Unifying the Parton Reggization approach and Auxiliary Parton method (UPRAP)

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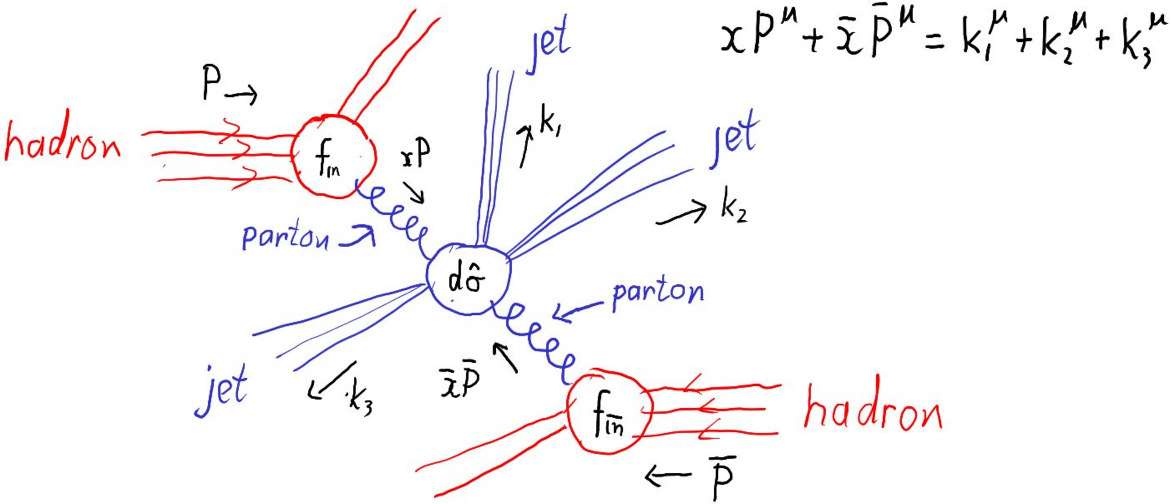
Collinear factorization in QCD at NLO

$$d\sigma^{LO} = \int \frac{dx_{in}}{x_{in}} \frac{d\bar{x}_{\overline{in}}}{\bar{x}_{\overline{in}}} f_{in}(x_{in})\, f_{\overline{in}}(\bar{x}_{\overline{in}})\, dB(x_{in}, \bar{x}_{\overline{in}})$$

general: $K^{\mu} = x_K P^{\mu} + \bar{x}_K \bar{P}^{\mu} + K_{\perp}^{\mu}$

one in-state: $k_{in}^{\mu} = x_{in} P^{\mu}$

other in-state: $k_{\overline{in}}^{\mu} = \bar{x}_{\overline{in}} \bar{P}^{\mu}$



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$$d\sigma^{NLO} \overset{?}{=} \int \frac{dx_{in}}{x_{in}} \frac{d\bar{x}_{\overline{in}}}{\bar{x}_{\overline{in}}} \left\{ f_{in}(x_{in})\, f_{\overline{in}}(\bar{x}_{\overline{in}}) \left[\frac{\alpha_s}{2\pi}\, dV(x_{in}, \bar{x}_{\overline{in}}) + \frac{\alpha_s}{2\pi}\, dR(x_{in}, \bar{x}_{\overline{in}}) \right] \right\}$$

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$$d\sigma^{NLO} = \int \frac{dx_{in}}{x_{in}} \frac{d\bar{x}_{\overline{in}}}{\bar{x}_{\overline{in}}} \Bigg\{ f_{in}(x_{in})\, f_{\overline{in}}(\bar{x}_{\overline{in}}) \left[\frac{\alpha_s}{2\pi} dV(x_{in}, \bar{x}_{\overline{in}}) + \frac{\alpha_s}{2\pi} dR(x_{in}, \bar{x}_{\overline{in}}) \right]_{\text{cancelling}-\text{divergences}}$$
$$+ \left[\quad f_{in}(x_{in}) \frac{-\alpha_s}{2\pi\epsilon} \int_{\bar{x}_{\overline{in}}}^1 d\bar{z}\, \mathcal{P}_{\overline{in}}(\bar{z}) f_{\overline{in}}\left(\frac{\bar{x}_{\overline{in}}}{\bar{z}}\right) \right.$$
$$\left. + f_{\overline{in}}(\bar{x}_{\overline{in}}) \frac{-\alpha_s}{2\pi\epsilon} \int_{x_{in}}^1 dz\, \mathcal{P}_{in}(z) f_{in}\left(\frac{x_{in}}{z}\right) \right] dB(x_{in}, \bar{x}_{\overline{in}})$$
$$+ \left[\frac{\alpha_s}{2\pi} f_{in}^{NLO}(x_{in})\, f_{\overline{in}}(\bar{x}_{\overline{in}}) + f_{in}(x_{in}) \frac{\alpha_s}{2\pi} f_{\overline{in}}^{NLO}(\bar{x}_{\overline{in}}) \right] dB(x_{in}, \bar{x}_{\overline{in}}) \Bigg\}$$

$$f_{in}^{NLO}(x_{in}) - \frac{1}{\epsilon} \int_{x_{in}}^1 dz\, \mathcal{P}_{in}(z) f_{in}\left(\frac{x_{in}}{z}\right) = \text{finite}$$

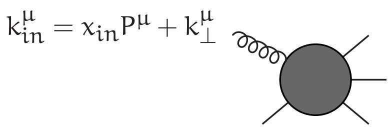
$$f_{\overline{in}}^{NLO}(\bar{x}_{\overline{in}}) - \frac{1}{\epsilon} \int_{\bar{x}_{\overline{in}}}^1 d\bar{z}\, \mathcal{P}_{in}(\bar{z}) f_{\overline{in}}\left(\frac{\bar{x}_{\overline{in}}}{\bar{z}}\right) = \text{finite}$$

Objective

Establish the same within hybrid k_T-factorization,
for which the LO cross section formula is

$$d\sigma^{LO} = \int \frac{dx_{in}}{x_{in}} \frac{d^2k_\perp}{\pi} \frac{d\bar{x}_{\overline{in}}}{\bar{x}_{\overline{in}}} F_{in}(x_{in}, k_\perp)\, f_{\overline{in}}(\bar{x}_{\overline{in}})\, dB^\star(x_{in}, k_\perp, \bar{x}_{\overline{in}})$$

- The amplitudes inside $B^\star(x_{in}, k_\perp, \bar{x}_{\overline{in}})$ depend explicitly on $k_\perp$
- They involve a space-like initial-state gluon with momentum $k_{in}^\mu = x_{in}P^\mu + k_\perp^\mu$



- Such amplitudes need care to be well-defined, to be gauge-invariant.
- We apply the auxiliary-parton method, and our objective is within this constraint.

Auxiliary parton method (tree-level)

$k_{in} = x_{in} P + k_{\perp}$

AvH, Kotko, Kutak 2013

We desire to obtain the matrix element with one space-like gluon for the process

$g^{\star}(k_{in})\ \omega_{\overline{in}}(k_{\overline{in}}) \ \to\ \omega_1(p_1)\ \omega_2(p_2)\ \cdots\ \omega_n(p_n)$ e.g. $g^{\star}(k_{in})\ g(k_{\overline{in}}) \ \to\ g(p_1)\ g(p_2)\ g(p_3)$

and do so by replacing the space-like gluon with an *on-shell auxiliary* quark pair

$q\big(k_1(\Lambda)\big)\ \omega_{\overline{in}}(k_{\overline{in}}) \ \to\ q\big(k_2(\Lambda)\big)\ \omega_1(p_1)\ \omega_2(p_2)\ \cdots\ \omega_n(p_n)$

with special momenta

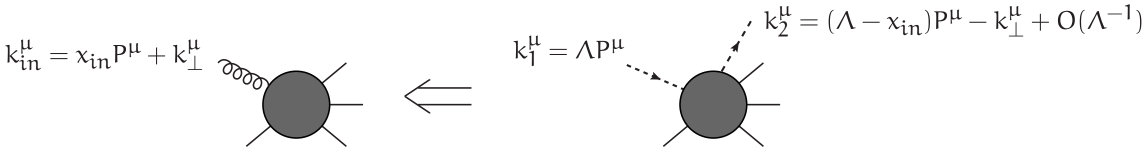
$$k_1^{\mu} = \Lambda P^{\mu} \quad , \quad k_2^{\mu} = p_{\Lambda}{}^{\mu} = (\Lambda - x_{in})P^{\mu} - k_{\perp}^{\mu} + \frac{|k_{\perp}|^2}{(\Lambda - x_{in})\nu^2}\,\bar{P}^{\mu}$$

such that, while individually on-shell, their difference is

$$k_1^{\mu} - k_2^{\mu} = x_{in}P^{\mu} + k_{\perp}^{\mu} + \mathcal{O}\big(\Lambda^{-1}\big) = k_{in}^{\mu} + \mathcal{O}\big(\Lambda^{-1}\big)$$

The matrix element with the space-like gluon is obtained by taking $\Lambda \to \infty$

$$\frac{1}{g_s^2 C_{aux}}\,\frac{x_{in}^2 |k_{\perp}|^2}{\Lambda^2}\,\big|\overline{\mathcal{M}}^{aux}\big|^2\big(\Lambda P, k_{\overline{in}}; p_{\Lambda}, \{p_i\}_{i=1}^{n}\big) \ \stackrel{\Lambda\to\infty}{\longrightarrow}\ \big|\overline{\mathcal{M}}^{\star}\big|^2\big(k_{in}, k_{\overline{in}}; \{p_i\}_{i=1}^{n}\big)$$



Auxiliary partons at one loop

Blanco, Giachino, AvH, Kotko, 2022, AvH, Motyka, Ziarko 2022

We recognize the following pattern:

$$dV^{\star} = dV^{\star\mathrm{fam}} + dV^{\star\mathrm{unf}}$$

$dV^{\star\mathrm{fam}}$ is independent of the type of auxiliary partons
has the correct regular on-shell limit
all $1/\epsilon^2, 1/\epsilon$ poles look as if the space-like gluon were on-shell

$$dV^{\star\mathrm{unf}} = a_\epsilon N_c \,\mathrm{Re}\big(\mathcal{V}_{\mathrm{aux}}\big)\, dB^{\star}$$ is proportional to Born result $a_\epsilon = \frac{\alpha_s}{2\pi}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}$

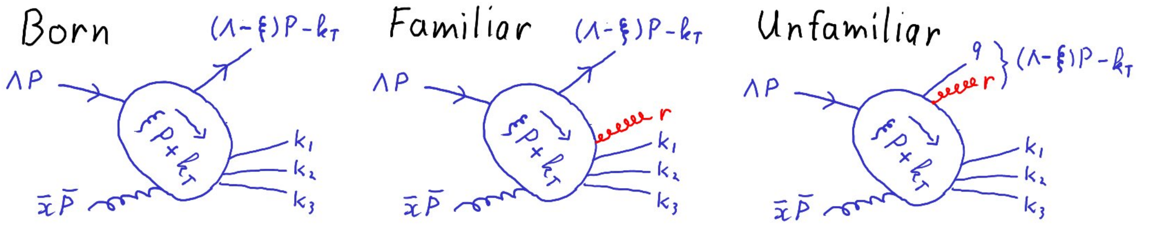
$$\mathcal{V}_{\mathrm{aux}} = \left(\frac{\mu^2}{|k_\perp|^2}\right)^{\epsilon}\left[\frac{2}{\epsilon}\ln\frac{\Lambda}{x_{\mathrm{in}}} - i\pi + \bar{\mathcal{V}}_{\mathrm{aux}}\right] + \mathcal{O}(\epsilon) + \mathcal{O}\big(\Lambda^{-1}\big)$$

$$\bar{\mathcal{V}}_{\text{aux-q}} = \frac{1}{\epsilon}\frac{13}{6} + \frac{\pi^2}{3} + \frac{80}{18} + \frac{1}{N_c^2}\left[\frac{1}{\epsilon^2} + \frac{3}{2}\frac{1}{\epsilon} + 4\right] - \frac{n_f}{N_c}\left[\frac{2}{3}\frac{1}{\epsilon} + \frac{10}{9}\right]$$

$$\bar{\mathcal{V}}_{\text{aux-g}} = -\frac{1}{\epsilon^2} + \frac{\pi^2}{3}$$

Real radiation with auxiliary partons

AvH, Motyka, Ziarko 2022



The differential phase space and the matrix element factorize for the *unfamiliar* case, where the radiative gluon participates in the consumption of Λ.

$$\frac{1}{C_{\text{aux}}}\left|\overline{\mathcal{M}}^{\text{aux}}\right|^2\big((\Lambda + x_{\text{in}})P, k_{\overline{\text{in}}}\,;x_r\Lambda P + r_\perp + \bar{x}_r\bar{P}\,,\, x_q\Lambda P + q_\perp + \bar{x}_q\bar{P}\,,\,\{p_i\}_{i=1}^n\big)$$

$$\stackrel{\Lambda\to\infty}{\longrightarrow}\ \mathcal{Q}_{\text{aux}}(x_q, q_\perp, x_r, r_\perp)\,\frac{\Lambda^2\left|\overline{\mathcal{M}}^\star\right|^2(x_{\text{in}}P - q_\perp - r_\perp, k_{\overline{\text{in}}}\,;\{p_i\}_{i=1}^n)}{x_{\text{in}}^2|q_\perp + r_\perp|^2}$$

$$\mathcal{Q}_{\text{aux}}(x_q, q_\perp, x_r, r_\perp) = x_q x_r\,\mathcal{P}_{\text{aux}}(x_q, x_r)\,|q_\perp + r_\perp|^2$$

$$\times\left[\frac{c_{\bar{q}}}{|q_\perp|^2|r_\perp|^2} + \frac{1}{x_r|q_\perp|^2 + x_q|r_\perp|^2 - x_q x_r|q_\perp + r_\perp|^2}\left(\frac{c_r\,x_r^2}{|r_\perp|^2} + \frac{c_q\,x_q^2}{|q_\perp|^2}\right)\right]$$

- Phase space also factorizes and the contribution can be calculated analytically.
- The result contains $\ln\Lambda$ and depends on the type of auxiliary partons.

Familiar real collinear singularity

AvH, Motyka, Ziarko 2022

Tree-level matrix elements with a space-like gluon still have a singularity when a radiative gluon becomes collinear to P.

$$\left|\overline{\mathcal{M}}^{\star}\right|^2\big(x_{\text{in}}P+k_\perp,k_{\overline{\text{in}}};r,\{p_i\}_{i=1}^n\big)$$
$$\xrightarrow{r\to x_rP}\frac{2N_c}{P\cdot r}\,\frac{x_{\text{in}}^2}{x_r(x_{\text{in}}-x_r)^2}\left|\overline{\mathcal{M}}^{\star}\right|^2\big((x_{\text{in}}-x_r)P+k_\perp,k_{\overline{\text{in}}};\{p_i\}_{i=1}^n\big)$$

Collinear splitting function with only the $1/z/(1-z)$ part.

Integrate over relevant phase space with restriction

$$\frac{\bar{x}_r}{\bar{x}_{\overline{\text{in}}}}<a\,\frac{x_r}{x_{\text{in}}}\quad\text{with}\quad a=\frac{|k_\perp-r_\perp|^2}{Sx_{\text{in}}\bar{x}_{\overline{\text{in}}}}\quad\text{and}\quad |r_\perp|^2=Sx_r\bar{x}_r\quad\Rightarrow\quad |r_\perp|<|k_\perp-r_\perp|\,\frac{x_r}{x_{\text{in}}}$$

which is the complement of the restriction on the unfamiliar phase space.

$$\int_0^1\frac{dx_{\text{in}}}{x_{\text{in}}}\int d^2k_\perp\,F(x_{\text{in}},k_\perp)\,dR^{\star\,\text{fam}}_{\text{coll}}\big(x_{\text{in}}P_A+k_\perp,k_{\overline{\text{in}}};\{p_i\}_{i=1}^n\big)$$
$$=\int_0^1\frac{dx_{\text{in}}}{x_{\text{in}}}\int d^2k_\perp\,\tilde{F}(x_{\text{in}},k_\perp)\,dB^\star\big(x_{\text{in}},k_\perp,\bar{x}_{\overline{\text{in}}};\{p_i\}_{i=1}^n\big)$$

$$\tilde{F}(x_{\text{in}},k_\perp)=\frac{2a_\epsilon N_c}{\pi_\epsilon\mu^{-2\epsilon}}\int_{x_{\text{in}}}^1\frac{dz}{z(1-z)}\int\frac{d^{2-2\epsilon}r_\perp}{|r_\perp|^2}\,\frac{|k_\perp|^2}{|k_\perp+r_\perp|^2}\,F\!\left(\frac{x_{\text{in}}}{z},k_\perp+r_\perp\right)\theta_{|r_\perp|<|k_\perp|(1-z)}$$

Essentially identical to formula from Nefedov 2020 for multi-Regge evolution.

A formula at NLO?

AvH, Motyka, Ziarko 2022

$$d\sigma^{\mathrm{NLO}} = \int \frac{dx_{\mathrm{in}}}{x_{\mathrm{in}}}\, d^2k_\perp \frac{d\bar{x}_{\overline{\mathrm{in}}}}{\bar{x}_{\overline{\mathrm{in}}}} \Big\{ F(x_{\mathrm{in}},k_\perp)\, f(\bar{x}_{\overline{\mathrm{in}}}) \Big[dV^\star(x_{\mathrm{in}},k_\perp,\bar{x}_{\overline{\mathrm{in}}}) + dR^\star(x_{\mathrm{in}},k_\perp,\bar{x}_{\overline{\mathrm{in}}}) \Big]_{\mathrm{cancelling}}$$
$$+ \Big[F^{\mathrm{NLO}}(x_{\mathrm{in}},k_\perp) + F(x_{\mathrm{in}},k_\perp)\Delta_{\mathrm{unf}}(x_{\mathrm{in}},k_\perp) + \Delta^\star_{\mathrm{coll}}(x_{\mathrm{in}},k_\perp) \Big] f(\bar{x}_{\overline{\mathrm{in}}})\, dB^\star(x_{\mathrm{in}},k_\perp,\bar{x}_{\overline{\mathrm{in}}})$$
$$+ \Big[f^{\mathrm{NLO}}(\bar{x}_{\overline{\mathrm{in}}}) + \Delta_{\overline{\mathrm{coll}}}(\bar{x}_{\overline{\mathrm{in}}}) \Big] F(x_{\mathrm{in}},k_\perp)\, dB^\star(x_{\mathrm{in}},k_\perp,\bar{x}_{\overline{\mathrm{in}}}) \Big\}$$

$$\Delta_{\overline{\mathrm{coll}}}(\bar{x}_{\overline{\mathrm{in}}}) = -\frac{a_\epsilon}{\epsilon}\int_{\bar{x}_{\overline{\mathrm{in}}}}^1 dz \left[\mathcal{P}^{\mathrm{reg}}_{\overline{\mathrm{in}}}(z) + \gamma_{\overline{\mathrm{in}}}\delta(1-z) \right] f\!\left(\frac{\bar{x}_{\overline{\mathrm{in}}}}{z}\right)$$

$$\Delta^\star_{\mathrm{coll}}(x_{\mathrm{in}},k_\perp) = -\frac{a_\epsilon}{\epsilon}\int_{x_{\mathrm{in}}}^1 dz \left[\frac{2N_c}{[1-z]_+} + \frac{2N_c}{z} + \gamma_g\delta(1-z) \right] F\!\left(\frac{x_{\mathrm{in}}}{z},k_\perp\right)$$

$$\Delta_{\mathrm{unf}}(x_{\mathrm{in}},k_\perp) = \frac{a_\epsilon N_c}{\epsilon}\left(\frac{\mu^2}{|k_\perp|^2}\right)^{\epsilon} \left[\mathrm{impactFactCorr} + \mathrm{couplingRenorm} - 2\ln\frac{2P\cdot\bar{P}x_{\mathrm{in}}}{|k_\perp|^2} \right]$$

$$f^{\mathrm{NLO}}(\bar{x}_{\overline{\mathrm{in}}}) + \Delta_{\overline{\mathrm{coll}}}(\bar{x}_{\overline{\mathrm{in}}}) = \mathrm{finite}$$

$$F^{\mathrm{NLO}}(x_{\mathrm{in}},k_\perp) + F(x_{\mathrm{in}},k_\perp)\Delta_{\mathrm{unf}}(x_{\mathrm{in}},k_\perp) + \Delta^\star_{\mathrm{coll}}(x_{\mathrm{in}},k_\perp) \overset{?}{=} \mathrm{finite}$$

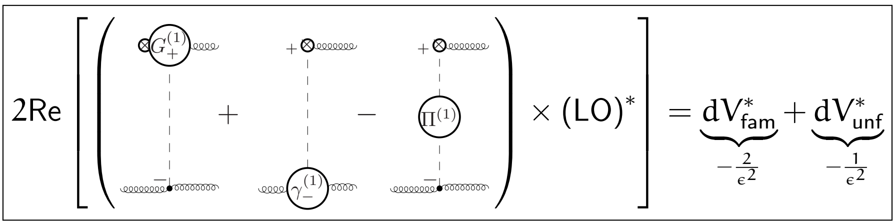
Parton Reggization approach

A method to directly obtain the high-energy limit of amplitudes.

Employs Lipatov's effective action to arrive at gauge invariant amplitudes with space-like gluons (reggeons) (Lipatov 1995, Lipatov, Vyazovsky 2001).

At tree level the auxiliary parton approach is completely equivalent.

At one loop the auxiliary parton appraoch gives the *complete* "high enery limit" of the amplitude, whereas the effective action rules give separate building blocks (Nefedov 2019).



We need to recognize the high-energy factorization in terms of impact factors and Green's function in the auxiliary parton expressions.

High-energy limit from auxiliary partons

To obtain the high-energy picture for the virtual contributions from the auxiliary partons, we just need to re-shuffle terms using an arbitrary scale μ_Y, to move a collinear divergence, and rapidity $\ln\lambda_1$, to move a "high-energy divergence".

$$
\begin{aligned}
dV^{\star}_{\text{targ}}(\epsilon,\lambda_1,\Lambda,\mu_Y) &= dV^{\star}_{\text{unf}}(\epsilon,\Lambda) - dB^{\star}\times a_\epsilon\left[\left(\frac{\mu^2}{\mu_Y^2}\right)^{\epsilon}\frac{N_c}{\epsilon^2}+\frac{\gamma_g}{\epsilon}+\left(\frac{\mu^2}{|k_\perp|^2}\right)^{\epsilon}\frac{2N_c\ln\lambda_1}{\epsilon}\right]\\
dV^{\star}_{\text{Green}}(\epsilon,\lambda_1) &= \qquad\qquad\quad dB^{\star}\times a_\epsilon\left[\left(\frac{\mu^2}{|k_\perp|^2}\right)^{\epsilon}\frac{2N_c\ln\lambda_1}{\epsilon}\right]\\
dV^{\star}_{\text{proj}}(\epsilon,\mu_Y) &= dV^{\text{fam}}_{\star\bar{\imath}n}(\epsilon) \quad + dB^{\star}\times a_\epsilon\left[\left(\frac{\mu^2}{\mu_Y^2}\right)^{\epsilon}\frac{N_c}{\epsilon^2}+\frac{\gamma_g}{\epsilon}\right].
\end{aligned}
$$

where

$$\gamma_g = \frac{11N_c - 4T_R n_f}{6}$$

Status and outlook

The same separation must be achieved for the real radiation contribution.

This is possible by dividing the radiative phase space into rapidity regions.

It involves yet another limit (than AvH, Motyka, Ziarko 2022) of the auxiliary-parton matrix elements.

Within collinear factorization, LO DGLAP evolution can be "re-discovered" by curing the remnant collinear divergence $\frac{-a_\epsilon}{\epsilon}\left[\mathcal{P}_{ij}\otimes f_i\right](x)$ via

$$f_i(x) \to f_i(x,\mu_F) + \frac{a_\epsilon}{\epsilon}\left(\frac{\mu^2}{\mu_F^2}\right)^{\epsilon}\sum_j\left[\mathcal{P}_{ij}\otimes f_j\right](x)$$

and demanding that the cross section is independent of μ_F.

Similarly, it looks like we can "re-discover" the CSS equation and the BFKL equation.