Ultra-high frequency primordial gravitational waves beyond the kHz: the case of cosmic strings.

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GdR “Cosmological Physics”,
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CLUSTER OF EXCELLENCE
QUANTUM UNIVERSE

Universität Hamburg

Ultra-high frequency gravitational waves

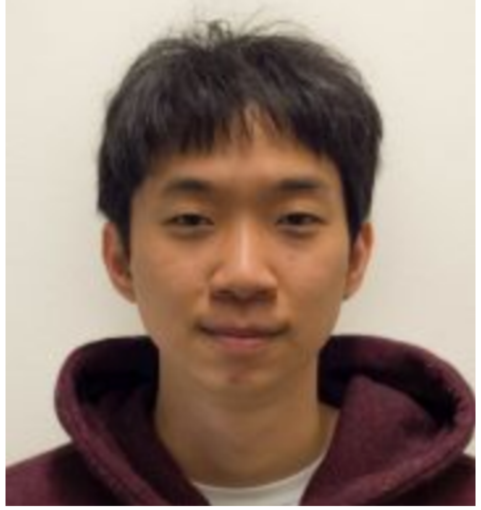
Question raised:

What are the largest predictions for primordial gravitational-wave backgrounds beyond the kiloHertz that leave no signal at LIGO/Virgo, ET, LISA, PTAs?

Talk based on

Phys. Rev. D 109, 103538 [arXiv: 2312.09281]

in collaboration with



Peera Simakachorn

Ultra-high frequency gravitational waves

For discussion on detection challenges,
See Ellis and Tito D’Agnolo [2412.17897]
& Aggarwal et al [2501.11723]

Gravitational Waves (GWs) & Particle Physics?

Gravitational Waves are not only a messenger to learn about

gravity, general relativity, astrophysics, black holes & compact objects but also potentially a project to learn about

new fundamental particle physics at energy scales which are not accessible at particle colliders.

Gravitational Waves (GWs) & Particle Physics?

Short-cut argument:

frequency of GWs of cosmological and macroscopic origin are proportional to H (Hubble rate) $\propto \sqrt{\rho_{tot}} \propto T^2$

Larger frequency —> larger H —> Larger temperatures & energy scales

High-frequency GWs : window on early universe particle physics at energy scales $\gg$ **electroweak scale**

This talk:

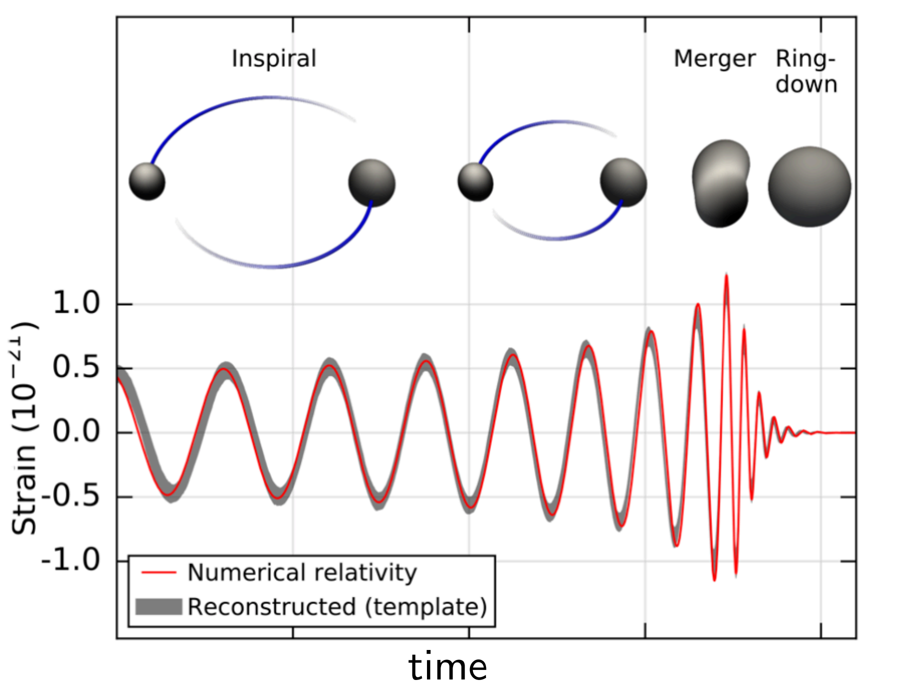
Within standard Einstein gravity (no modified gravity!)

**Non-standard physics comes from particle physics,
not from the gravity side**

Two types of gravitational-wave (GW) signals

- **Astrophysical signals (in the late universe)**

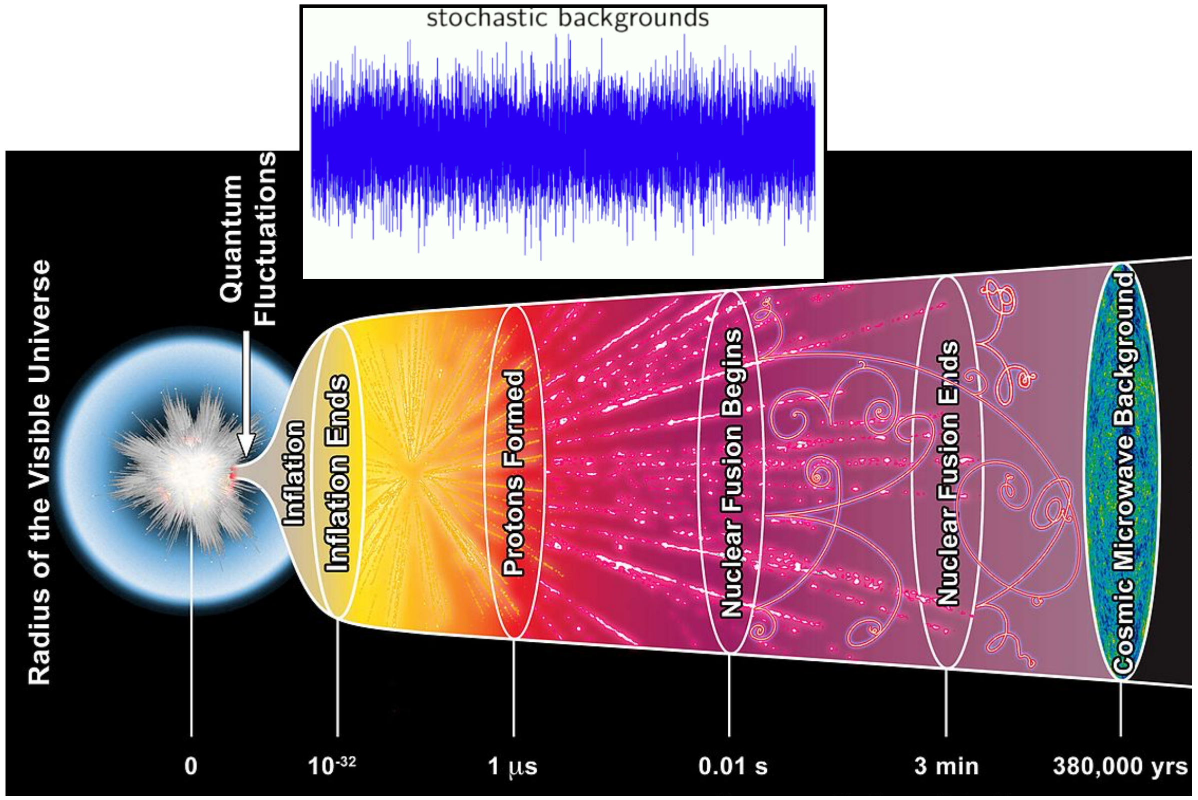
✔ discovered



LIGO&Virgo, arXiv:1602.03841

- **Cosmological background filling the whole universe (a relic from the early universe)**

✘ not yet discovered

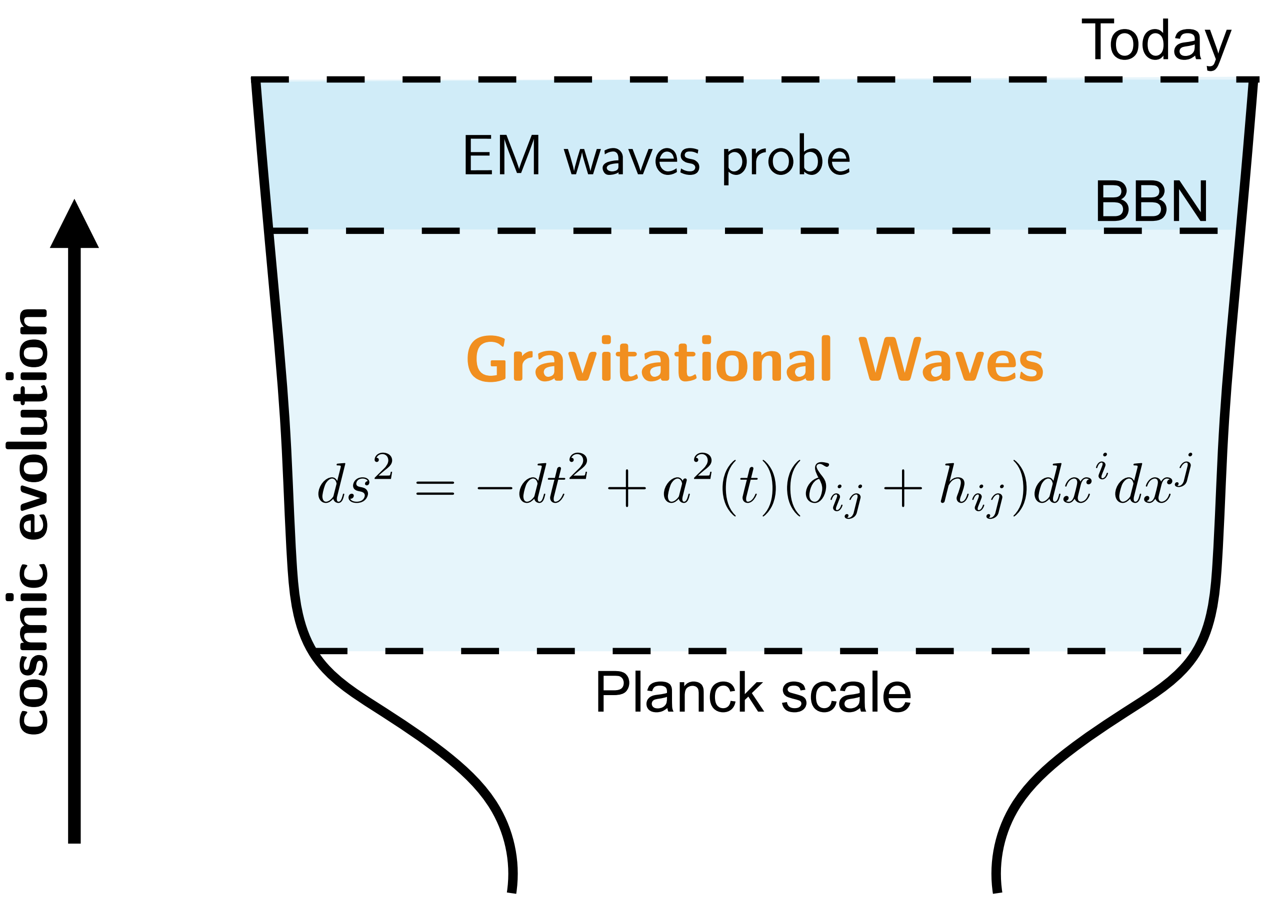


Note: Astrophysical signals can lead to a stochastic background if they cannot be resolved.

Primordial gravitational waves: Fossil radiation .

superposition of GW generated by an enormous number of causally independent sources, arriving at random times and from random directions.

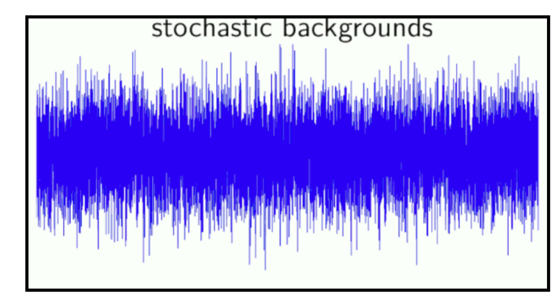
Individual waves are not detectable, sources can not be resolved but instead we can only observe a Stochastic GW Background. For most of the cosmological sources, it is homogeneous, isotropic, gaussian and unpolarized and appears as a noise in the detector.



Random h_{ij}

$\Downarrow$

Stochastic GW background (SGWB) characterized by energy density



$$\rho_{\rm GW} = \frac{\langle \dot{h}_{ij} \dot{h}^{ij} \rangle}{32\pi G}$$

Ensemble average

$\Updownarrow$

time/space average

Probing high-energy physics with gravitational waves .

Interaction rate of GW →

Expansion rate →

$$\frac{\Gamma_{\rm GW}(T)}{H(T)} \sim \frac{G^2 T^5}{T^2/M_{\rm pl}} = \left(\frac{T}{M_{\rm pl}}\right)^3 \ll 1$$

GW produced below the Planck scale are decoupled: They propagate freely in the universe until today.

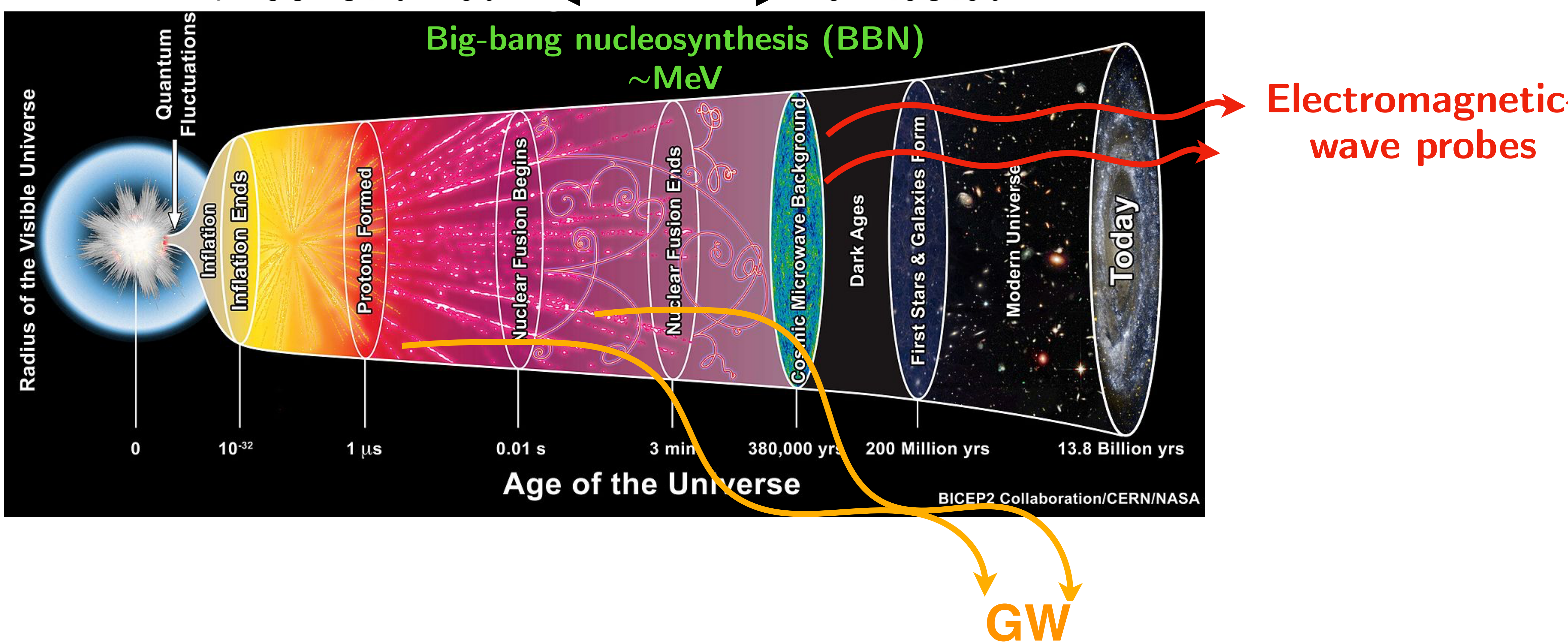
They do not loose memory of conditions when produced.

They retain spectral shape, typical frequency and intensity characteristic of production mechanism, encoding information about particle physics at high-energy scales that cannot be probed by colliders.

Probing high-energy physics with gravitational waves .

High energies **Low energies**

unconstrained ⟵ ⟶ **well-tested**



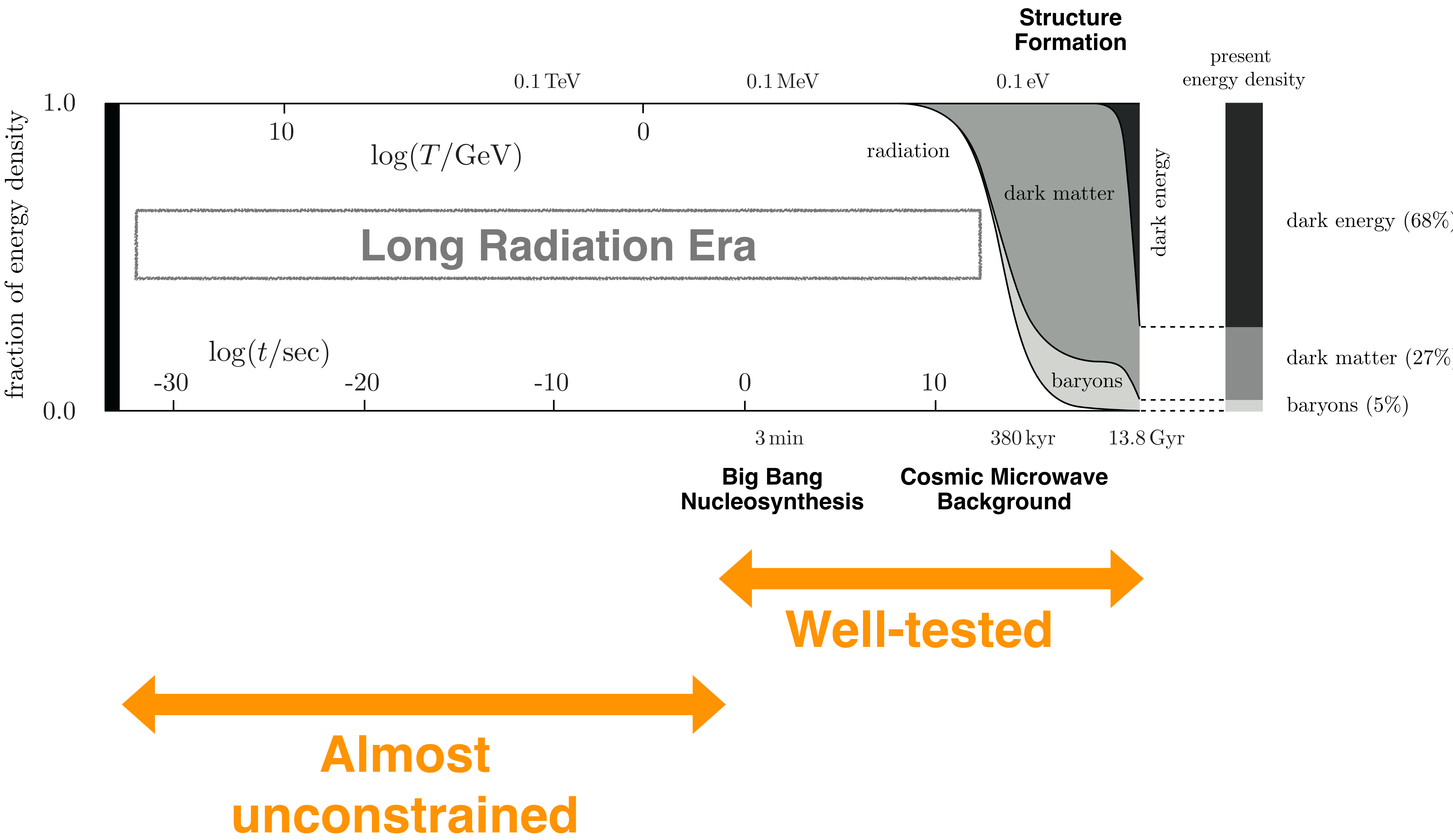
Energy density of GW background: $$\rho^{\rm GW}_{\rm today} = \rho^{\rm GW}_{\rm prod}\left(\frac{a_{\rm prod}}{a_{\rm today}}\right)^4$$

The universe is expanding. ⇨ **Probes non-standard cosmological evolutions**

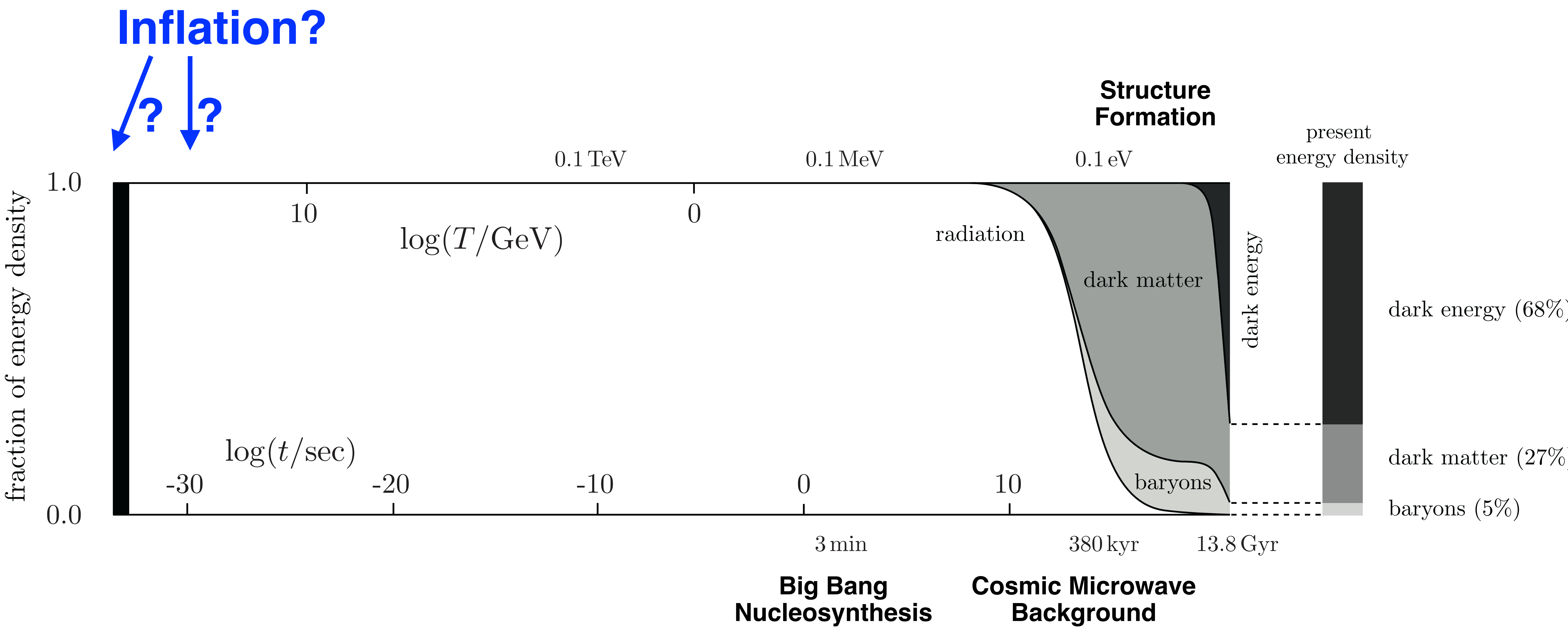
its production mechanism ⇨ **particle physics beyond the Standard Model (BSM)**

What can we learn on particle physics and cosmological history from primordial gravitational waves?

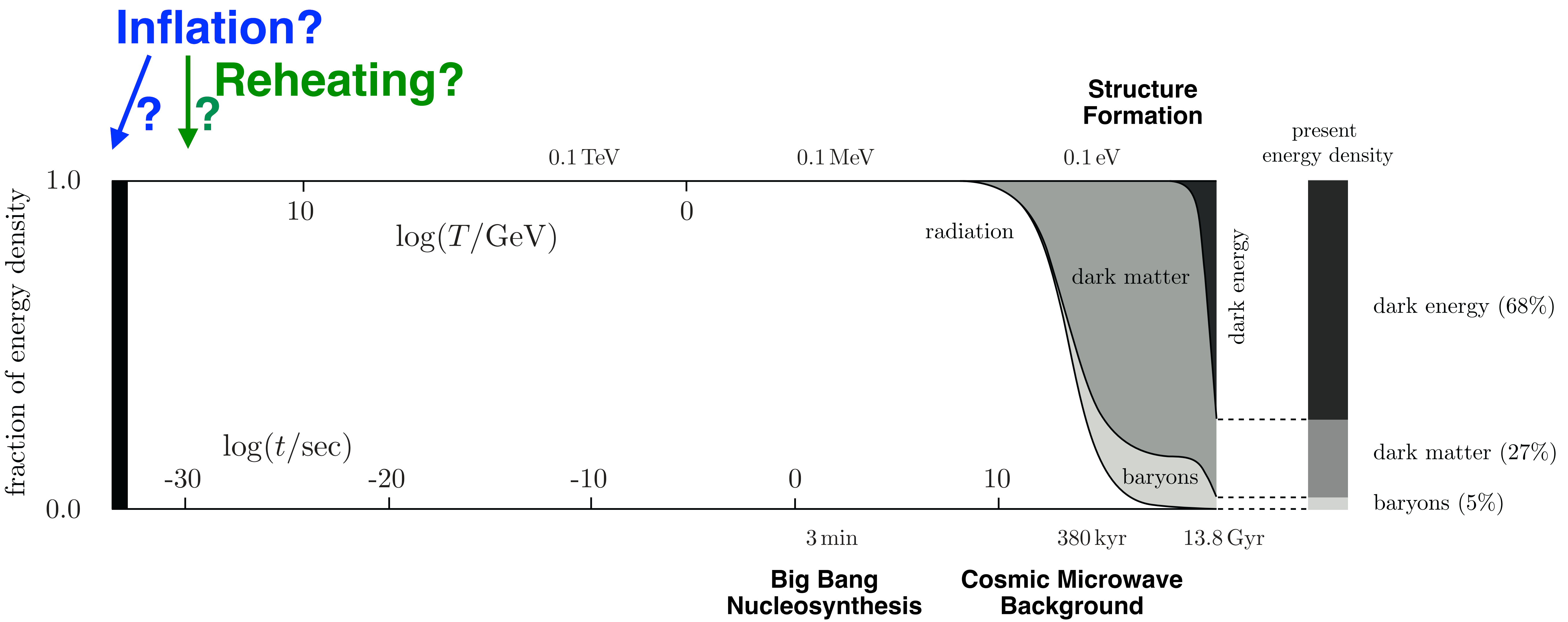
Standard Cosmological History



Cosmological History

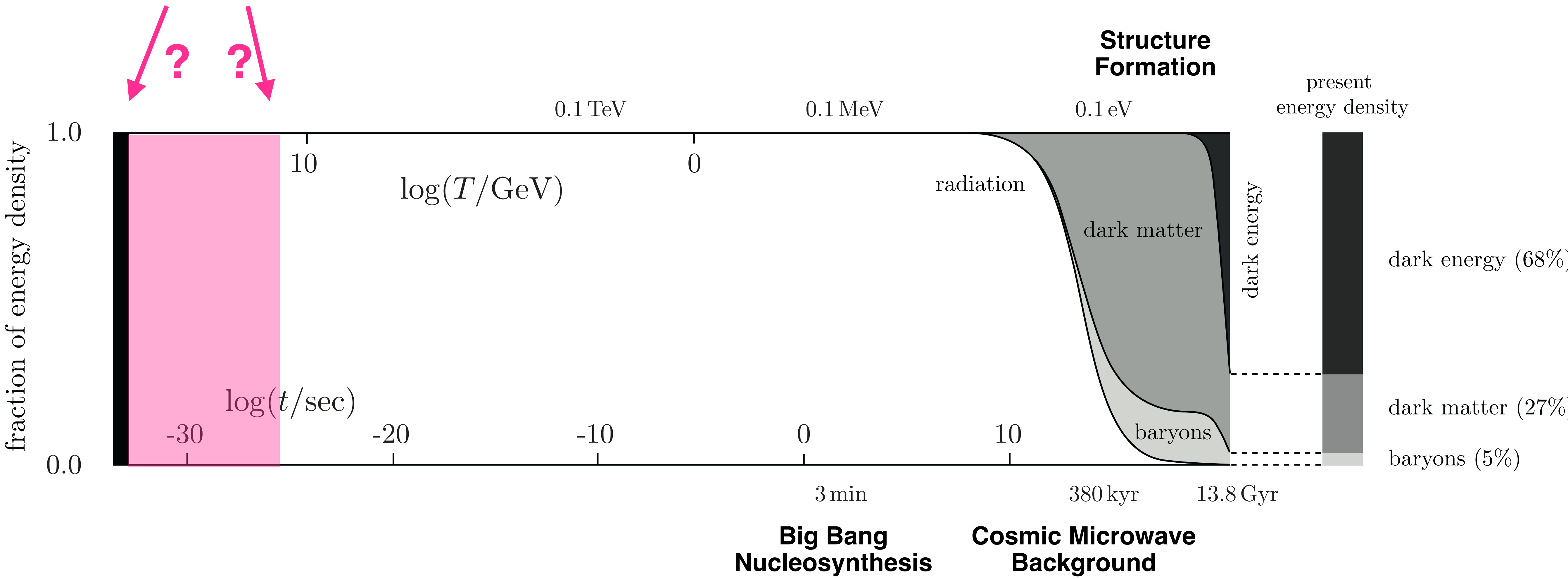


Cosmological History



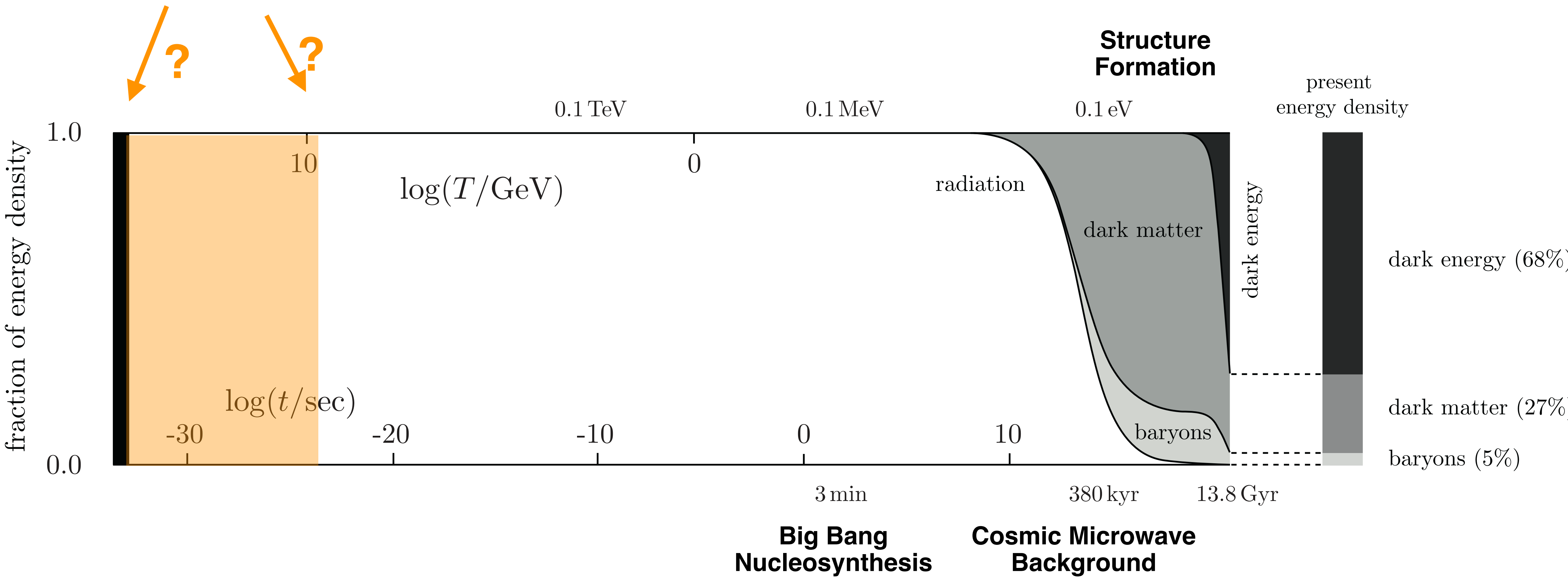
Cosmological History

Kination after inflation?



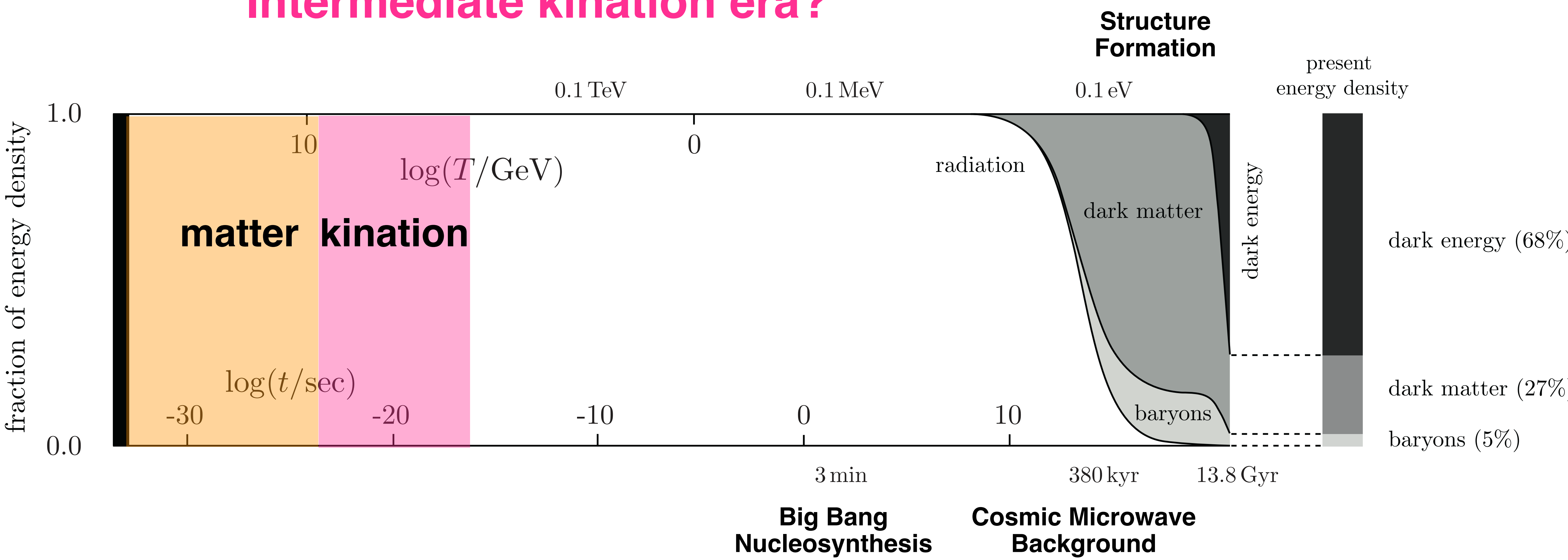
Cosmological History

Early Matter era after inflation?

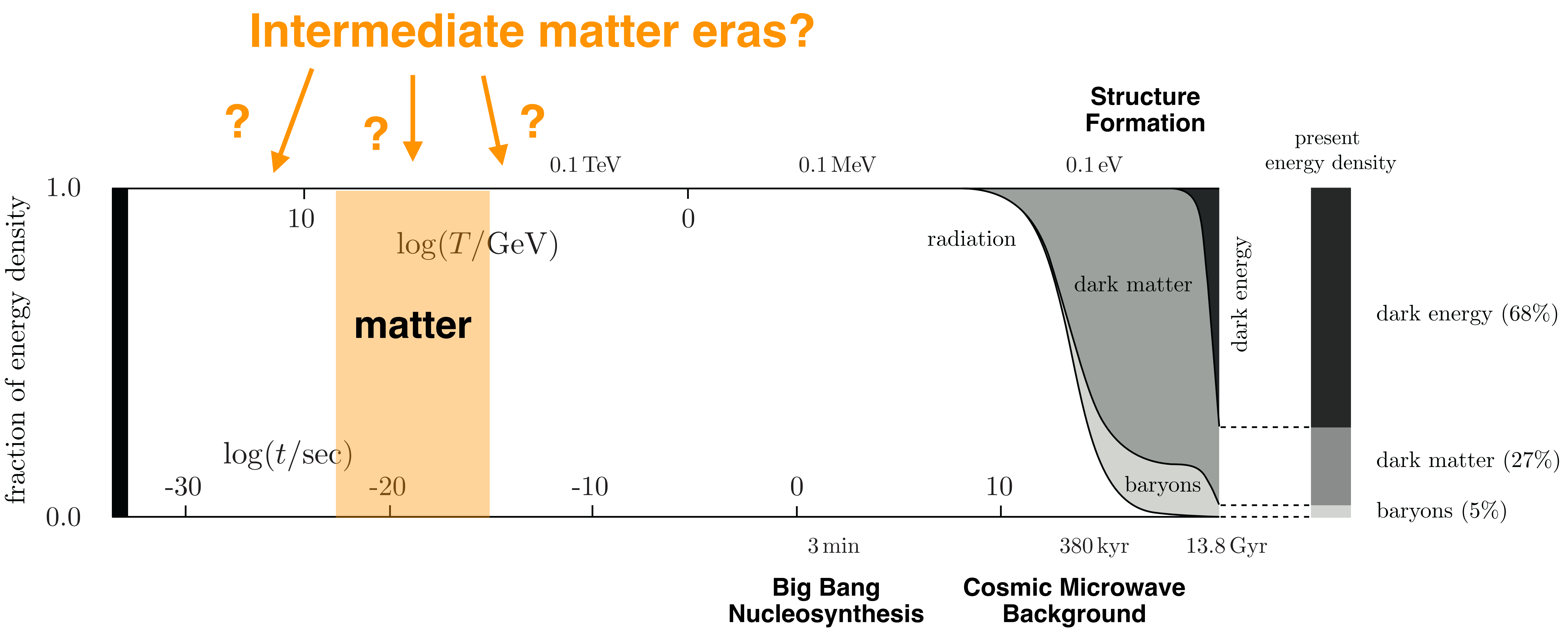


Cosmological History

Intermediate kination era?

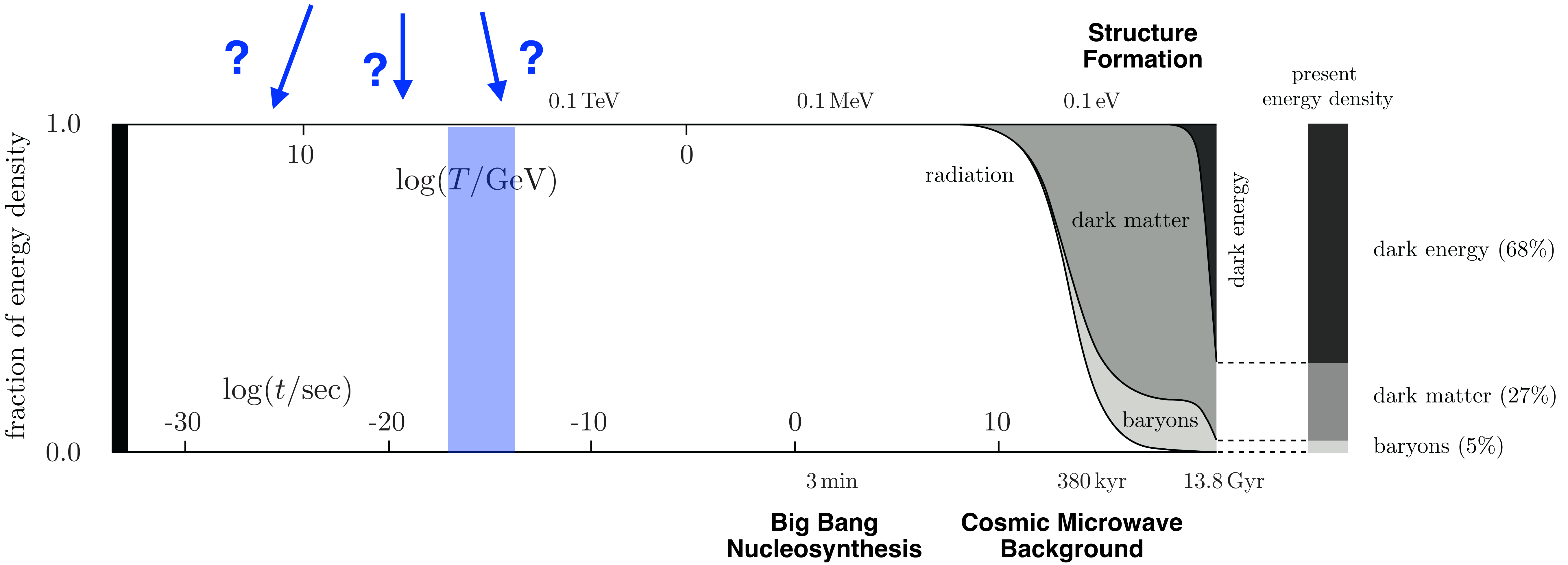


Cosmological History

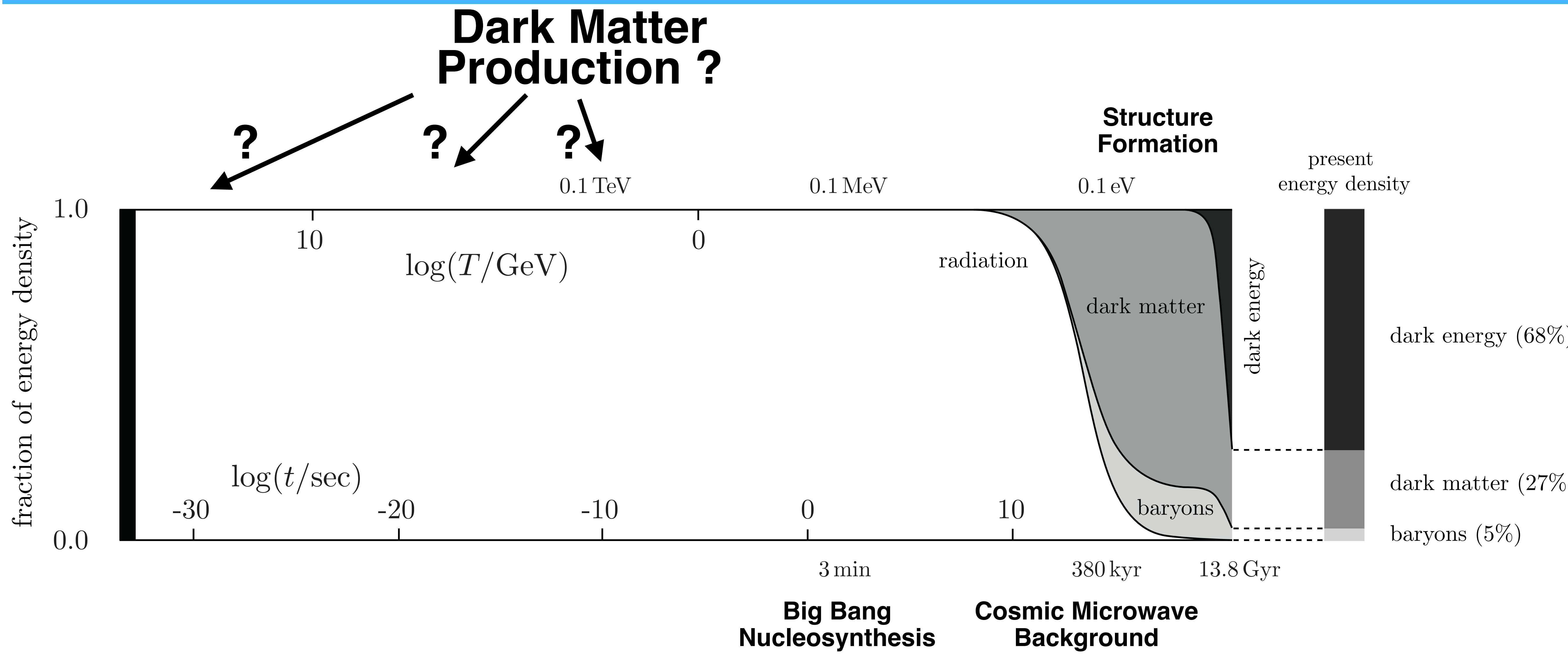


Cosmological History

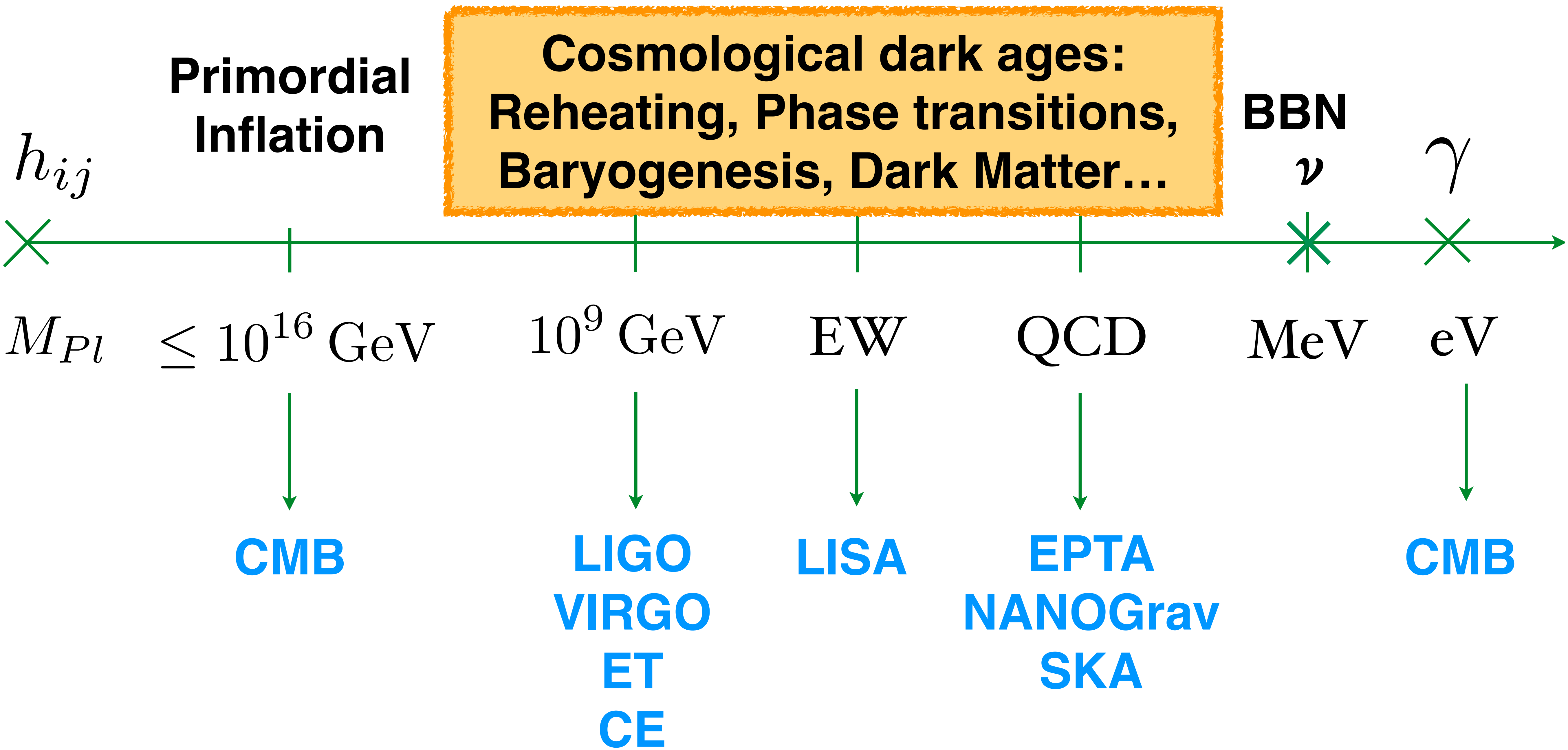
Secondary intermediate late inflation eras?



Cosmological History

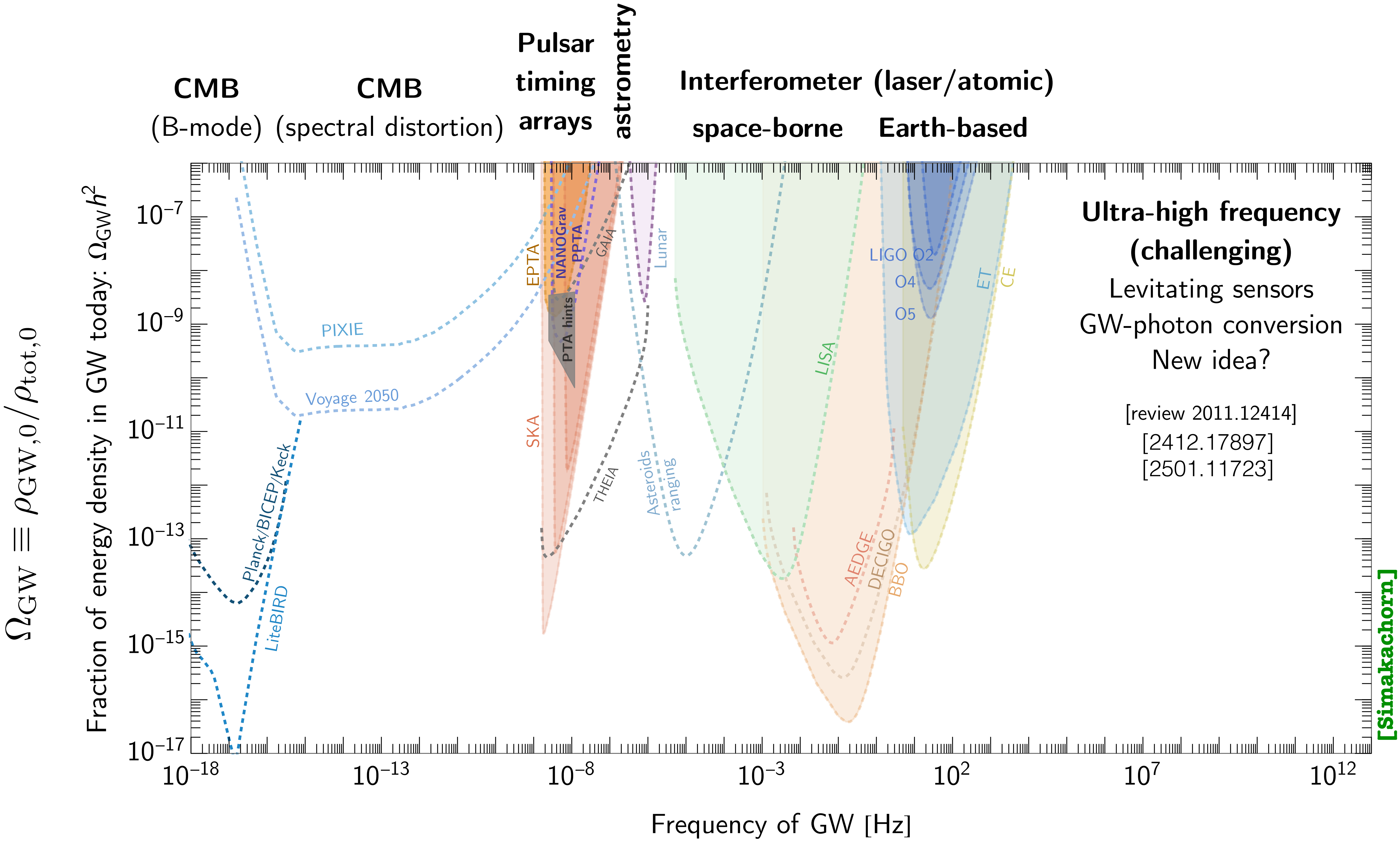


Probing the cosmological history with Gravitational Waves:



Current and future GW experiments constitute a new avenue of investigation in particle physics and cosmology.

The landscape of current & future GW experiments.



Primordial GW

Tensor perturbations of Friedmann-Robertson-Walker metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

Wave equation:

$$\ddot{h}_{ij} + 3H\,\dot{h}_{ij} + k^2\,h_{ij} = 16\pi G\,\Pi_{ij}^{TT}$$

Source:

Tensor anisotropic stress

=Transverse Traceless component
of the energy-momentum tensor of the source $= (P_{il}P_{jm} - \frac{1}{2}P_{ij}P_{lm})T_{lm}$

$P_{ij} = \delta_{ij} - \hat{k}_i\hat{k}_j$

Well-known cosmological sources .

-> Cosmological Phase Transitions (Short-lasting sources) (C. Caprini's talk tomorrow)

-> Cosmic Strings (Long-lasting sources) (This talk)

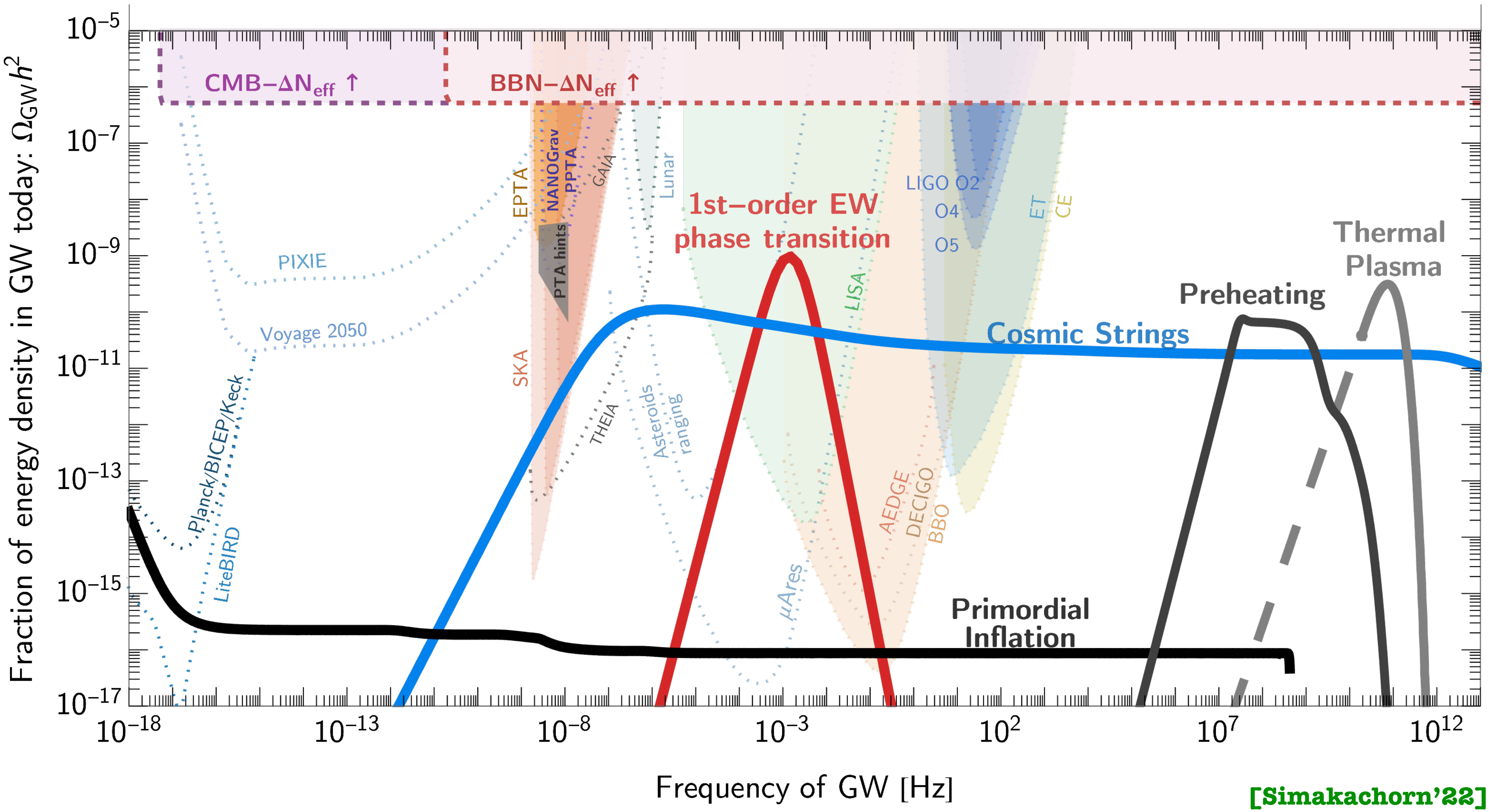
-> Inflation

-> Reheating of the universe

see
-review 1801.04268
-1912.02569 (cosmic strings)
-PhD thesis P. Simakachorn

Beyond-the-Standard Model benchmark sources.

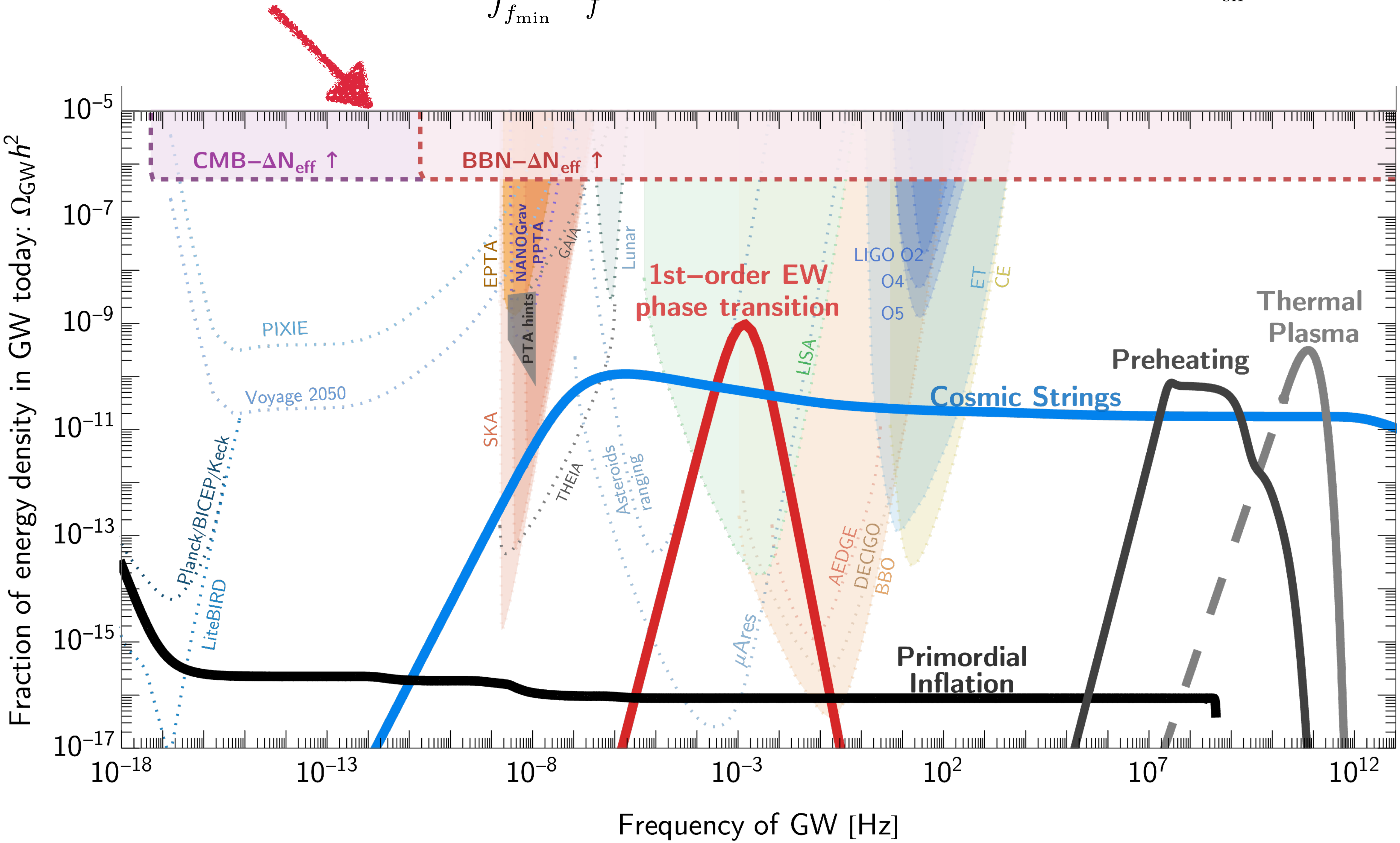
Preheating, first-order phase transitions, cosmic strings



[Simakachorn'22]

Upper theoretical bound.

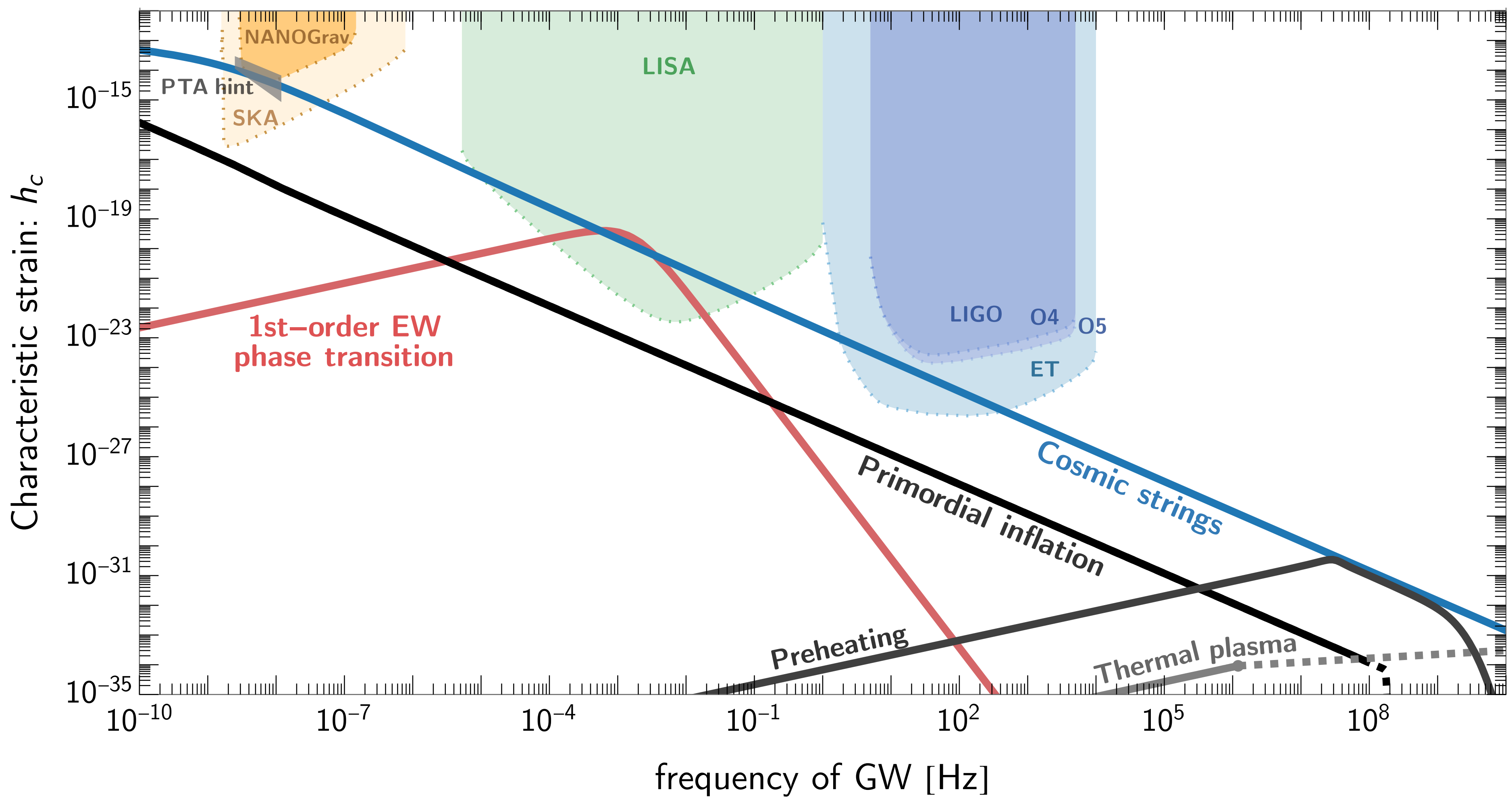
GW as extra radiation: $\int_{f_{\min}}^{f_{\max}} \frac{df}{f}\, \Omega_{\rm GW}(f) \lesssim 0.23\, \Omega_{\rm rad,0} \Delta N_{\rm eff}$ where $\Delta N_{\rm eff}^{\rm BBN,CMB} \lesssim 0.2$



[Simakachorn'22]

In strain units.

$$h_c \simeq 1.26 \times 10^{-18} (\mathrm{Hz}/f_{\mathrm{GW}}) \sqrt{\Omega_{\mathrm{GW}} h^2}.$$



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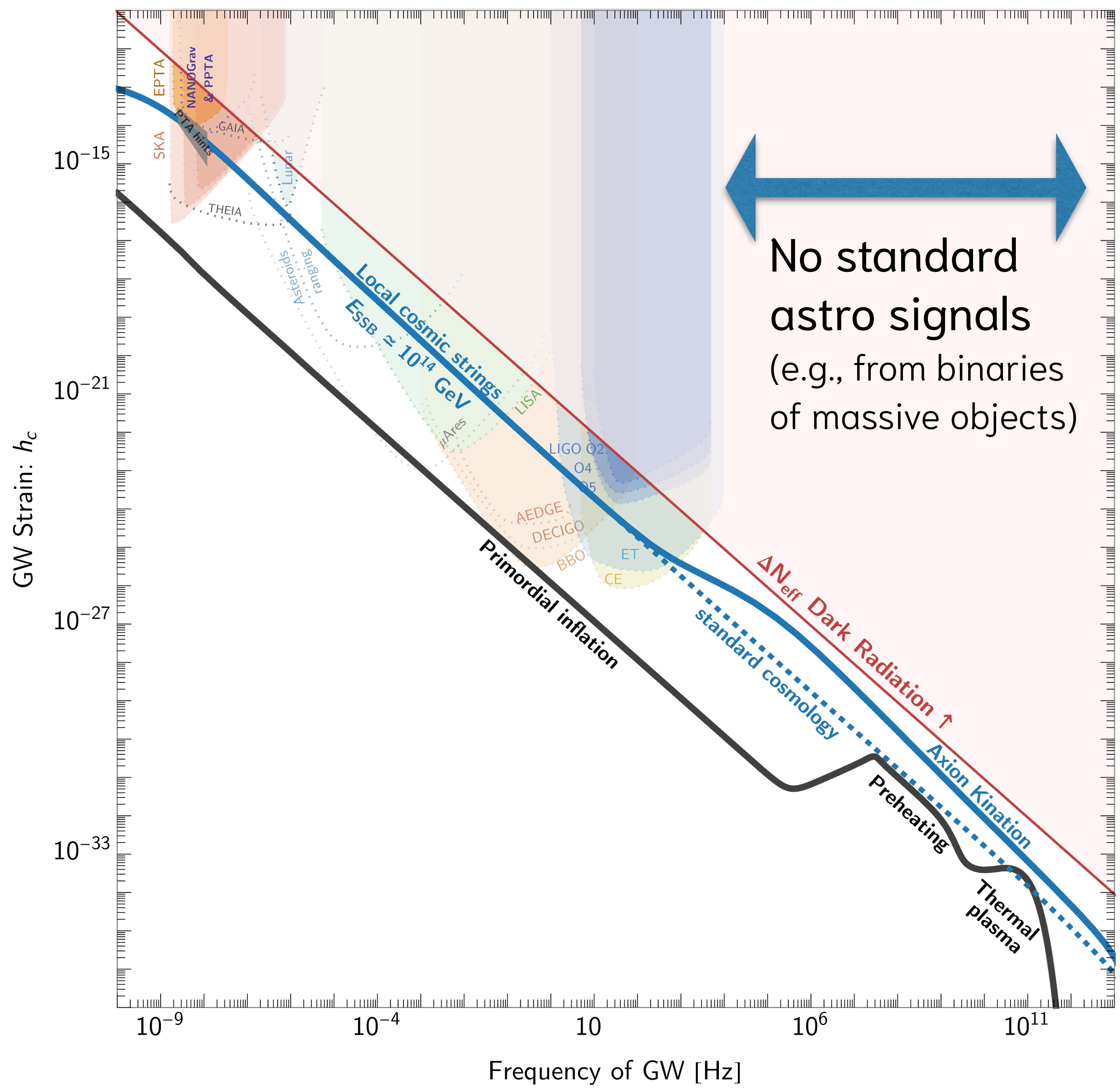


Characteristic Frequencies for causal (and short-lasting) sources .

T_* **temperature of the universe at time of emission**

f_* **frequency at time of emission**

observed frequency: $f \sim f_* \frac{T_0}{T_*} \sim \mathcal{O}(H_*) \frac{T_0}{T_*} \sim \frac{T_*}{M_{Pl}} T_0 \sim T_* \times 10^{-13} \ 10^{-19}$ **GeV**

H_* = Hubble rate at T_*

If T_* **~ 100 GeV:** (Electroweak scale)

$$f \sim 10^{-30}\,\mathrm{GeV} \sim 10^{-30} \times 10^{25}\ \mathrm{Hz} \sim 10^{-5}\ \mathrm{Hz}$$

LISA !

If T_* **~** 10^{10} **GeV:** (Peccei-Quinn scale)

$f_* \sim \frac{1}{10} \mathcal{O}(H_*)$ **–>** **f ~ 100 Hz** ET !

Reading the history of the universe.

GW frequency $f_{GW,0} \simeq \lambda_{GW}^{-1}\left(\frac{a_{prod}}{a_0}\right)$

Low-freq. lim

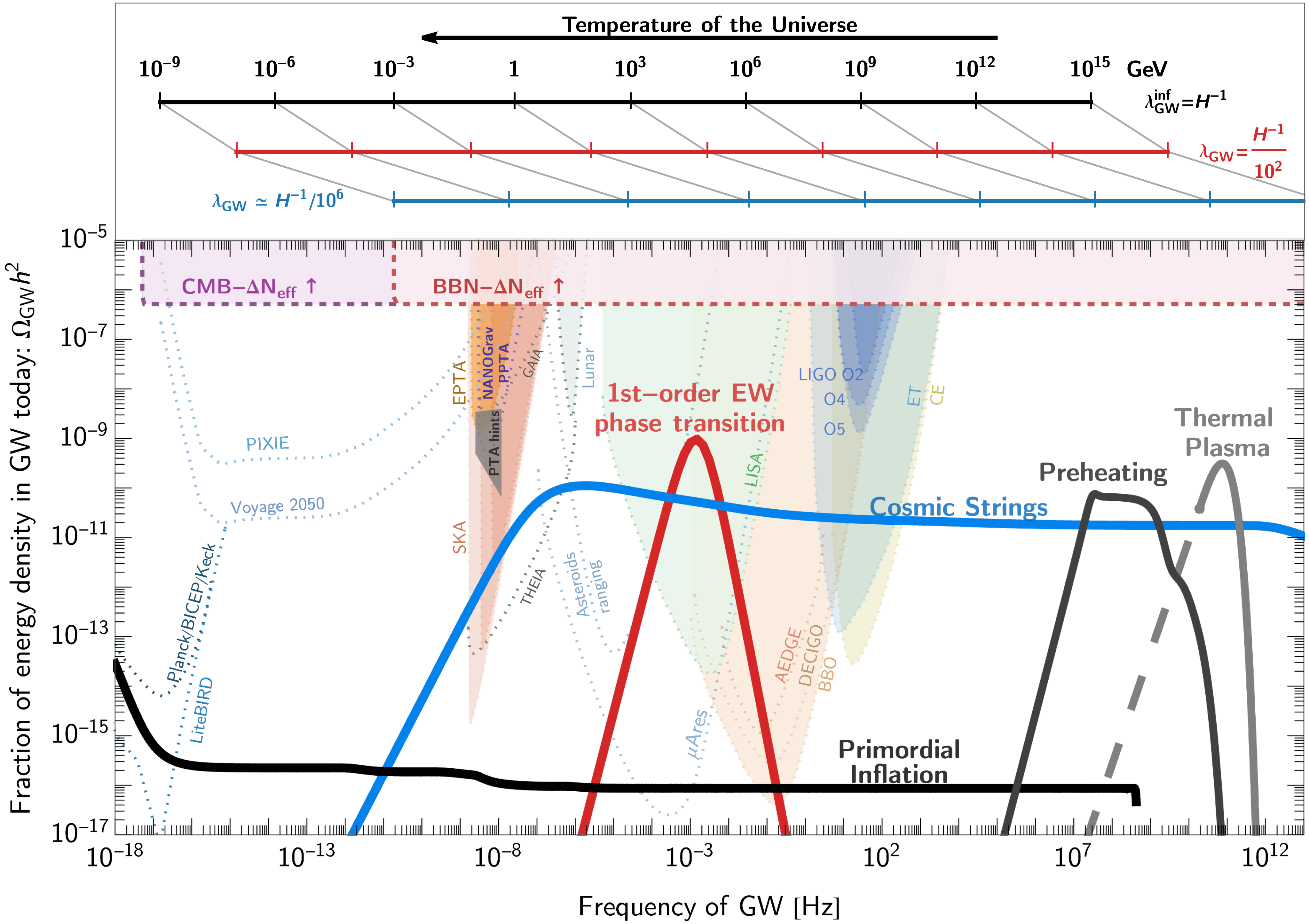
$f_{GW}^{min} \simeq H_0^{-1}$

$\simeq 10^{-18}$ Hz

High-freq. limit

$f_{GW}^{max} \simeq 10^{13}$ Hz

$(\lambda_{GW} \sim H^{-1} \sim M_{pl}^{-1})$



[Simakachorn]

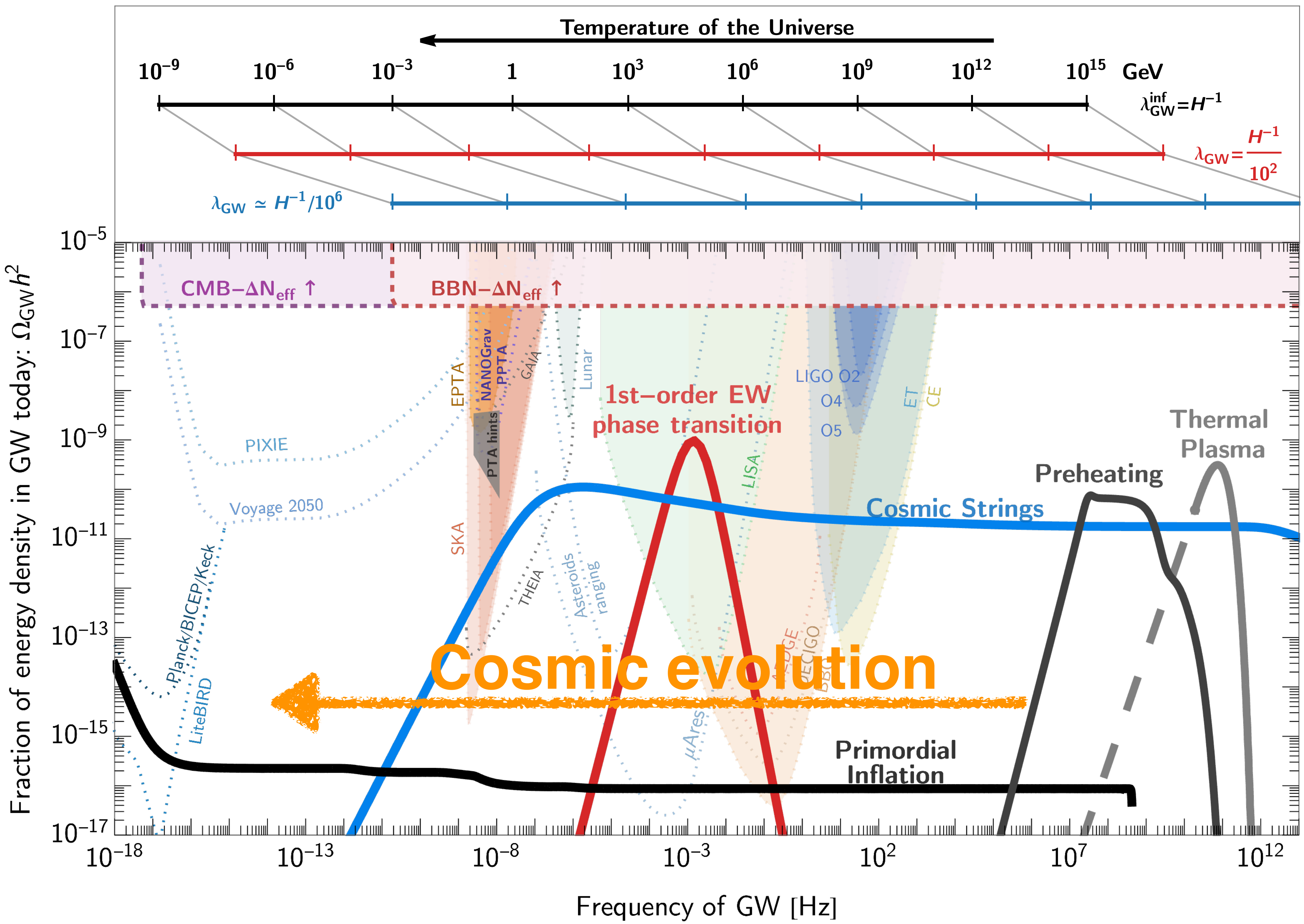
Reading the history of the universe.

GW frequency $f_{\mathrm{GW},0} \simeq \lambda_{\mathrm{GW}}^{-1} \left(\dfrac{a_{\mathrm{prod}}}{a_0} \right)$

Low-freq. lim

$f_{\mathrm{GW}}^{\mathrm{min}} \simeq H_0^{-1}$

$\simeq 10^{-18}$ Hz



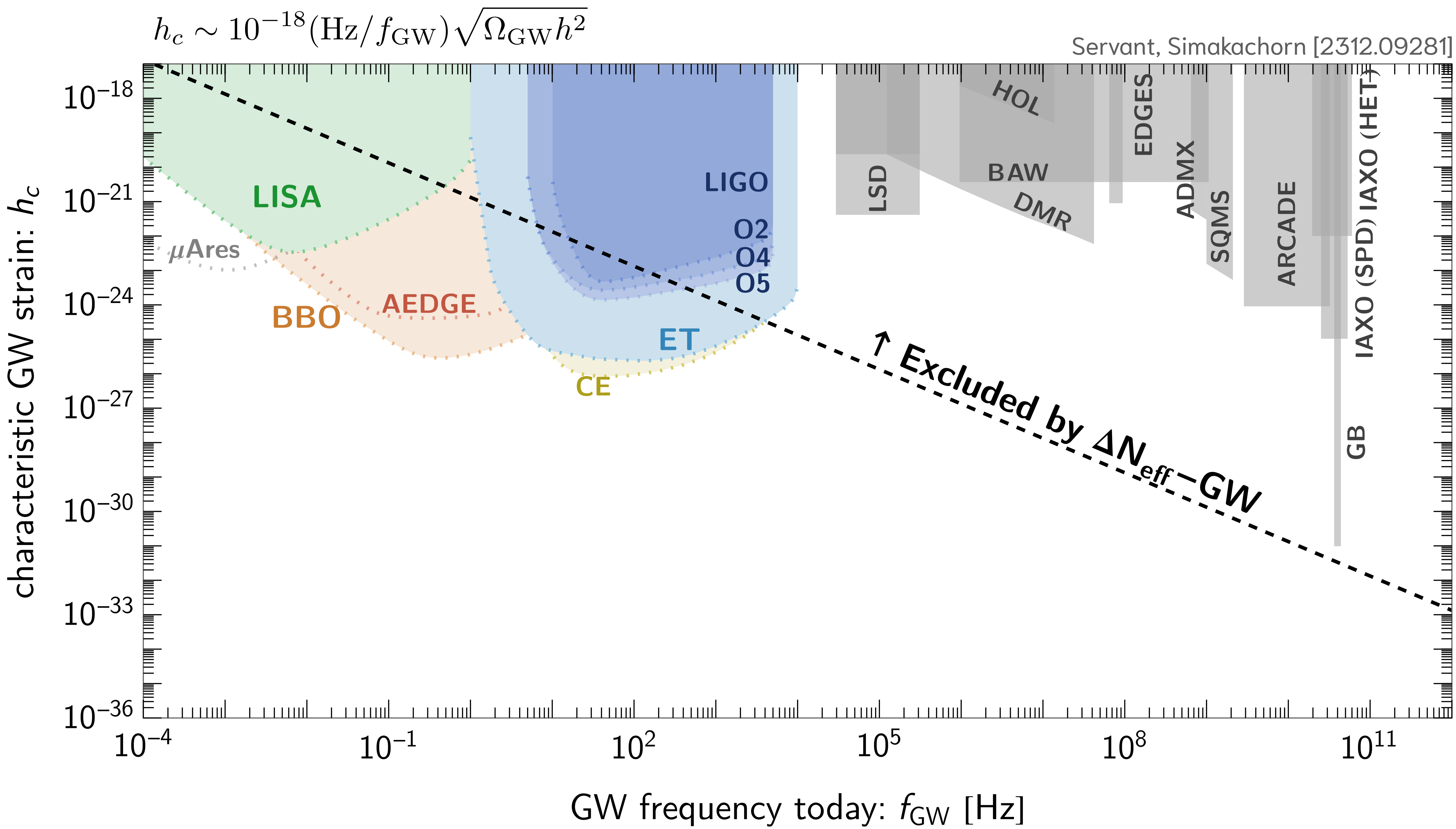
High-freq. limit

$f_{\mathrm{GW}}^{\mathrm{max}} \simeq 10^{13}$ Hz

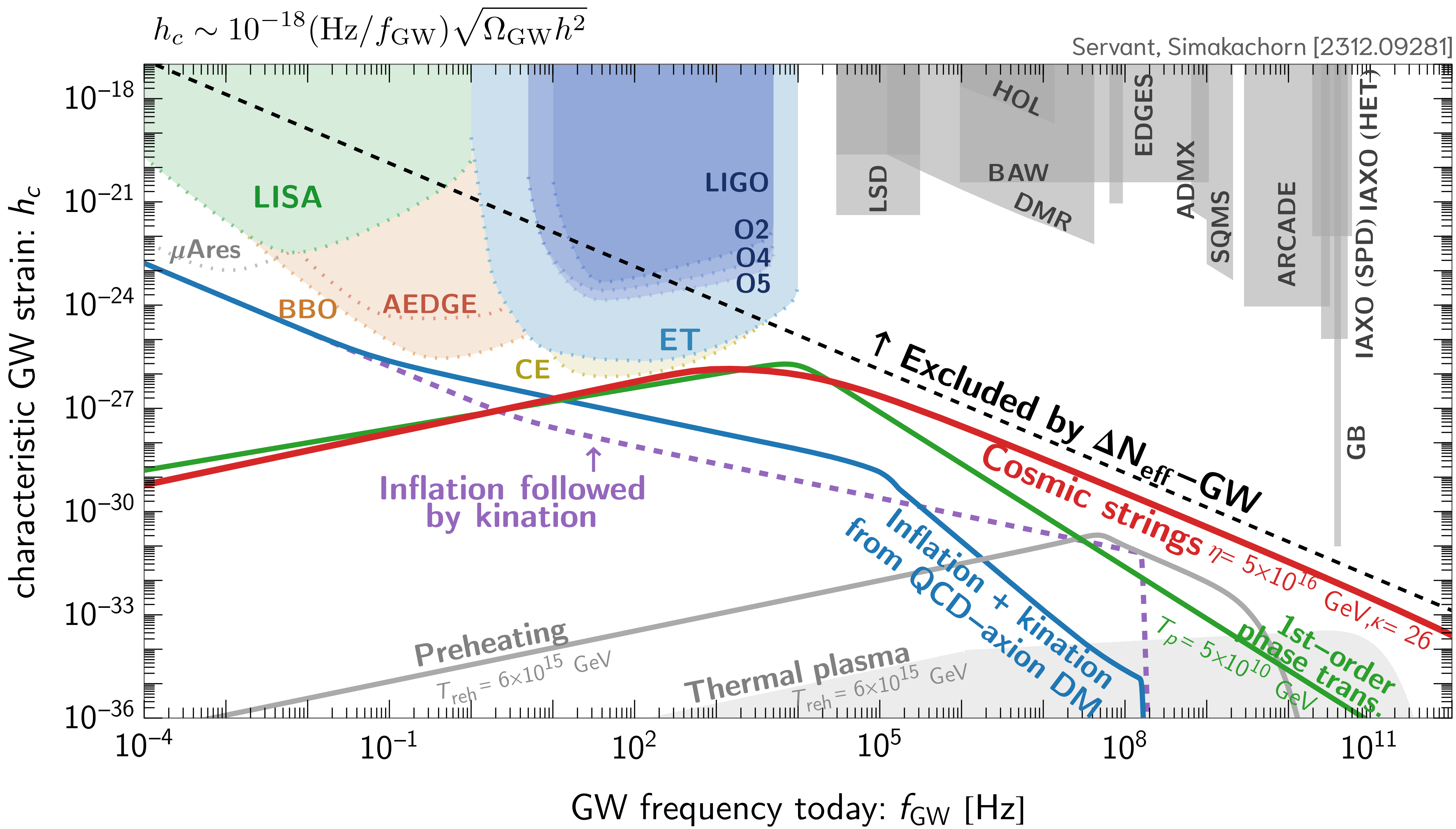
$(\lambda_{\mathrm{GW}} \sim H^{-1} \sim M_{\mathrm{pl}}^{-1})$

[Simakachorn]

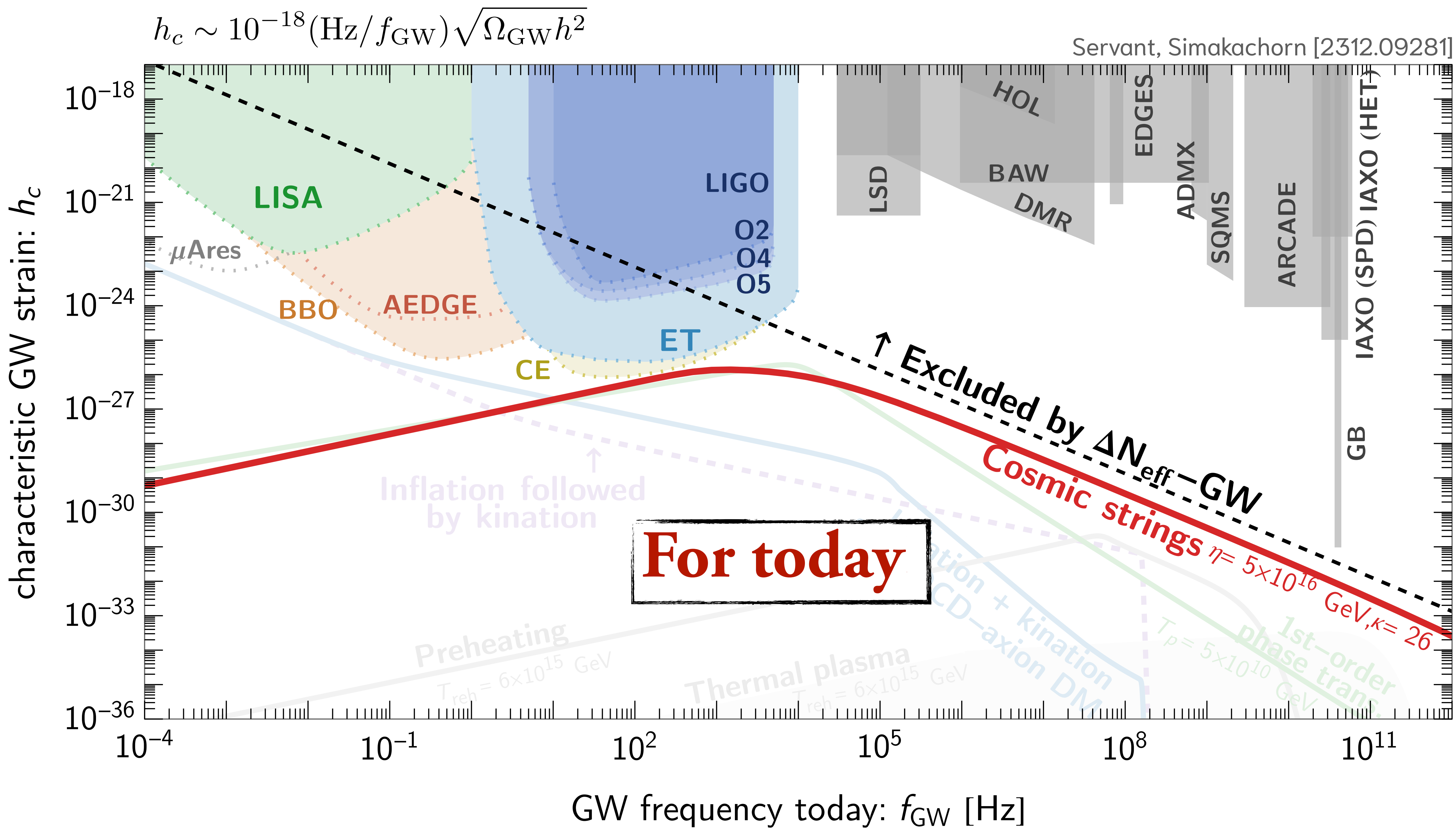
Ultra-high frequency GWs: current constraints



Ultra-high frequency GWs: primordial signals

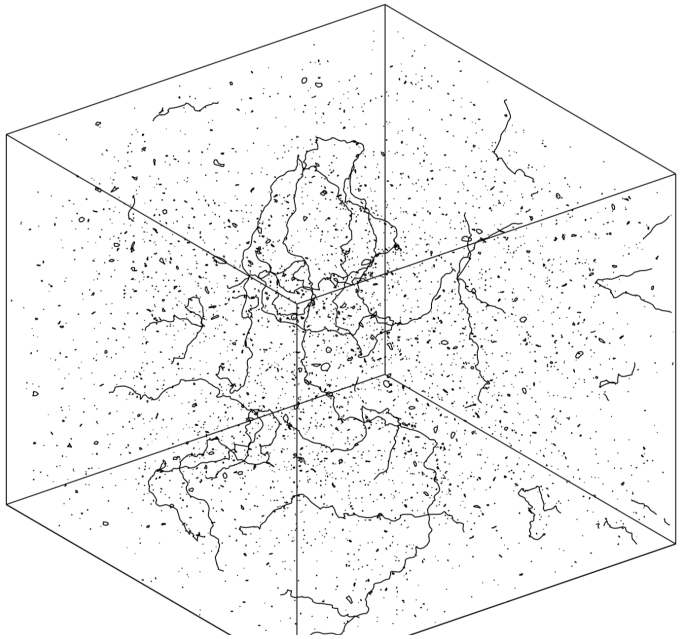


Ultra-high frequency GWs from cosmic strings



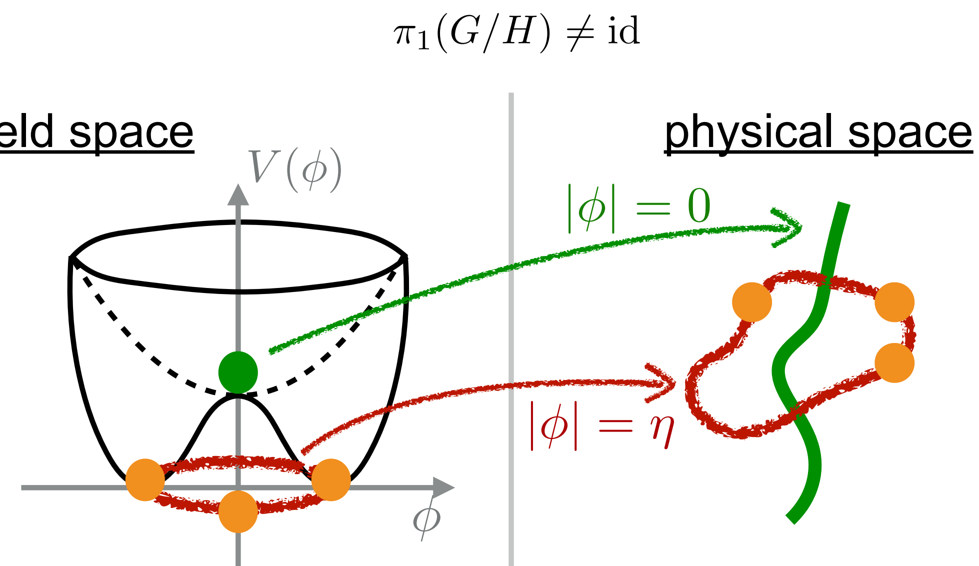
Gravitational Waves from cosmic strings.

recent reviews: **[1909.00819, 1912.02569]**



Network of cosmic strings
[Allen & Shellard, 1990]

Gravitational Waves from cosmic strings.



e.g., $V(\Phi) \simeq \lambda(|\Phi|^2 - \eta^2)^2$

Cosmic string: Line-like topological defect arising after spontaneous U(1) symmetry breaking at some energy scale η.

The broken symmetry can be either local or global;

—> local or global (axionic) cosmic strings.

string tension: $\mu \sim \eta^2$

$$\mu \sim \eta^2 \times \begin{cases} 1, & \textbf{local} \\ \ln(m_\phi/H), & \textbf{global} \end{cases}$$

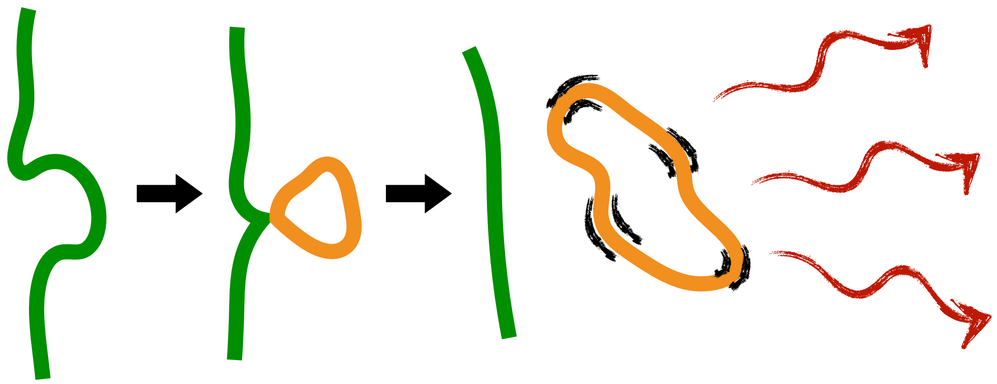
String tension: $G\mu \simeq \left(\frac{\eta}{m_{\mathrm{Pl}}}\right)^2 \simeq 7 \times 10^{-7} \left(\frac{\eta}{10^{16}\ \mathrm{GeV}}\right)^2$

Axionic strings: $\eta = f_a$

Loop formation & scaling regime.

After the network formation, the string network keeps producing loops.

String intercommutation: loop formation depletes energy from the network.



Cosmic strings do not overclose the universe.

The energy density of the network tracks the total energy density of the Universe

Scaling regime: $\rho_{\rm net}(t) \simeq \mu/t^2 \simeq \ G\mu\rho_{\rm tot}(t)$

$$\rho_\infty \propto t^{-2} \propto \begin{cases} a^{-3} & \text{for matter} \\ a^{-4} & \text{for radiation} \\ a^{-6} & \text{for kination} \end{cases}$$

GW from cosmic strings.

Cosmic strings: Long-lasting source of GW

The produced loops decay into particles and GW.

Local-string loops decay dominantly into GW while global-string loops decay dominantly into Goldstone radiation.

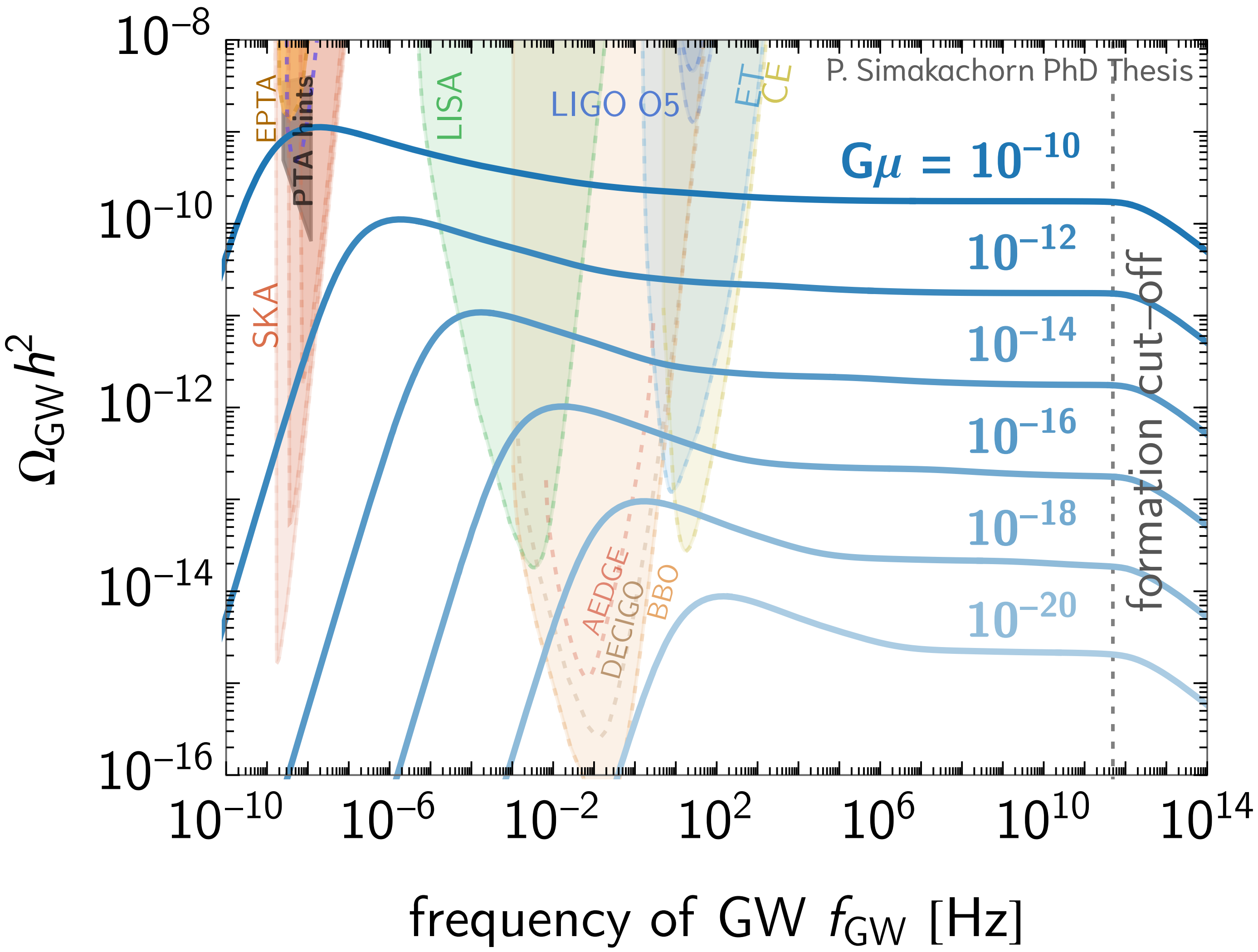
GW from local cosmic strings.

Superposition of many loop populations producing GW at time $\tilde{t}$ and of many oscillation k^{th}-modes,

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{1}{\rho_{c,0}} \sum_{k=1}^{k_{\max}} \frac{2k}{f_{\rm GW}} \cdot \Gamma^{(k)} G\mu^2 \int_{t_{\rm form}}^{t_0} n_{\rm loop}(\tilde{t}) \left[\frac{a(\tilde{t})}{a(t_0)}\right]^5 d\tilde{t},$$

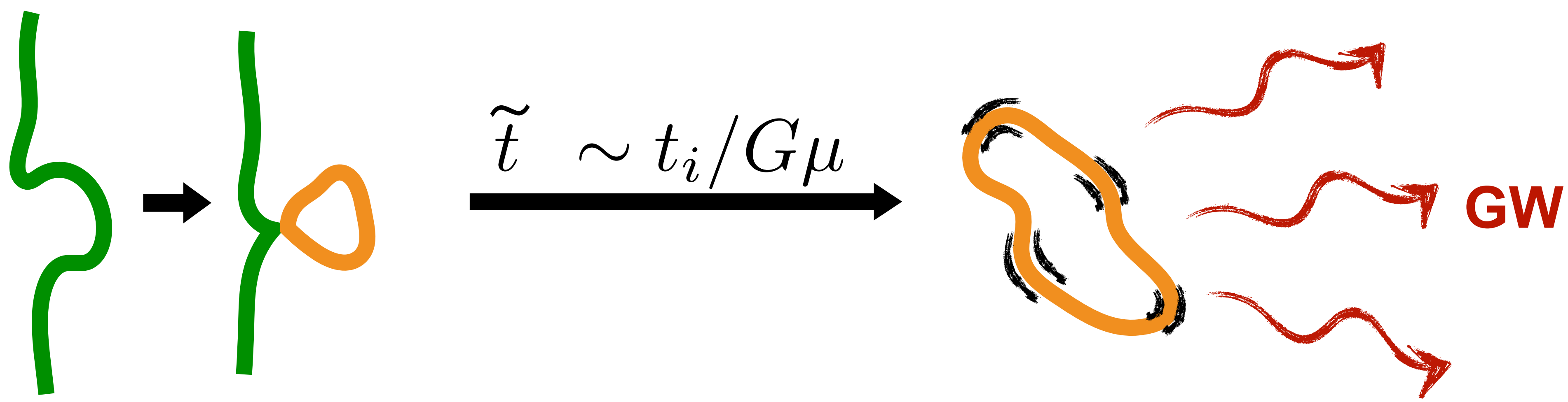
GW from a loop

# of loops produced along cosmic history (from production time until today)



cosmic string (local)

loop production @ t_i loop emission @ $\tilde{t}$



Relation between observed frequency & Hubble radius at emission in radiation era

$$f \approx H_i \left(\frac{a_i}{a_0}\right) \left(\frac{1}{G\mu}\right)^{1/2}$$

(for loops produced and emitting GW in the radiation era)

(the delayed emission happens at $a/a_i = (t/t_i)^{1/2} = 1/\sqrt{(G\mu)}$)

Relation between observed frequency & Hubble radius at emission.

The broad band GW spectrum is the result of the superposition of GW generated by many populations of loops produced at different temperatures.

Each emits GW at frequency $f_{\rm GW}^{\rm emit} \simeq 2k/l$

k : GW mode number of loop oscillation

l: loop's size

The GW contribution at higher frequencies comes from smaller loops produced at higher energy scales.

Time of GW emission for local strings: $\tilde{t} \simeq \alpha/(2\Gamma G\mu)t_i$

In contrast, global strings quickly emit GW after loop production: $\tilde{t} \simeq t_i$

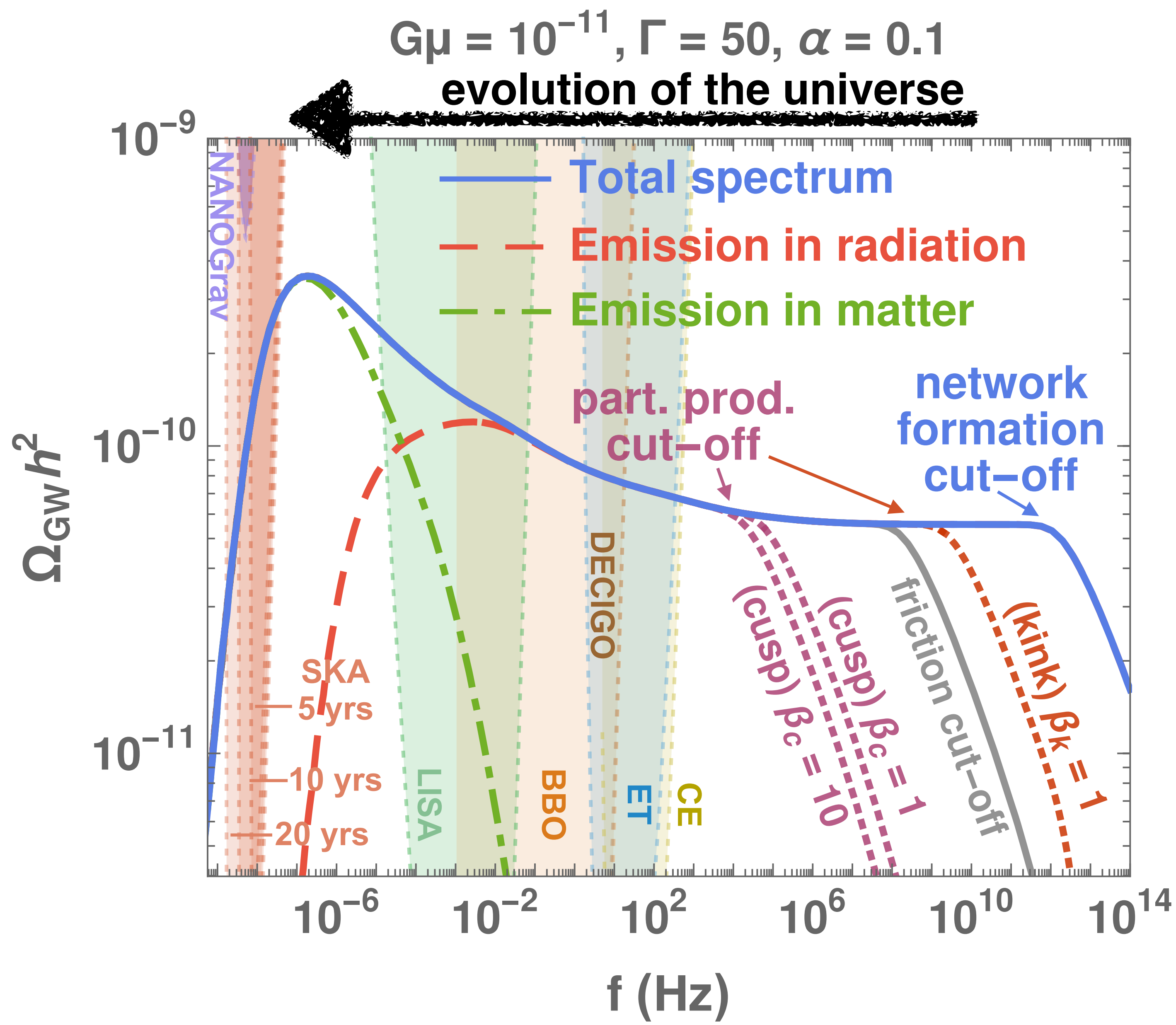
t_i : time of loop formation

For a loop population created at temperature T, the GW spectrum is sourced maximally at a GW frequency today that is higher for local strings compared to global strings.

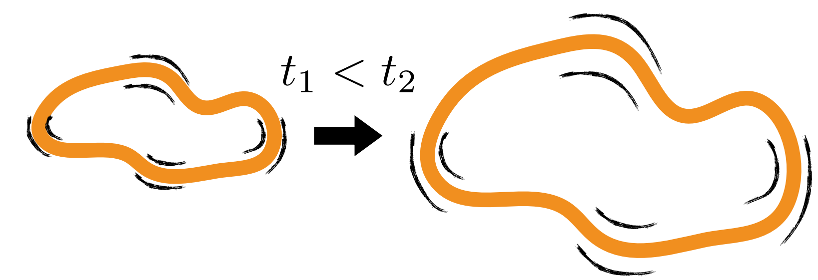
Gravitational Waves from Cosmic strings.

[1912.02569]

(long-lasting sources).



Higher f ⇔ Earlier emission



smaller loop ⇔ higher oscillation f

@ earlier t_i

more GW from more loops
but more red-shift

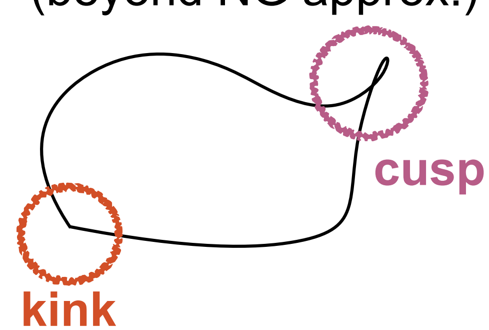
⇨ Flat during radiation

−.−.−

contribution from loops produced in radiation era but emitting GW today.

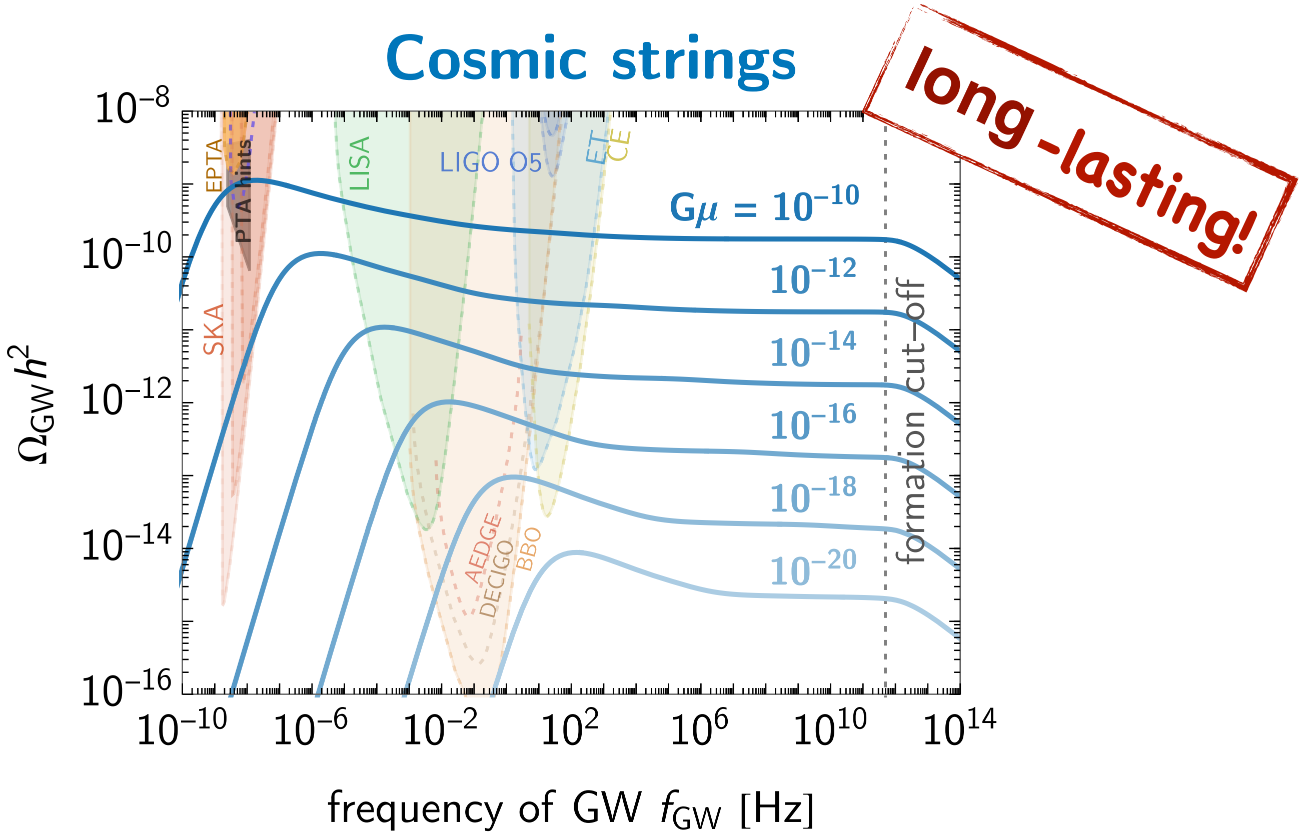
$$f_{\rm low}^{\rm stable} = \frac{2}{\Gamma G\mu t_0} \simeq (1.48 \times 10^{-7}\ {\rm Hz}) \left(\frac{50 \times 10^{-11}}{\Gamma G\mu}\right)$$

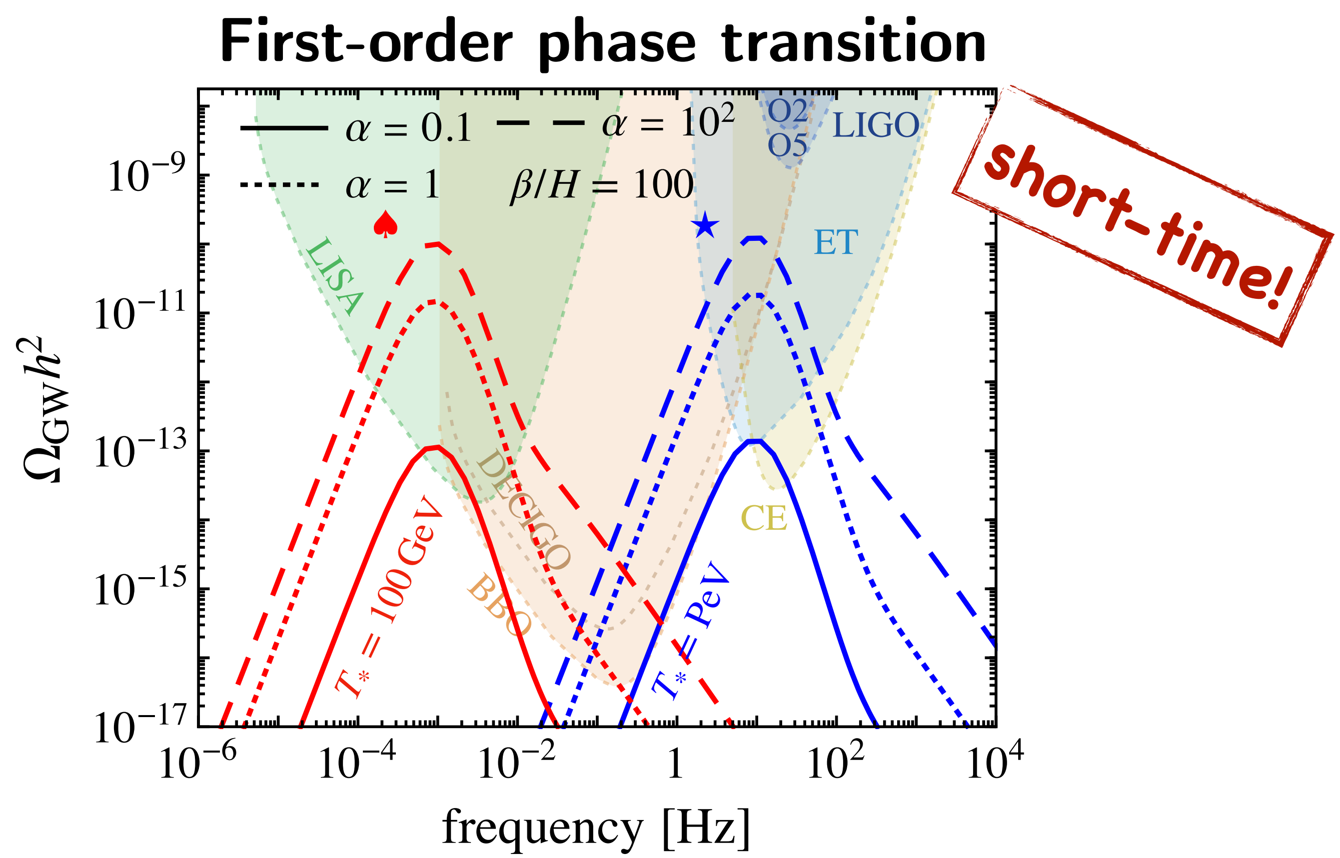
singular structures on loop
(beyond NG approx.)



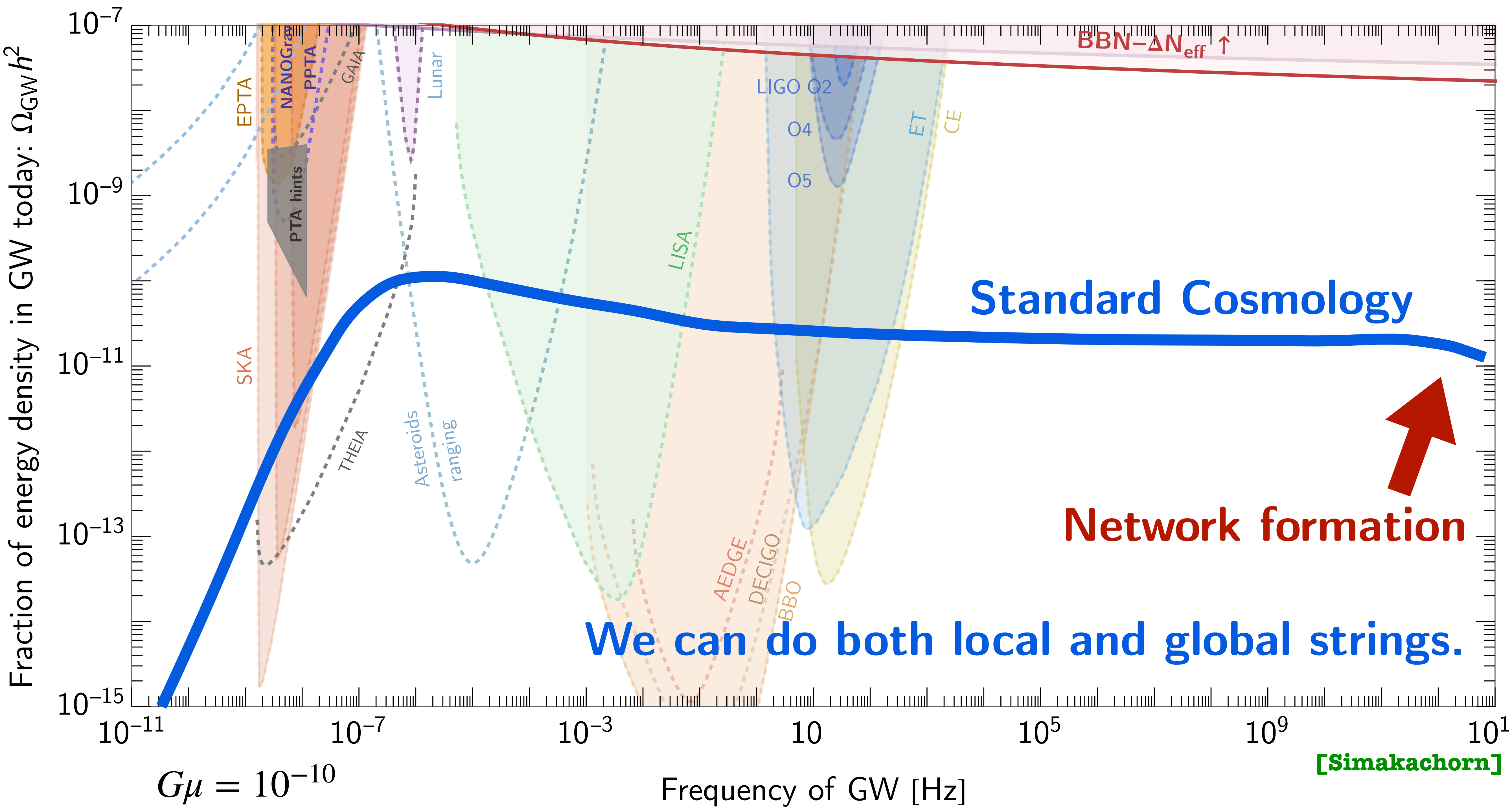
lead to particle emission

Short-lasting vs long-lasting primordial sources.





Gravitational Waves from cosmic strings.



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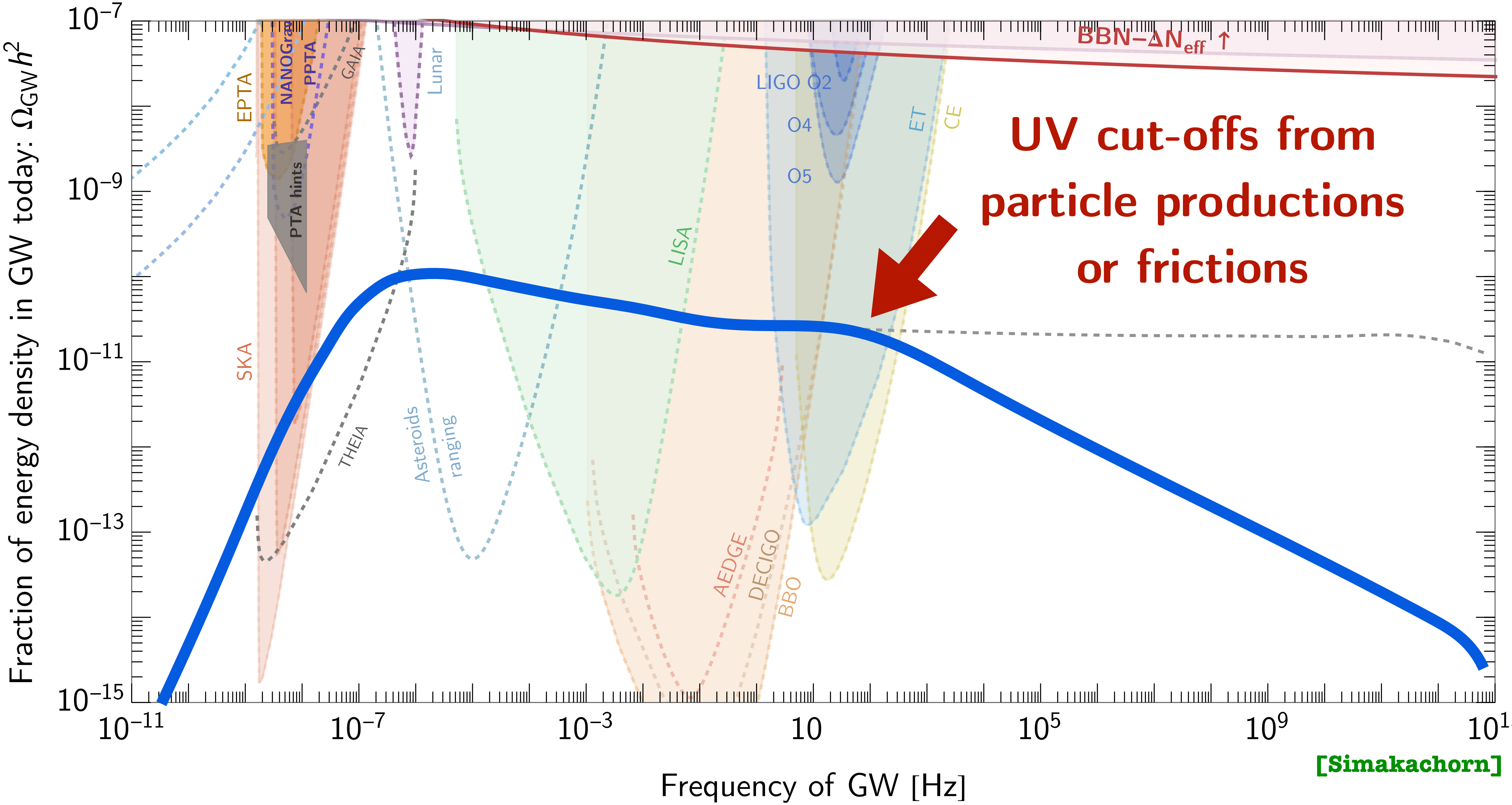

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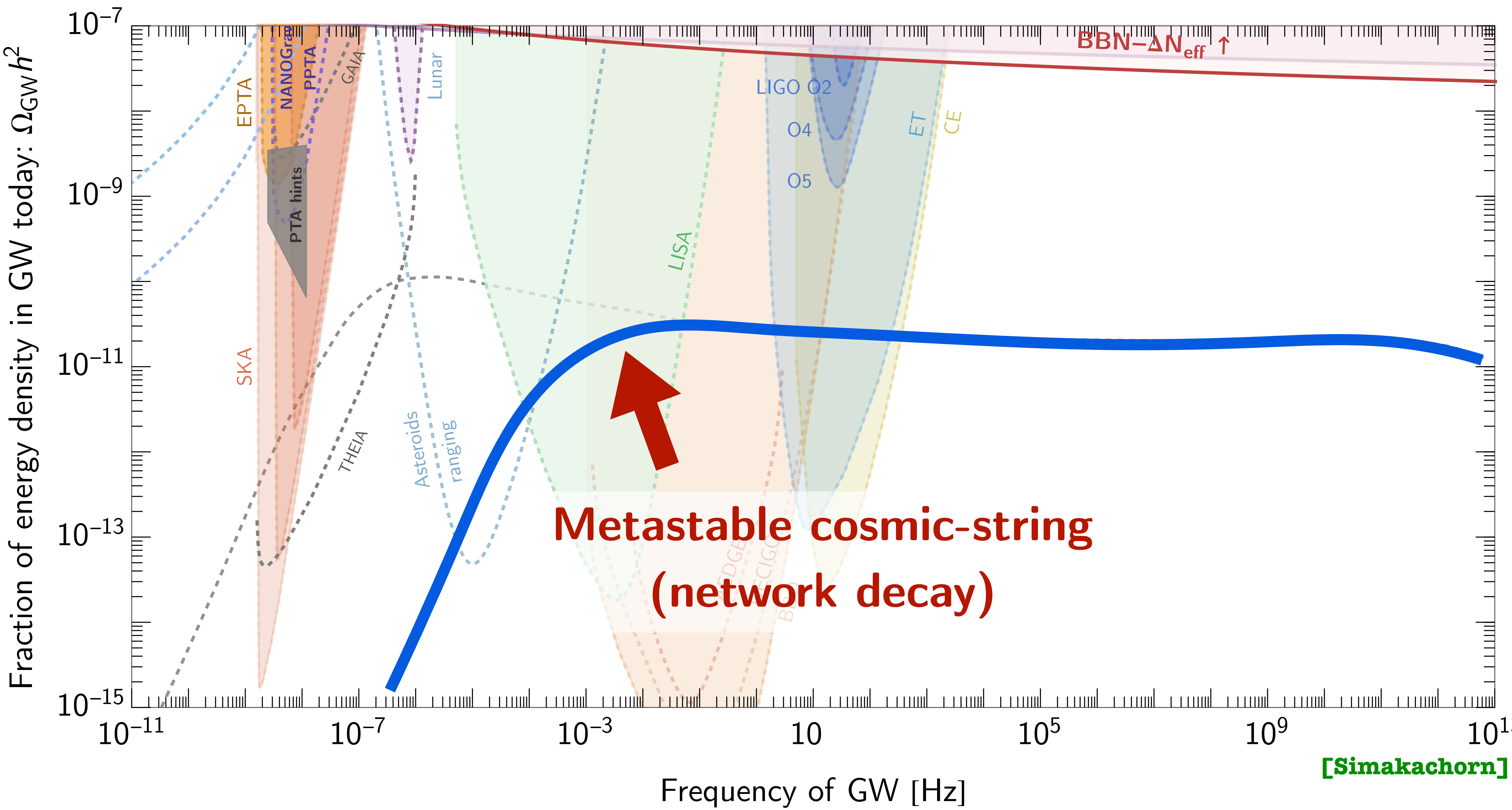
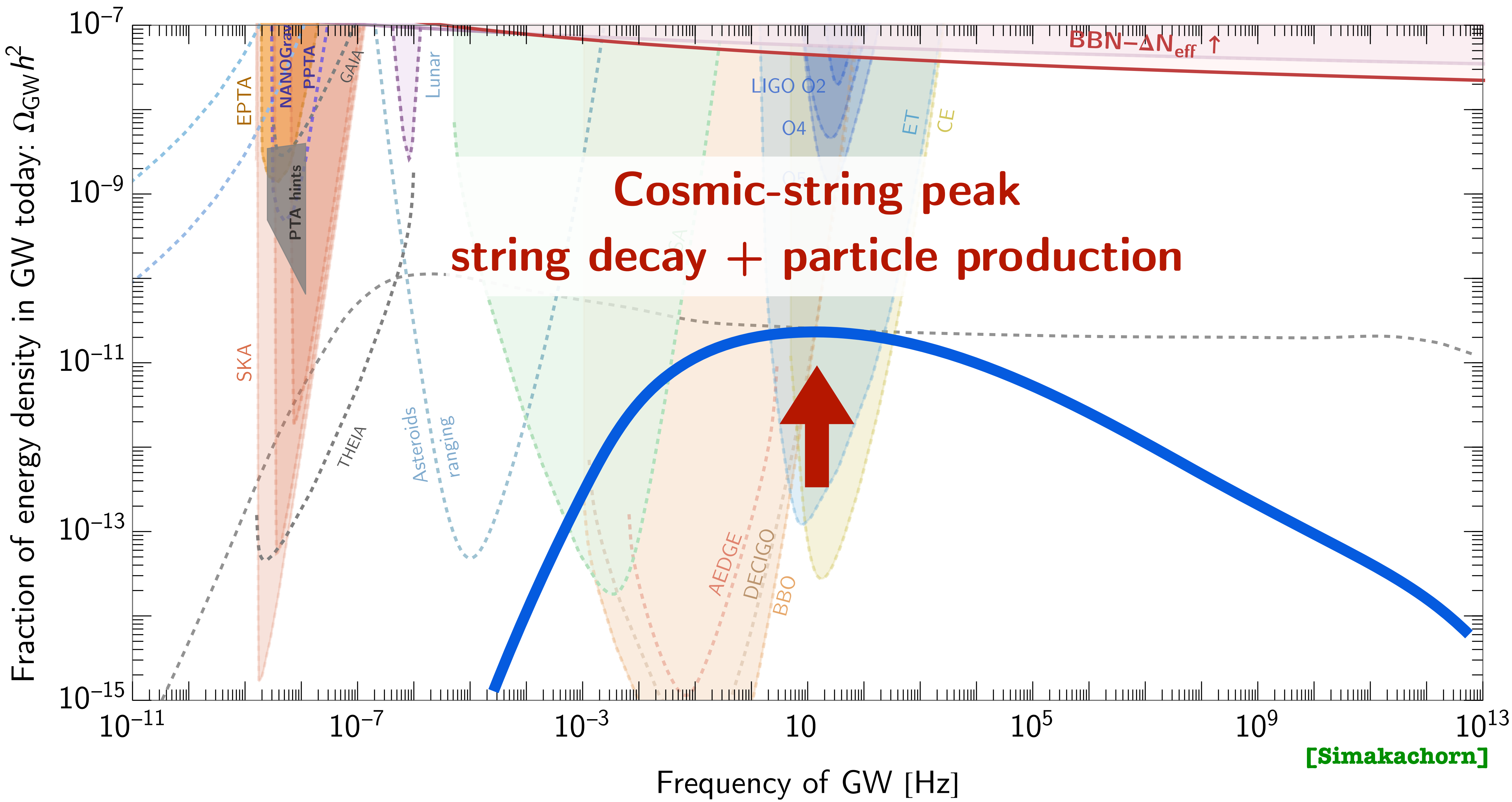


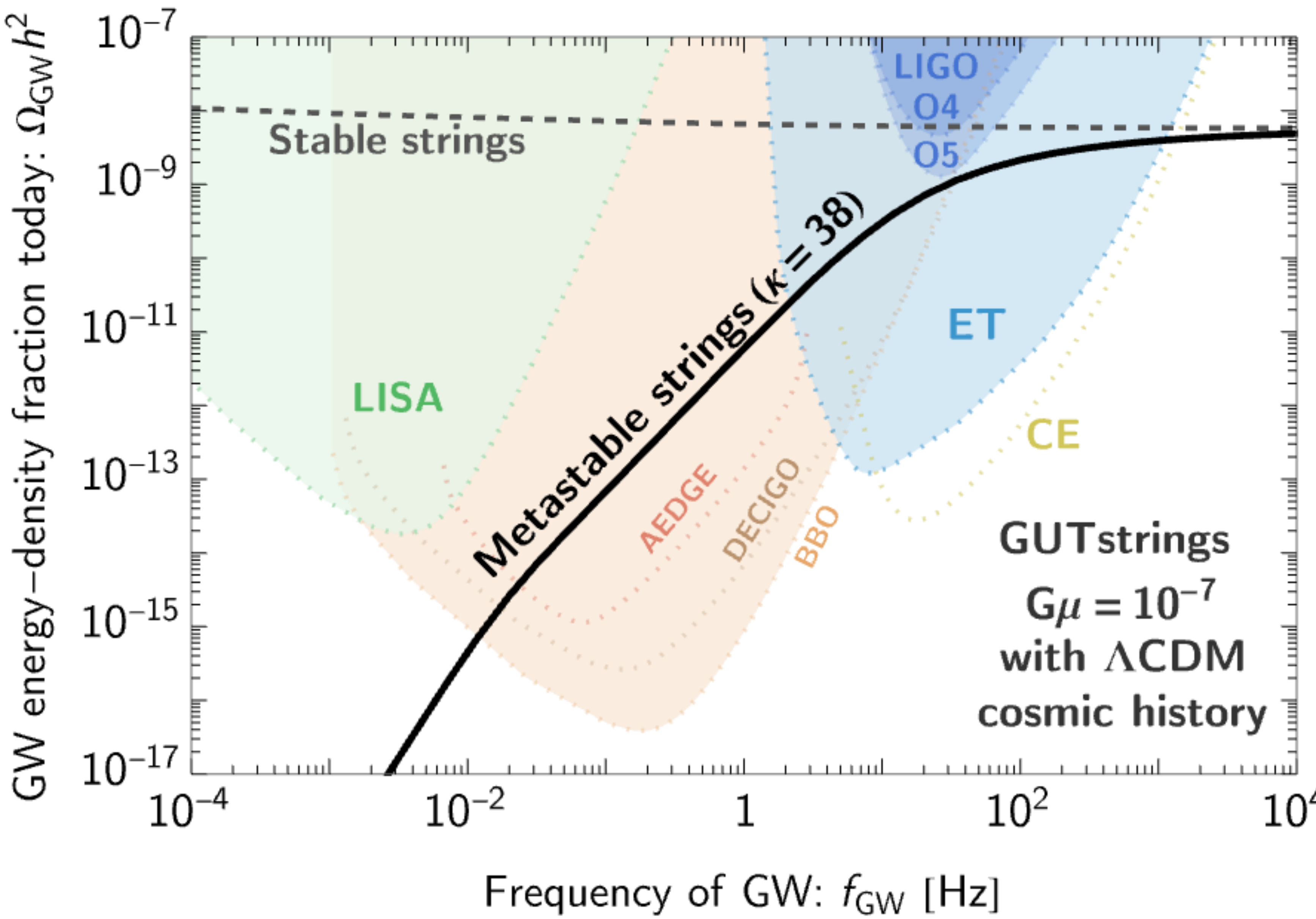
[Simakachorn]

Gravitational Waves from cosmic strings.



GWs from Grand Unified Theory metastable cosmic strings.

Missed at LIGO/Virgo and at LISA, visible at ET



$$\kappa = \left(\frac{m_M}{\eta}\right)^2$$

$\eta \sim 3 \times 10^{15}$ GeV

LOCAL STRINGS
vs
GLOBAL STRINGS.

See comparison in Appendix F of [1912.02569] .

Loops from global strings : short-lived

Loops from local strings : long-lived.

—> different GW spectra in both frequency and amplitude.

LOCAL STRINGS vs GLOBAL STRINGS.

Global strings: no gauge field, instead massless Goldstone mode, with logarithmically-divergent gradient energy.

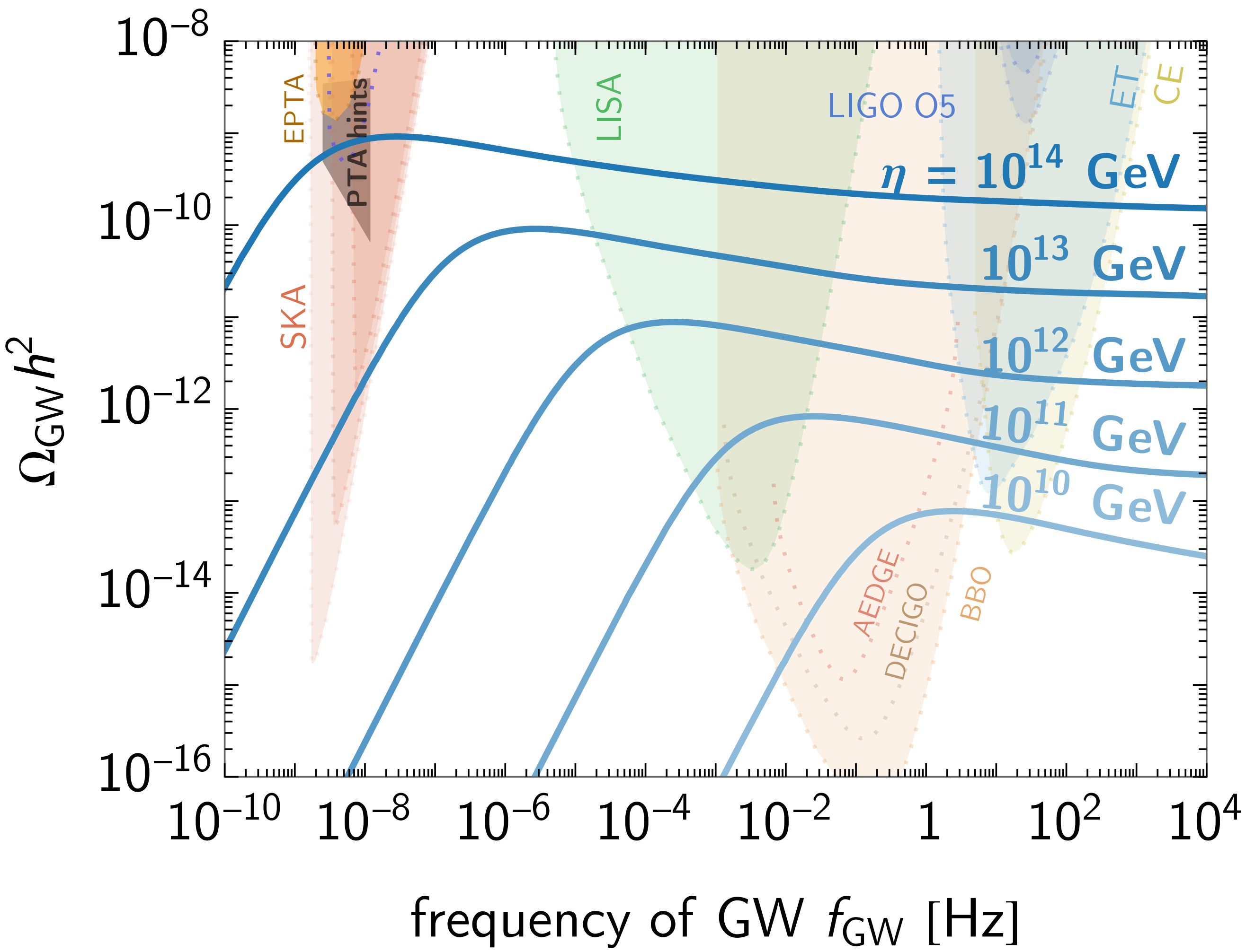
Loops quickly decay into axion particles.

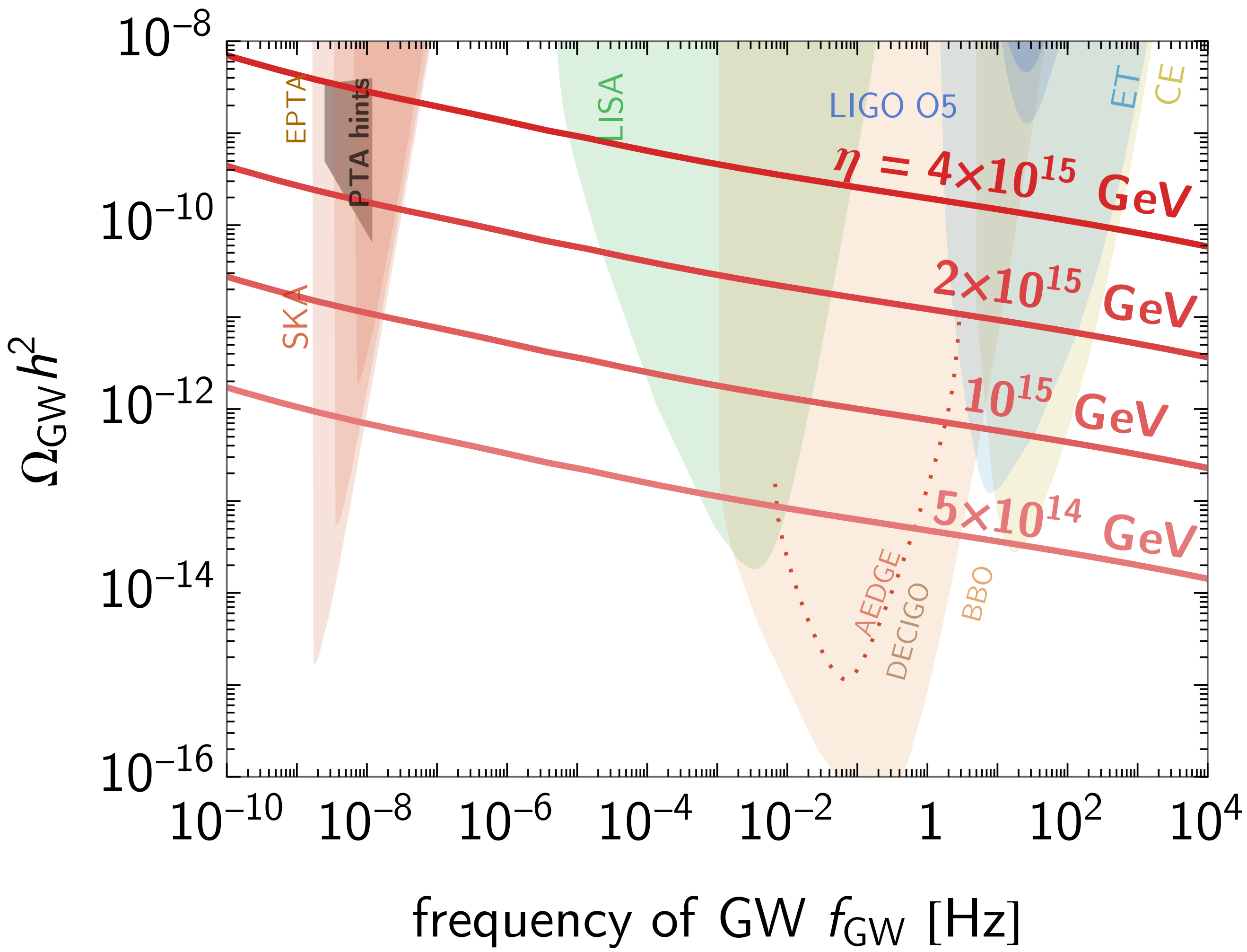
GW are mainly produced at the time of the loop production.

$$\Omega_{\rm GW}^{\rm local} \simeq \Omega_r \frac{\eta}{M_{\rm pl}}, \qquad \Omega_{\rm GW}^{\rm global} \simeq \Omega_r \left(\frac{\eta}{M_{\rm pl}}\right)^4 \log^3(\eta t_i).$$

$\eta = f_a$

LOCAL STRINGS versus GLOBAL STRINGS (m_a=0)





spectral shape changes with η

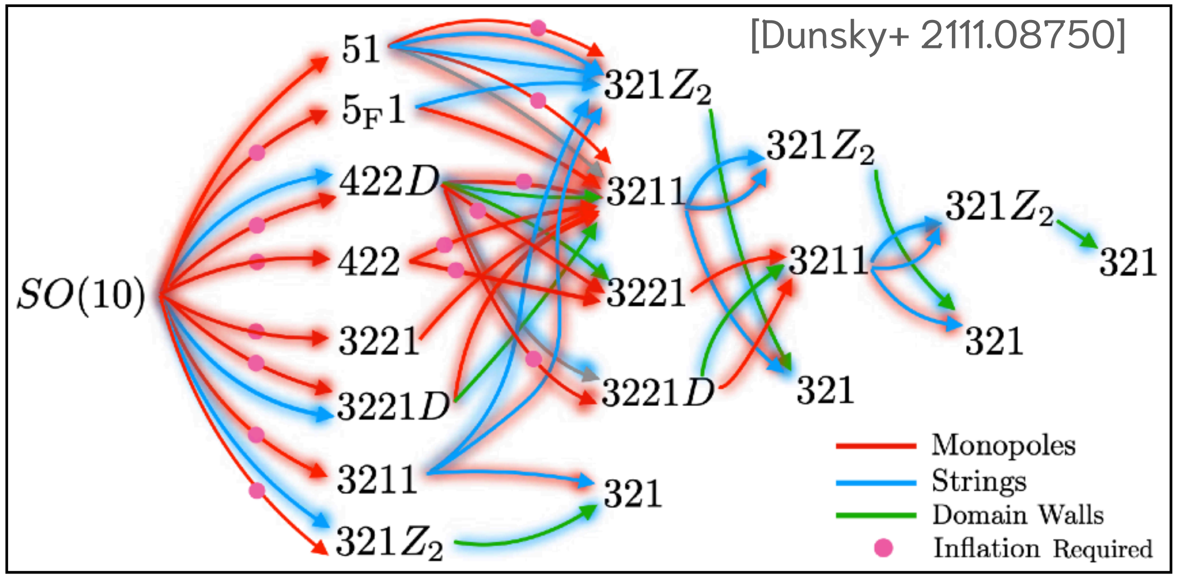
local loops live longer before decaying (& lifetime depends on η)

global loops decay fast.

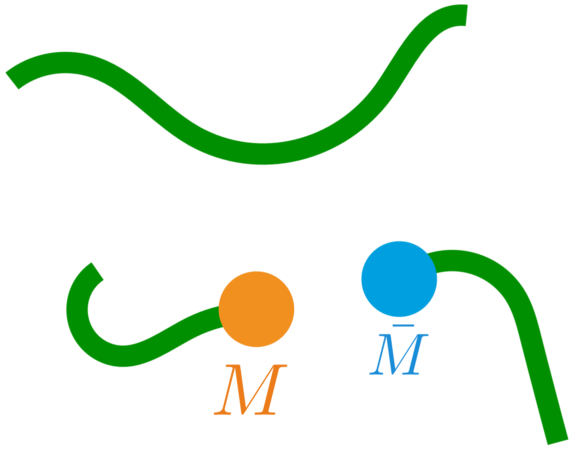
To reach the same amplitude as the local strings, global strings need a larger η since GW production is not the leading energy loss.

Metastable local cosmic strings.

Multiple-step phase transitions —> Hybrid defects



string-monopole



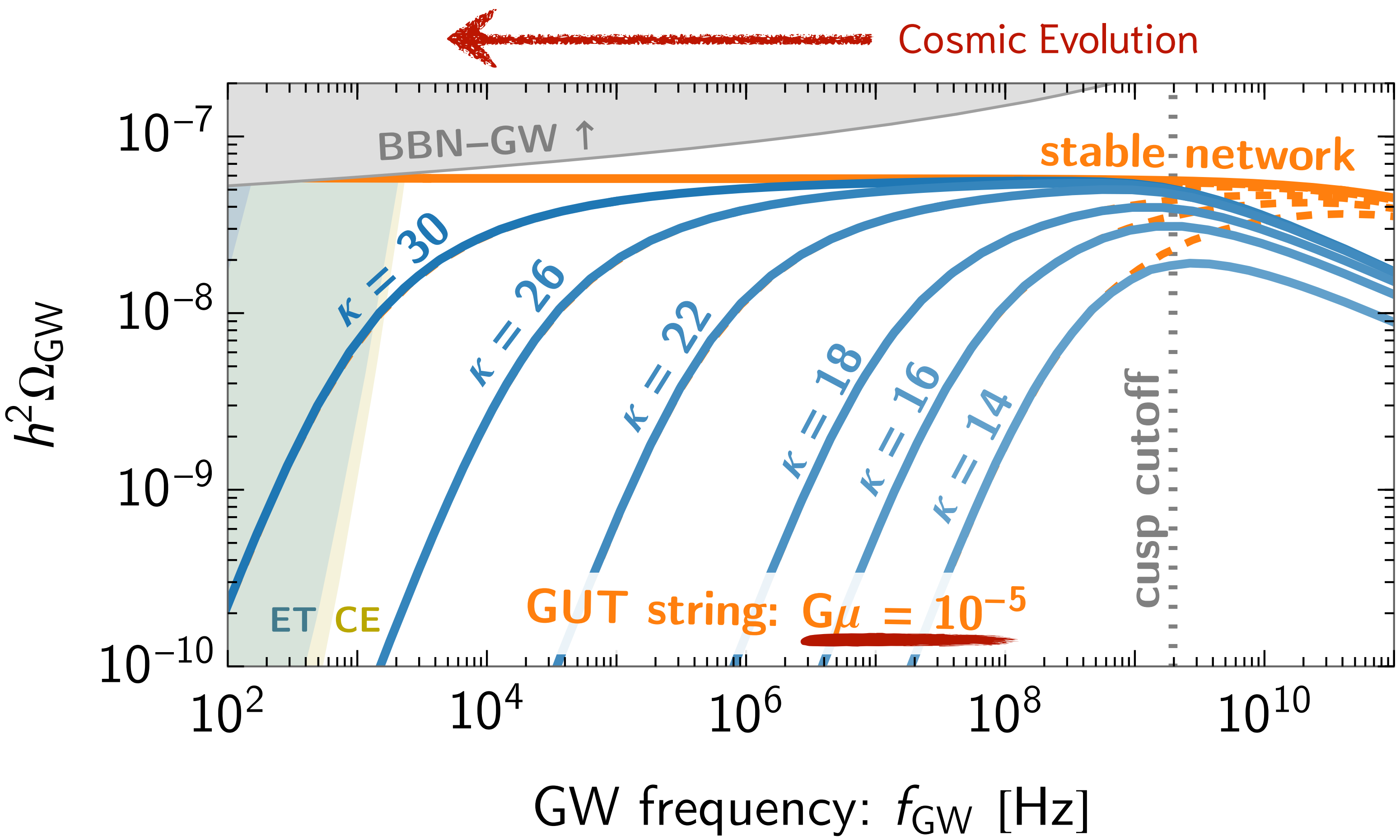
String network late decay due to nucleation of monopole-antimonopole pairs

$$\sqrt{\kappa} \equiv \frac{m_M}{\eta} \left[\frac{\text{monopole scale}}{\text{string scale}} \right] > 1$$

Large GW backgrounds @ Ultra-high frequencies from metastable local cosmic strings.

Infrared (IR) cutoff:

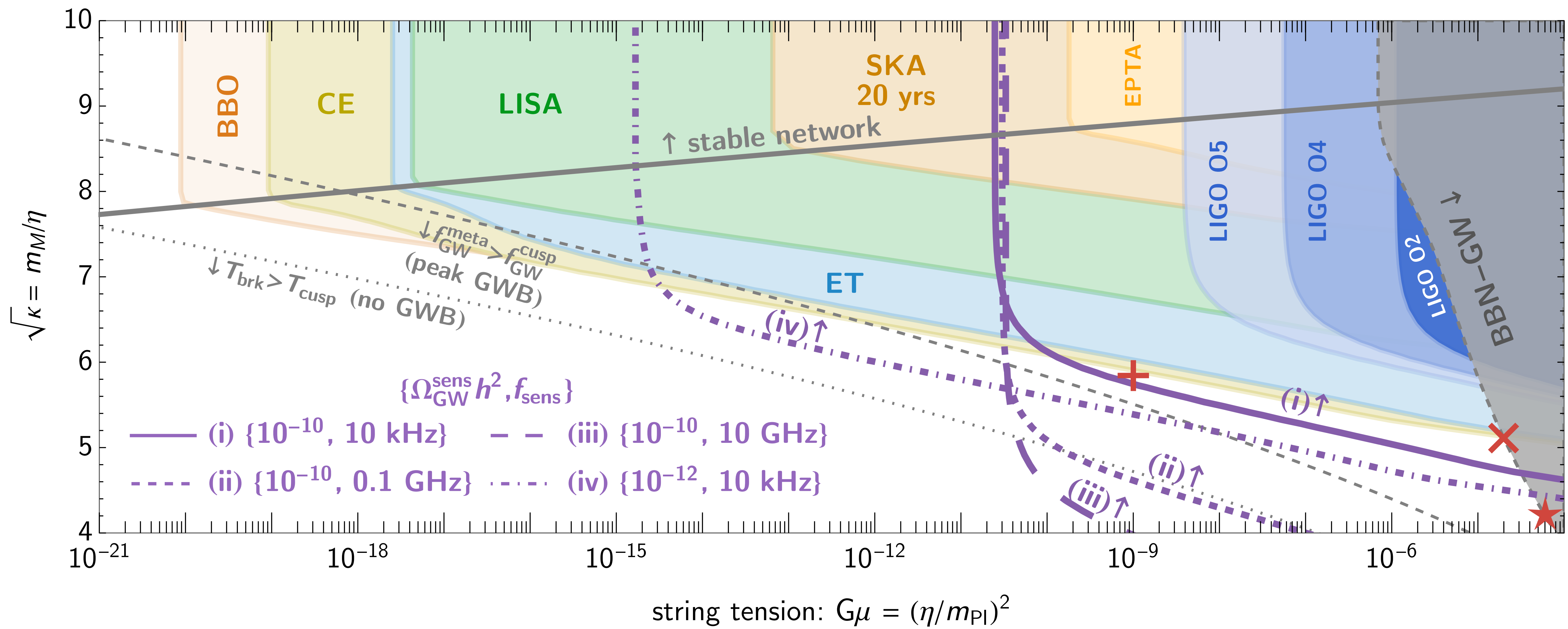
$$f_{\rm GW}^{\rm meta} \simeq {\rm MHz}\left(10^{-5}/G\mu\right)^{\frac{1}{2}} e^{-\frac{\pi}{4}(\kappa-25)}$$

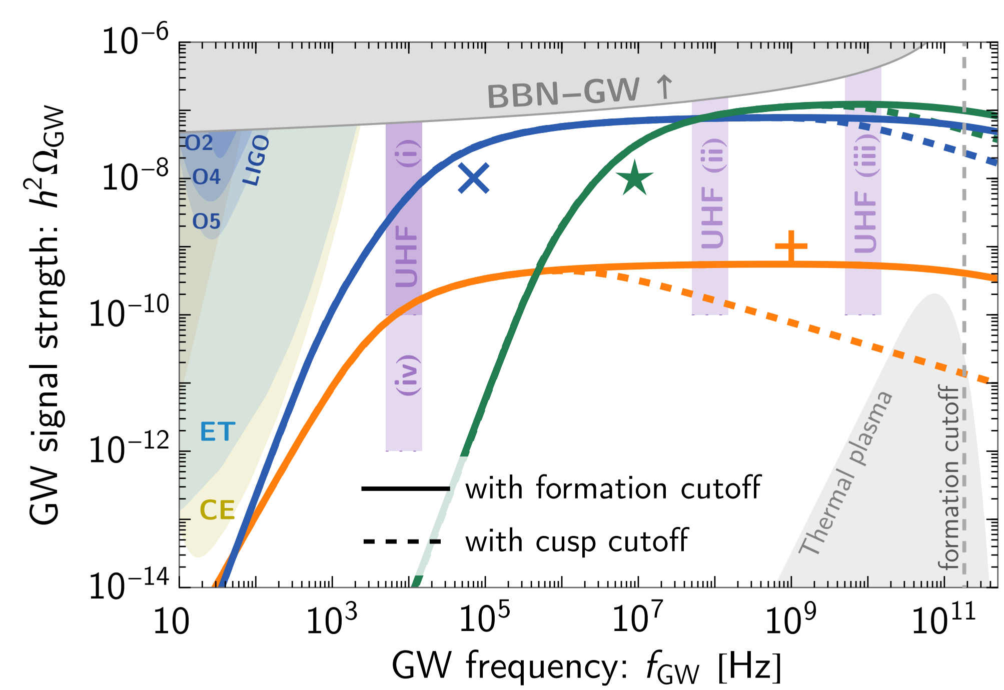


Any UHF experiment with sensitivity

slightly better than $\Delta N_{\rm eff}$–GW bound probes into GUT–scale physics.

Detectability of GWs from metastable local strings.





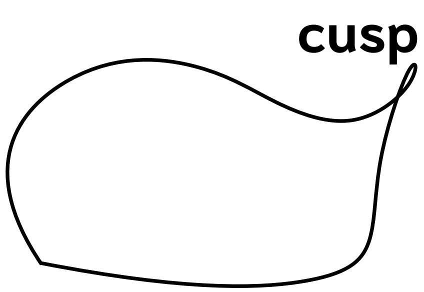
UV cutoffs.

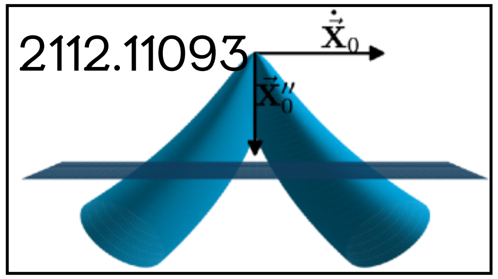
UHF signal from smaller loops at earlier times.

$$f_{\rm GW} \simeq 50\ {\rm kHz} \left[\frac{T}{10^{10}\ {\rm GeV}}\right] \left[\frac{10^{-5}}{G\mu}\right]^{\frac{1}{2}}$$

$$l_{\rm loop} \sim H^{-1} \sim 10^{-10}\ {\rm GeV} \left[\frac{10^{14}\ {\rm GeV}}{T}\right]^{2}$$

1D description breaks e.g.,
small-scale structures (e.g., cusp and kink).





$$\left.\frac{dE}{dt}\right|_{\rm cusp} \sim \lambda^{-\frac{1}{4}} \frac{\mu^{3/4}}{l_{\rm loop}^{1/2}}$$

E.g. GW emission is inefficient due to particle production from cusps if

$$f_{\rm GW} > f_{\rm GW}^{\rm cusp} \simeq {\rm GHz}\ \lambda^{1/8} \left(\frac{G\mu}{10^{-5}}\right)^{3/4}$$

scalar self-coupling

$$V(\Phi) \simeq \lambda(|\Phi|^2 - \eta^2)^2$$

$$w \simeq m_\Phi^{-1} \simeq (\sqrt{\lambda}\eta)^{-1}$$

UV cutoffs.

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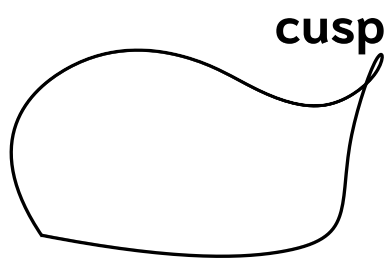

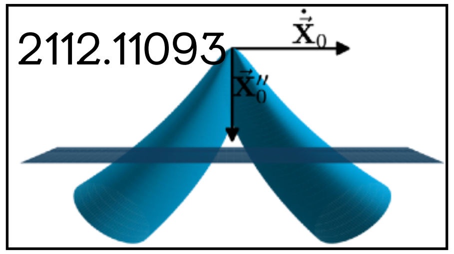

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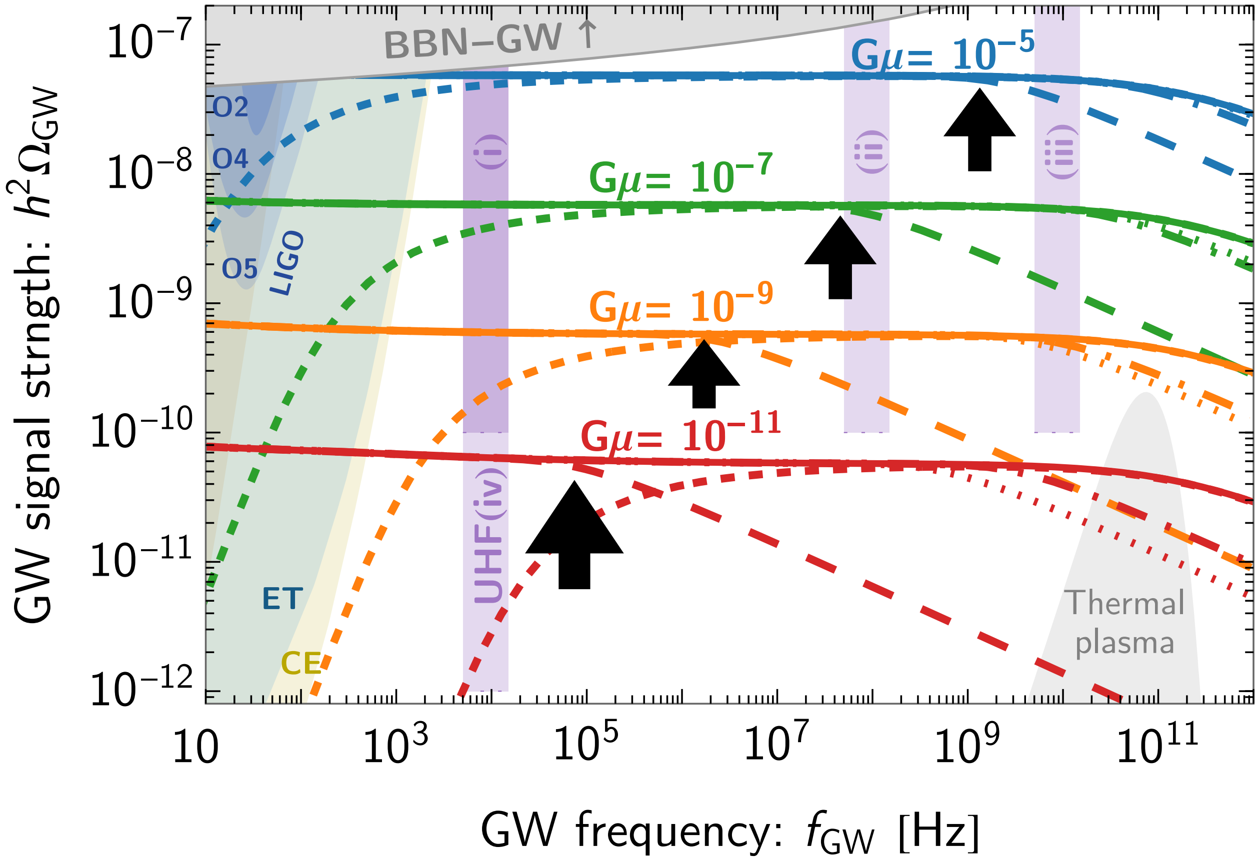
Servant, Simakachorn [2312.09281]



UV cutoff in the GW spectrum.

UV cutoffs.

Depend on scalar potential parameters

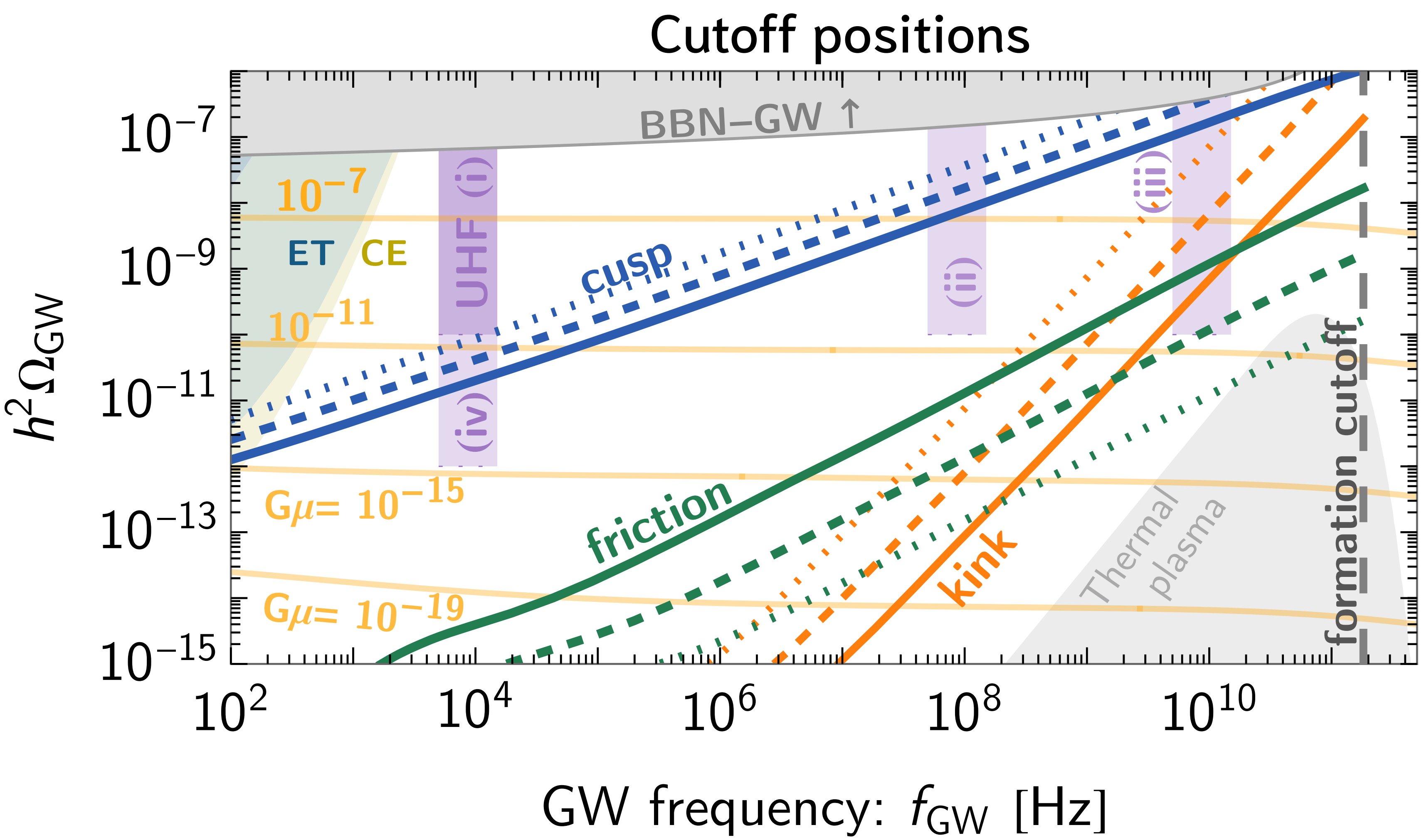
$$f_{\rm GW}^{\rm form} \simeq 182~{\rm GHz}\sqrt{\lambda}\left[\frac{g_*(T_{\rm form})}{106.75}\right]^{\frac{1}{4}}$$

$$f_{\rm GW}^{\rm cusp} \simeq 62.3~{\rm kHz}\sqrt{\frac{1}{\beta_c}}\left(\frac{G\mu}{10^{-11}}\right)^{3/4}$$

$$f_{\rm GW}^{\rm fric} \simeq 0.45~{\rm GHz}~\beta_{\rm fric}^{-1}\left(\frac{G\mu}{10^{-11}}\right)^{\frac{1}{2}}\left[\frac{g_*(T)}{106.75}\right]^{\frac{3}{4}}$$

$$f_{\rm GW}^{\rm kink} \simeq 2.79~{\rm GHz}\sqrt{\frac{1}{\beta_k}}\left(\frac{G\mu}{10^{-11}}\right)^{1/4}$$

$$\beta_c \simeq N_c\lambda^{-1/4},~~\text{and}~~\beta_k \simeq N_{kk}\lambda^{-1/2}$$



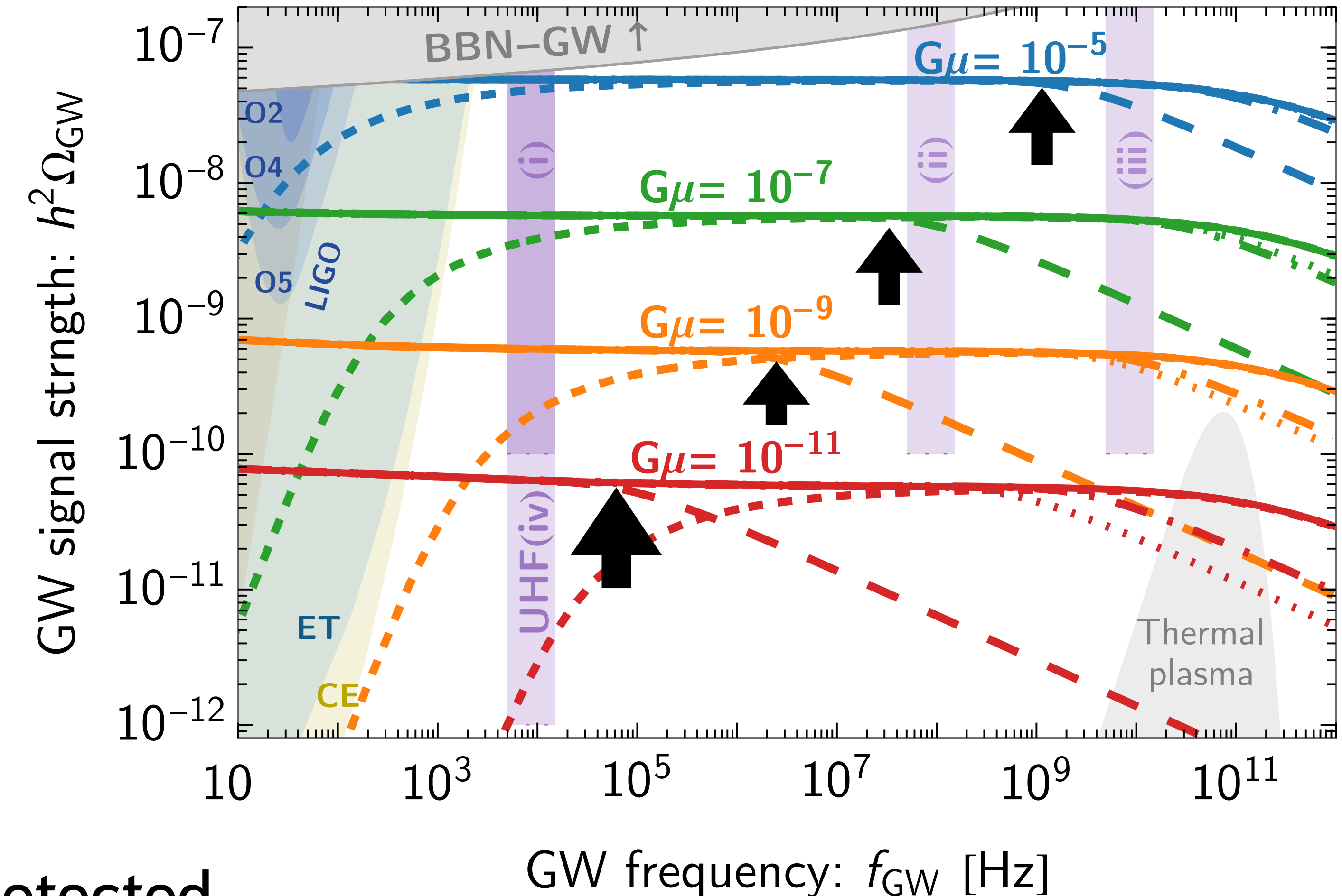
Reconstruction of the scalar potential via GWs.

UV cutoff in the GW spectrum
e.g., from cusp,
moves with the string's fatness

$w \simeq m_\Phi^{-1} \simeq (\sqrt{\lambda}\eta)^{-1}$

$$f_{\rm GW}^{\rm cusp} \simeq {\rm GHz}\, \lambda^{1/8} \left(\frac{G\mu}{10^{-5}}\right)^{3/4}$$

$$V(\Phi) \simeq \lambda(|\Phi|^2 - \eta^2)^2$$

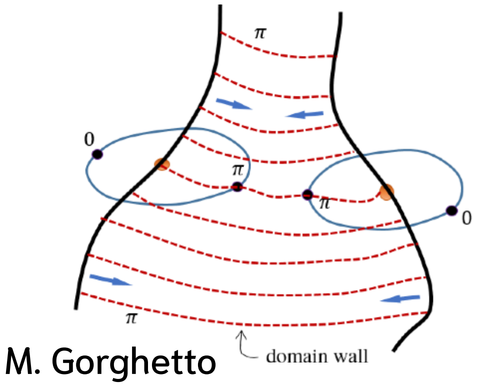


How to extract the UV cutoff if GWB is detected.

- Detect directly the cutoff (need some luck)
- Several detectors at different frequencies.
  Detect the flat part and the UV slope, ⇒ UV cutoff at the intersection (more generic)

Global (axionic) strings:
IR cutoff of GW spectrum fixed by axion mass .

String–domain wall



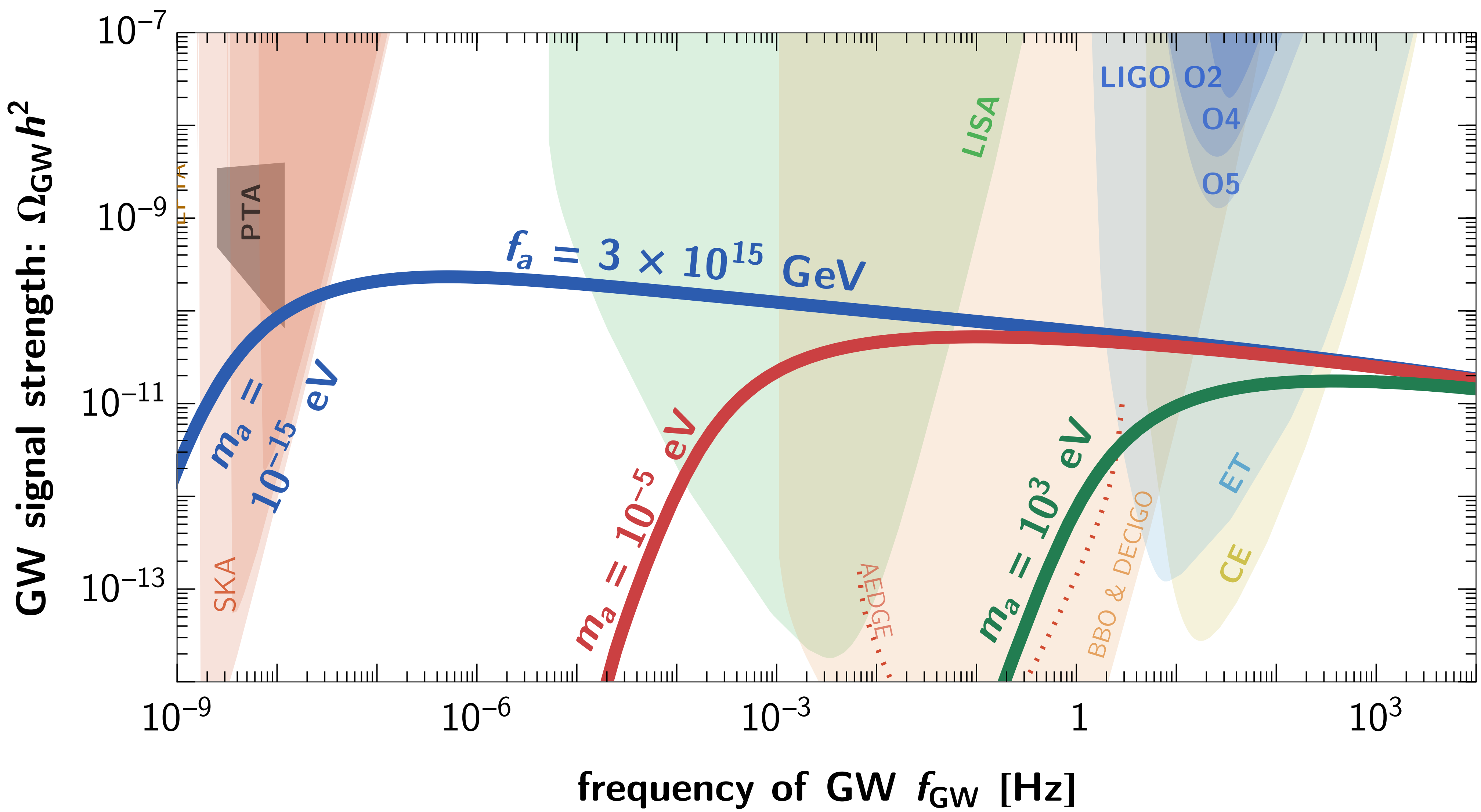
M. Gorghetto

Strings attached to domain walls -> collapse

Network decays when H ~ m_a

Corresponding IR cutoff frequency:

$$f_{\rm GW}^{\rm cs}(m_a) \simeq 9.4\ {\rm nHz} \left(\frac{\alpha}{0.1}\right) \left(\frac{m_a}{10^{-15}{\rm eV}}\right)^{\frac{1}{2}} .$$



UHFGWs from Global (axionic) strings:

Light axion ($m_a \lesssim 10^{-22}$ eV)

$\Rightarrow$ ~stable strings

Small UHF signal

- $\Delta N_{\rm eff}$–Goldstone bound

$$f_a \lesssim \mathcal{O}(1-3)\times 10^{15}\ \mathrm{GeV}$$

Cui, Chang '21, Hardy, Nicoleuscu, Gorghetto '21

- Pulsar–timing arrays

$$f_a \lesssim 2.8\times 10^{15}\ \mathrm{GeV}$$

Servant, Simakachorn [2307.03121]

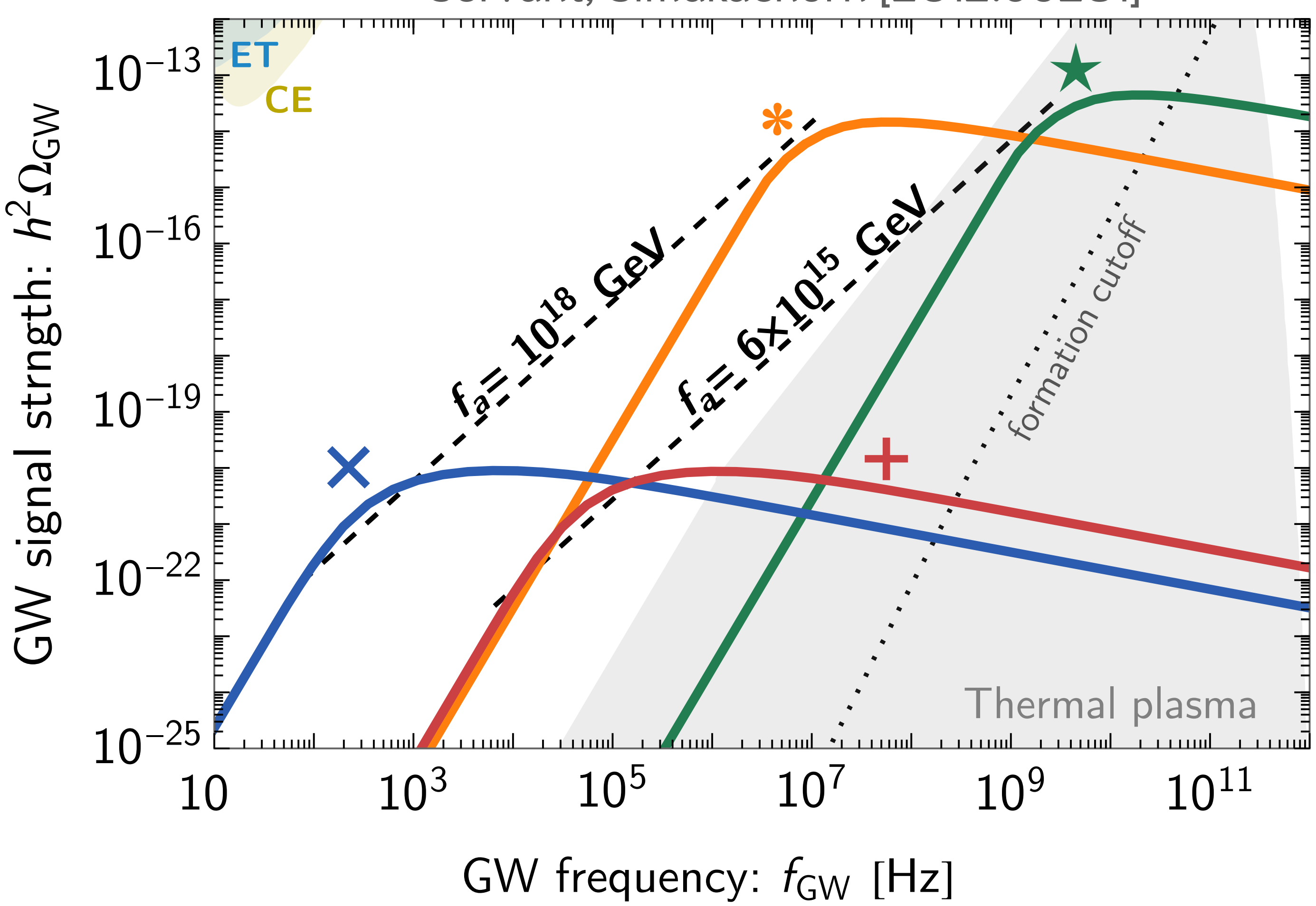
Heavy axion ($m_a \gtrsim$ GeV)

$\Rightarrow$ IR cutoff in UHF

Small signal, even for **large** f_a.

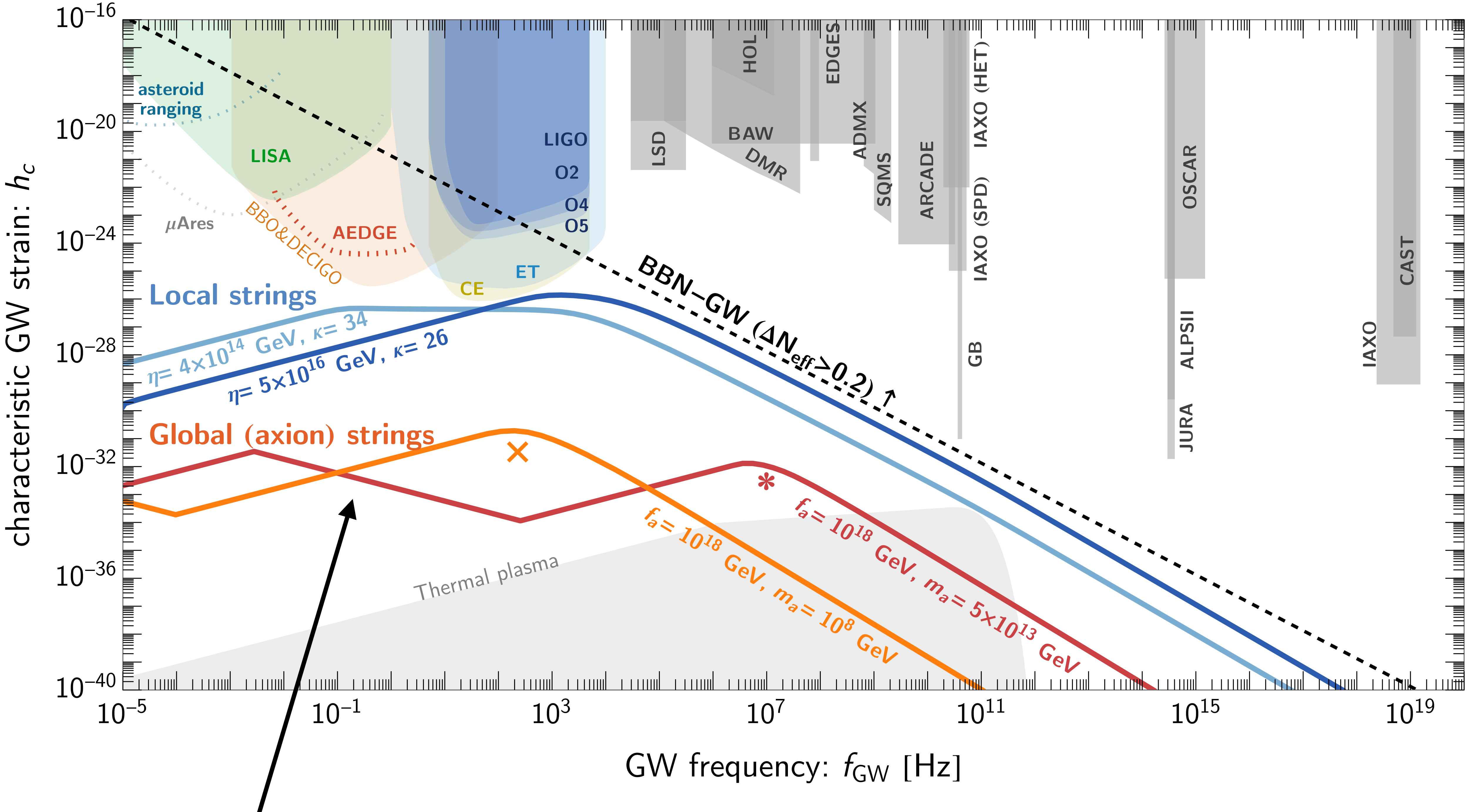
GWB is diluted by matter domination from axions produced from string collapse.

Servant, Simakachorn [2312.09281]



UHFGWs from local and global (axionic) strings:

(Best cases)



Low–frequency slope is changed by the modified causality tail during the axion matter domination.

Conclusion.

Ultra-High frequency regime (beyond kHz) : A window to explore new physics above 10^{10} GeV.

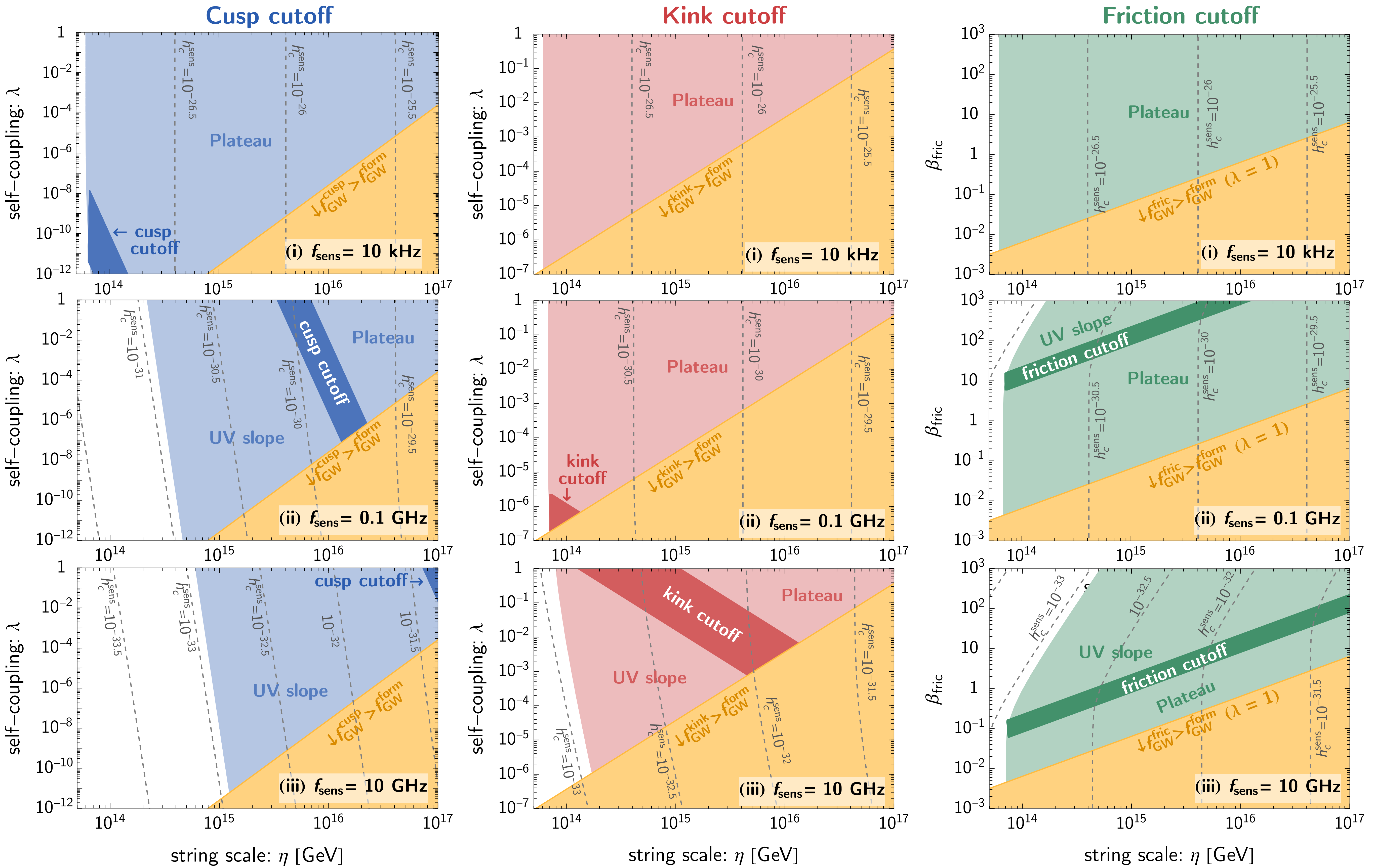
An experiment slightly more sensitive than the ΔN_{eff} bound could probe the Universe at ultrahigh energy scales.

Signals from metastable strings: access the scalar potential (e.g. self-coupling λ) via the observation of the UV cutoff in the spectrum.

Signals from axionic strings are suppressed and would require exquisite sensitivity.

Extra material.

Reconstruction of the scalar potential via GWs.



Frequencies limits .

The lowest frequency of GW is that of those GW produced today with the largest possible source size, i.e., the Hubble horizon today:

$$f_{\rm GW,lowest} = H_0 \simeq 10^{-18}\ {\rm Hz}.$$

The highest frequency of typical primordial GW arises from the highest energy scale $H_{prod} \simeq M_P$

$$f_{\rm GW,highest} \simeq 10^{13}\ {\rm Hz}.$$

(For experts: It is possible to find primordial GWs which go around this argument and have larger frequencies)

Largest possible amplitude .

Early produced GW act as an effective number of neutrino relics which is strongly constrained by Cosmic Microwave Background (CMB) measurement and by Big Bang Nucleosynthesis (BBN) predictions

$$\int_{f_{\mathrm{BBN,CMB}}}^{f_{\max}} \frac{df}{f}\Omega_{\mathrm{GW}}(f) \leq 5.6\times 10^{-6}\Delta N_\nu$$

$$\Delta N_\nu \leq 0.2$$

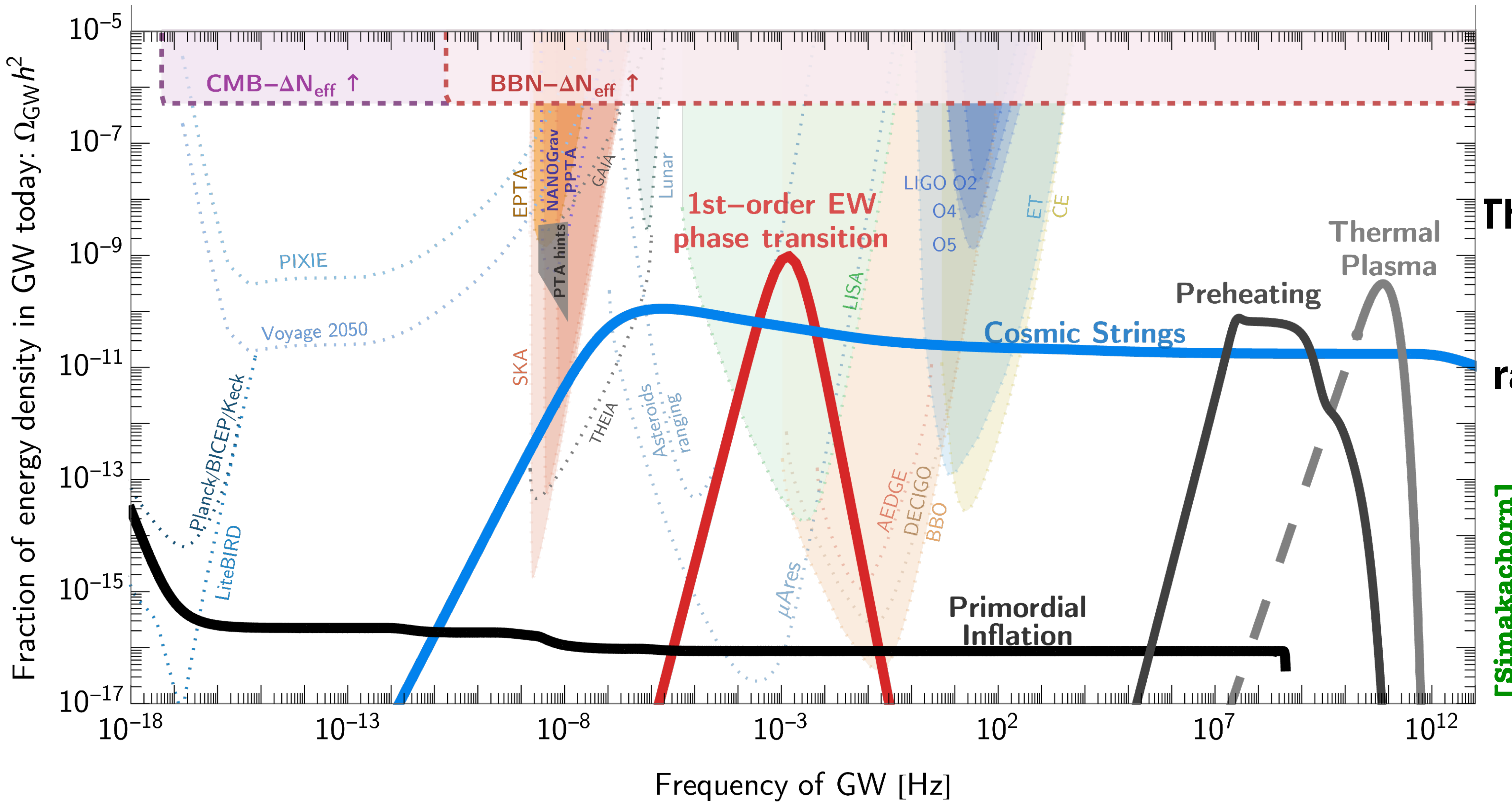
$$\Omega_{\mathrm{GW},*} \leq 5.6\times 10^{-6}\Delta N_\nu \cdot \begin{cases} \log^{-1}\left[\frac{f_{\max}}{\max(f_{\mathrm{ref}},f_{\min})}\right] & \text{for flat spectrum,} \\ \beta\left[1-\left(\frac{f_{\mathrm{ref}}}{f_{\mathrm{peak}}}\right)^{\beta}\right]^{-1} & \text{for peak,} \end{cases}$$

$\Omega_{\mathrm{GW}} = \Omega_{\mathrm{GW},*}$ for $f_{\min} < f < f_{\max}$,

$\Omega_{\mathrm{GW}}(f) = \Omega_{\mathrm{GW},*}(f/f_{\mathrm{peak}})^{\beta}$

GW spectra are sensitive to the cosmological history.

frequency $f_{\mathrm{GW},0} \simeq \lambda_{\mathrm{GW}}^{-1}\left(\frac{a_{\mathrm{prod}}}{a_0}\right)$ **energy density** $\rho_{\mathrm{GW},0} \simeq \rho_{\mathrm{GW}}^{\mathrm{prod}}\left(\frac{a_{\mathrm{prod}}}{a_0}\right)^4$



These spectra assume a Standard Model radiation era at high energies

What if the universe is not radiation-dominated at high energies?

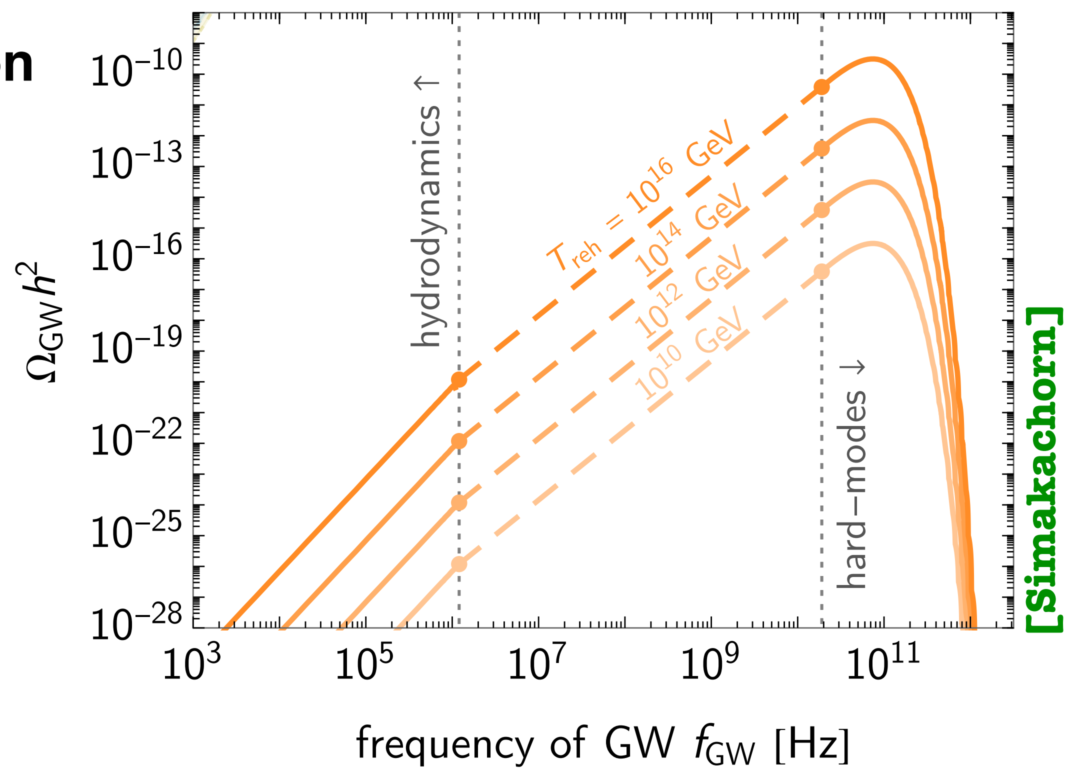
GWs from the Standard Model plasma.

T_* **temperature of the universe at time of emission**

f_* **frequency at time of emission**

observed frequency: $f \sim f_* \frac{T_0}{T_*} \sim \mathcal{O}(H_*) \frac{T_0}{T_*}$

Instead $f_* \sim T_*$ —-> cancellation



$f_* \sim T_0/2\pi \sim 5.10^{10}$ Hz

- J. Ghiglieri and M. Laine, 1504.02569
- A.Ringwald, J.Schütte-Engel and C.Tamarit, 2011.04731
- A.Ringwald and C.Tamarit, 2203.00621

GWs from inflation.

a: scale factor of the universe

w: equation of state of the universe

$$\Omega_{GW}^{inflation} \sim a^{1-3\omega} \sim f^{-2\frac{(1-3\omega)}{(1+3\omega)}}$$

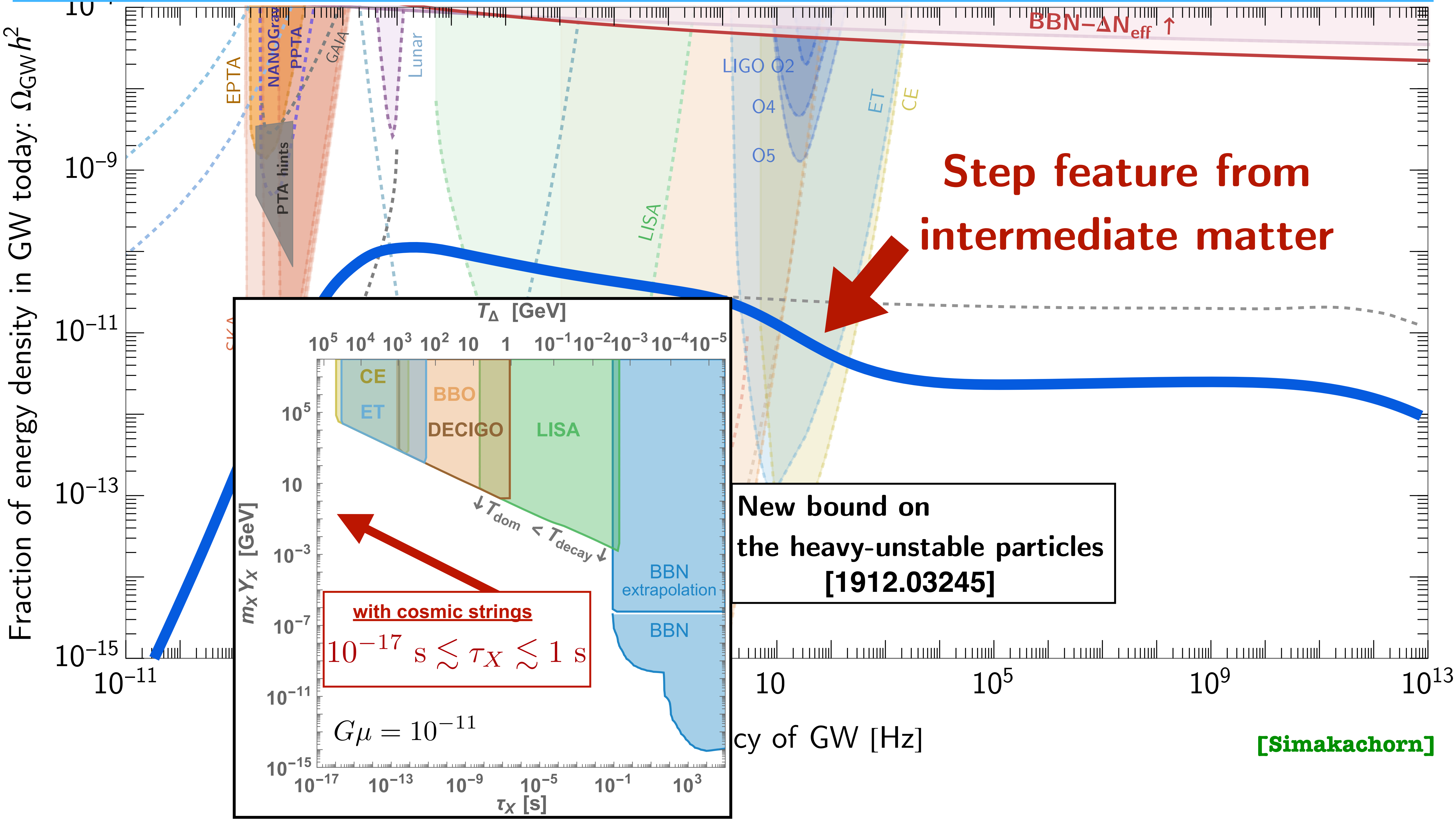
w=1/3, radiation era —> scale-invariant spectrum

w=0, matter era —> $\Omega_{GW}^{inflation} \sim f^{-2}$

w=1, kination —> $\Omega_{GW}^{inflation} \sim f$

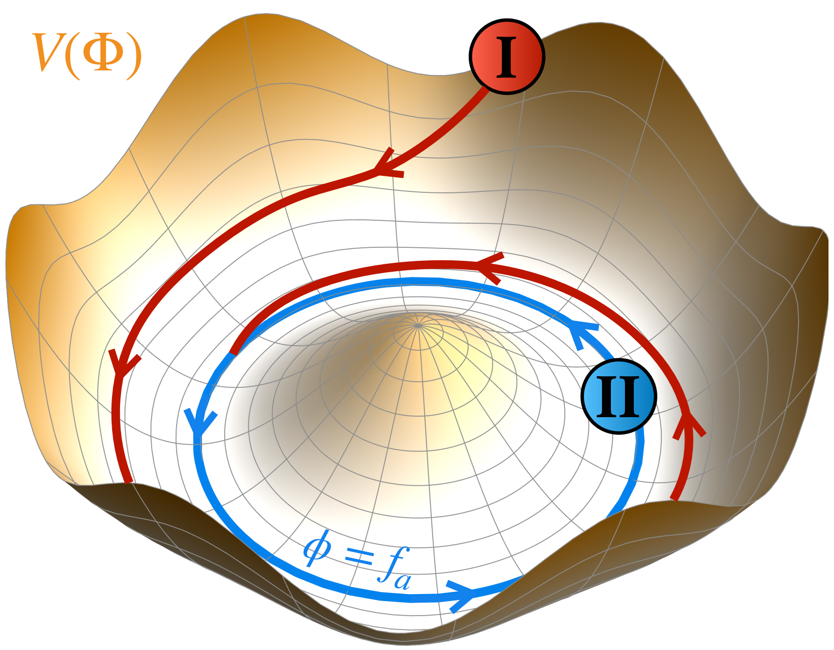
The GW spectrum from inflation encodes the full cosmological evolution of the equation of state of the universe!

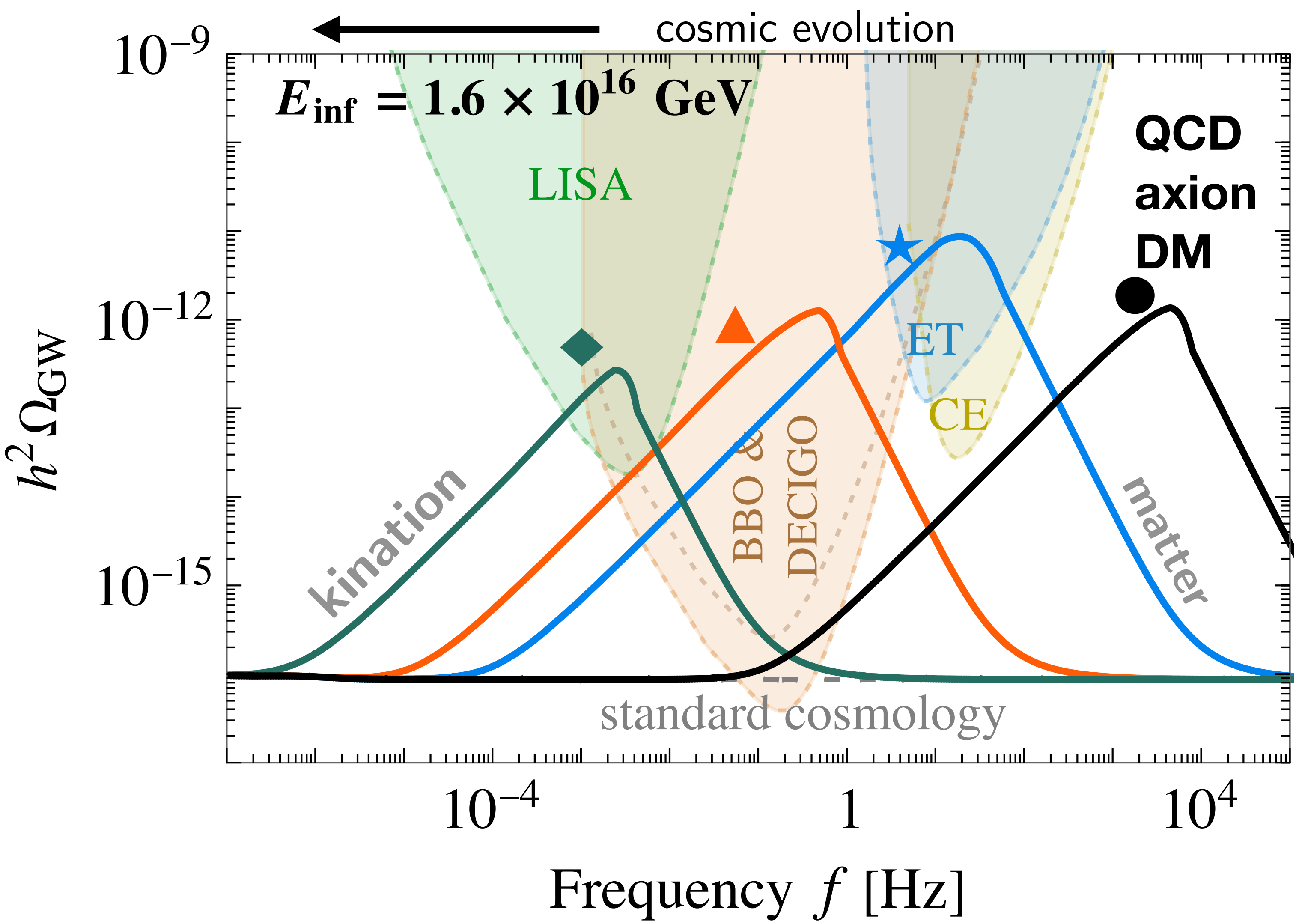
Gravitational Waves from cosmic strings in non-standard cosmology.



[Simakachorn]

Amplification of inflationary GW from axion-induced kination era.

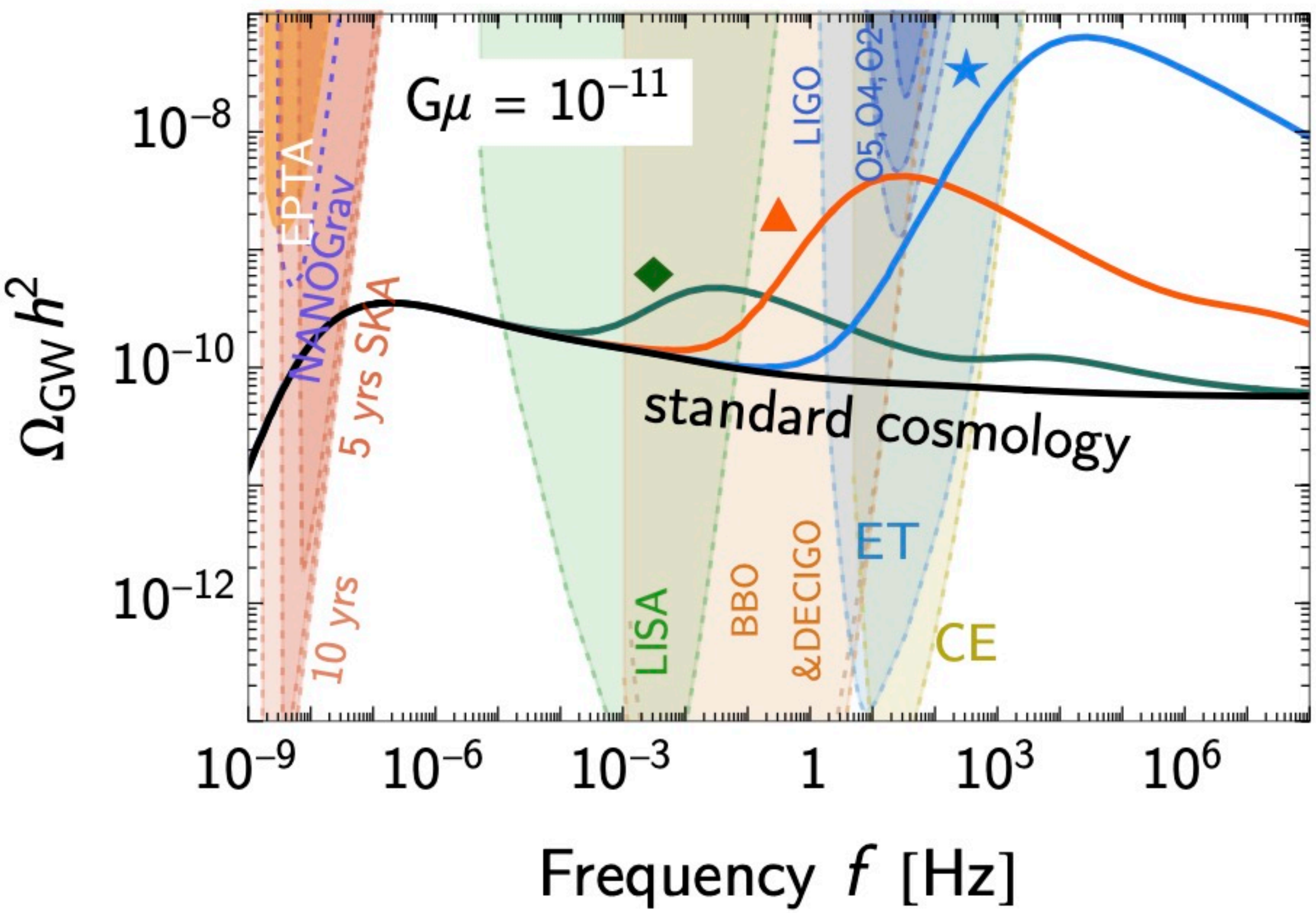




$f_a \sim 10^{10}$GeV
$N_{KD} = 5$

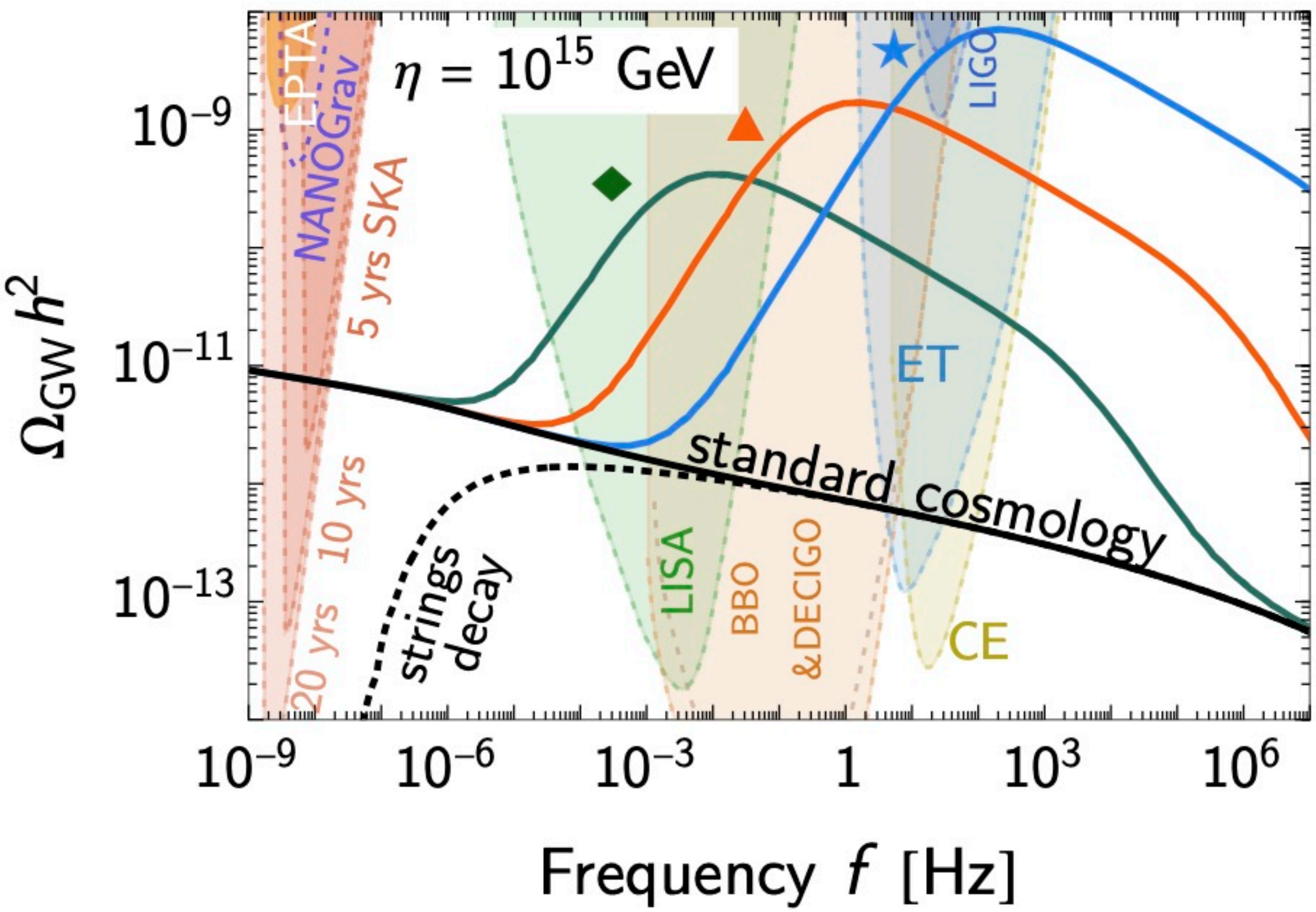
[Gouttenoire et al 2108.10328 & 2111.01150]

Amplification of GW from local cosmic strings due to an axion-induced kination era.



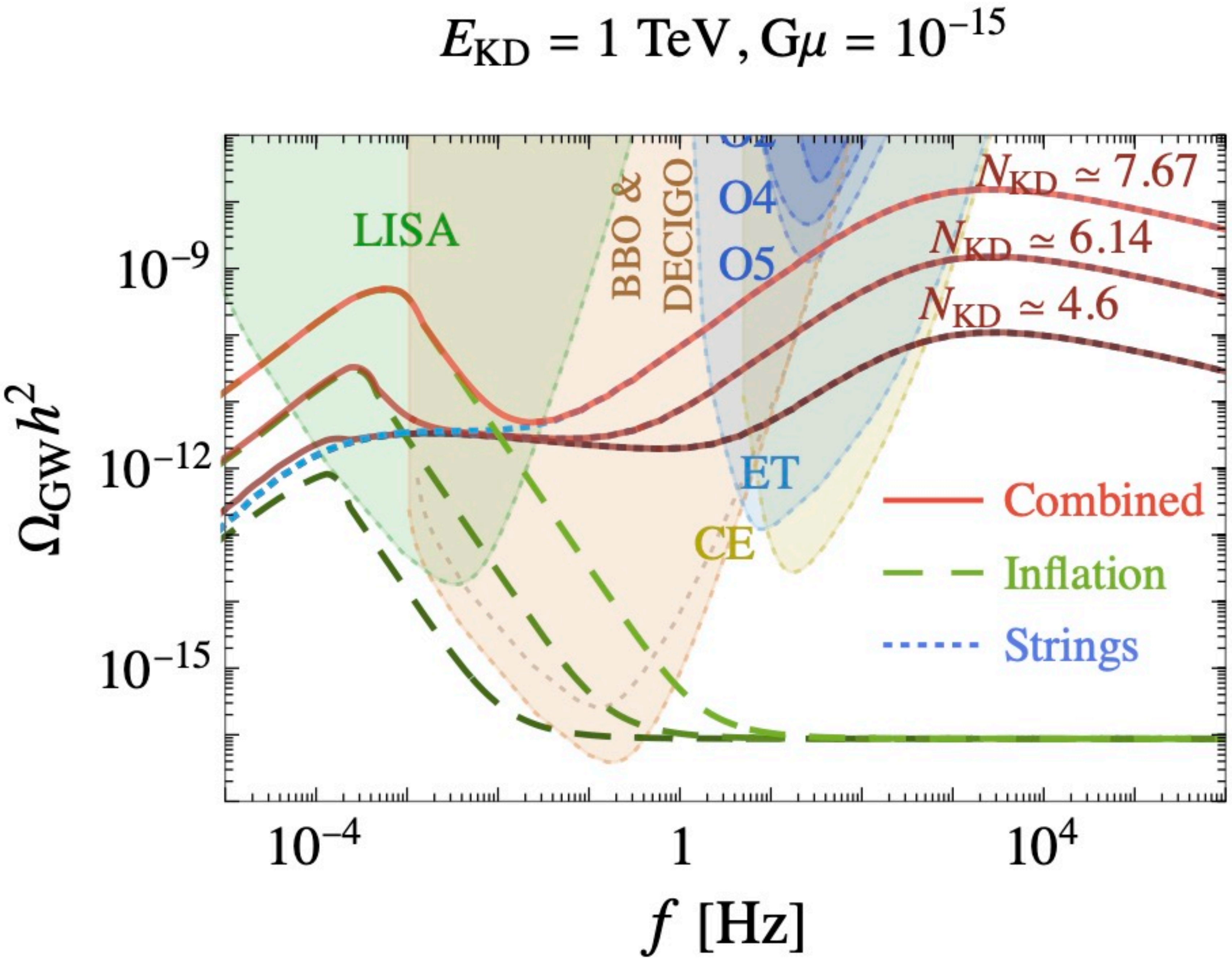
[2111.01150]

Amplification of GW from global cosmic strings due to an axion-induced kination era.



[2111.01150]

Gravitational Waves from inflation & local cosmic strings in non-standard cosmology induced by rotating axions.



[2111.01150]

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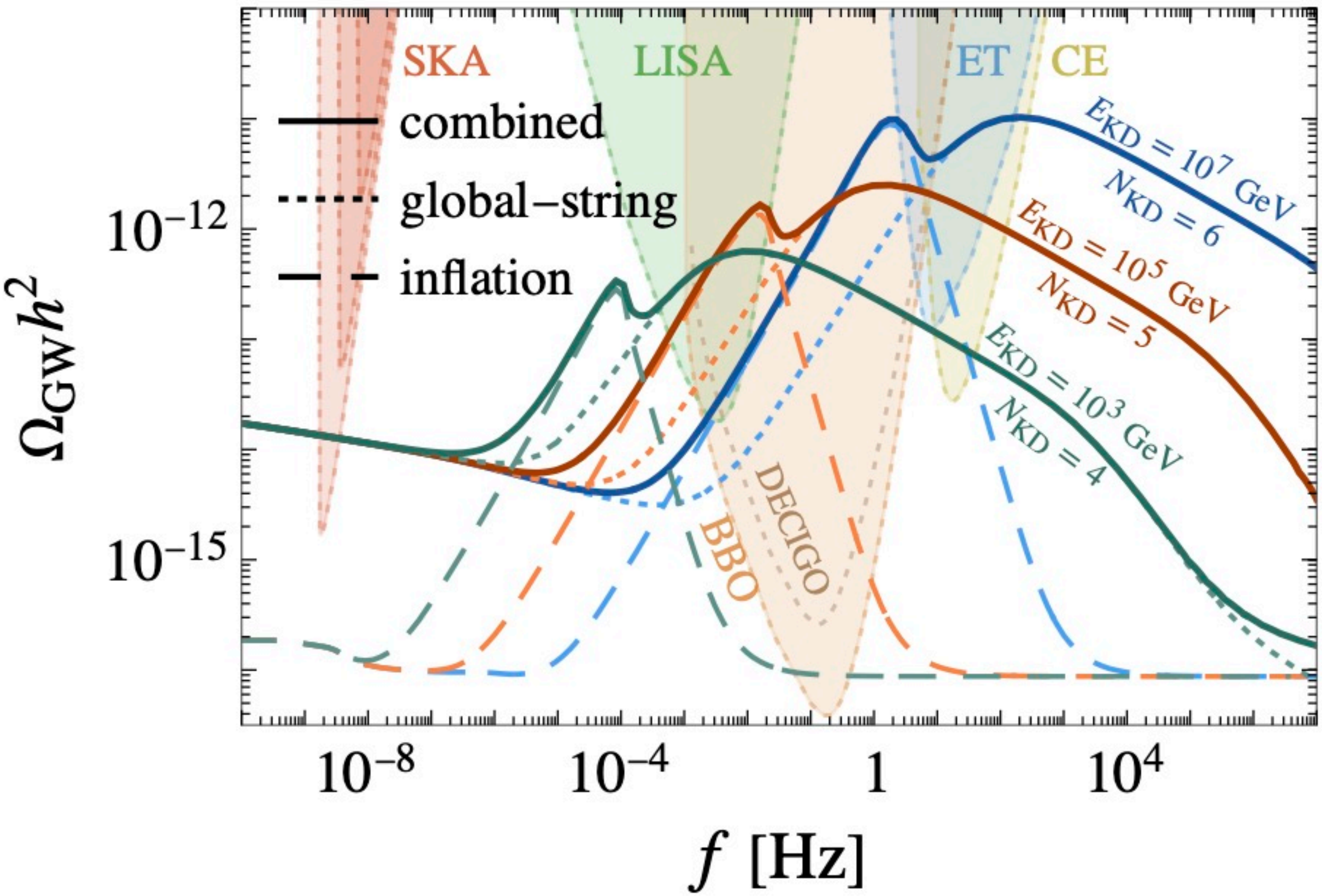

[2111.01150]

Stochastic GW background of primordial origin.

Consider the cosmological perturbation theory on the isotropic-homogeneous expanding Universe, described by the Friedmann-Robertson-Walker metric,

$$ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j,$$

GW: tensor perturbation satisfying the transverse traceless condition

The equation-of-motion follows from the linearized Einstein equation,

$$\ddot{h}_{ij}(\mathbf{x},t) + 3H\dot{h}_{ij}(\mathbf{x},t) - \frac{\nabla^2}{a^2}h_{ij}(\mathbf{x},t) = 16\pi G\Pi_{ij}^{\mathrm{TT}}(\mathbf{x},t),$$

transverse-traceless part of the anisotropic stress tensor defined by

$$a^2\Pi_{ij} = T_{ij} - pa^2(\delta_{ij} + h_{ij})$$

Fourier decomposition:

$$\ddot{h}_{ij}(\mathbf{k},t) + 3H\dot{h}_{ij}(\mathbf{k},t) + \frac{k^2}{a^2}h_{ij}(\mathbf{k},t) = 16\pi G\Pi_{ij}^{\mathrm{TT}}(\mathbf{k},t),$$

2 limits:

$$h_\lambda(\mathbf{k},\tau) = \begin{cases} \frac{A_\lambda(\mathbf{k})}{a(\tau)}e^{ik\tau} + \frac{B_\lambda(\mathbf{k})}{a(\tau)}e^{-ik\tau}, & \text{for } k \gg aH \text{ (sub-horizon)}, \\ A_\lambda(\mathbf{k}) + B_\lambda(\mathbf{k})\int^\tau \frac{d\tau'}{a^2(\tau')}, & \text{for } k \ll aH \text{ (super-horizon)}, \end{cases}$$

oscillatory behavior redshifted by expansion

stays frozen and later re-enters the horizon and starts oscillating

$d\tau \equiv dt/a$ is the conformal time

Stochastic GW background of primordial origin.

Early-universe production process operates within a causal patch ($\lambda_{GW} \leq H^{-1}$), much smaller than the horizon size today,

$$\frac{\lambda_{\rm GW,0}}{H_0^{-1}} \leq \frac{H^{-1}_{\rm prod}}{H_0^{-1}} \left[\frac{a_0}{a_p}\right] \simeq \Omega_{r,0}^{-1/2} \left[\frac{T_0}{T_p}\right] \simeq 2 \cdot 10^{-13} \left[\frac{100~\rm GeV}{T_p}\right],$$

Primordial GW sources from many uncorrelated patches randomize the amplitude of h_{ij} (x, t) observed today and contribute to the *stochastic GW background*.

For an isotropic, homogeneous, unpolarized, stationary, and gaussian background, the correlation function reads $\langle h_{ij}(\mathbf{x},\tau)\, h_{ij}(\mathbf{x},\tau)\rangle = 2 \int d(\log k)\, h_c^2(k,\tau)$

dimensionless characteristic strain

$$\rho_{\rm GW} = \frac{\left\langle \dot{h}_{ij}(\mathbf{x},t)\dot{h}_{ij}(\mathbf{x},t)\right\rangle}{32\pi G} = \frac{\left\langle h'_{ij}(\mathbf{x},\tau)h'_{ij}(\mathbf{x},\tau)\right\rangle}{32\pi G a^2}.$$

$$\rho_{\rm GW} = \int d(\log k)\frac{k^2 h_c^2(k,\tau)}{16\pi G a^2(\tau)} \equiv \int d(\log k)\frac{d\rho_{\rm GW}}{d\log k},$$

$$\Omega_{\rm GW,0}(f) = \frac{k^2 h_c^2(k,\tau_0)}{16\pi G a_0^2} = \frac{\rho_{\rm GW}^{\rm prod}(f)}{\rho_{\rm tot,0}}\left(\frac{a_{\rm prod}}{a_0}\right)^4,$$

$$h_c \simeq 1.26\times 10^{-18}({\rm Hz}/f_{\rm GW})\sqrt{\Omega_{\rm GW}h^2}.$$

Due to $h_c^2 \sim a^{-2}$ for sub-horizon mode, *the GW energy density of some mode* k *redshifts as radiation* a^{-4}.