Theoretical predictions for wavelet ℓ_1-norm

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Institut Pascal, Orsay, France

slides at vilasinits.github.io/Talks/COLOURS/

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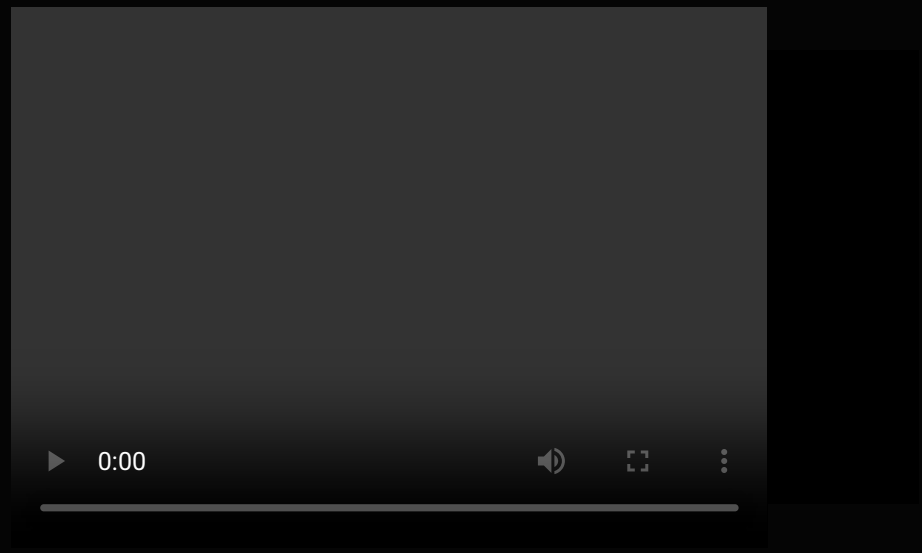
Introduction – Weak Lensing

- **Weak Lensing (WL)**: An observational technique in cosmology used to study the distribution of matter in the universe.
- **Principle:** Light from distant galaxies is deflected by gravitational fields, leading to subtle distortions in their observed images.
- **Weak Lensing Effect:** Small, coherent distortions in the shapes of background galaxies.

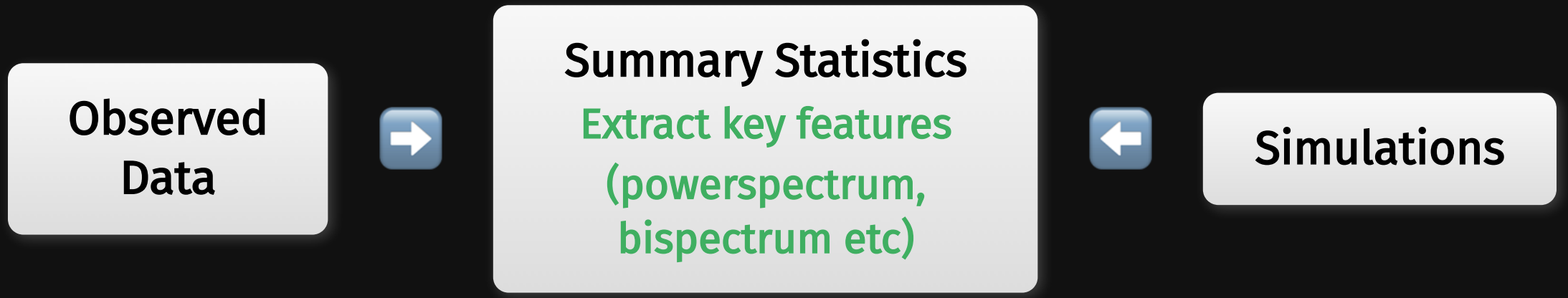


- WL provides a direct measurement of gravitational distortions.
- It allows us to probe cosmic structure, study dark matter, and constrain cosmological parameters.

Recap

Why use HOS?

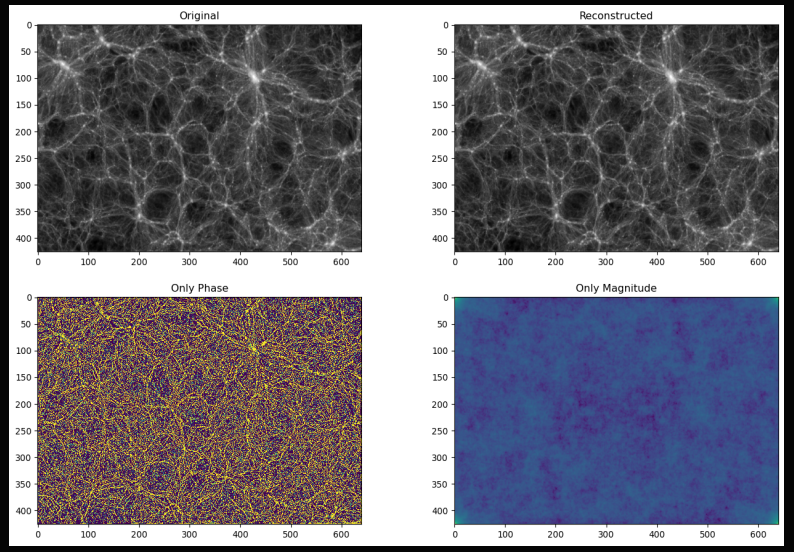
Cosmology Inference Pipeline



General approach: 2 point statistics

- Has a theory prediction

  But...not all the information is captured!



- Cannot extract non-Gaussian statistics

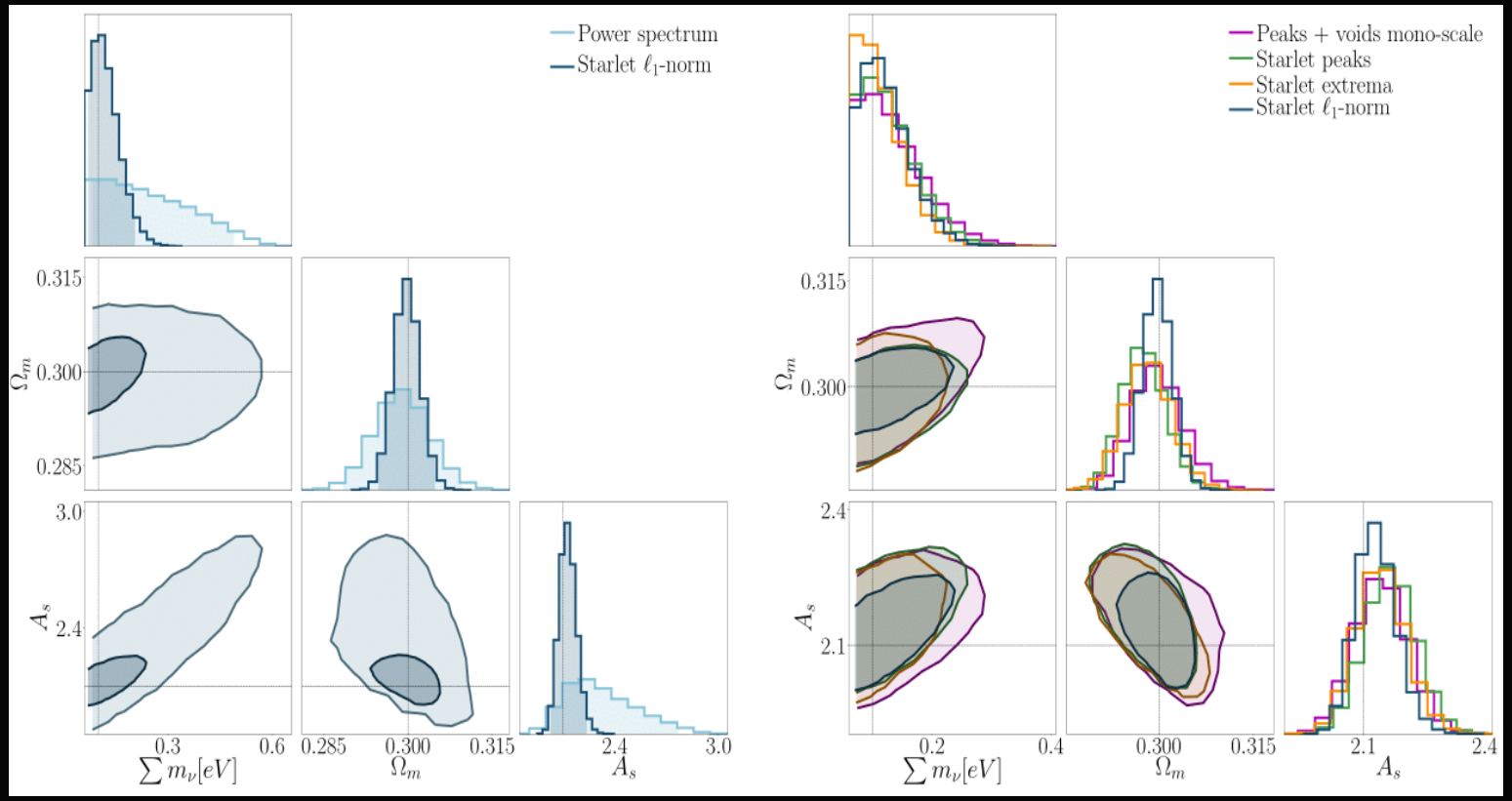
Alternative approach: Higher order statistics

- Can extract non-Gaussian statistics
- Several HOS exists
  - Bispectrum
  - Minkowski functionals
  - Betti numbers
  - PDF
  - Wavelet ℓ_1-norm
- Many do not have a theory prediction
- Need simulations

Why wavelet ℓ_1-norm?

$$l_1^{j,i} = \sum_{u=1}^{coef(Sj,i)} |S_{j,i}[u]|$$

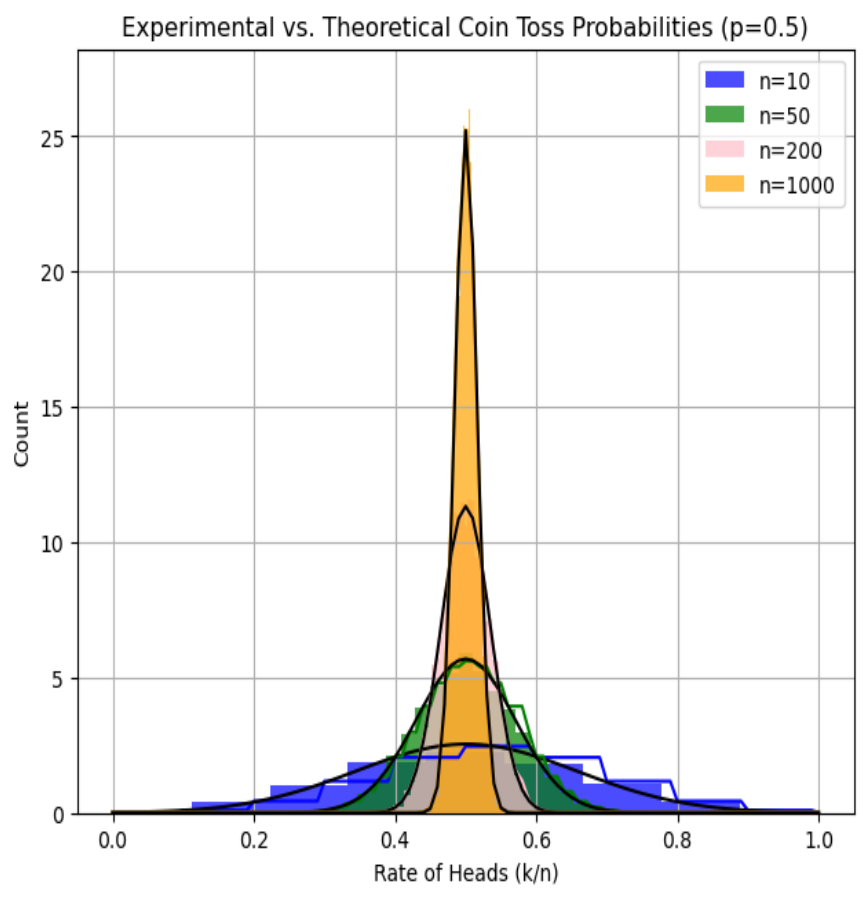
- the wavelet ℓ_1-norm carries the information encoded in all pixels of the map.
- It is shown in that it remarkably outperforms commonly used summary statistics, such as the power spectrum or the combination of peak and void counts.

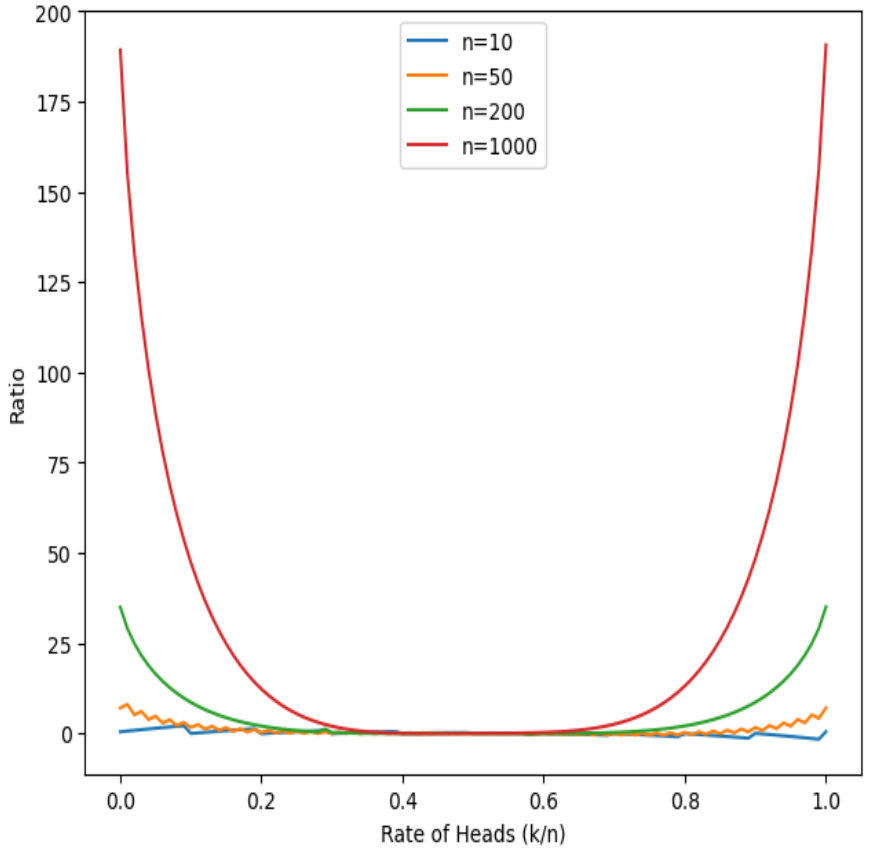


Ajani et al. (2021)

Working principle

Intuition for large deviation theory

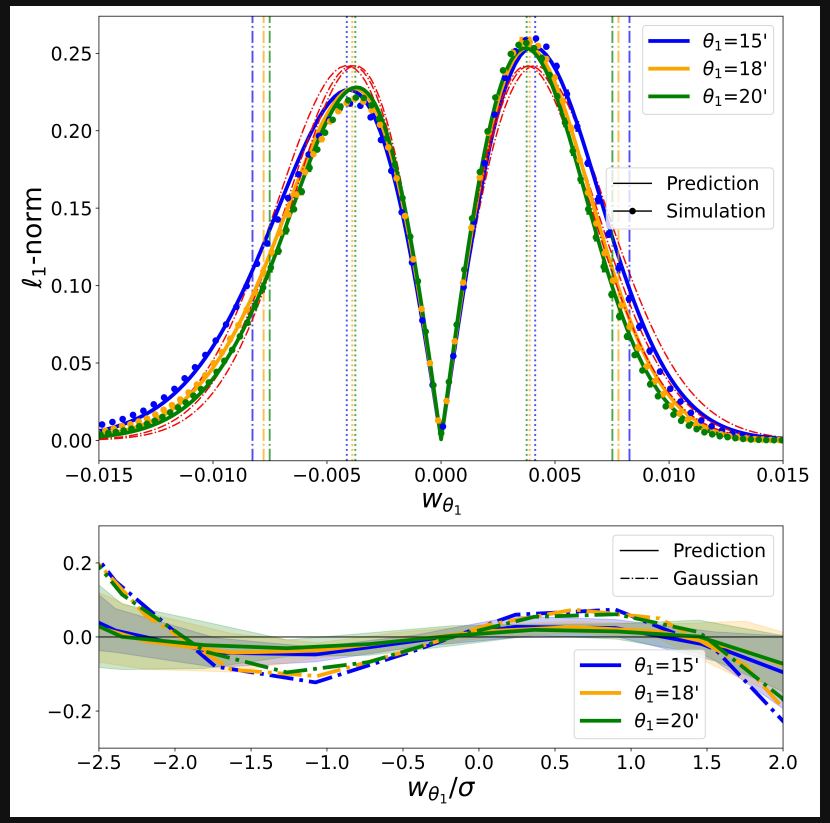


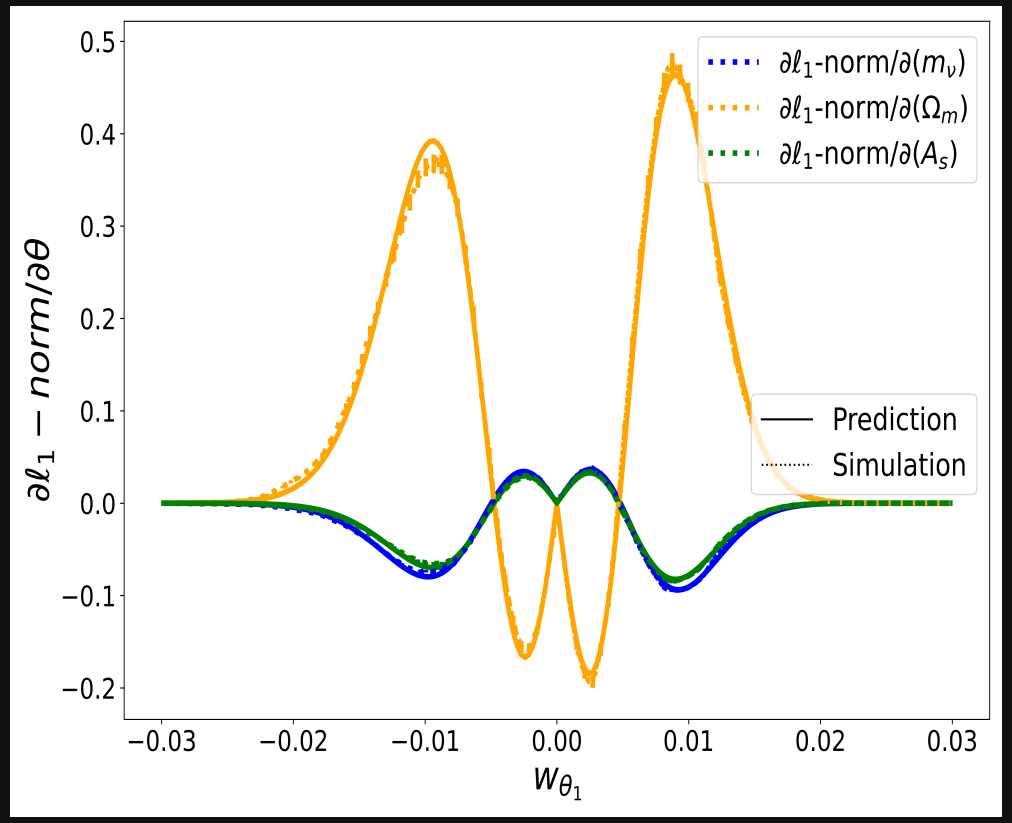


Theoretical Pipeline Overview

- **Start:** From the first principles of cosmology → *Spherical Collapse Model*
  (One-to-one mapping between initial and final densities)
- **LDT Framework:** used to predict the probability distribution
- **Final:** Connect the distribution to the ℓ_1-norm

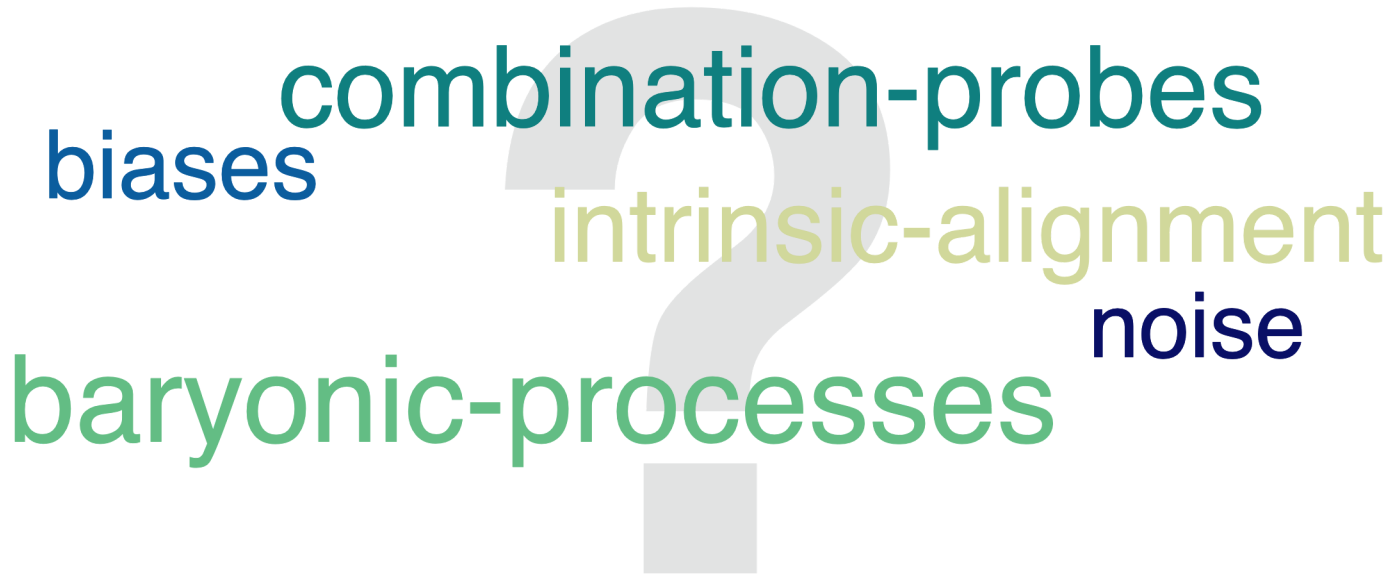
Results



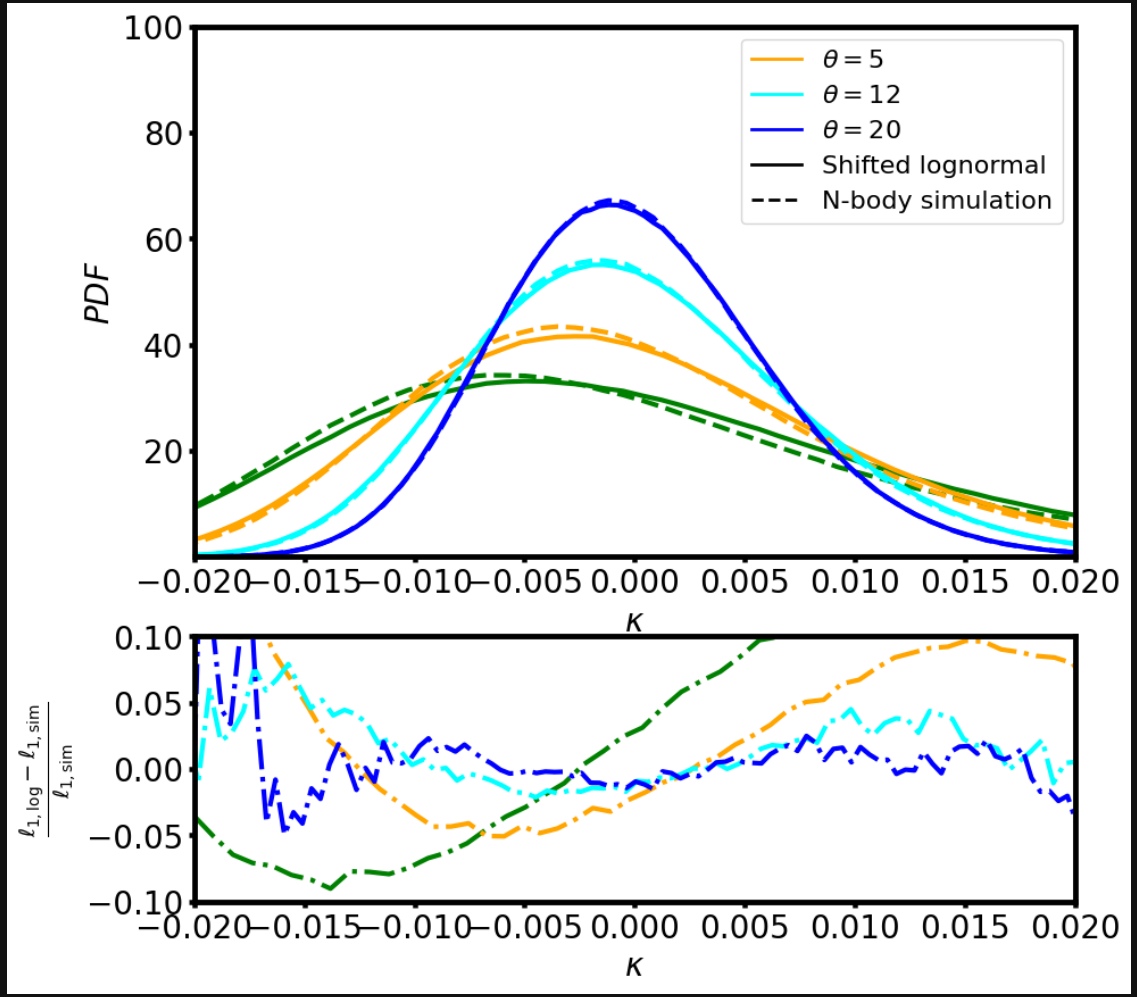


Sreekanth, V. T., Codis, S., Barthelemy, A., & Starck, J.-L. 2024, A&A, 691,A80

What about...



Is an N-body simulation still the only way? 🤔



Current emulators for weak-lensing convergence maps does not encode accurate HOS!

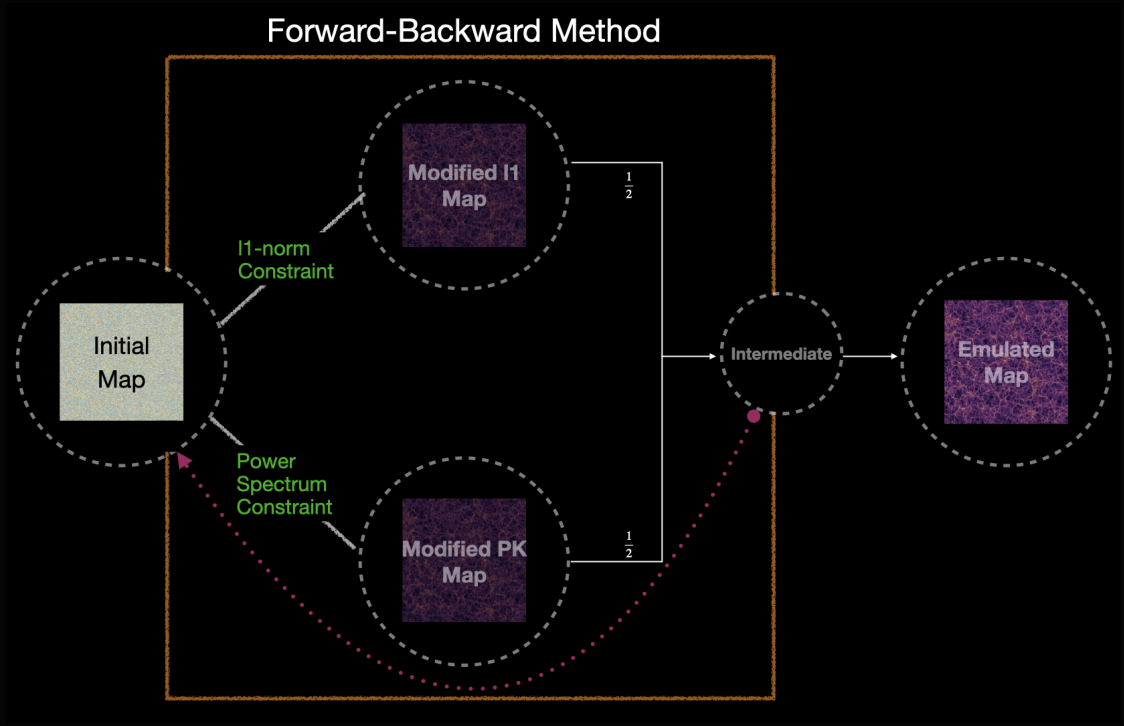
Generative Process of Weak Lensing Convergence Map

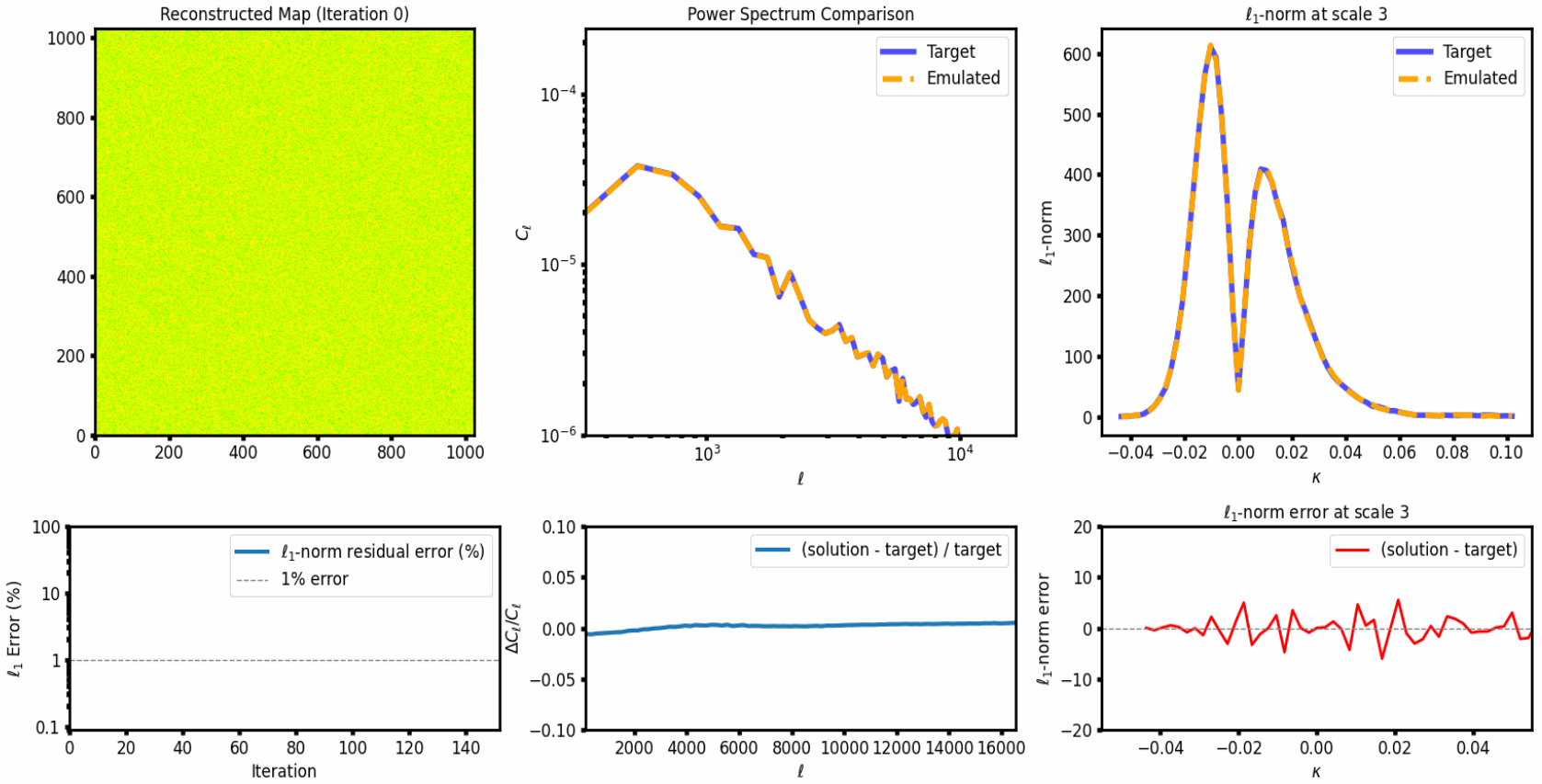
Gaussian Initial Map → Iterative Generalised Forward–Backward Method → Converged κ Map

GFB → decompose the optimisation into simpler sub-problems associated with each constraint and alternate between them

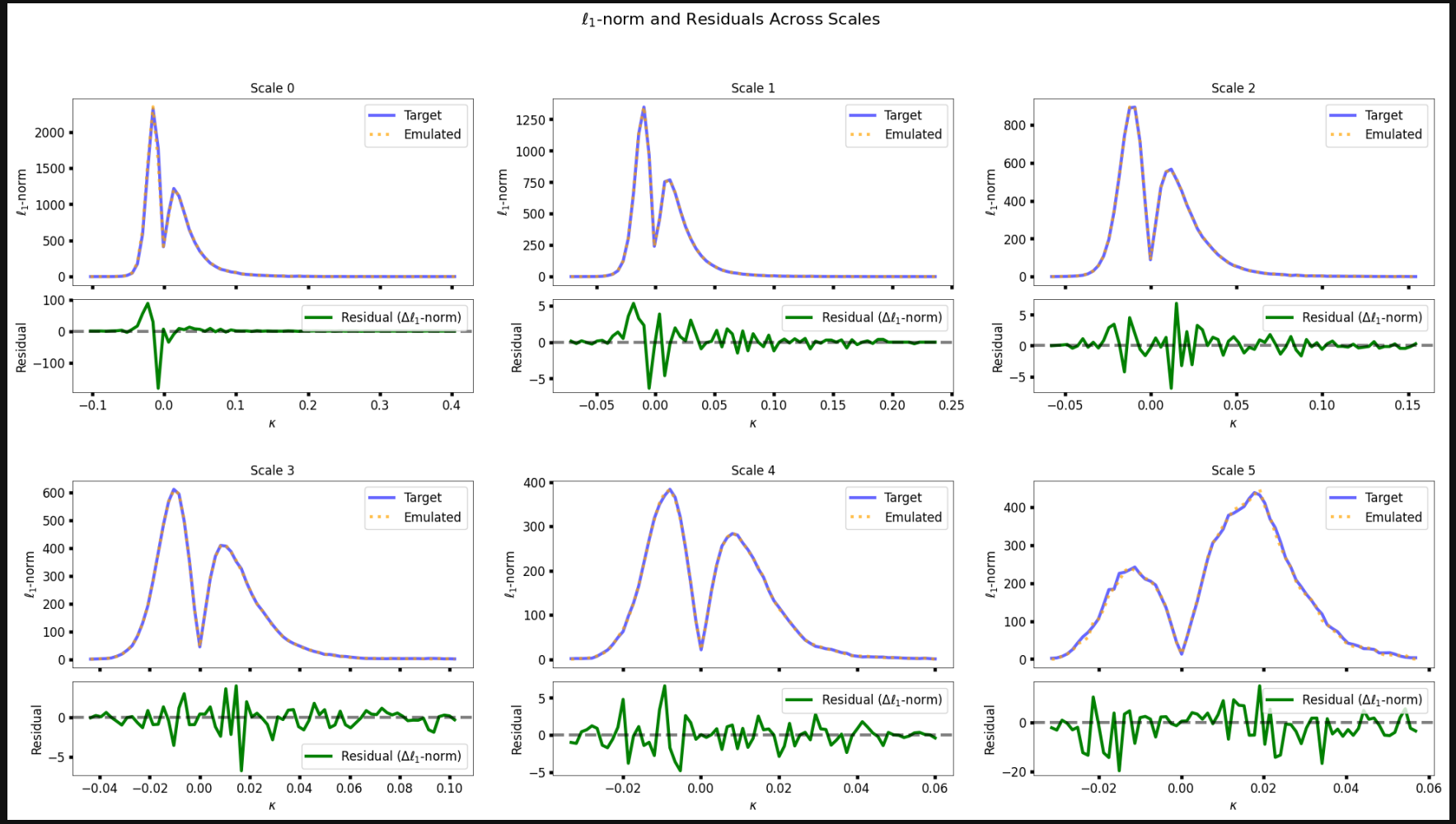
- $\|\mathrm{P}[\kappa] - \mathrm{P}_{target}\|^2{}_2 \;;\quad \| \ \ell_1[\kappa] - \ell_1[\kappa]_{target} \ \|^2$

where P is the power spectrum operator and $\ell_1[\kappa]$ is the wavelet ℓ_1 norm of the map κ.

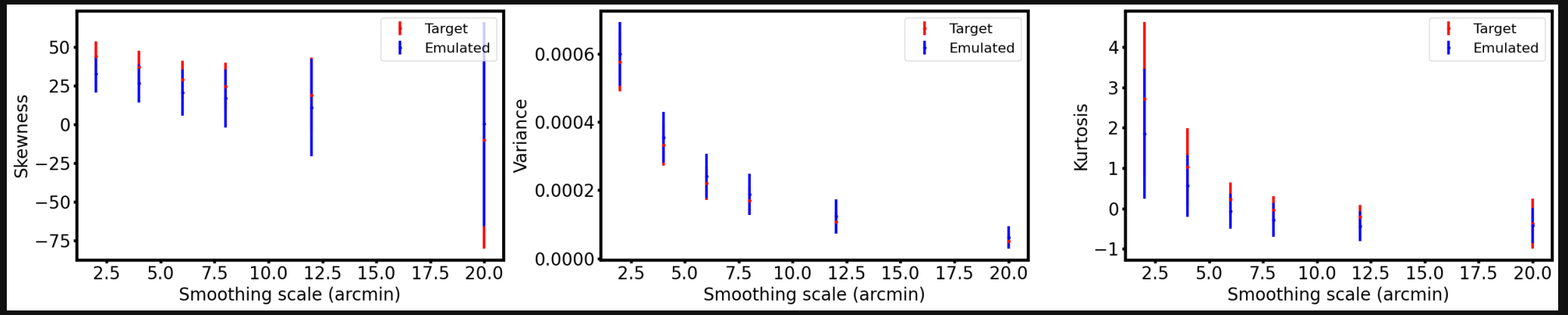

Forward-Backward Method
Modified l1 Map
l1-norm Constraint
Initial Map
Power Spectrum Constraint
Modified PK Map
1/2
1/2
Intermediate
Emulated Map

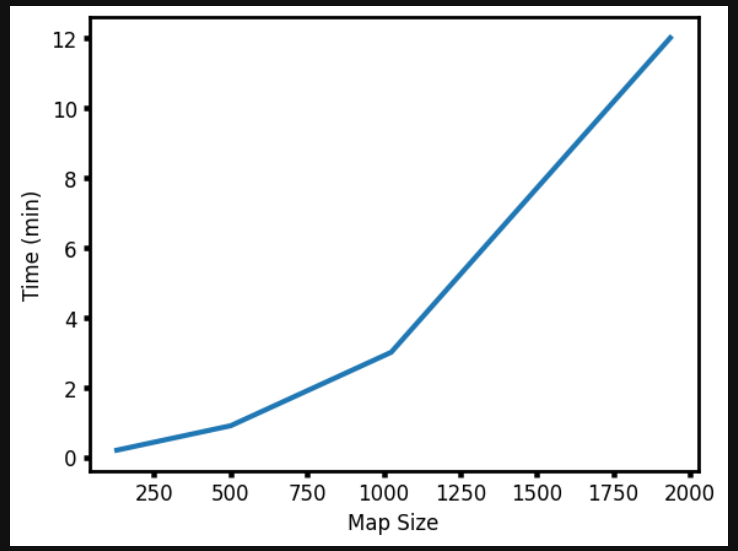


Submitted to, A&A


ℓ_1-norm and Residuals Across Scales
Scale 0
Scale 1
Scale 2
Scale 3
Scale 4
Scale 5
Target
Emulated
Residual ($\Delta\ell_1$-norm)
ℓ_1-norm
Residual
κ

Benchmarking





Conclusion

- Need HOS to extract non-gaussian information.
- We have developed a theoretical prediction tool for the wavelet ℓ_1-norm using large deviation theory.
- The prediction is validated against N-body simulations and is robust.
- Developed an emulator for kappa maps using forward-backward method.
- The emulator is relatively fast and can be used to generate kappa maps with desired properties.

Future Work

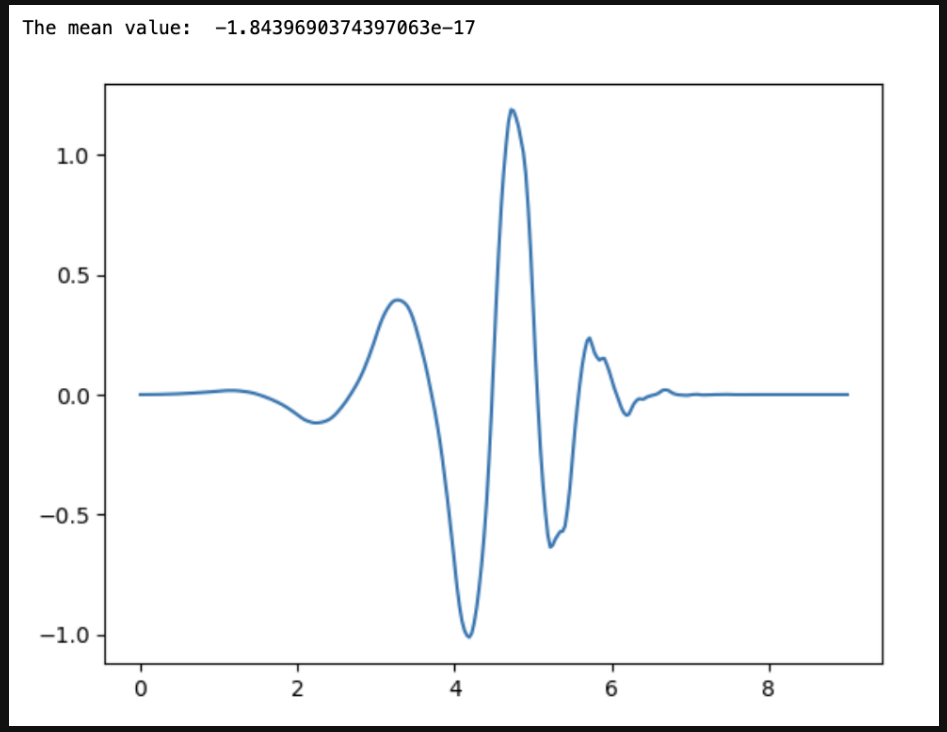
- Include stochasticity in the prediction tool.
- Investigate the potential of using other filtering schemes in wavelet ℓ_1-norm.

Thank you!

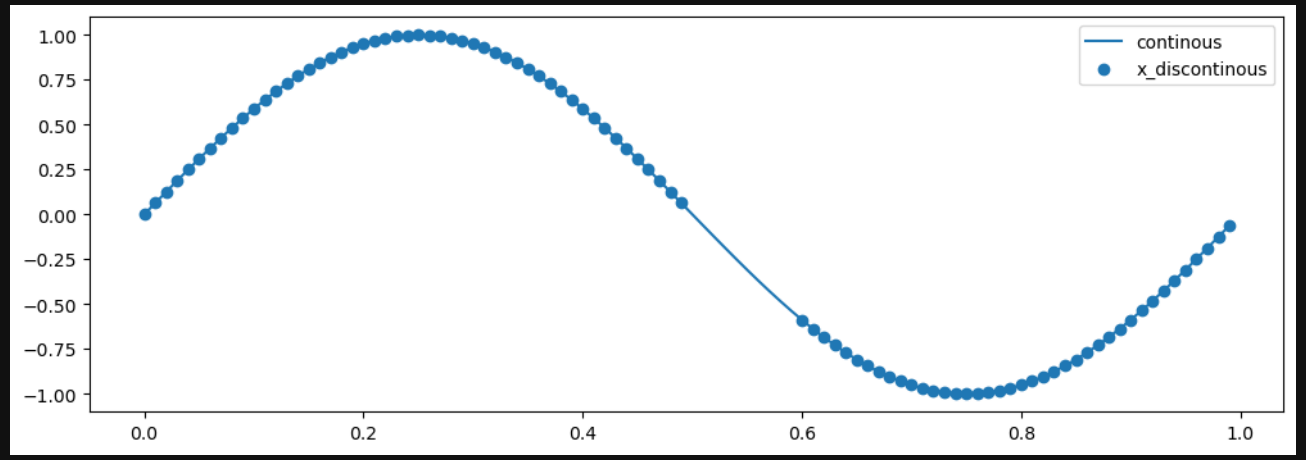
Extra slides

Wavelets

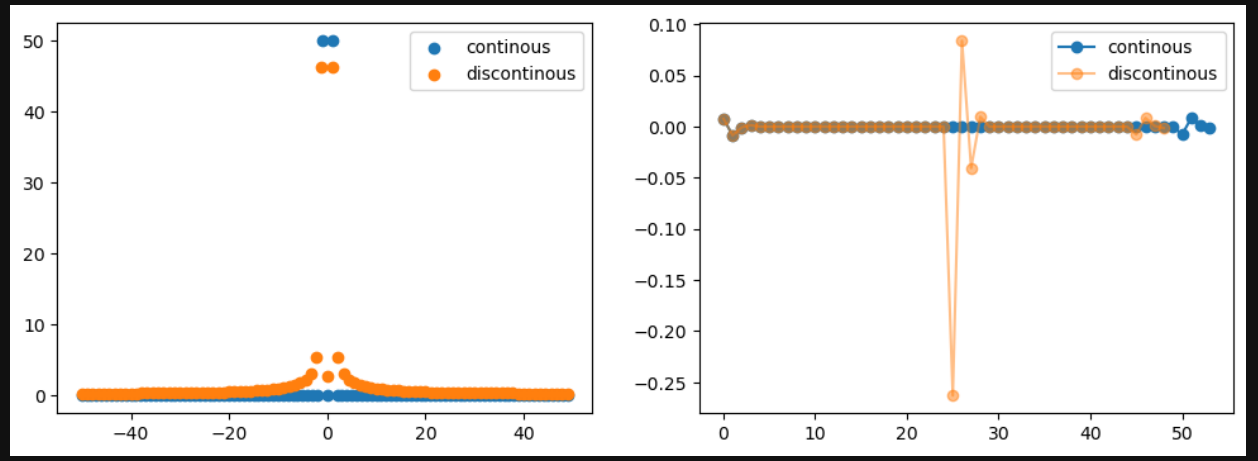
A wavelet is a waveform of effectively limited duration that has an average value of zero and nonzero norm.



This is an example of a DB5 wavelet



The coefficients from Fourier transform and wavelet trandform respectively



Especially useful in cases where we have discontinuties, trends etc that other techniques often tend to miss