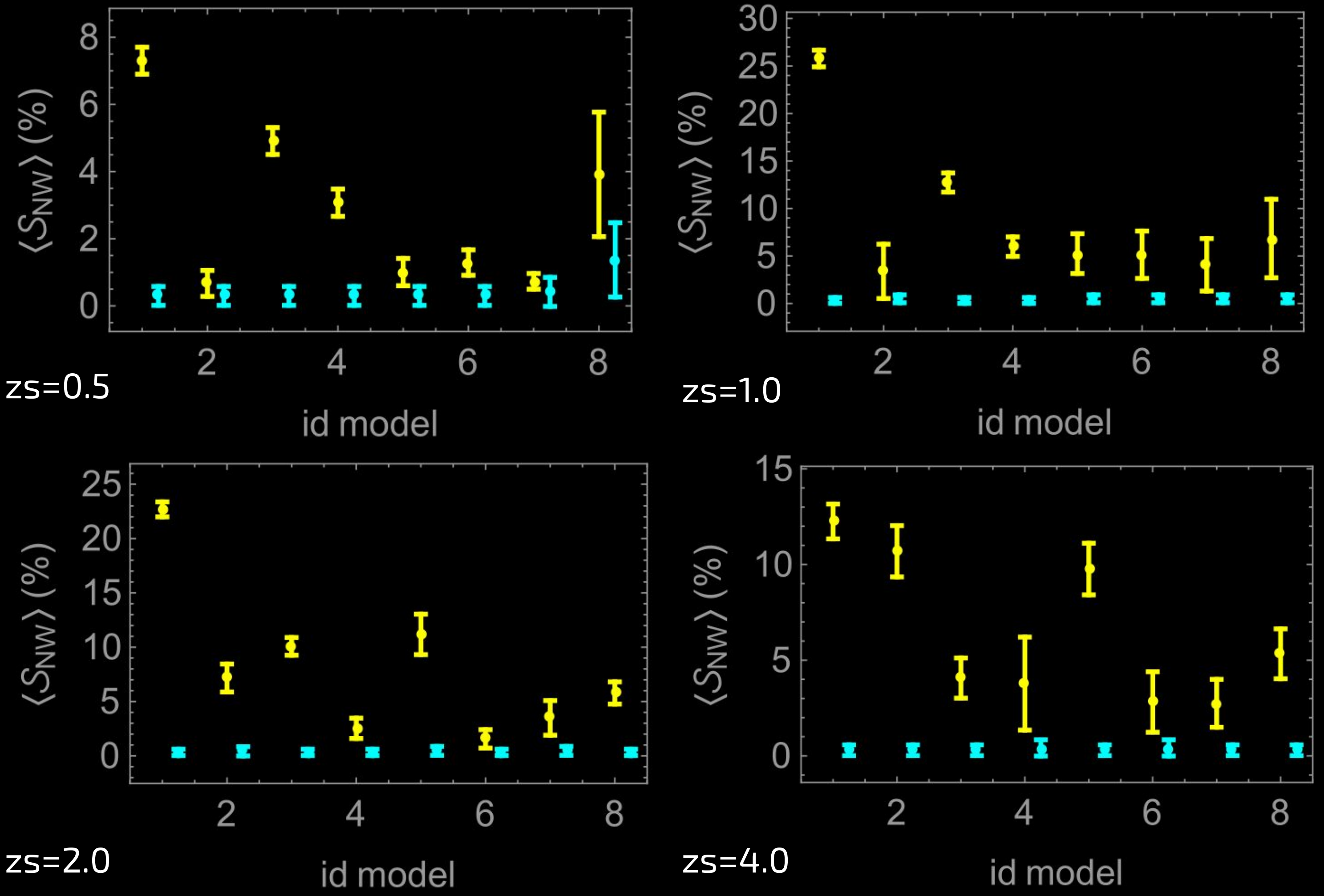
7 - fR6 m_ν = 0.06eV

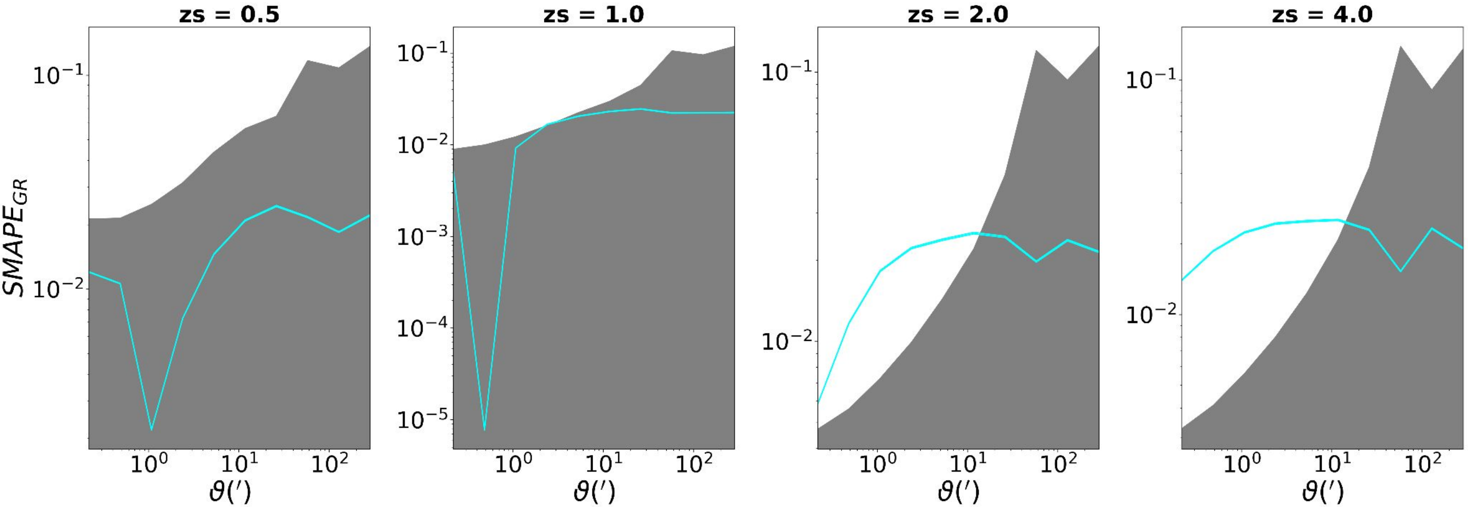
8 - fR6 m_ν = 0.1eV

GR - MG

[2500 realizations]

PRELIMINARY

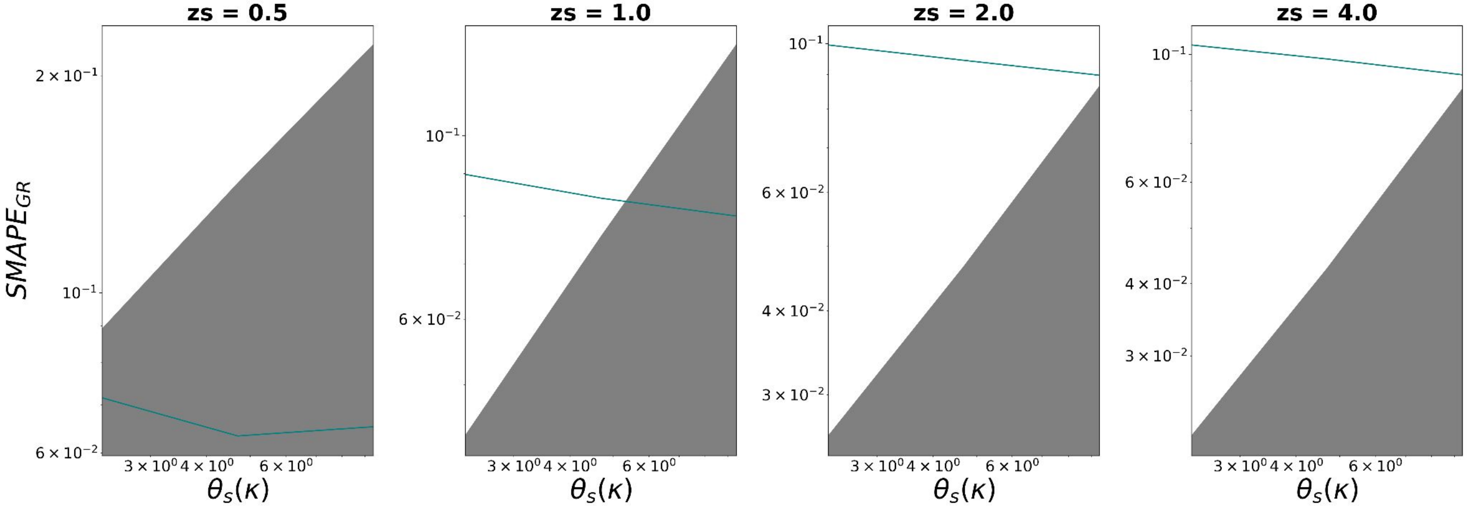




2PCF

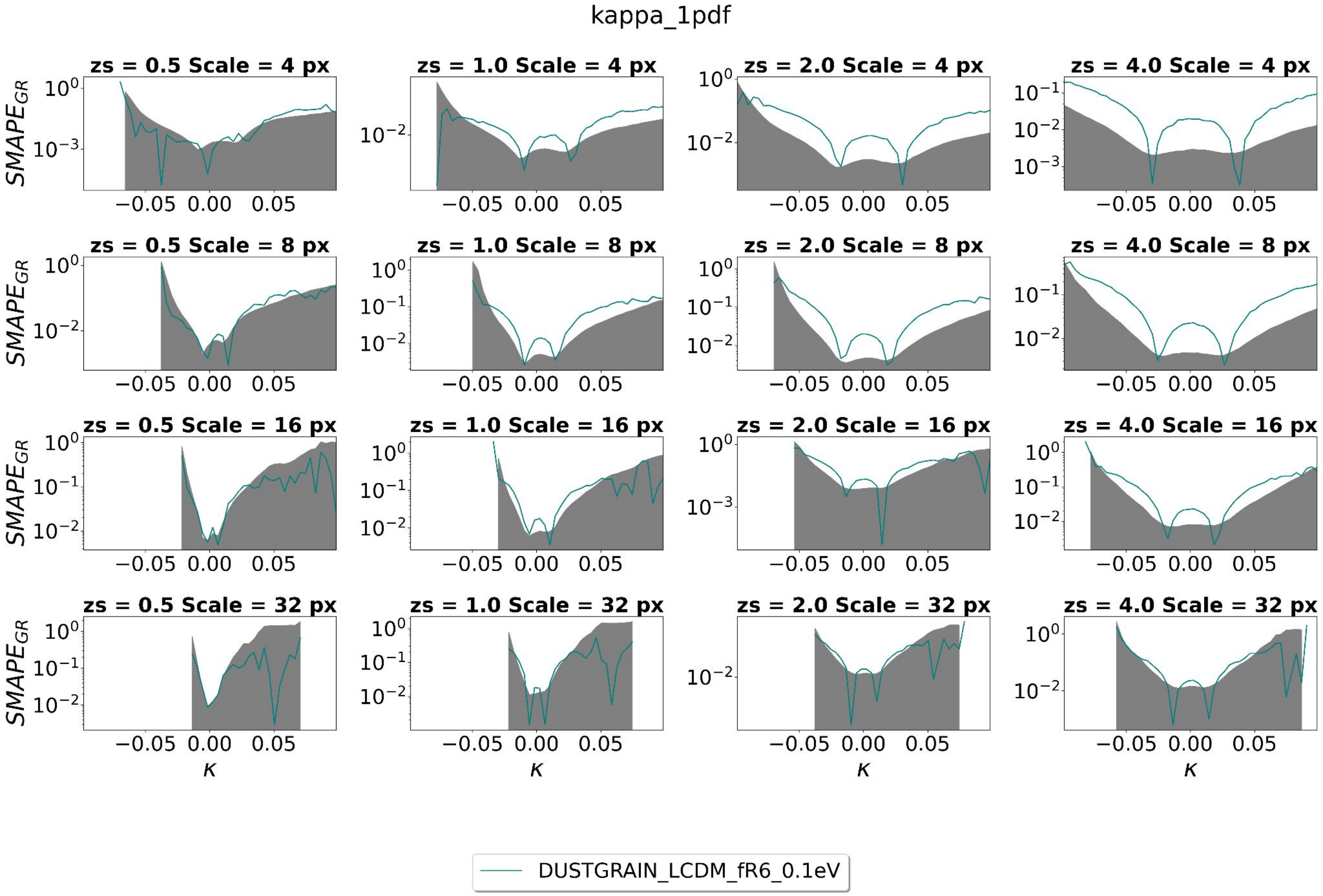
fR6

$m_\nu = 0.1$ eV



< κ^3 >

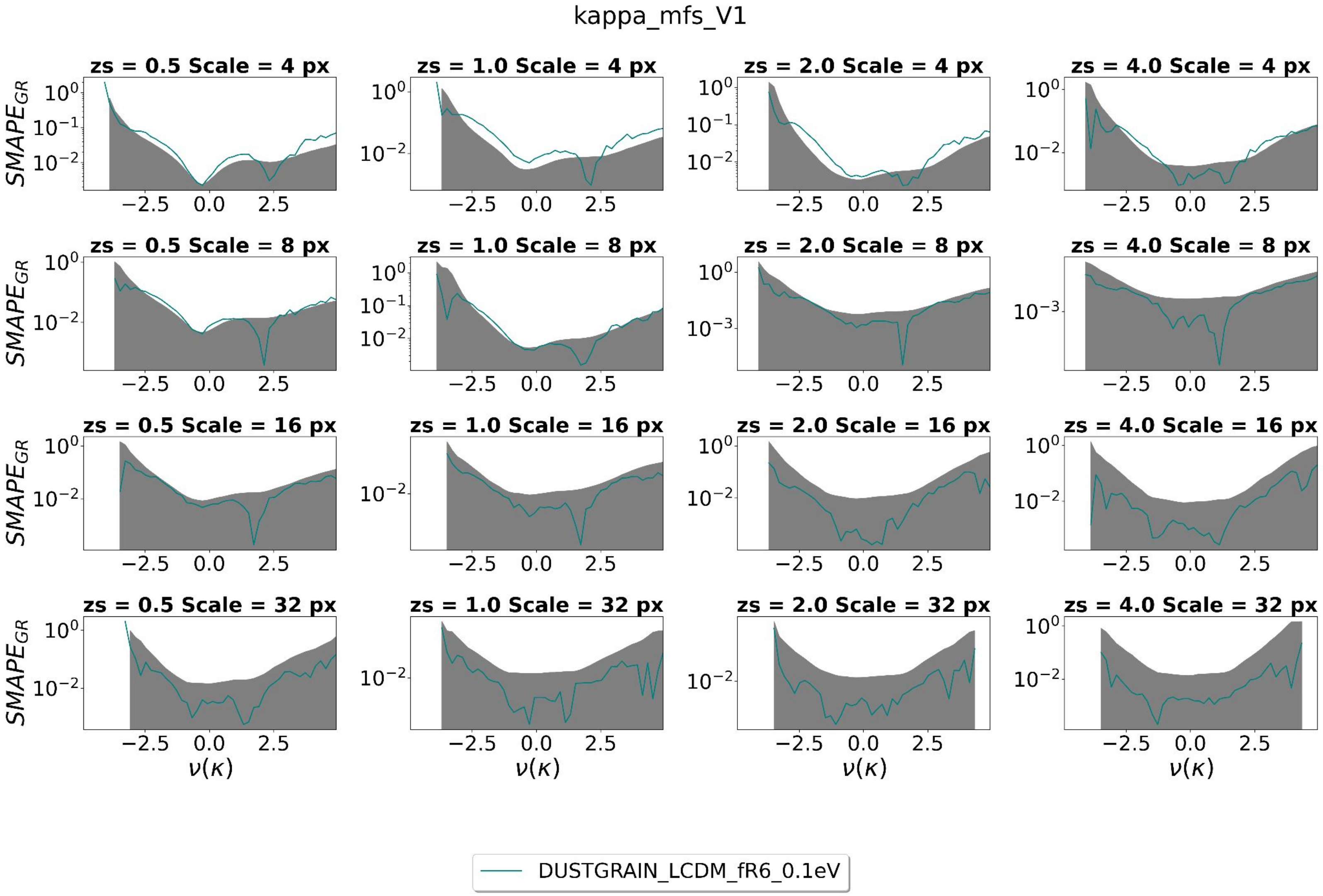
Thanks Simone!



fR6

m_ν = 0.1 eV

κ-PDF



fR6

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MF V_1