Field-level inference lecture 1: Probability theory and signal processing

Cosmology Beyond the Analytic Lamppost course (2025)

Florent Leclercq

www.florent-leclercq.eu

Institut d'Astrophysique de Paris
CNRS & Sorbonne Université

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Miles Canyon, Whitehorse, Yukon, Canada

Programme

01 Probability theory and signal processing

02 Monte Carlo techniques

03 Bayesian hierarchical models

04 State-of-the-art

Course overview

Homepage: http://florent-leclercq.eu/teaching.php

- Lecture 1: Probability theory and signal processing — Monday 16 June 14:00-15:30
  - Probability theory
    - Introduction
    - Basic principles of probability theory
  - Signal processing
    - Gaussian random fields
    - Bayesian signal processing and Wiener filtering (denoising)
    - Wiener filtering for field-level inference
  - Practical work: Wiener filtering for Field-level inference
- Lecture 2: Monte Carlo techniques — Monday 16 June 16:00-17:30
  - Monte Carlo techniques
    - Sufficient statistics and Blackwell-Rao estimators
    - Markov Chain Monte Carlo
    - Markov Chain Monte Carlo Beyond Metropolis-Hastings
  - Practical work: Field-level inference with non-linear models
- Lecture 3: Bayesian hierarchical models — Tuesday 17 June 14:00-15:30
  - Bayesian decision theory and experimental design
  - Bayesian hierarchical models
  - Practical work: Bayesian hierarchical models for joint field-parameter sampling
- Lecture 4: State-of-the-art — Tuesday 17 June 16:00-17:30
  - Introduction to State-of-the-art field-level inference
  - Field-level inference for the Cosmic Microwave Background
  - Field-level inference for Galaxy clustering
  - Field-level inference for other/Joint cosmological probes
  - Machine Learning for Field-level inference

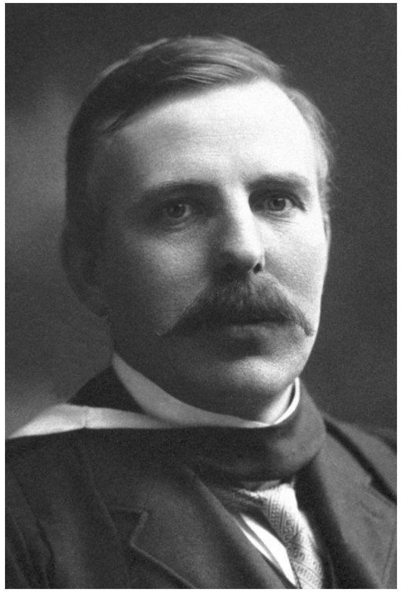
01 PROBABILITY THEORY

INTRODUCTION

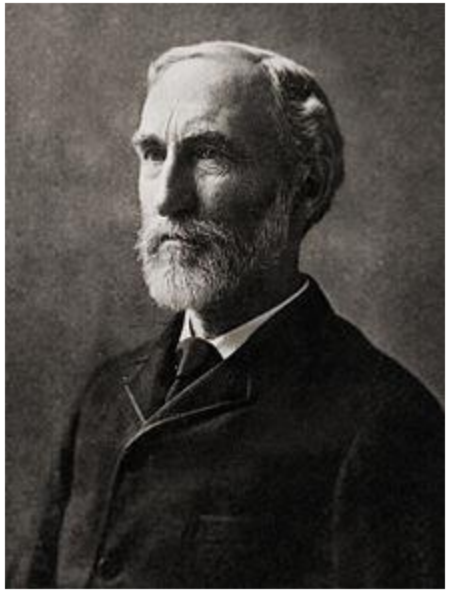
Why proper statistics matter

An historical example: the Gibbs paradox

> If your experiment needs statistics, you ought to have done a better experiment.
> Ernest Rutherford



Ernest Rutherford
(1871-1937)



J. Willard Gibbs
(1839-1903)

- Gibbs's canonical ensemble and grand canonical ensembles, derived from the maximum entropy principle, fail to correctly predict thermodynamic properties of real physical systems.
- The predicted entropies are always larger than the observed ones... there must exist additional microphysical constraints:
  - Discreteness of energy levels: radiation: Planck (1900), solids: Einstein (1907), Debye (1912), Ising (1925), individual atoms : Bohr (1913)...
  - ...Quantum mechanics: Heisenberg, Schrödinger (1927)

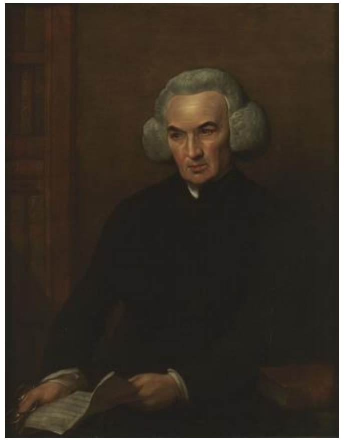
The first clues indicating the need for quantum physics were uncovered by seemingly "unsuccessful" application of statistics.

How to reason rationally in the presence of uncertainty: Bayes' Theorem

- How do we measure a quantity? How do we verify a theory? More broadly, how does knowledge progress?
- Bayes' Theorem (1763): a mathematical statement about how we analyse evidence and change our minds when we obtain new information.
- But why should we use it?
  - Bayes' theorem is trivial and outdated.
  - It measures belief. It says we can learn even from missing or incomplete data, from approximations, from ignorance. It runs counter to the conviction that science requires objectivity and precision.
  - After Laplace's death, it was pronounced dead and buried.

This is (probably!) not the right person

Thomas Bayes (1701-1761)



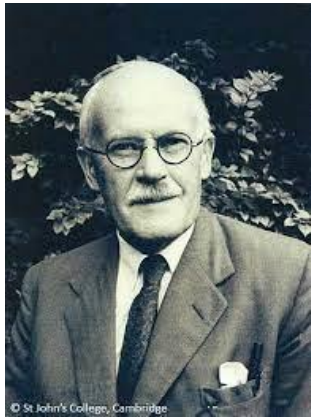
Richard Price (1723-1791)

Pierre-Simon de Laplace (1749-1827)

Pictures taken at Bunhill Fields Burial Ground, City of London, 2021

Frequentism versus Bayesianism in the 19th and 20th century

- Two conceptions of the nature of probabilities and scientific questions:
  - “Objective” probabilities linked to the frequency of repetitive random phenomena. Questions related to determined and reproducible experiments.
  - “Subjective” probabilities linked to the certainty given to a measurement or a theory. Questions related to phenomena and choices that do not involve the idea of repetition.

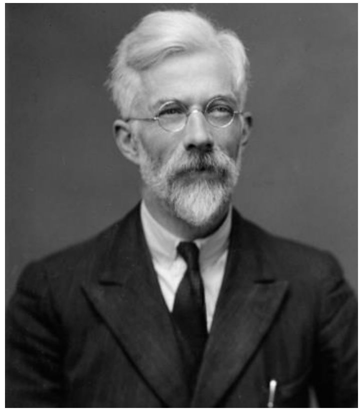


Harold Jeffreys
(1891-1989)

Leonard J. Savage
(1917-1971)

> [Fisher] sometimes published insults that only a saint could entirely forgive.
> Savage 1976, Annals of Statistics

Karl Pearson
(1857-1936)



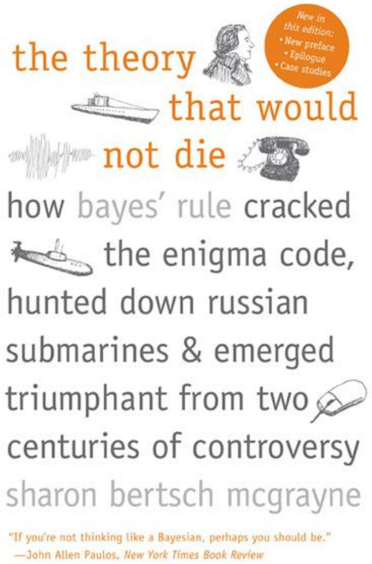
Ronald Aylmer Fisher
(1890-1962)

Jerzy Neyman
(1894-1981)

- Frequentist and Bayesian techniques yield the same results when working with large samples. It is only with small numbers and low occurrences that frequentist estimation and Bayesian induction differ.

The theory that would not die

- And yet, Bayes’ theorem helped in many practical situations:
  - Exonerate Alfred Dreyfus from miscarriage of justice (Henri Poincaré, 1899-1906),
  - Save the Bell Telephone system from financial panic (Edward C. Molina, 1907),
  - Predict earthquakes and tsunamis (Harold Jeffreys, 1930-1940),
  - Direct Allied artillery fire and locate German submarines (1939-1945),
  - Break the German navy’s Enigma cipher (Alan Turing, 1940-1944),
  - Prove that smoking causes lung cancer (Jerome Cornfield, 1951)
  - Search for a lost H-bomb and then a submarine at sea (John P. Craven, 1966-1968)...

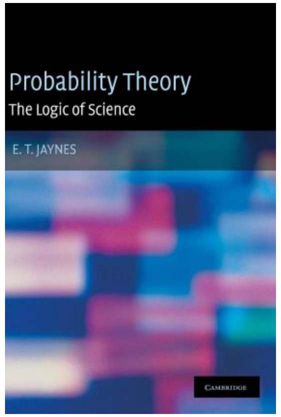


Sharon Bertsch McGrayne (2012)

- The scientific battle lasted for 150 years, until computers arrived.

> The superiority of Bayesian methods is now a thoroughly demonstrated fact in a hundred different areas. One can argue with a philosophy; it is not so easy to argue with a computer printout, which says to us: “Independently of all your philosophy, here are the facts of actual performance.”
> Jaynes (2002), Probability Theory — The logic of science

- Cox-Jaynes theorem (1946): Any system to manipulate “plausibilities”, consistent with Cox’s desiderata, is isomorphic to Bayesian probability theory.

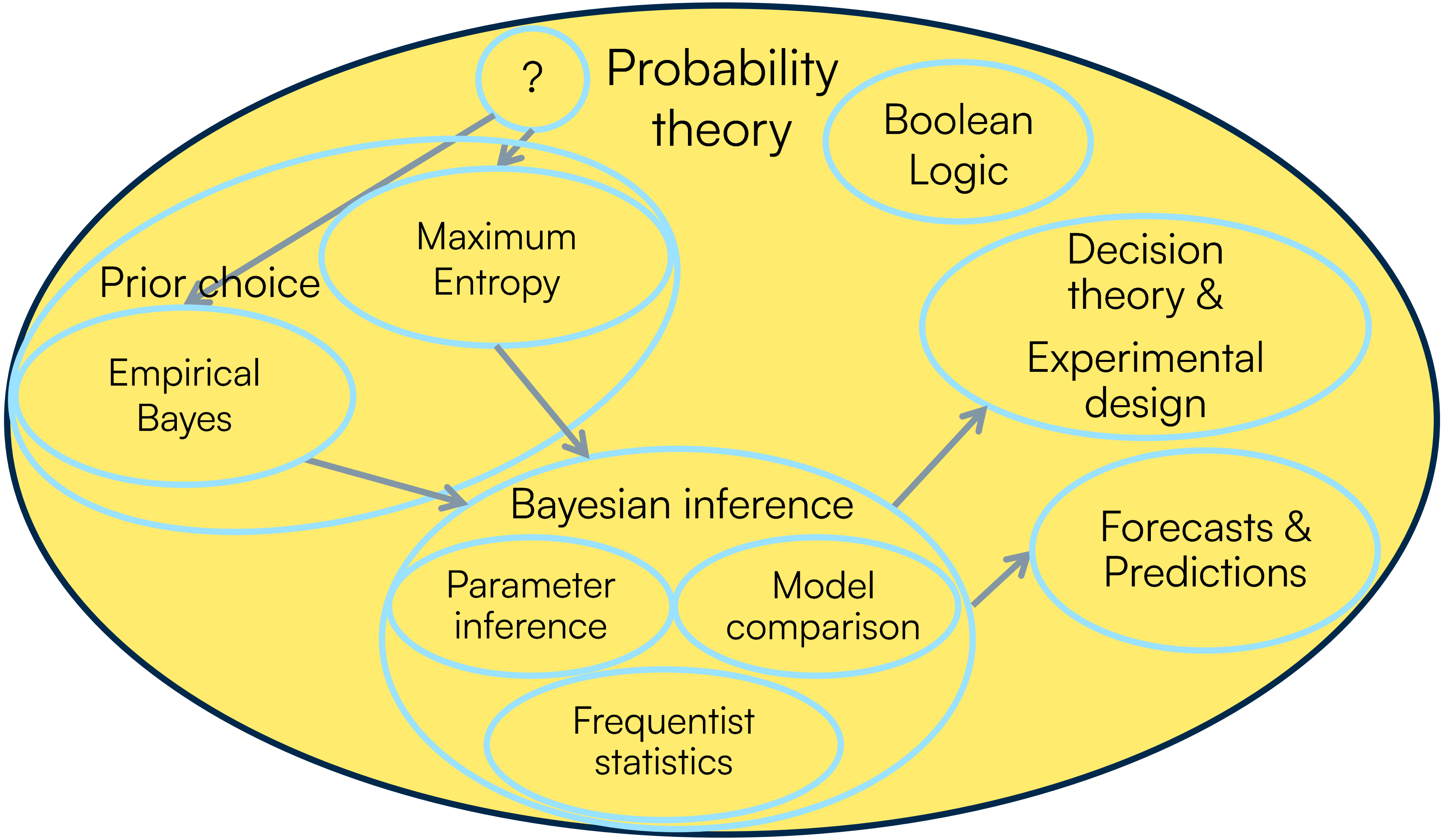


Jaynes (2002)

Richard Threlkeld Cox (1898-1991)

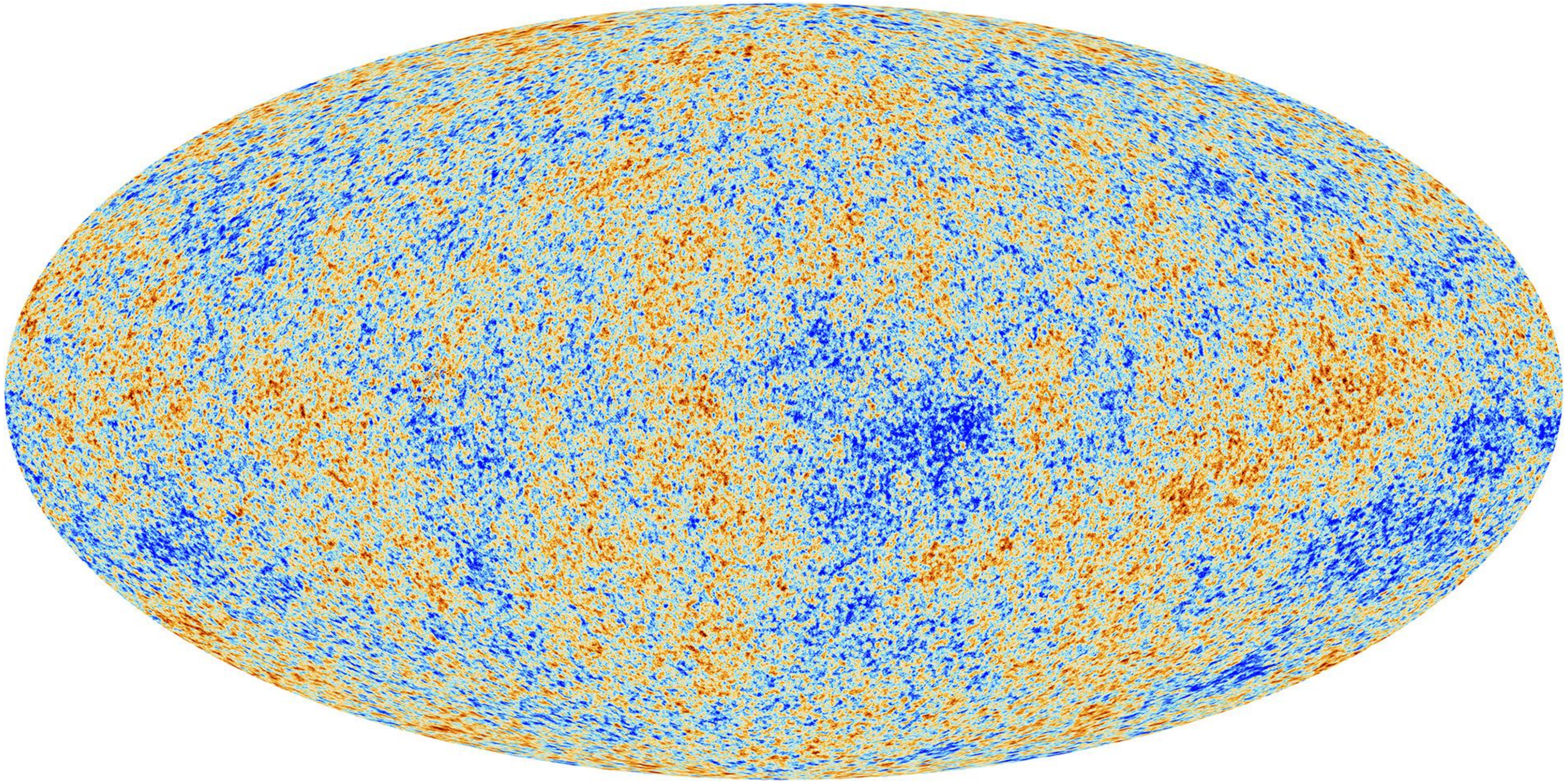
Edwin Thompson Jaynes (1922-1998)

Jaynes's "probability theory": an extension of ordinary Boolean logic



BASIC PRINCIPLES OF PROBABILITY THEORY

Typical problem: analysis of the Cosmic Microwave Background

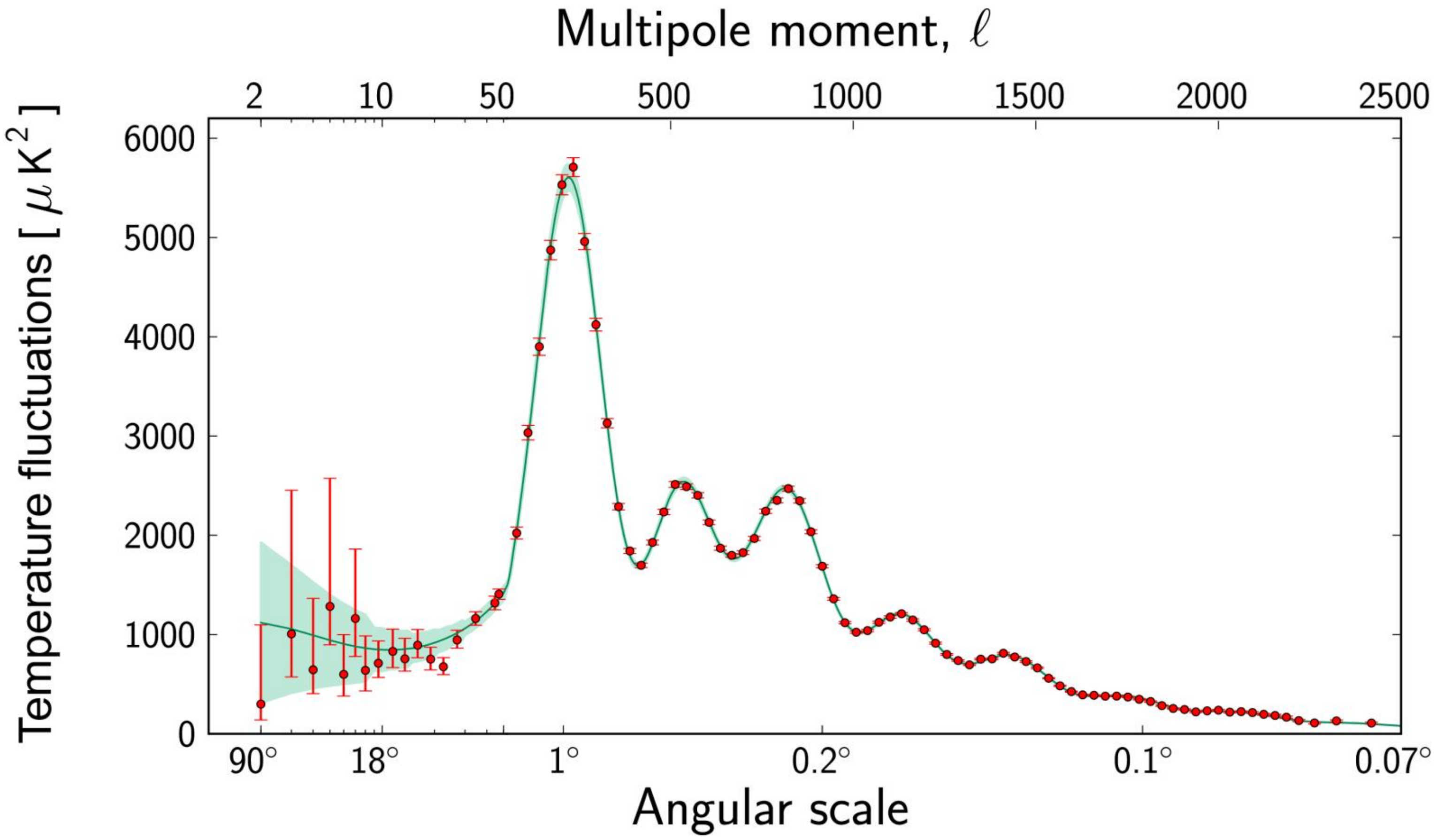


Usually we compress the data into some "summary statistics", such as the correlation function of temperature fluctuations, or the power spectrum

Planck 2018

Typical problem: analysis of the Cosmic Microwave Background

ΛCDM fits the Planck data well.



Planck 2018 V, 1907.12875

Vocabulary: direct and inverse problems

- “Synthesis problems” are direct problems: given some theory/model of a physical process (a generative model), we want to predict the result of an experiment. This is also called forward modelling.
- “Analysis problems” are inverse problems: given some data, we want to infer something about the process that generated the data.
- Inverse problems are generally harder than predicting an outcome, given a physical process.
- Typical classes of inverse problems:
  - Parameter inference
  - Hypothesis testing
  - Model comparison

What kind of questions do we want to answer?

- Parameter inference:
  - I have a set of (x, y) pairs, with errors. If I assume $y = mx + c$, what are the values of m and c?
  - I have detected 5 X-ray photons from a source at known distance in my lab. What is the luminosity of the source and its uncertainty?
  - By analysing the velocity distribution of stars in a galaxy, what can be inferred about the galaxy's mass distribution and the presence of dark matter?
  - Given LIGO/Virgo gravitational wave data, what are the masses of the inspiralling objects?

What kind of questions do we want to answer?

- Hypothesis testing:
  - Is the cosmic microwave background radiation consistent with (initially) Gaussian fluctuations?
  - Do high-energy cosmic rays exhibit correlations with astrophysical sources?
  - Are neutron star mergers the dominant source of heavy elements (e.g., r-process elements) in the universe?
  - Are the statistical properties of exoplanet populations consistent with standard planet formation models?

What kind of questions do we want to answer?

- Model comparison:
  - Do observations of young stellar objects with protoplanetary disks better support core accretion or disk instability models for planet formation?
  - Based on historical climate data, which climate model more accurately predicts global temperature changes—those incorporating strong feedback mechanisms or those with moderate feedback?
  - Do LHC data support the existence of the Higgs boson, or no Higgs boson?
  - Do observations of Type Ia supernovae better support the cosmological constant model of dark energy or dynamic dark energy models such as quintessence?

The meaning of probability

- Frequentist view: probability describes the relative frequency of outcomes in infinitely long trials.
- Bayesian view: probability describes a degree of belief.
- But!
  - The first view (regarding repetitive phenomena) can be included in the second one.
  - It is misleading when, in 2025, the debate between frequentists and Bayesians is still referred to as if it were ongoing, since it was settled in the second half of the 20th century.
- The Bayesian view expresses what we often want to know, e.g.
  - Given the Planck CMB data, what is the probability that the density parameter of cold dark matter Ω_m is between 0.3 and 0.4?
  - and not: given a fictitious infinite population of universes, what is the probability that 95% of them have Ω_m between 0.3 and 0.4?
- Logical proposition: a statement that could be true or false.
- Conditional probability: $p(A|B)$ is the degree to which truth of a logical proposition B implies that proposition A is also true.

Basic principles of probability theory

- Joint probabilities and conditional probabilities: $p(A,B) = p(A|B)p(B)$
- Marginal probabilities: $p(A) = \int p(A,B)\,\mathrm{d}B$
- Normalisation rule: $p(A) + p(\bar{A}) = 1$ or $\int p(A)\mathrm{d}A = 1$
- Product rule (logical "and"): $p(A,B|C) = p(A|B,C)p(B|C)$
- Sum rule (logical "or"): $p(A+B|C) = p(A|C) + p(B|C) - p(A,B|C)$
- Bayes Theorem (parameter inference):

posterior, likelihood, prior, evidence

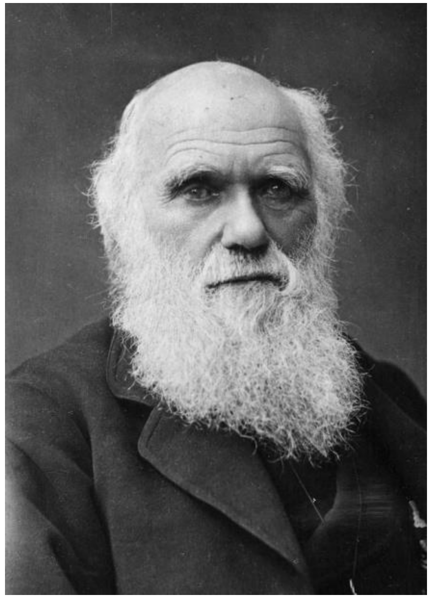
$$p(s|d) = \frac{p(d|s)\,p(s)}{p(d)}$$

- Bayes factor (model comparison): $\mathcal{B}_{12} = \frac{p(d|\mathcal{M}_1)}{p(d|\mathcal{M}_2)}$ where $p(d|\mathcal{M}_i) = \int p(d|s_i,\mathcal{M}_i)p(s_i|\mathcal{M}_i)\,\mathrm{d}s_i$

  The ratio of evidences (not of likelihoods) takes into account the effect of "Occam's razor".

Conditional probabilities

- Avoid the "probability 101" mistake: $p(B|A)$ is not the same as $p(A|B)$!
- Example:



Charles Darwin (1809-1882), photographed by Herbert Rose Barraud (1881)

  - A = is male; B = has beard
  - $p(B|A) \approx 0.1$ (?)
  - $p(A|B) \approx 1$

Parameter inference in practice

- Notations: data d, model $\mathcal{M}$, parameters θ
- First rule: write down what you want to know. Usually, it is the probability distribution for the parameters, given the data, and assuming a model: $p(\theta|d, \mathcal{M})$. This is the posterior.
- To compute it we use Bayes' theorem: $p(\theta|d, \mathcal{M}) = \dfrac{p(d|\theta, \mathcal{M})p(\theta|\mathcal{M})}{p(d|\mathcal{M})}$
  - where the likelihood is $L(\theta) = p(d|\theta, \mathcal{M})$ (an unnormalised function of θ, while d is fixed)
  - and the prior is $\pi(\theta) = p(\theta|\mathcal{M})$
- Dropping the dependence on $\mathcal{M}$: $p(\theta|d) = \dfrac{L(\theta)\pi(\theta)}{p(d)} \propto L(\theta)\pi(\theta)$
  - The evidence $p(d|\mathcal{M})$ is irrelevant for parameter inference.
  - In practice, in your code, you will write: $\ln p(\theta|d) = \ln L(\theta) + \ln \pi(\theta) + \text{const.}$
- So you need to analyse the problem:
  - What is the data? $\rightarrow d$
  - What is the model for the data? } $\rightarrow L(\theta)$
  - What are the parameters? } $\rightarrow L(\theta)$, $\pi(\theta)$
  - What is the prior distribution? } $\rightarrow \pi(\theta)$
- After the experiment, the posterior may act as the prior for the next experiment: we "update the prior" with the information from the experiment

Some conceptual considerations regarding Bayesian inference

- Self-consistency? Yes.
  - Consider data from 2 experiments. We can do one of 3 things:
    - Define prior; obtain posterior from dataset 1; update the prior, then analyse dataset 2
    - As above, but swap datasets 1 and 2
    - Define prior; obtain posterior from datasets 1 and 2 combined
  - These have to (and do) give the same answer.

- Subjective? Yes and No. Jaynes (2002), Probability Theory — The logic of science, Chap 2, p. 39
  - Any probability assignment is necessarily subjective in the sense that it describes only a state of knowledge, and not anything that could be measured in a physical experiment. But *whose* state of knowledge? Answer is always: any rational thinker. Anyone who has the same information but comes to a different conclusion is necessarily violating one of the Cox-Jaynes desiderata.
  - Probability assignments completely objective in the sense that they are independent of the personality of the user (e.g. in parameter inference the posterior is determined unambiguously from the prior and the likelihood). It is objectivity in this sense that is needed for a "scientifically respectable theory of inference".

Some conceptual considerations regarding Bayesian inference

- Priors? Yes, they are needed.
  - The No-Free-Lunch theorem for optimisation problems:
    - When searching for the local extremum of a target function (the likelihood in our case) in a finite space, the average performance of algorithms (that do not resample points) across all possible problems is identical (Wolpert & Macready 1997).
    - In other words: if no assumptions are made about the data, there is NO reason to prefer one model over any other. This means, without prior knowledge we have to test all models!
    - Important implication: there exists no algorithm that performs equally well on all inference tasks (Ho & Pepyne 2002); prior information should always be used to match procedures to problems.

> It appears to be a quite general principle that, whenever there is a randomized way of doing something, then there is a nonrandomized way that delivers better performance but requires more thought.
> Jaynes (2002), Probability Theory — The logic of science

  - Prior specification is for model comparison a key ingredient of the model building step. If the prior cannot be meaningfully set, then the physics in the model is probably not good enough.

Wolpert & Macready (1997); Ho & Pepyne (2002)

01 SIGNAL PROCESSING

GAUSSIAN RANDOM FIELDS

Gaussian random fields

- Definition: any random vector x with pdf

$$p(x|\mu, C) = \frac{1}{\sqrt{|2\pi C|}} \exp\left[-\frac{1}{2}(x-\mu)^\intercal C^{-1}(x-\mu)\right]$$
$$-2\ln p(x|\mu, C) = (x-\mu)^\intercal C^{-1}(x-\mu) + \ln|2\pi C|$$

for any vector μ (the mean) and any symmetric positive-definitive matrix C (the covariance matrix).

- Property: one can check that the mean is actually μ and the covariance matrix is actually C, i.e.

$$\langle x \rangle = \int_{-\infty}^{+\infty} x p(x|\mu, C)\,\mathrm{d}x = \mu \quad \text{and} \quad \langle (x-\mu)(x-\mu)^\intercal \rangle = \int_{-\infty}^{+\infty} (x-\mu)(x-\mu)^\intercal p(x|\mu, C)\,\mathrm{d}x = C$$

- Trick for doing Gaussian integrals: integration by differentiation.
  - Since the Gaussian pdf has only one global maximum, the mean of the pdf is its mode (its maximum) and it can be found by maximising the exponent. Therefore, μ is found by maximising $\partial_x \ln p(x|\mu, C)$:

$$-\partial_x \ln p(x|\mu, C)|_{x_\mathrm{max}} = \partial_x \left[\frac{1}{2}(x-\mu)^\intercal C^{-1}(x-\mu)\right]\Bigg|_{x_\mathrm{max}} = C^{-1}(x-\mu)|_{x_\mathrm{max}} = 0 \quad \text{gives} \quad x_\mathrm{max} = \mu$$

  - It's easy to verify that $\partial_x \partial_{x^\intercal} \ln p(x|\mu, C) = -C^{-1}$. Therefore, C is found by $C = -\left[\partial_x \partial_{x^\intercal} \ln p(x|\mu, C)\right]^{-1}$ (look at the exponent, select the coefficient matrix of all quadratic terms in the variable, invert, and multiply by -1).

Gaussian random fields

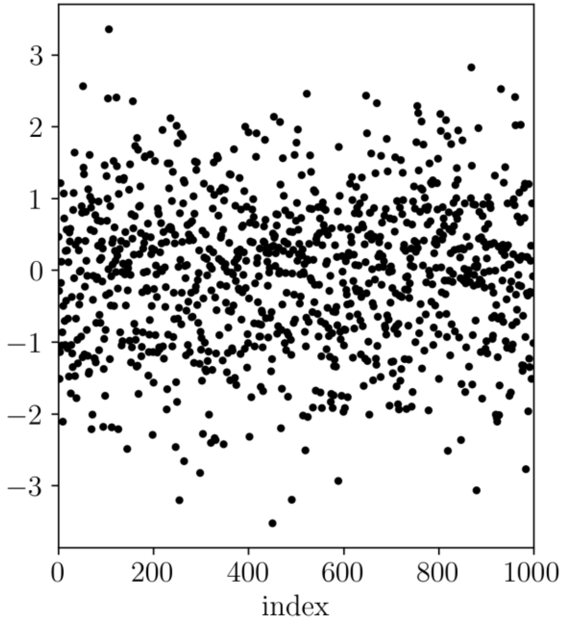
- Generating a Gaussian random field x :
  - Draw a white noise vector ξ (uncorrelated zero-mean unit-variance Gaussian variables)
  - Find the matrix square-root of C : $\sqrt{C}$ (any such matrix works)
  - Compute $x = \sqrt{C}\xi + \mu$
- Then x will be a sample from the desired pdf:

$$p(x|\mu, C) = \frac{1}{\sqrt{|2\pi C|}} \exp\left[-\frac{1}{2}(x-\mu)^{\intercal} C^{-1} (x-\mu)\right]$$

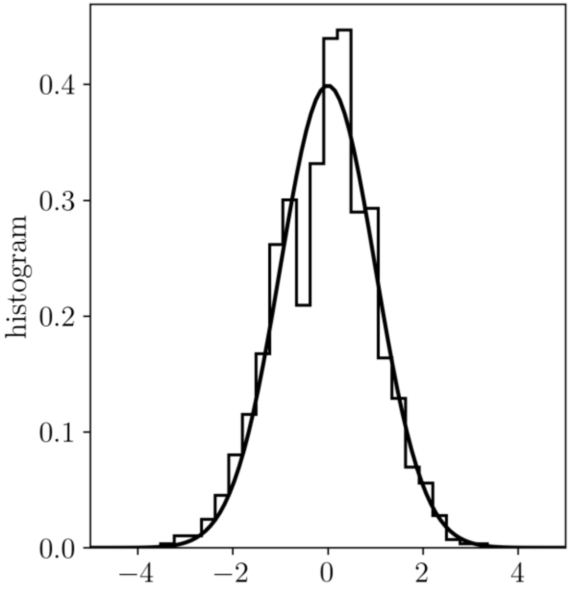
$$-2\ln p(x|\mu, C) = (x-\mu)^{\intercal} C^{-1} (x-\mu) + \ln|2\pi C|$$

Gaussian random fields: examples

white noise



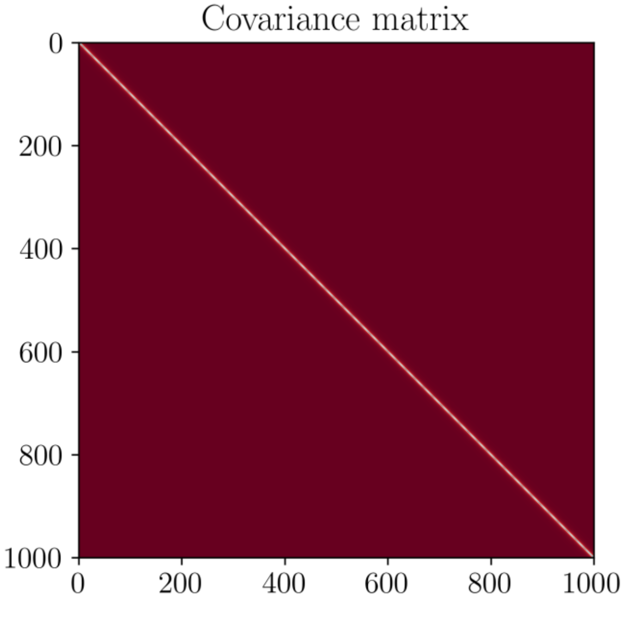
histogram

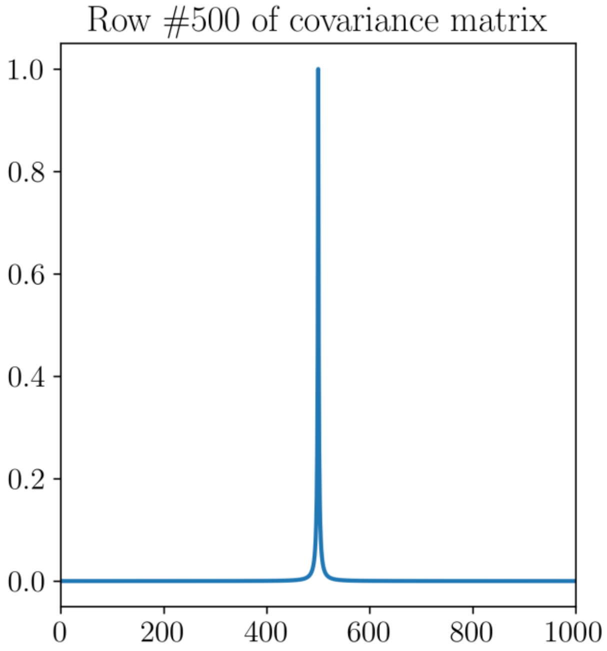


Gaussian random fields: examples

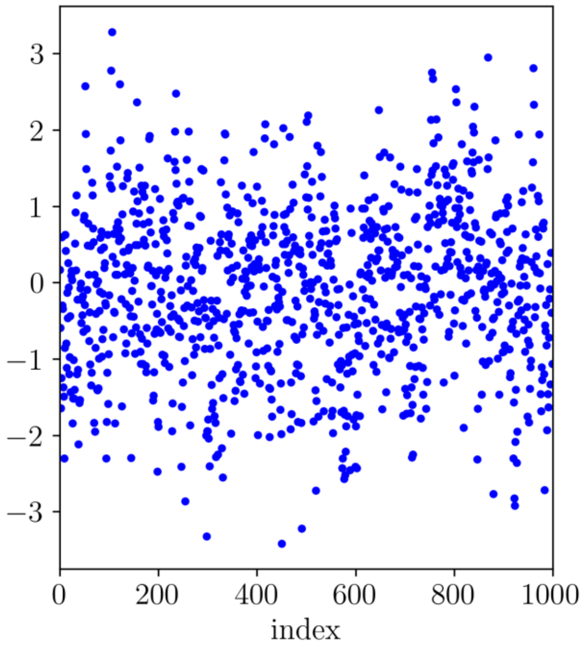
$$C_{ij} = \frac{1}{(1 + |i - j|/2)^2}$$

covariance matrix

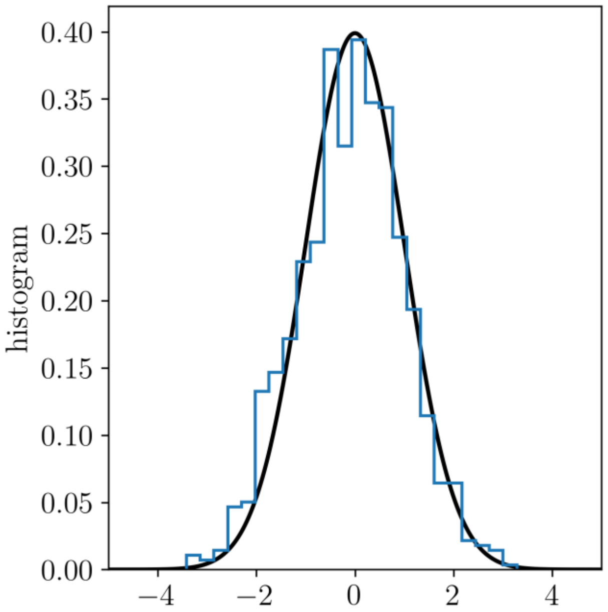




GRF



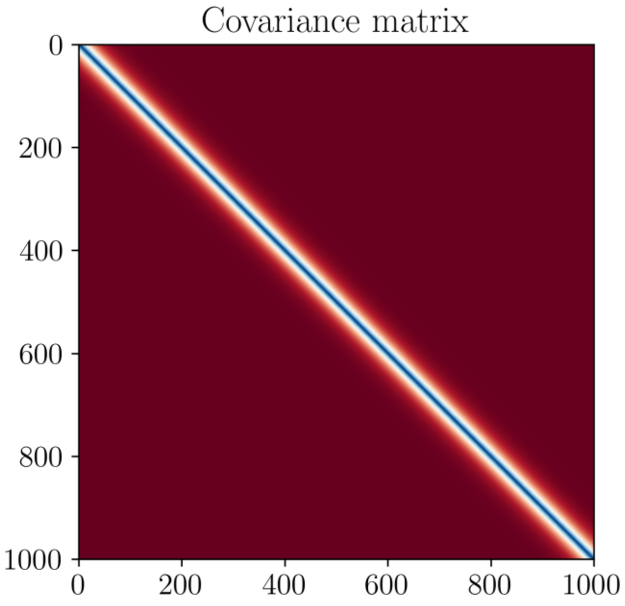
histogram

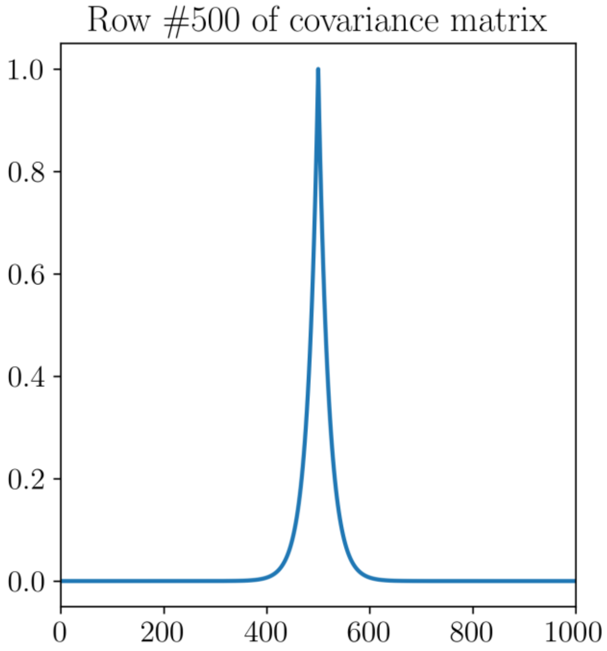


Gaussian random fields: examples

$$C_{ij} = \exp\left(-\frac{|i-j|}{20}\right)$$

covariance matrix





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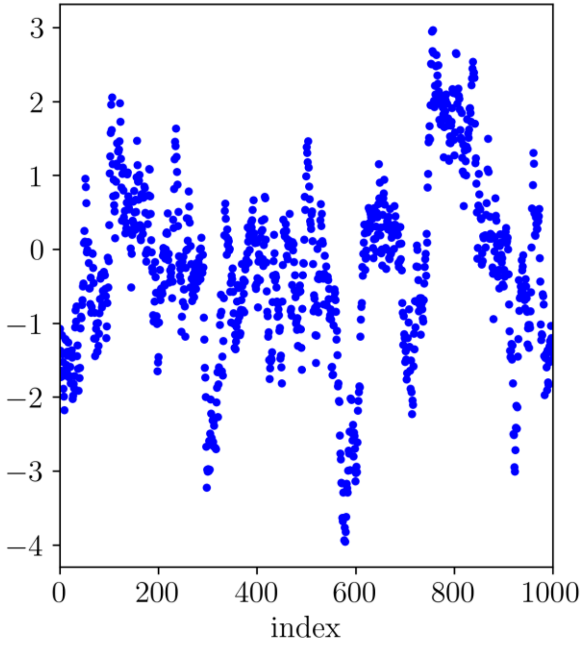
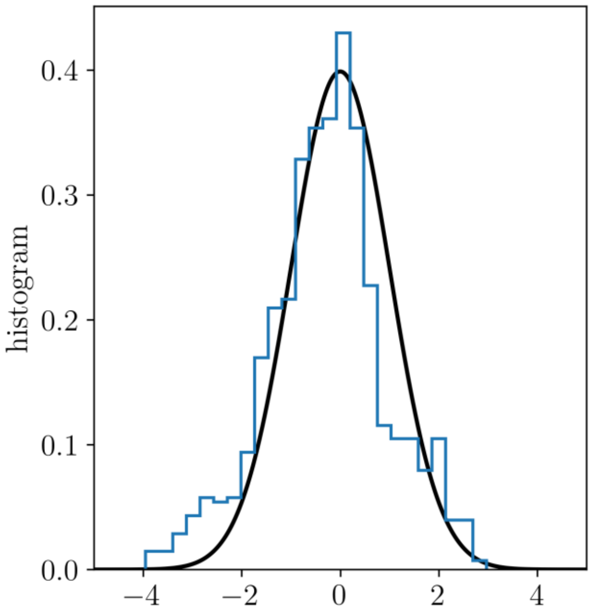


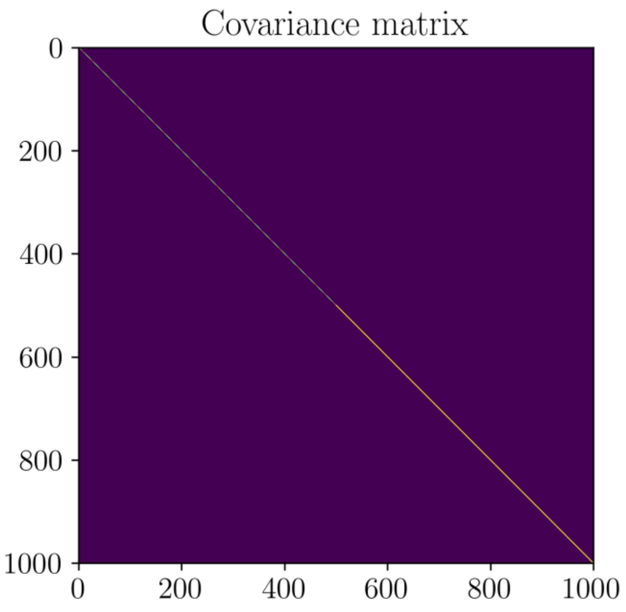
histogram

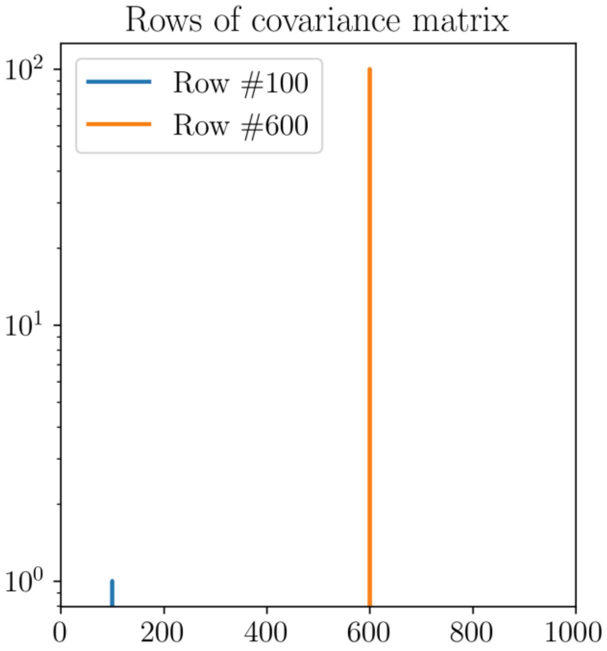


Gaussian random fields: examples

$$C_{ii} = \begin{cases} 1 & \text{if } i < N/2 \\ 100 & \text{otherwise} \end{cases}$$
$$C_{ij} = 0 \quad \text{for} \quad i \neq j$$

covariance matrix





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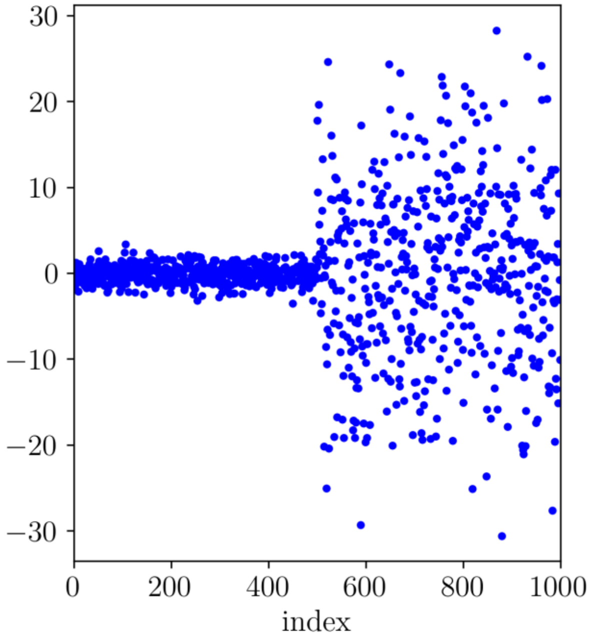
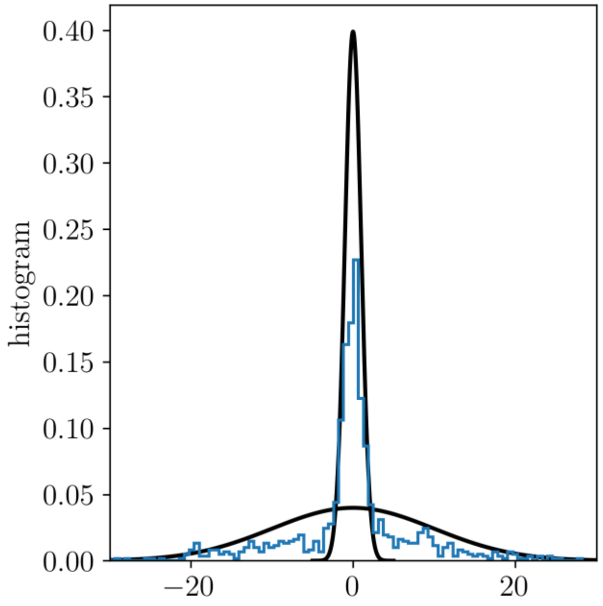


histogram



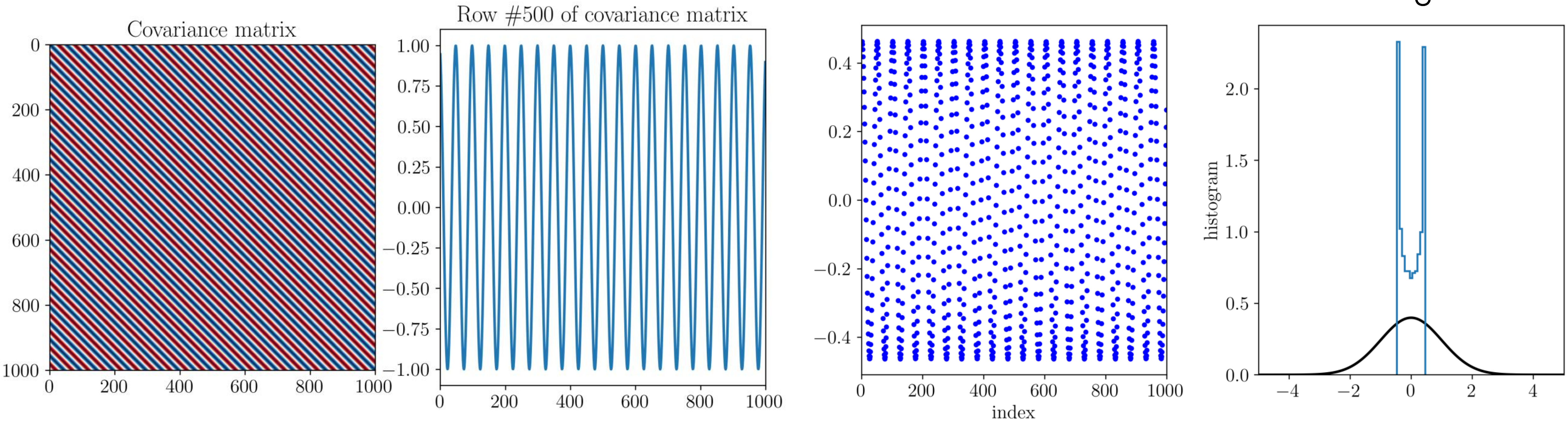
Gaussian random fields: examples

$$C_{ij} = \cos\left(\frac{i-j}{8}\right)$$

covariance matrix

GRF

histogram

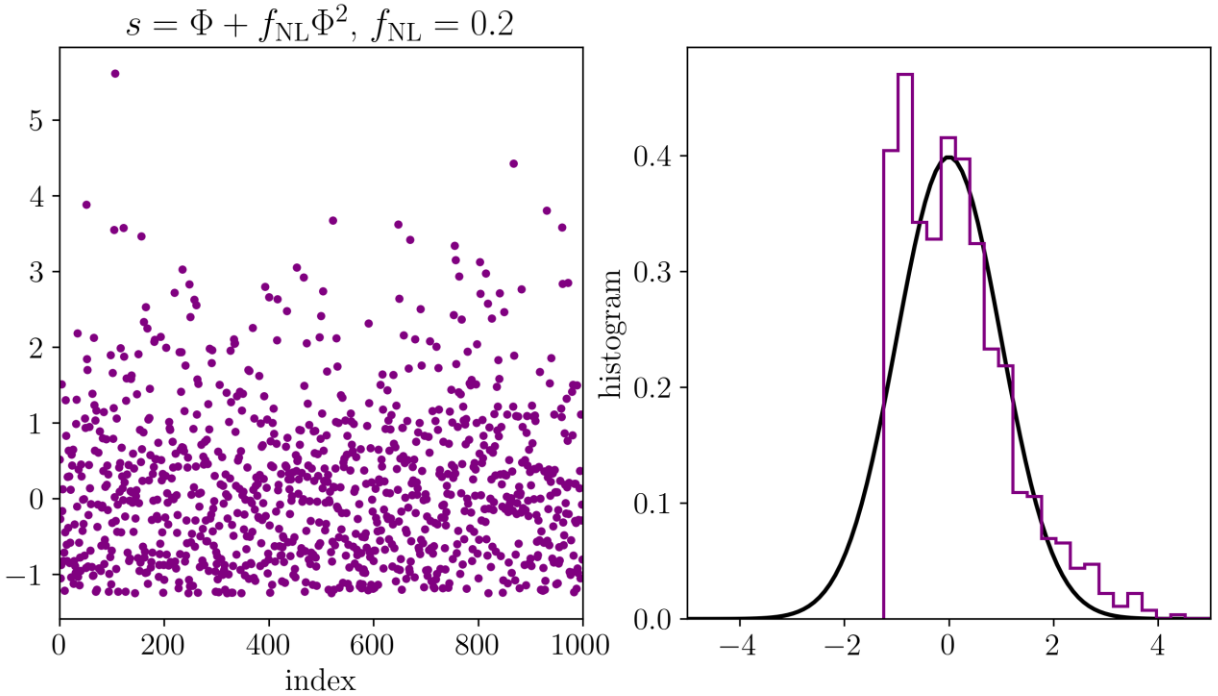


Histograms of Gaussian random fields are not always Gaussian!

Example of a non-Gaussian signal

- $s = \Phi + f_{\rm NL}\Phi^2$ where Φ is a GRF.
- In cosmology, this is called "local-type" non-Gaussianity
- Primordial non-Gaussianity contains information on cosmological inflation.

signal histogram



- The one-point pdf is skewed.

Marginals and conditionals of Gaussian random fields

- We work with a "joint" Gaussian random field $\begin{pmatrix} x \\ y \end{pmatrix}$

with Mean: $\mu = \begin{pmatrix} \mu_x \\ \mu_y \end{pmatrix}$ Covariance: $C = \begin{pmatrix} C_{xx} & C_{xy} \\ C_{yx} & C_{yy} \end{pmatrix}$

- Theorem: The marginal ($p(x)$) and the conditional ($p(x|y)$) pdfs are also Gaussian, with means and variances given below.
- Marginals: $$\langle x \rangle_{p(x)} = \int x p(x,y)\,\mathrm{d}y = \mu_x$$ $$\langle (x-\mu_x)(x-\mu_x)^\intercal \rangle_{p(x)} = \int (x-\mu_x)(x-\mu_x)^\intercal p(x,y)\,\mathrm{d}y = C_{xx}$$

i.e. the marginal mean and covariance are just the corresponding parts of the joint mean and covariance.

- Conditionals: $$\langle x \rangle_{p(x|y)} \equiv \mu_{x|y}$$ $$\langle (x-\mu_x)(x-\mu_x)^\intercal \rangle_{p(x|y)} \equiv C_{x|y}$$ where

Mean: $$\mu_{x|y} = \mu_x + C_{xy} C_{yy}^{-1} (y - \mu_y)$$

Covariance: $$C_{x|y} = C_{xx} - C_{xy} C_{yy}^{-1} C_{yx}$$

For a proof, see e.g. Leclercq (2015), appendix A.

BAYESIAN SIGNAL PROCESSING & WIENER FILTERING

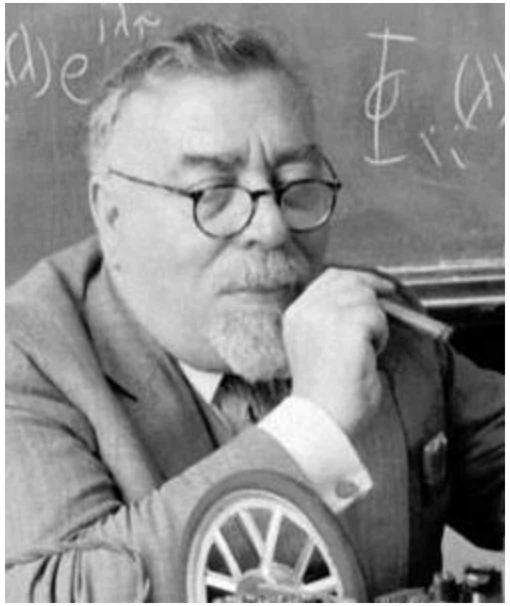
Bayesian denoising (Wiener filtering)

- Data model: $d = s + n$ where $\begin{pmatrix} s \\ d \end{pmatrix}$ is jointly Gaussian.
- Solution:

$$\mu_{s|d} = \mu_s + C_{sd} C_{dd}^{-1} (d - \mu_d)$$
$$C_{s|d} = C_{ss} - C_{sd} C_{dd}^{-1} C_{ds}$$

- Notations: $C_{ss} \equiv S$ and $C_{nn} \equiv N$.
- Assumption: $C_{sn} = C_{ns} = 0$. Then

$$C_{dd} = S + N$$
$$C_{sd} = C_{ss} + C_{sn} = C_{ss} = S$$



Norbert Wiener
(1894-1964)

- Final expressions:

mean: $\mu_{s|d} = \mu_s + S(S+N)^{-1}(d - \mu_d) = \mu_s + (S^{-1} + N^{-1})^{-1} N^{-1} (d - \mu_d)$

covariance: $C_{s|d} = S - S(S+N)^{-1} S = (S^{-1} + N^{-1})^{-1}$

Wiener filtering: derivation

- The canonical expression for a Gaussian is:

  $$-2\ln p(x|\mu,C) = \ln|2\pi C| + (x-\mu)^\intercal C^{-1}(x-\mu) = \ln|2\pi C| + \eta^\intercal \Lambda^{-1}\eta - 2\eta^\intercal x + x^\intercal \Lambda x$$

  where $\Lambda \equiv C^{-1}$ and $\eta \equiv C^{-1}\mu$

- Assuming $\mu_s = \mu_d = 0$ (it's easy to put the mean back in when necessary), we have for the Wiener filtering problem:

  $$-2\ln p(s) = \ln|2\pi S| + s^\intercal S^{-1} s$$

  $$-2\ln p(d|s) = \ln|2\pi N| + (d-s)^\intercal N^{-1}(d-s)$$

  $$= \ln|2\pi N| + \eta^\intercal N \eta - 2\eta^\intercal s + s^\intercal N^{-1} s$$ with $\eta \equiv N^{-1}d$

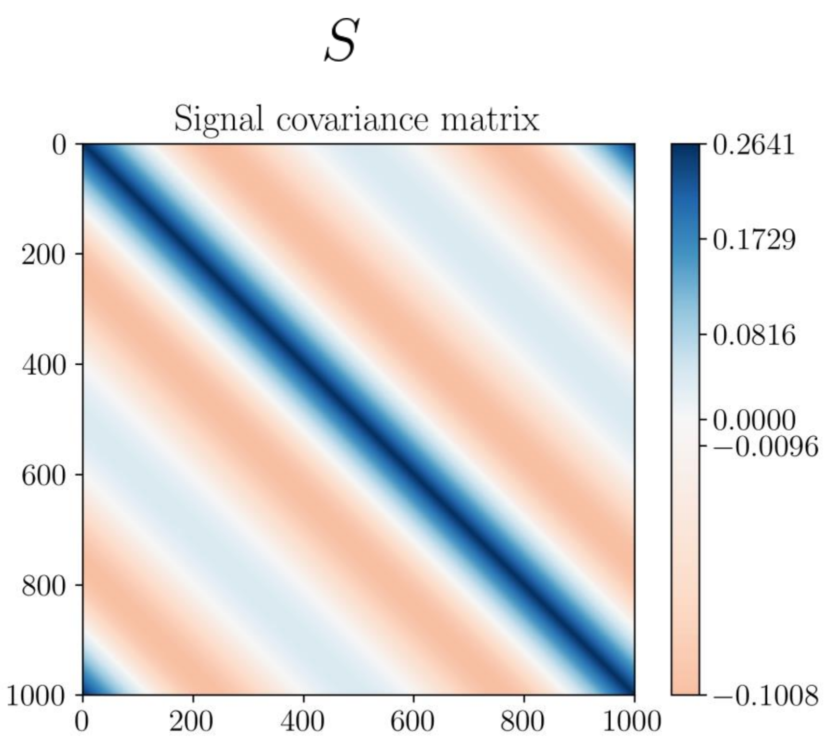
- Therefore:

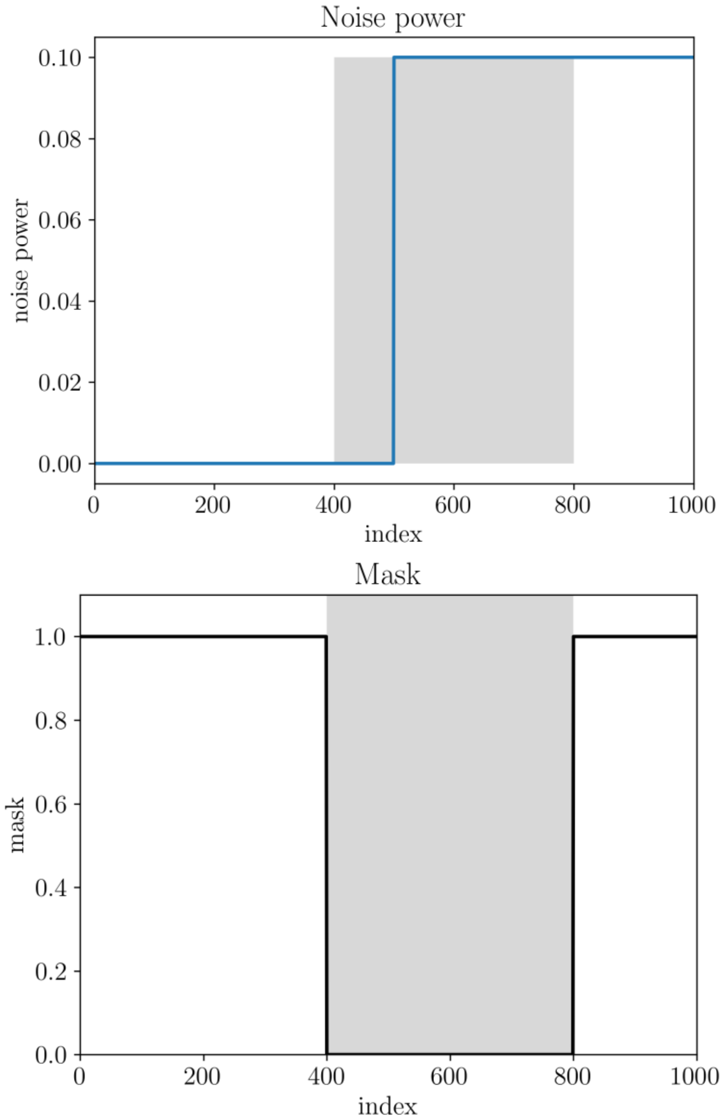
  $$-2\ln p(s|d) = \text{const} - 2\eta^\intercal s + s^\intercal (S^{-1} + N^{-1}) s$$

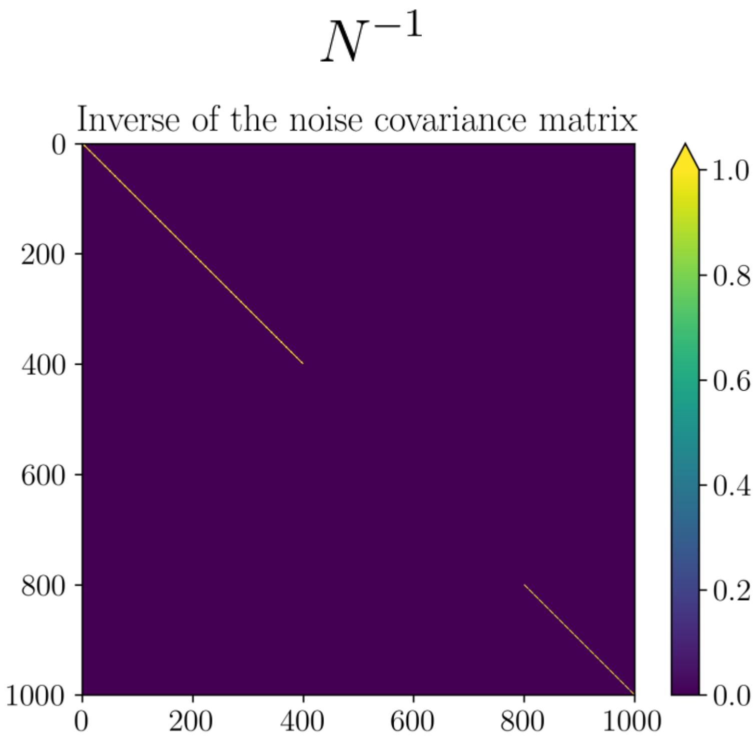
- The posterior has the canonical expression of a Gaussian with covariance matrix $W = (S^{-1} + N^{-1})^{-1}$, and its mean is $W\eta = (S^{-1} + N^{-1})^{-1} N^{-1} d$.

Bayesian denoising (Wiener filtering): example

- Setup signal and noise covariance matrices



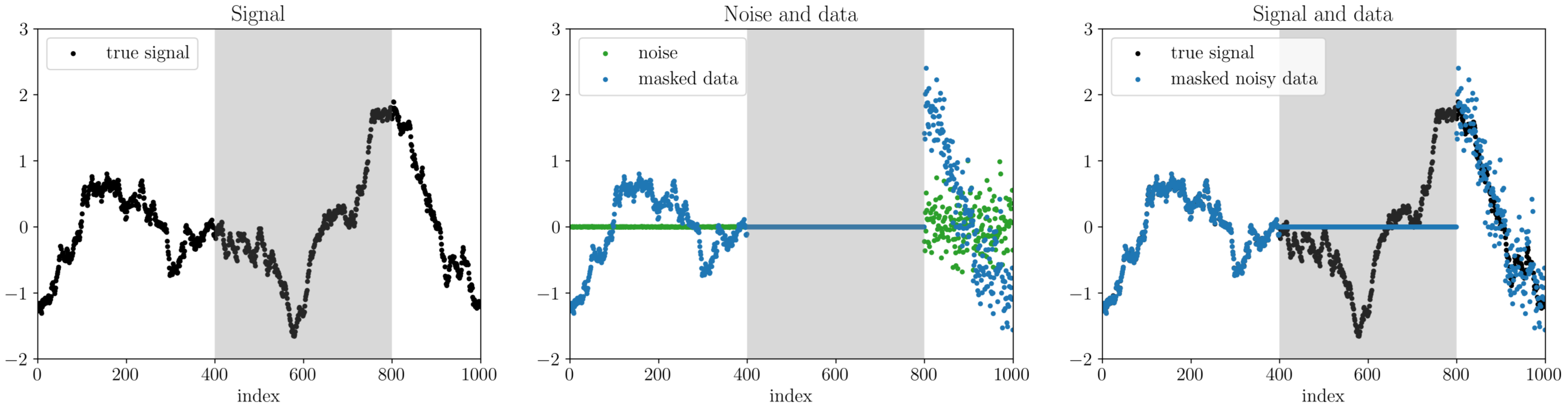




Bayesian denoising (Wiener filtering): example

- Generate signal and mock data

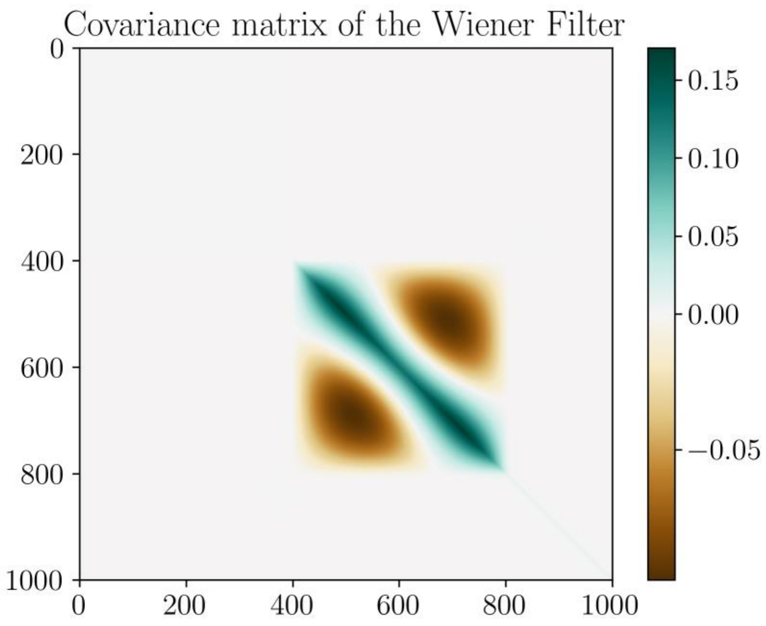
$$d = s + n$$



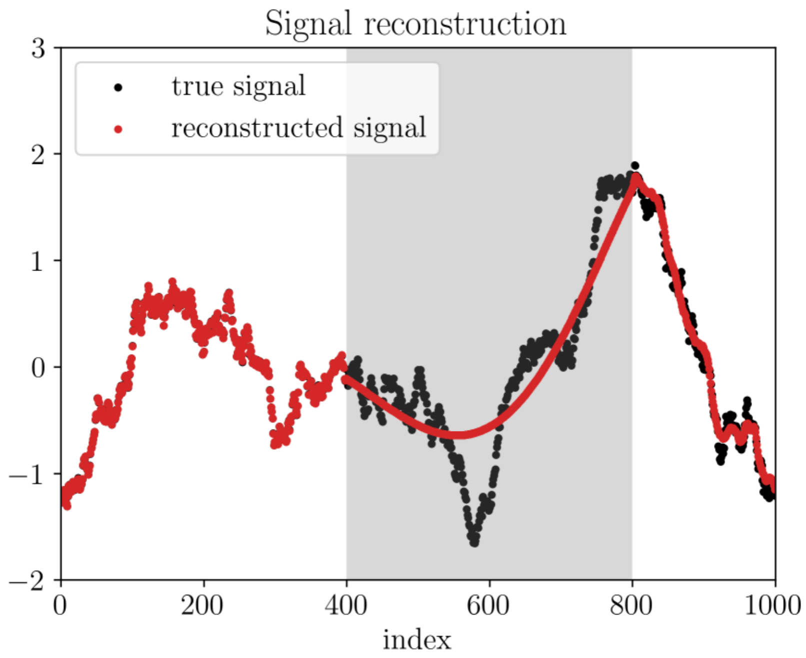
Bayesian denoising (Wiener filtering): example

- Perform Wiener filtering
- The mean of the reconstruction corresponds to the maximum a posteriori

$$C_{s|d} = (S^{-1} + N^{-1})^{-1}$$



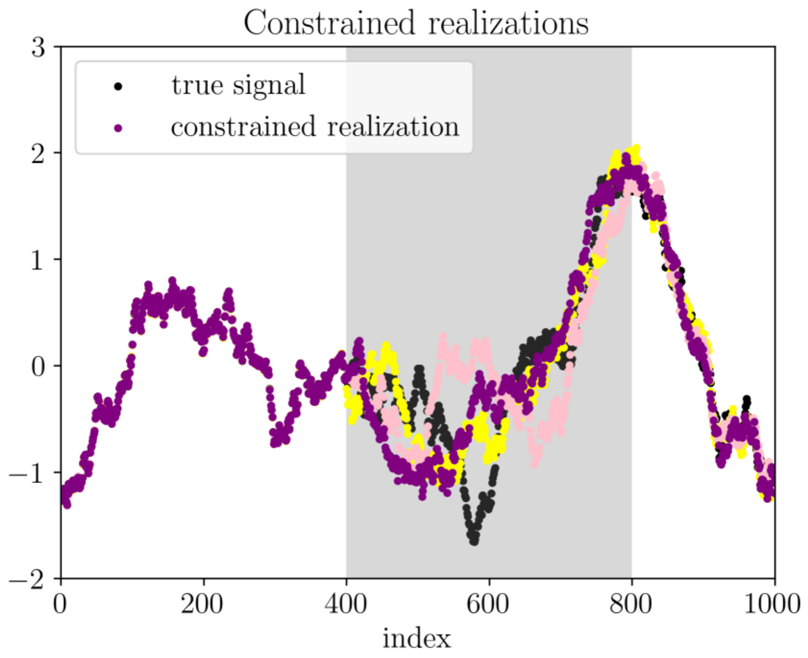
$$\mu_{s|d} = \mu_s + C_{s|d} N^{-1} (d - \mu_d)$$



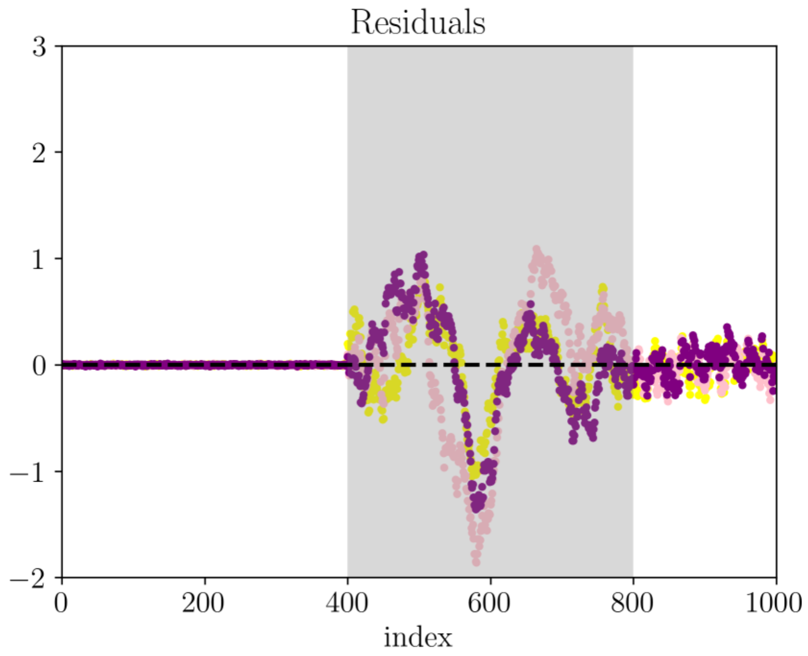
Bayesian denoising (Wiener filtering): example

- Draw constrained realisations of the denoised signal

$$s_{\text{sim}} = \mu_{s|d} + \sqrt{C_{s|d}}\,\xi$$



$$s_{\text{sim}} - s_{\text{true}}$$

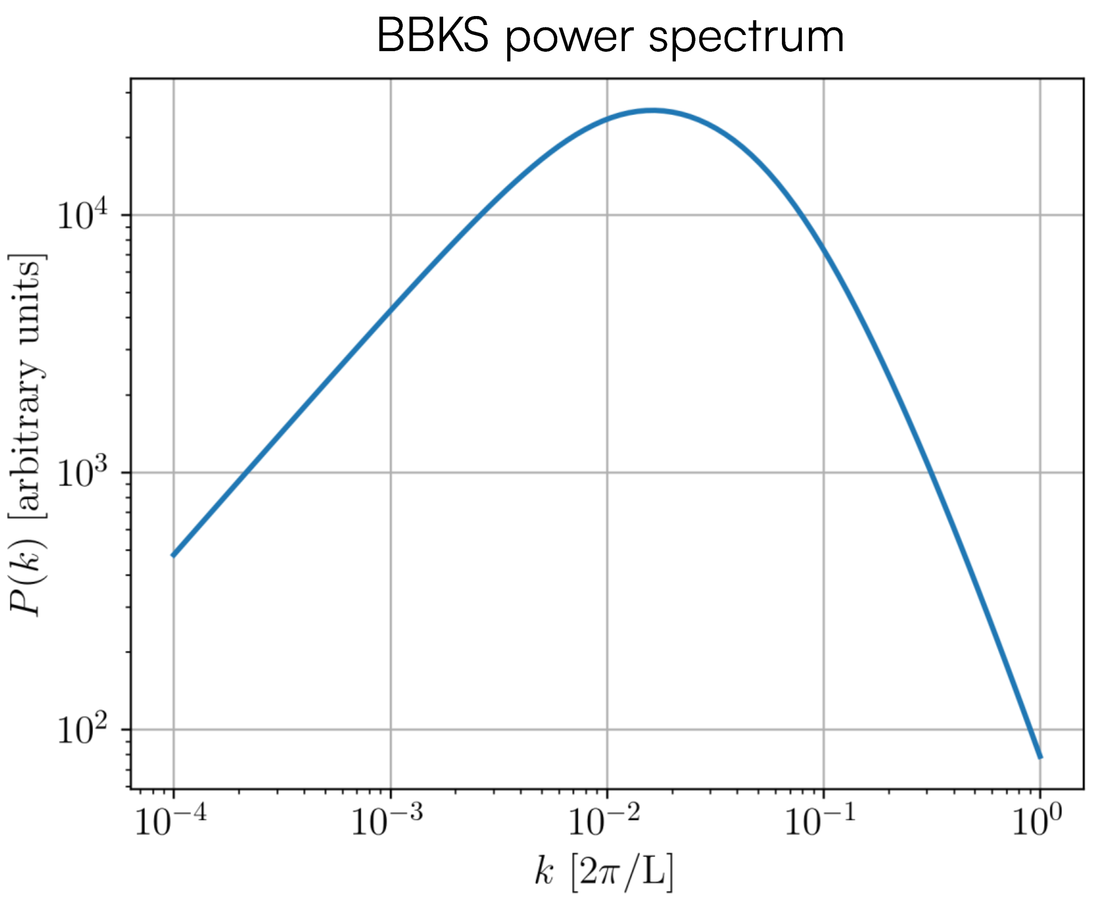


WIENER FILTERING FOR FIELD-LEVEL INFERENCE

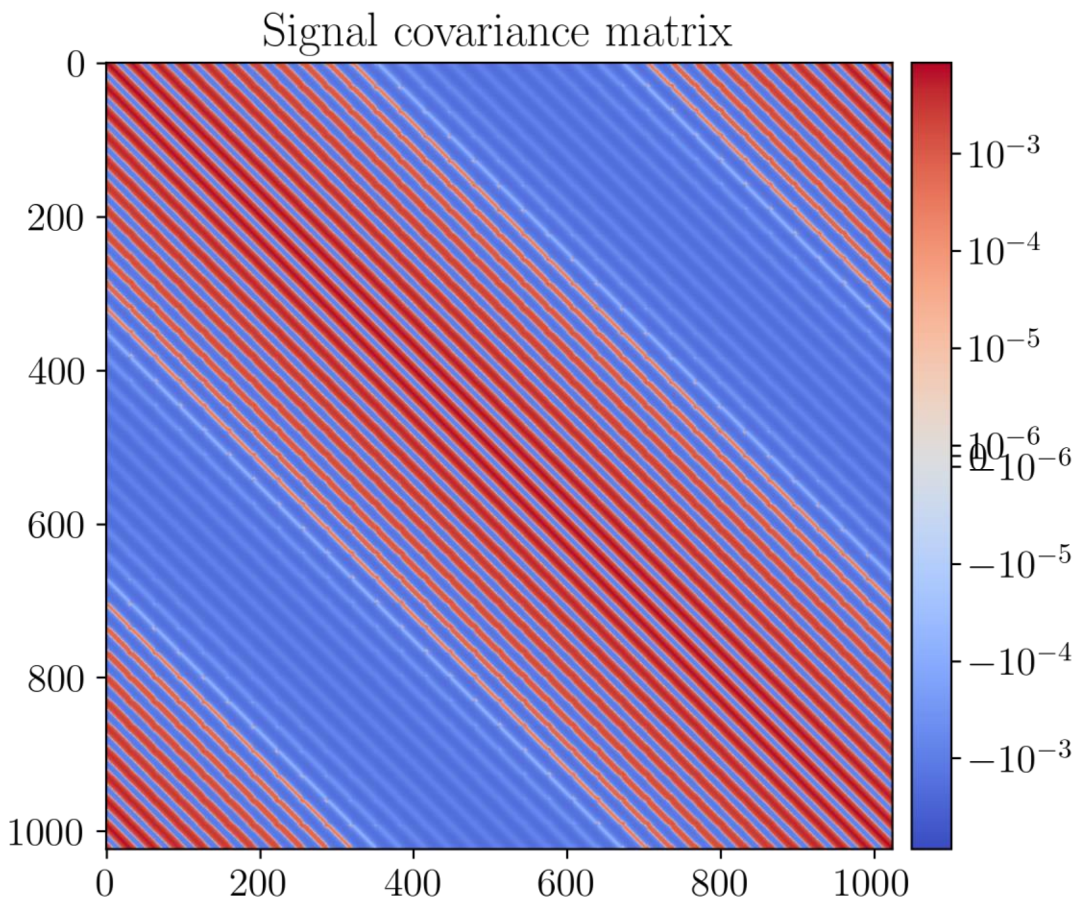
A linear field-level model

Exercise: Wiener filtering

- Setup signal covariance matrix (diagonal in Fourier space)

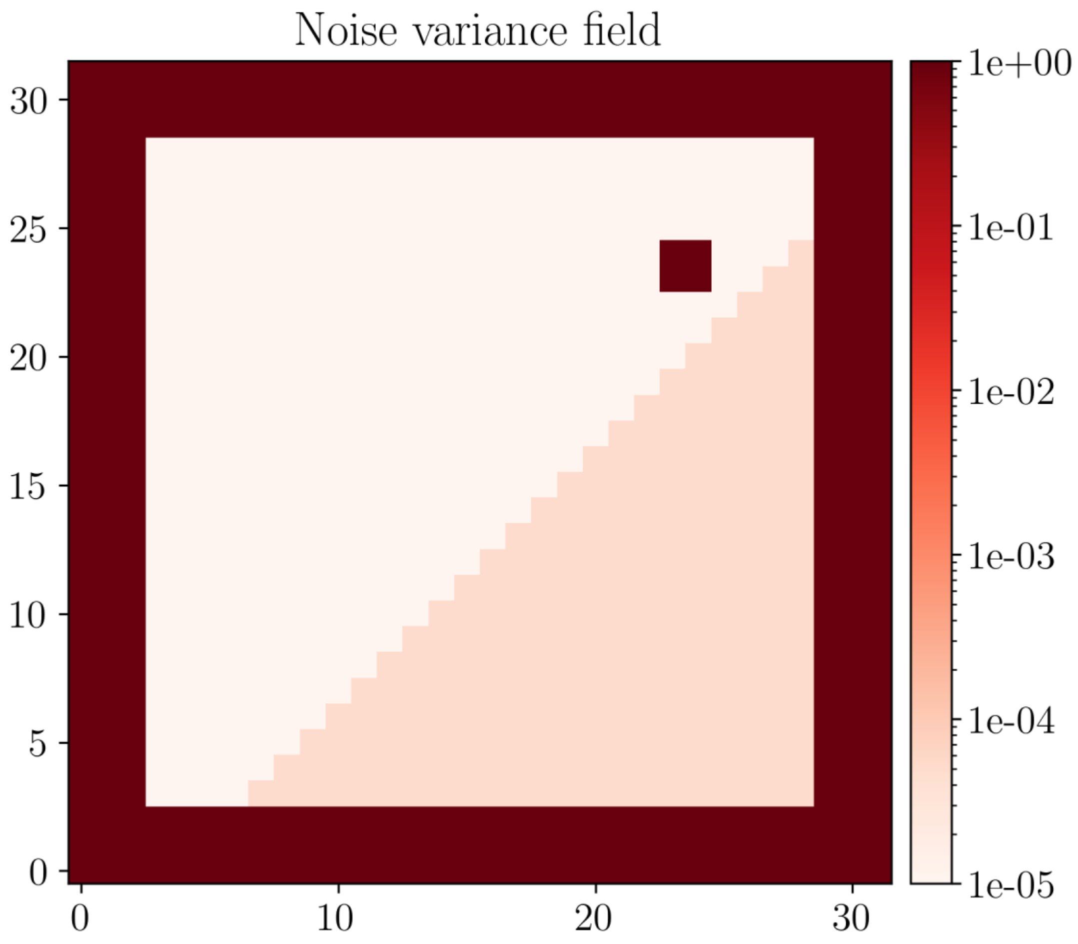


S

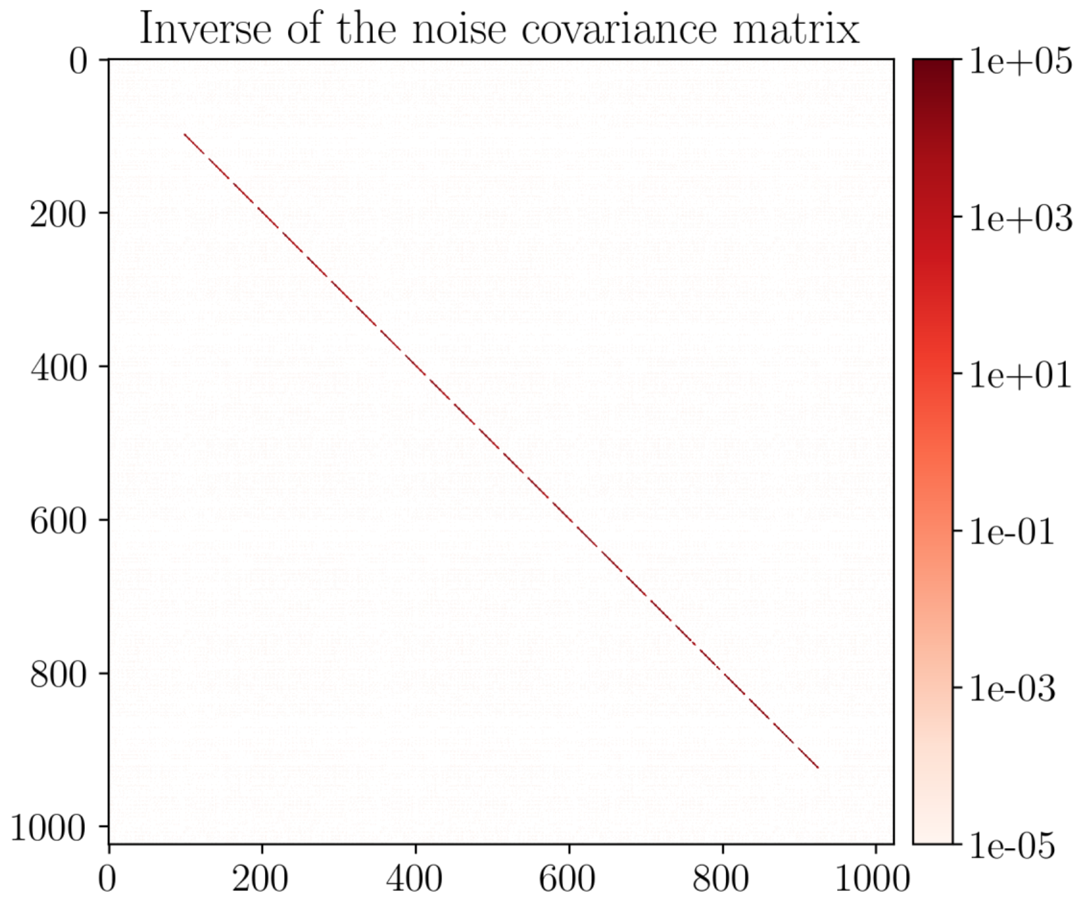


A linear field-level model

- Setup noise covariance matrix (diagonal in pixel space)



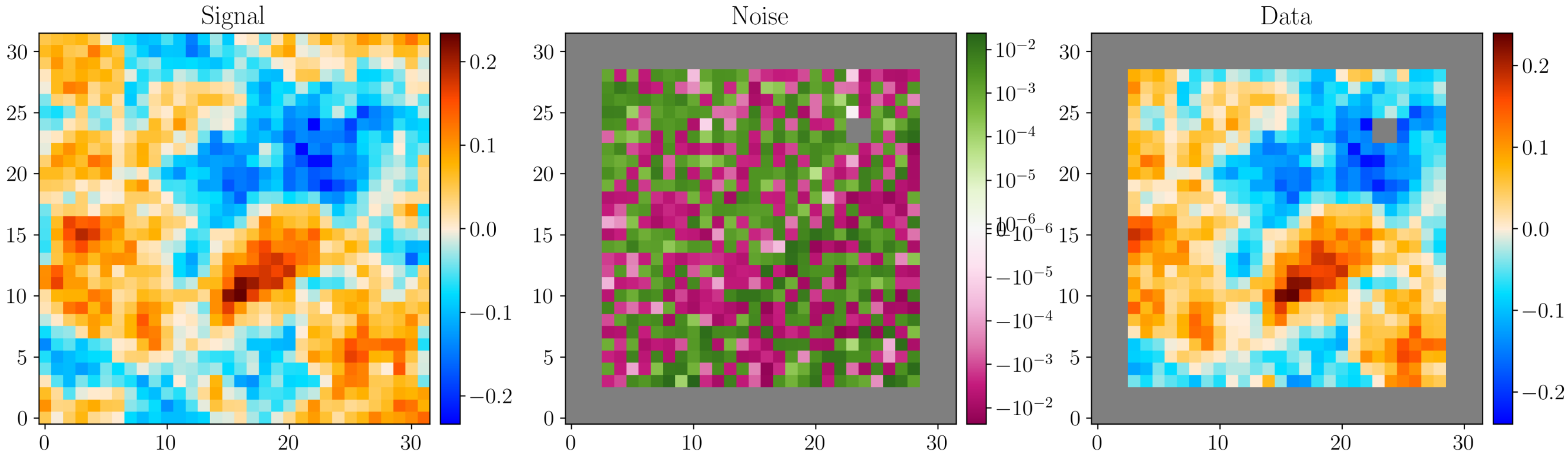
N^{-1}



A linear field-level model

- Generate signal and mock data

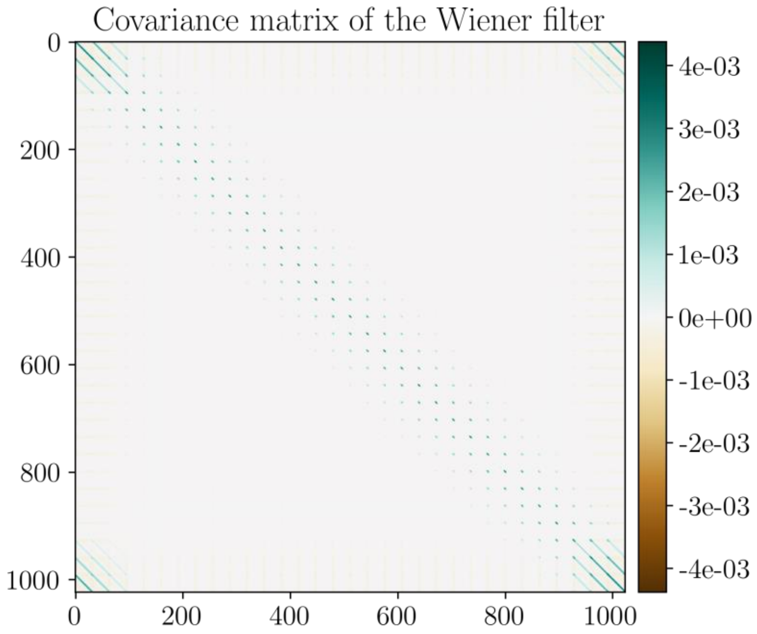
$$d = s + n$$



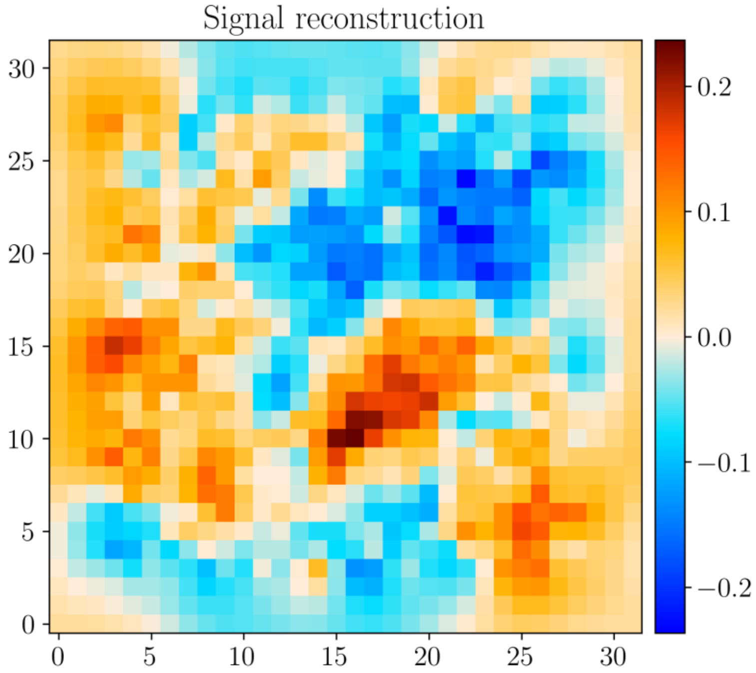
Wiener filtering for field-level inference

- Perform Wiener filtering: compute the Wiener covariance matrix and Wiener filter mean field

$$C_{s|d} = (S^{-1} + N^{-1})^{-1}$$

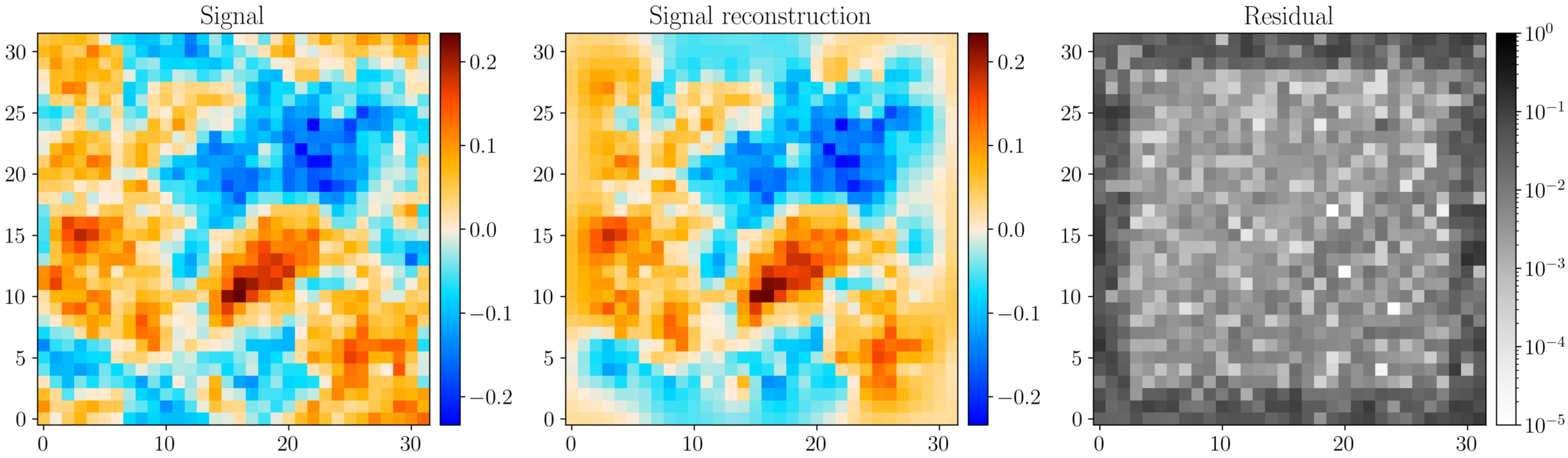


$$\mu_{s|d} = C_{s|d} N^{-1} d$$



Wiener filtering for field-level inference

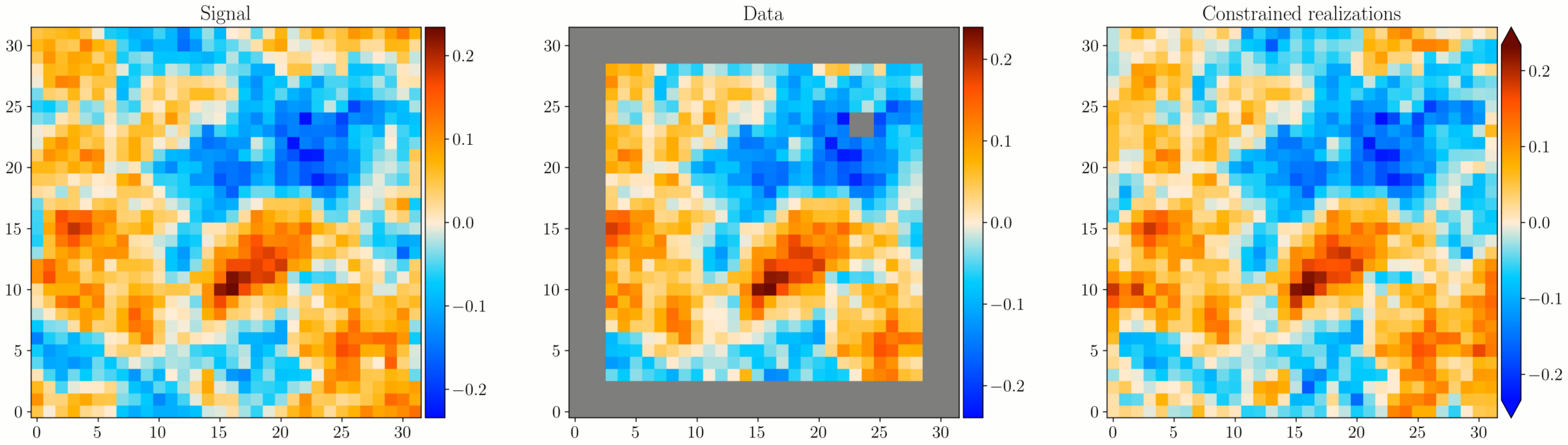
- The mean of the reconstruction corresponds to the maximum a posteriori.



Wiener filtering for field-level inference

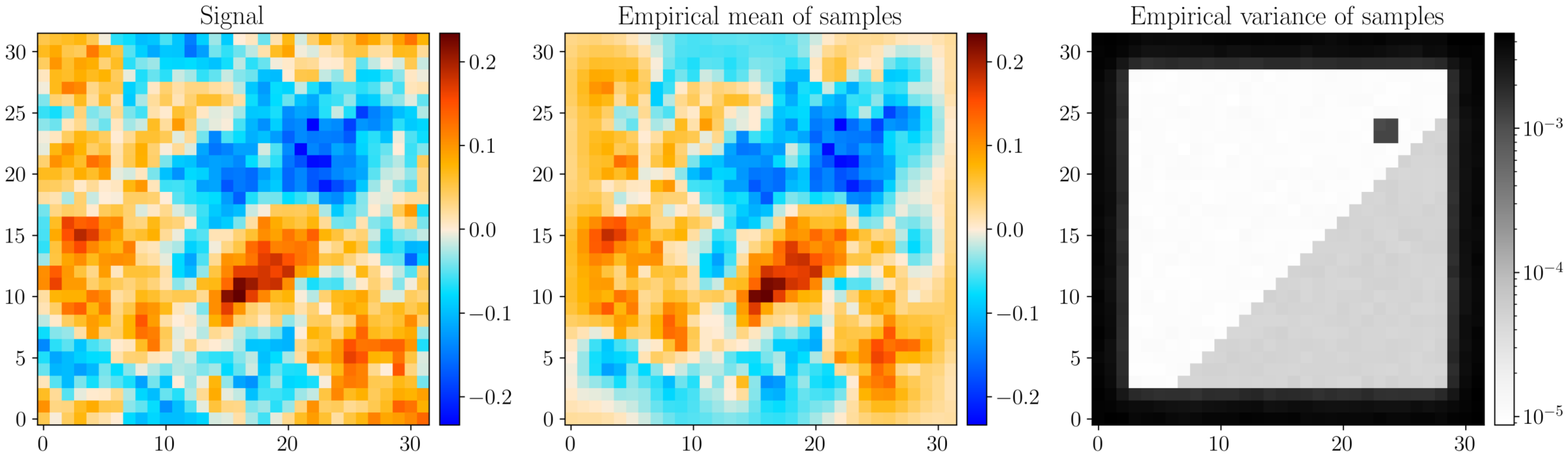
- Draw constrained realisations of the denoised signal

$$\tilde{s} = \mu_{s|d} + \sqrt{C_{s|d}}\,\xi$$



Wiener filtering for field-level inference

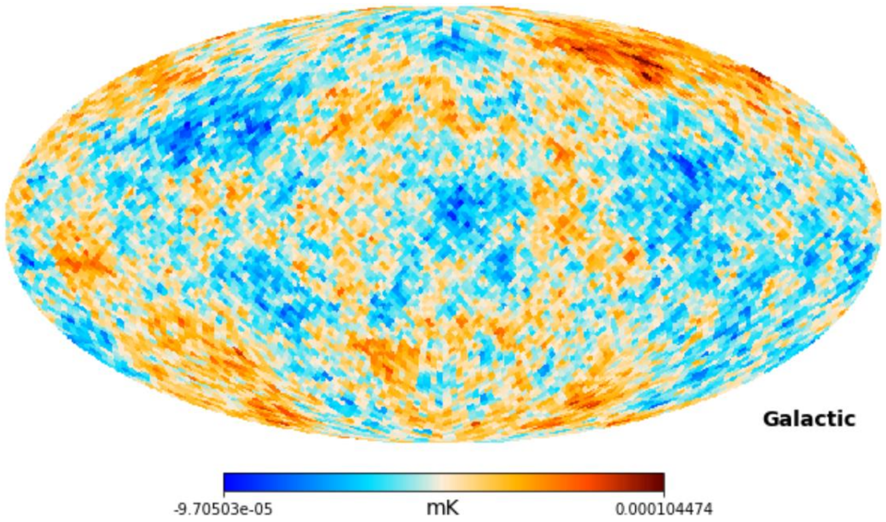
- Given a set of samples of the posterior, we usually show their empirical mean and empirical variance (or standard deviation).



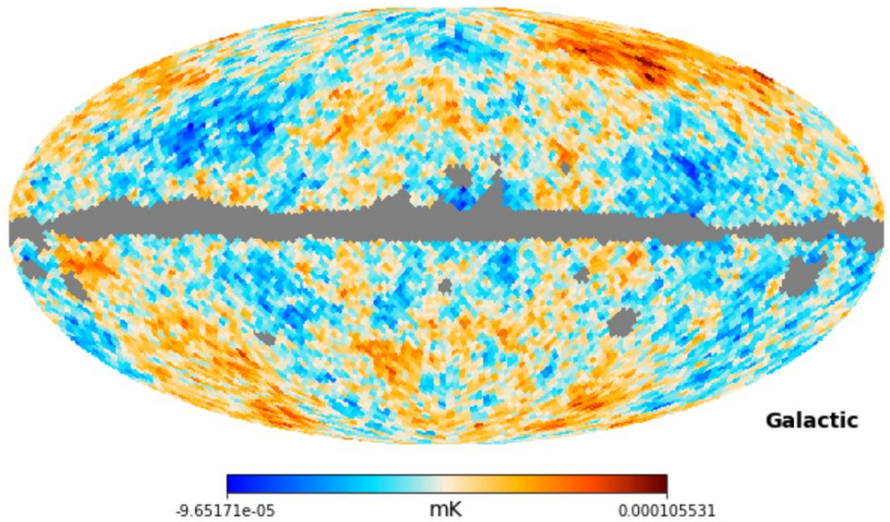
Wiener filtering for field-level inference: CMB example

Implementation at Github:florent-leclercq/Bayes_InfoTheory/Wiener_filter_denoising_CMB.ipynb

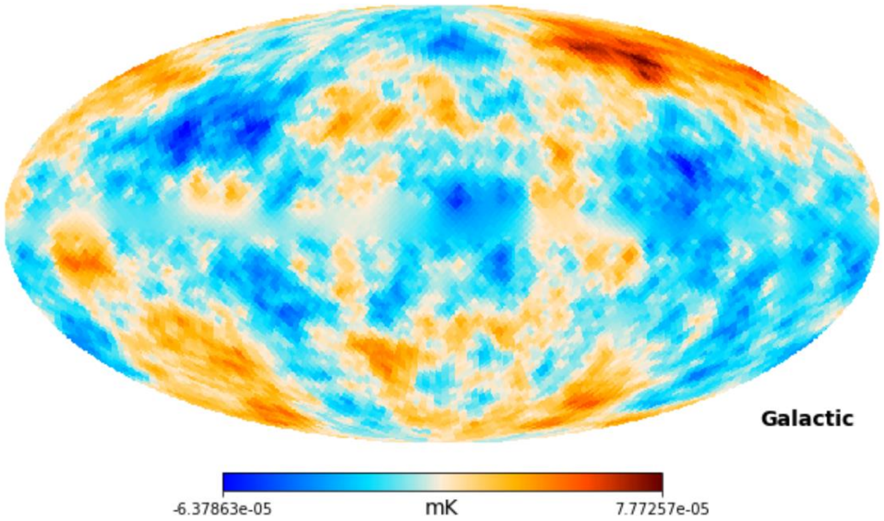
True signal



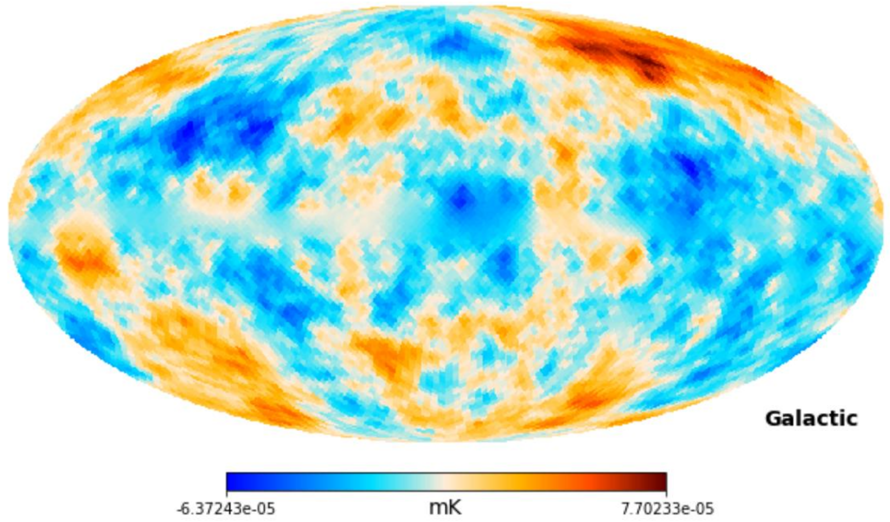
Simulated data d



Wiener filtered data (posterior mean)



One simulated signal



References and further reading

References:

- S. B. McGrayne (2012), *The Theory That Would Not Die: How Bayes' Rule Cracked the Enigma Code, Hunted Down Russian Submarines, and Emerged Triumphant from Two Centuries of Controversy*
- E. T. Jaynes (2002), *Probability Theory: The Logic of Science*

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