Phase-space approximation for three-flavor neutrino oscillations: challenging quantum algorithms

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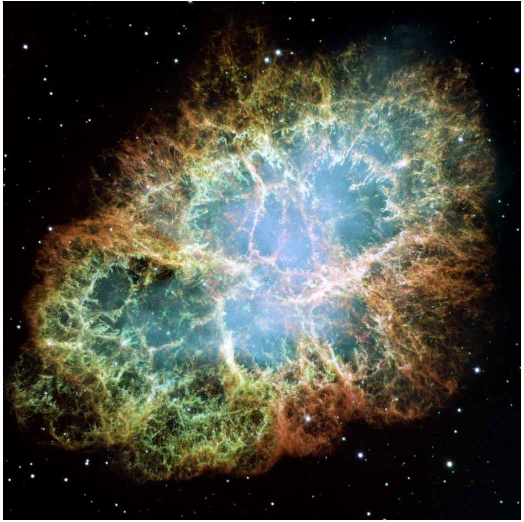
Advanced Quantum Algorithms for Many-Body systems (AQAM-2025)

Outline

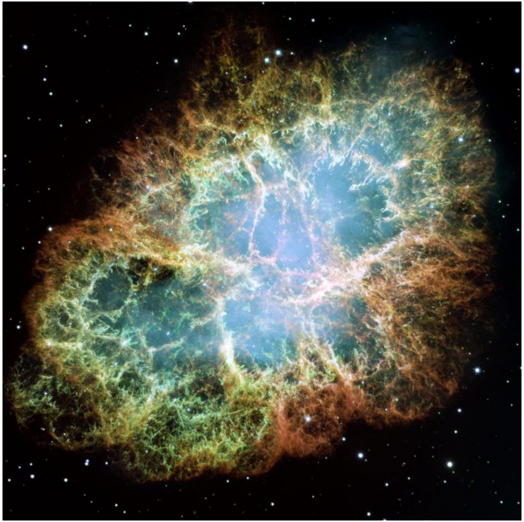
The Physics Context

- Neutrino oscillations in astrophysical environments involve many physical processes and large interacting ensembles.
- Simplified models but still face exponential complexity of the Hilbert space.
- Paradigmatic many-body quantum problem



Challenges

- Exact solutions: limited to $N \lesssim 10$ for three flavors.
- Mean-field approximation: often fails to capture entanglement and to describe quantum many-body correlations.
- Tensor networks: few tens of neutrinos, but not suited for systems with all-to-all interactions.
- Quantum computing: promising but so far restricted to relatively small numbers of neutrinos and short evolution times.
- Need for a new method to simulate hundreds of neutrinos in the three-flavor case.



Three-Flavor Hamiltonian in the mass basis

System of N three-flavor neutrinos

$$H = \sum_{\alpha=1}^{N} \vec{B} \cdot \vec{Q}(\alpha) + \sum_{\alpha \neq \beta}^{N} \mu_{\alpha\beta} \vec{Q}(\alpha) \cdot \vec{Q}(\beta). \tag{1}$$

- One-body term: describing vacuum oscillations.
- Two-body term: neutrino-neutrino interactions.
- $\vec{B}$: eight-vector containing neutrino masses
- α: neutrino label
- $\mu_{\alpha\beta}$: two-body interaction strength
- $\vec{Q}$: eight-vector defined as:

$$Q_m(\alpha) = \frac{1}{2} \sum_{i,j=1}^{3} \underbrace{a_i^\dagger(\alpha)}_{\text{Creation operator}} \Big(\underbrace{\lambda_m}_{\text{Gell-Mann matrix}} \Big)_{ij} \underbrace{a_j(\alpha)}_{\text{Annihilation operator}} \qquad i = 1, 2, 3,\ m = 1, \ldots, 8$$

PMNS Mixing and Parameters

- Three types of neutrinos: flavor eigenstates (ν_e, ν_μ, ν_τ), or mass eigenstates (ν_{m_1}, ν_{m_2}, ν_{m_3})
  Flavor and mass bases related via PMNS 3×3 matrix U_{PMNS}.
- Parameters from NuFIT (2024):

Parameter	Values
δm^2	7.41×10^{-17} MeV^2
Δm^2	2.505×10^{-15} MeV^2
θ_{12}	33.67^o
θ_{13}	8.58^o
θ_{23}	42.3^o
δ	232^o

[1] I. Esteban, M. C. González-García, M. Maltoni, T. Schwetz, and A. Zhou, JHEP 09, 178, arXiv:2007.14792 [hep-ph].
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Phase Space Approximation (PSA) Idea

- **Phase Space Approach (PSA):** map quantum many-body problem evolution to a statistical set of "simple" trajectories.
- Long history in low energy nuclear physics, successfully applied to systems with all-to-all interactions.
- Success in two-flavor neutrino systems → goal: extend to three flavors.

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A stochastic mean-field approach for nuclear dynamics

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Abstract

We propose a microscopic stochastic approach to improve description of nuclear dynamics beyond the mean-field approximation at low energies. It is shown that, for small amplitude fluctuations, the proposed model gives a result for the dispersion of a one-body observable that is identical to the result obtained previously through a variational approach. Furthermore, by projecting the proposed stochastic mean-field evolution on a collective path, a generalized Langevin equation is derived for collective variable, which incorporates one-body dissipation and one-body fluctuation mechanism in accordance with quantal fluctuation–dissipation relation.

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Phase-space methods for neutrino oscillations: Extension to multibeams

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PSA Concept

Maps the complex evolution of interacting particles onto a statistical ensemble of independent, simpler trajectories, each initialized with stochastically sampled initial conditions.

(i) Choice of the "simple" trajectories:
$\longrightarrow$ Mean-field equations of motion.

(ii) Initial sampling strategy
$\longrightarrow$ Not unique
$\longrightarrow$ Initial fluctuations sampled to reproduce first and second quantum moments.

(i) Mean field equations of motion....

Ehrenfest theorem for the set of one-body operators Q_m. Setting $\vec{P}(\alpha) = 2\langle\vec{Q}(\alpha)\rangle$

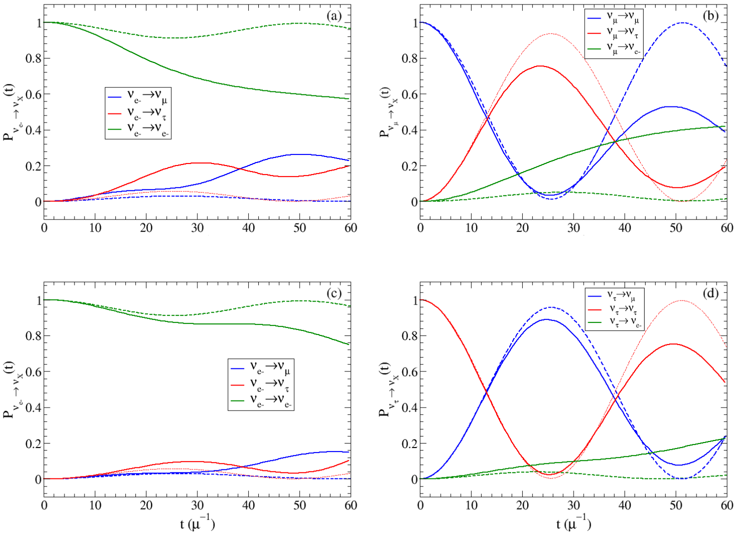
$$\frac{d}{dt}\vec{P}(\alpha) \quad = \quad \vec{B} \times \vec{P}(\alpha) + \frac{1}{2}\sum_{\beta\neq\alpha}\mu_{\alpha\beta}\vec{P}(\beta) \times \vec{P}(\alpha). \tag{2}$$

- $\vec{P}_i$: 8-component polarization vector.
- $\times$: generalized SU(3) cross product.

For N neutrinos, the mean-field evolution reduces to a set of $8N$ non-linear coupled equation that can easily be solved even for large N

...and how they fail...

- N=8 interacting neutrinos
- Initialized to $|\nu_e \nu_\mu \nu_e \nu_\tau \nu_\tau \nu_e \nu_\mu \nu_e\rangle$
- Flavor evolution computed exactly (solid lines) vs. mean-field approximation (dashed lines).



(ii) Initial sampling

The equation of motion for $\vec{P}(\alpha)$ are solved with random initial variables.

$\Longrightarrow$ deterministic problem replaced by a stochastic one

- Sampling not unique, but constrained to reproduce first and second moments at $t = 0$.

Initial quantum problem

- State described by a density D(t)
- For any observable $\hat{A}$, quantum mean and variance given by:

$$\langle \hat{A} \rangle = \mathrm{Tr}\big(\hat{A} D(t)\big),$$

$$\sigma_A^2 = \mathrm{Tr}\big(\hat{A}^2 D(t)\big) - \Big(\mathrm{Tr}\big(\hat{A} D(t)\big)\Big)^2.$$

Ensemble (sampling) estimates

$$\langle \hat{A} \rangle \approx \frac{1}{N_{\mathrm{evt}}} \sum_{\lambda=1}^{N_{\mathrm{evt}}} A^{(\lambda)},$$

The ensemble variance is estimated as:

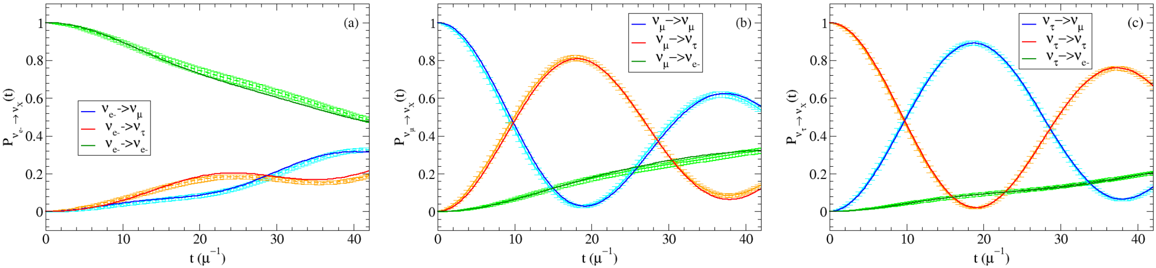
$$\Sigma_A^2 = \frac{1}{N_{\mathrm{evt}}} \sum_{\lambda=1}^{N_{\mathrm{evt}}} \left(A^{(\lambda)}\right)^2 - \left(\frac{1}{N_{\mathrm{evt}}} \sum_{\lambda=1}^{N_{\mathrm{evt}}} A^{(\lambda)}\right)^2$$

"PSA recipe"

1. Specify the initial many-body quantum state.
   For example: each neutrino α is initially in a definite flavor ν_e, ν_μ, or ν_τ
2. Compute the one-body means and variances at $t = 0$
   For simple initial states (uncorrelated, Slater-determinant like) these computations are straight-forward.
3. Choose a sampling strategy for the ensemble.
   Gaussian distributions with means and variances computed at step 2.
4. Initialize each event.
5. Evolve each event independently using mean-field equations.
   Standard Runge-Kutta methods
6. Compute averages and fluctuations of observables over the ensemble.
   Jackknife method.

Validation: small system comparisons

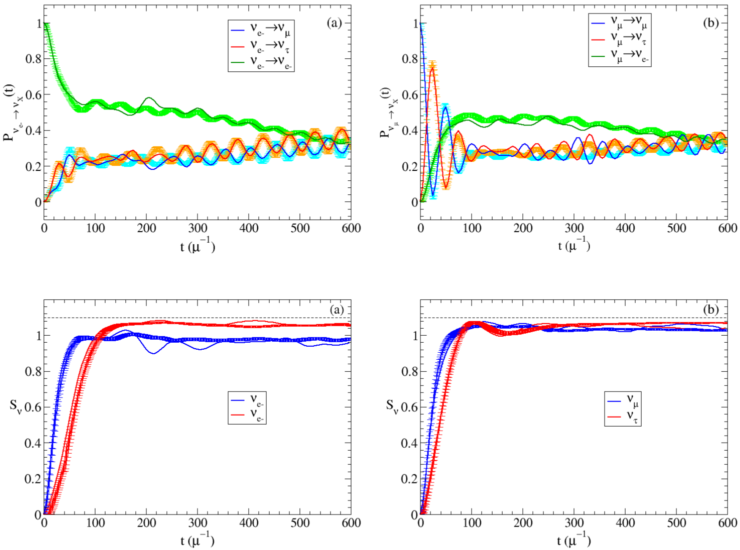
System of 6 interacting neutrinos, initialized to $|\nu_e\nu_\mu\nu_\tau\nu_\tau\nu_\mu\nu_e\rangle$.



Flavor population evolutions of three selected neutrinos. Exact algorithm (solid line, dark colors), and PSA (dashed lines with error bars, light colors).

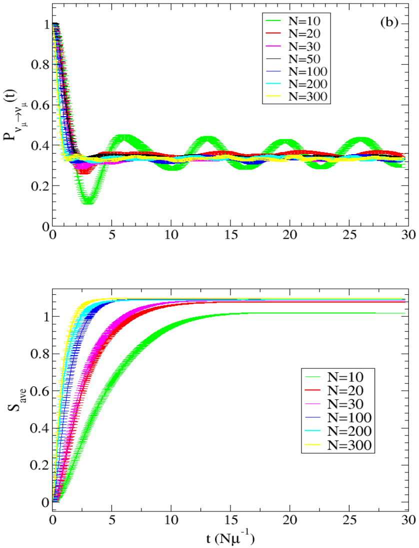
Long time evolution:

- 8 interacting neutrinos
- Initialized to $|\nu_e\nu_\mu\nu_e\nu_\tau\nu_\tau\nu_e\nu_\mu\nu_e\rangle$
- Flavor evolutions of two selected neutrinos (top).
- One-neutrino entropy for first four neutrinos (bottom).
- Exact algorithm (solid line), and PSA (dashed lines with error bars).



Large Ensemble Results

- Results for up to 300 interacting neutrinos
- Initialized to $|\nu_\mu \nu_e \nu_\tau \ldots\rangle$
- Oscillation probabilities of one neutrino embedded within a set of $N-1$ ensemble:
  - *whatever N, P_{ν_μ} reaches asymptotically in average an equiprobable partition of the neutrino flavors (probability 1/3).*
  - *the larger N, the shorter the time to reach the equipartition between flavors.*
- Average one-neutrino entropy as a function time.
  - *entropy increases more rapidly as N increases.*



Scaling Advantages

PSA method especially powerful when the number of neutrinos is large:

- The cost scales with number of events $\times$ cost of mean-field evolution, rather than exponentially in particle number.
- Enables simulations with $N \sim 300$ neutrinos on a single CPU.
- A large enough number of events required to keep statistical noise small.
- Each trajectory independent $\Rightarrow$ obvious parallelism can be implemented.
- On a quantum computer: simulating 300 neutrinos requires 300 qutrits, or 476 qubits (assuming efficient encoding).

Advantages

- Computationally very efficient
- Captures dissipation, equilibration, entropy growth
- Easily applicable to various Hamiltonians (time-dependent, including matter effects,...)
- Easily applicable to various initial states (simple uncorrelated or thermal/entangled)

$\Longrightarrow$ Highly competitive tool for studying the dynamics of neutrino systems

Summary

- PSA applied to three-flavor neutrinos oscillations.
- Validated against exact results for small N.
- Very high predictive power for describing neutrino oscillations involving all three flavor components.
- Applicable to large-N systems (hundreds or more), computationally efficient and naturally suited to parallelization.
- Benchmark for quantum simulators
- Serious competitor and/or a classical computing reference for future applications of neutrino physics using quantum computers.

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Thank you!