Multi-messenger (astro)physics theory

IDPASC

Saclay, 23/07/2025

Enrico Peretti
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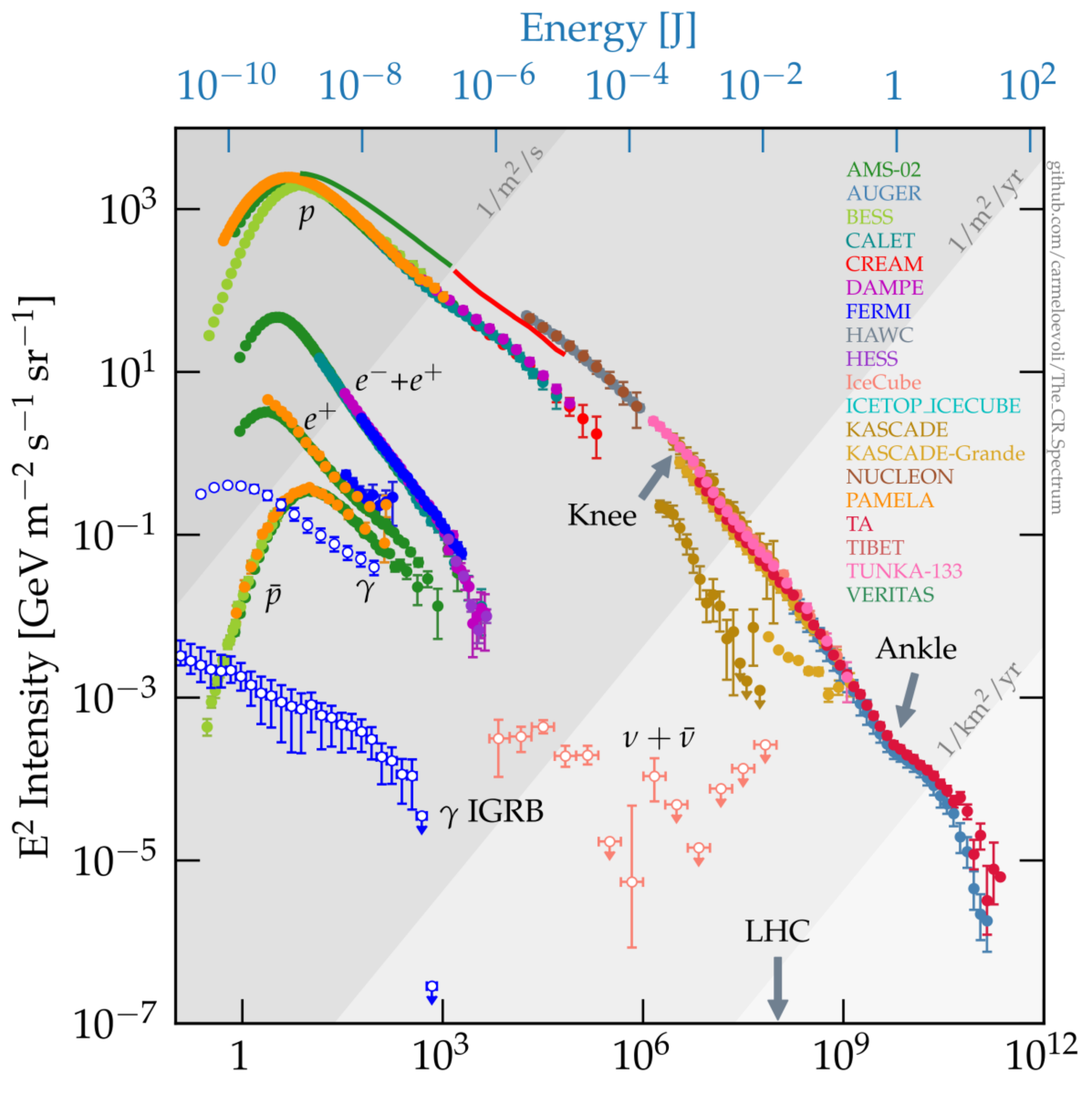
The multi-messenger cosmic flux



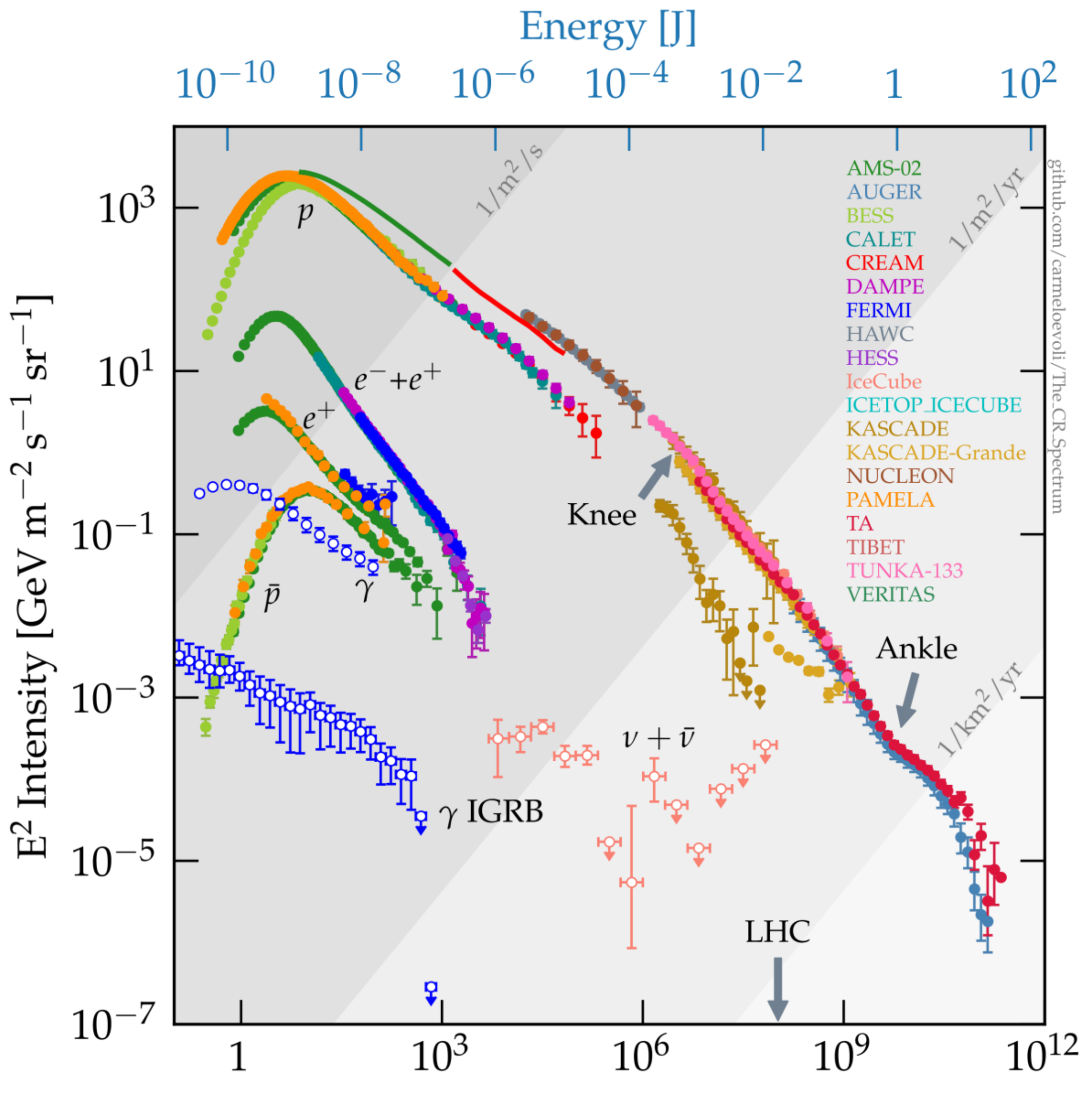
Credit: Evoli

◈ **Cosmic rays** → 90% H, 9% He, 1% other;

Galactic origin: below the Knee

Extragalactic origin: above the Ankle

The multi-messenger cosmic flux



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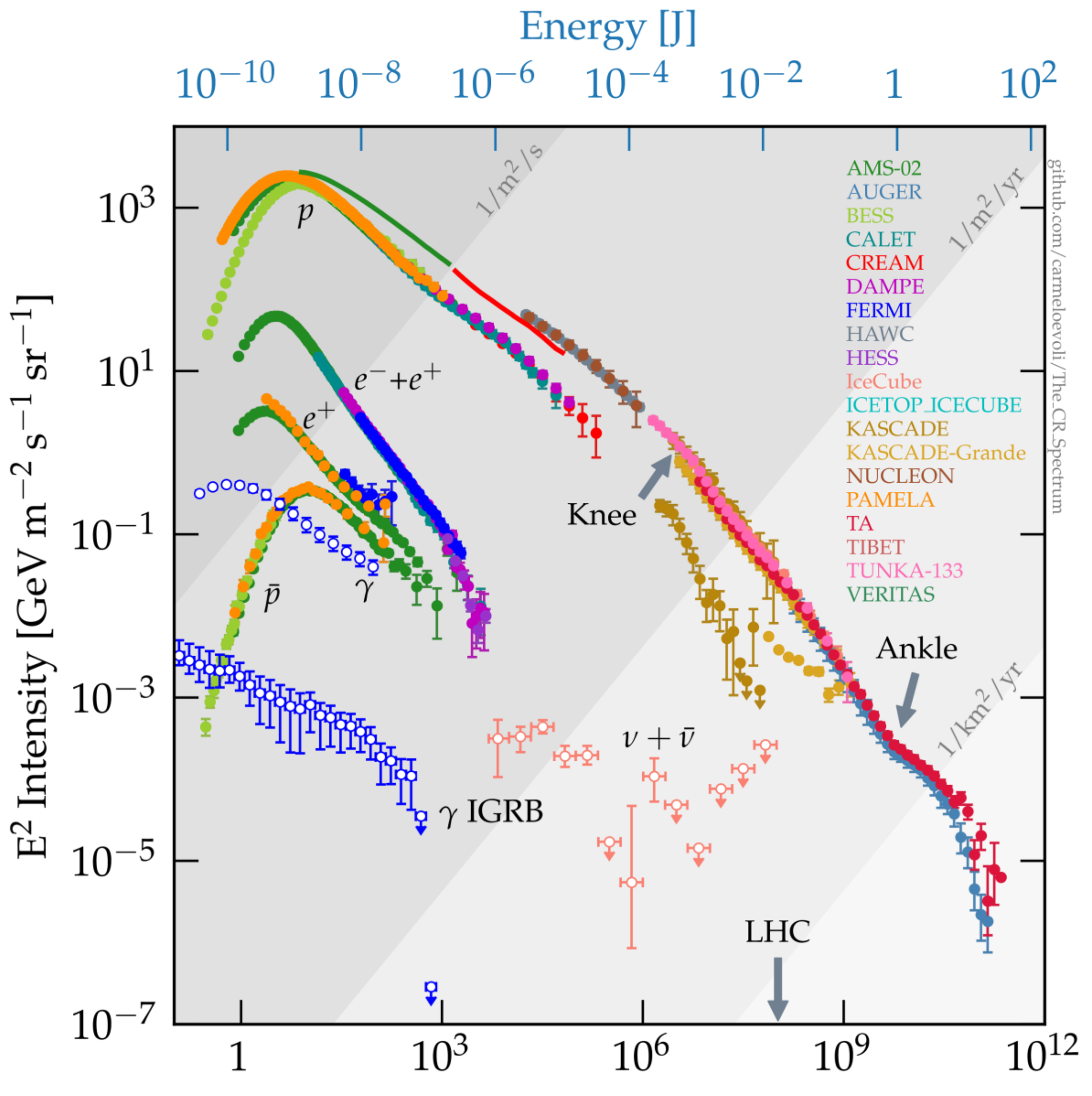
  Galactic diffuse: Coincident with Galactic disc

  Isotropic diffuse: Probably extragalactic

The multi-messenger cosmic flux



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- **Gamma rays** → Two diffuse components:

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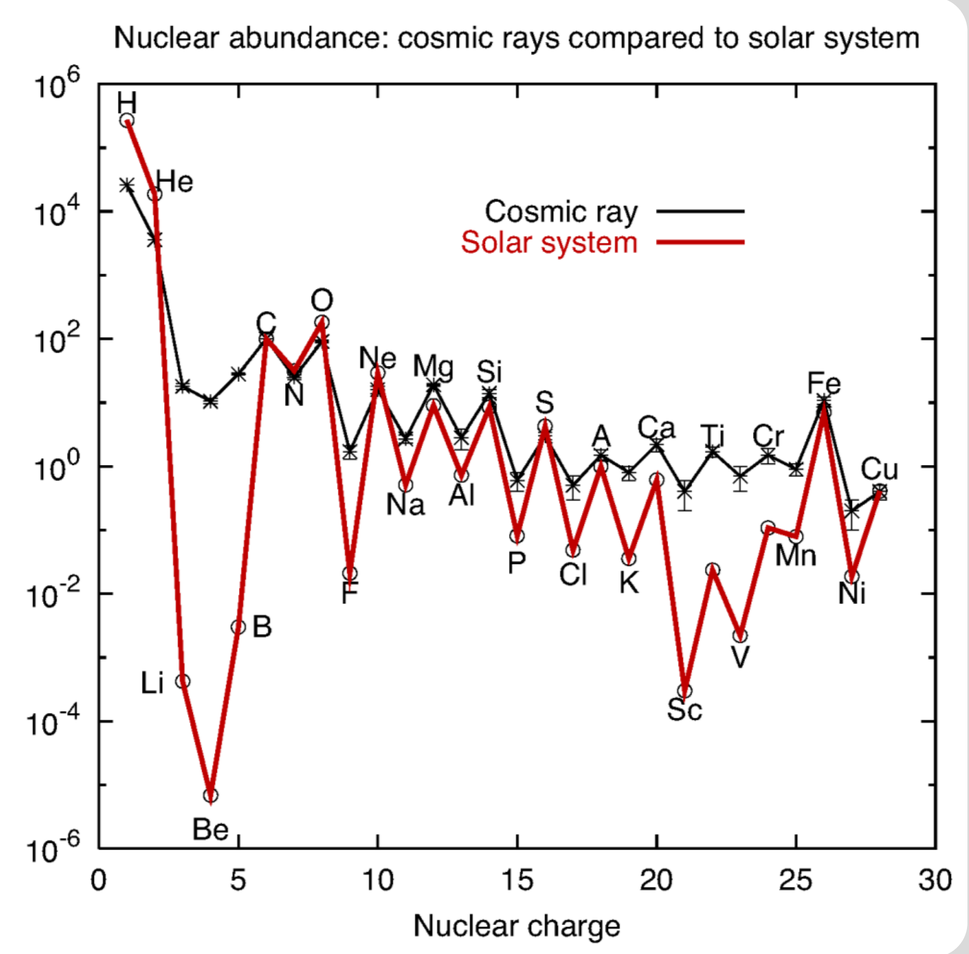
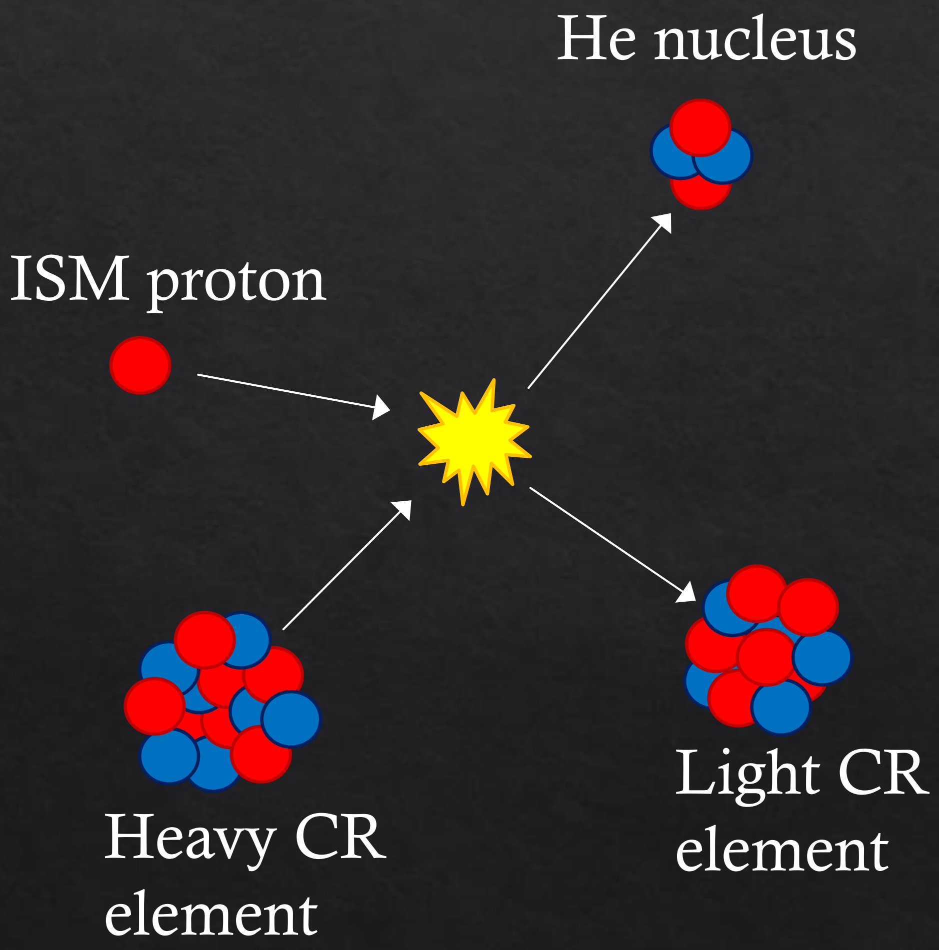
- **Neutrinos** → Isotropic and likely extragalactic

Galactic cosmic rays

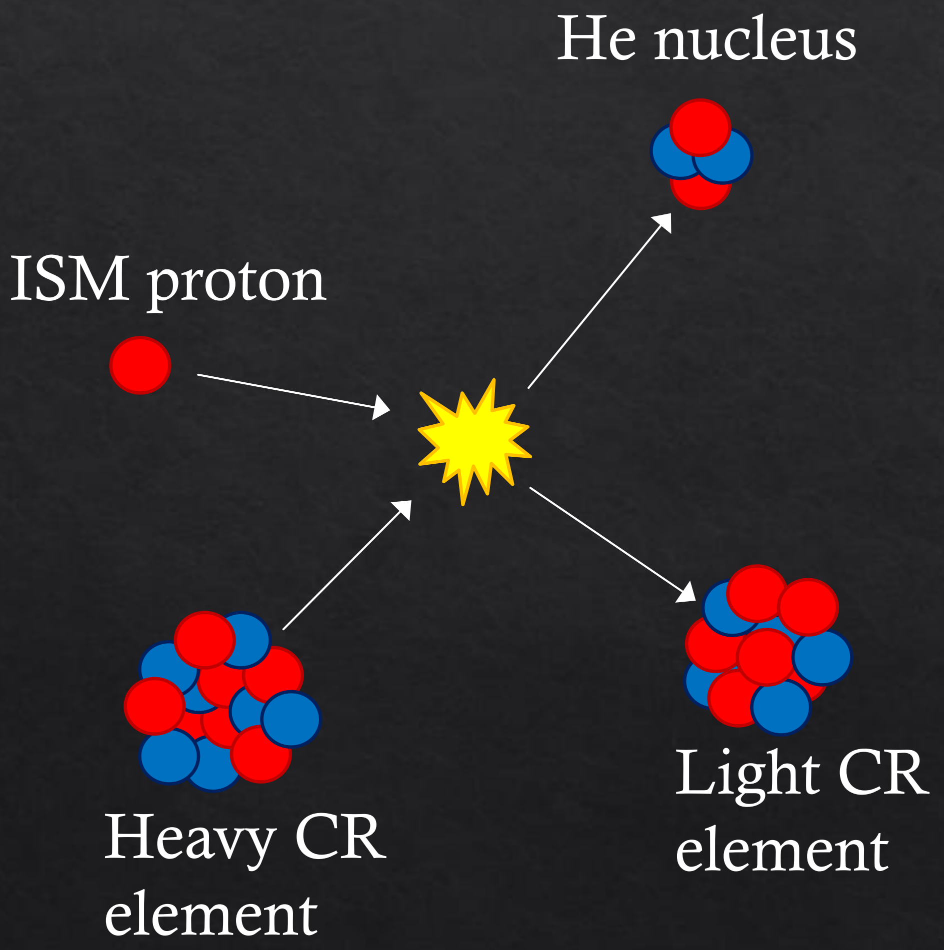


- The nuclear abundances of cosmic rays approximately follow those found in the Solar system
- The major difference is found in the relative abundancies of Li, Be and B and sub-Fe

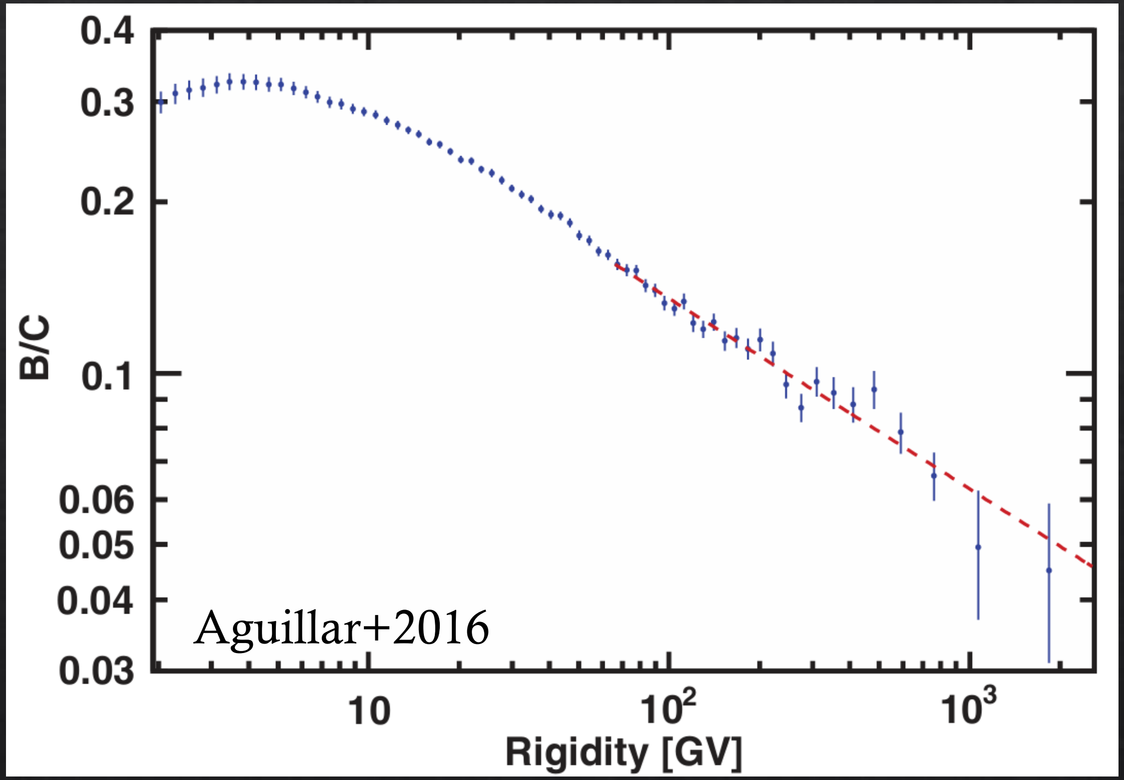
Spallation and cosmic clocks



Spallation and cosmic clocks

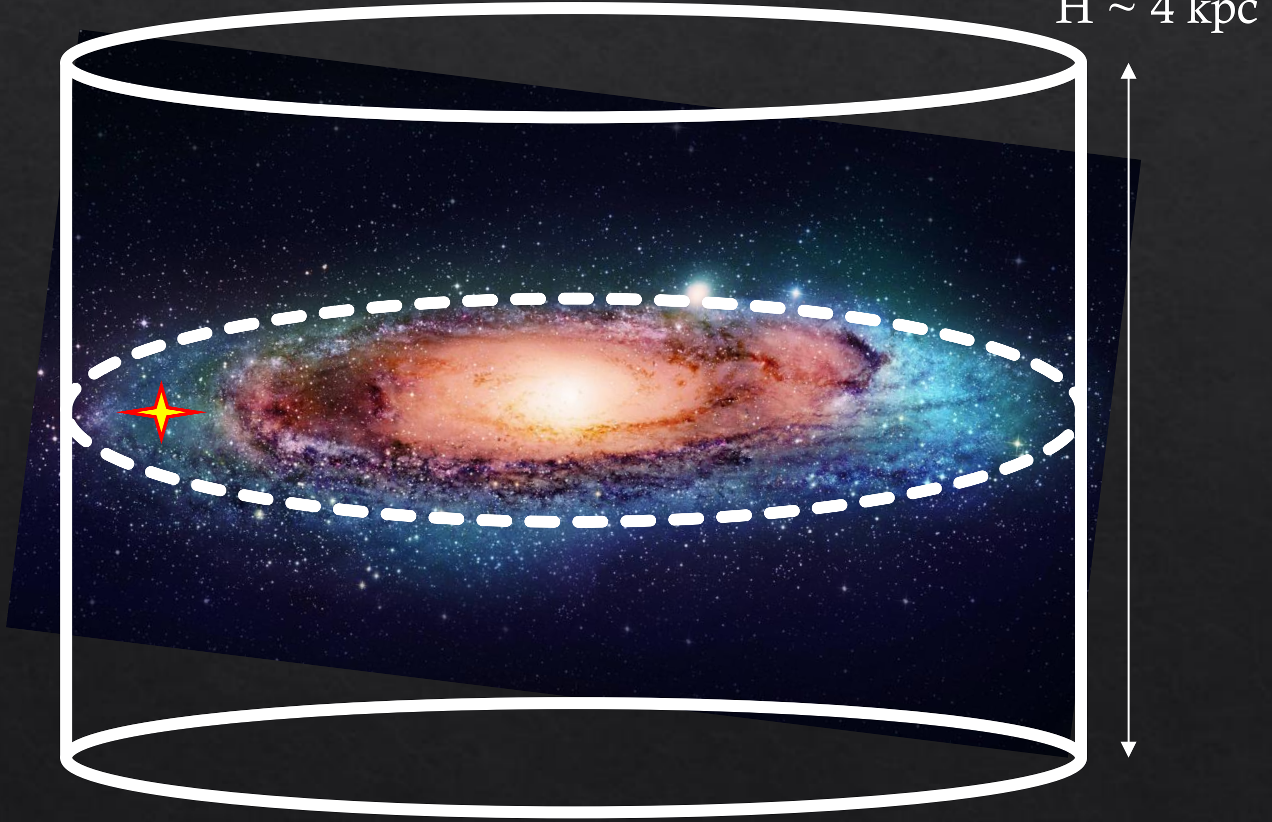


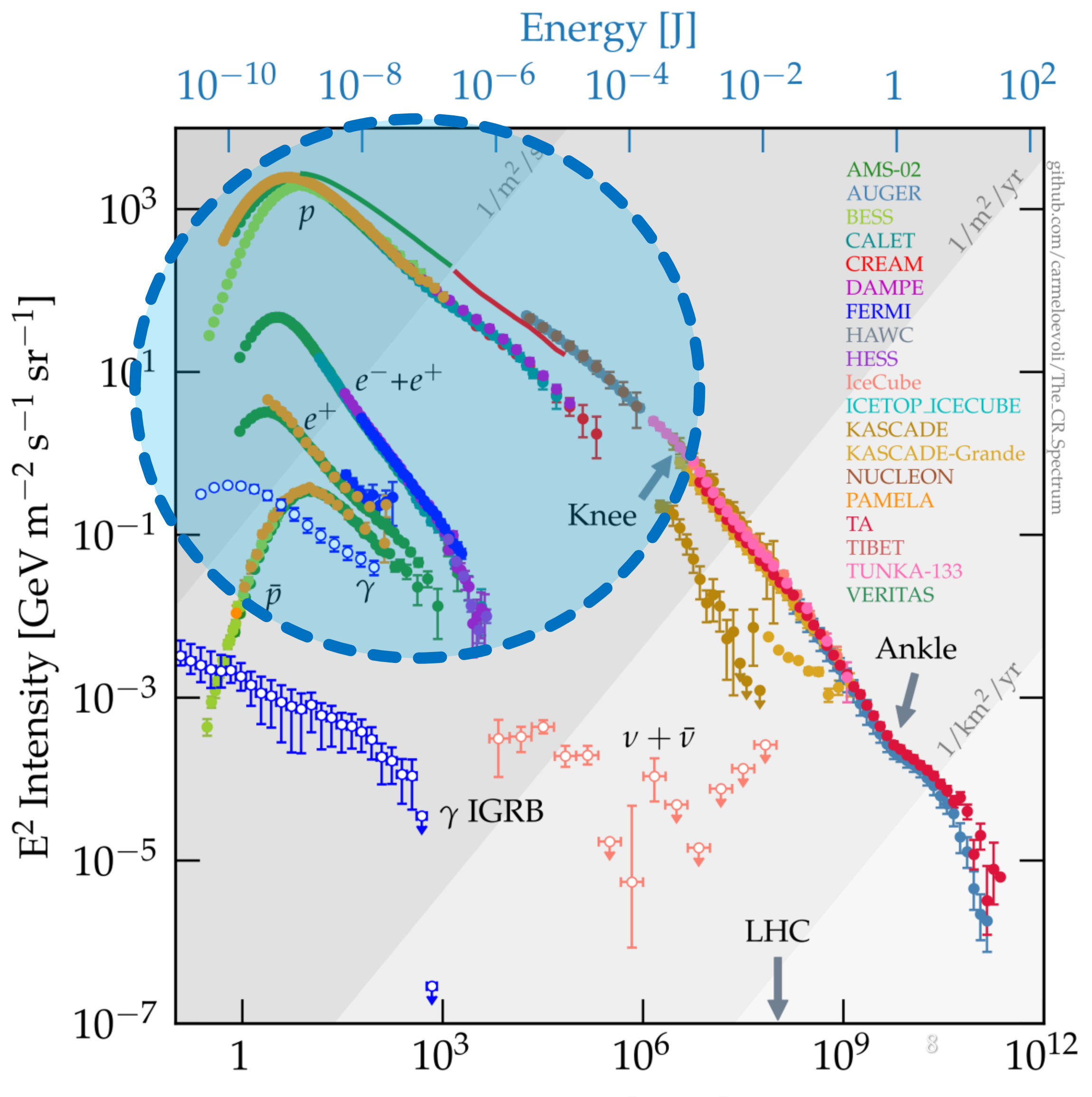
- Li, Be and B result from spallation of heavier cosmic rays (C, N, O, Ne)



- B/C → $t_{res} \gtrsim 10\ Myr$

Galactic cosmic rays





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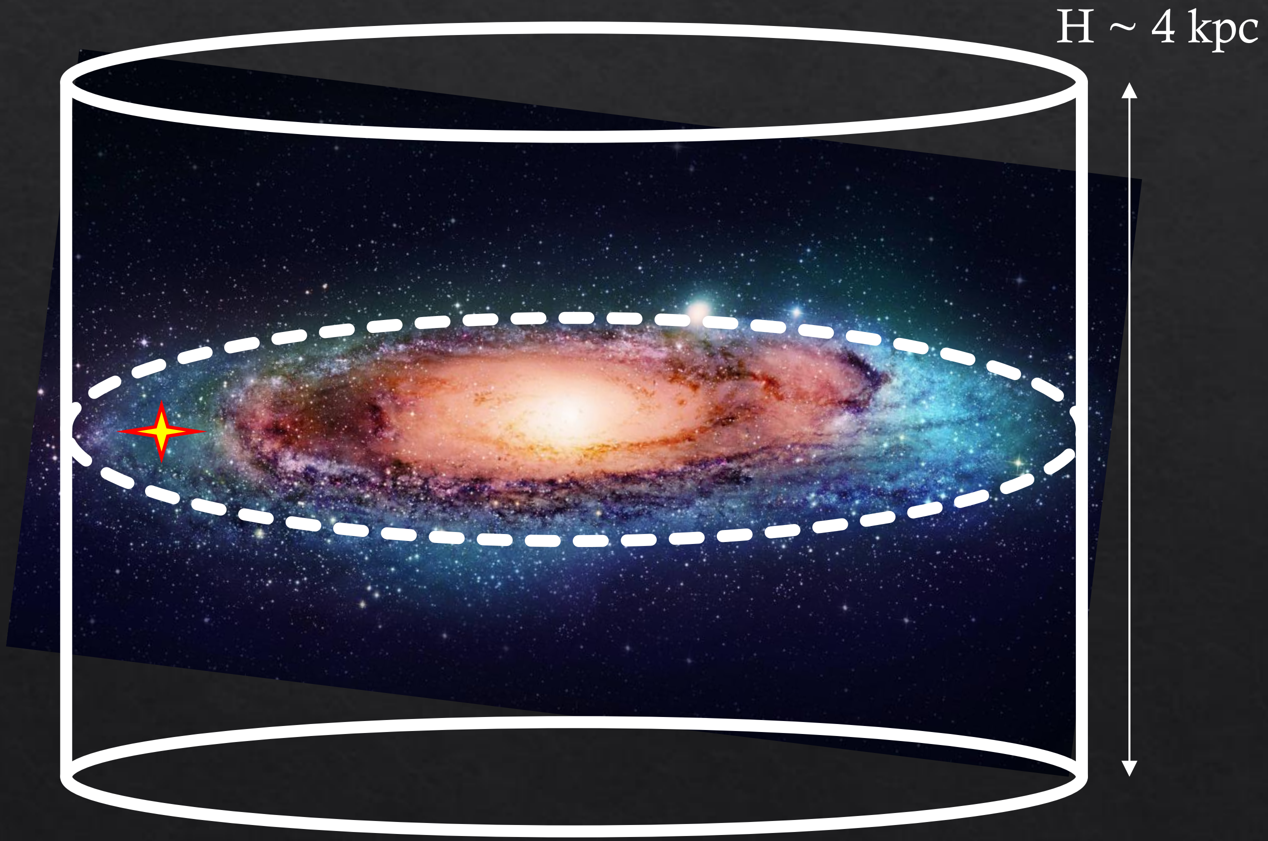
Galactic cosmic rays

- $t_{ball} \approx \frac{H}{c} \approx 1.3 \cdot 10^4 \, yr \, H_{4kpc}$



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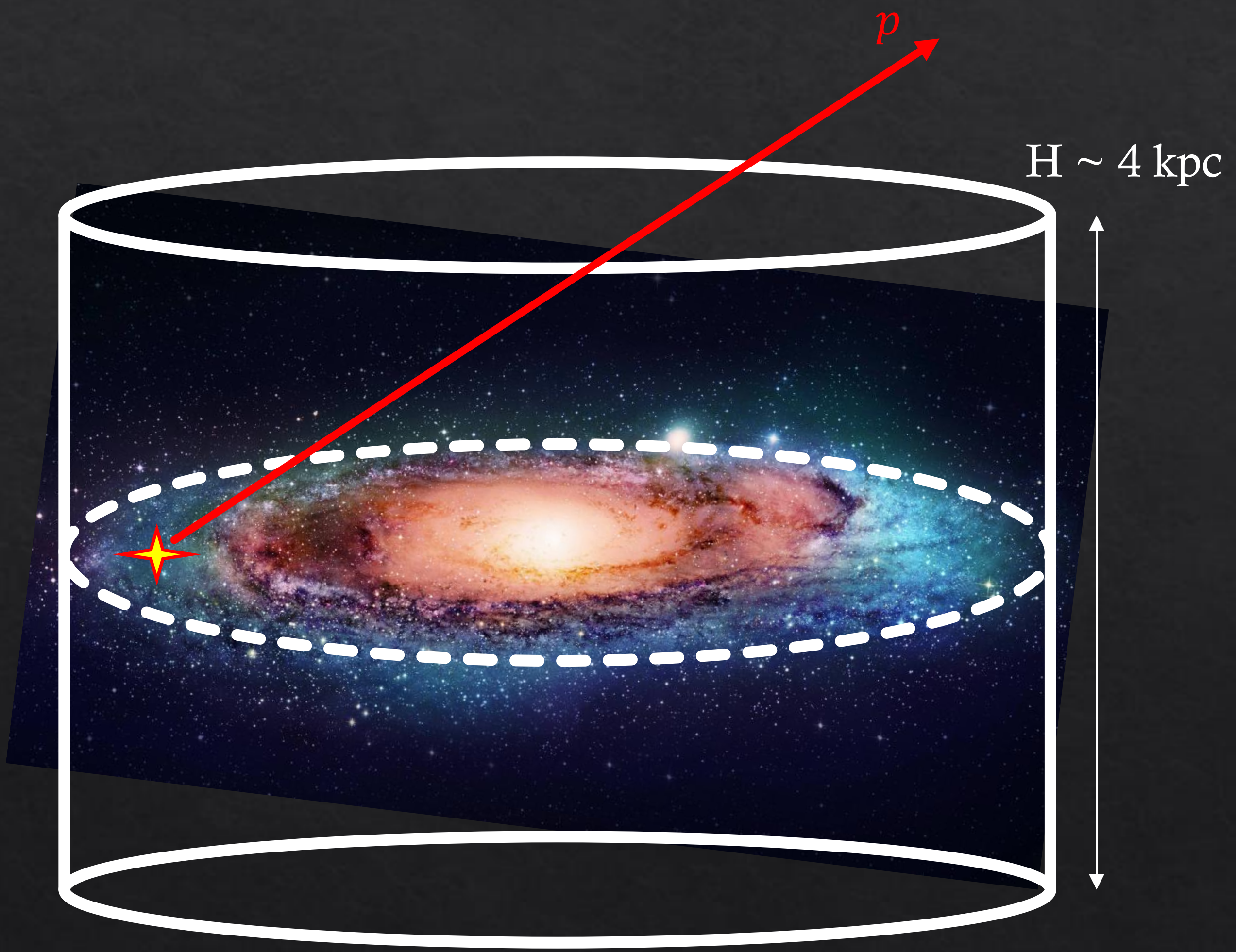

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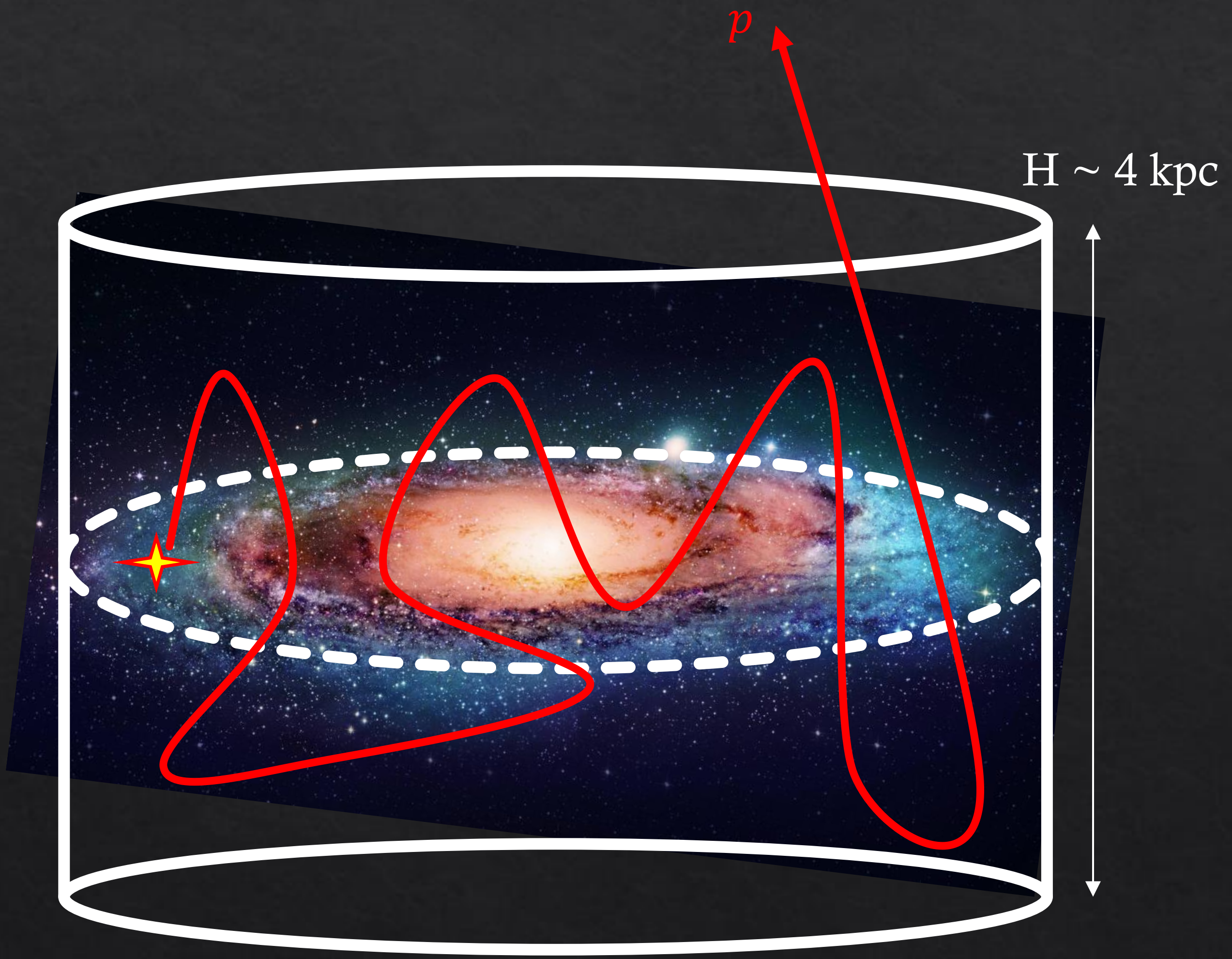

- $t_{ball} \approx \frac{H}{c} \approx 1.3 \cdot 10^4 \; yr \; H_{4kpc}$
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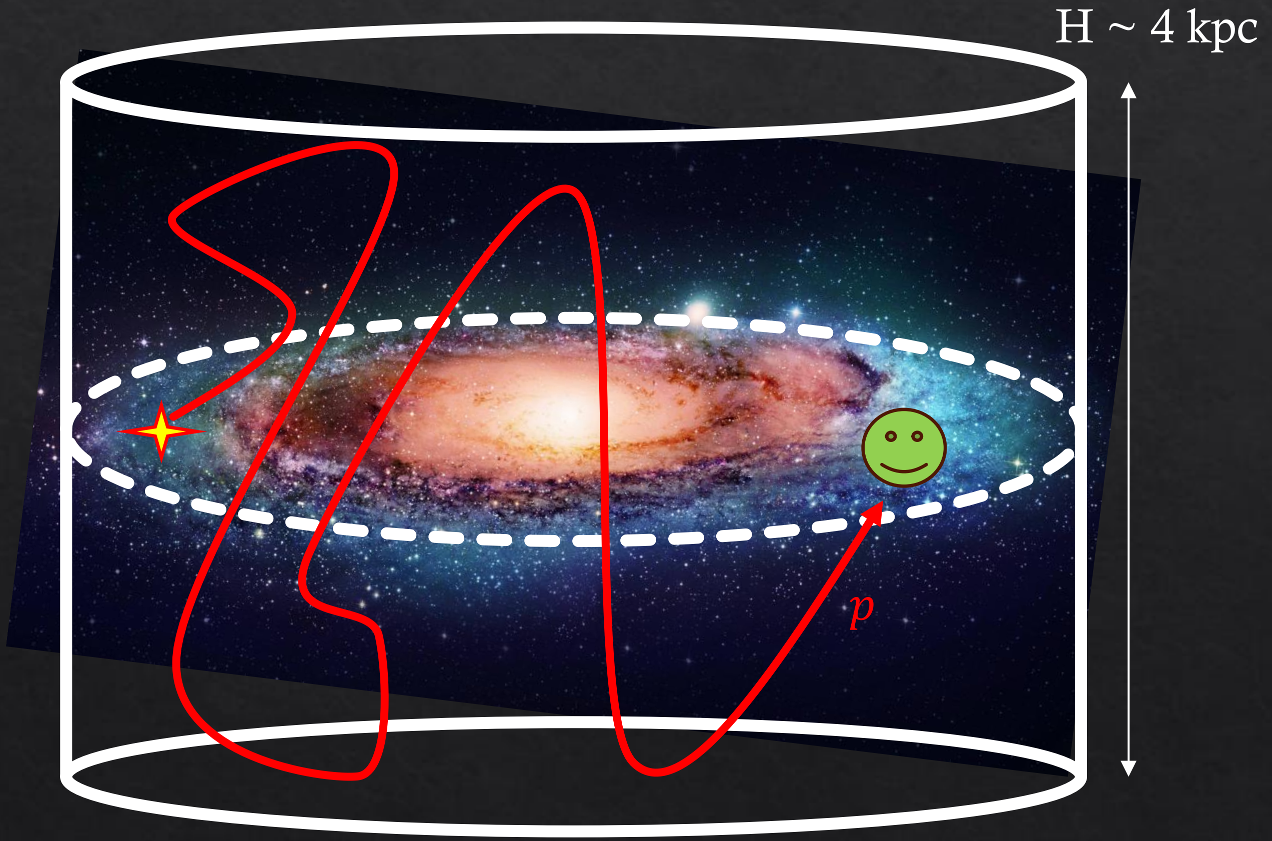
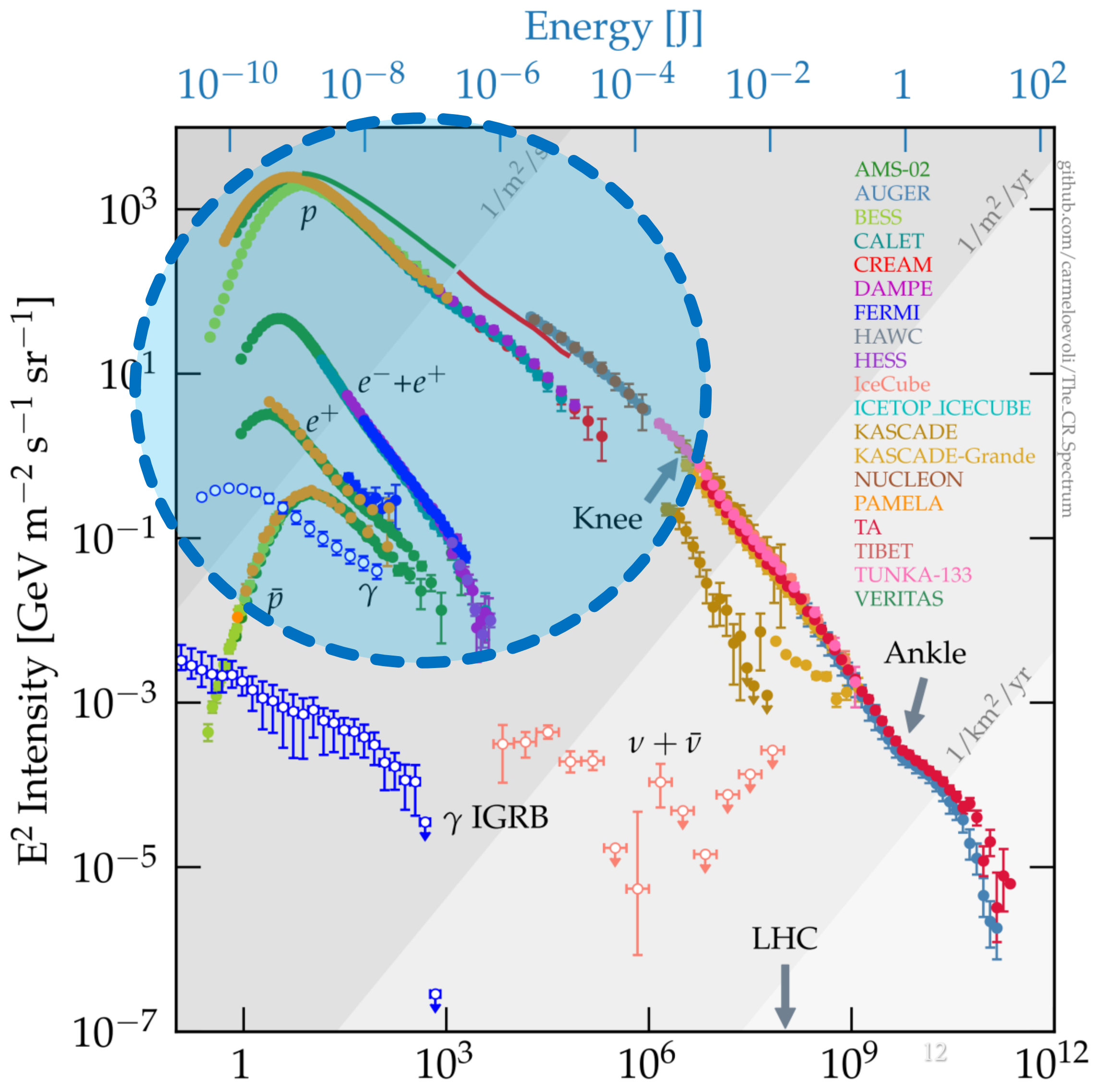
- The motion of cosmic rays in the magnetized Galactic halo cannot be ballistic

Galactic cosmic rays





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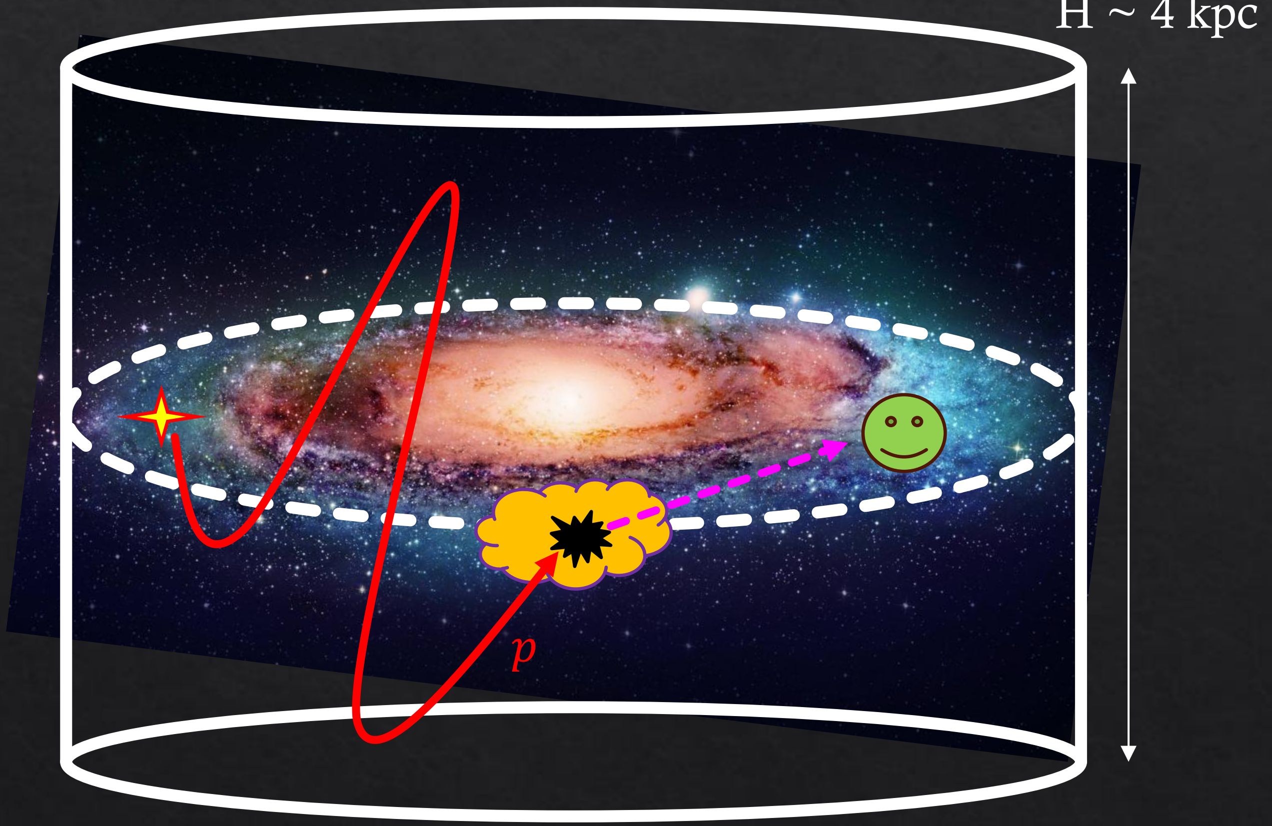
Galactic cosmic rays



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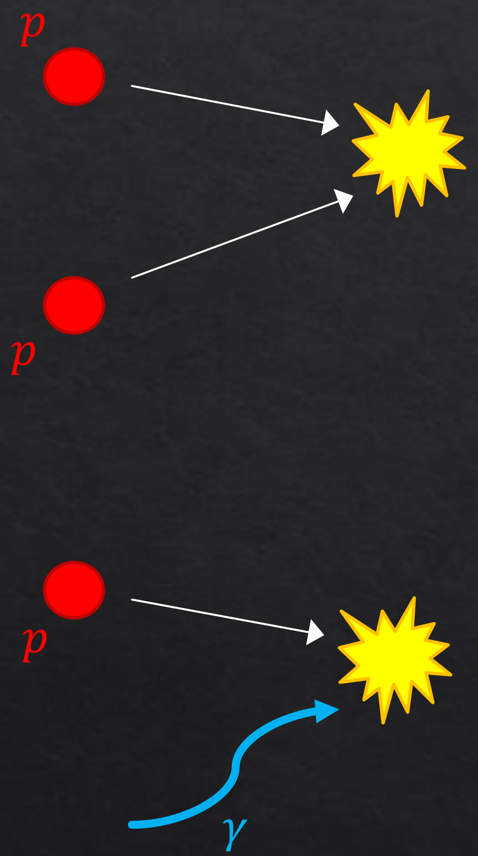
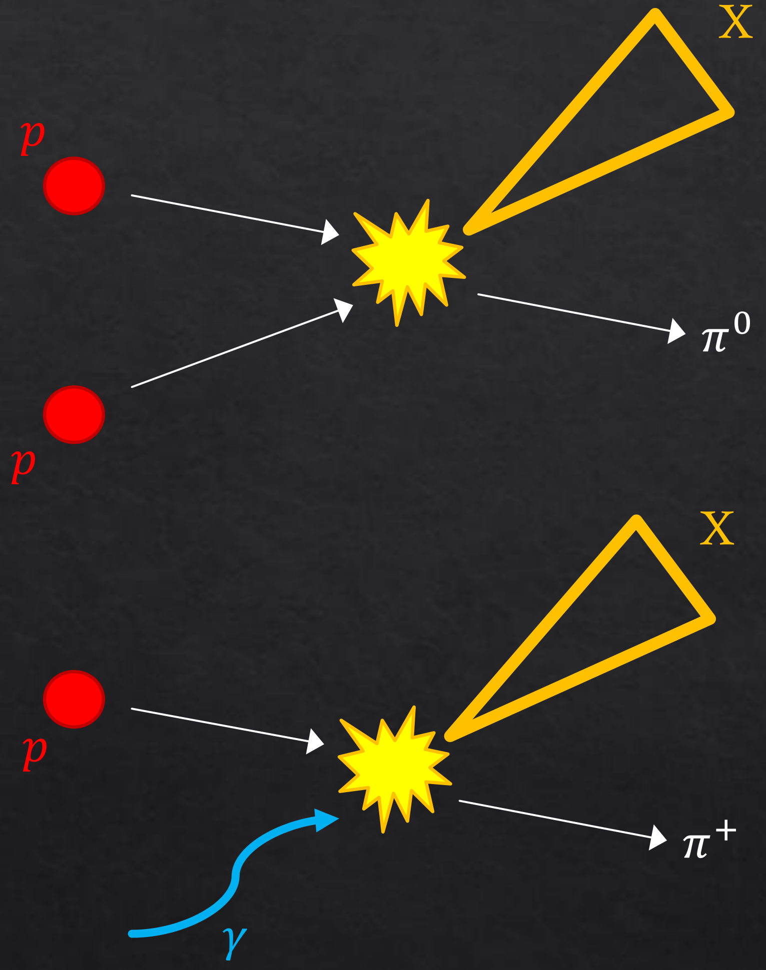
- The motion of cosmic rays in the magnetized Galactic halo cannot be ballistic
- Cosmic rays can be detected directly or indirectly […]

Non-thermal radiation – hadronic processes



- Inelastic collisions with gas or radiation are the most common interaction mechanism of high-energy protons

Non-thermal radiation – hadronic processes



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- The typical result is a leading pion (retaining ~20% of the parent proton energy)

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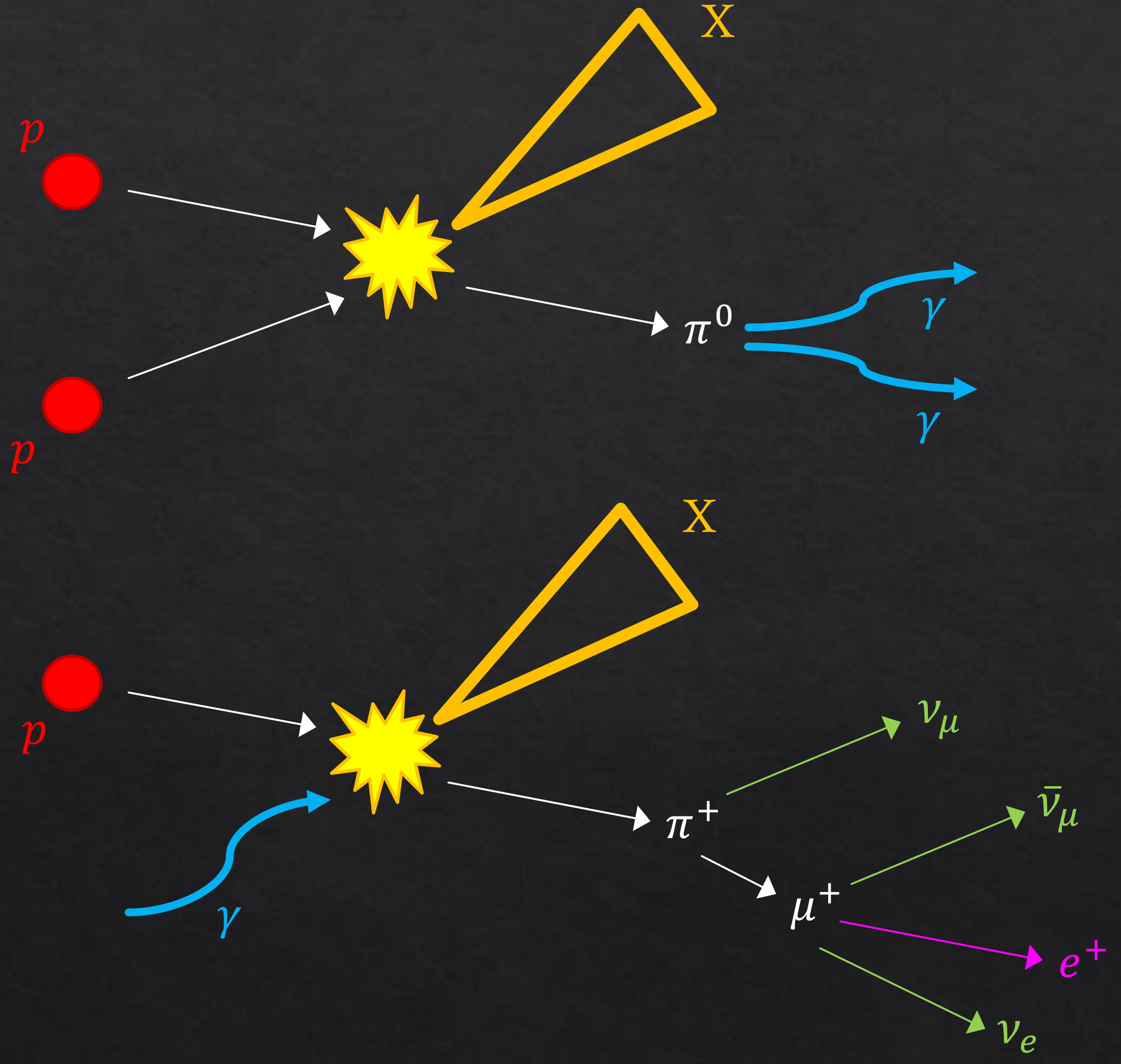

- Inelastic collisions with gas or radiation are the most common interaction mechanism of high-energy protons
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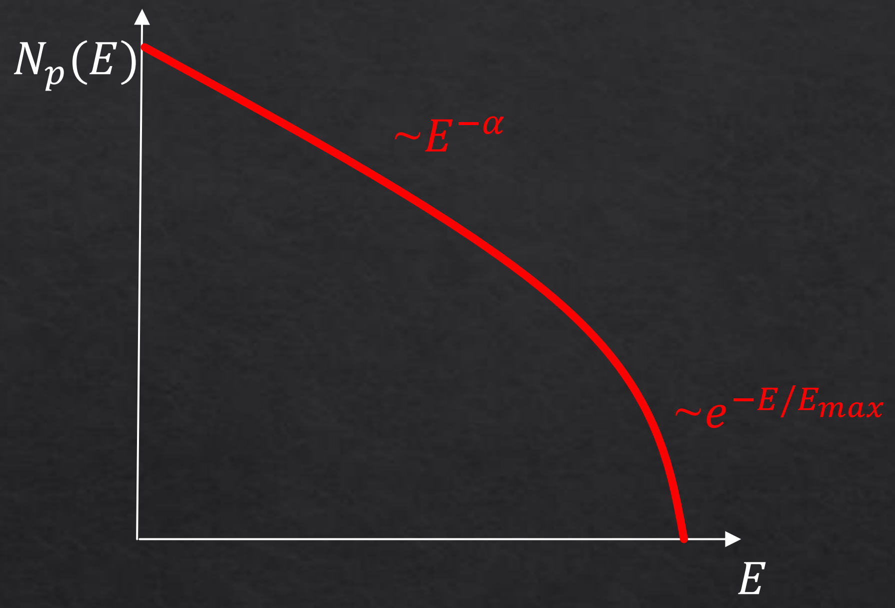
- Gamma rays and neutrino are byproducts of inelastic cosmic-ray interactions

Non-thermal radiation – hadronic processes



- High-energy particles in sources typically have non-thermal energy distributions – power-laws

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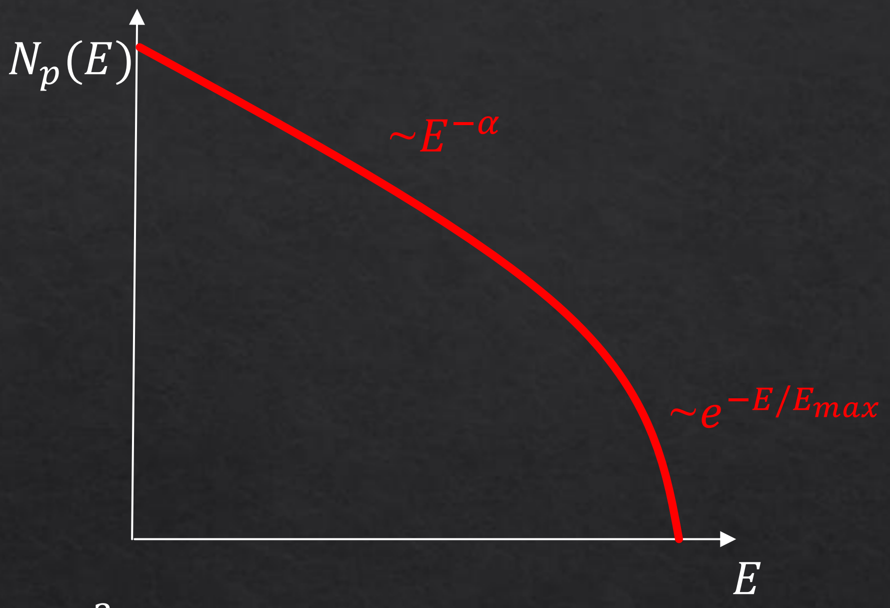

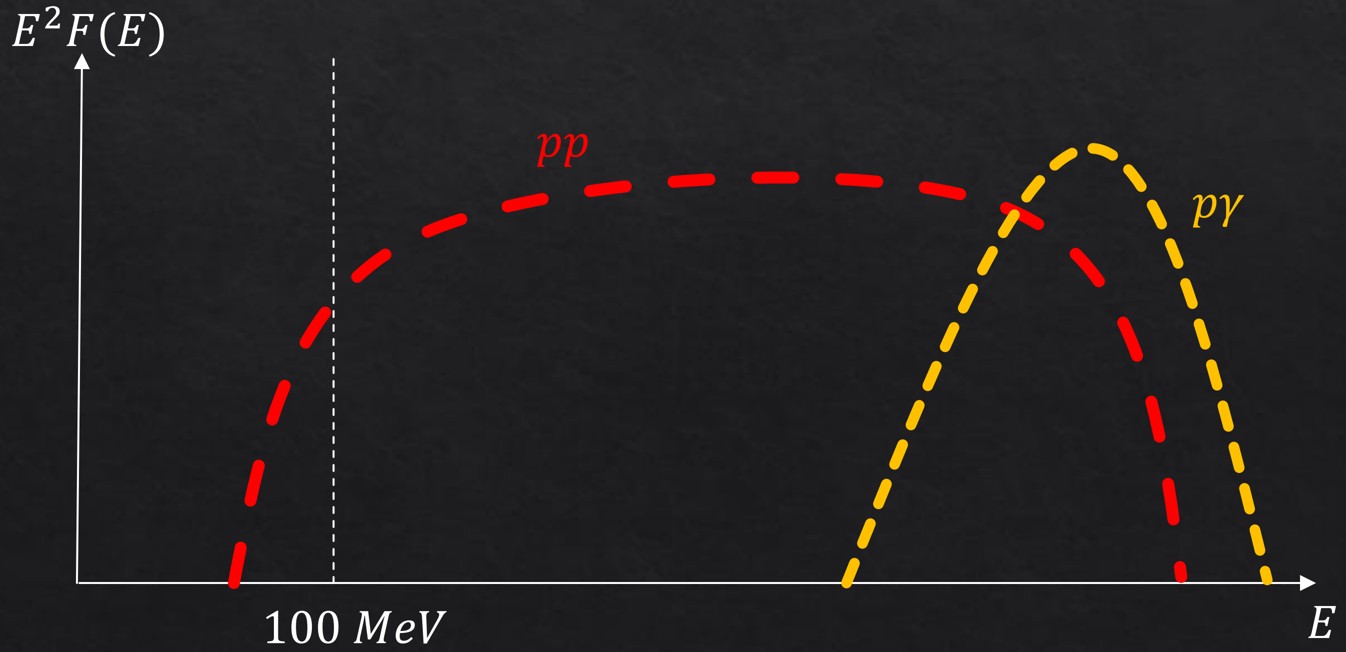

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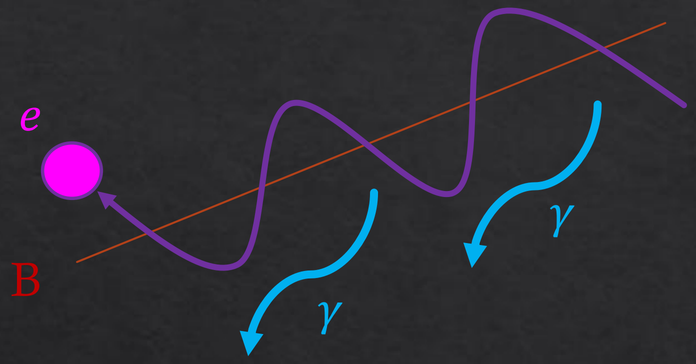
- The photon spectra from hadronic and photo-hadronic interactions are typically observed in gamma-ray above 100 MeV

Non-thermal radiation – leptonic processes



- Synchrotron: high-energy electrons in a magnetic field (radio to X-ray)

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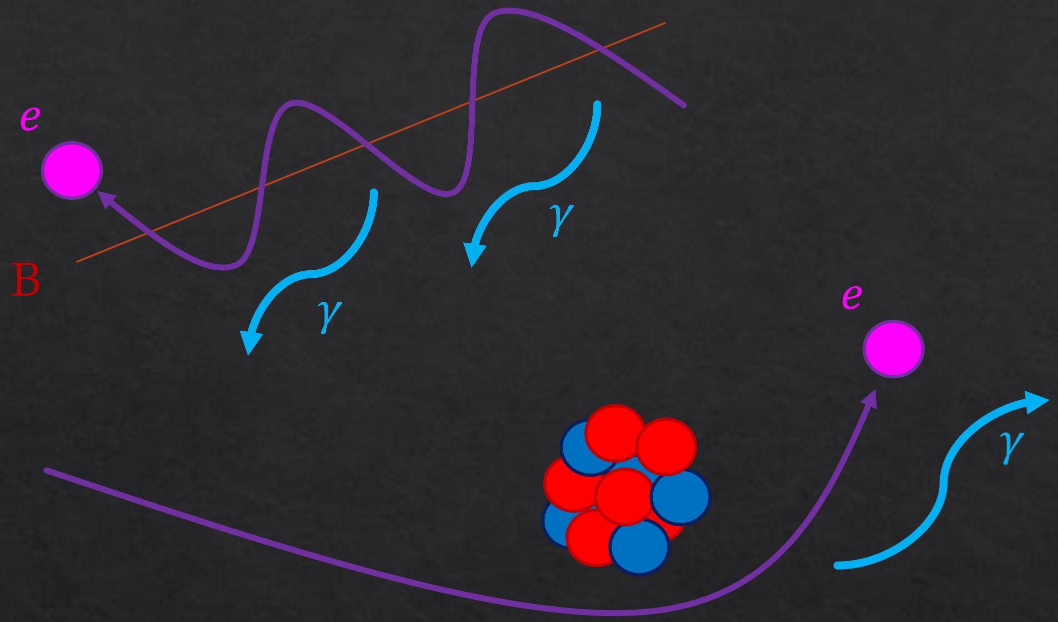

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- Bremsstrahlung: high-energy electrons emit photons when deflected by the electric field of a nucleus (hard X-ray to soft gamma rays)

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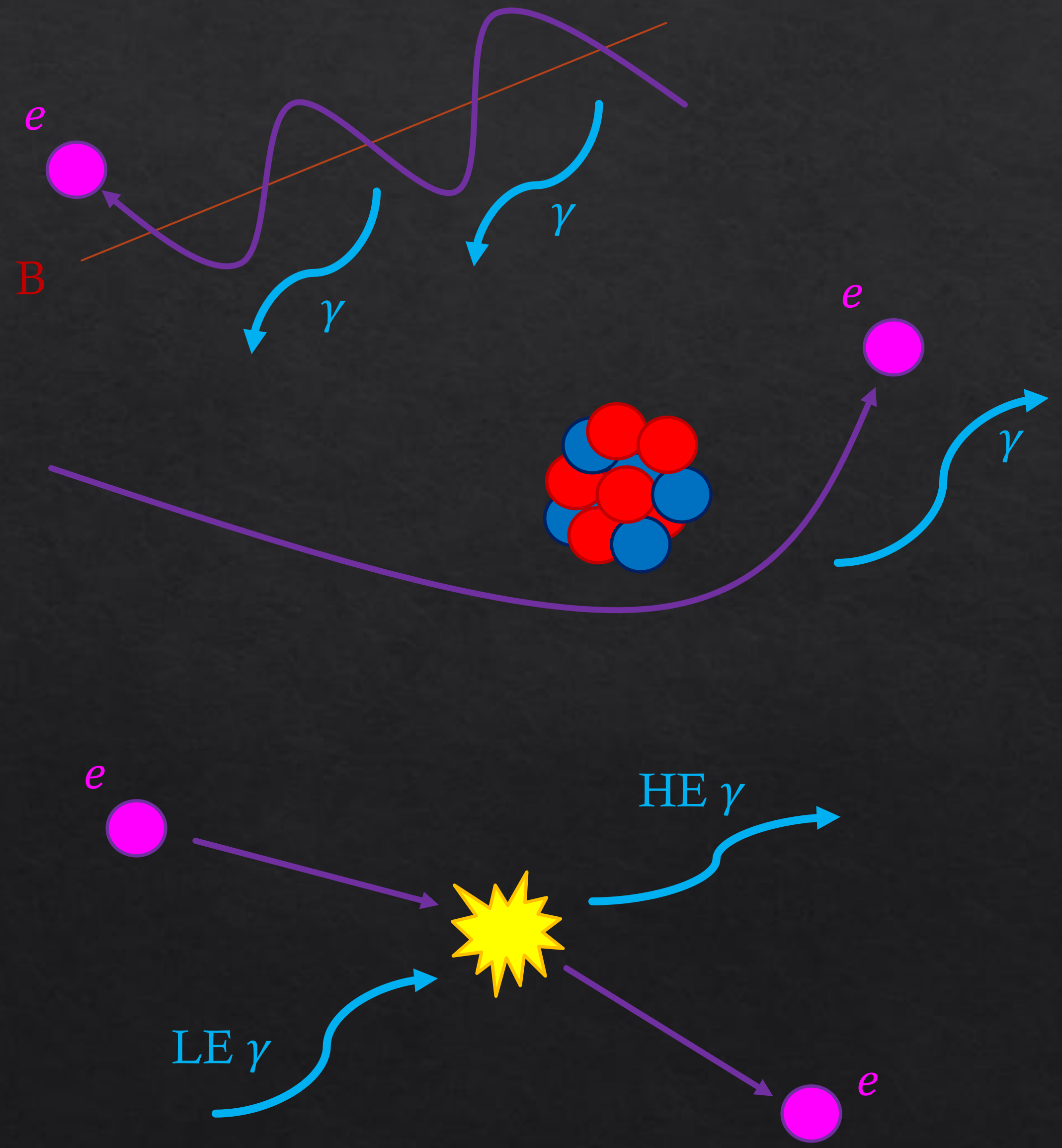

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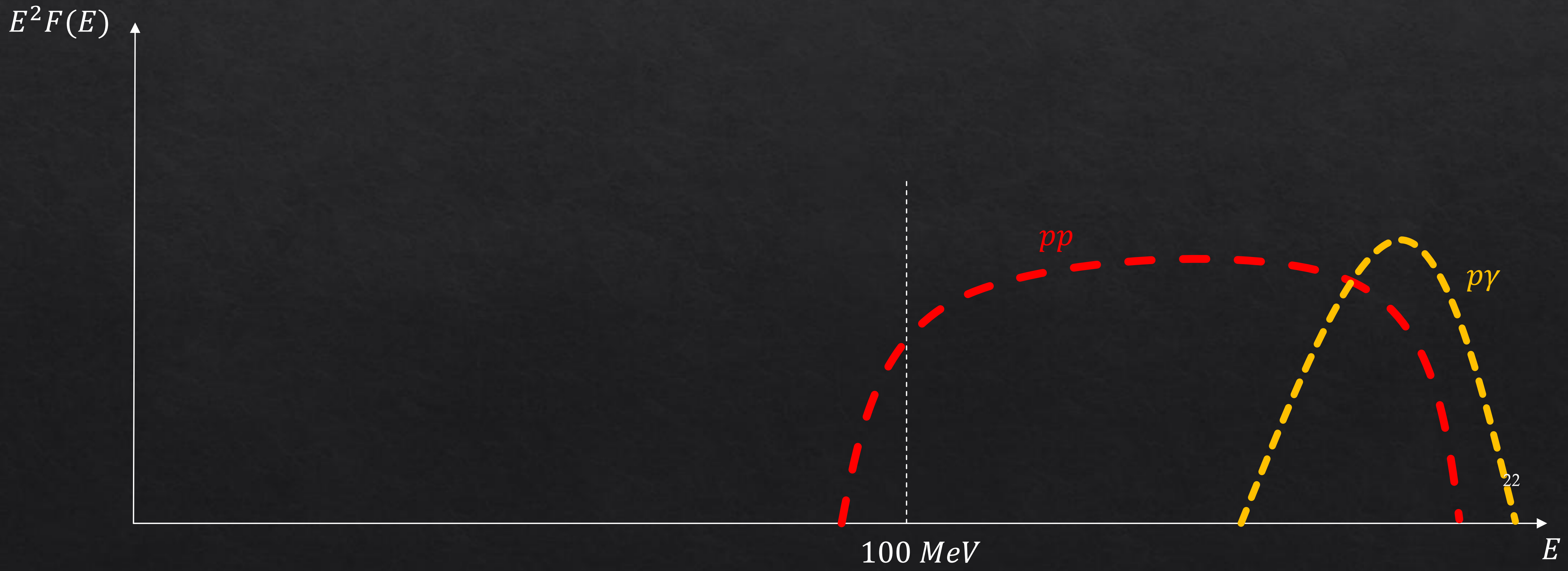
- Bremsstrahlung: high-energy electrons emit photons when deflected by the electric field of a nucleus (hard X-ray to soft gamma rays)
- Inverse-Compton: high energy electrons upscatter low energy radiation (gamma-ray domain)

Non-thermal radiation – multiwavelength



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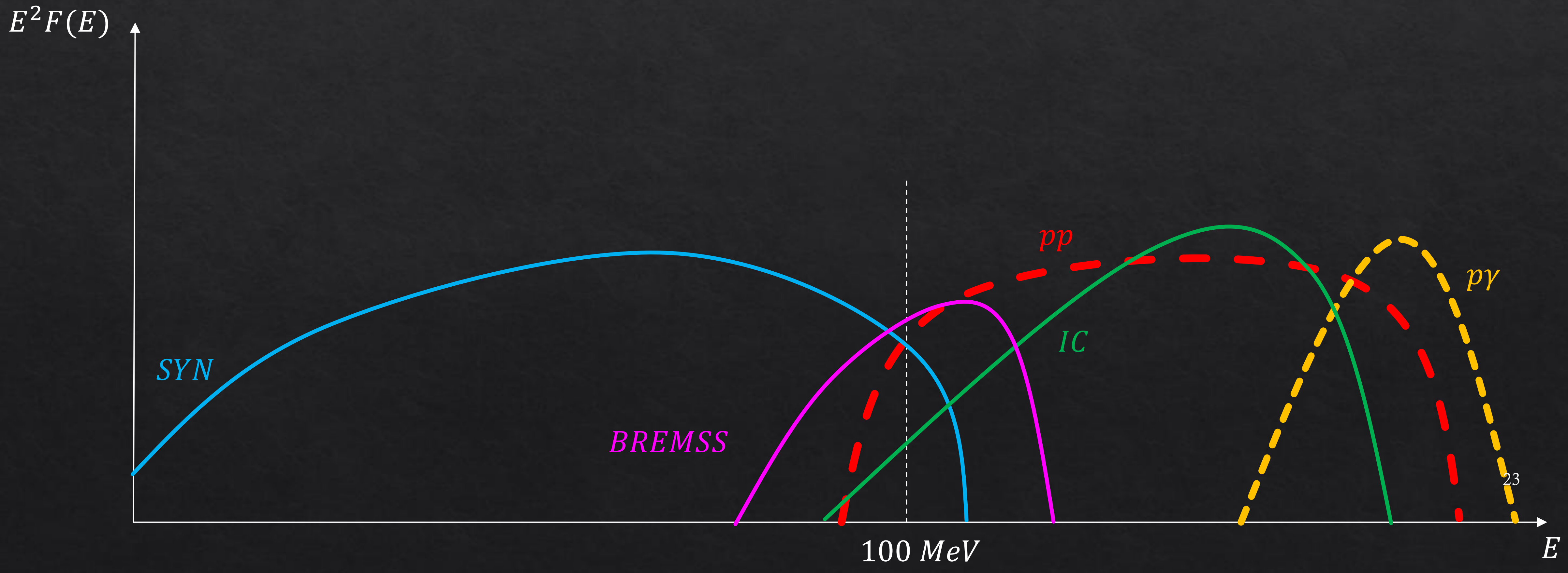
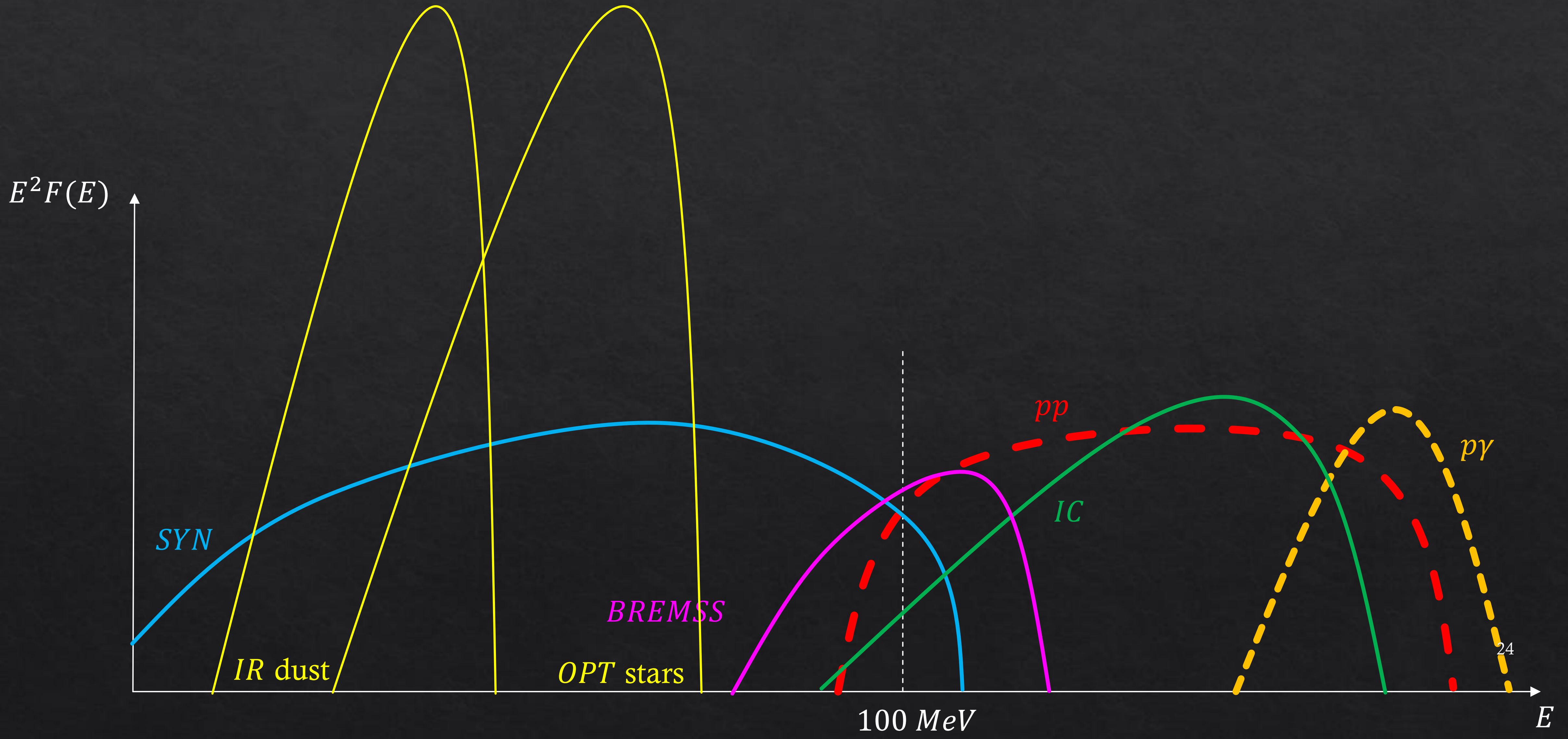
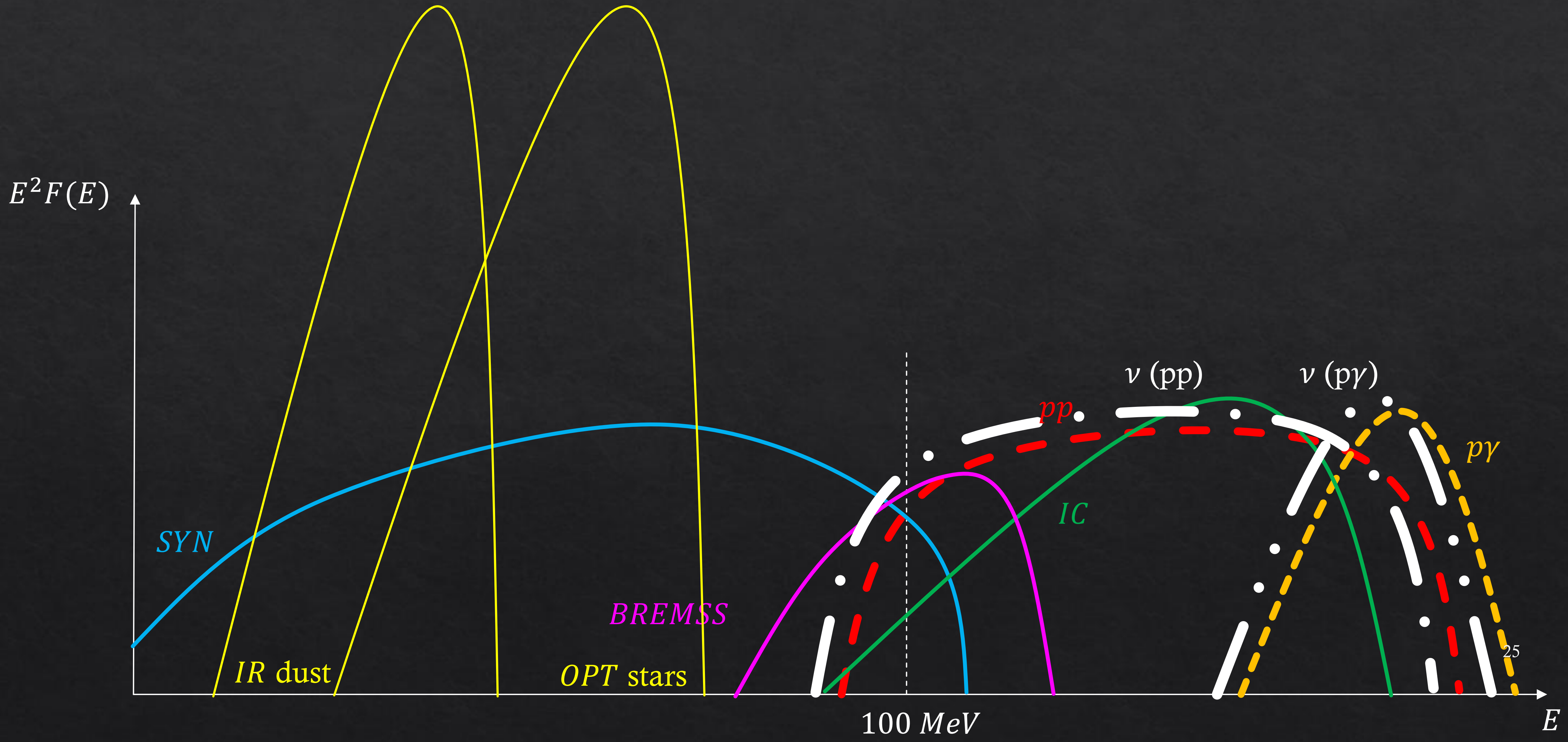


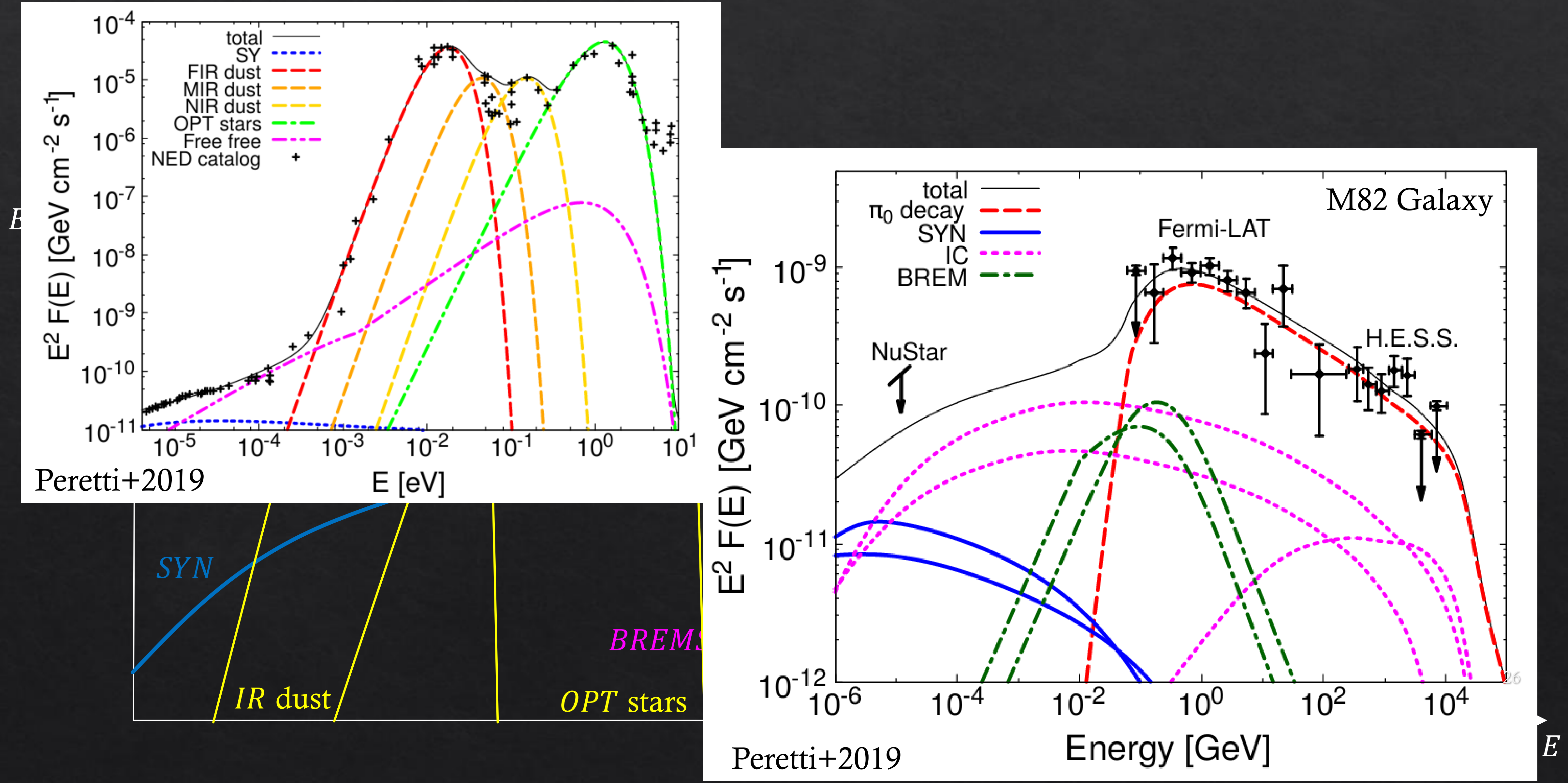
Non-th+th radiation – multiwavelength



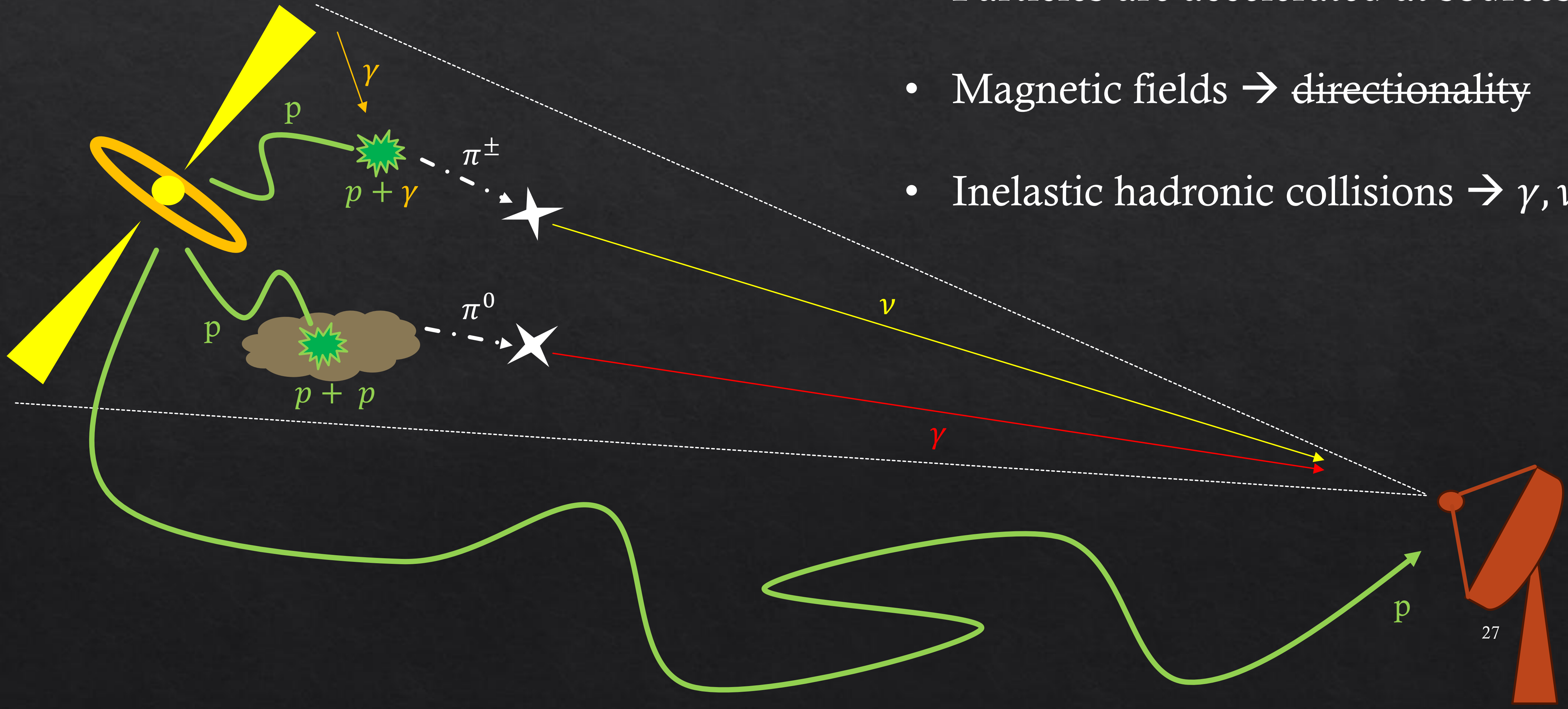
Multi-messenger – photons and neutrinos



Example: Starburst galaxy M82



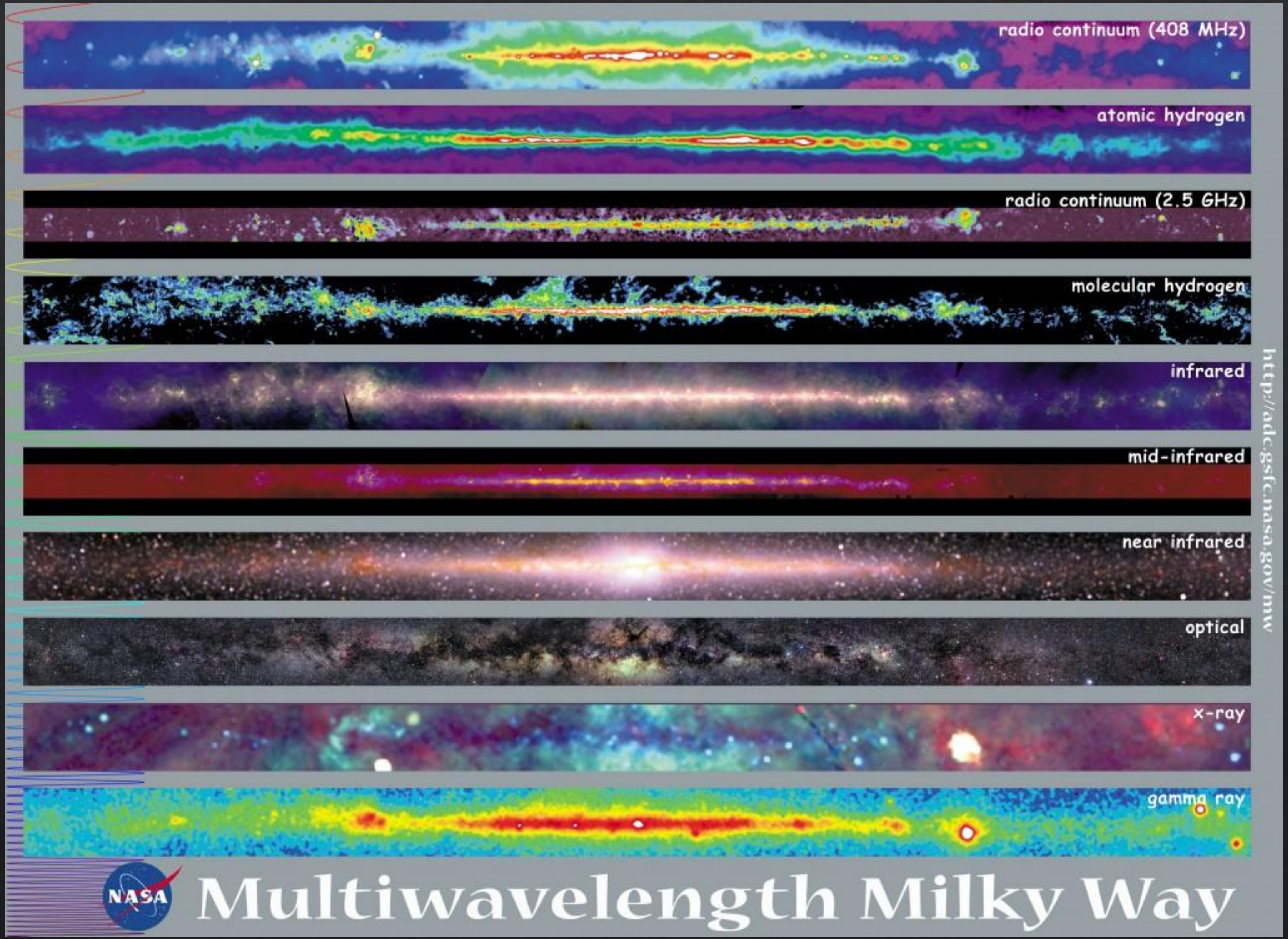
Multi-messenger astronomy: the basic idea



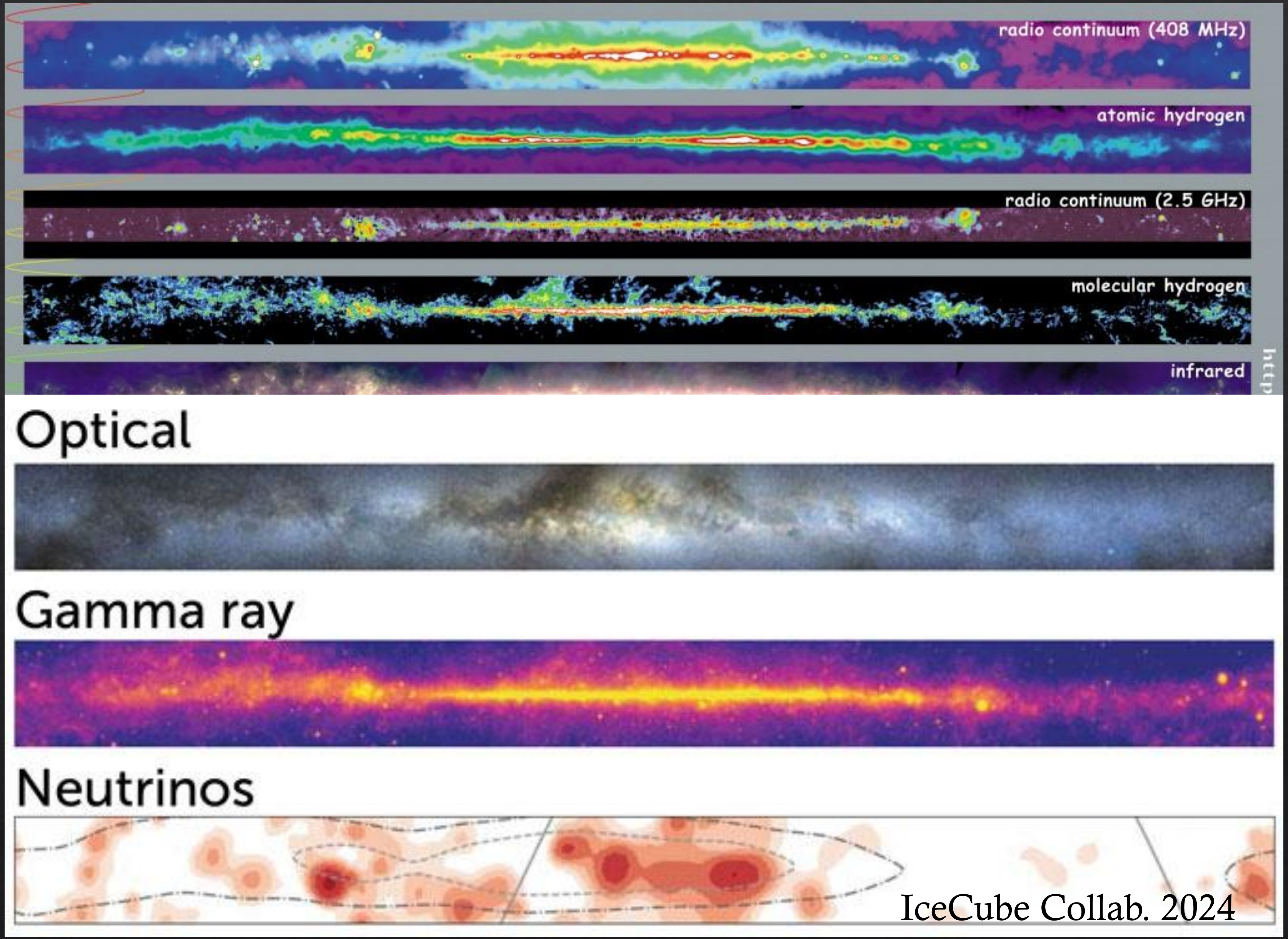
- Particles are accelerated at sources
- Magnetic fields → ~~directionality~~
- Inelastic hadronic collisions → γ, ν

Multi-messenger Galaxy

Galactic multiwavelength emission



Galactic high-energy neutrinos



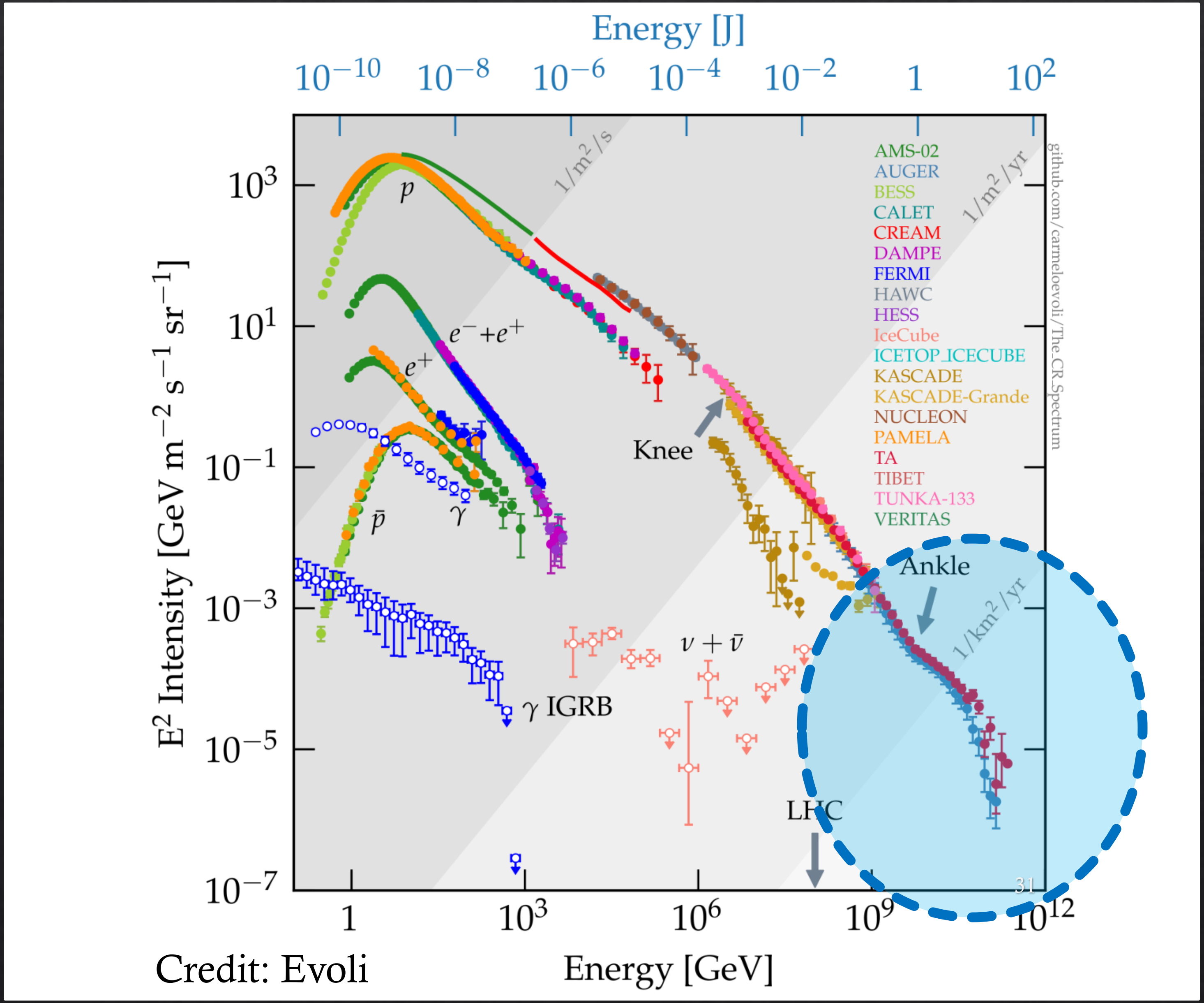
Extragalactic cosmic rays

- Larmor radius of EeV particles comparable with Galactic scales

$$r_L(E) = 1\; E_{EeV}\; B_{\mu G}^{-1} kpc$$

- No evidence of anisotropy along the Galactic plane

- Lack of observation of multi-PeV accelerators



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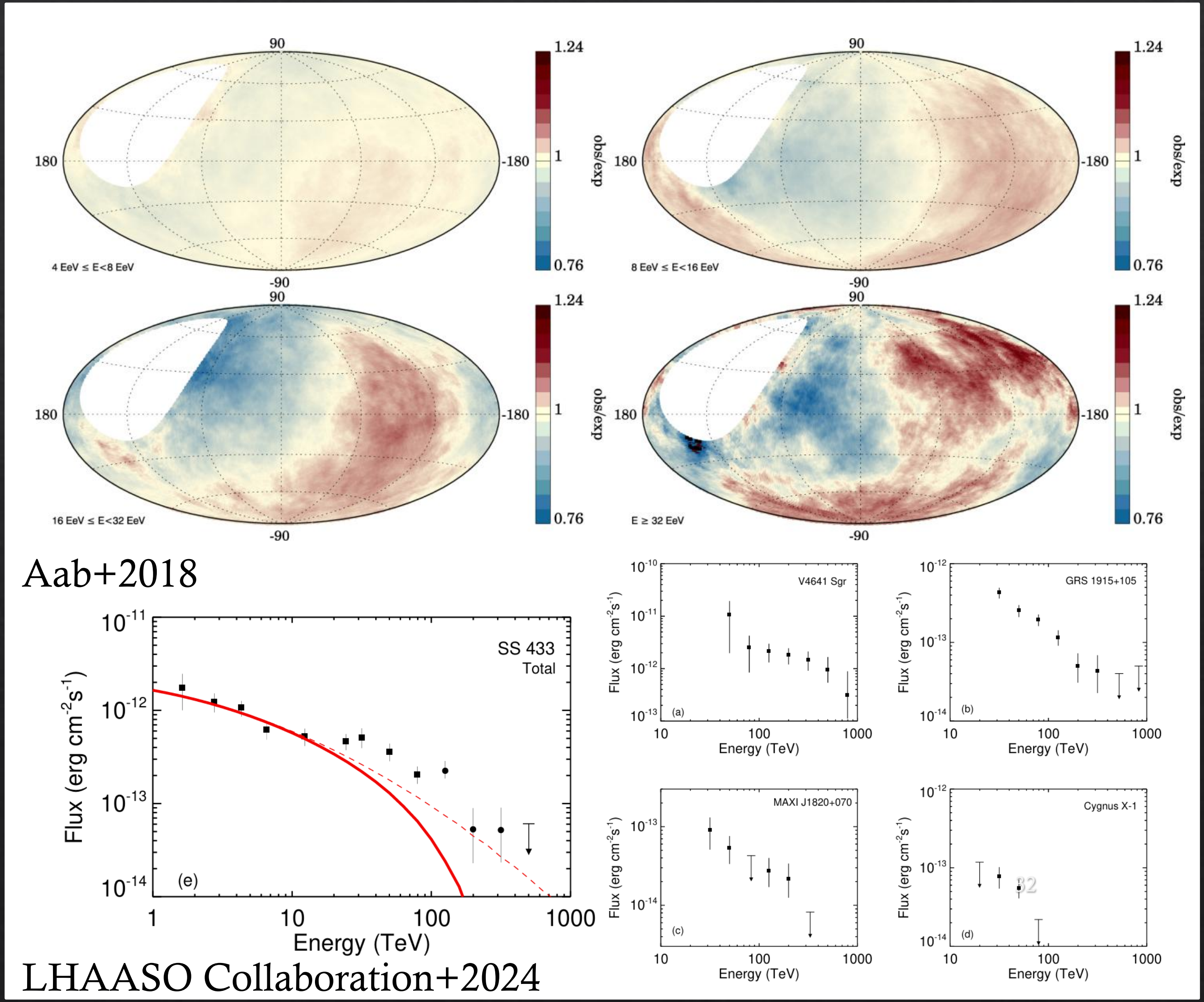

Outline

- Fundamentals of paricle transport in astrophysical plasma
- Particle acceleration (diffusive shock acceleration)
- Studying and modeling cosmic sources

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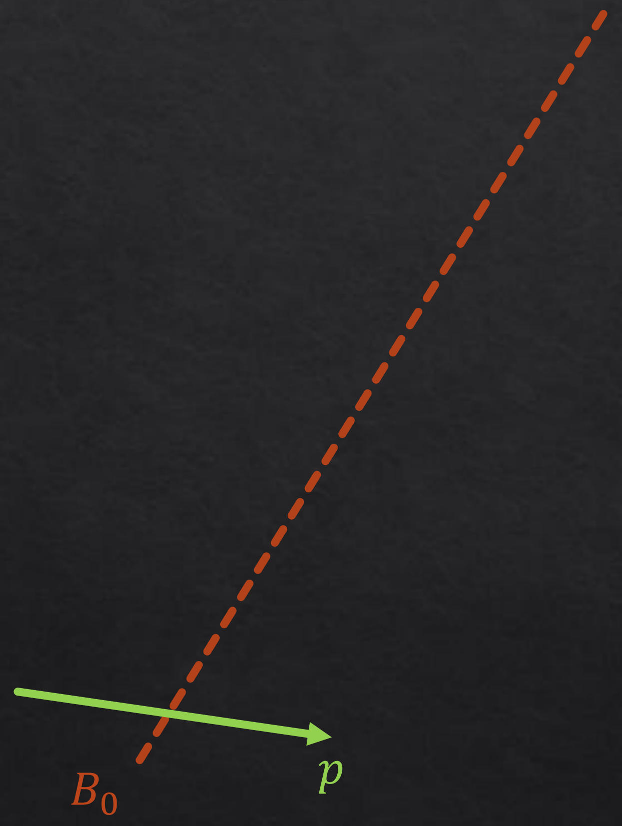
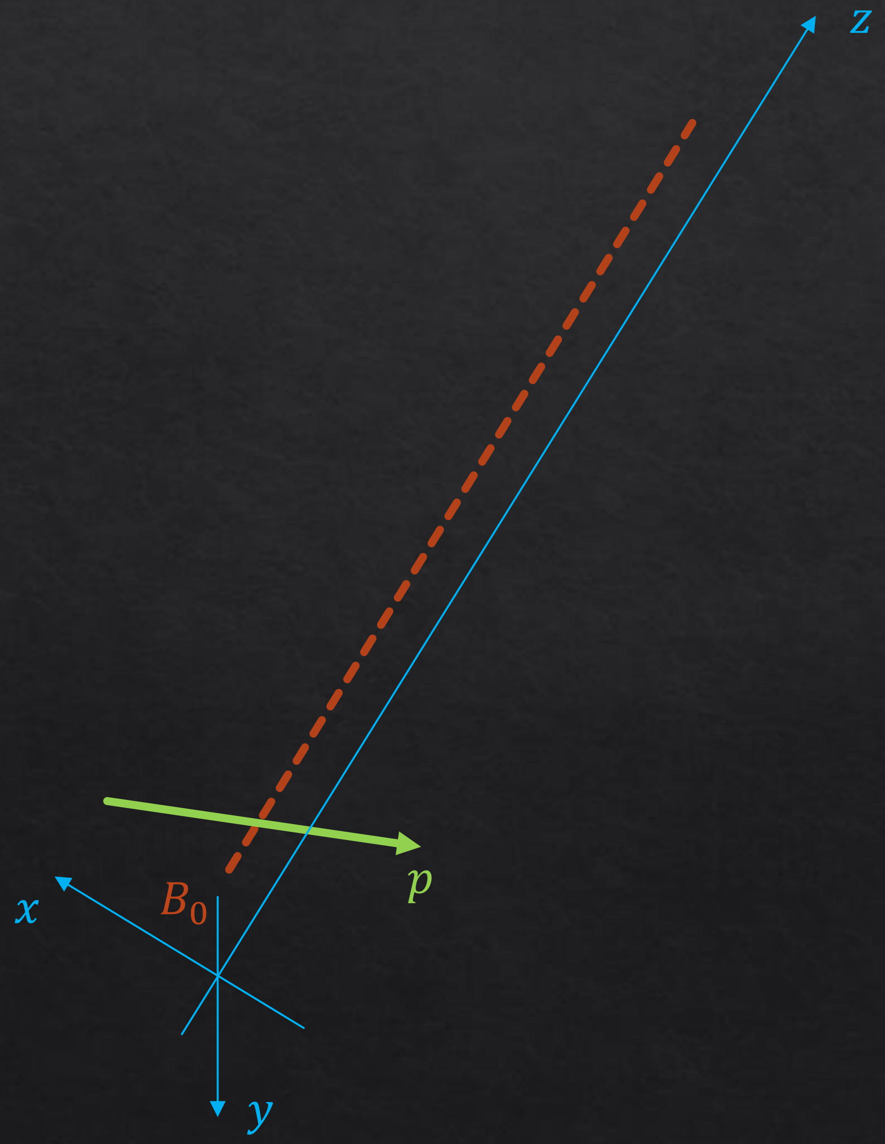
Charged particles in magnetic fields - 1

- A charged particle in presence of a magnetic field experiences the Lorentz force



$$\frac{d\bar{p}}{dt} = q\,\frac{\bar{v}}{c} \times \bar{B}_0$$

Charged particles in magnetic fields - 1



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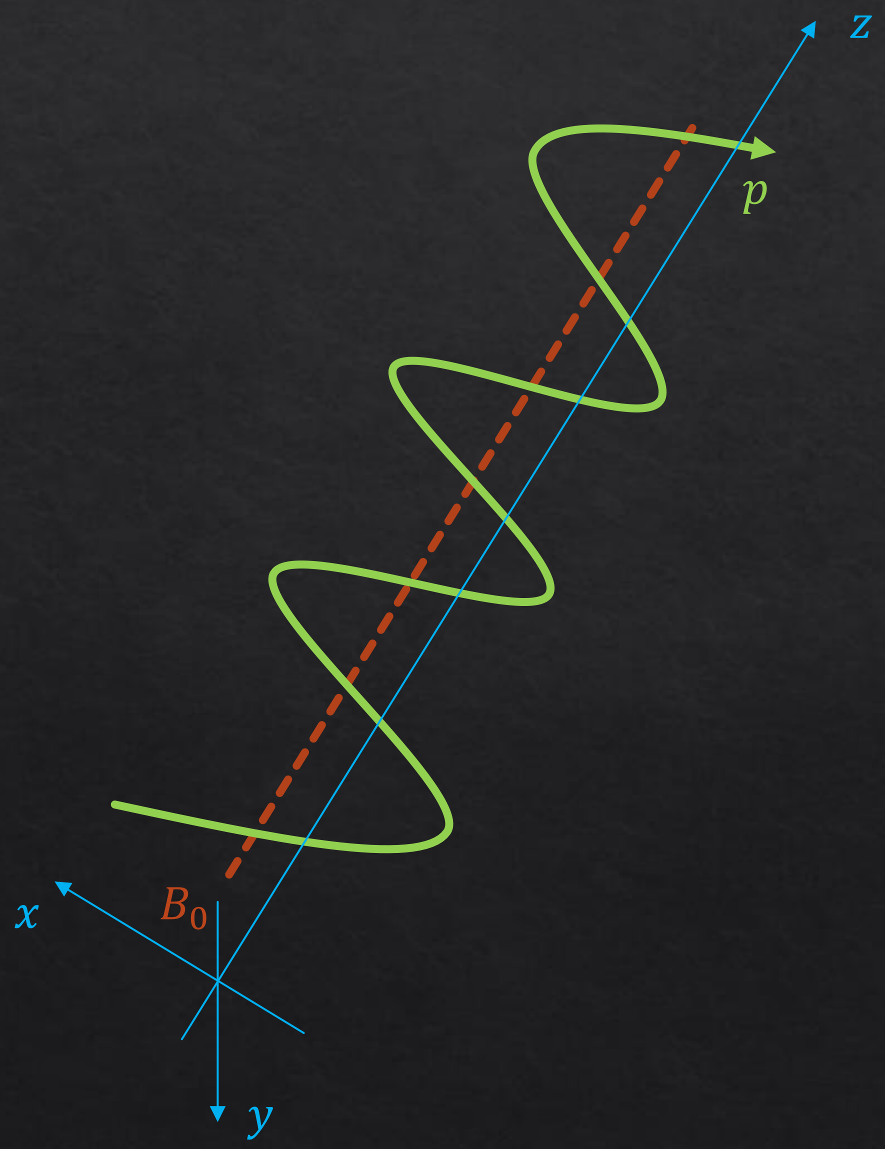
- Projecting the motion on the axis

$$\frac{dv_x}{dt} = \frac{qB_0}{m\gamma c}\, v\, \sin\vartheta = \frac{qB_0}{m\gamma c} v_y$$

$$\dot{v}_z = 0$$

Charged particles in magnetic fields - 1



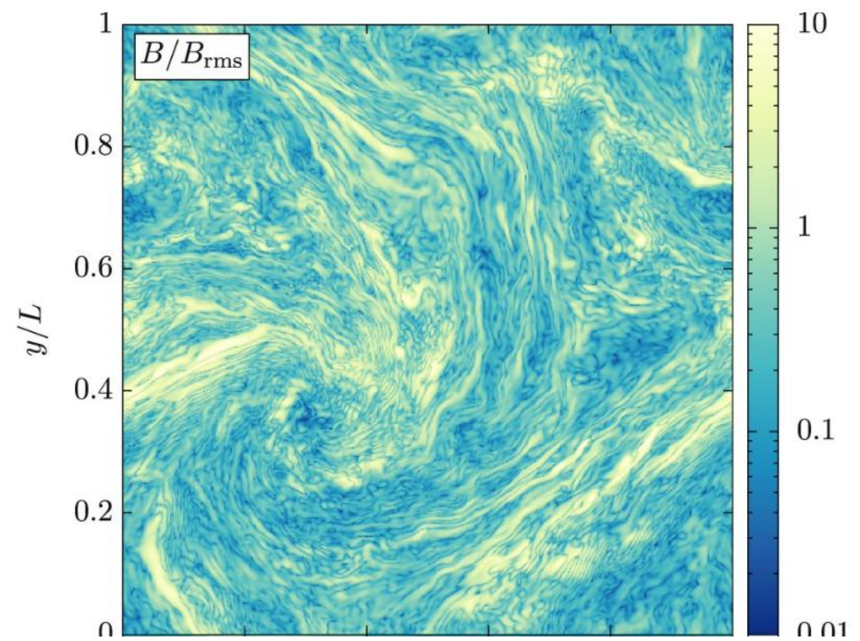
- The equations of motion describe an helics

$$\begin{cases} v_x(t) = \ |\bar{v}|\,(1-\mu^2)^{1/2}\cos(\omega t) \\ v_y(t) = -|\bar{v}|\,(1-\mu^2)^{1/2}\sin(\omega t) \\ v_z(t) = \ |\bar{v}|\,\mu \end{cases}$$

- $\mu = \cos\theta$ and $\omega = \frac{qB_0}{m\gamma c}$

- μ and $|\bar{v}|$ are constant with time

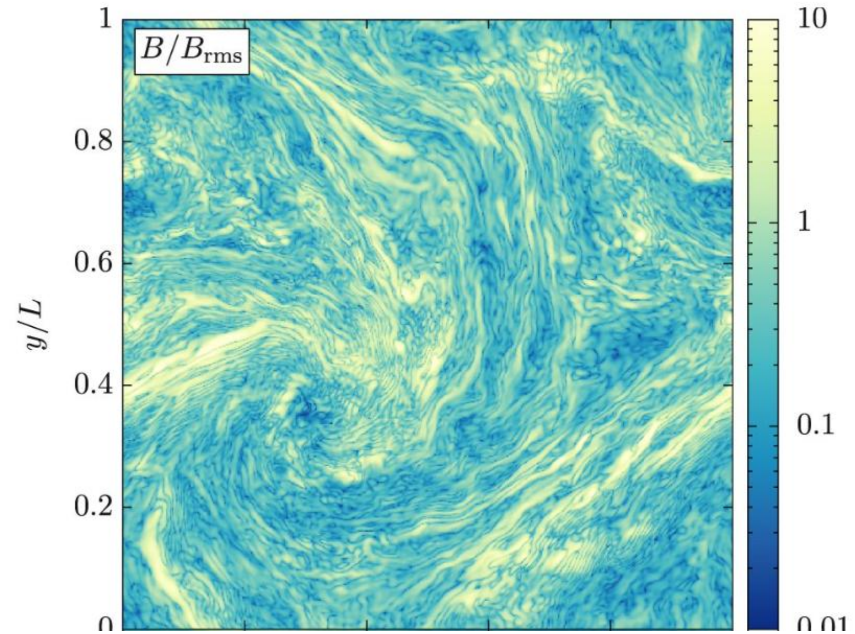
Astrophysical plasma, turbulence and Alfven waves



Credit: plasma.princeton.edu

- Regular magnetic fields are not typical of realistic astrophysical plasma

Astrophysical plasma, turbulence and Alfven waves



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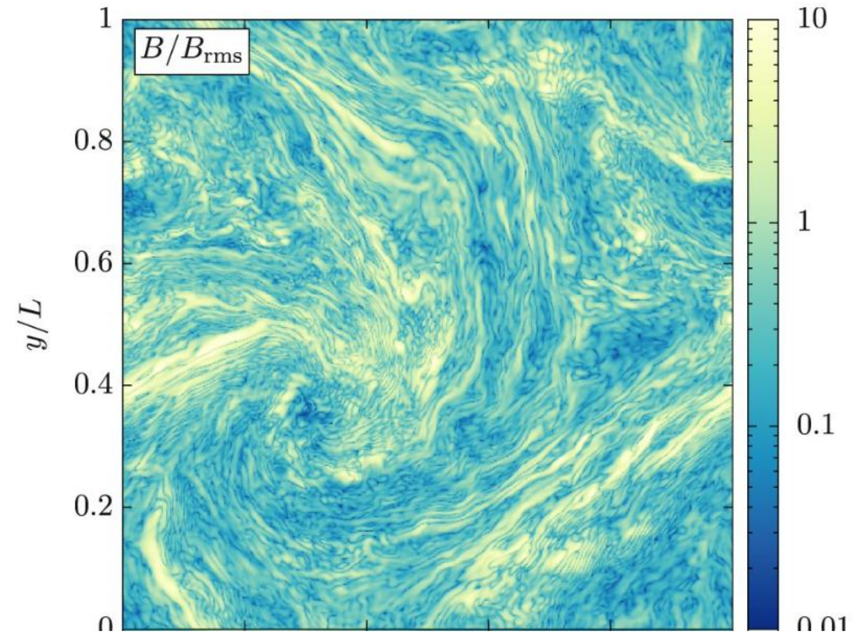
- Turbulence is present in fluids and plasma

Astrophysical plasma, turbulence and Alfven waves



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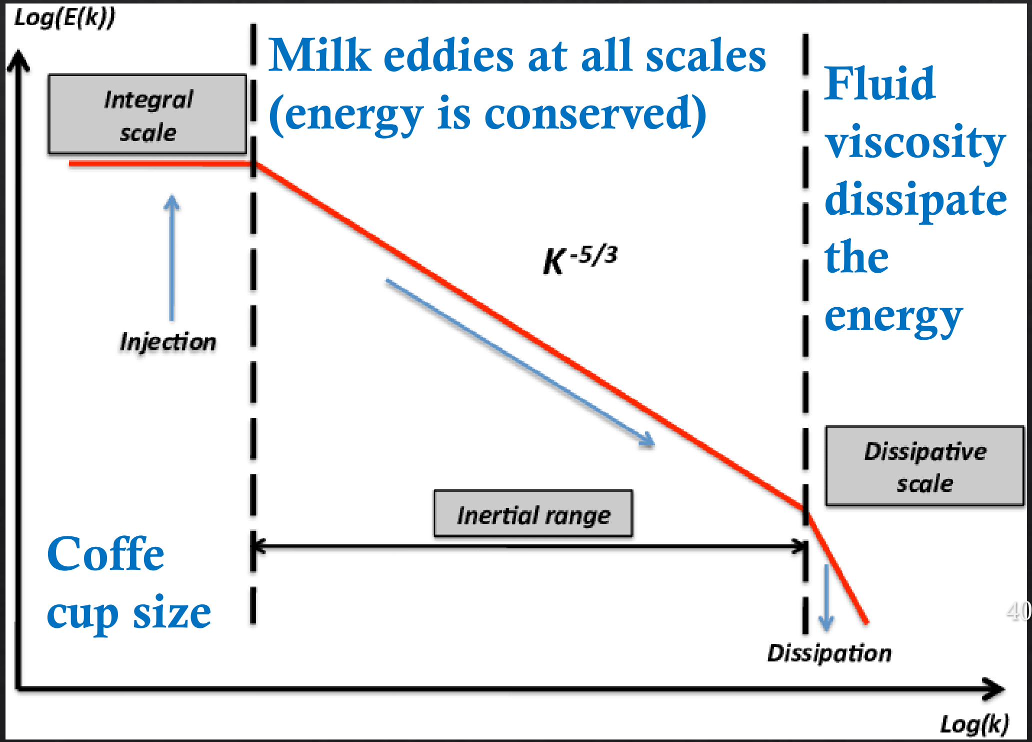
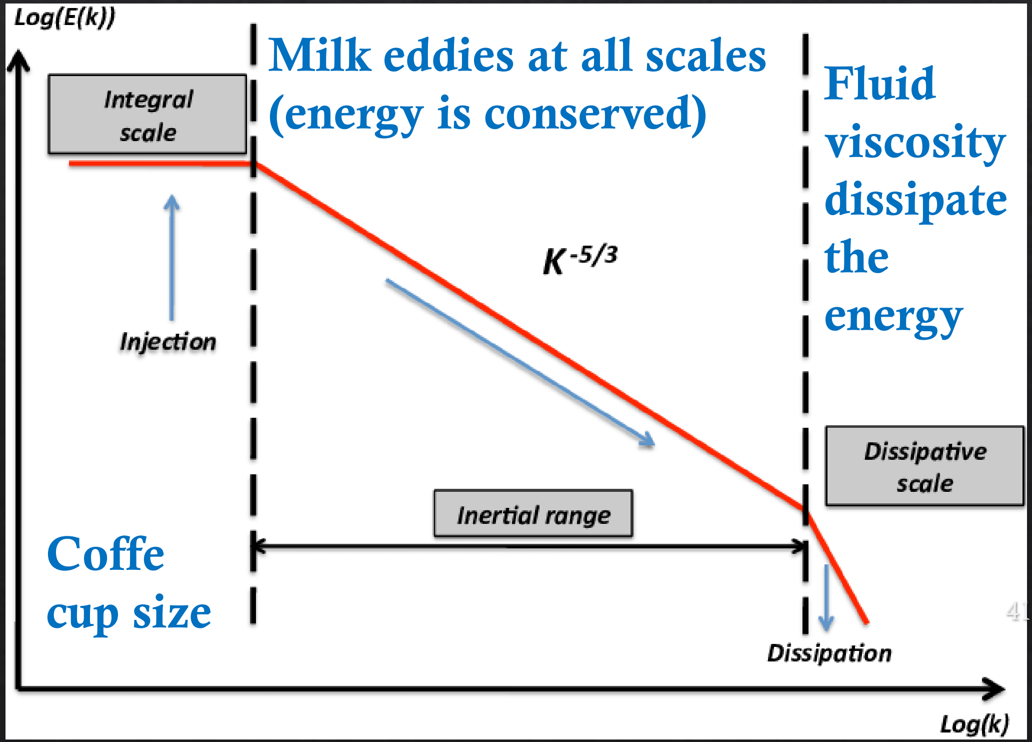


Astrophysical plasma, turbulence and Alfven waves

Alfven waves in MHD plasma

- MHD cascade is characterized by Alfven waves which transfer the energy at smaller scales
- Alfven waves travel along B_0
- $v_A = \frac{B_0}{\sqrt{4\pi\rho}}$

- Regular magnetic fields are not typical of realistic astrophysical plasma
- Turbulence is present in fluids and plasma



Charged particles in magnetic fields - 2

- Let's now take our regular field B_0 and perturb it with a single Alfven wave of amplitude δB

$$\frac{d\bar{p}}{dt} = q\,\frac{\bar{v}}{c} \times \left(\bar{B}_0 + \overline{\delta B}\right) = \frac{q}{c}\begin{vmatrix} i & j & k \\ v_x & v_y & v_z \\ \delta B & \delta B & B_0 \end{vmatrix}$$

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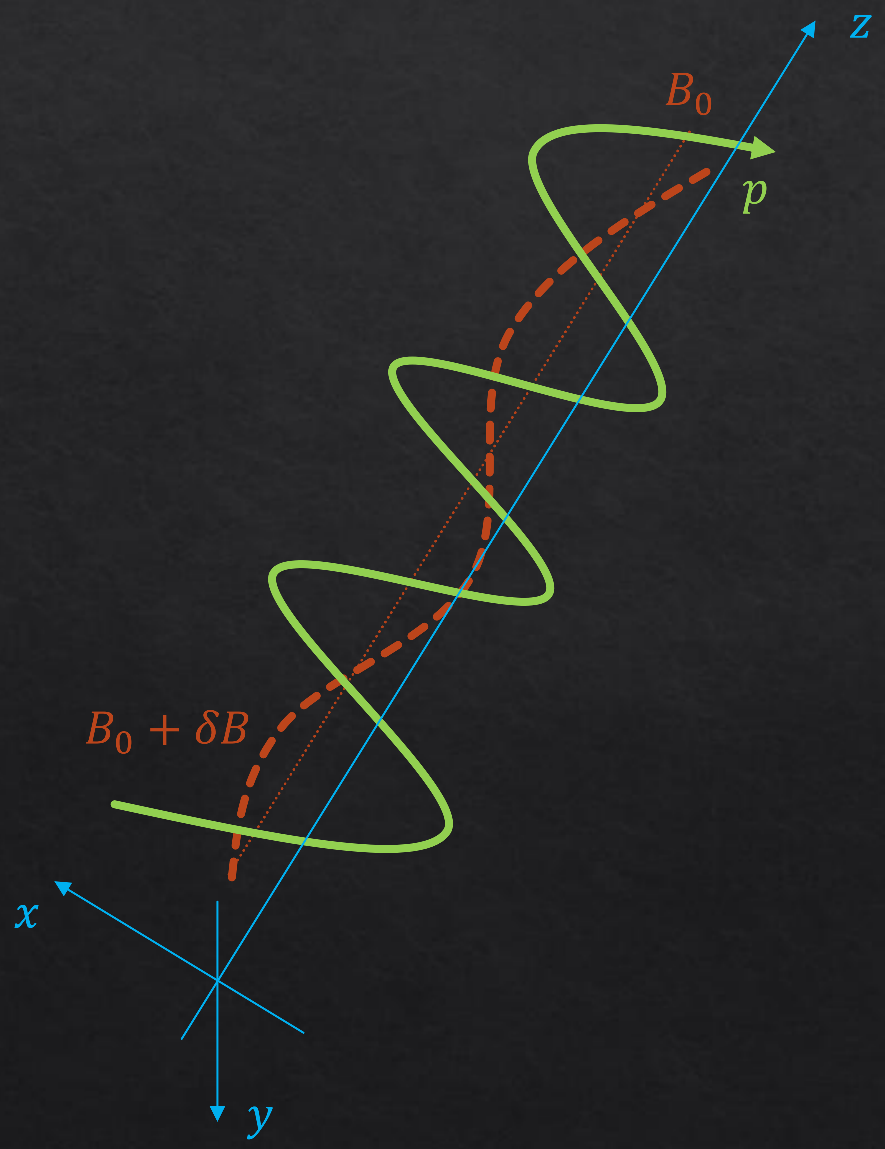

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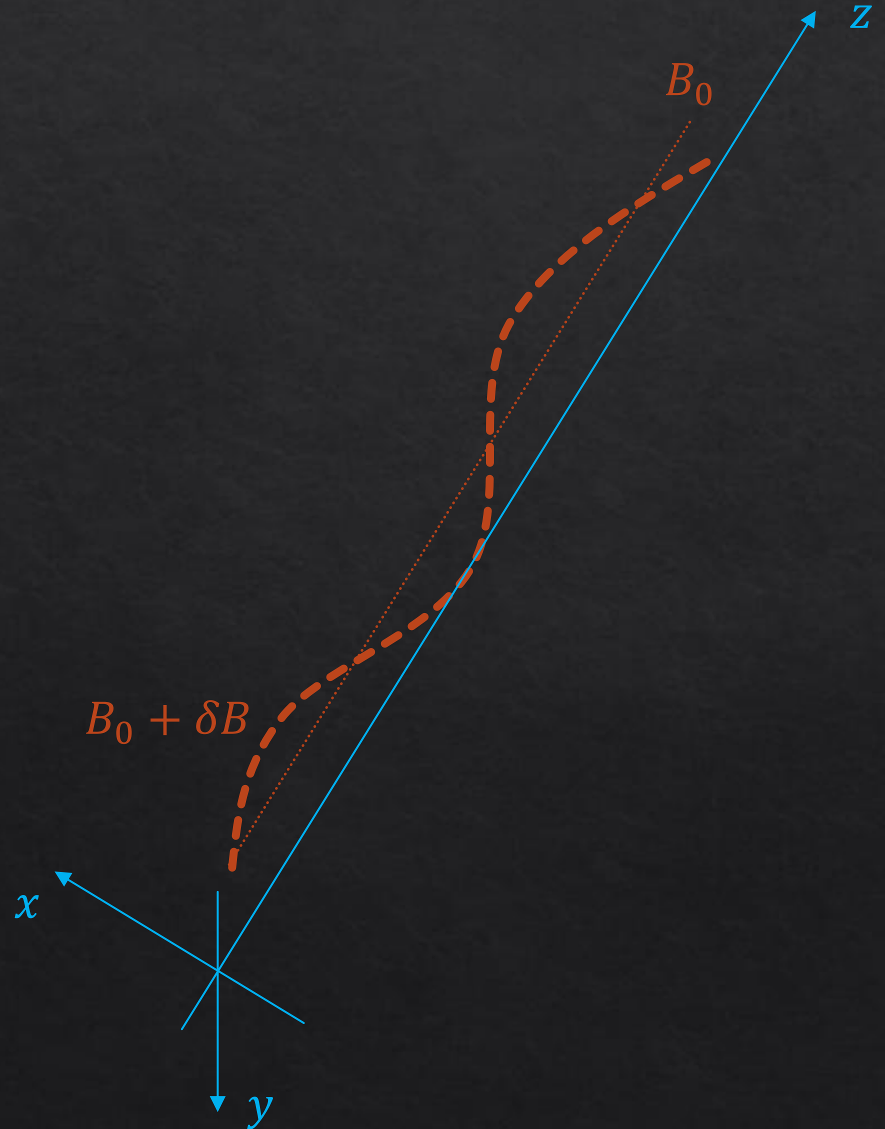
- The motion along x and y is not heavily perturbed

$$\frac{dv_x}{dt} = \frac{qB_0}{m\gamma c} v_y - \frac{qB_0}{m\gamma c}\, v_z \left(\frac{\delta B}{B_0}\right)$$

Charged particles in magnetic fields - 2



- The motion along z has interesting features

$$\frac{dv_z}{dt} = \frac{qB_0}{m\gamma c}\left[v_x\left(\frac{\delta B_y}{B_0}\right) - v_y\left(\frac{\delta B_x}{B_0}\right)\right]$$

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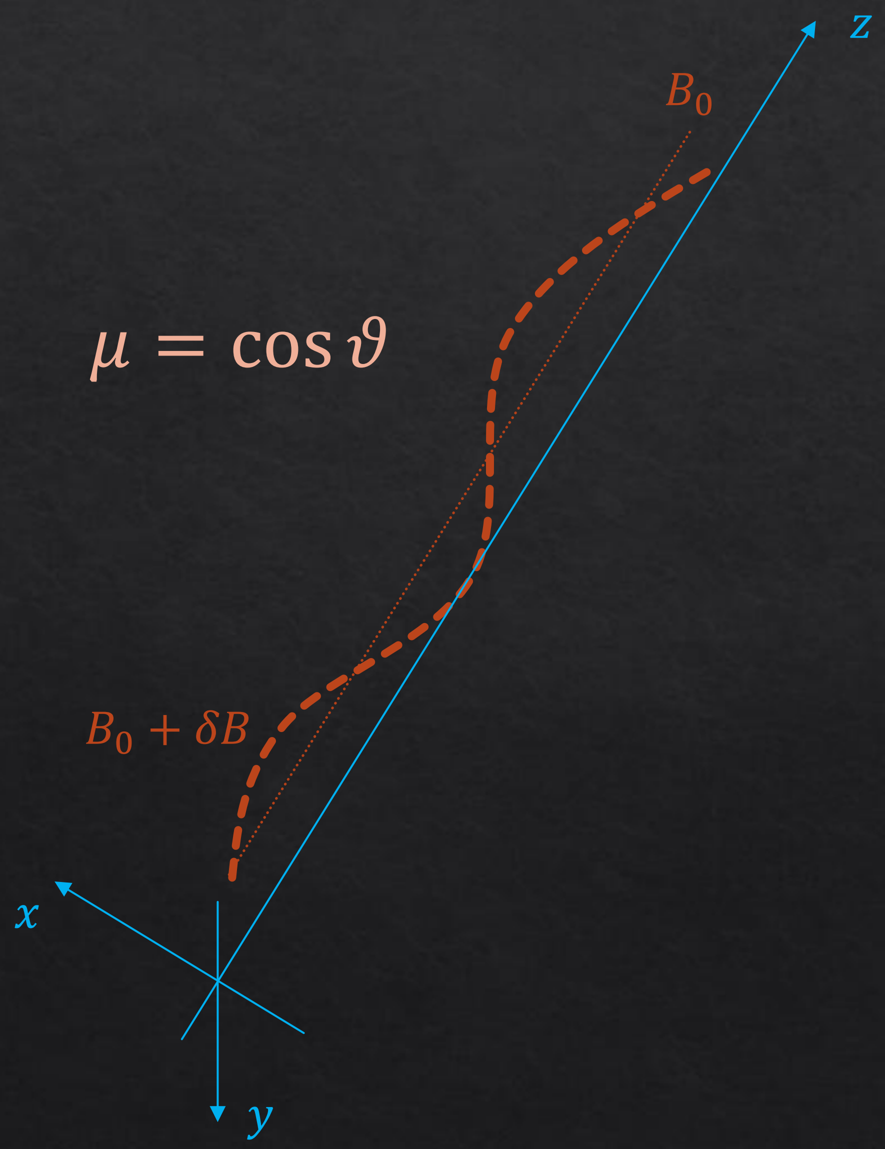

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- Recalling that $v_z = |\bar{v}|\,\mu$ the above eq. gives

$$\frac{d\mu}{dt} = \omega\left(\frac{\delta B}{B_0}\right)\sqrt{1-\mu^2}\cos(\omega t - kz + \varphi)$$

Charged particles in magnetic fields - 2

Pitch angle time average

$$\left\langle \frac{\Delta\mu}{\Delta t} \right\rangle = \frac{1}{\Delta t} \int_0^{\Delta t} dt \left(\frac{d\mu}{dt} \right) = 0$$

- The time average of the pitch angle variation is zero

- The motion along z has interesting features

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- **Similar to the unperturbed case?**

- The motion along z has interesting features

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Charged particles in magnetic fields - 2

Pitch angle time average 2

$$\left\langle \frac{\Delta\mu}{\Delta t}\frac{\Delta\mu}{\Delta t} \right\rangle_{\varphi,t} = \frac{1}{2\pi\,\Delta t^2}\int_0^{\Delta t} dt' \int_0^{\Delta t} dt \int_0^{2\pi} d\varphi \;\times$$

$$\times \cos(\omega t - kz + \varphi)\cos(\omega t' - kz + \varphi)$$

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$$\times \cos(\omega t - kz + \varphi)\cos(\omega t' - kz + \varphi)$$

- We can recognize a delta function:

$$\langle \Delta\mu\Delta\mu \rangle_{\varphi,t} = \omega^2 \left(\frac{\delta B}{B_0}\right)^2 (1-\mu^2)\, 2\pi\, \Delta t\, \delta[\omega - |\bar{v}| k\mu]$$

- The motion along z has interesting features

$$\frac{dv_z}{dt} = \frac{qB_0}{m\gamma c}\left[v_x\left(\frac{\delta B_y}{B_0}\right) - v_y\left(\frac{\delta B_x}{B_0}\right)\right]$$

- Recalling that $v_z = |\bar{v}|\,\mu$ the above eq. gives

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Charged particles in magnetic fields - 2

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$$\boldsymbol{k^{-1} = |\bar{v}|\, \mu\, \omega^{-1} = r_L !!!}$$

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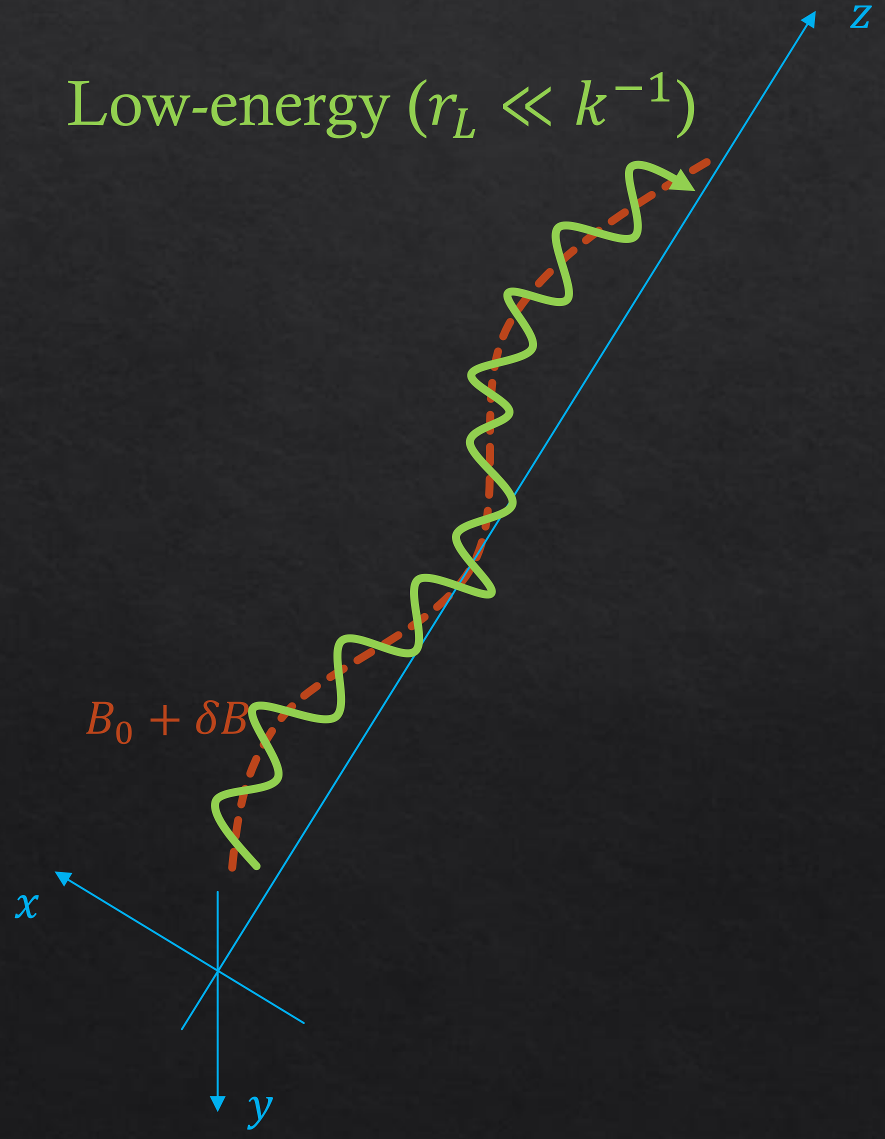
Diffusion coefficient

$$D_{\mu\mu} \equiv \frac{1}{2} \left\langle \frac{\Delta\mu\,\Delta\mu}{\Delta t} \right\rangle$$

$$= \omega \left(\frac{\delta B}{B_0}\right)^2 (1-\mu^2)\, \pi k_{res} \delta[k \pm k_{res}]$$

- We considered a single Alfven wave of wavenumber k
  - $k_{res} = r_L^{-1}$

Diffusion in pitch angle



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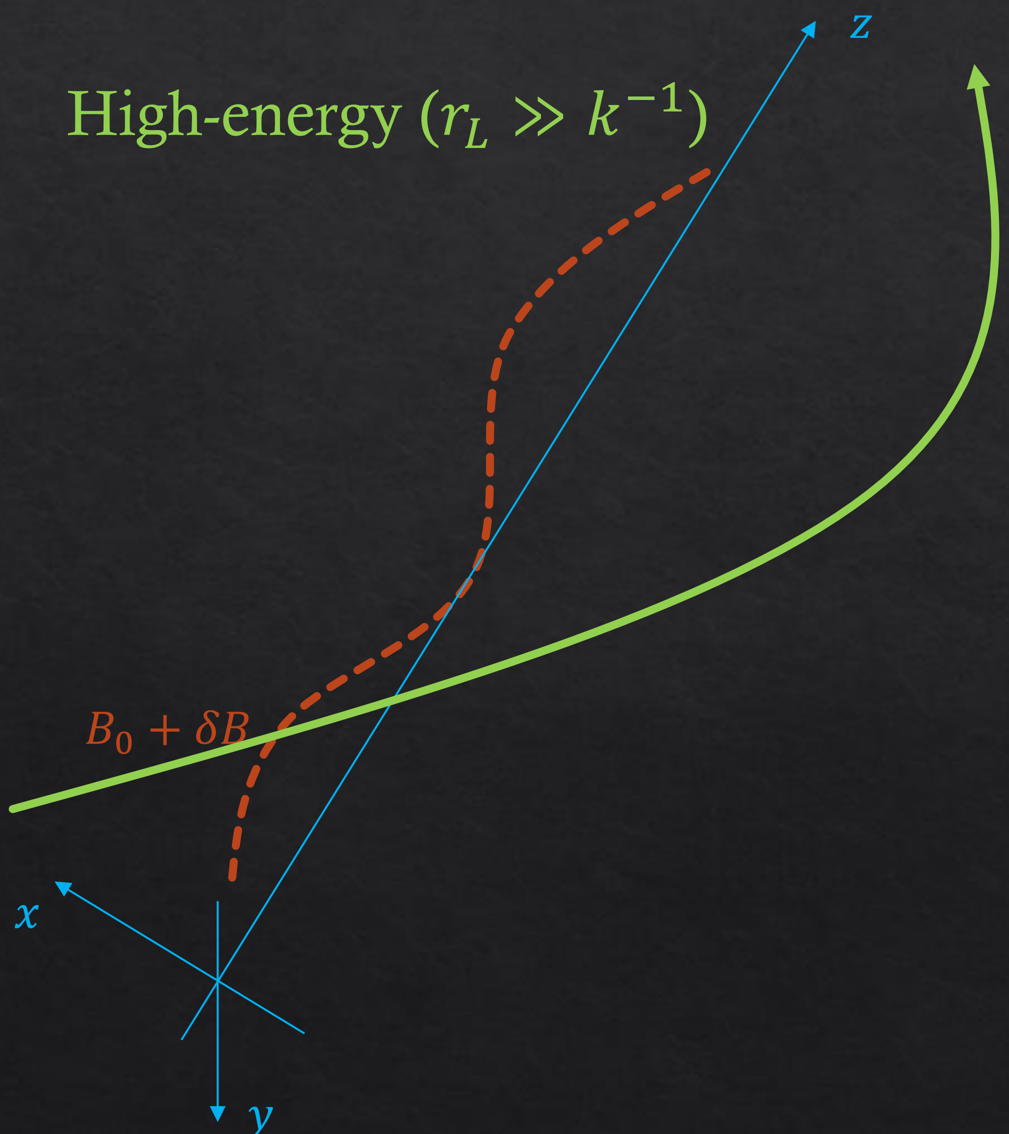

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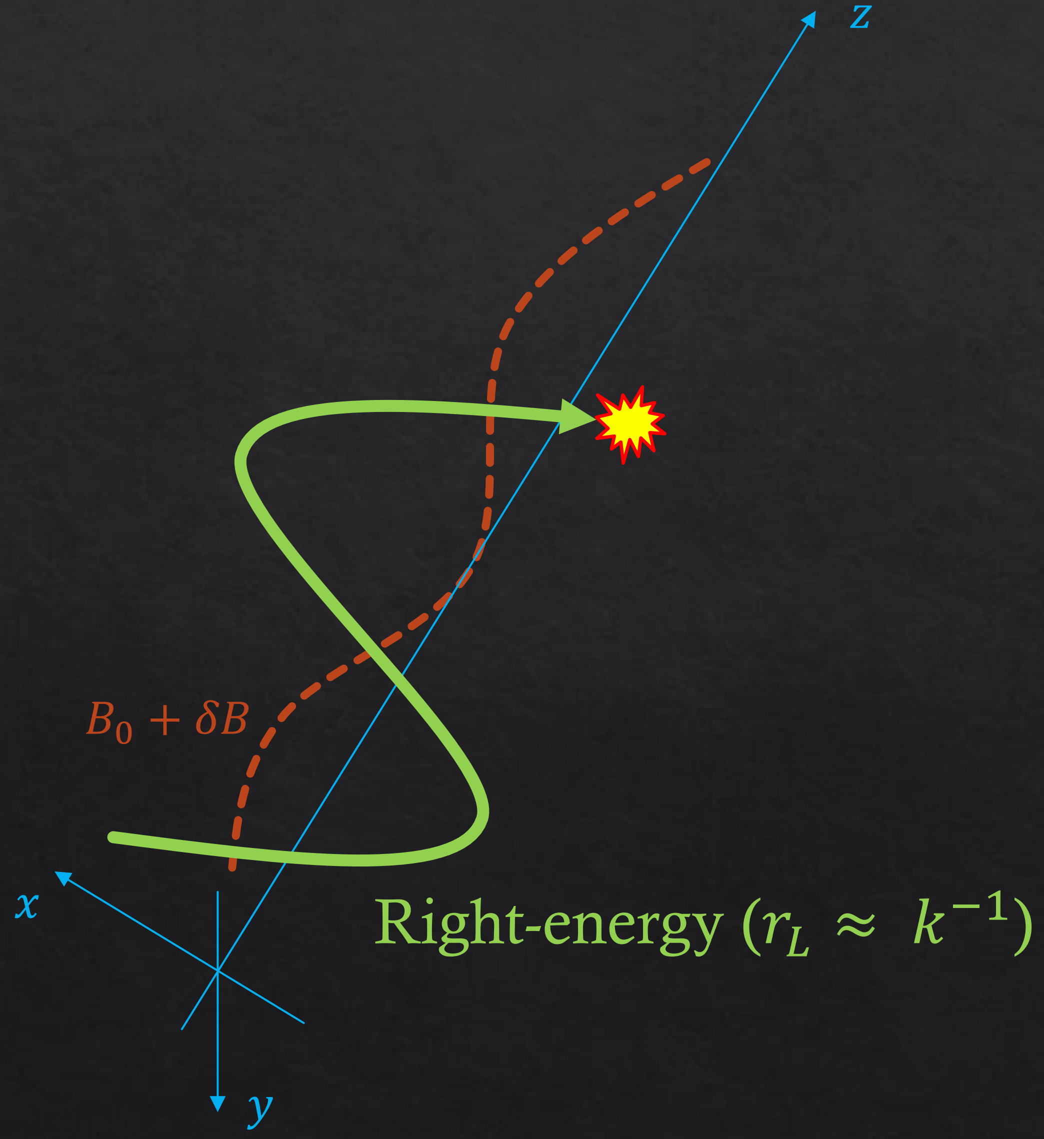

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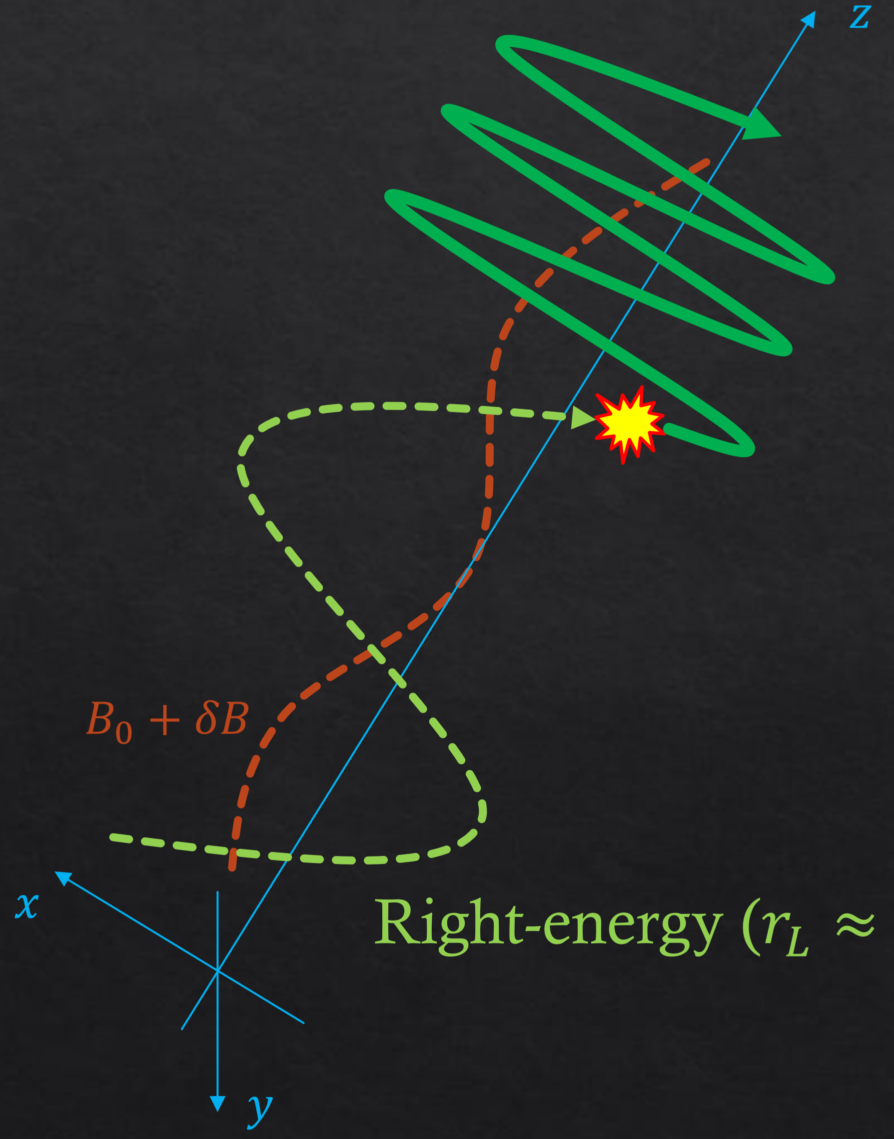

Right-energy ($r_L \approx k^{-1}$)

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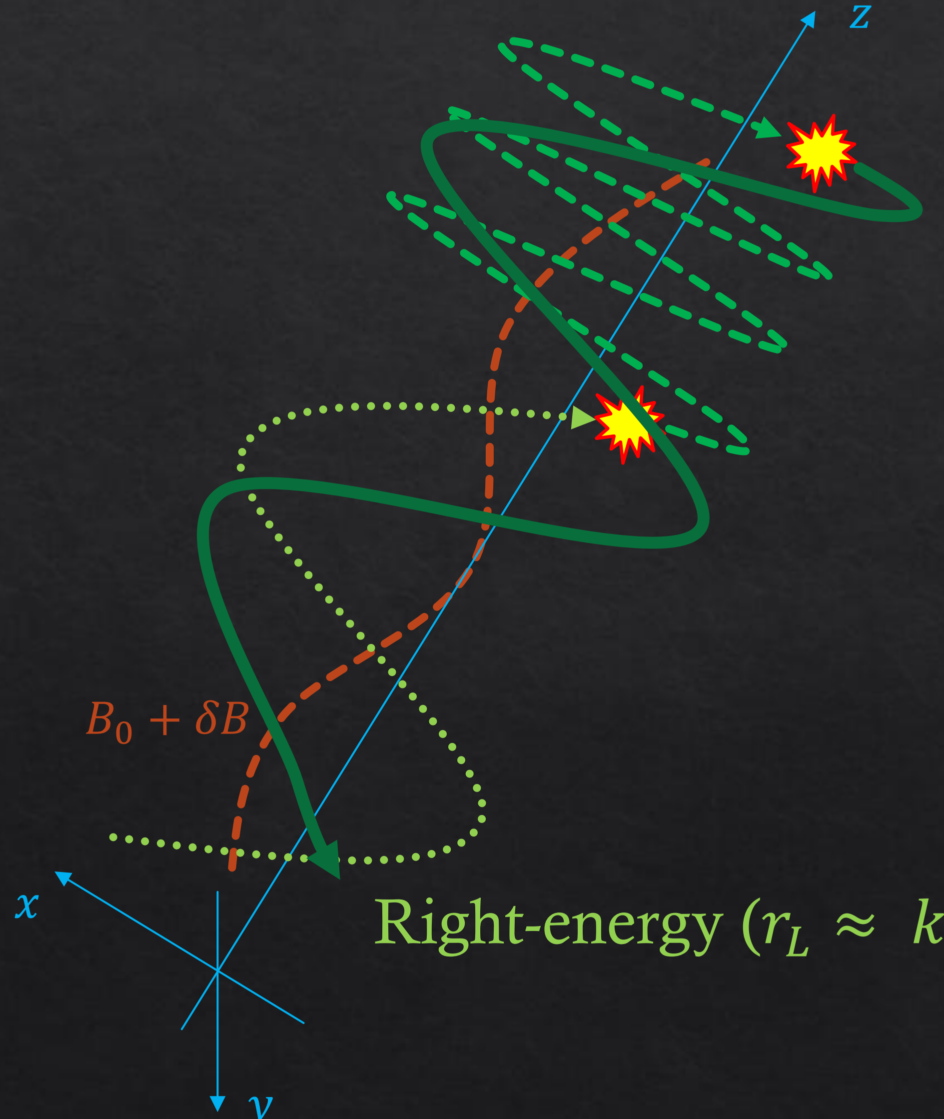

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Cosmic-ray diffusion - 1

From one k to P(k)

- It is possible to consider an entire power spectrum of Alfven waves

$$\left[{}^{\delta B(k)}/_{B}\right]^2 = P(k)\,{}^{dk}/_{k}$$

$$D_{\mu\mu} = \omega\,\pi\,P(k_{res})(1-\mu^2)$$

From μ to ϑ

- I can get rid of $\mu = \cos\vartheta$

$$D_{\vartheta\vartheta} = \frac{1}{2}\left\langle\frac{\Delta\vartheta^2}{\Delta t}\right\rangle = \pi\omega\,P(k_{res})$$

Cosmic-ray diffusion - 2

From μ to ϑ

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$$D_{\vartheta\vartheta} = \frac{1}{2}\left\langle\frac{\Delta\vartheta^2}{\Delta t}\right\rangle = \pi\omega\, P(k_{res})$$

Pitch angle reversal

- $\Delta\vartheta = 1.57$ rad → particles invert their motion along z
- When $\Delta\vartheta \approx O(1\, rad)$

$$\Delta t_1 \approx [\pi\omega\, P(k_{res})]^{-1}$$

Cosmic-ray diffusion - 3

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Spatial diffusion

- $\lambda = |\bar{v}|\, \Delta t_1$ is the mean free path of particles before reversal

- Spatial diffusion coefficient

$$D_{zz} = \frac{v\lambda}{3} = \frac{1}{3}\,\frac{r_L\, v}{P(k_{res})}$$

Cosmic-ray diffusion - 3

Diffusion scenarios

- Depending on $(\delta B/B_0)$, and the cohrence length we can have different norm of $P(k)$
- $P(k)$ can have different slopes, δ, in the inertial range (typical $\delta = \frac{5}{3}; \frac{3}{2}$)
- $P(k) = 1$ is the most extreme scenario and it is known as Bohm diffusion

Spatial diffusion

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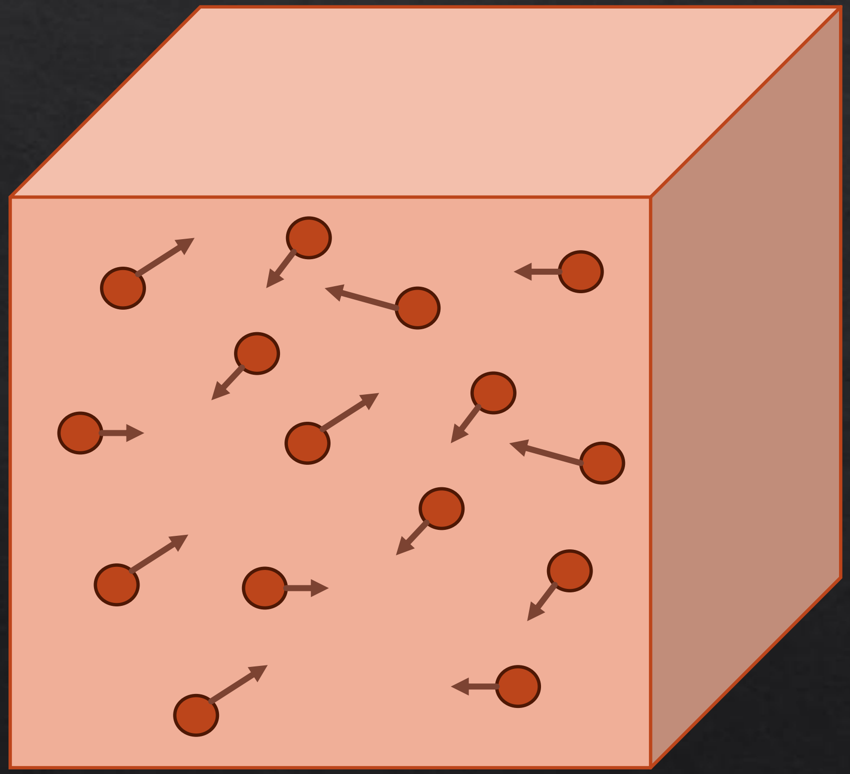
$$D_{zz} = \frac{v\lambda}{3} = \frac{1}{3}\frac{r_L\, v}{P(k_{res})}$$

Transport of cosmic rays



- We rely on statistical physics and describe cosmic rays with a distribution function $f(z, p, t)$
- $f(z, p, t) = \; dN/dV \; dp^3 \; dt$
- Cosmic rays scatter and diffuse on magnetic field waves

Cosmic-ray transport equation

- The transport equation describes the evolution of the distribution function

$$\frac{\partial f}{\partial t} = DIFFUSION + ADVECTION + ADIABATIC\ LOSSES + INJECTION + LOSSES$$

Cosmic-ray transport equation

- The transport equation describes the evolution of the distribution function

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] - u\frac{\partial f}{\partial z} + \frac{1}{3}\,p\,\frac{\partial f}{\partial p}\,\frac{\partial u}{\partial z} + Q - \frac{1}{p^2}\frac{\partial}{\partial p}\left[\frac{dp}{dt}\,p^2\,f\right]$$

- 1D cosmic-ray transport equation for high-energy particles

The leaky box apprximation -1

- The transport equation can be simplified with a set of assumptions

1. Steady state → $\partial f/\partial t = 0$ → Good assumption if $t_{dyn} \gg \min[\tau_{esc}, \tau_{loss}]$
2. Homogeneity → f constant in the whole volume
3. Global escape timescale $\tau_{esc} = [\tau_{diff}^{-1} + \tau_{adv}^{-1}]^{-1}$
4. Losses are equal everywhere in the volume with a typical timescale τ_{loss}

The leaky box apprximation -2

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] - u\frac{\partial f}{\partial z} + \frac{1}{3}p\ \frac{\partial f}{\partial p}\ \frac{\partial u}{\partial z} + Q - \frac{1}{p^2}\frac{\partial}{\partial p}\left[\frac{dp}{dt}\ p^2\ f\right]$$

- Advection: $\tau_{adv} = L/U$
- Diffusion: $\tau_{diff} = L^2/D$

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Take home message 1

- Cosmic rays diffuse in the interstellar medium
- We can model cosmic-ray transport with the advection-diffusion equation
- We can often adopt a nice simplified approach – the leaky box

Open issue

- Non linearity: CRs leaving a certain environment might develop currents affecting their own transport

→ $D[t,z,p; f(t,z,p)]$

Outline

- Fundamentals of paricle transport in astrophysical plasma
- Particle acceleration (diffusive shock acceleration)
- Studying and modeling cosmic sources

Particle acceleration in space

- Injection → extraction of non-thermal tail from the plasma Maxwellian

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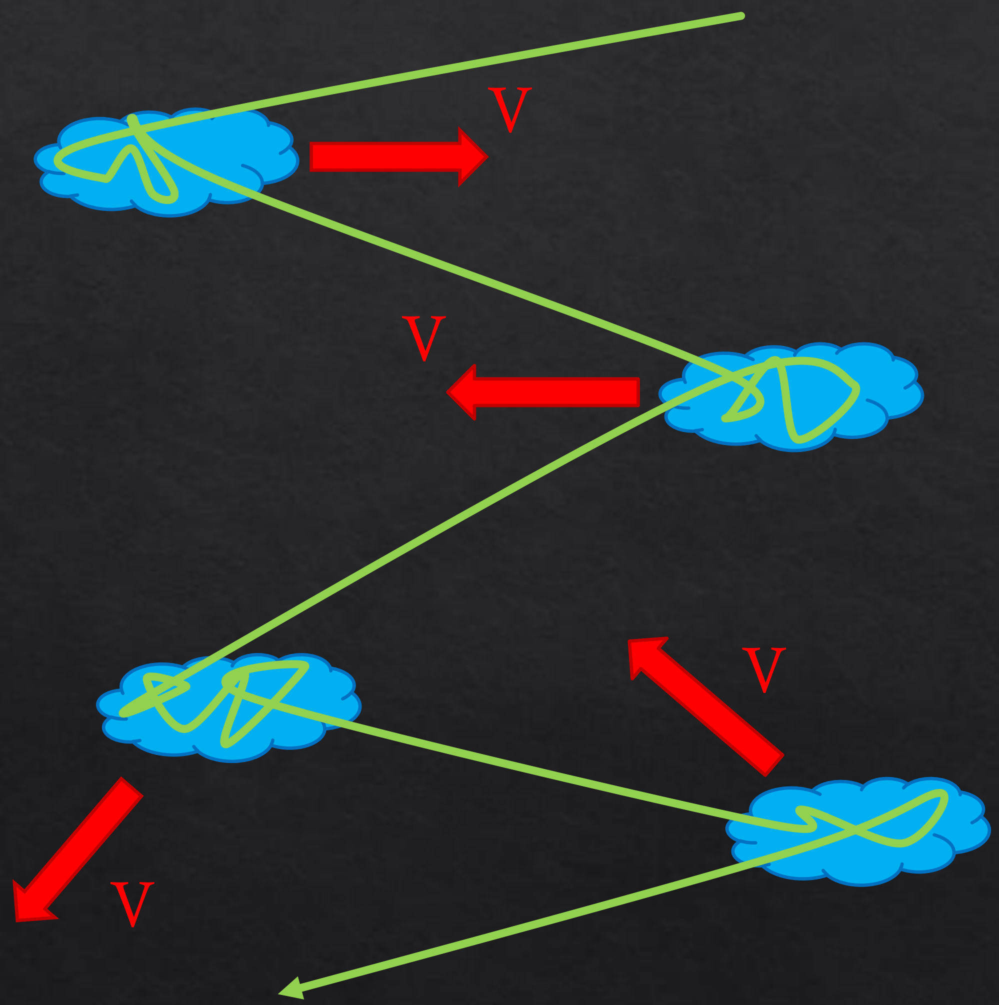
- Electric fields cannot last long because plasma self-regulate with currents

- Particle acceleration can still take place with different mechanisms:
  1. Turbulence (second order Fermi)
  2. Diffusive shock acceleration
  3. Magnetic reconnection (not today ☹)

The original idea of 1949 (2nd order Fermi)



- Charged particles are scattered by magnetized clouds (can be waves as well) in random motion

$$\frac{dp_z}{dt} \propto \omega\, p_\perp \frac{\delta B}{B_0} \sim \omega\, p \frac{V_A}{c}$$

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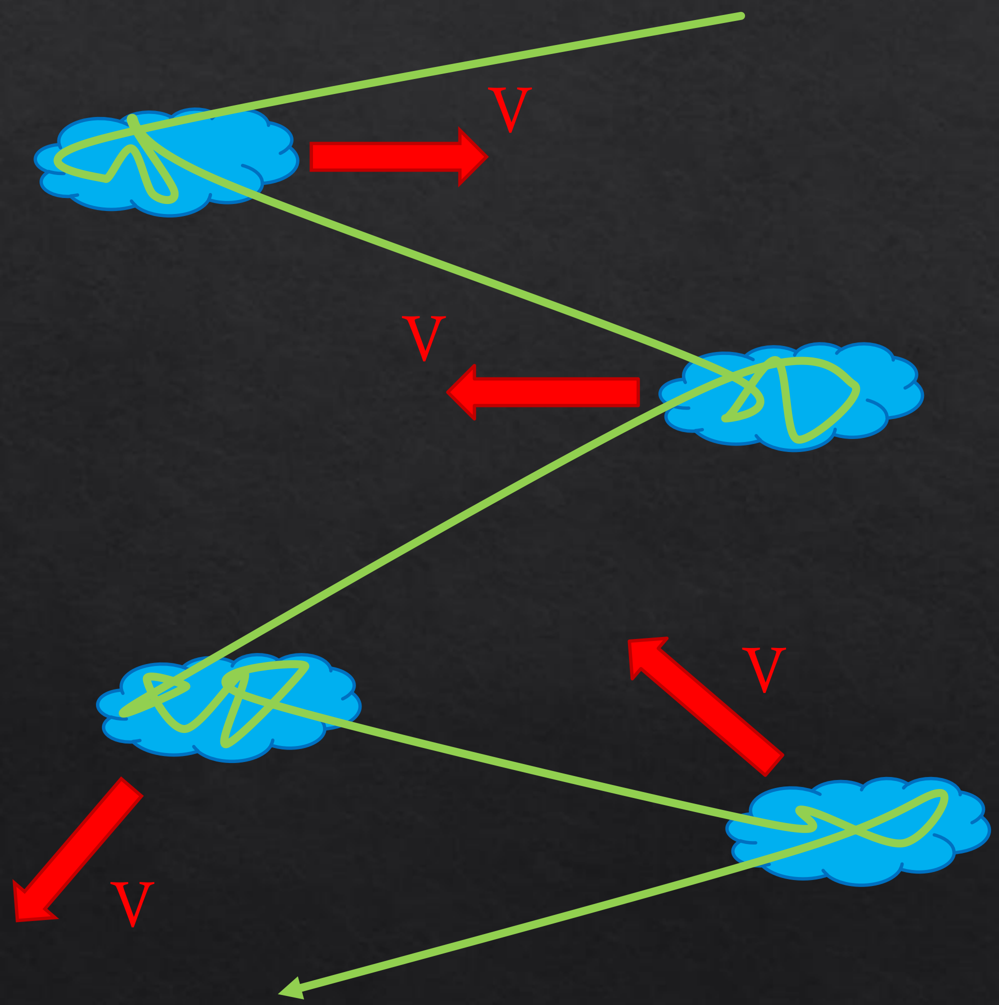

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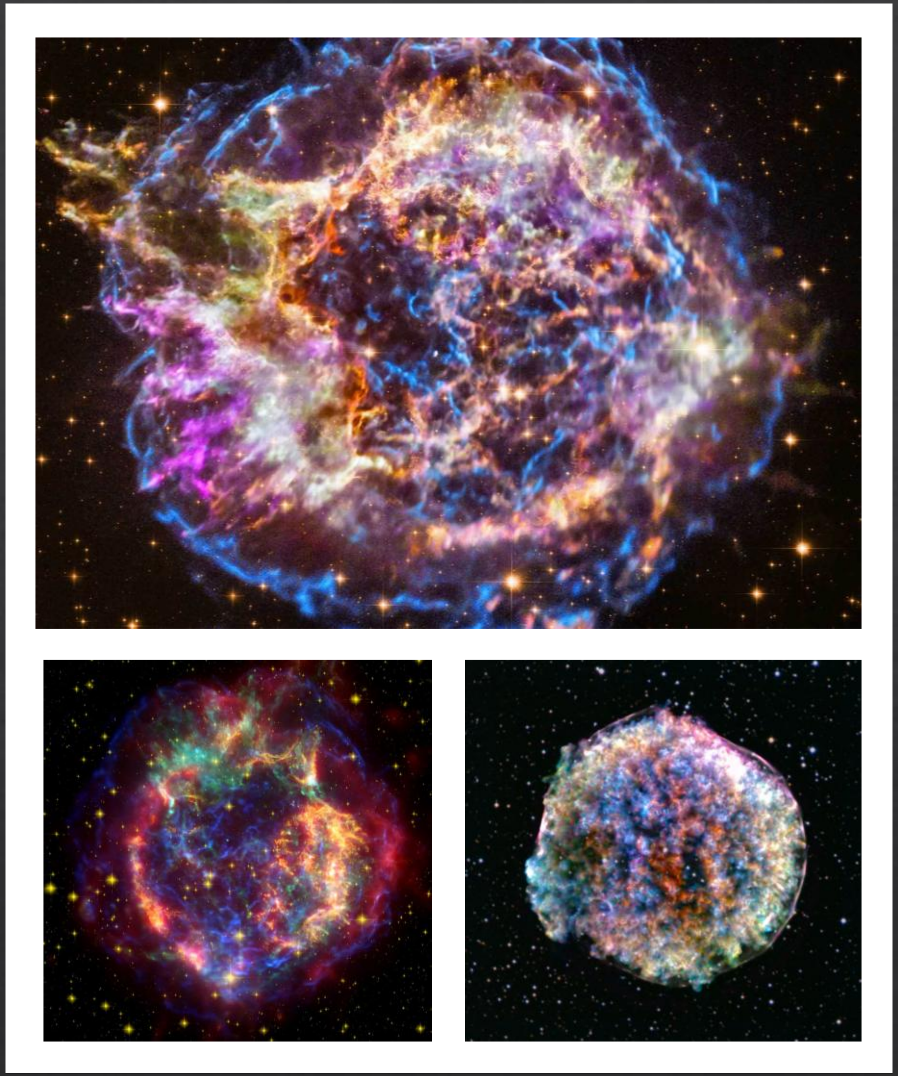
$$\frac{dp_z}{dt} \propto \omega\, p_\perp \frac{\delta B}{B_0} \sim \omega\, p \frac{V_A}{c}$$

$$D_{pp} = \left\langle \frac{\Delta p\, \Delta p}{\Delta t} \right\rangle \propto \left(\frac{V_A}{c} \right)^2$$

- Diffusion in momentum is inefficient!

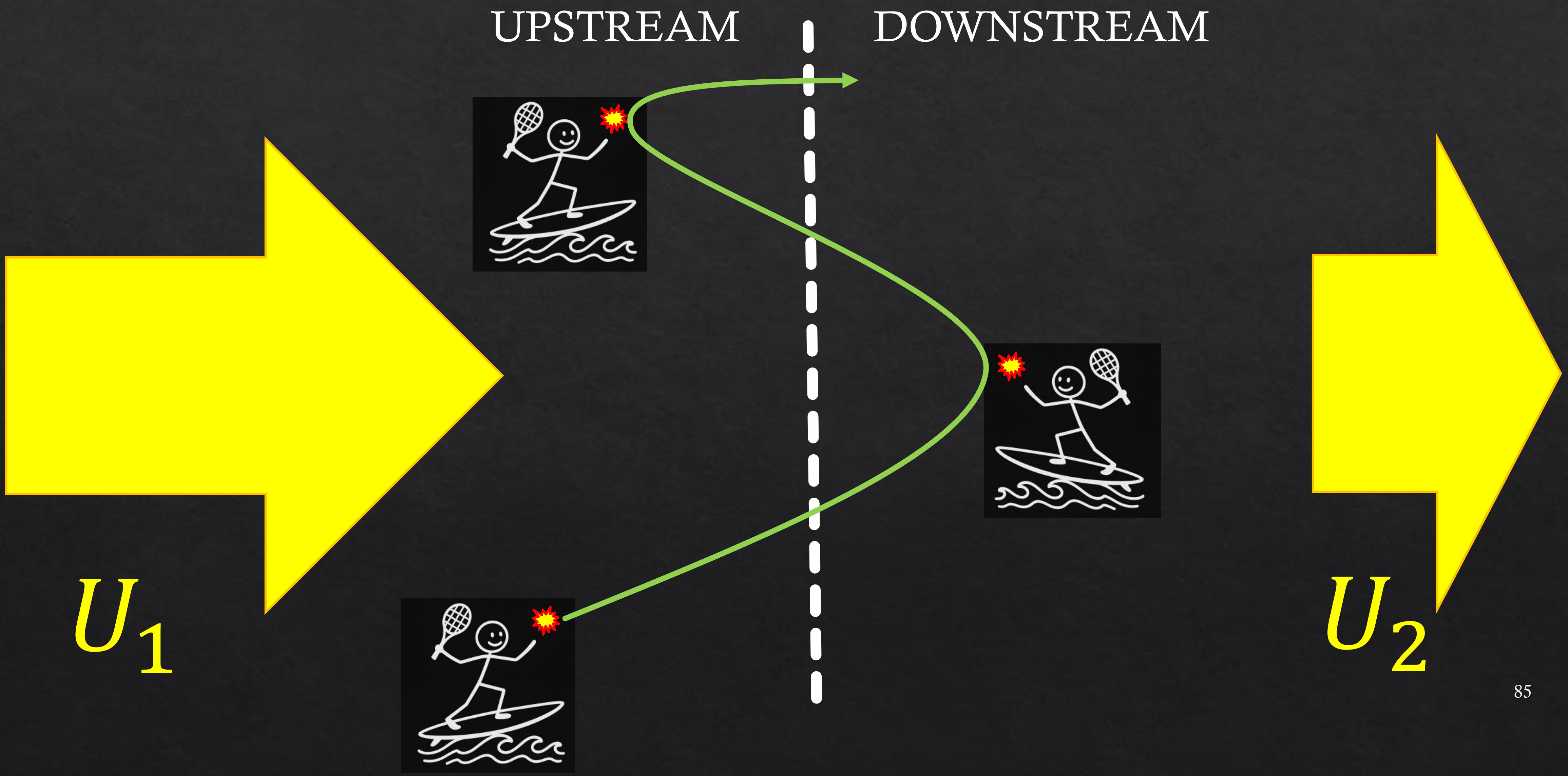
Shock waves

- In the interstellar medium (warm) the sound speed is about $c_s \approx 10\ km/s$
- Supernova blast waves are launched at $10^4\ km/s$ meaning Mach number ~ 10^3
- Shocks are collisionless → mediated by instabilities



Krymskii 1977, Bell 1978, Blandford & Ostriker 1978

Diffusive shock acceleration – textbook approach



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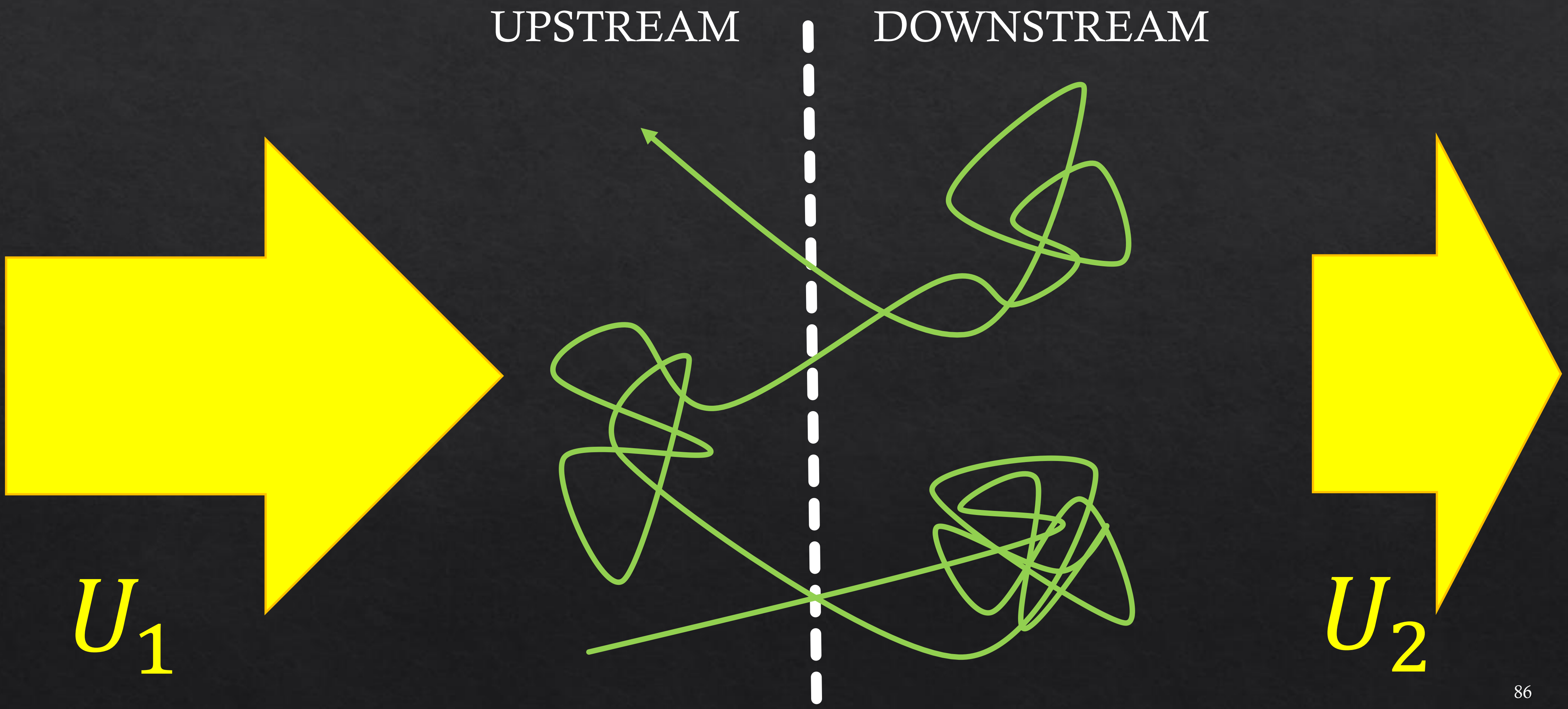

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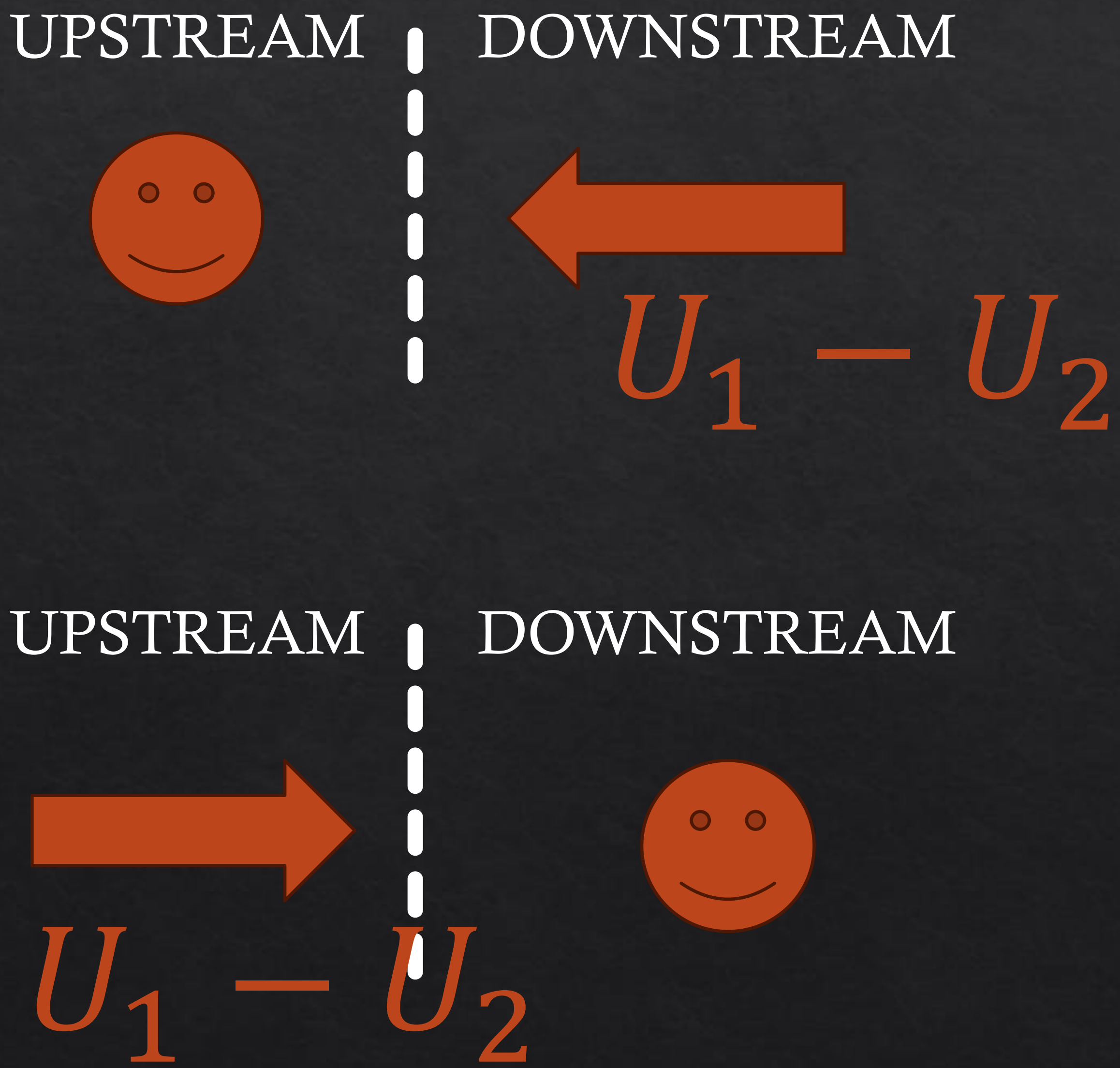


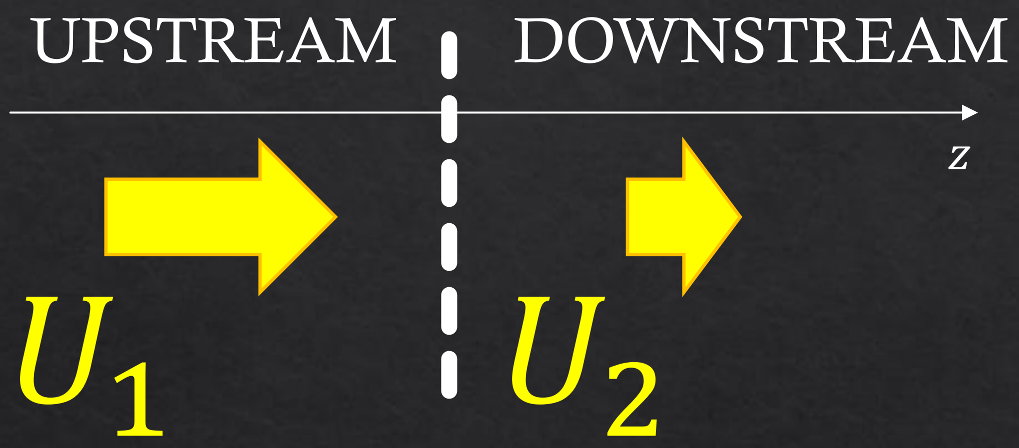
- Particles are gaining energy because, when crossing the shock, they enter a reference frame with the right motion for an energy boost

$$\frac{\Delta E}{E} = \frac{4}{3}\left(\frac{U_1 - U_2}{c}\right)$$

- The particle spectrum does not depend on microphysics

$$f(p) \propto p^{-4} \leftrightarrow f(E) \propto E^{-2} \; (pc \approx E)$$

Transport approach to DSA



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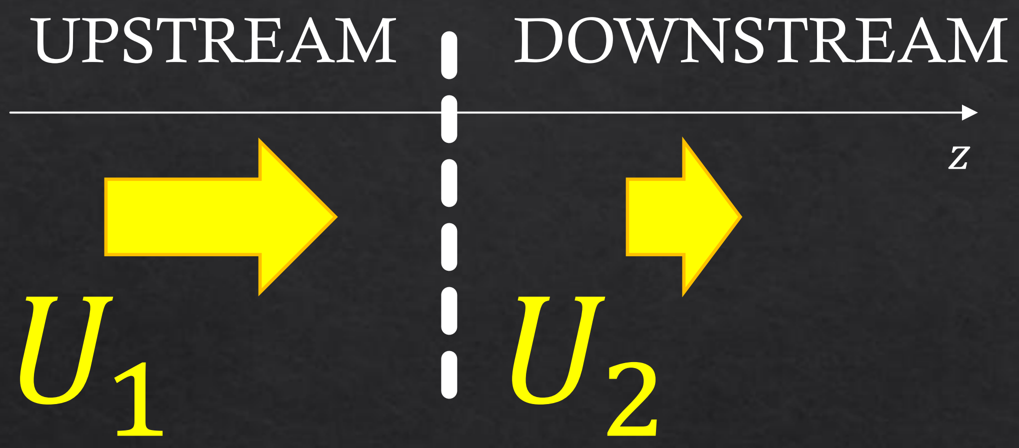

- Let's solve the 1D transport equation

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] - u\frac{\partial f}{\partial z} + \frac{1}{3}p\frac{\partial f}{\partial p}\frac{\partial u}{\partial z} + Q$$

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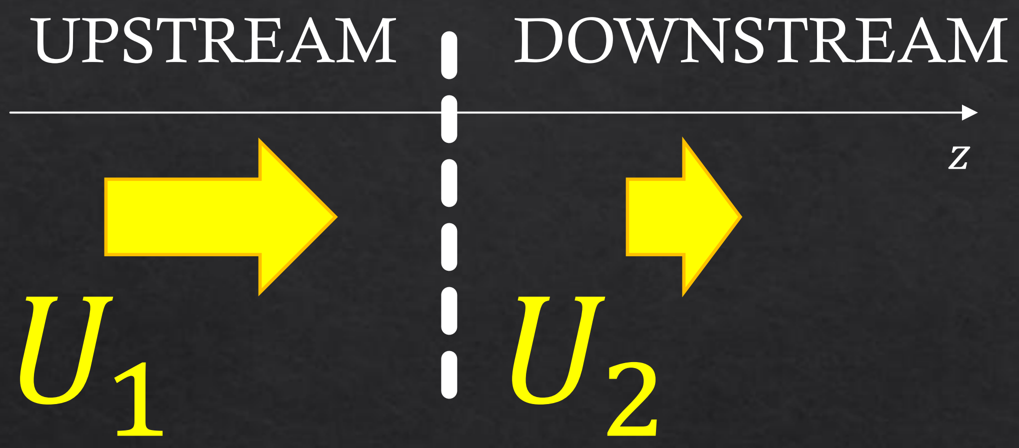

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- Assumptions and boundary conditions:

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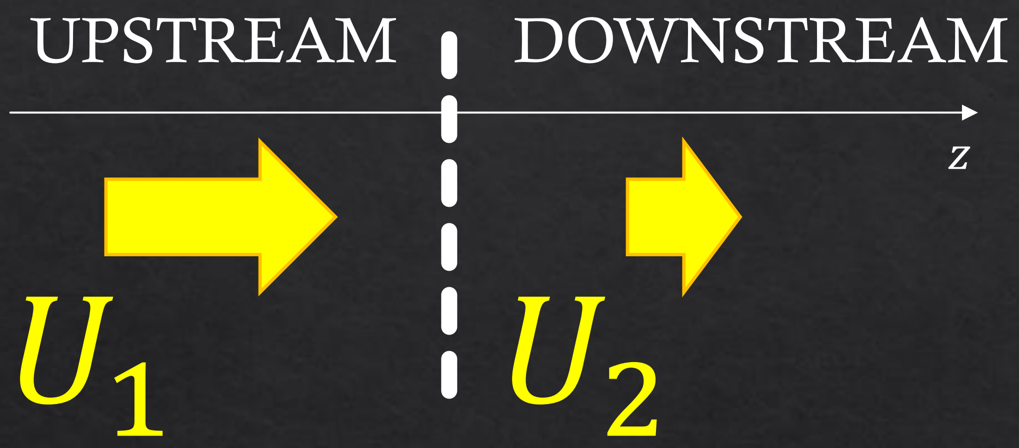

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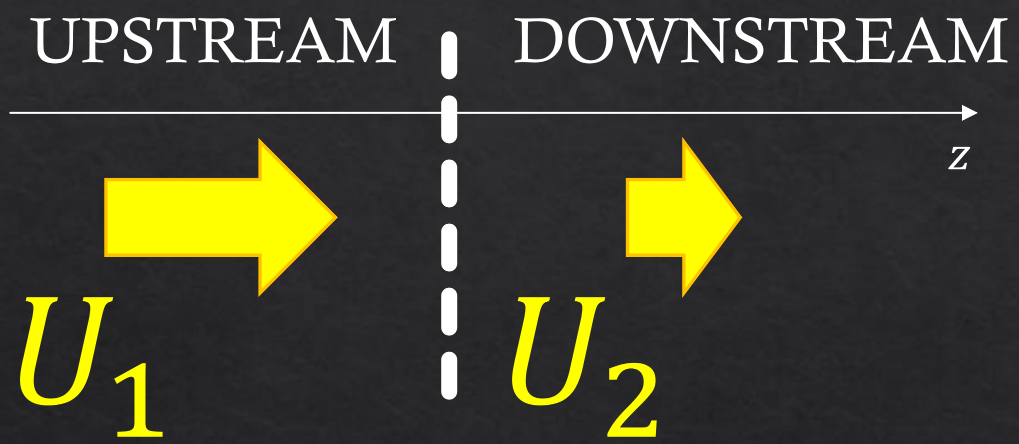

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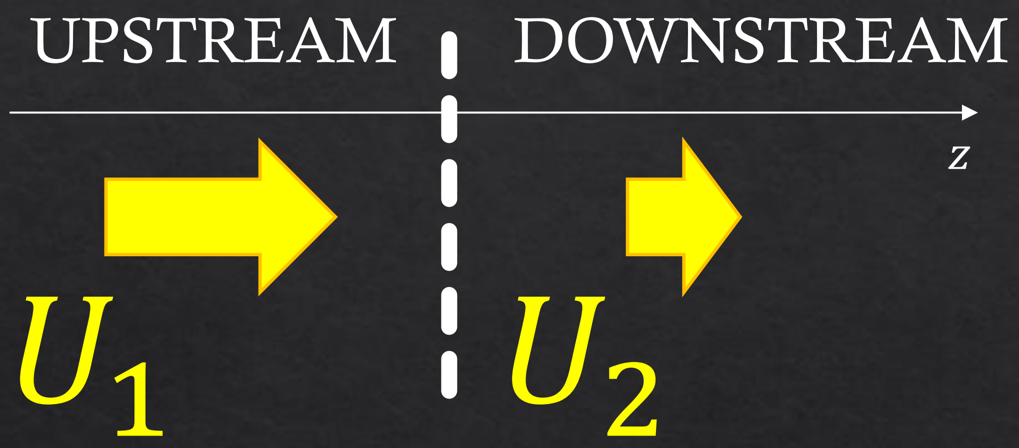

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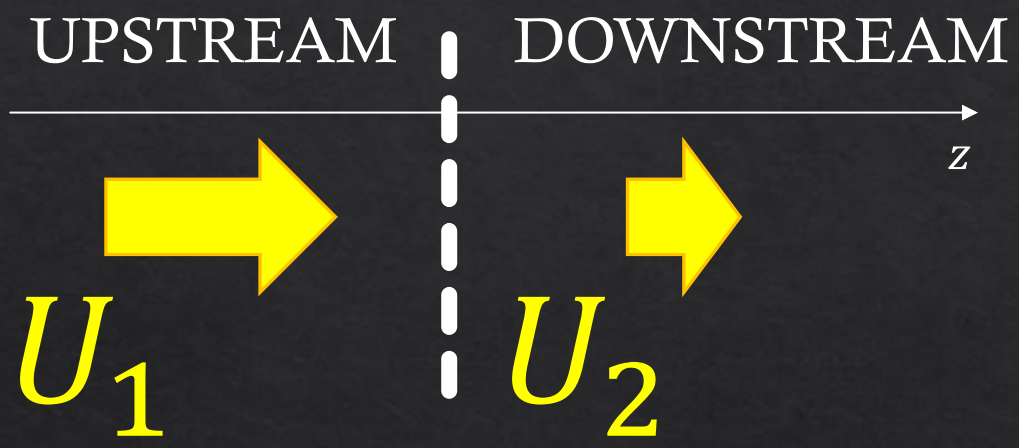

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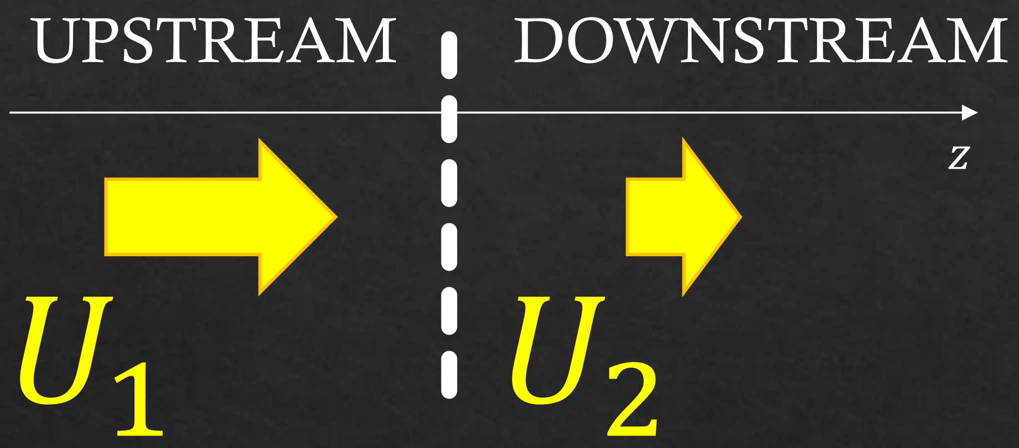
Transport approach to DSA



Injection

$$Q(z,p) = \frac{\eta\, n_1 U_1}{4\pi p^2}\, \delta\left[p - p_{inj}\right] \delta[z]$$

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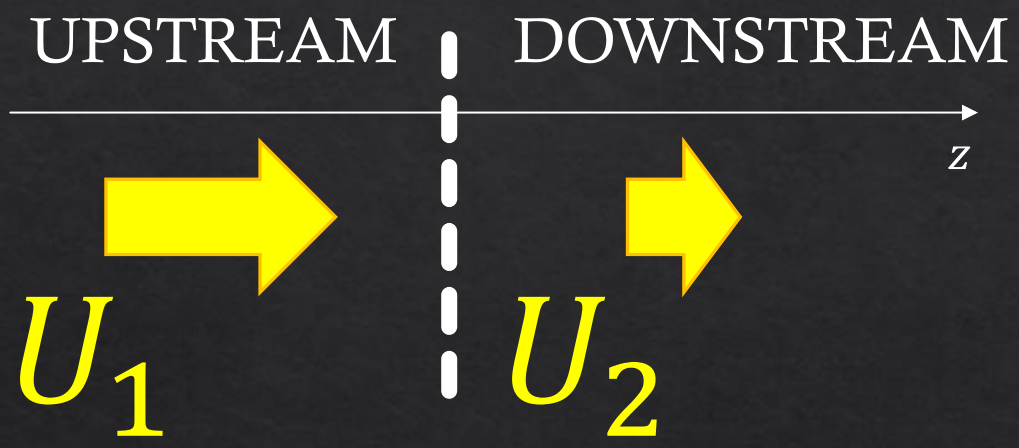
Transport approach to DSA



Injection

Fraction η of the arrival flux at the shock

$$Q(z,p) = \frac{\eta\, n_1 U_1}{4\pi p^2} \delta[p - p_{inj}]\, \delta[z]$$

- Let's solve the 1D transport equation

$$0 = \frac{\partial}{\partial z}\left[D \frac{\partial f}{\partial z}\right] - u \frac{\partial f}{\partial z} + \frac{1}{3}\, p\, \frac{\partial f}{\partial p}\, \frac{\partial u}{\partial z} + Q$$

- Assumptions and boundary conditions:
  1. $f(z > 0) = const.$
  2. $f(z \to -\infty) = \partial_z f(z \to -\infty) = 0$
  3. Negligible energy losses
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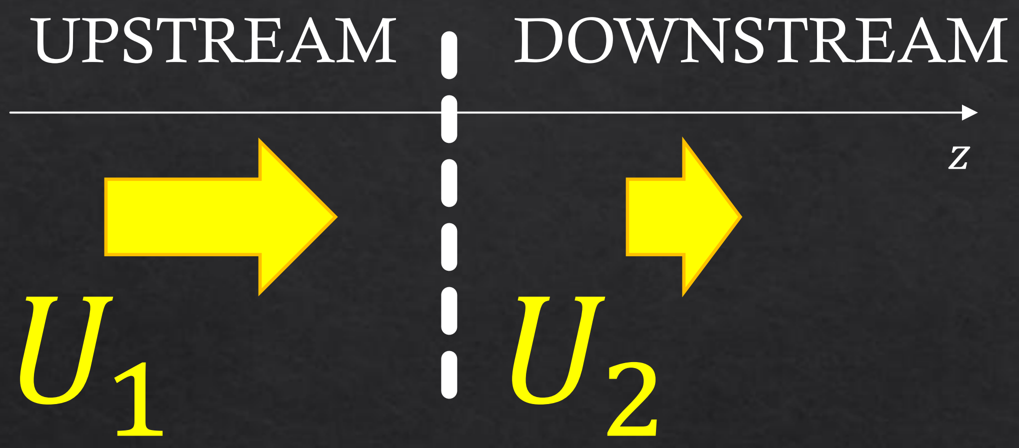

Injection

Energization only at the shock

$$Q(z,p) = \frac{\eta\, n_1 U_1}{4\pi p^2}\, \delta[p - p_{inj}]\, \delta[z]$$

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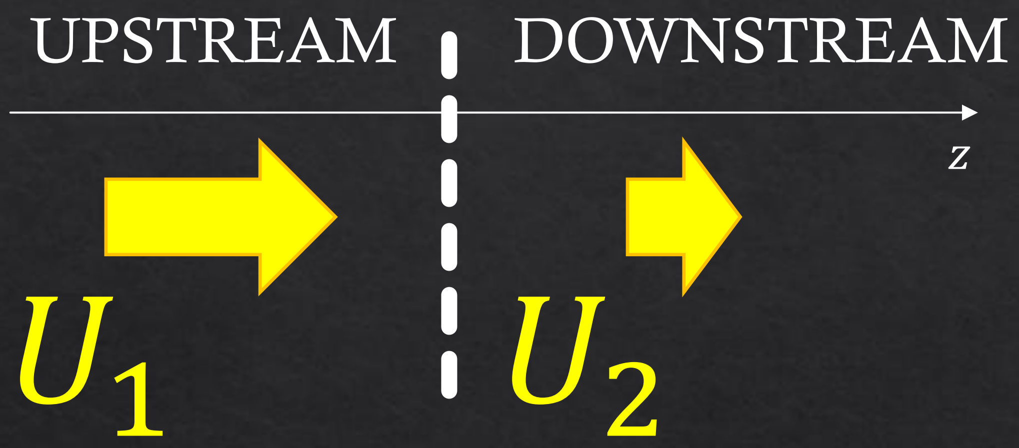

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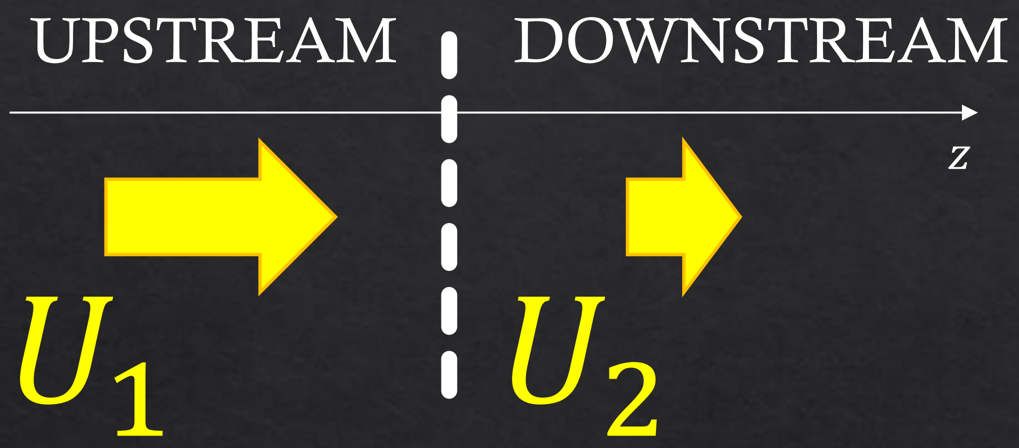
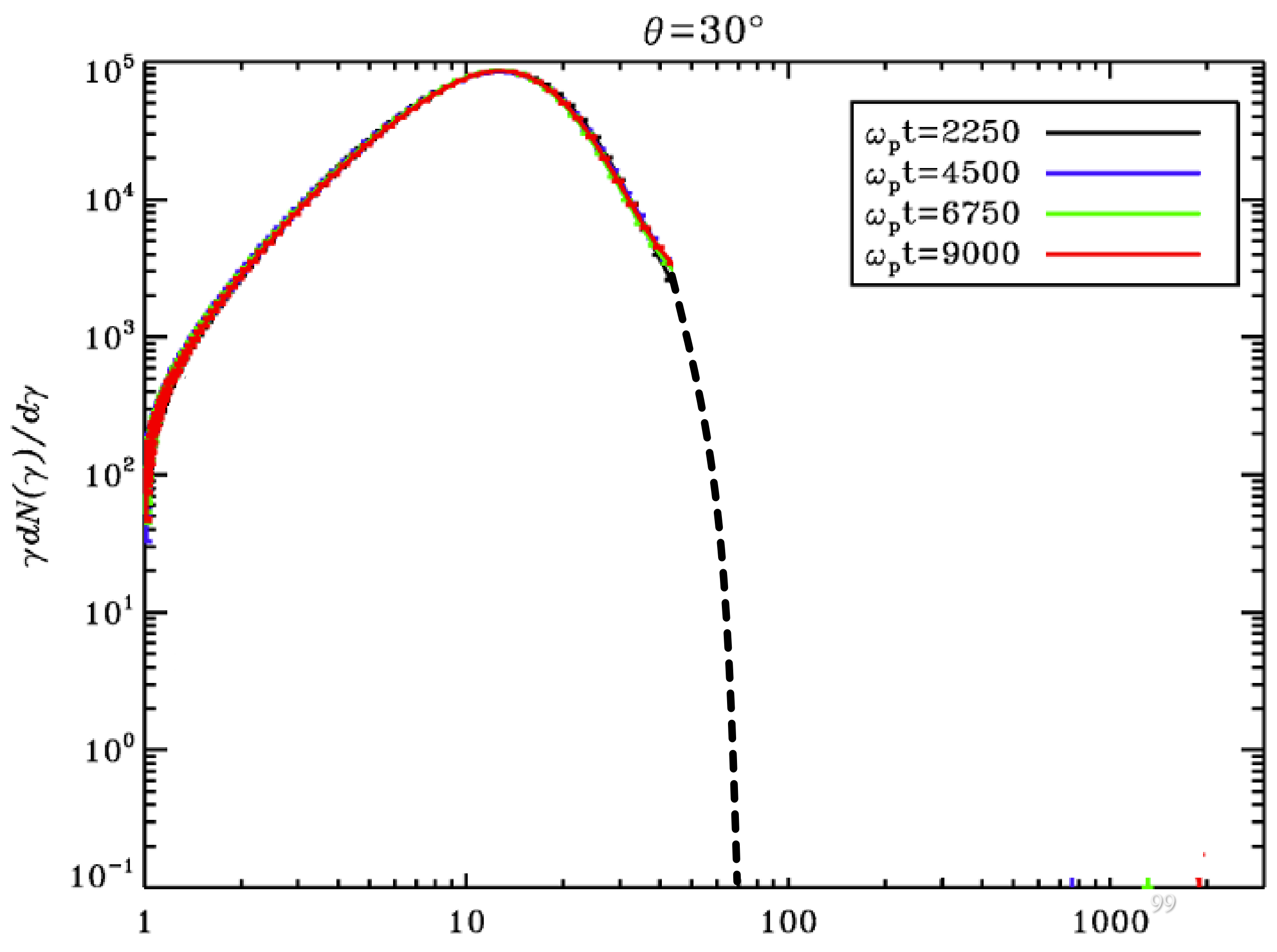
Injection

Minimum momentum to enter DSA

$$Q(z,p) = \frac{\eta\, n_1 U_1}{4\pi p^2}\delta[p - p_{inj}]\delta[z]$$

Transport approach to DSA





Spitkovsky & Sironi 2009

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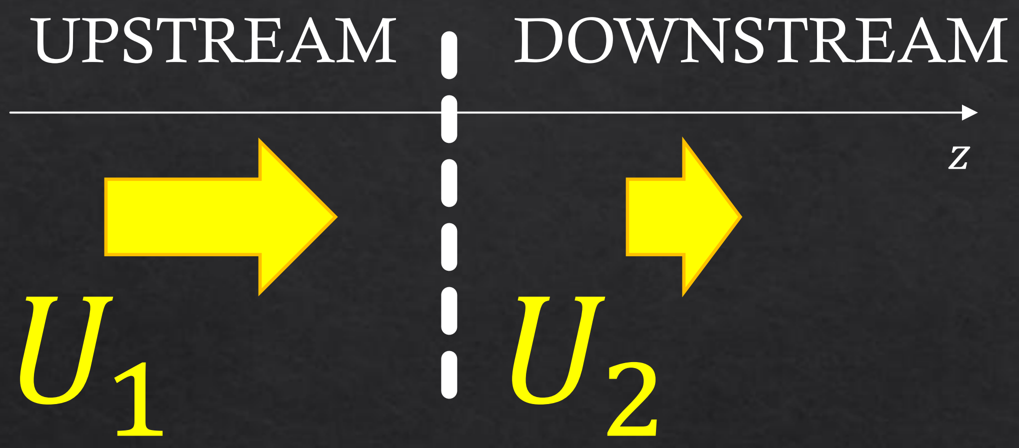

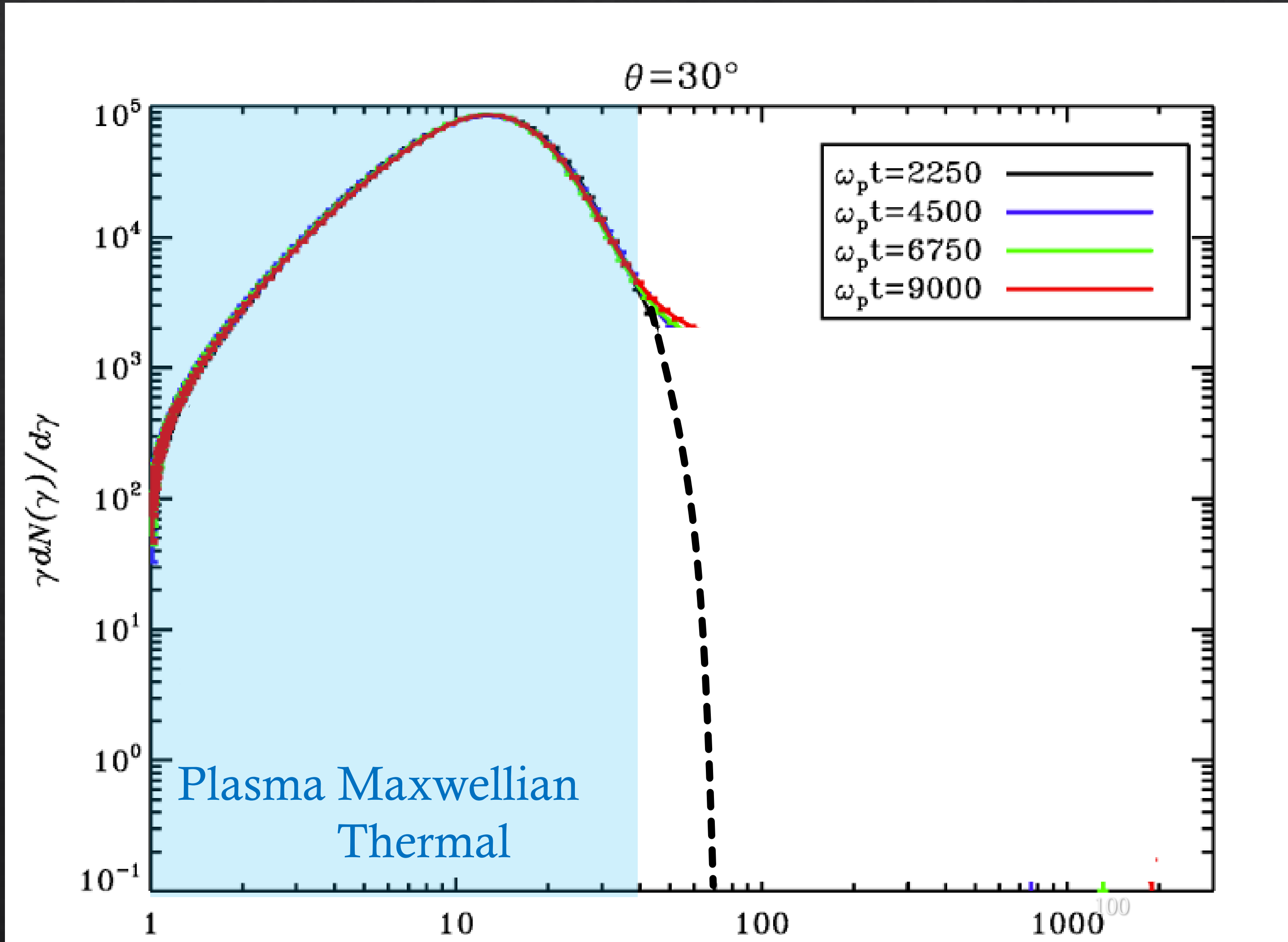

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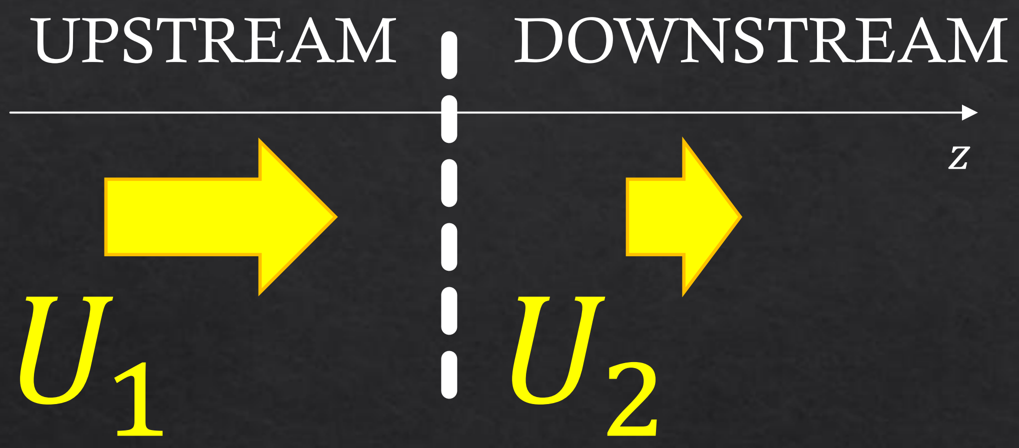

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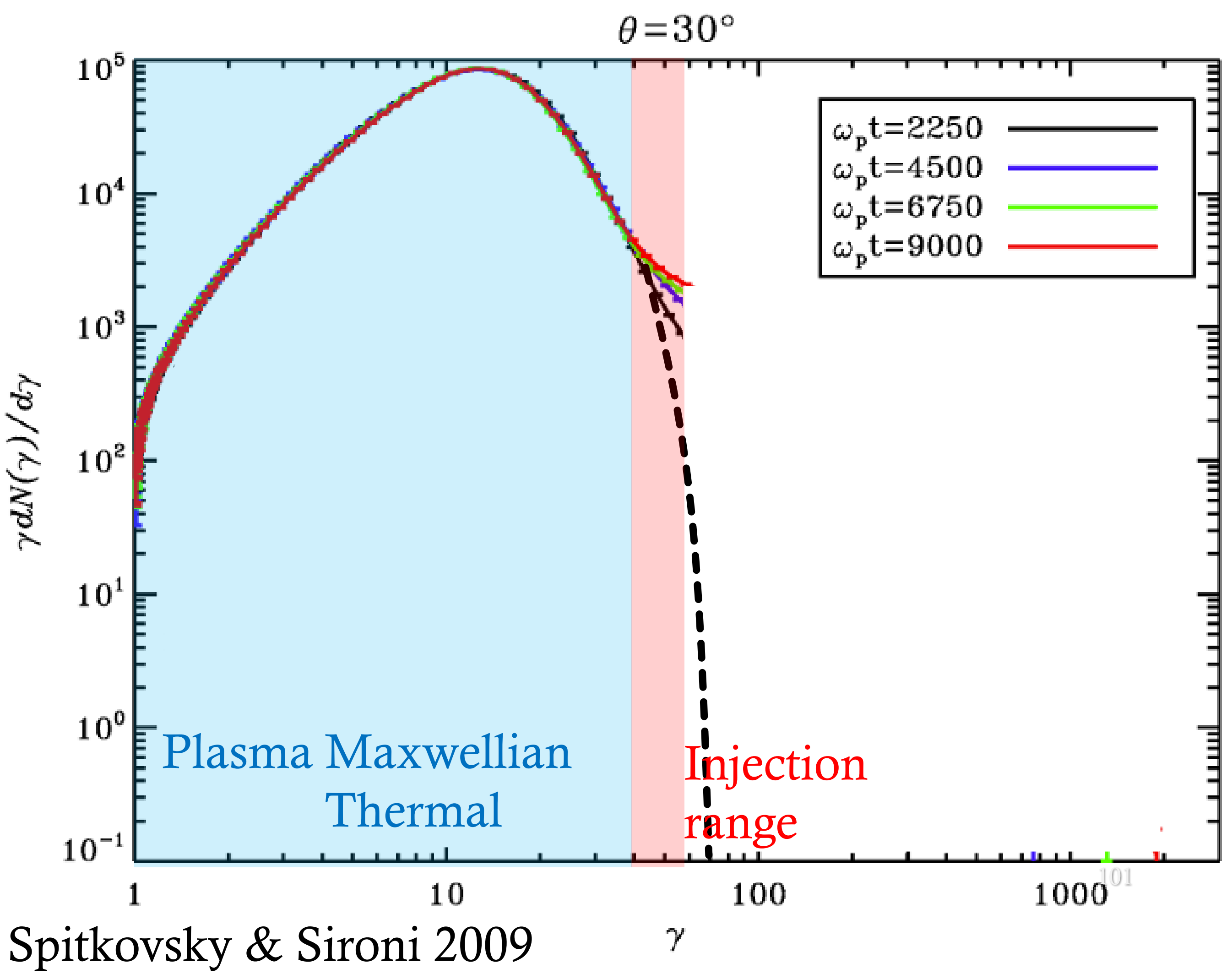

Spitkovsky & Sironi 2009

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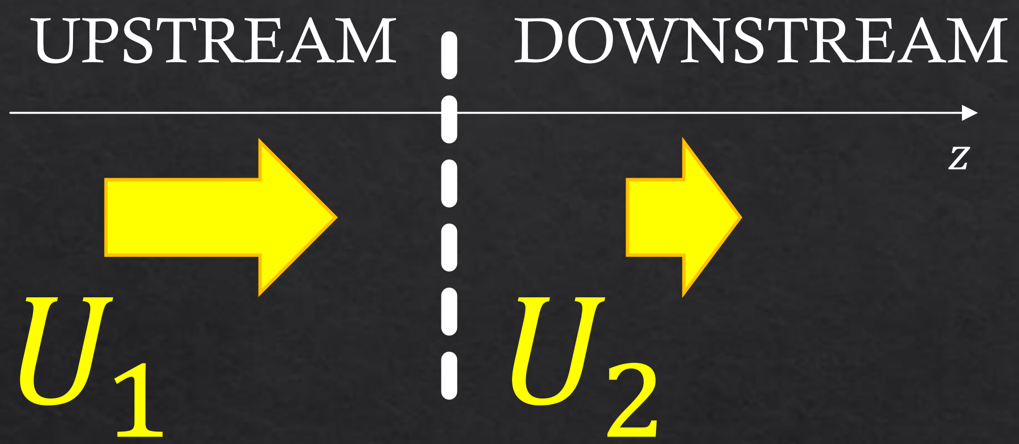

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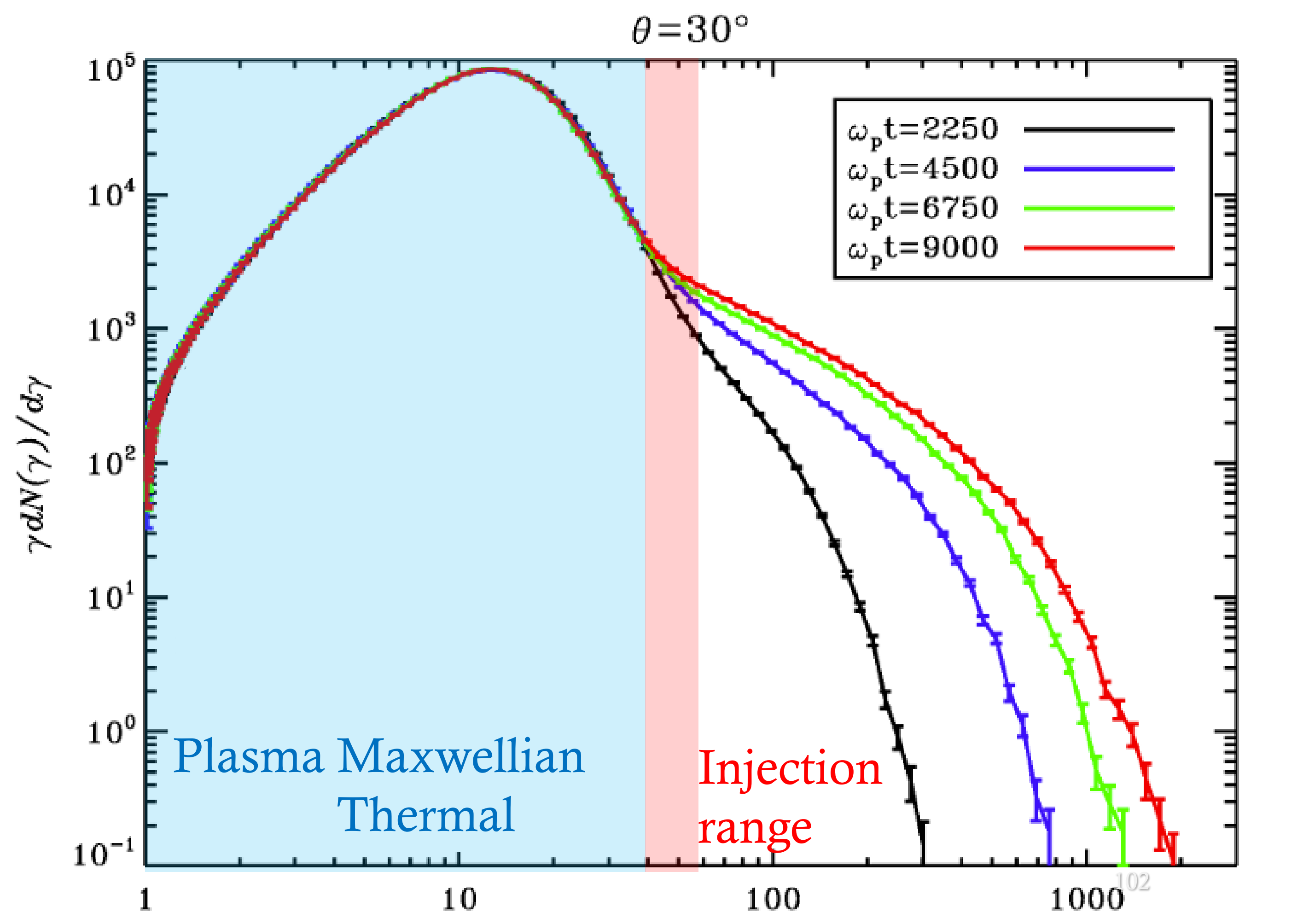

Spitkovsky & Sironi 2009

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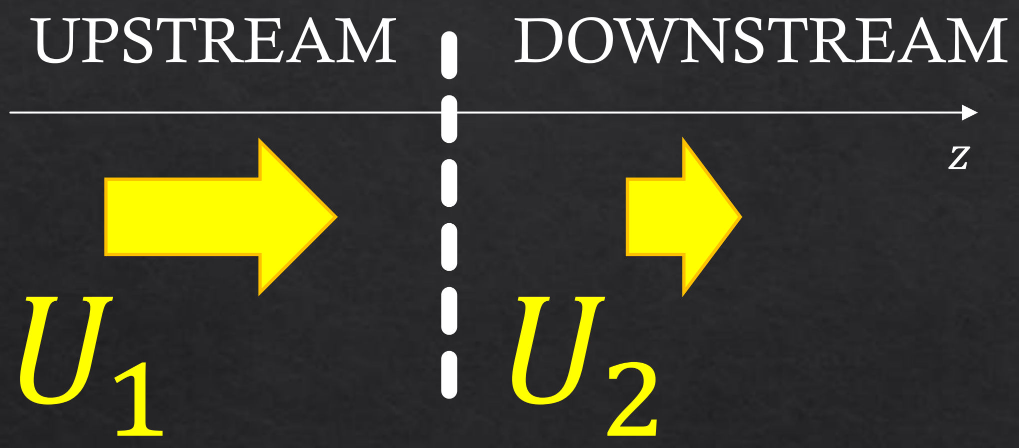

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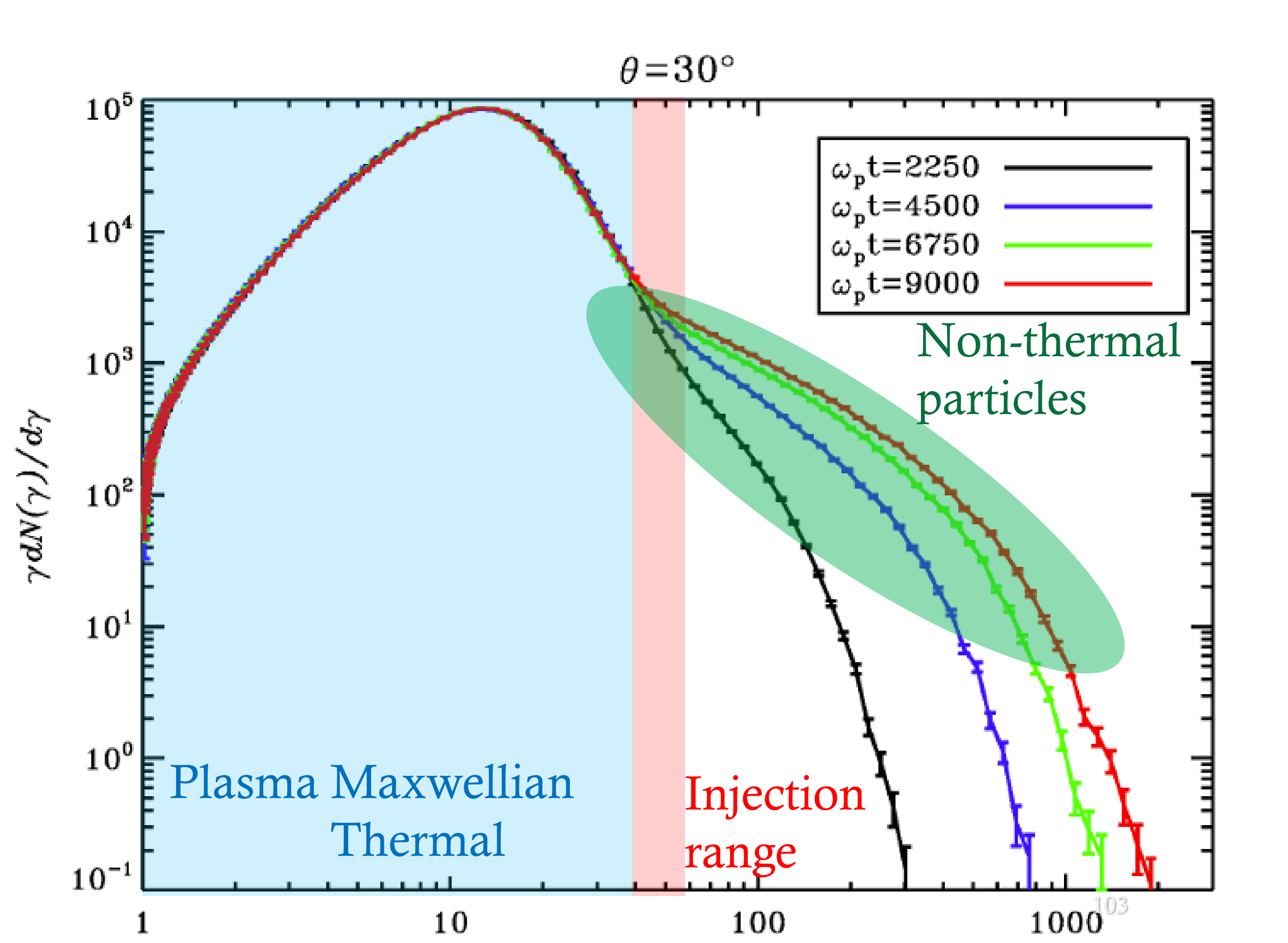
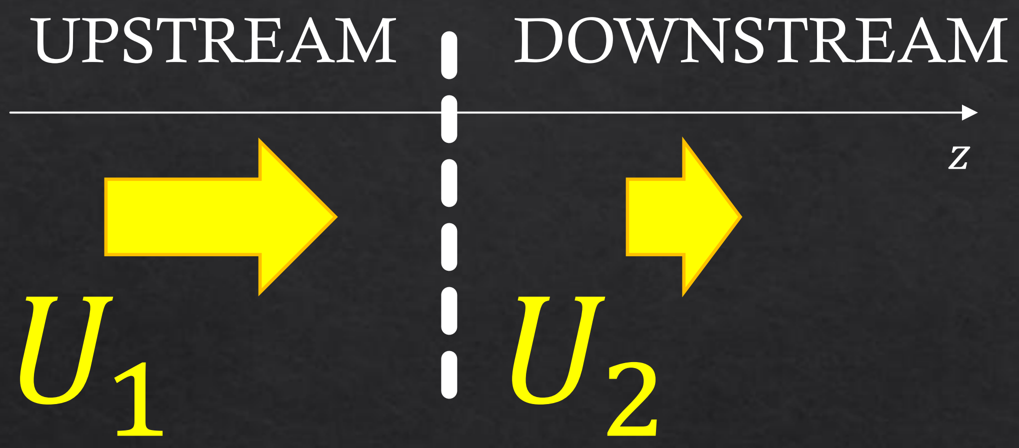


Spitkovsky & Sironi 2009

Transport approach to DSA: upstram



- We solve from $-\infty$ to 0

$$0 = \frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] - u\frac{\partial f}{\partial z} + \frac{1}{3}p\frac{\partial f}{\partial p}\frac{\partial u}{\partial z} + Q$$

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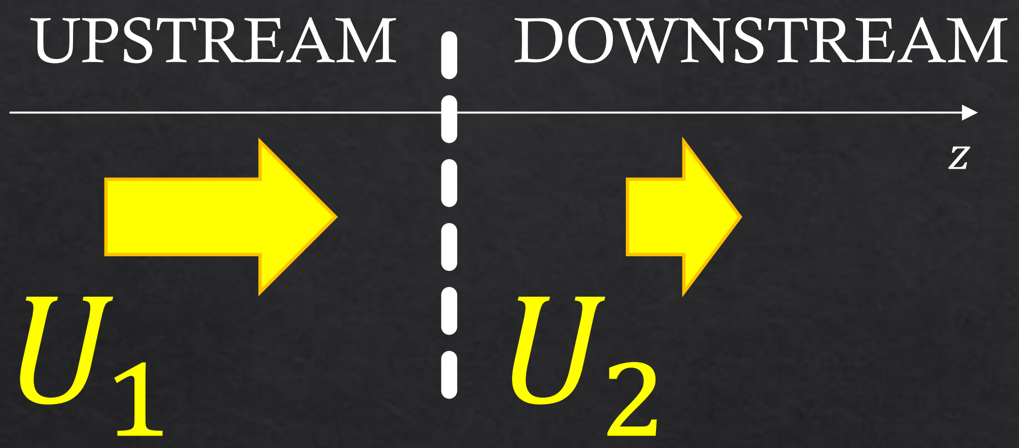

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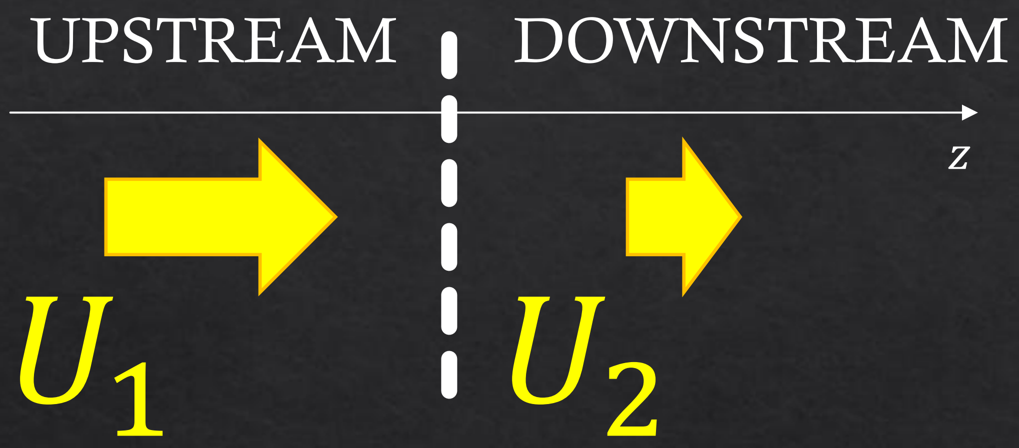

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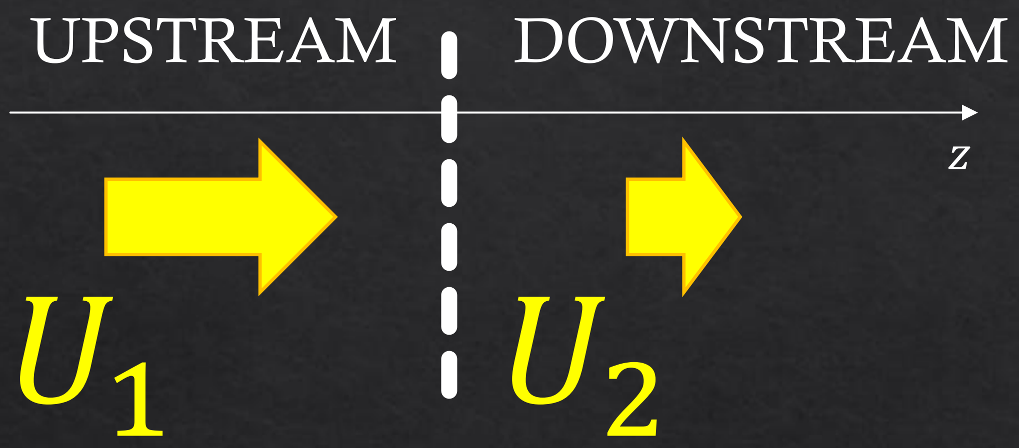

Upstream solution

Flux conservation equation

$$D\frac{\partial f}{\partial z} = U_1 f$$

- We solve from $-\infty$ to 0

$$0 = \frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] - u\frac{\partial f}{\partial z} + \frac{1}{3}p\frac{\partial f}{\partial p}\frac{\partial u}{\partial z} + Q$$

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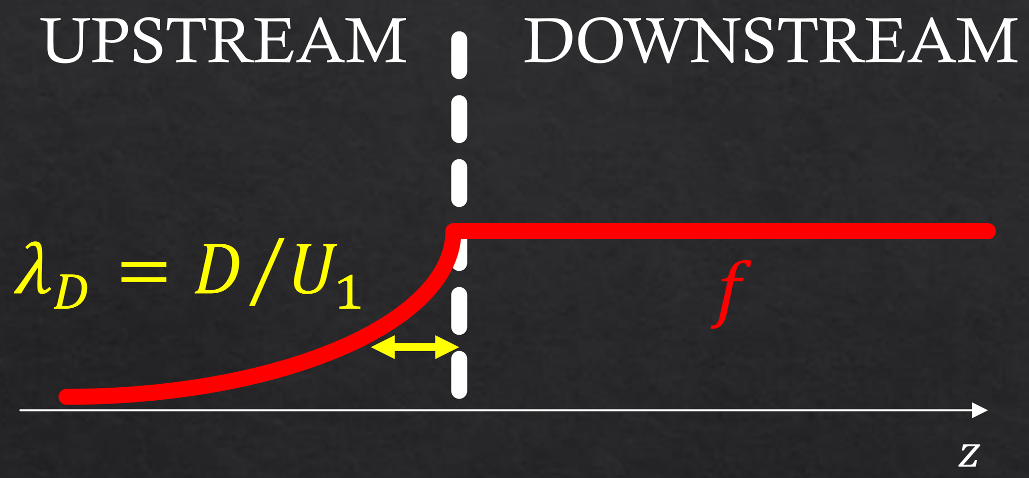

Upstream solution

Exponential drop

$$f(z,p) = f_0(p)\, e^{-|z|U_1/D(p)}$$

- We solve from $-\infty$ to 0

$$0 = \frac{\partial}{\partial z}\left[D\frac{\partial f}{\partial z}\right] - u\frac{\partial f}{\partial z} + \frac{1}{3}p\frac{\partial f}{\partial p}\frac{\partial u}{\partial z} + Q$$

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Transport approach to DSA: solution at the shock

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- Integrate the transport eqution from $0-\varepsilon$ up to $0+\varepsilon$

Transport approach to DSA: solution at the shock

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Upstream flux conservation

$$0 = -U_1 f_0 + 0 - \frac{(U_1 - U_2)}{3}p\frac{\partial f_0}{\partial p} + Q_0(p)$$

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Upstream flux conservation

Continuity

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$\partial_z \theta[z] = \delta[z]$

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Where, in the LHS, I can recognize a total derivative of $p^s f_0(p)$

Transport approach to DSA: solution at the shcok

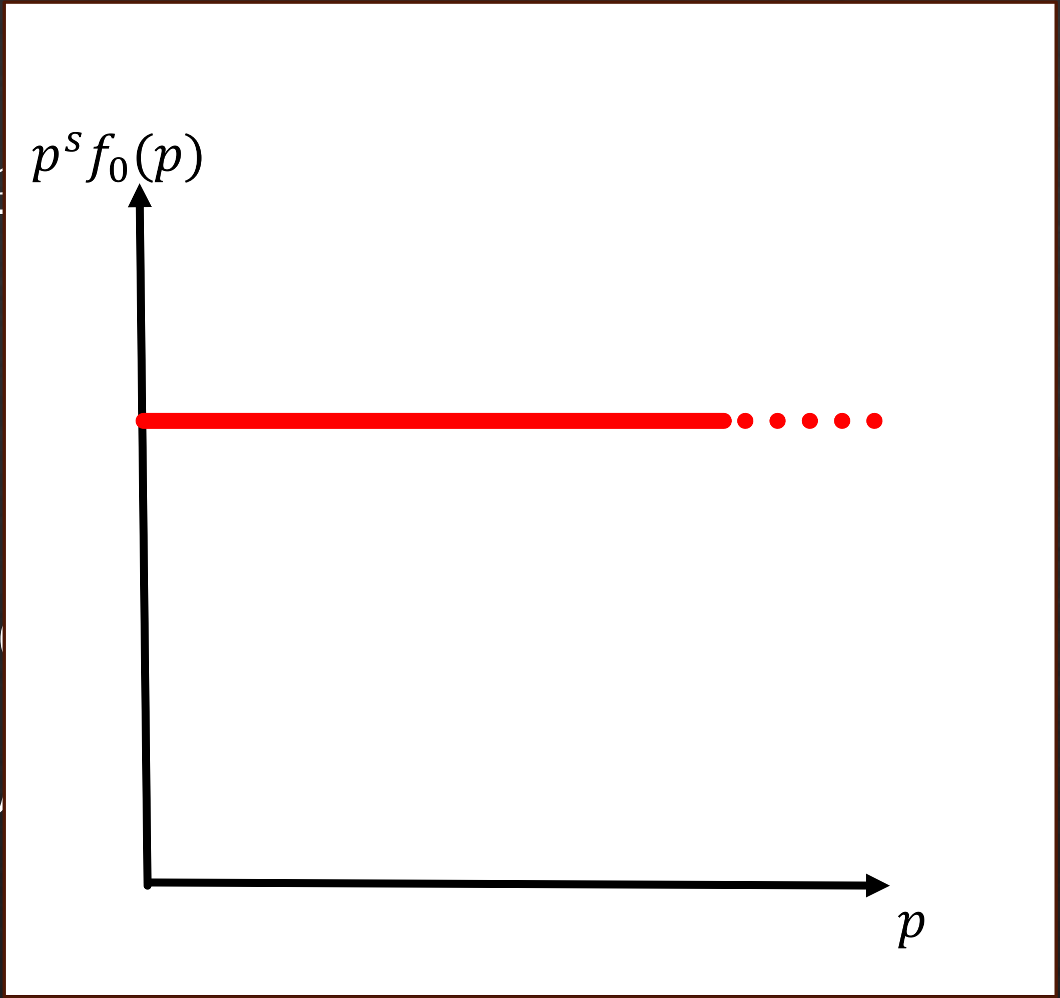
Shock solution

$$f_0(p) = \frac{s}{U_1 p_{inj}} Q_0(p_{inj}) \left(\frac{p_{inj}}{p}\right)^s$$

$) - \frac{(U_1}{}$

$p\, \partial_p f_0(p) + s f_0($

$s = 3U_1/(U$



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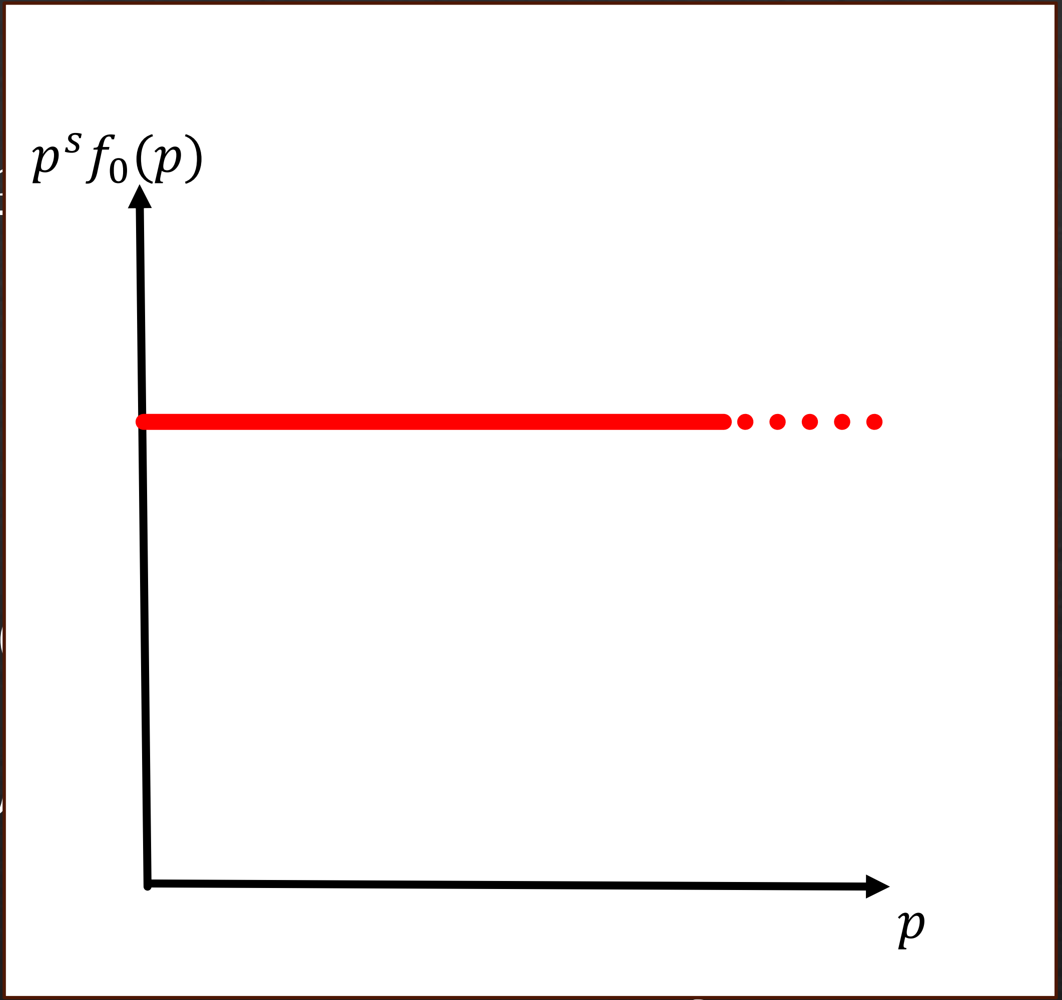
- We found a power-law solution in momenutm with index s = 4



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Energetic consideration

- As in the toy-model we end up with no constraints on the maximum energy

$$U = \int dp \; 4\pi \, p^2 \, T(p) \, f_0(p) \rightarrow \ln \frac{p}{p_{inj}}$$

Where, in the LHS, I can recognize a total derivative of $p^s f_0(p)$

What are we missing?

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There are two directions to make the model more realistic

1. Drop the steady-state assumption

2. Constrain the size of the accelerator

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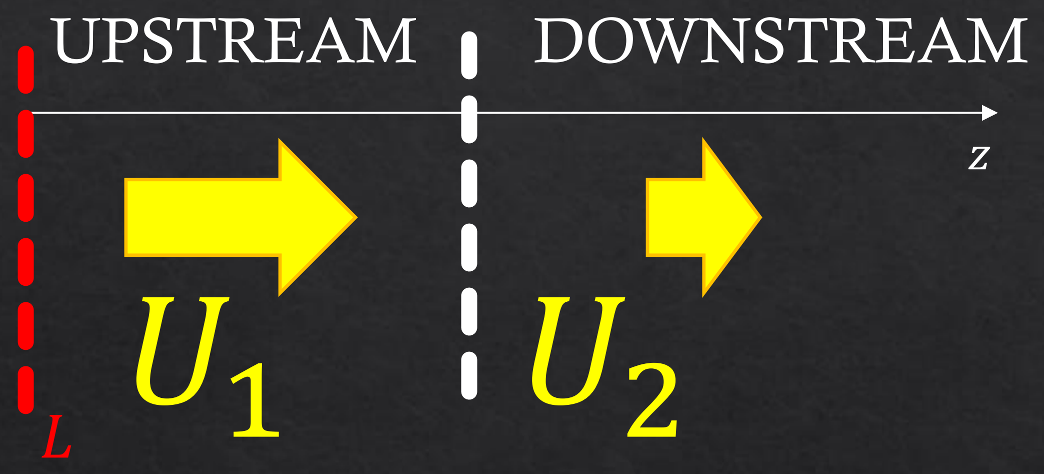
NICE EXERCISE

$$f(z = L) = 0$$

Transport approach to DSA with modified bc

FEB → $f(L,p) = 0$



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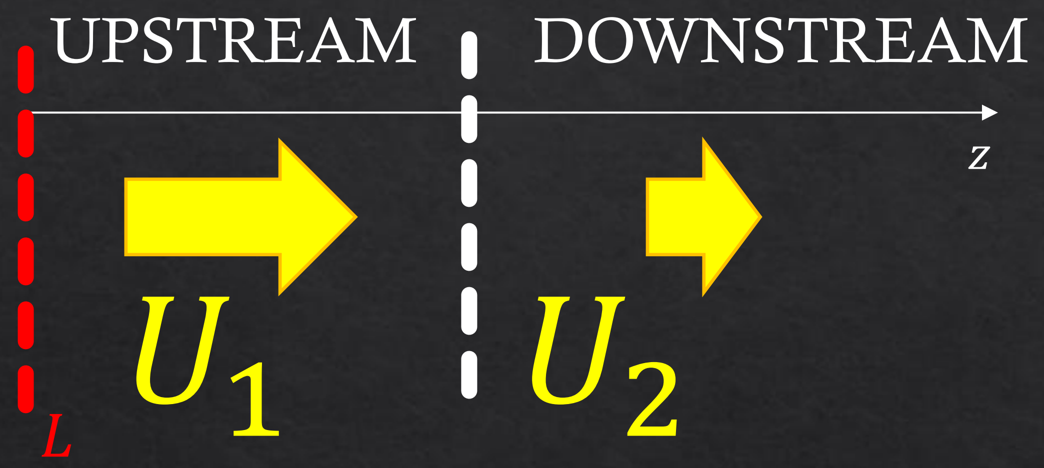

New upstream solution

$$\partial_z[D\partial_z f] = U_1\partial_z f \rightarrow D\partial_z f - U_1 f = j_{esc}$$

$$f(z,p) = f_0(p)\,\frac{e^{-(L-z)U_1/D(p)} - 1}{e^{-LU_1/D(p)} - 1}$$

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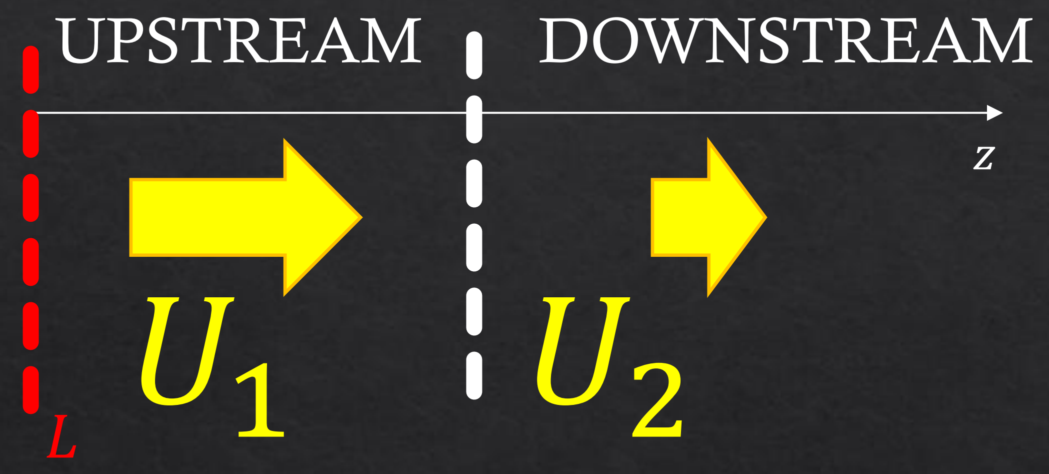

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New solution at the shock

$$f_0(p) \propto p^{-s} \exp\{\int_{p_{inj}}^{p} \frac{dp'}{p'}\,\frac{-s}{\exp[-LU_1/D(p')] - 1}\}$$

Transport approach to DSA with modified bc

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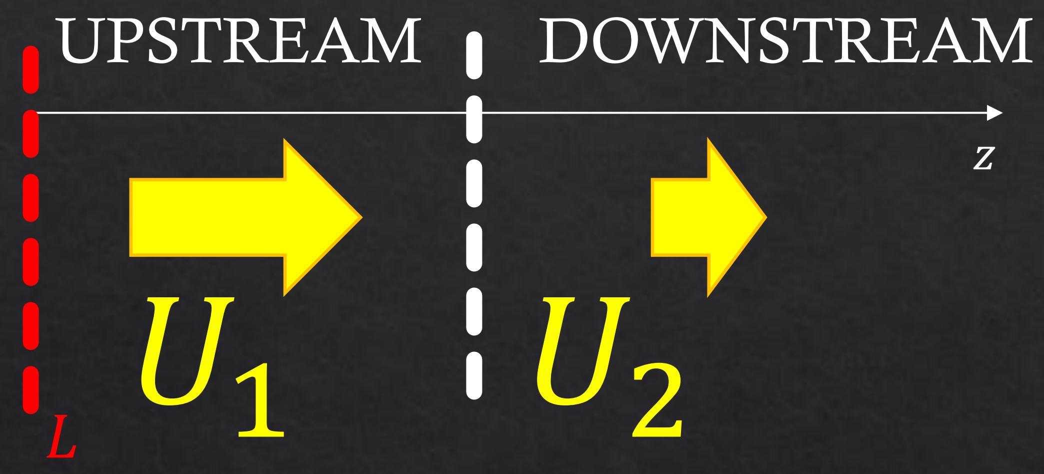

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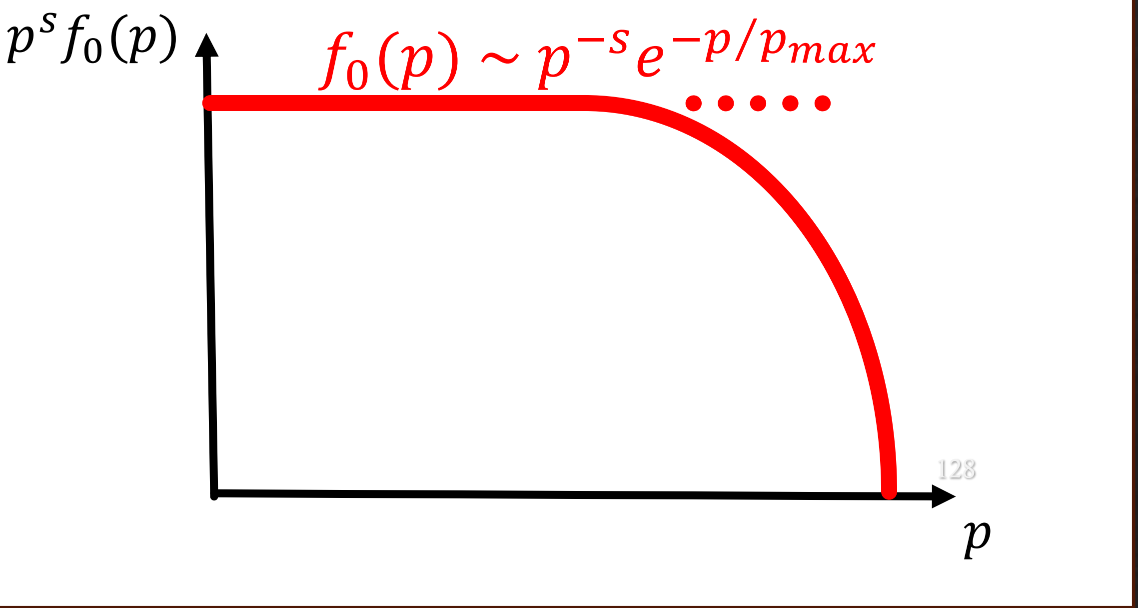

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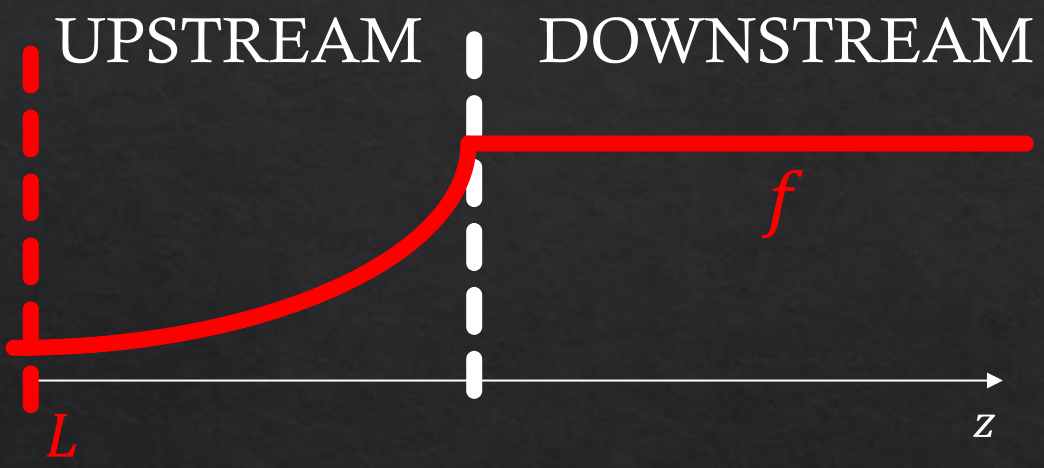

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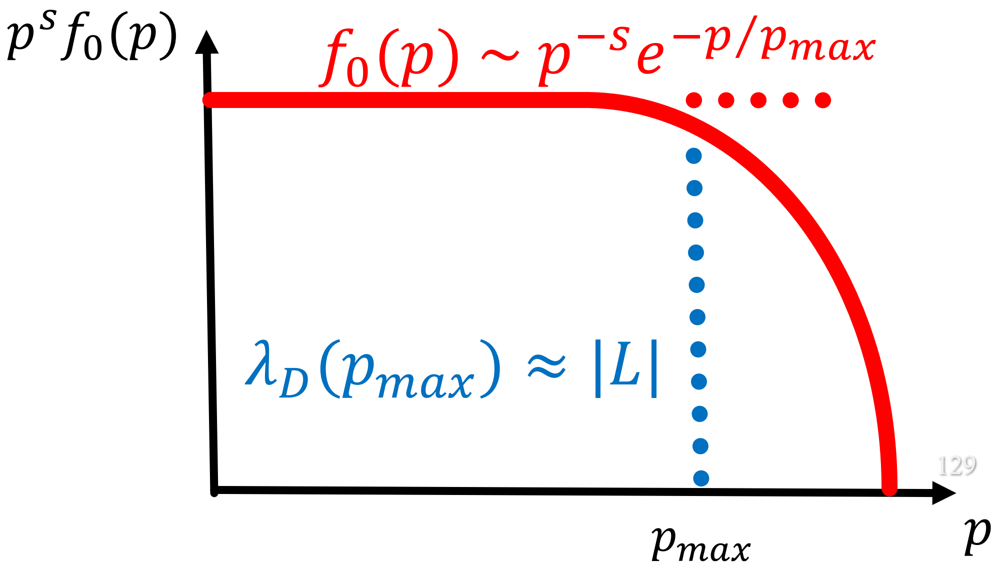


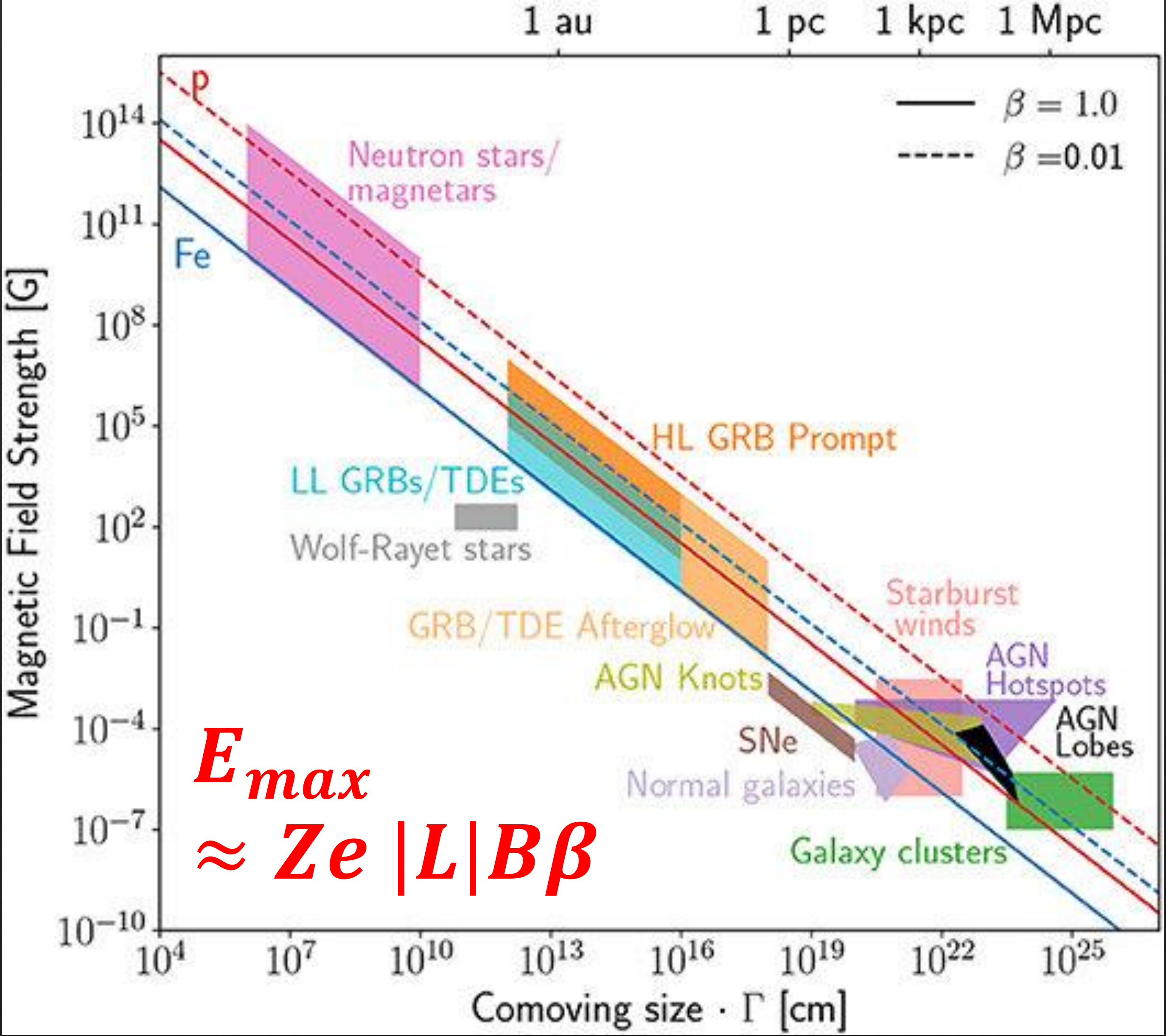
Comments on the maximum energy

- Including a free escape boundary condition in DSA results in the appearence of an exponential suppression in the spectrum

$$D(E_{max}) \approx |L|\, U_1 \;\rightarrow\; E_{max} \approx Ze\, |L| B\, (U_1/c)$$

- This is known as Hillas criterium and it is found assuming the most optimistic diffusion scenario (Bohm ←→ $P(k) = 1$)

The Hillas plot and the origin of UHECRs

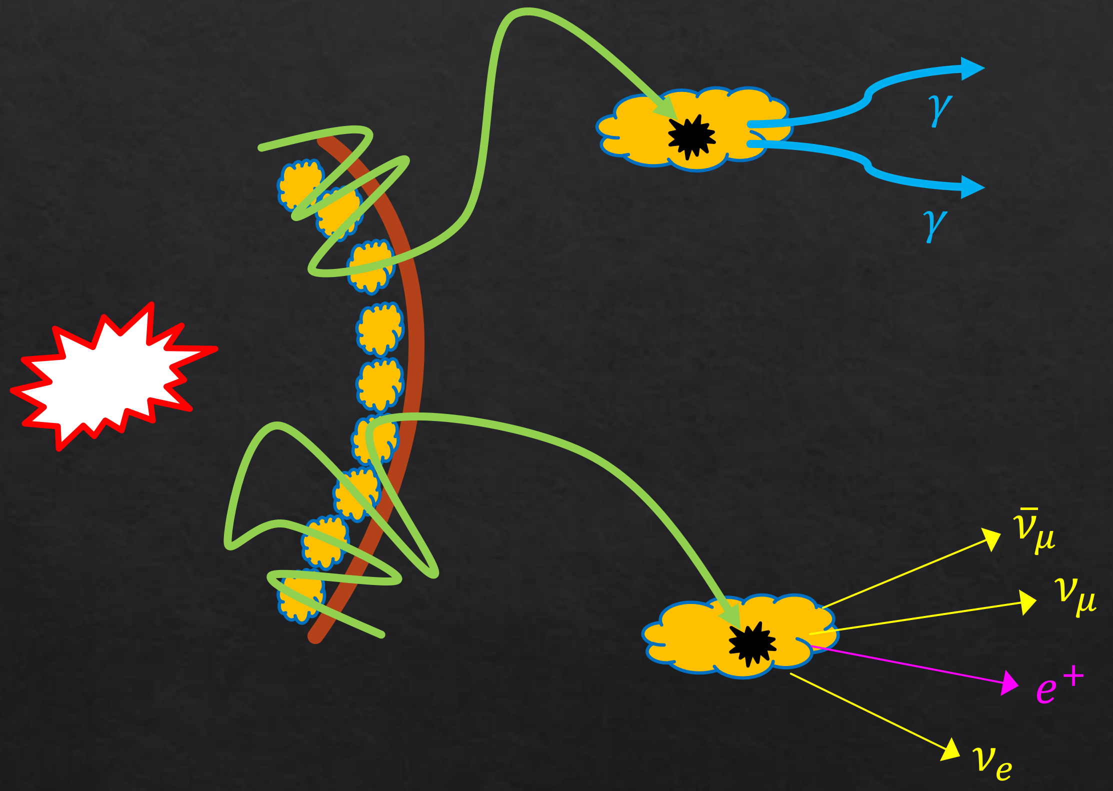


$$E_{max} \approx Ze\,|L|B\beta$$

Supernova remnants at work

- Protons are accelerated at supernova remnant shocks via DSA (inferred spectra $\sim p^{-\alpha}$ with $\alpha \gtrsim 4$)

Supernova remnants at work



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- Protons can encounter dense gas downstream or clouds in the remnant vicinity where target material is higher

Supernova remnants at work

Spectra of pp byproducts

The pion production rate reads

$$dN_\pi(E_\pi)/dt \propto n\, c\, \sigma_{pp}(E_p) f(E_p)$$

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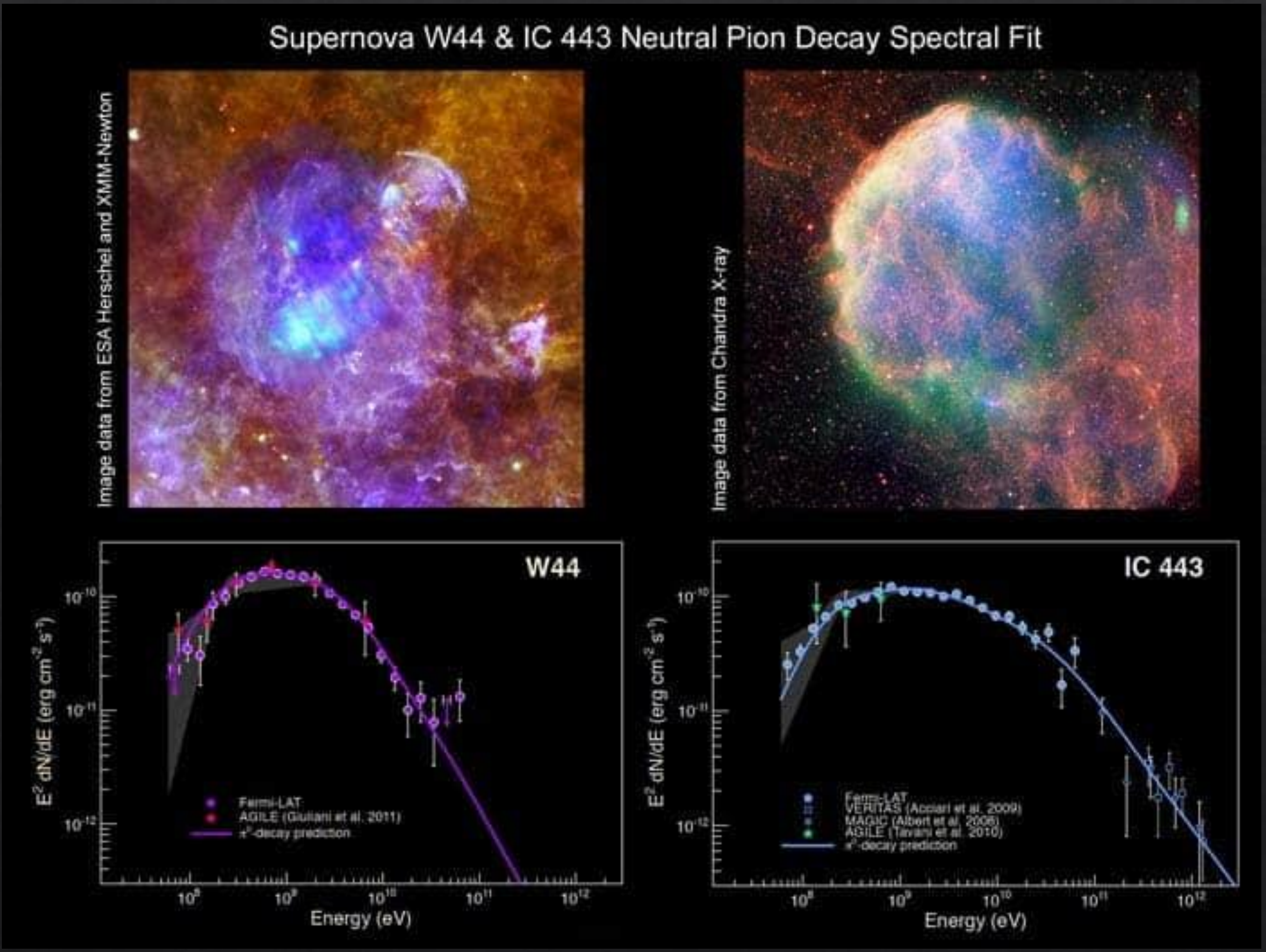
$$\boldsymbol{f(E_p) \sim E_p^{-\alpha} \rightarrow N_{\gamma,\nu}(E_{\gamma,\nu}) \sim E_{\gamma,\nu}^{-\alpha}}$$

- Protons are accelerated at supernova remnant shocks via DSA (inferred spectra $\sim p^{-\alpha}$ with $\alpha \gtrsim 4$)
- Protons can encounter dense gas downstream or clouds in the remnant vicinity where target material is higher
- Gamma rays and neutrinos preserve the power-law index of their parent protons with $E_\gamma \approx 0.1\, E_p$ and $E_\nu \approx 0.05\, E_p$

Supernova remnants at work



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Take home message 2

- Cosmic rays are efficiently accelerated at strong shocks
- The maximum energy is a natural outcome of considering systems with a finite size (or with a limited acceleration time)
- Gamma rays can be expected from the source surroundigs as a result of pp interactions
- Spectral slope of gamma rays (often also the cut off) resemble the spectrum of the accelerated parent protons

Open issues

- Non linearity: CRs leaving a source might develop currents affecting their own confinement

- CRs might modify the shock structure and B waves might not be at reast with the fluid

Outline

- Fundamentals of paricle transport in astrophysical plasma
- Particle acceleration (diffusive shock acceleration)
- Studying and modeling cosmic sources

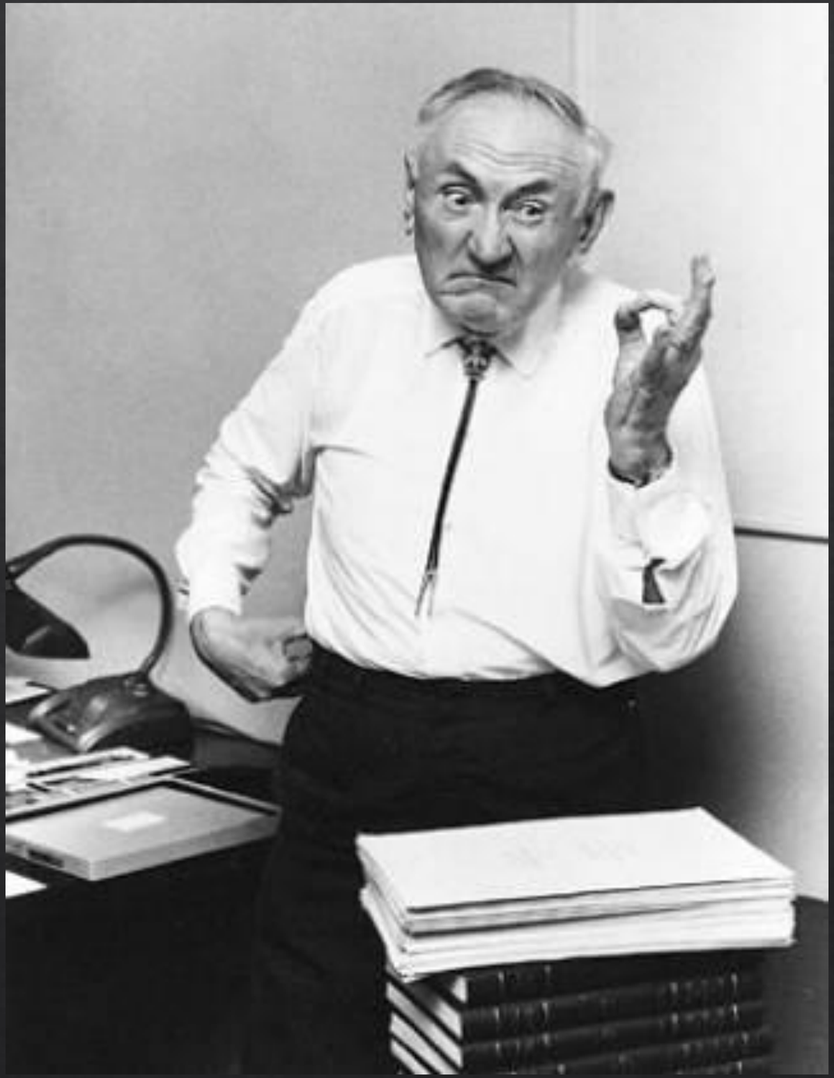
Multi-messenger model of our Galaxy

- Radius: 15 kpc
- Magnetized halo: 4 kpc
- Disk height: 0.1 kpc

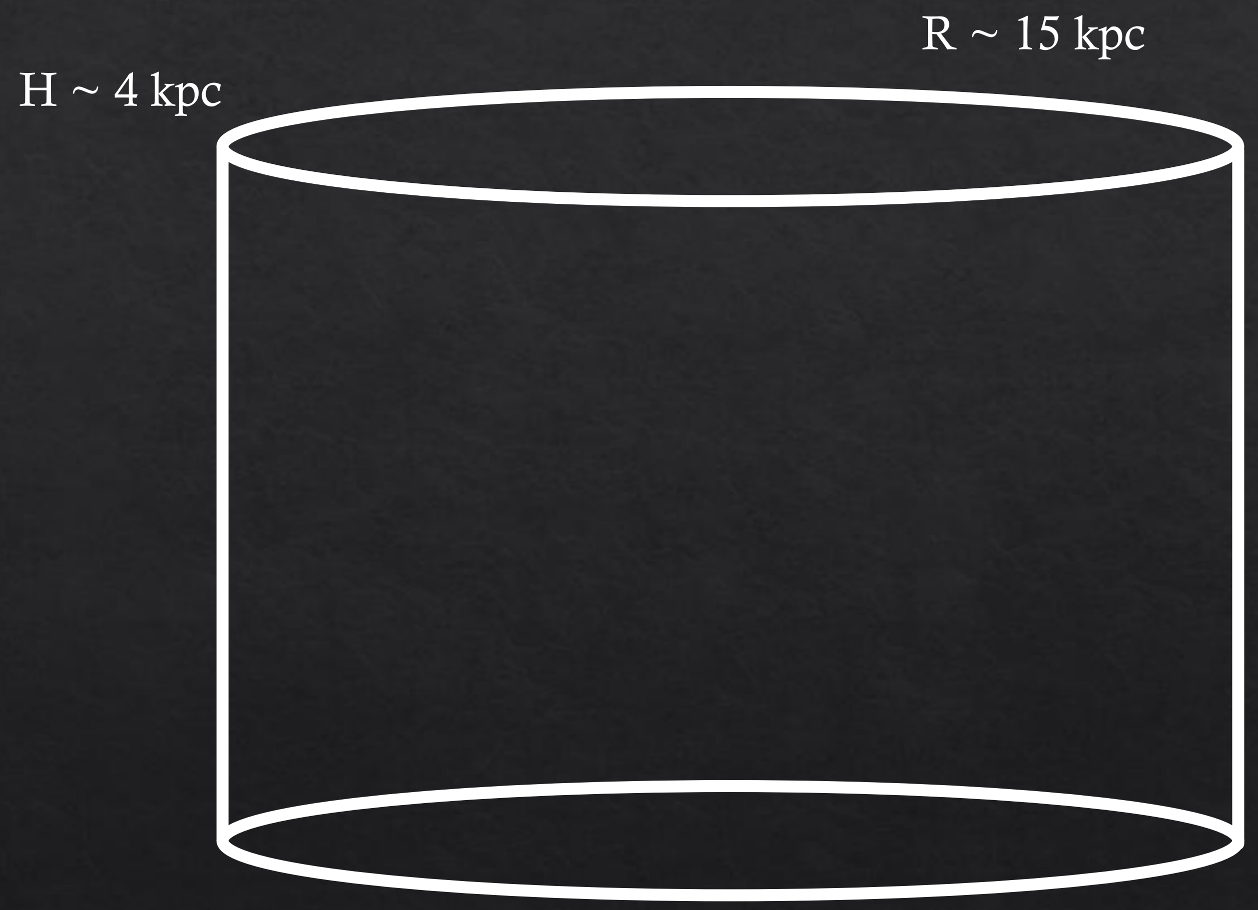
- $E_{SN} = 10^{51}\ erg$
- $\mathcal{R}_{SN} = 0.01\ yr^{-1}$
- $\xi_{CR} = 0.1$

The supernova paradigm

- Energy density of cosmic rays: $U_{CR} \approx 1\ eV/cm^3$
- Total CR energy in the Galaxy → $E_{CR} \approx 10^{56}\ erg$
- Total CR power → $P_{CR} \approx E_{CR}/\tau_{diff} \approx 10^{41}\ erg/s$
- Total power supernovae → $P_{SN} = \mathcal{R}_{SN}\ E_{SN} \approx\ 10^{42}\ erg/s$
- DSA can transfer $\xi_{CR} \approx 0.1$ of P_{SN} into P_{CR}



Leaky-box approach



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- Advection:

  $U = 0 \rightarrow \tau_{adv} = \infty$

- Proton-proton energy losses:

  $n_{ISM} = 1\ cm^{-3}, n_h = 10^{-3}\ cm^{-3}$

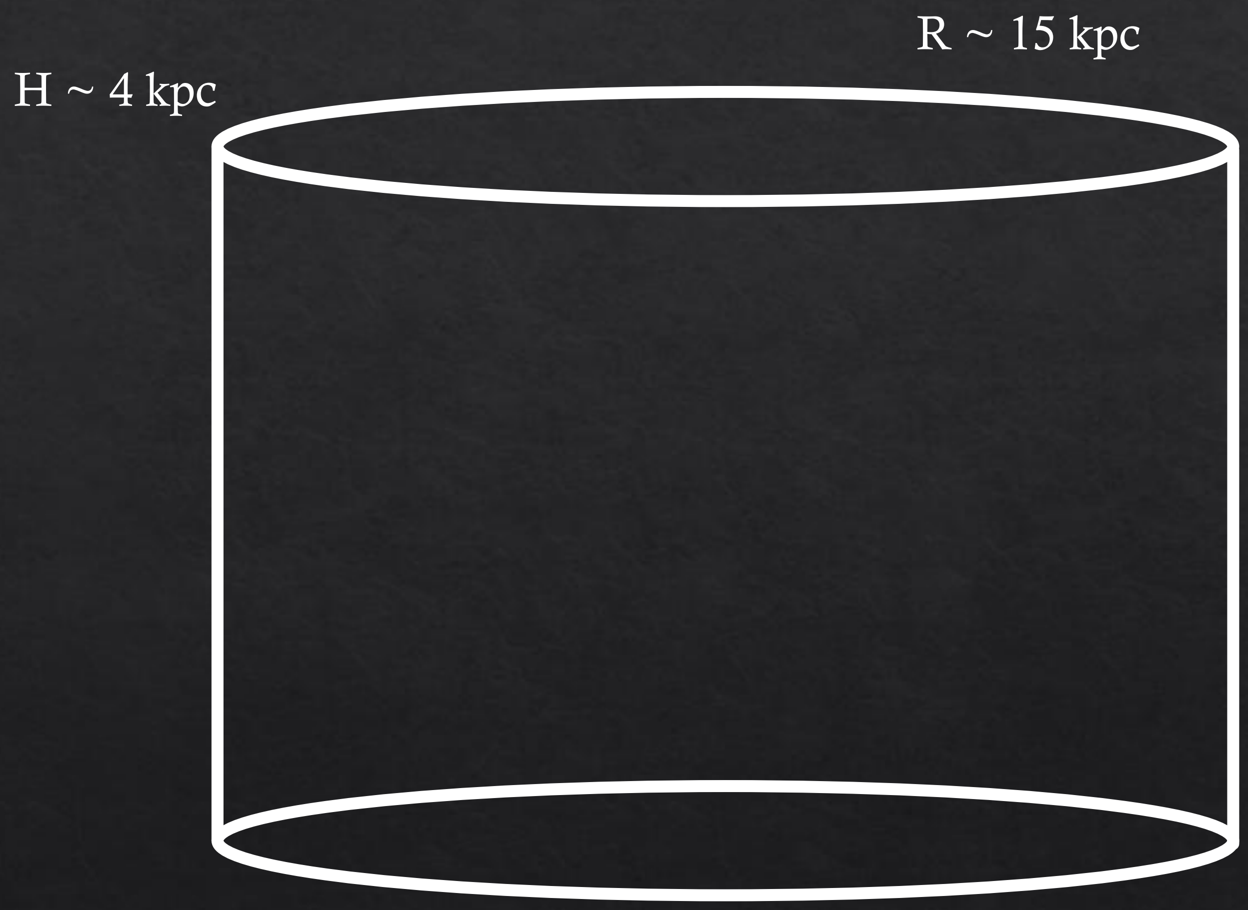
  $\tau_{pp,d} \approx 53\ n_{cm^{-3}}^{-1}$ Myr

  $\tau_{pp,h} \approx 53\ n_{cm^{-3}}^{-1}$ Gyr

- Diffusion:

  D(1 GeV)$\approx 10^{28}\frac{cm^2}{s}$

  $\tau_{diff} \approx 480\ H_{4kpc}^2\ E_{1\ GeV}^{-\delta}\ Myr$



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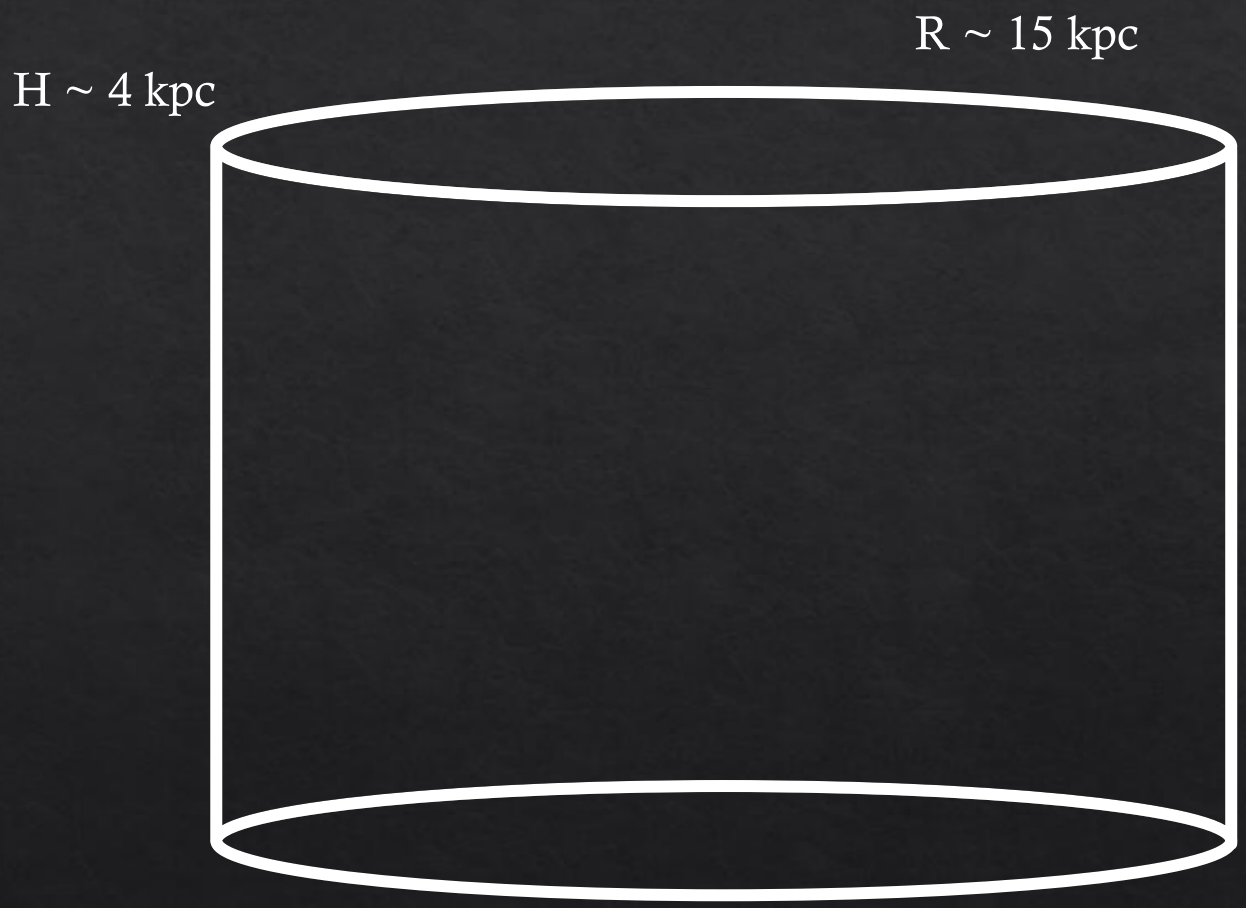

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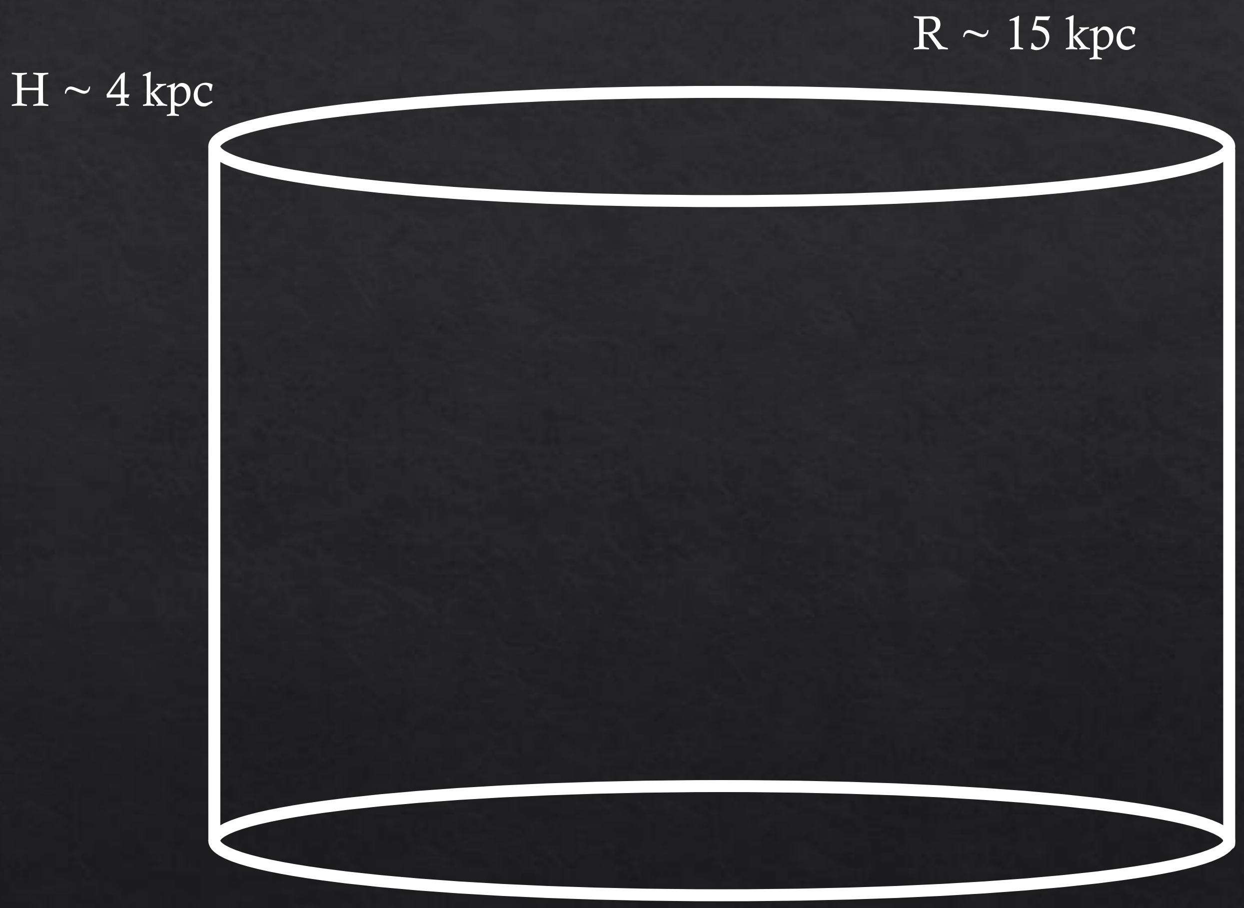

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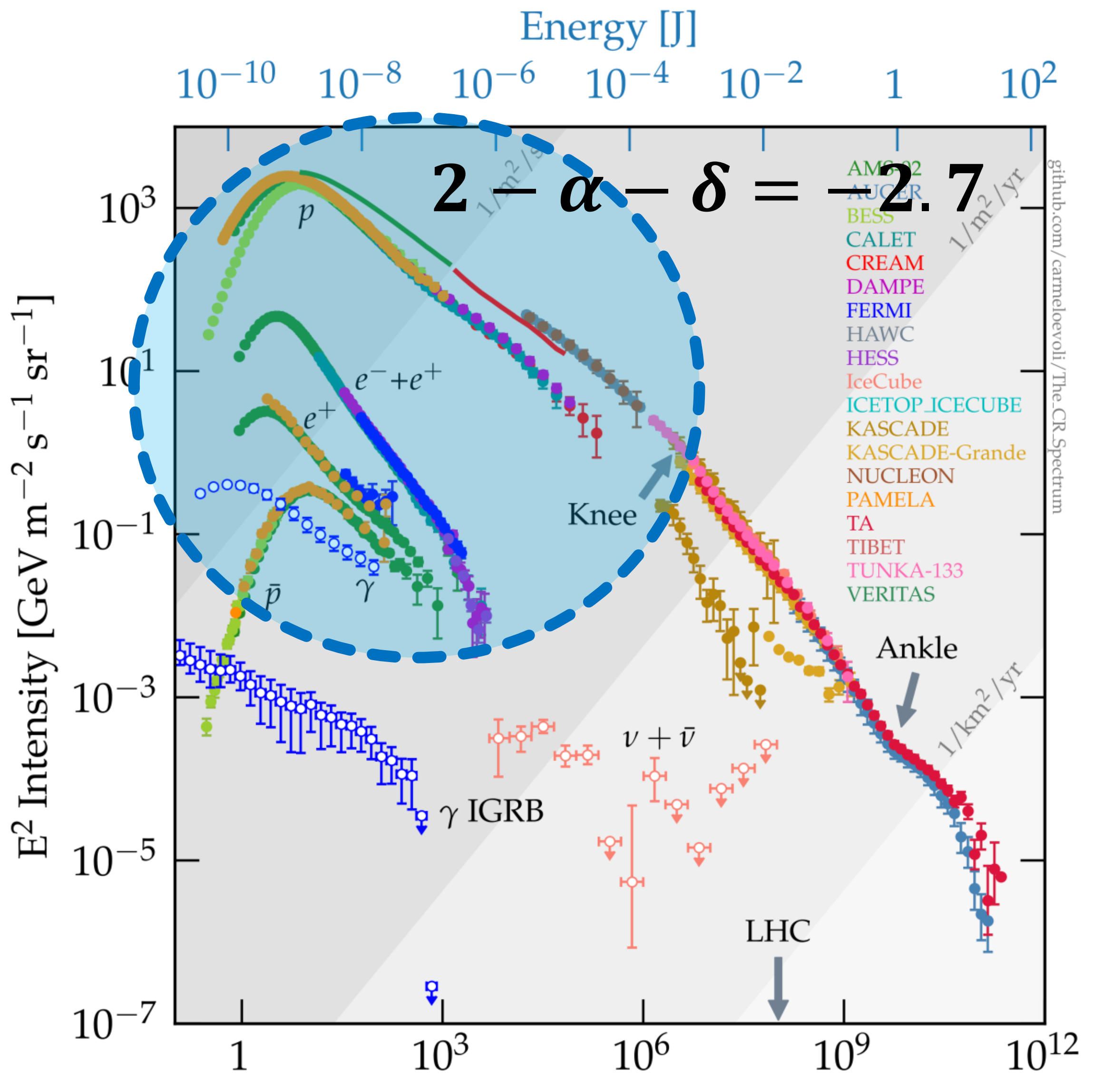

Credit: Evoli

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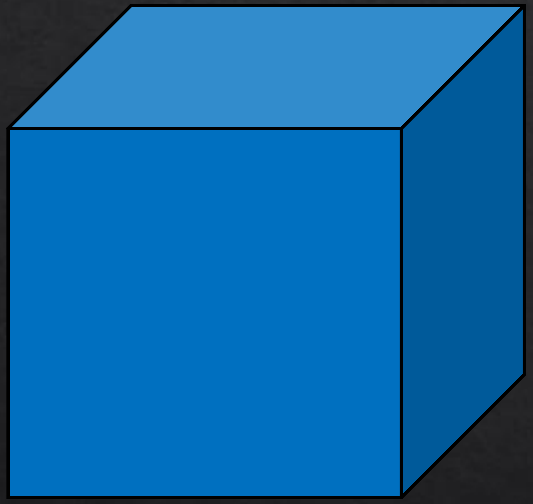
Which gives:

$$L_\gamma^{(d)} \approx 10^{41} erg/s\ \&\ L_\gamma^{(h)} \approx 10^{38} erg/s$$

Results of the leaky box



Halo density
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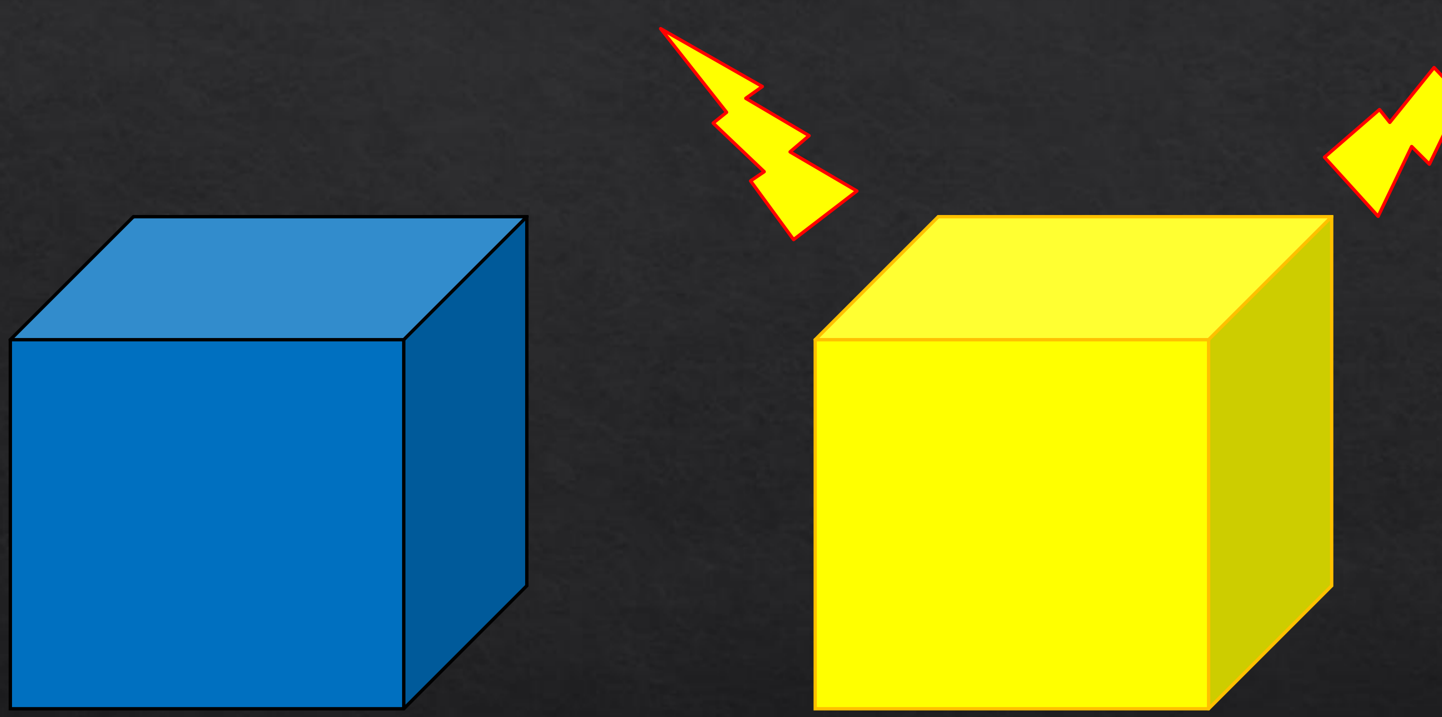

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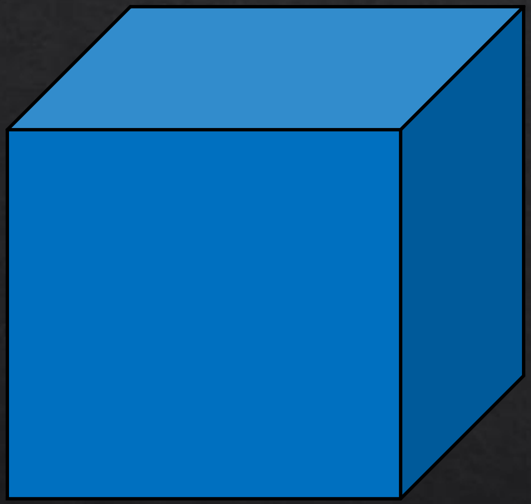

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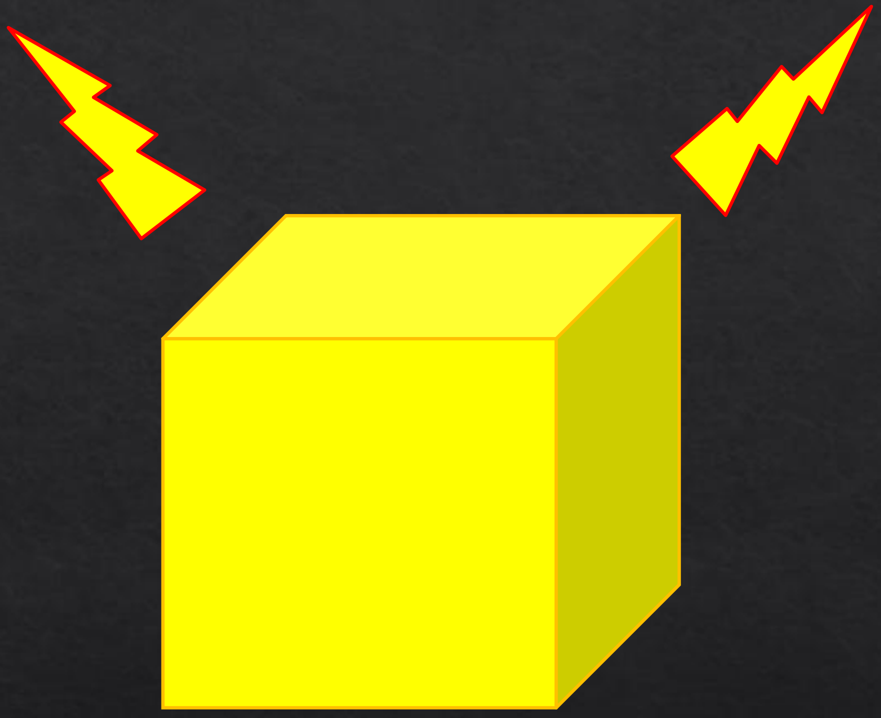

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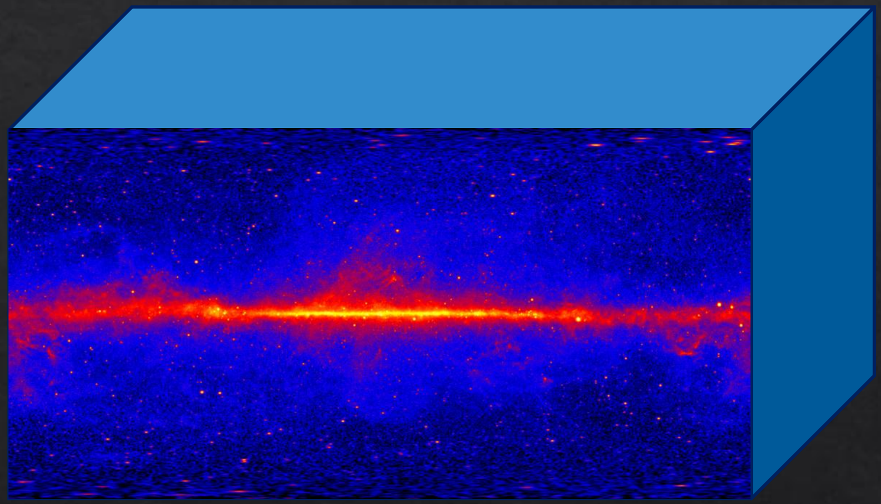

Fermi-LAT sky
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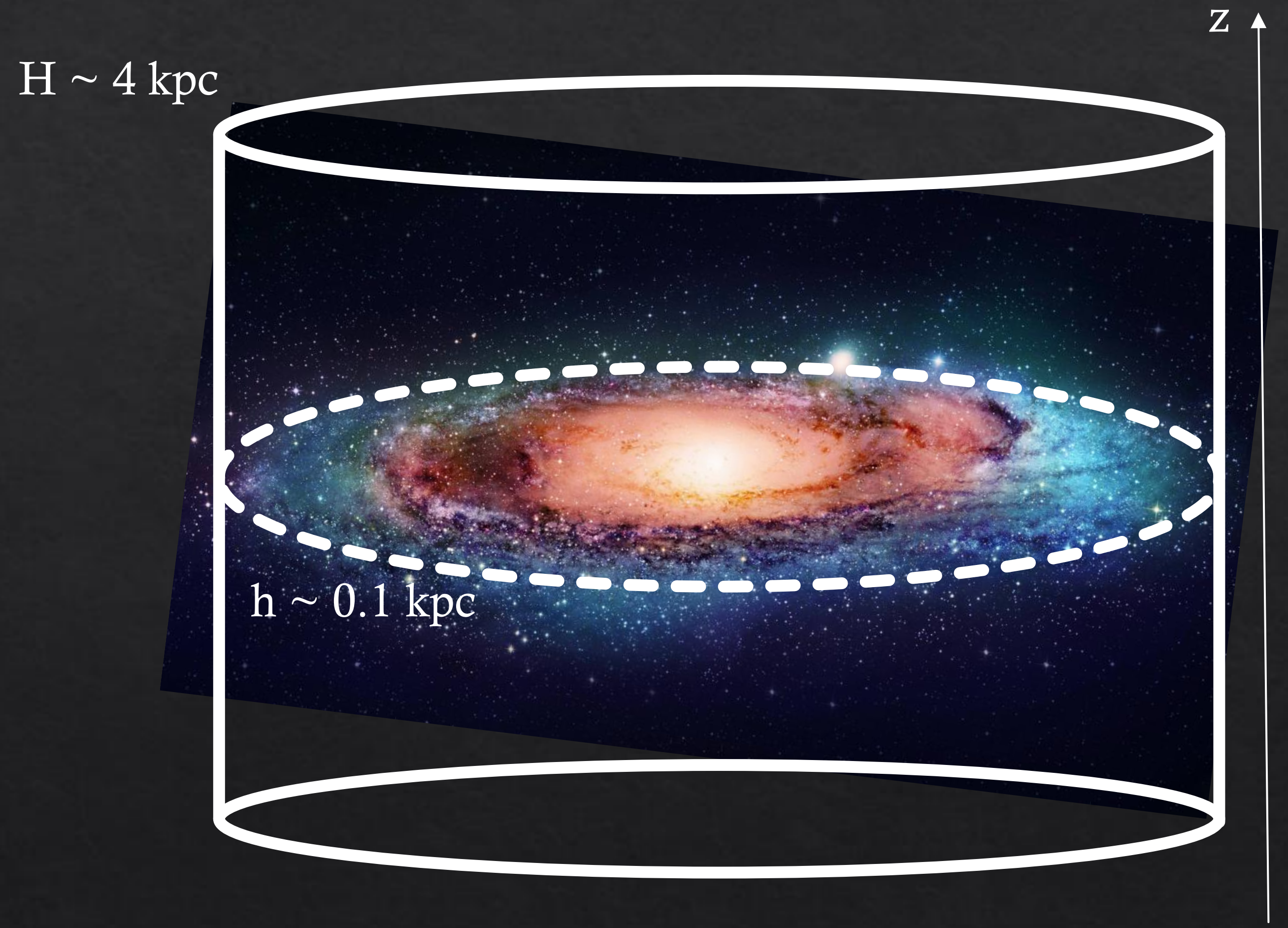
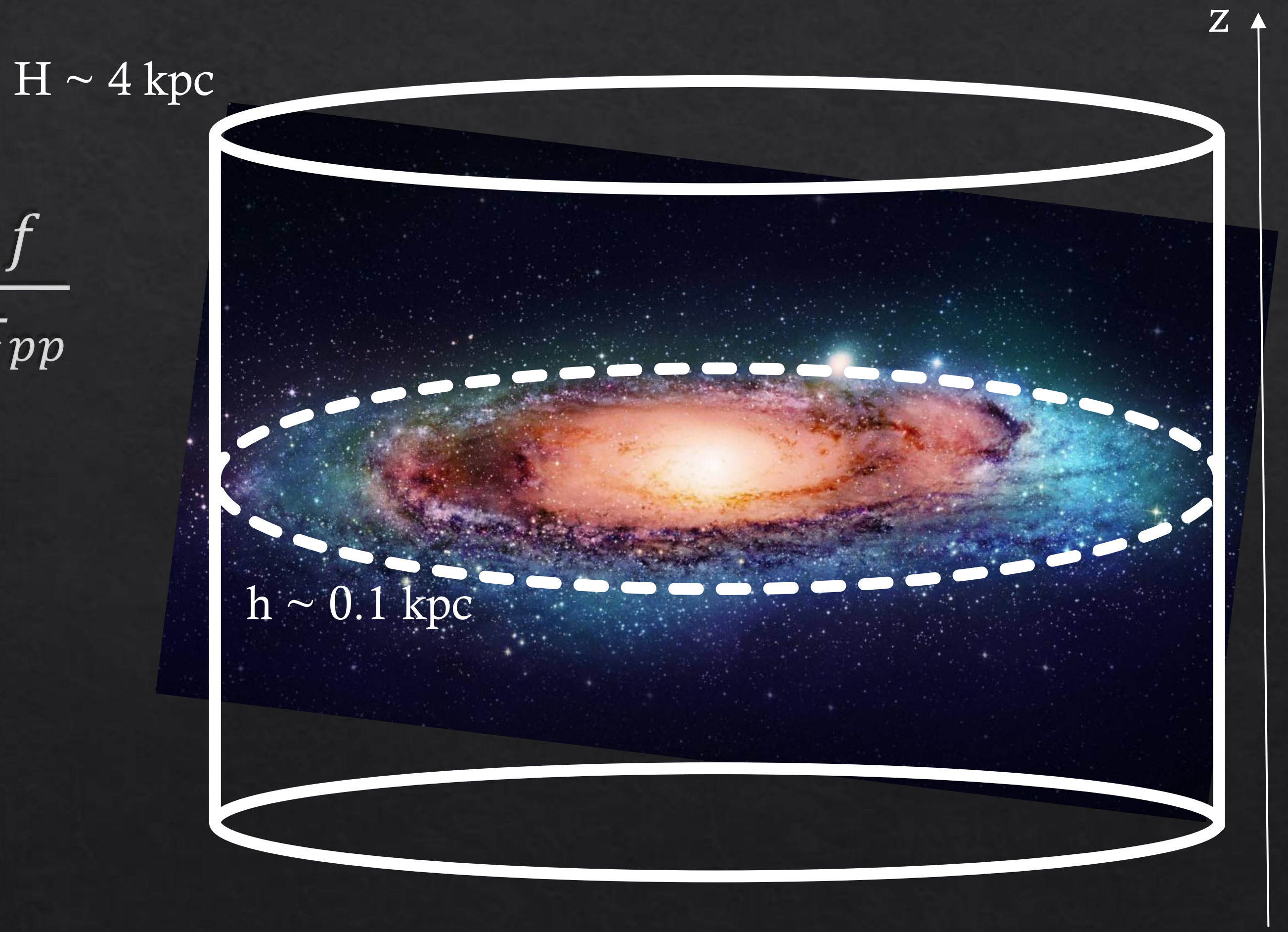
Transport approach to Galactic cosmic rays



Transport approach to Galactic cosmic rays

- Let's solve the 1D transport equation

$$u \frac{\partial f}{\partial z} = \frac{\partial}{\partial z}\left[D \frac{\partial f}{\partial z}\right] + \frac{1}{3} p \frac{\partial f}{\partial p} \frac{\partial u}{\partial z} + Q - \frac{f}{\tau_{pp}}$$



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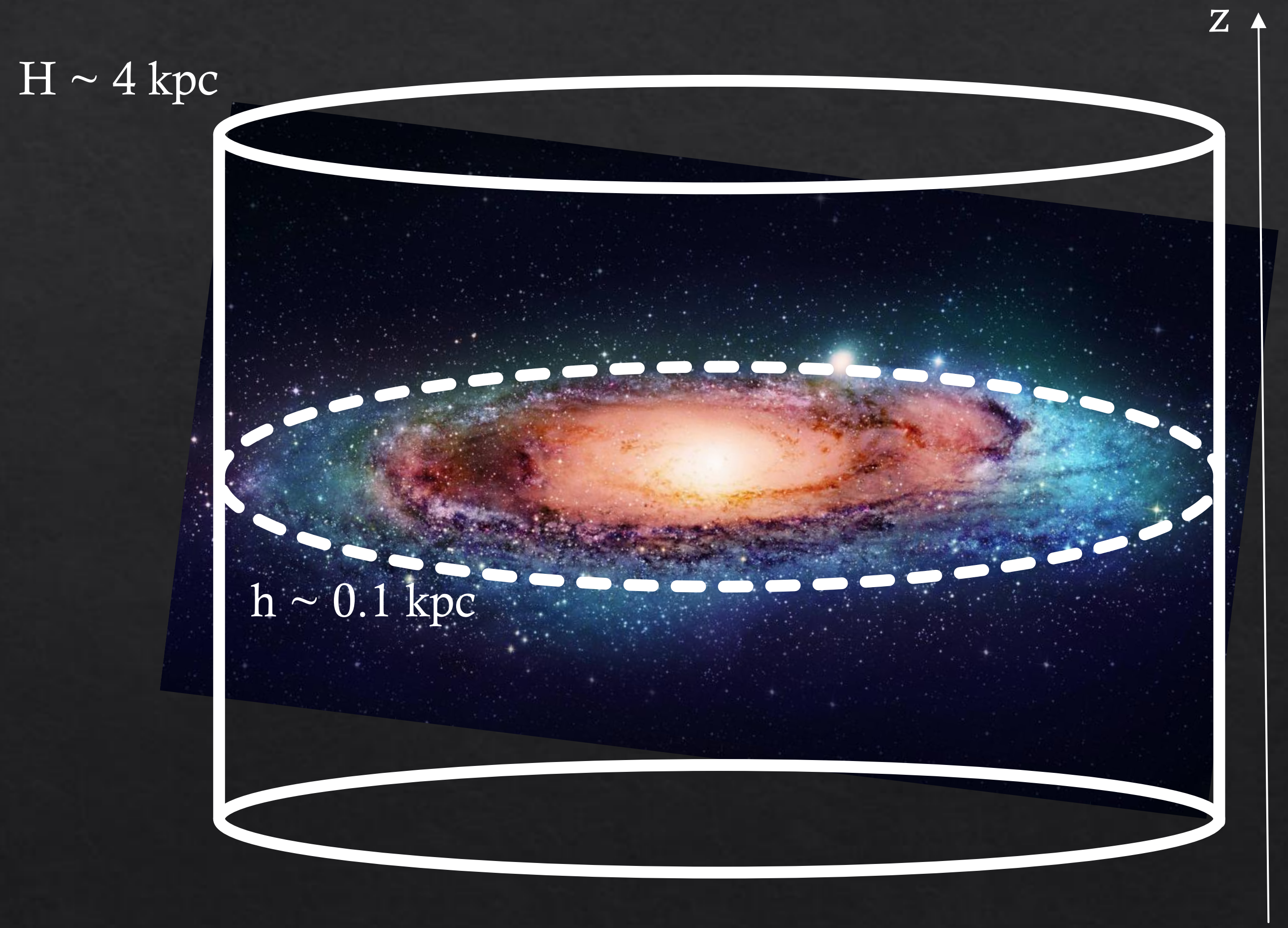

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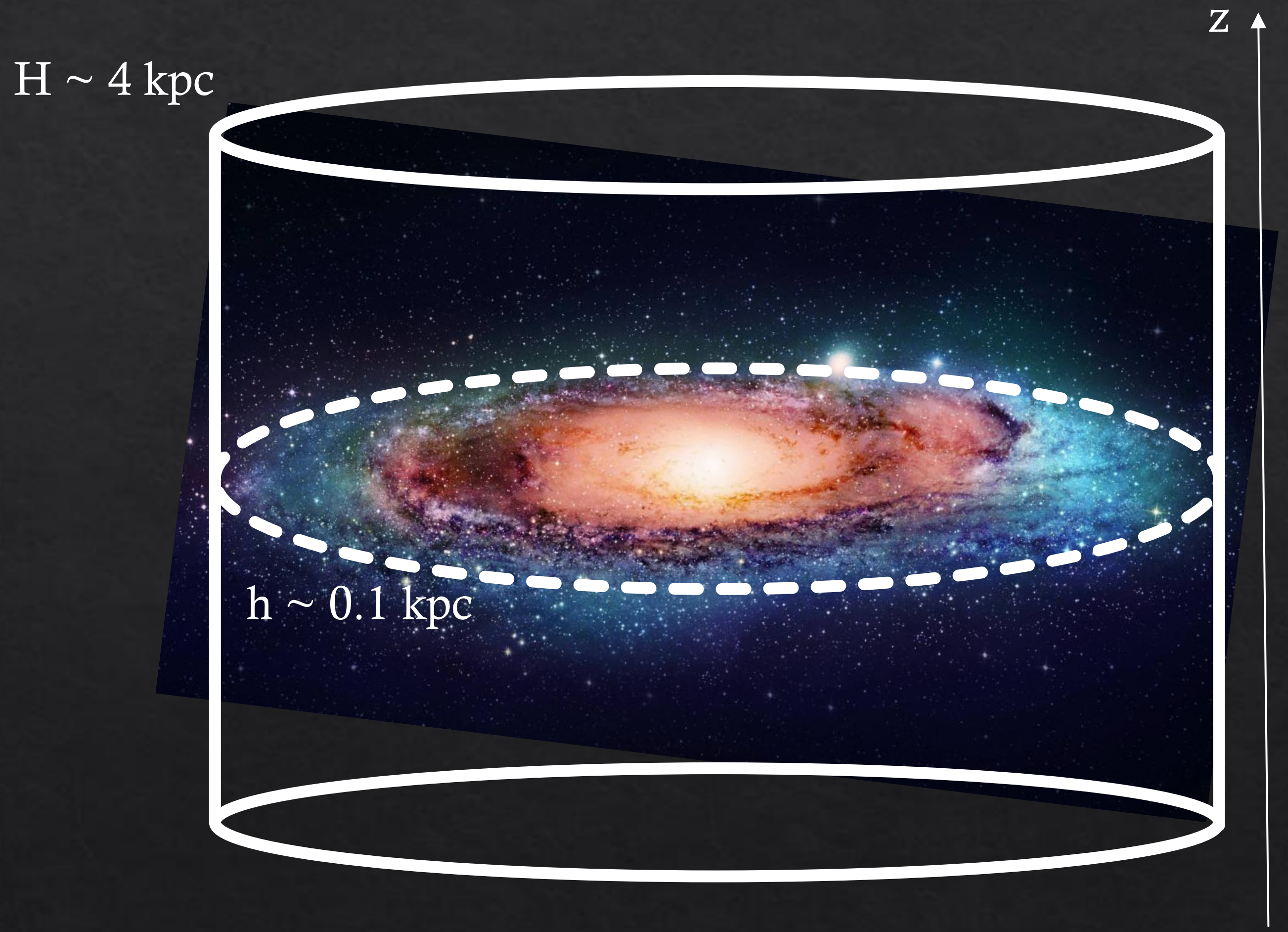
4. $Q(z, p) \propto h\,\delta[z]$ and $\tau_{pp} \propto h\,\delta[z]$



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Transport approach to Galactic cosmic rays

Gamma-ray luminosity

- We have now a space-dependent model where interactions practically take place only in the disc

$$L_\gamma = \xi_{CR} \mathcal{R}_{SN} E_{SN} \tau_{diff} {\tau_{pp}^{*}}^{-1}$$

Where now the effective density reads

$$n_{eff} = n_d (h/H)$$

$\rightarrow L_\gamma \approx 10^{39}\ erg/s$

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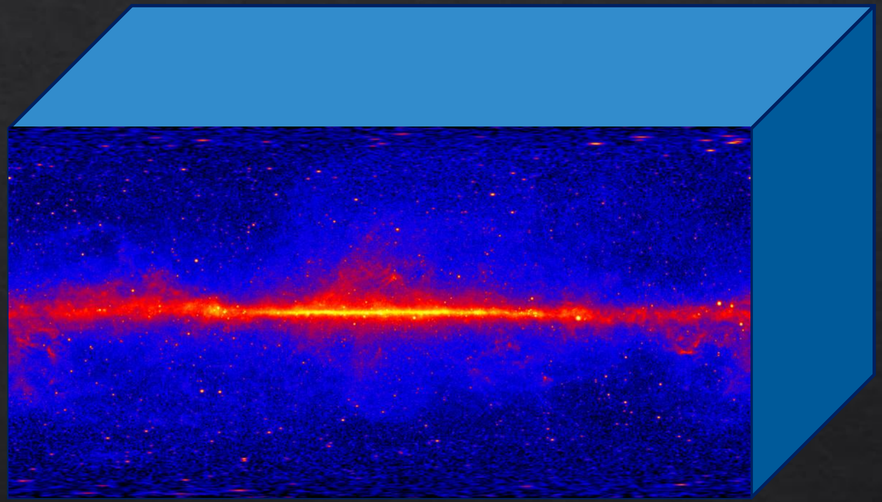
Results of the transport model

Result of transport



Fermi-LAT sky

Take home message 3

- Supernova remnants are powerful cosmic-ray factories – perhaps accounting for the bulk of cosmic rays observed in our Galaxy
- The leaky box approximation is extremely useful for quantitative estimates but it must be adopted with some grain of salt
- The spatial dependent transport shall be adopted to properly model morphology

Open issues

- Advection
- Space-dependent diffusion
- Time-dependent features

Thank you!