Perturbative QCD in Quarkonium Production (PQQP)

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- regular meetings on ZOOM
- visit by I. Babiarz and W. Schäfer in Orsay, 23.6.-28.6. 2024, funded by IFJ PAN
- second visit by W.S., 17.9. - 21.9. 2024 as referee for PhD defence of Y. Yedelkina, visit funded by J.-P. Lansberg.
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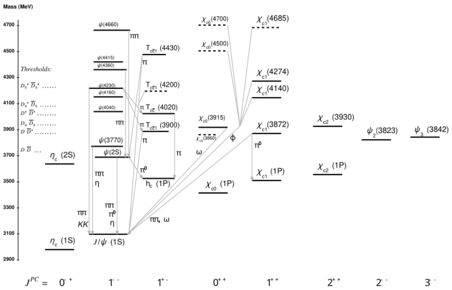
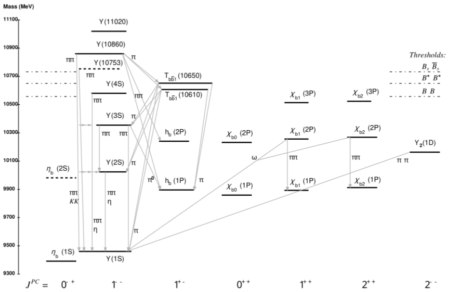
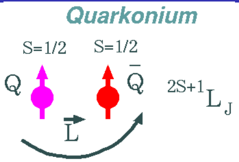
$\gamma^*\gamma^*$ transition form factors of pseudoscalar quarkonia: relativistic and next-to-leading order α_s corrections

I. Babiarz, C. Flett, M. A. Ozcelik, W. Schäfer and A. Szczurek,
paper in preparation

Introduction



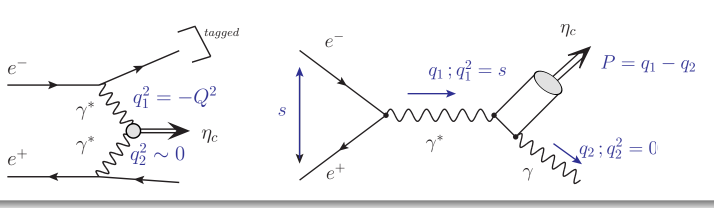




- Quarkonia $\equiv$ bound states of heavy quark and antiquark ($c\bar{c}$ or $b\bar{b}$).
- Spectra and many properties of states below open heavy flavor thresholds are well described by potential models of $Q\bar{Q}$ interactions.
- The ground state of the quarkonium system is a pseudoscalar $Q\bar{Q}$-state with vanishing orbital angular momentum (S–wave) in the spin–singlet.

$$\eta_Q : {}^1S_0, \quad J^{PC} = 0^{-+}$$

- The positive C-parity implies decay/coupling to two photons.



- We are interested in the coupling of η_Q to two (off-shell) photons

$$\mathcal{M}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_Q) = i\,4\pi\alpha_{\rm em}\,\varepsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu\epsilon_2^\nu q_1^\alpha q_2^\beta \underbrace{\mathcal{F}_{\eta_Q}(t_1,t_2)}_{\text{transition FF}} \qquad t_1 = \frac{q_1^2}{m_Q^2}, \quad t_2 = \frac{q_2^2}{m_Q^2}\ .$$

- the $\gamma^*\gamma^*$ transition form factor describes the following observables:

1. **on-shell**: $t_1 = t_2 = 0$: the decay width $\eta_Q \to \gamma\gamma$.
2. **space-like region**: $t_1 < 0, t_2 = 0$: exclusive production of η_Q in *single-tagged* e^+e^- collisions. $t_1 < 0, t_2 < 0 \to$ *double tagged* e^+e^- collisions.
3. **time–like region**: $t_1 > 0, t_2 = 0$: exclusive production of $\eta_Q\gamma$ in e^+e^- annihilation; Dalitz decay $\eta_Q \to \gamma\ell^+\ell^-$ or $\eta_Q \to 4\ell$.

Non-Relativistic Quantum Chromo-Dynamics (NRQCD)

- a good approximation to the potential between heavy quarks has a Coulomb and linear term (Cornell potential):

$$V(r) = -\frac{C_F \alpha_s}{r} + \sigma r$$

- From Ehrenfest's relation, we have for the relative velocity v of quarks in the bound state, $m_Q v^2 \sim V(r)$. For a Coulomb bound state using $r \sim 1/(m_Q v)$, we obtain $v \sim \alpha_s$. Empirically for $c\bar{c}$, $v^2 \sim \alpha_s \sim 0.3$.
- the **effective field theory** derived from QCD that allows for a systematic expansion in the small parameters v and α_s is NRQCD (Bodwin, Braaten & Lepage (1995)).
- The η_Q–state has an expansion, with a power counting in v motivated by a multipole expansion

$$\begin{aligned} |\eta_Q\rangle &= \mathcal{O}(v^0)\Big|[Q\bar{Q}](^1S_0^{[1]})\Big\rangle + \mathcal{O}(v)\Big|[Q\bar{Q}](^1P_1^{[8]})g\Big\rangle + \mathcal{O}(v^2)\Big|[Q\bar{Q}](^3S_1^{[8]})g\Big\rangle \\ &+ \mathcal{O}(v^2)\Big|[Q\bar{Q}](^1S_0^{[8]})gg\Big\rangle + \dots . \end{aligned}$$

  also operators made of quark field ψ and antiquark field χ have a definite power counting in v.
- short distance degrees of freedom $r < 1/m_Q$ are “integrated out” and described by perturbative QCD.

Relativistic corrections to the transition form factor

- The NRQCD factorization formula for the transition FF to relative order v^2 reads:

$$\mathcal{F}_{\eta_Q}(t_1,t_2) = \frac{C_0(t_1,t_2)}{m_Q^2}\Big\langle \eta_Q\Big|\psi^\dagger\chi\Big|0\Big\rangle + \frac{D_0(t_1,t_2)}{m_Q^4}\Big\langle \eta_Q\Big|\psi^\dagger\Big(-i\frac{\mathbf{D}}{2}\Big)^2\chi\Big|0\Big\rangle + \dots .$$

- **long distance physics** is contained in the matrix elements (LDMEs):
  1. wave function at origin

$$\langle \eta_Q|\psi^\dagger\chi|0\rangle = \sqrt{2M_{\eta_Q}}\langle \eta_Q|\psi^\dagger\chi|0\rangle_{\rm BBL} = \sqrt{2M_{\eta_Q}}\sqrt{\frac{N_c}{2\pi}}R_{\eta_Q}(0)\,.$$

  2. effective expansion parameter:

$$\langle v^2\rangle_{\eta_Q} = \frac{\Big\langle \eta_Q\Big|\psi^\dagger\Big(-i\frac{\mathbf{D}}{2}\Big)^2\chi\Big|0\Big\rangle}{m_Q^2\Big\langle \eta_Q\Big|\psi^\dagger\chi\Big|0\Big\rangle}\,.$$

- **short distance physics** is contained in the short-distance coefficients $C_0(t_1,t_2)$ and $D_0(t_1,t_2)$ which carry the dependence on photon virtualities t_1, t_2 and are calculable in perturbative QCD.
- as we assign the same smallness to v^2 and α_s, we should evaluate coefficient $C_0(t_1,t_2)$ to one–loop order in pQCD.

Short distance coefficients in the v^2 expansion

- To extract short distance coefficients C_0, D_0, one replaces the η_Q by a plane-wave $Q\bar{Q}$ state coupled to ${}^1S_0^{[1]}$ quantum numbers.
- calculate $\gamma^*\gamma^* \to [Q\bar{Q}]({}^1S_0^{[1]})$ in perturbative QCD
  1. We write the quark and antiquark momenta as
     $$p_Q = \frac{1}{2}P + k\,, \quad p_{\bar{Q}} = \frac{1}{2}P - k\,, \quad Q\bar{Q}\ \text{rest frame}: P = (2E, \vec{0})\,, \quad k = (0, \vec{k})\,, \quad E = \sqrt{m_Q^2 + \vec{k}^2}\,.$$
     and introduce
     $$\tau = -\frac{t_1 + t_2}{4}\,, \quad \omega = \frac{t_1 - t_2}{t_1 + t_2}\,.$$
  2. the pQCD form factor has the same short distance coefficients as in the NRQCD expansion:
     $$\mathcal{F}_{{}^1S_0}(t_1, t_2) = \frac{C_0(t_1, t_2)}{m_Q^2}\Big\langle [Q\bar{Q}]({}^1S_0^{[1]})\Big|\psi^\dagger\chi\Big|0\Big\rangle + \frac{D_0(t_1, t_2)}{m_Q^4}\Big\langle [Q\bar{Q}]({}^1S_0^{[1]})\Big|\psi^\dagger\Big(-i\frac{\mathbf{D}}{2}\Big)^2\chi\Big|0\Big\rangle + \dots$$
  3. expanding in $\vec{k}^2/m_Q^2$, we obtain:
     $$C_0(t_1, t_2) = \frac{e_f^2}{2m_Q}\frac{1}{1+\tau}\,, \quad D_0(t_1, t_2) = \frac{\omega^2\tau^2 - 3\tau^2 - 7\tau - 5}{3(1+\tau)^2}\,C_0(t_1, t_2)\,.$$

Matching to NRQCD, transition FF of η_Q

$$\tau = -\frac{t_1 + t_2}{4}\,, \quad \omega = \frac{t_1 - t_2}{t_1 + t_2}\,.$$

LO TFF of η_Q

$$\mathcal{F}_0(t_1, t_2) = \frac{e_f^2}{2m_Q^3}\frac{1}{1+\tau}\left\langle \eta_Q \middle| \psi^\dagger \chi \middle| 0 \right\rangle .$$

η_Q TFF up to order v^2

$$\begin{aligned} \mathcal{F}_{\eta_Q}(t_1, t_2) &= \frac{1}{m_Q^2}\left\langle \eta_Q \middle| \psi^\dagger \chi \middle| 0 \right\rangle \Big(C_0(t_1, t_2) + D_0(t_1, t_2)\langle v^2 \rangle_{\eta_Q} + \dots \Big) \\ &= \mathcal{F}_0(t_1, t_2)\Big(1 + \frac{\omega^2\tau^2 - 3\tau^2 - 7\tau - 5}{3(1+\tau)^2}\langle v^2 \rangle_{\eta_Q} + \dots \Big) \end{aligned}$$

- for $t_1 = t_2 = 0$ we can calculate the $\eta_Q \to \gamma\gamma$ decay width. We checked that up to order v^4 we agree with the result of Bodwin & Petrelli (2009).
- the value of $\langle v^2 \rangle_{\eta_Q}$ must be determined by data or potential models.

All–order summation of v^2 corrections

- phenomenologically, in the charm sector v^2 corrections turn out to be large ($\sim 30 \div 40\%$ for the $\eta_c \to \gamma\gamma$ width). As suggested by Bodwin et al. (2008), one can sum up v^2–corrections that stem from a certain class of operators

$$\mathcal{F}_{\eta_Q}(t_1, t_2) = \sum_n c_n(t_1, t_2) \Big\langle \eta_Q \Big| \psi^\dagger \Big(-i\frac{\mathbf{D}}{2}\Big)^{2n} \chi \Big| 0 \Big\rangle = \Big\langle \eta_Q \Big| \psi^\dagger \chi \Big| 0 \Big\rangle \sum_n c_n(t_1, t_2) \, \langle \vec{k}^{2n} \rangle_{\eta_Q} \, .$$

- We exploit the fact, that we can obtain the pQCD transition–FF $\mathcal{F}_{^1S_0}(t_1, t_2)$ for $\gamma^*\gamma^* \to [Q\bar{Q}](^1S_0^{[1]})$ to all orders in $\vec{k}^2/m_Q^2$, so that we determine:

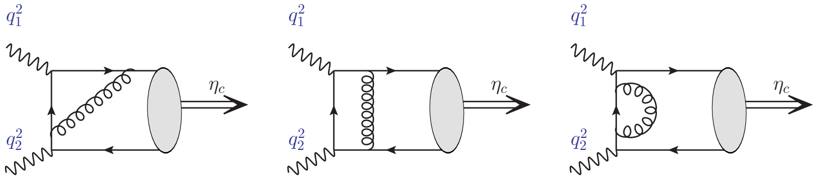
$$c_n(t_1, t_2) = \frac{1}{2\sqrt{2N_c}} \frac{1}{n!} \Big(\frac{\partial^n}{\partial \vec{k}^{2n}} \frac{\mathcal{F}_{^1S_0}(t_1, t_2)}{E(\vec{k}^2)} \Big) \Big|_{\vec{k}^2 = 0} \, .$$

- following Bodwin et al., we assume $\langle \vec{k}^{2n} \rangle_{\eta_Q} = \langle \vec{k}^2 \rangle^n_{\eta_Q} = (m_Q^2 \langle v^2 \rangle_{\eta_Q})^n$, so that we can sum up the series to

all-order v^2 TFF

$$\mathcal{F}_{\eta_Q}(t_1, t_2) \quad = \quad \frac{\langle \eta_Q | \psi^\dagger \chi | 0 \rangle}{2\sqrt{2N_c}} \frac{\mathcal{F}_{^1S_0}(t_1, t_2, \langle \vec{k}^2 \rangle_{\eta_Q})}{E(\langle \vec{k}^2 \rangle_{\eta_Q})} , \quad \langle \vec{k}^2 \rangle_{\eta_Q} = m_Q^2 \langle v^2 \rangle_{\eta_Q}$$

Perturbative α_s corrections



We can express the **perturbative correction** to the form-factor as follows

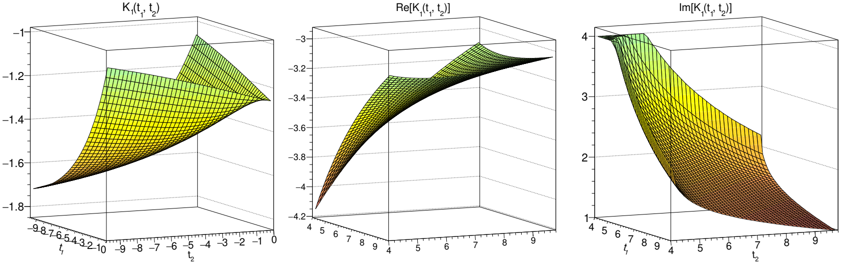
$$\mathcal{F}(t_1, t_2) = \mathcal{F}_0(t_1, t_2)\left(1 + \frac{\alpha_s}{\pi} C_F K_1(t_1, t_2)\right) + \mathcal{O}\left(\alpha_s^2\right), \quad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}.$$

- Utilizing packages `FeynArts`, `FeynCalc`, `Apart`, `FIRE` we obtain a compact expression for $K_1(t_1, t_2)$ in terms of multiple polylogarithms

$$G(a_1, ..., a_n; x) = \int_0^x \frac{dt}{t - a_1} G(a_2, ..., a_n; t).$$

- We reproduce the well known on-shell-limit $K_1(0,0) = \frac{\pi^2}{8} - \frac{5}{2}$, as well as $K_1(t, 0)$ in the space-like (Sang & Chen, 2010) and time–like (Feng et al. (2015)) regions.
- N.b.: on-shell (Abreu et al. (2023)) and for one virtual photon (Feng et al. (2010), numerically), the TFF is known to two–loops.

perturbative correction $K_1(t_1, t_2)$



- the perturbative correction as a function of *two virtualities* is obtained for the first time.
- For **spacelike** virtualities K_1 is a real function. It is negative in the whole domain.
- In the **timelike** region, for $t_i > 4$, we obtain also a nonvanishing imaginary part. It is a completely new result.
- As the Born result $\mathcal{F}_0(t_1, t_2)$ is *purely real*, both in spacelike and timelike domain, the imaginary part enters formally only at α_s^2.
- For charmonium, $\alpha_s \sim 0.3$, so that function K_1 enters with prefactor $\alpha_s C_F/\pi \sim 0.127$

Decay width for $\eta_c \to \gamma\gamma$

Table: Transition form factor at the on-shell point $F_{\eta_c}(0,0)$ and radiative decay width for two sets of parameters: (1) $|\mathrm{R}(0)|^2 = 0.9089$ GeV^3 and $< v^2 >= 0.226$, $m_c = 1.4$ GeV, (2) $|\mathrm{R}(0)|^2 = 0.881$ GeV^3 and $< v^2 >= 0.3$, $m_c = 1.5$ GeV . We used $\alpha_s = 0.3$ everywhere.

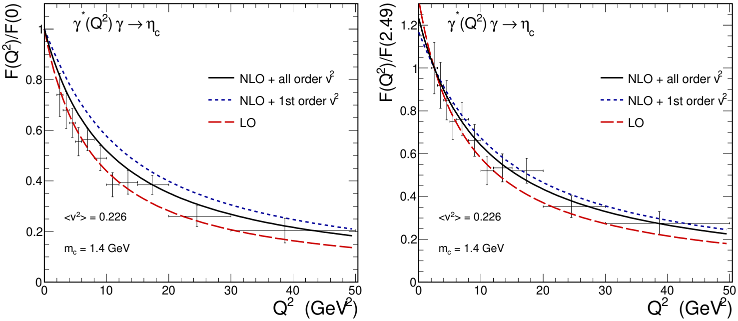
	F(0,0) [GeV^{-1}]	$\Gamma_{\eta_c \to \gamma\gamma}$ [keV]	F(0,0) [GeV^{-1}]	$\Gamma_{\eta_c \to \gamma\gamma}$ [keV]
LO	0.13	18.9	0.10	12.09
LO + 1st corr. v^2	0.081	7.33	0.052	3.02
LO + all order v^2	0.093	9.55	0.067	5.03
NLO	0.106	12.4	0.087	8.51
NLO + 1st corr. v^2	0.056	3.58	0.035	1.39
NLO + all order v^2	0.068	5.17	0.05	2.83
PDG		5.1 ± 0.4		5.1 ± 0.4

- two-photon decay width obtained from FF at on-shell point

$$\Gamma_{\eta_c \to \gamma\gamma} = \frac{\pi}{4} \alpha_{\mathrm{em}}^2 M_{\eta_c}^3 |\mathcal{F}_{\eta_c}(0,0)|^2$$

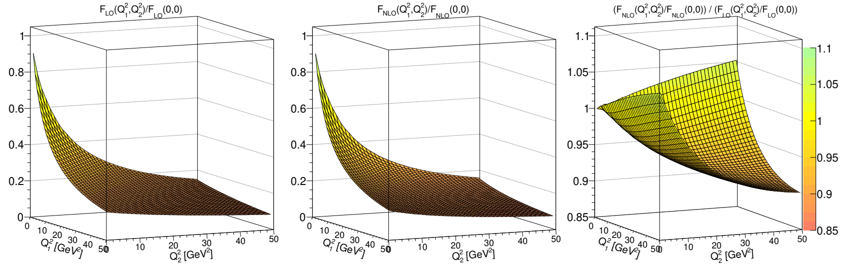
- large $\mathcal{O}(v^2)$ corrections are somewhat mitigated by all-order v^2 summation.

$\mathcal{F}_{\eta_c}(t,0) \equiv F(Q^2)$ from single-tagged e^+e^- collisions



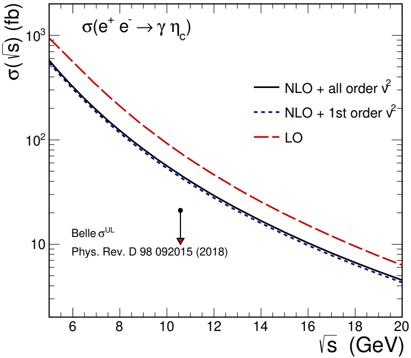
- The Belle collaboration published data for the ratio $F(Q^2)/F(0)$, thus normalizing to the rate for η_c production in *untagged* collisions.
- 1st order NLO and v^2 corrections worsen the description of data! Some mitigation through all-order v^2 resummation.
- We can get rid of the influence of untagged data, by normalizing data wrt. to the value in the first measured bin. Agreement with data is restored.

NLO vs LO in the spacelike region for η_c



- Combined v^2 and α_s corrections for the two-dimensional FF are presented for the first time.
- Predictions can be checked in future double–tagged e^+e^- experiments.

Cross section for exclusive $e^+e^- \to \eta_c\gamma$



$$\sigma = \frac{2\pi^2\alpha_{em}^3}{3}|\mathcal{F}(t,0)|^2\left(1-\frac{M^2}{s}\right)^3, \quad t = \frac{s}{m_Q^2}.$$

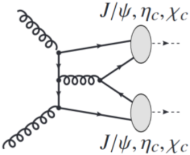
- at high enough $\sqrt{s}$, outside the resonance region, photons in the final state have large momentum and are emitted in the perturbative phase of the reaction $\longrightarrow$ calculable in terms of our transition FF.
- Measurements up to now exist only in the region of $\psi(4040), \psi(4415)$ resonances. At high energies $\sqrt{s} \sim 10$ GeV , the Belle collaboration cites an upper limit for the cross section of about 20 fb.

Summary

- We have studied the $\gamma^*\gamma^*$ transition form factor of η_Q in NRQCD factorization including perturbative α_s corrections as well as relativistic corrections in the v^2 expansion.
- We include for the first time NLO α_s and all-order v^2 corrections.
- Our results are in agreement with those in the literature in the appropriate limits.
- In contrast to the literature, our result also retains the imaginary part in the timelike region.
- Comparison with single-tag e^+e^- Belle data for one virtual photon points to a problem with the normalization wrt. untagged data. After normalizing to the lowest bin of single-tagged data, we restore agreement of theory with data. The inclusion of all-order v^2 corrections has a major impact on the decay width.
- In the studied energy range, NLO and v^2–corrections to the exclusive cross section for $e^+e^- \to \gamma\eta_c$ are negative and move theory prediction closer to Belle upper limit.

Outlook

- further study of TFF including mixed $\alpha_s v^2$ corrections.
- Pair production of S–wave quarkonia $\gamma^*\gamma^* \to \mathcal{Q}_1\mathcal{Q}_2$, with $\mathcal{Q}_i = \eta_c, J/\psi$.



- For pair production NLO corrections and $\mathcal{O}(v^2)$ corrections are available for real photons. It would be interesting to obtain all–order v^2 summed results. During our work we have found a frame–independent way to perform the v^2 expansion, which may simplify calculations.
- v^2–corrections to exclusive diffractive reactions like $\gamma p \to J\psi p, \gamma p \to \psi(2S)p$ are also of topical interest.
- Move to hadronic reactions with initial state gluons, e.g. $gg \to J/\psi\Upsilon$.