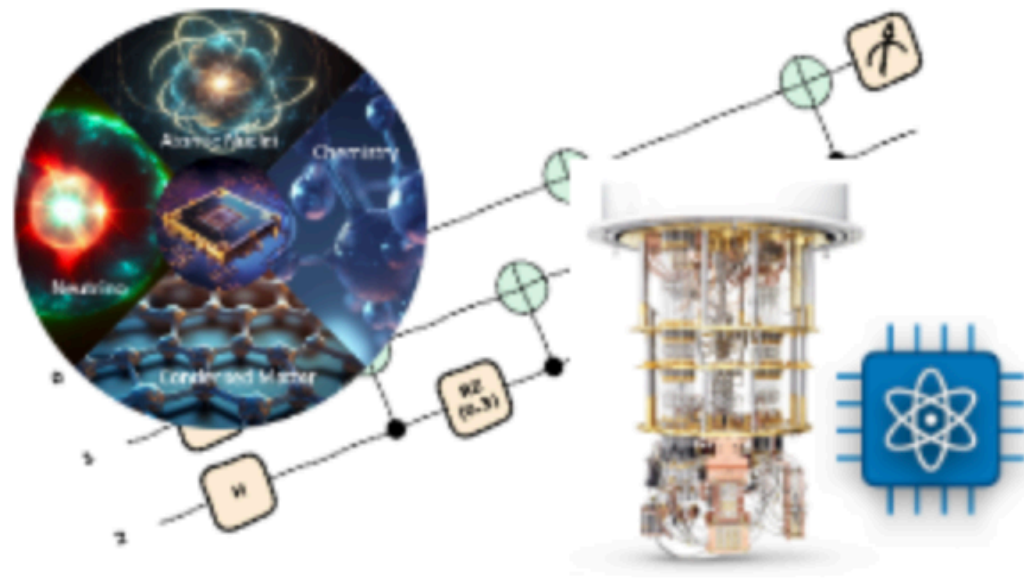




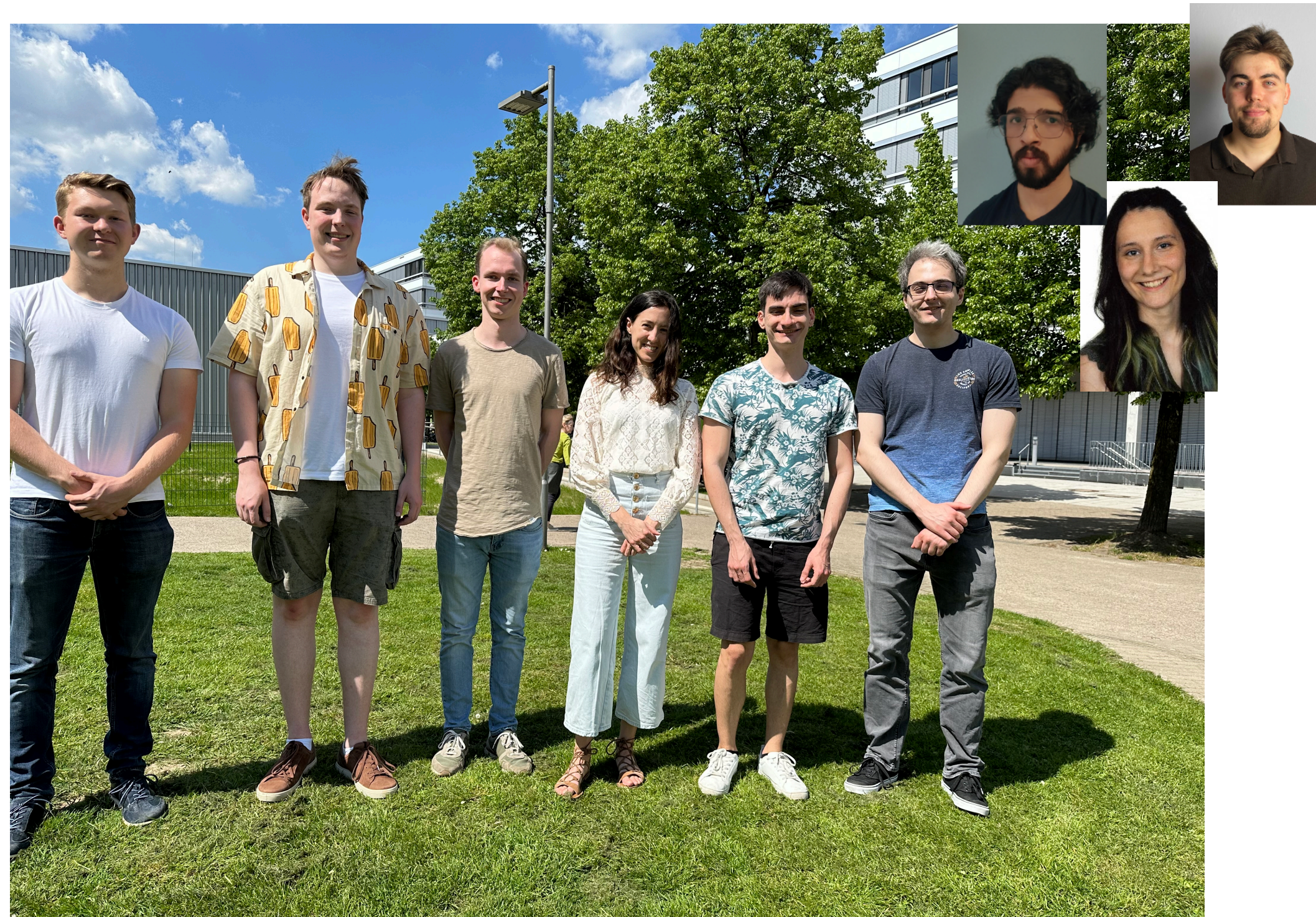
Workshop on
Advanced Quantum Algorithms for Many-Body Systems
AQAM-2025
Institut Botanique, Montpellier

QUANTUM SIMULATIONS DRIVEN BY MANY-BODY COMPLEXITY

CAROLINE ROBIN

THANKS TO COLLABORATORS!

Uni Bielefeld group



From left to right:

Erik Müller, Florian Brökemeier, Momme Hengstenberg, CR,
Federico Rocco, James Keeble, Niranjan Jayakrishnan,
Elisabeth Hahm, Felix Schunder

IQuS @ UW Seattle



Martin Savage

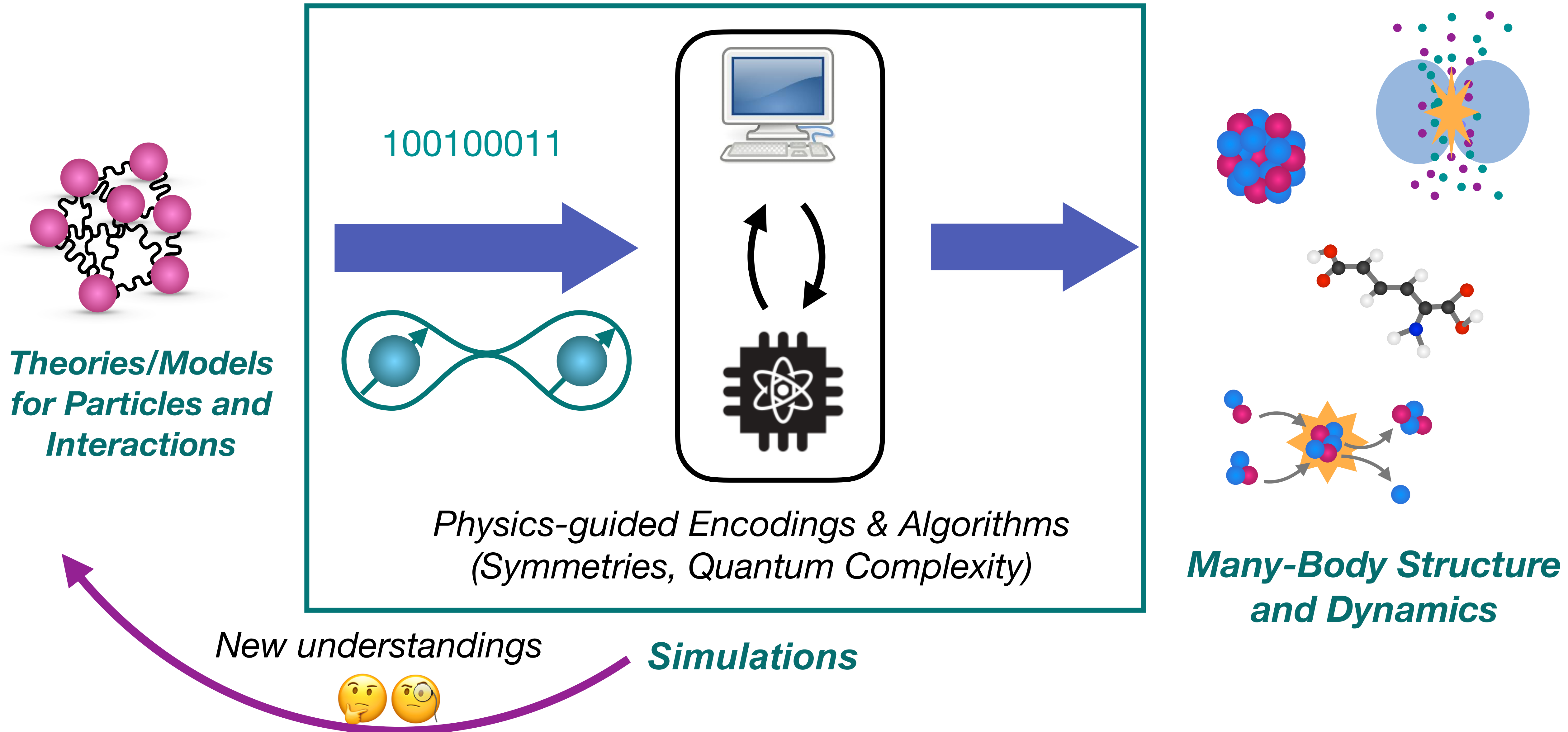


Marc Illa
(Now at PNNL)



Ivan Chernyshev
(Now at LANL)

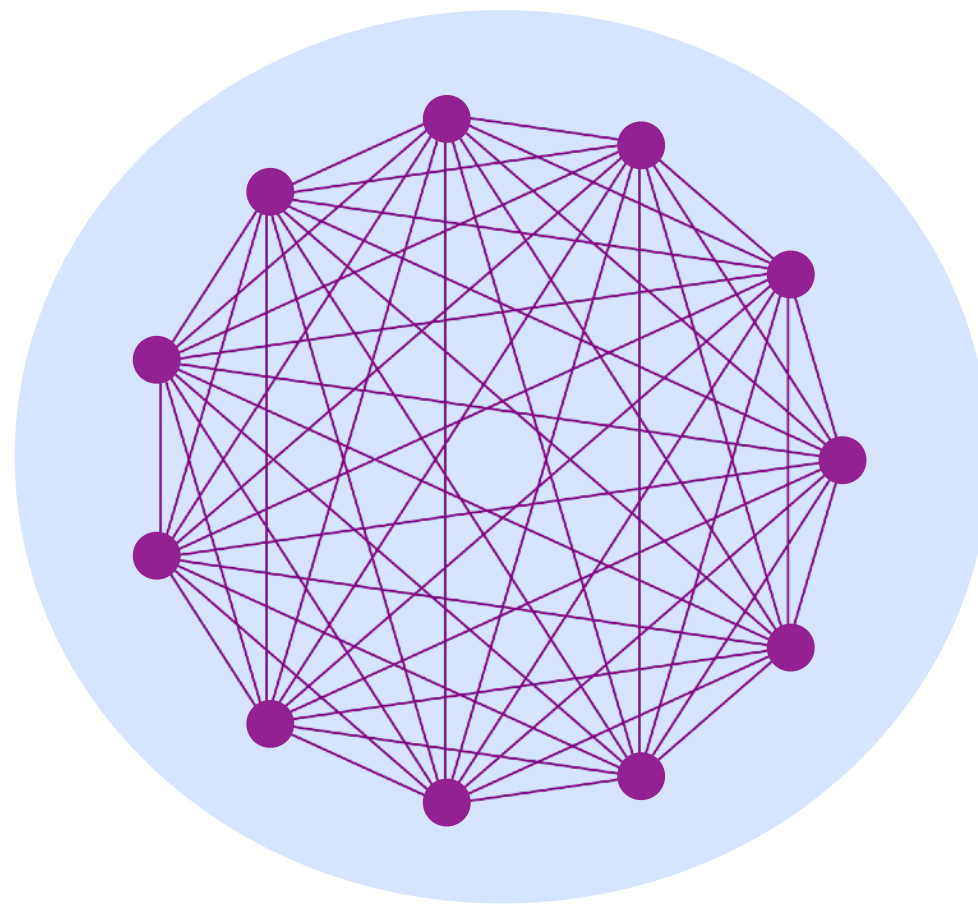
Towards Quantum Simulations of Quantum Many-Body Systems



Many-Body Complexity

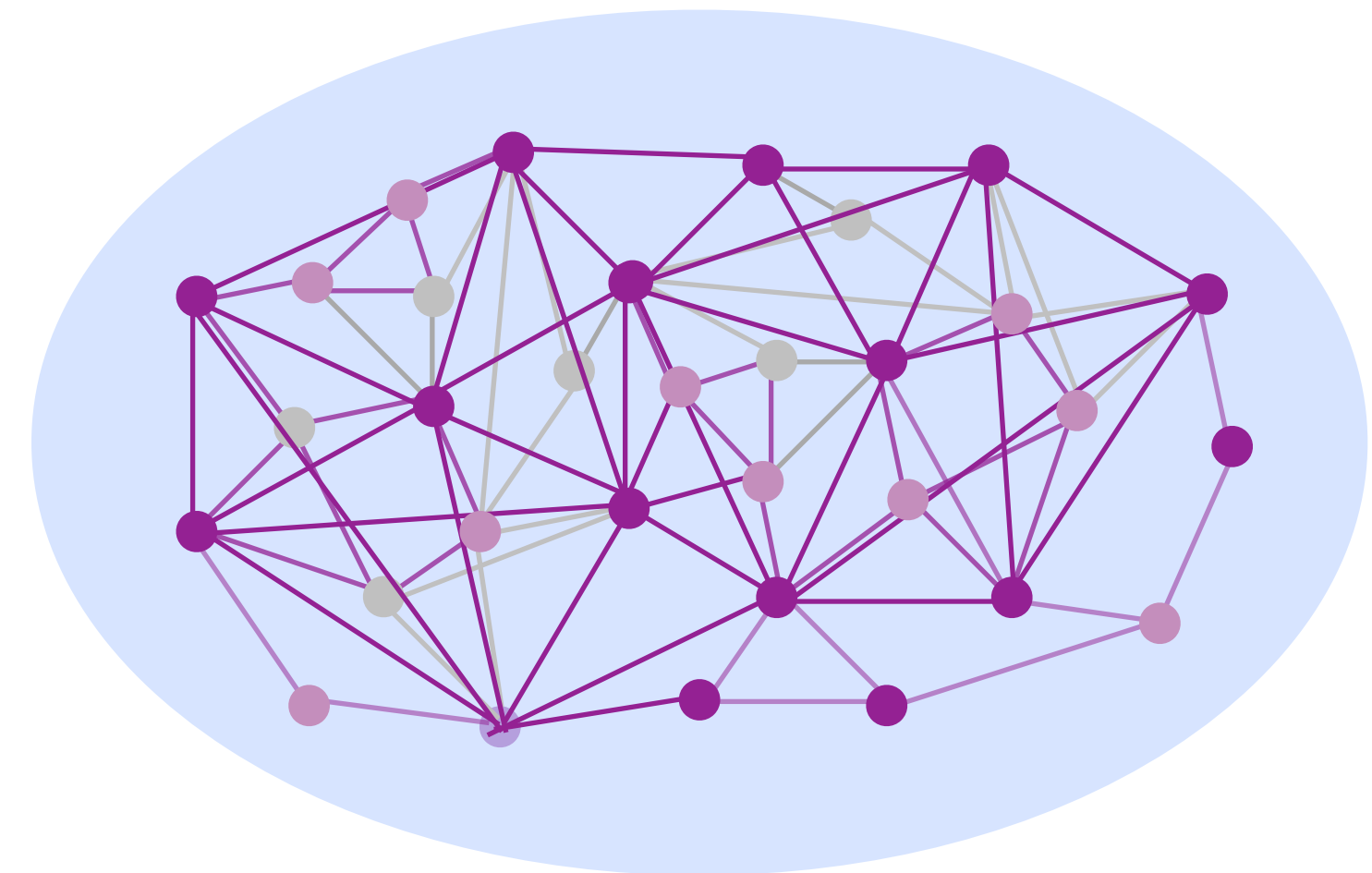
Complexity of a quantum state $\longleftrightarrow$ amount of information needed to represent it

(I) Quantum Entanglement + **(II) Non-Stabilizerness (Quantum “Magic”)**



→ highly-entangled “stabilizer states”
~ graph states ~ N^2 bits

“Simple”



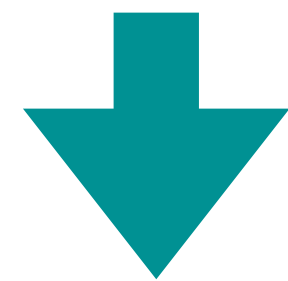
Arbitrary magic & entangled state:
~ $\exp(N)$ bits

“Complex”

Quantum Computational Complexity

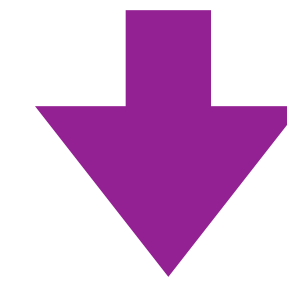
Universal Quantum Gate Set (Clifford+T gate resource theory):

$$\left(H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad \text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \right) + \left(T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \right)$$



Entanglement

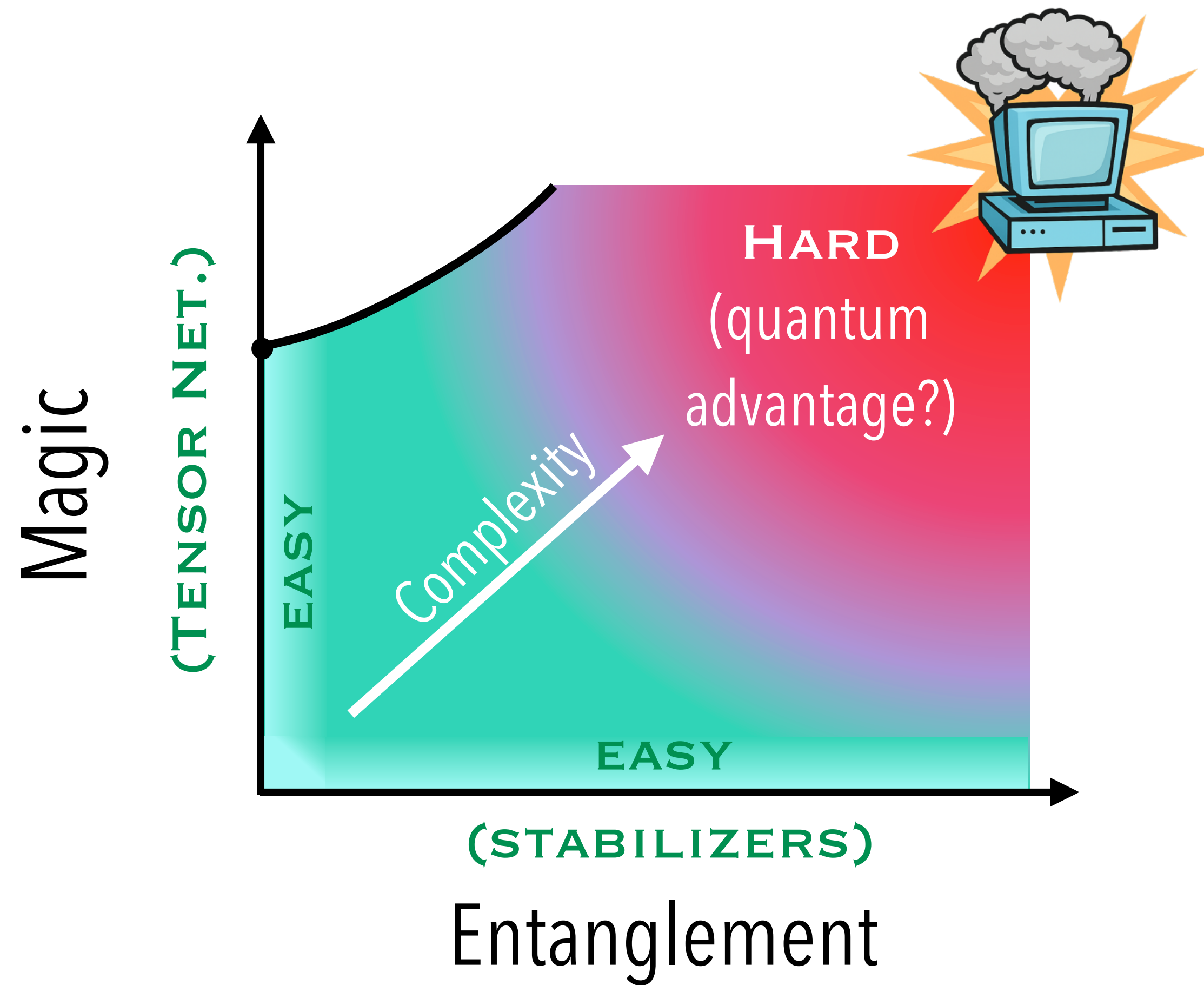
- Efficient classically — *Gottesman Knill (1998)*
- Cheap fault-tolerant implementations
- Non-universal



Non-stabilizerness (Magic)

- Classical resources scale $\sim \exp(\# \text{ T gates})$
— *Aaronson & Gottesman (2004)*
- Expensive fault-tolerant implementations
- Needed for universality & quantum advantage

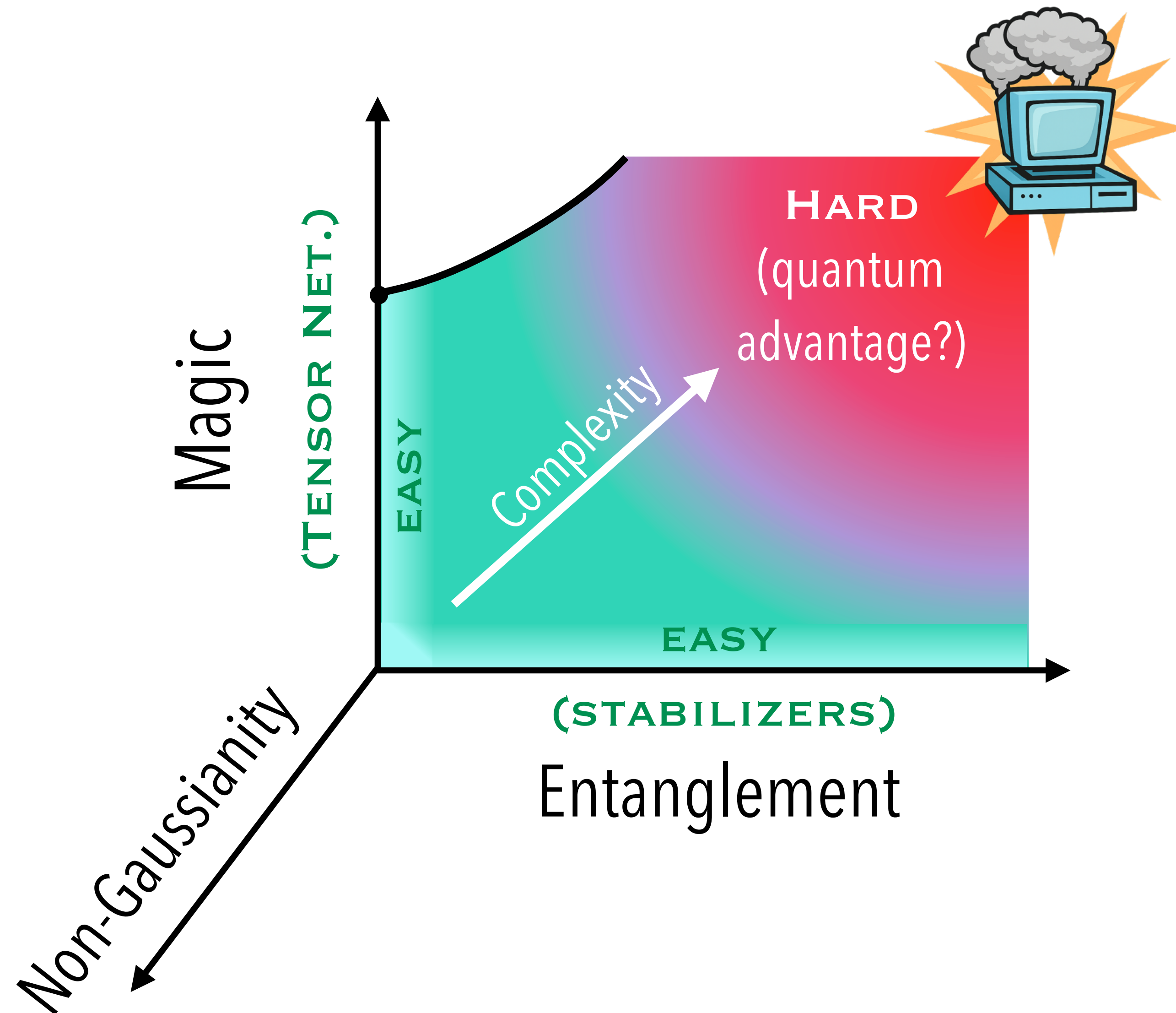
Quantum Simulations — when is a Quantum Computer required?



See works by Hamma, Leone, Oliviero, Gu, Haug, Tarabunga, Sierant, Turkeshi, Tirrito, Collura, Dalmonte, Lami, ...

Inspired by Alioscia Hamma, Qmeets seminar,
https://www.youtube.com/watch?v=UrOCCGpL_go

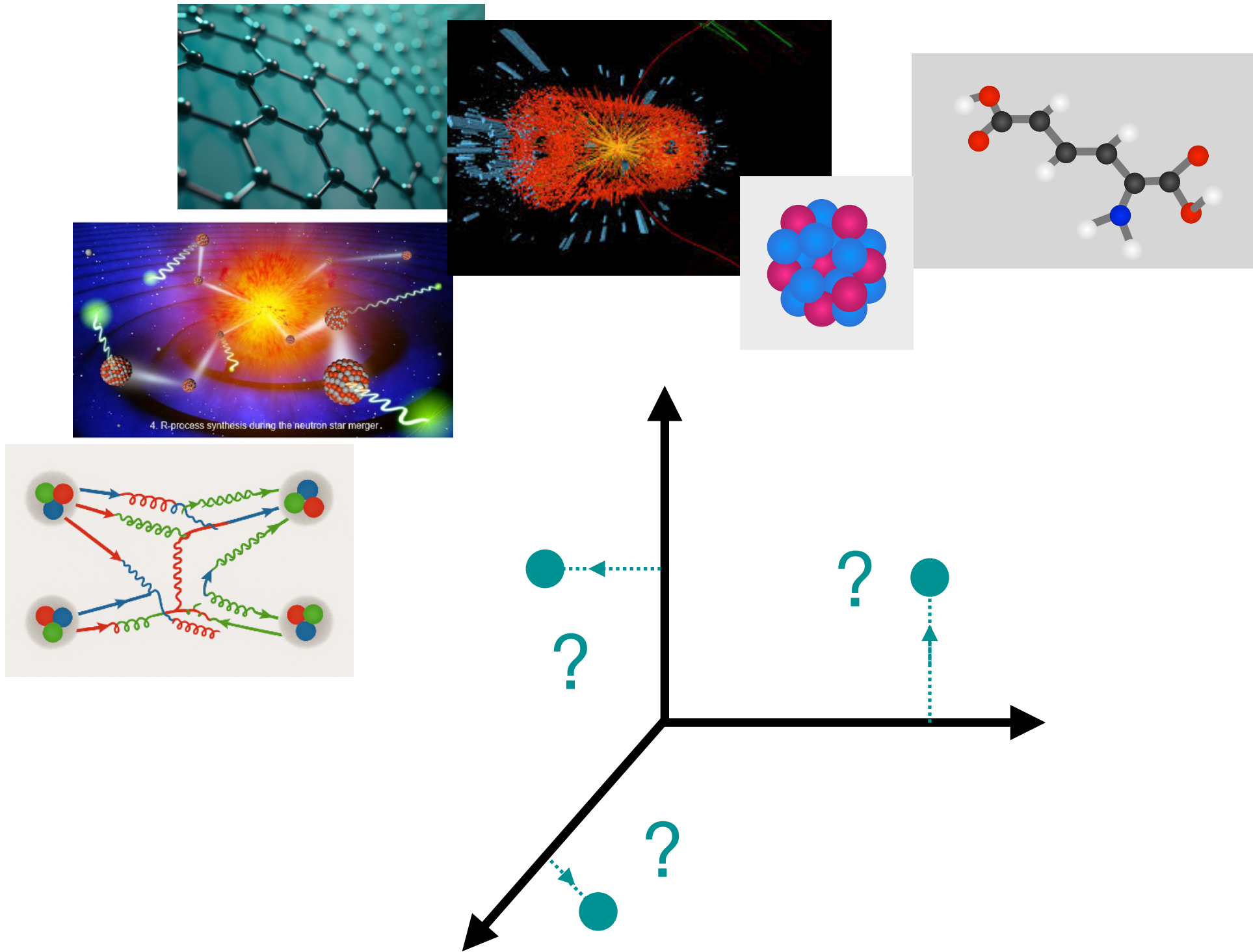
Quantum Simulations — when is a Quantum Computer required?



See works by Hamma, Leone, Oliviero, Gu, Haug, Tarabunga, Sierant, Turkeshi, Tirrito, Collura, Dalmonte, Lami, ...

Inspired by Alioscia Hamma, Qmeets seminar,
https://www.youtube.com/watch?v=UrOCCGpL_go

Motivational Questions & Goals



- Where do physical states of interest stand in the quantum complexity phase diagram?
- How is quantum complexity generated in dynamical processes?

- *Optimal paths, architectures and algorithms for targeted quantum simulations*
- *Search for quantum advantage*
- *How are many-body phenomena are rooted in quantum information*

Outline

★ Quantum complexity and the emergence of collectivity in many-body systems

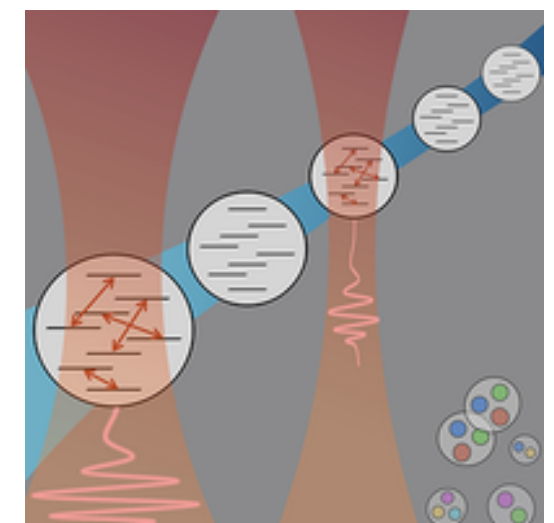
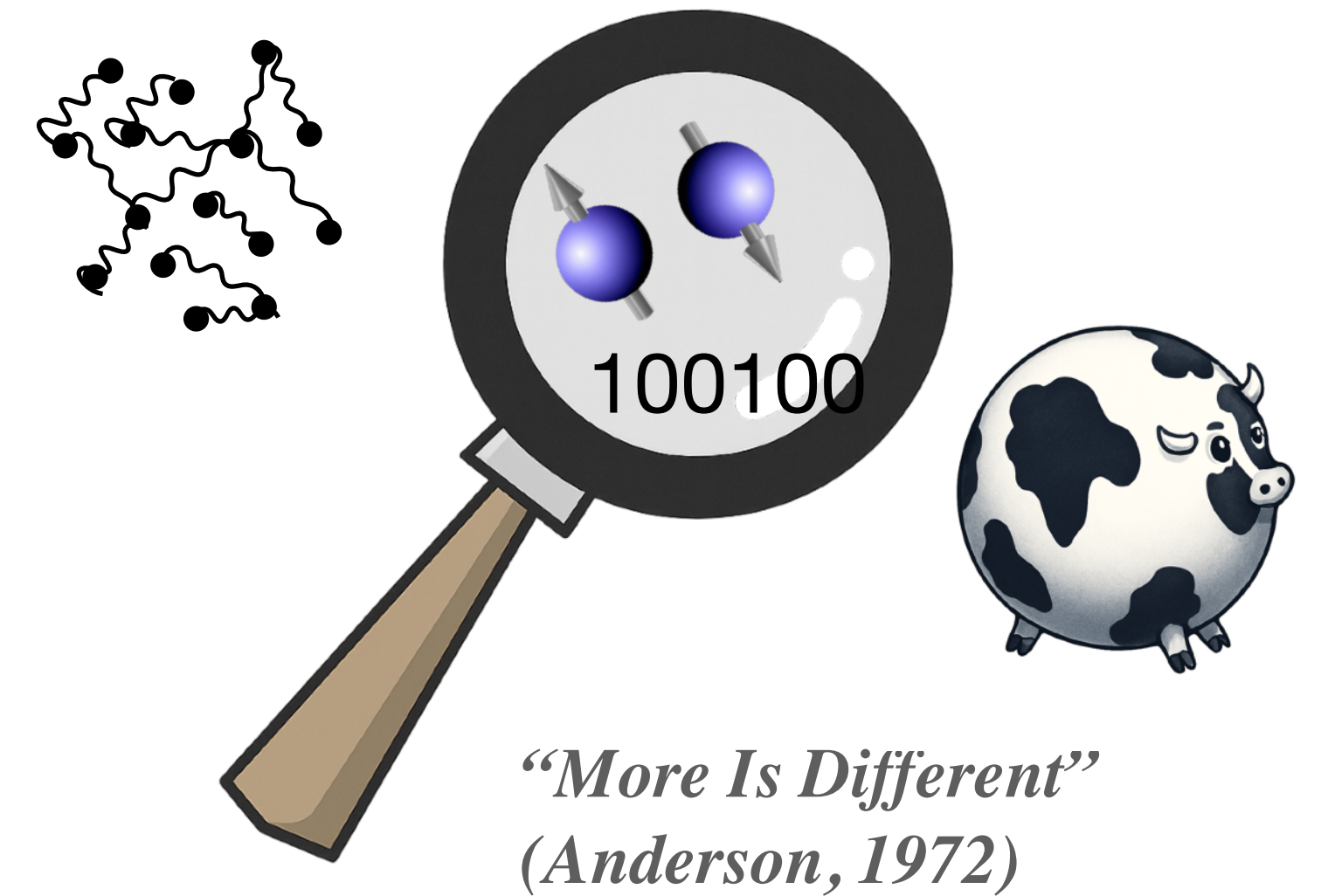
→ *Entanglement, non-stabilizerness and deformation in nuclear systems*

★ Complexity-guided ground-state finding algorithms

→ *Stabilizer-accelerated many-body ground state estimations*

→ *Information and complexity rearrangement with Hamiltonian-learning-VQE*

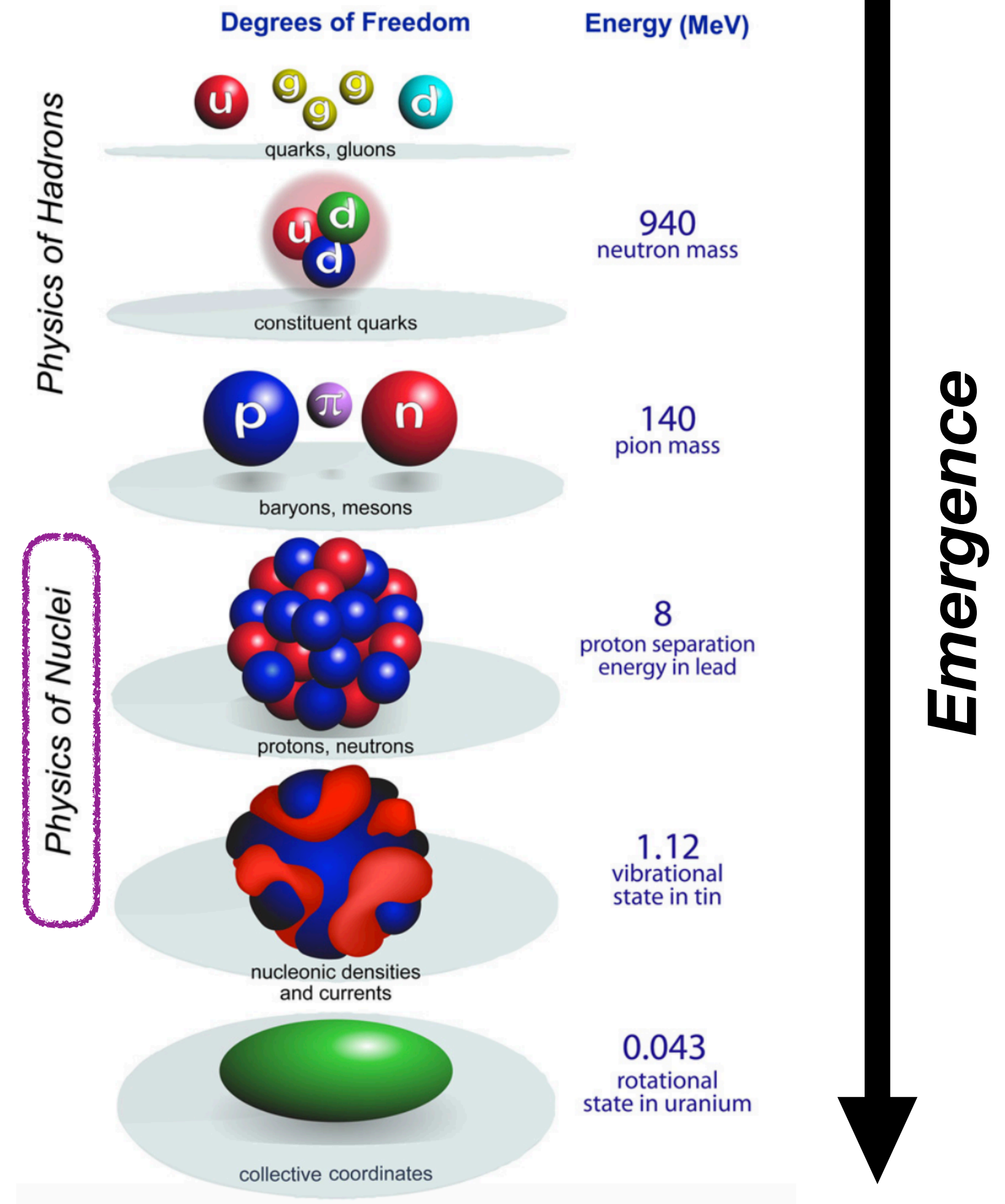
★ Many-body and gate complexity of many-body dynamics with qudits



Atomic Nuclei as unique quantum many-body systems

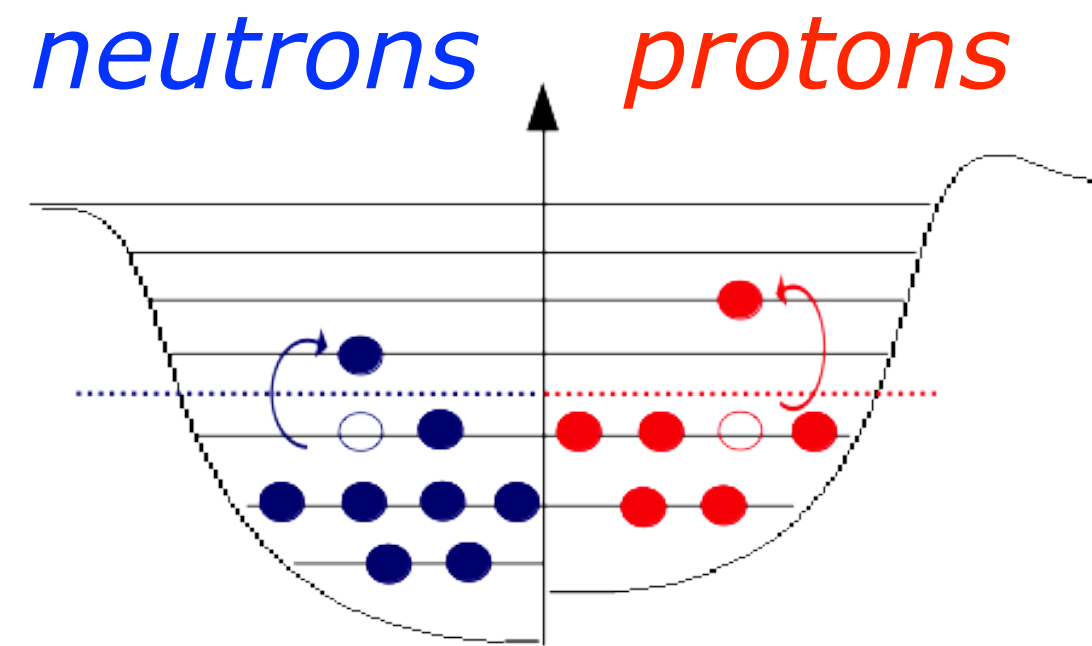
- two-specie mesoscopic self-bound systems composed of 2 to ~300 protons and neutrons
- Strong non-perturbative many-body nuclear forces
- various phenomena: single-particle shells, clustering, halos, emergent collective phenomena: superfluidity, vibrations, rotations...

Scales of nuclear physics



Adapted from Bertsch, Dean, Nazarewicz SciDAC review (2007)

Multi-Body Entanglement and Magic in Nuclei



Investigations of bipartite entanglement in pure and mixed states:

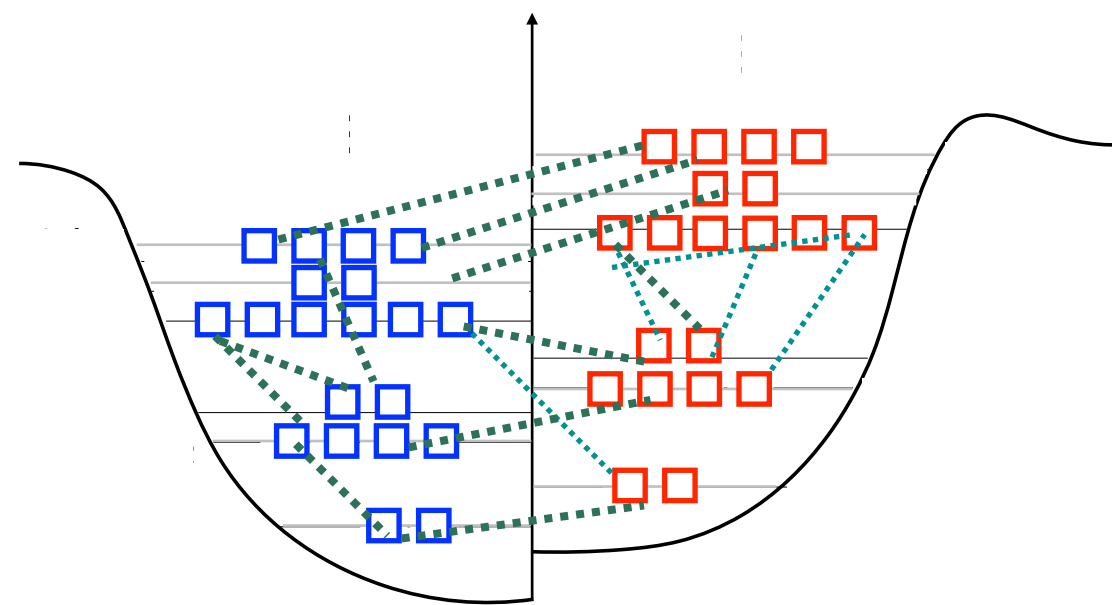
See e.g. Papenbrock & Dean (2003), Gorton & Johnson (2023), Legeza+ (2015), CR+ (2020), Tichai+ (2022), Pérez-Obiol+ (2023)

→ Proton-neutron von Neumann entropy; Two-Orbital Mutual Information, Negativity...

→ see talk by Arnau Rios

Here: Multi-body entanglement to investigate emergent collectivity:

Brökemeier, CR+, PRC 111, 034317 (2025)



Multi-Partite entanglement via n -tangles*

*Wong, Christensen, PRA 63, 044301 (2001)

$$\tau_{(i_1 \dots i_n)}^{(n)} = \left| \langle \Psi | \hat{\sigma}_y^{(i_1)} \otimes \dots \otimes \hat{\sigma}_y^{(i_n)} | \Psi^* \rangle \right|^2$$

Orbital-to-qubit JW mapping

$$a_i^\dagger \rightarrow \left(\prod_{j < i} \hat{\sigma}_z^{(j)} \right) (\hat{\sigma}_x^{(i)} - i \hat{\sigma}_y^{(i)}) / 2$$

⇒ n -tangles are related to $n/2$ -body entanglement

Multi-Body Entanglement and Magic in Nuclei

How to quantify magic?

$$\hat{\rho} = |\Psi\rangle\langle\Psi| = \frac{1}{d} \sum_P \langle\Psi|\hat{P}|\Psi\rangle \hat{P}$$

$\swarrow$ Pauli string

$d = 2^{n_{\text{qubits}}}$

Stabilizer states have:

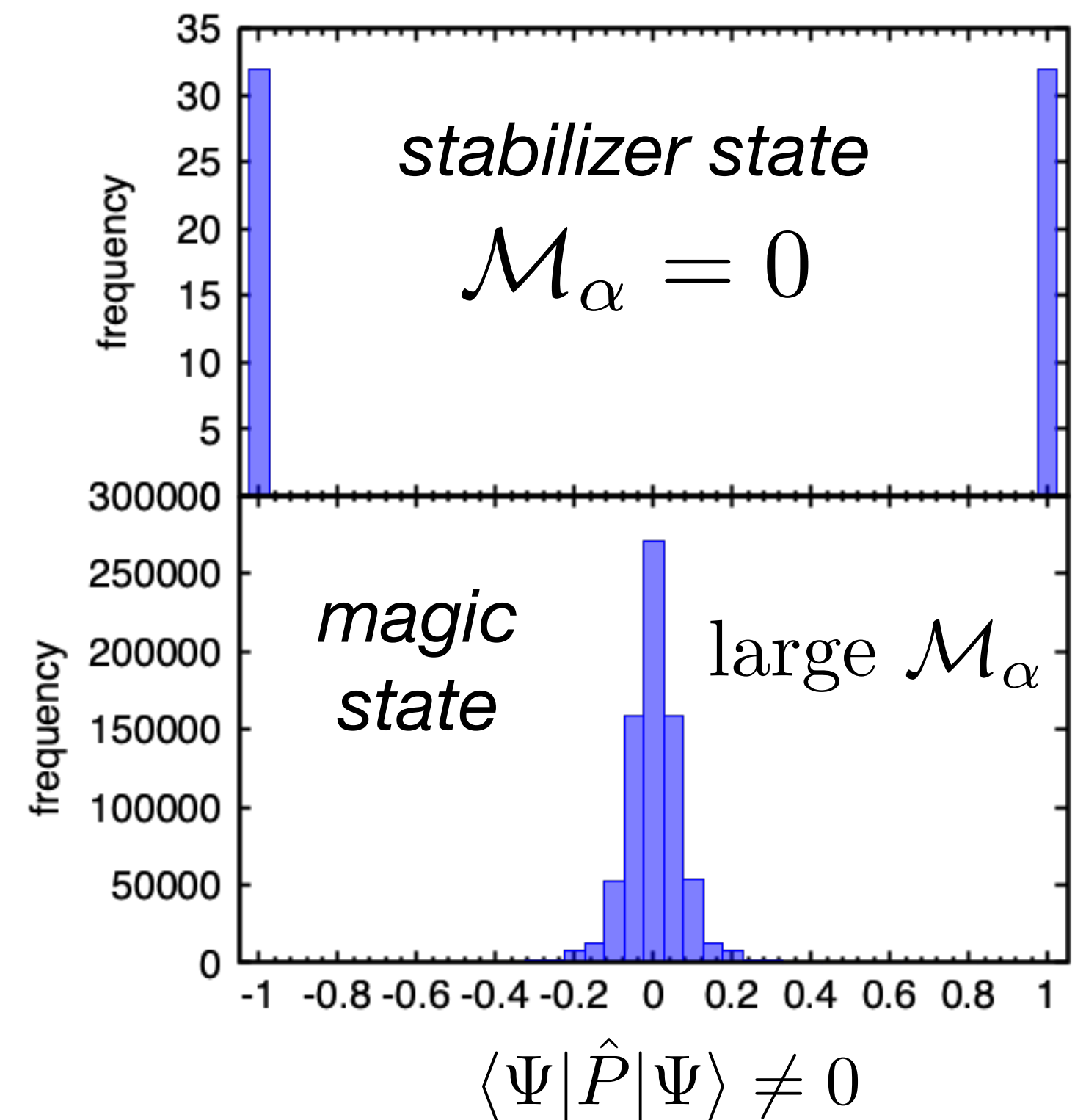
$$\begin{aligned} \langle\Psi|\hat{P}|\Psi\rangle &= \pm 1 && \text{for } d \text{ Pauli strings} \\ &= 0 && \text{for the rest} \end{aligned}$$

Stabilizer Rényi Entropy*:

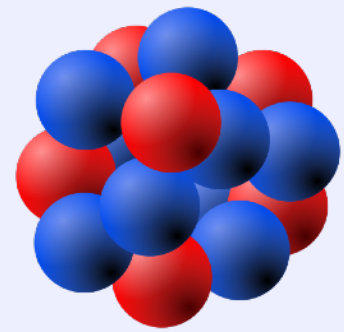
**Leone, Oliviero, Hama, PRL 128, 050402 (2022)*

$$\mathcal{M}_\alpha(|\Psi\rangle) = -\log(d) + \frac{1}{1-\alpha} \log \left(\sum_P \frac{\langle\Psi|\hat{P}|\Psi\rangle^{2\alpha}}{d^\alpha} \right)$$

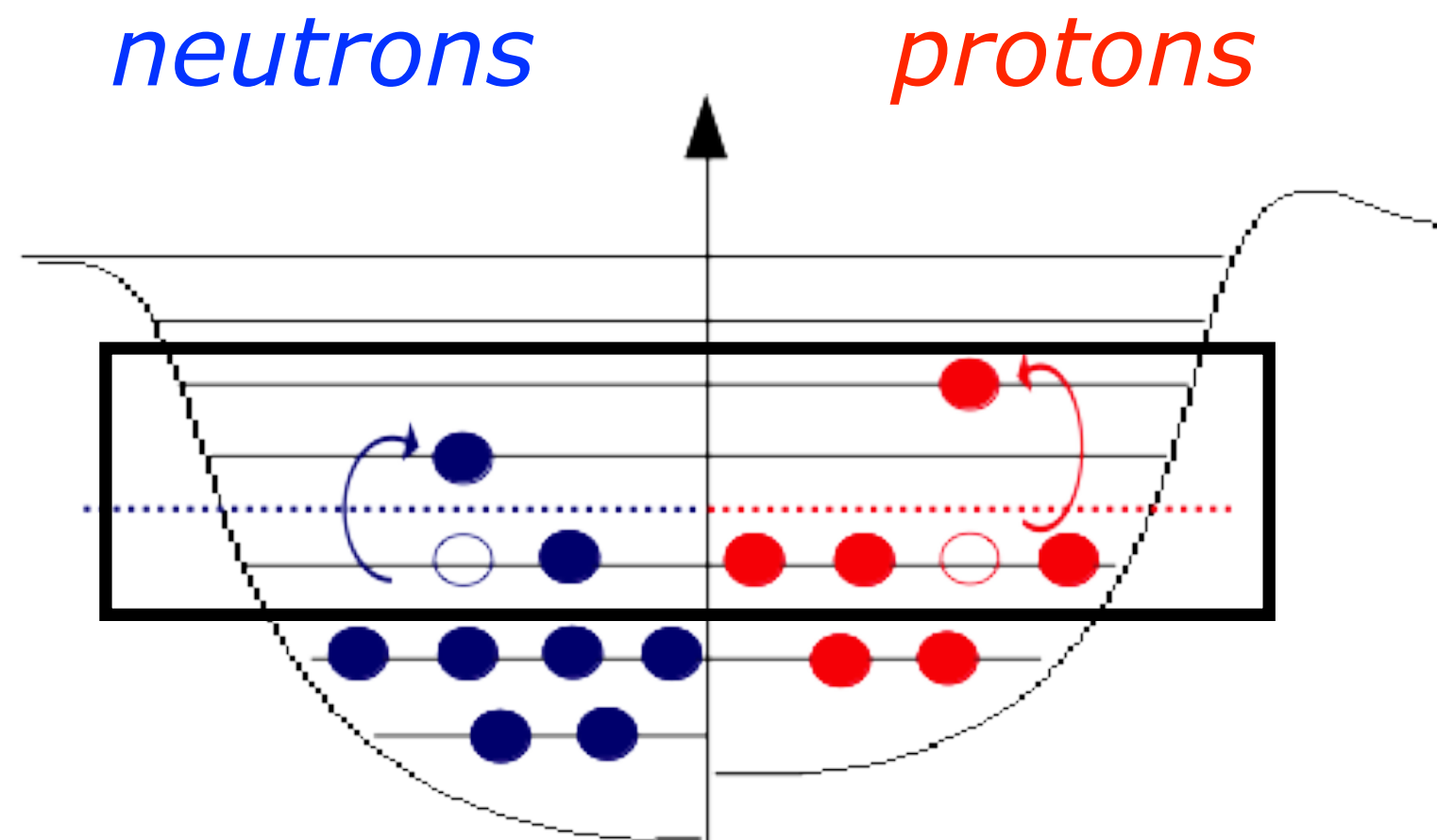
$\Rightarrow \sim$ distance to the nearest stabilizer state



Multi-Body Entanglement and Magic in Nuclei

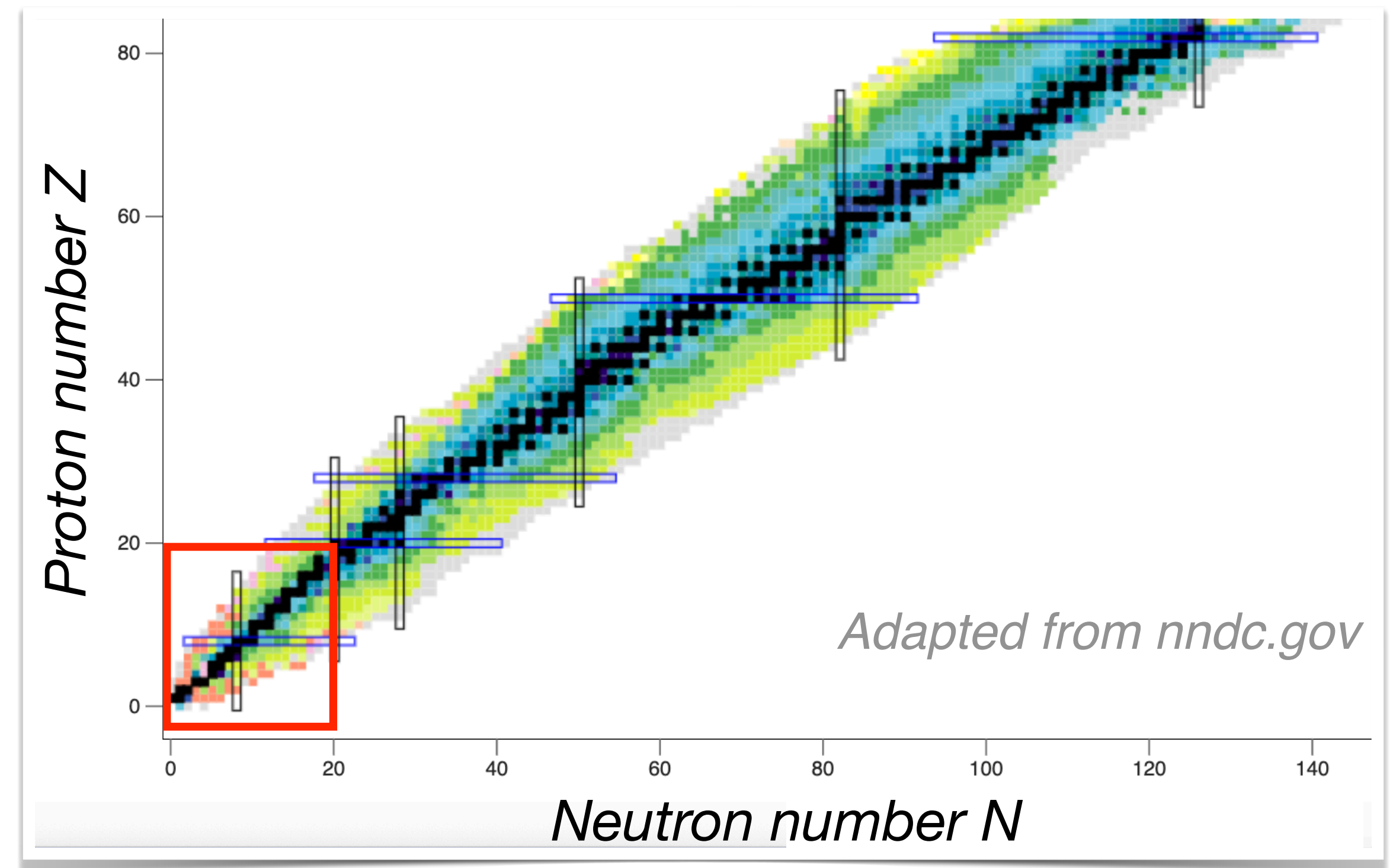


$$|\Psi\rangle = \sum_{\pi\nu} C_{\pi\nu} |\phi_{\pi}\rangle \otimes |\phi_{\nu}\rangle$$



active-space calculations

$$\hat{H} = \sum_{ij} \varepsilon_i a_i^{\dagger} a_j + \frac{1}{4} \sum_{ijkl} \bar{v}_{ijkl} a_i^{\dagger} a_j^{\dagger} a_l a_k$$

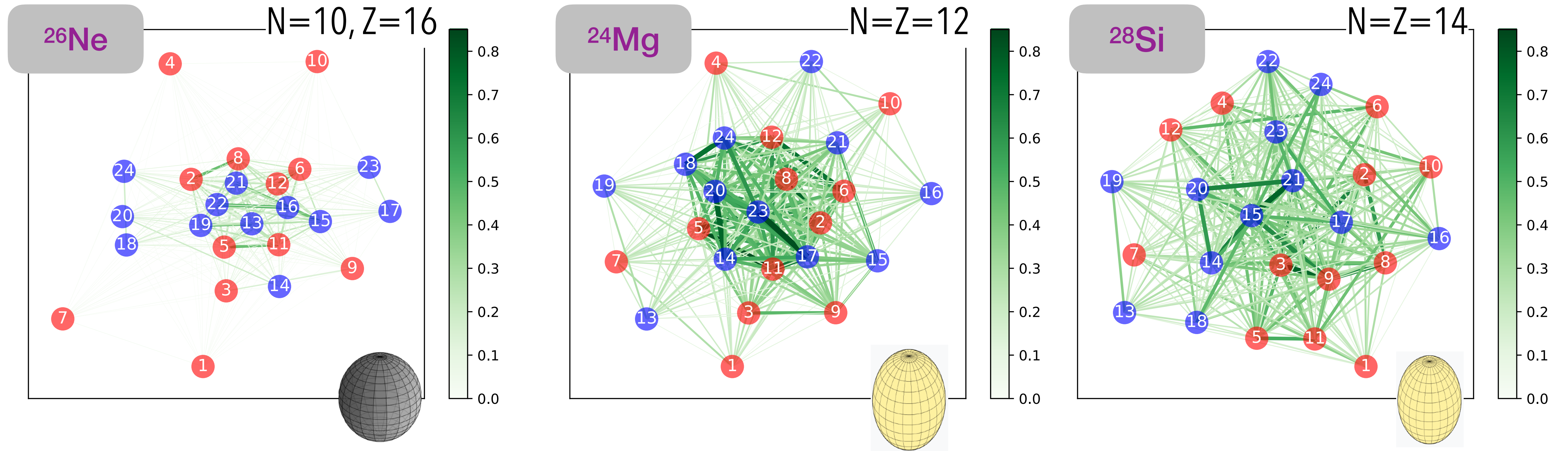


p-and sd-shell nuclei $2 < Z, N < 20$
 → mapped onto 12 and 24 qubits

Multi-Body Entanglement in sd-shell (24 qubits) Nuclei

Brökemeier, CR+, PRC 111, 034317 (2025)

Collective entanglement networks (8-tangles)



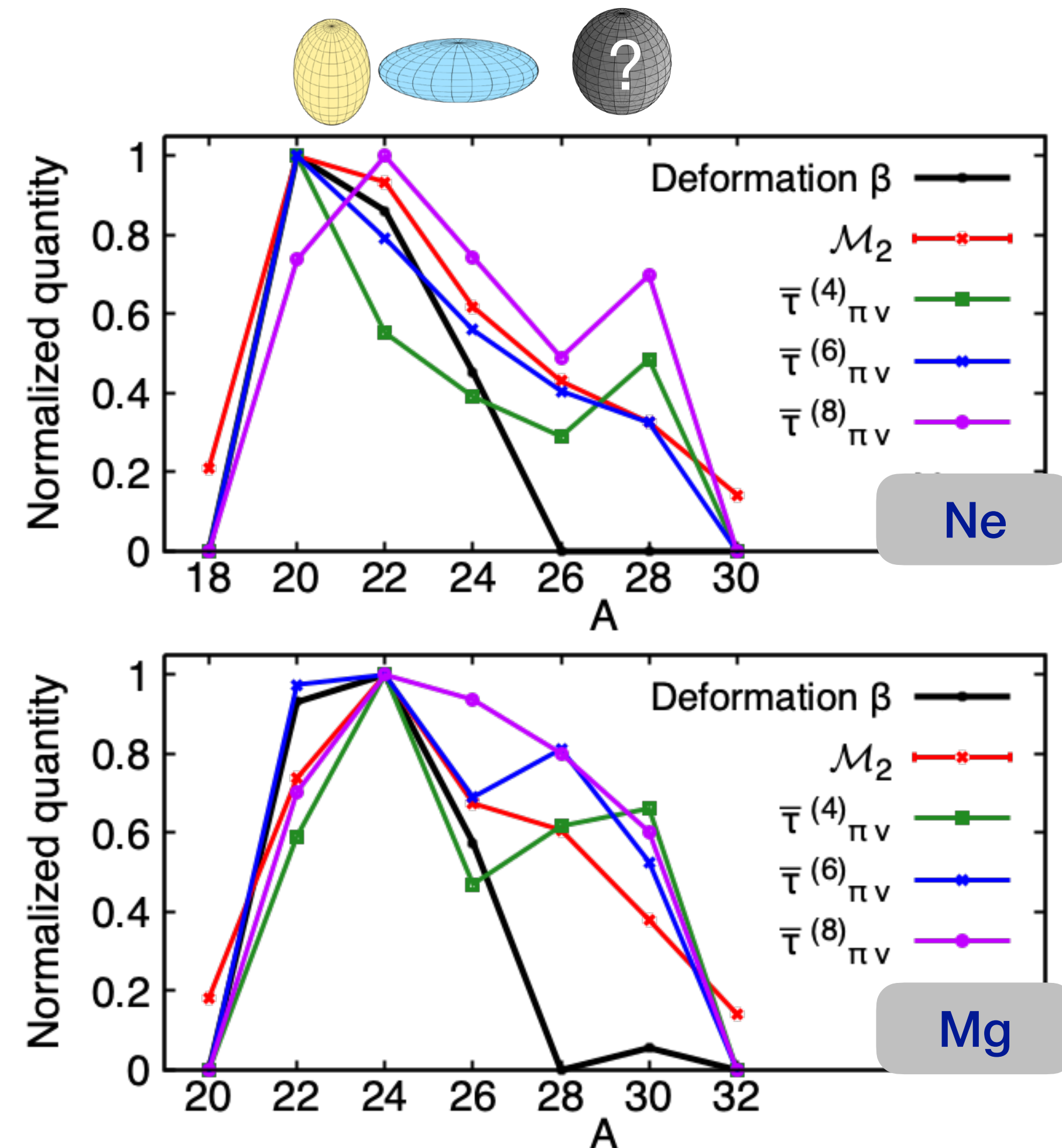
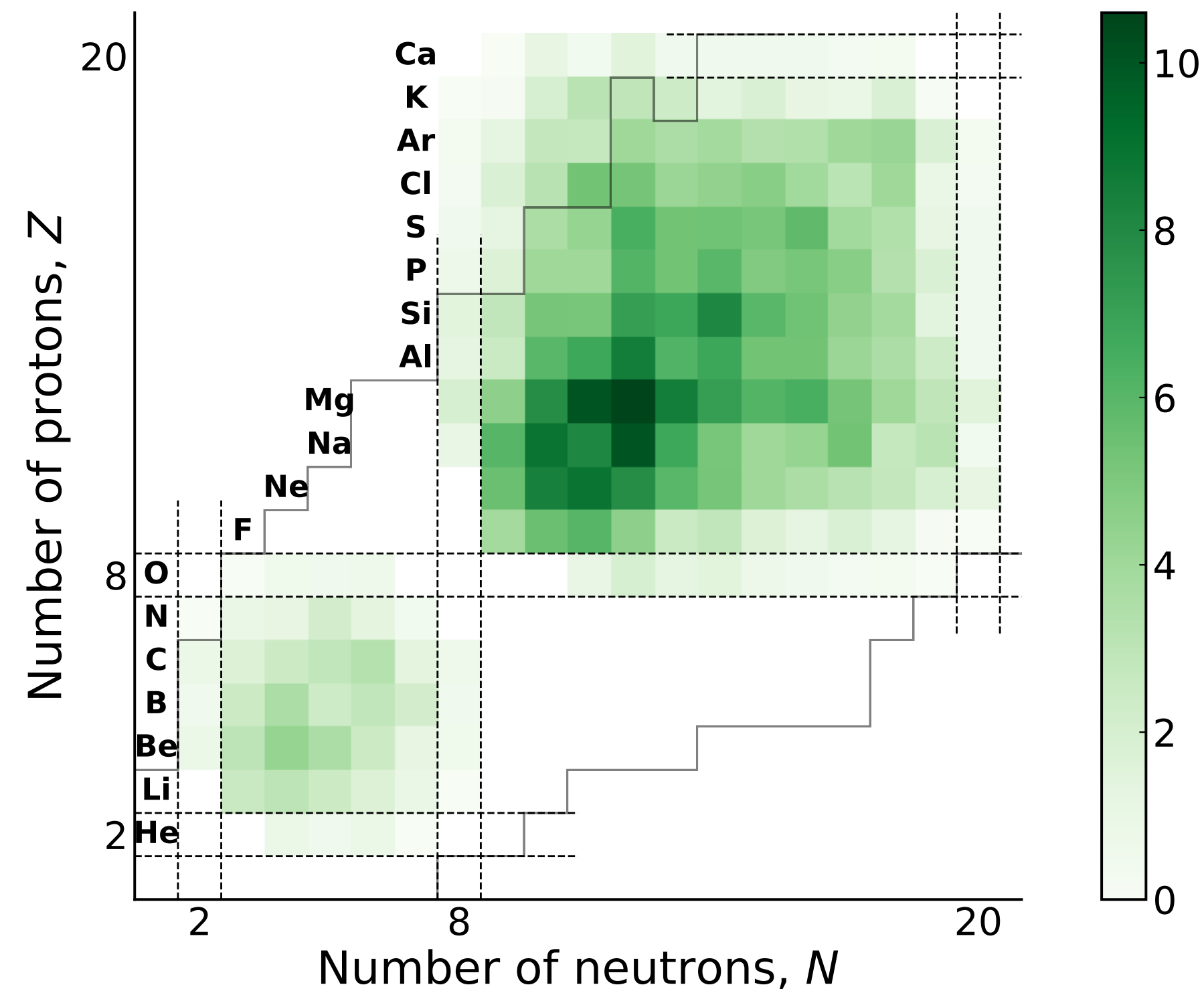
Edge values

$$e_{i_1 i_2}^{(8)} = \sum_{i_3 < i_4 < i_5 < i_6 < i_7 < i_8} \tau_{(i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8)}^{(8)}$$

Signature of Shape Collectivity in Magic and Multi-pn Entanglement

Stabilizer 2-Rényi Entropy $\mathcal{M}_2$

Exact and MCMC computations



- Maximal magic and proton-neutron tangles coincide with maximal deformation in nuclei
- Magic and tangles also persist in the region where axial deformation vanishes (shape co-existence, other correlations?)

Singling out Deformation: the Lipkin-Meshkov-Glick model

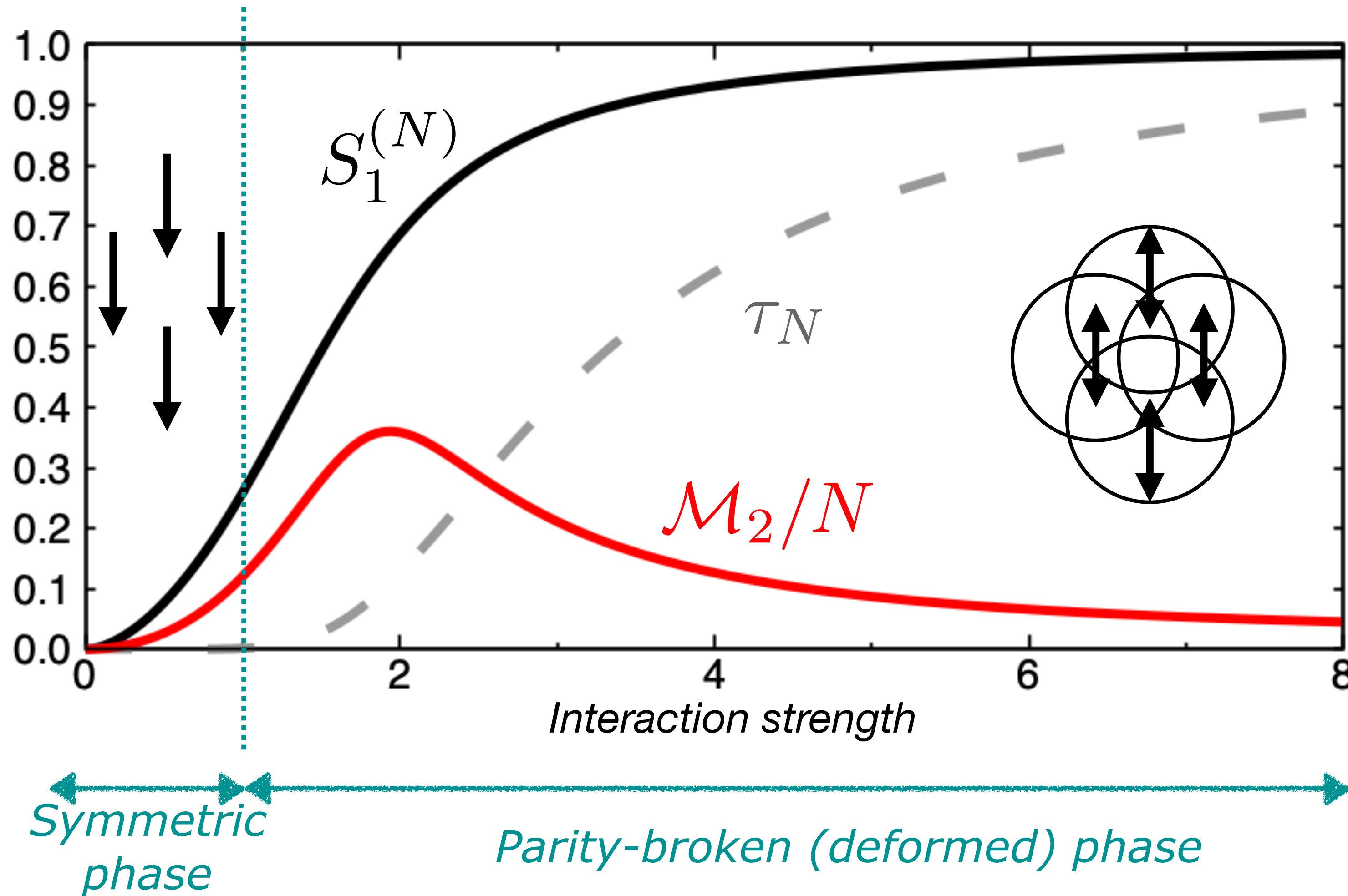


$$\hat{H} = \varepsilon \hat{J}_z - V_x (\hat{J}_x^2 + \chi \hat{J}_y^2)$$

$$\hat{J}_\alpha = \frac{1}{2} \sum_{i=1}^N \hat{\sigma}_\alpha^{(i)}, \quad \alpha = x, y, z \quad \chi \in [-1, 0)$$

- * Phase Transition: doubly degenerate ground state of mixed parity ($N \rightarrow \infty$)
- * Relevance for nuclear physics, condensed matter, trapped-ion quantum computing, quantum sensing/metrology...

Singling out Deformation: the Lipkin-Meshkov-Glick model



$S_1^{(N)}$ = One-spin entanglement entropy

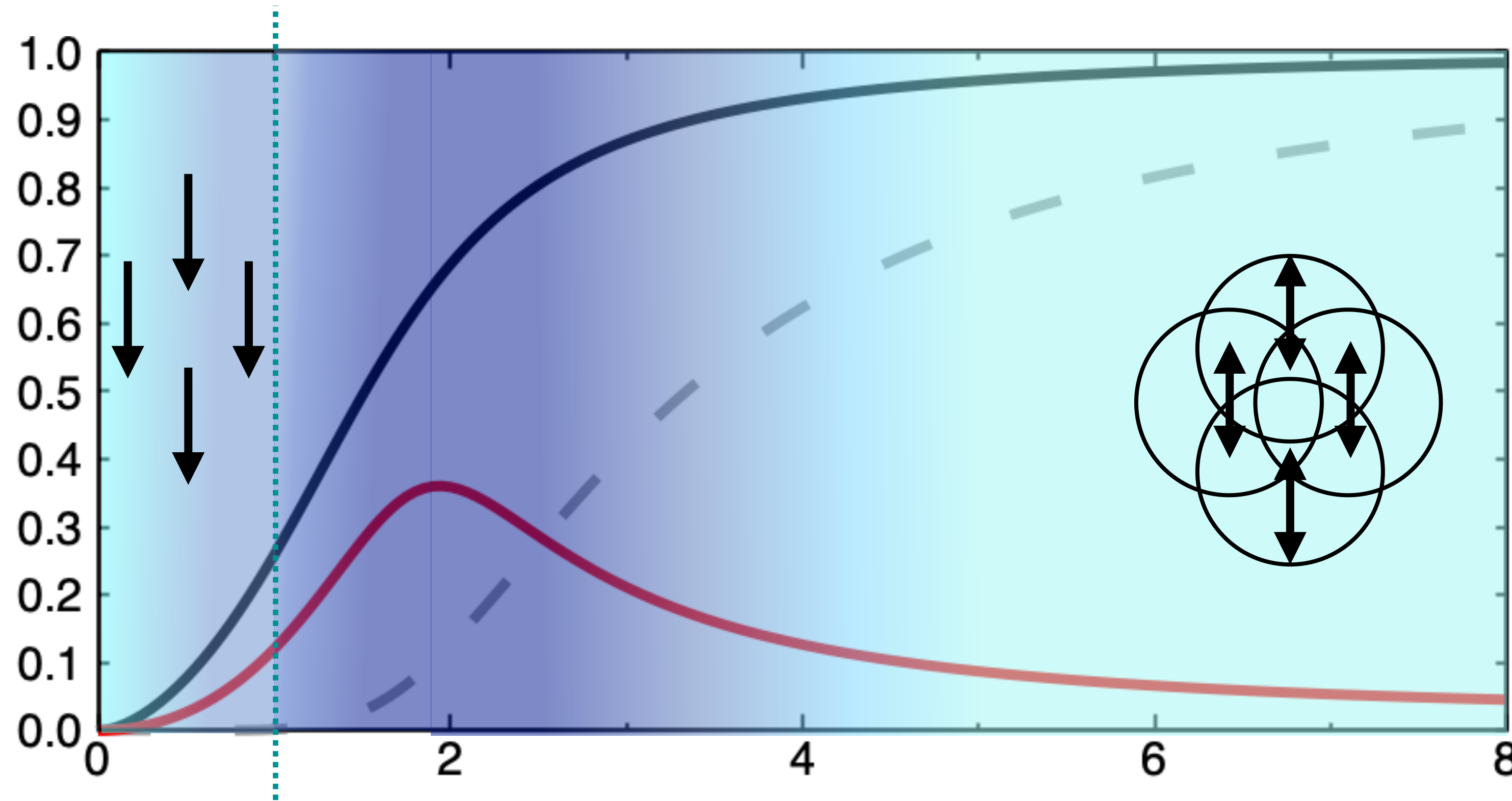
τ_N = N-tangle

$\mathcal{M}_2$ = Stabilizer 2-Rényi entropy

N=8

Latorre+ (2005); Hengstenberg, CR+ (2023),
Passarelli+ (2024), CR PRA 112, 052408 (2025)..
..

Singling out Deformation: the Lipkin-Meshkov-Glick model



Simple (trivial)

Complex

Simple (entangled)



Can stabilizer states
capture deformation
in an efficient &
symmetry-preserving way?

CR PRA 112, 052408 (2025)

Outline

★ Quantum complexity and the emergence of collectivity in many-body systems

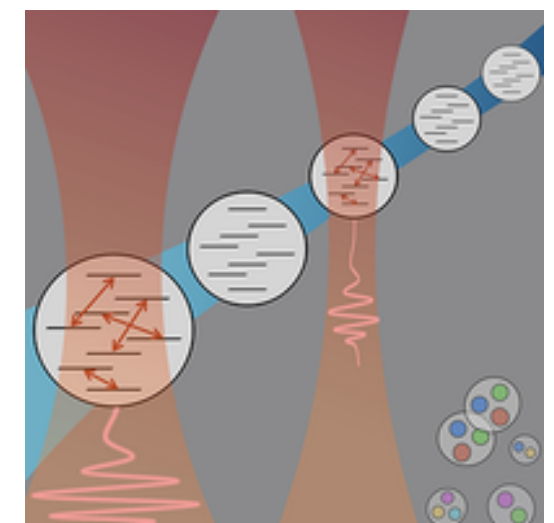
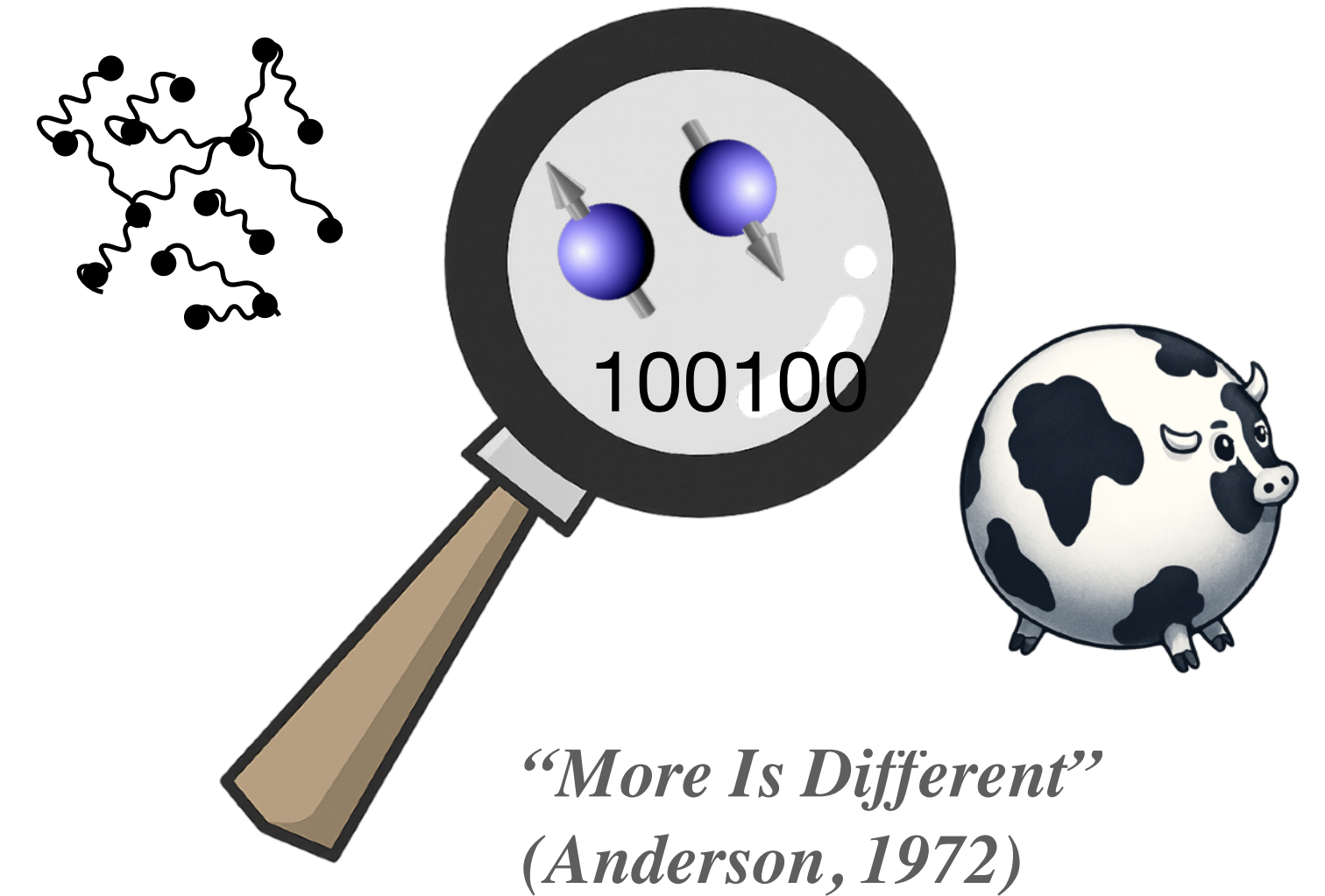
→ *Entanglement, non-stabilizerness and deformation in nuclear systems*

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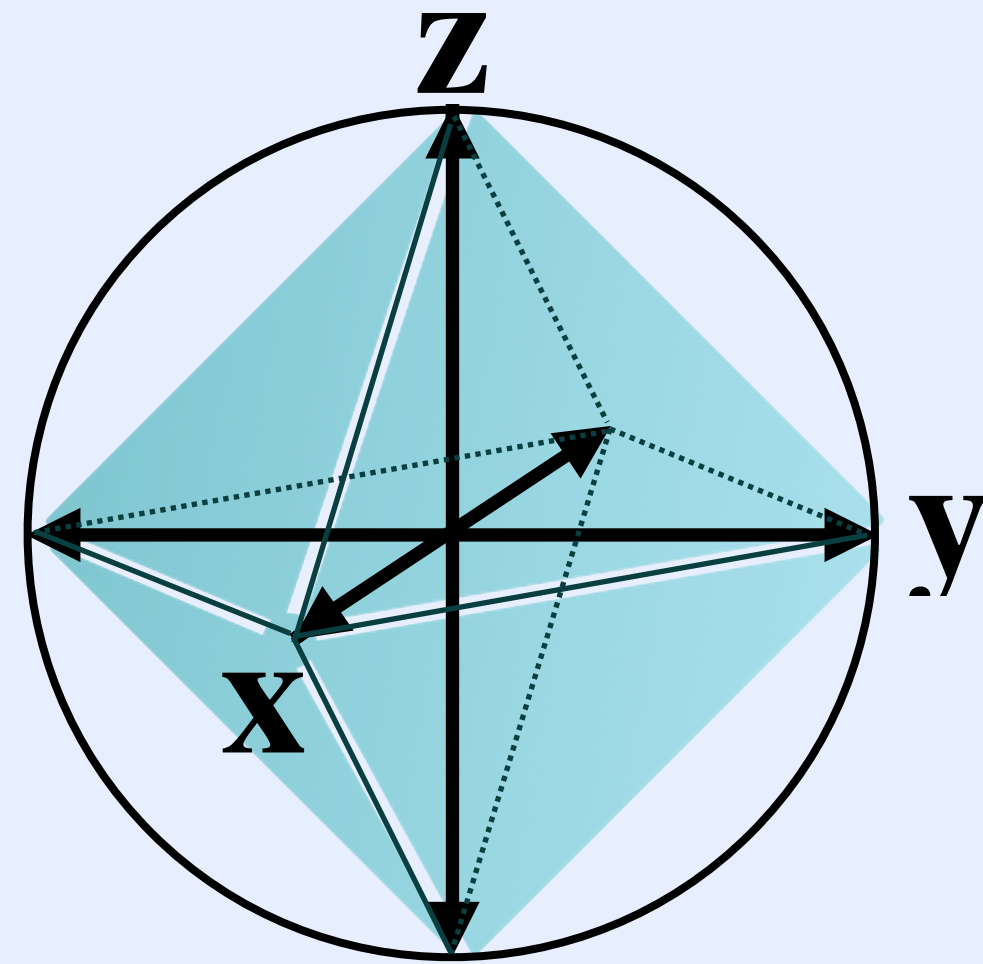
★ Many-body and gate complexity of many-body dynamics with qudits



Stabilizer Ground States

- one qubit:

6 stabilizer states



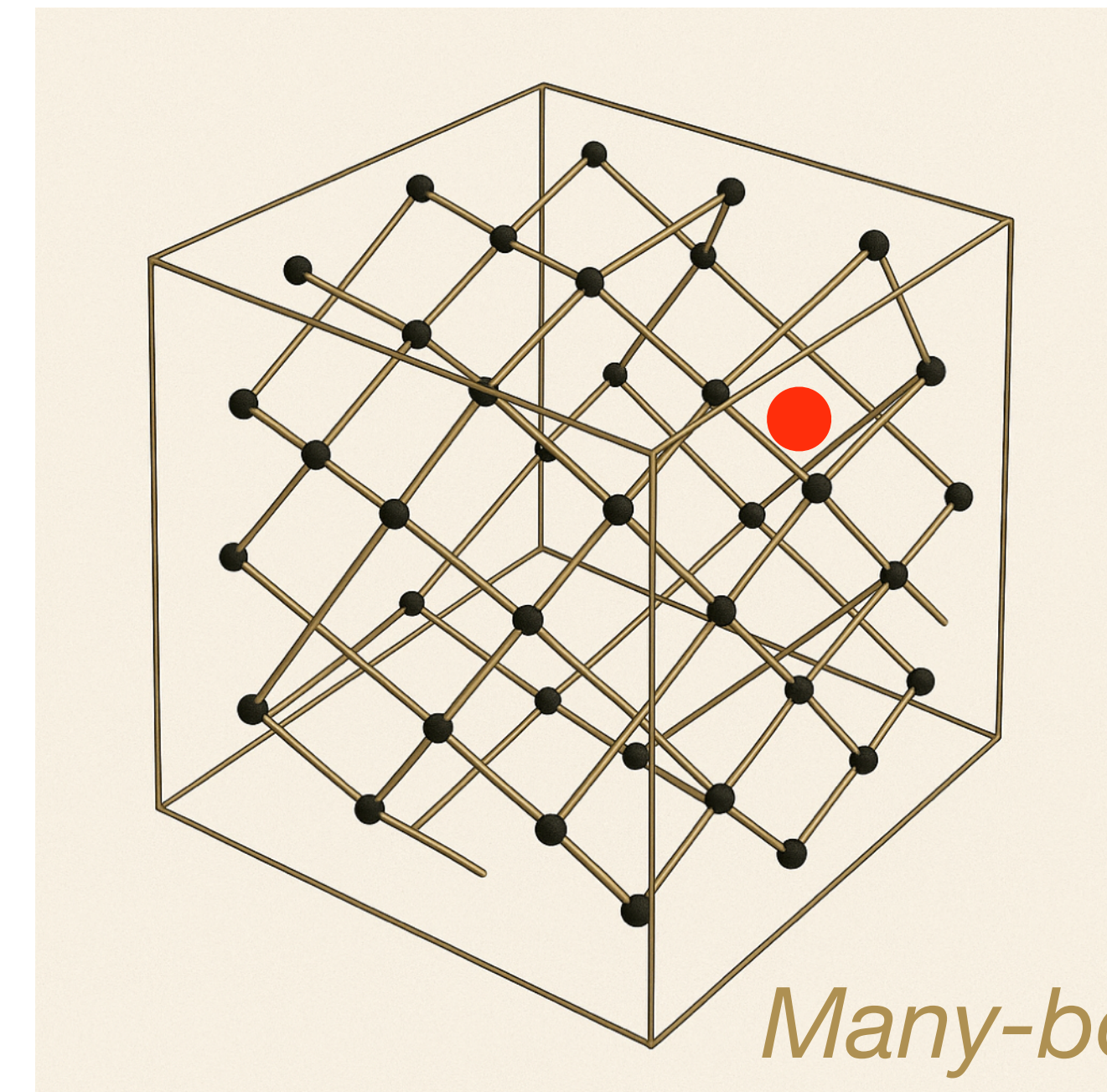
- two qubits: 60 stabilizer states
incl. 24 entangled states

e.g. $\frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$, $\frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$

- Three qubits: 1080, Four qubits 36720...

- n qubits: $N_{stab} = 2^{(1/2+o(1))n^2}$

Aaronson Gottesman 2004



*Many-body Hilbert
space*

→ How to determine the
closest stabilizer state to the
physical ground state?

Stabilizer Ground-State Method

👉 *Strategy:*

*CR PRA 112, 052408
(2025)*

$$\hat{H} = \underbrace{\sum_{P \in \mathcal{S}} a_P \hat{P}}_{\hat{H}_{stab}} + \underbrace{\sum_{P \in \mathcal{G}_N(H) \setminus \mathcal{S}} a_P \hat{P}}_{\hat{W}},$$

Stabilizer part

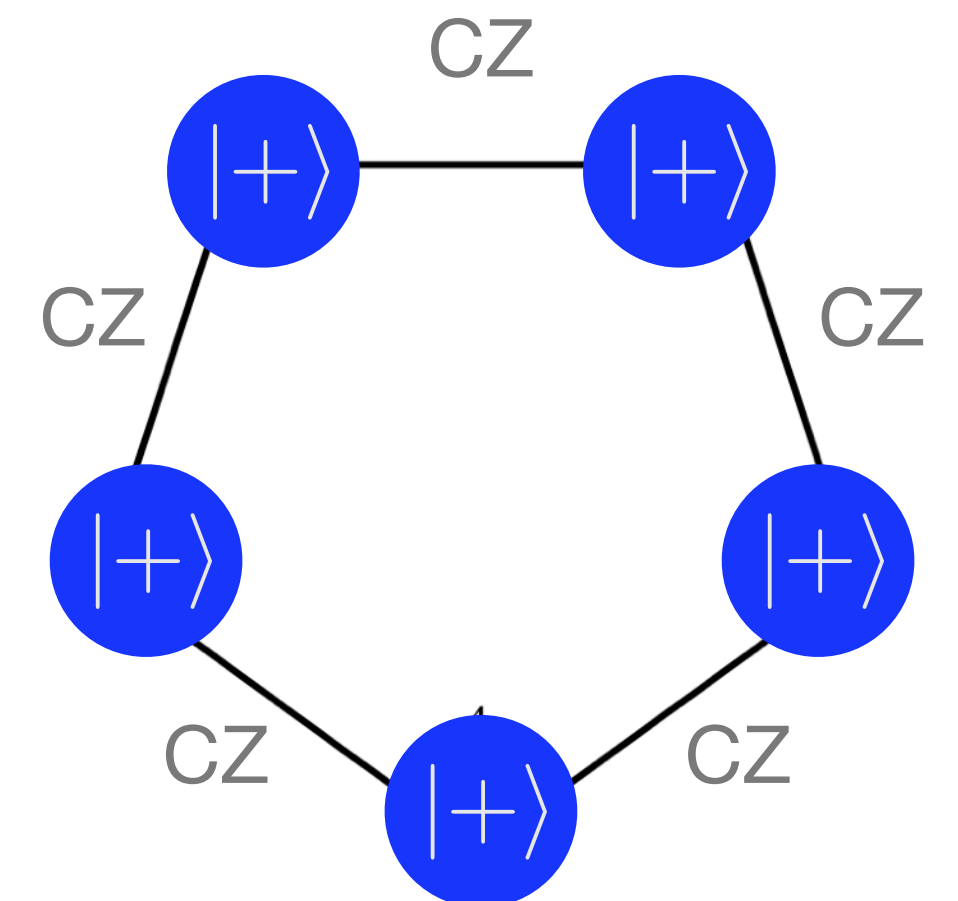
Magic-inducing part

*See also Sun+ (2024), Gu+
(2024), Weaving+ (2022)...*

$\mathcal{S} = \langle g_1, g_2, \dots, g_N \rangle$
Stabilizer group

- Explore all partitionings, and pick the energy-minimizing solution
- The stabilizer ground state can be prepared using graph states

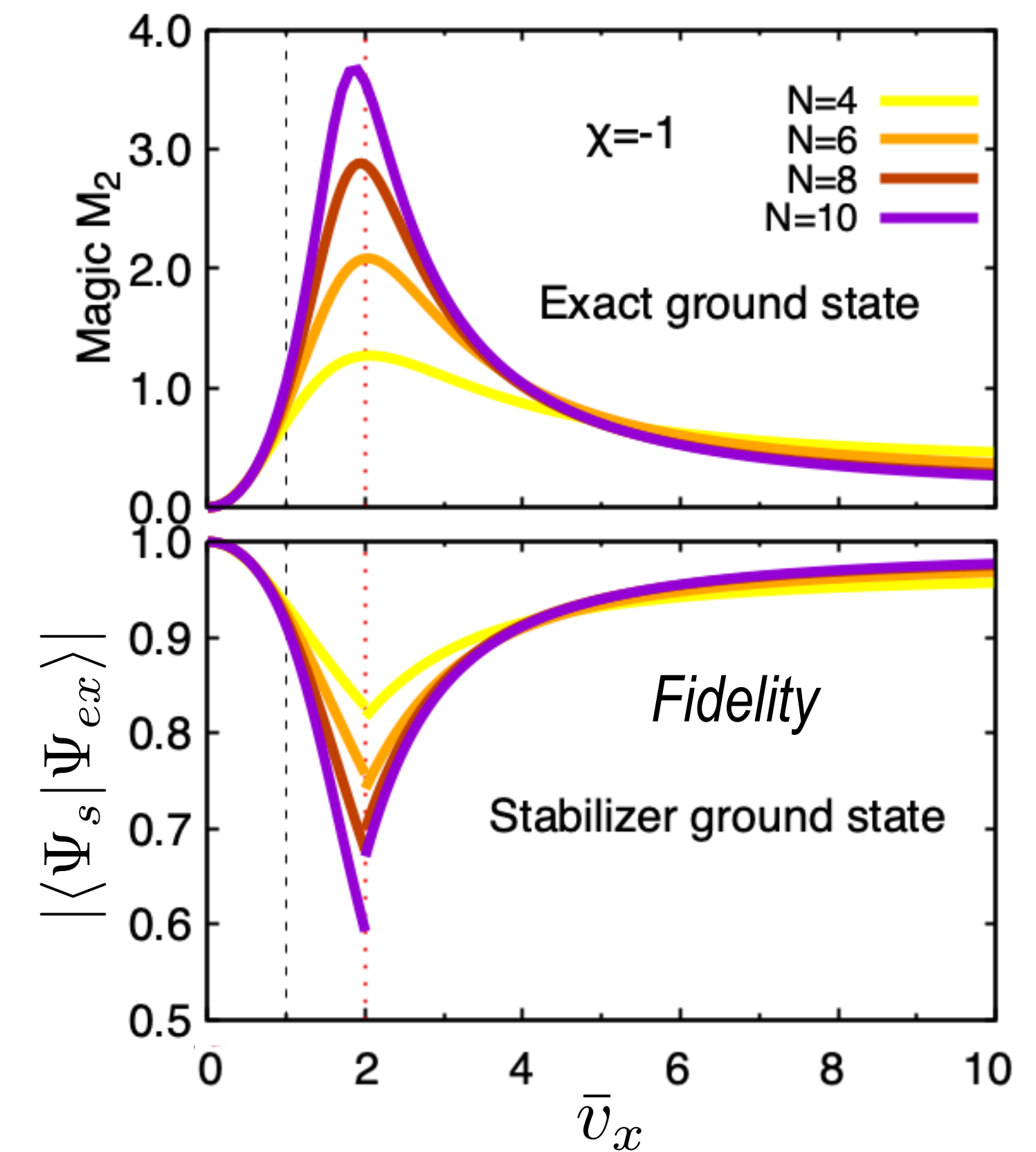
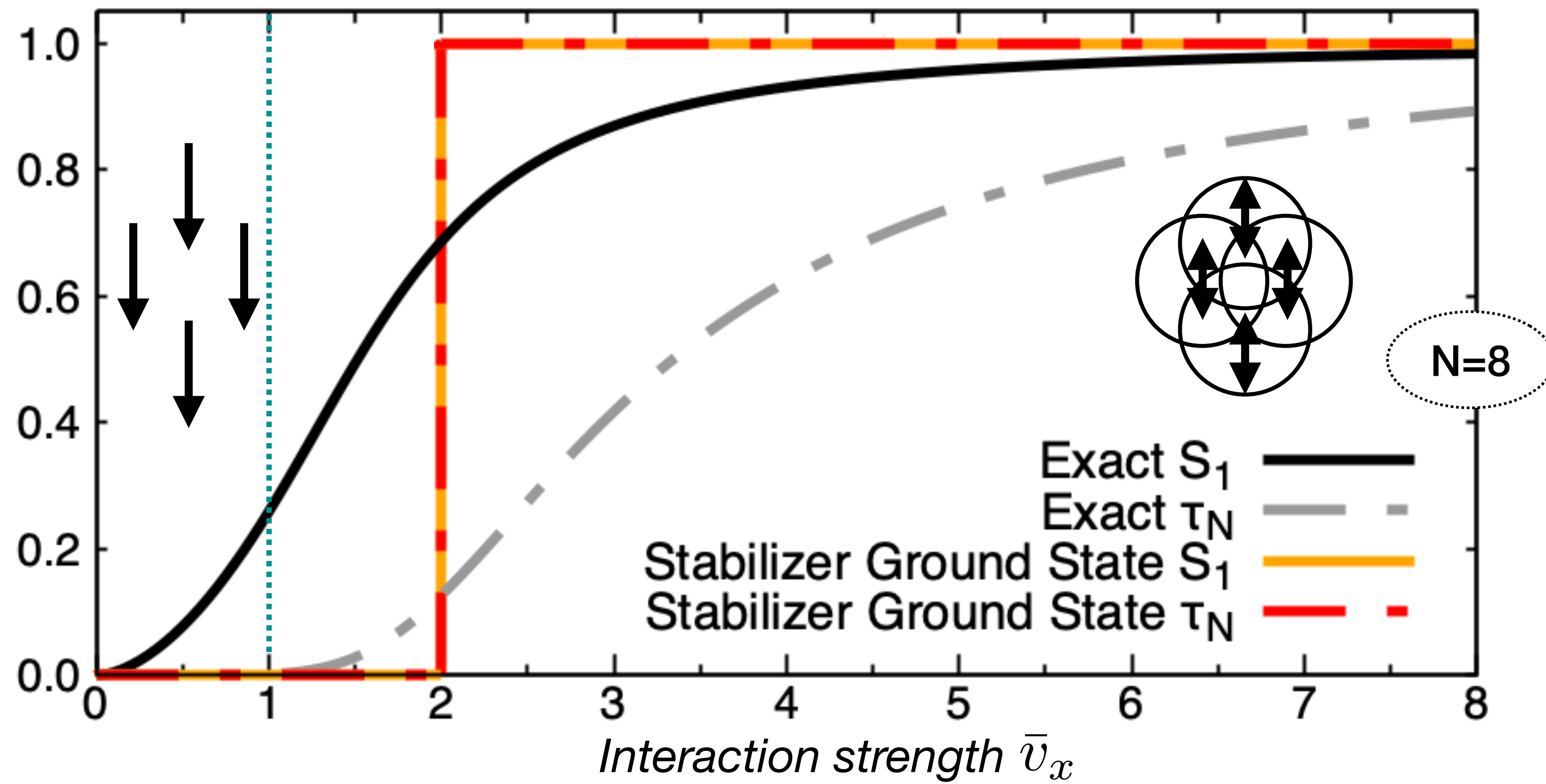
$$|\Psi_s\rangle = \prod_{i=1}^N C_i |G\rangle \quad \text{where} \quad |G\rangle = \left(\prod_{e \in E} CZ_e \right) |+\rangle^{\otimes N}$$



The 1-qubit Clifford C_i 's can be determined efficiently, using stabilizer tableaus

see e.g. Van den Nest+ PRA 69, 022316 (2004).

Stabilizer Ground State in the LMG model

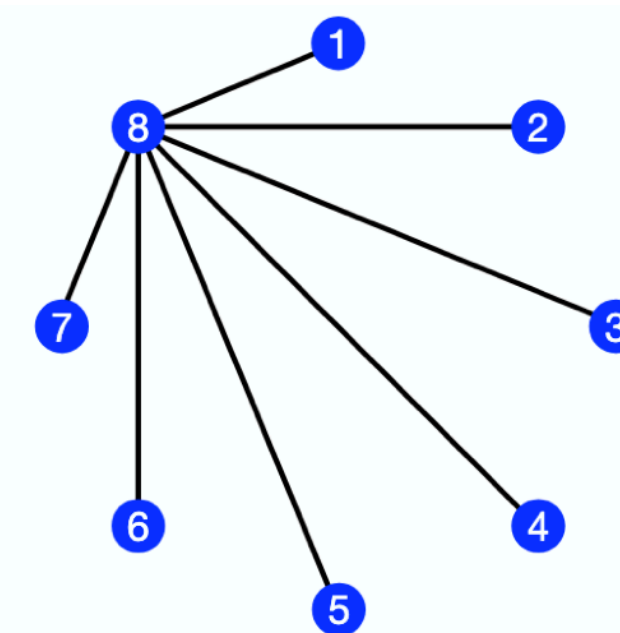


$$|\Psi_s\rangle = |\downarrow\downarrow \dots \downarrow\rangle \quad |\Psi_s\rangle = (X_1)^{N \bmod 2} H_N |G\rangle$$

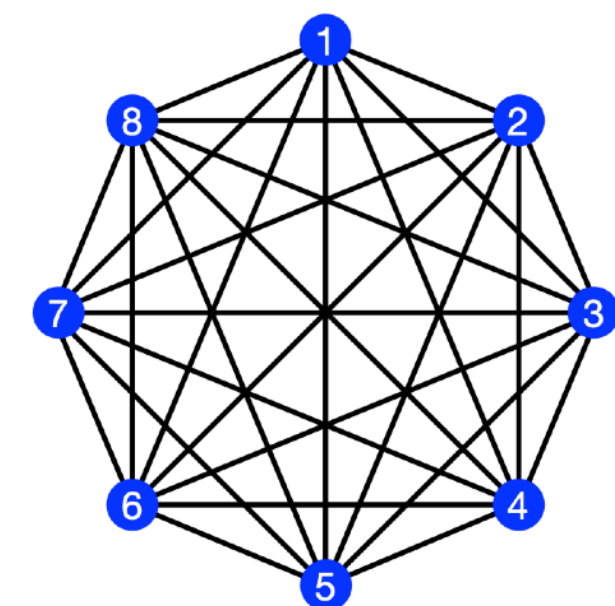
Beyond stabilizer ground state?
(Q)ITE, ADAPT-VQE...

CR PRA 112, 052408 (2025)

$$|G\rangle =$$



LC

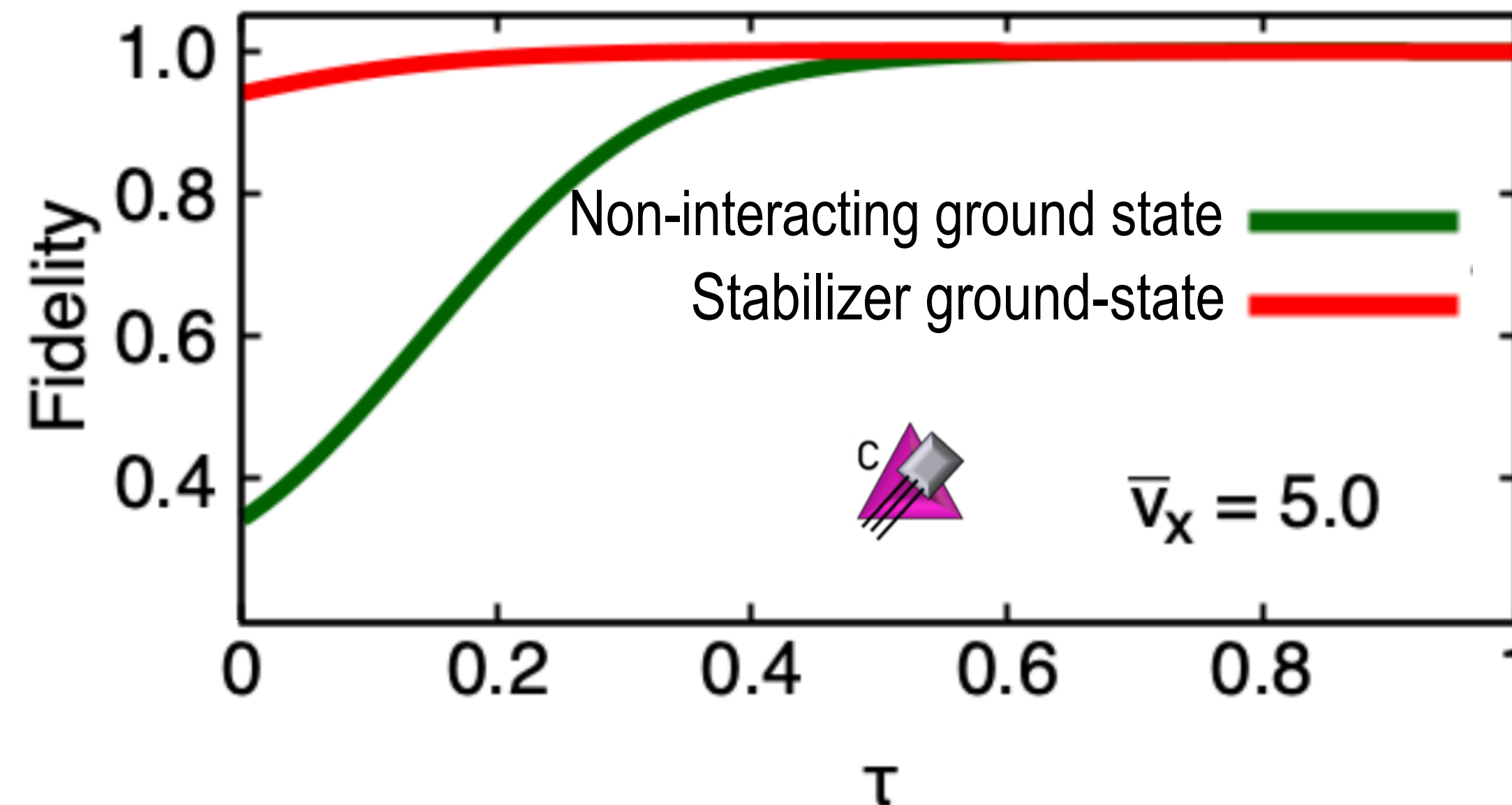


Beyond Stabilizer Ground State with Magic Injection

★ Example: (Quantum) Imaginary Time Evolutions

$$e^{-(\hat{H}-\bar{E}_0)\tau} |\eta\rangle = \sum_n e^{-(E_n-\bar{E}_0)\tau} \langle \Phi_n | \eta \rangle |\Phi_n\rangle \xrightarrow{\tau \rightarrow \infty} \propto |\Phi_0\rangle$$

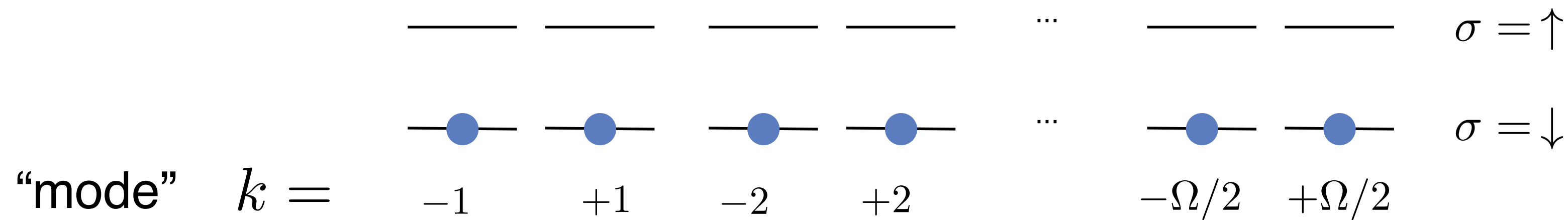
Convergence with different initial states:



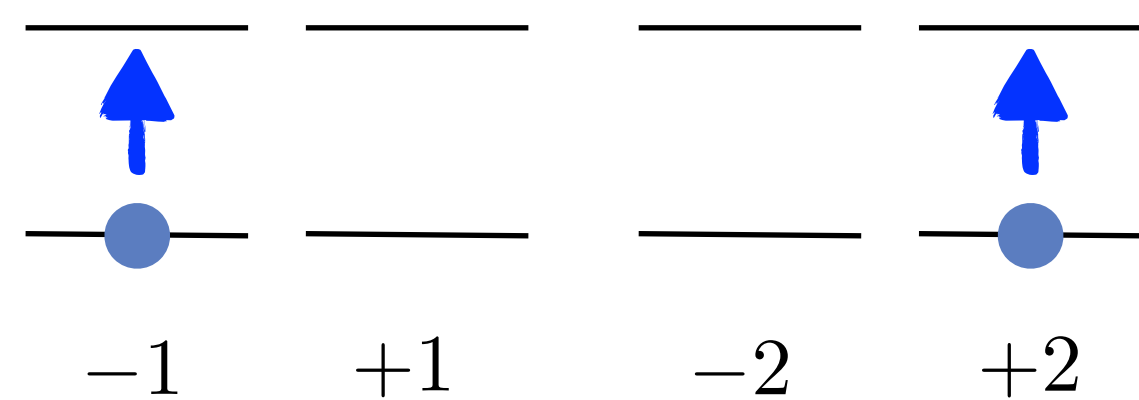
Many-Body Complexity, Deformation and Superfluid pairing

★The Agassi model

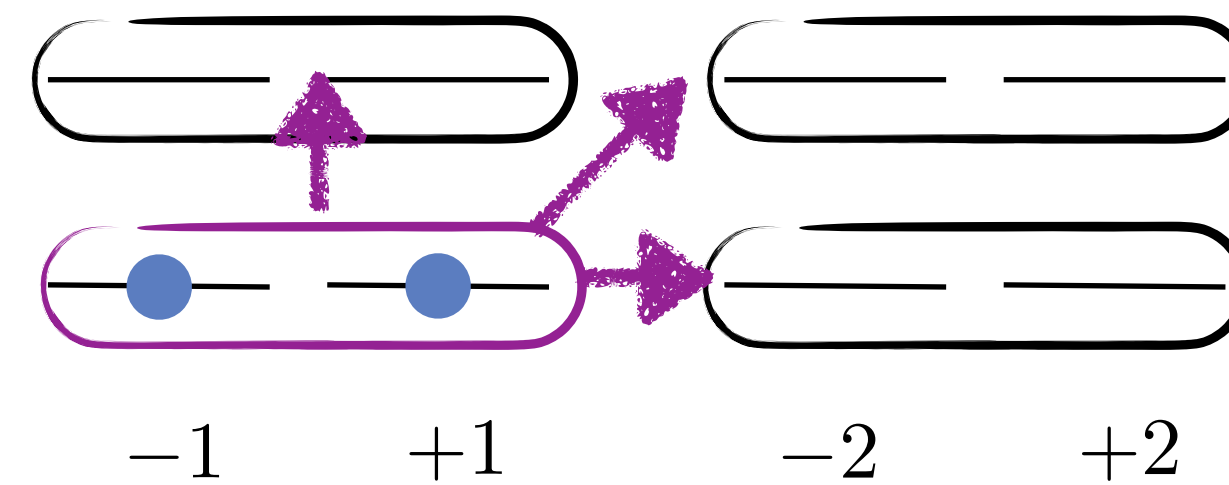
= extension of the LMG model with superfluid pairing



$$\hat{H} = \varepsilon \hat{J}_z - \frac{V}{2} \left(\hat{J}_+^2 + \hat{J}_-^2 \right) - g \sum_{\sigma\sigma'} \hat{B}_\sigma^\dagger \hat{B}_{\sigma'}$$

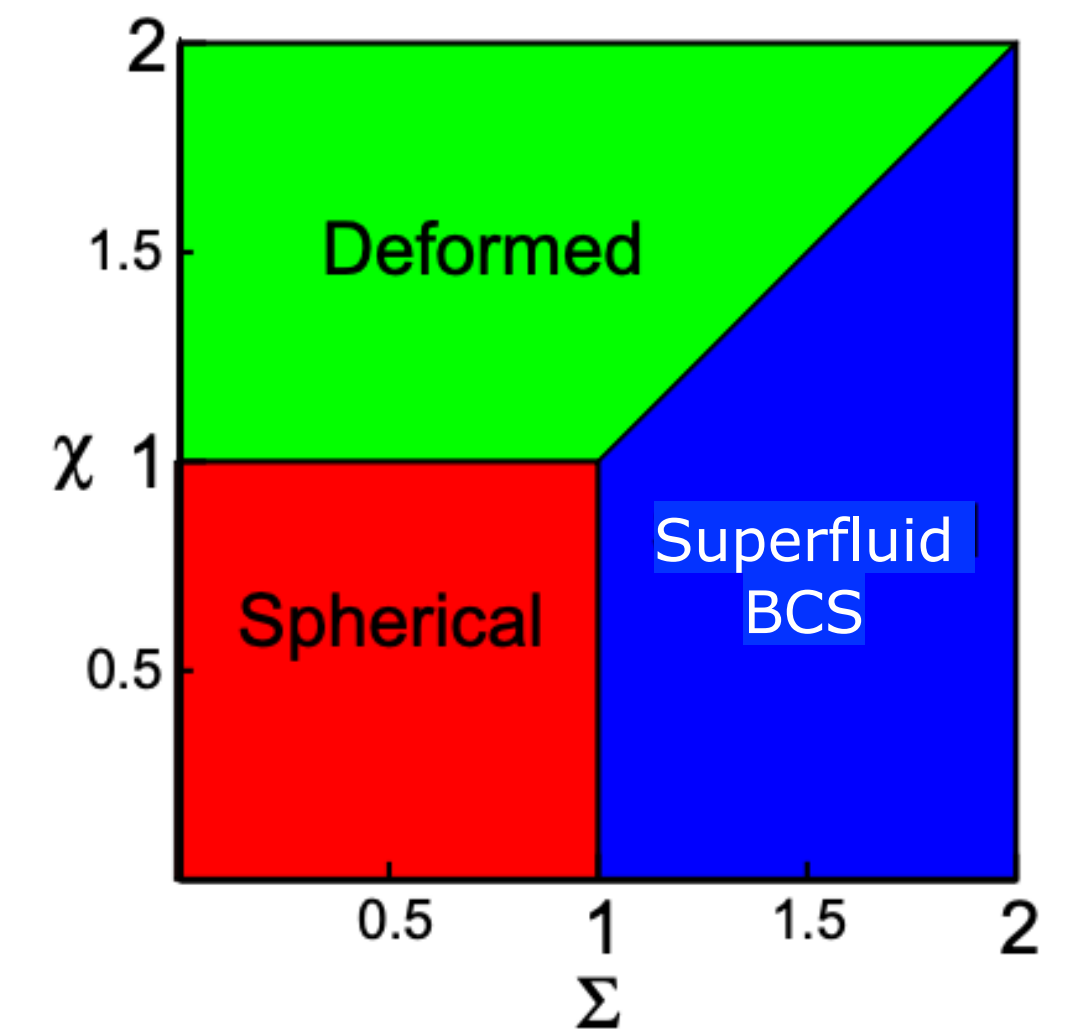


particle-hole interaction V



pairing interaction g

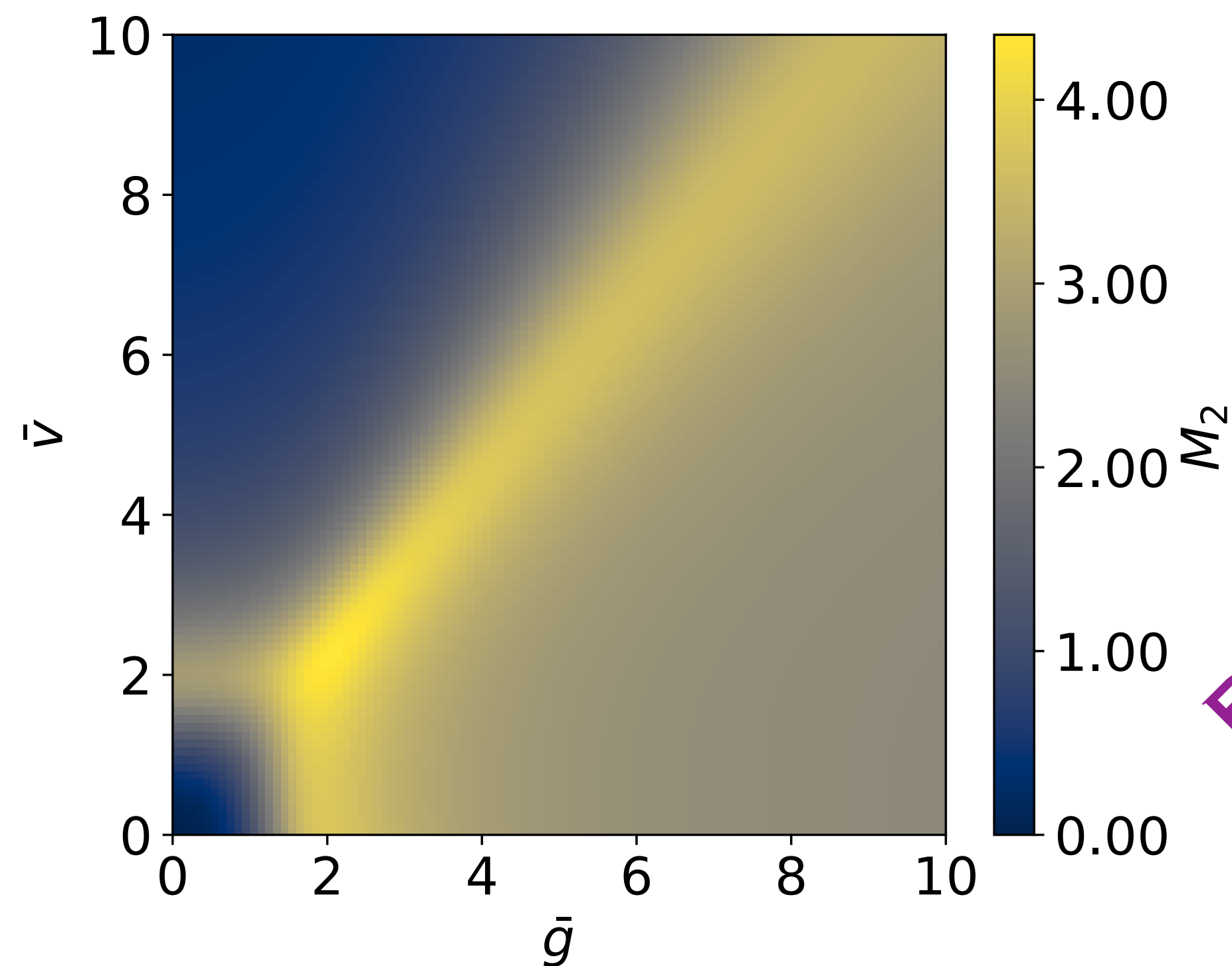
*D. Agassi, *Nucl. Phys. A* 116, 49 (1968)



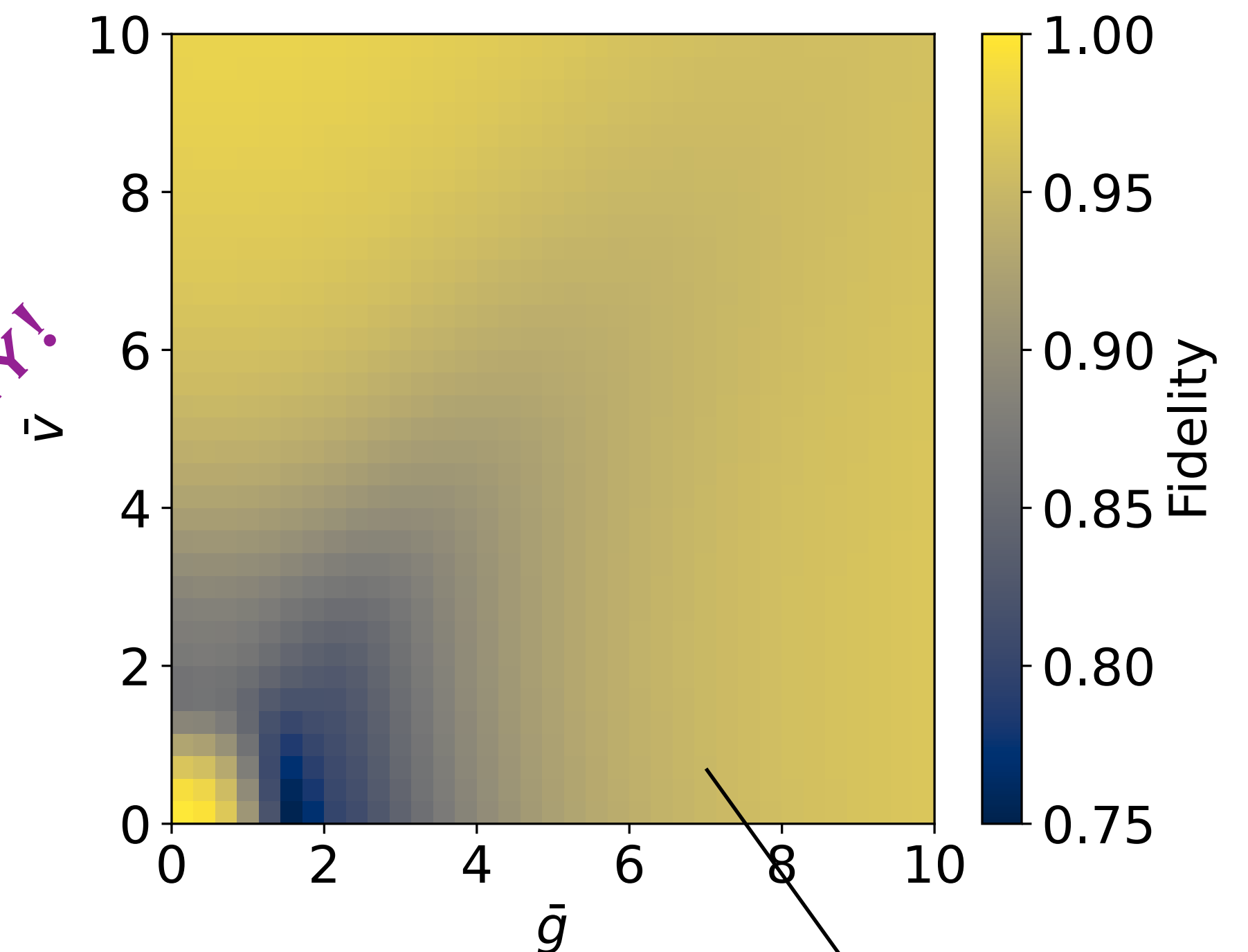
[Pérez-Fernández+ PLB
829 137133 (2022)]

Many-Body Complexity, Deformation and Superfluid pairing

Magic (Stabilizer Rényi Entropy $\mathcal{M}_2$)



Stabilizer ground state fidelity



PRELIMINARY!

Low-rank stabilizer

Outline

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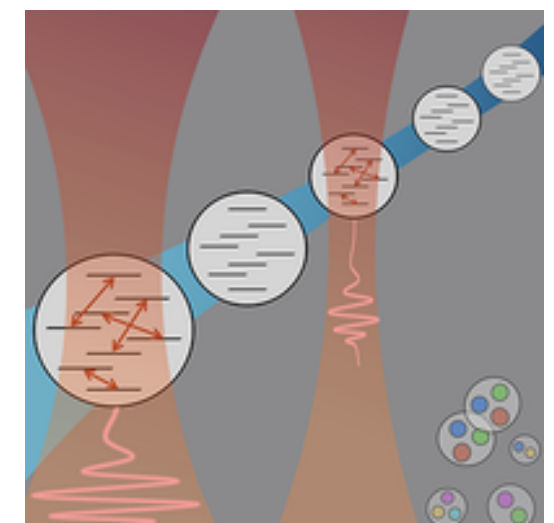
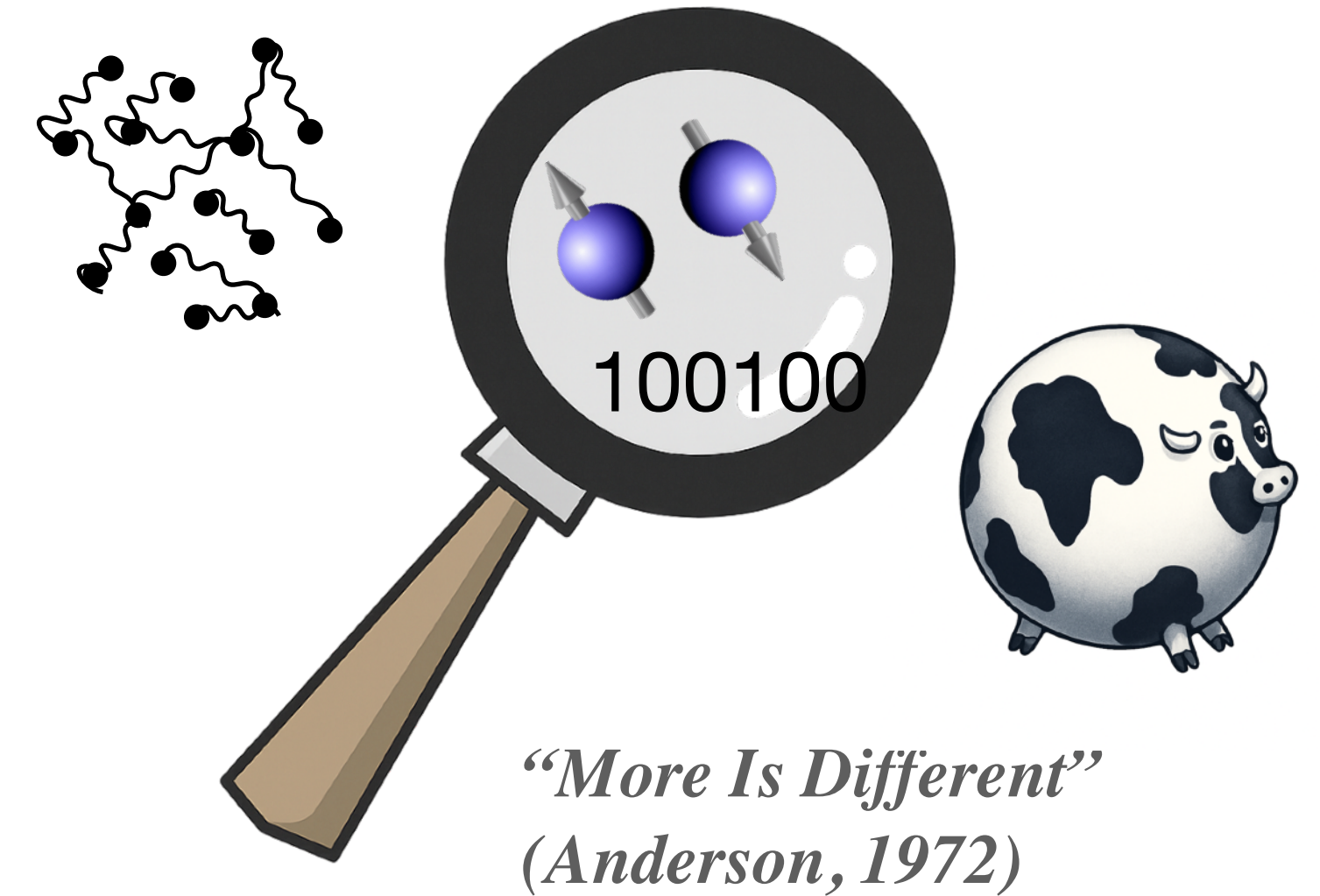
→ *Entanglement, non-stabilizerness and deformation in nuclear systems*

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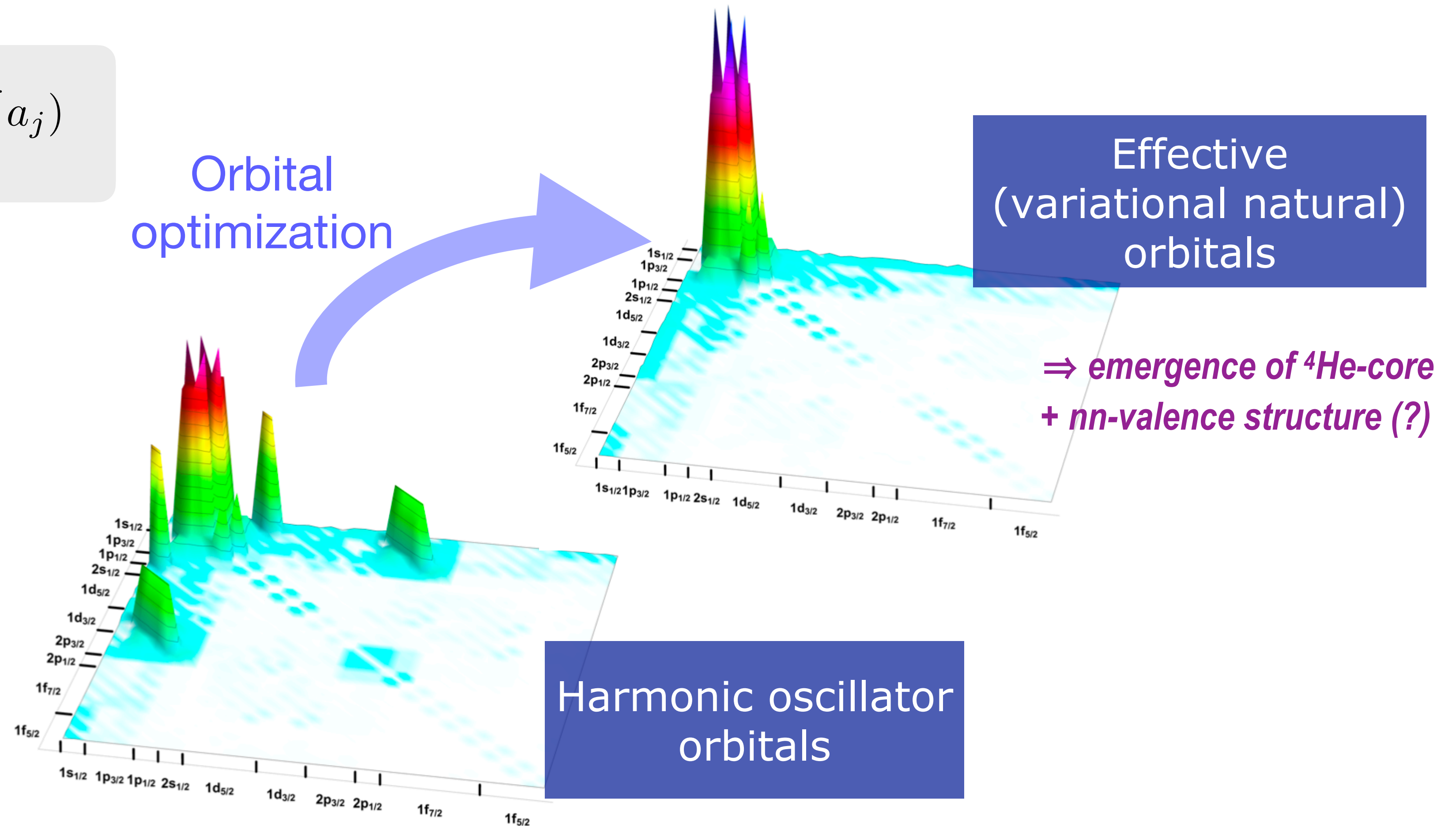
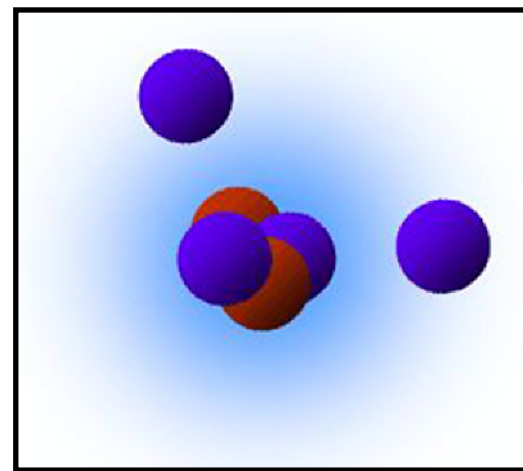
Information and Entanglement Rearrangement in Truncated Model Spaces

Basis optimization/Effective Hamiltonian techniques $\hat{H} \rightarrow \hat{U}(\vec{\beta})\hat{H}\hat{U}(\vec{\beta})^\dagger$

★ In nuclei: $\hat{U} = \exp(\sum_{ij} T_{ij} a_i^\dagger a_j)$

Orbital
optimization

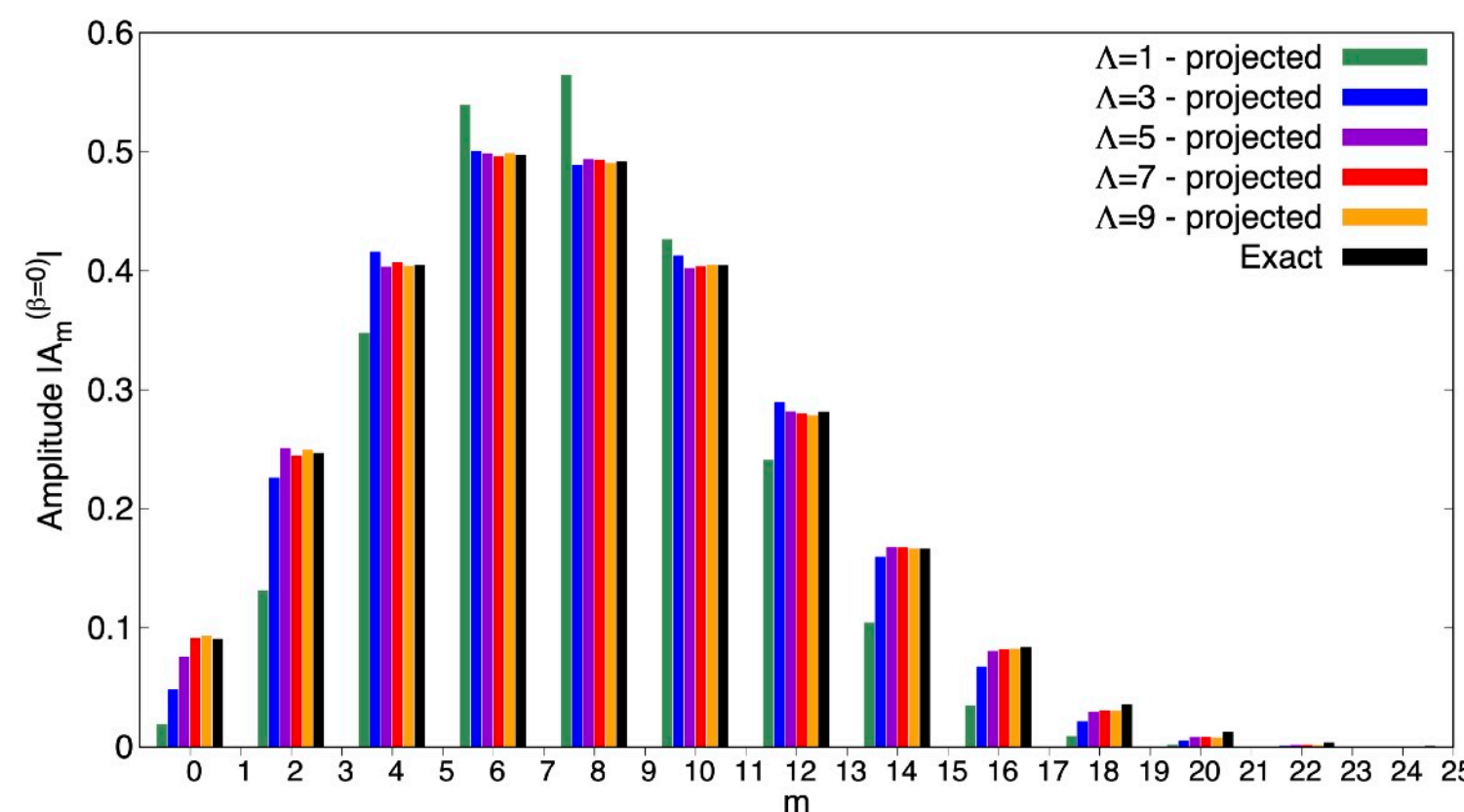
**Neutron-neutron
mutual information**
(${}^6\text{He}$, *ab-initio*, *no-core CI*)



Information and Entanglement Rearrangement in Truncated Model Spaces

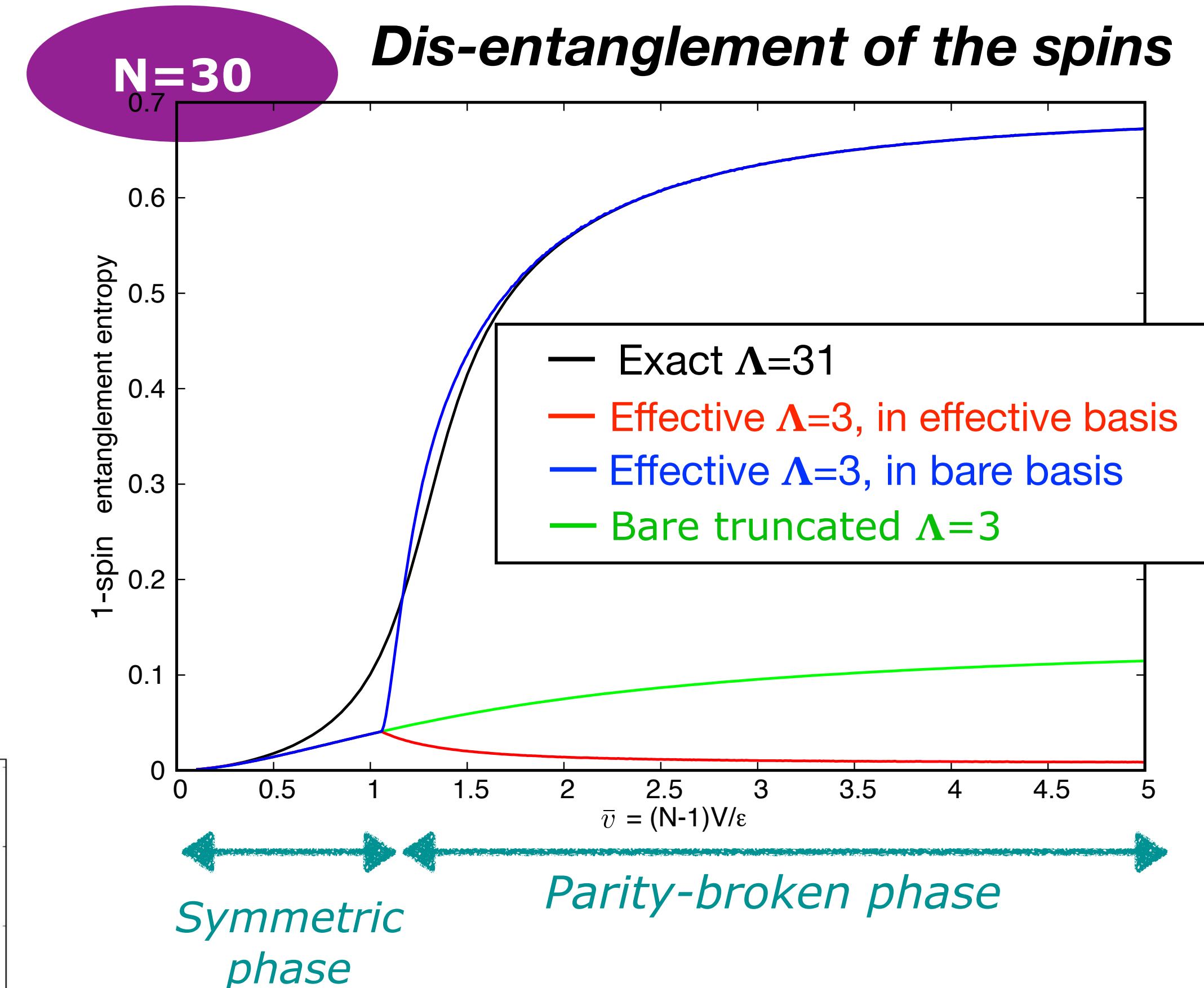
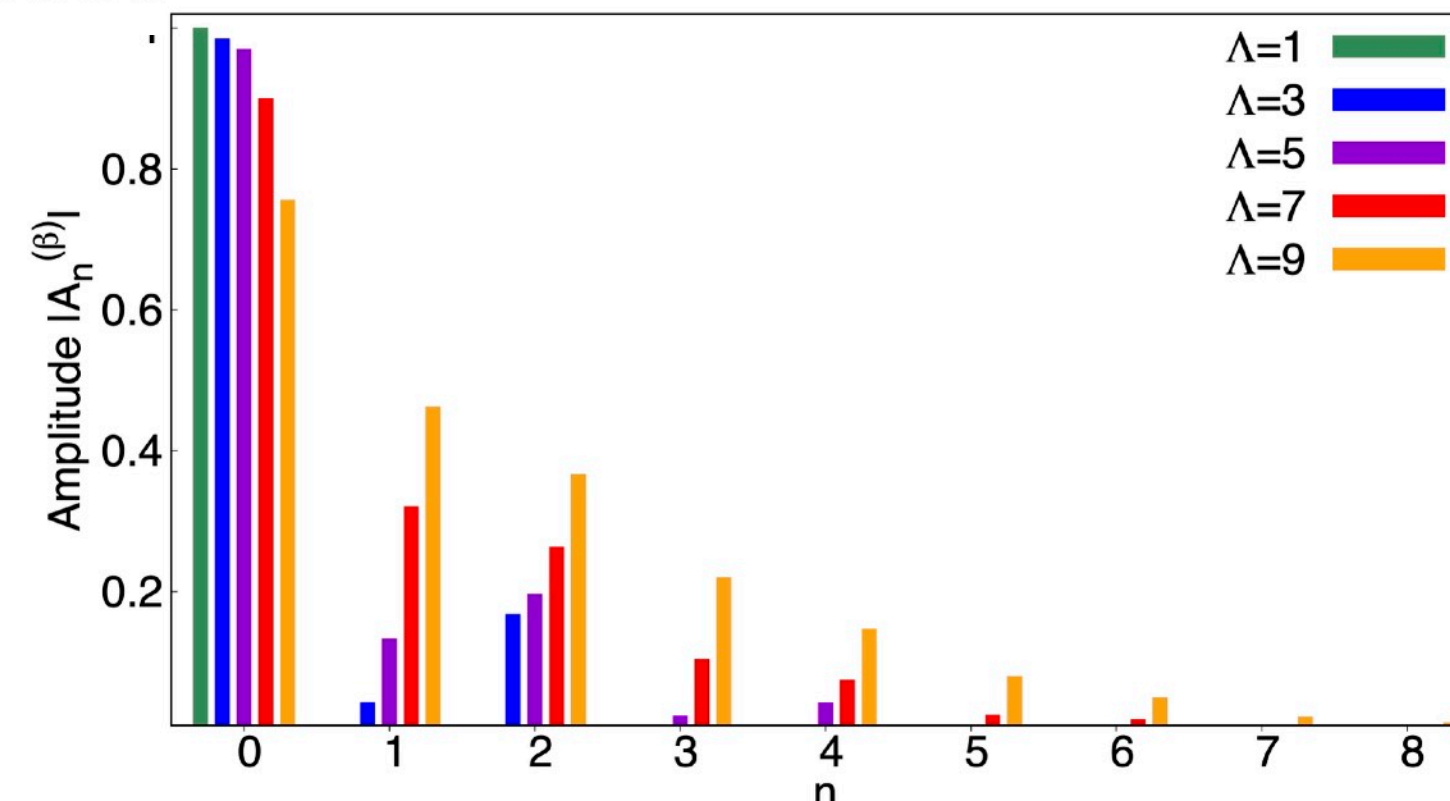
Basis optimization/Effective Hamiltonian techniques $\hat{H} \rightarrow \hat{U}(\vec{\beta})\hat{H}\hat{U}(\vec{\beta})^\dagger$

★ In the LMG model: $\hat{U}(\beta) = \prod_i \exp(-i\sigma_y \beta)$



Localization of the many-body wave function in the Hilbert space

Λ = cut off on many-body configurations



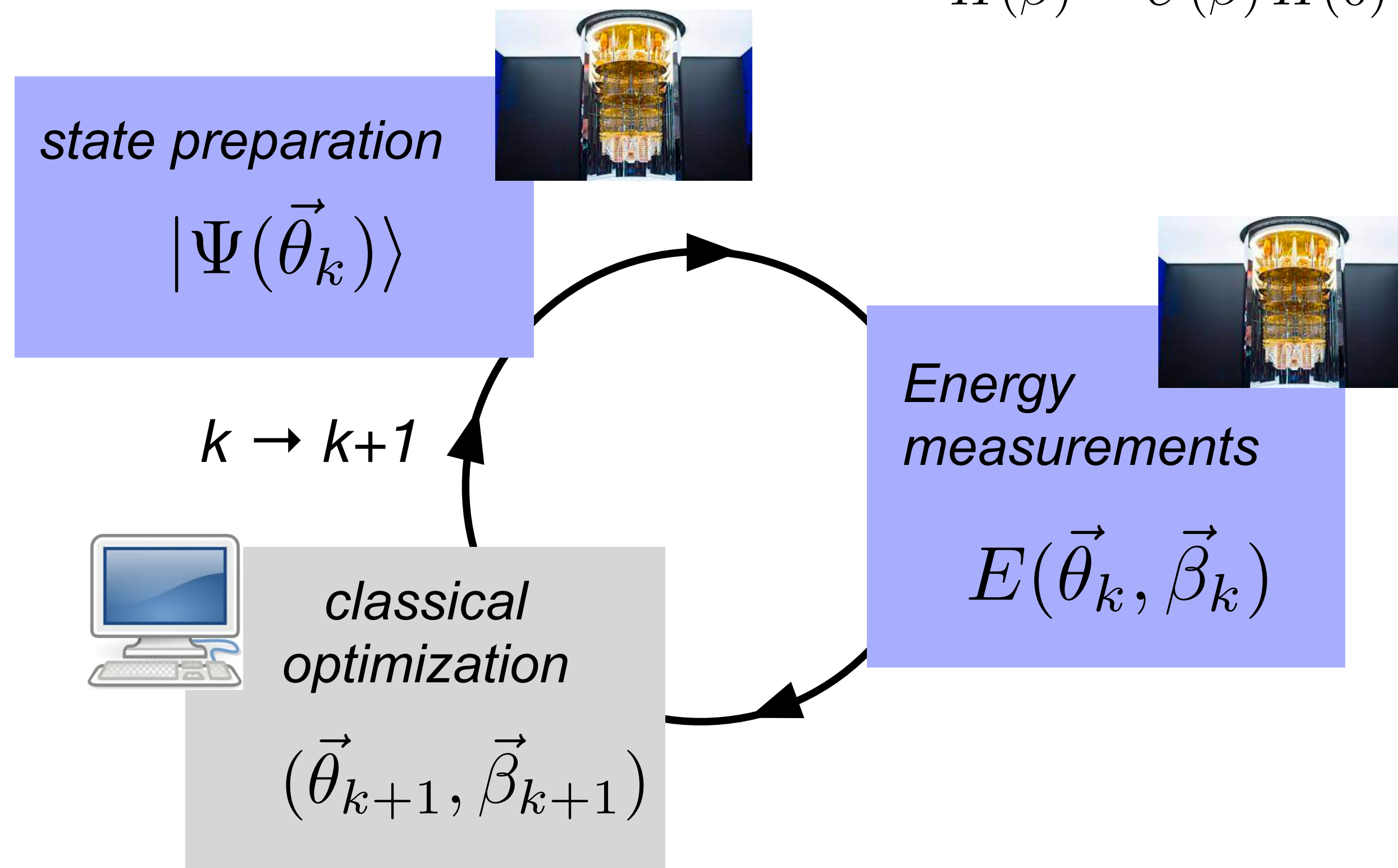
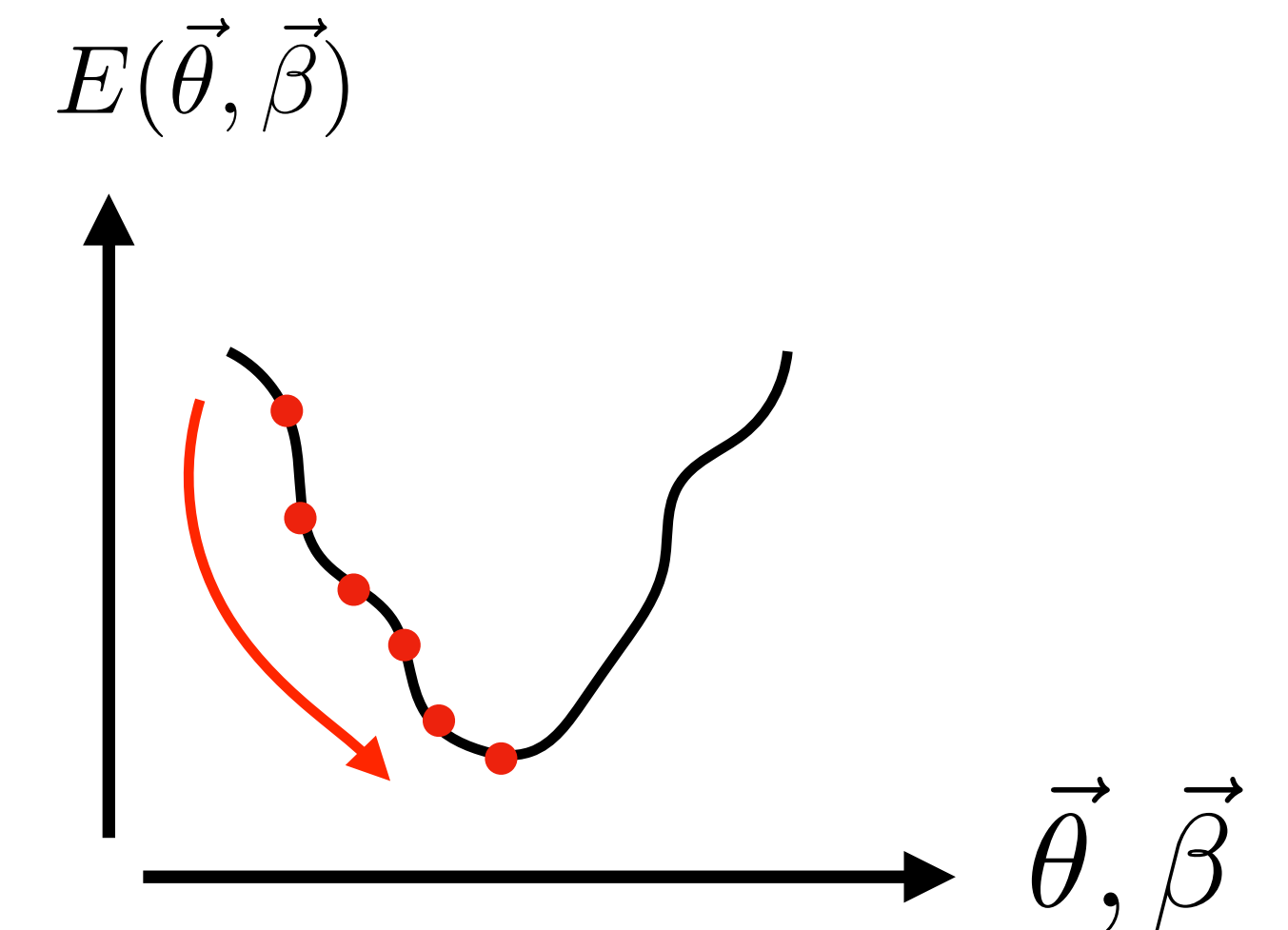
Hengstenberg, CR, Savage EPJA 59, 231 (2023)
CR, Savage PRC 108, 024313 (2023)

Information Rearrangement in Classical and Quantum Computations

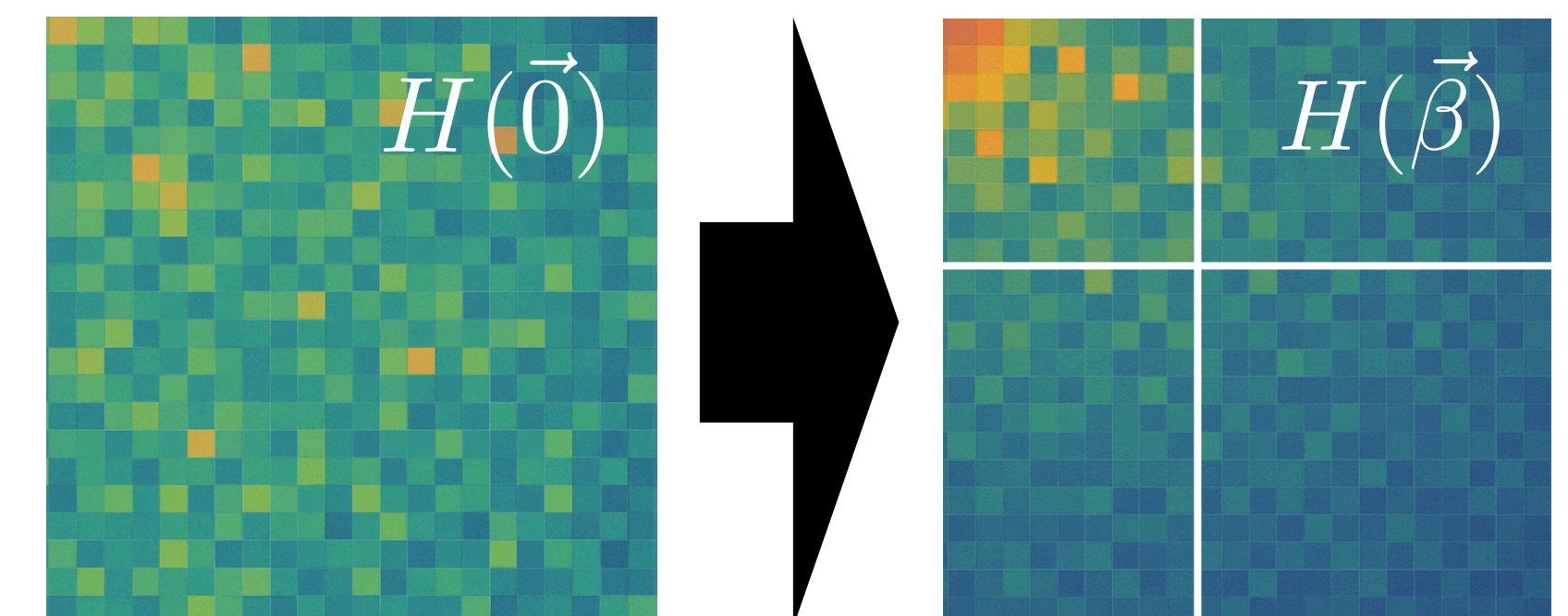
★ *Hamiltonian-Learning-VQE Algorithm for ground-state finding:*

CR, Savage PRC 108, 024313 (2023)

$$E(\vec{\theta}, \vec{\beta}) = \langle \Psi(\vec{\theta}) | \hat{H}(\vec{\beta}) | \Psi(\vec{\theta}) \rangle$$
$$\hat{H}(\vec{\beta}) = \hat{U}(\vec{\beta}) \hat{H}(\vec{0}) \hat{U}^\dagger(\vec{\beta})$$



Information rearrangement within a truncated model space (schematic)



⇒ learns the effective Hamiltonian and identifies the associated ground state simultaneously

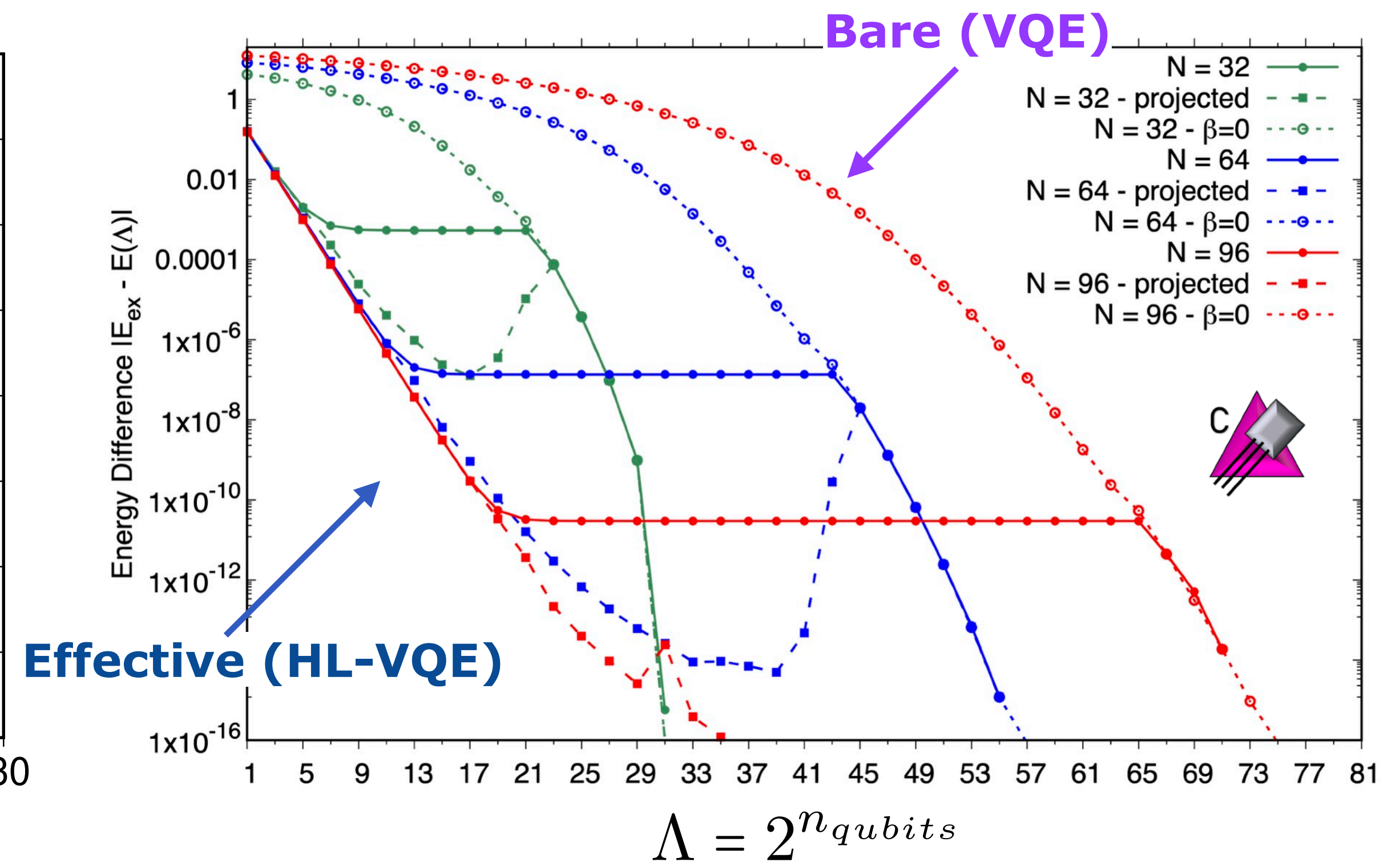
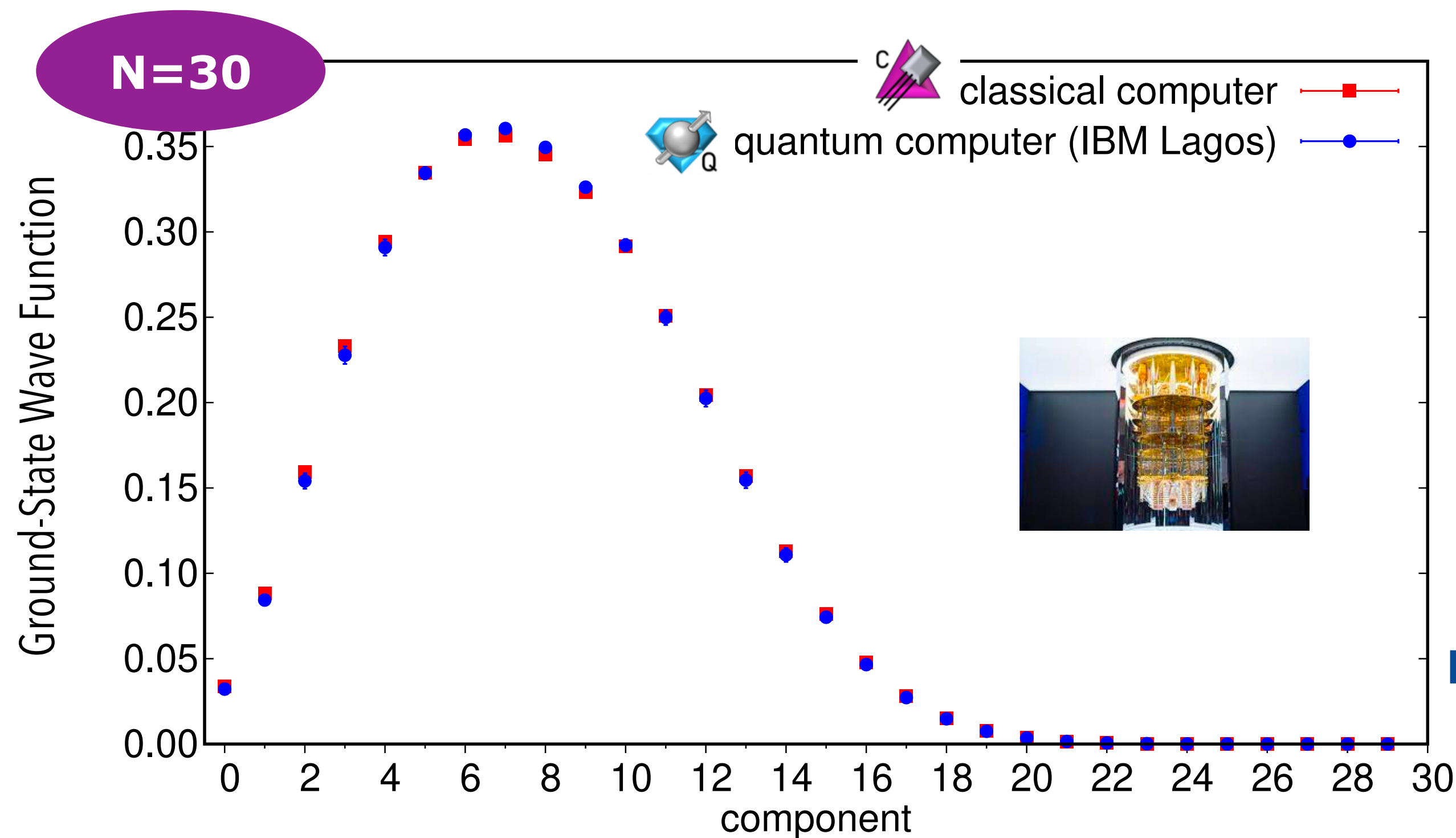
Information Rearrangement in Classical and Quantum Computations

→ *Application to the LMG model:*

CR, Savage PRC 108, 024313 (2023)

Wave function extracted from IBM quantum computer

Energy convergence (classical)



→ *Exponentially less resources are needed for a given accuracy*

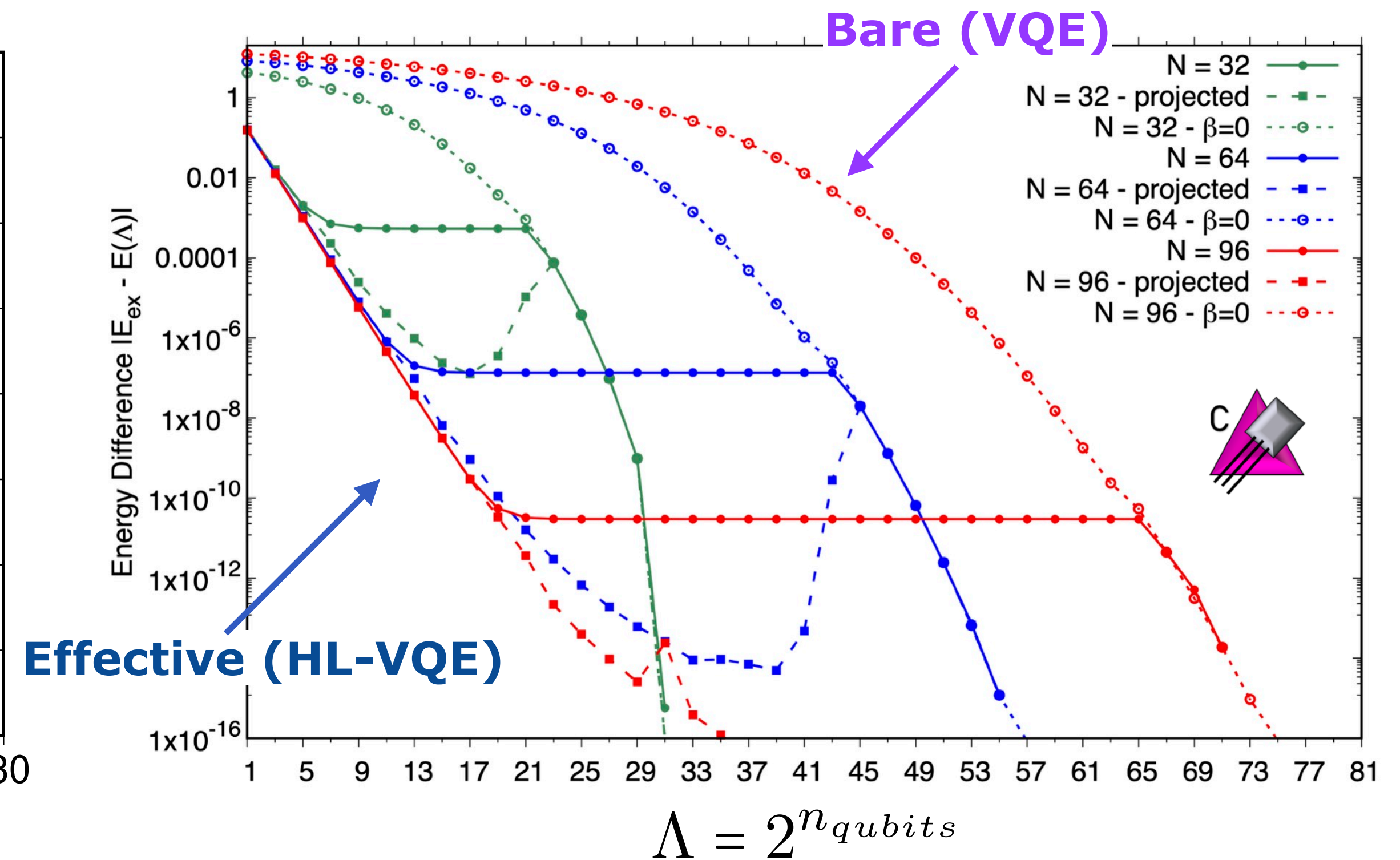
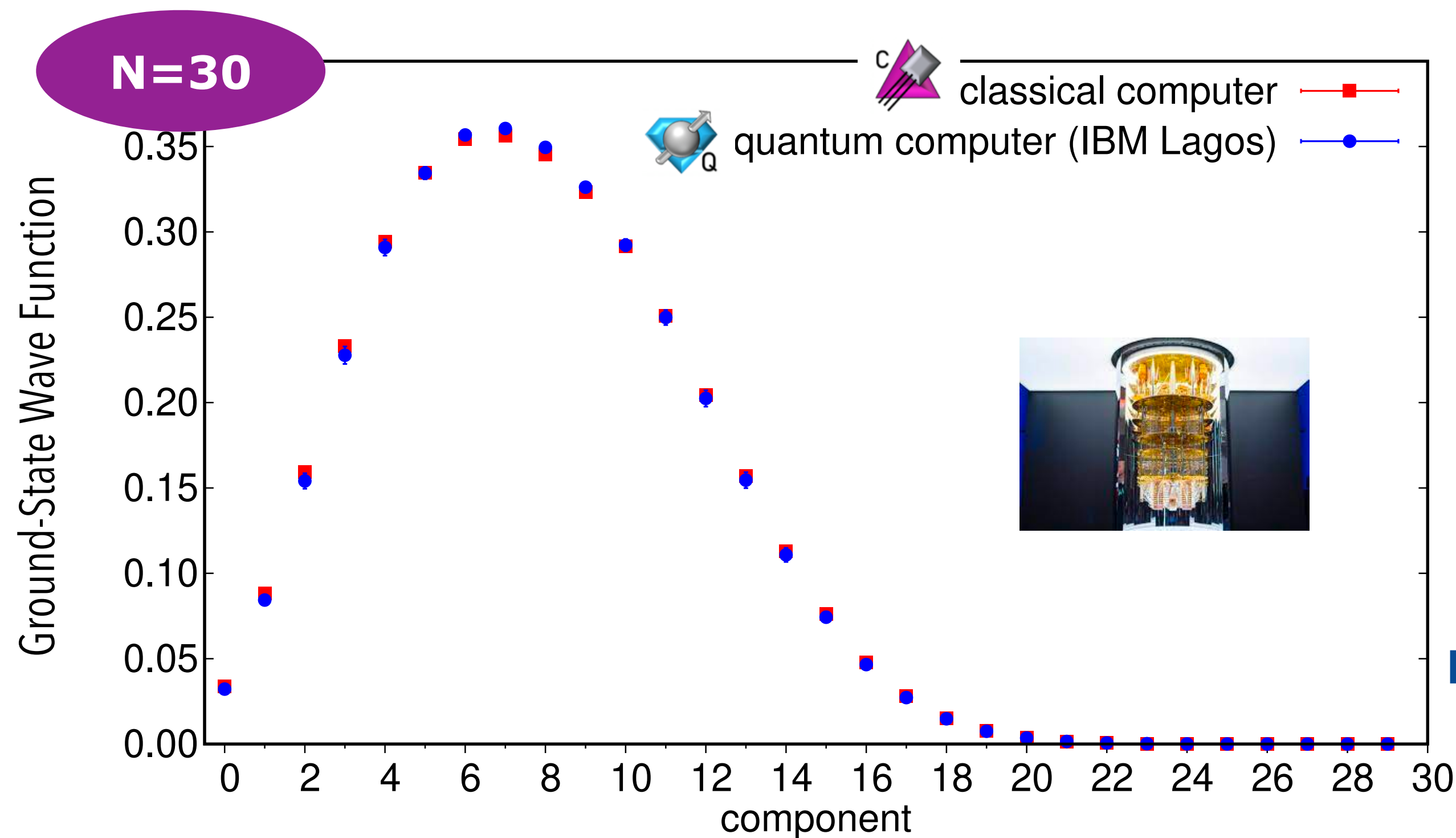
Information Rearrangement in Classical and Quantum Computations

→ **Application to the LMG model:**

CR, Savage PRC 108, 024313 (2023)

Wave function extracted from IBM quantum computer

Energy convergence (classical)



→ also see talk by Pauline Ollitrault for orbital optimization in quantum chemistry algorithms

→ Exponentially less resources are needed for a given accuracy

Outline

★ Quantum complexity and the emergence of collectivity in many-body systems

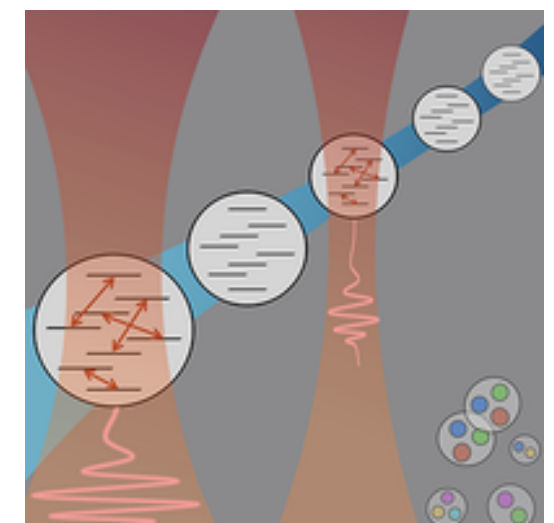
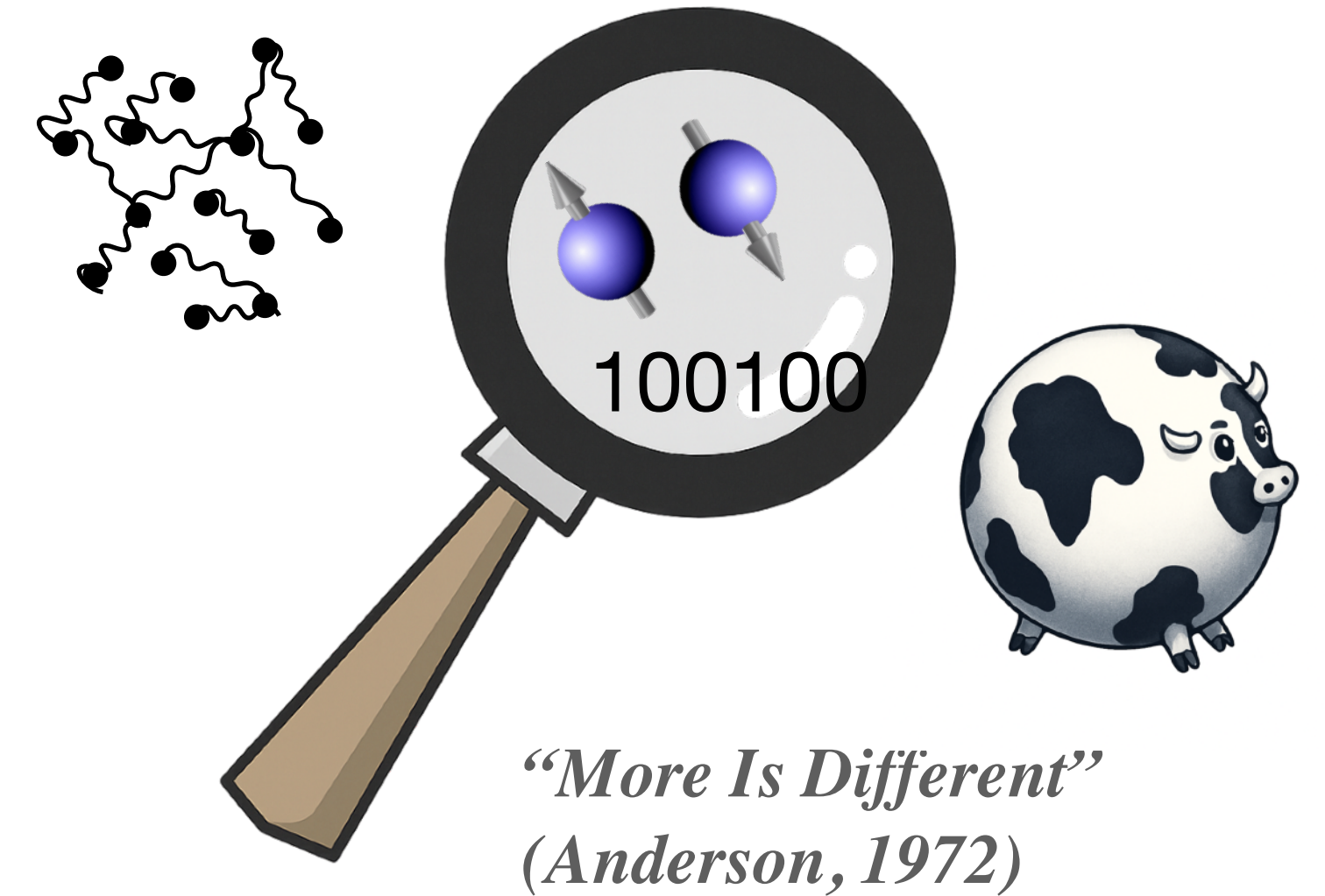
→ *Entanglement, non-stabilizerness and deformation in nuclear systems*

★ Complexity-guided ground-state finding algorithms

→ *Stabilizer-accelerated many-body ground state estimations*

→ *Information and complexity rearrangement with Hamiltonian-learning-VQE*

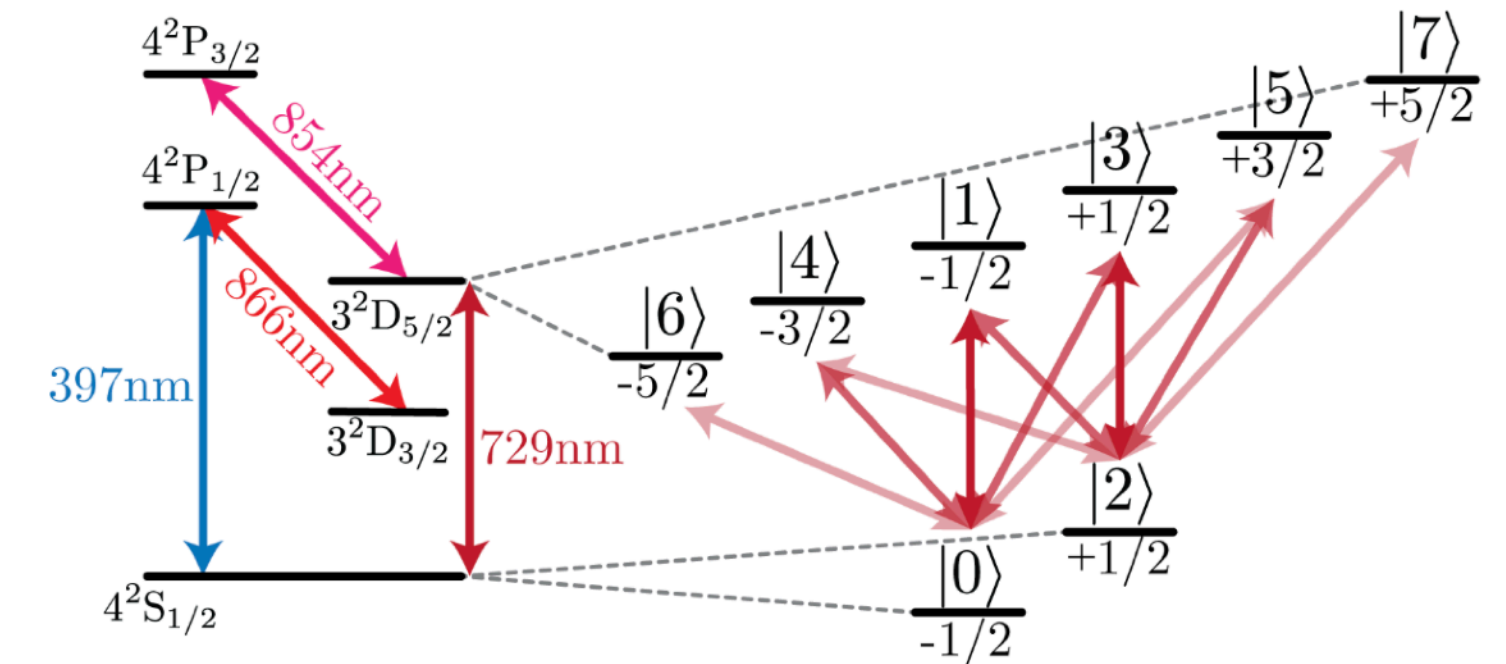
★ Many-body and gate complexity of many-body dynamics with qudits



Quantum Simulations with Qudits

Quantum Computers with qudits ($d > 2$ -level quantum systems) allow for naturally mapping symmetries of the systems of interest

Developed with trapped ions, superconducting devices and more
— see e.g. Wang+, *Front. Phys.* **8**, 589504 (2020)



Trapped ion system Innsbruck

<https://www.quantumoptics.at/en/research/qudits.html>

Can qudits reduce computational (gate) complexity?

e.g. Meth+ *Nature Physics* **21**, 570–576 (2025), Calajó+ *PRX Quantum* **5**, 040309 (2024), Illa, CR, Savage, *PRC* **108**, 064306 (2023), & *PRD* **110**, 014507 (2024), Kürkçüoğlu+ *arXiv:2410.16414*, Fromm+ *EPJ Quant. Technol.* **12**, 92 (2025), Turro+ *PRD* **111**, 043038 (2025)...

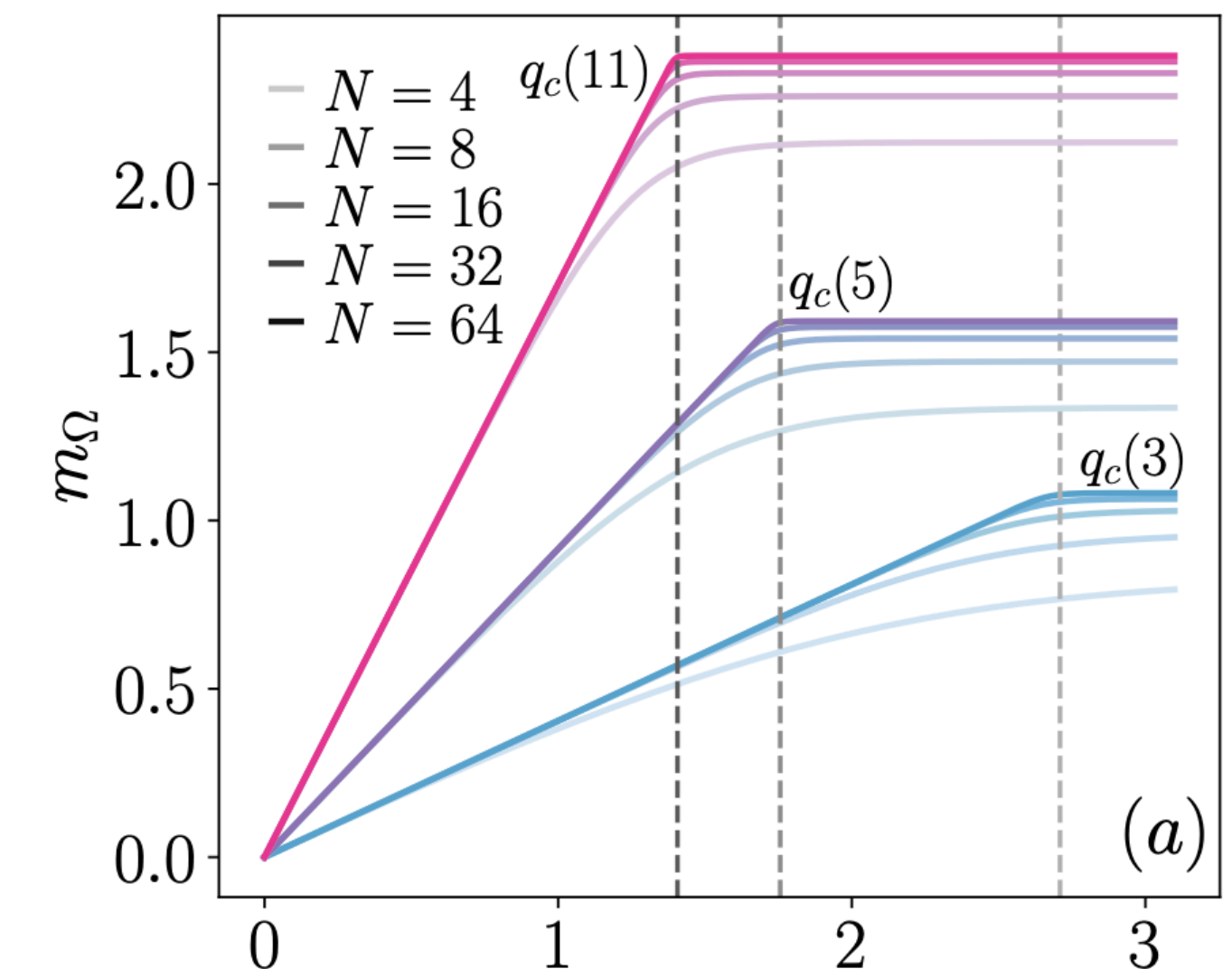
→ Also see talk by Even Chiari

Are qudits more efficient at generating/spreading many-body complexity?

Studies of magic in random circuits, e.g.:

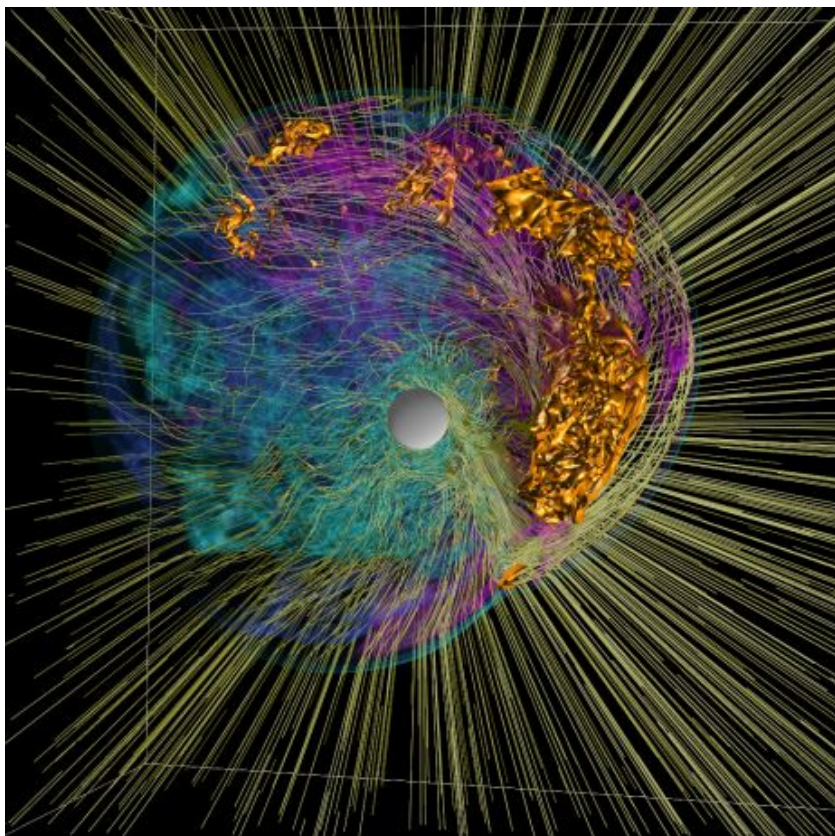
Turkeshi, Tirrito, Sierant, *Nature Comm.* **16**, 2575 (2025)

Magni, Turkeshi *arXiv:2506.02127*



q Magni+, 2506.02127

Neutrino Dynamics with Qutrits

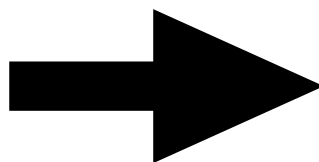


Neutrinos from core-collapse supernovae
Wikimedia commons

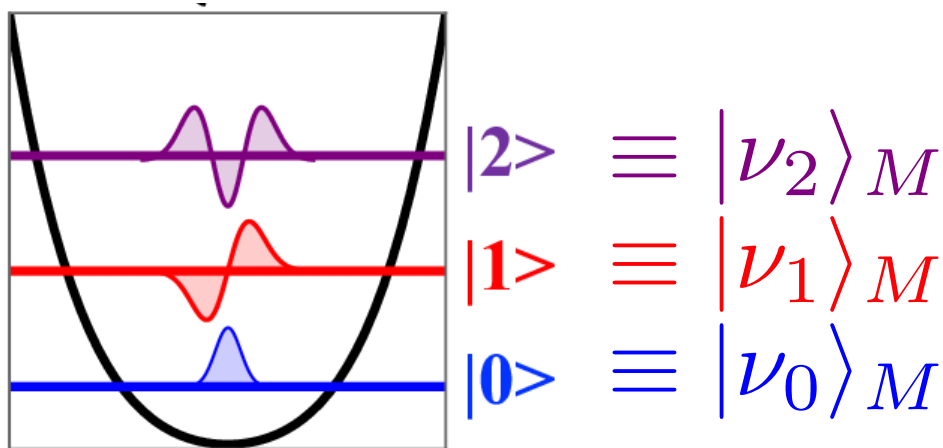
Neutrinos = weakly-interacting, strongly-correlated particles

3 favours of neutrinos $|\nu\rangle_e, |\nu\rangle_\mu, |\nu\rangle_\tau$

related to mass eigenstates: $|\nu\rangle_F = U_{PMNS} |\nu\rangle_M$



Mapping onto qutrits:



Gate complexity
Qubits vs qudit mappings:

Turro+ PRD 111, 043038 (2025)

Qudit	Circuit	All-to-all		Linear chain	
		2-q gate count	2-q gate depth	2-q gate count	2-q gate depth
Qutrit	Fig. 1	4	4	4	4
Qubit	A (Fig. 3)	24	13	42	31
	B (Fig. 4)	18	12	30	25

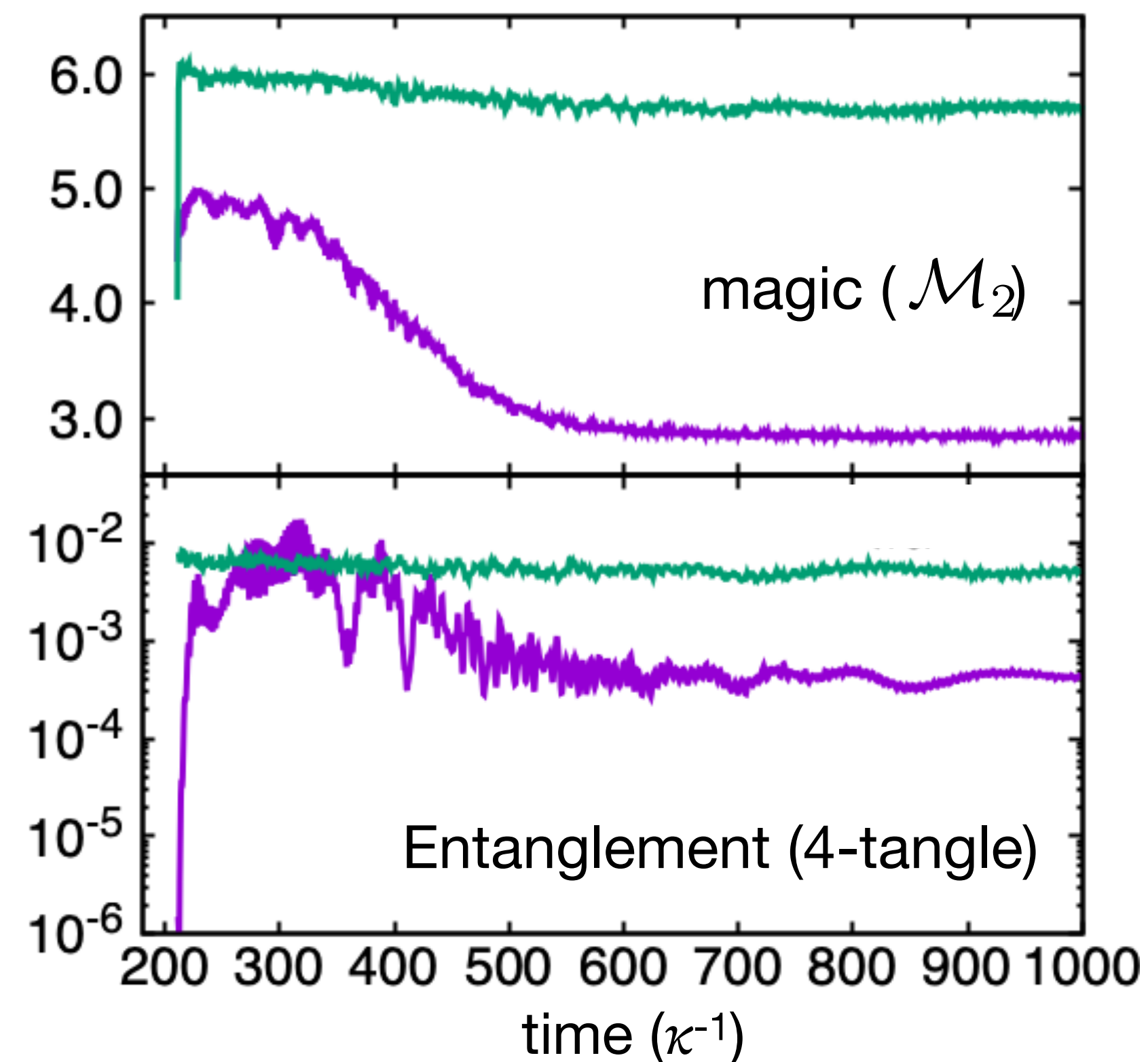
TABLE II. The two-qudit entangling gate count and depth for the two-neutrino quantum circuits proposed, involving two qutrits or four qubits.

See also works by Alessandro Roggero & Denis Lacroix, see talk by Mariane Mangin Brinet

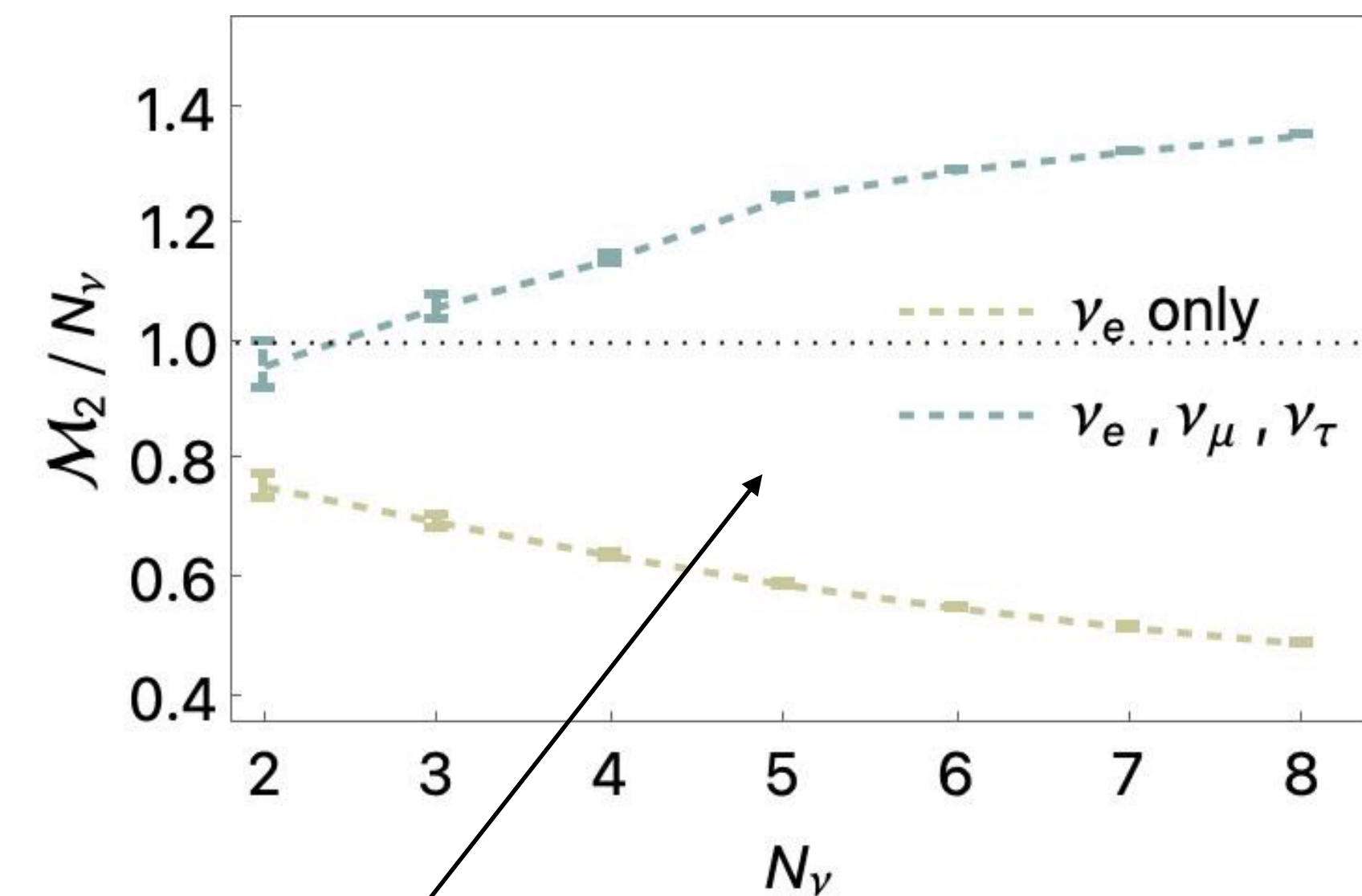
Magic and Entanglement in Neutrino Dynamics

Chernyshev, CR, Savage PRR 7 023228 (2025)

1-flavour $|\nu_e \nu_e \nu_e \nu_e \nu_e\rangle$ vs 3-flavour $|\nu_e \nu_e \nu_\mu \nu_\mu \nu_\tau\rangle$
initial state:



Asymptotic values:



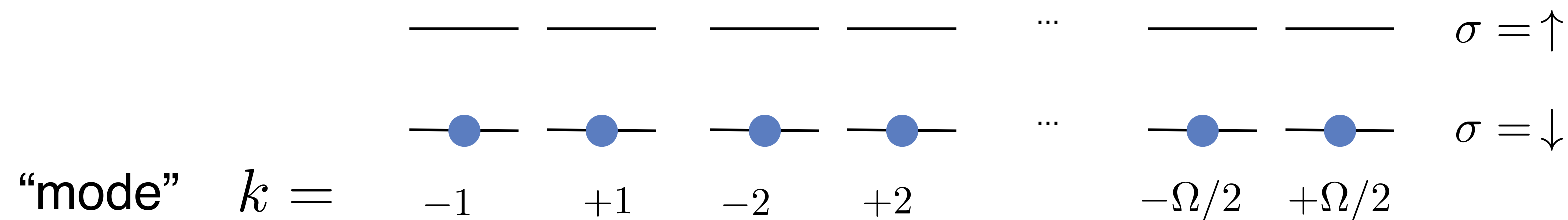
Max value for a tensor-product state

👉 Flavour mixing enhances complexity during the evolution

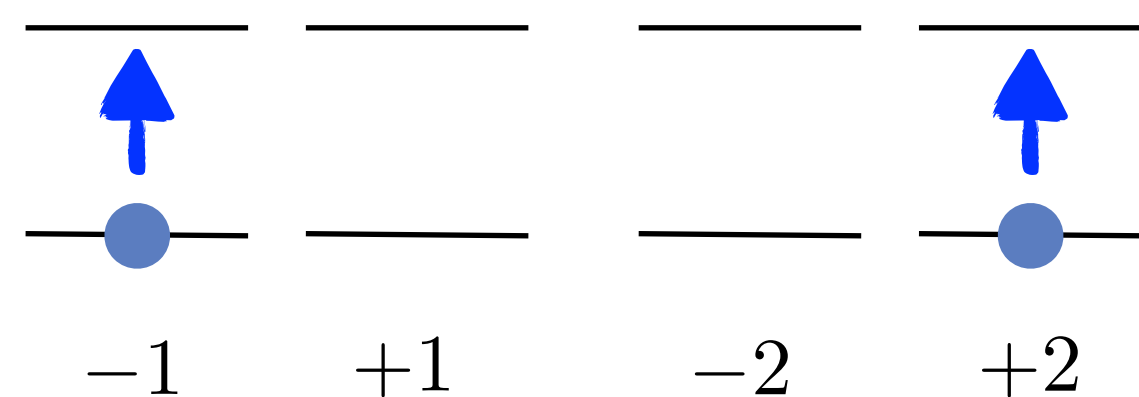
Symmetry-guided mapping of the Agassi model onto qudit systems

★The Agassi model

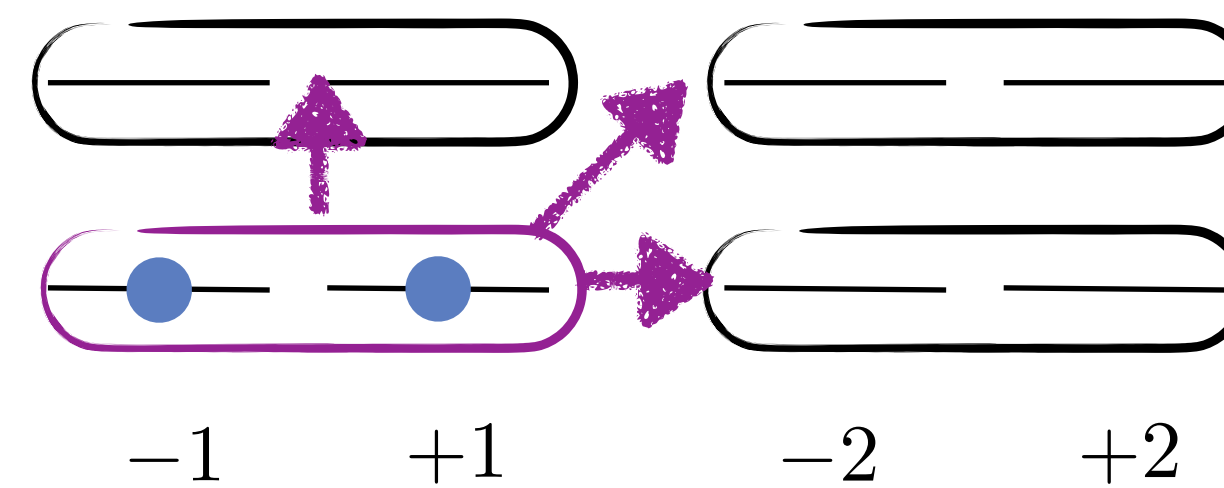
= extension of the LMG model with superfluid pairing



$$\hat{H} = \varepsilon \hat{J}_z - \frac{V}{2} \left(\hat{J}_+^2 + \hat{J}_-^2 \right) - g \sum_{\sigma\sigma'} \hat{B}_\sigma^\dagger \hat{B}_{\sigma'}$$

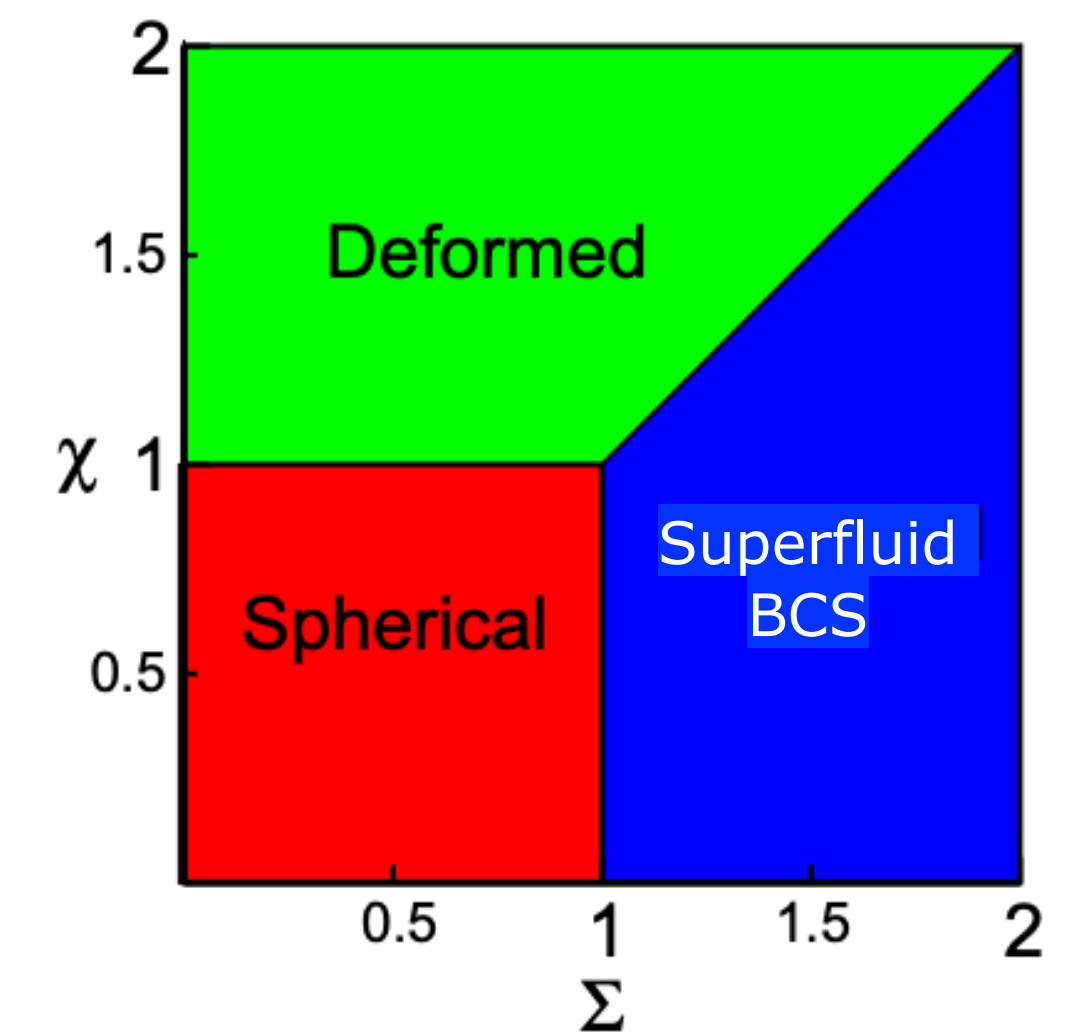


particle-hole interaction V



pairing interaction g

*D. Agassi, *Nucl. Phys. A* 116, 49 (1968)

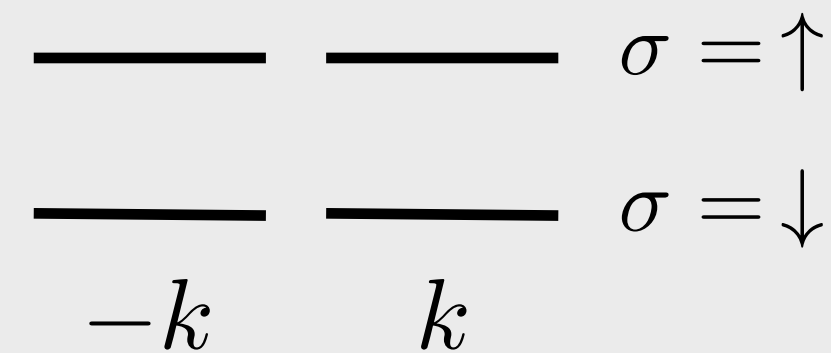


[Pérez-Fernández+ PLB
829 137133 (2022)]

Symmetry-guided mapping of the Agassi model onto qudit systems

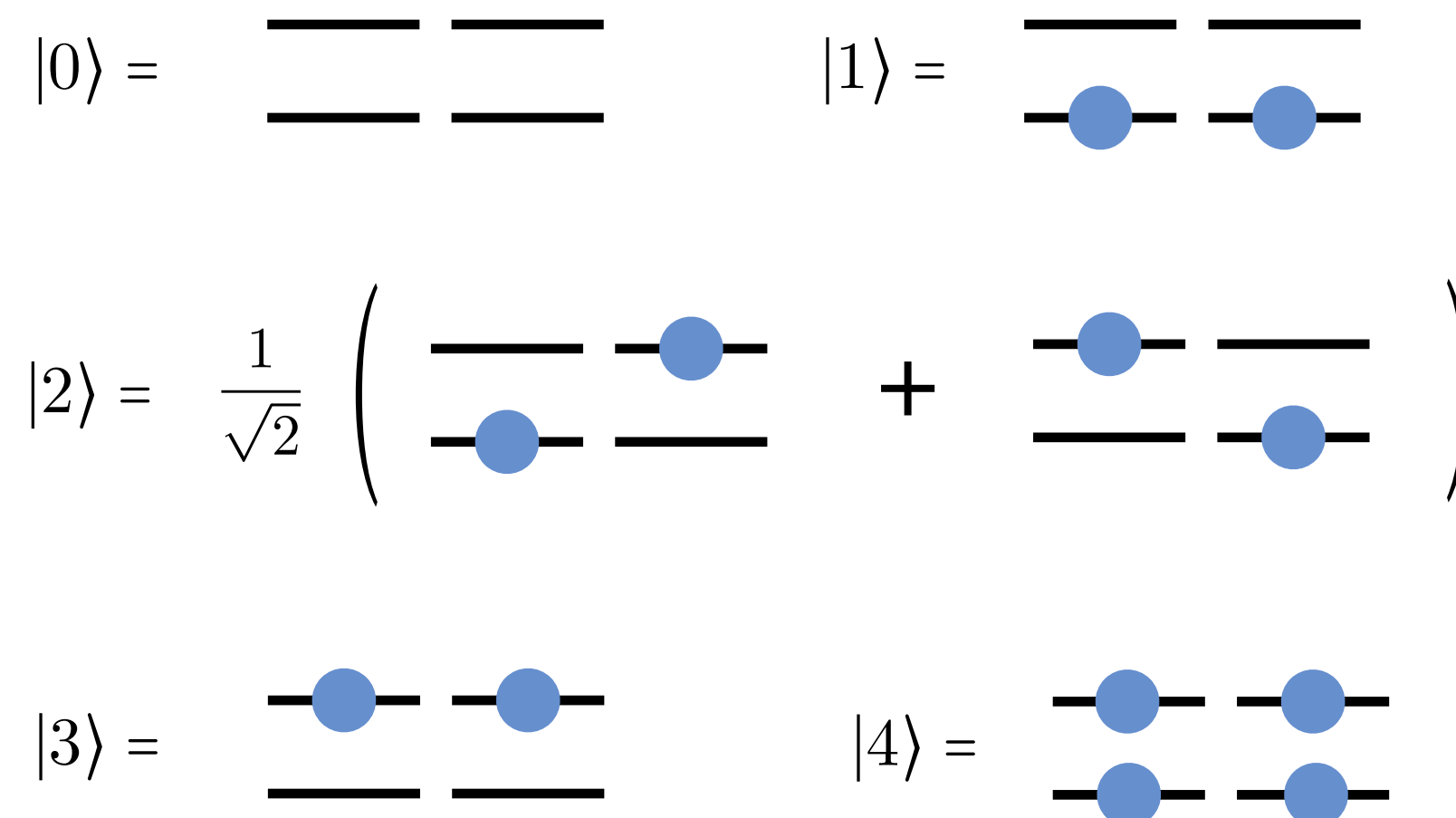
* Make use of the $SO(5)$ symmetry:

Degrees of freedom = pairs of modes

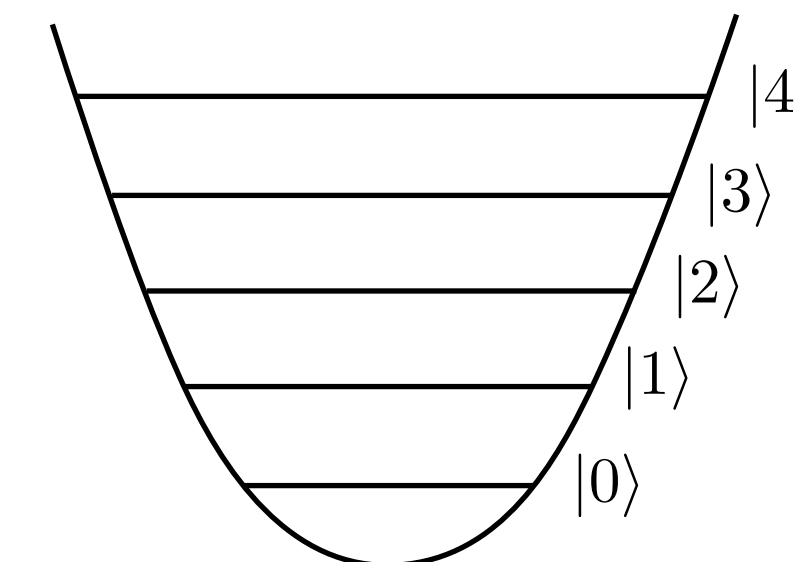


$J_z, J_{\pm}, B_{\uparrow,\downarrow}, B_{\uparrow,\downarrow}^{\dagger}$
 = generators of $SO(5)$

$\Rightarrow$ 5 states:



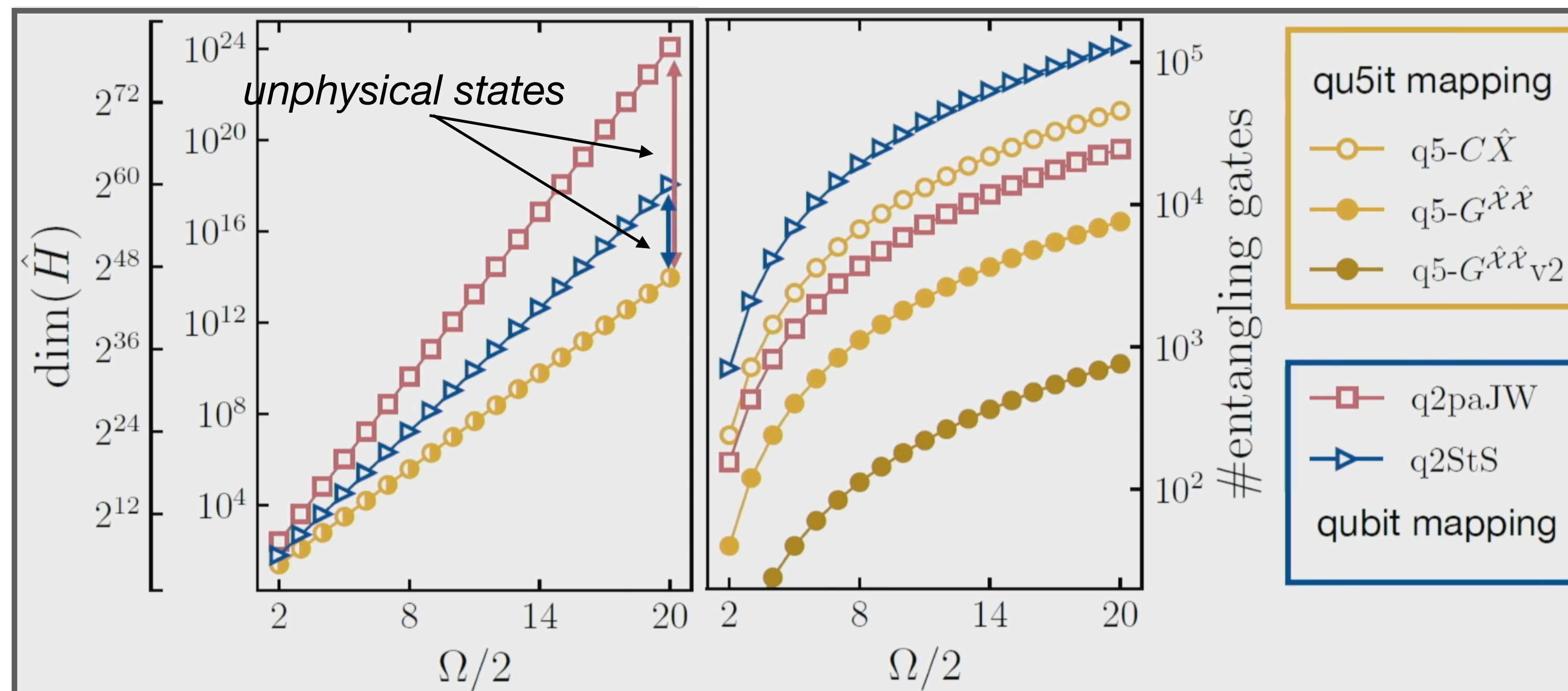
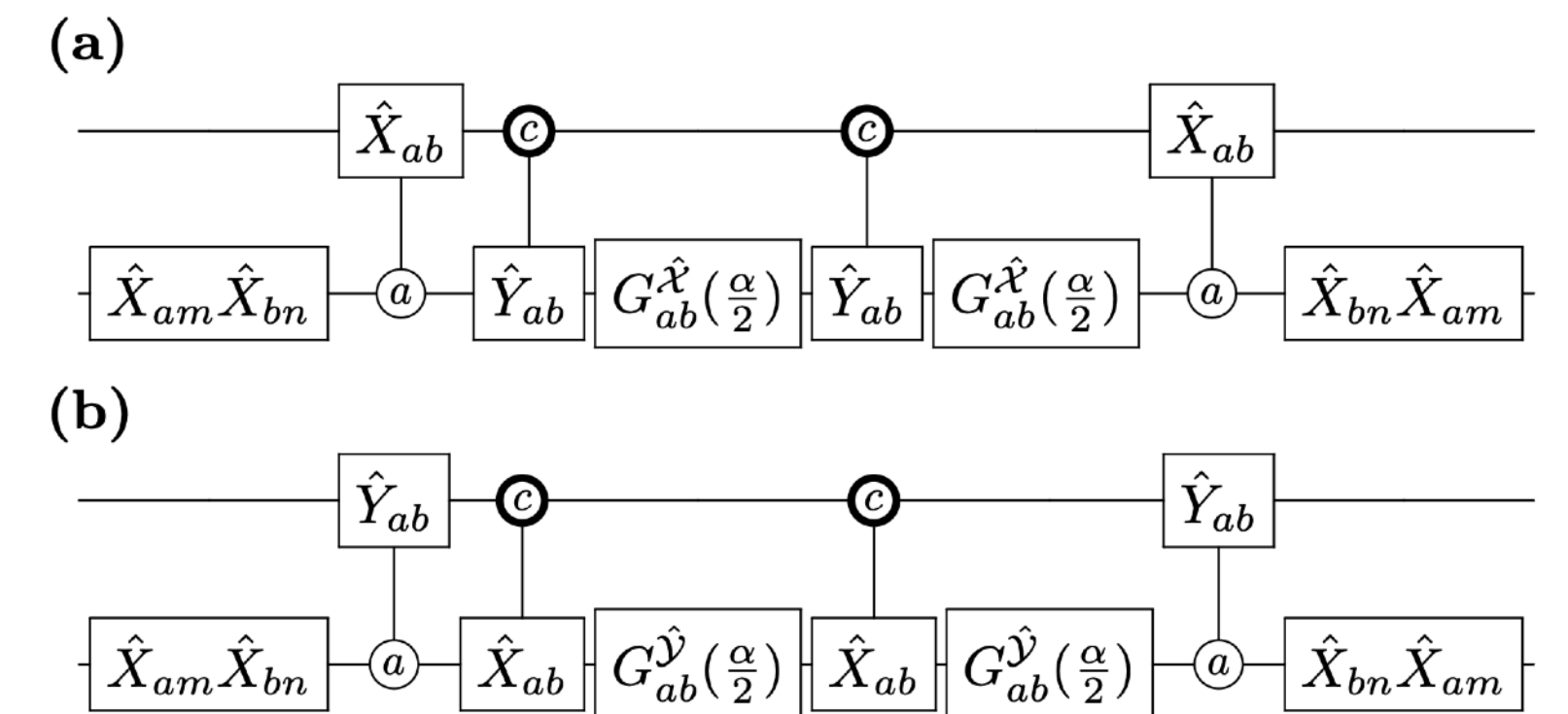
Naturally maps onto “qu5its”
[qudits with $d=5$]



Symmetry-guided Quantum Simulations with Qudits

Developed a qudit-system classical simulator for unitary dynamics using Google's *cirq*

Errors & Resource Requirements for Simulating Real-Time Dynamics (LO Trotter):

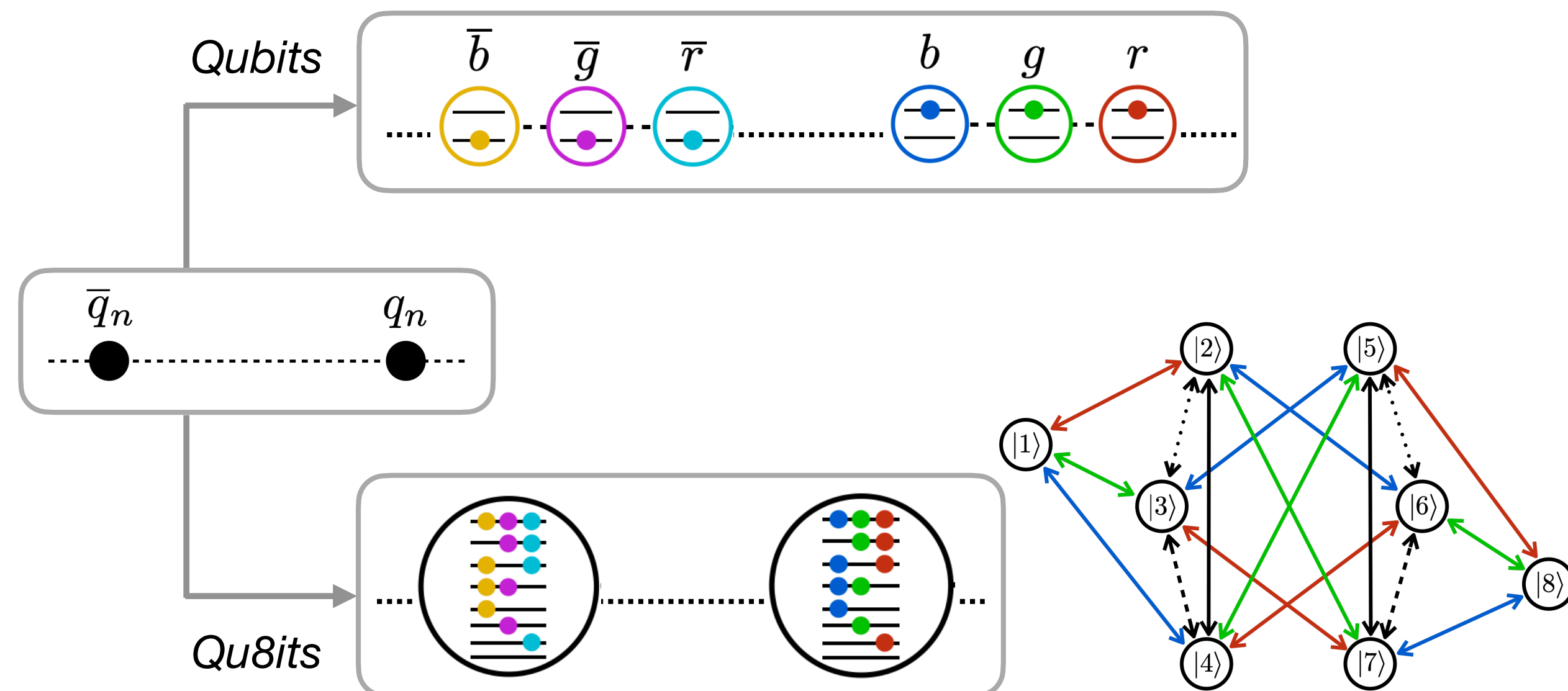
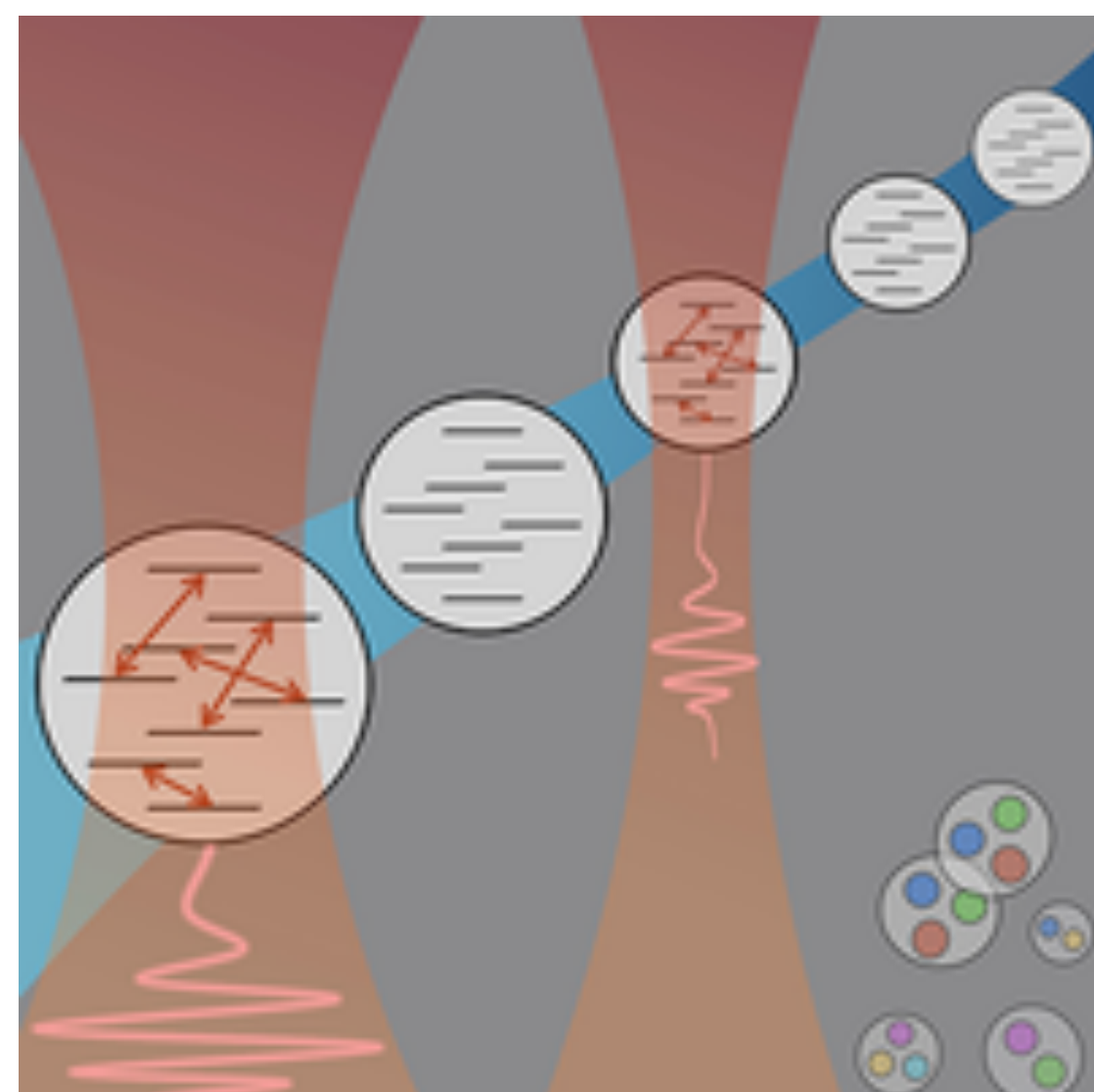


"Physics-aware" JW mapping to qubits
qu5it-state to qubit-states mapping

Qudits for Quantum Simulations of 1+1D $SU(3)$ Lattice QCD

Illa, CR, Savage, PRD 110, 014507 (2024)

Editor's suggestion



Resource for time evolution (single Trotter step):

Qudits	Number of qudits	U_{kin} ent. gates	U_{el} ent. gates
Qubit ($d = 2$)	$6N_f L$	$6N_f(8L - 3) - 4$	$N_f(2L - 1)[23N_f(2L - 1) - 17]$
Qu8it ($d = 8$)	$2N_f L$	$6N_f(2L - 1)$	$4N_f(2L - 1)[N_f(2L - 1) - 1]$
Reduction in resources ($L \rightarrow \infty$)	3	4	5.75

Conclusions

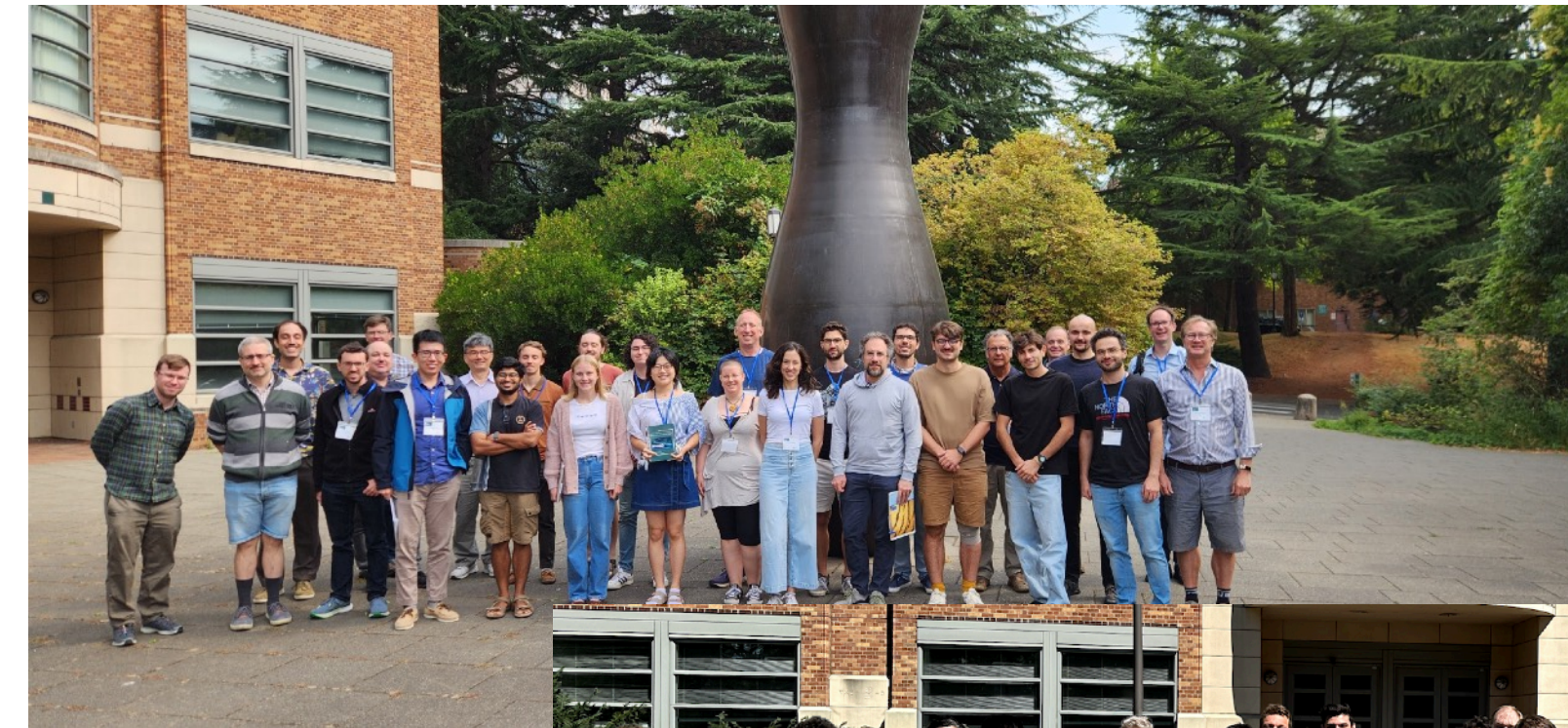
- ★ Concepts of QIS bring new insights into many-body phenomena and underlying forces
 - *Better understand the underlying mechanisms governing the emergence of collective behaviours and other phenomena*
- ★ Entanglement, Magic and Symmetries are key ingredients for designing resource efficient quantum computations of many-body structure and real-time dynamics
 - *Combine these aspects to develop efficient algorithms for NISQ and fault-tolerant quantum computers*
- ★ Exchanges of ideas and techniques between fields of QMB physics and QIS is essential

Workshops on “Many-Body Quantum Magic”



2024: Abu Dhabi

<https://mbqm.tii.ae/index.php>



2025: IQUS,
UW, Seattle

<https://iqus.uw.edu/events/iqus-workshop-2025-2/>



2026: ECT* in Trento,
October 5-16



Organizers: Tobias Haug, CR, Martin Savage, Emanuele Tirrito, Xhek Turkishi

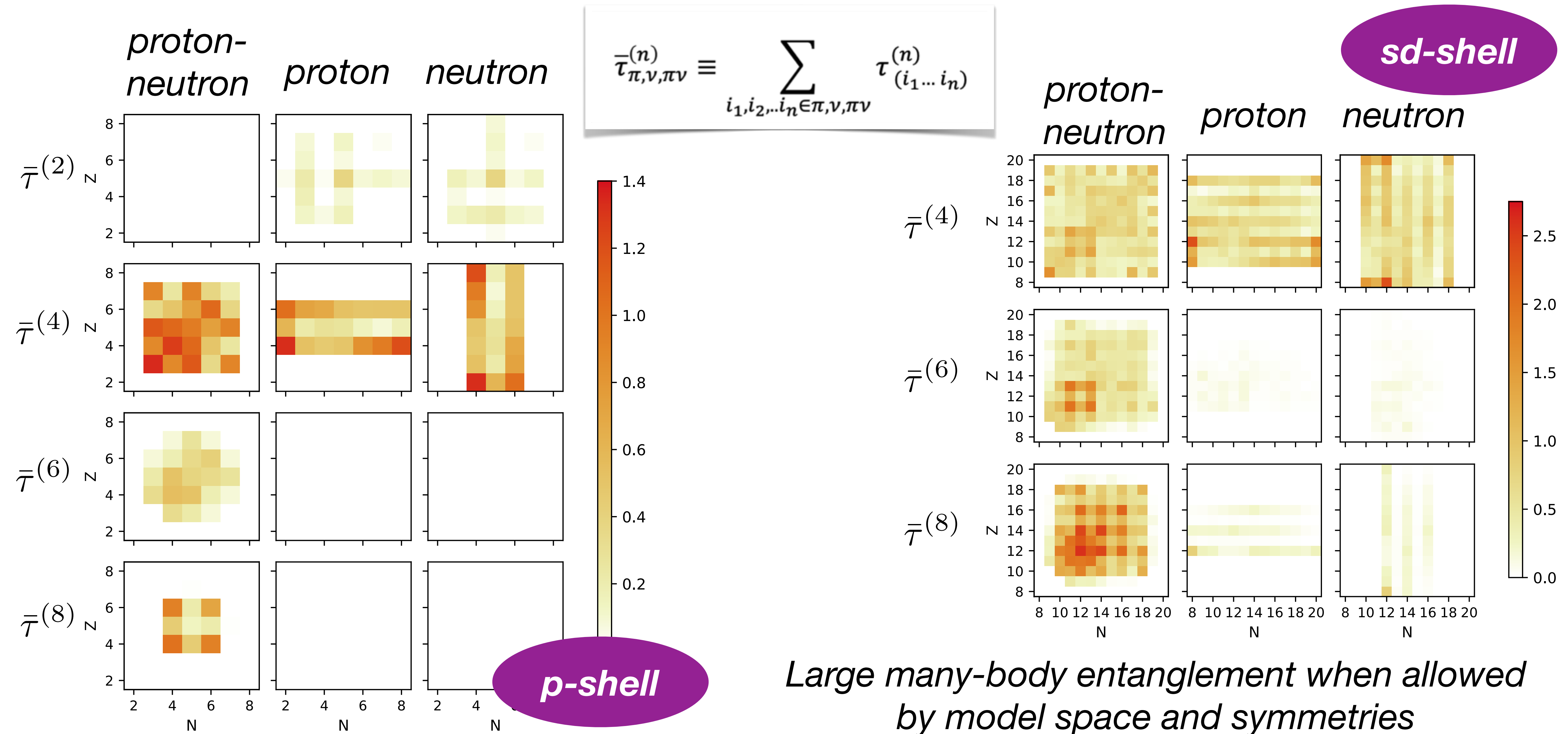
<https://www.ectstar.eu/workshops/many-body-quantum-magic-mbqm-2026/>

Looking for sponsors!

THANK You!

BACKUP

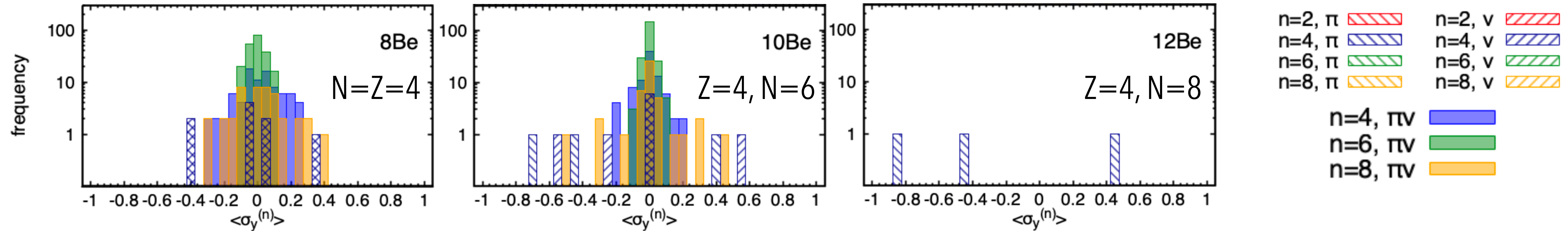
Multi-Partite Entanglement in Shell-Model Nuclei



Multi-Body Entanglement in p -shell (12-qubit) Nuclei

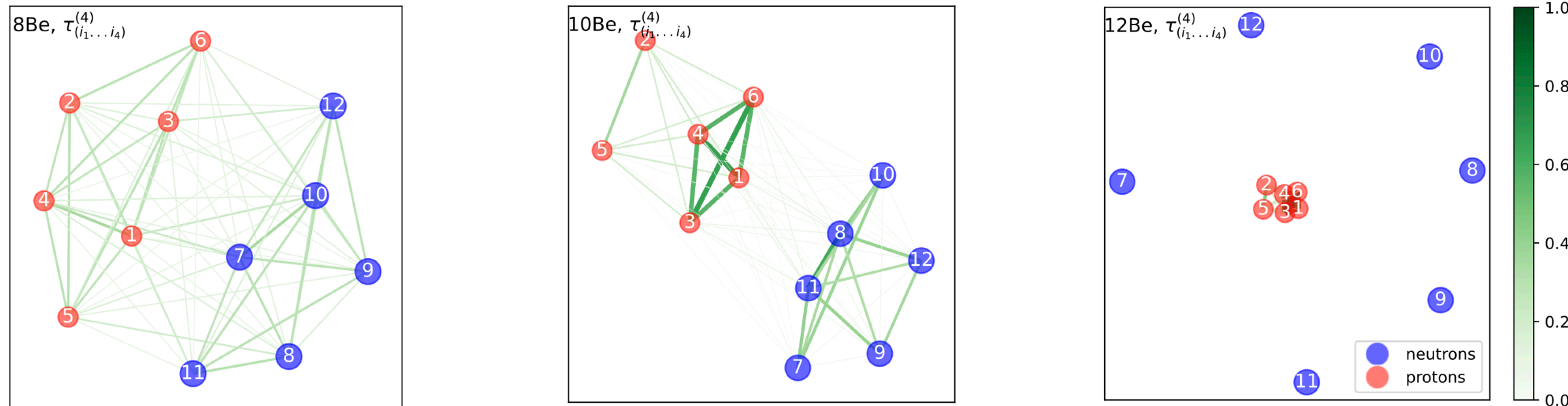
Brökemeier, CR+, PRC 111, 034317 (2025)

*Distribution of the $\sigma_y^{\otimes n}$ Pauli strings expectation values



- proton-neutron entanglement is more collective than pure proton or neutron entanglement
- large proton-neutron 8-tangles $\rightarrow$ hint of alpha correlations?

*Network plots

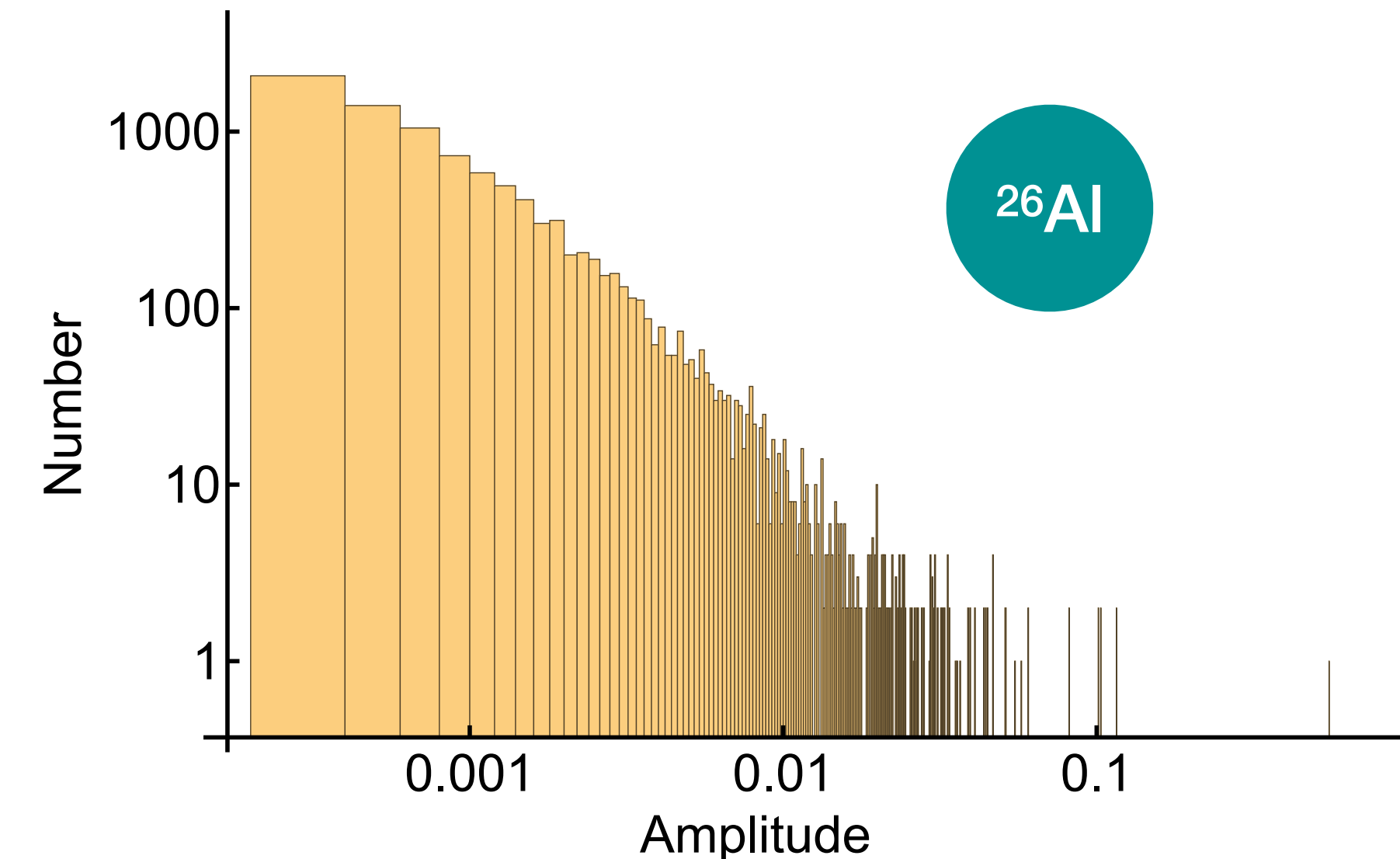


$$e_{i_1 i_2}^{(4)} = \sum_{i_3 < i_4} \tau_{(i_1, i_2, i_3, i_4)}^{(4)}$$

Protons become more entangled as neutrons are added $\rightarrow$ in accordance with Pérez-Obiol+ Eur. Phys. J. A 59, 240 (2023)

Magic in sd-shell nuclei

- SREs still require $d^2 = 4^{n_{qubits}}$ expectation values
- MCMC techniques can be used to compute SREs in large systems *Tarabunga et al PRX Quantum 4, 040317 (2023)*
- However the distribution of amplitudes in the wave function of collective nuclei slows down the convergence of MCMC

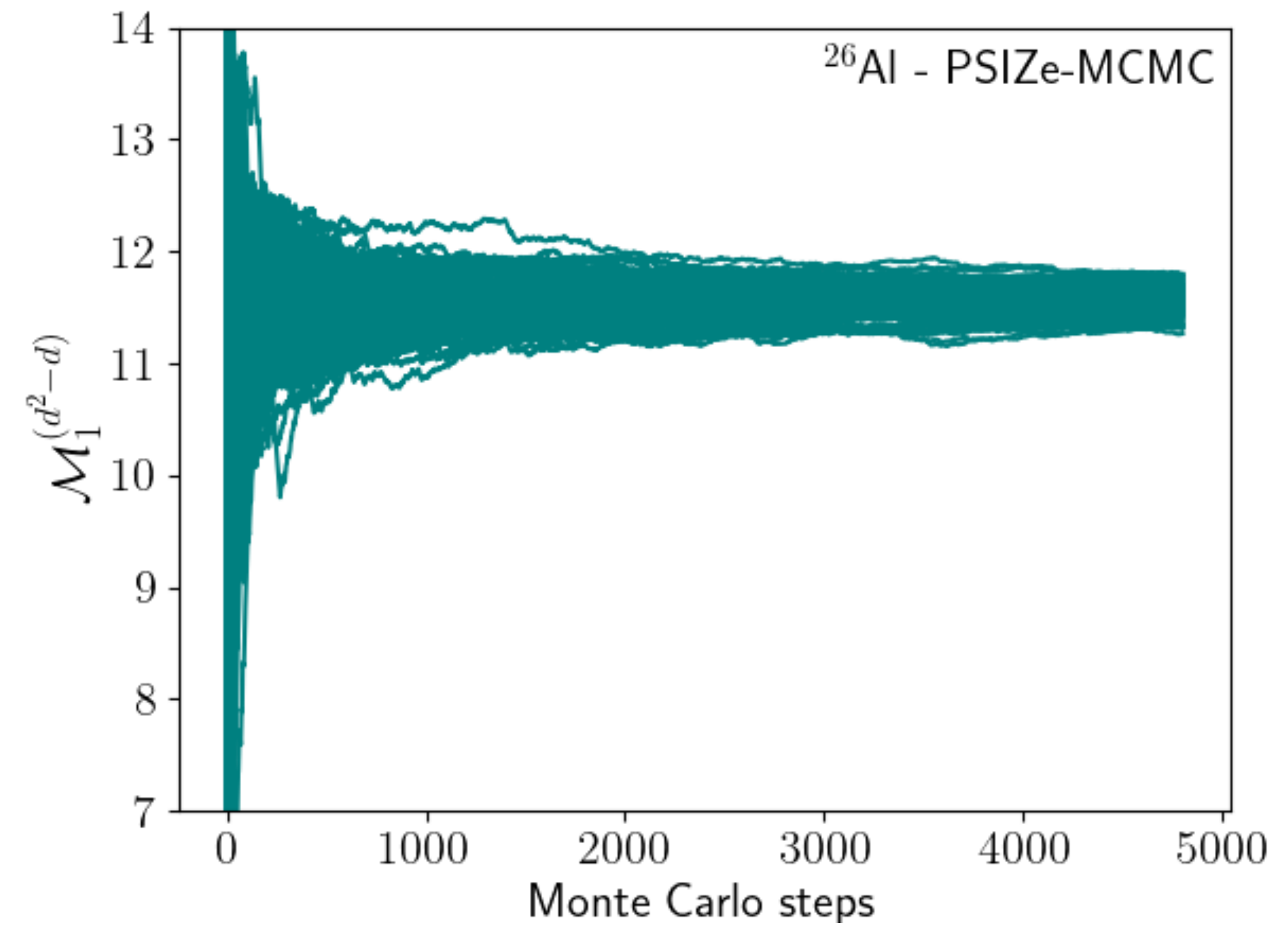
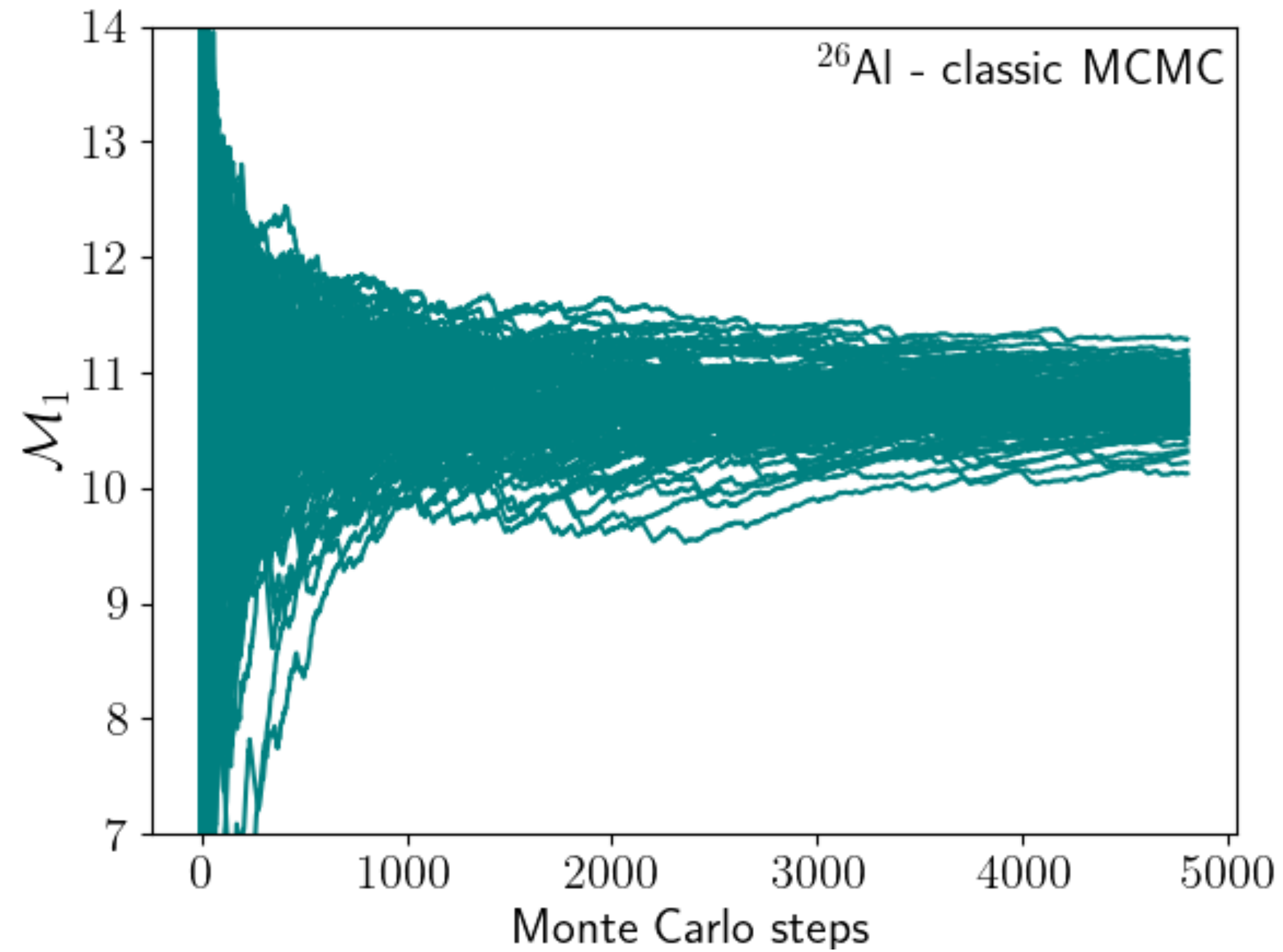


➡ “Pauli-String IZ exact MCMC” (PSIZE-MCMC) algorithm:

Expectation values of IZ strings computed exactly, MCMC samples to remaining space

Brökemeier, Hengstenberg, Keeble, CR, Rocco & Savage, arXiv:2409.12064

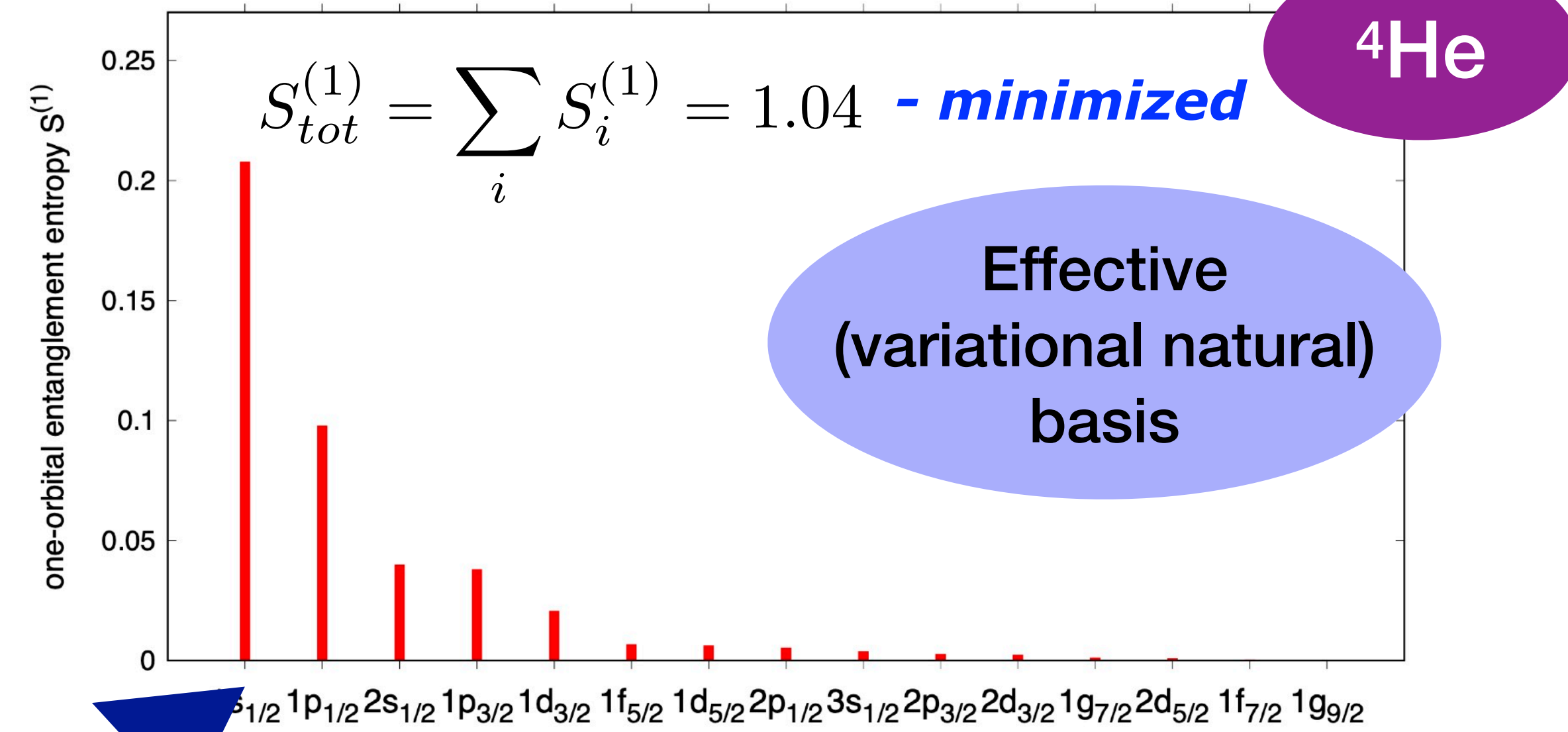
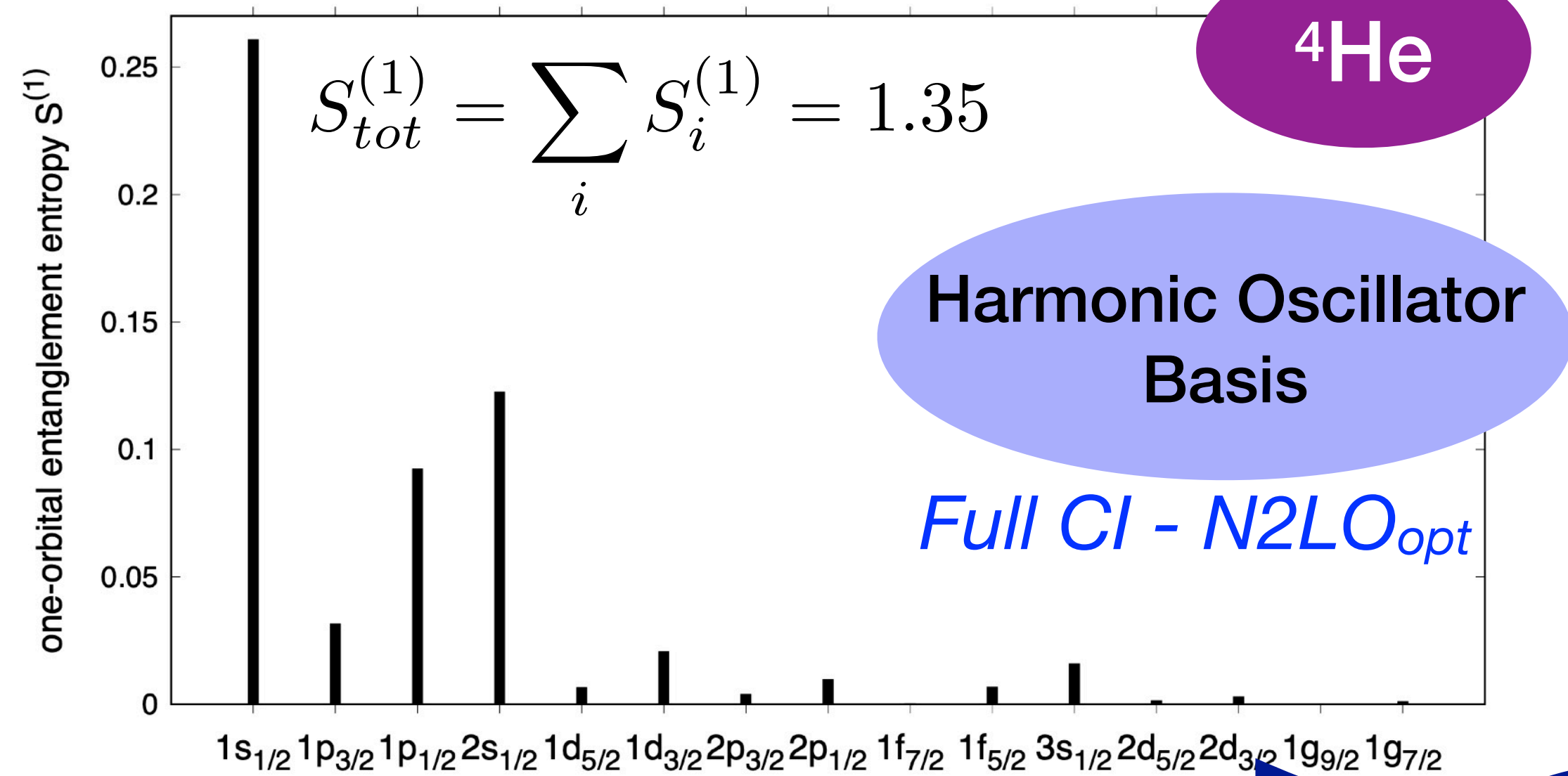
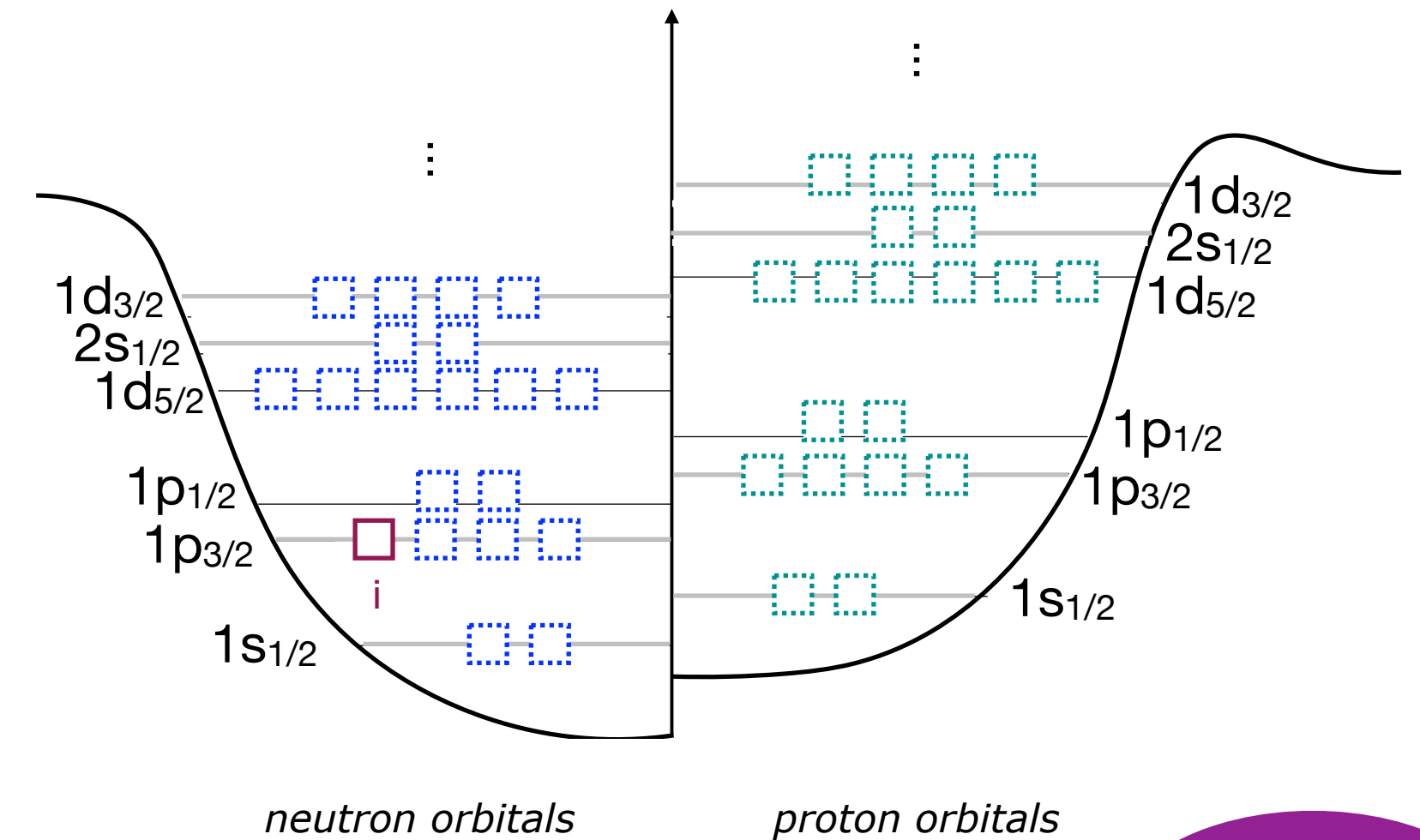
Magic in Nuclei: PSIZe-MCMC algorithm



Entanglement Rearrangement In Nuclei

Single-orbital Von Neumann entropy:

$$S_{(i)}^{(1)} = -\text{Tr} [\rho^{(i)} \ln \rho^{(i)}]$$



$$\hat{U} = e^{i\hat{T}}$$

$$\hat{T} = \sum_{ij} T_{ij} a_i^\dagger a_j$$

Beyond stabilizer Ground State with Magic Injection

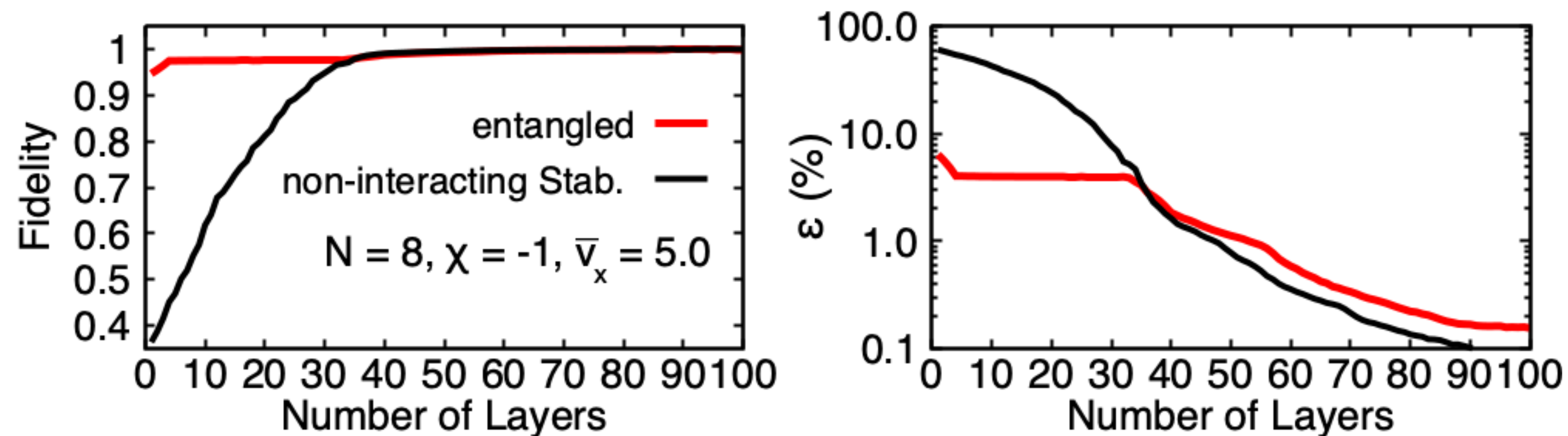
* Grimsley+ Nature Comm. 10, 3007 (2019)

★ ADAPT-VQE *

$$|\Phi(\theta_1, \dots, \theta_L)\rangle = \prod_{l=1}^L e^{i\theta_l \hat{T}_l} |\phi\rangle \quad \frac{\partial E}{\partial \theta_l} \Big|_{\theta_l=0} = -i \langle \Phi_{l-1} | [\hat{T}_l, \hat{H}] | \Phi_{l-1} \rangle$$

For the LMG model: $\hat{T}_{ij}^{\pm} = X_i Y_j \pm Y_i X_j$

CR arXiv:2505.02923



- Long plateaux due to very small energy gradients for many iterations
- Destroys the coherence, and rebuilds it

Symmetry-guided mapping of the Agassi model onto qudit systems

★ Time evolution – circuits for simulations using qu5its

• Hamiltonian mapping to qu5its:

$$\hat{H} = \sum_{j=1}^{\Omega/2} \sum_{j' \neq j=1}^{\Omega/2} H_{(jj')}^{(2)}$$

Acts on 2 qu5its j, j'

• Trotter decomposition at leading order:

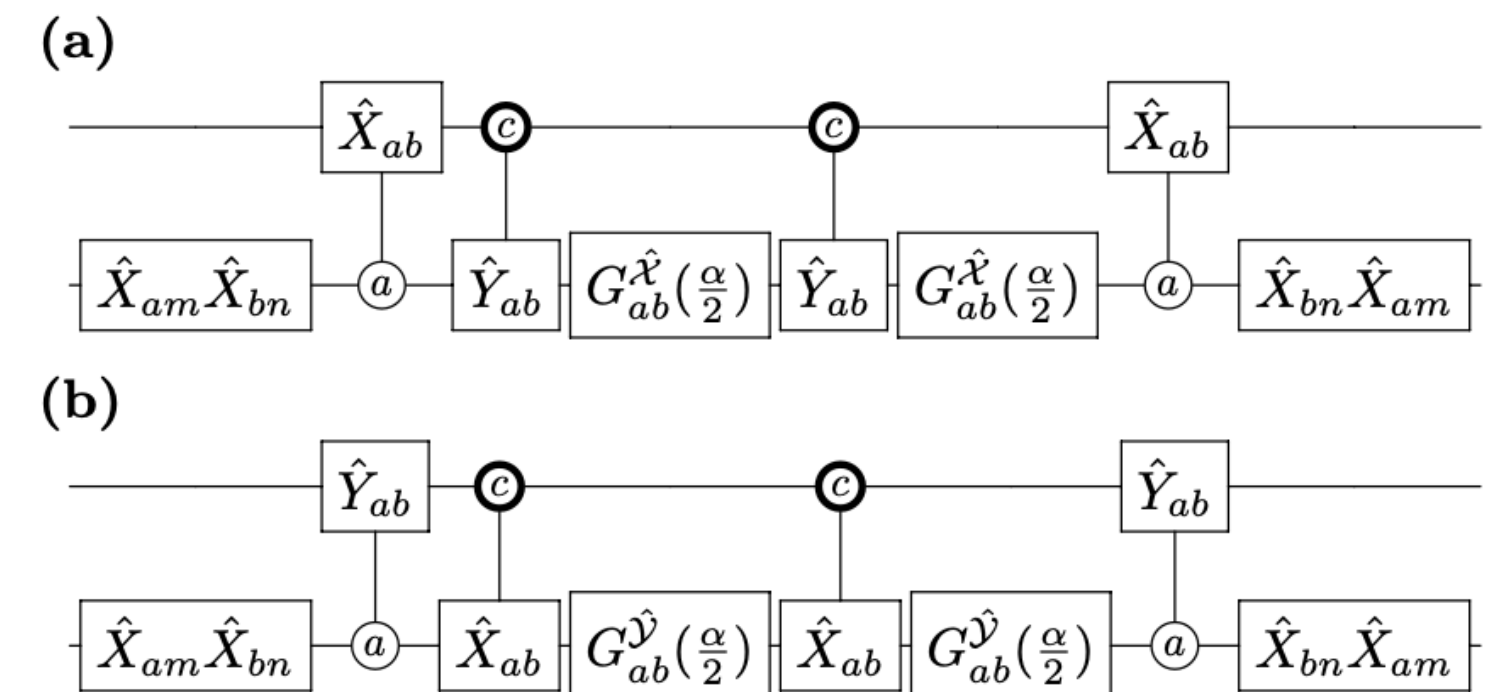
$$\hat{U}(t) = e^{-i\hat{H}t} \simeq \left(e^{-i\hat{H}\Delta t} \right)^{n_{Trot}}$$

$$e^{-i\hat{H}\Delta t} = e^{-i \sum_{jj'} \hat{H}_{jj'}^{(2)} \Delta t} \simeq \prod_{jj'} \prod_a e^{-i\hat{H}_{jj'}^{(2,a)} \Delta t}$$

$$\begin{aligned} H^{(2)} &\equiv \sum_a \hat{H}^{(2,a)} \\ &= \left[\varepsilon \hat{j}_z - (V + g) \hat{\mathcal{X}}_{13} - g \hat{N}_{\text{pairs}} \right] \otimes \hat{I}_5 \\ &\quad + \hat{I}_5 \otimes \left[\varepsilon \hat{j}_z - (V + g) \hat{\mathcal{X}}_{13} - g \hat{N}_{\text{pairs}} \right] \\ &\quad - V \sum_{r,s \in \{(12), (23)\}} \left(\hat{\mathcal{X}}_r \otimes \hat{\mathcal{X}}_s - \hat{\mathcal{Y}}_r \otimes \hat{\mathcal{Y}}_s \right) \\ &\quad - \frac{g}{2} \sum_{r,s \in \{(01), (03), -(14), -(34)\}} \left(\hat{\mathcal{X}}_r \otimes \hat{\mathcal{X}}_s + \hat{\mathcal{Y}}_r \otimes \hat{\mathcal{Y}}_s \right) \end{aligned}$$

generators of Givens rotations

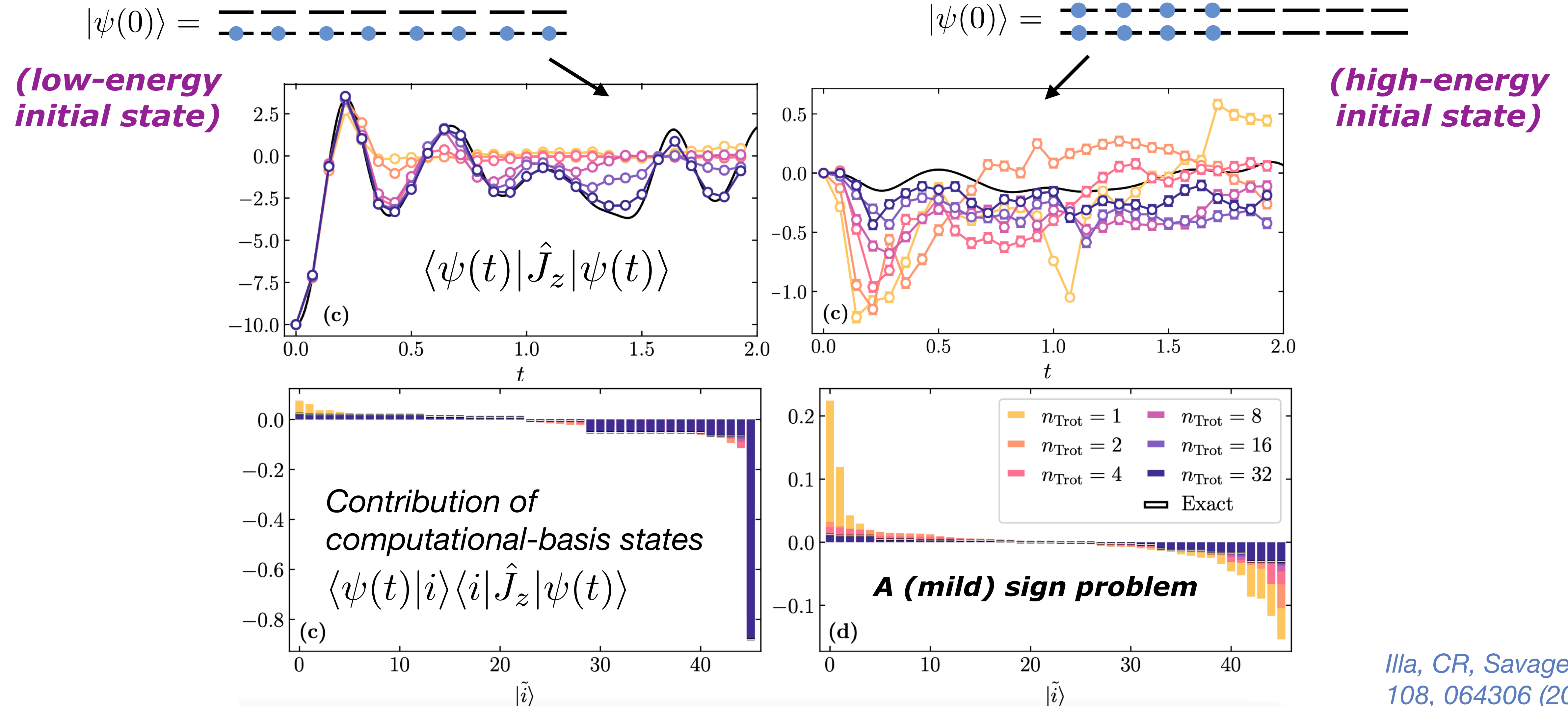
$$G_{abmn}^{\mathcal{X}\mathcal{X}}(\alpha) \quad G_{abmn}^{\mathcal{Y}\mathcal{Y}}(\alpha)$$



Circuits for $G_{abmn}^{\mathcal{X}\mathcal{X}}(\alpha)$ and $G_{abmn}^{\mathcal{Y}\mathcal{Y}}(\alpha)$

Dynamics simulations of the Agassi model: low- vs high-energy initial states

★ Trotterized time evolution on a qudit-system simulator using Google's *cirq* software:

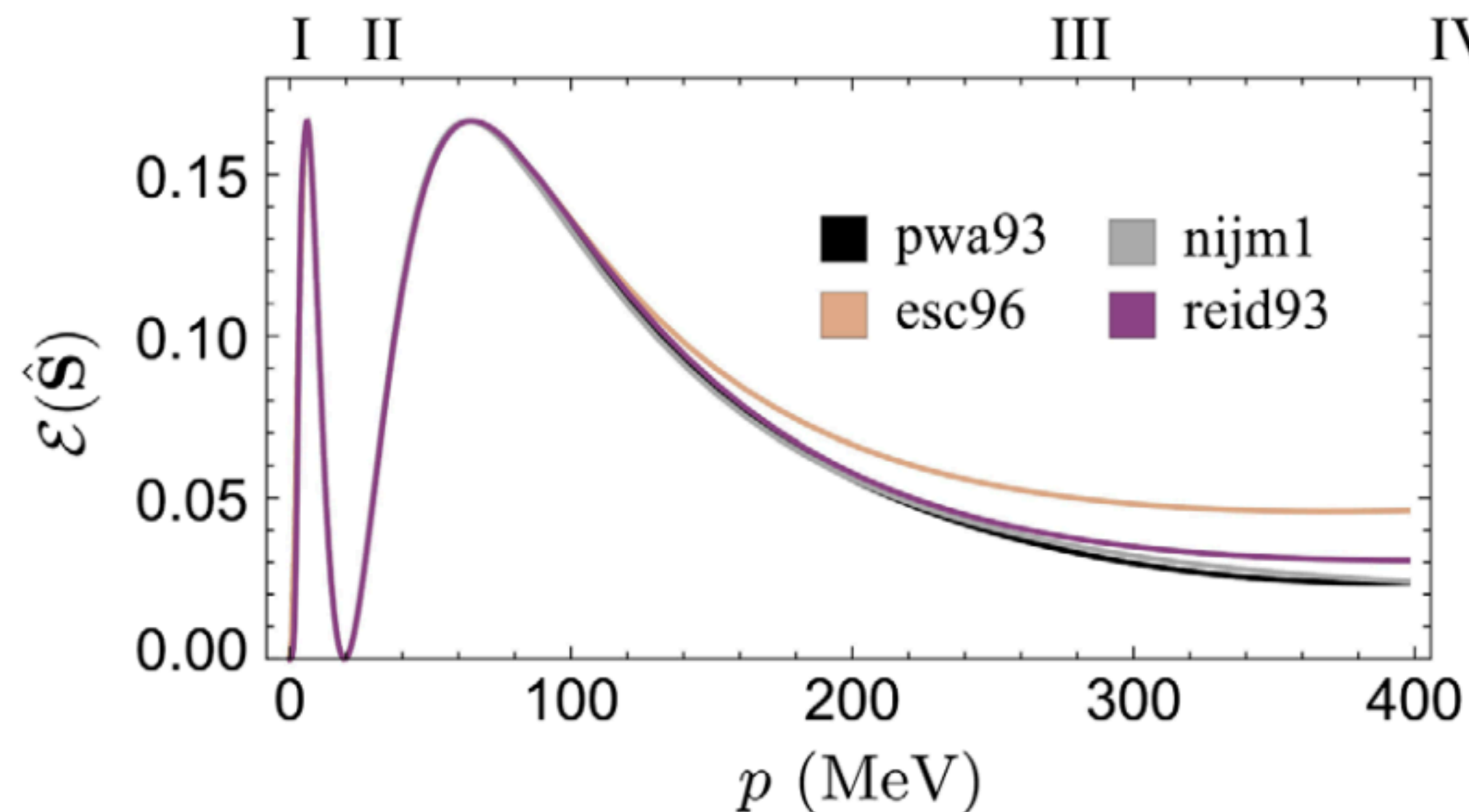


The Magic Power of Nuclear and Hyper-Nuclear Forces

Beane, Kaplan, Klco, Savage PRL 122, 102001 (2019)

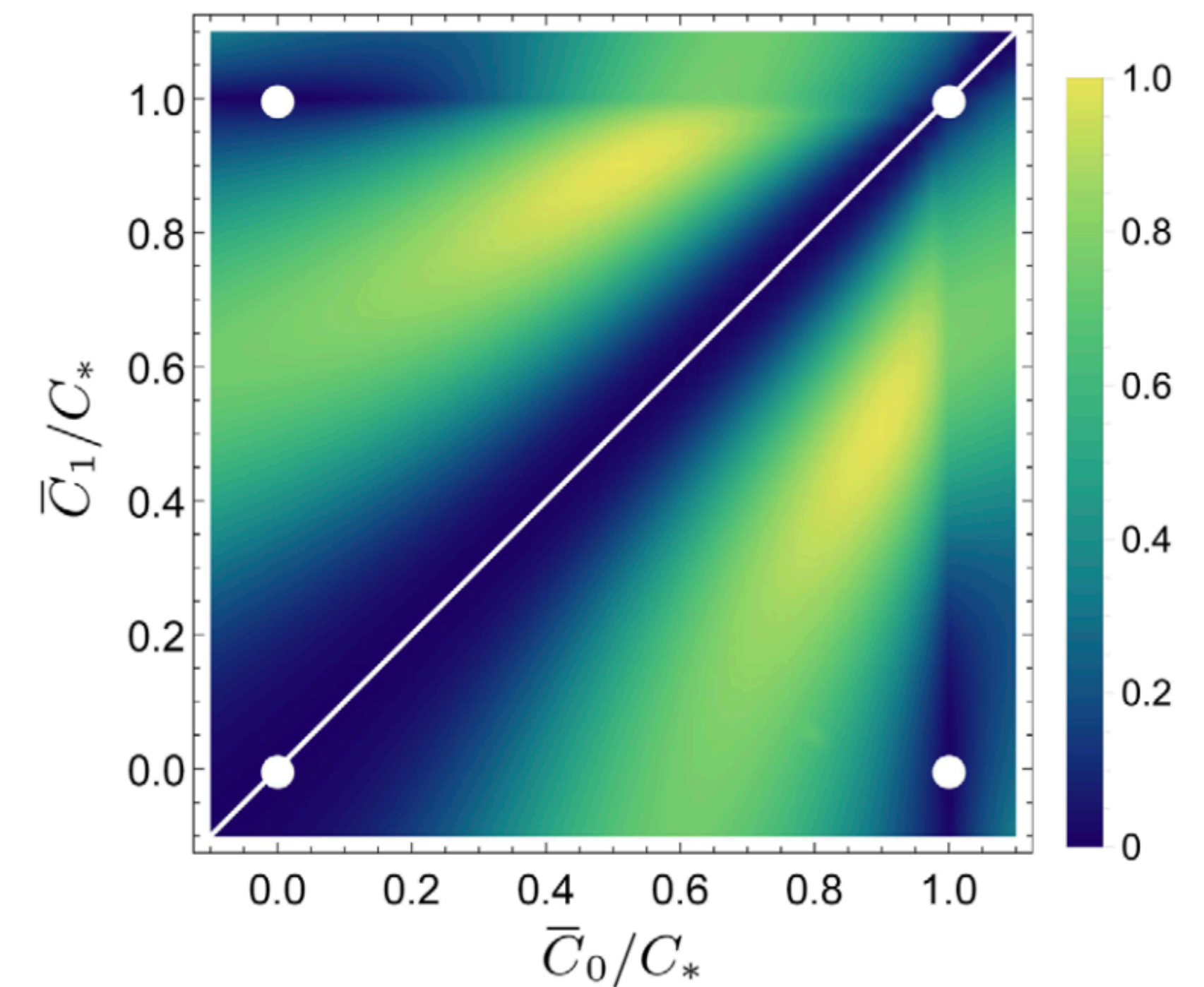
See also Liu+ PRC 107, 025204 (2023); PLB 856, 138899 (2024) ;
Kirchner+ Few Body Syst. 65, 29 (2024)

Entanglement Power $\mathcal{E}(\hat{\mathbf{S}}) = 1 - \int \frac{d\Omega_1}{4\pi} \frac{d\Omega_2}{4\pi} \text{Tr}_1[\hat{\rho}_1^2]$



S-wave NN scattering

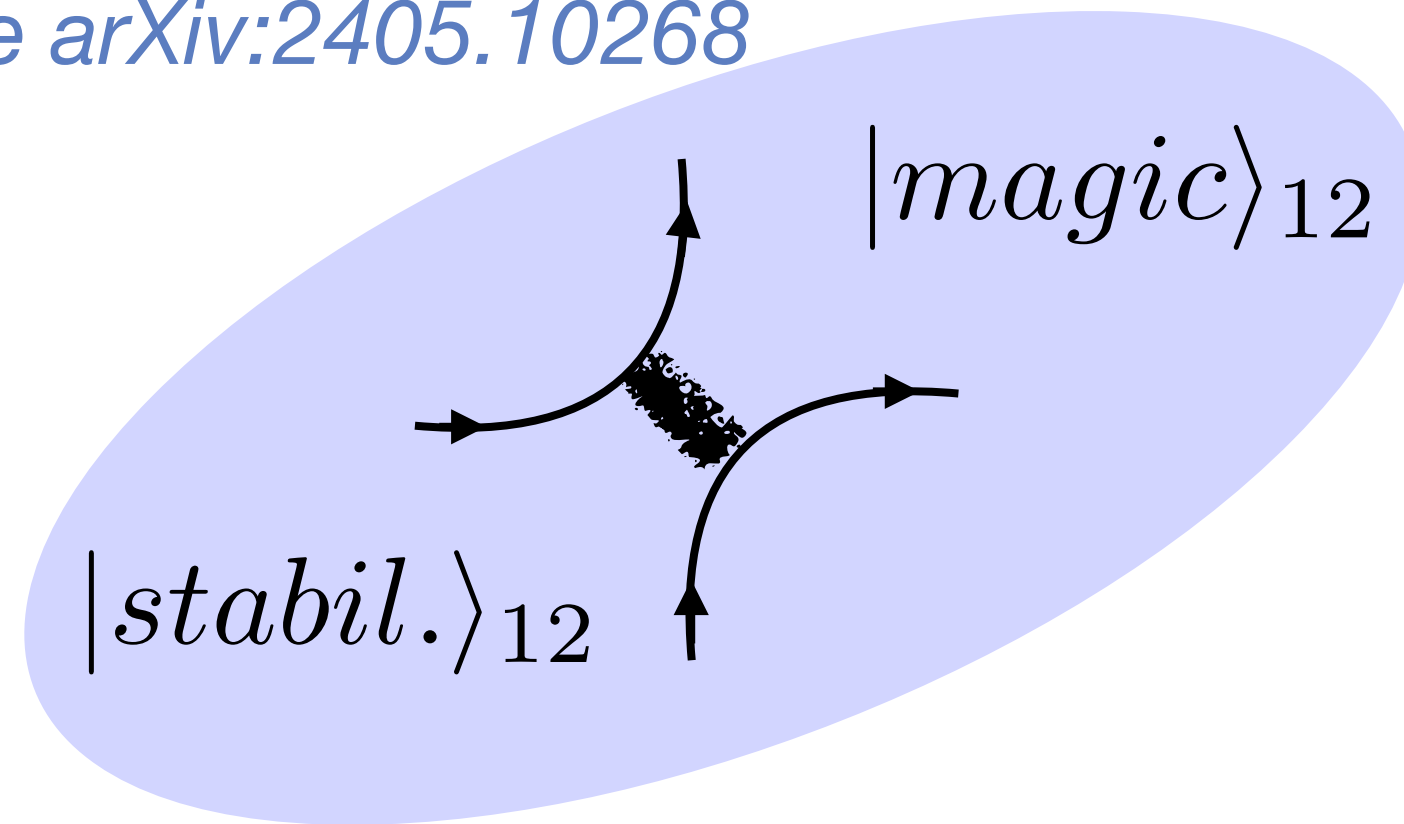
$$\hat{\mathbf{S}} = \frac{1}{4} (3 e^{2i\delta_1} + e^{2i\delta_0}) \hat{\mathbf{1}} + \frac{1}{4} (e^{2i\delta_1} - e^{2i\delta_0}) \hat{\boldsymbol{\sigma}} \cdot \hat{\boldsymbol{\sigma}}$$



vanishing entanglement power occurs at
points of emergent global symmetries

The Magic Power of Nuclear and Hyper-Nuclear Forces

CR & M. J. Savage arXiv:2405.10268



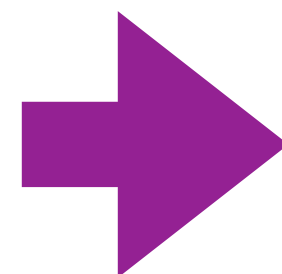
Magic power of the S-matrix:

$$\overline{\mathcal{M}}(\hat{\mathbf{S}}) \equiv \frac{1}{\mathcal{N}_{ss}} \sum_{i=1}^{\mathcal{N}_{ss}} \mathcal{M}(\hat{\mathbf{S}} |\Psi_i\rangle)$$

Average fluctuations in magic induced by the S-matrix

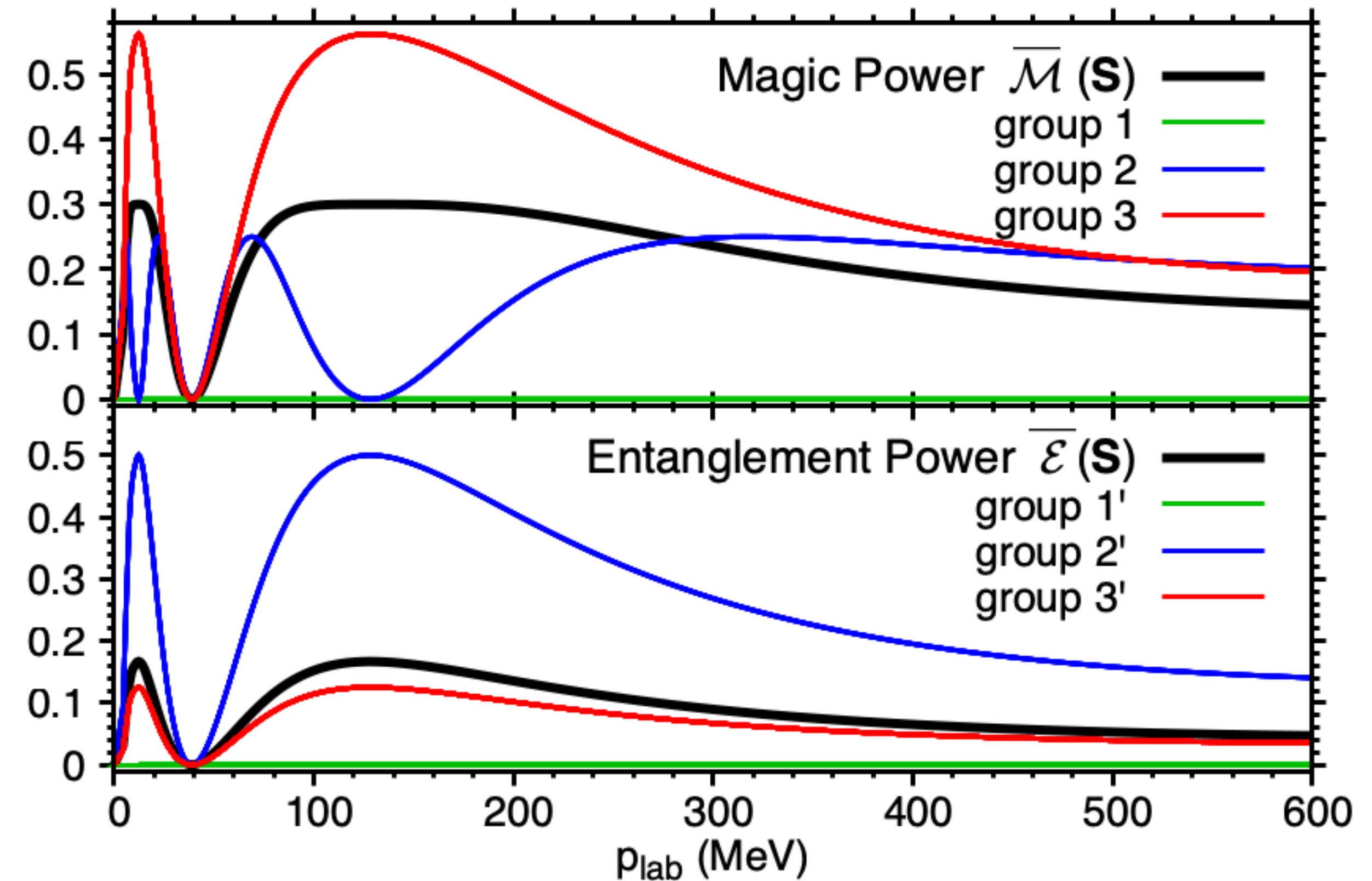
Entanglement power of the S-matrix

$$\overline{\mathcal{E}}(\hat{\mathbf{S}}) \equiv \frac{1}{\mathcal{N}_{ss}^{TP}} \sum_{i=1}^{\mathcal{N}_{ss}^{TP}} \mathcal{E}(\rho_i^{(1)}(\hat{\mathbf{S}}))$$



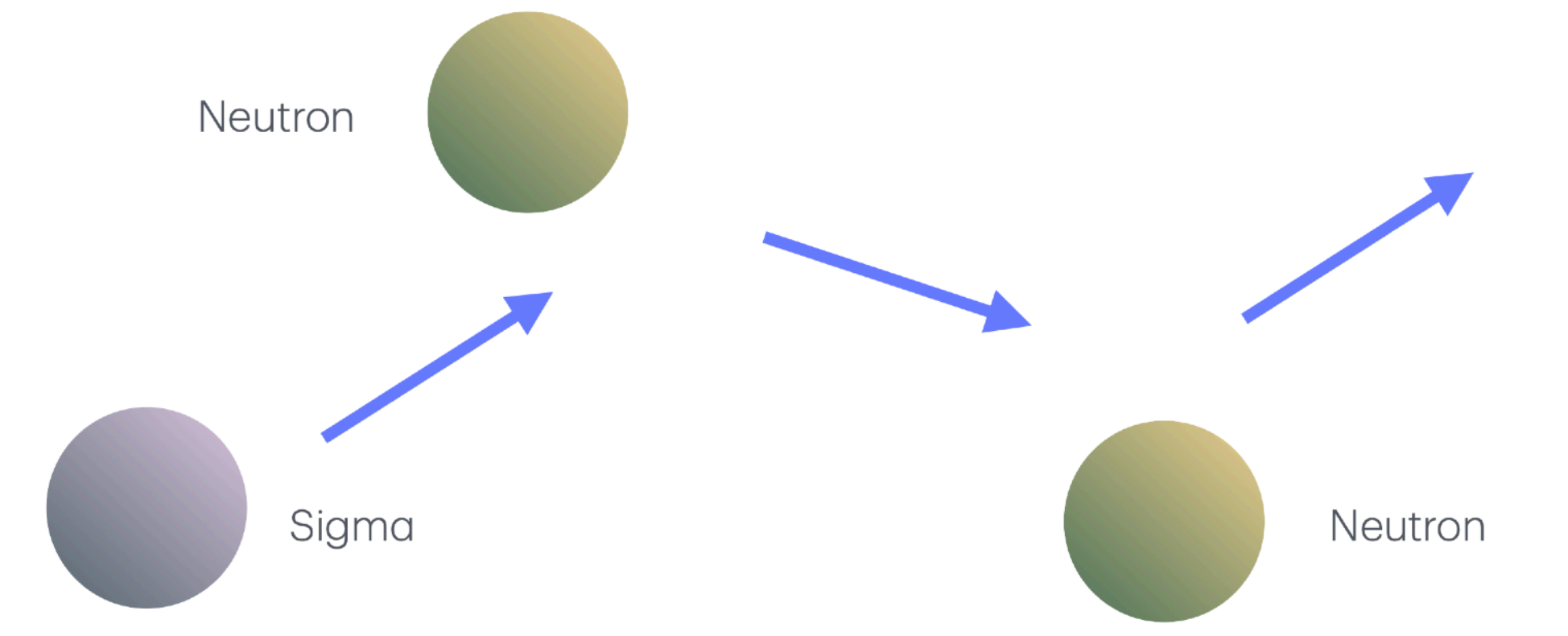
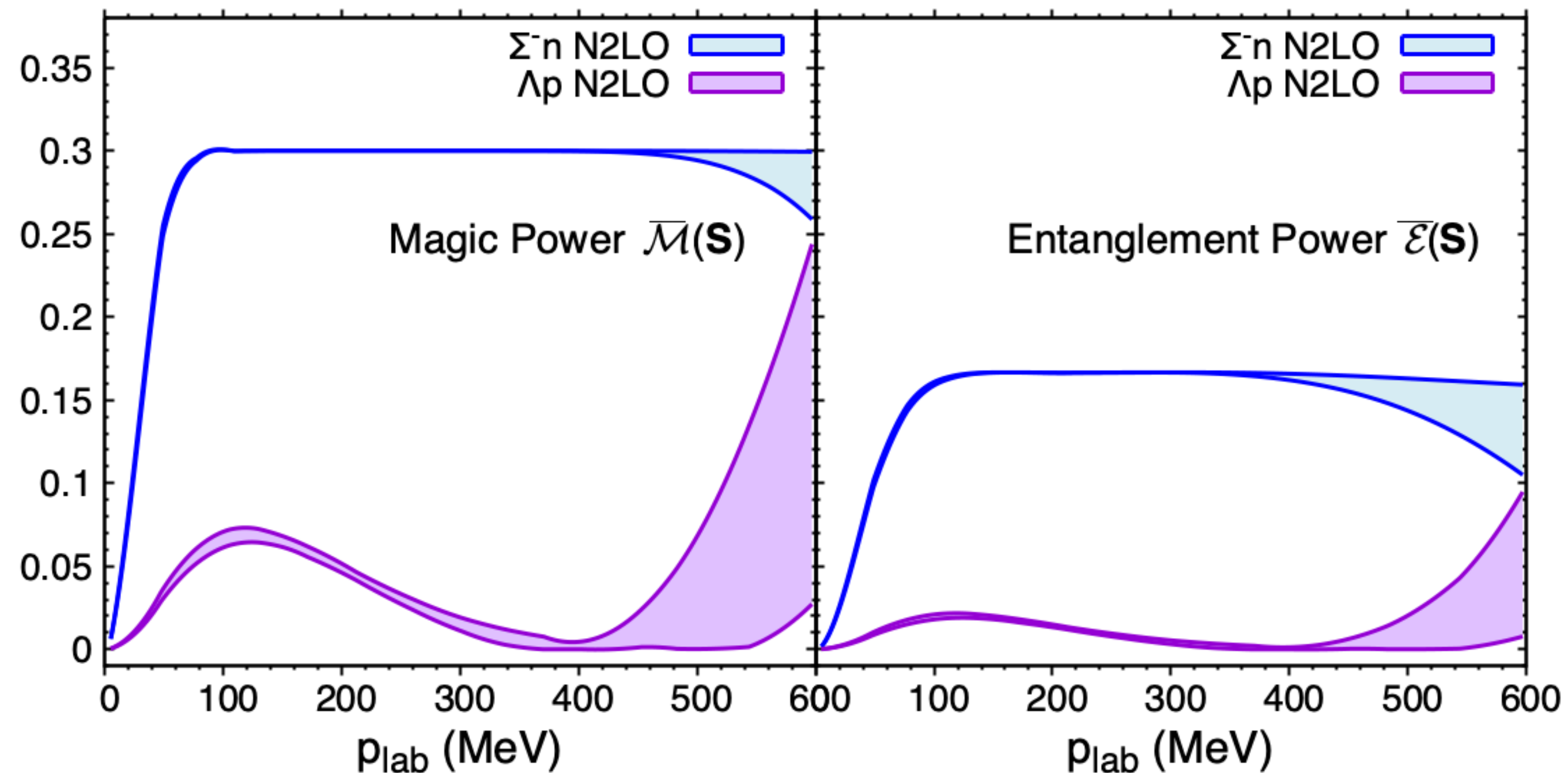
Same results as in Beane+ PRL 122, 102001 (2019) with continuous integration over spin orientations of initial tensor-product states

$$\overline{\mathcal{M}}(\hat{\mathbf{S}}) = \frac{3}{20} \left(3 + \cos(4 \Delta\delta) \right) \sin^2(2 \Delta\delta) \quad \overline{\mathcal{E}}(\hat{\mathbf{S}}) = \frac{1}{6} \sin^2(2 \Delta\delta)$$

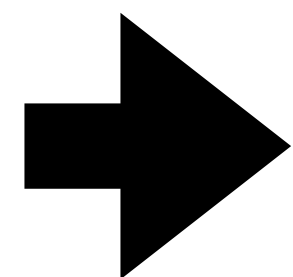


The Magic Power of Nuclear and Hyper-Nuclear Forces

CR & M. J. Savage arXiv:2405.10268



Magic between neutrons is induced by successive Σ -n scatterings (assumed no decoherence)



Σ -baryon is identified as a potential candidate catalyst for enhanced spreading of magic and entanglement in dense matter

Entanglement power also in Beane+ PRL 122, 102001 (2019); Liu+ PLB 856, 138899 (2024)