The Standard Model of particle physics… and beyond!

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The 14th IDPASC school

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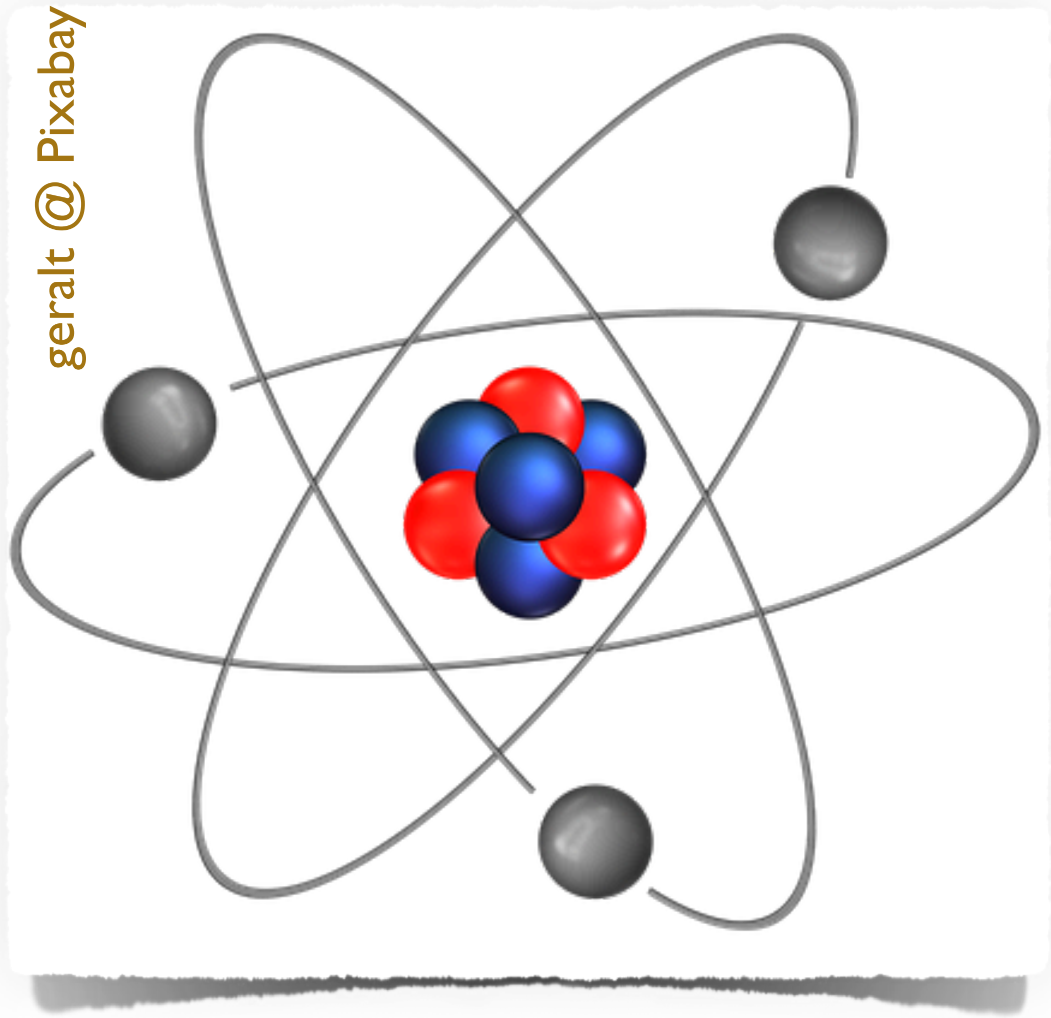
A brief history of 100 years of particle physics

A window to the infinitely small

Atom compositeness: 3 entities to explain matter (in the early 1900s)

- Rutherford's model (1911)
    - ⋆ Protons
    - ⋆ Neutrons (Chadwick, 1932)
    - ⋆ Electrons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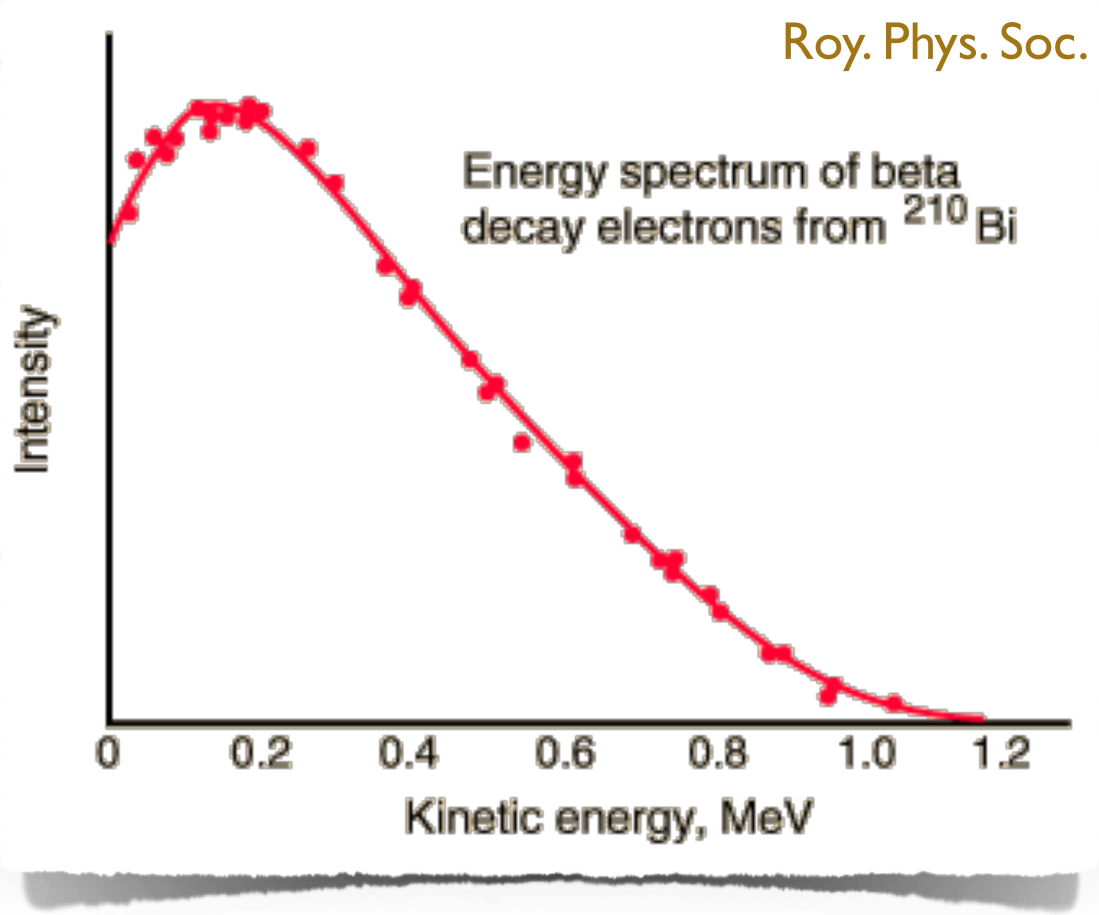
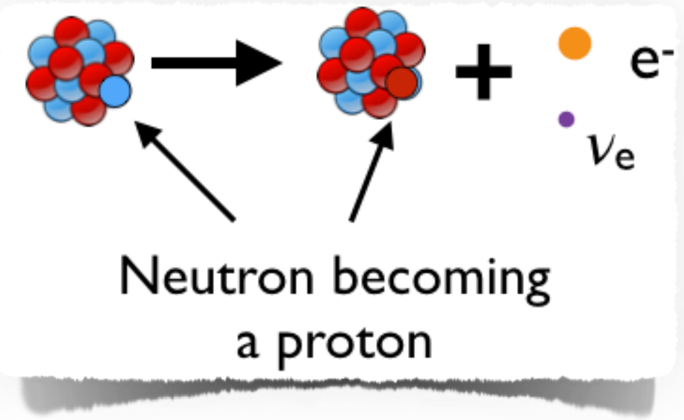


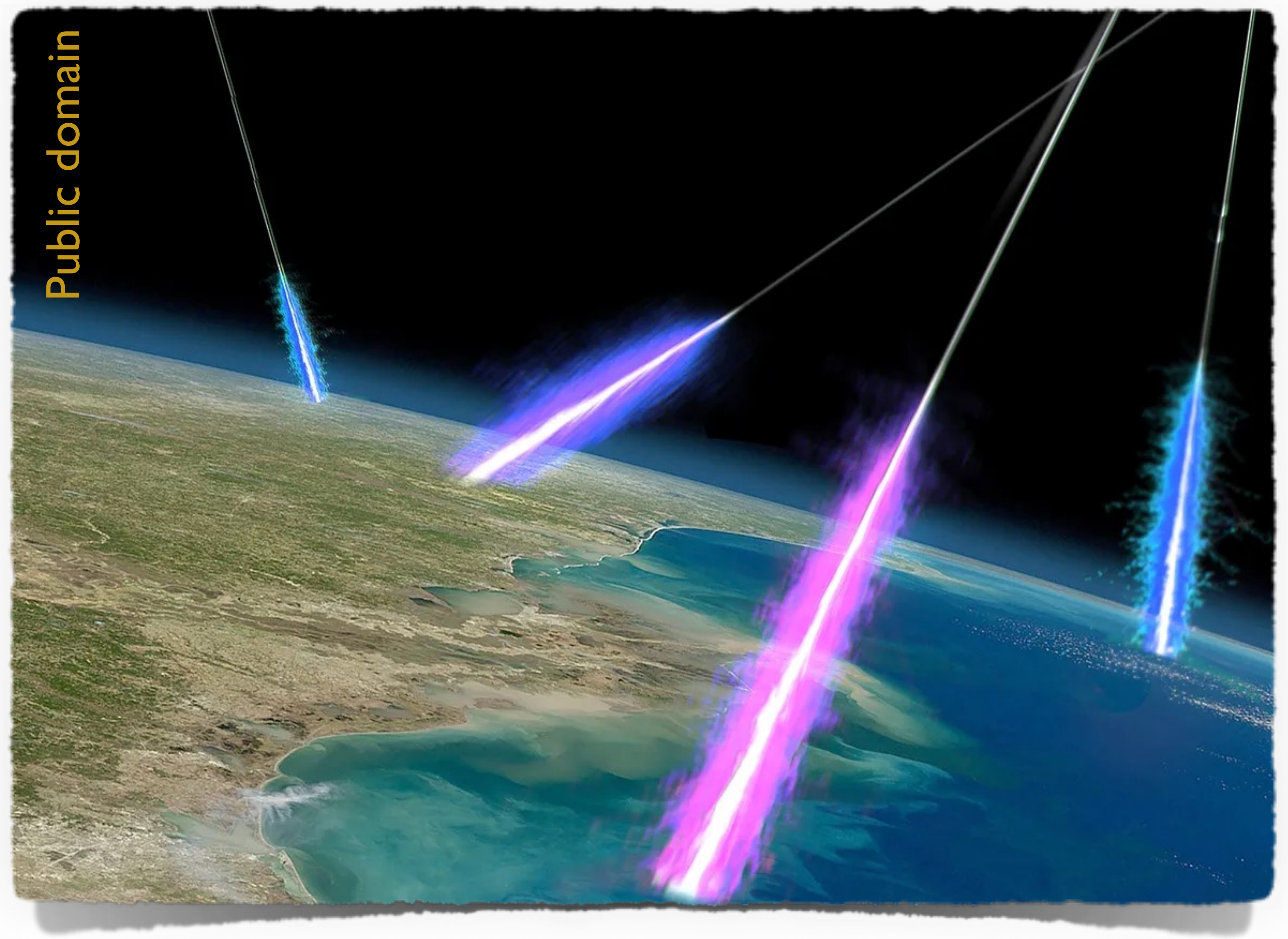
First radioactivity studies

- Need for a neutrino (4th building bloc of matter)
- Missing energy in decay processes



Neutron becoming a proton

Hadronic physics and the quark model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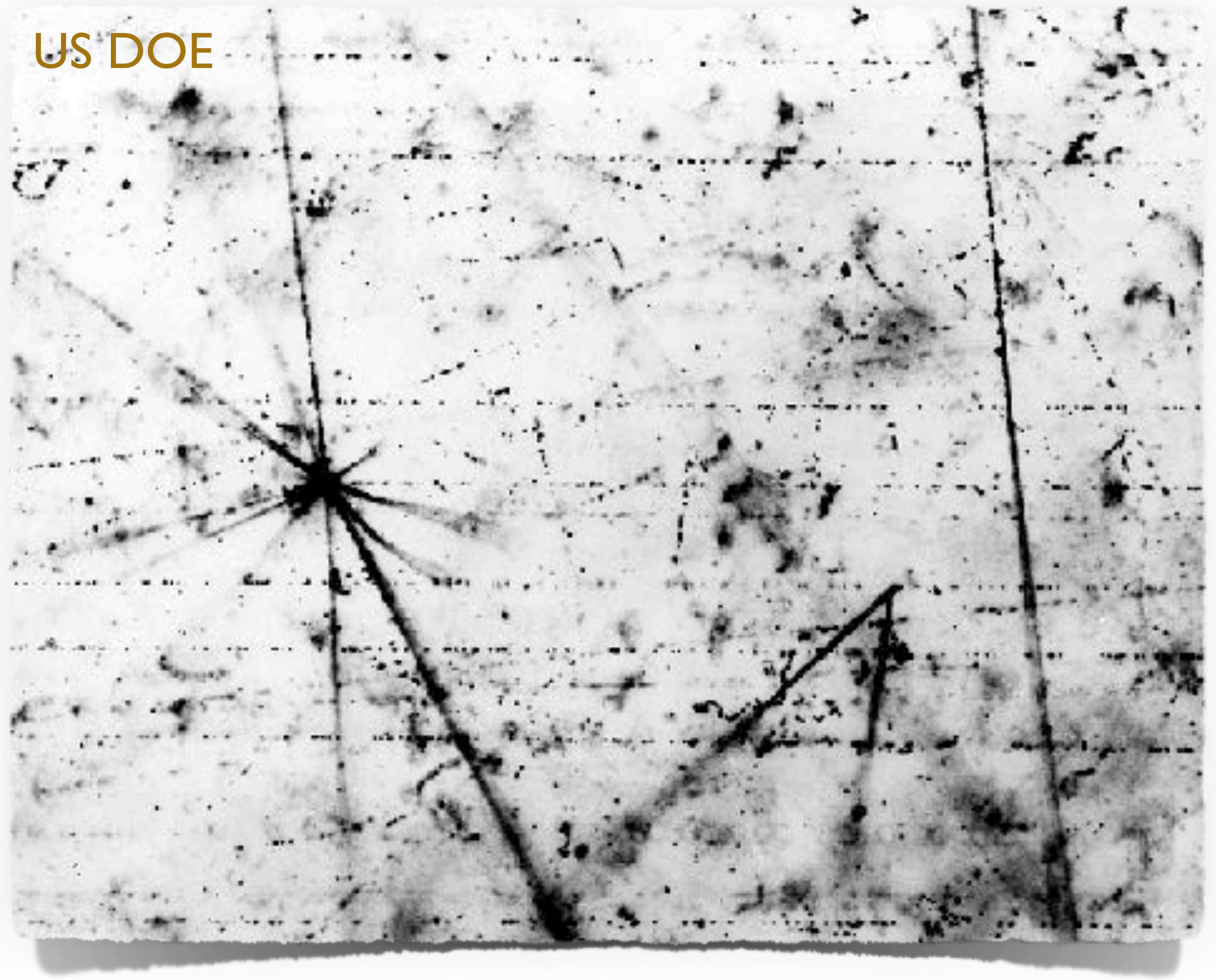
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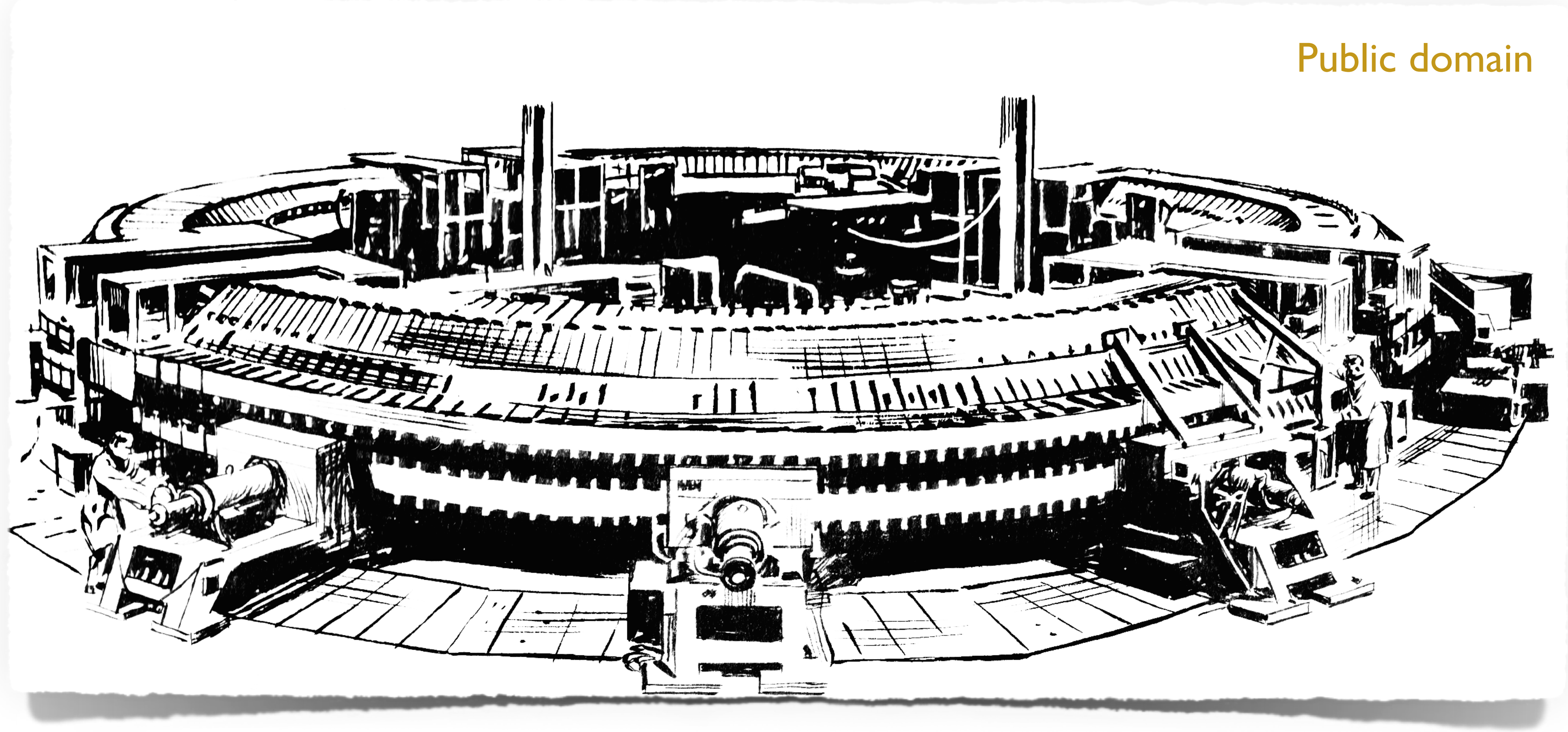
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p π^+ Ξ

n Δ π^0 ρ

e^- ν_e η

Ω Λ_c^+ Λ^0

η'

ω Λ_b^0

ν_μ π^- Σ μ^-

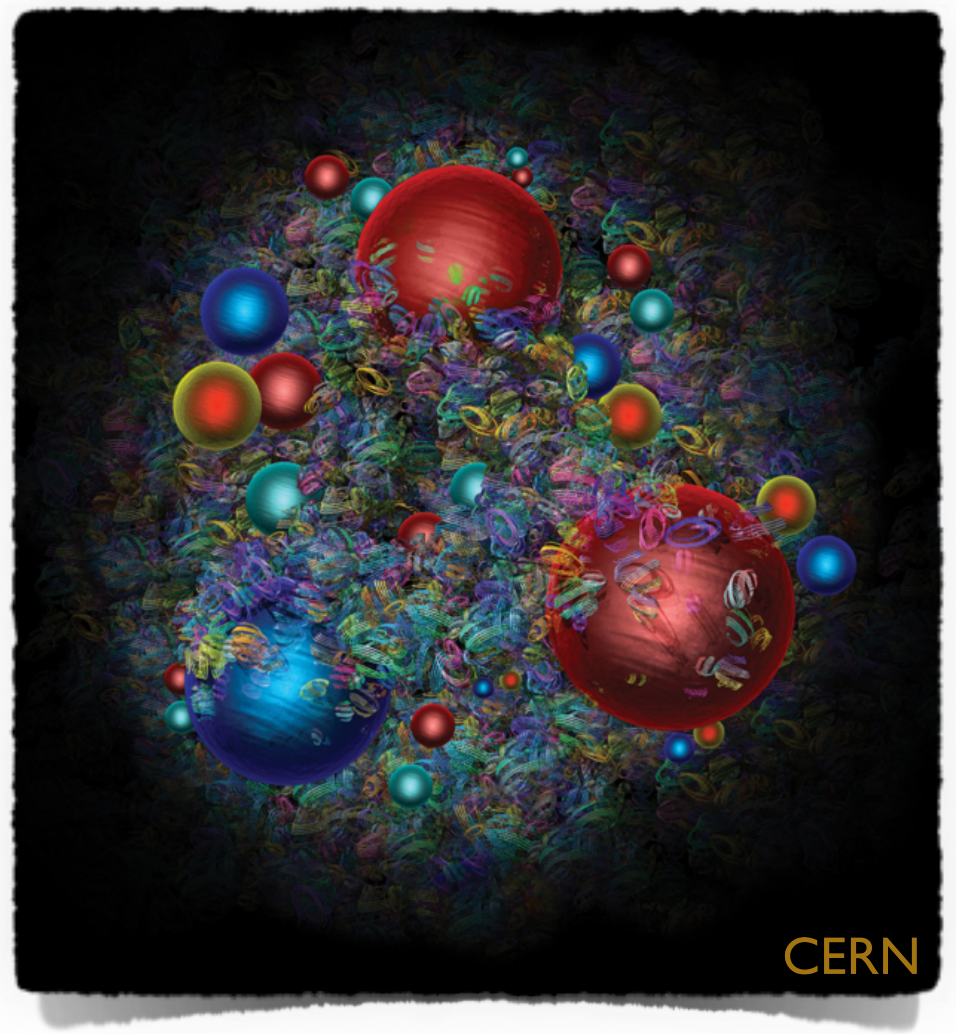
Particle accelerators from the 50s/60s

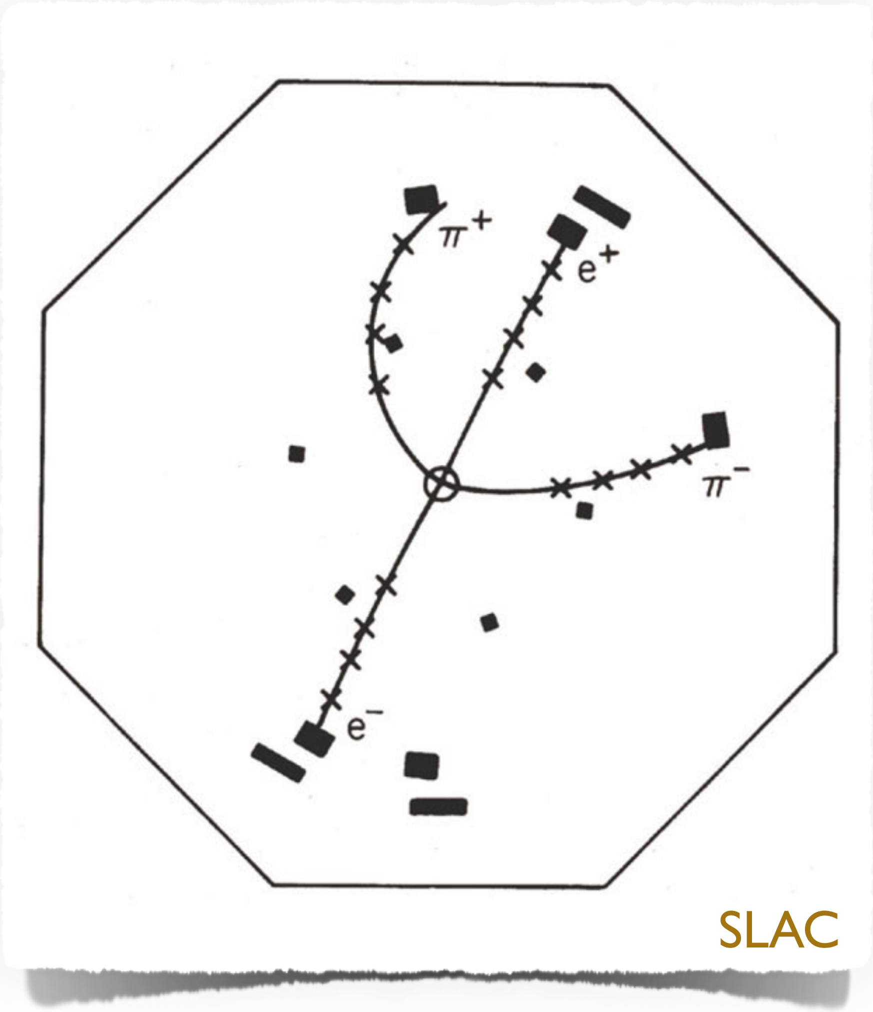
- Cosmic rays
  - ⋆ High-altitude detectors (photo emulsions)
  - ⋆ Charged particle tracks
- Human-made accelerators
  - ⋆ Electromagnetic fields
- Production of new subatomic particles
  - ⋆ Muons, pions, kaons, *etc*.

The quark model (1964)

- Hadrons non elementary
  (Gell-Mann, Zweig, Glashow et Bjorken)
- Made of fundamental quarks/antiquarks
  (among 4 flavours: up, down, strange and charm)
- The lepton sector: electron, muon and neutrinos
- Organisation of matter from few entities

Towards the Standard Model



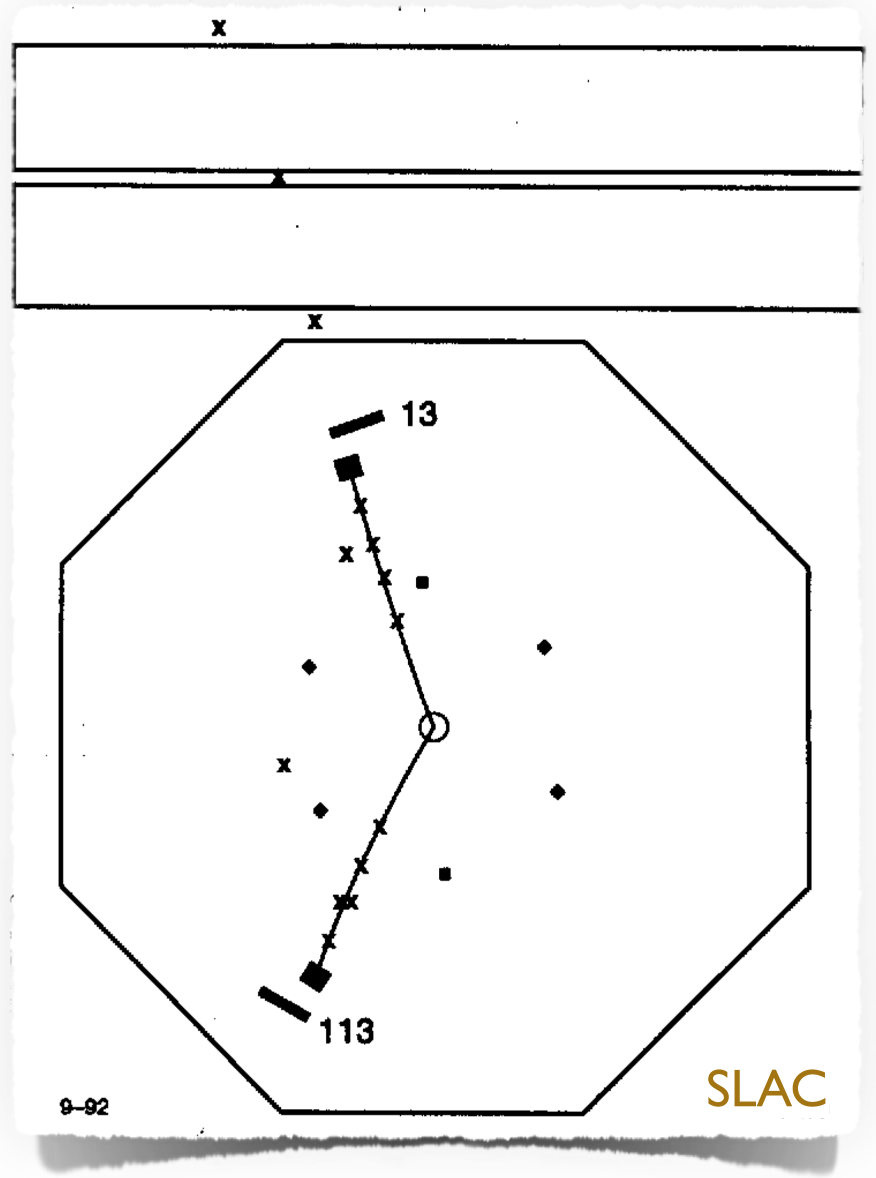


The proton substructure (1968)

- Discovery of the up and down quarks
- Indirect validation of the strange quark hypothesis

The November Revolution (1974)

- Discovery of the charm



The third generation (1970 − 2000)

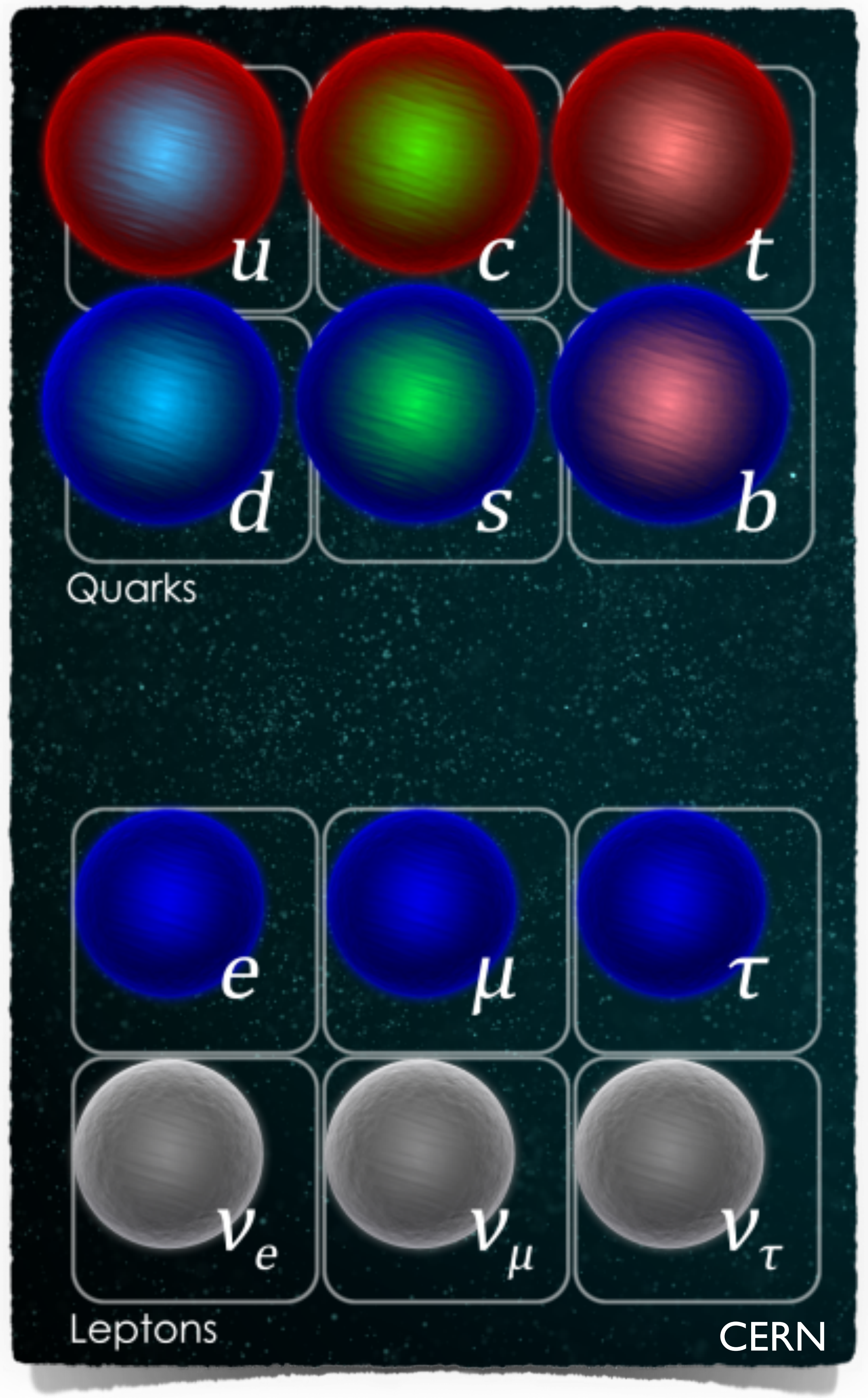
- The τ proposed by Tsai (1971)
  - ➙ Discovered at SLAC/Berkeley (1975 − 1978)

$$e^+e^- \to \tau^+\tau^- \to e^-\mu^+ + E_T^{\rm miss}$$

- Weak interactions: need for a tau neutrino (Fermilab, 2000)
- CP-violating kaon decays: need for top and bottom quarks (GIM − 1970, KM − 1973)
  - ➙ Proposed by Harari (1975)
  - ➙ Discovered at Fermilab (1977, 1995)

Three generations of fermions

Fundamental constituents of matter: 3 families of particles/antiparticles



Particle content

- 3 up-type quarks and 3 down-type quarks
- 3 charged leptons and 3 associated neutrinos

1st generation

- Sufficient to describe all matter

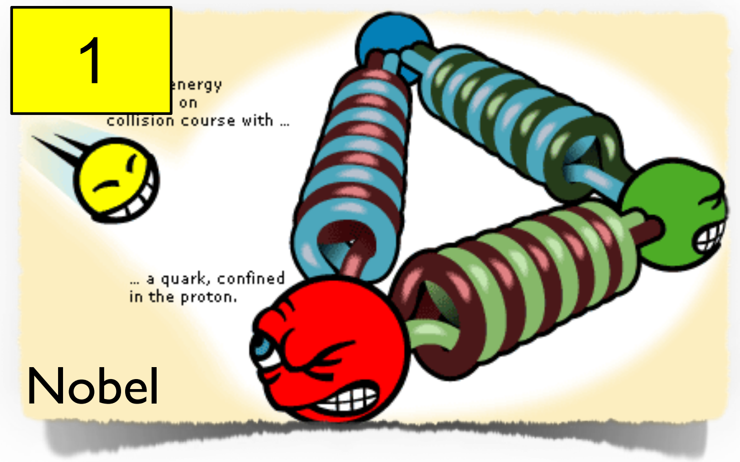
2nd and 3rd generations

- More massive with identical quantum properties
- Unstable
- Observed in high-energy collisions (accelerators, cosmic rays)

3+1 fundamental interactions

Strong interaction ⚛️

- Interactions between coloured particles (quarks)
- Mediated by massless gluons g (spin 1)
  ➙ Self interactions
- Hadrons and mesons made of quarks and gluons
  ➙ Stability of atomic nuclei

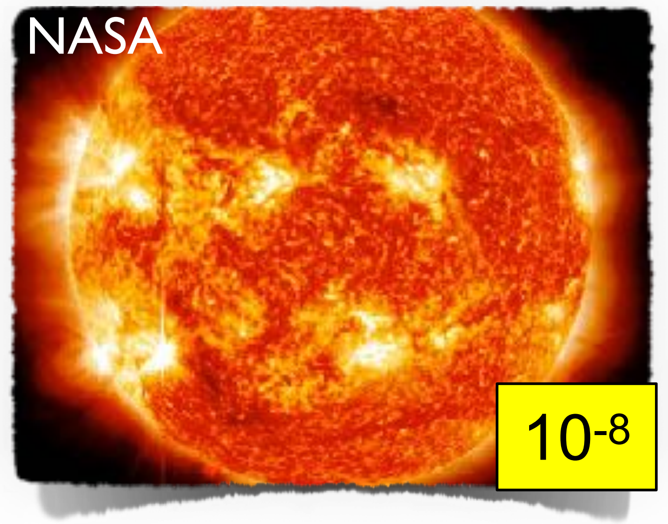


Electromagnetism ⚡

- Interactions between charged particles (quarks and charged leptons)
- Mediated by massless photons (spin 1)
- Large-scale phenomena: atomic stability, electricity, chemistry, biology, *etc*.

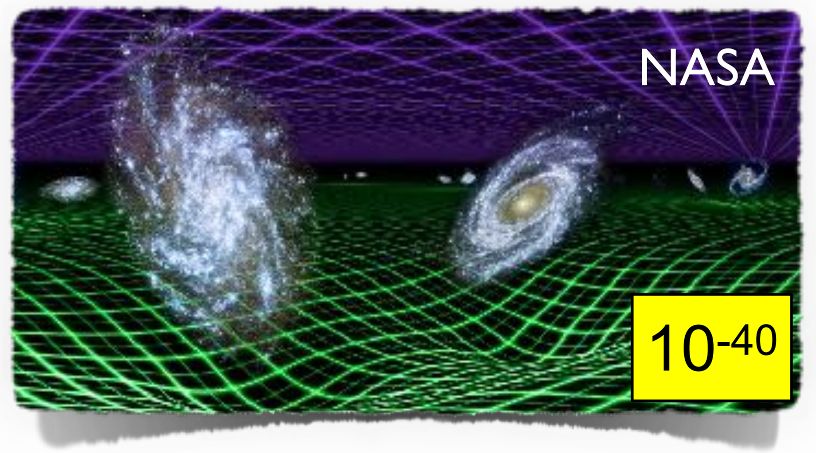
Weak interaction ☢️

- Interactions between 'left-handed' fermions
- Mediated by massive weak *W* and *Z* bosons (spin 1)
  ➙ Self interactions, also with photons
- Large set of phenomena: radioactivity, star evolution, *etc*.



Gravitation 🍏

- Feeble interactions (of strength 10^{-40}) between all particules
- Mediated by the (non-observed) massless graviton (spin 2)
- Not described by the Standard Model ➙ superstrings, M-theory, quantum loop gravity, *etc*.



Theoretical framework

The Standard Model - generalities

Symmetry principles ⬌ elementary particles and their interactions

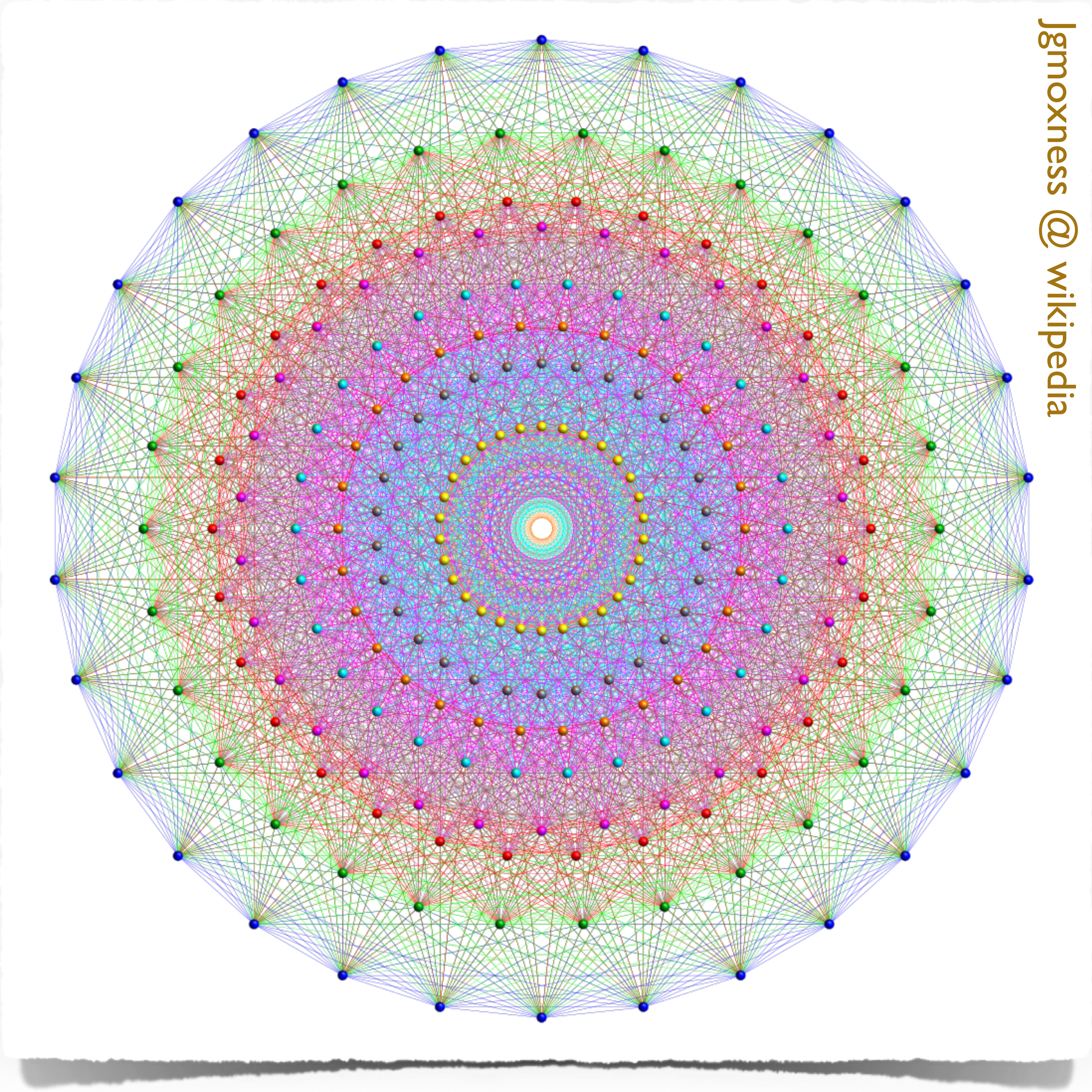
- Compatible with special relativity
  - ⋆ Invariance of the speed of light
  - ⋆ Physics independent of the inertial reference frame
    - ➙ Minkowski spacetime (metric, scalar products, *etc.*)
- Compatible with quantum mechanics
  - ⋆ Probabilistic interpretation
- Quantum field theory
  - ⋆ Quantisation: harmonic/fermionic oscillators
- Interactions: gauge theories

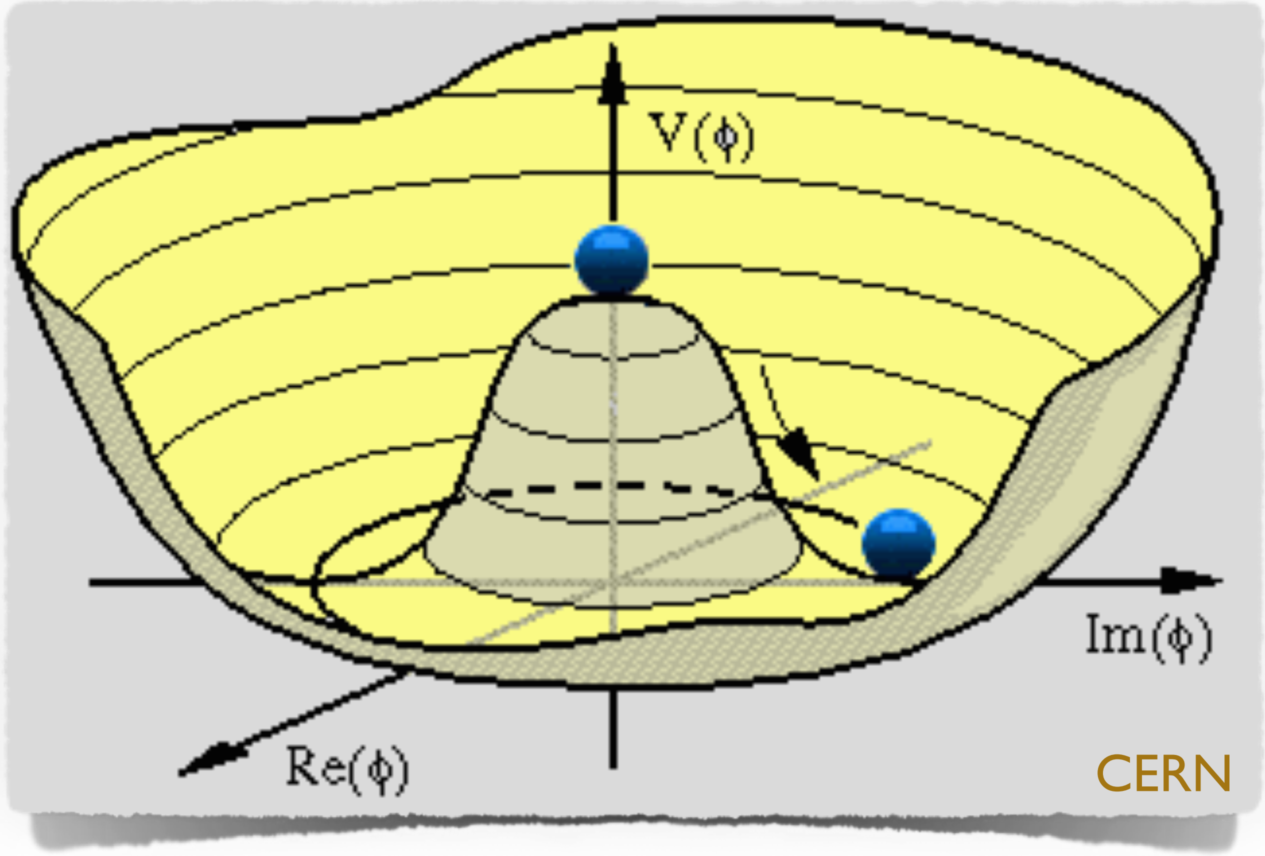
Pedro @ Flickr

Symmetries

Why symmetries ?

- Symmetry ≡ operation leaving the laws of physics unchanged
  - ⋆ External spacetime symmetries: rotations, boosts, *etc.*
  - ⋆ Internal symmetries: phase in quantum mechanics ($|\Psi\rangle \to e^{i\alpha}|\Psi\rangle$)
- Nœther theorem
  - ⋆ Conserved charged / symmetry
  - ⋆ Examples:
    - ➙ Spacetime symmetries: energy, spin, *etc.*
    - ➙ Internal symmetries: electric charge, fundamental interactions, *etc.*


Jgmoxness @ wikipedia



CERN

Extra tool: symmetry breaking

- Anomalies ⬌ quantum numbers
- Masses if the gauge bosons

More on symmetries

Euler-Lagrange equations

- Physics encoded in a Lagrangian density $\mathscr{L}$
  - ⋆ Dependance on quantum fields ϕ (spin 0), ψ (spin 1/2), A^μ (spin 1)
    - ➙ Particle creations and annihilations
- Dynamics dictated by the principle of least action

$$\frac{\partial \mathscr{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathscr{L}}{\partial(\partial_\mu \phi)} = 0$$

  - ⋆ Conserved current and charge

Symmetric transformations

- Wigner: (anti-)unitary operators $\{G_i\}$

$$\phi(x) \to G_i\,\phi(x)$$

- Hermitian symmetry generators $\{g_i\}$

$$G_i = \exp\left[i\omega^i g_i\right]$$

  - ⋆ Symmetry algebra

$$[g_i, g_j] = i f_{ij}{}^k\, g_k$$

    - ➙ Examples: $\mathfrak{iso}(1,3)$, $\mathfrak{su}(2)$, $\mathfrak{su}(3)$, …

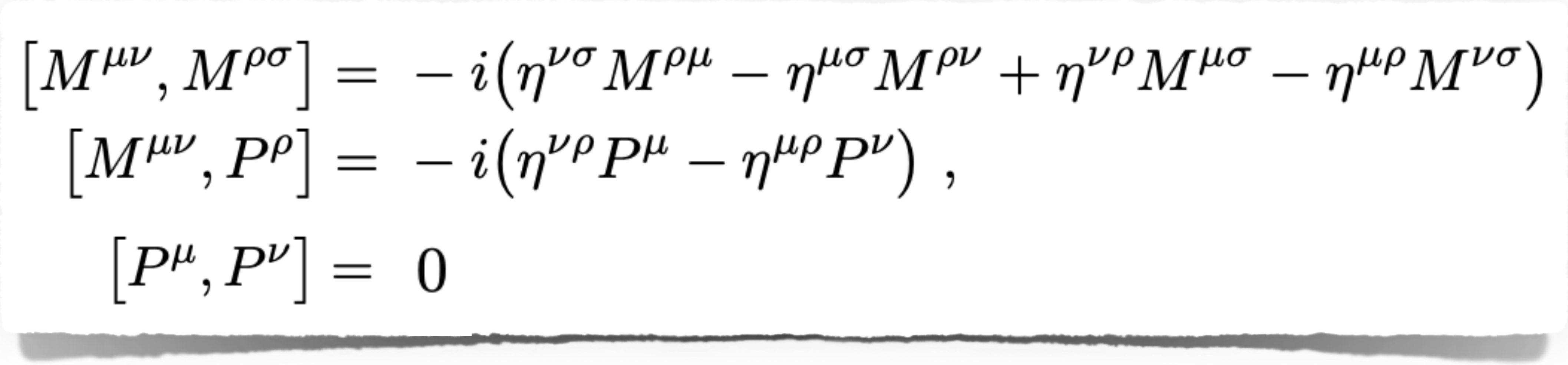
      ≡ mass, spin, fundamental interactions

- Lagrangian invariance: $\mathscr{L} \to \mathscr{L} + \partial_\mu(\ldots)$
  - ➙ Conserved quantity from each symmetry

Mass and spin

The Poincaré algebra $\mathfrak{iso}(1,3)$

- Space-time translations, rotations and boosts
- Invariance of Maxwell equations + all relativistic wave equations



$$\begin{aligned}\left[M^{\mu\nu}, M^{\rho\sigma}\right] =&\ -i\left(\eta^{\nu\sigma} M^{\rho\mu} - \eta^{\mu\sigma} M^{\rho\nu} + \eta^{\nu\rho} M^{\mu\sigma} - \eta^{\mu\rho} M^{\nu\sigma}\right)\\ \left[M^{\mu\nu}, P^{\rho}\right] =&\ -i\left(\eta^{\nu\rho} P^{\mu} - \eta^{\mu\rho} P^{\nu}\right) ,\\ \left[P^{\mu}, P^{\nu}\right] =&\ 0\end{aligned}$$

Poincaré invariance

- Quadratic Casimir operator P^2: particle masses
- Lorentz algebra isomorphic to $\mathfrak{so}(3) \oplus \mathfrak{so}(3)$: spin
  - ⋆ (0,0) ≡ scalars
  - ⋆ (1/2,0)⊕(0,1/2) ≡ fermions/spinors
  - ⋆ (1/2,1/2) ≡ vectors
- Representation characterised by mass/spin quantum numbers
- Relativistic wave equations and Lagrangians

  ➙ Klein-Gordon, Dirac, Yang-Mills

Relativistic wave equations

Scalar fields: $\phi(x) \to \phi'(x') = \phi(x)$

- (0,0) representation of the Lorentz algebra
- Klein-Gordon equation from the mass-energy relation

$$P^2 = m^2 \quad \Leftrightarrow \quad \left(\Box + m^2\right)\phi = 0$$

- Problem in the 1920s: solution with *E<0*
- Lagrangian for a free field

$$\mathscr{L}_{\rm KG} = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi$$

- In the Standard Model: the Higgs field

Spinors: $\psi(x) \to \psi'(x') = \Lambda_{1/2}\,\psi(x)$

- (1/2,0)⊕(0,1/2) representation of the Lorentz algebra
- Attempt to solve the *E<0* solutions of the KG equation

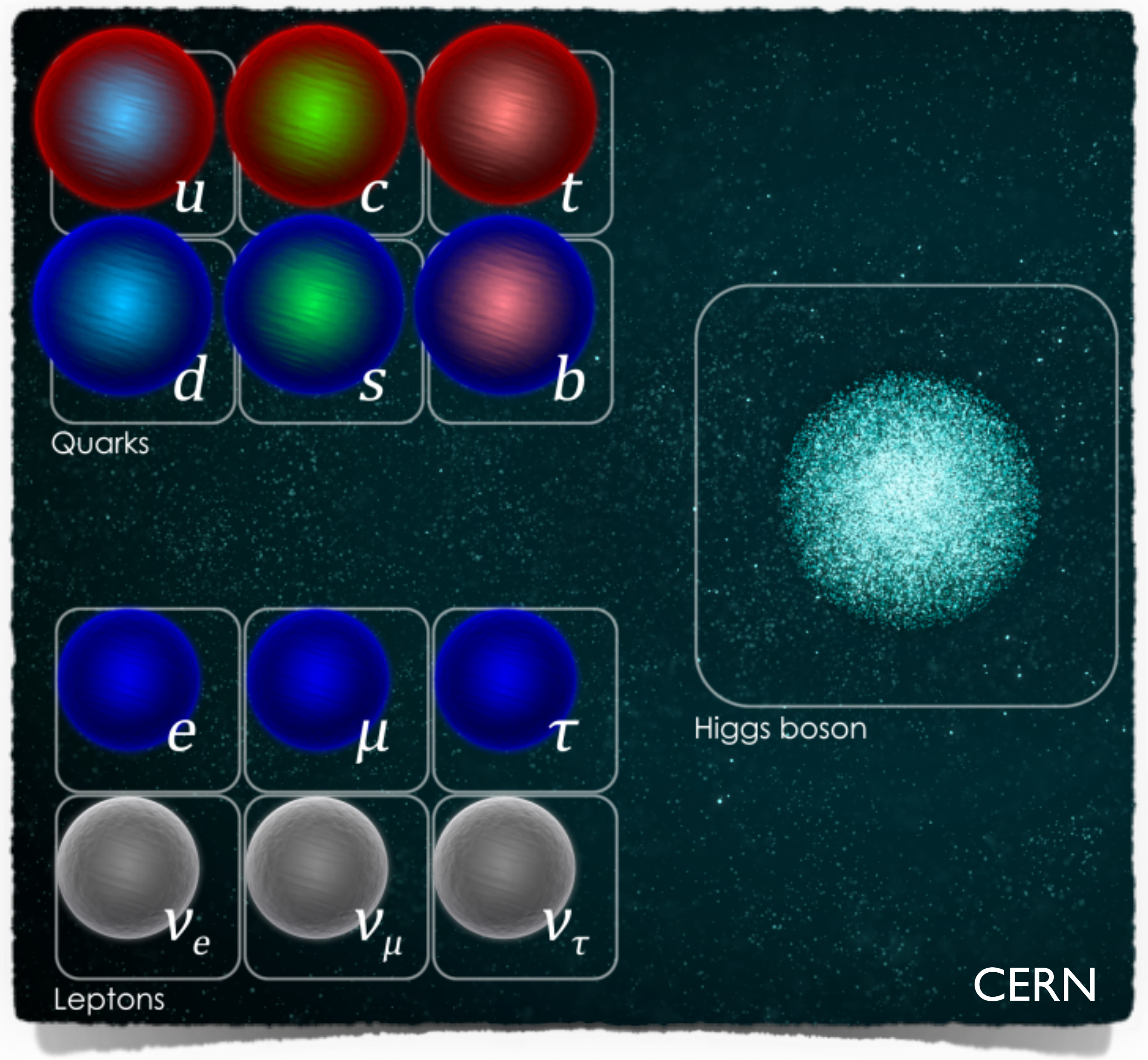
  → Dirac equation: $\left(i\partial\!\!\!/ - m\right)\psi = 0$

- Dirac Lagrangian for a free field

$$\mathscr{L}_{\rm Dirac} = i\bar\psi\partial\!\!\!/\psi - m\bar\psi\psi$$

  → Mass terms: mix left- and right-handed spinors

- In the Standard Model: the matter fields



Relativistic wave equations

Vector fields: $A^\mu(x) \to A^{\mu'}(x') = \Lambda^\mu{}_\nu \, A^\nu(x)$

- (0,0) representation of the Lorentz algebra
- Maxwell / Yang-Mills equations ➙ free Lagrangian

$$\mathscr{L}_{\rm YM} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a - A^a_\mu \, j^\mu_a$$

➙ Coupling to a current

➙ Self-interactions if non-Abelian

➙ Massless states

- In the Standard Model: gauge bosons

Gauge symmetries − fundamental interactions

- Example *SU(N)*, *U(1)*
  - ⋆ Association of a coupling constant *g*
  - ⋆ Representations given to matter fields ≡ charges
  - ⋆ Yang-Mills /Maxwell Lagrangian for the vector field dynamics

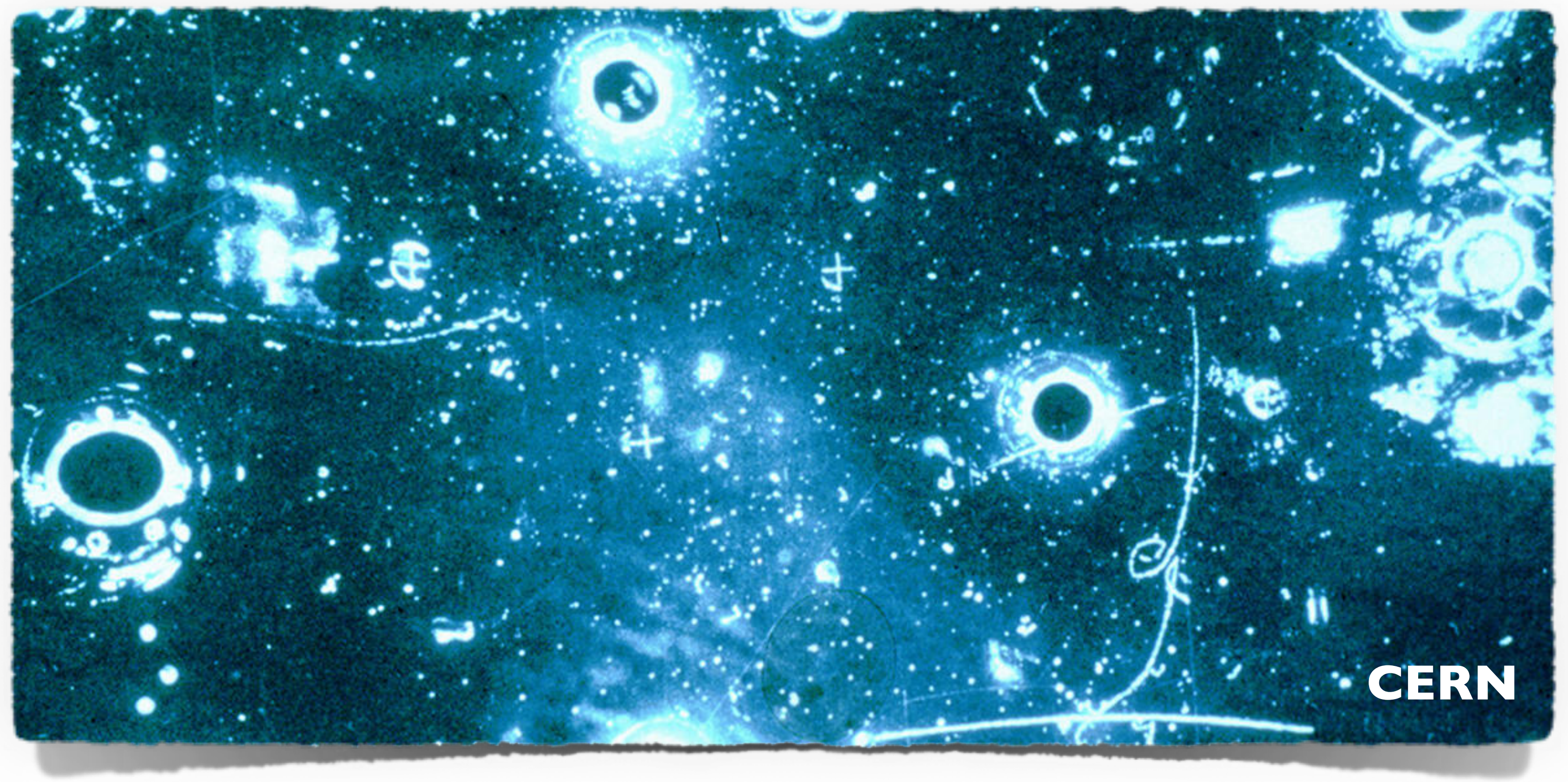
$$\mathscr{L}_{\rm QED} = \sum_{j=e,\nu_e,u,d} \bar\Psi_j \, i\gamma^\mu D_\mu \Psi^j - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

with $D_\mu = \partial_\mu - ieqA_\mu \ , \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

$$\mathscr{L}_{\rm QCD} = \sum_{j=u,d} \bar\Psi_j \, i\gamma^\mu D_\mu \Psi^j - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$$

with $D_\mu = \partial_\mu - ig_s T_a G^a_\mu \ , \qquad F_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_s f_{bc}{}^a G^b_\mu G^c_\mu$

From a free theory to an interacting one



Gauge invariance of the Lagrangian

- Free fields at the beginning
- SU(N) gauge interactions ➙ field in some representation

  $$\psi(x) \to \psi'(x) = U\psi(x) = e^{i\,g\,T_a\,\omega^a}\,\psi(x)$$
- Step1: constant { ω }
- Step2: local { $\omega(x)$ }

Constant transformation parameters

- Transformation laws

$$\partial_\mu\psi(x) \to U\partial_\mu\psi(x)\,, \qquad \bar\psi(x)\partial_\mu\psi(x) \to \bar\psi(x)U^\dagger U\partial_\mu\psi(x) = \bar\psi(x)\partial_\mu\psi(x)$$

$$\Rightarrow \mathscr{L} \to \mathscr{L}$$

Local transformation parameters

- Transformation laws

$$\partial_\mu\psi(x) \not\to U(x)\partial_\mu\psi(x)$$

➙ Lagrangian not invariant

A vector field to save the day

- Modifications of the derivative $\partial_\mu \to D_\mu = \partial_\mu - igA^a_\mu T_a$
  - ⋆ Introduction of a vector field with *ad hoc* transformation rules
  - ⋆ Lagrangian invariance preserved

$$A^\mu(x) \to U(x)\Big[A^\mu(x) + \frac{i}{g}\partial^\mu\Big]U^\dagger(x) \qquad \Rightarrow \qquad D_\mu\psi(x) \to U(x)D_\mu\psi(x)$$

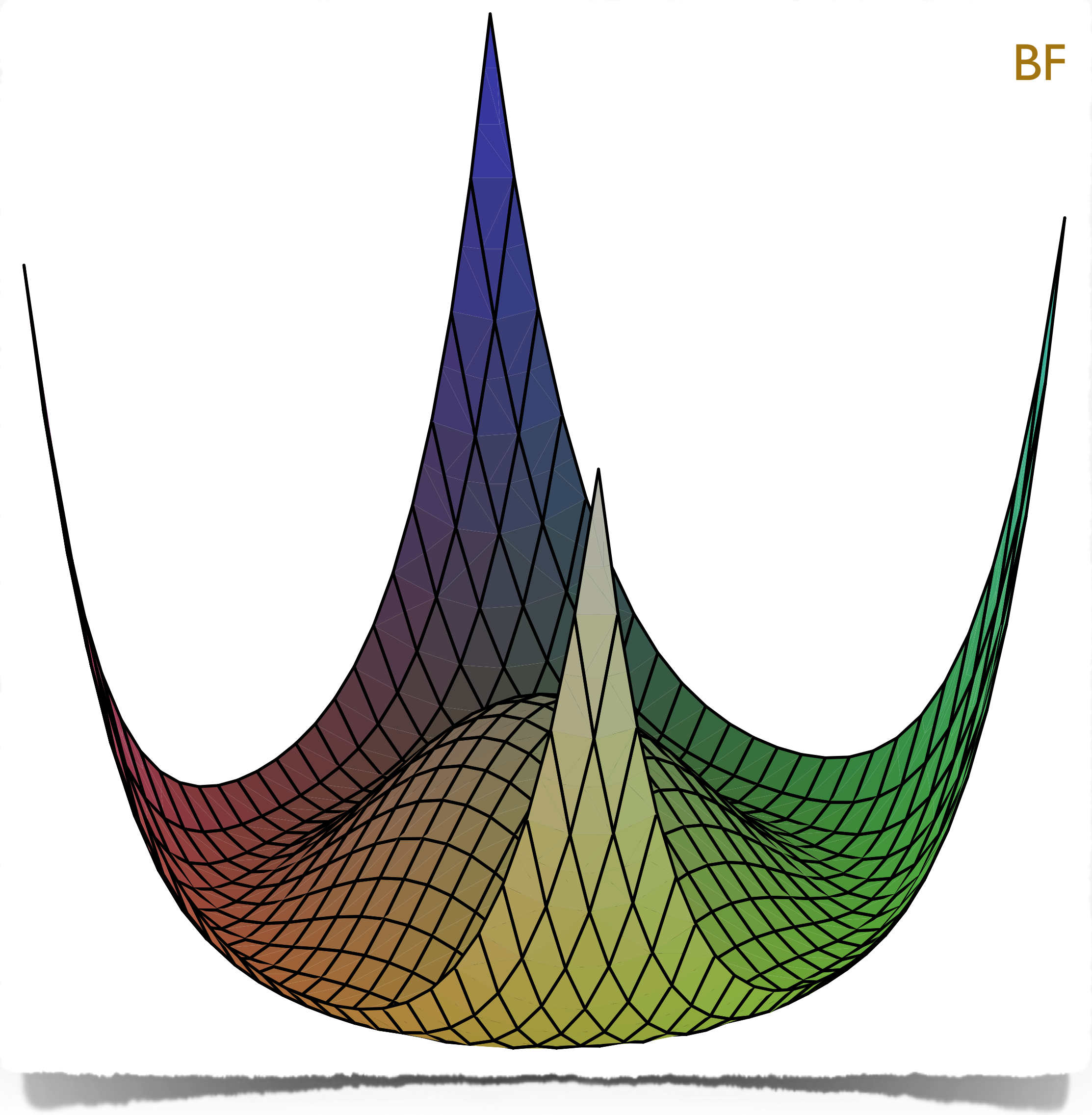
$$\bar\psi(x)D_\mu\psi(x) \to \bar\psi(x)U^\dagger(x)U(x)D_\mu\psi(x) = \bar\psi(x)D_\mu\psi(x) \qquad \Rightarrow \qquad \mathscr{L} \to \mathscr{L}$$

Symmetry breaking

Toy theoretical setup with a *U(1)*$_X$ gauge symmetry

- Gauge boson X^μ + gauge coupling g_X
- Scalar field φ charged under the *U(1)*$_X$-symmetry
- Free Lagrangian:

$$\mathscr{L}_{\rm toy} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} - D_\mu\varphi^\dagger D^\mu\varphi$$



Interactions: everything not forbidden by the symmetry

- Scalar potential: $V = -\mu^2\varphi^\dagger\varphi + \lambda\big(\varphi^\dagger\varphi\big)^2$
- Minimisation of the potential and shift of φ around the vev

$$\frac{\partial V}{\partial \varphi} = 0 \quad \Leftrightarrow \quad \varphi = \frac{1}{\sqrt{2}}\big[v + s^0 + ia^0\big]$$

➙ Generation of a scalar mass term for s^0 ≡ Higgs boson

➙ The pseudo-scalar a^0 massless ≡ Goldstone boson

➙ Generation of a multiscalar interactions between s^0 and a^0

Mass generation (the Brout-Englert-Higgs mechanism)

- Generation of a gauge boson mass term

$$D_\mu\varphi^\dagger D^\mu\varphi \quad \Rightarrow \quad \frac{1}{2} g_x^2 v^2 X^\mu X_\mu$$

➙ The Goldstone boson eaten by the gauge boson (3 degrees of freedom)

- Bonus: fermion mass terms from Yukawa interactions

$$-y\bar\psi\psi\varphi \to -\frac{yv}{\sqrt{2}}\bar\psi\psi$$

Summary so far

The Nœther procedure in a nutshell

- Choose a gauge group
- Set up matter fields (and representations)
- Write the free Lagrangian
- Promote derivative to covariant derivatives
- Add kinetic terms for the gauge boson

Bonus items

- Interactions are dictated by geometry
- Gauge group and matter content not predicted
- The gauge symmetry can be eventually broken
- The theory must be anomaly-free

The Standard Model

Towards the Standard Model

Electromagnetism

- Gauge group: $U(1)_{em}$
- Gauge boson: the photon
- Quantum numbers from data

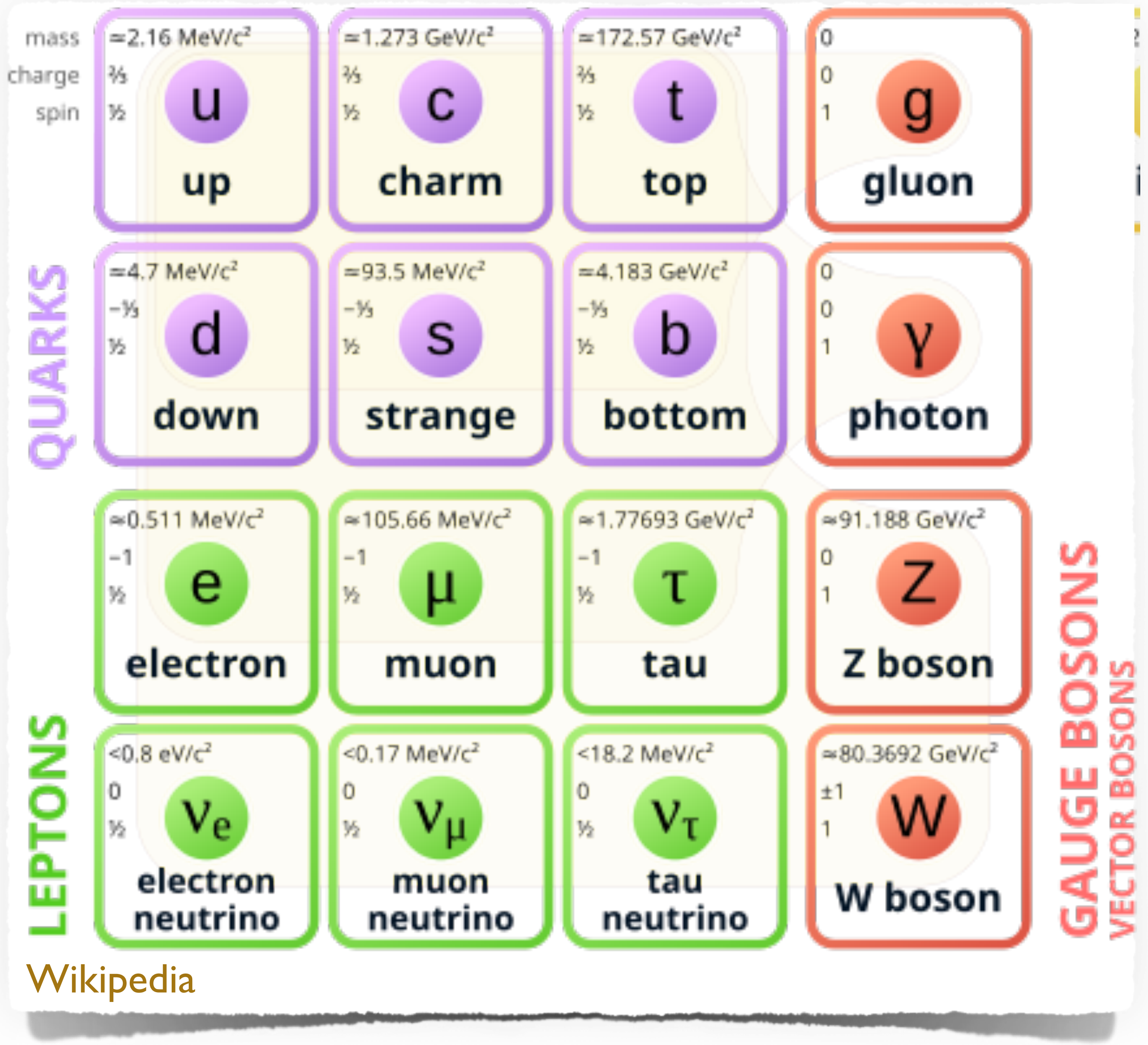
Weak interactions

- Fermi theory in the 1930s
  - → $SU(2)_L$ doublets
  - → Left-handed fermions
- Gauge bosons: W/Z bosons
- Lagrangian:

$$\mathscr{L} = \bar{L}\left[i\not{D}\right]L + \bar{e}_R[i\not{\partial}]e_R - \frac{1}{4}W^i_{\mu\nu}W_i^{\mu\nu}$$

Strong interaction

- Gauge group: $SU(3)_C$
- Gauge boson: gluons
- Quarks carry colour
  - → Corroborated by baryon spectrum



Wikipedia

Problem:

1. Massless matter and gauge bosons
2. W/Z boson different from the W_i bosons

→ Symmetry breaking

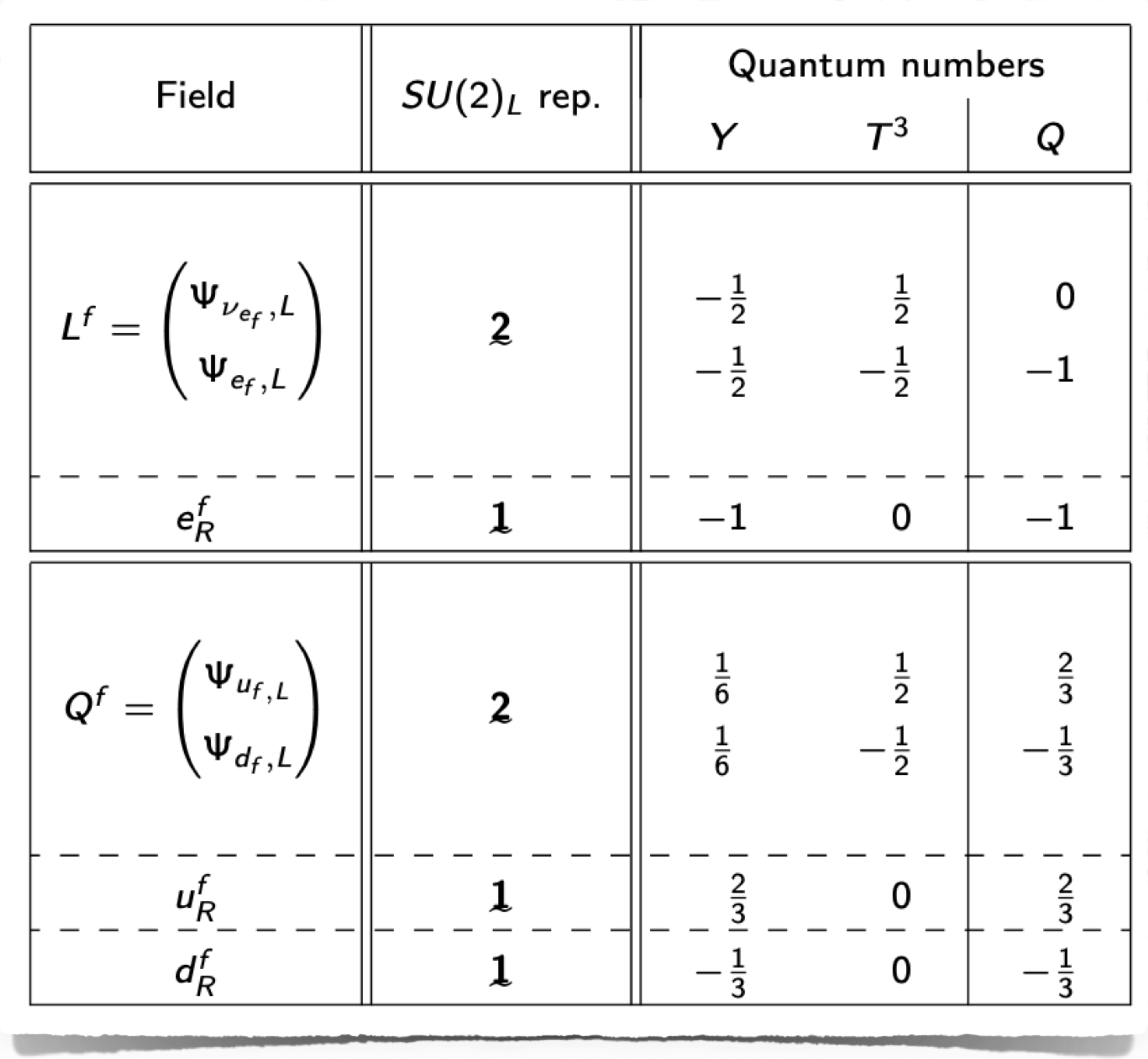
The Standard Model

Introduction of an *ad hoc* hypercharge and weak interaction group

- Gauge group: $SU(2)_L \times U(1)_Y$
- Addition of a scalar field ($SU(2)_L$ Higgs doublet)
- Symmetry breaking

$$SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$$

- Hypercharge quantum numbers to recover the electric charges ($Y+T^3 = Q$)



Field	$SU(2)_L$ rep.	Quantum numbers		
		Y	T^3	Q
$L^f = \begin{pmatrix} \Psi_{\nu_{e_f},L} \\ \Psi_{e_f,L} \end{pmatrix}$	$\underset{\sim}{\mathbf{2}}$	$-\frac{1}{2}$ $-\frac{1}{2}$	$\frac{1}{2}$ $-\frac{1}{2}$	0 -1
e^f_R	$\underset{\sim}{\mathbf{1}}$	-1	0	-1
$Q^f = \begin{pmatrix} \Psi_{u_f,L} \\ \Psi_{d_f,L} \end{pmatrix}$	$\underset{\sim}{\mathbf{2}}$	$\frac{1}{6}$ $\frac{1}{6}$	$\frac{1}{2}$ $-\frac{1}{2}$	$\frac{2}{3}$ $-\frac{1}{3}$
u^f_R	$\underset{\sim}{\mathbf{1}}$	$\frac{2}{3}$	0	$\frac{2}{3}$
d^f_R	$\underset{\sim}{\mathbf{1}}$	$-\frac{1}{3}$	0	$-\frac{1}{3}$

Electroweak symmetry breaking

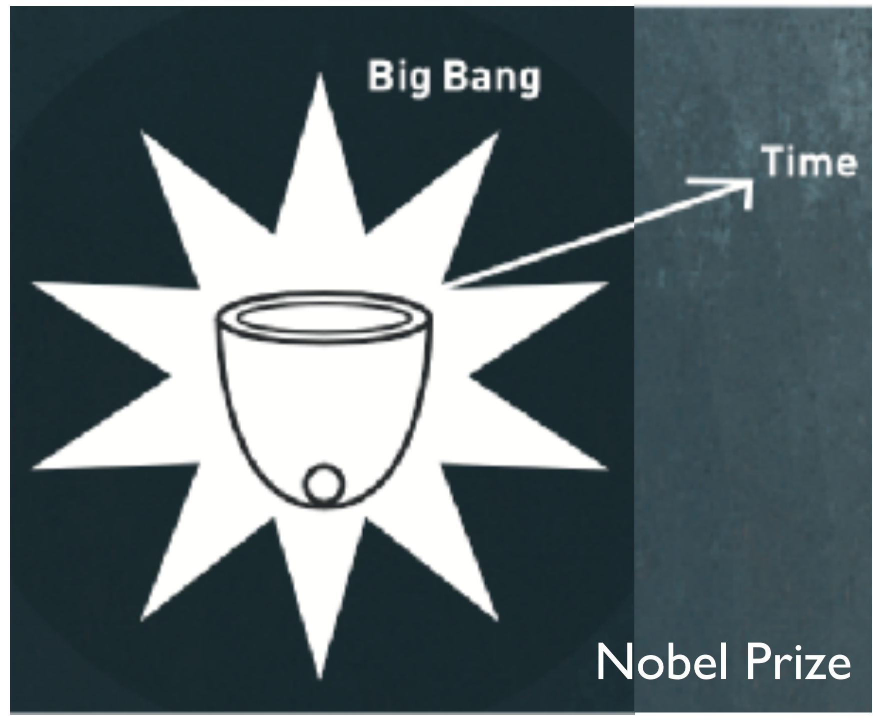
- Higgs doublet (4 degrees of freedom) + scalar potential
- Mixing in the electroweak sector
  - ⋆ Neutral gauge boson mixing: W^3/B ➞ photon/Z
  - ⋆ W/Z boson masses (three Goldstones)
- One extra degree of freedom: a Higgs boson

[Englert & Brout (PRL`64); Higgs (PRL`64)]

The Higgs boson revisited

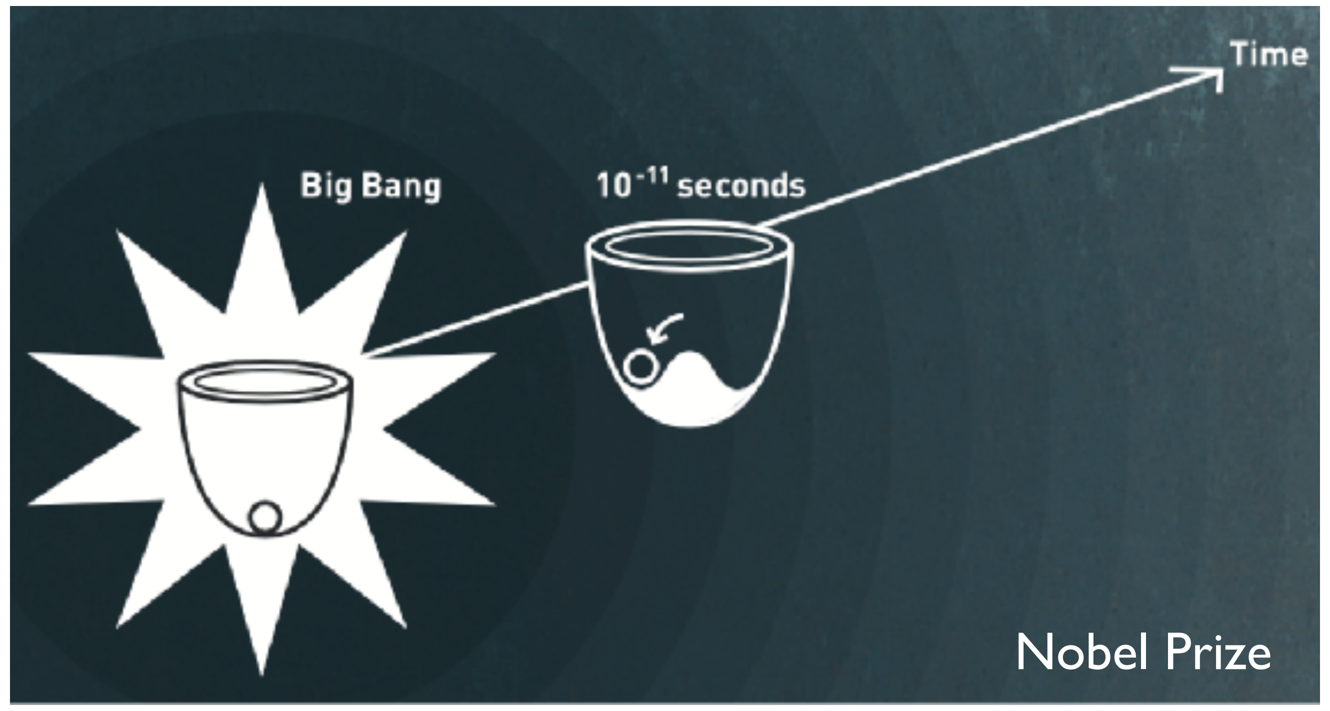
Gauge symmetries and interactions

- Incredible description of nature
- Massless particules ➝ contradiction wrt data



Symmetry behind our world hidden

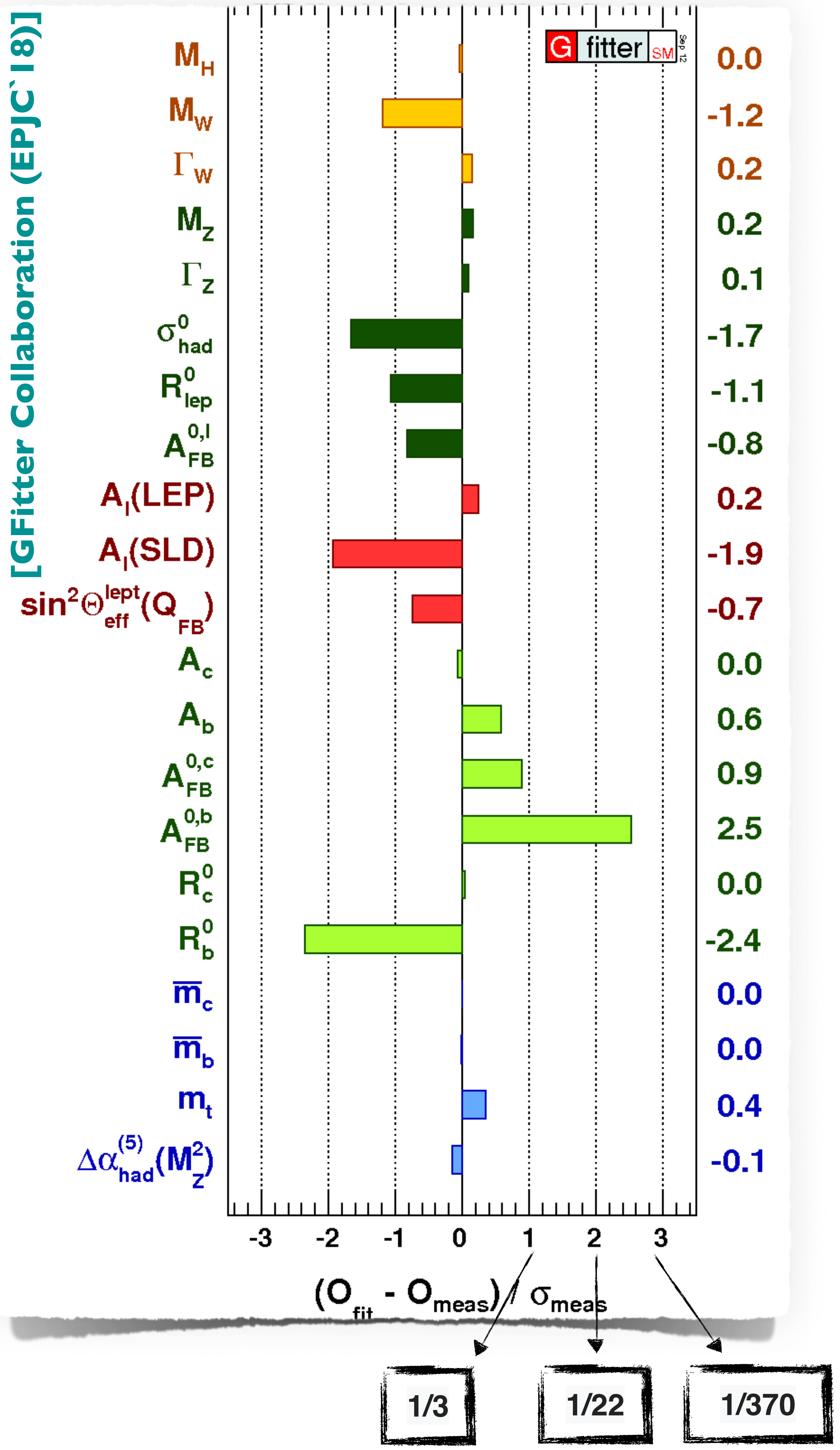
- Universe born gauge-symmetric
  ➝ Massless states and unified forces
- Analogy: bowl+marble
  ➝ The marble goes back to the fundamental state



At *10*$^{-11}$ second: a bump

- The marble far from the centre
- System still symmetric, but symmetry hidden

60 years of experimental tests



[GFitter Collaboration (EPJC`18)]

1/3

1/22

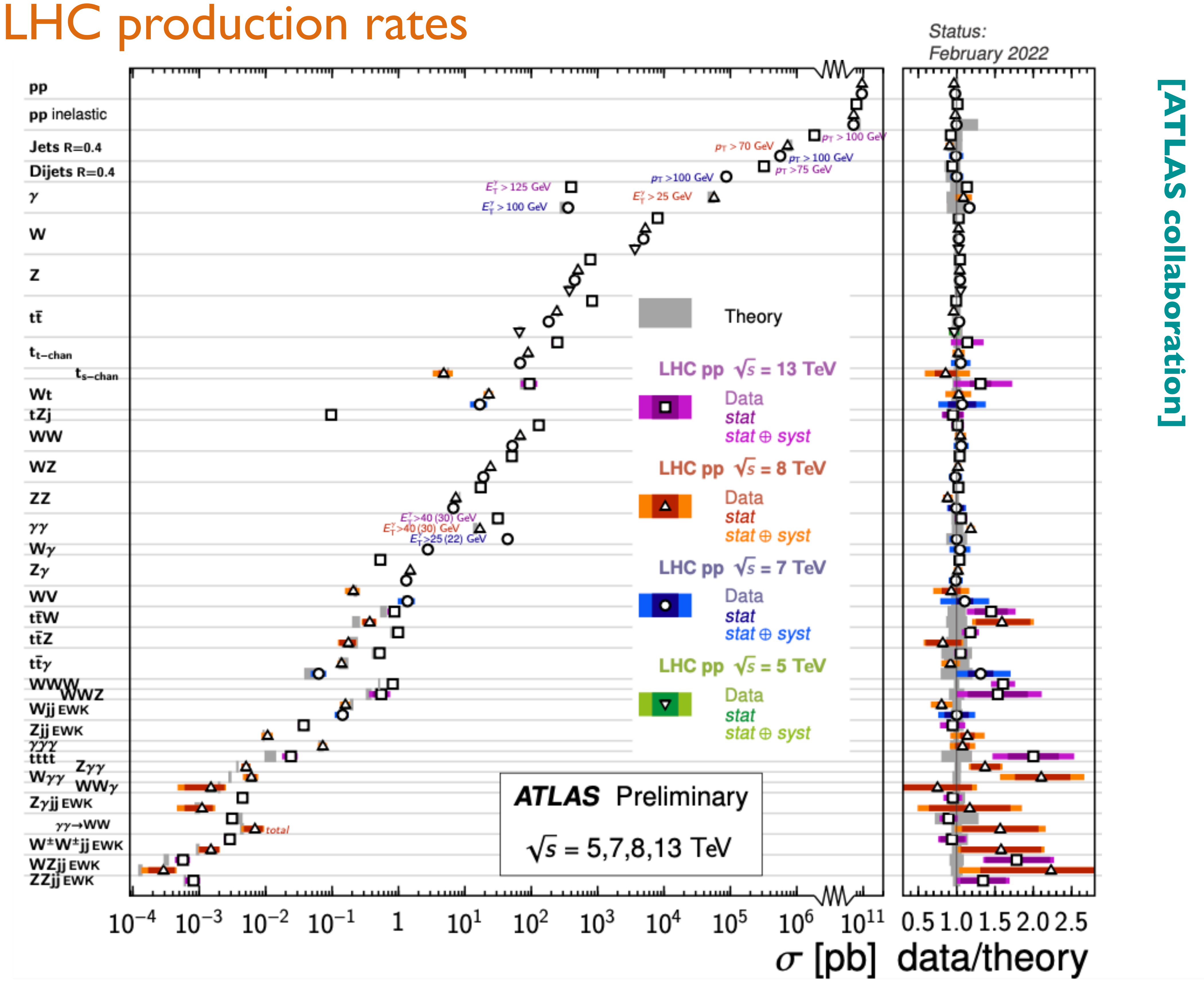
1/370

Standard Model fit

- Theory-experiment disagreement < 3σ (no new physics)

  → 5σ necessary for a discovery (1/1,750,000)

LHC production rates

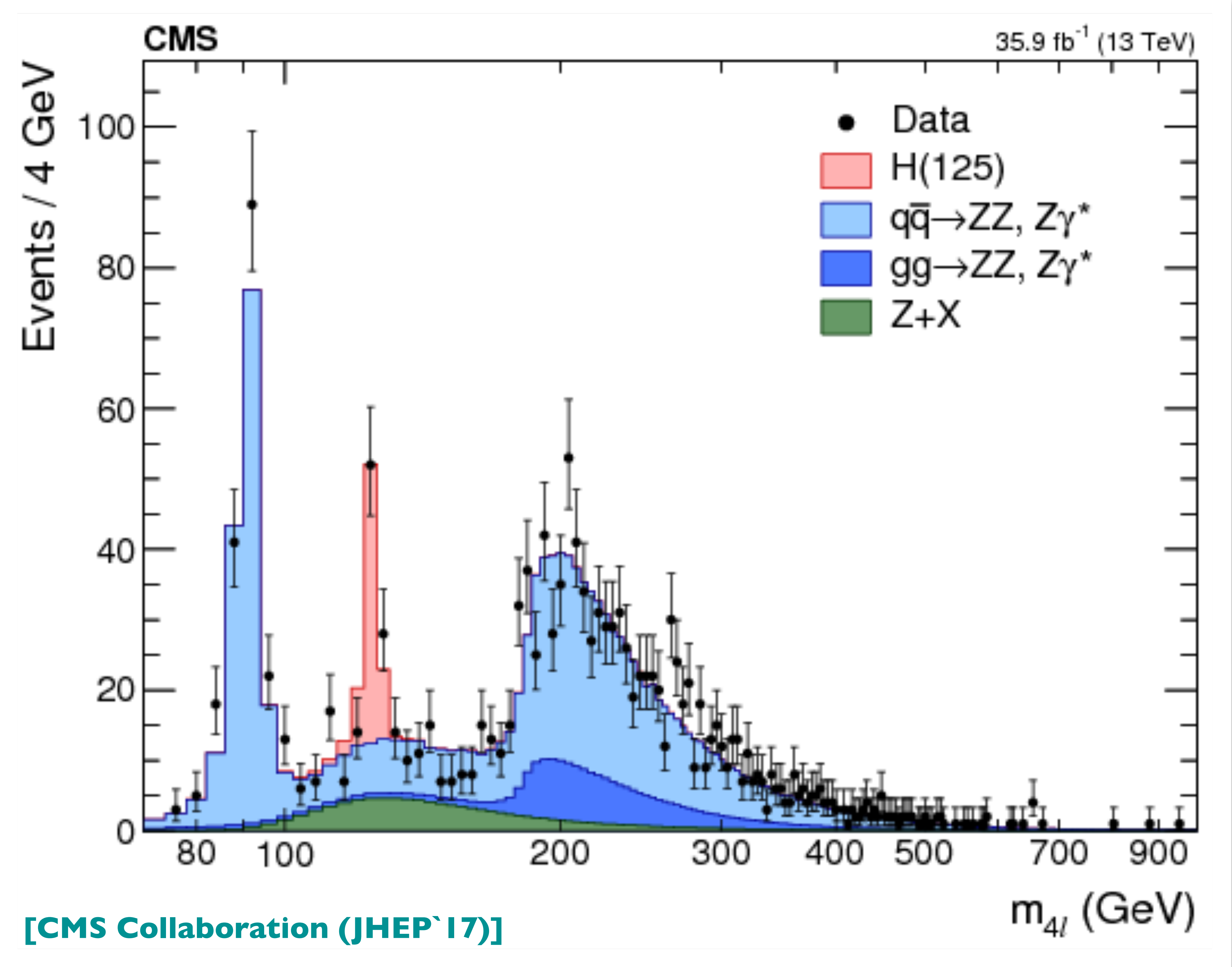


[ATLAS collaboration]

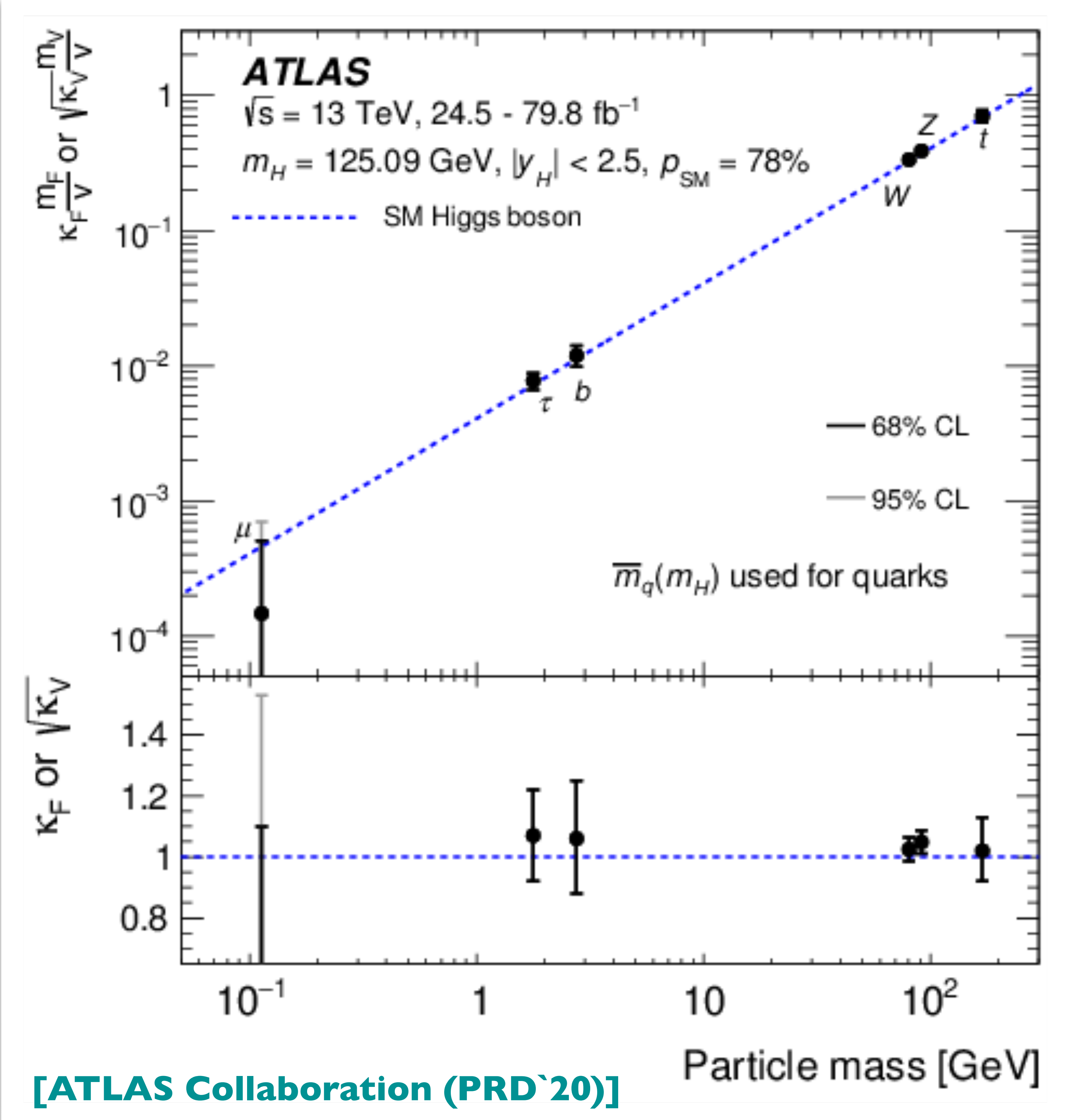
Experimental proof of the Higgs boson

Observation

- Validation of the Brout-Englert-Higgs mechanism
  - ➙ mass of the *W* et *Z* boson + existence of the Higgs boson
- Quark and lepton masses ≡ interactions with the Higgs boson
  - ➙ Couplings proportional to mass

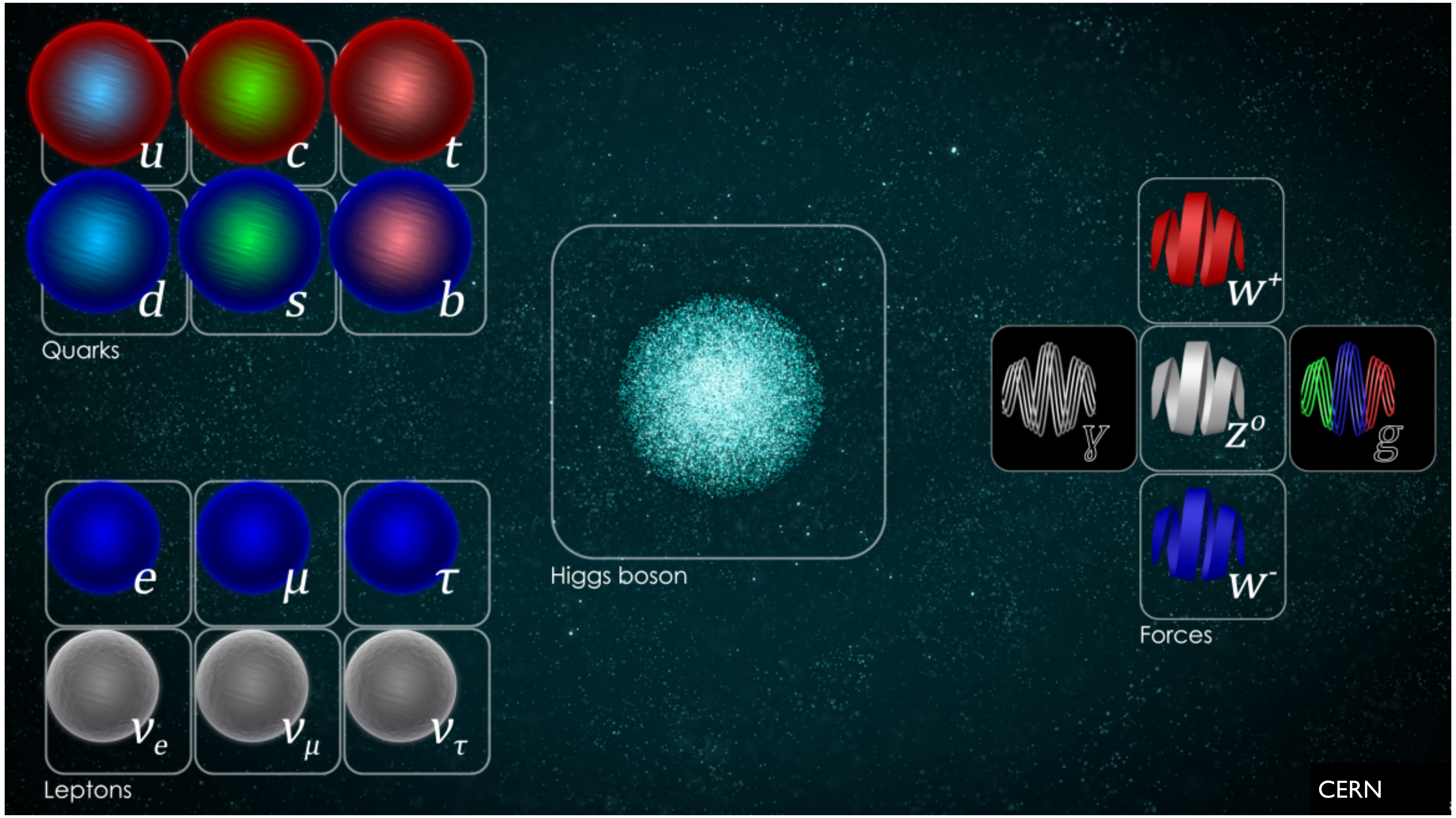


[CMS Collaboration (JHEP`17)]



[ATLAS Collaboration (PRD`20)]

The Standard Model: summary



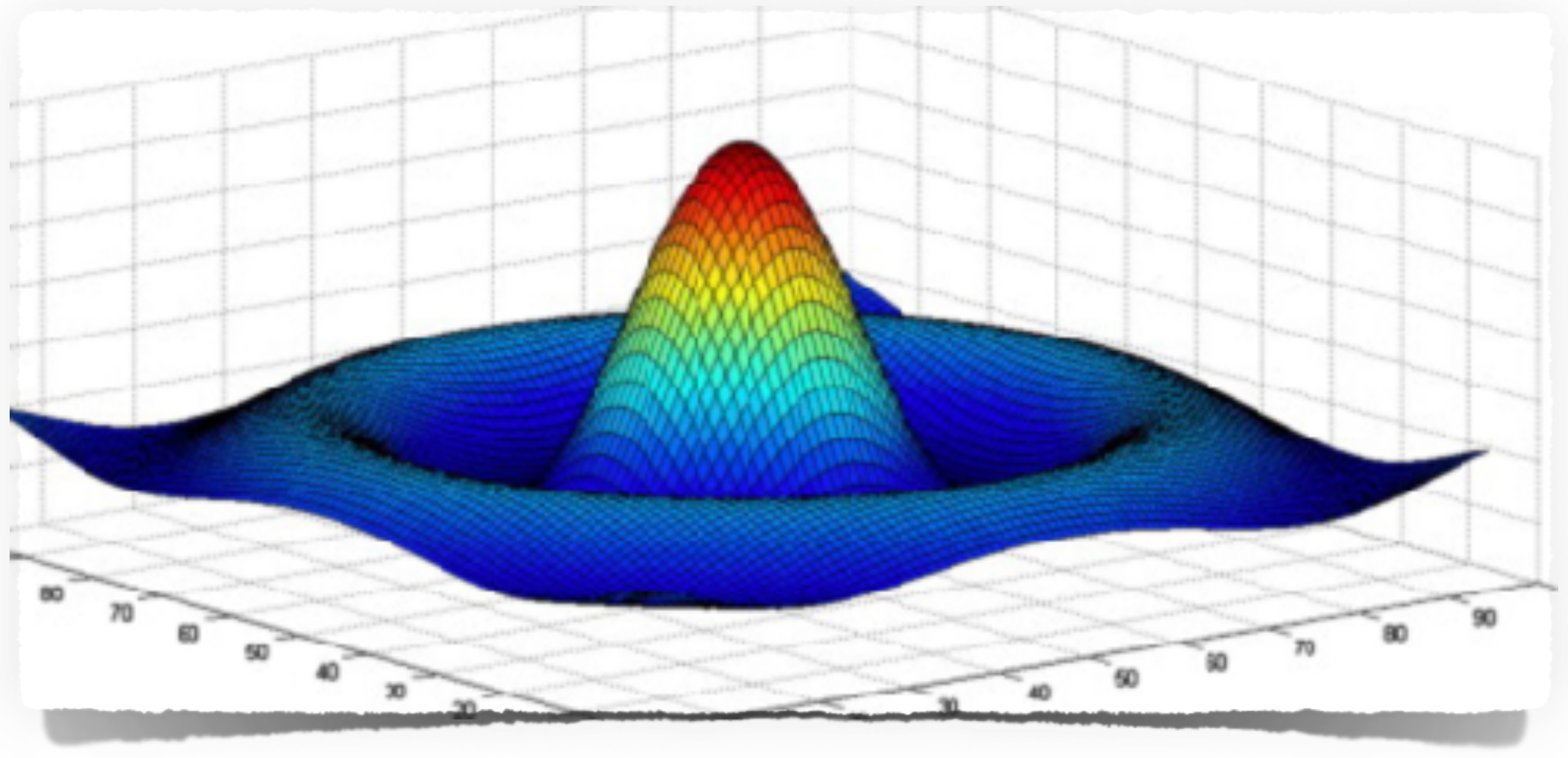
All particles found

- Higgs boson (2012)
- Tau neutrino (2000)
- Top quark (1995)
- *etc.*

Tested over 30 orders of magnitude

- 10^{-18} eV : bounds on the photon mass
- 10^{+13} eV : LHC energy

Success and failures of the Standard Model



SM extremely successful

- Agreement with a century of data
- No clear sign of new phenomena

Why going beyond ?

- Practical limitations
- Conceptual issues
- Few experimental anomalies

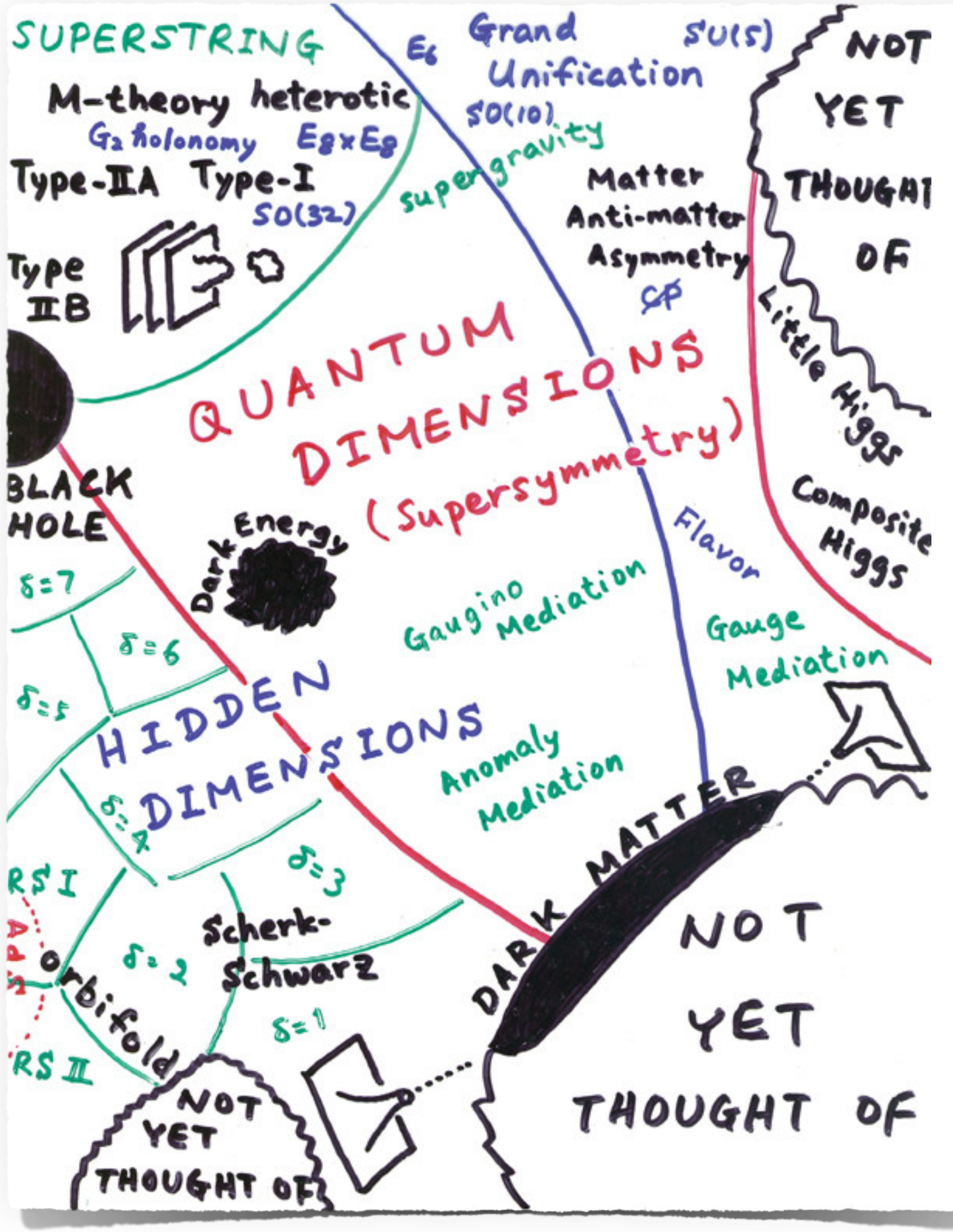
Some issues of the SM

- Naturalness of the theory / hierarchy problem
- Neutrino masses
- Dark matter
- Gauge structure of the theory
- Origins of CP violation
- Matter/antimatter asymmetry
- Lepton/quark mixings and masses
- Dark energy
- Four-dimensionality of space-time
- Gravity

Going beyond the Standard Model?

SM ≡ tip of the iceberg

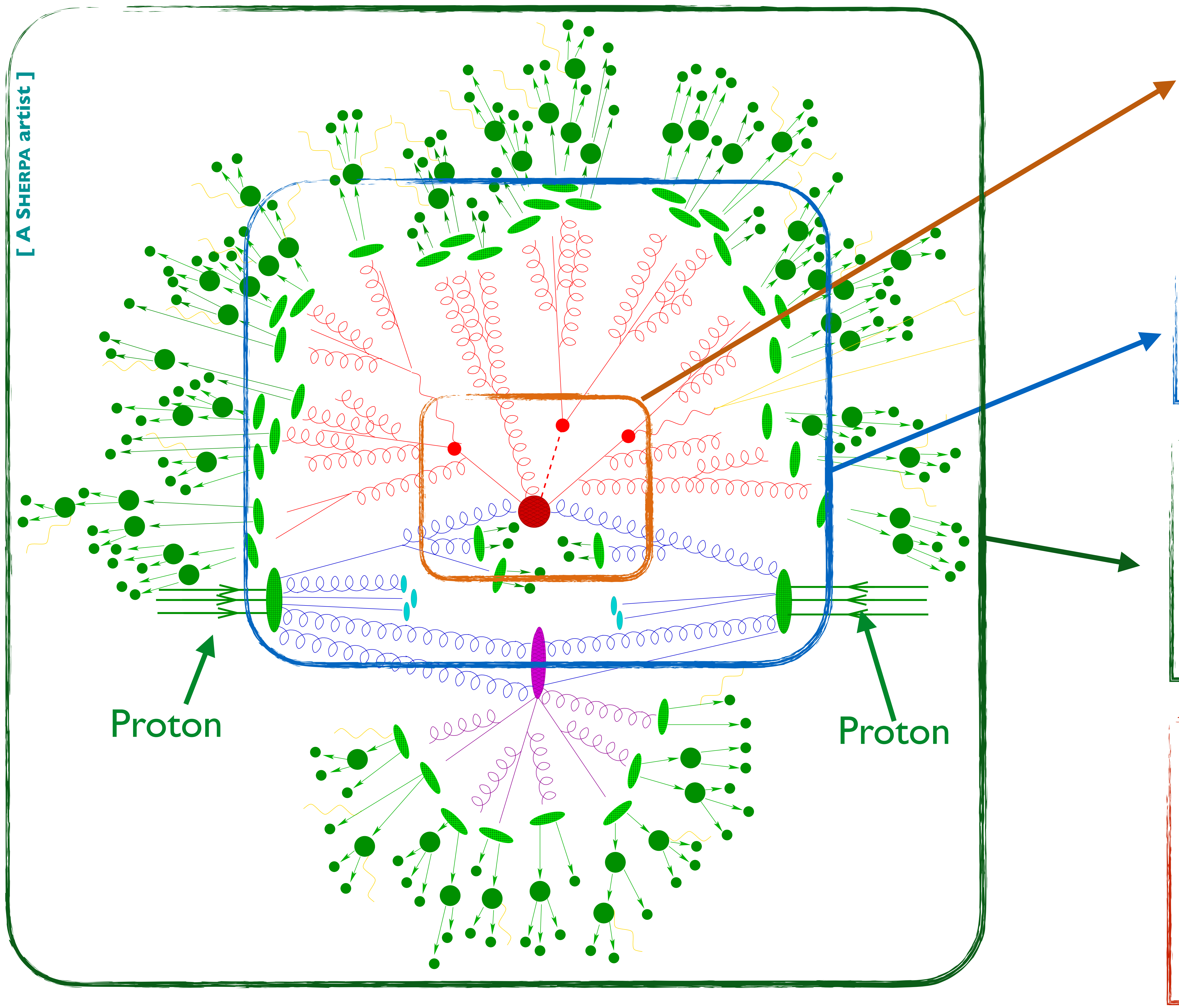
- BSM exploration to solve its issues
- No clear path
  - ➙ Dark matter ≡ (excellent) way
  - ➙ Collider signals comprising missing transverse energy



[Hitoshi Murayama]

Predictions at colliders!

Deciphering a proton collision



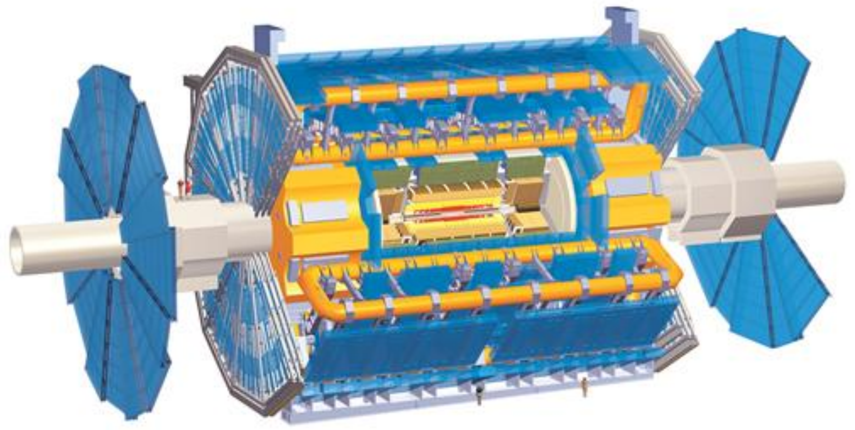
Hard process (0.1 − 1 TeV scale)
- Depends on the model (SM/BSM)
- Perturbative calculations

Parton showering (1 − 100 GeV)
- Universal (QCD)

Hadronisation (sub-GeV)
- Model-based, universal

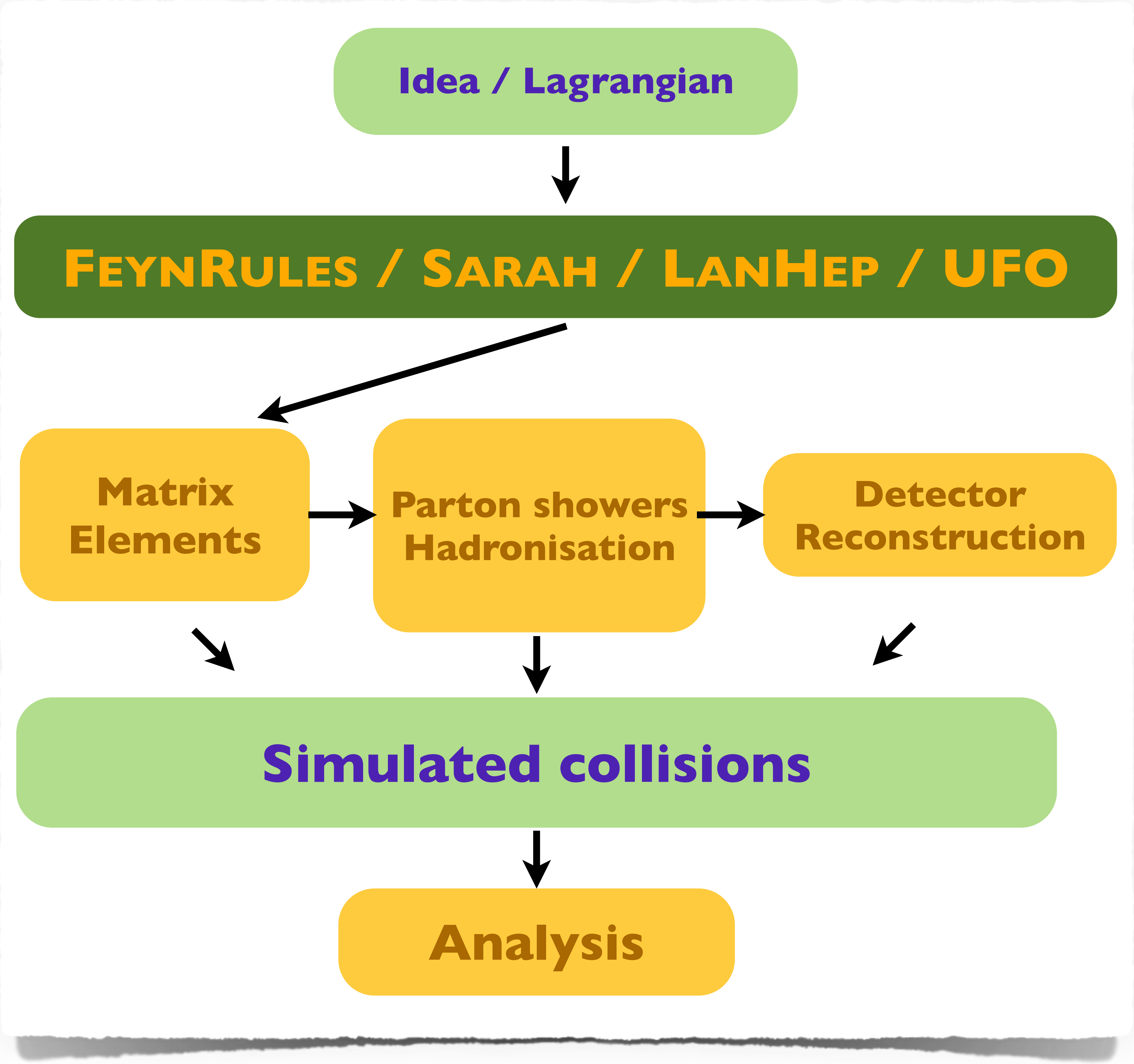
Underlying event (sub-GeV)
- Model-based, non-universal

Detector simulation (sub-MeV)



Connecting ideas to simulations…

[Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC`11)]



Idea / Lagrangian

FeynRules / Sarah / LanHep / UFO

Matrix Elements

Parton showers Hadronisation

Detector Reconstruction

Simulated collisions

Analysis

- Model building → FeynRules, LanHep, Sarah UFO
- Hard scattering
  - ⋆ Feynman diagram and amplitude generation
  - ⋆ Monte Carlo integration
  - ⋆ Event generation

  → CalcHep, Herwig++, MG5_aMC, Sherpa, Whizard, …
- QCD environment
  - ⋆ Parton showering
  - ⋆ Hadronisation
  - ⋆ Underlying event

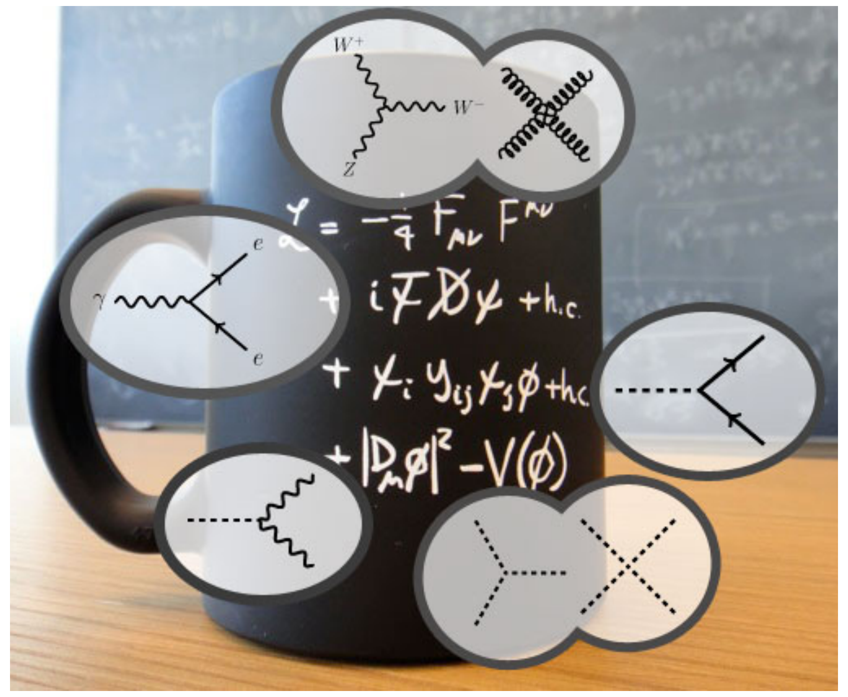
  → Herwig, Pythia, Sherpa
- Detector simulation
  - ⋆ Simulation of the detector response
  - ⋆ Object reconstruction

  → Delphes Rivet / MadAnalysis 5 – SFS
- Event analysis
  - ⋆ Signal/background analysis
  - ⋆ LHC recasting

  → Rivet / MadAnalysis 5

Model implementations

The role of the Lagrangian



$$\begin{gathered}
-\frac12\partial_\nu g^a_\mu\partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac14 g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \\
\frac12 i g_s^2(\bar q^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu + \bar G^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar G^a G^b g^c_\mu - \partial_\nu W^+_\mu\partial_\nu W^-_\mu - \\
M^2W^+_\mu W^-_\mu - \frac12\partial_\nu Z^0_\mu\partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac12\partial_\mu A_\nu\partial_\mu A_\nu - \frac12\partial_\mu H\partial_\mu H - \\
\frac12 m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \frac12\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\frac{2M^2}{g^2} + \\
\frac{2M}{g}H + \frac12(H^2+\phi^0\phi^0+2\phi^+\phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - \\
W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - \\
W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \frac12 g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \\
\frac12 g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + \\
g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \\
\frac18 g^2\alpha_h[H^4+(\phi^0)^4+4(\phi^+\phi^-)^2+4(\phi^0)^2\phi^+\phi^-+4H^2\phi^+\phi^-+2(\phi^0)^2H^2] - \\
gMW^+_\mu W^-_\mu H - \frac12 g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac12 ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - \\
W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac12 g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \\
\phi^+\partial_\mu H)] + \frac12 g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) +
\end{gathered}$$

Implementation of a new physics model in an MC programme

- Definition: particles, parameters and vertices (≡ Lagrangian)
  - ➙ translated in some programming language
- Tedious, time-consuming, error prone
- Beware of restrictions/conventions

⋆Highly redundant (each tool, each model)

⋆No-brainer tasks (from Feynman rules to codes)

Systematisation
Automation

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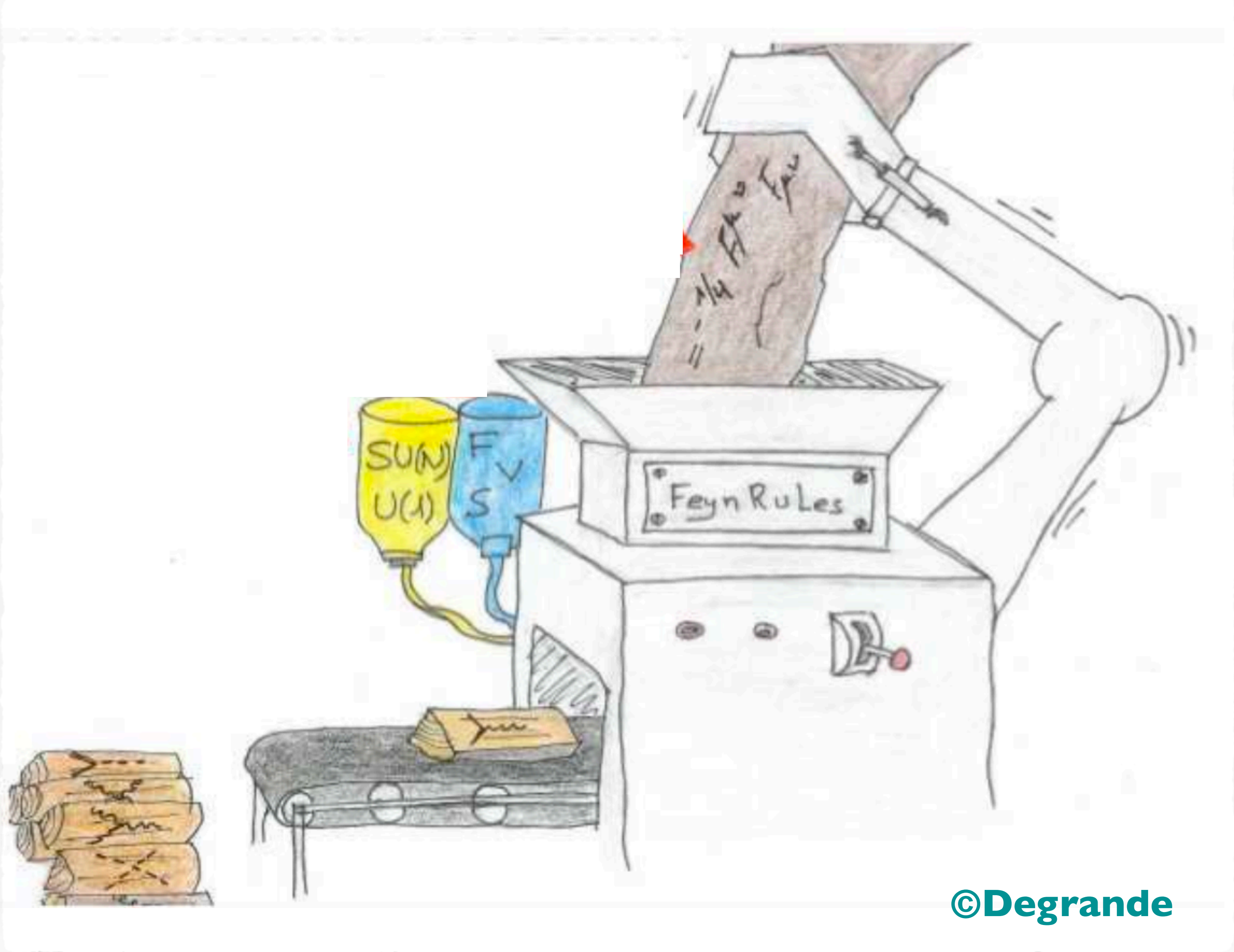

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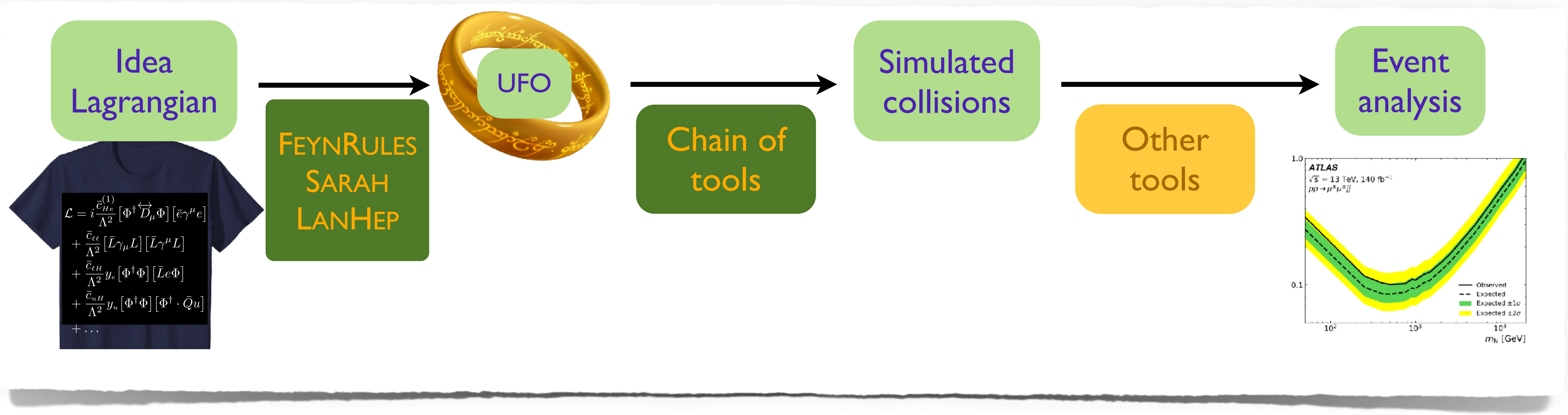
Interfacing Lagrangians and event generators

Linking a Lagrangian to a Monte Carlo tool

- Derivation of the model's Feynman rules (vertices, particle content, *etc.*)
- Interface to event generators
  - ⋆ Removal of vertices not compliant with the tool (colour and Lorentz structures)
  - ⋆ Translation to a specific format and programming language

- The UFO − one format to rule them all
  - ⋆ Design of a unique intermediate layer



Idea Lagrangian

FeynRules
SaRAH
LanHEP

UFO

Chain of tools

Simulated collisions

Other tools

Event analysis

$$\mathcal{L} = i\frac{\bar{c}^{(1)}_{He}}{\Lambda^2}\left[\Phi^\dagger \overleftrightarrow{D}_\mu \Phi\right]\left[\bar{e}\gamma^\mu e\right] + \frac{\bar{c}_{\ell\ell}}{\Lambda^2}\left[\bar{L}\gamma_\mu L\right]\left[\bar{L}\gamma^\mu L\right] + \frac{\bar{c}_{\ell H}}{\Lambda^2} y_e \left[\Phi^\dagger\Phi\right]\left[\bar{L}e\Phi\right] + \frac{\bar{c}_{uH}}{\Lambda^2} y_u \left[\Phi^\dagger\Phi\right]\left[\Phi^\dagger\cdot\bar{Q}u\right] + \ldots$$

The Universal FEYNRULES Output

[Degrande et al. (CPC ’12)]
[Darmé et al. (EPJC`24)]

The UFO in a nutshell

- UFO ≡ Universal Feynman Output
  - ⋆ Universal as not tied to any specific programme
- Set of PYTHON files to be linked to any code with full information
  - ⋆ Generic colour and Lorentz structures
  - ⋆ Restrictions on acceptable colour/Lorentz structures enforced at the software level

Initially designed as the MG5aMC model format, UFOs now standard

FEYNRULES SARAH LANHEP

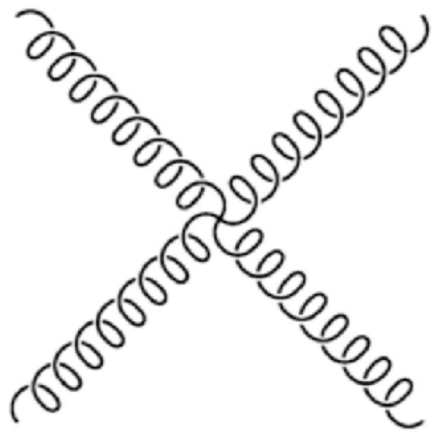
ACHILLES COMIX CONTUR GOSAM HERWIG 7 MADANALYSIS 5

MADDM MADGRAPH5_aMC@NLO RECOLA SHERPA WHIZARD

The UFO: strategy for interactions

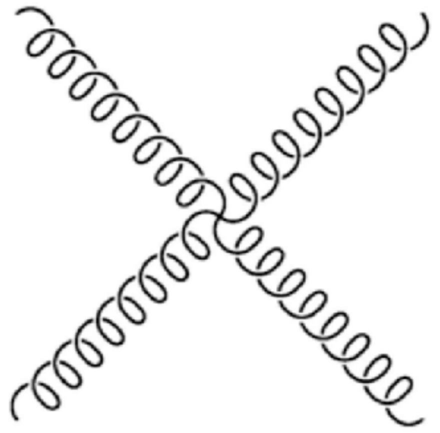
Decomposition in a spin x colour basis (coupling strengths ≡ coordinates)

- Example: the quartic gluon vertex



$$\begin{aligned} & ig_s^2\, f^{a_1a_2b} f^{ba_3a_4}\, (\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4}) \\ & + ig_s^2\, f^{a_1a_3b} f^{ba_2a_4}\, (\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}) \\ & + ig_s^2\, f^{a_1a_4b} f^{ba_2a_3}\, (\eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}) \end{aligned}$$

- UFO version



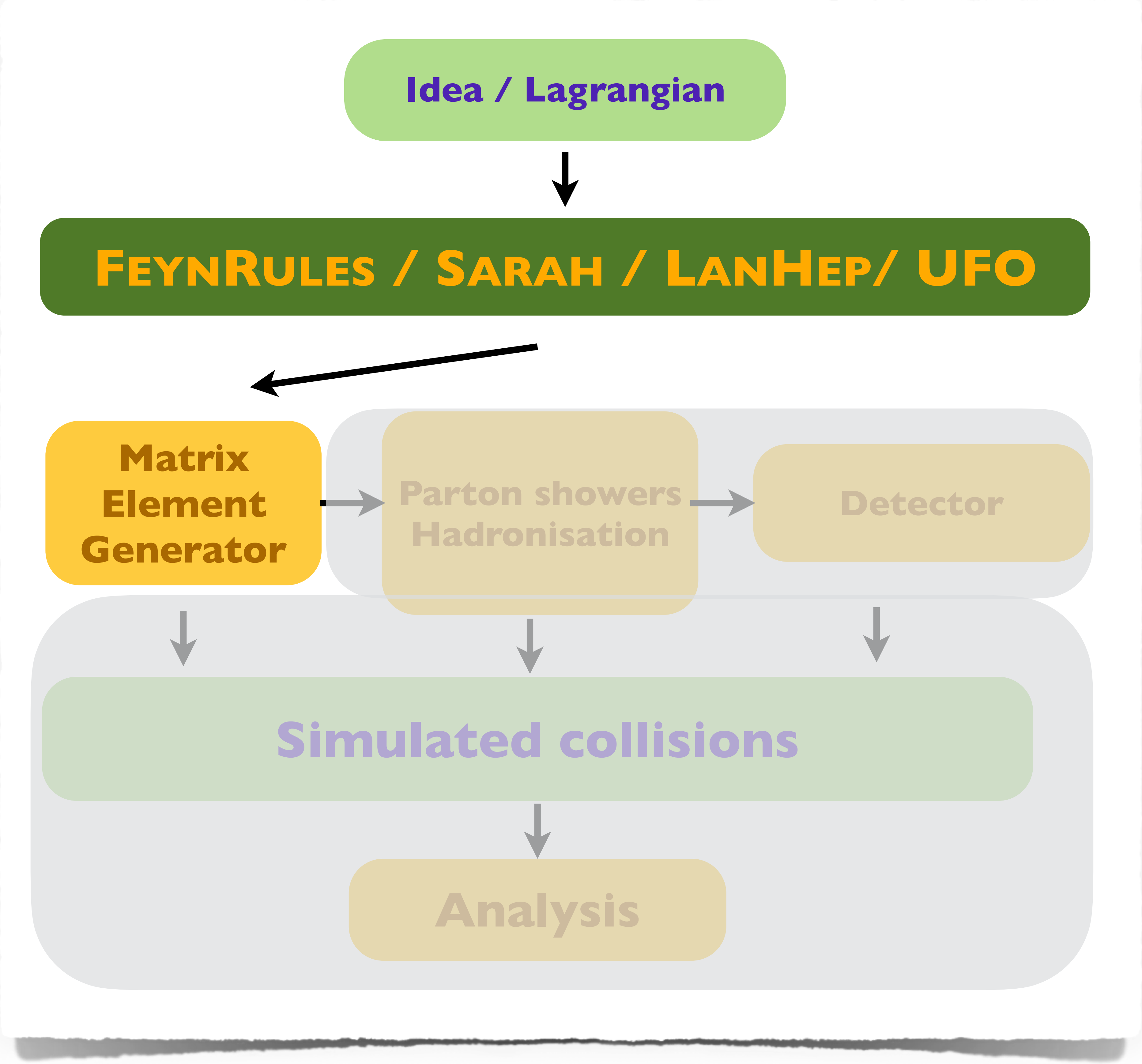
$$\begin{aligned} & \left(f^{a_1a_2b} f^{ba_3a_4},\ f^{a_1a_3b} f^{ba_2a_4}, f^{a_1a_4b} f^{ba_2a_3}\right) \\ & \times \begin{pmatrix} ig_s^2 & 0 & 0 \\ 0 & ig_s^2 & 0 \\ 0 & 0 & ig_s^2 \end{pmatrix} \begin{pmatrix} \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} \\ \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \\ \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \end{pmatrix} \end{aligned}$$

- ⋆ 3 elements for the colour basis
- ⋆ 3 elements for the spin (Lorentz structure) basis
- ⋆ 9 coordinates (6 are zero, only 1 encoded)
- ⋆ Efficient coding in PYTHON

Connecting ideas to simulations…

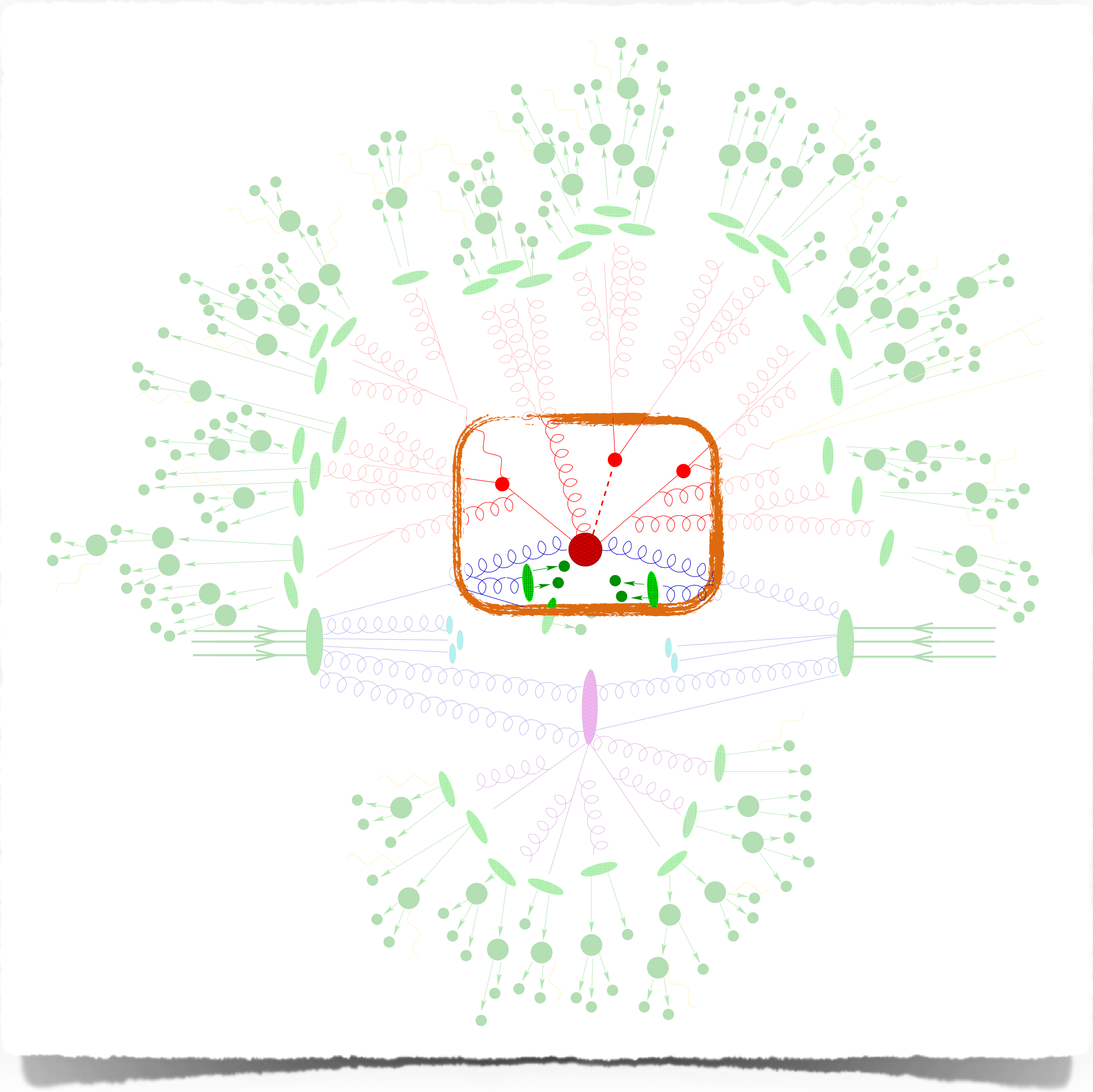
[Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC`11)]

Connecting ideas to simulations (and cross section calculations)



- Model building

- Hard scattering
  - ⋆ Feynman diagram and amplitude generation
  - ⋆ Monte Carlo integration
  - ⋆ Event generation

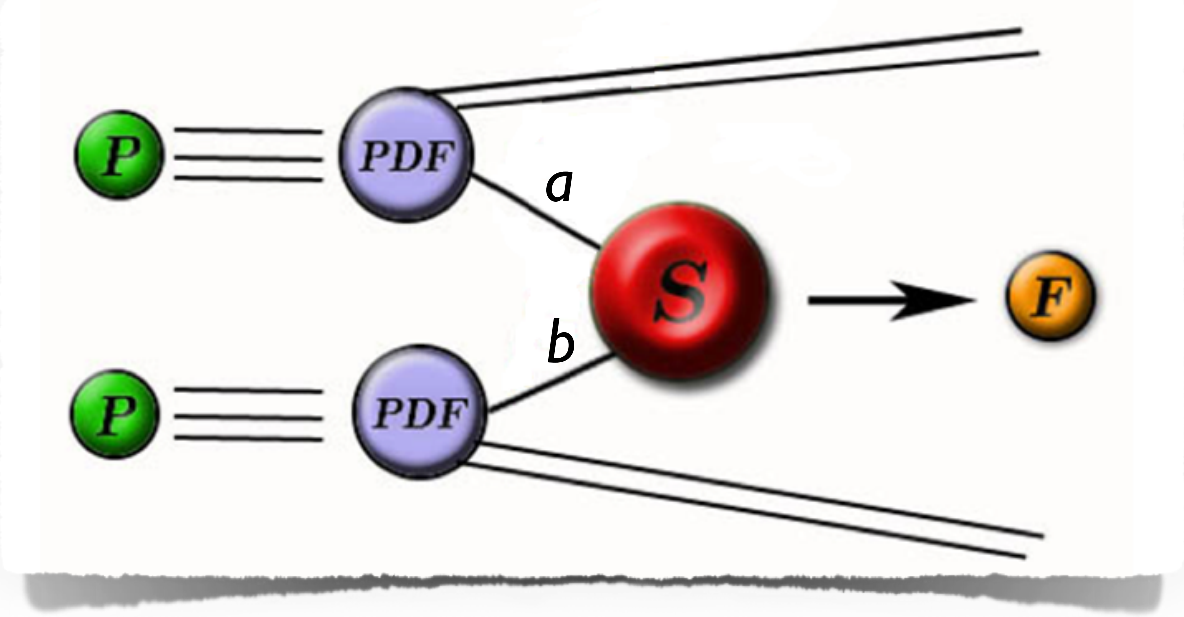


Goals:
- ➝ cross section calculations
- ➝ event generation

Feynman diagrams

QCD 101: predictions at the LHC

Distribution of an observable ω: the QCD factorisation theorem



$$\frac{d\sigma}{d\omega} = \sum_{a,b} \int dx_a\, dx_b\, d\Phi_n\, f_{a/p}(x_a;\mu_F)\, f_{b/p}(x_b;\mu_F)\, \frac{1}{2s} \overline{|\mathcal{M}|}^2(s;\mu_F,\mu_R)\, \mathcal{O}_\omega(\Phi_n)$$

- Long distance physics: the parton densities
- Short distance physics: the differential parton cross section $d\sigma_{ab}$
- Separation of both regimes through the factorisation scale μ_F
  - ⋆ Choice of the scale → theoretical uncertainties
- Sum over all final state configurations and all options for the initial state
  - ⋆ Phase space integration of the matrix element
    - → Highly-dimensional integral ($3n$-2 integrals ≡ n-body final state)
    - → Phase space structure ≡ analytical calculations hopeless

3 ingredients for cross section calculations

- Parton densities
- Matrix element
- Numerical Integration

Parton densities

Long distance physics: parton densities

- Relation of protons to their quark and gluon content
- Dependence on the momentum fraction x of the parton in the proton
- Dependence on a scale *Q* → new physics at large scale

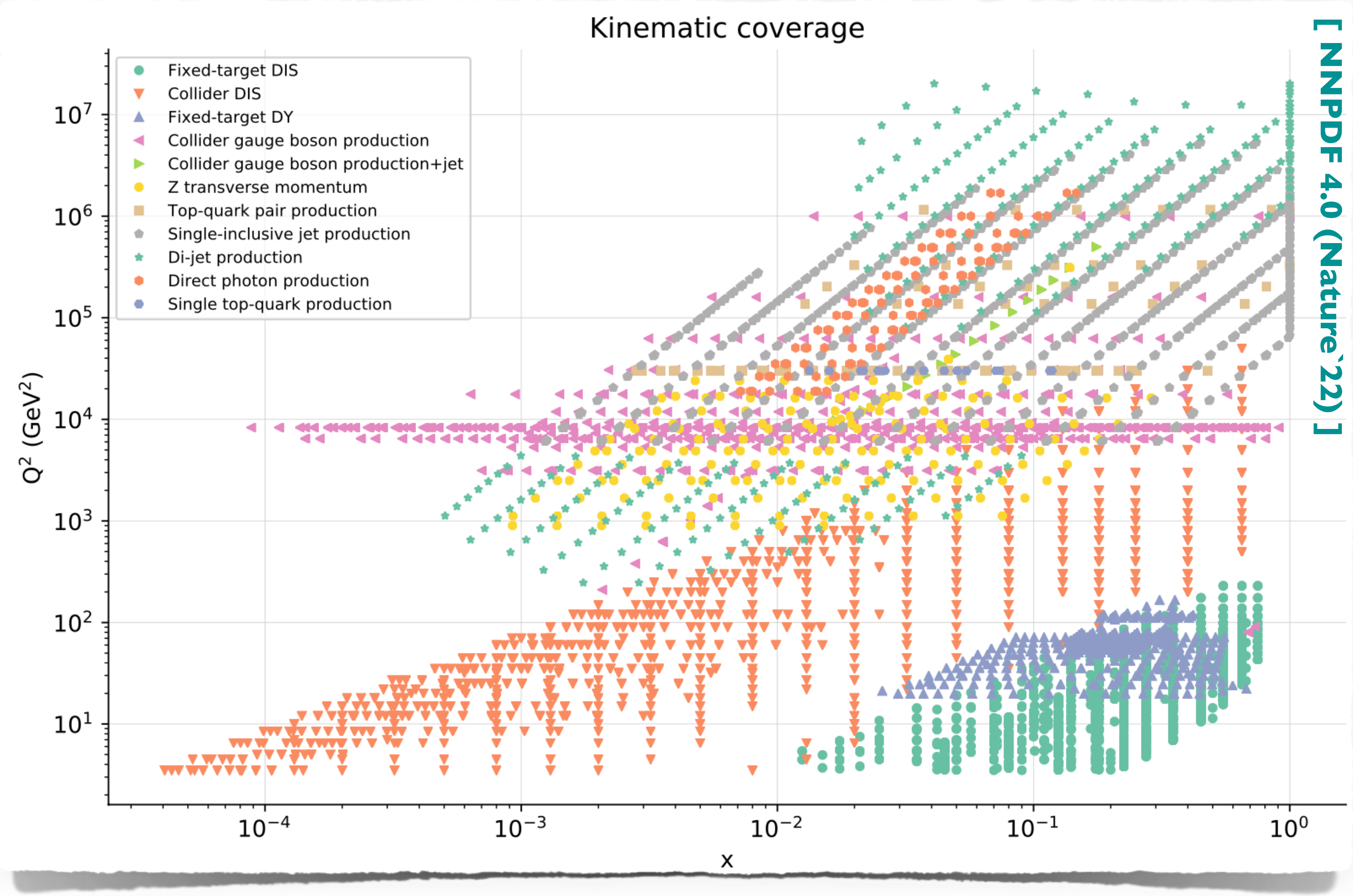
In practice

- Fitted from experimental data [in some regime *(x,Q)*]
- Evolution driven by QCD (DGLAP/BFKL)

→ PDFs for any *(x, Q)* obtained from the fit

PDF ⇔ dominant initial-states

- Huge gluon luminosity at small *x* (LHC)
- Large valence luminosity at large *x* (LHC)



[NNPDF 4.0 (Nature`22)]

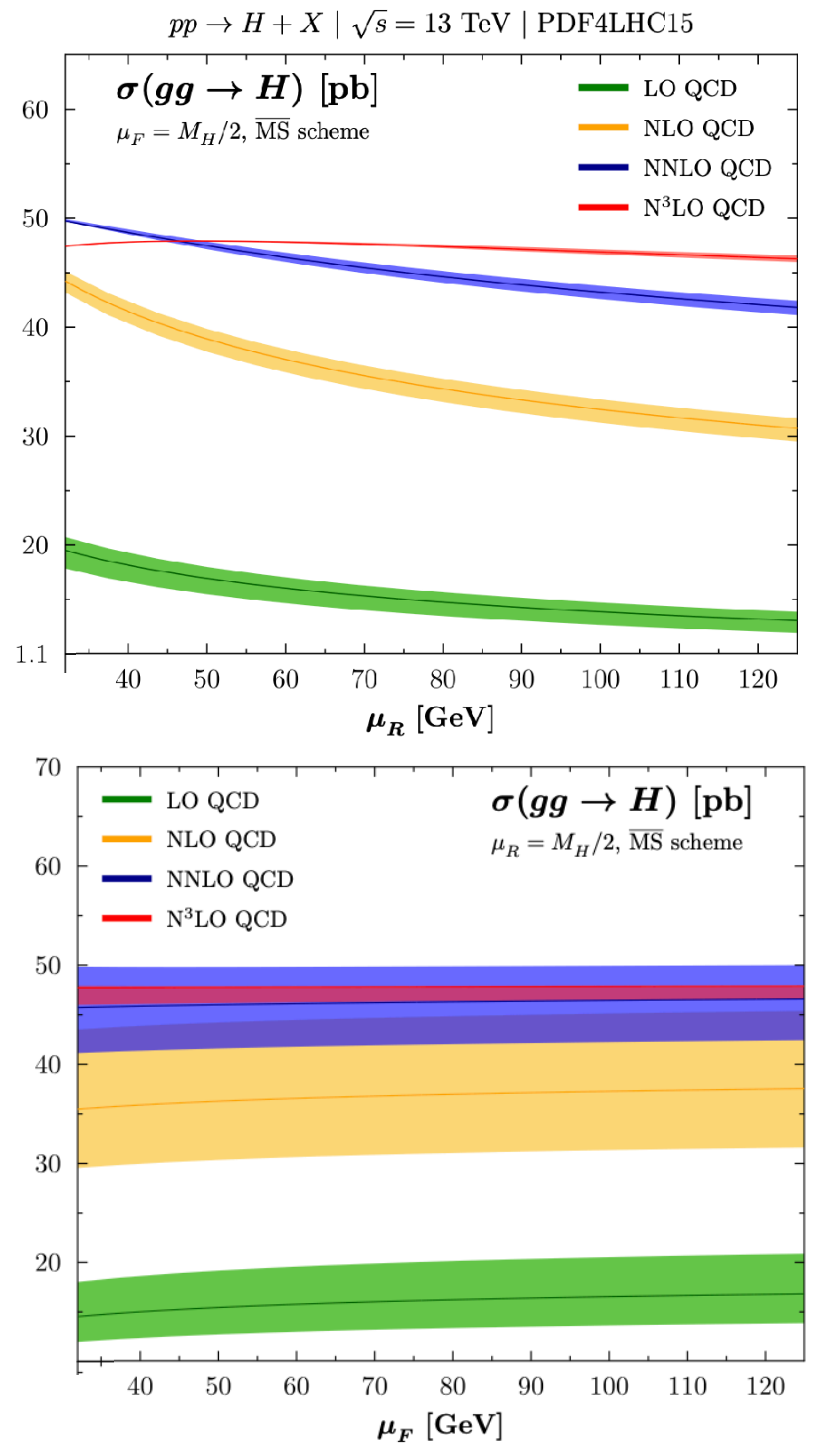
Partonic cross sections

Short distance physics: the partonic cross section

- Calculated order by order in perturbative QCD: $d\sigma = d\sigma^{(0)} + \alpha_s \, d\sigma^{(1)} + \ldots$
  - ⋆ Perturbative series: coupling ≡ expansion parameter
  - ⋆ More orders included ➙ more precise predictions
  - ⋆ Truncation of the series and α_s ➙ theoretical uncertainties [renormalisation and factorisation scales]

Predictions at leading order (LO)

- Uncertainties mainly on the normalisation (total cross section)
  ➙ reduction of the uncertainties through NLO/NNLO corrections
- Good enough for shapes (at least after using multiparton merging)

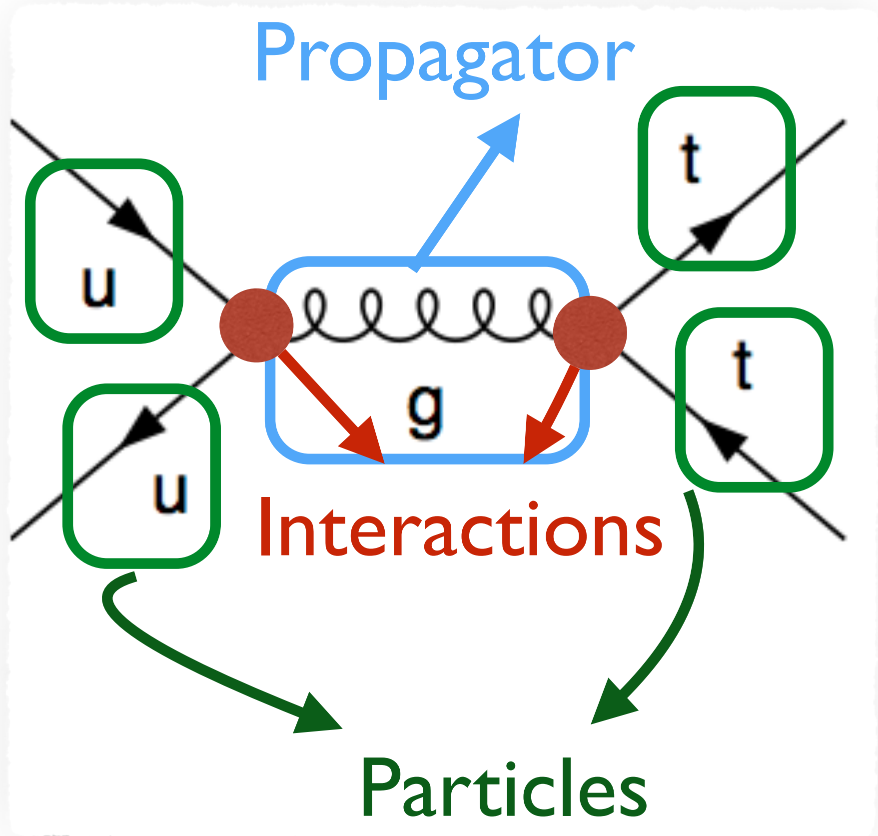


[Baglio, Duhr, Mistlberger & Szafron (JHEP`22)]

Feynman diagram calculations

Matrix elements computed from Feynman diagrams (amplitudes)

- Drawing of all diagrams for a given process
- Extraction of the amplitude from the Feynman rules



A simple example: top-antitop production from quarks

- One single diagram

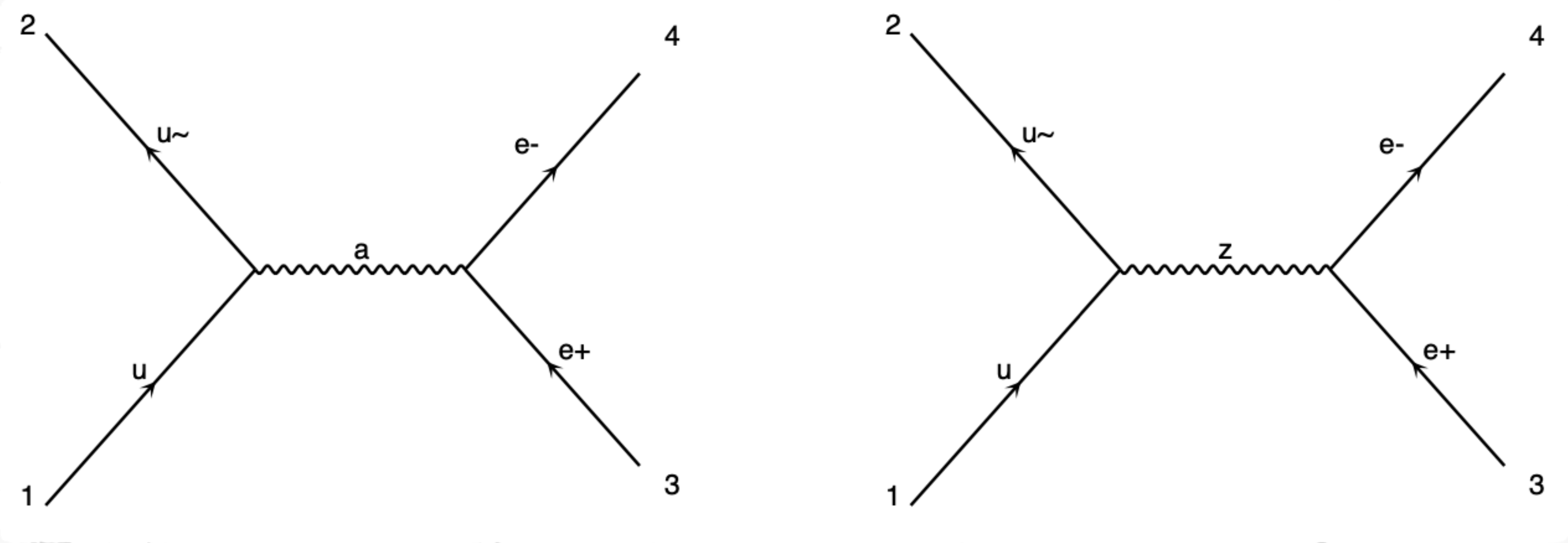
$$i\mathcal{M} = i\underline{g_s^2}\,\left[\boxed{\bar{v}_2}\underline{\gamma^\mu}\boxed{u_1}\right]\boxed{\frac{\eta_{\mu\nu}}{s}}\left[\boxed{\bar{u}_3}\underline{\gamma^\nu}\boxed{v_4}\right]\underline{T^a_{c_2c_1}T^a_{c_3c_4}}$$

- Squaring with the conjugate amplitude
- Algebraic calculation (colour and Lorentz structures)
- Sum/average over the external states

$$\overline{|\mathcal{M}|^2} = \frac{1}{36}\frac{2g_s^4}{s^2}\text{Tr}\left[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu\right]\left[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu\right]$$
$$= \frac{16g_s^4}{9s^2}\Big[(p_1\cdot p_3)(p_2\cdot p_4)+(p_1\cdot p_4)(p_2\cdot p_3)\Big]$$

Standard calculation followed by an efficient phase space integration (compact integrand)

- Drell-Yan process

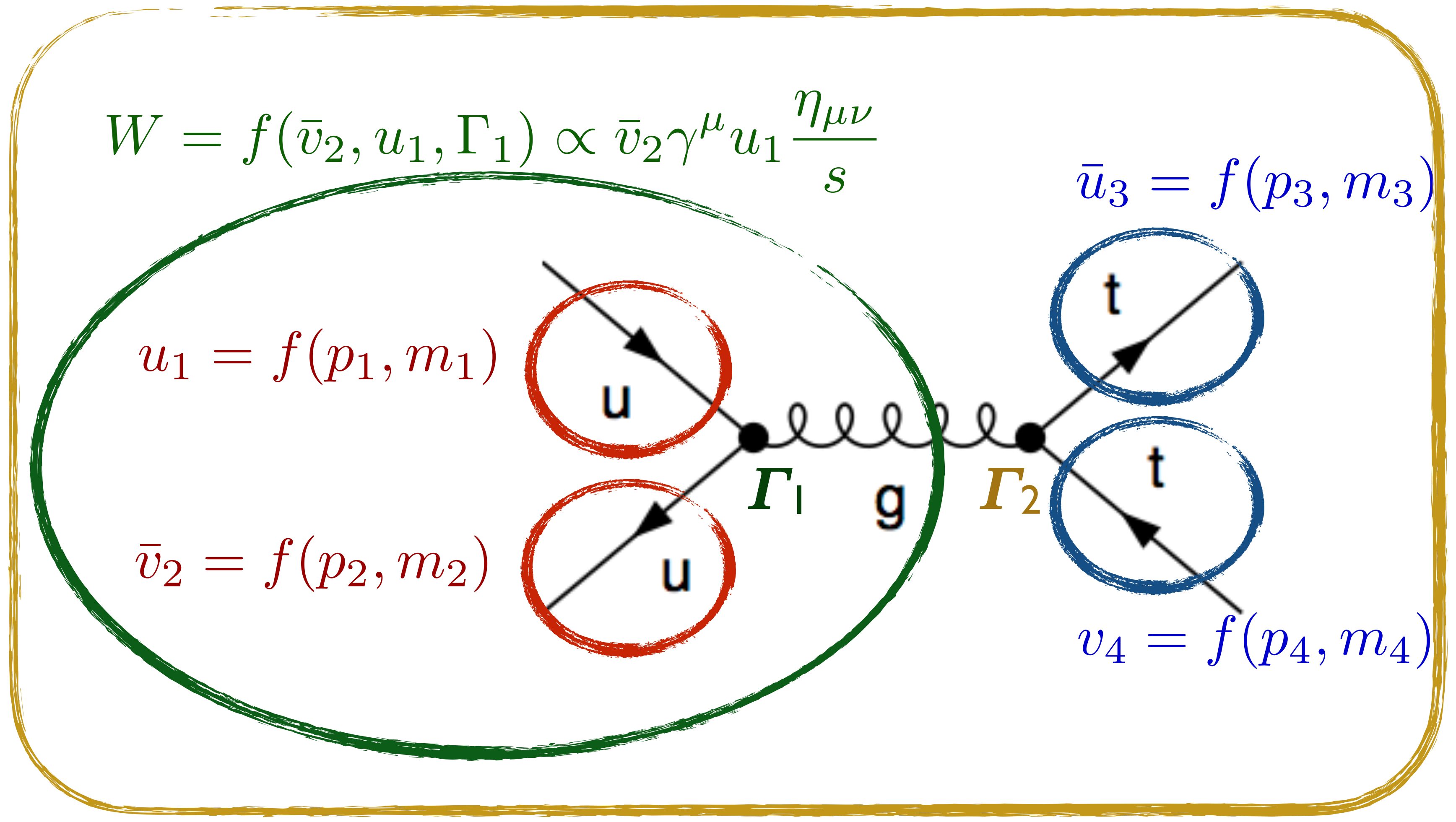


- To compute: $|M|^2 = |M_\gamma|^2 + |M_Z|^2 + 2\Re\{M_\gamma^* M_Z\}$
- The number of diagrams increases with the number of final-state particles
  ➞ Complexity as N^2
- Any 2-to-4 calculation and beyond ≡ a problem

Helicity amplitudes

Principle

- Evaluation of the amplitude for fixed external helicities

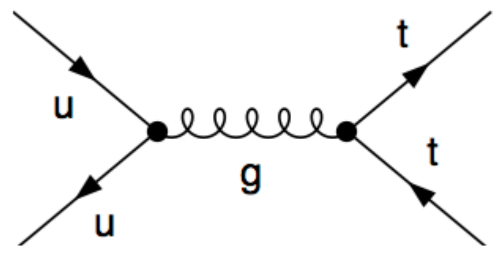


$i\mathcal{M} = f(\bar{u}_3, v_4, W, \Gamma_2)$

1. External incoming particles (numbers)
   ➙ For fixed helicity and momentum
2. Wave function of the gluon propagator
3. External outgoing particles
4. Full amplitude (complex number)

- Add all amplitudes ➙ single complex number
- Squaring
- Sum/average over the external states

The building blocks ≡ so-called HELAS functions



$u_1 = f(p_1, m_1)$
$\bar{v}_2 = f(p_2, m_2)$
$\bar{u}_3 = f(p_3, m_3)$
$v_4 = f(p_4, m_4)$
$W = f(\bar{v}_2, u_1, \Gamma_1)$
$i\mathcal{M} = f(\bar{u}_3, v_4, W, \Gamma_2)$

- HELAS ≡ HELicity Amplitude Subroutine
- One routine / Lorentz structure (Γ_i)
  - ⋆ SM [Murayama, Watanabe & Hagiwara (KEK-91-11)]
  - ⋆ MSSM [Cho, Hagiwara, Kanzaki, Plehn, Rainwater & Stelzer (PRD`06)]
  - ⋆ HEFT [Frederix (2007)]
  - ⋆ Spin 2 [Hagiwara, Kanzaki, Li &Mawatari (EPJC`08)]
  - ⋆ Spin 3/2 [Mawatari & Takaesu (EPJC`11)]
  - ⋆ Everything (ALOHA) [de Aquino, Link, Maltoni, Mattelaer & Stelzer (CPC`12)]

Comparison

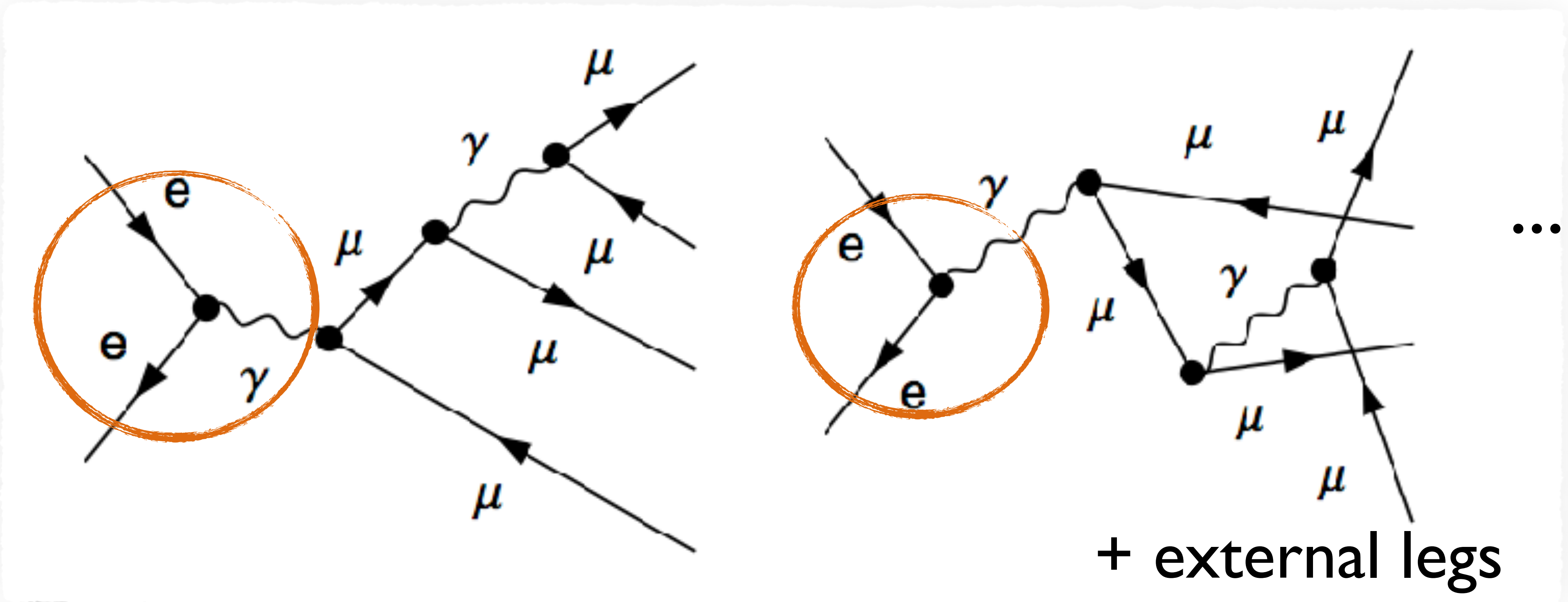
	For *M* diagrams	For *N* particles	2 → 6 example
Analytical	M^2	$(N!)^2$	10^9
Helicity	M	$N!\ 2^N$	10^7

Still a problem…
Can we do better ?

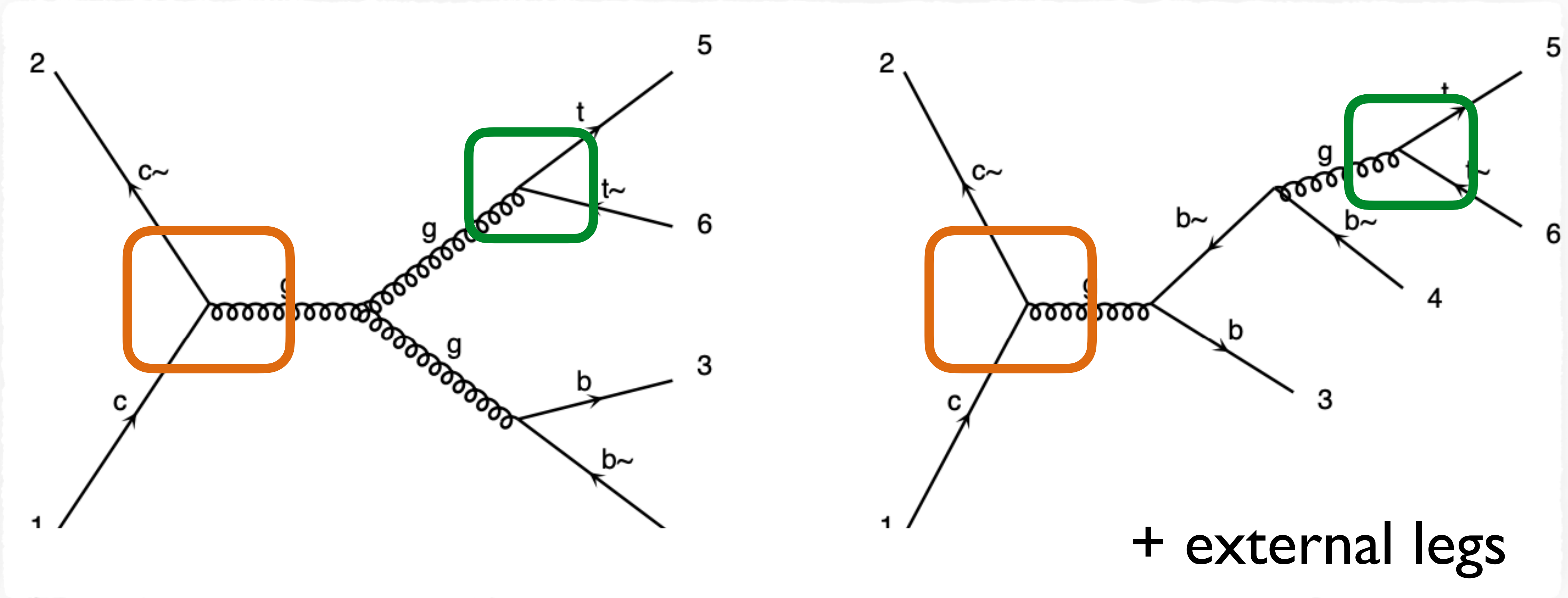
Recycling

Recycling: reusing pieces from one diagram to another

- Not recalculating what is already calculated



+ external legs



+ external legs

- Significative gain in computing time

	For *M* diagrams	For *N* particles	2 →6 example
Analytical	M^2	$(N!)^2$	10^9
Helicity	M	$N!\ 2^N$	10^7
Recycling	M	$(N-1)!\ 2^{N-1}$	$5\text{x}10^5$

Other potential optimisation methods

- Recursion relations, 5D wave functions, *etc.*
- Several new optimisations in MG5aMC (*e.g.* helicity recycling)

[Mattelaer & Ostrolenk (EPJC`21)]

Now this may numerically work!

Phase space integration

Observable calculations

The QCD factorisation theorem

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} = \sum_{a,b} \int \mathrm{d}x_a\ \mathrm{d}x_b\ \mathrm{d}\Phi_n\ f_{a/p}(x_a;\mu_F)\ f_{b/p}(x_b;\mu_F)\ \frac{1}{2s}\overline{|\mathcal{M}|}^2(s;\mu_F,\mu_R)\ \mathcal{O}_\omega(\Phi_n)$$

- Any observable ➙ integral calculation needed
- The phase space
  - ➙ highly-dimensional integral (*3n-2* integrals ≡ *n*-body final state)
  - ➙ complex structure ≡ analytical calculations hopeless

- The integrand is a very peaked function (propagators)
  - ➙ Need for general and flexible numerical methods
- Bonus: integration one thing, events another!

Numerical integration − methods

- Standard methods like trapezium/Simpson very good in 4dims, not in D-dims
  - ➙ Trapezium: precision in $1/N^{2/d}$
  - ➙ Simpson: precision in $1/N^{4/d}$
- Monte Carlo integration saves the day
  - ➙ Precision in $1/\sqrt{N}$

Monte Carlo integration: the method

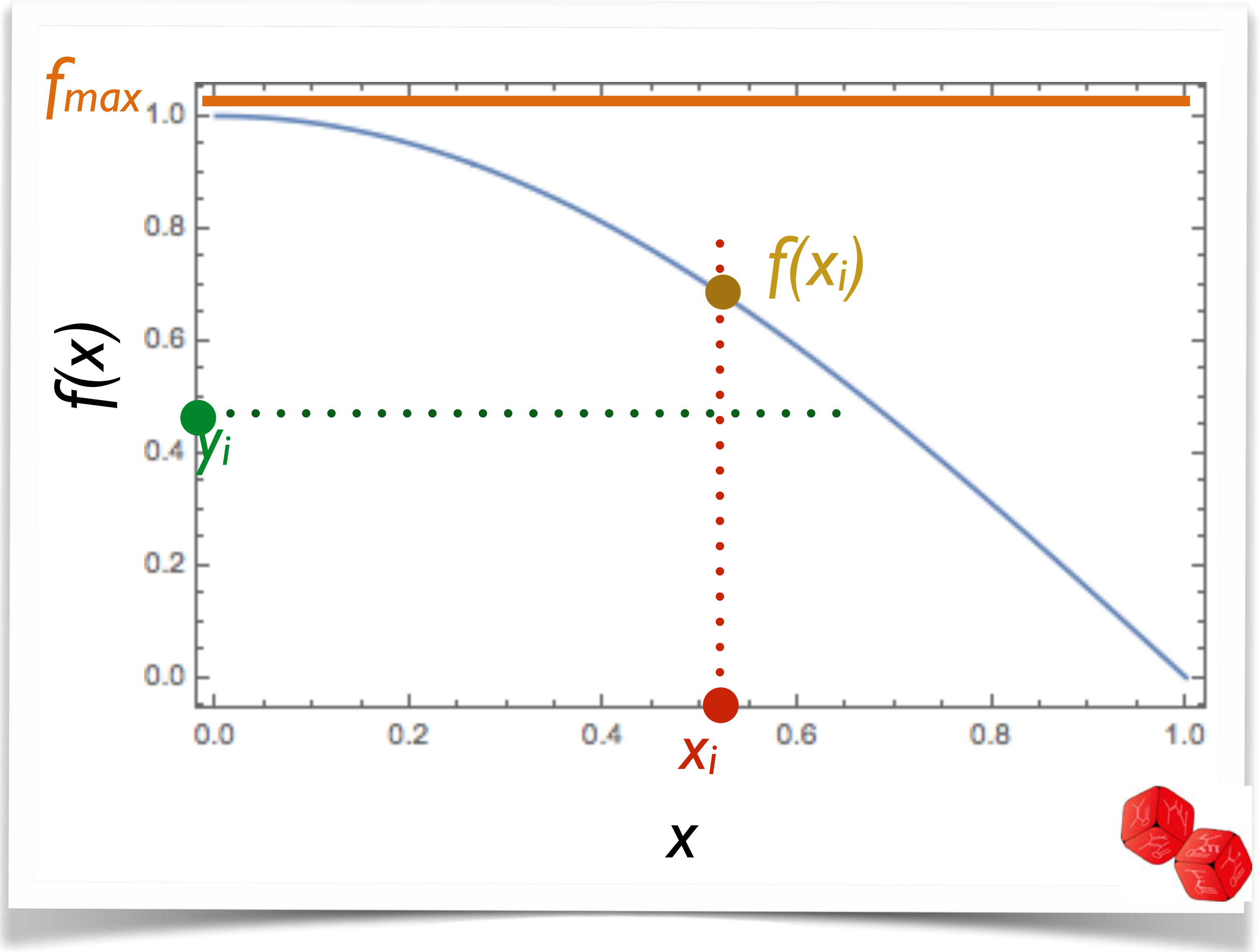
The one-dimensional example: evaluate the integral *I*

$$I = \int_a^b \mathrm{d}x \ f(x)$$

1. Determine $f_{max} > f(x) \ \forall \ x \in [a,b]$
2. At a given step *i*,
   - ⋆ pick a random point $x_i \in [a,b]$
   - ⋆ pick a random number $y_i < f_{max}$
3. Compare with $f(x_i)$
   - ⋆ If $y_i > f(x_i)$: reject the point
   - ⋆ If $y_i < f(x_i)$: accept the point
4. Evaluate the integral

$$I_N = \frac{N_{\rm accepted}}{N_{\rm total}} \ \mathcal{V}$$

integration volume



Monte Carlo integration: the error

The mean value theorem

- If *f(x)* is continuous:

$$\exists\xi \in [a,b] : I = \int_a^b \mathrm{d}x\ f(x) = (b-a)f(\xi) = (b-a)\langle f\rangle$$

- We can approximate $\langle f\rangle$ by an averaged sum

$$I = \int_a^b \mathrm{d}x\ f(x)\ \approx I_N = \frac{b-a}{N}\sum_{n=1}^N f(x_n)$$

★ $\langle f\rangle$ ≡ sampling the integrand at random points

★ MC method: random choice of points

The error on the integral ≡ the variance

- Independent of the number of dimensions
- Minimisable

$$V = (b-a)\int_a^b \mathrm{d}x\ f^2(x) - I^2\ \approx V_N = \frac{(b-a)^2}{N}\sum_{n=1}^N f^2(x_n) - I_N^2$$

Result

$$I = I_N \pm \sqrt{\frac{V_N}{N}}$$

- Error easy to estimate and independent of the number of dimensions
- Improvement possible by minimising V_N (ideal case: *f(x)* = *cst* ⇔ $V=V_N=0$)

➙ Change of variables to flatten the integrand

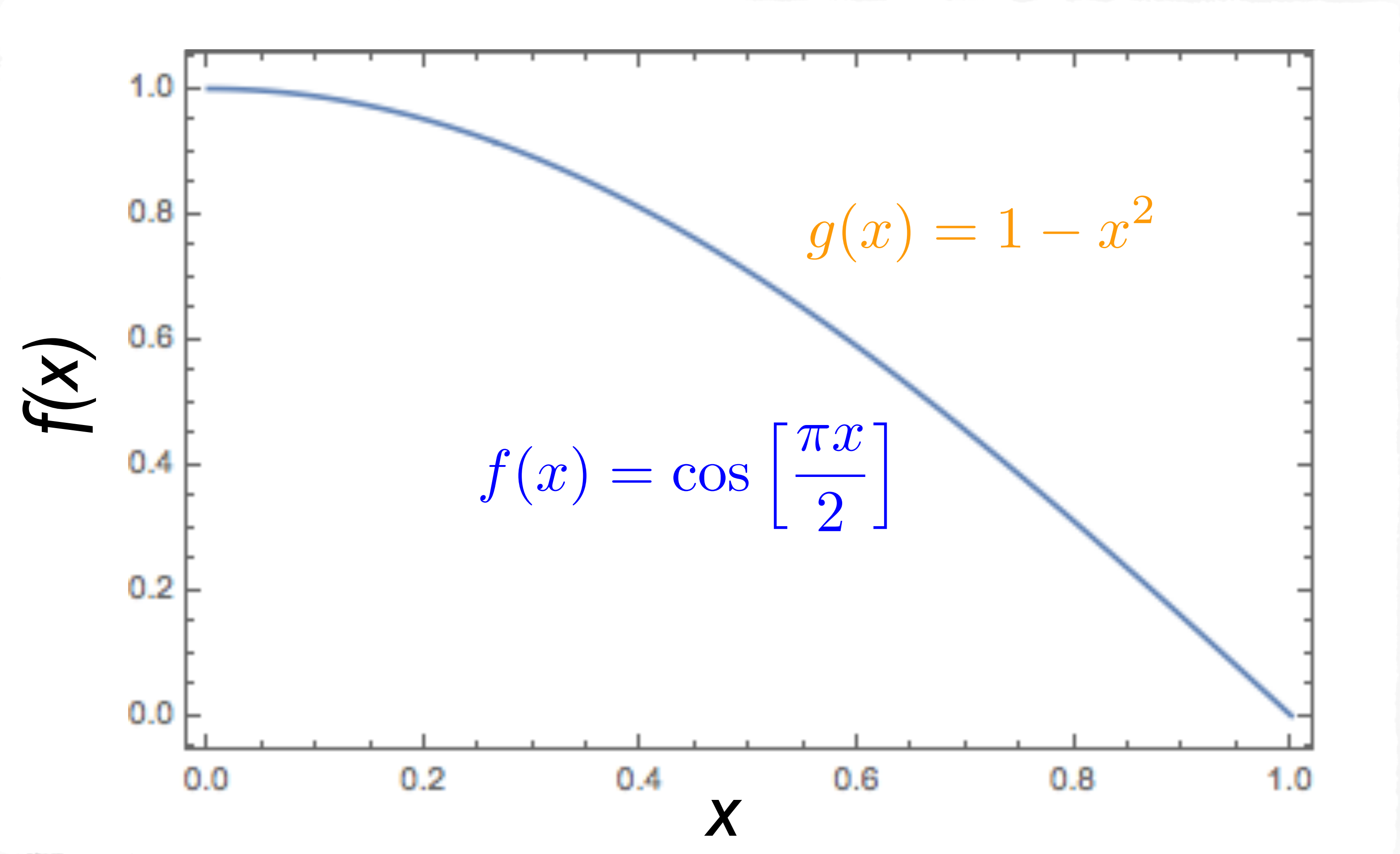
Importance sampling: a practical example

Integral to calculate

$$I = \int_0^1 \mathrm{d}x \;\; \cos\left[\frac{\pi x}{2}\right] = \frac{2}{\pi} \approx 0.6366$$

$$I_N = 0.637 \pm \frac{0.307}{\sqrt{N}}$$

- Convergence slow
- Precision ⇒ large N
- Strength: scalability with $N_{\rm dim}$



Clever change of variable ➝ reduction of the variance

- The ratio *f(x)/g(x)* ≃ *1* (ideal case)

$$I = \int_0^1 \mathrm{d}x \;\, (1-x^2) \frac{\cos\left[\frac{\pi x}{2}\right]}{1-x^2} = \int_{\xi_1}^{\xi_2} \mathrm{d}\xi \boxed{\frac{\cos\left[\frac{\pi x(\xi)}{2}\right]}{1-x(\xi)^2}} \simeq 1 \quad \text{with} \quad \xi = x - \frac{1}{3}x^3$$

$$I_N = 0.637 \pm \boxed{\frac{0.031}{\sqrt{N}}}$$

➝ Faster convergence

Phase space parametrisation crucial

- Better convergence

Importance sampling in action

The one-dimensional example: evaluate the integral *I*

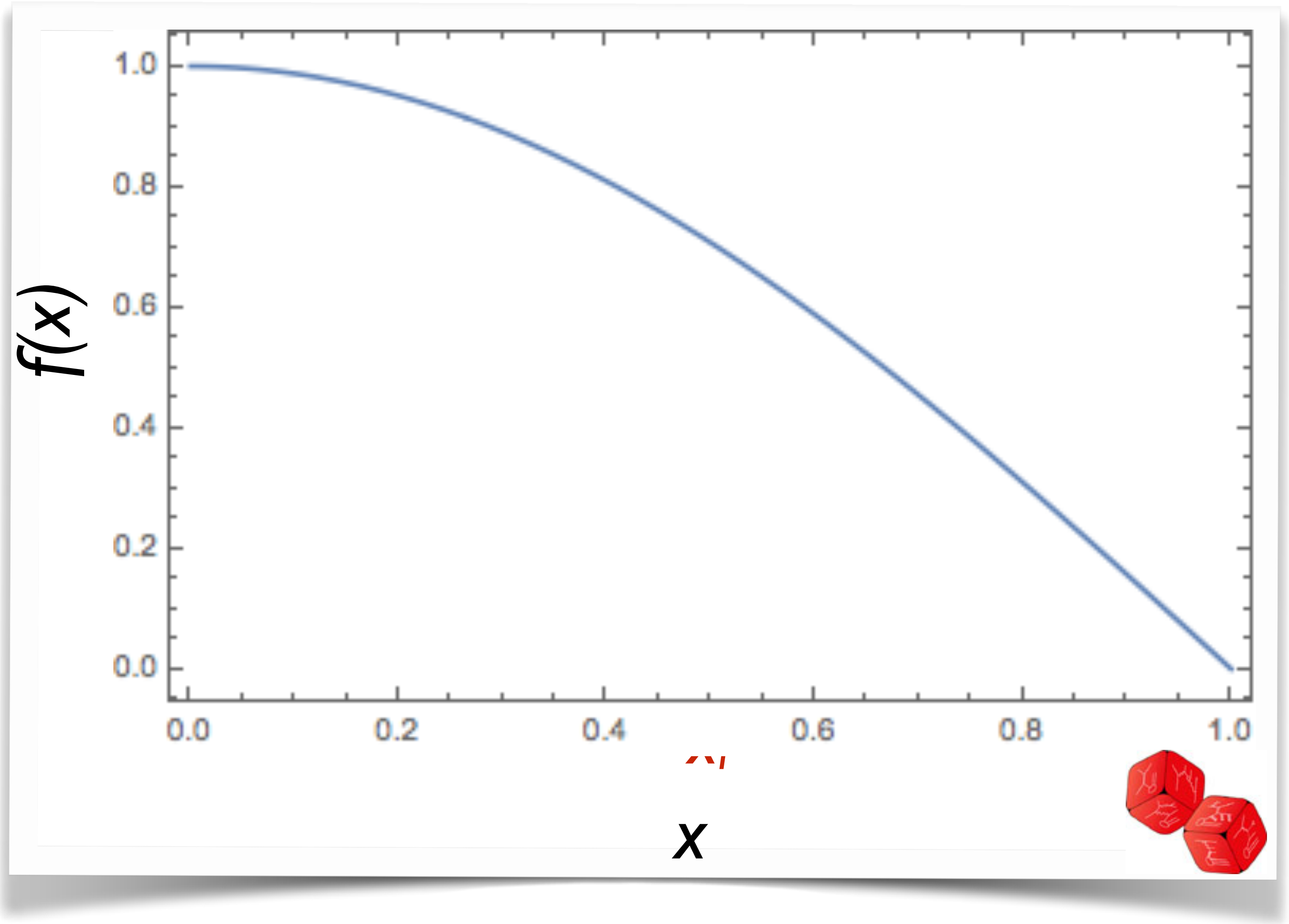
$$I = \int_a^b \mathrm{d}x \; f(x)$$

1. Find *g(x) so that g(x)* > *f(x)* ∀ *x* ∈ *[a,b]*

2. At a given step *i*,
   - ⋆ pick a random point x_i distributed as *g(x)*
   - ⋆ pick a random number $y_i < g(x_i)$

3. Compare with $f(x_i)$
   - ⋆ If $y_i > f(x_i)$: reject the point
   - ⋆ If $y_i < f(x_i)$: accept the point

4. Evaluate the integral $$I_N = \frac{N_{\rm accepted}}{N_{\rm total}} \; \mathcal{V}$$



More points are sampled where the function is the largest

Problem of a peaked integrand

The QCD factorisation theorem

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} = \sum_{a,b} \int \mathrm{d}x_a\ \mathrm{d}x_b\ \mathrm{d}\Phi_n\ f_{a/p}(x_a;\mu_F)\ f_{b/p}(x_b;\mu_F)\ \frac{1}{2s}\overline{|\mathcal{M}|}^2(s;\mu_F,\mu_R)\ \mathcal{O}_\omega(\Phi_n)$$

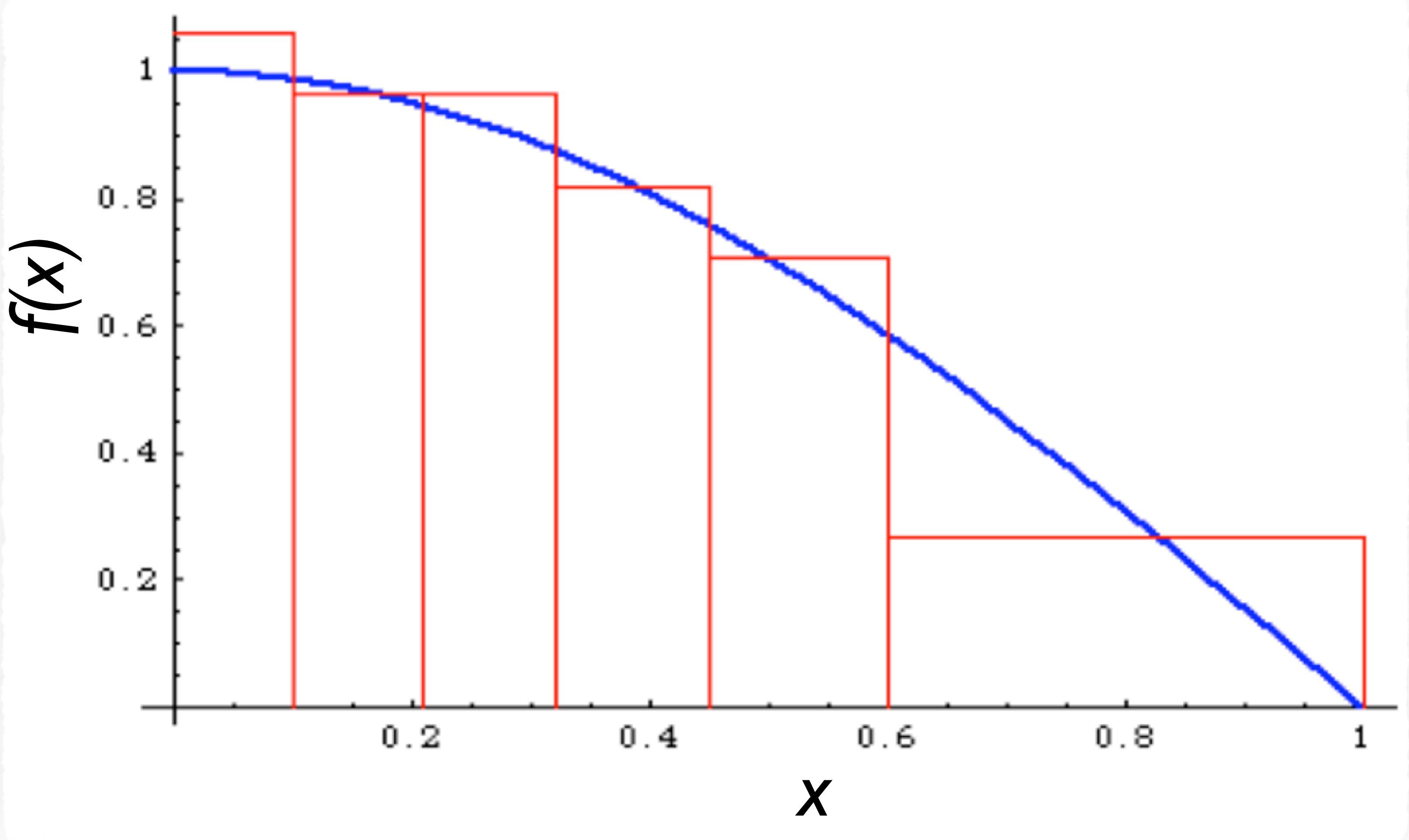
- For each point, we have a weight given by $f_{a/p}(x_a;\mu_F)\ f_{b/p}(x_b;\mu_F)\ \frac{1}{2s}\overline{|\mathcal{M}|}^2(s;\mu_F,\mu_R)$
- Interpretation: each sampled momentum configuration ➝ a weight

Problem: a peaked integral is peaked (➝ propagators)

- Random phase space points: very little chance to contribute
  ➝ Few points carry the bulk of the integral
- Flattening the integrand ≡ change of variables (importance sampling)
  ➝ Knowledge about the integrand (to find the supremum function)

Construction of an approximative function of the integrand

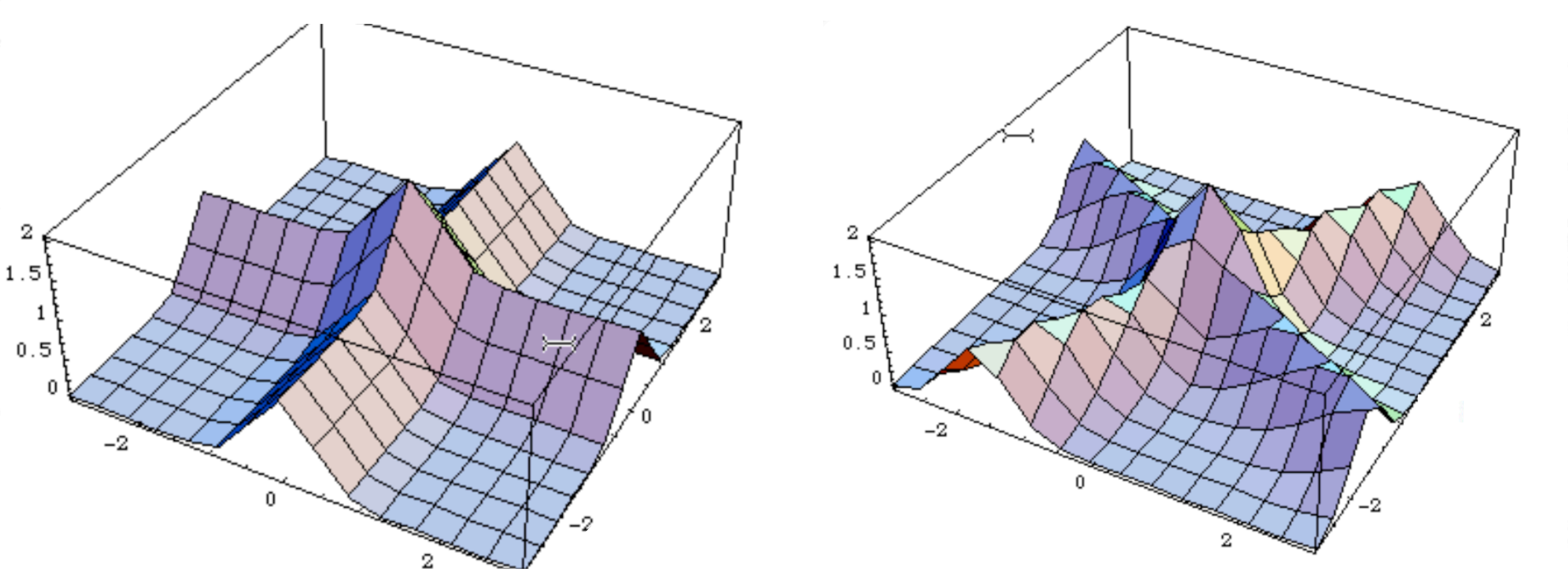
- Division of the integration domain in sub-domains (variable bin-size)
  - ⋆ Adjustment: identical variance in each bin
    ➝ Many bins where the function is large
  - ⋆ Minimisation of the overall variance
- More bins where the integrand fluctuates more
  - ⋆ The binned function ≡ approximation *g(x)* of the integrand



The VEGAS algorithm

The VEGAS algorithm

- Relies on 1D integration
- Projections on the various axes to align the integrand
  - ➙ Factorisation of integration variable by variable
- Adaptive sampling ➙ points in ‘*interesting*’ domains
- Finding a good rotation crucial
  - ➙ Rarely the case
  - ➙ Multi-channeling



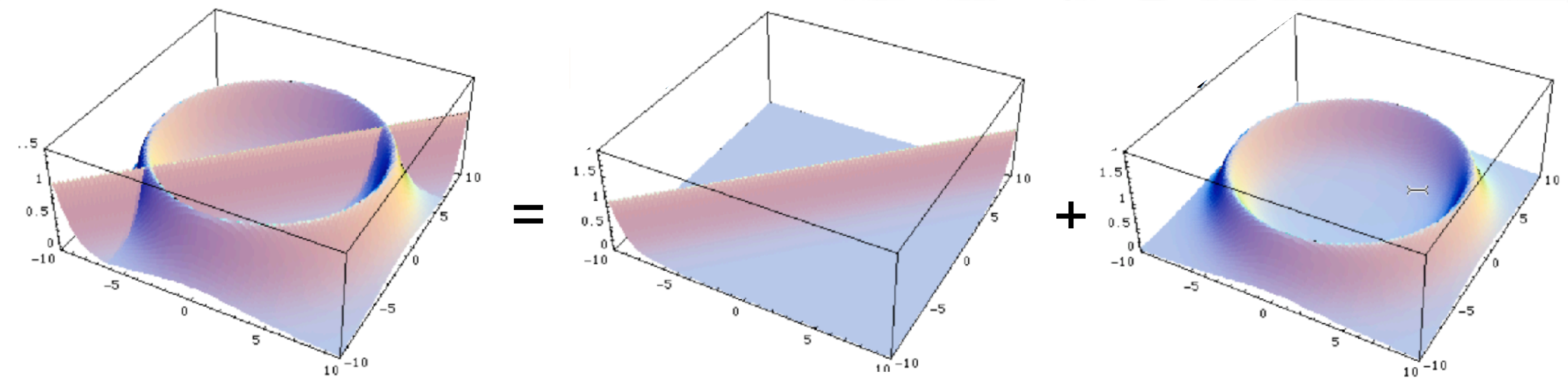
integrand well aligned

rotation needed

Multi-channeling

- One rotation per channel

$$g(x) = \sum_{\text{channels}} \alpha_i g_i(x) \quad \text{with} \quad \sum_{\text{channels}} \alpha_i = 1$$



- Each g_i takes care of one single peak of the integrand

$$I = \int \mathrm{d}x\ f(x) = \int \mathrm{d}x \boxed{\frac{f(x)}{g(x)}}^{\simeq 1} g(x) = \sum_i \alpha_i \int \mathrm{d}x\ \frac{f(x)}{g(x)} g_i(x)$$

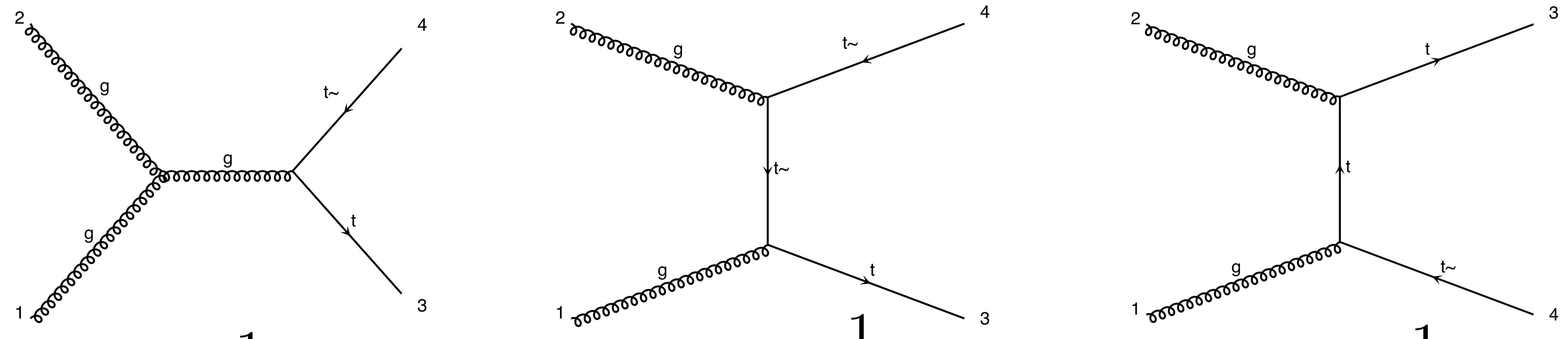
importance sampling

multi-channeling

⋆ Knowledge of the matrix element

Multi-channel integration: an example

Top-antitop production: 3 diagrams



$$\mathcal{A}_s \propto \frac{1}{s} \qquad \mathcal{A}_t \propto \frac{1}{t-m_t^2} \qquad \mathcal{A}_u \propto \frac{1}{u-m_t^2}$$

$$g(x) = \sum_{\text{channels}} \alpha_i g_i(x)$$

- Three different pole structures

$$I = \int \mathrm{d}\Phi_2 \, \big|\mathcal{A}_s + \mathcal{A}_t + \mathcal{A}_u\big|^2 = \sum_{i=s,t,u} \int \mathrm{d}\Phi_2 \, \frac{\overset{g_i(\Phi)}{\big|\mathcal{A}_i\big|^2}}{\underbrace{\big|\mathcal{A}_s\big|^2 + \big|\mathcal{A}_t\big|^2 + \big|\mathcal{A}_u\big|^2}_{g(\Phi)}} \underbrace{\big|\mathcal{A}_s + \mathcal{A}_t + \mathcal{A}_u\big|^2}_{f(\Phi)}$$

- ⋆ *f(Φ) / g(Φ) ≃ 1*
  - ➞ Integration easy
  - ➞ Integration of one single diagram at a time (pole structure known)
- ⋆ Multi-channeling on the basis of the different diagrams

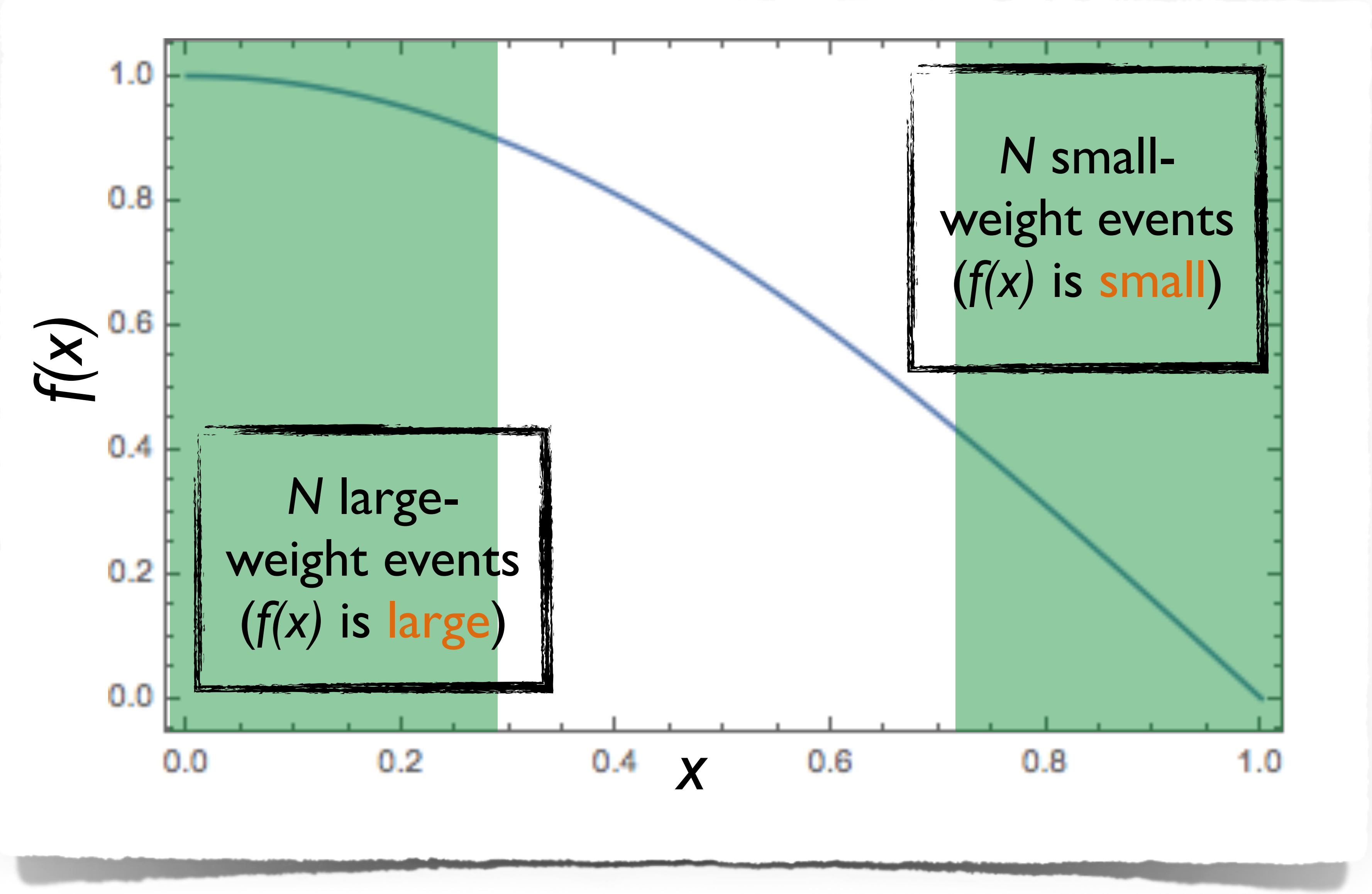
Event generation

Weighted and unweighted events

Accepted points ➙ event generation

- One point ≡ one event
- Integrand value ≡ event weight
- Not all events are equal

Weighted events

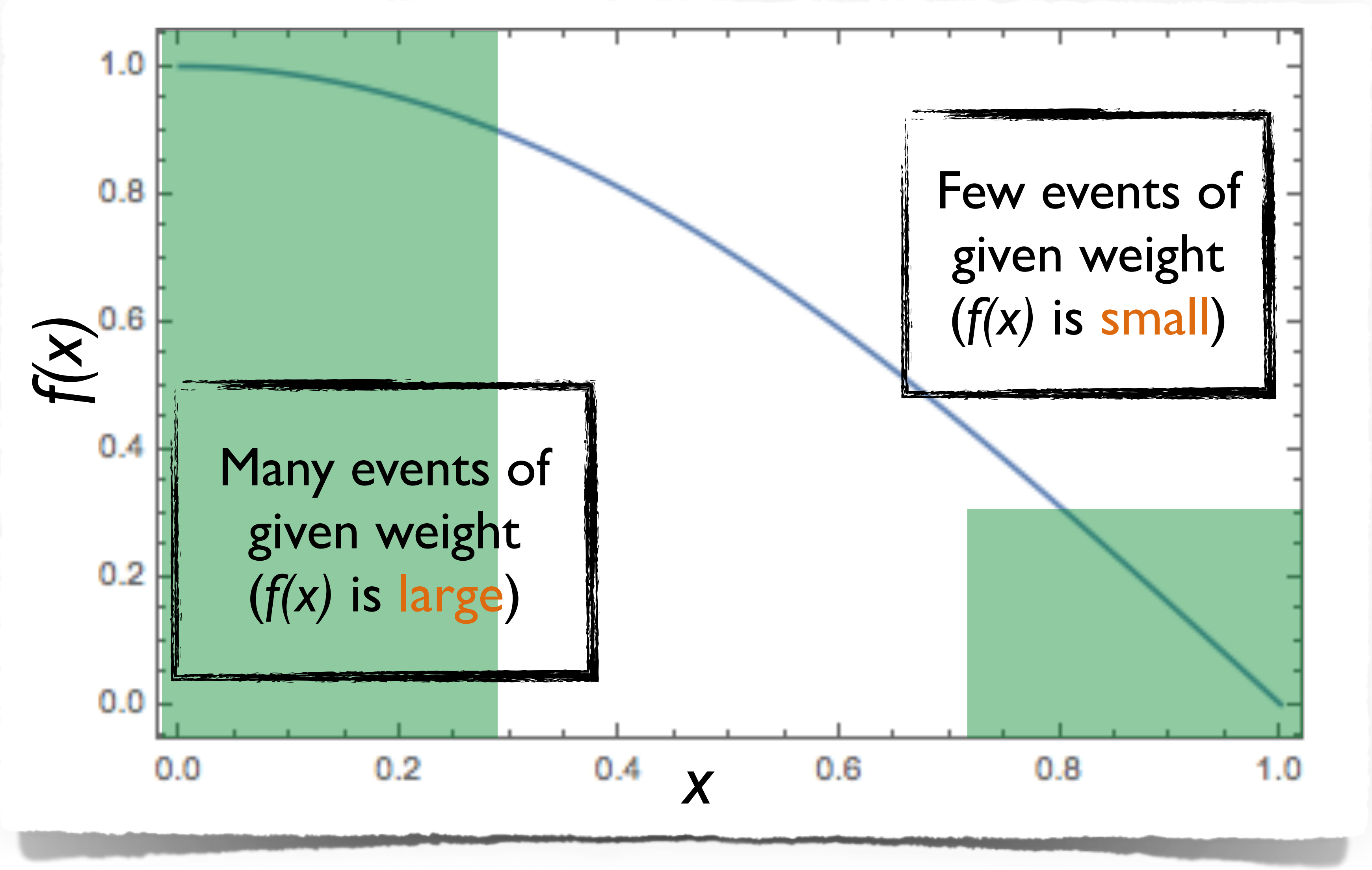


Enforcing equal-weight events

- Distributed as occurring in nature
- All events are equal
- Weight value: recovering the total rate

➙ $\langle\omega\rangle = \sigma_{tot}/N_{events}$

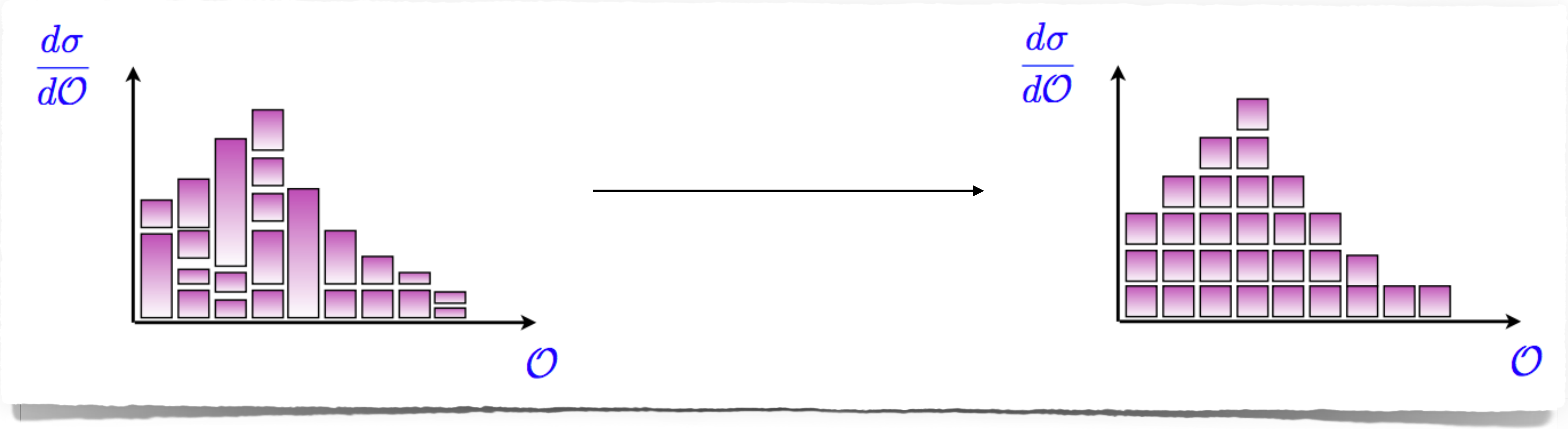
Unweighted events



Unweighted events in practice

Principle of unweighting

- Determination of a threshold during the integration phase ω_{max}
- Determination of the average weight $\langle\omega\rangle = \sigma_{tot}/N_{events}$
- Accept/reject: acceptation with a probability $\omega(\Phi)$ / ω_{max}
- Each event is assigned the weight $\langle\omega\rangle$



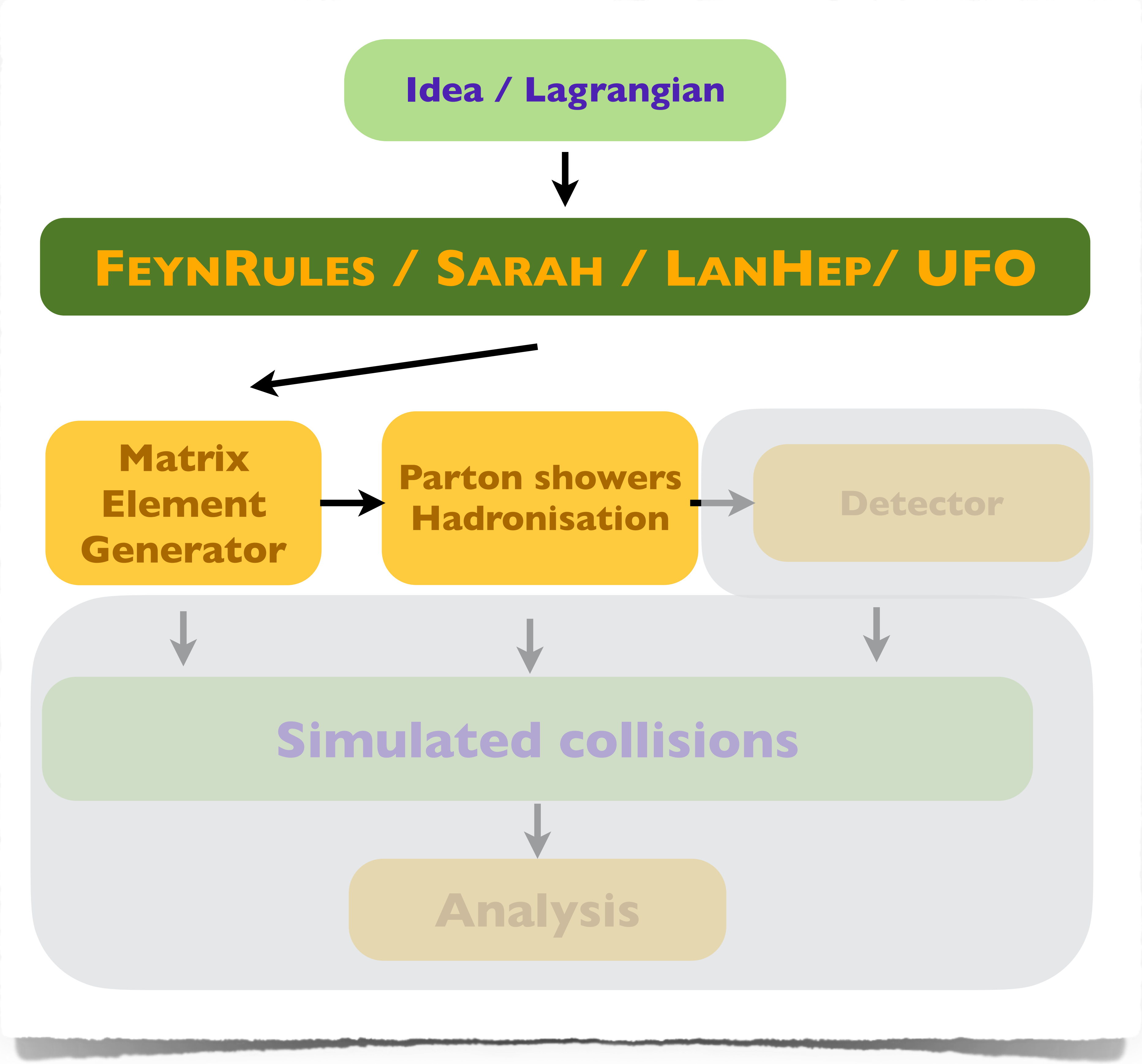
Requirements

- Integrand bounded from above (ω_{max} must exist)
- Integrand positive-definite (bypassable)

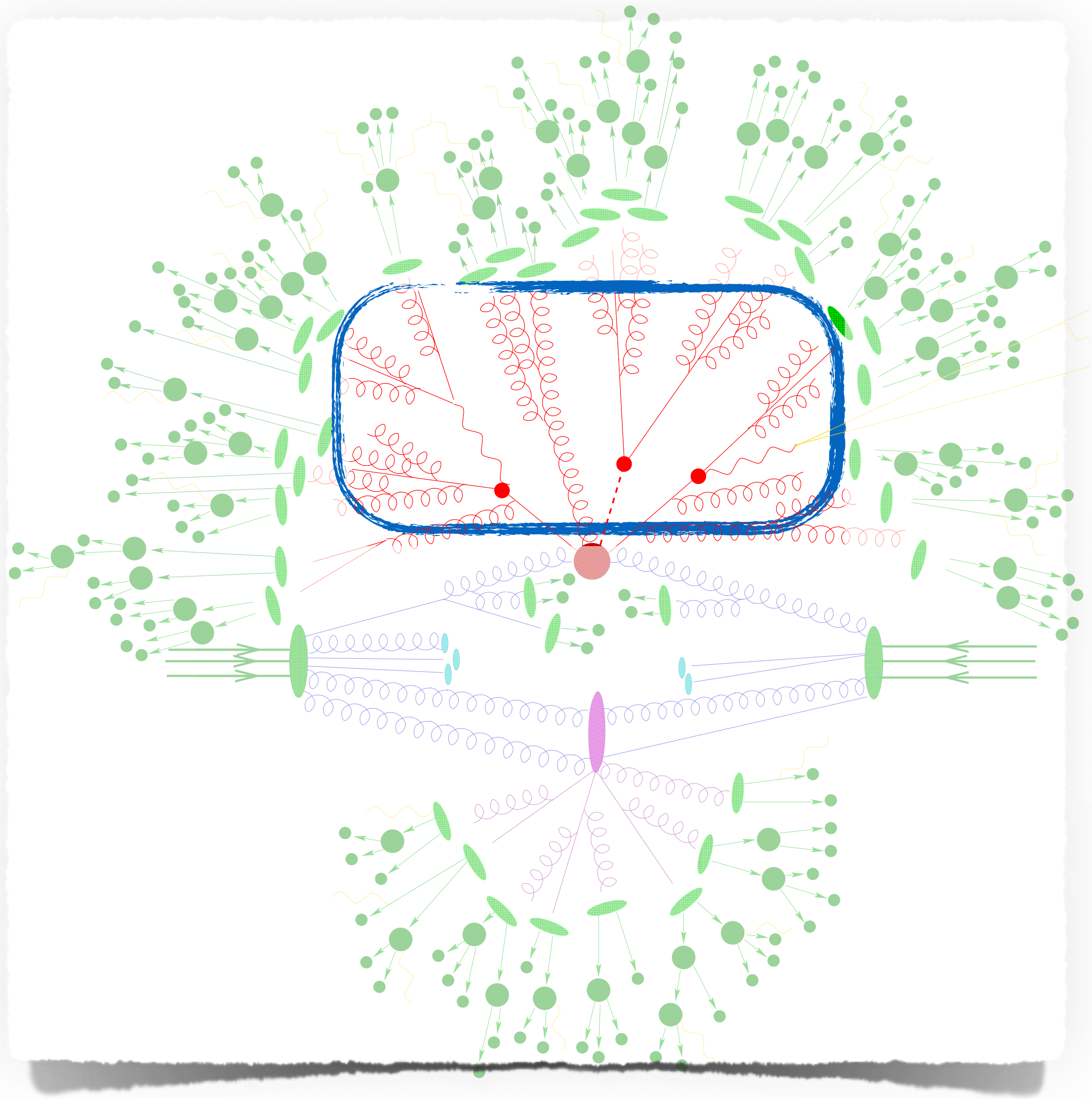
Connecting ideas to simulations…

[Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC`11)]

Connecting ideas to simulations (and cross section calculations)



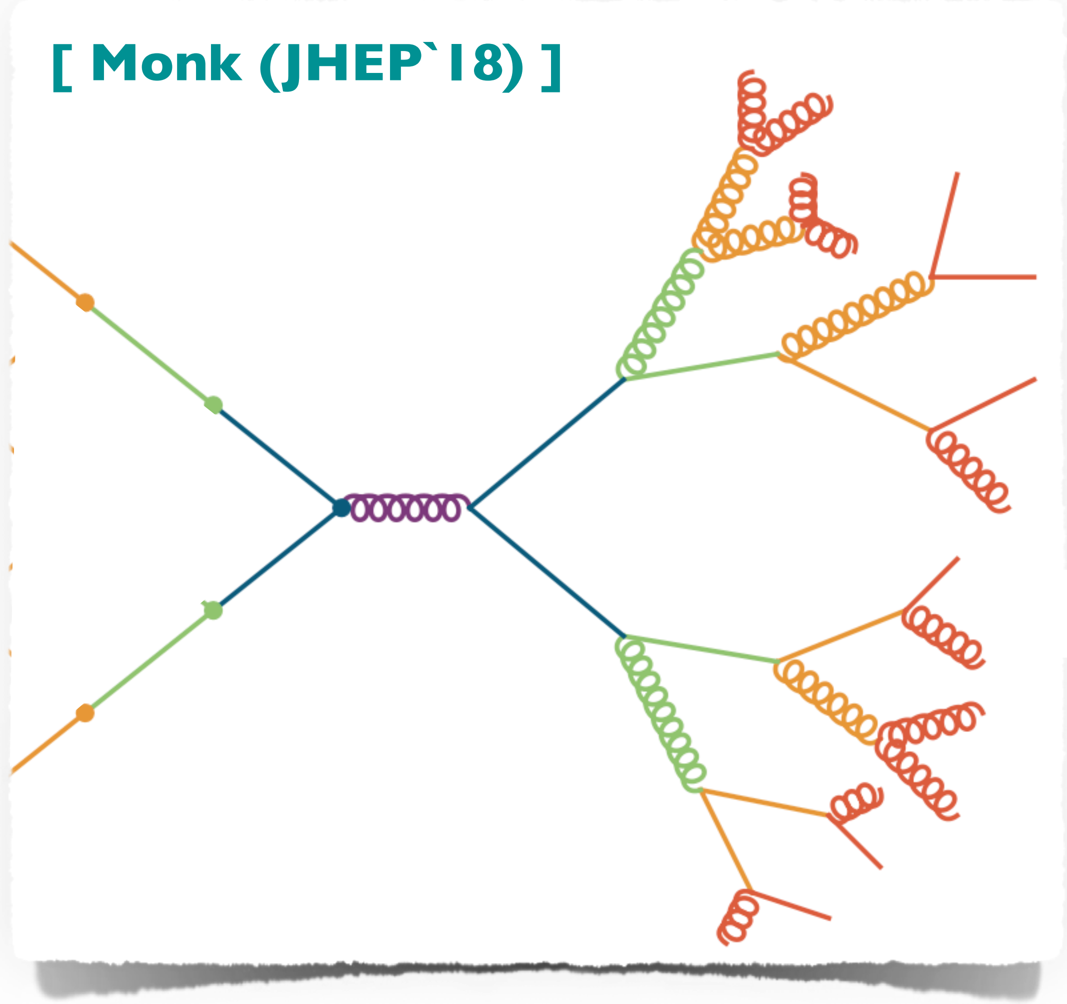
- QCD environment
  - ⋆ Parton showering
  - ⋆ Hadronisation
  - ⋆ Underlying event



Parton evolution - generalities

Radiation of accelerated charges

- TeV scale: large moment transfers → lot of radiation
- QED radiation
  - ⋆ Electrically-charged particles radiate photons
  - ⋆ Photons can split into a (charged) fermion-antifermion pair
- QCD radiation
  - ⋆ Quarks can radiate gluons
  - ⋆ Gluons can split into a quark-antiquark or a gluon pair (QCD is non-Abelian)

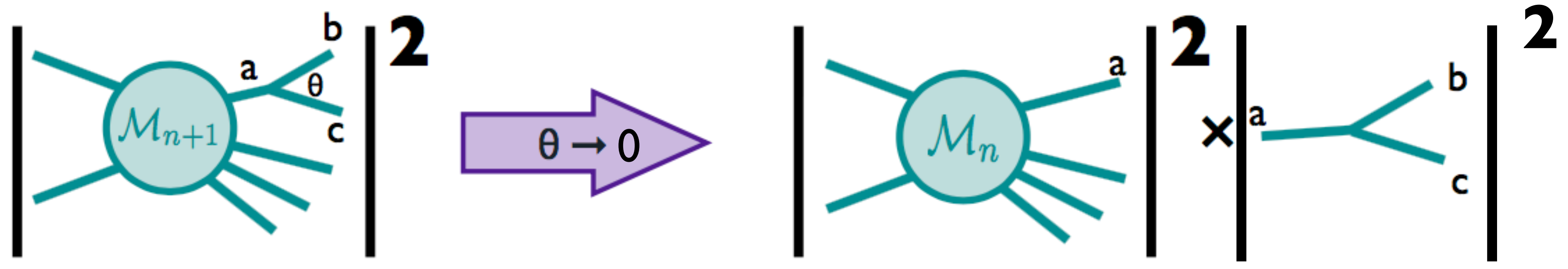


[Monk (JHEP`18)]

Parton showering

- Dressing of each high-energetic coloured particle
- Arbitrary large number of partons (multiple radiation)
  - ⋆ Radiated partons radiate → cascade of radiations
- QCD factorisation properties
  - ⋆ Universal soft and collinear radiation
    - → The *n*+*1* emission independent of the history
    - → Markov chain

$$\mathrm{d}\sigma_{2\to 3} \propto \sigma_{2\to 2} \sum_{i=q,\bar{q}} C_F \frac{\alpha_s}{2\pi} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{1+z^2}{1-z}\, \mathrm{d}z$$



The parton shower algorithm

$$\phi_a(t,t_0) = \Delta_a(t,t_0) + \sum_b \int_{t_0}^{t} \frac{\mathrm{d}t'}{t'} \mathrm{d}z\ \Delta(t,t') \frac{\alpha_s(t')}{2\pi} P_{ab}(z)\ \phi_b(t', z t_0)\phi_c(t', (1-z)t_0)$$

Two ingredients

- The splitting functions P_{ab}
- The Sudakov form factor Δ (no-emission probability)

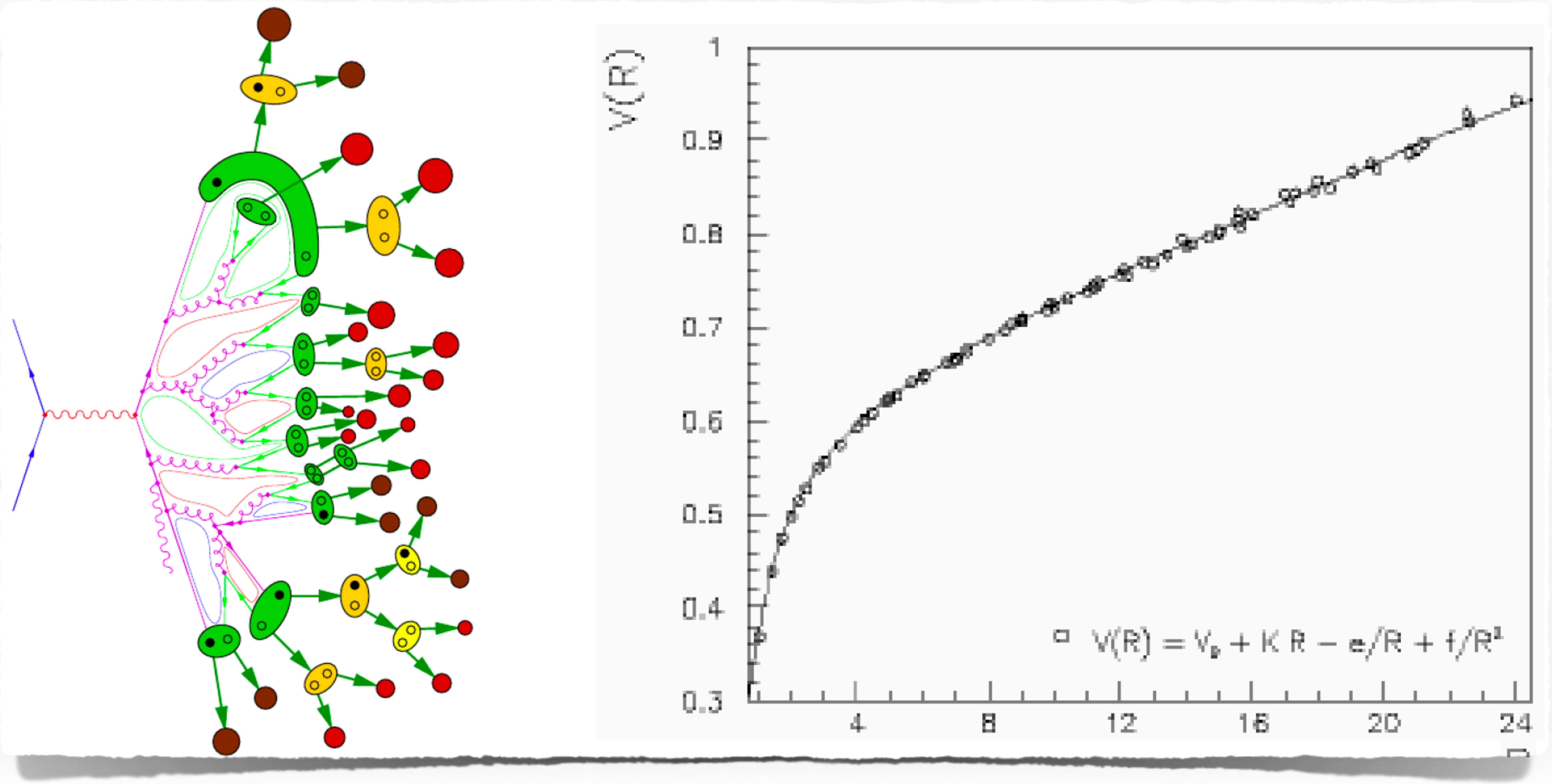
The algorithm

- Starting point: a parton a scale t_0
- Generation of an emission scale t_I from the Sudakov probability $\Delta_a(t_0,t_I)$
  - ⋆ If t_I < t_{cut}: stop (t_{cut} ≡ break down of perturbative QCD)
  - ⋆ If t_I > t_{cut}: generation of z_I according to $P_{ab}(z)$
    → one extra final-state parton
- Iteration until stop for all partons

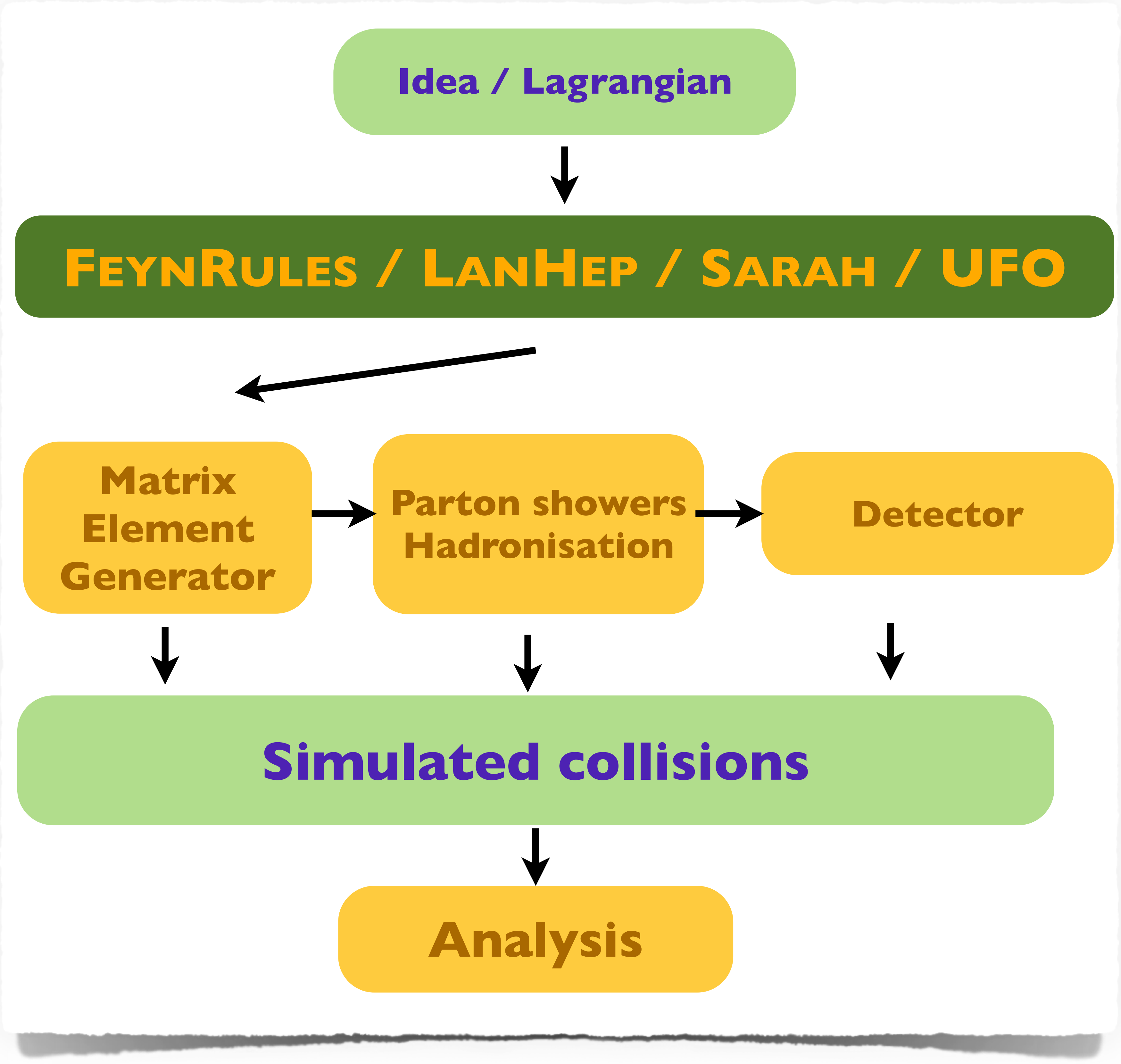
Hadronisation

Models

- Break down of perturbative QCD around 1 GeV
- Non-perturbative models: from partons to hadrons
  - ⋆ The Lund string model **[Andersson, Gustafson, Ingelmanm & Sjöstrand (PR'83)]**
    - ➙ Uniform string tension between colour charges
    - ➙ Pair creation at large distance
    - ➙ String broken down into hadrons
  - ⋆ The cluster model **[Webber (NPB'84)]**
    - ➙ Pre-confinement: colour singlet from colour-adjacent partons
    - ➙ Non-perturbative gluon splittings in $q\bar{q}$ pairs
- Inclusion of hadron decays



Predictions at colliders



Idea / Lagrangian

FeynRules / LanHEP / SARAH / UFO

Matrix Element Generator

Parton showers Hadronisation

Detector

Simulated collisions

Analysis

Event simulation is a complex process

- Factorised it into separate parts
- Event simulation performed step-by-step

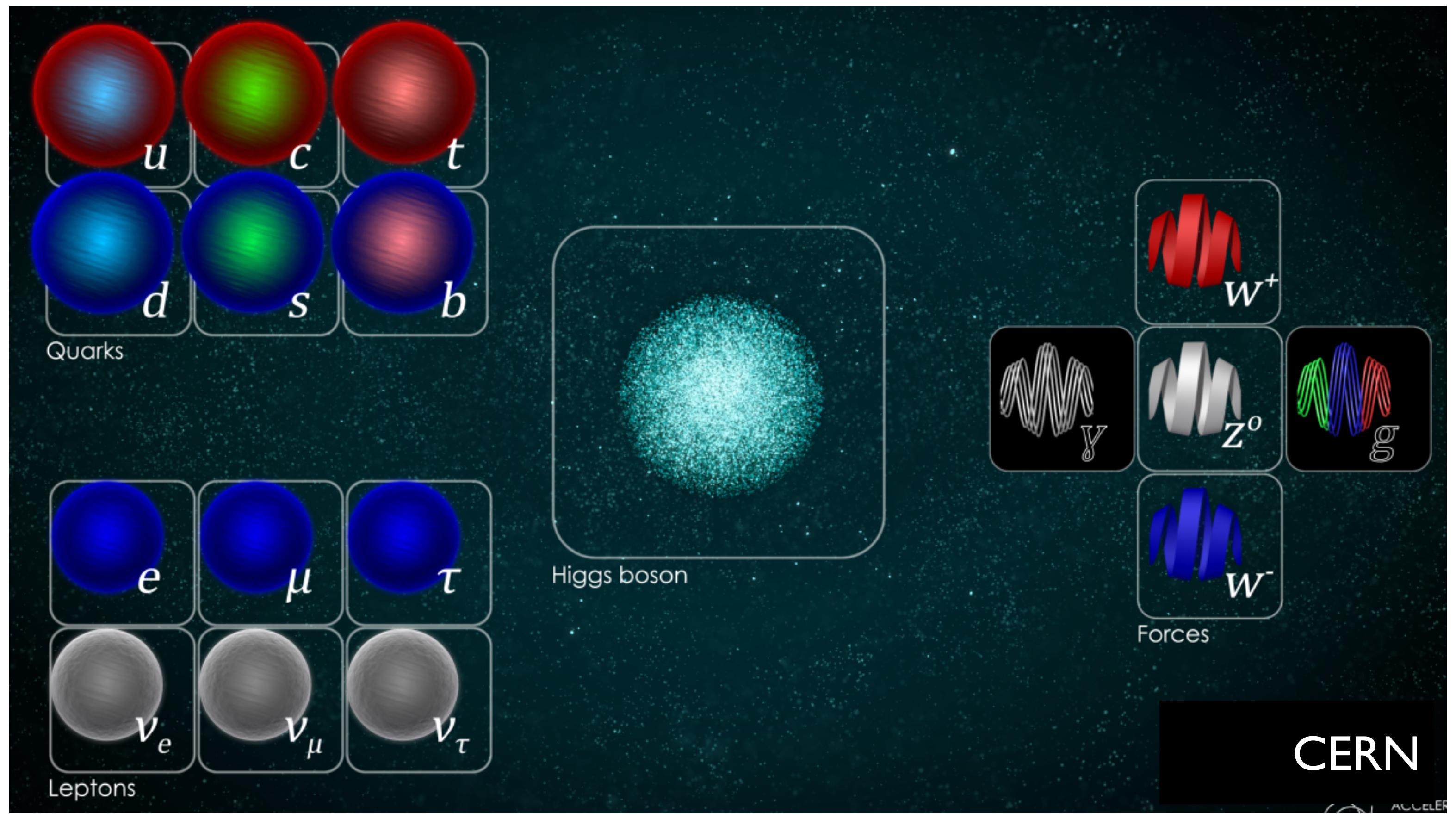
1st parts of the chain

- Connecting models (Lagrangians) to tools
- Generation of matrix elements
- Heavy particle decays
- Cross section calculations
- Event generation

Next steps

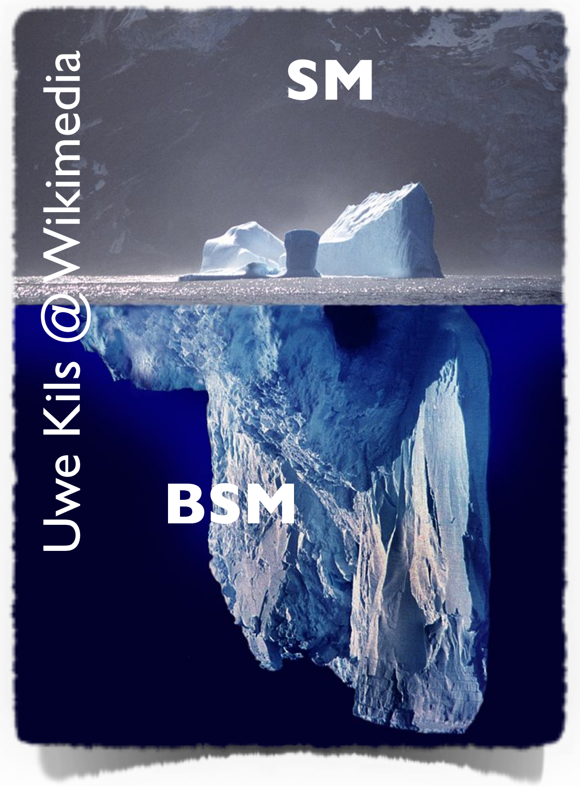
- QCD environment: parton showering, hadronisation
- Detector simulation
- Signal analysis
- Comparison with data / phenomenological study

Summary



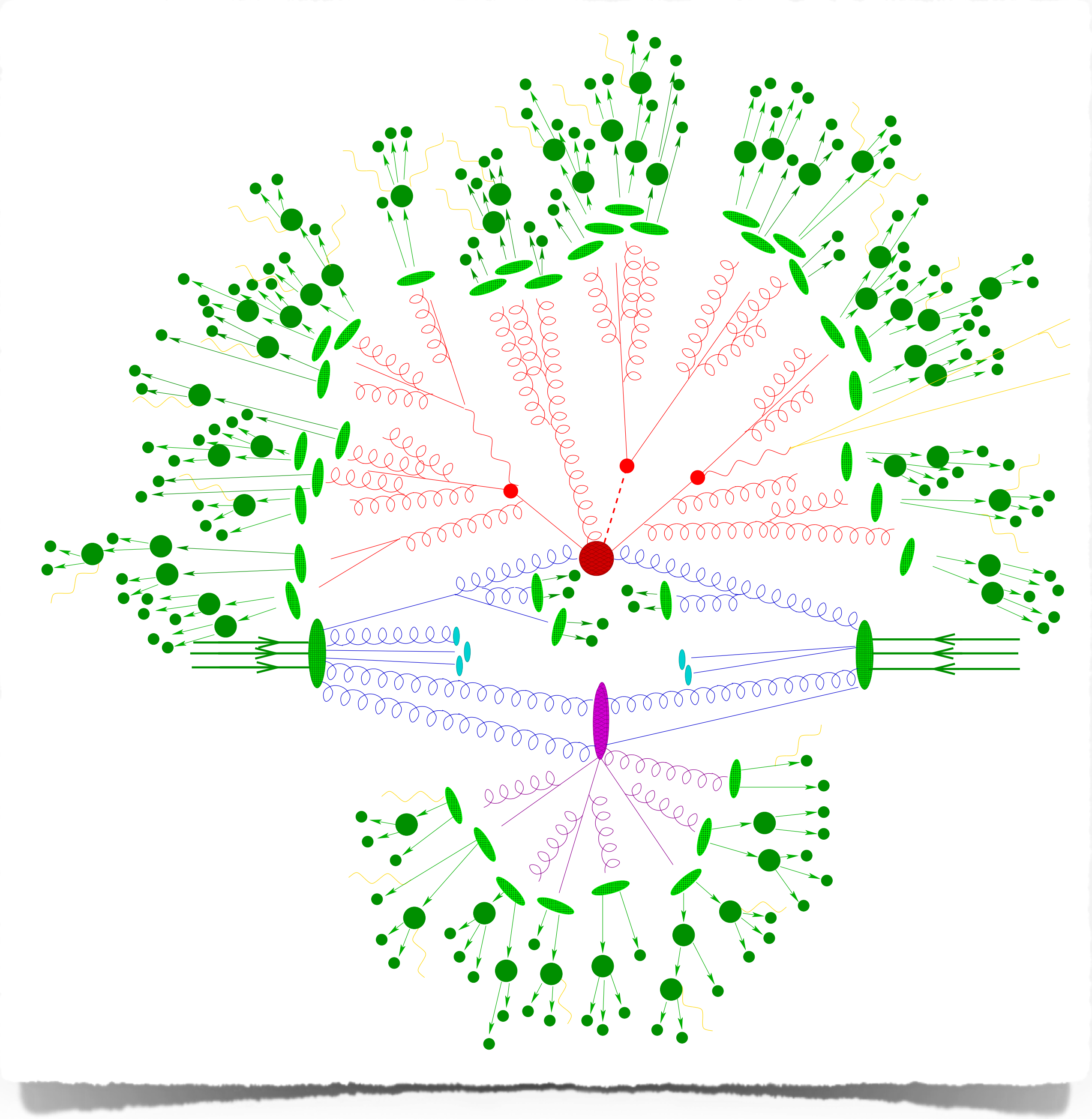
The Standard Model ≡ incredible description of the microscopic world

- Hunts for anomalies and deviations
- Parts of the model untested for decades
  → New phenomena



Standard Model ≡ tip of the iceberg

- Dark matter, neutrinos, hierarchy problem, *etc.*
- New physics = core of the high-energy research programme (theory and experiment)



Collider event simulation is one way

- Advanced tools to relate theory and experiment
- Next week: how to relate experiment to theory!