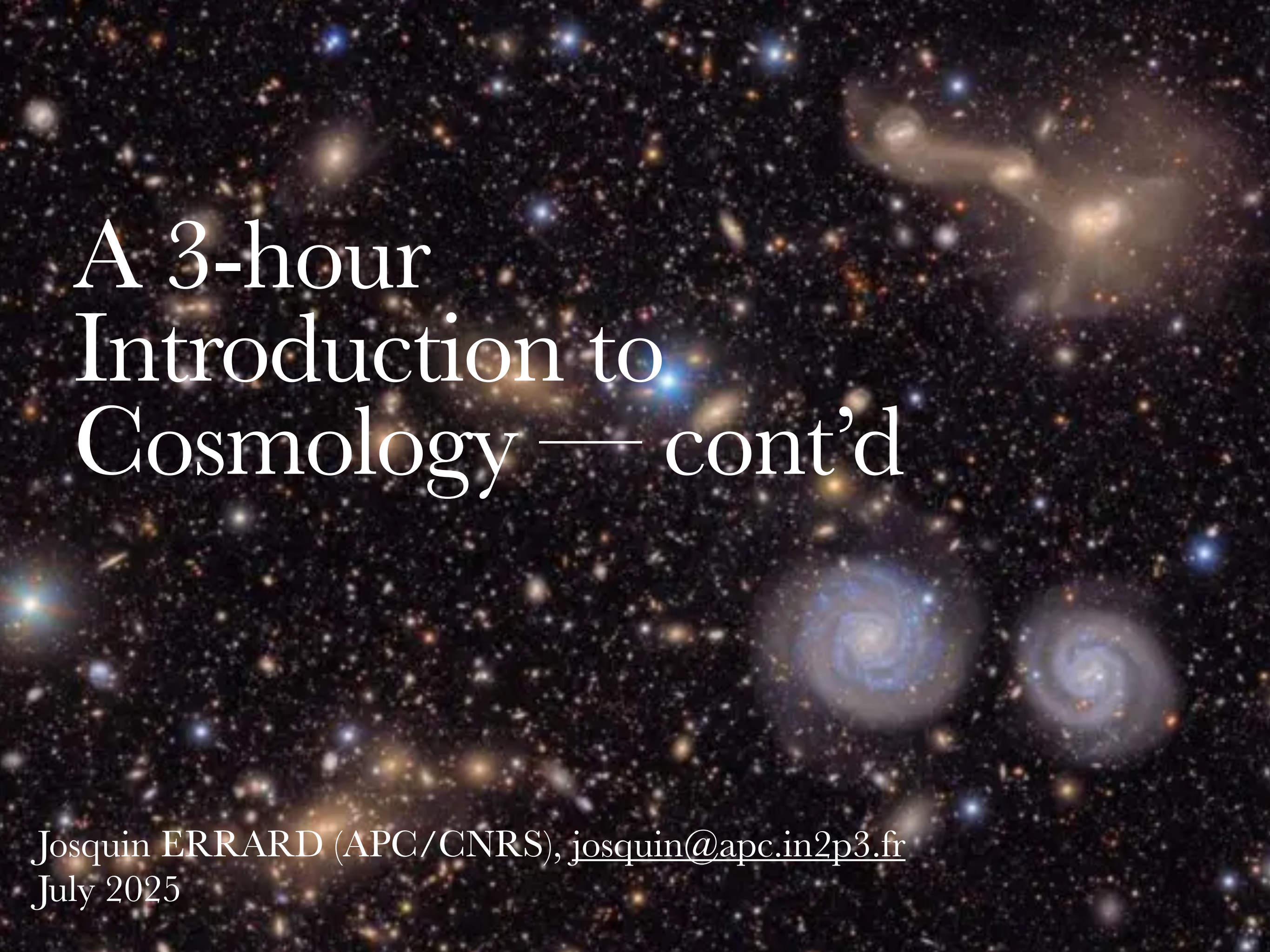


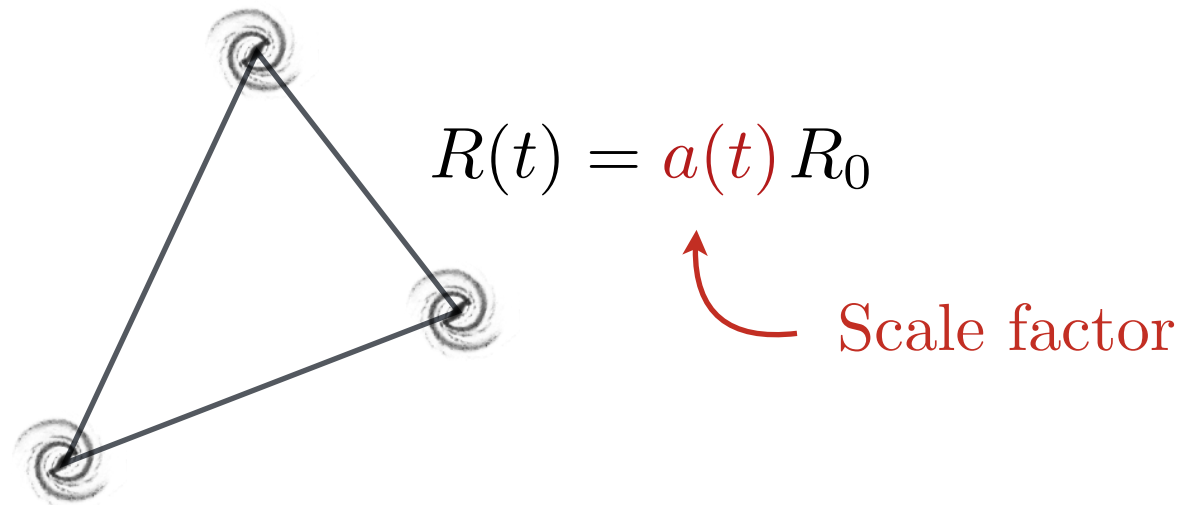


A 3-hour Introduction to Cosmology — cont'd

Josquin ERRARD (APC/CNRS), josquin@apc.in2p3.fr
July 2025

Recap of Lecture 1

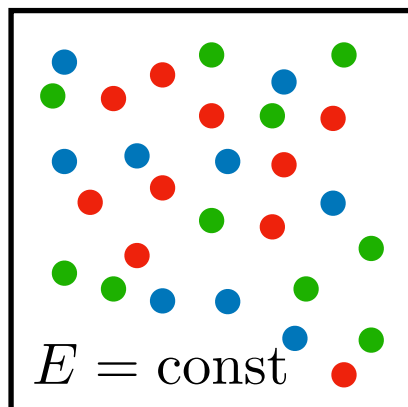
- The Universe is expanding:



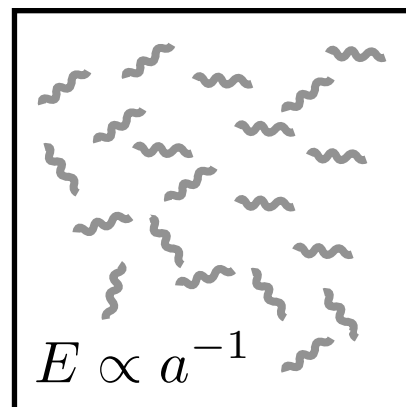
- The rate of expansion is determined by the Friedmann equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho$$

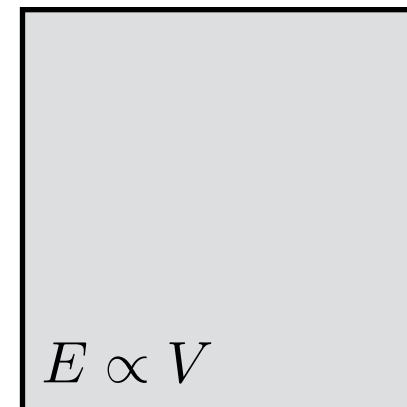
- The energy density of matter, radiation and dark energy dilutes as

$$\rho_m \propto a^{-3}$$


$E = \text{const}$

$$\rho_r \propto a^{-4}$$


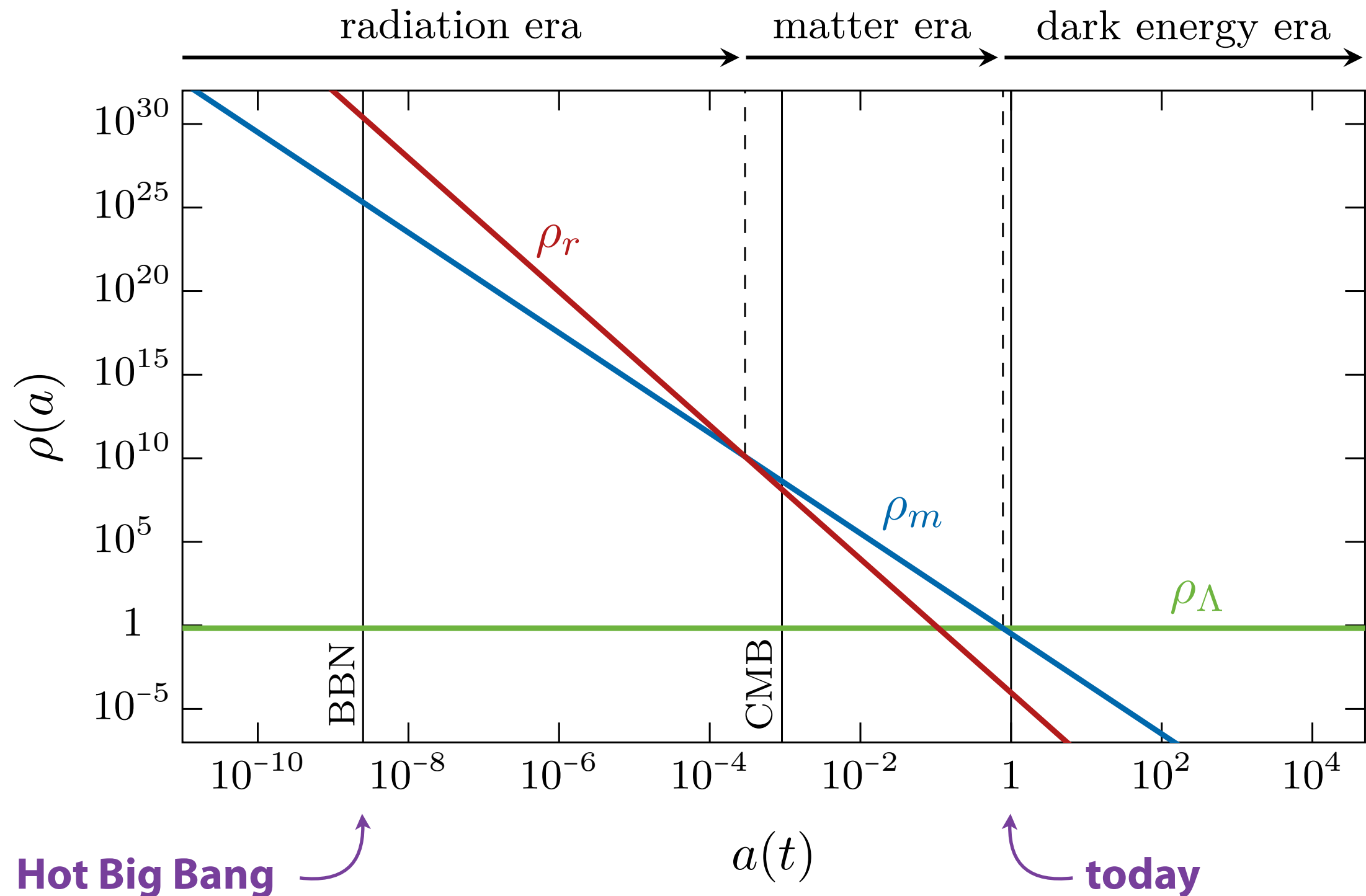
$E \propto a^{-1}$

$$\rho_\Lambda \propto a^0$$


$E \propto V$

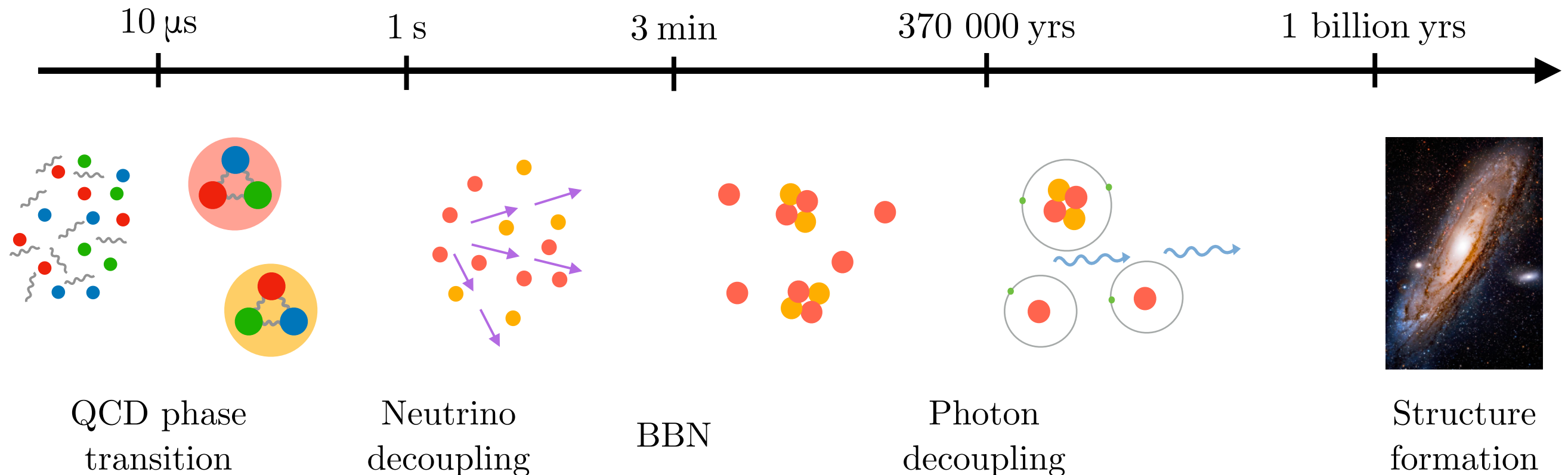
Recap of Lecture 1

- The Universe started hot and dense, but then cooled and diluted:

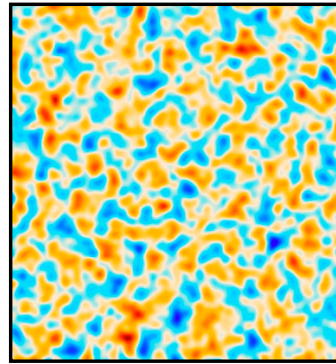
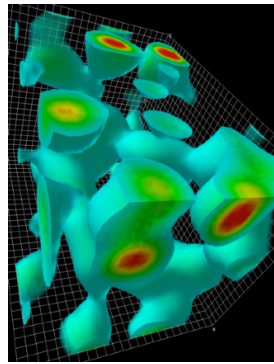
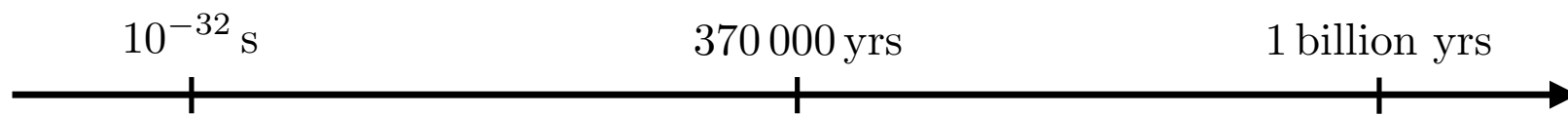


Recap of Lecture 1

This history of the Universe is an observational fact:



- The basic picture has been confirmed by many independent observations.
- Many precise details are probed by measurements of the CMB.



Recap of Lecture 1

Quantum fluctuations



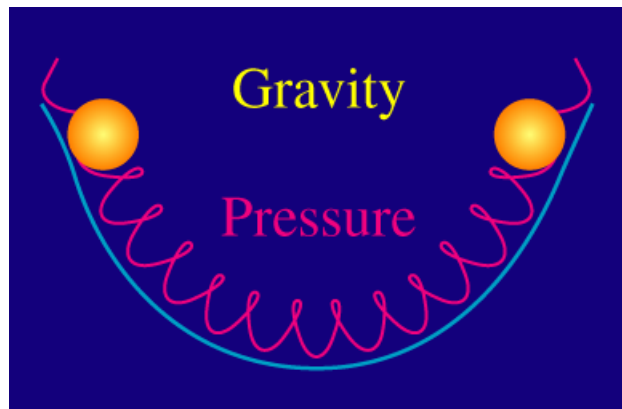
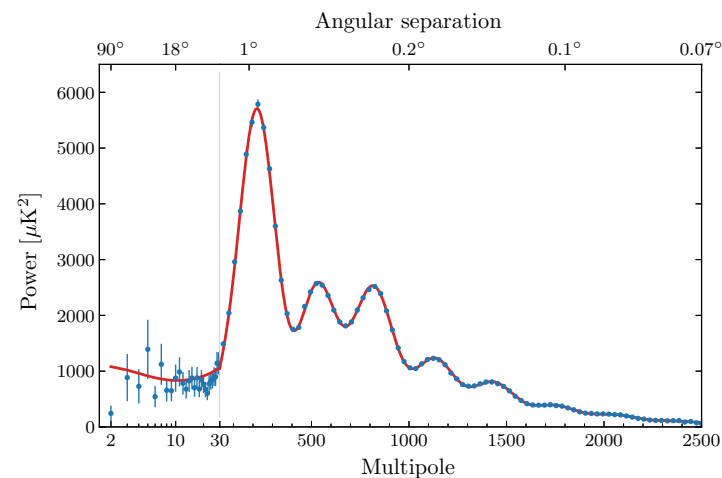
CMB fluctuations



Galaxies

Gravitational clustering

Cosmic sound waves



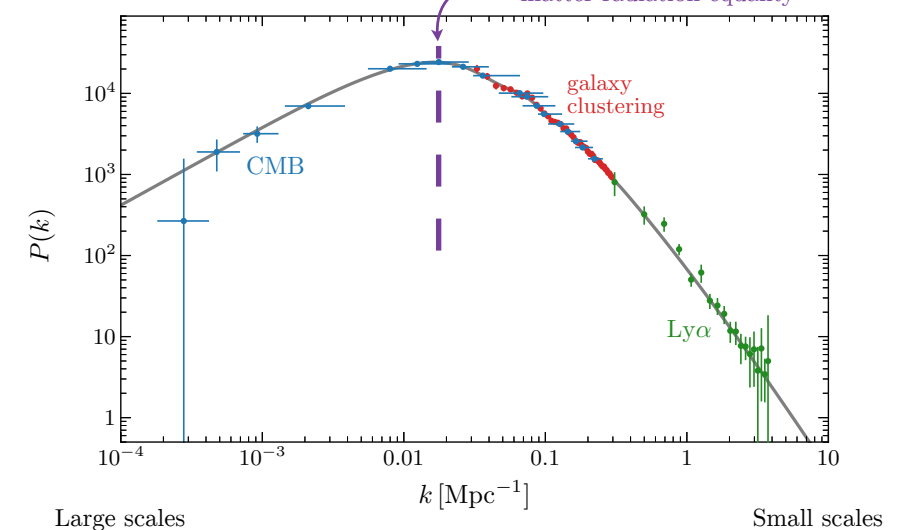
$$\ddot{\delta} + 2H\dot{\delta} = (4\pi G\bar{\rho})\delta$$

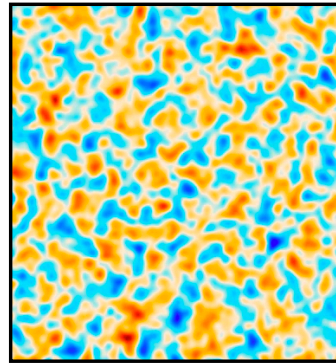
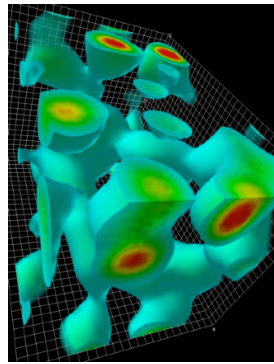
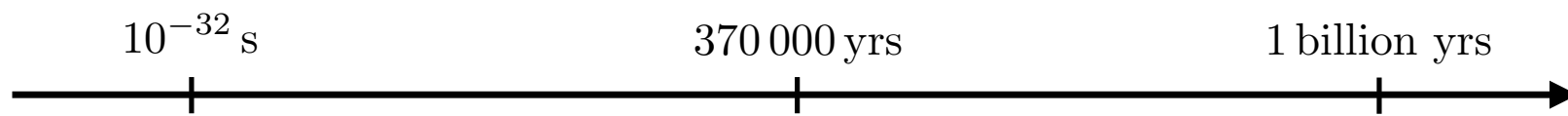
Gravitational force

Hubble friction

Power Spectrum

The observed **matter power spectrum** is





Recap of Lecture 1

Quantum fluctuations



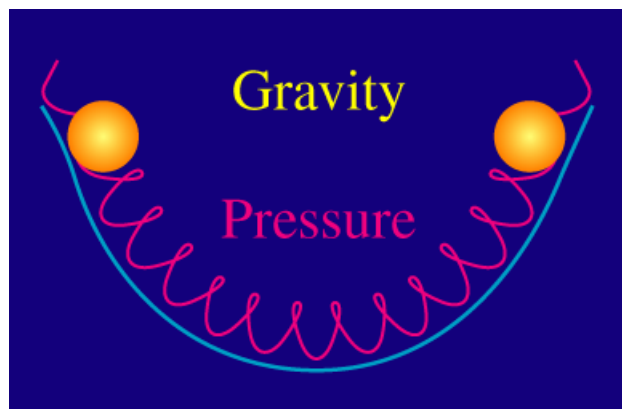
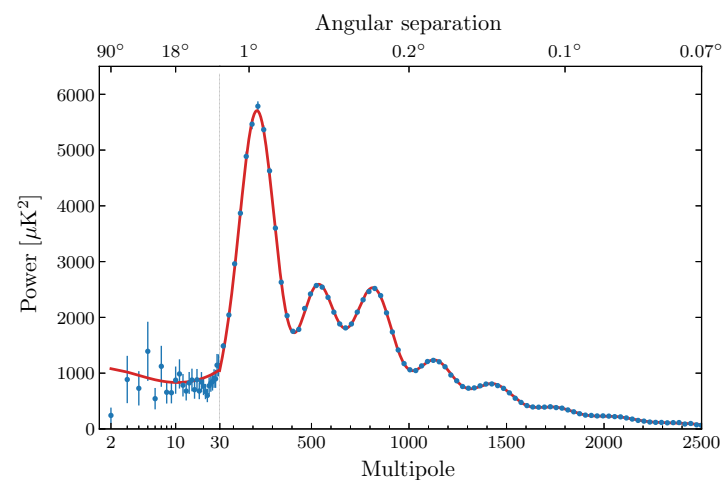
CMB fluctuations



Galaxies

Gravitational clustering

Cosmic sound waves



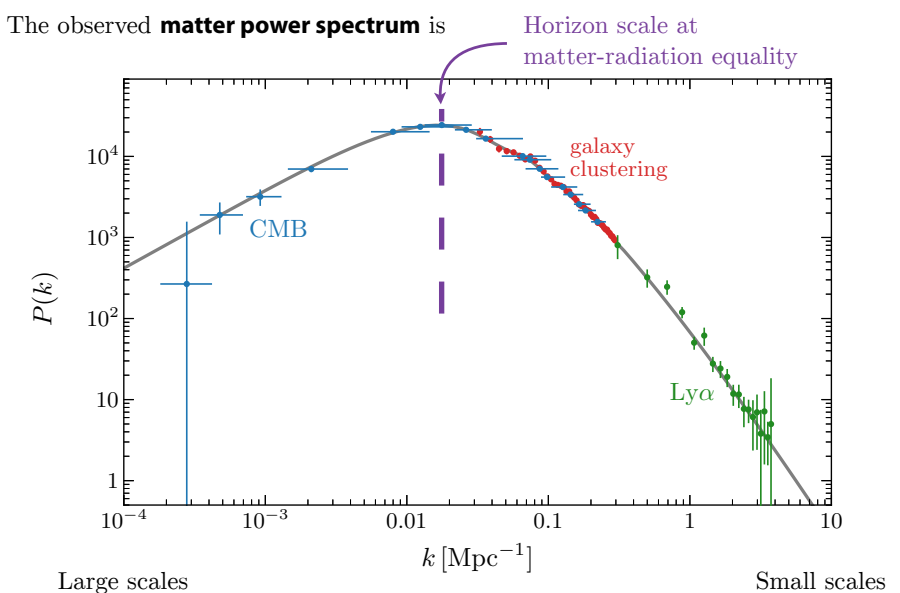
$$\ddot{\delta} + 2H\dot{\delta} = (4\pi G\bar{\rho})\delta$$

Gravitational force

Hubble friction

Power Spectrum

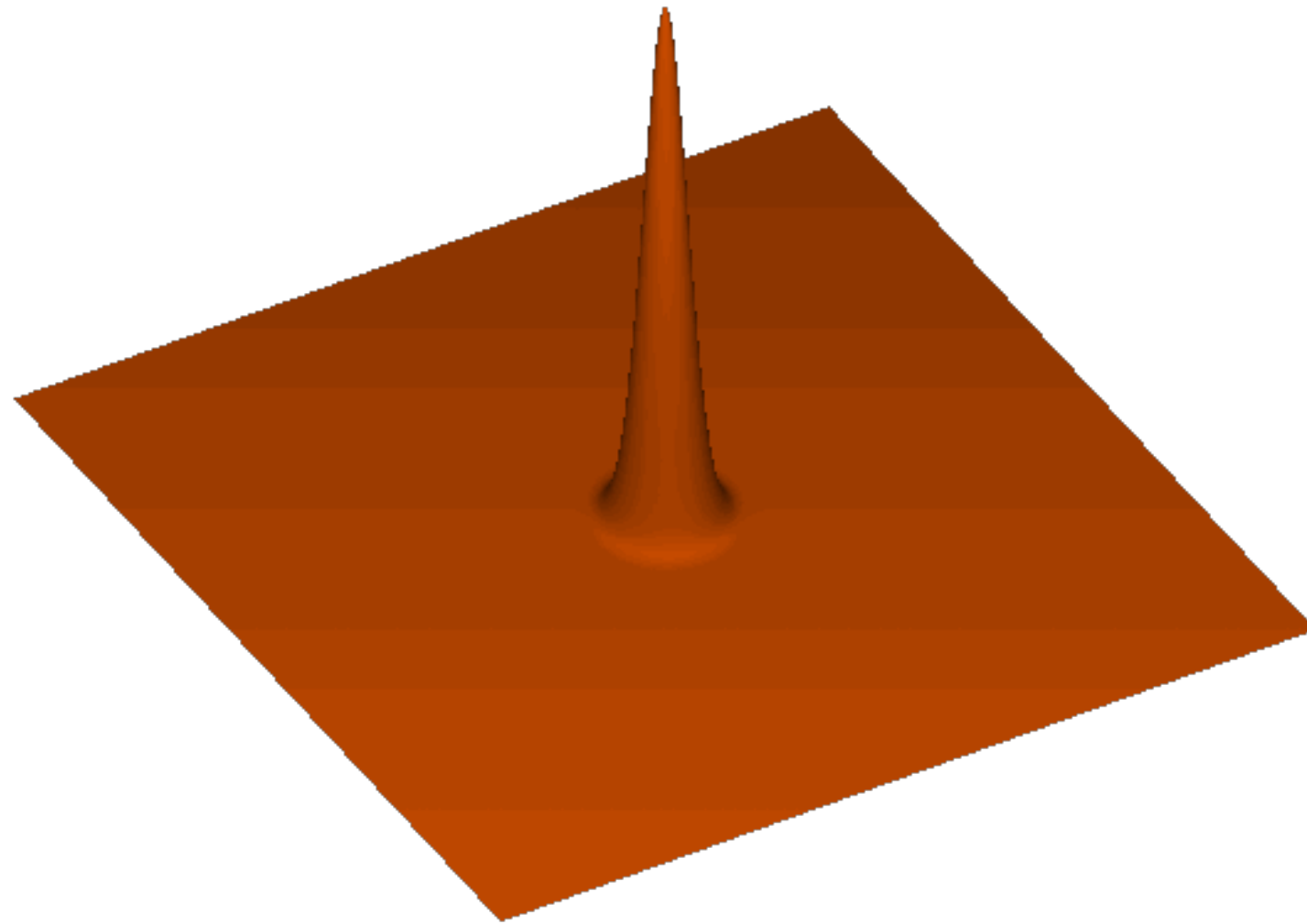
The observed **matter power spectrum** is



Questions?

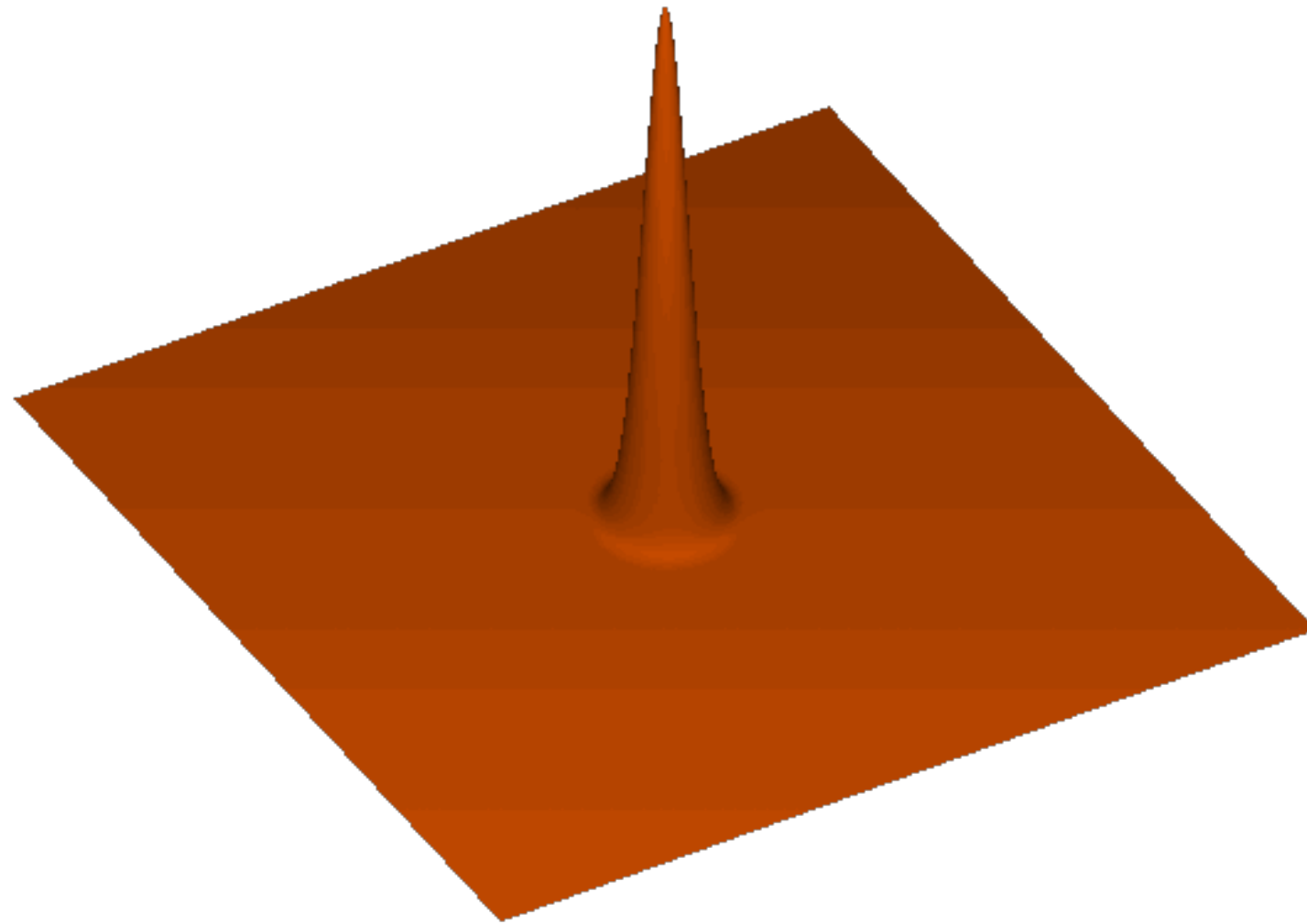
Cosmic Sound Waves

Consider the evolution of a single localized density fluctuation:



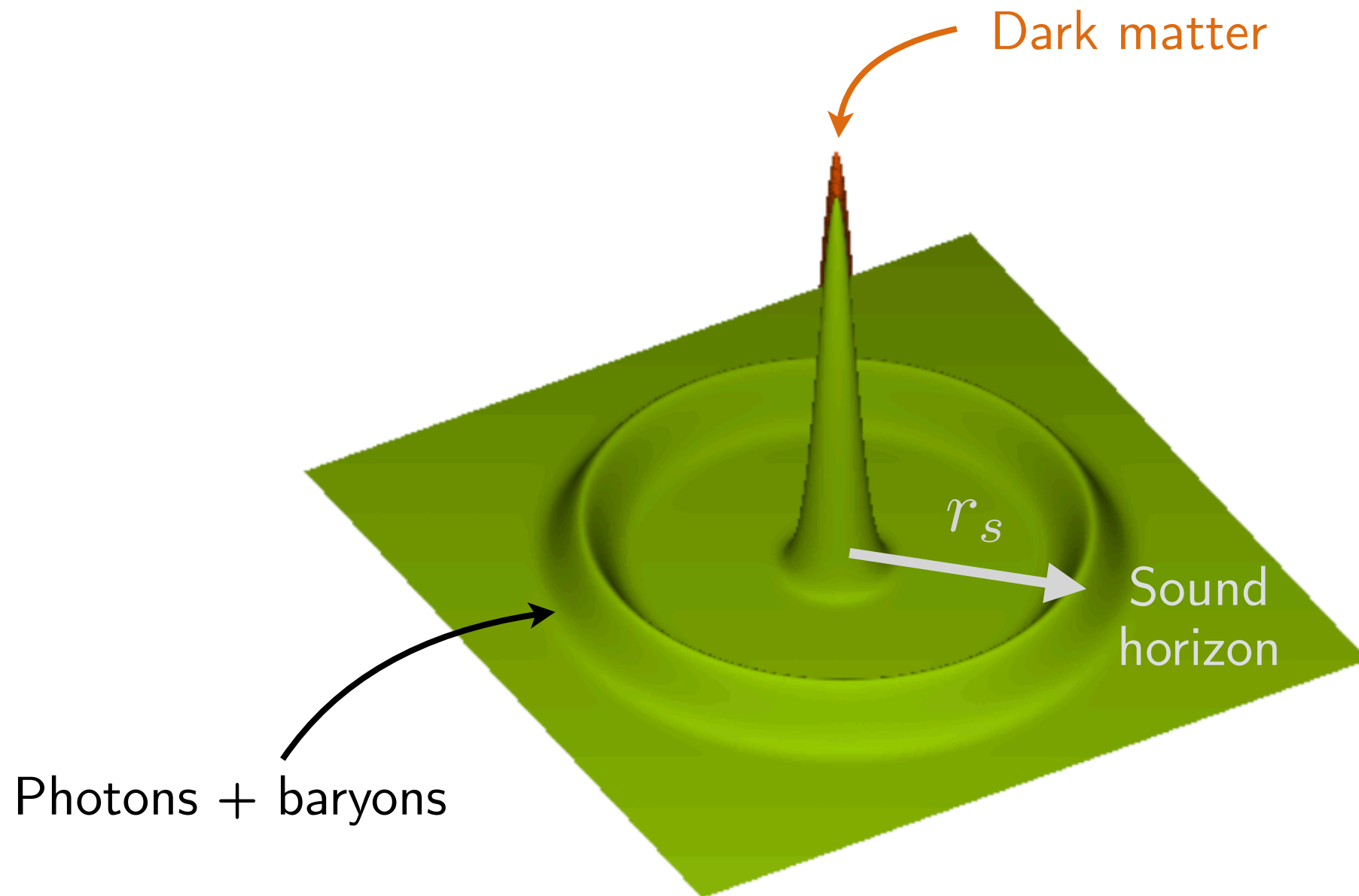
Cosmic Sound Waves

Consider the evolution of a single localized density fluctuation:



Cosmic Sound Waves

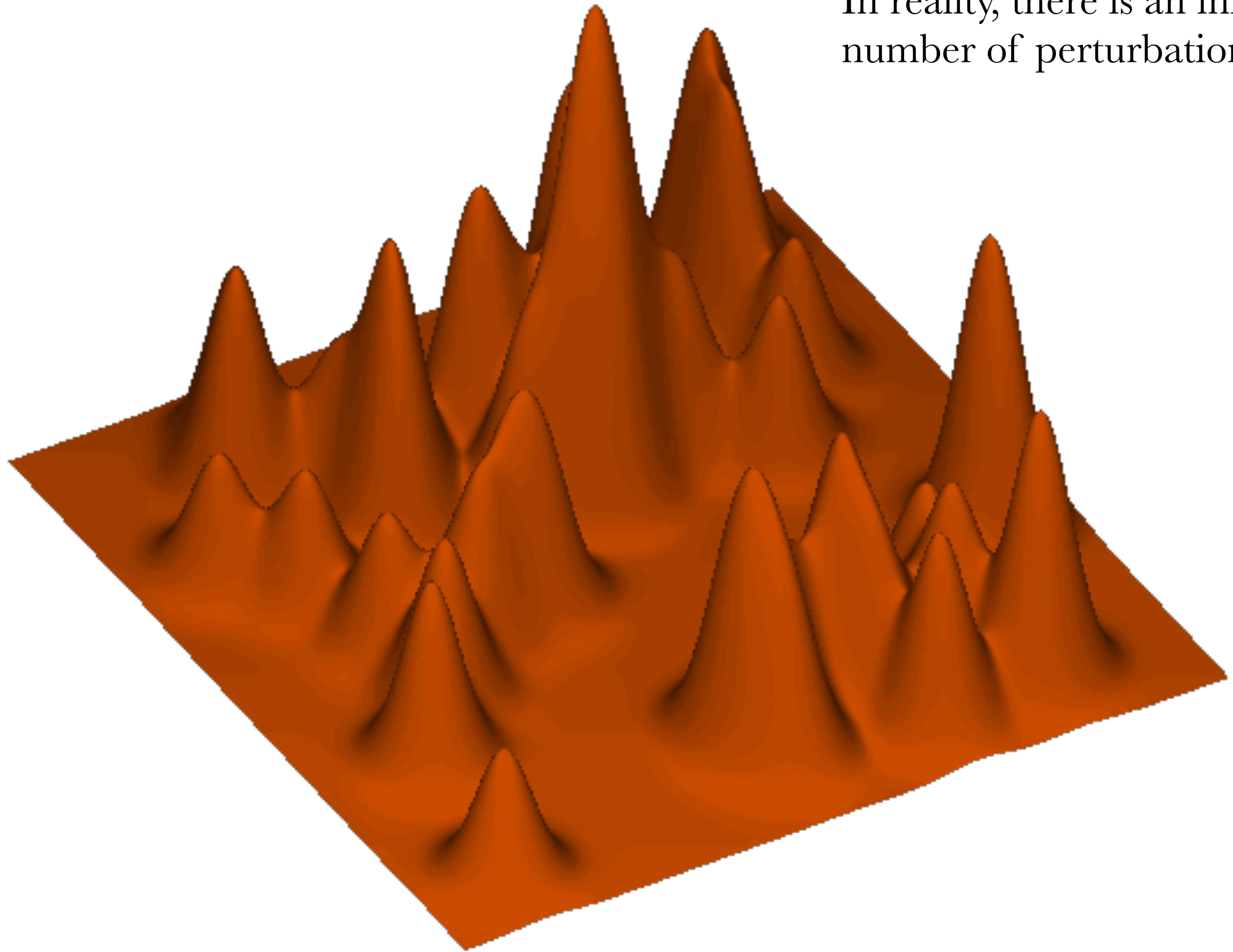
This creates a radial sound wave in the photon-baryon fluid:



The wave travels a distance of 50 000 light-years (called the **sound horizon**) before the Universe becomes transparent to light.

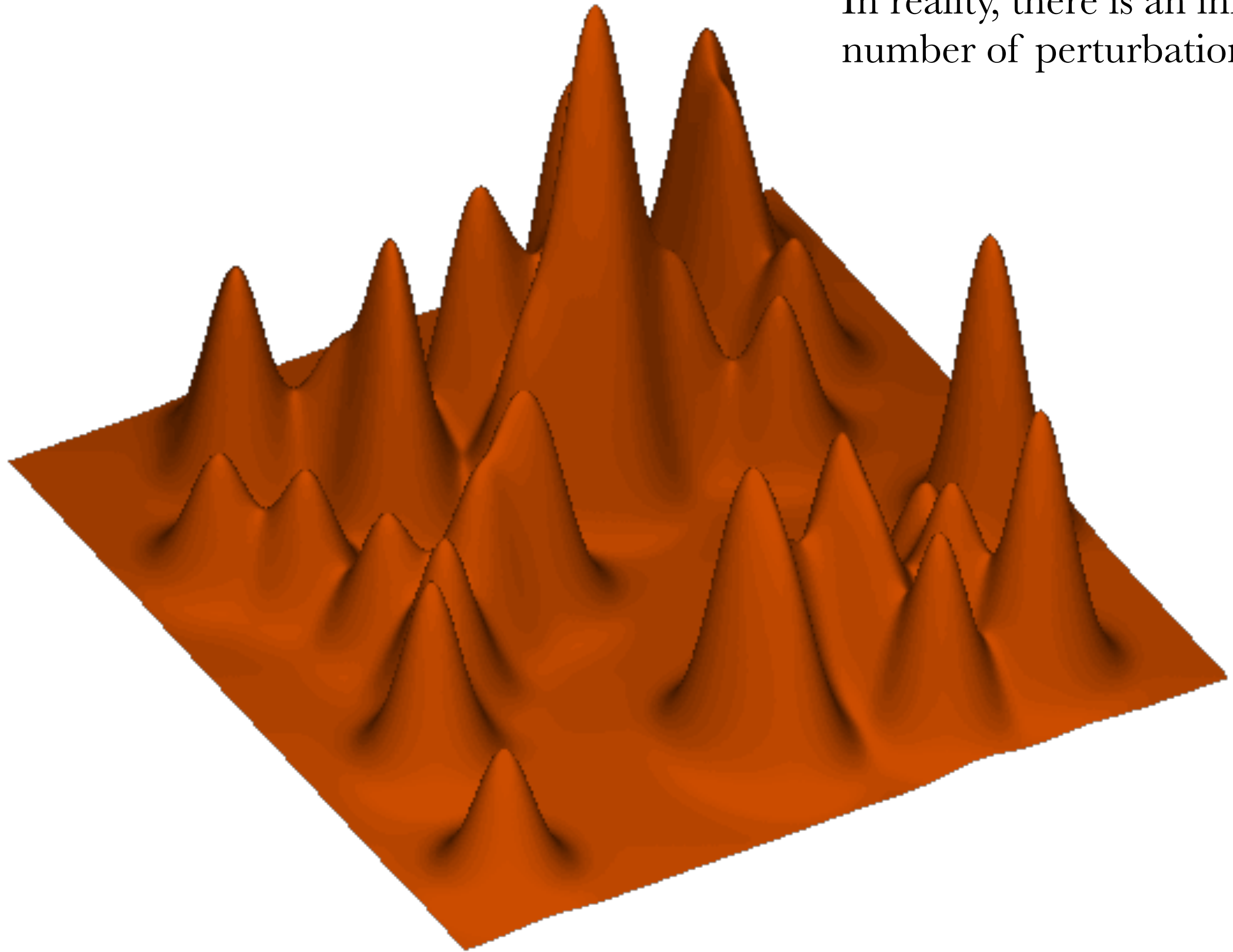
Cosmic Sound Waves

In reality, there is an infinite number of perturbations ...



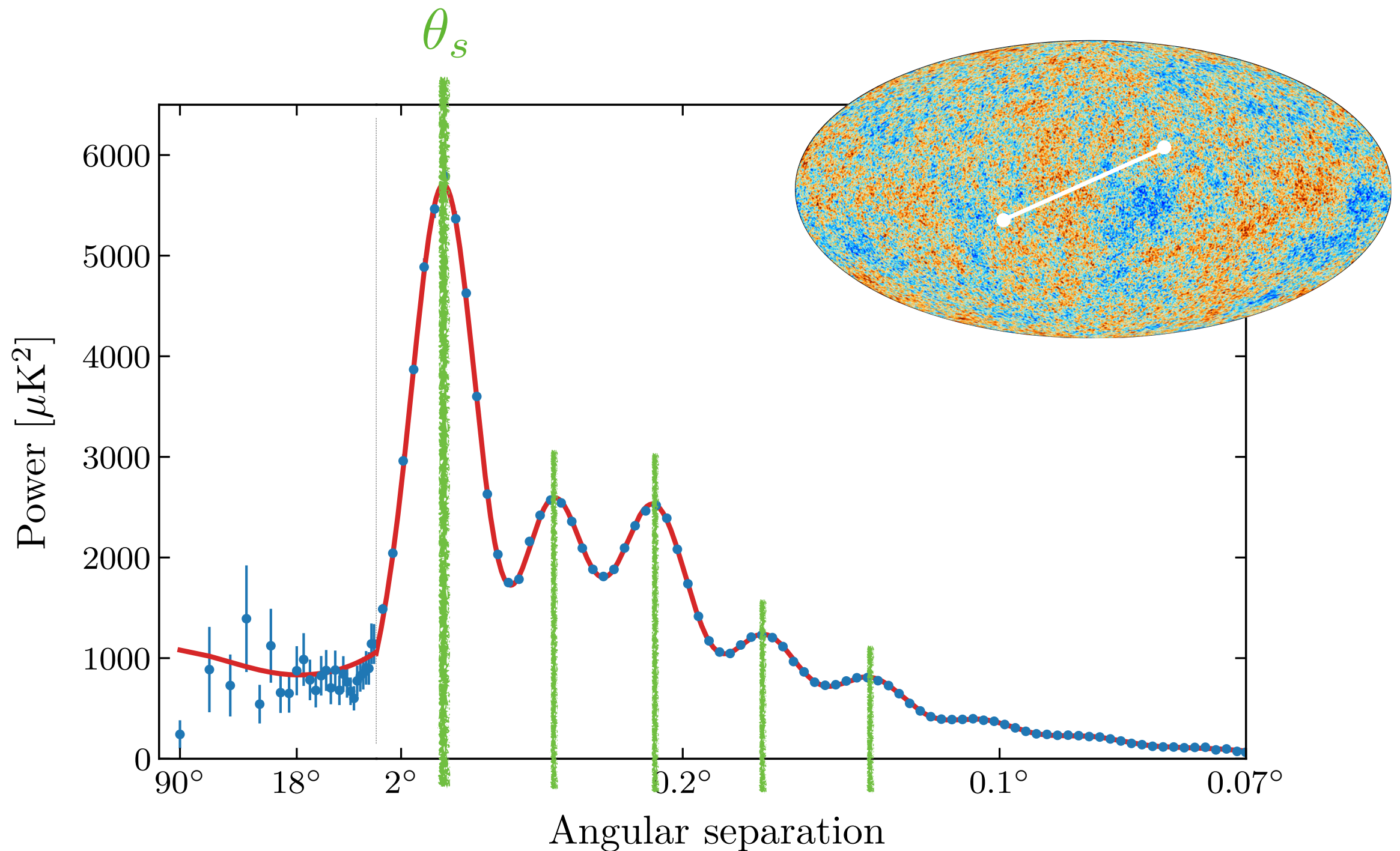
Cosmic Sound Waves

In reality, there is an infinite number of perturbations ...



CMB Anisotropies

This sound horizon is imprinted in the pattern of CMB fluctuations:



Photon–Baryon Fluid and Decoupling

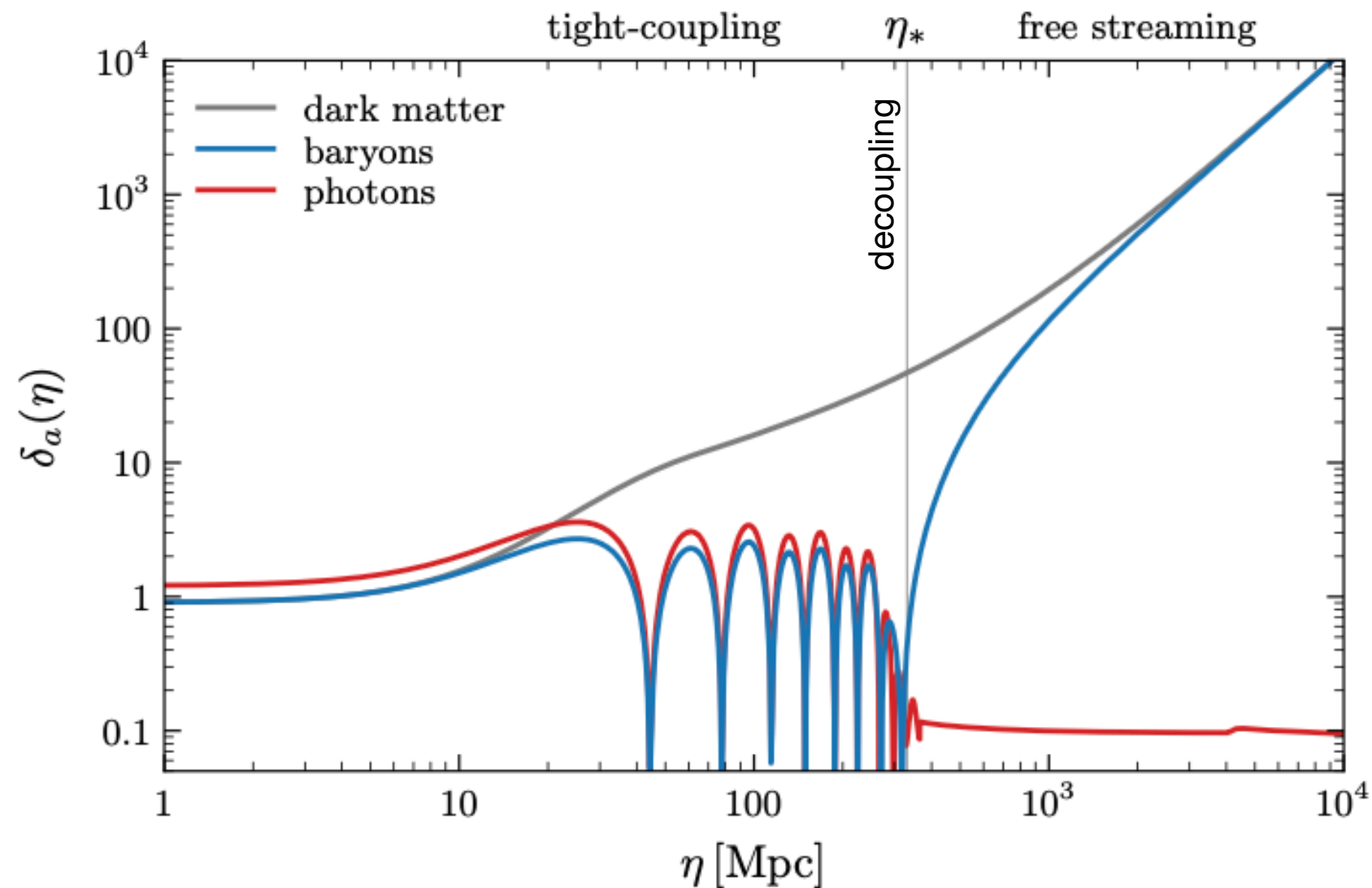


Fig. 6.8

Evolution of the density contrasts of dark matter (*gray*), baryons (*blue*) and photons (*red*) for $k = 0.25 h \text{ Mpc}^{-1}$. Note that around the time of decoupling the fluid approximation breaks down and the evolution is *not* captured by the equations described in this chapter. Instead, this figure was produced by solving the Boltzmann equations for the distribution functions of each species numerically.

Transfer Function of The Photon Density

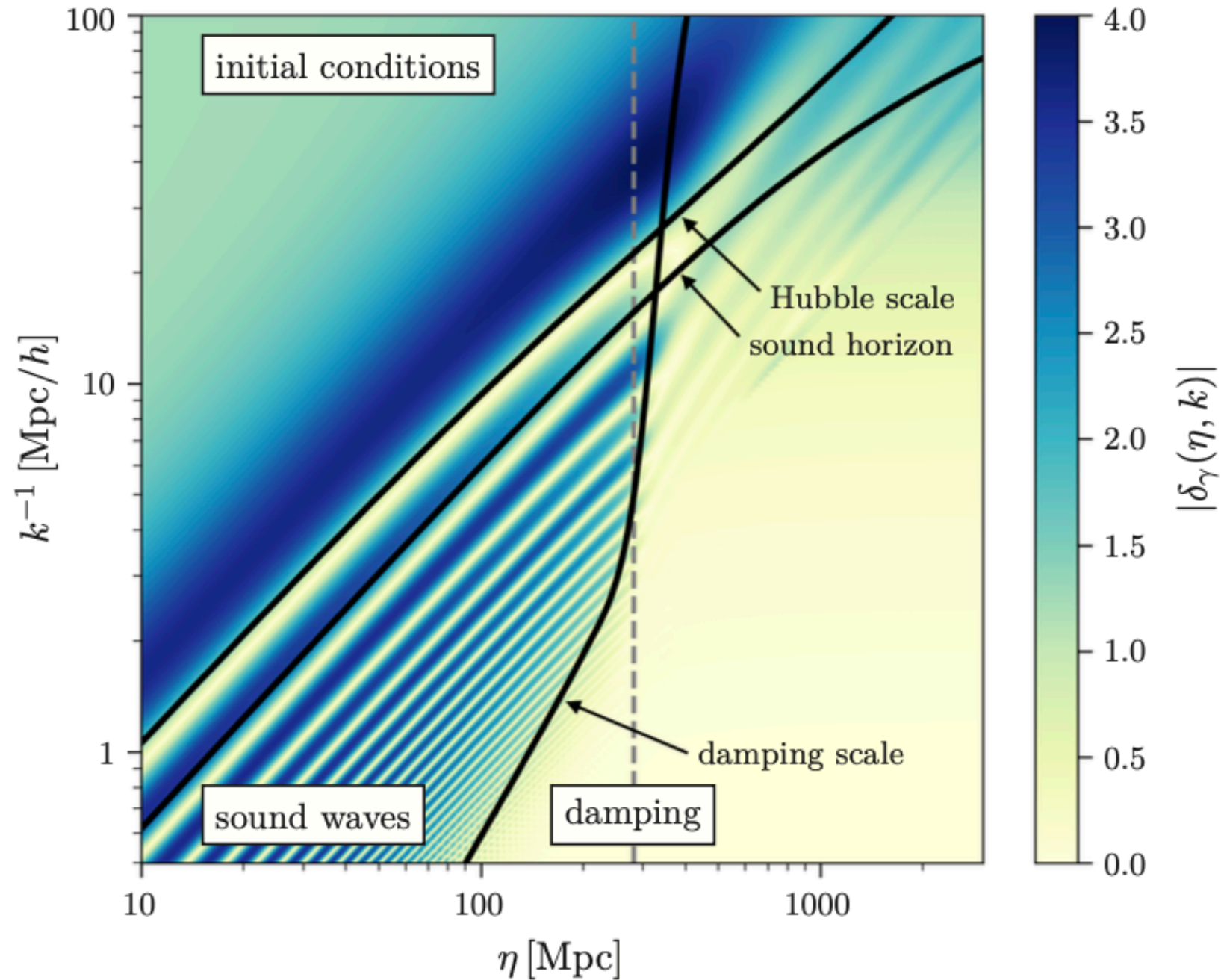


Fig. 7.10

Transfer function of the photon density contrast as a function of k and η , computed with CLASS [12]. Illustrated are also the evolution of the Hubble scale, the sound horizon and the damping scale, and the moment of recombination (dashed line). We see that sound waves propagate below the sound horizon and are suppressed below the damping scale.

Acoustic Peaks

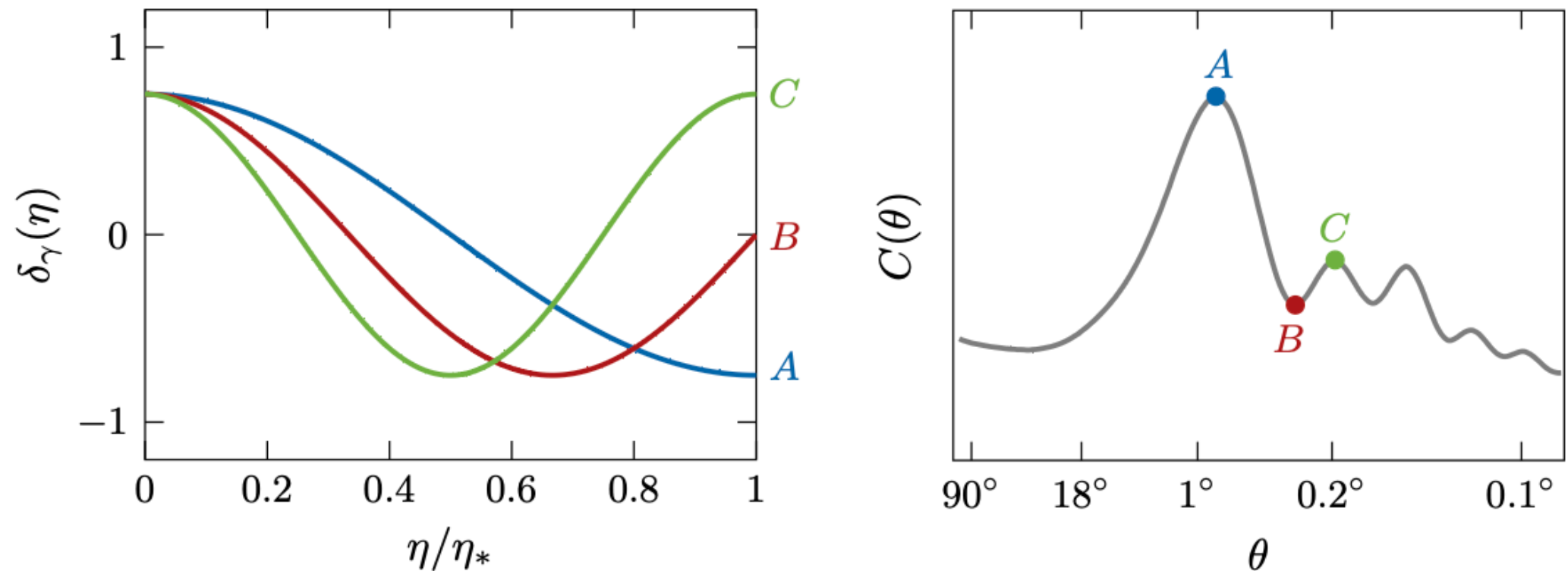


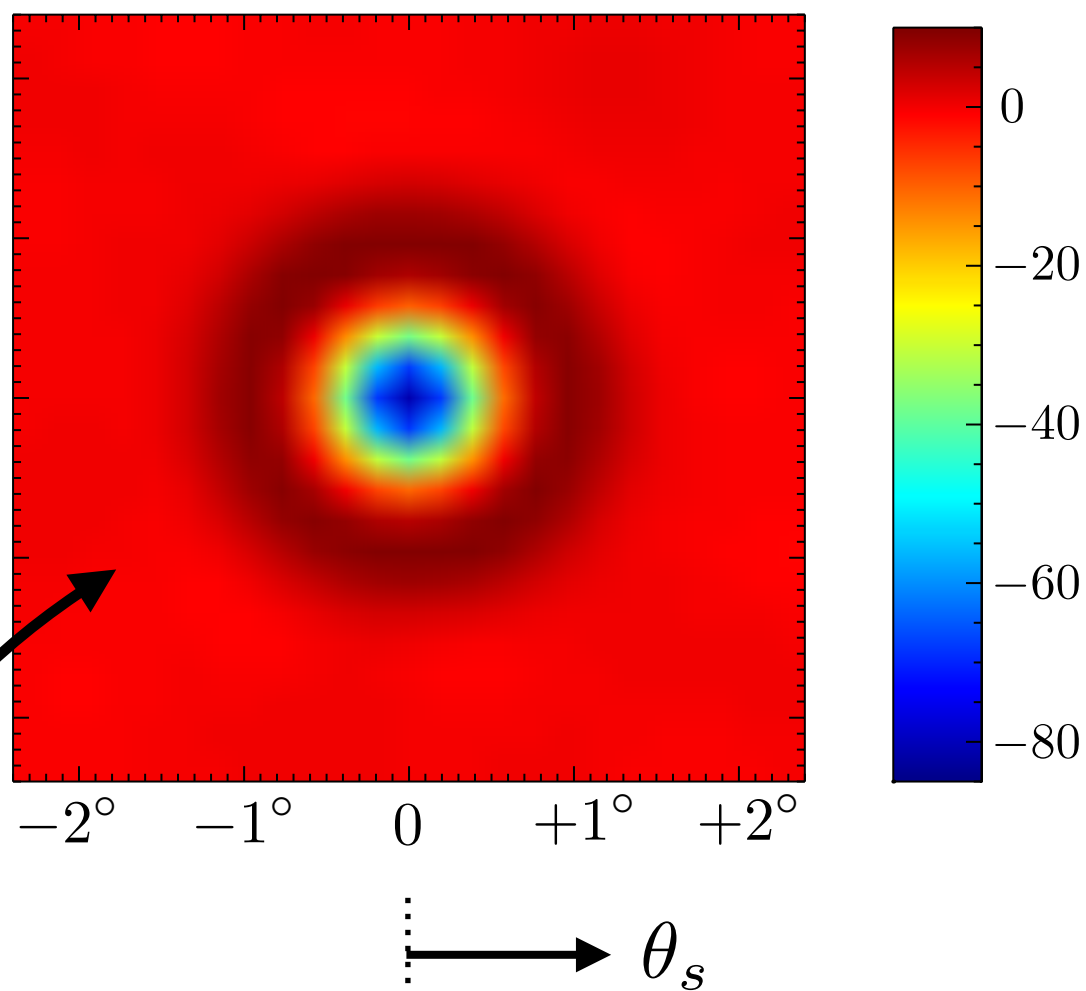
Fig. 6.10

Cartoon illustrating the origin of the peaks in the CMB power spectrum. *Left:* Fluctuations of different wavelengths are captured at different moments in their evolution and therefore have different amplitudes at decoupling. *Right:* Since the square of the amplitude determines the power on a given length scale, waves that are captured at an extremum (*A* or *C*) produce the peaks in the CMB spectrum, while waves that are captured with zero amplitude (*B*) produce the troughs.

CMB Anisotropies

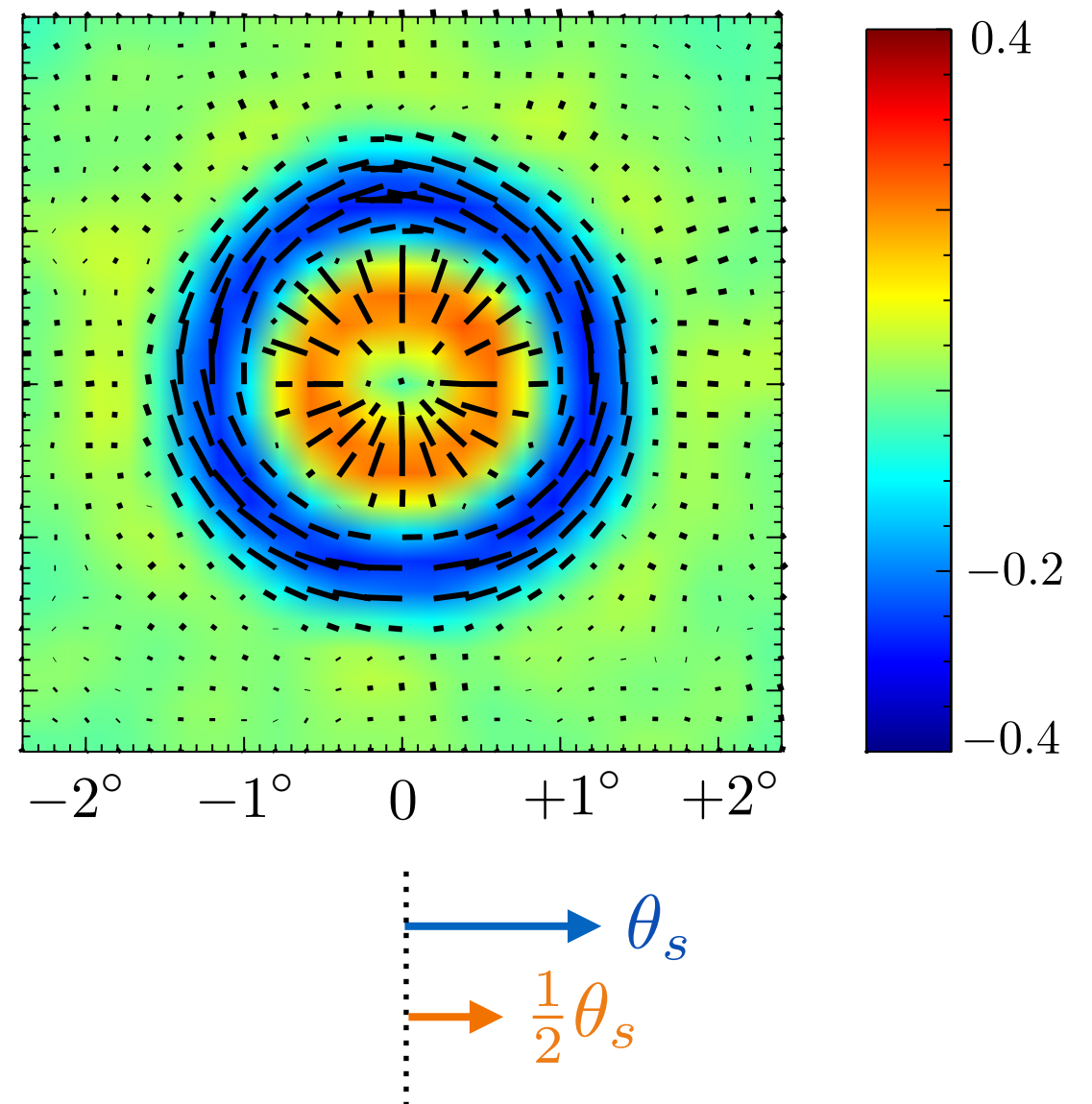
This sound horizon is imprinted in the pattern of CMB fluctuations:

Temperature



intensity of 11396 cold spots

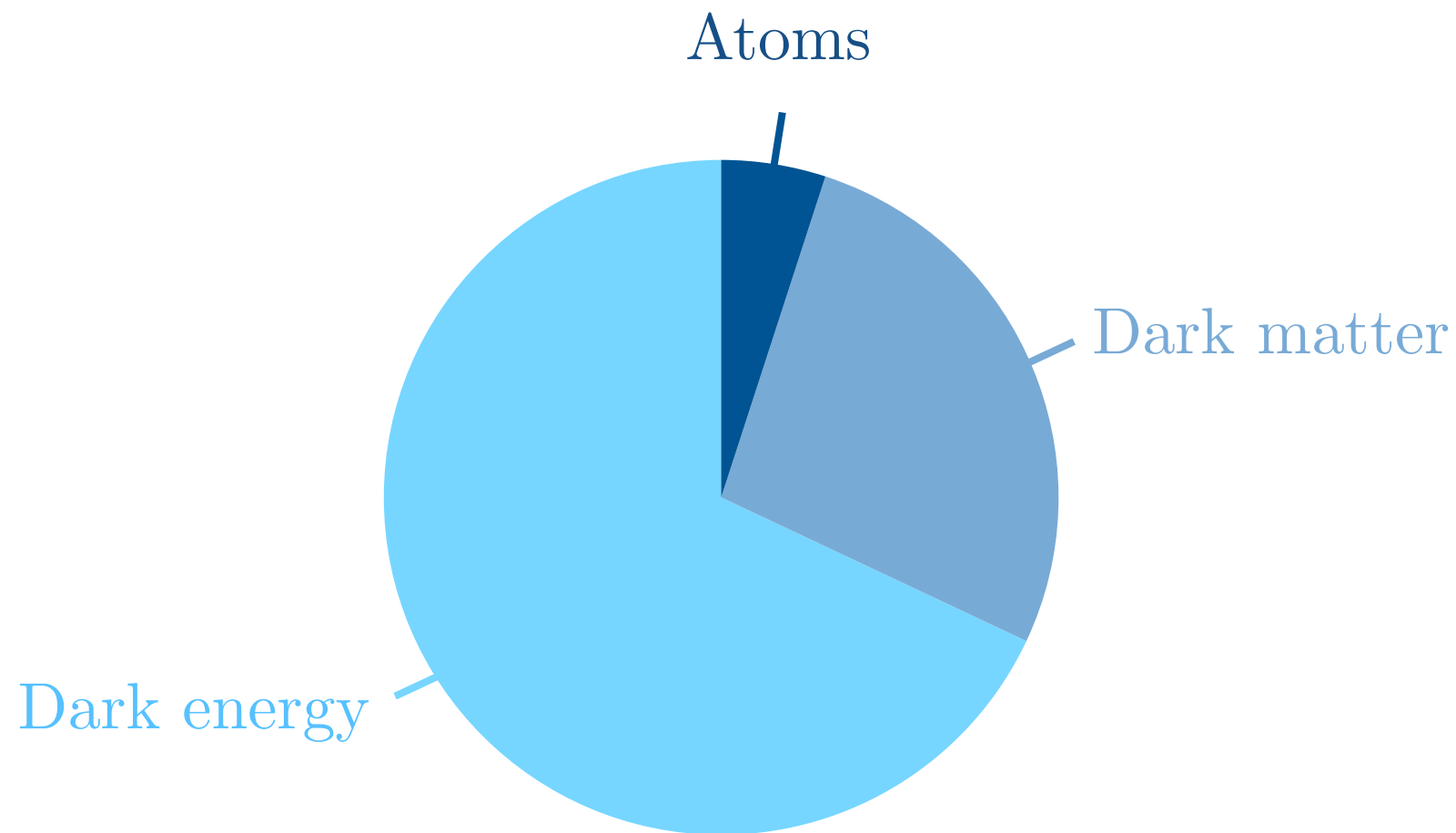
Polarization



Planck (2015)

CMB Anisotropies

The precise pattern of the CMB fluctuations depends on the composition of the Universe (and its initial conditions):

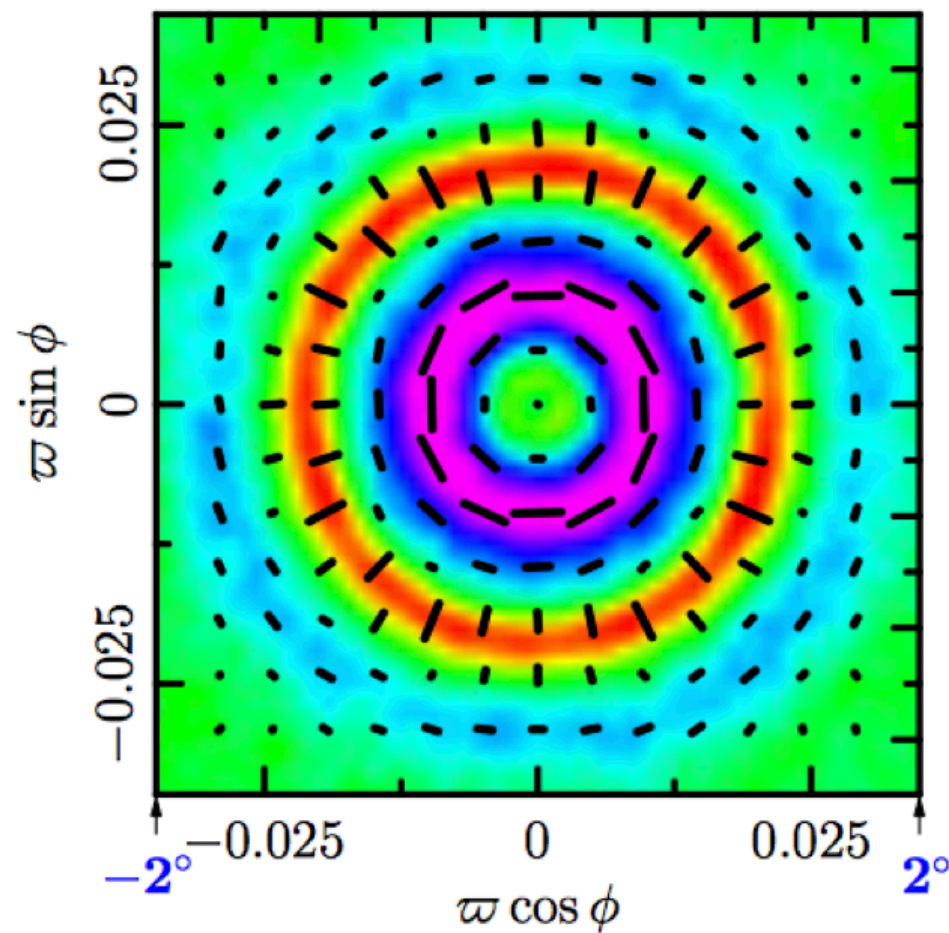


Observations of the CMB have therefore allowed us to determine the parameters of the cosmological standard model.

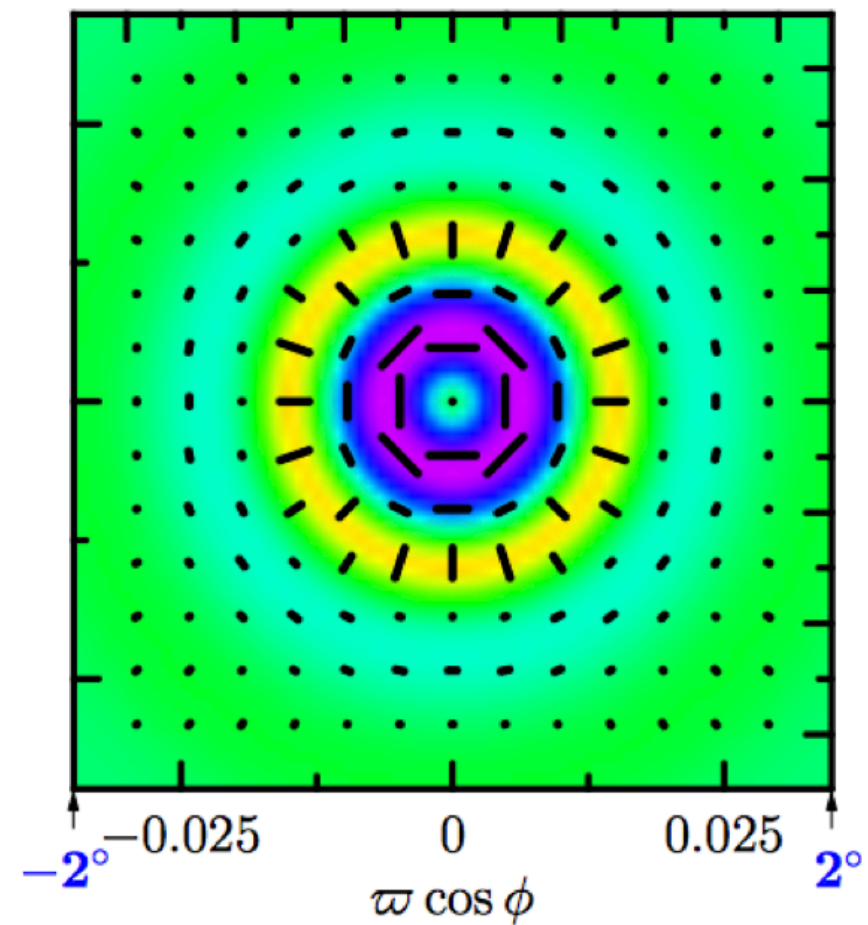
Dark Matter

Without dark matter, the data would look very different:

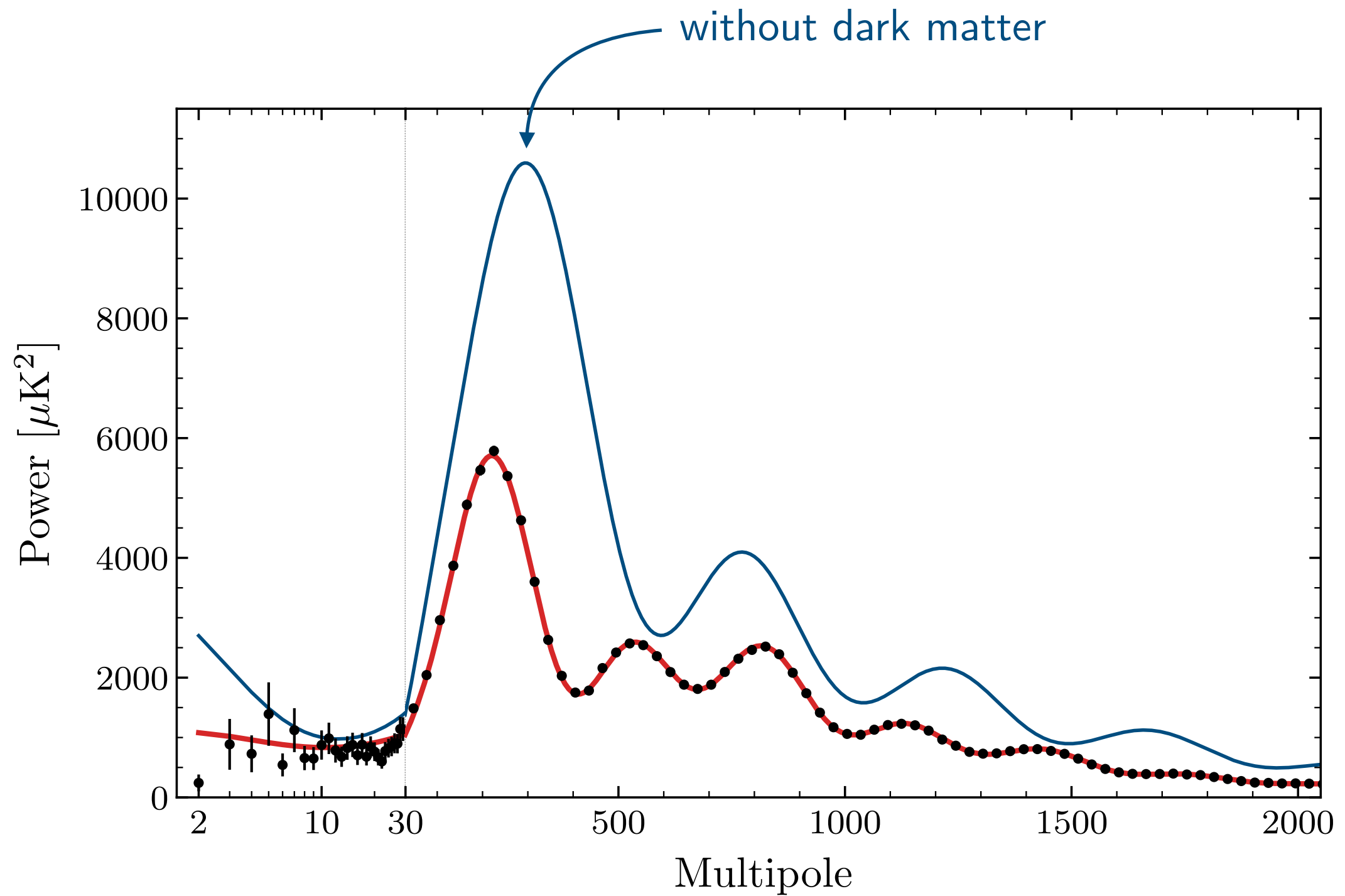
Data



No Dark Matter



This can also be seen in the power spectrum: $\Omega_m = 0.32$

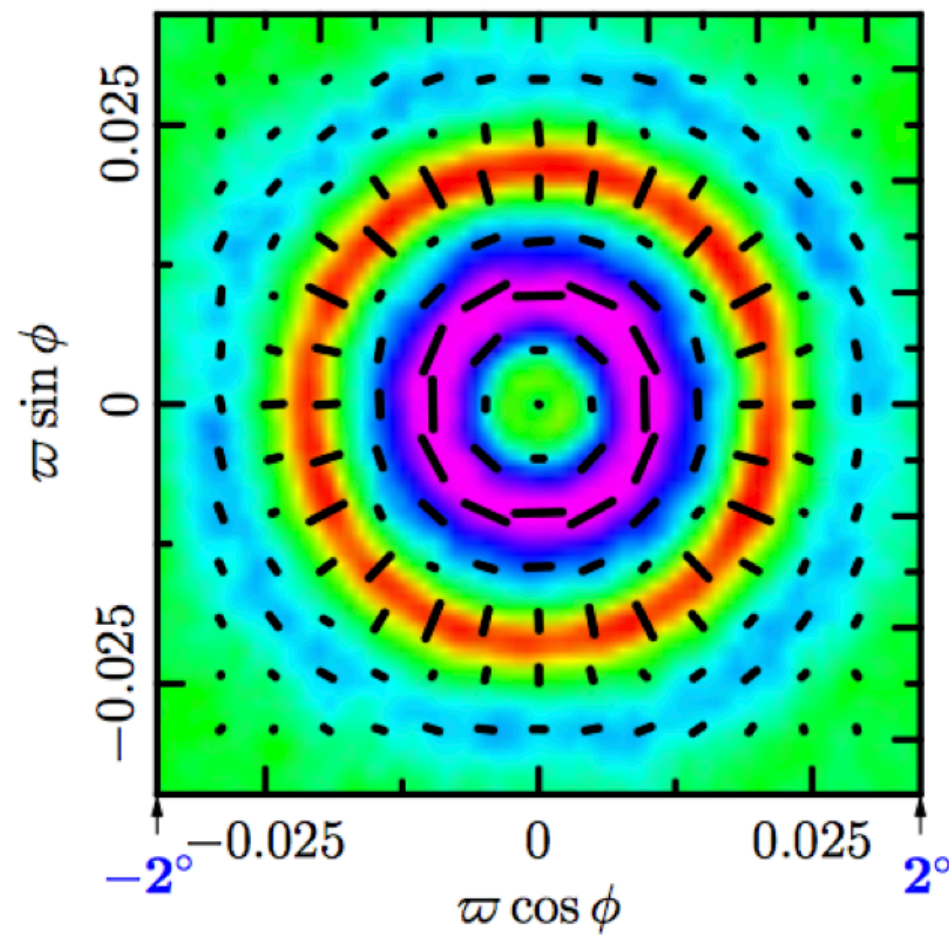


Planck (2018)

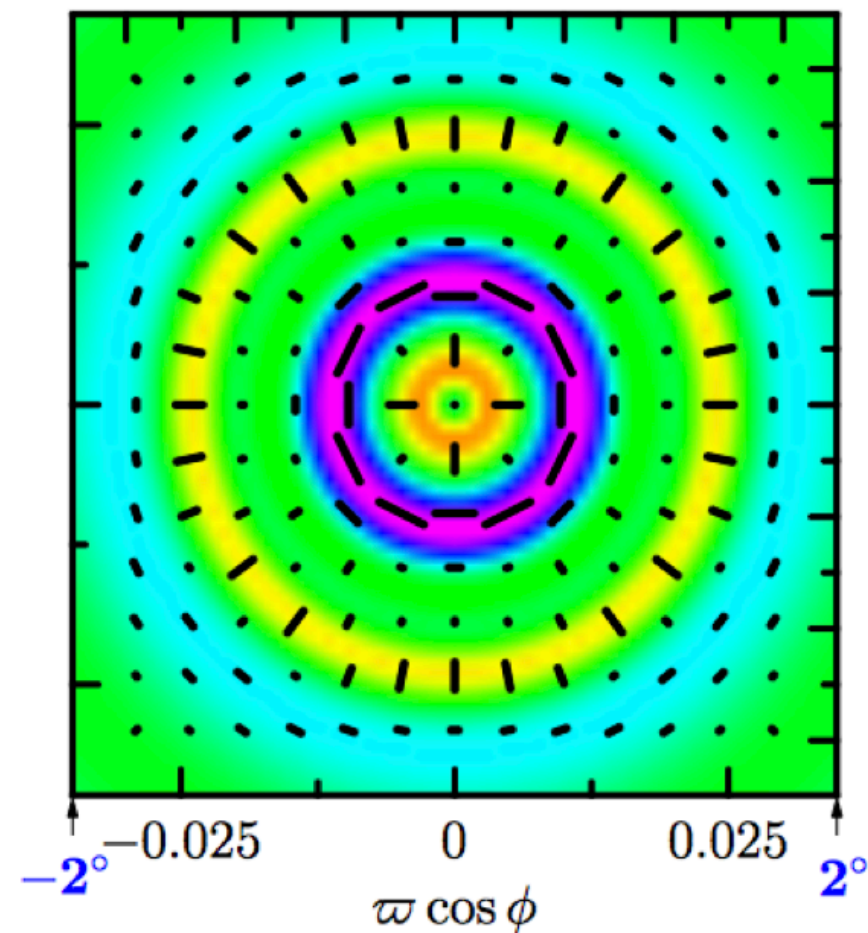
Dark Energy

Without dark energy, the data would look very different:

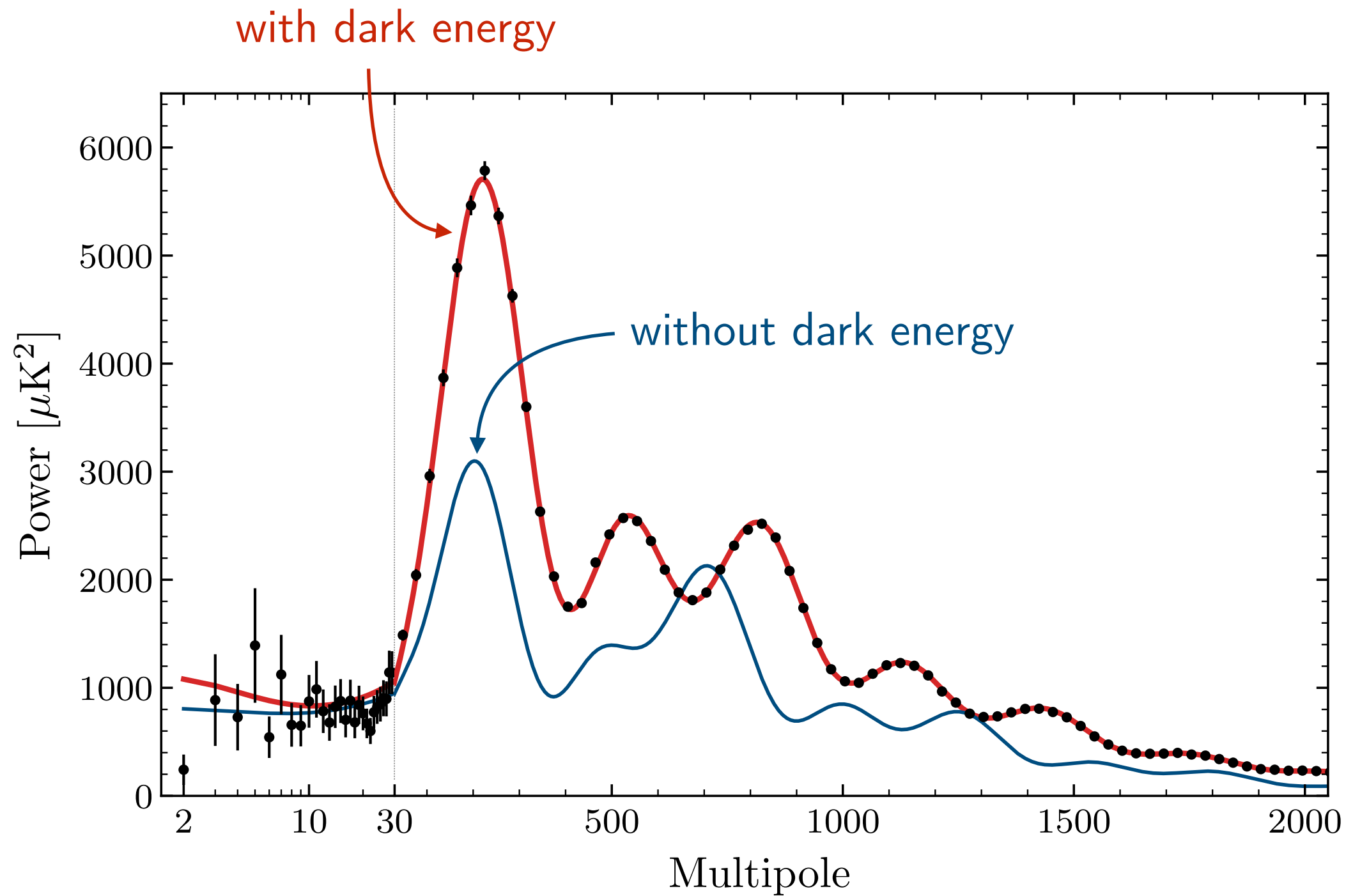
Data



No Dark Energy

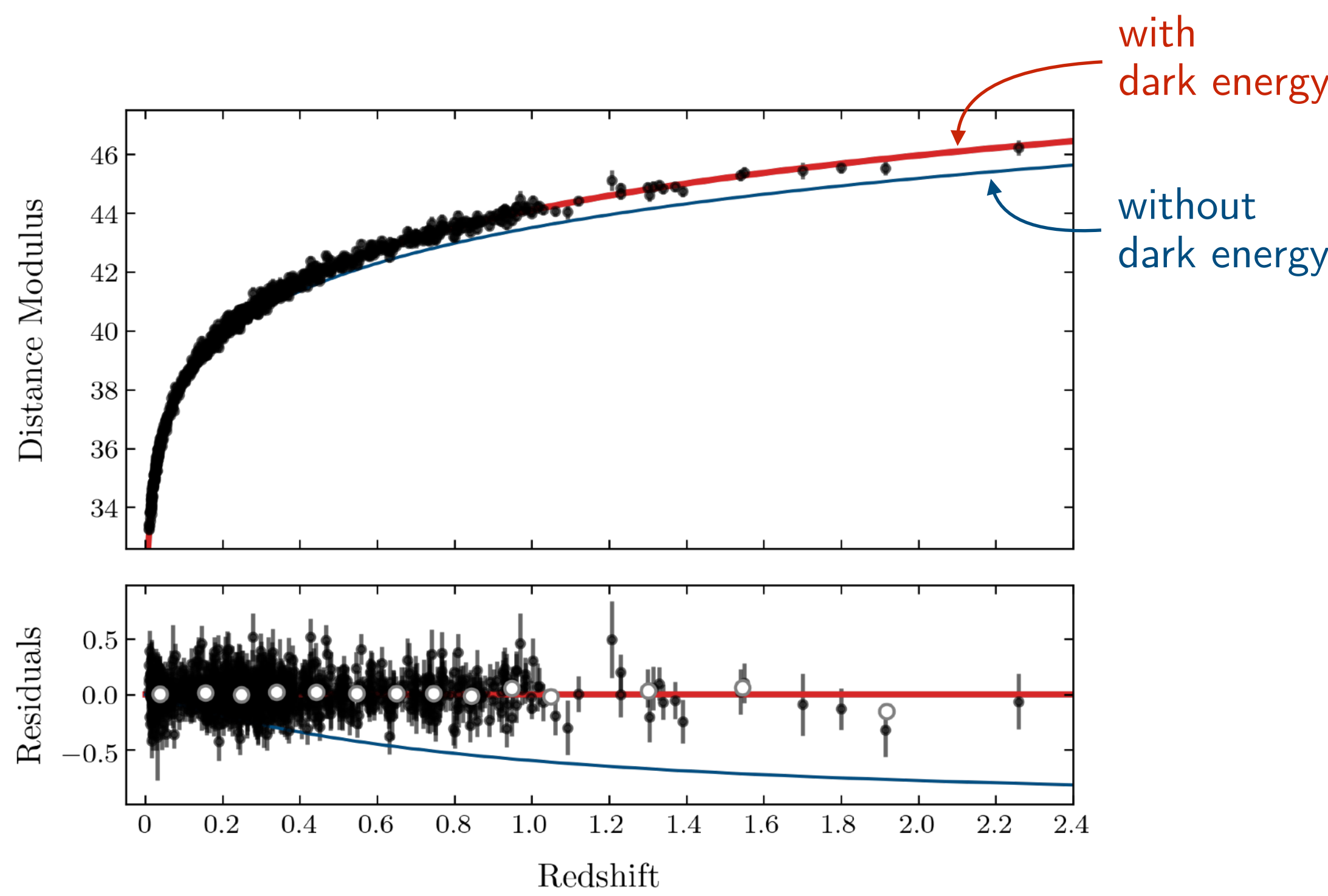


This can also be seen in the power spectrum: $\Omega_{\Lambda} = 0.68$



Planck (2018)

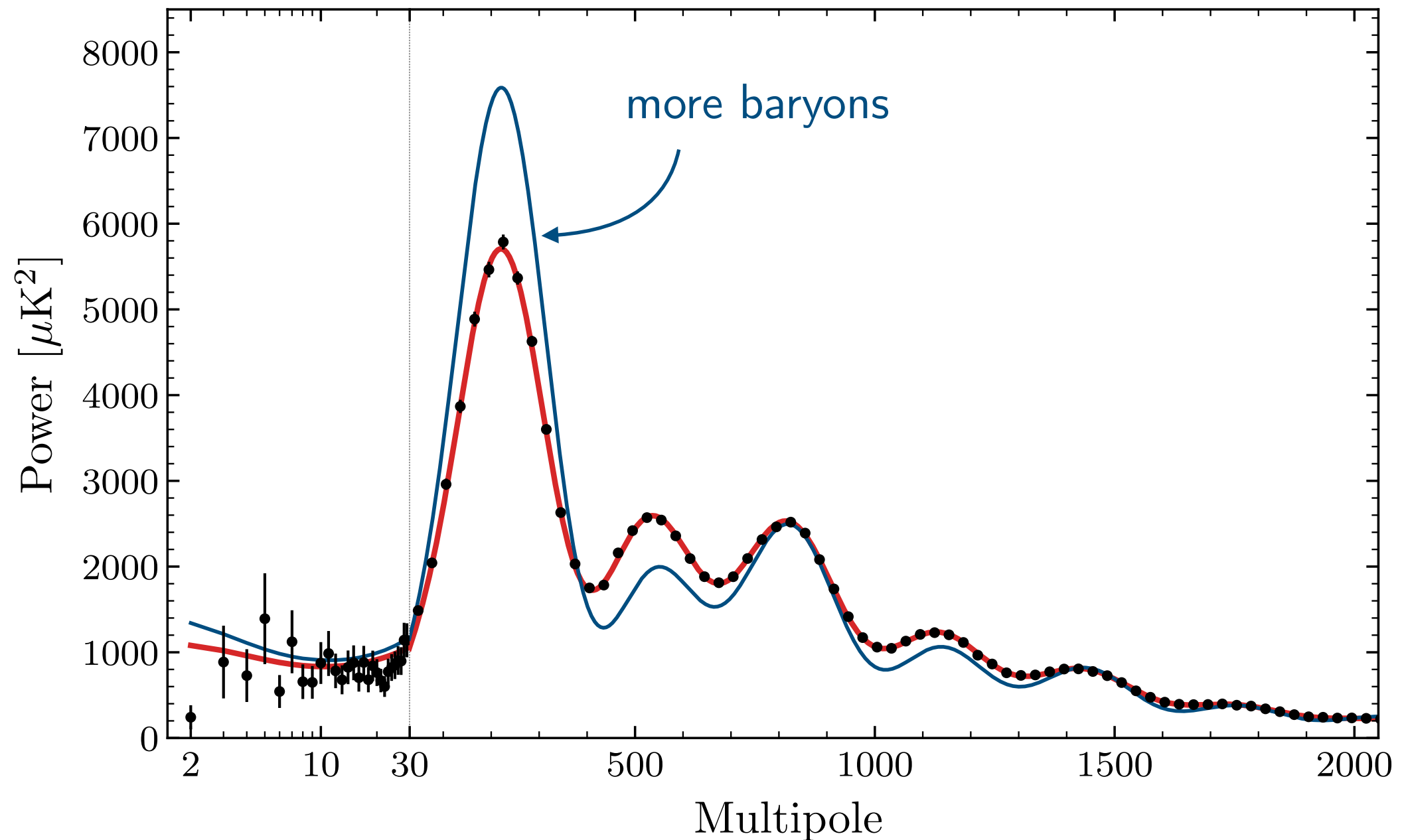
This is consistent with the direct observation of **dark energy** from supernova observations:



Riess et al (1998)
Perlmutter et al (1998)

Baryons

The peak heights depend on the baryon density: $\Omega_b = 0.04$

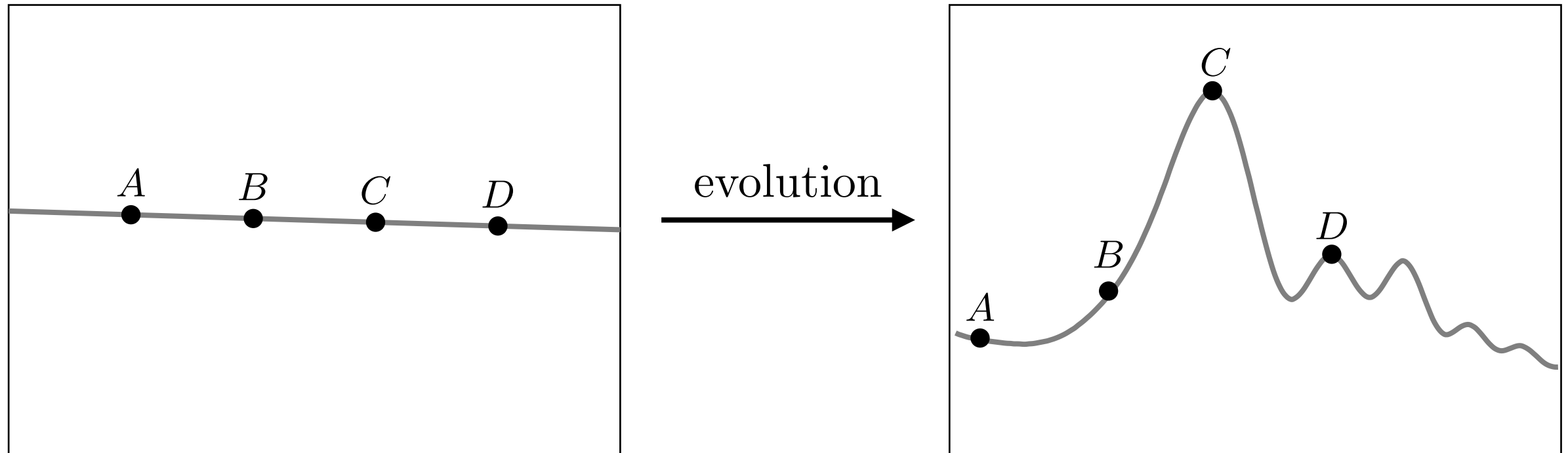


The measured baryon density is consistent with BBN.

Planck (2018)

Initial Conditions

The CMB power spectrum also probes the initial conditions:



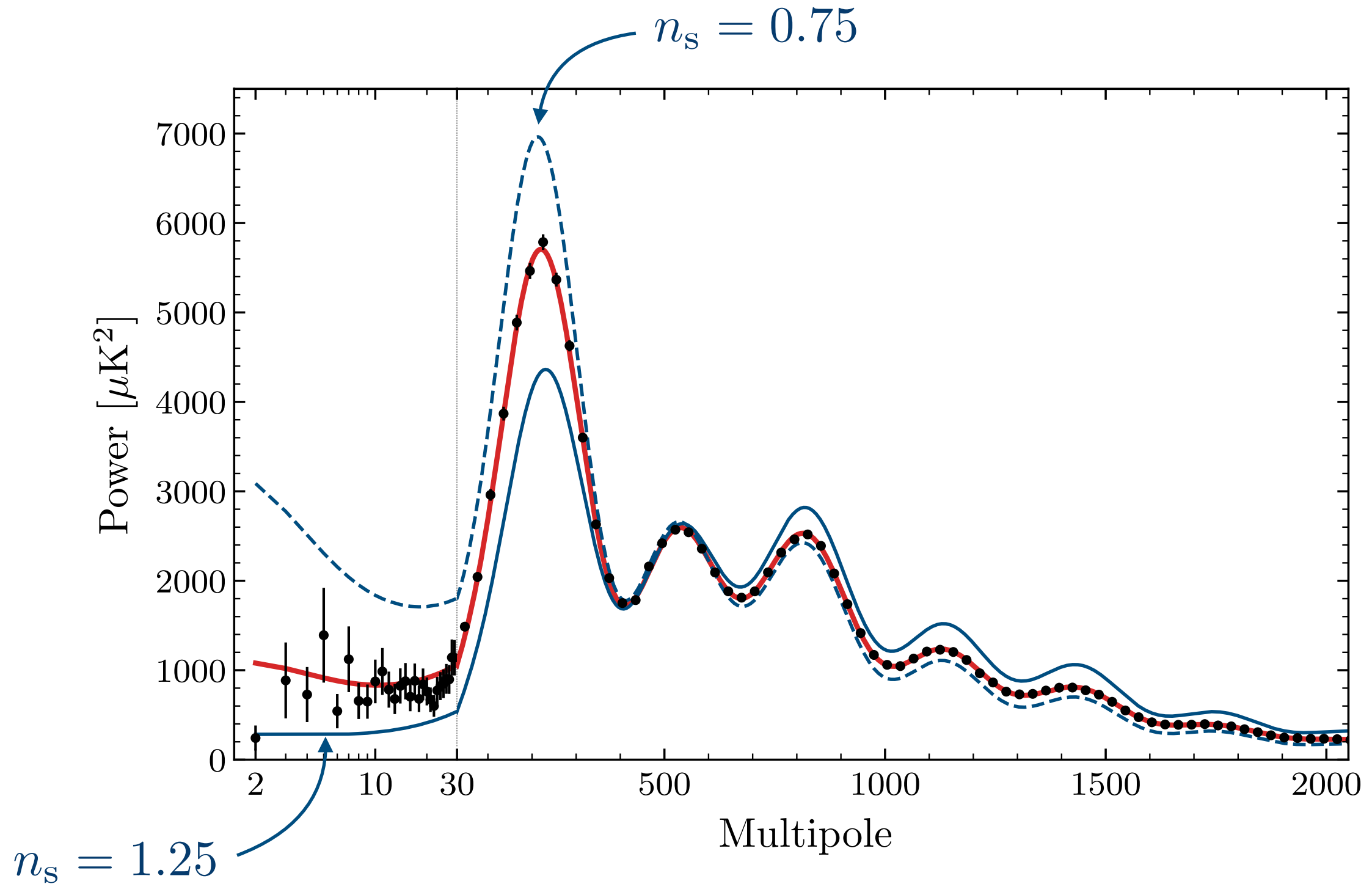
$$A_s \left(\frac{k}{k_0} \right)^{n_s - 1}$$

Amplitude

scale-dependence

$$A_s = 2.20 \times 10^{-9}$$

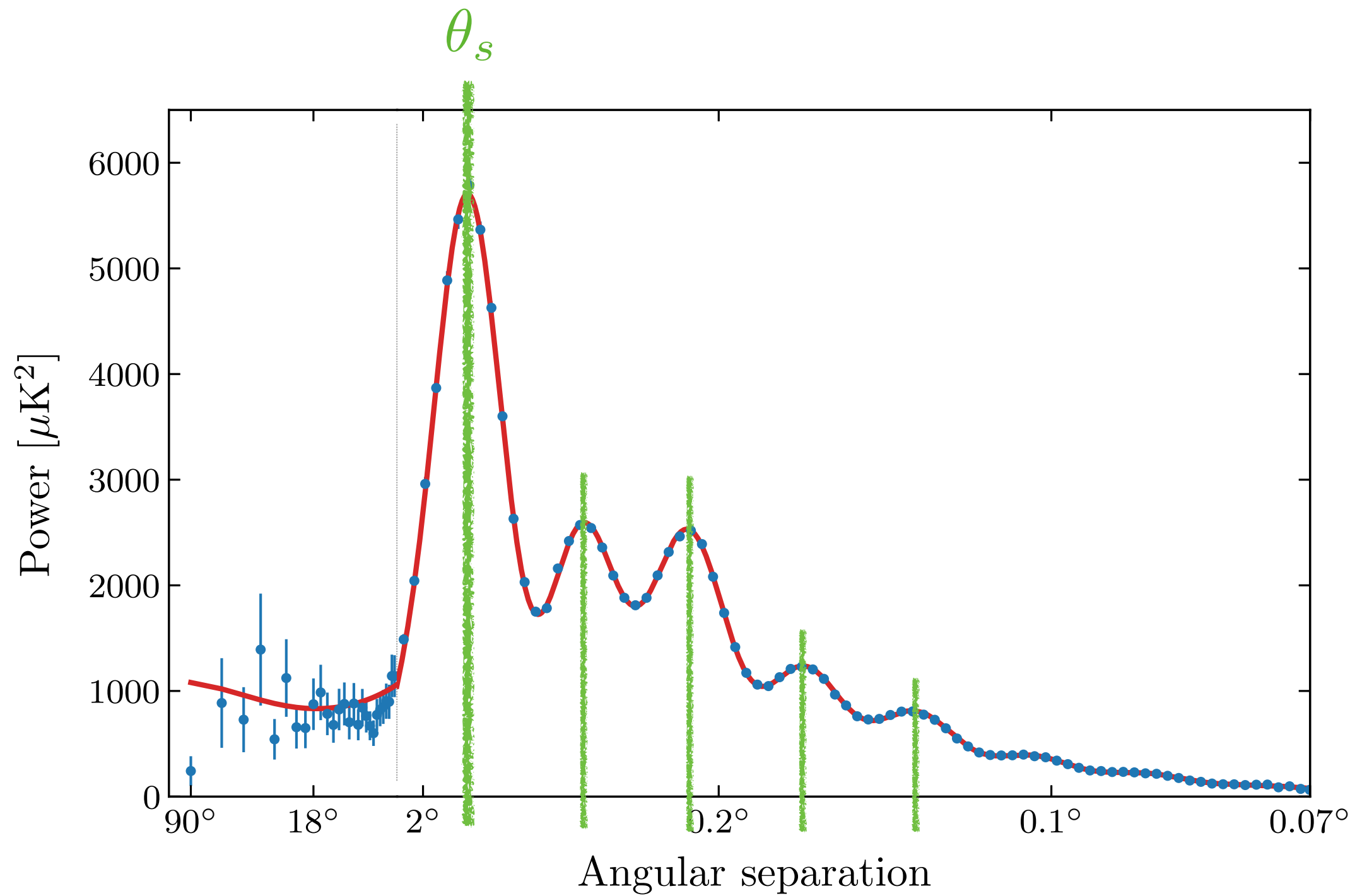
The primordial power spectrum is close to scale invariant: $n_s = 0.96$

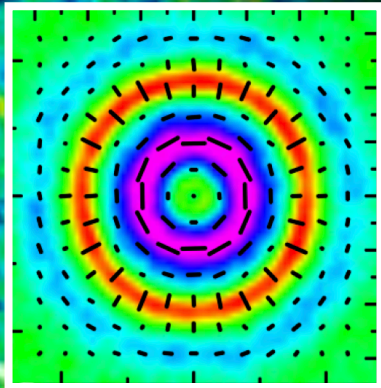


The observed deviation from scale invariance is significant.

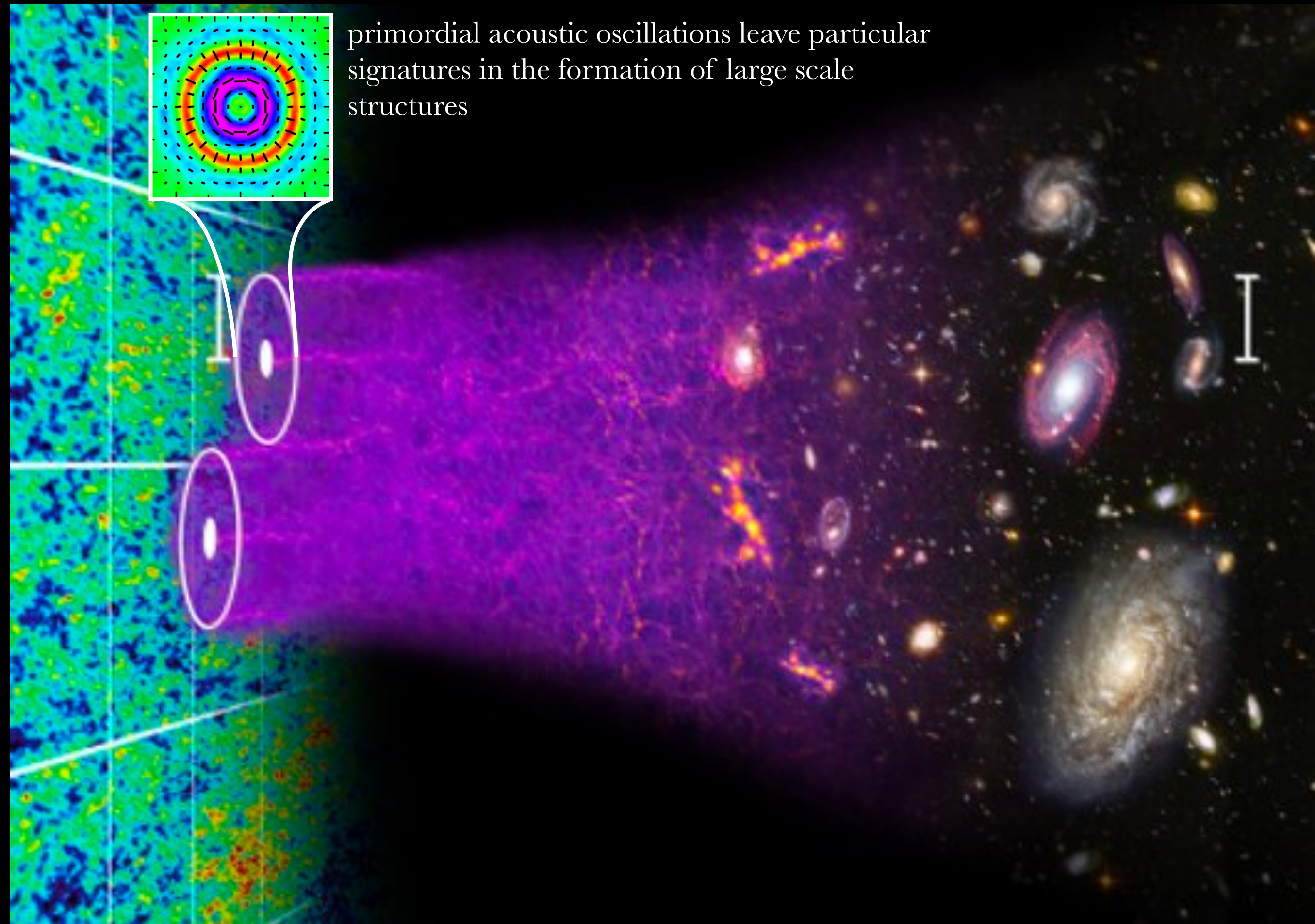
WMAP (2009)
Planck (2018)

primordial acoustic oscillations leave particular
signatures in the formation of large scale
structures

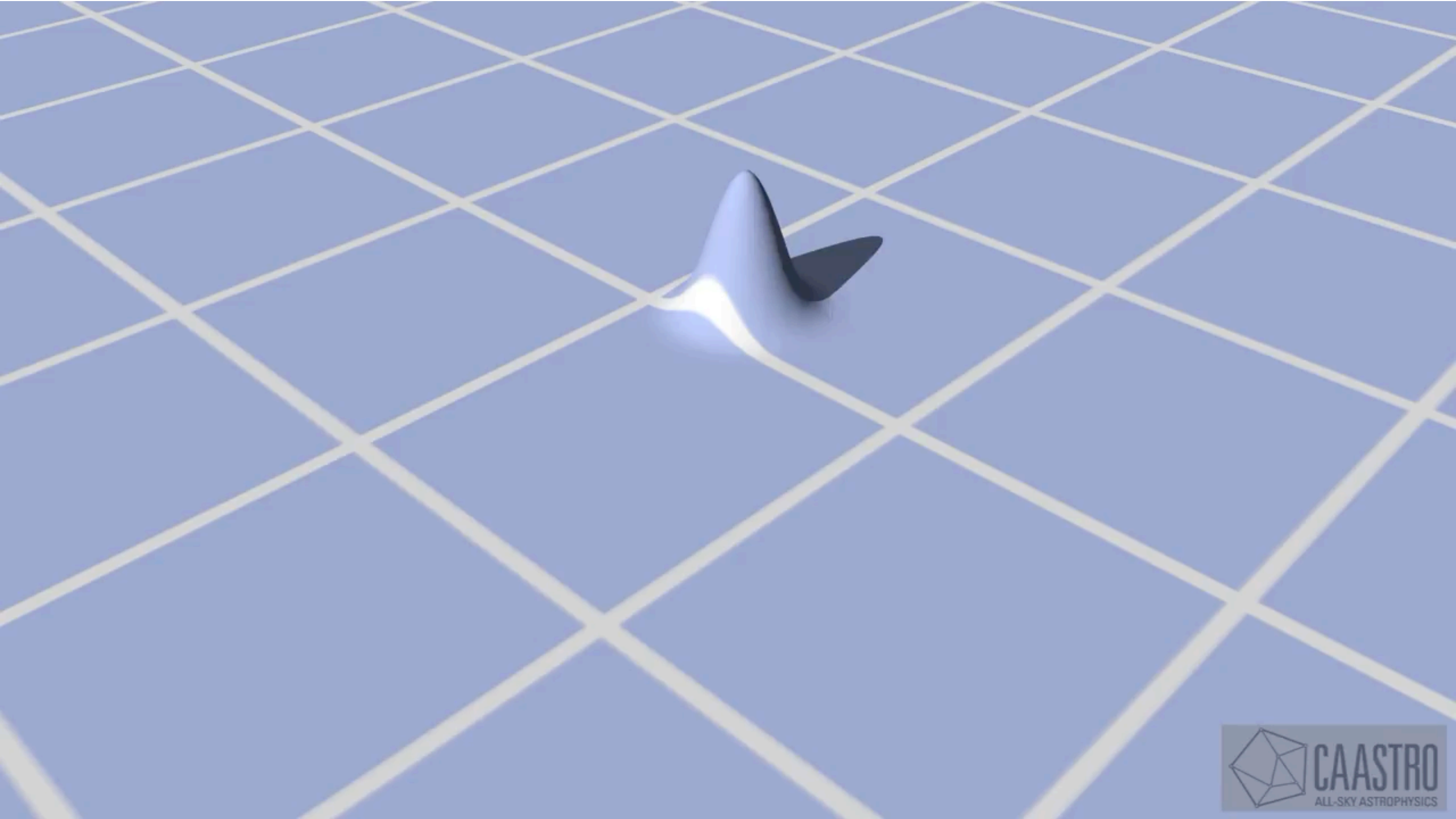




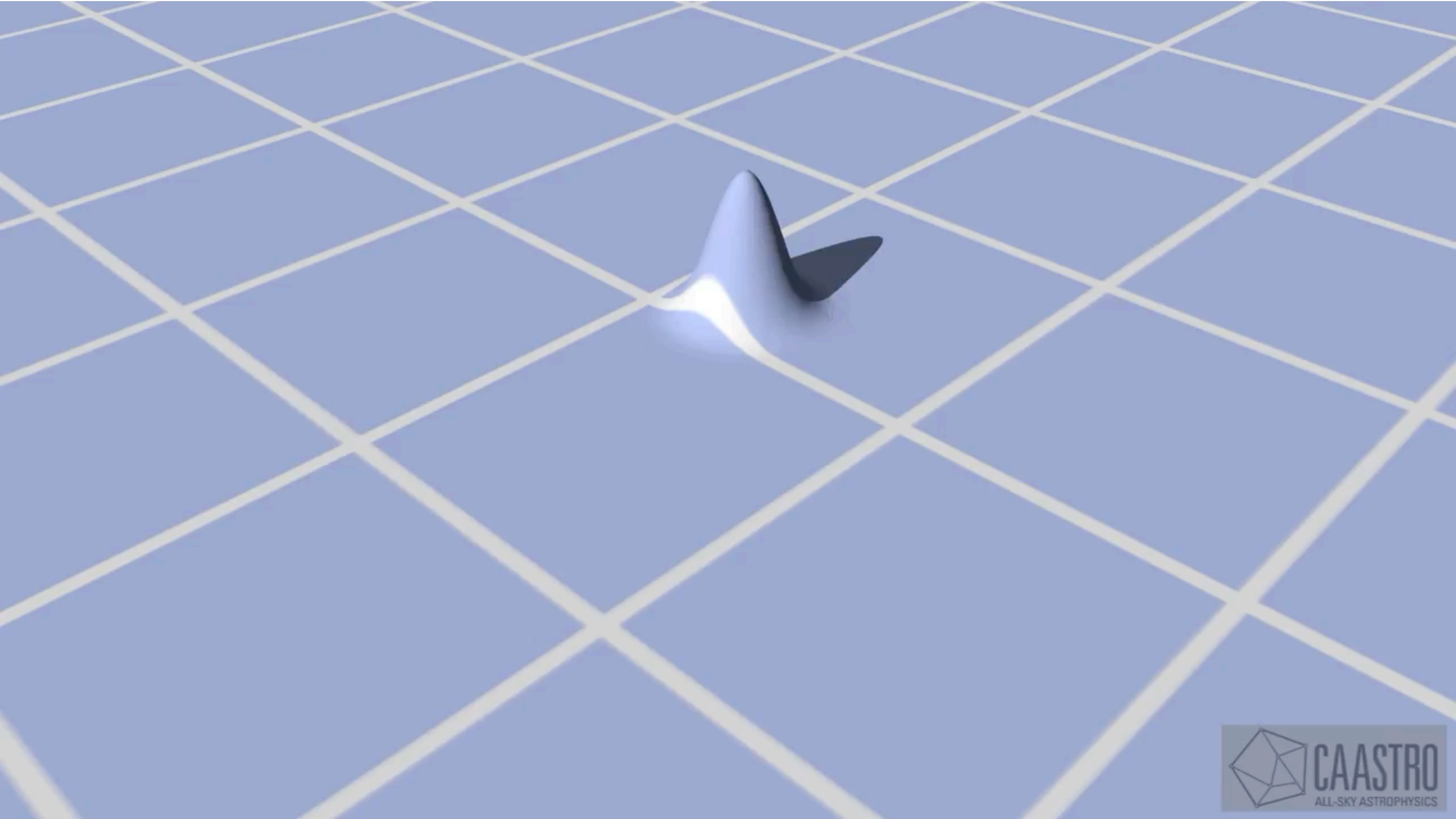
primordial acoustic oscillations leave particular signatures in the formation of large scale structures



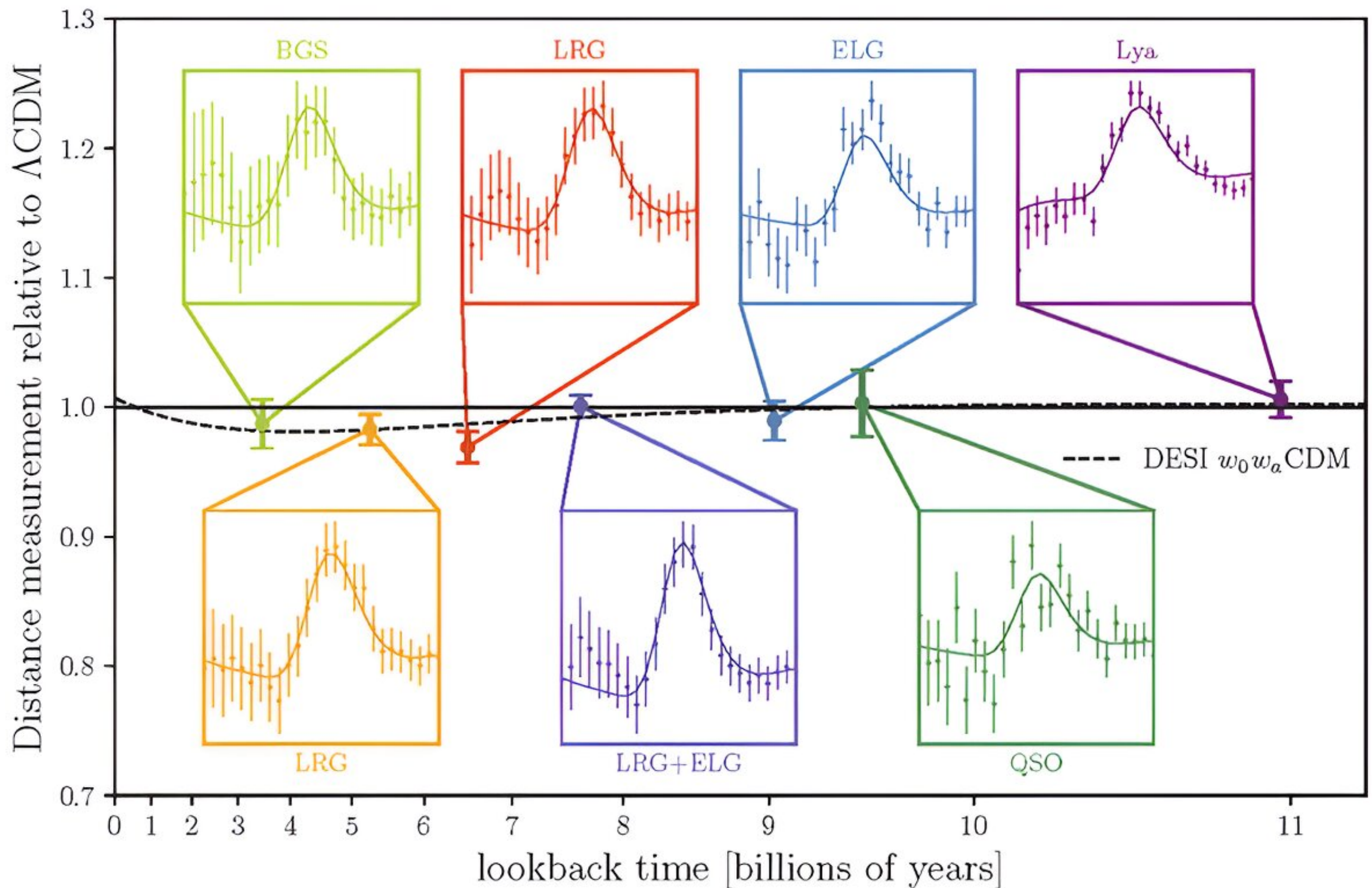
from primordial perturbations to the statistical study of galaxies' positions
[cf. C. Yèche's lecture]



from primordial perturbations to the statistical study of galaxies' positions
[cf. C. Yèche's lecture]



from primordial perturbations to the statistical study of galaxies' positions
[cf. C. Yèche's lecture]



A simple 5-parameter model fits all observations:

$$\Omega_{\text{b}} = 0.04 \quad \text{Amount of ordinary matter}$$

$$\Omega_{\text{m}} = 0.32 \quad \text{Amount of dark matter}$$

$$\Omega_{\Lambda} = 0.68 \quad \text{Amount of dark energy}$$

$$10^9 A_{\text{s}} = 2.20 \quad \text{Amplitude of density fluctuations}$$

$$n_{\text{s}} = 0.96 \quad \text{Scale dependence of the fluctuations}$$

A key challenge of modern cosmology is to explain these numbers:

$$\Omega_b = 0.04$$



Why is there more matter than antimatter?

$$\Omega_m = 0.32$$



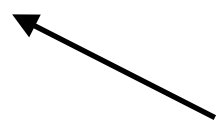
What is the dark matter?

$$\Omega_\Lambda = 0.68$$



What is the dark energy?

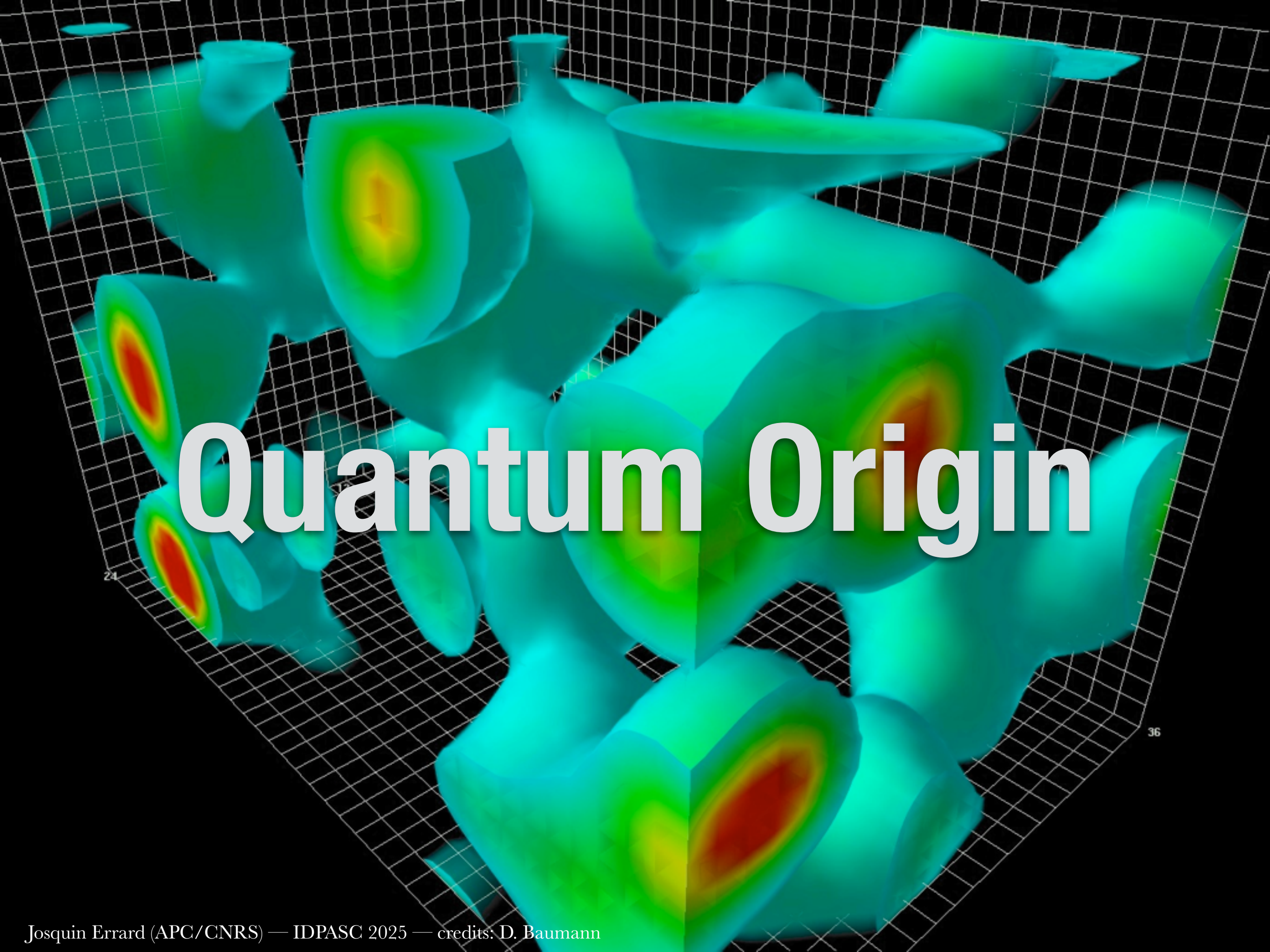
$$10^9 A_s = 2.20$$



What was the origin of the fluctuations?

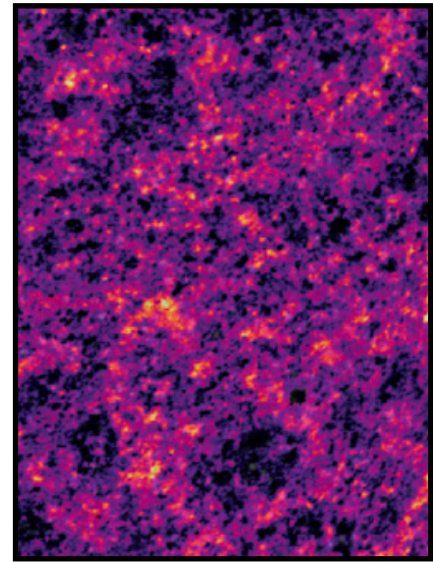
$$n_s = 0.96$$



A 3D visualization of a quantum state, likely a wavefunction or probability density, plotted on a 3D grid. The surface is colored with a gradient from cyan to red, indicating different values or phases. The grid lines are visible, and the overall shape is complex and multi-lobed. The text "Quantum Origin" is overlaid in the center.

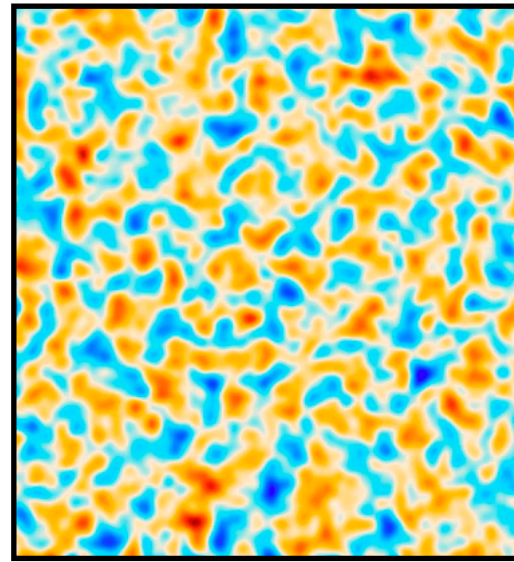
Quantum Origin

So far, we have described the evolution of fluctuations in the hot Big Bang and the formation of the large-scale structure of the Universe:



Primordial
fluctuations

→
Cosmic
sound waves



CMB
fluctuations

→
Gravitational
clustering

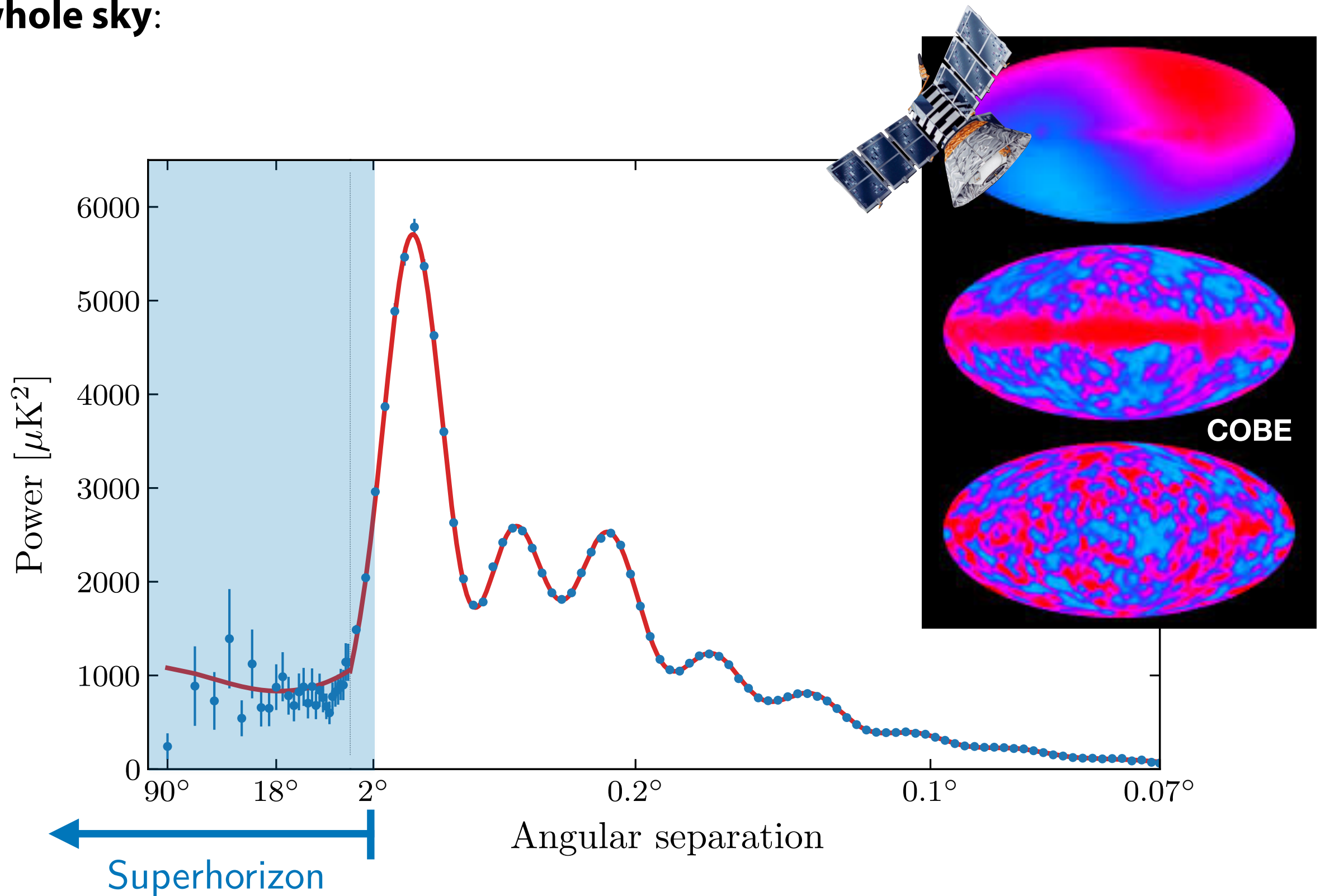


Galaxies

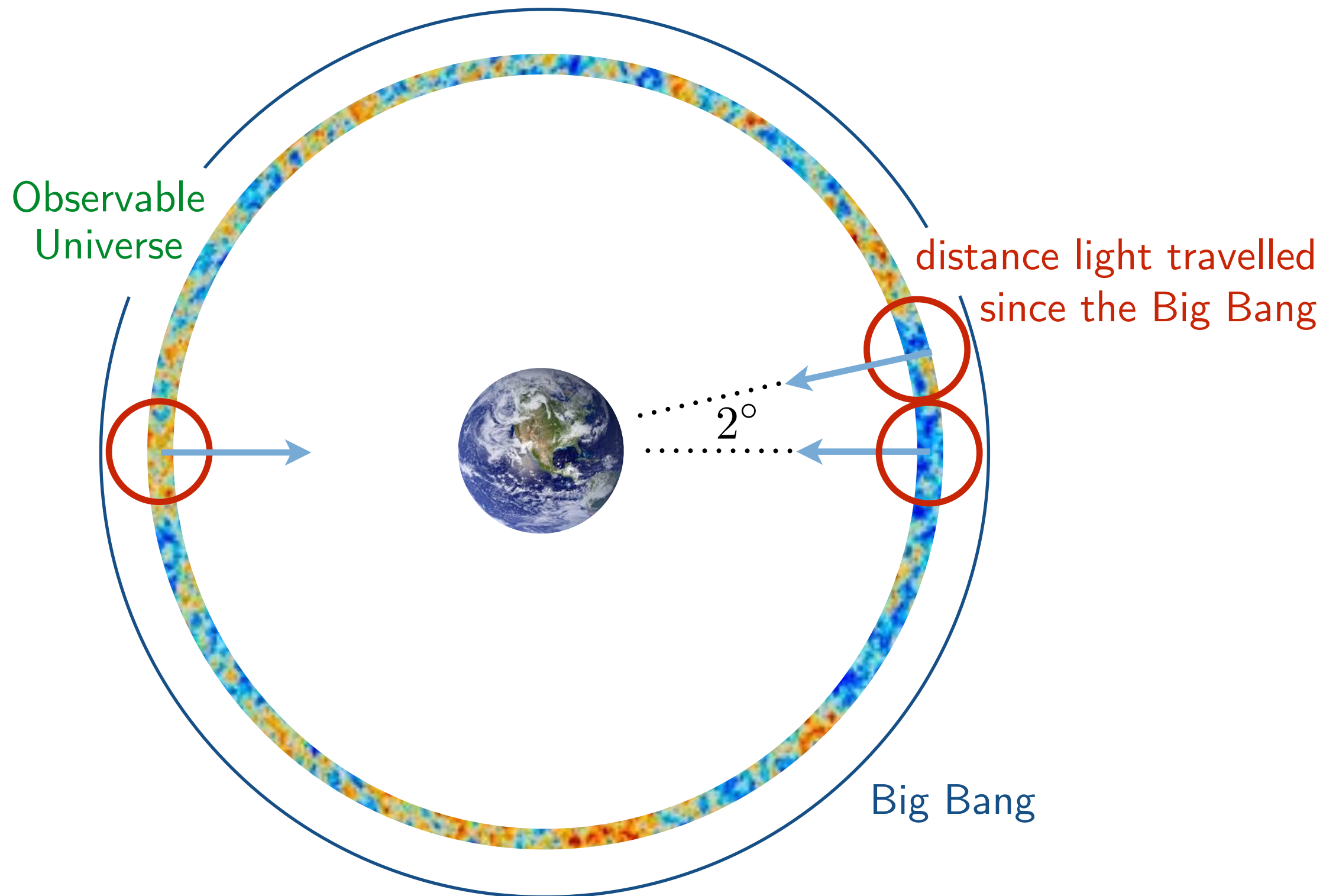
We now want to ask:

What created the primordial fluctuations?

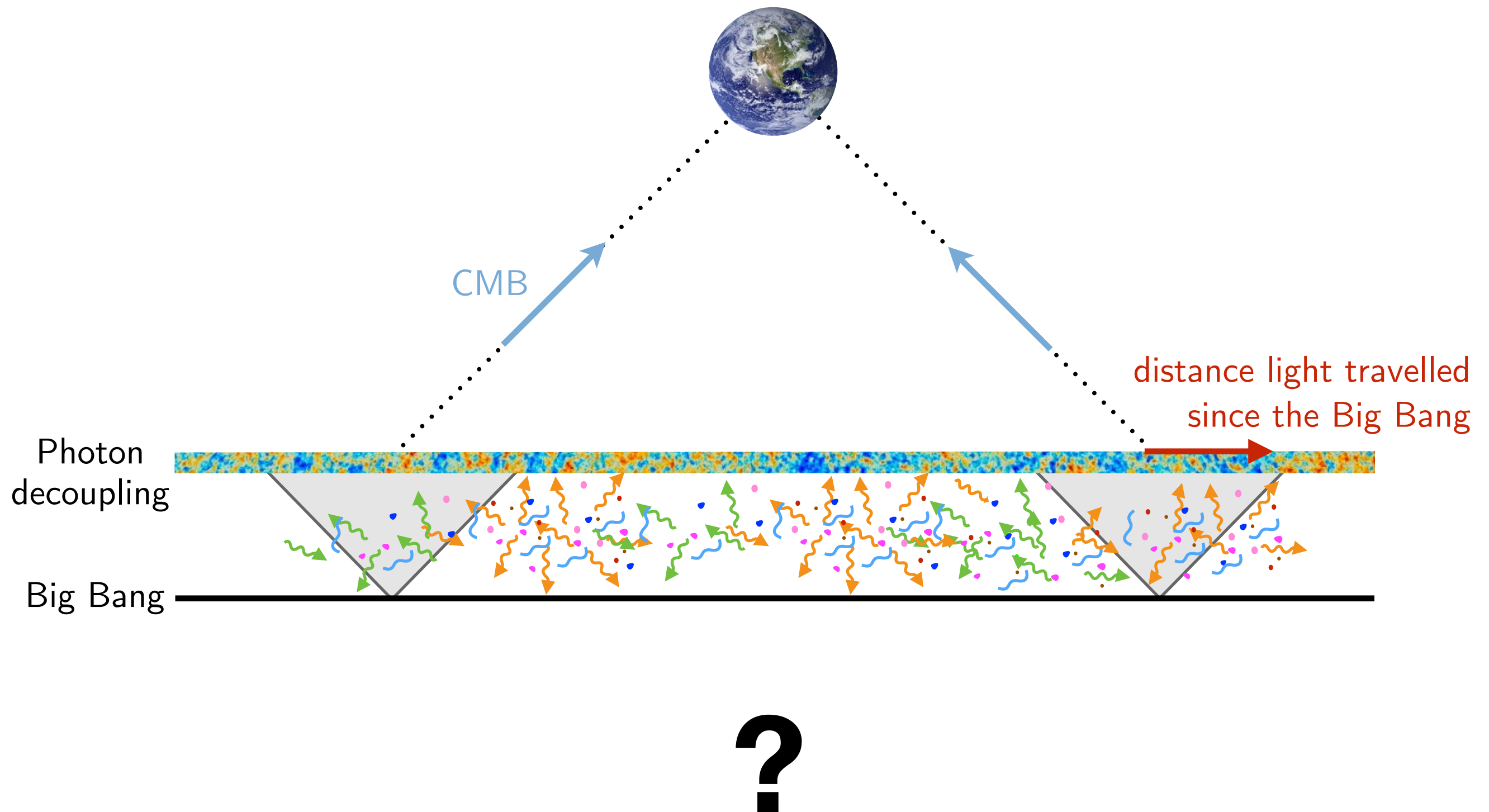
An important clue is the fact that the CMB fluctuations are **correlated over the whole sky**:



In the standard hot Big Bang theory, this is **impossible**:

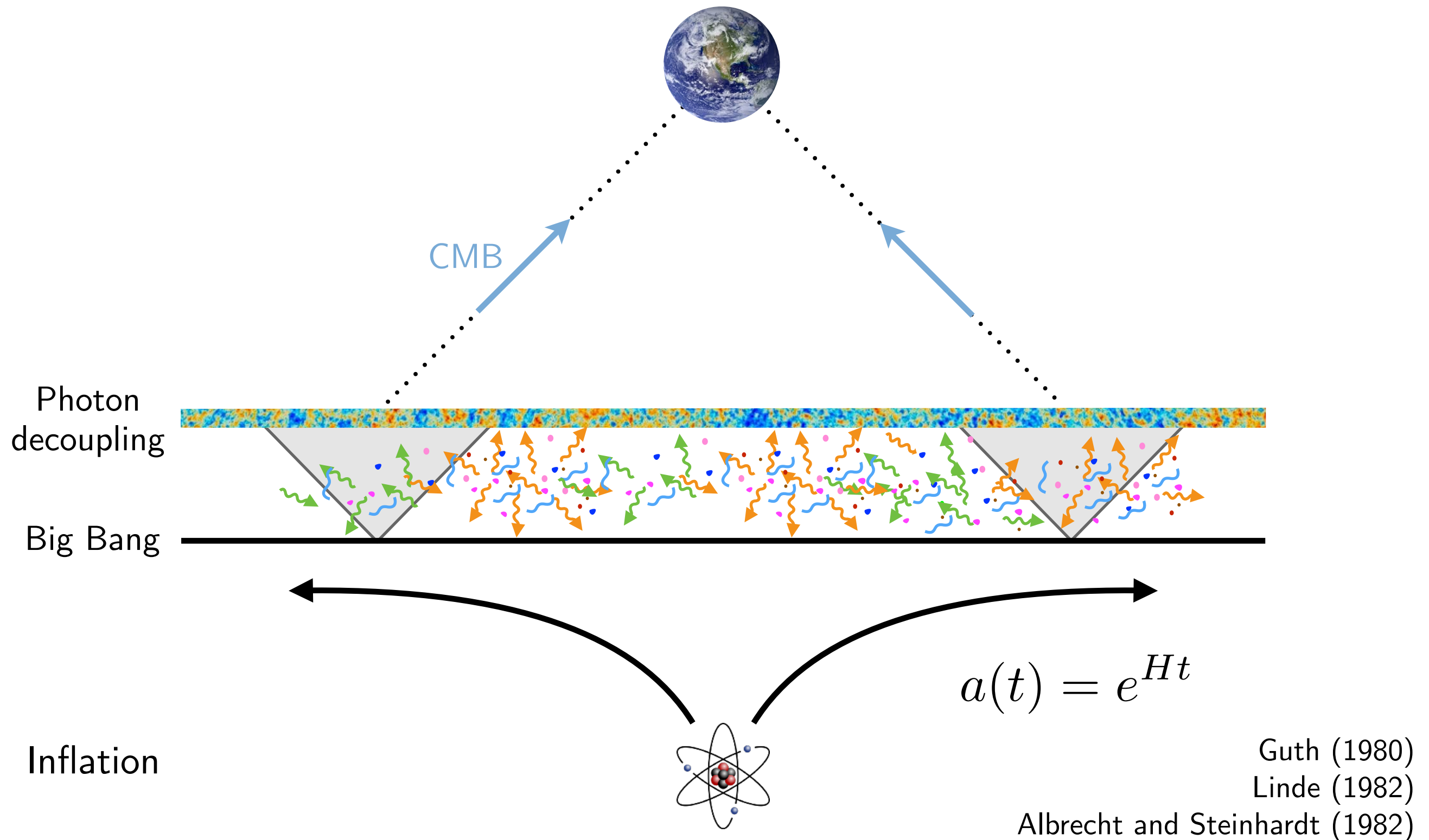


The correlations must have been created **before the hot Big Bang**:



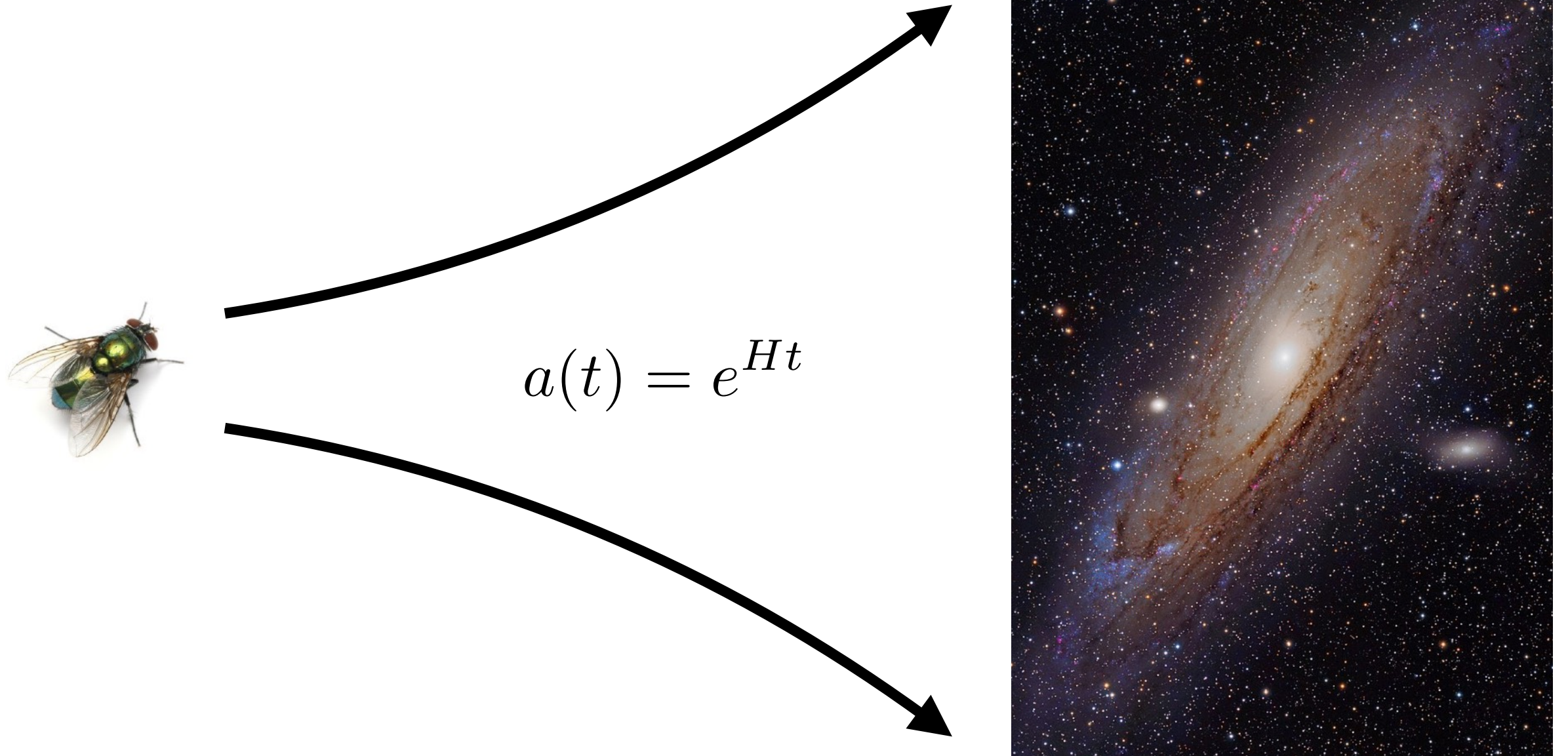
Inflation

Inflation solves the problem by invoking a period of **superluminal expansion**:



Inflation

In less than 10^{-32} seconds, the Universe doubled in size at least 80 times:



The entire observable Universe then originated from a microscopic, causally connected region of space.

Inflation

To achieve inflation requires a substance with a nearly **constant energy density** (like **dark energy**):

$$P > -\frac{1}{3}\rho$$

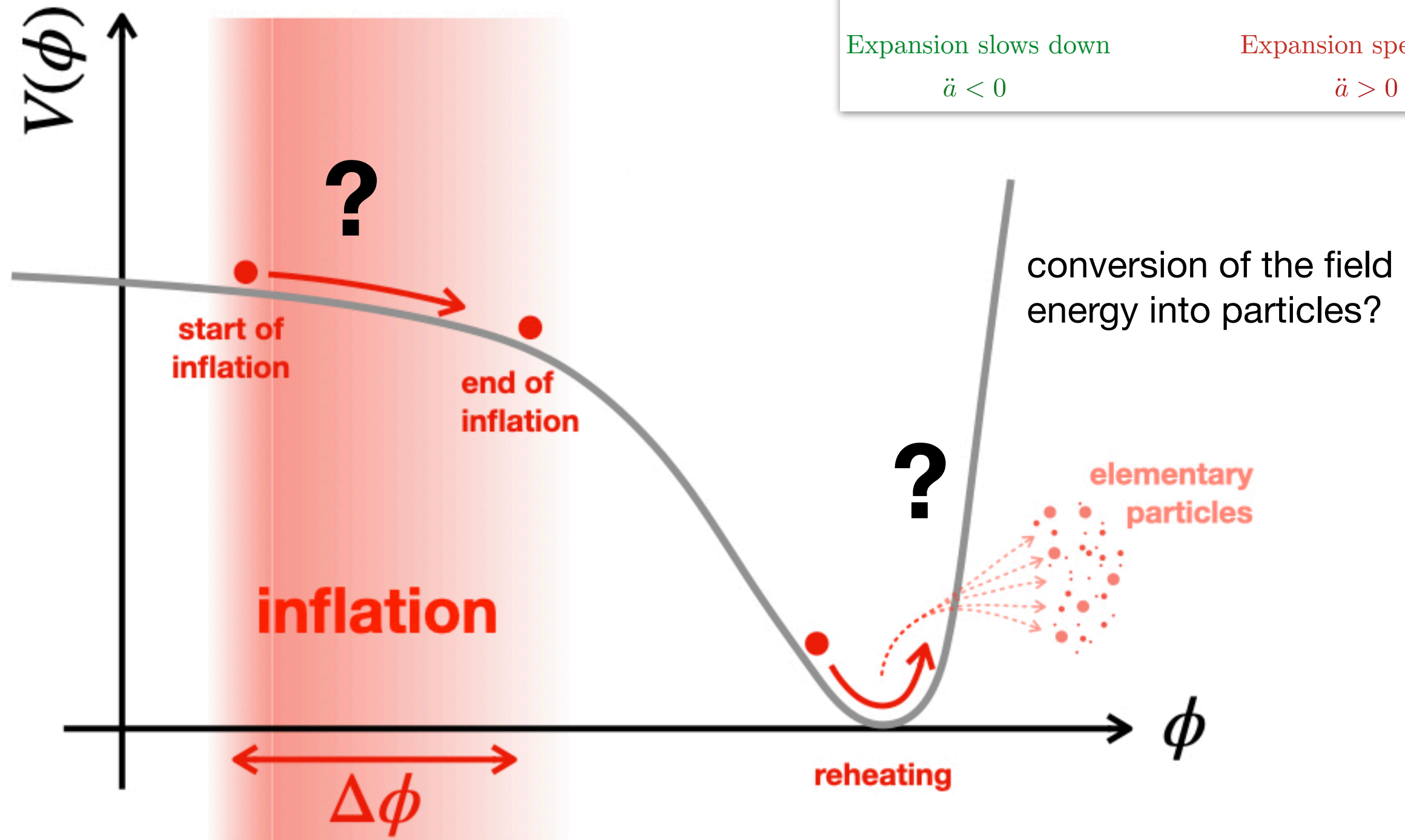
Expansion slows down

$$\ddot{a} < 0$$

$$P < -\frac{1}{3}\rho$$

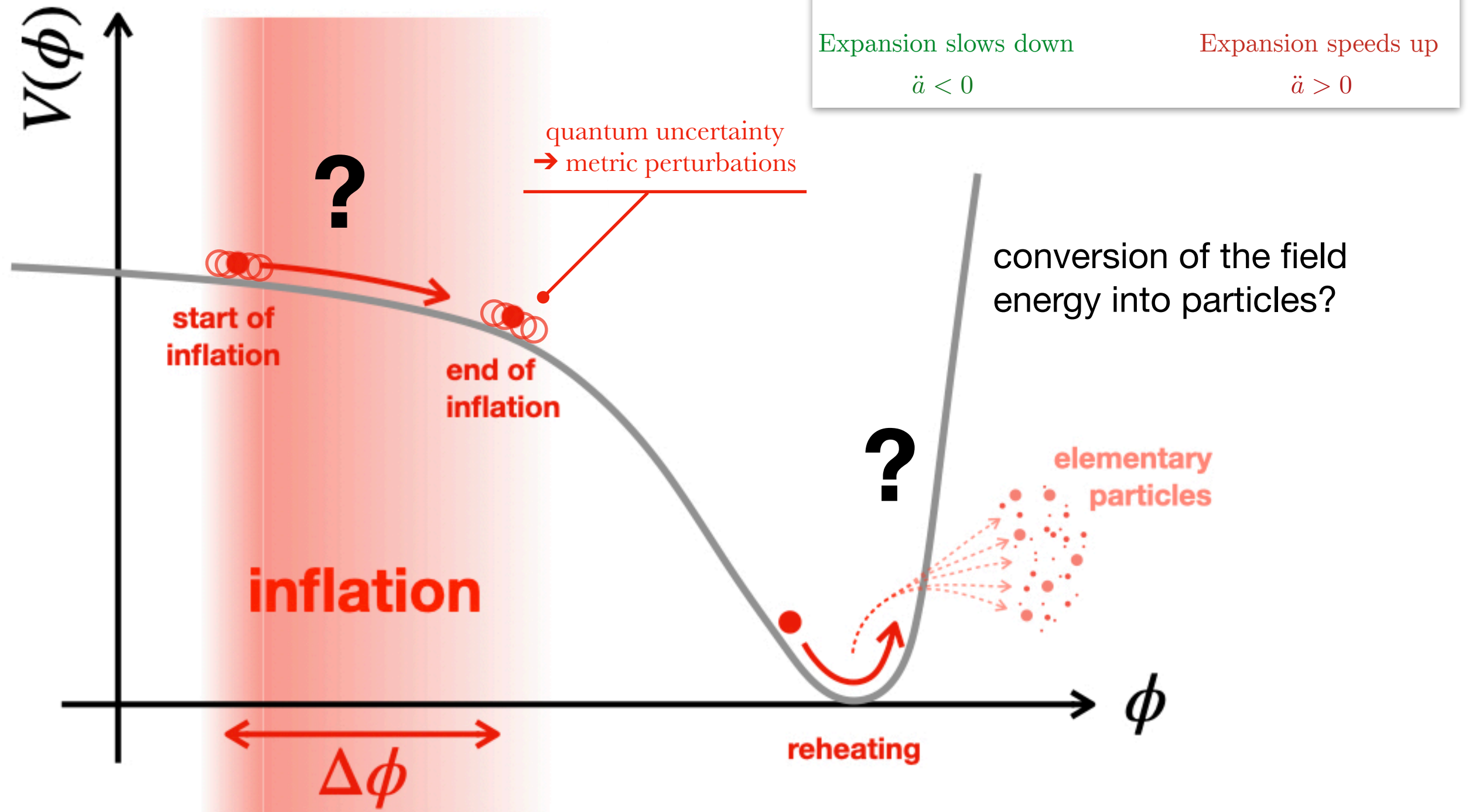
Expansion speeds up

$$\ddot{a} > 0$$



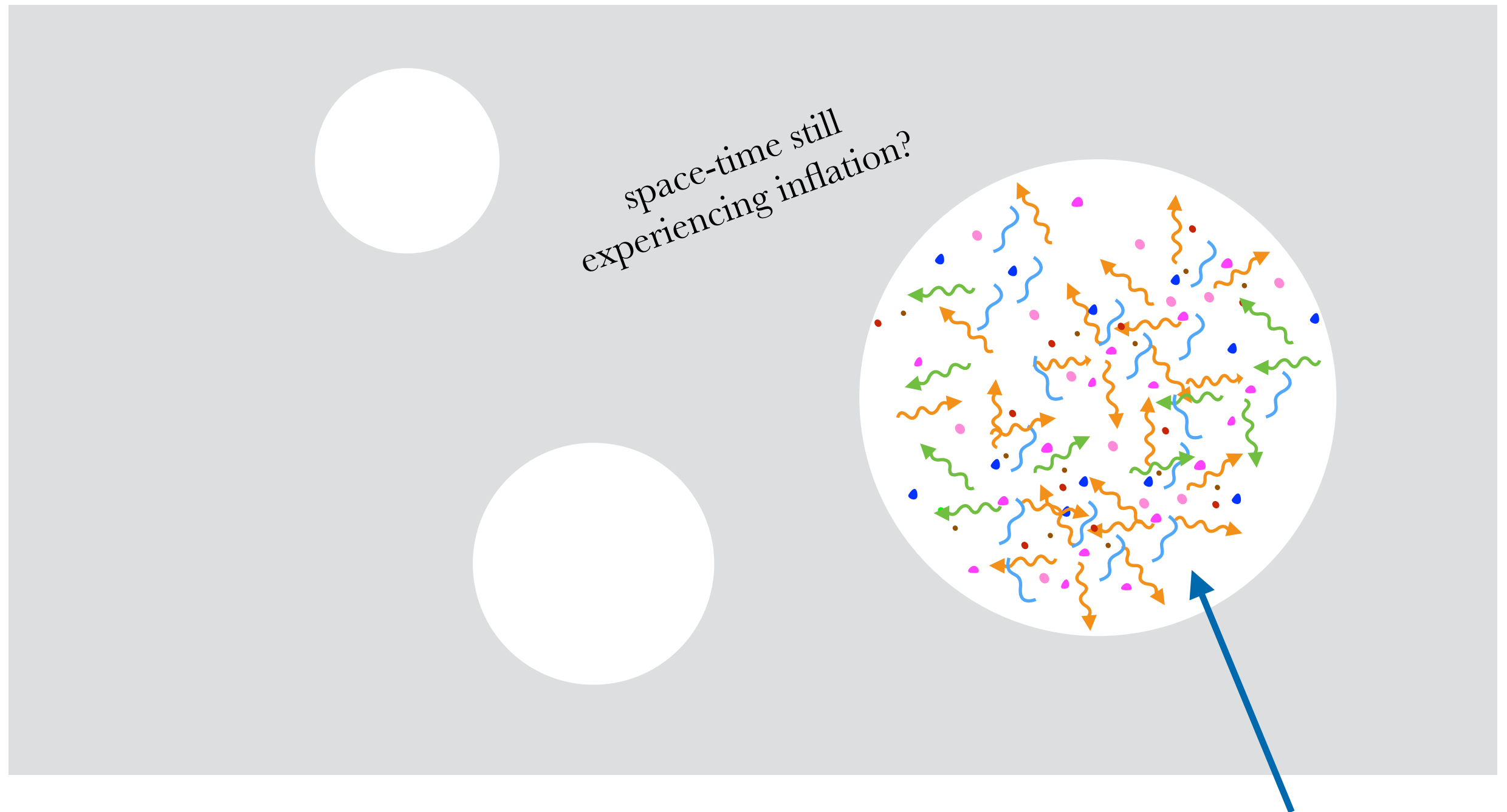
Inflation

To achieve inflation requires a substance with a nearly **constant energy density** (like **dark energy**):



Inflation

To end inflation, this substance must **decay** (like a radioactive material):

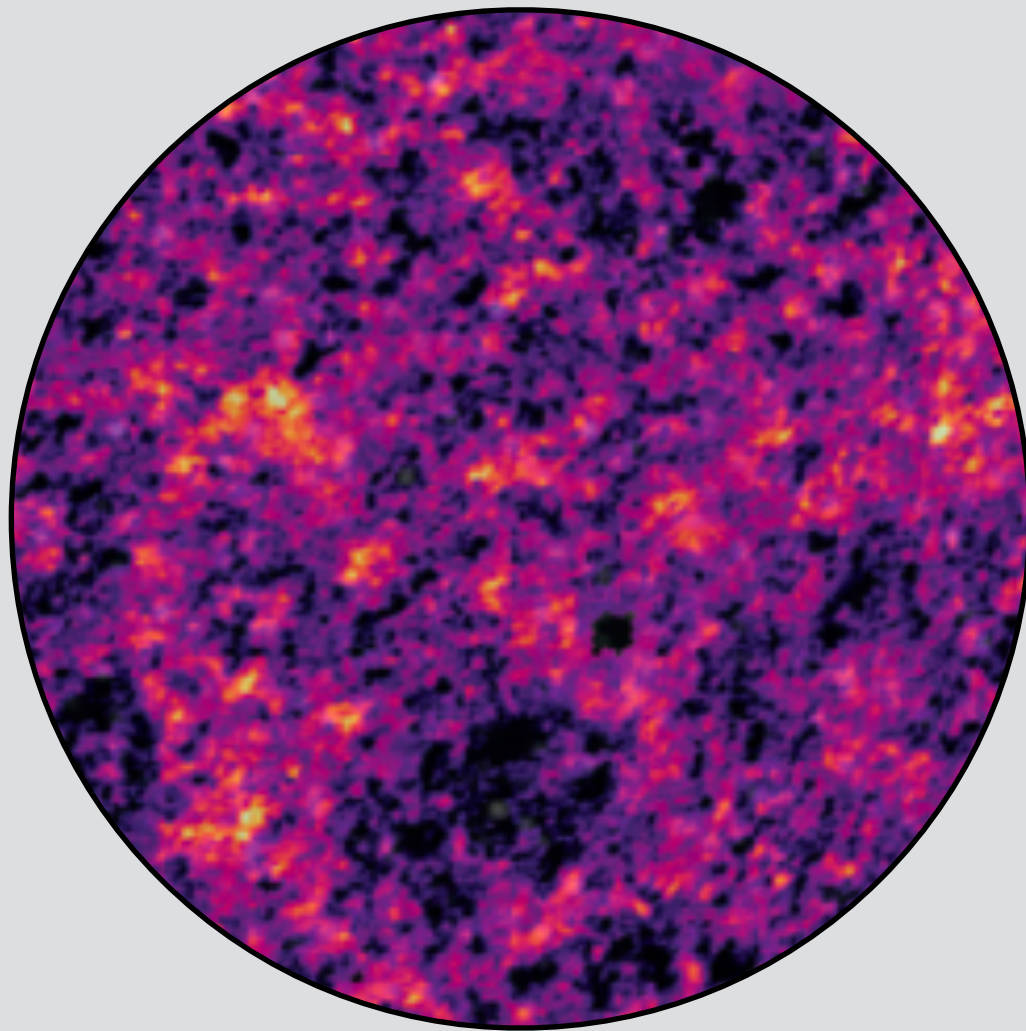


The product of this decay is the **hot Big Bang**.

One of these bubbles is our Universe.

Inflation

In quantum mechanics, the end of inflation is **probabilistic** and varies throughout space:

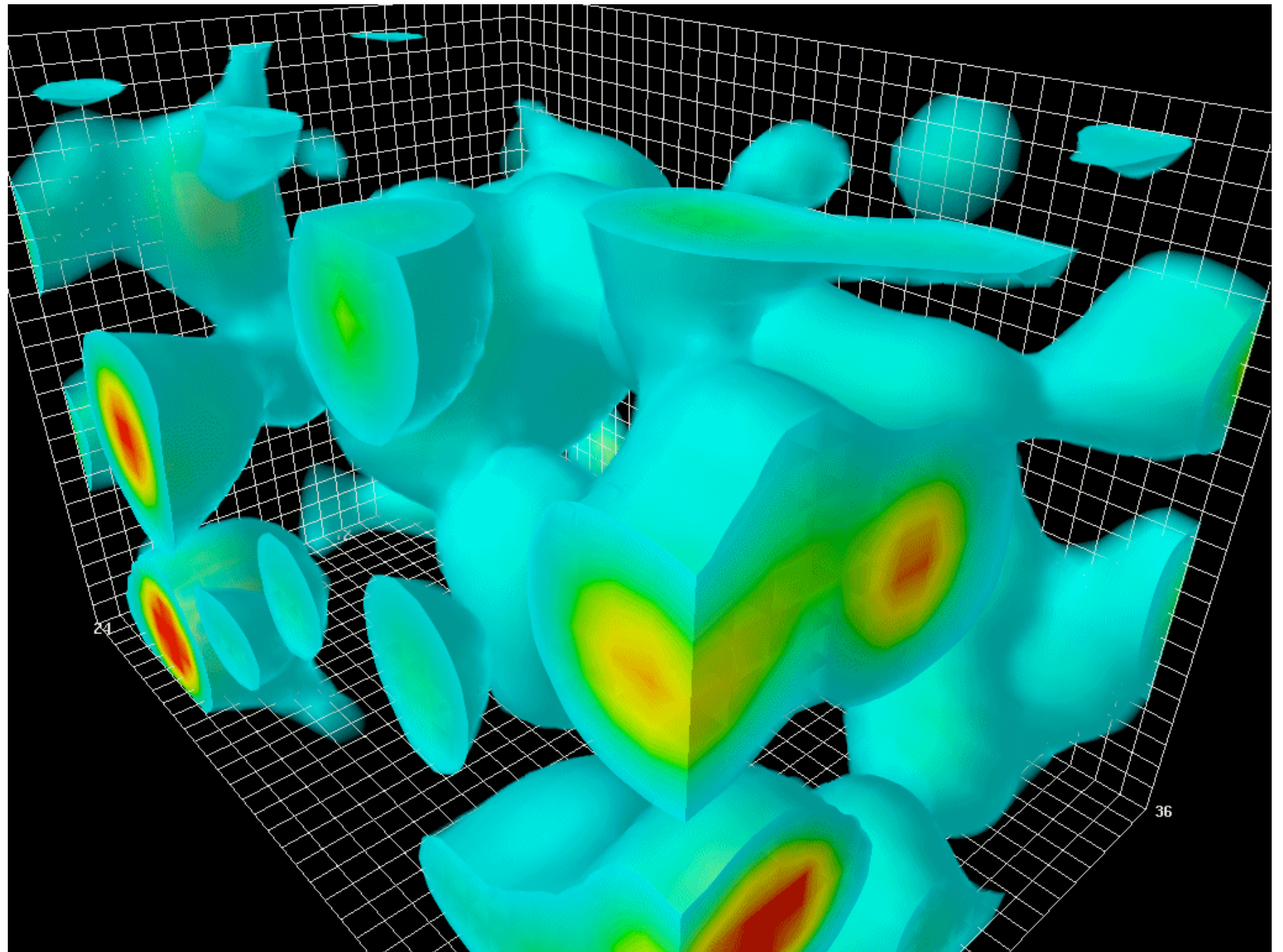


This creates the **primordial density fluctuations**.

Quantum Fluctuations

In quantum mechanics, empty space is full of violent fluctuations:

$$\Delta x \Delta p \geq \hbar$$

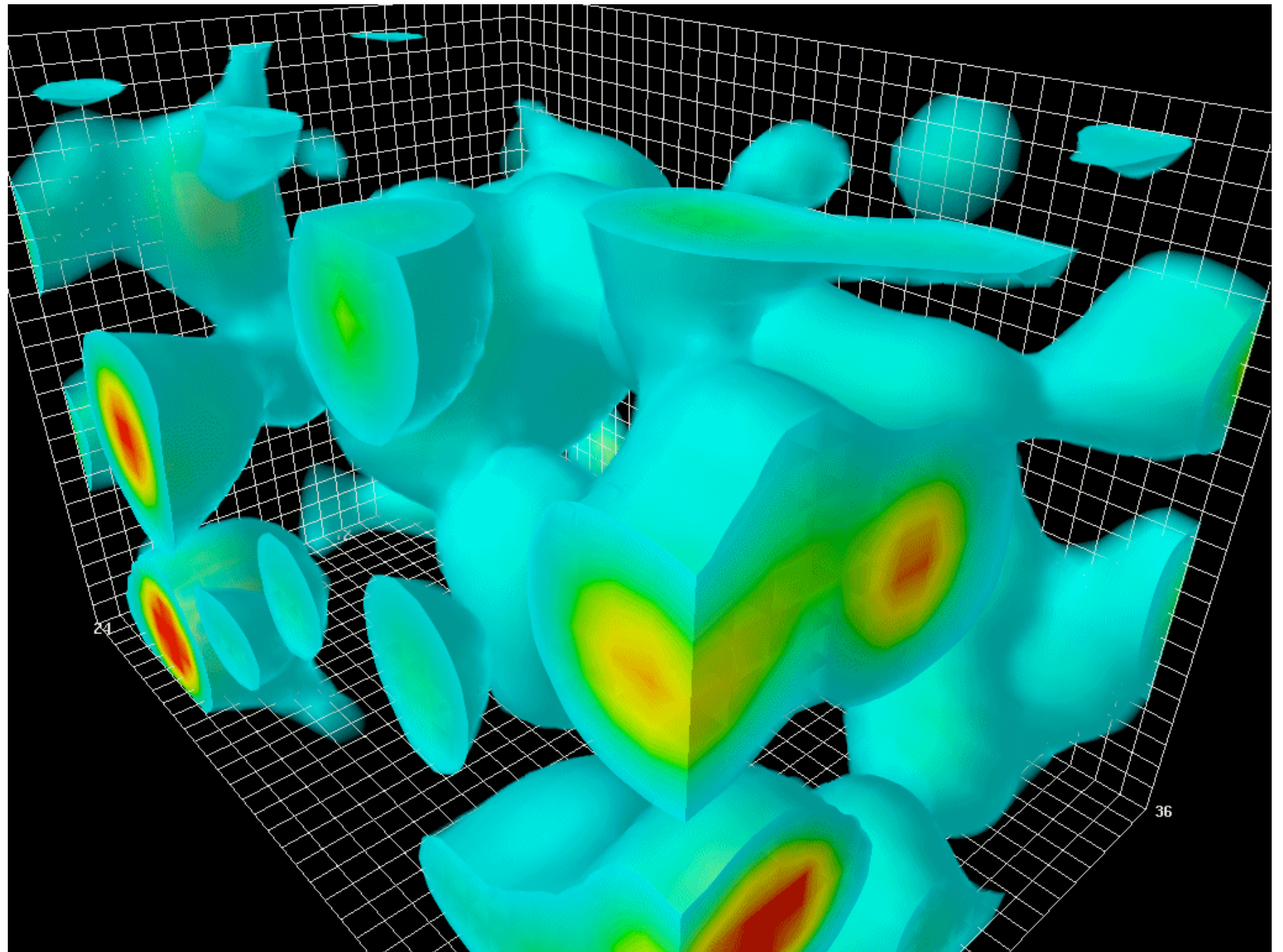


credits: Wikipedia. The animation illustrates the typical four-dimensional structure of gluon-field configurations averaged over in describing the vacuum properties of QCD. The volume of the box is 2.4 by 2.4 by 3.6 fm, big enough to hold a couple of protons. Contrary to the concept of an empty vacuum, QCD induces chromo-electric and chromo-magnetic fields throughout space-time in its lowest energy state. After a few sweeps of smoothing the gluon field (50 sweeps of APE smearing), a lumpy structure reminiscent of a lava lamp is revealed.

Quantum Fluctuations

In quantum mechanics, empty space is full of violent fluctuations:

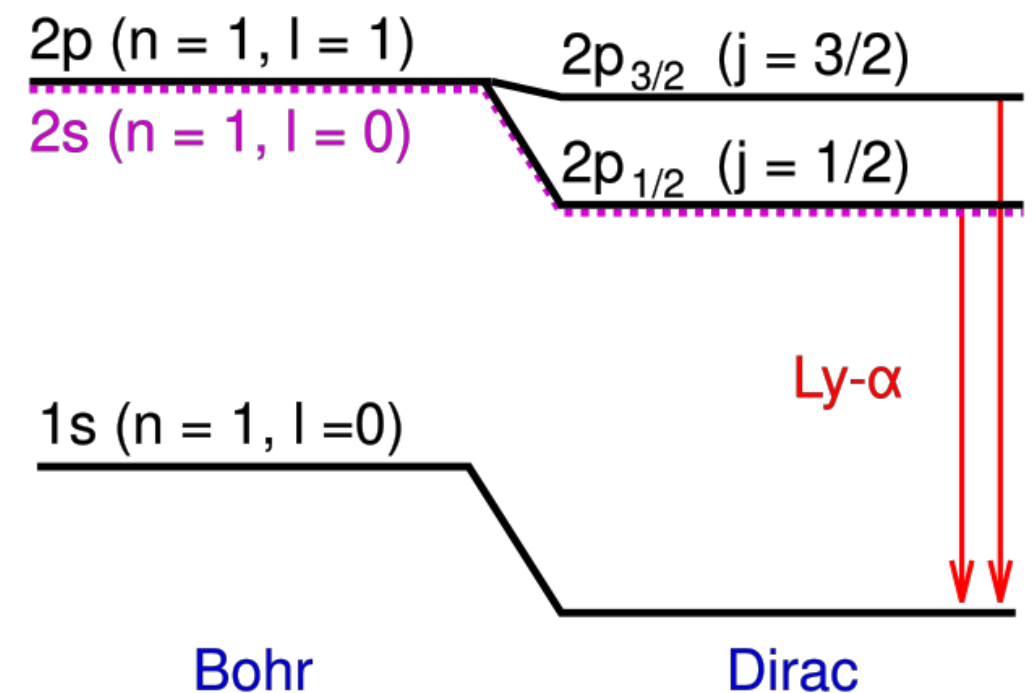
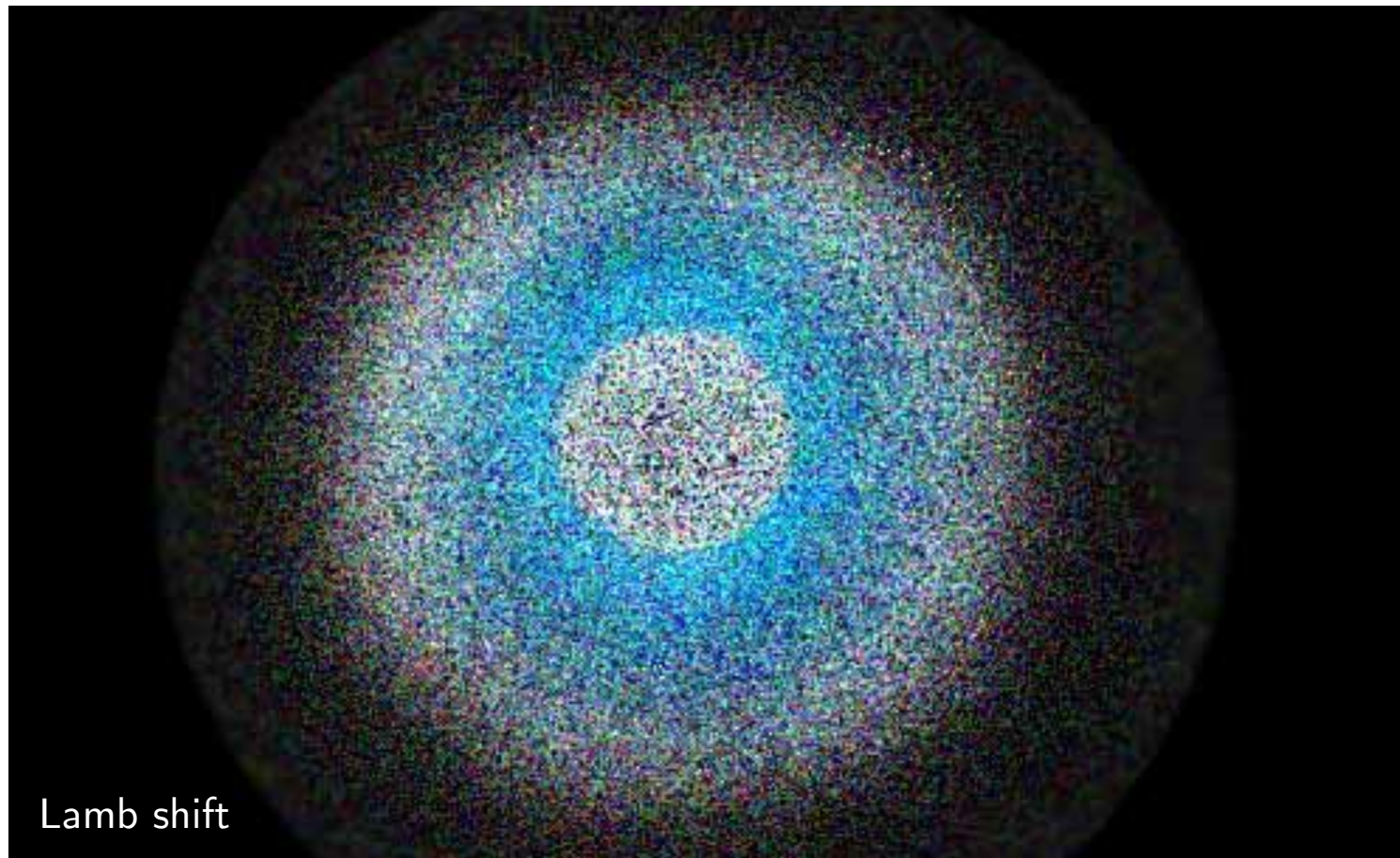
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Quantum Fluctuations

These quantum fluctuations are real, but usually have small effects:



The electron interacts with the fluctuating quantum vacuum $\rightarrow$ its effective position “jitters” (Lamb called this the electron’s “self-field”).

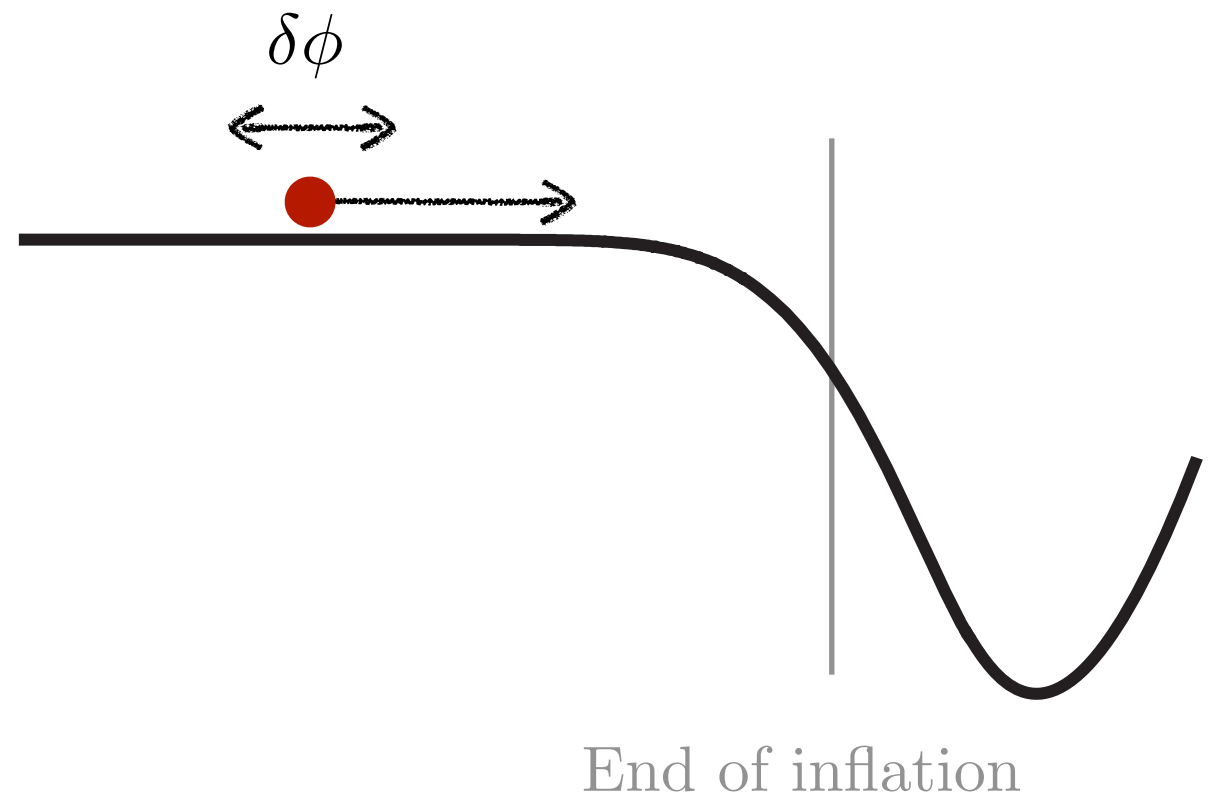
Fluctuations during Inflation

Each Fourier mode of the inflaton fluctuations satisfies:

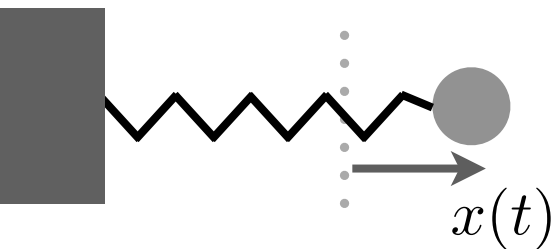
$$\ddot{\delta\phi} + 3H\dot{\delta\phi} + \omega_k^2(t)\delta\phi = 0$$

Hubble friction $k^2/a^2(t)$

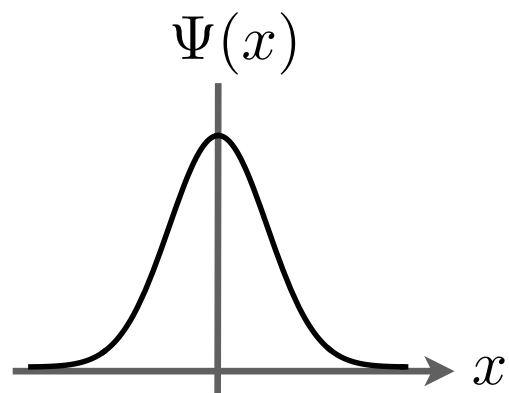
This is the equation of a time-dependent harmonic oscillator.



Quantum Harmonic Oscillators



$$\ddot{x} + \omega^2 x = 0$$



$$\langle x^2 \rangle = \frac{\hbar}{2\omega}$$

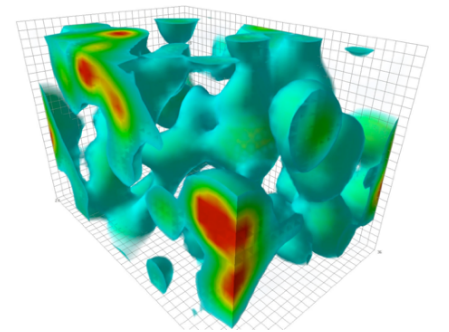
Zero-point fluctuations

$$\ddot{\delta\phi} + 3H\dot{\delta\phi} + \omega_k^2(t)\delta\phi = 0$$

$$\delta\phi_c \equiv a^{3/2}\delta\phi$$

$$\ddot{\delta\phi}_c + \omega_k^2(t)\delta\phi_c = 0$$

$$\langle (\delta\phi)_c^2 \rangle = \frac{1}{2\omega_k(t)}$$



$$\langle (\delta\phi)^2 \rangle = \frac{1}{2} \frac{1}{a^3(t)} \frac{1}{k/a(t)}$$

Hollands and Wald (2002)

Quantum Fluctuations during Inflation

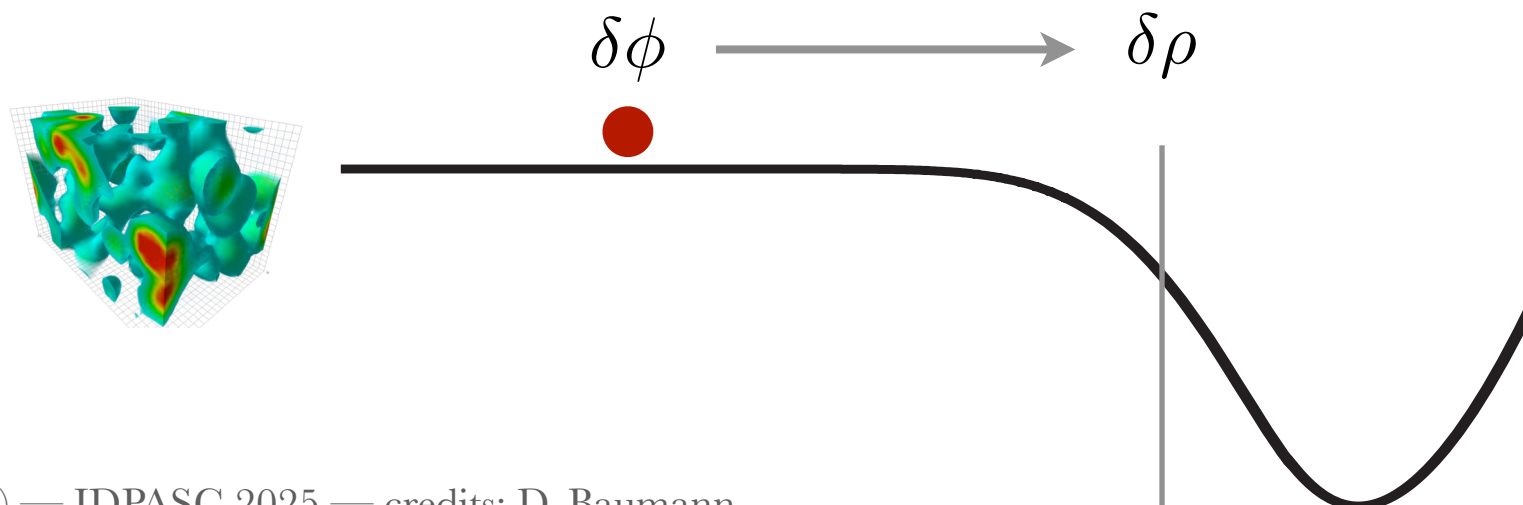
This holds as long as the mode evolves adiabatically (inside the horizon):

$$\langle (\delta\phi)^2 \rangle = \frac{1}{2} \frac{1}{k^3} \left(\frac{k}{a(t)} \right)^2$$

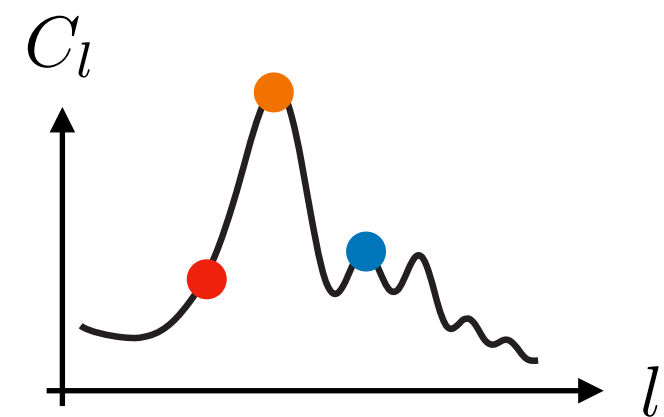
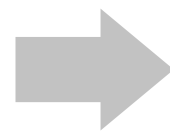
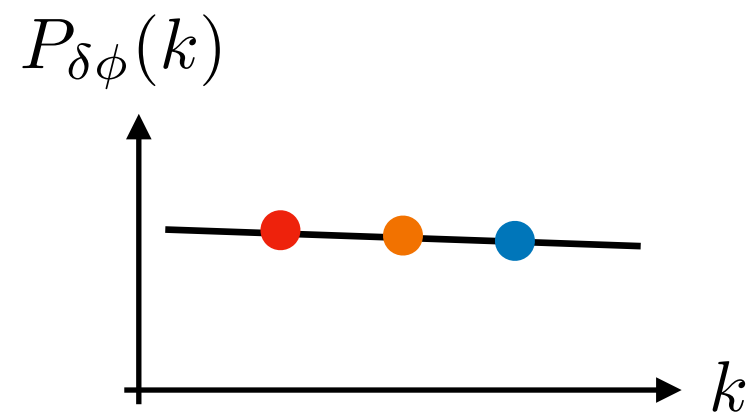
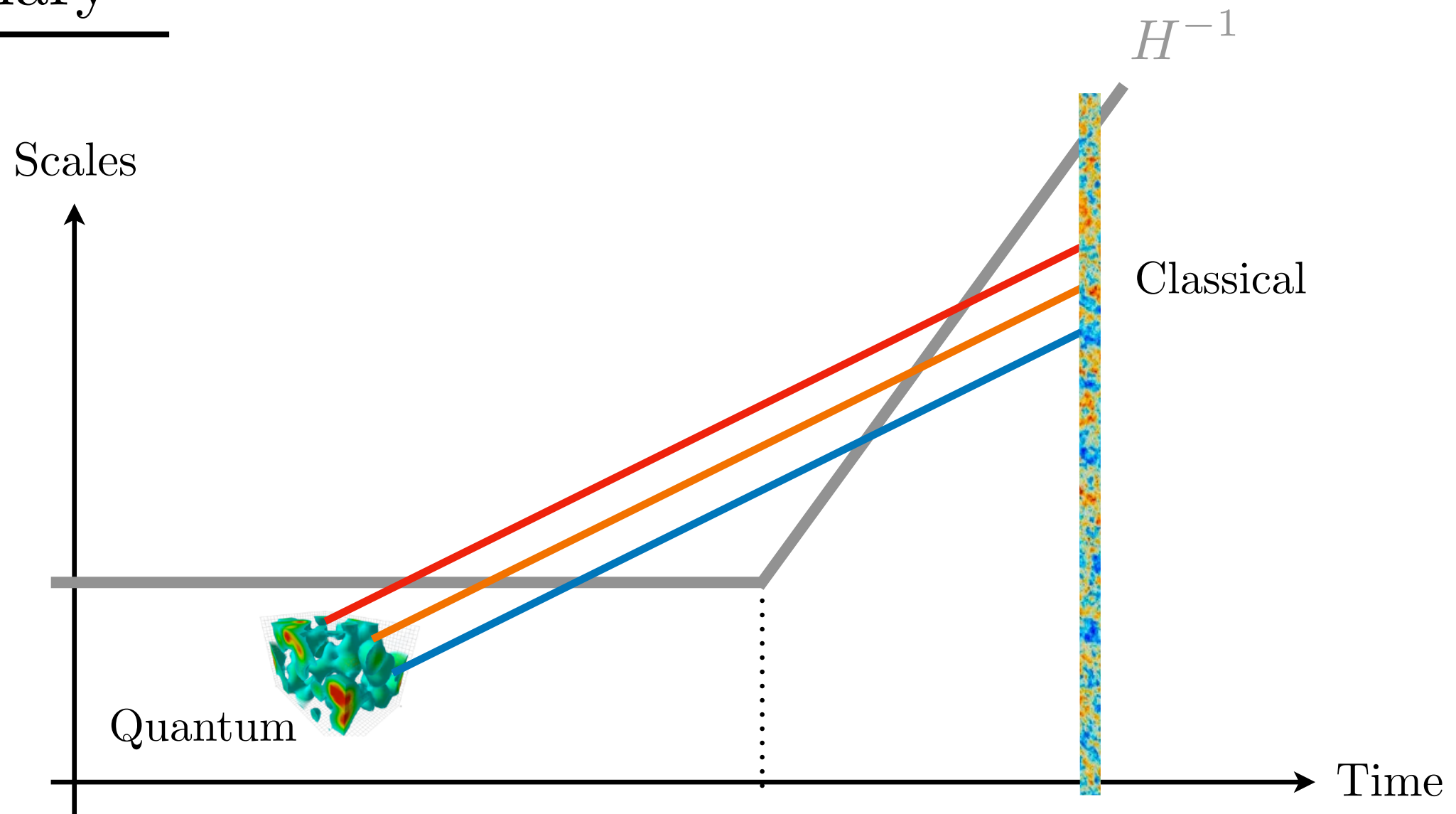
At horizon crossing, $k/a(t_*) = H(t_*)$, the fluctuations freeze and we get

$$P_{\delta\phi}(k) \equiv \frac{k^3}{2\pi^2} \langle (\delta\phi)^2 \rangle_* = \left(\frac{H}{2\pi} \right)^2 \approx \text{const}$$

After inflation, these become nearly scale-invariant density fluctuations.

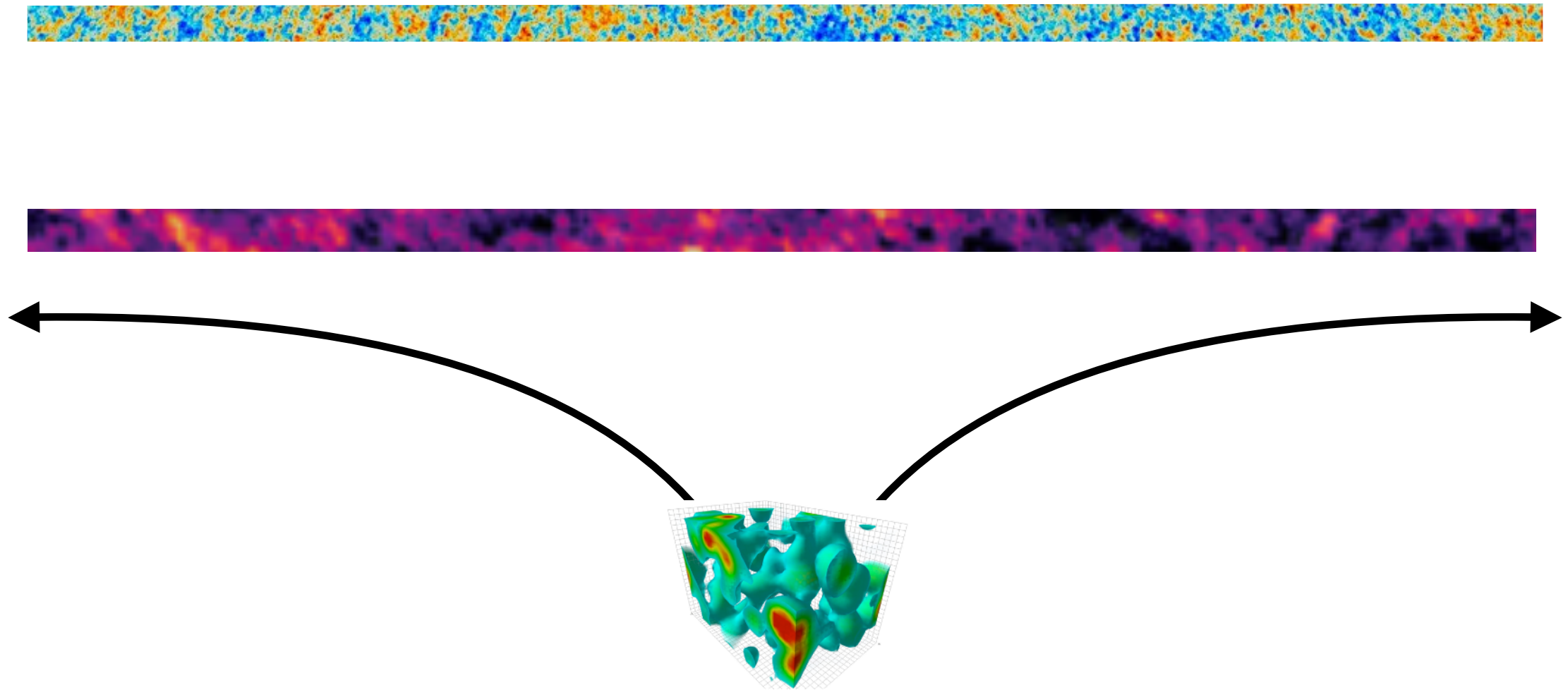


Summary



Quantum Fluctuations

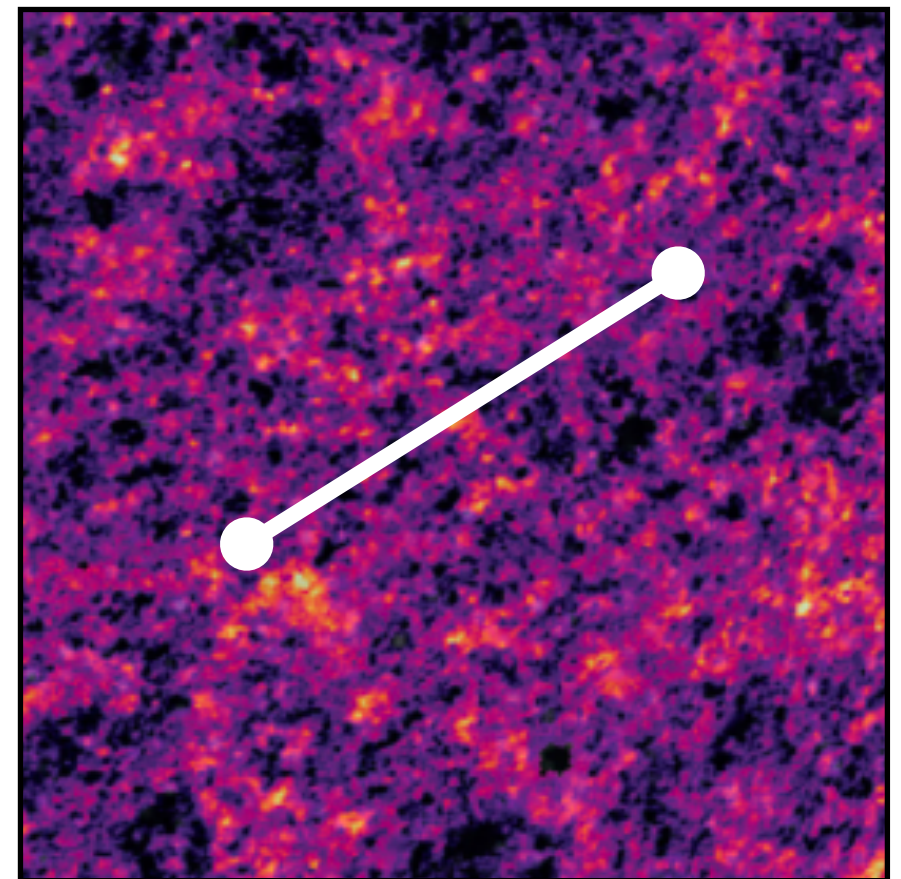
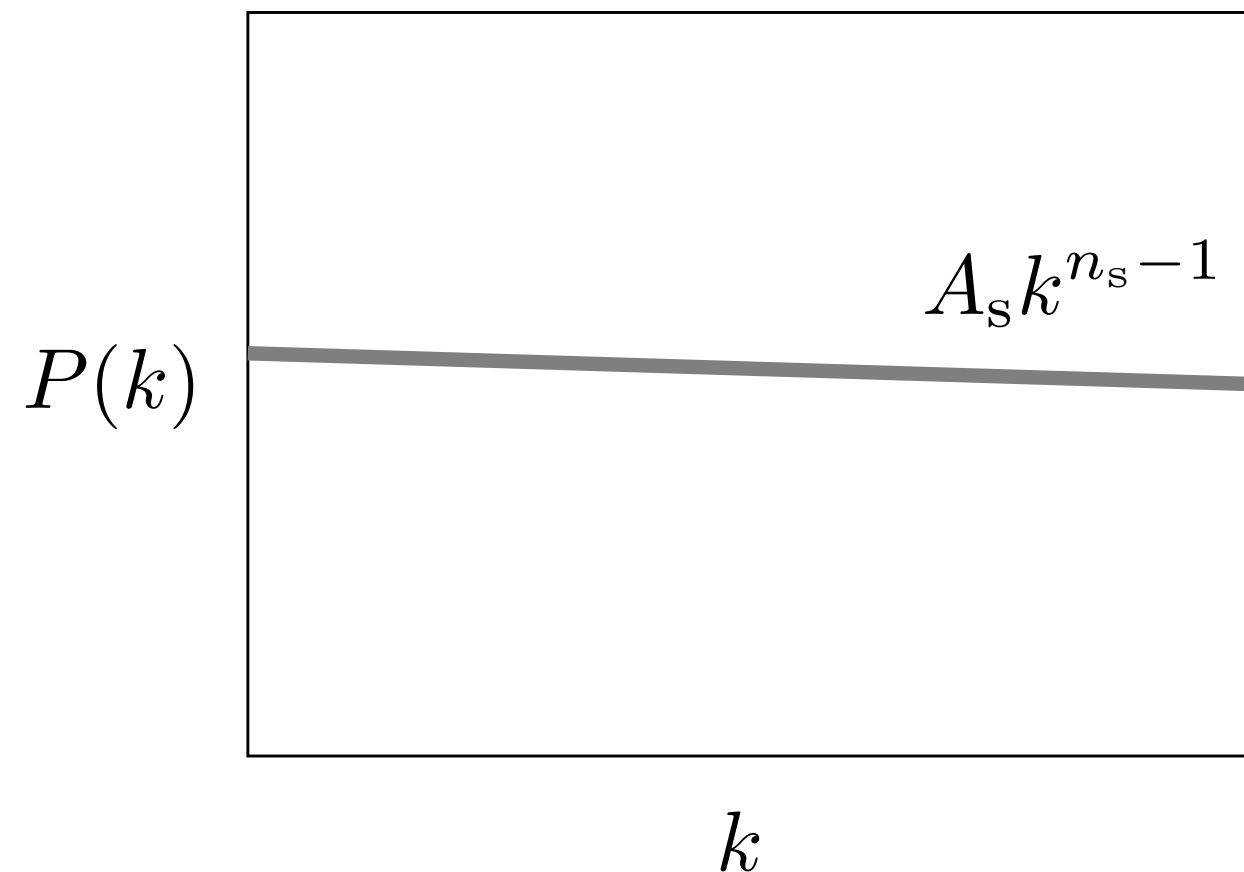
During inflation, these quantum fluctuations get amplified and stretched:



After inflation, these fluctuations become the large-scale density fluctuations.

Primordial Correlations

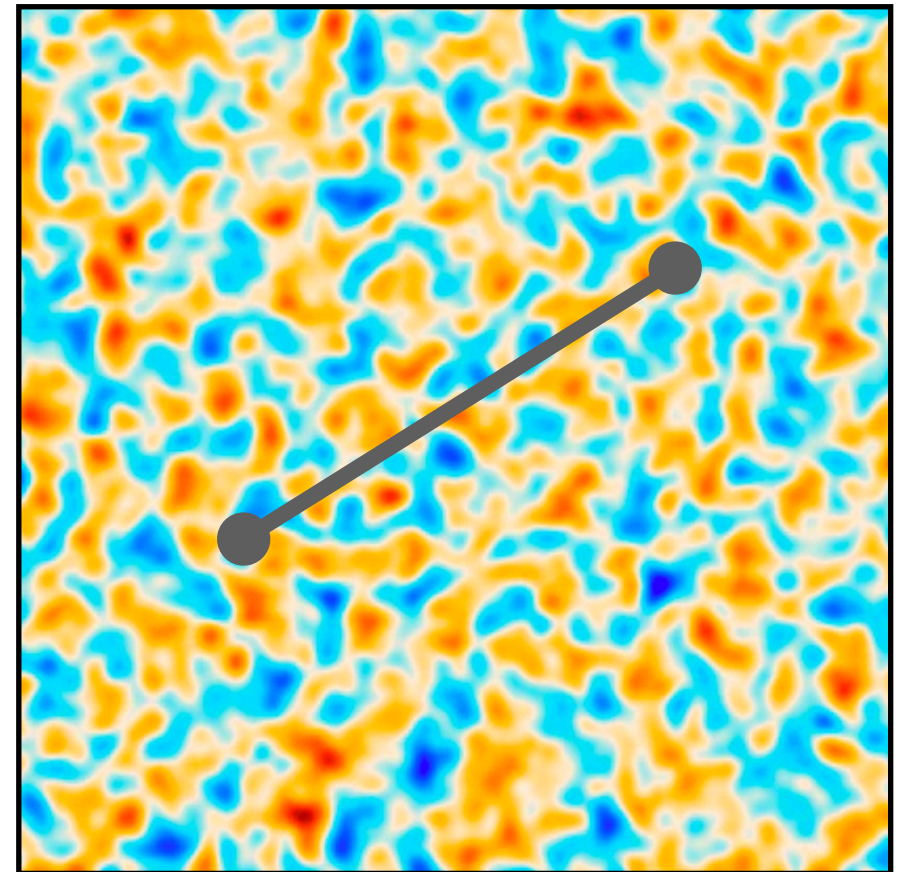
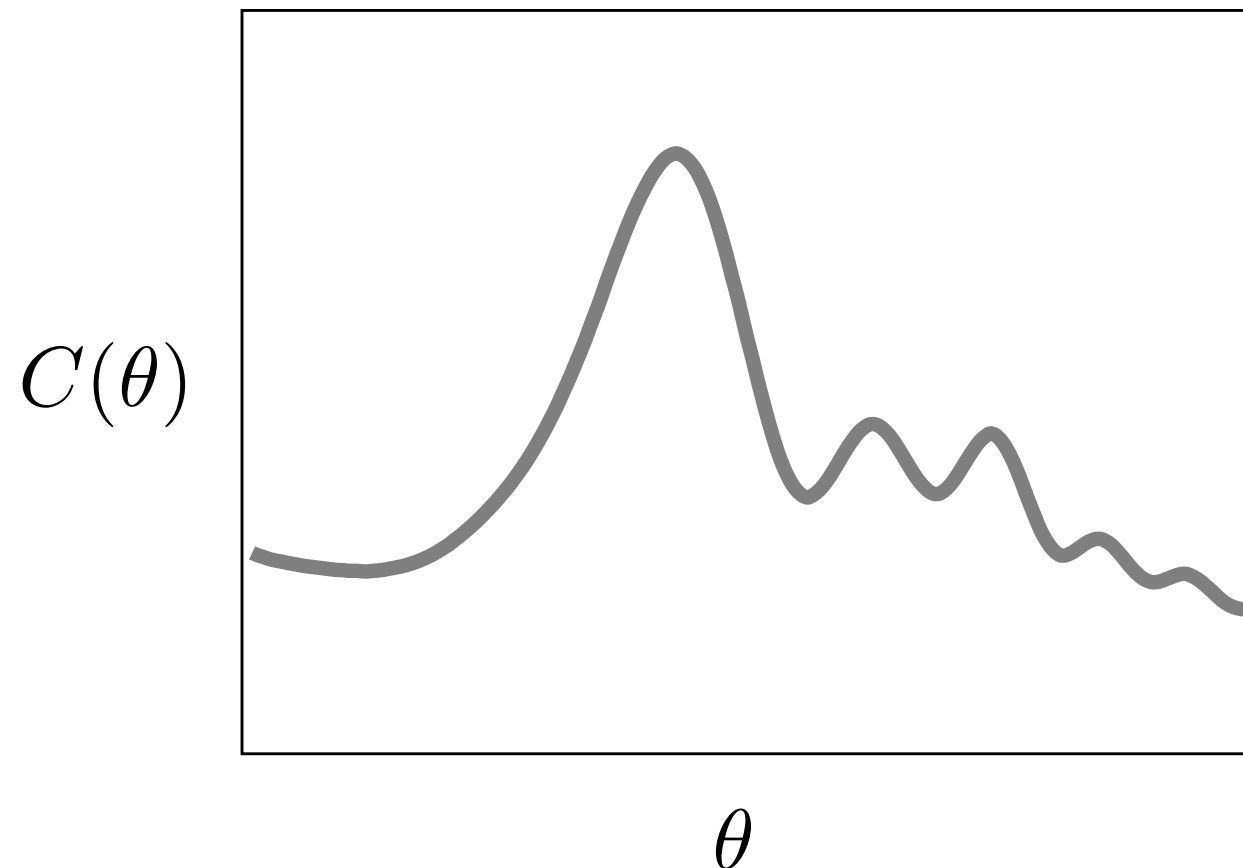
The nearly constant inflationary vacuum energy leads to an approximately **scale-invariant** power spectrum:



The slow decay of the inflationary energy predicts slightly more power on large scales: $n_s < 1$

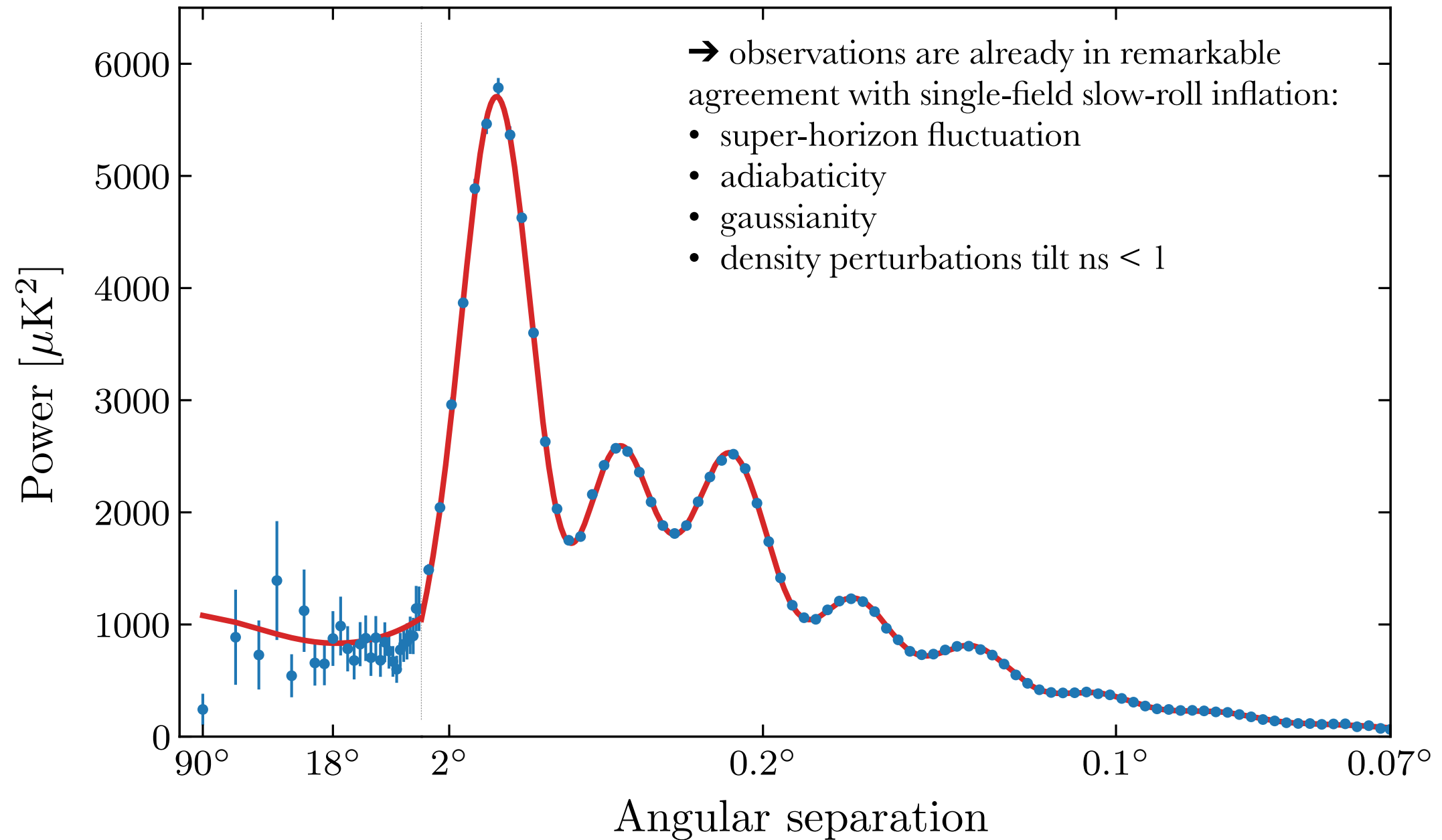
CMB Anisotropies

The well-understood physics of the photon-baryon fluid turns these primordial correlations into correlations of the CMB anisotropies:



CMB Anisotropies

The predicted correlations are in remarkable agreement with the data:

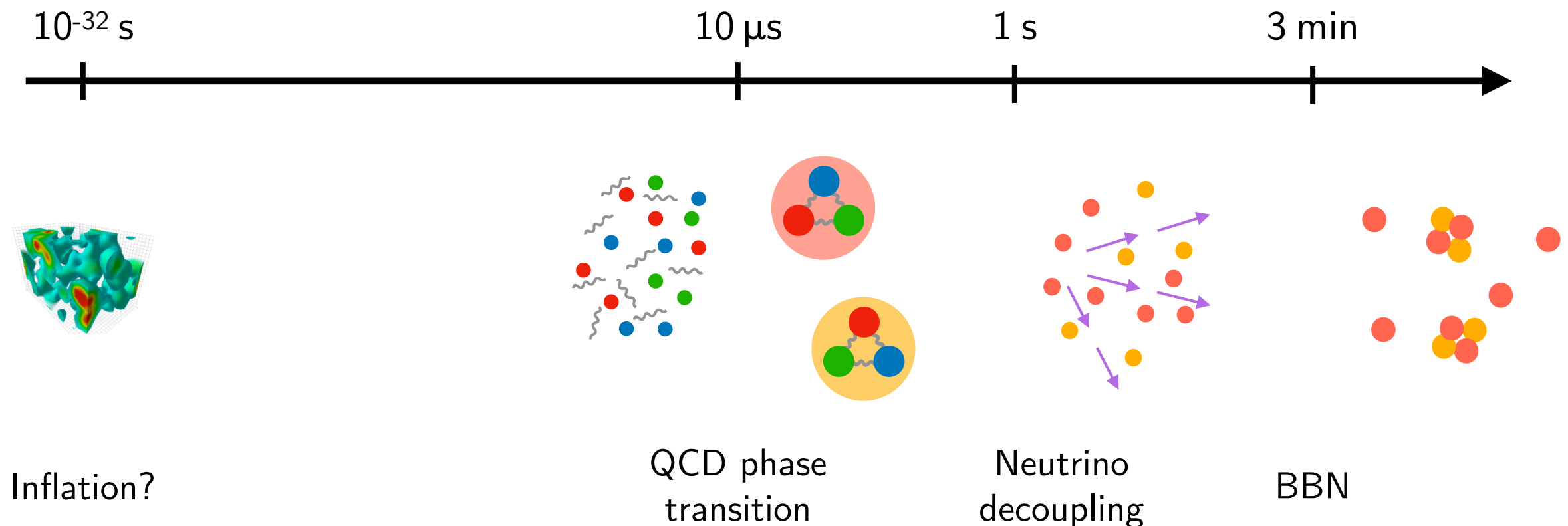


“Extraordinary claims require extraordinary evidence”

Carl Sagan

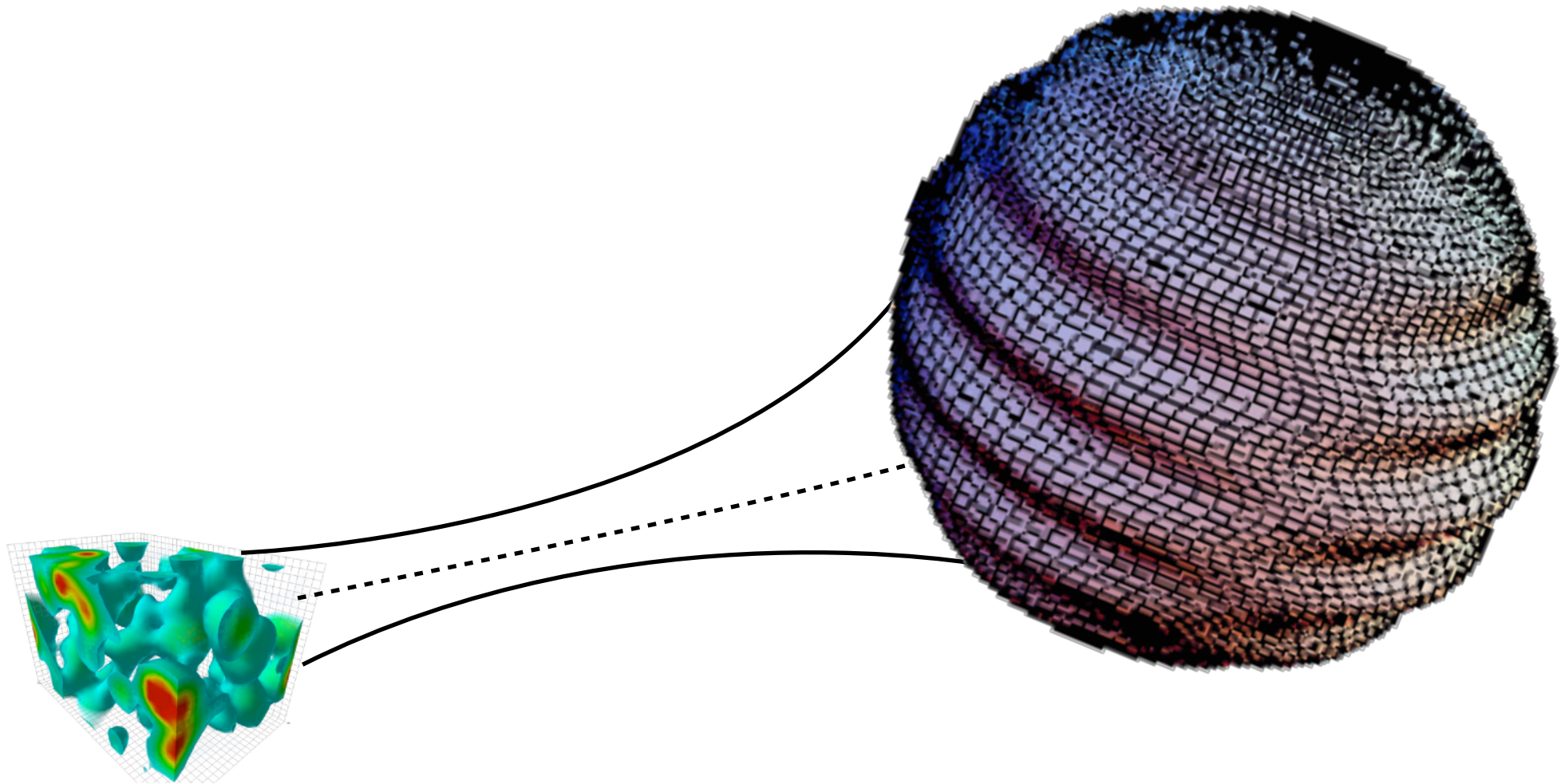


How can inflation become part of the standard history of the Universe with the same level of confidence as BBN?



Primordial Gravitational Waves

Besides density fluctuations, inflation predicts **gravitational waves**:



The strength of the signal depends on the energy scale of inflation, which may be as high as 10^{16} GeV.

Inflation

- According to single field, slow-roll inflationary scenario, quantum vacuum fluctuations excite cosmological scalar and tensor perturbations

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_0} \right)^{n_s - 1} \quad \text{scalar}$$

$$\mathcal{P}_{\mathcal{T}}(k) = A_t \left(\frac{k}{k_0} \right)^{n_t} \quad \text{tensor}$$

- With the definition of the tensor-to-scalar ratio “r” $r = A_t / A_s$ which characterizes the amplitude of GW and gives direct constraints on

- the shape of the potential $V^{1/4}(\phi) \simeq 10^{16} \text{ GeV} \left(\frac{r}{0.01} \right)^{1/4}$

- energy scale of inflation $\frac{\Delta\phi}{M_P} \simeq \mathcal{N}_* \left(\frac{r_*}{8} \right)^{1/2} \simeq \left(\frac{r}{0.001} \right)^{1/2}$

- inflaton field excursion $r = 8M_{\text{Pl}}^2 \left(\frac{V_\phi}{V} \right)^2$

- derivative of the potential $n_s - 1 \equiv \frac{d \ln \mathcal{P}_\zeta}{d \ln k} \simeq -3M_{\text{Pl}}^2 \left(\frac{V_\phi}{V} \right)^2 + 2M_{\text{Pl}}^2 \frac{V_{\phi\phi}}{V}$

CMB polarization and its description

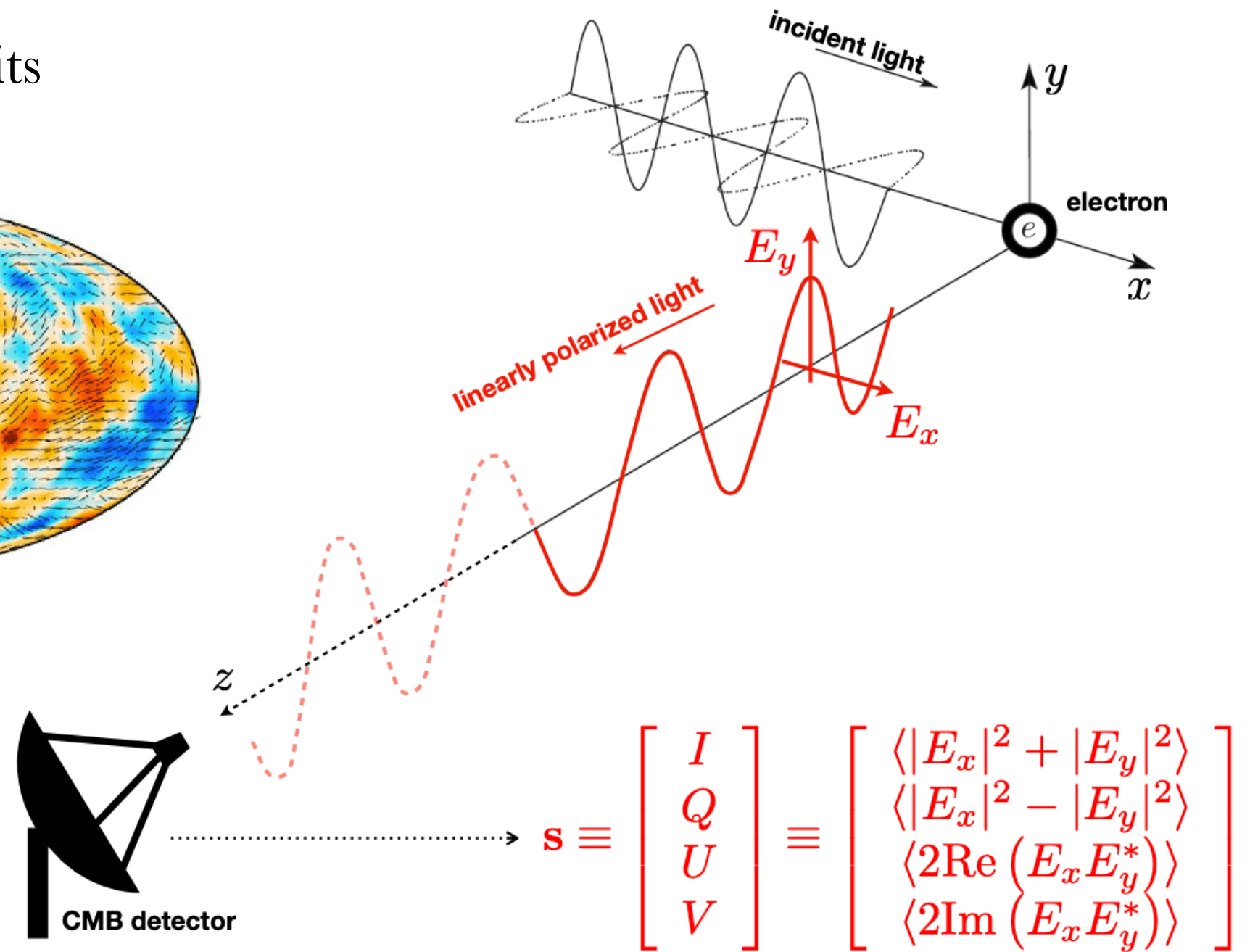
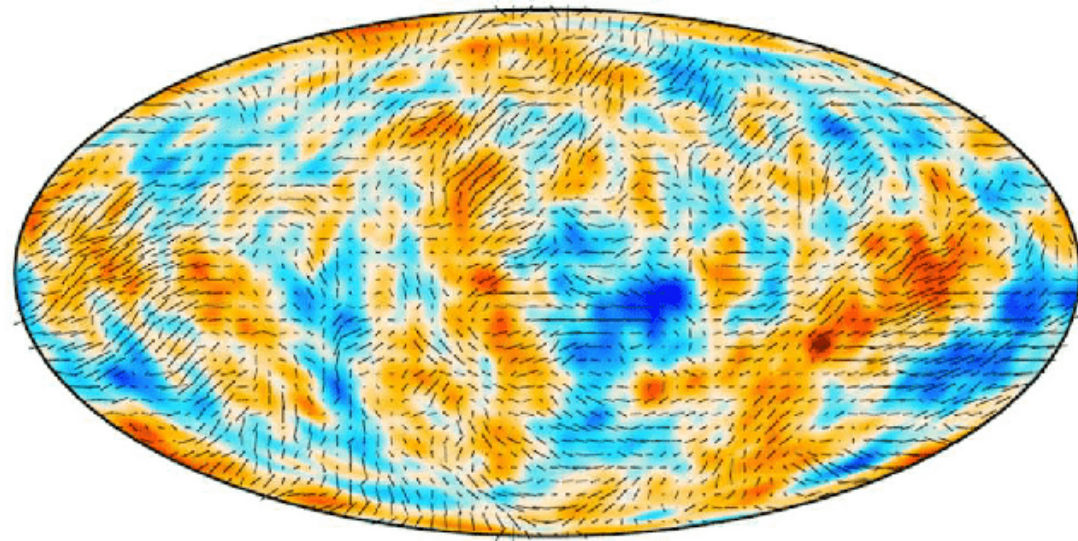


Figure 1.5: Thomson scattering, adapted from [186], and definition of the observed Stokes parameters. I corresponds to the total intensity of a given light beam, Q and U are the two Stokes parameters describing the linear polarization angle and amplitude, and V is for the circular polarization. In practice in this manuscript, the last dimension will be dropped as CMB light is, as far as it has been characterized, not circularly polarized (although atmospheric emission may be through the Zeeman splitting of O_2 molecules, e.g. [234]).

polarization needs anisotropic light + reflection



NO POLARIZER



WITH POLARIZER

polarization needs anisotropic light + reflection

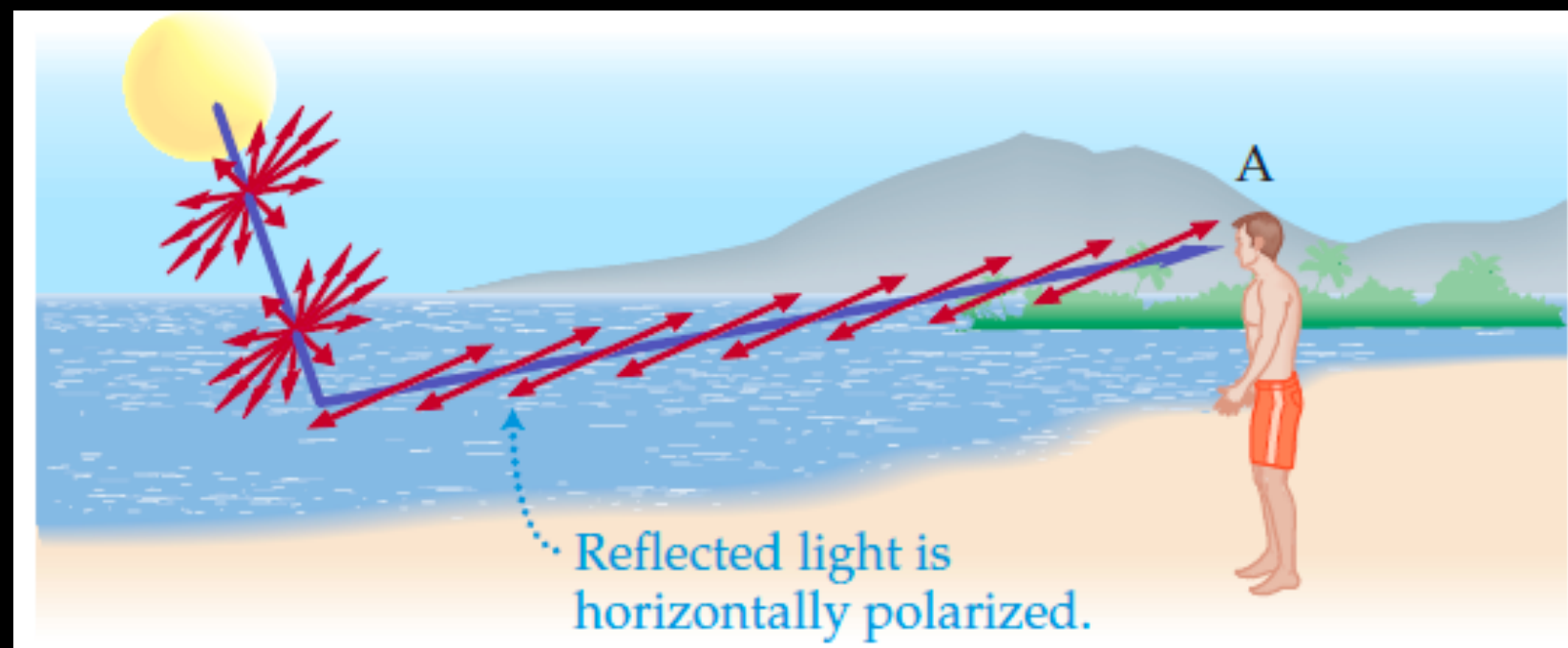


horizontal polarization

NO POLARIZER



WITH POLARIZER



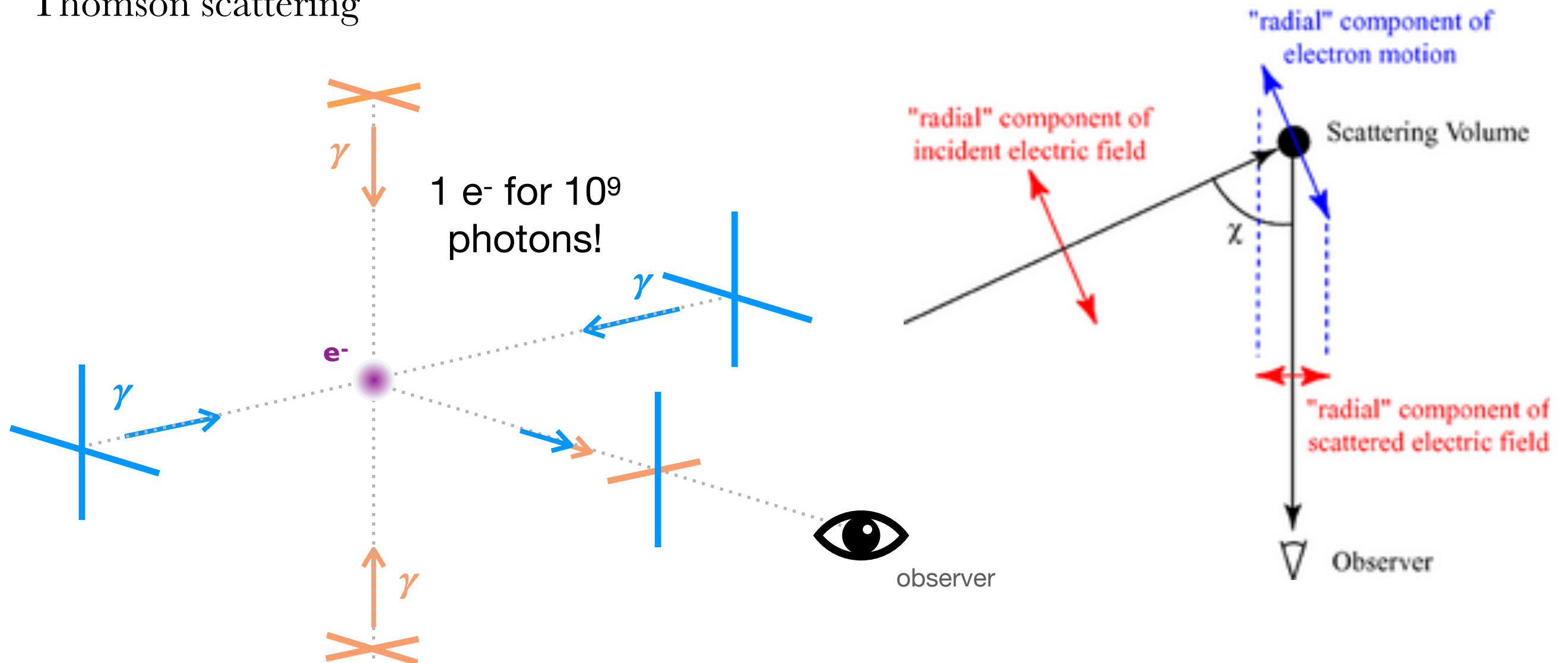
Reflected light is horizontally polarized.

From CMB anisotropies to polarization

$$\frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{8\pi} |\vec{\epsilon} \cdot \vec{\epsilon}'|^2$$

Angular dependence leads to linear polarization.

Thomson scattering

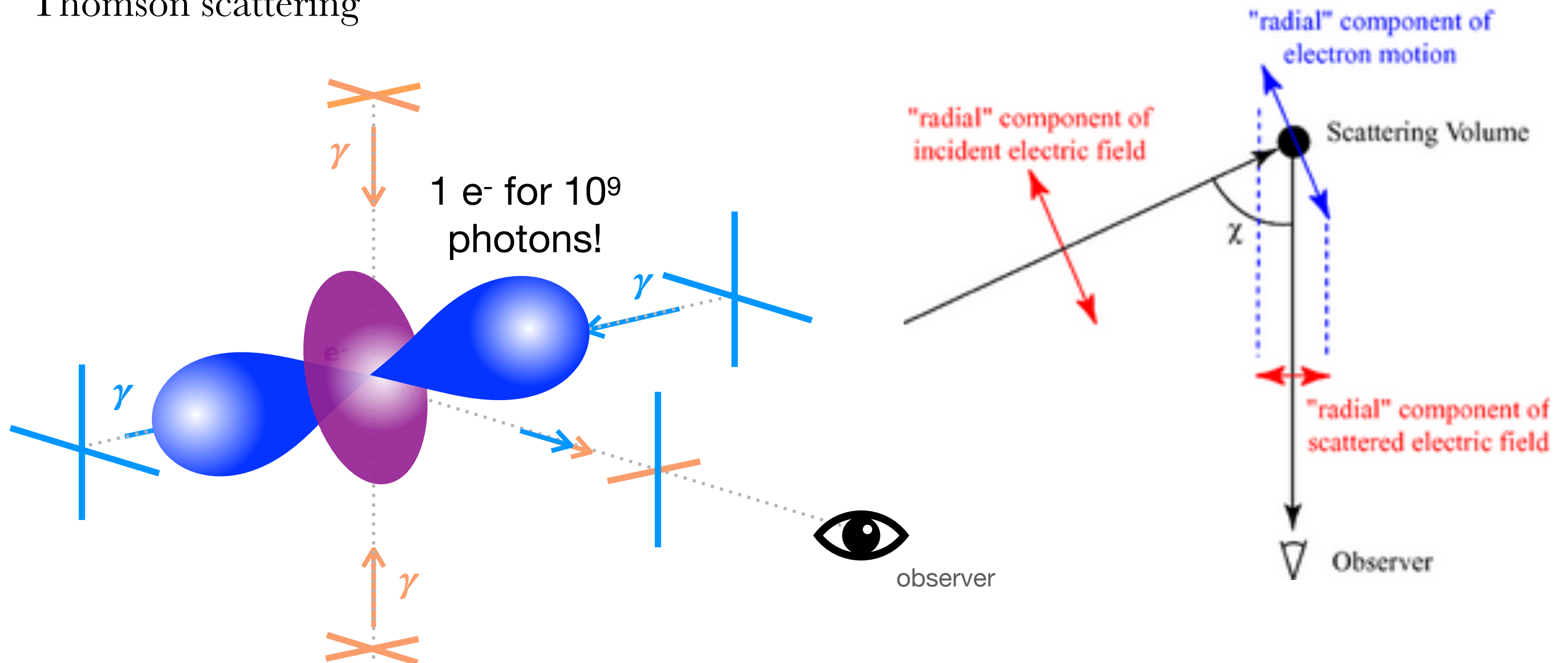


From CMB anisotropies to polarization

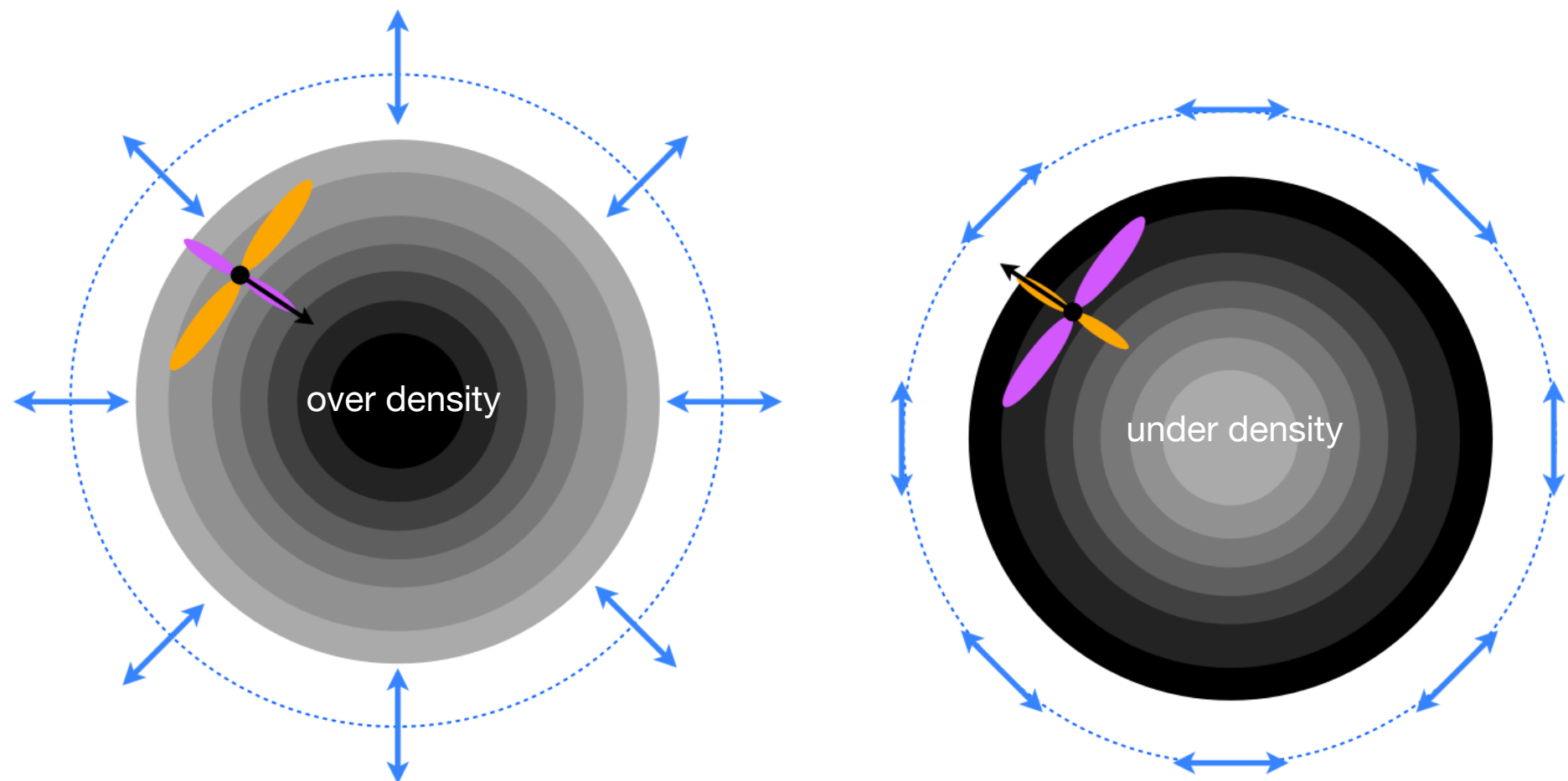
$$\frac{d\sigma}{d\Omega} = \frac{3\sigma_T}{8\pi} |\vec{\epsilon} \cdot \vec{\epsilon}'|^2$$

Angular dependence leads to linear polarization.

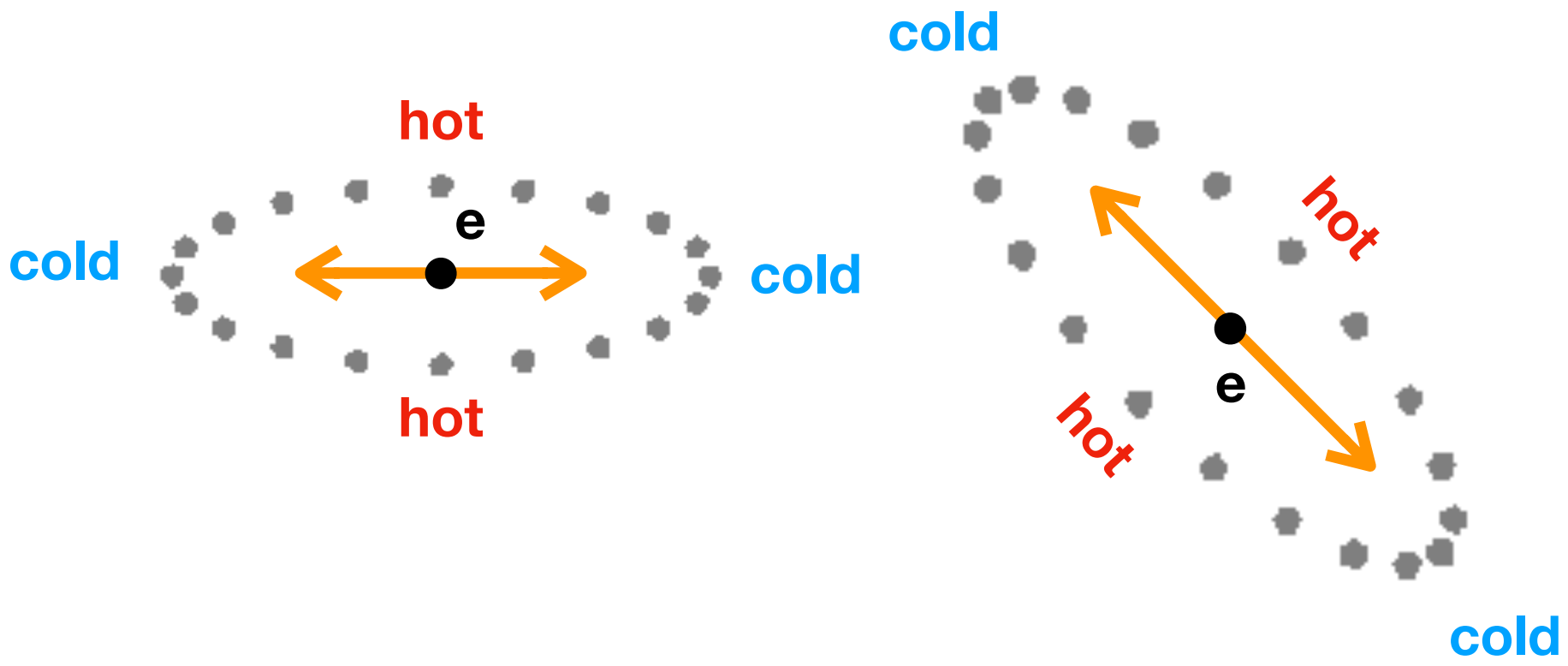
Thomson scattering



Thomson scattering + quadrupolar anisotropies = linear polarization

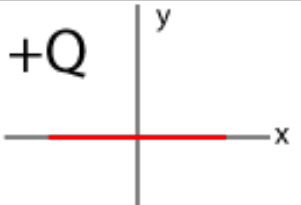
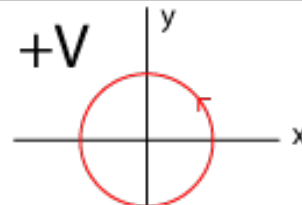
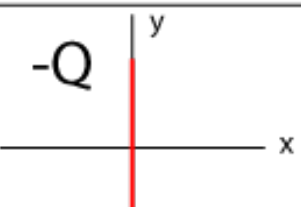
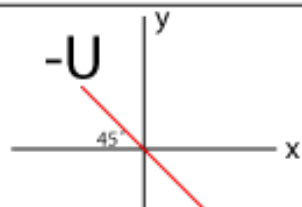
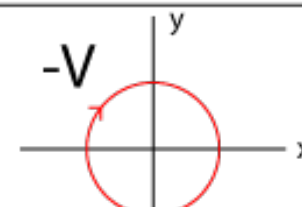


Thomson scattering
in stretched space



Quantifying polarization

One could describe the polarization by means of standard Stokes Q and U parameters, but that will make their value dependent of the choice of x- and y-axes.

100% Q	100% U	100% V
+Q  $Q > 0; U = 0; V = 0$ (a)	+U  $Q = 0; U > 0; V = 0$ (c)	+V  $Q = 0; U = 0; V > 0$ (e)
-Q  $Q < 0; U = 0; V = 0$ (b)	-U  $Q = 0; U < 0; V = 0$ (d)	-V  $Q = 0; U = 0; V < 0$ (f)

$$I = |E_x|^2 + |E_y|^2,$$

$$Q = |E_x|^2 - |E_y|^2,$$

$$U = 2\text{Re}(E_x E_y^*),$$

$$V = -2\text{Im}(E_x E_y^*),$$

$$\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Linearly polarized (horizontal)

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Right-hand circularly polarized

$$\begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

Linearly polarized (vertical)

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}$$

Left-hand circularly polarized

$$\begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

Linearly polarized (+45°)

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

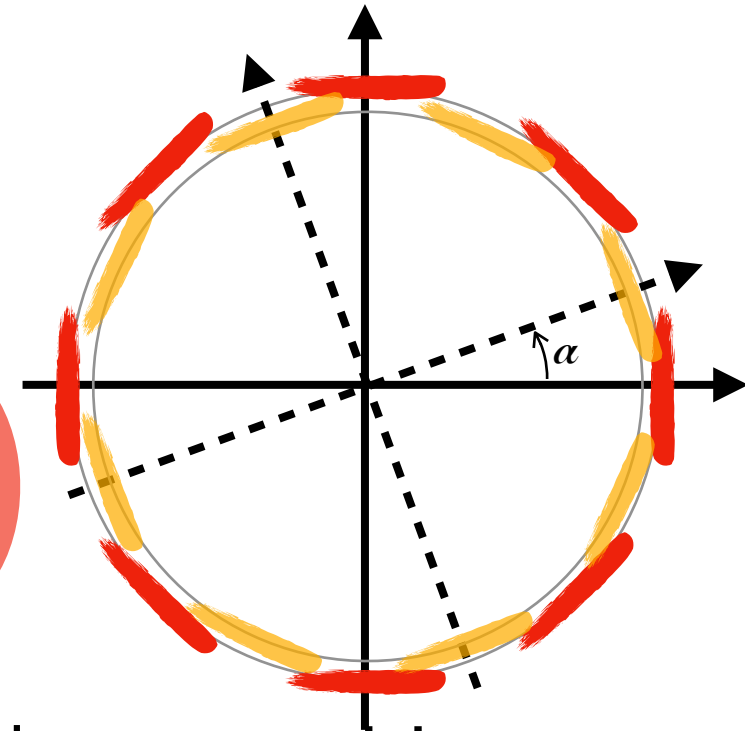
Unpolarized

$$\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$$

Linearly polarized (−45°)

Coordinate dependence of Q and U

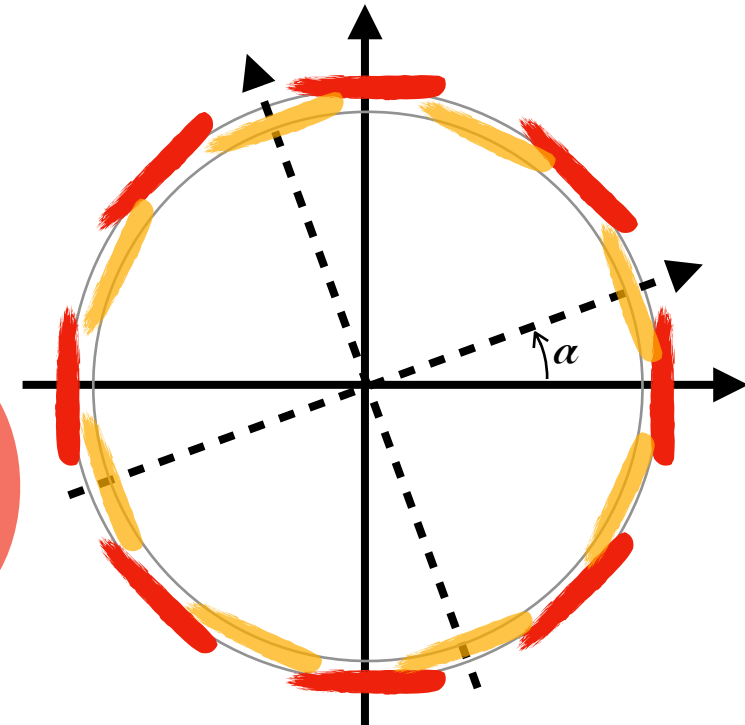
$$\begin{pmatrix} Q' \\ U' \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix} \begin{pmatrix} Q \\ U \end{pmatrix}$$



➔ Motivates decomposition into coordinate-independent quantities.

Coordinate dependence of Q and U

$$\begin{pmatrix} Q' \\ U' \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix} \begin{pmatrix} Q \\ U \end{pmatrix}$$



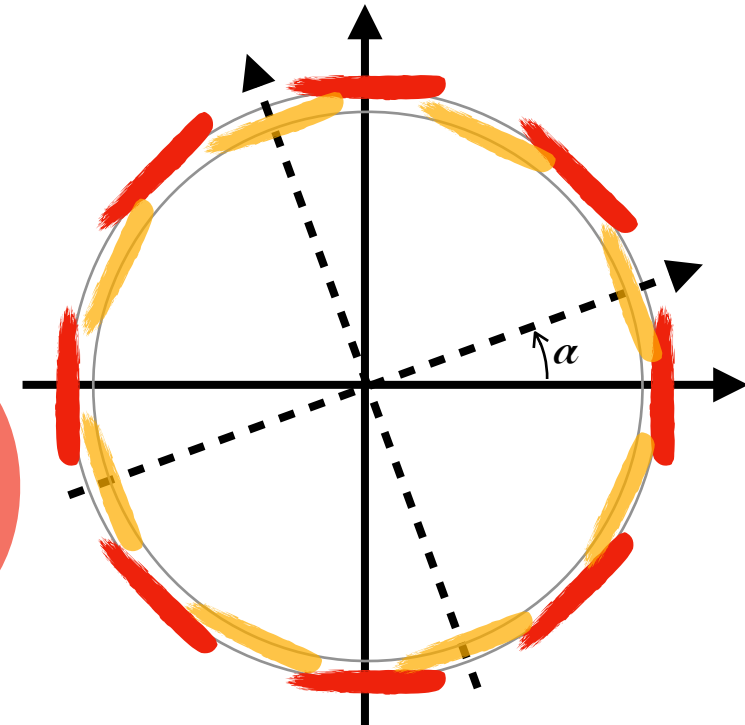
➔ Motivates decomposition into coordinate-independent quantities.

To do that, we write the polarization as a complex vector, which behaves as a spin-2 field. (We will then choose special spin-2 spherical harmonic coefficients which has the rotation term in-built, which **cancels out** the rotation of $Q + iU$ vector.)

$$\begin{aligned} P &\equiv \sqrt{Q^2 + U^2} \\ \frac{U}{Q} &\equiv \tan \alpha \end{aligned} \quad Q + iU = P e^{2i\alpha}$$

Coordinate dependence of Q and U

$$\begin{pmatrix} Q' \\ U' \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix} \begin{pmatrix} Q \\ U \end{pmatrix}$$



➔ Motivates decomposition into coordinate-independent quantities.

To do that, we write the polarization as a complex vector, which behaves as a spin-2 field. (We will then choose special spin-2 spherical harmonic coefficients which has the rotation term in-built, which **cancels out** the rotation of $Q + iU$ vector.)

$$P \equiv \sqrt{Q^2 + U^2}$$

$$\frac{U}{Q} \equiv \tan \alpha$$

$$Q + iU = P e^{2i\alpha}$$

under rotation: $\alpha \rightarrow \alpha + \psi \longrightarrow Q \pm iU \rightarrow e^{\mp 2i\psi} (Q \pm iU)$

P remains invariant ... The magnitude of $Q \pm iU$ is preserved, but its phase changes

We thus write:

$$Q(\mathbf{n}) \pm iU(\mathbf{n}) = \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2} {}_{\mp 2}a_{\boldsymbol{\ell}} \exp(\pm 2i\phi_{\boldsymbol{\ell}} + i\boldsymbol{\ell} \cdot \mathbf{n})$$

and defining:

$${}_{\pm 2}a_{\boldsymbol{\ell}} \equiv - \left(E_{\boldsymbol{\ell}} \pm iB_{\boldsymbol{\ell}} \right)$$

We thus write:

$$Q(\mathbf{n}) \pm iU(\mathbf{n}) = \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2} \mp 2a_{\boldsymbol{\ell}} \exp(\pm 2i\phi_{\boldsymbol{\ell}} + i\boldsymbol{\ell} \cdot \mathbf{n})$$

and defining:

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$$Q(\mathbf{n}) \pm iU(\mathbf{n}) = \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2} (E_{\boldsymbol{\ell}} \pm iB_{\boldsymbol{\ell}}) \exp(\pm 2i\phi_{\boldsymbol{\ell}} + i\boldsymbol{\ell} \cdot \mathbf{n})$$

by construction $E_{\boldsymbol{\ell}}$ and $B_{\boldsymbol{\ell}}$ do not pick up a factor of $\exp(2i\phi)$ under coordinate rotation. 🥳

$E_{\boldsymbol{\ell}}$ and $B_{\boldsymbol{\ell}}$ are invariant under rotation.

What kind of polarization patterns do these quantities represent?

Q and U produced by E and B modes are given by:

$$Q(\mathbf{n}) = \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2} \left(E_{\boldsymbol{\ell}} \cos(2\phi_{\boldsymbol{\ell}}) - B_{\boldsymbol{\ell}} \sin(2\phi_{\boldsymbol{\ell}}) \right) \exp(i\boldsymbol{\ell} \cdot \mathbf{n})$$

$$U(\mathbf{n}) = \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2} \left(E_{\boldsymbol{\ell}} \cos(2\phi_{\boldsymbol{\ell}}) + B_{\boldsymbol{\ell}} \sin(2\phi_{\boldsymbol{\ell}}) \right) \exp(i\boldsymbol{\ell} \cdot \mathbf{n})$$

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$$U(\mathbf{n}) = \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2} \left(E_{\boldsymbol{\ell}} \cos(2\phi_{\boldsymbol{\ell}}) + B_{\boldsymbol{\ell}} \sin(2\phi_{\boldsymbol{\ell}}) \right) \exp(i\boldsymbol{\ell} \cdot \mathbf{n})$$

Let's consider Q and U that are produced by a single Fourier mode

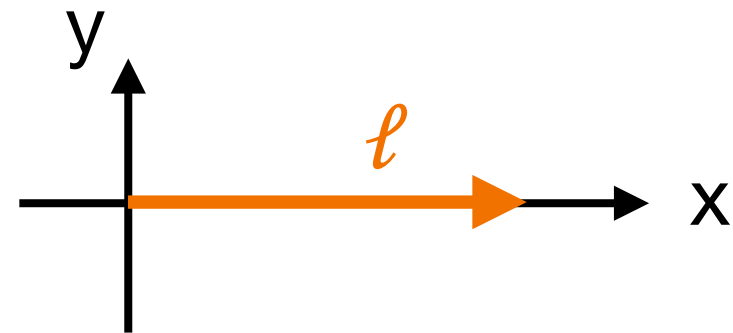
Q and U produced by E and B modes are given by:

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Let's consider Q and U that are produced by a single Fourier mode

Taking the x-axis to be the direction of a wave vector, we obtain



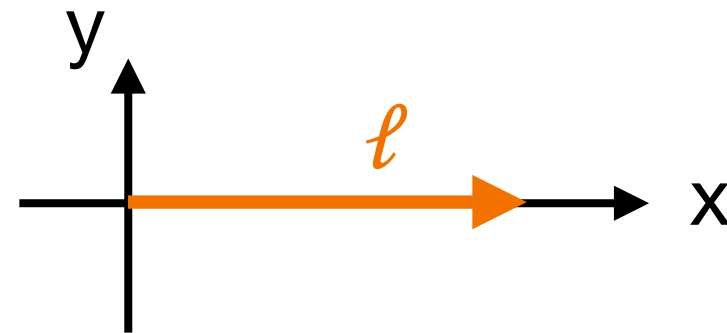
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$$U(\mathbf{n}) = \int \frac{d^2\boldsymbol{\ell}}{(2\pi)^2} \left(E_{\boldsymbol{\ell}} \cos(2\phi_{\boldsymbol{\ell}}) + B_{\boldsymbol{\ell}} \sin(2\phi_{\boldsymbol{\ell}}) \right) \exp(i\boldsymbol{\ell} \cdot \mathbf{n})$$

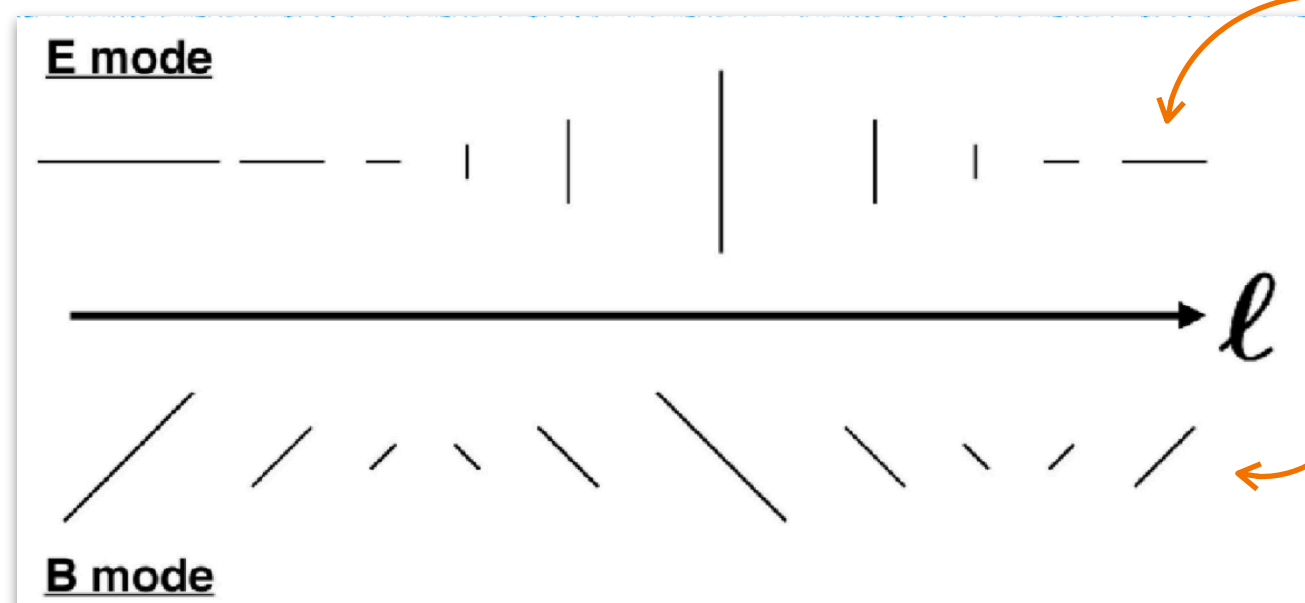
Let's consider Q and U that are produced by a single Fourier mode

Taking the x-axis to be the direction of a wave vector, we obtain



$$Q(\mathbf{n}) = \text{Re} \left(E_{\boldsymbol{\ell}} \exp(i\boldsymbol{\ell} \cdot \mathbf{n}) \right)$$

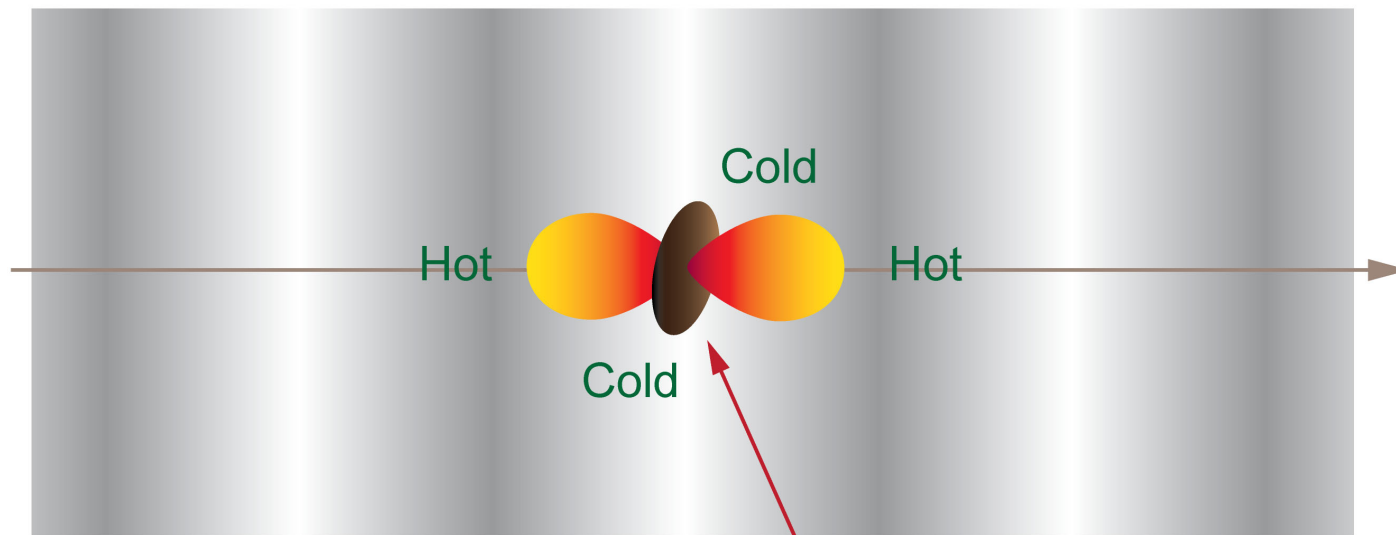
$$U(\mathbf{n}) = \text{Re} \left(B_{\boldsymbol{\ell}} \exp(i\boldsymbol{\ell} \cdot \mathbf{n}) \right)$$



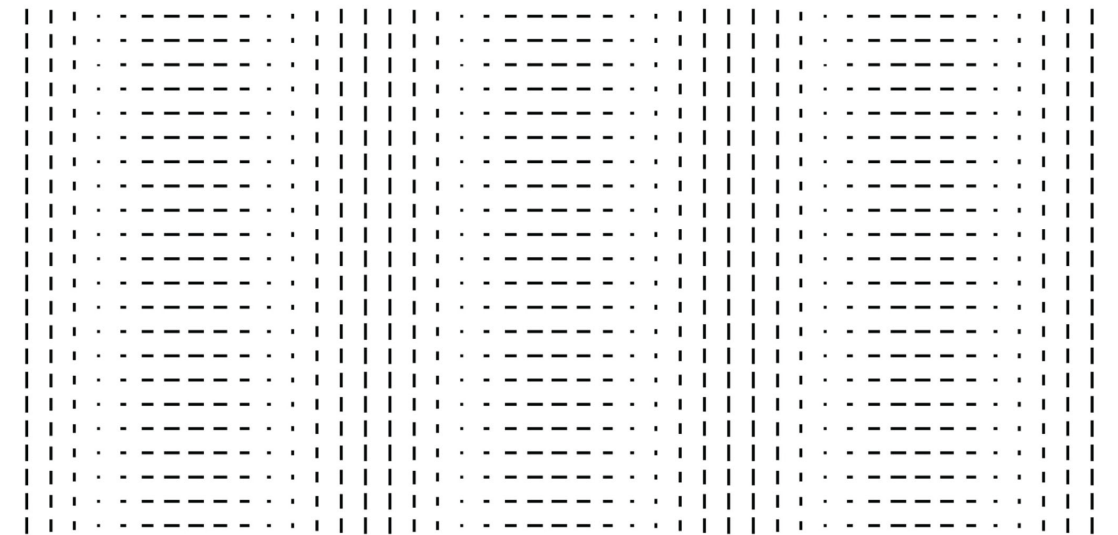
parallel or
perpendicular
to wave vector
direction

45 deg tilted
wrt wave
vector
direction

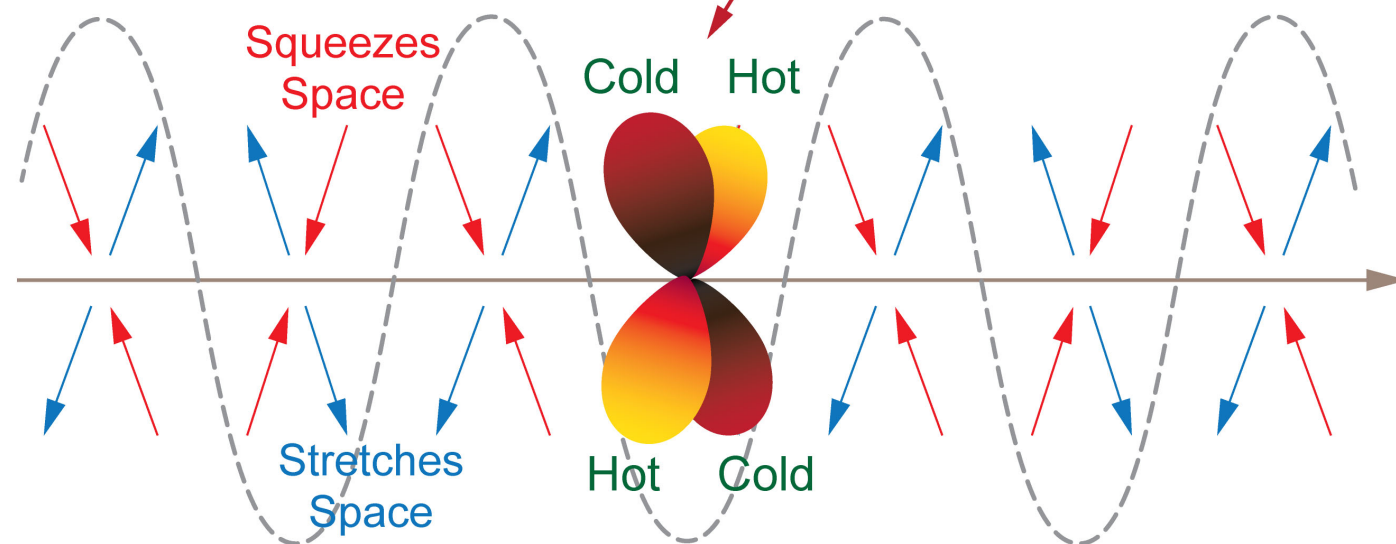
Density Wave



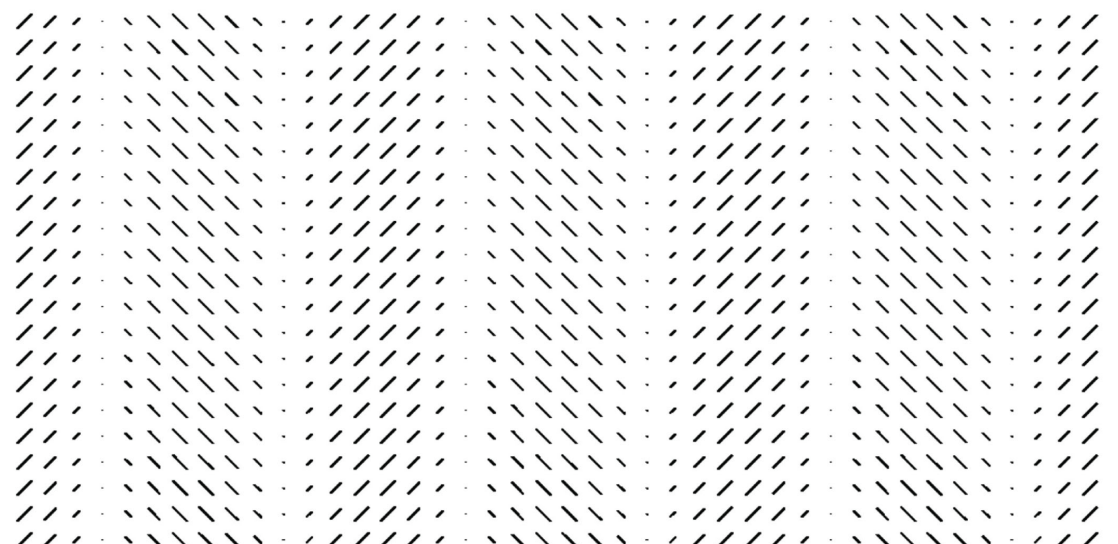
E-Mode Polarization Pattern



Gravitational Wave



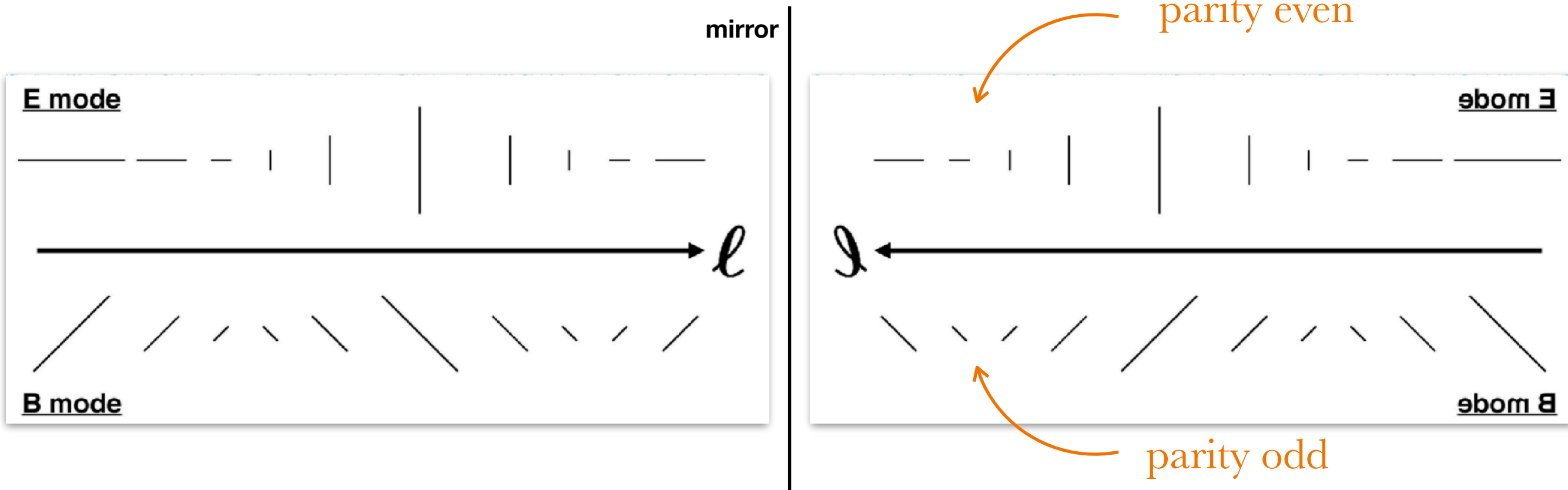
B-Mode Polarization Pattern



Images credit: BICEP collaboration

- E-modes are Stokes Q defined wrt to ℓ axis
- B-modes are Stokes U defined wrt to ℓ axis

coordinate-
independent
statement!

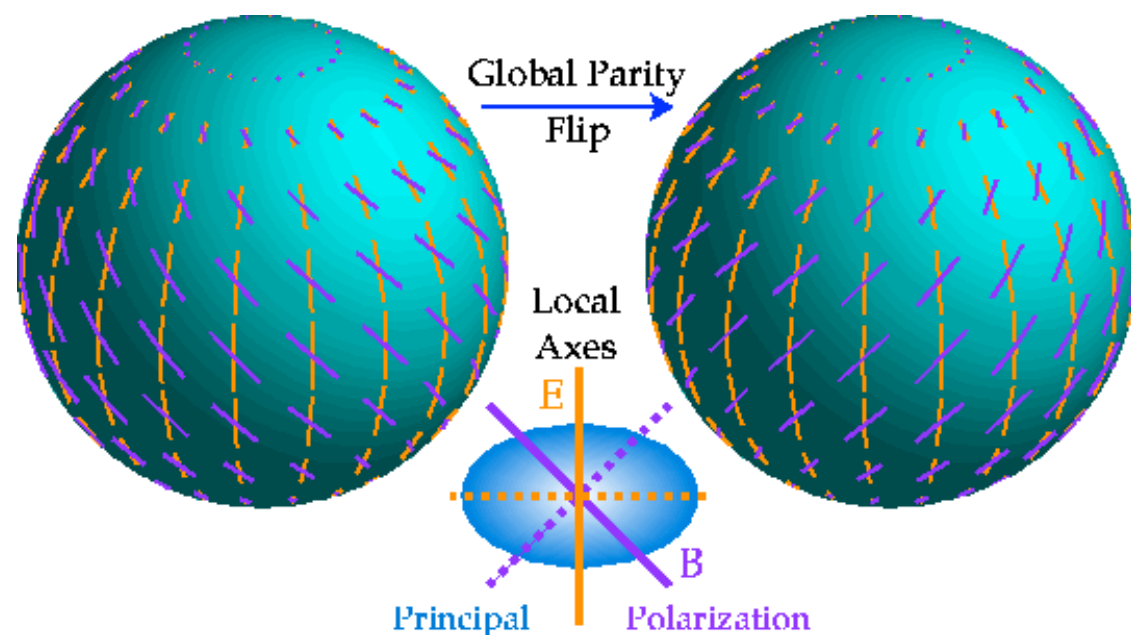
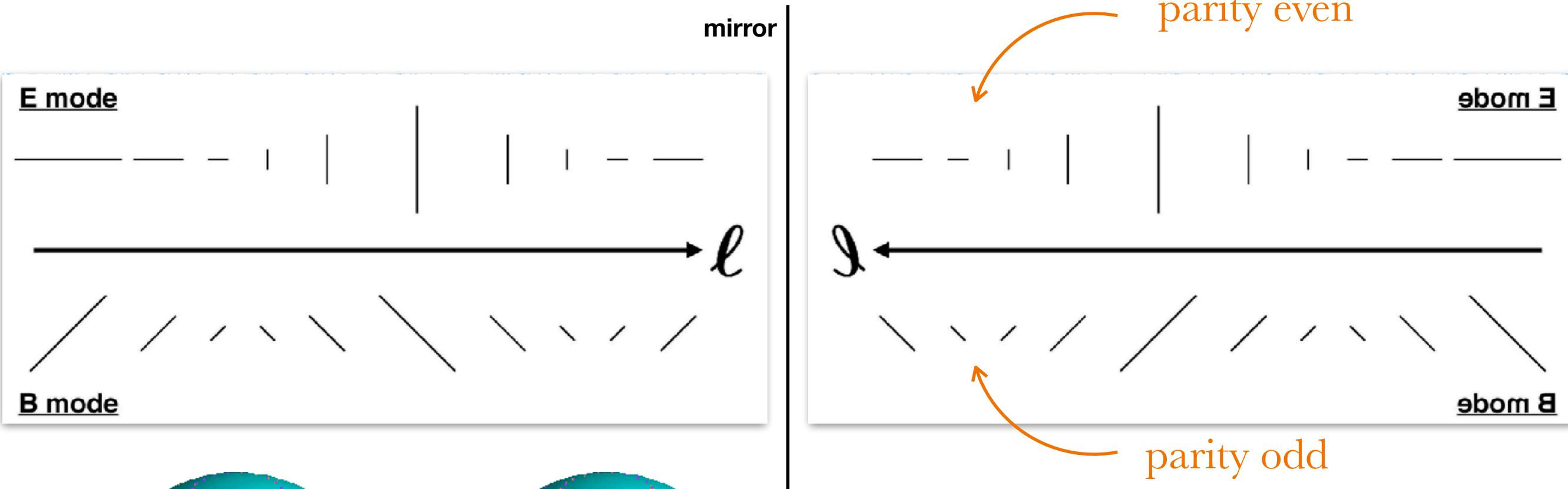


- E-modes are Stokes Q defined wrt to ℓ axis
- B-modes are Stokes U defined wrt to ℓ axis

coordinate-
independent
statement!

parity even

parity odd



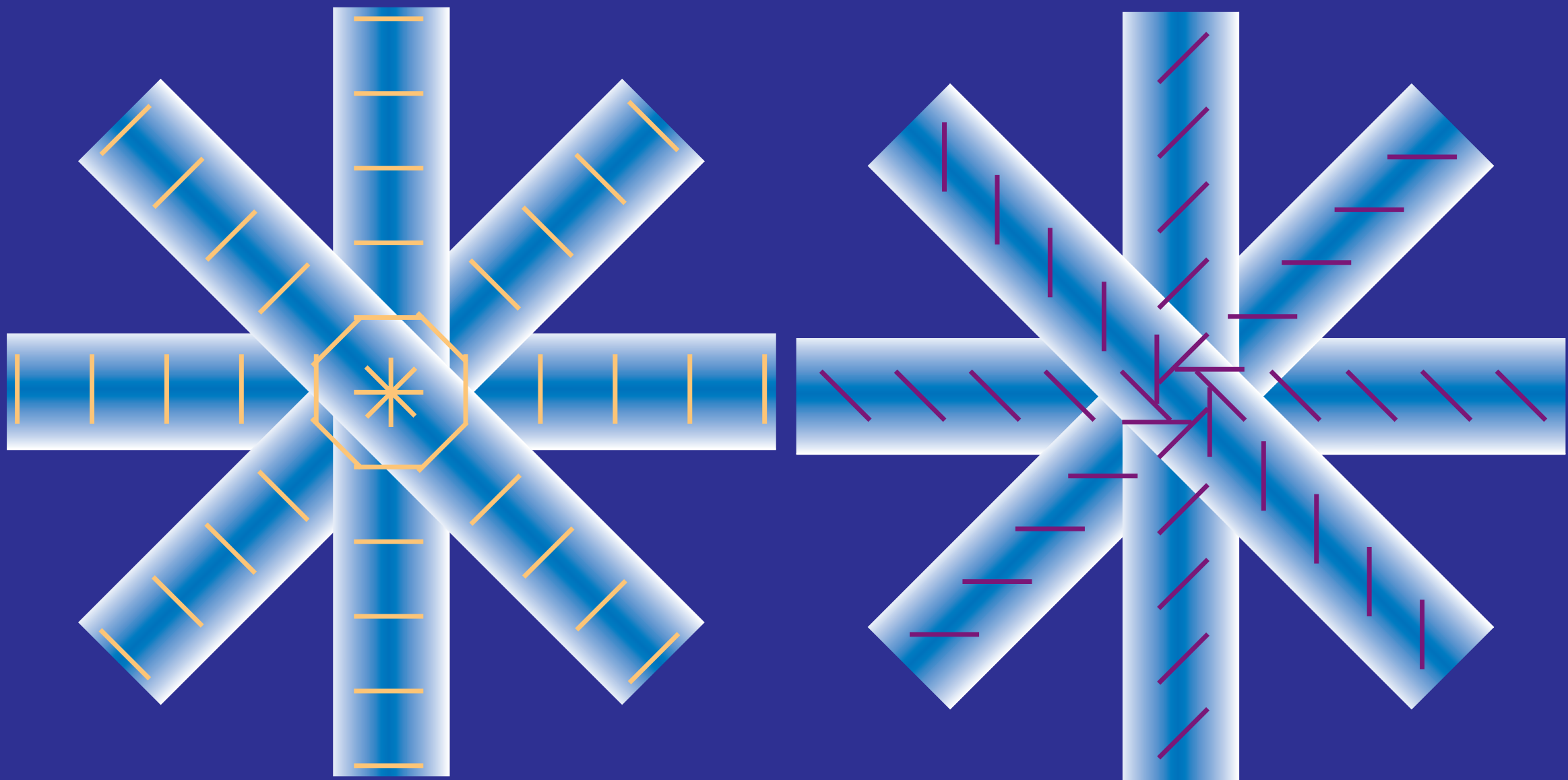
The E- and B-modes are distinguished by their behavior under a parity transformation $\mathbf{n} \rightarrow -\mathbf{n}$.

Images credit: Wayne Hu

E and B modes

credits:
Wayne Hu

- Superimposing wavevectors
- B-modes have handedness or odd parity

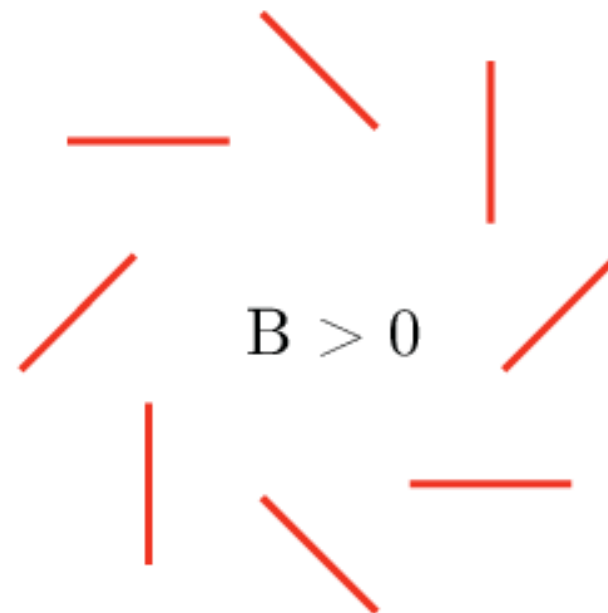
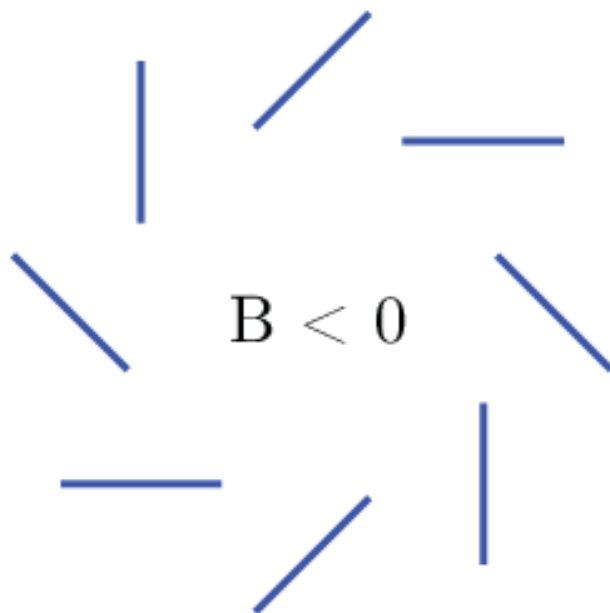
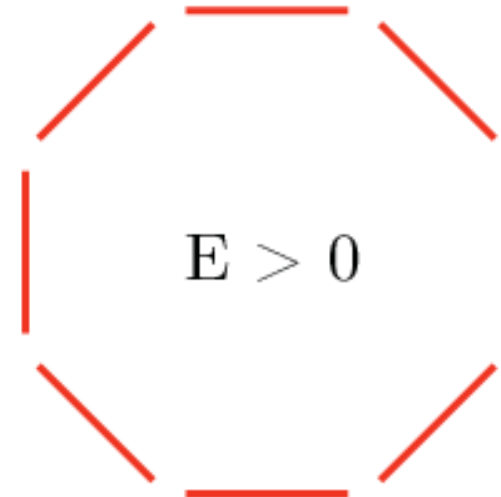
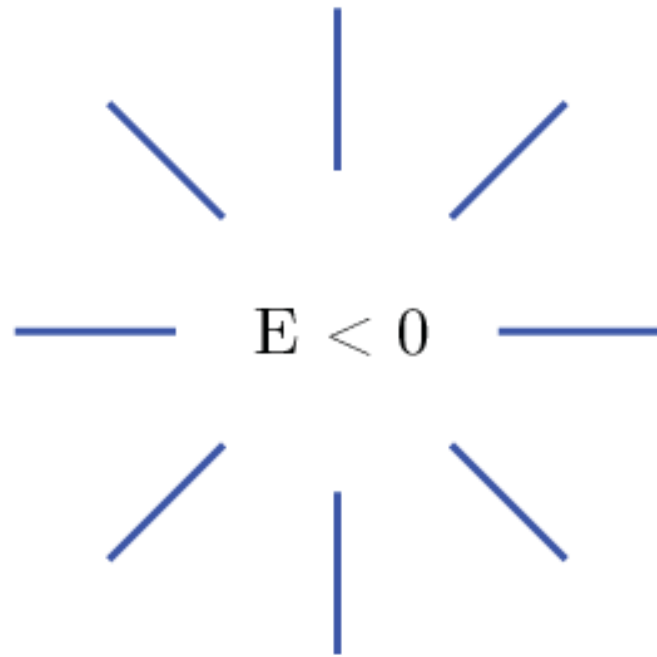


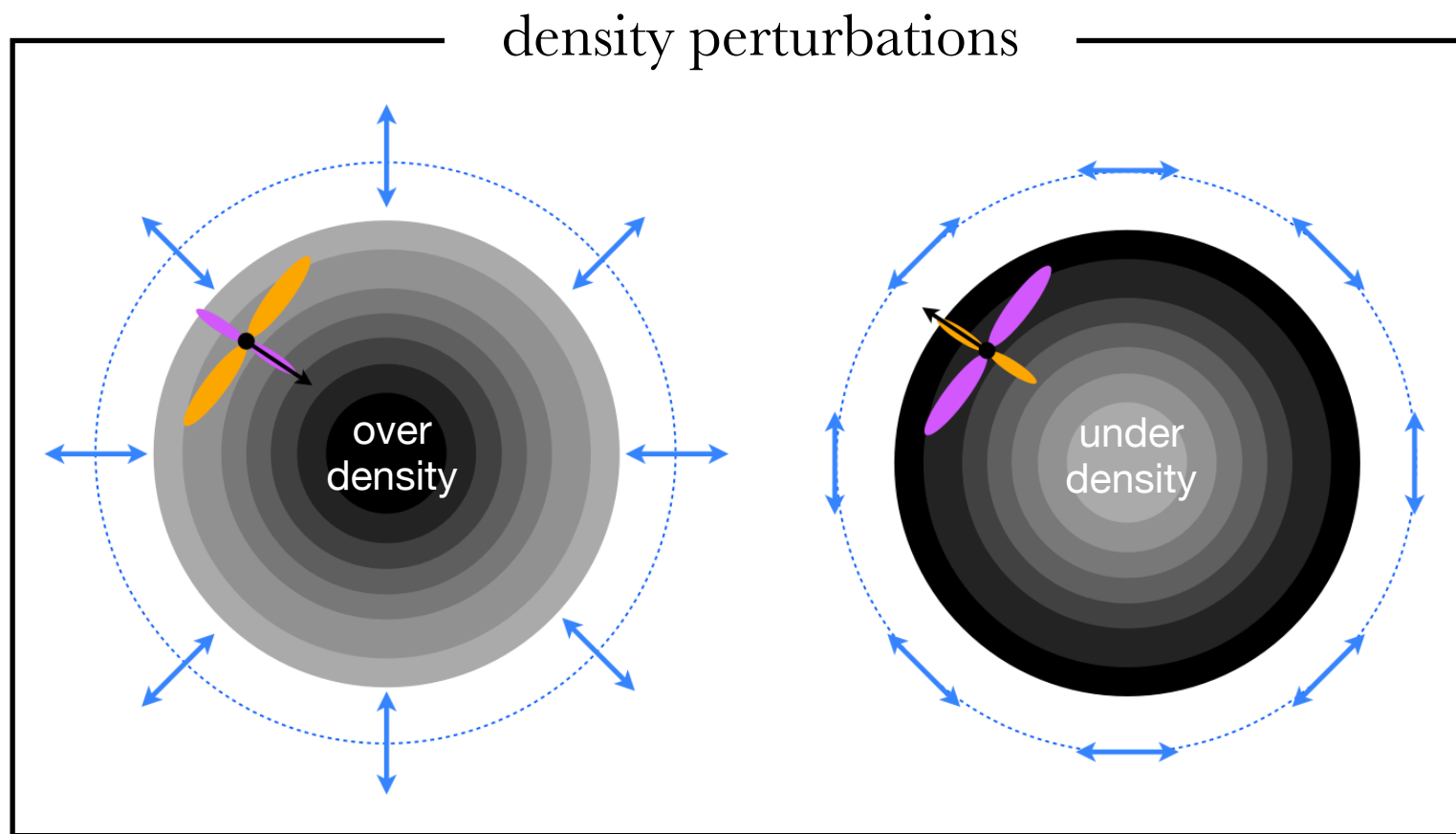
E-modes

B-modes

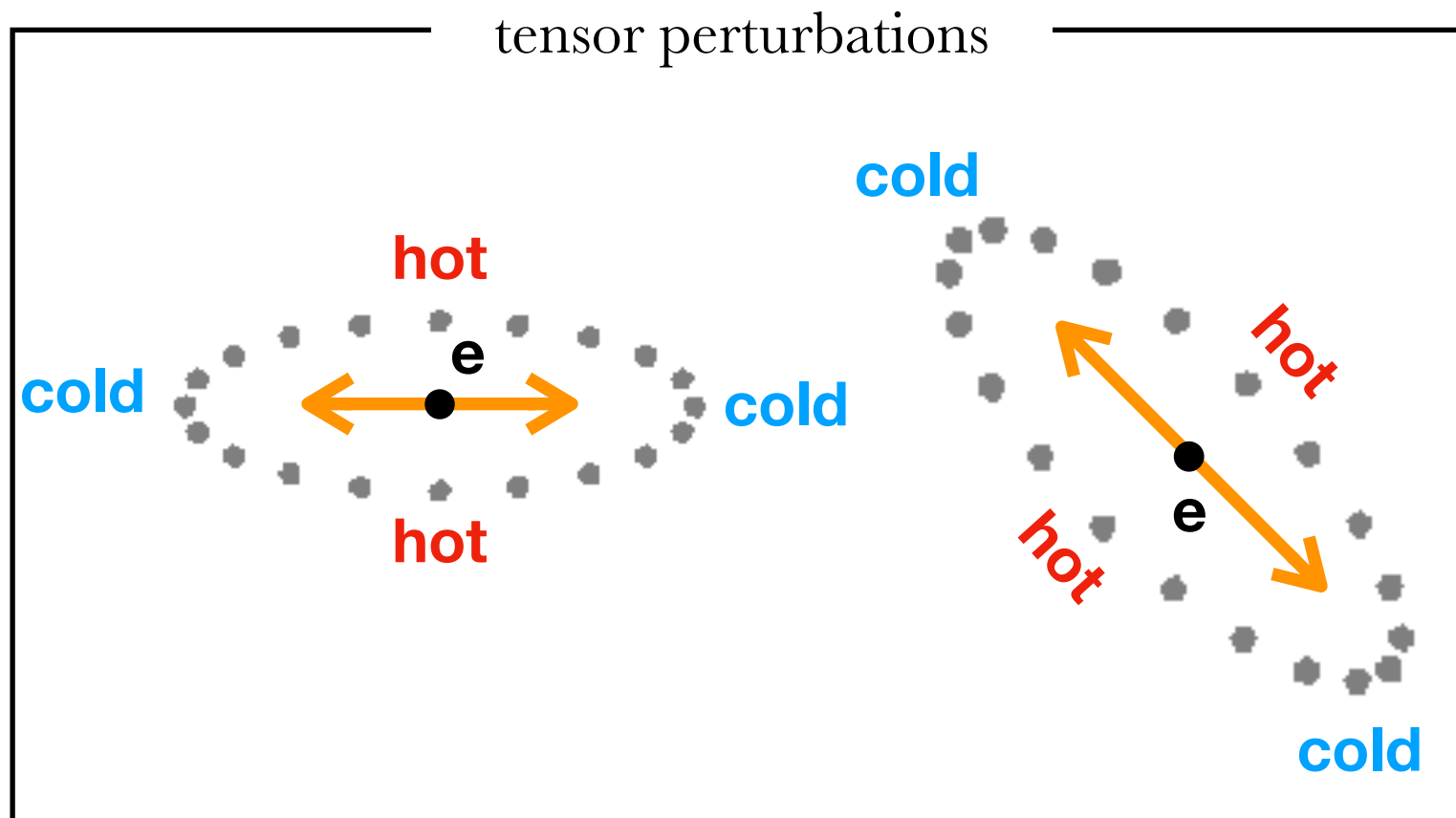
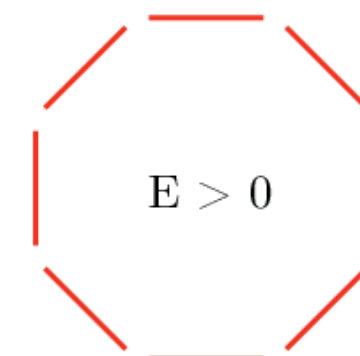
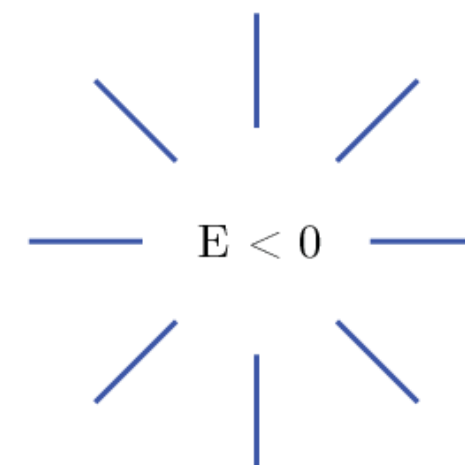
E-modes = Q in $\mathbf{k}$ coordinates

B-modes = U in $\mathbf{k}$ coordinates

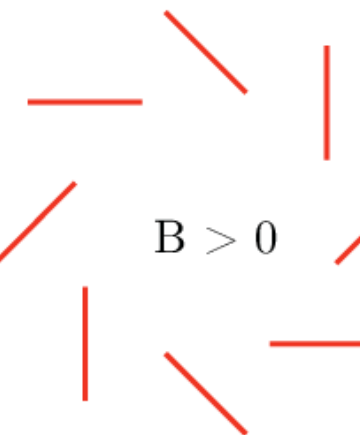
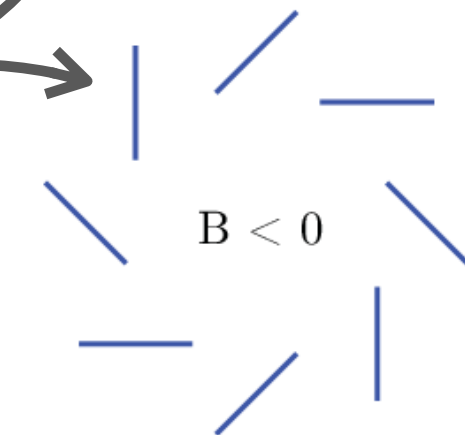




only

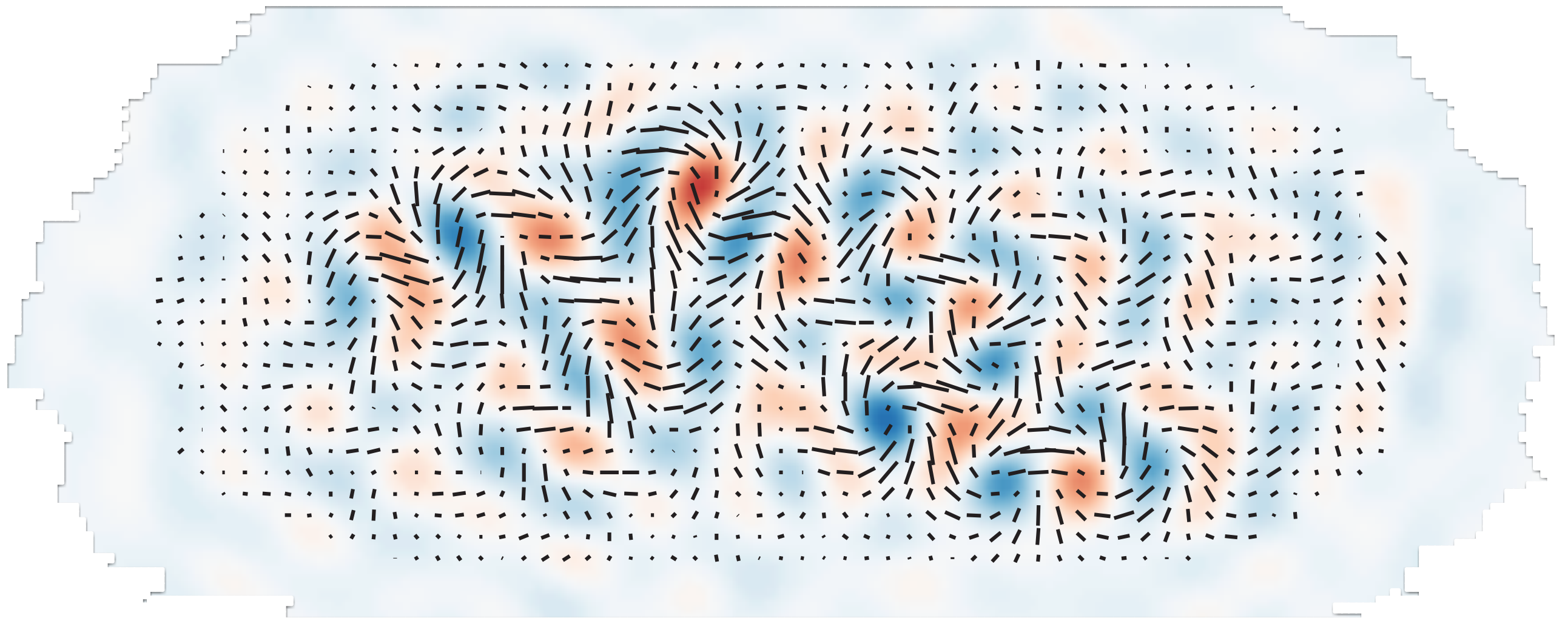


both



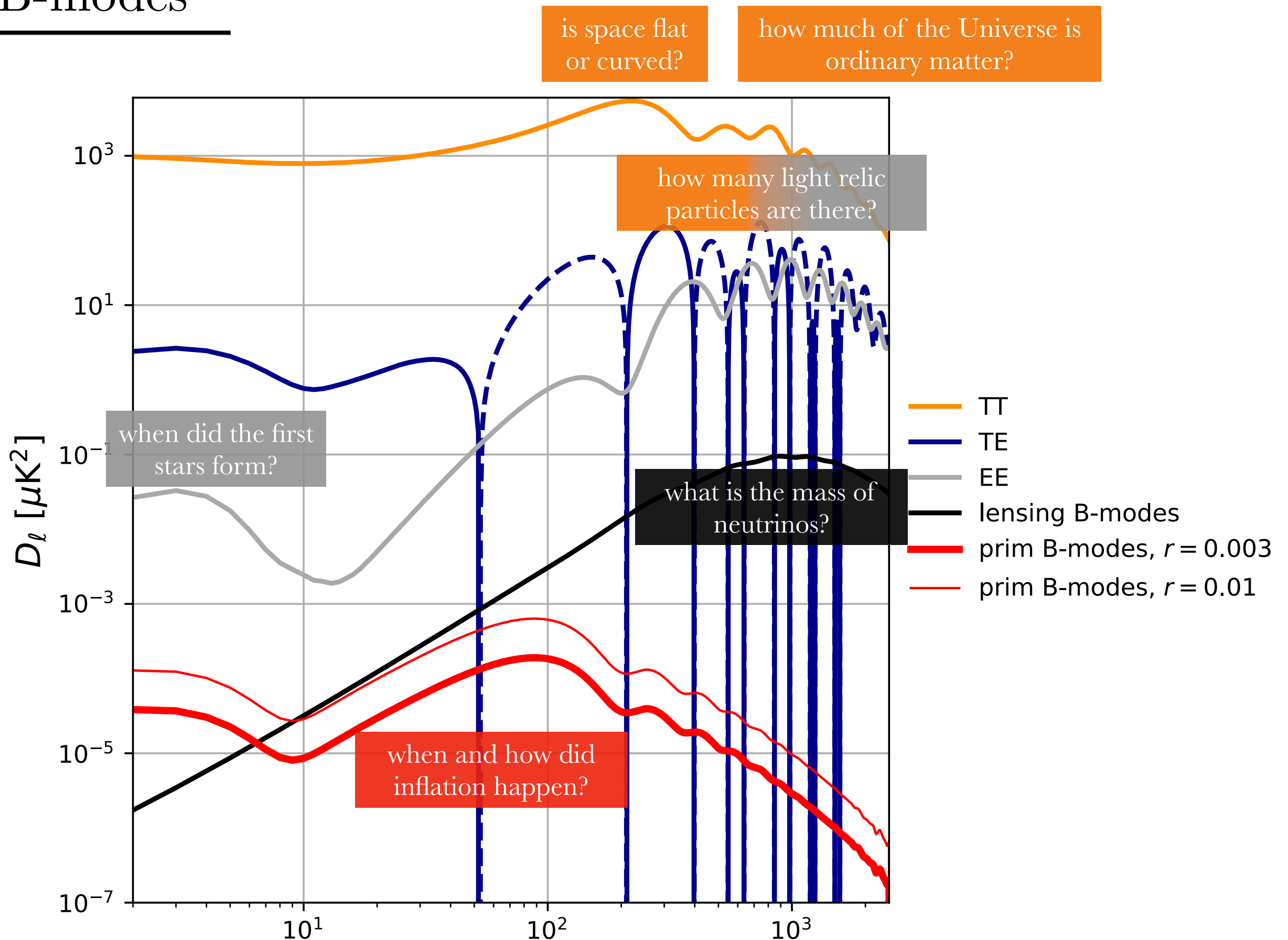
B-modes

These gravitational waves would produce a characteristic swirl pattern (called **B-modes**) in the polarization of the CMB:



Detecting these B-modes is a central goal of observational cosmology.

B-modes



Past and On-going experiments

CLASS



POLARBEAR



CMB Stage III

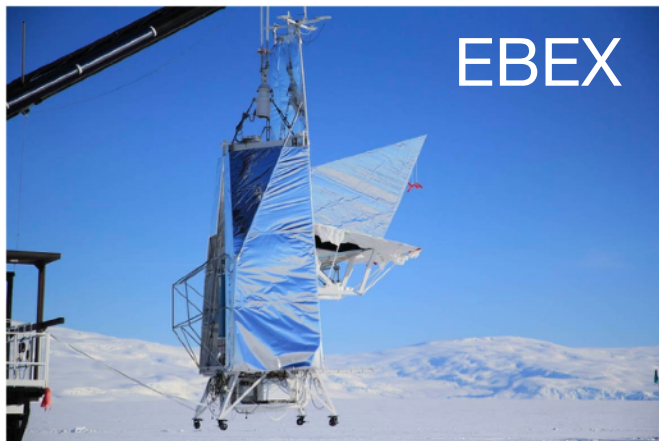
ACTPol



SPIDER



EBEX



ABS

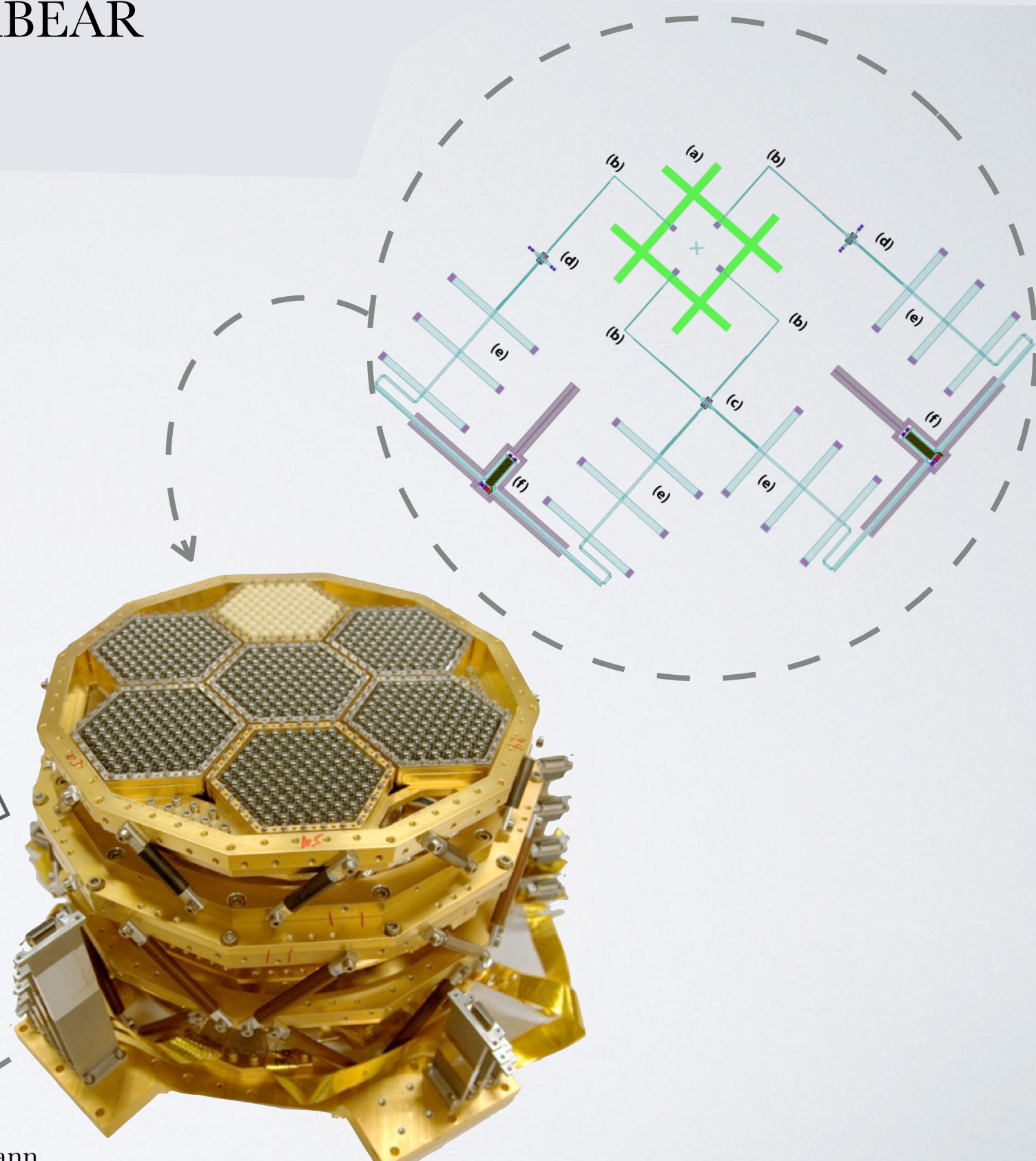
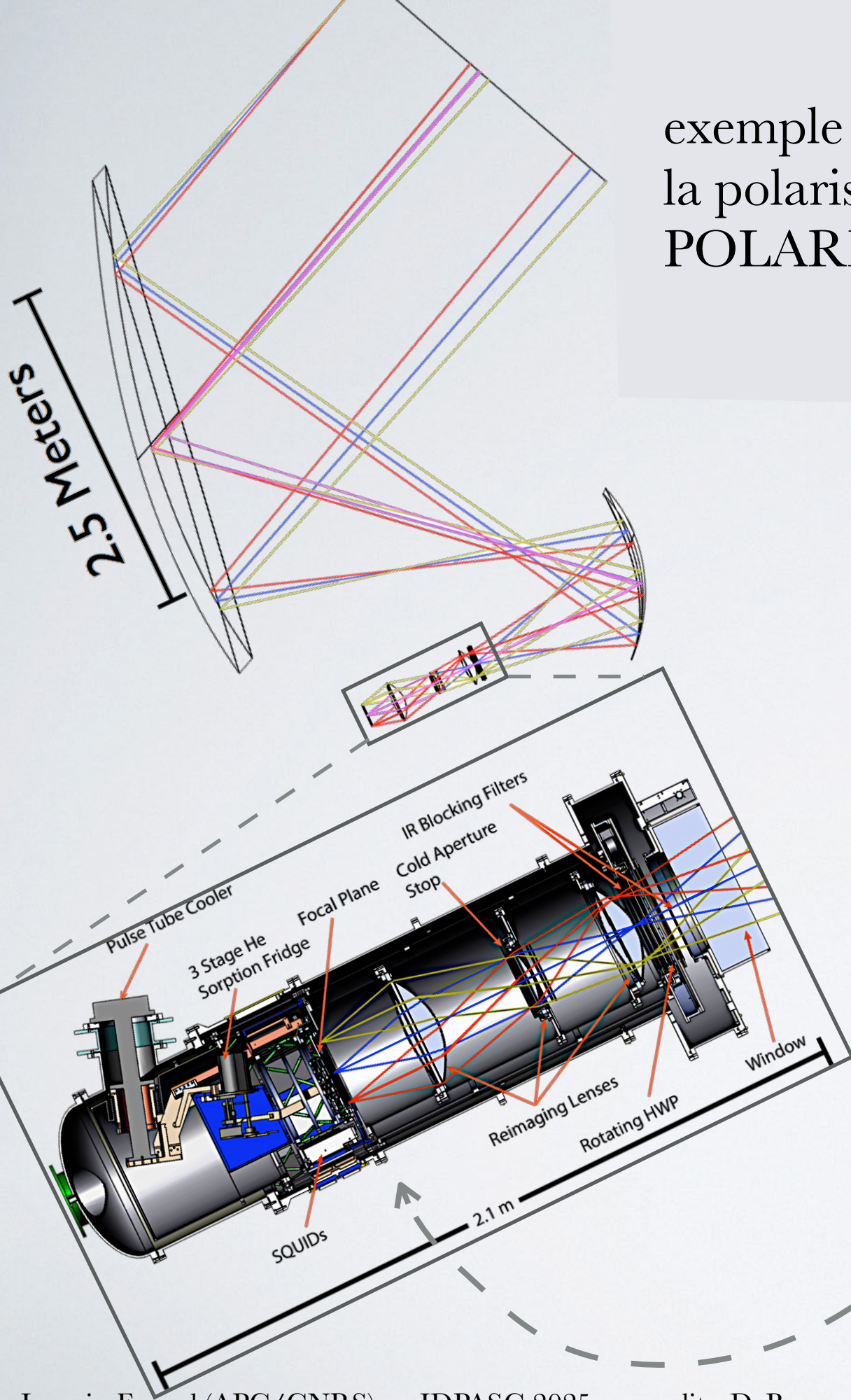


BICEP

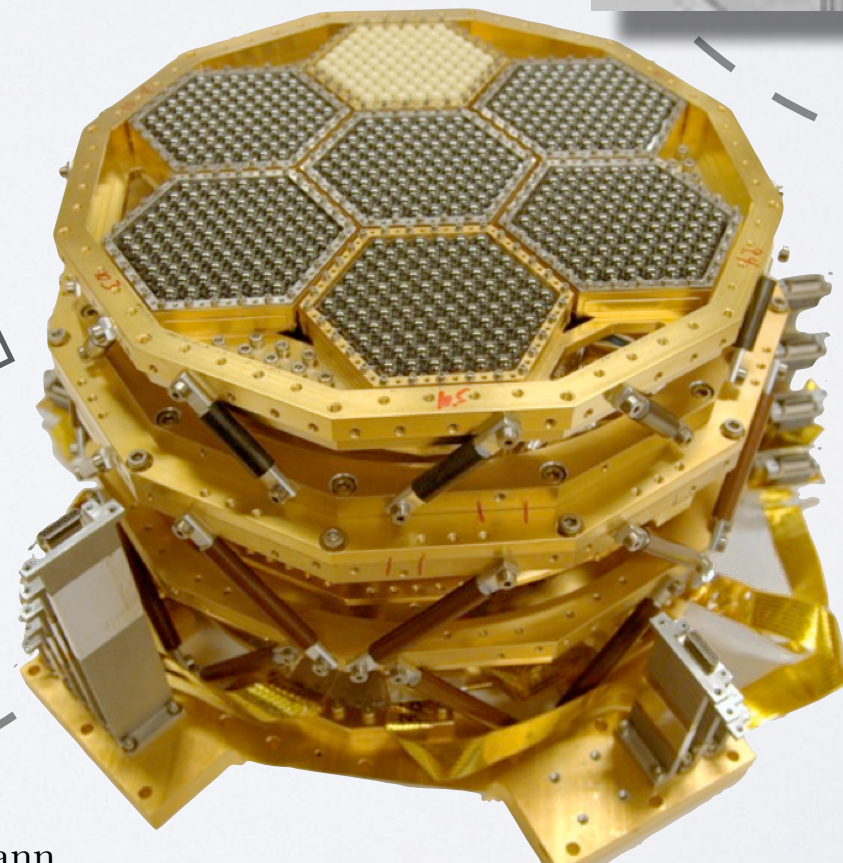
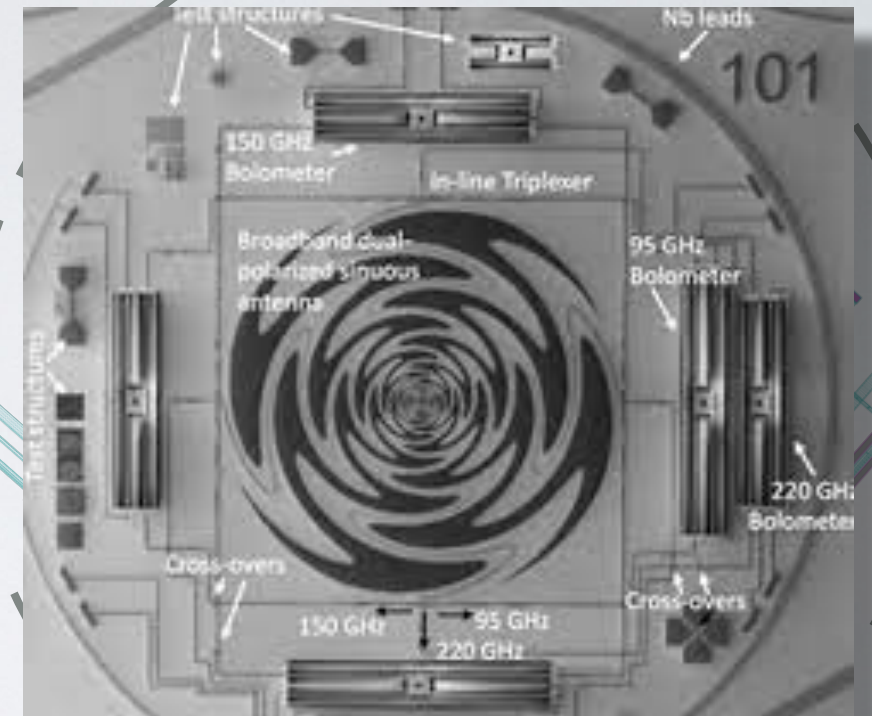
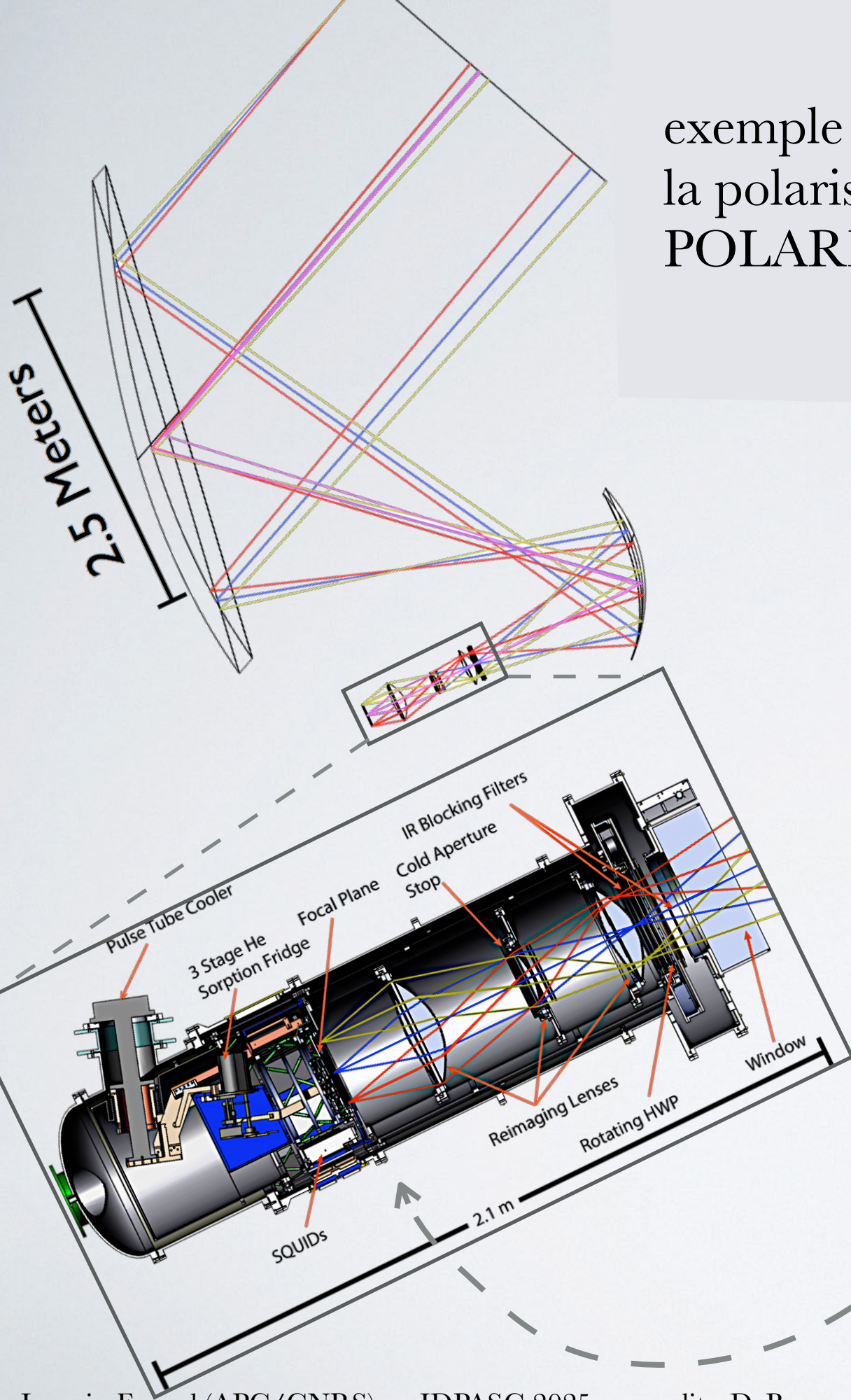


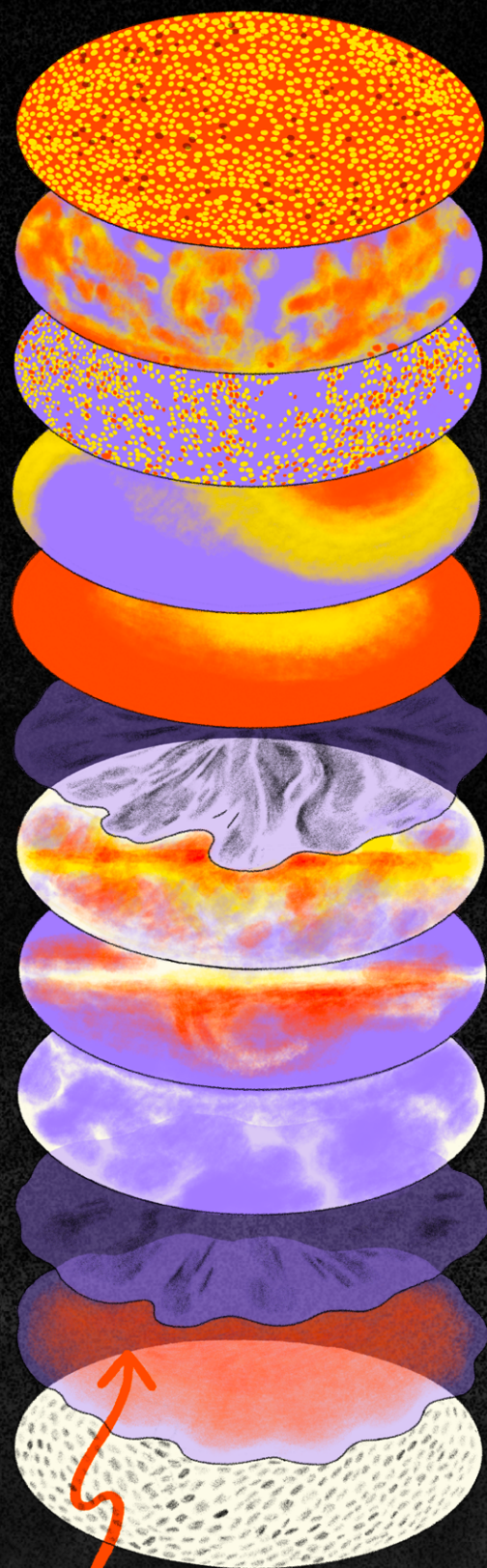
SPTPol

exemple d'observatoire de
la polarisation du CMB :
POLARBEAR



exemple d'observatoire de
la polarisation du CMB :
POLARBEAR





B-modes

E-modes

Intensity
anisotropies

Dipole

Monopole

Gravitational
lensing

Galactic and
extra-galactic
foregrounds

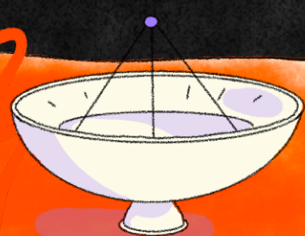
Atmosphere

Systematics

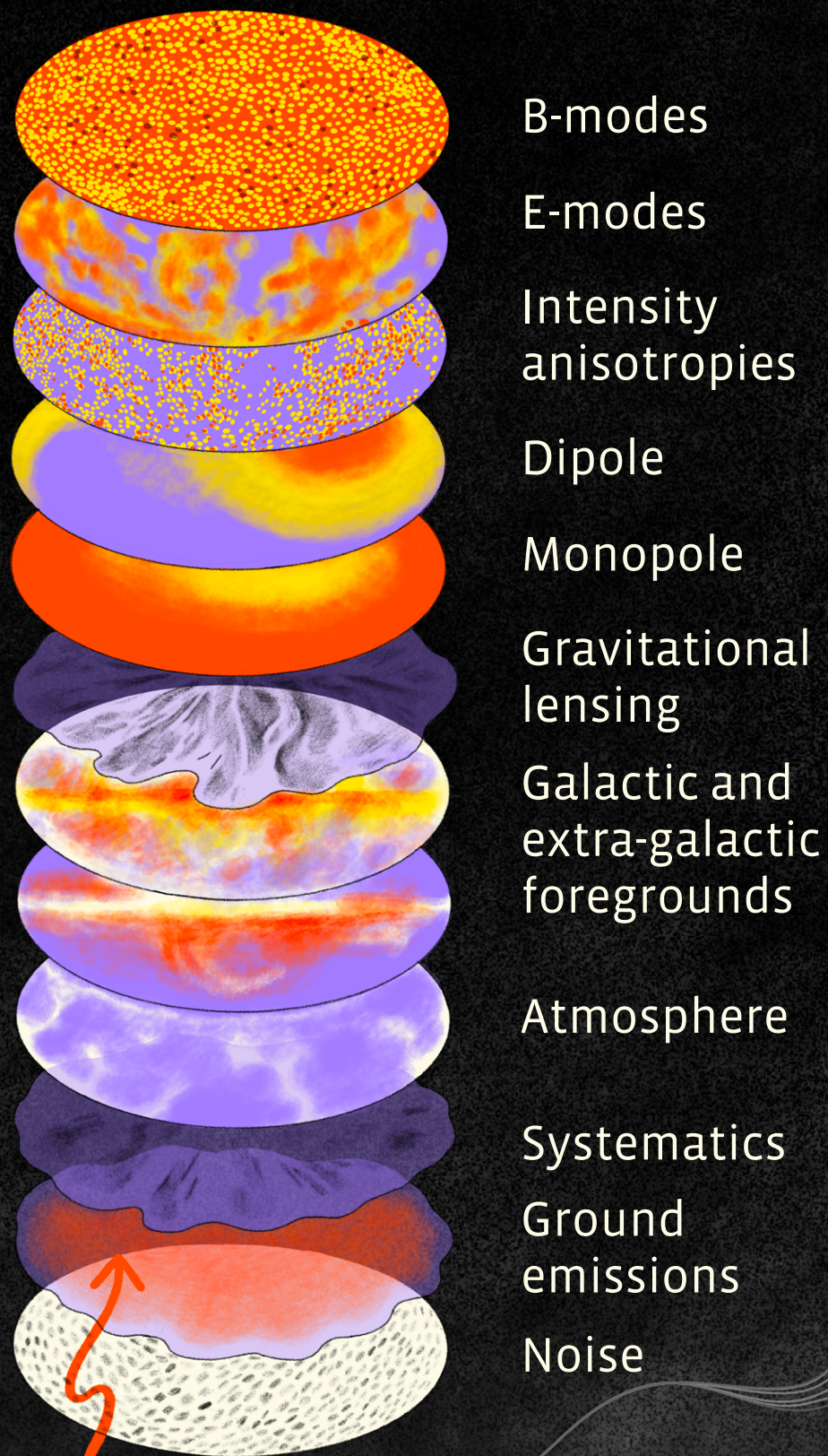
Ground
emissions

Noise

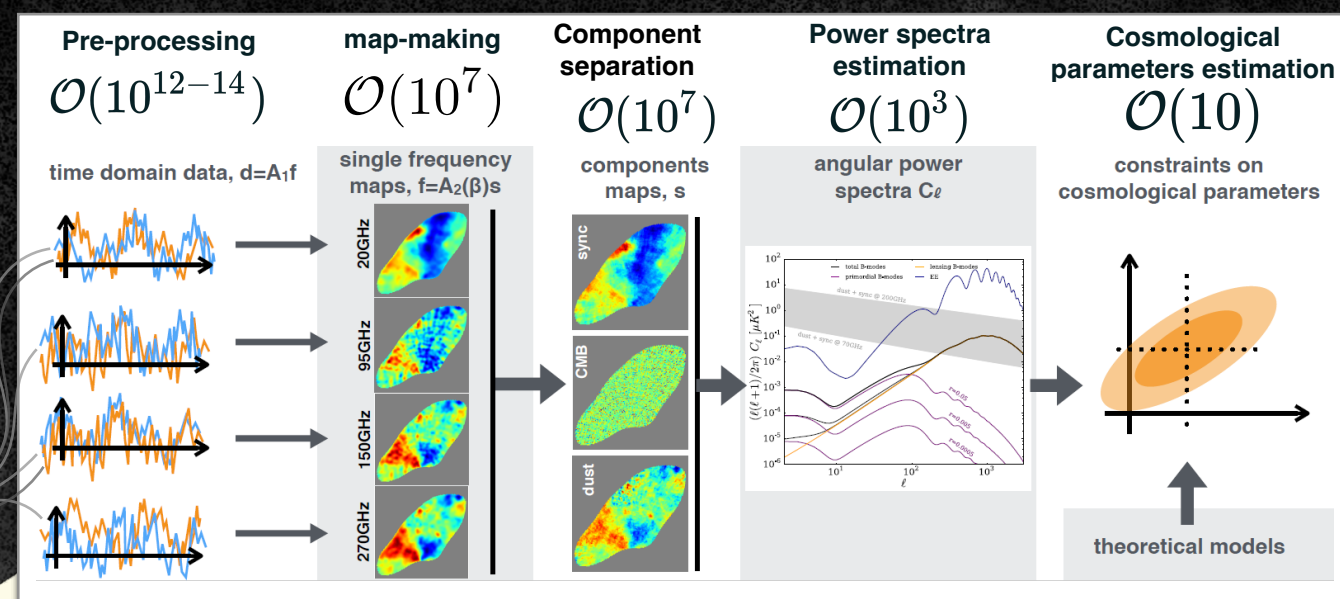
CMB

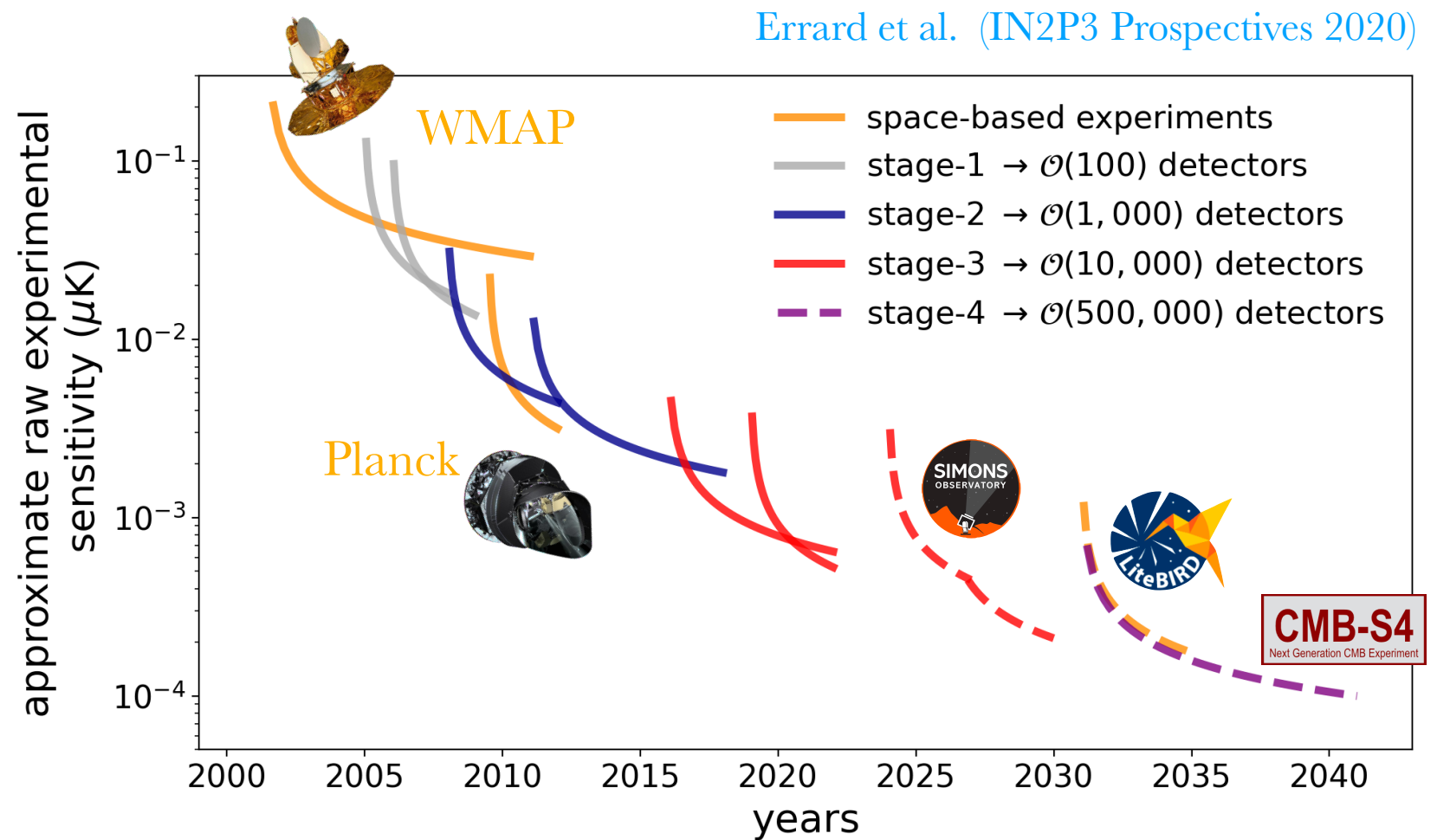


Design: Ève Barlier & Josquin Errard,
funded by ERC Scipol No.~101044073,
CNRS, 2025. All rights reserved.

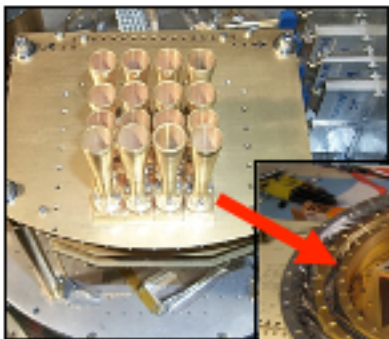


CMB

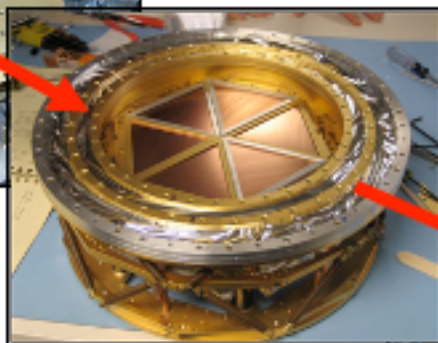




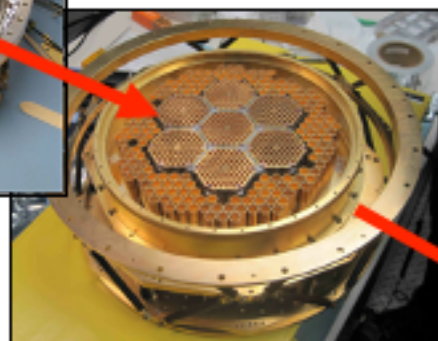
2001: ACBAR
16 detectors



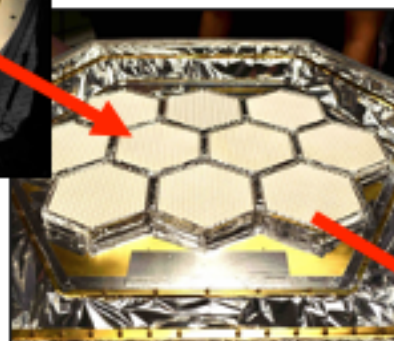
2007: SPT
960 detectors



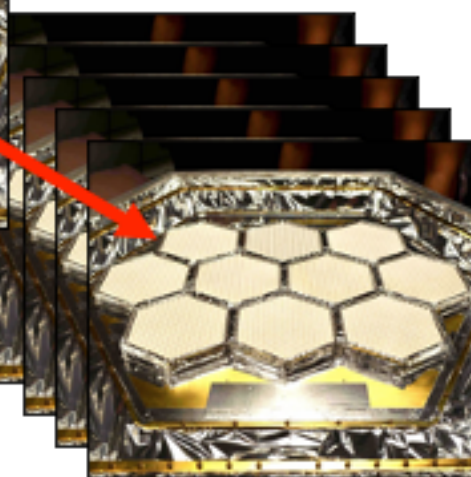
Stage-2
2012: SPTpol
~1600 detectors



Stage-3
2016: SPT-3G
~16,000 detectors



Stage-4
202: CMB-S4
500,000 detectors



Detector sensitivity has been limited by photon “shot” noise for last ~15 years; further improvements are made only by making more detectors!

credits: Nils Halverson

Zoom on an On-going experiment: the Simons Observatory



15+ Countries, 60+ Institutions, 375+ Researchers



F2F Chicago, July 2024



Zoom on an On-going experiment: the Simons Observatory



15+ Countries, 60+ Institutions, 375+ Researchers

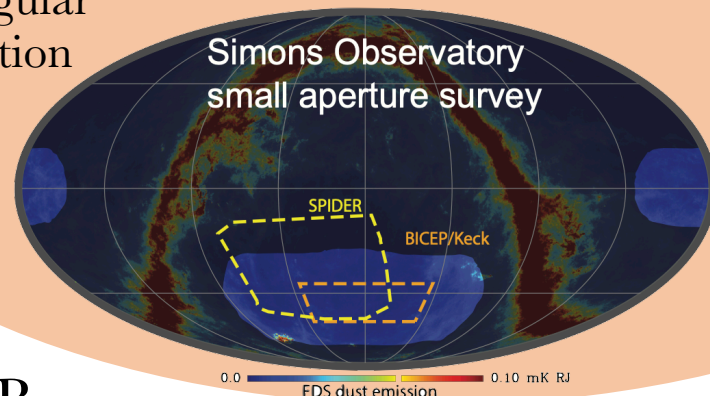




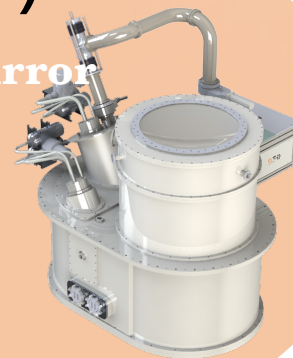
Two surveys are needed to measure large- and small-angular scales

Small aperture telescope (SAT)

Deep maps with low angular resolution



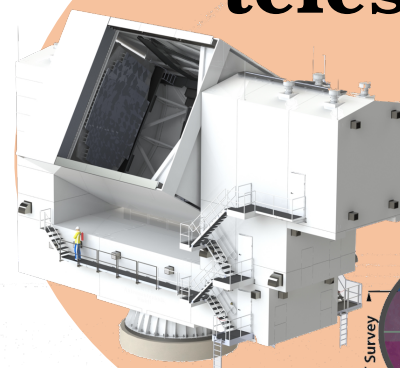
0.5m mirror



Excellent site for CMB observations:

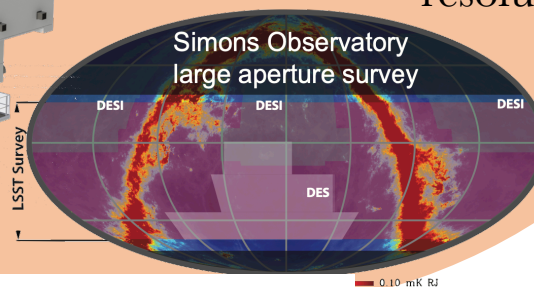
- PWV ~ 1.2 mm (good but 3x South Pole)
- Low latitude (-23°) gives high fsky

Large aperture telescope (LAT)



6m mirror

Wide maps with high angular resolution



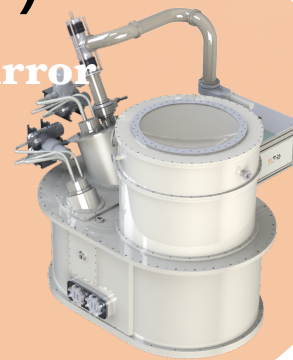
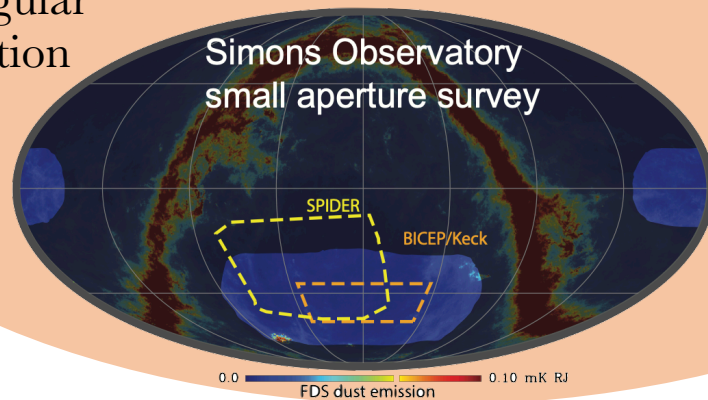
From: Gabriele Coppi, Rolando Dunner, Federico Nati, Matias Rojas



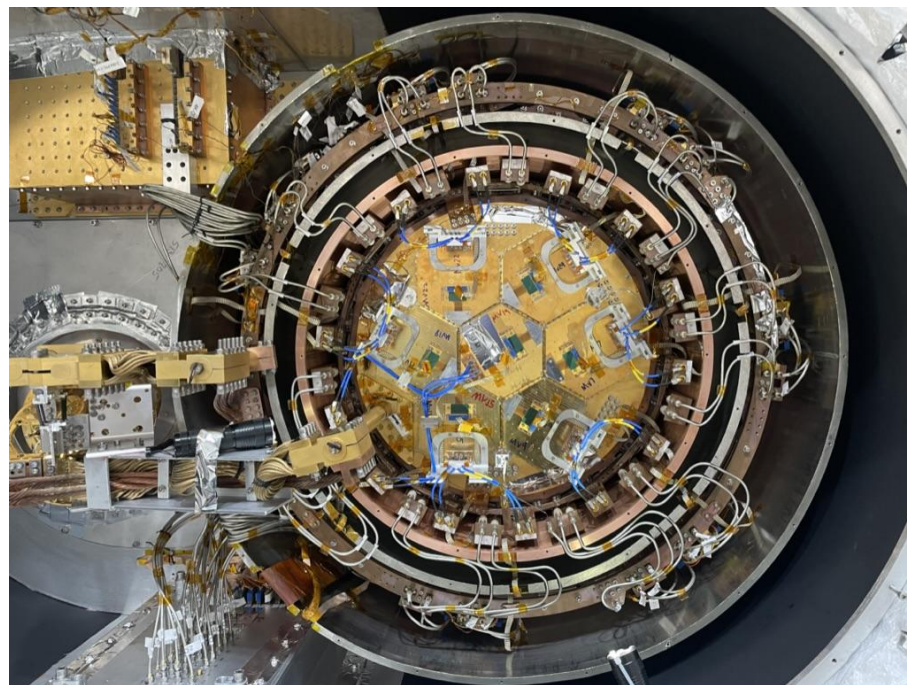
Small aperture telescope (SAT)

Deep maps with low angular resolution

0.5m mirror



3x SATs each with 12,000 detectors
Measuring large-scale correlations



SAT focal plane with 7 detector arrays

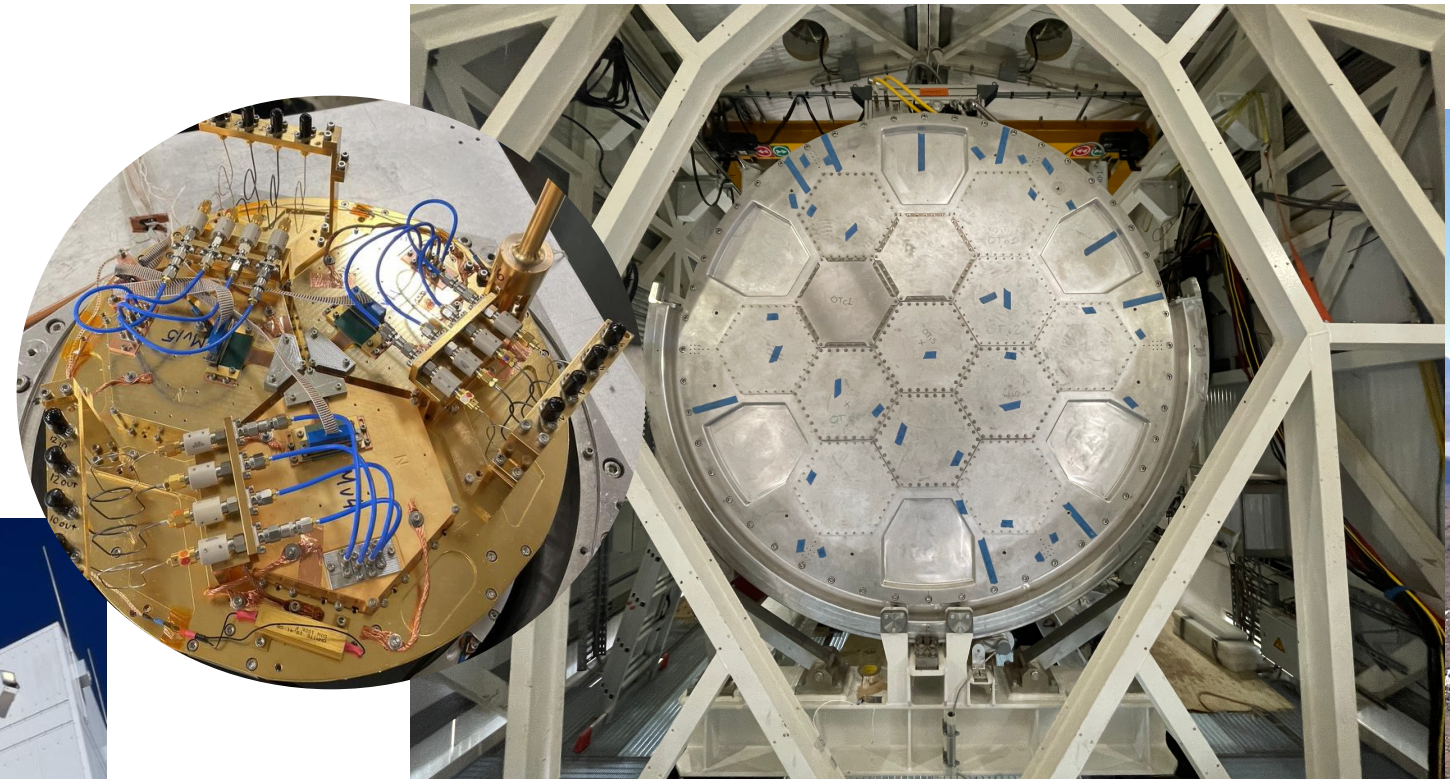


From: Gabriele Coppi, Rolando Dunner,
Federico Nati, Matias Rojas



Over 30,000 detectors
in six tubes
Targeting small scale
correlations

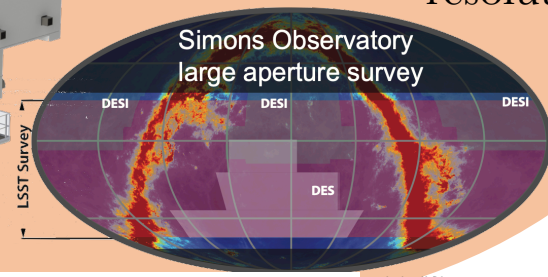
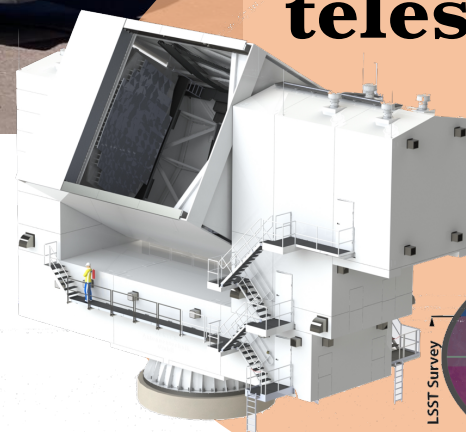
3x detector arrays per tube



**Large aperture
telescope (LAT)**

6m mirror

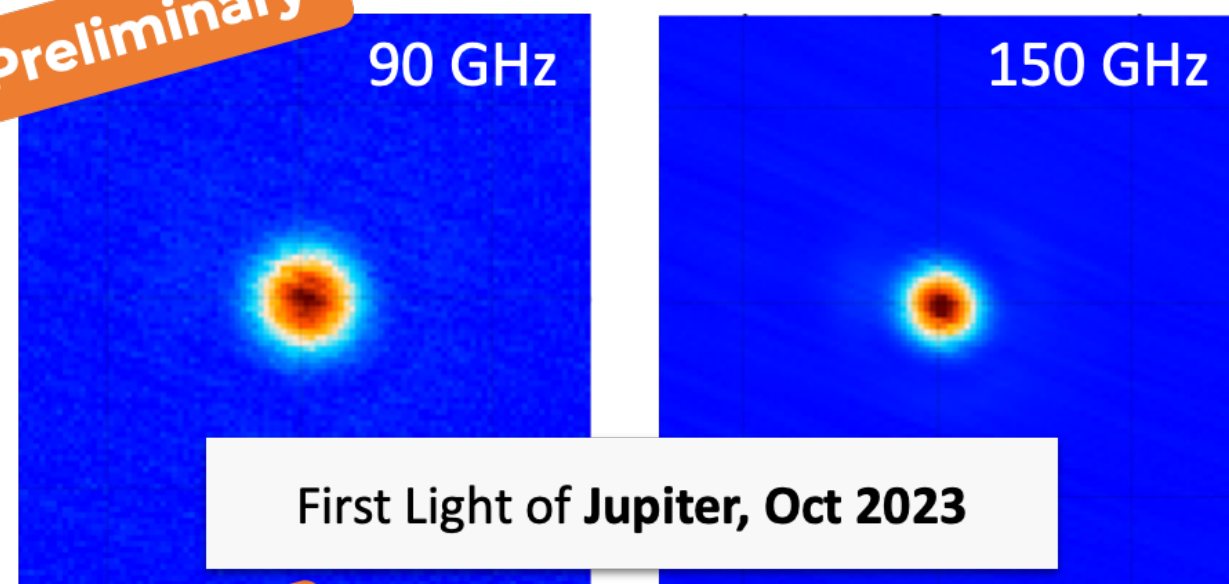
Wide maps with
high angular
resolution



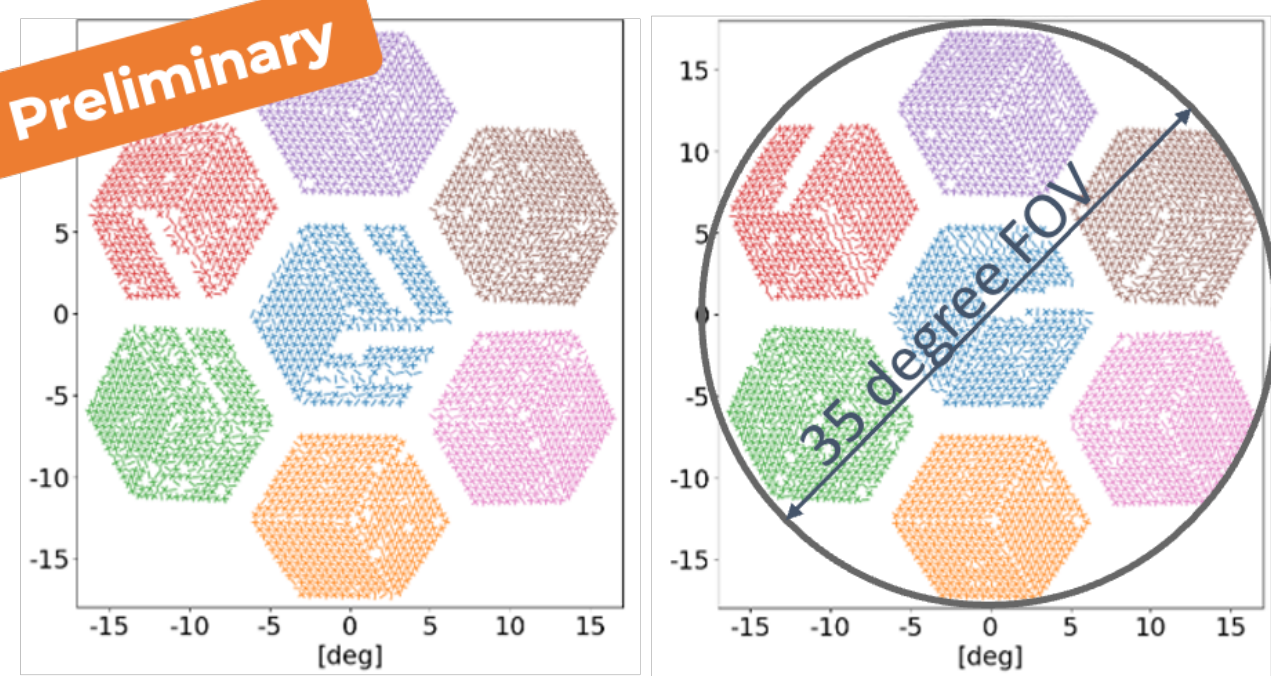
From: Gabriele Coppi, Rolando Dunner,
Federico Nati, Matias Rojas



Preliminary



Preliminary



First Light Maps from SATs

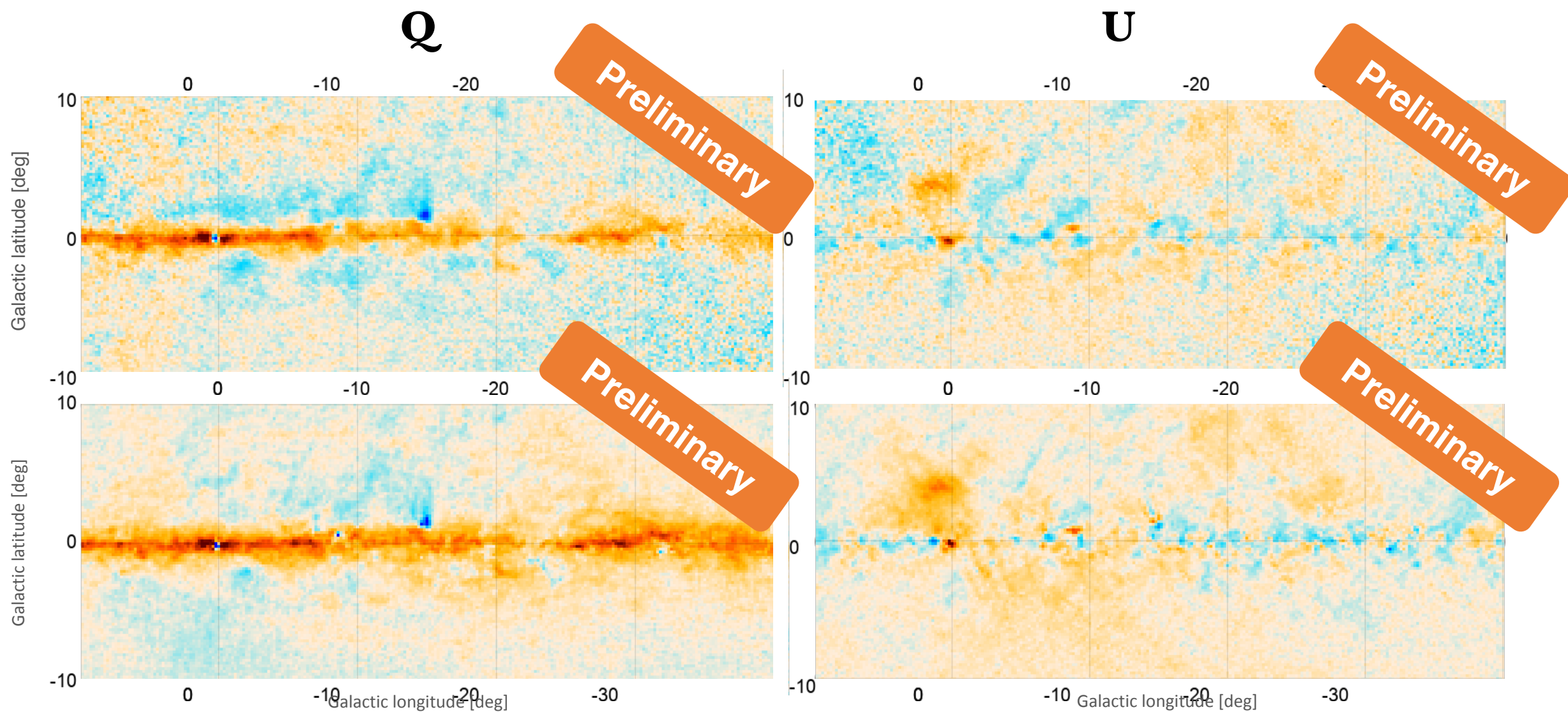




Galaxy maps

SO

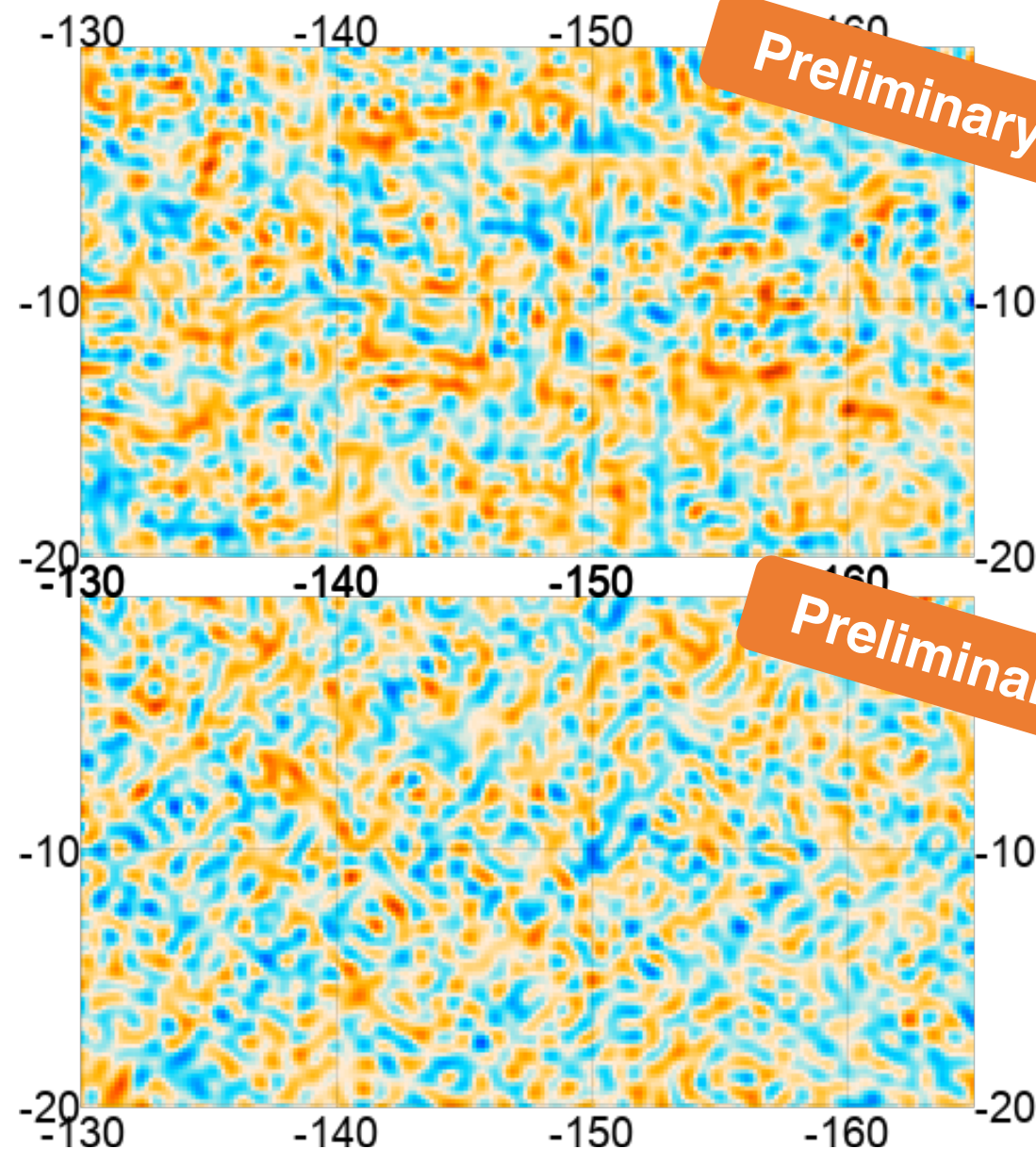
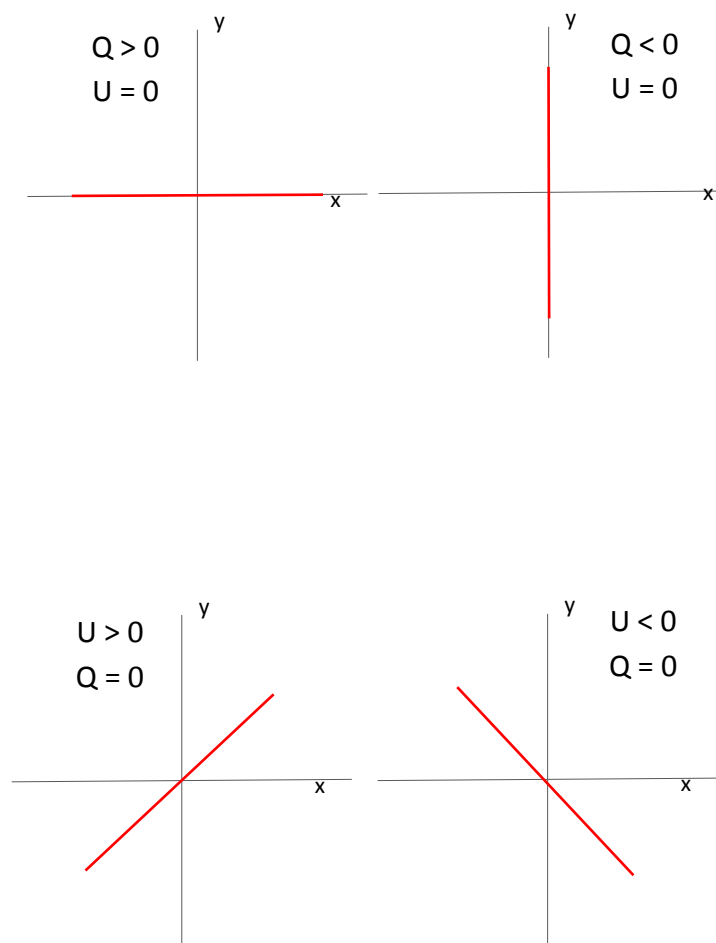
PLANCK



Galaxy center maps in comparison with Planck demonstrate instrument performance and larger scale recovery.



CMB maps



Started mapping the sky with two MF SATs.

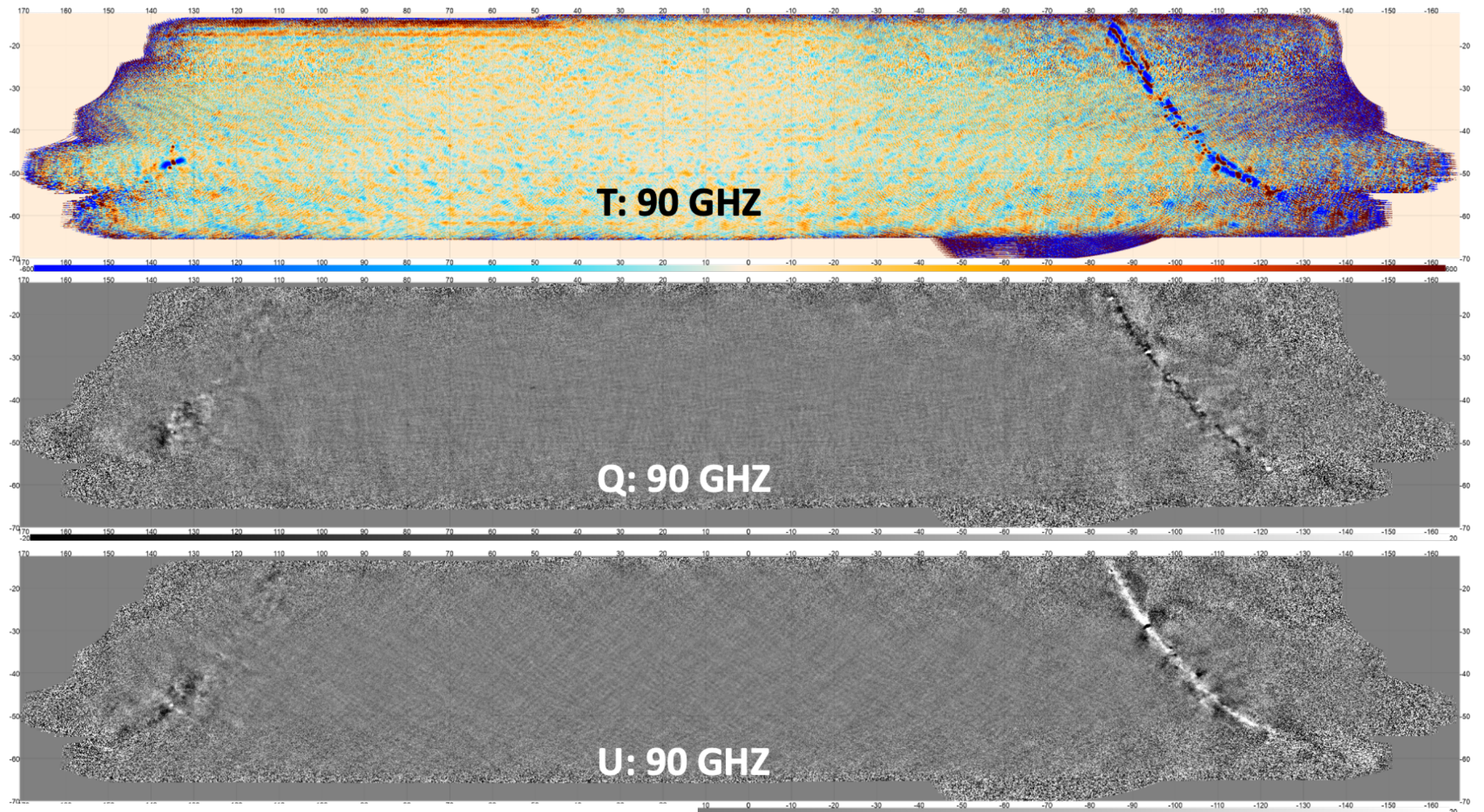
Applied low-pass filter to maps. → **Zoom-in**

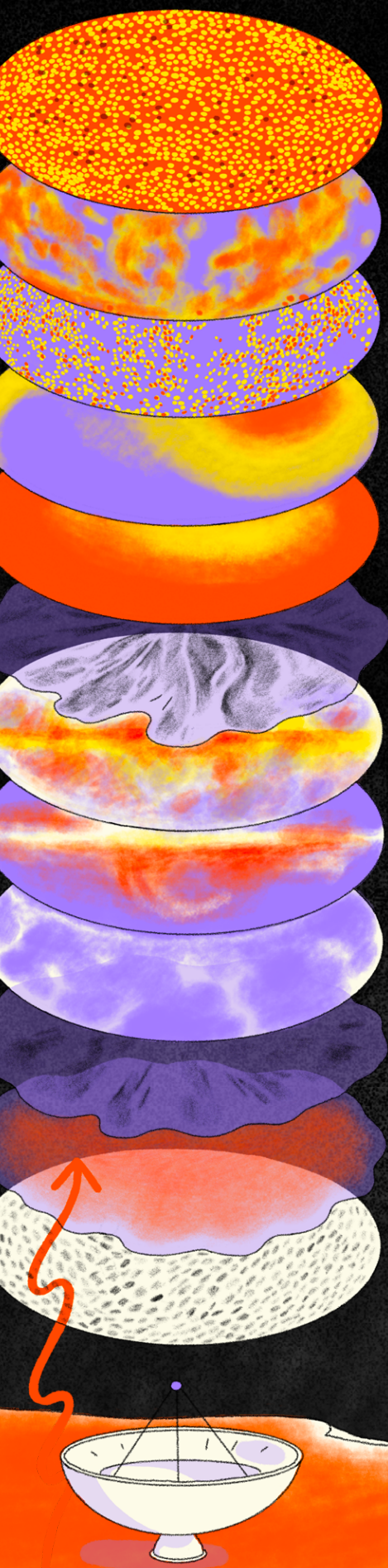
Q/U polarization patterns start being visible in the targeted SAT regions.



CMB maps

Preliminary SAT Maps: 90 GHz from One Telescope





B-modes

E-modes

Intensity
anisotropies

Dipole

Monopole

Gravitational
lensing

Galactic and
extra-galactic
foregrounds

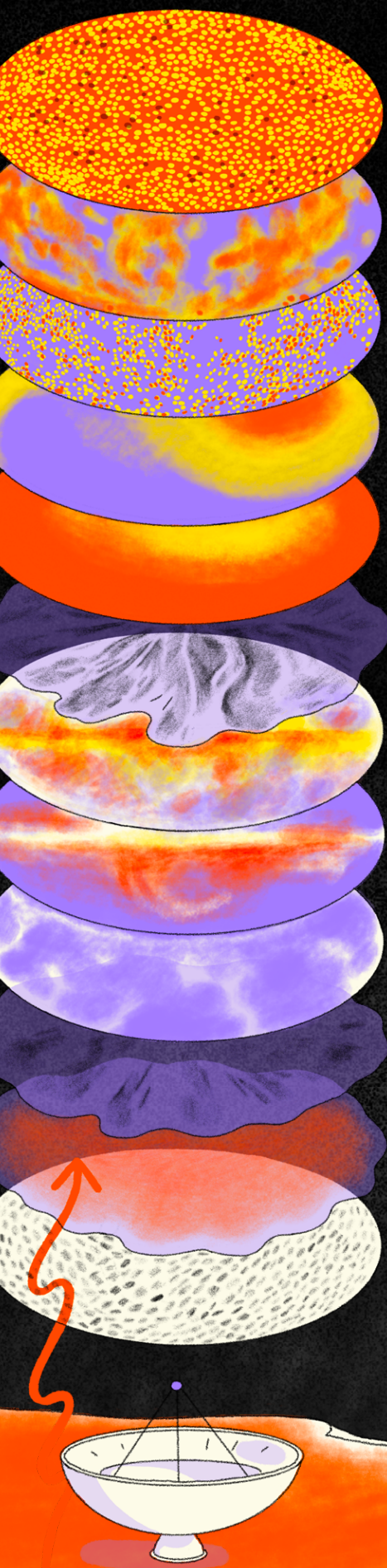
Atmosphere

Systematics

Ground
emissions

Noise

CMB



B-modes

E-modes

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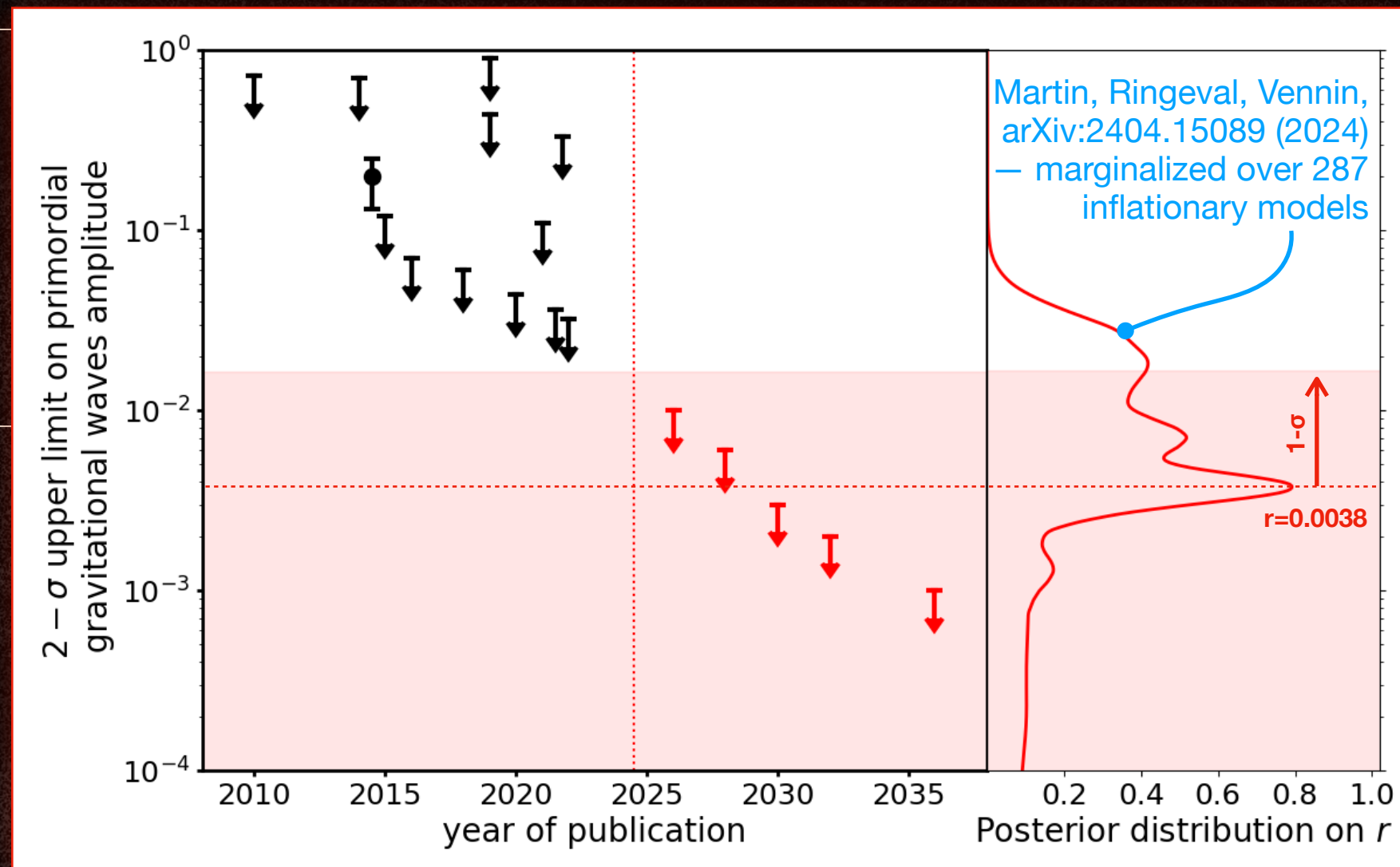
Galactic and
extra-galactic
foregrounds

Atmosphere

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Ground
emissions

Noise



LiteBIRD is the next-generation CMB satellite selected by JAXA as a Strategic Large Mission to be launched in 2029

4700 multichroic TES detectors

50x Planck sensitivity on large angular scales

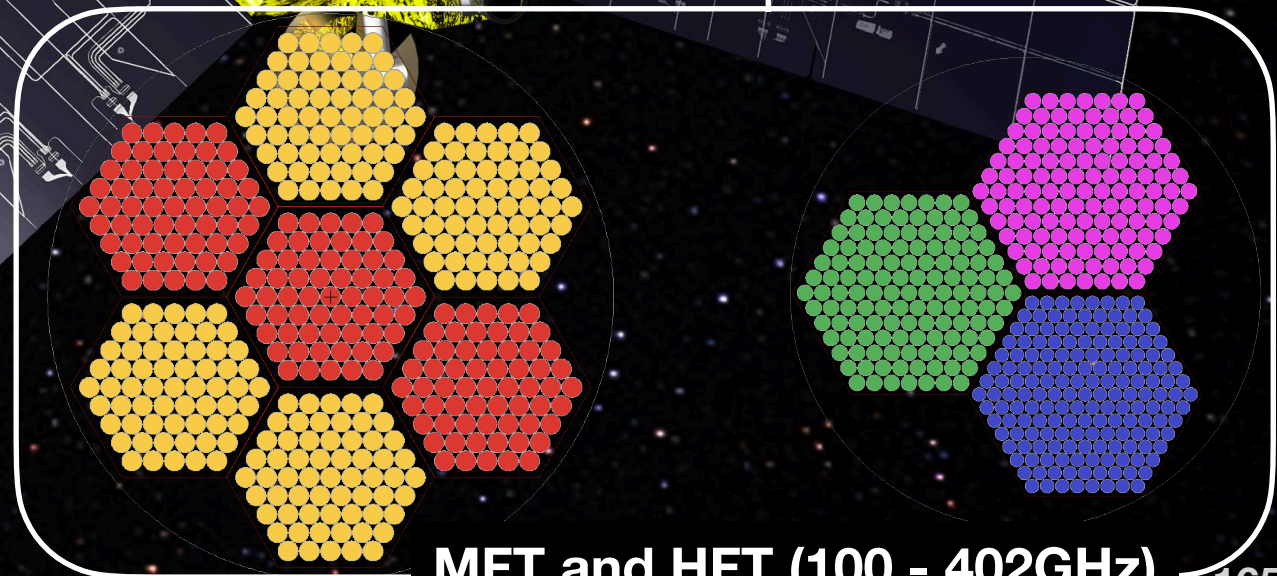
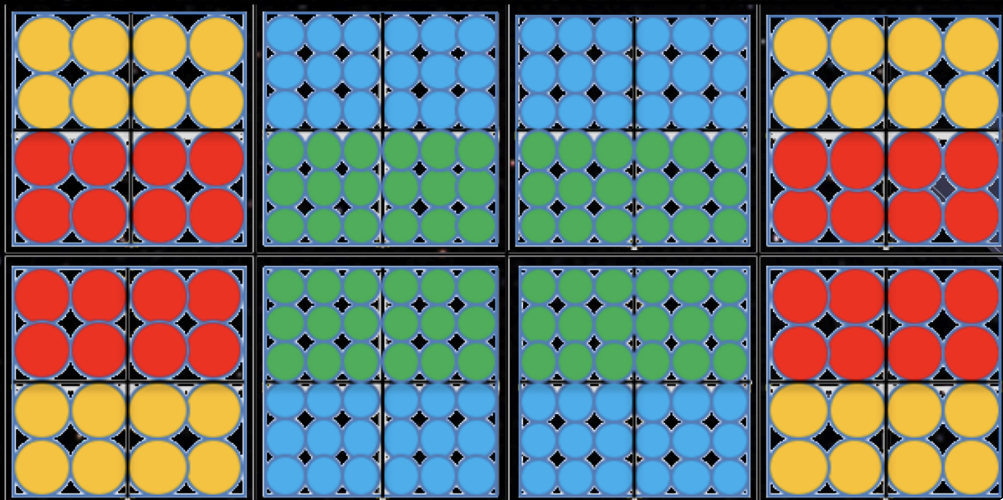
15 frequency bands

$40 \leq \nu \leq 402 \text{ GHz}$

3 telescopes + 3 instruments
rotating half-wave plates
year observation at L2

$$\sigma(r) \sim 5.7 \times 10^{-4}$$

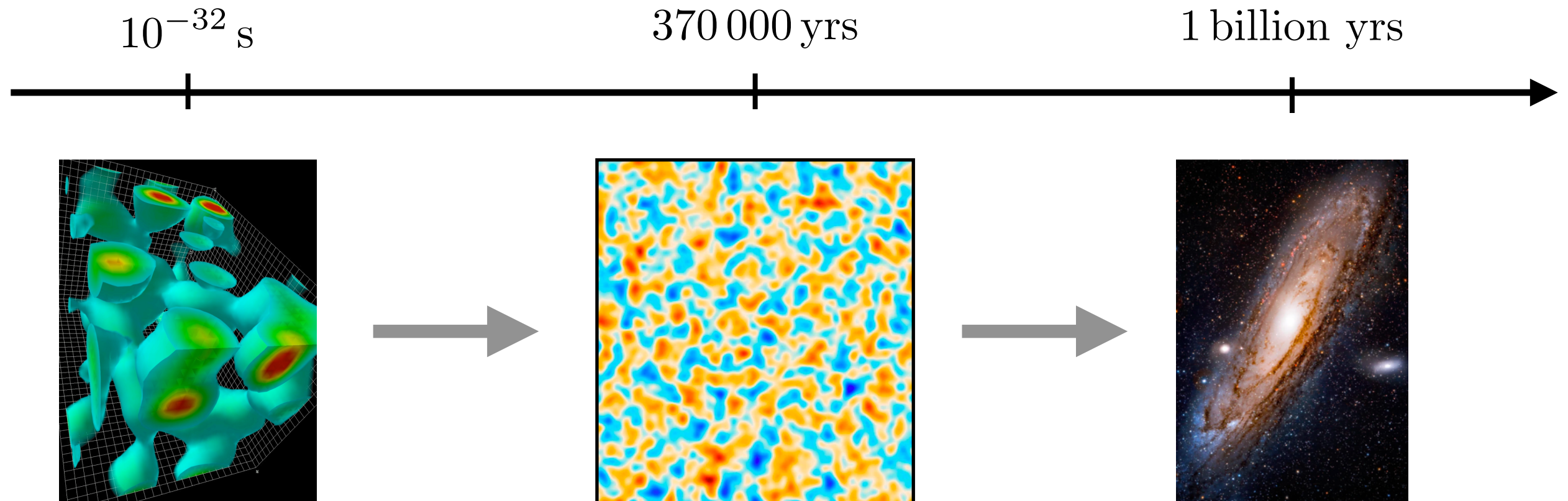
LFT (40 - 140GHz)



MFT and HFT (100 - 402GHz)

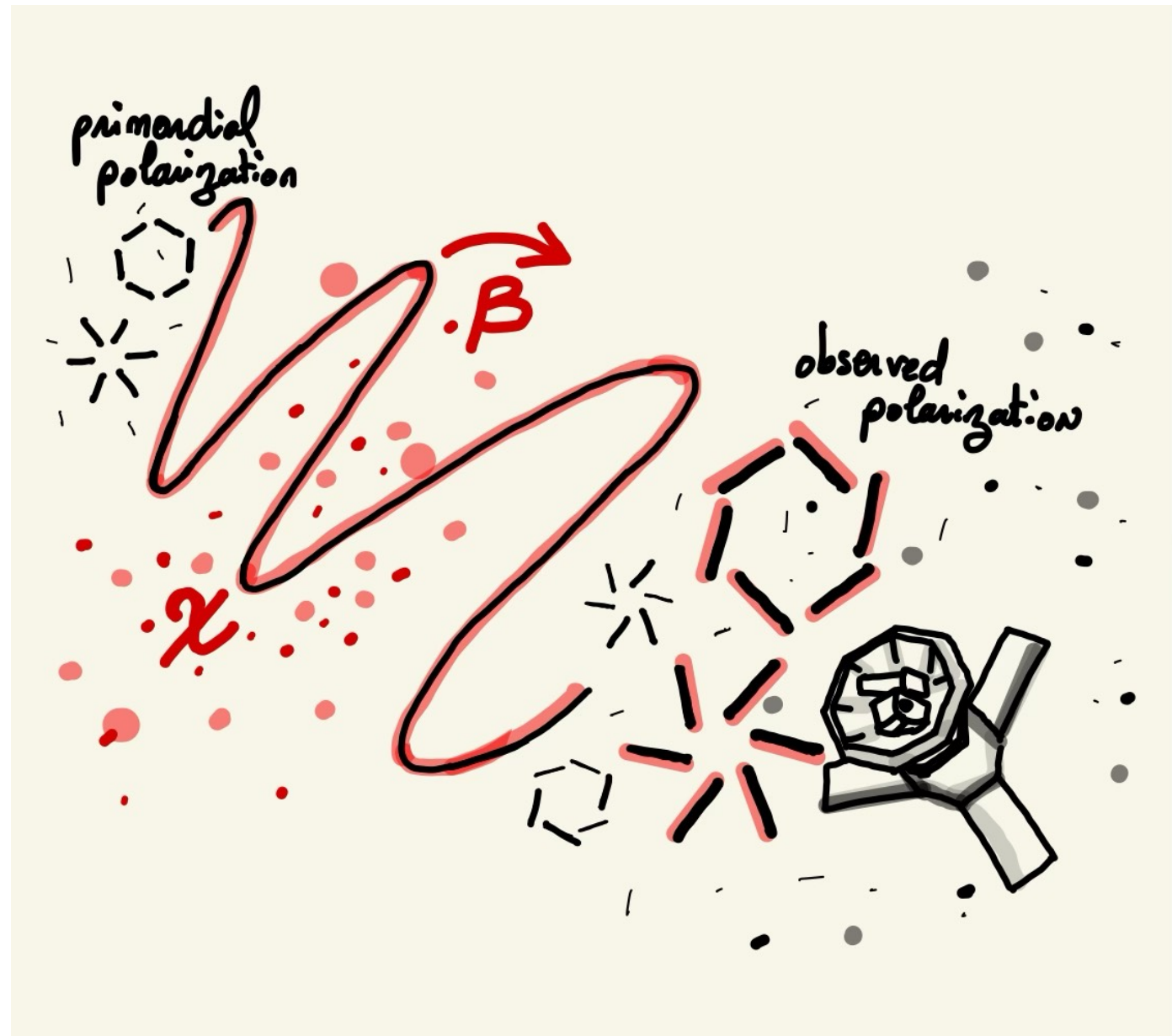


A B-mode detection would be a milestone towards a complete understanding of the origin of all structure in the Universe:



Cosmic Birefringence

In classical electromagnetism *birefringence* refers to the optical property where polarization states experience a different refractive index and propagate in different paths.



In a cosmological context — The interaction with new particles or fields beyond the standard model, potentially linked to dark matter or dark energy, would rotate the polarization plane of CMB photons as they travel through space.

This effect is called cosmic birefringence. It creates specific correlations in CMB polarization that would be zero otherwise and a B-mode signal not due to primordial gravitational waves. A detection would provide evidence for new physics and valuable insights into the dark components of the Universe.

Scalar field DM/DE coupled to the CS term

DM = Dark Matter; DE = Dark Energy; CS = Chern Simons

$$I = \int d^4x \sqrt{-g} \left[-\frac{1}{2} (\partial\chi)^2 - V(\chi) - \frac{1}{4} F^2 - \frac{\alpha}{4f} \chi F \tilde{F} \right]$$

- χ is a neutral pseudoscalar field (spin 0).
- Why consider χ as a good DM/DE candidate?

We wrote

$$\theta = \frac{\chi}{f}$$

- *Why not?* We have an example in the Standard Model: a neutral pion.
- We expect $\alpha \simeq \alpha_{\text{EM}} \simeq 10^{-2}$ and $f < M_{\text{Pl}} \simeq 2.4 \times 10^{18}$ GeV.
- χ can be composed of fermions like a pion, or a fundamental pseudoscalar like an “axion” field.



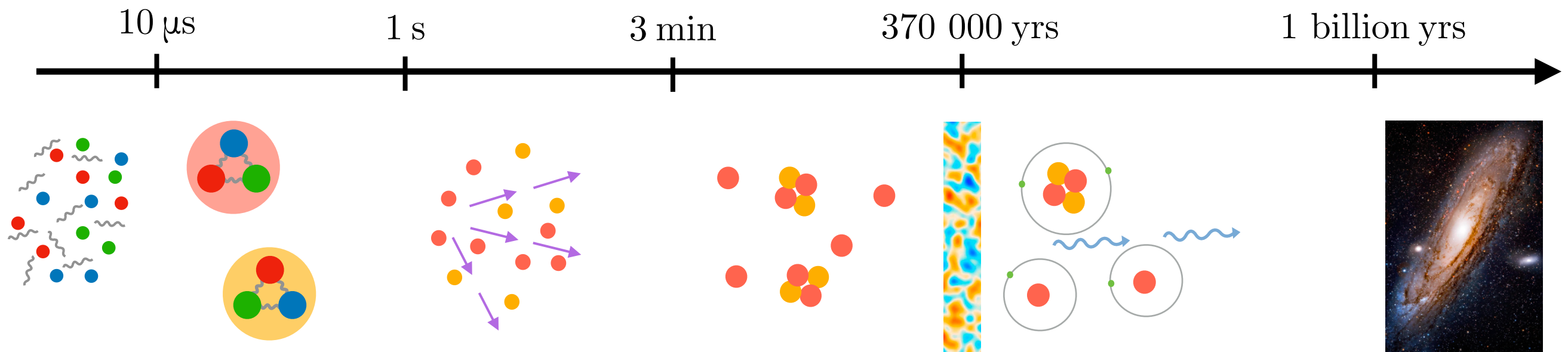
Jim Simons in 2023

Alternative solution: artificial calibrator

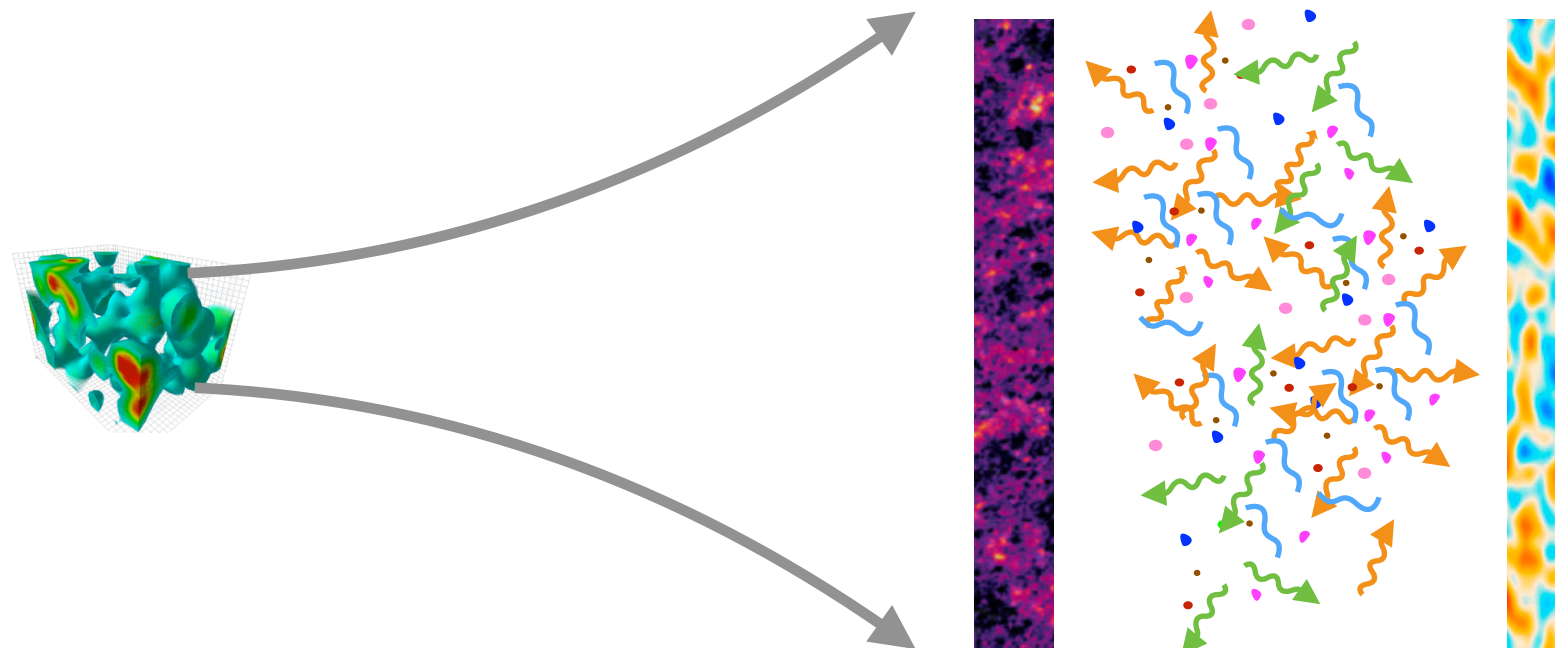


Conclusions

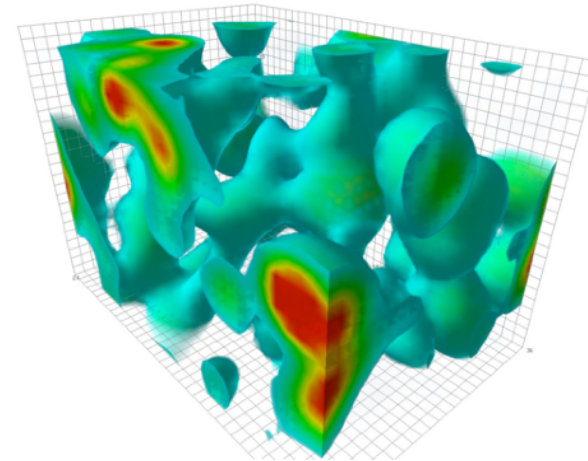
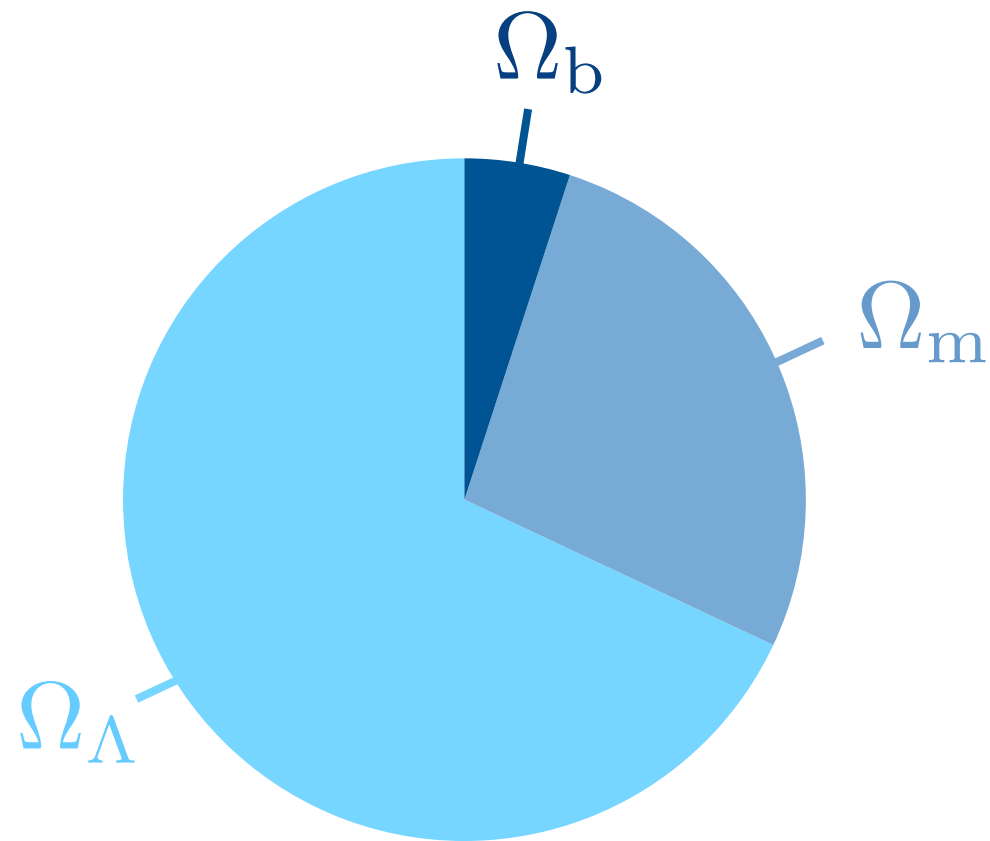
We have a remarkably consistent picture of the history of the Universe from fractions of a second after the Big Bang until today:



We also have tantalizing evidence that the primordial seed fluctuations for the formation of structure were created during a period of inflation:



Observations of the CMB have revolutionized cosmology:

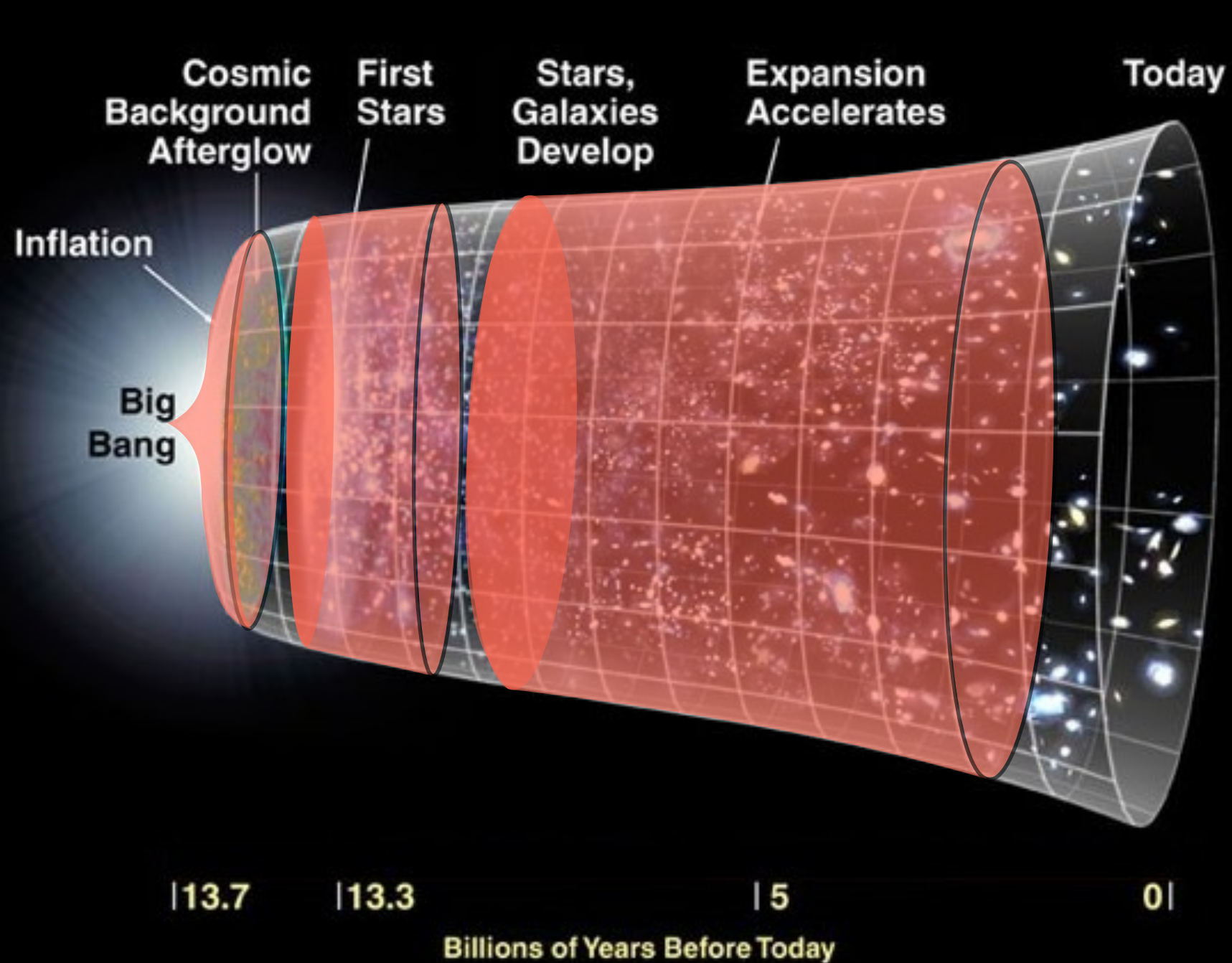


A_s, n_s

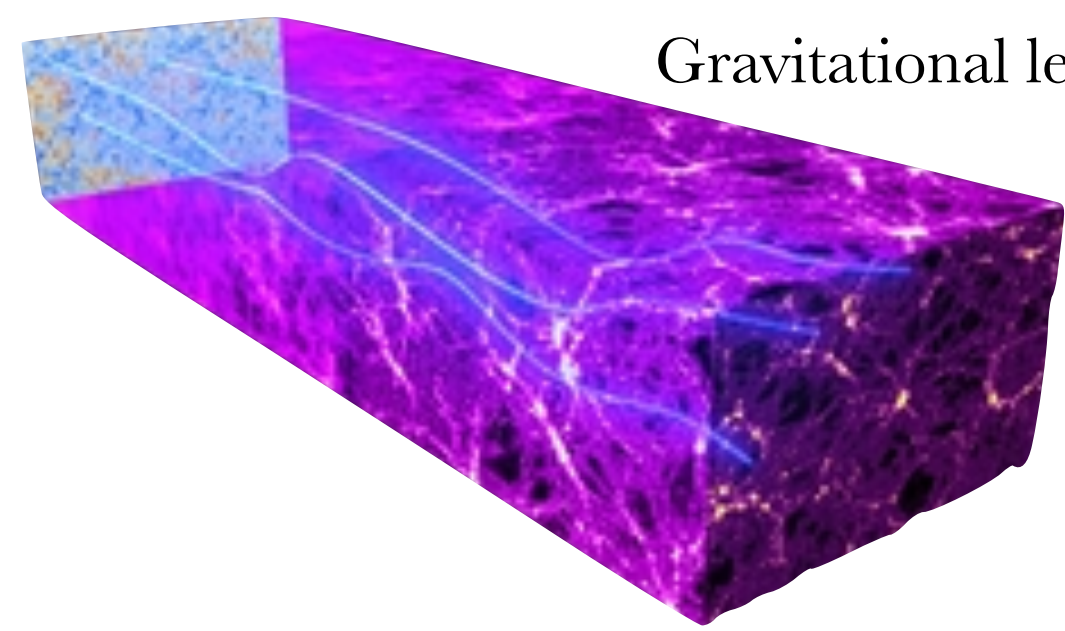
Yet, many fundamental questions remain:

- What is dark matter and dark energy?
- Did inflation really occur? And what was driving it?
- What is the origin of the matter-antimatter asymmetry?

We hope that future observations will shed light on these questions.



Gravitational lensing



Planck

