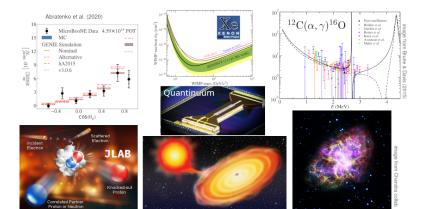
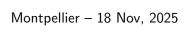
Towards quantum simulation of nuclear reactions

Alessandro Roggero









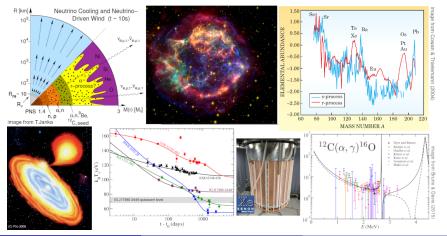


The need for ab-initio many-body dynamics in NP

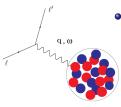
- $m{\nu}$ scattering for supernovae explosion and NS cooling
- capture reactions for crust heating and nucleosynthesis

- cross sections for dark-matter discovery and neutrino physics
- transport properties of neutron star matter for X-ray emission

1/18



Inclusive cross section and the response function

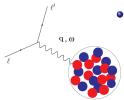


cross section determined by the response function

$$R_O(\omega) = \sum_f \left| \langle f | \hat{O} | \Psi_0 \rangle \right|^2 \delta \left(\omega - E_f + E_0 \right)$$

ullet excitation operator \hat{O} specifies the vertex

Inclusive cross section and the response function

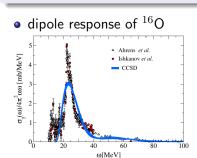


cross section determined by the response function

$$R_O(\omega) = \sum_f \left| \langle f | \hat{O} | \Psi_0 \rangle \right|^2 \delta \left(\omega - E_f + E_0 \right)$$

 \bullet excitation operator \hat{O} specifies the vertex

Extremely challenging classically for strongly correlated quantum systems



quasi-elastic EM response of ¹²C

Bacca et al. PRL(2013) LIT+CC

Lovato et al. PRL(2016) GFMC+Laplace

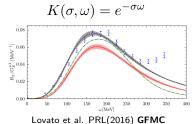
Many body dynamics with Integral Transforms

A possible way out with integral transform techniques

Efros (1989), Carlson & Schiavilla (1992), Efros, Leidemann & Orlandini (1994)

$$T(\sigma) = \int d\omega K(\sigma, \omega) R_O(\omega) = \langle 0 | \hat{O}^{\dagger} K \left(\sigma, \hat{H} - E_0 \right) \hat{O} | 0 \rangle$$

Laplace



Lorentz

$$K(\sigma,\omega;\Gamma) = \frac{\Gamma}{\Gamma^2 + (\sigma - \omega)^2}$$

Bacca et al. PRL(2013) LIT+CC

3/18

PROBLEM: the inversion procedure is often ill-posed, difficult to assign error bars on the reconstructed response function

Alessandro Roggero Nuclear reactions Montpellier - 18 Nov, 2025

Many body dynamics with Integral Transforms II

A possible way out with integral transform techniques

Efros (1989), Carlson & Schiavilla (1992), Efros, Leidemann & Orlandini (1994)

$$T(\sigma) = \int d\omega K(\sigma, \omega) R_O(\omega) = \langle 0 | \hat{O}^{\dagger} K \left(\sigma, \hat{H} - E_0 \right) \hat{O} | 0 \rangle$$

Fourier

$$K(\sigma, \omega) = e^{-i\sigma\omega}$$

$$T(\sigma) = \langle 0|\hat{O}^{\dagger} \exp\left(-i\sigma(\hat{H} - E_0)\right) \hat{O}|0\rangle = \langle 0|\hat{O}^{\dagger}(\sigma)\hat{O}(0)|0\rangle$$

The transformation is unitary so the inversion is "easy"

Many body dynamics with Integral Transforms II

A possible way out with integral transform techniques

Efros (1989), Carlson & Schiavilla (1992), Efros, Leidemann & Orlandini (1994)

$$T(\sigma) = \int d\omega K(\sigma, \omega) R_O(\omega) = \langle 0 | \hat{O}^{\dagger} K \left(\sigma, \hat{H} - E_0 \right) \hat{O} | 0 \rangle$$

Fourier

$$K(\sigma,\omega) = e^{-i\sigma\omega}$$

$$T(\sigma) = \langle 0|\hat{O}^{\dagger} \exp\left(-i\sigma(\hat{H} - E_0)\right) \hat{O}|0\rangle = \langle 0|\hat{O}^{\dagger}(\sigma)\hat{O}(0)|0\rangle$$

The transformation is unitary so the inversion is "easy"

PROBLEM: we don't really have efficient and unbiased methods to do time evolution for interacting many-particle systems

ADVANTAGE: if we did, we could do more than linear response!

Many body dynamics with Integral Transforms II

A possible way out with integral transform techniques

Efros (1989), Carlson & Schiavilla (1992), Efros, Leidemann & Orlandini (1994)

$$T(\sigma) = \int d\omega K(\sigma, \omega) R_O(\omega) = \langle 0 | \hat{O}^{\dagger} K \left(\sigma, \hat{H} - E_0 \right) \hat{O} | 0 \rangle$$

Fourier

$$K(\sigma,\omega) = e^{-i\sigma\omega}$$

$$T(\sigma) = \langle 0|\hat{O}^{\dagger} \exp\left(-i\sigma(\hat{H} - E_0)\right) \hat{O}|0\rangle = \langle 0|\hat{O}^{\dagger}(\sigma)\hat{O}(0)|0\rangle$$

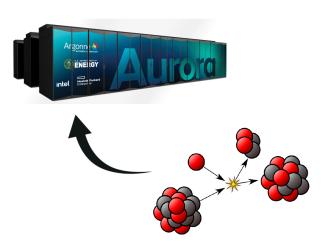
The transformation is **unitary** so the inversion is "easy"

PROBLEM: we don't really have efficient and unbiased classical methods to do time evolution for interacting many-particle systems

ADVANTAGE: if we did, we could do more than linear response!

Simulations of nuclear dynamics

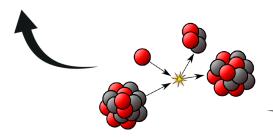
Classical simulations



Simulations of nuclear dynamics

Classical simulations





Quantum simulations



IBM



Simulations of nuclear dynamics

Classical Quantum simulations simulations Argonne 4 ENEIGY Blume-Kohout et al. (2013)

Inclusive cross section from QC

For inclusive scattering seems reasonable to get real-time correlators

$$R_O(\omega) = \int dt e^{i\omega t} C(t)$$
 with $C(t) = \langle \Psi_0 | O(t) O(0) | \Psi_0 \rangle$

• Can be done "easily" using one additional qubit (Somma et al. (2001))

Inclusive cross section from QC

For inclusive scattering seems reasonable to get real-time correlators

$$R_O(\omega) = \int dt e^{i\omega t} C(t) \quad \text{with} \quad C(t) = \langle \Psi_0 | O(t) O(0) | \Psi_0 \rangle$$

• Can be done "easily" using one additional qubit (Somma et al. (2001))

Turns out it is much more convenient to compute moments

 $Somma(2019),\ AR\ et\ al.(2020),\ AR(2020),\ Rall(2020),\ Baroni\ et\ al.(2021),\ AR\&Sobczyk(2022),\ Kiss\ et\ al.(2023)$

$$M_F(t) = \langle \Psi_0 | Oe^{-itH}O | \Psi_0 \rangle \qquad \qquad M_C(n) = \langle \Psi_0 | O\mathsf{T}_n\left(H\right)O | \Psi_0 \rangle$$

• Chebyshev Polynomials T_n appear naturally [e.g. in QSP/QSVT]

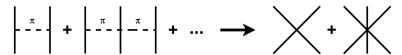
$$f(H) |\Phi\rangle = \sum_{n=0}^{\infty} c_k \mathsf{T}_n(H) |\Phi\rangle \approx \sum_{n=0}^{M} c_k \mathsf{T}_n(H) |\Phi\rangle$$

 Very popular recently for early fault-tolerant ground state energy estimation (and preparation)
 [Lin & Tong (2022), Dong et al. (2022), Wan et al. (2022)]

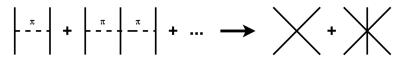
Alessandro Roggero Nuclear reactions Montpellier - 18 Nov, 2025

6/18

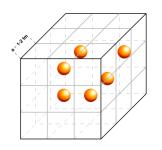
As a starting point we can use simple nuclear interactions without pions



As a starting point we can use simple nuclear interactions without pions



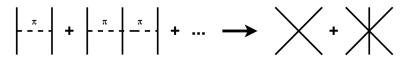
We then place nucleons on a spatial lattice to regularize them



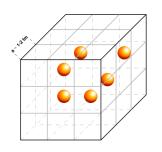
Minimal setup

- 10^3 lattice with spacing $a \approx 1 2fm$
- 4 spin-isospin states for each particle
 → we need at least 4000 orbitals
- for energy resolution $\Delta\omega$ we need total evolution time $T\approx 1/\Delta\omega$

As a starting point we can use simple nuclear interactions without pions



We then place nucleons on a spatial lattice to regularize them



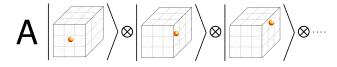
Minimal setup

- 10^3 lattice with spacing $a \approx 1 2fm$
- 4 spin-isospin states for each particle \longrightarrow we need at least 4000 orbitals
- for energy resolution $\Delta\omega$ we need total evolution time $T\approx 1/\Delta\omega$

7 / 18

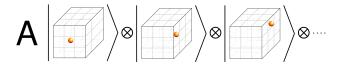
- $\bullet~10^{11}-10^{12}$ operations and ≈ 4000 qubits [Roggero et al. PRD (2020)]
- 10^9-10^{11} operations and ≈ 6000 qubits [J.Watson et al. arXiv:2312.05344]

For nuclear reactions we want large volumes V with few particles N inside \to IDEA: why not use first quantization instead?



Should cost $N\log(V)$ qubits and hopefully not too many more gates

For nuclear reactions we want large volumes V with few particles N inside → IDEA: why not use first quantization instead?



Should cost $N \log(V)$ qubits and hopefully not too many more gates

Long history of first-quantization methods for quantum simulations of Chemistry Abrams & Lloyd (1997), Kassal et al. (2008), Jones et al. (2012), Babbush et al. (2019),...

- anti-symmetrization costs $\mathcal{O}(N\log(N)\log(V))$ [D.Berry et al. (2018)]
- kinetic energy simple in momentum space costing $\mathcal{O}(N\log^2(V))$

The main bottleneck is evolving under the potential energy

$$e^{-it\hat{U}}|x\rangle = e^{-itU(x)}|x\rangle$$

8 / 18

The main bottleneck is evolving under the potential energy

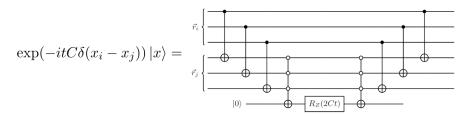
$$e^{-it\hat{U}}|x\rangle = e^{-itU(x)}|x\rangle$$

9/18

In a contact theory this is not necessarily true anymore. For instance

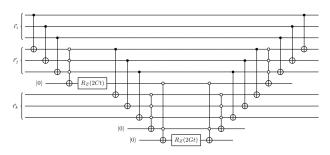
$$\hat{U}|x\rangle = \left[\sum_{ij}^{N} U_{ij}(x_i, x_j)\right]|x\rangle = \left[C\sum_{ij}^{N} \delta(x_i - x_j)\right]|x\rangle$$

but then all pairs commute with each other and for one pair



Spagnoli, Lissoni, Roggero, arXiv:2507.22814

The same idea can be extended to three-body contacts too



With this implementation, the step cost is $\mathcal{O}(N^3 \log(V))$

Turns out 1st quant. can beat 2nd quant. in both memory and gates

$$C_{2nd}^T = \mathcal{O}\left(\frac{t^{3/2}}{\sqrt{\epsilon}}\sqrt{N}V\right) \qquad C_{1st}^T = \mathcal{O}\left(\frac{t^{3/2}}{\sqrt{\epsilon}}N^{7/2}\log(V)\right)$$

Spagnoli, Lissoni, Roggero, arXiv:2507.22814

Can we do better using Quantum Signal Processing? [Low & Chuang (2017)]

In order to use QSP we need a **block-encoding** of the Hamiltonian

$$\langle 0|U_H|0\rangle = H/\Lambda$$

Can we do better using Quantum Signal Processing? [Low & Chuang (2017)]

In order to use QSP we need a **block-encoding** of the Hamiltonian

 $\langle 0|U_H|0 \rangle = H/\Lambda$ for time evolution we use it $\mathcal{O}\left(\Lambda t + \log(1/\epsilon)\right)$ times

Can we do better using Quantum Signal Processing? [Low & Chuang (2017)]

In order to use QSP we need a **block-encoding** of the Hamiltonian

$$\langle 0|U_H|0\rangle=H/\Lambda$$
 for time evolution we use it $\mathcal{O}\left(\Lambda t+\log(1/\epsilon)\right)$ times

• efficient block-encodings of the kinetic energy known [Su et al. (2021)]

Can we do better using Quantum Signal Processing? [Low & Chuang (2017)]

In order to use QSP we need a block-encoding of the Hamiltonian

$$\langle 0|U_H|0 \rangle = H/\Lambda$$
 for time evolution we use it $\mathcal{O}\left(\Lambda t + \log(1/\epsilon)\right)$ times

- efficient block-encodings of the kinetic energy known [Su et al. (2021)]
- for the potential we exploit again the fact it is a contact

$$U(x) = C \sum_{i < j} \delta(x_i - x_j) + G \sum_{i < j < k} \delta(x_i - x_j) \delta(x_i - x_k)$$
$$= \frac{3C - G}{6} \sum_{i} \left(\sum_{j \neq i} \delta(x_i - x_j) \right) + \frac{G}{6} \sum_{i} \left(\sum_{j \neq i} \delta(x_i - x_j) \right)^2$$

for any given i, the sum in parenthesis is bounded by 4.

Can we do better using Quantum Signal Processing? [Low & Chuang (2017)]

In order to use QSP we need a **block-encoding** of the Hamiltonian

$$\langle 0|U_H|0 \rangle = H/\Lambda$$
 for time evolution we use it $\mathcal{O}\left(\Lambda t + \log(1/\epsilon)\right)$ times

- efficient block-encodings of the kinetic energy known [Su et al. (2021)]
- for the potential we exploit again the fact it is a contact

$$U(x) = C \sum_{i < j} \delta(x_i - x_j) + G \sum_{i < j < k} \delta(x_i - x_j) \delta(x_i - x_k)$$
$$= \frac{3C - G}{6} \sum_{i} \left(\sum_{j \neq i} \delta(x_i - x_j) \right) + \frac{G}{6} \sum_{i} \left(\sum_{j \neq i} \delta(x_i - x_j) \right)^2$$

for any given i, the sum in parenthesis is bounded by 4. But then that can be done in $\mathcal{O}(N\log(V))$ and outer sum another $\mathcal{O}(N)$

$$C_{2nd}^T = \mathcal{O}\left(\frac{t^{3/2}}{\sqrt{\epsilon}}\sqrt{N}V\right) \qquad C_{1st}^T = \mathcal{O}\left(\frac{t^{3/2}}{\sqrt{\epsilon}}N^{7/2}\log(V)\right)$$

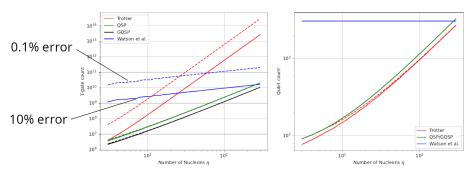
We can do even better using Quantum Signal Processing [Low & Chuang (2017)]

$$C_{1st}^{QSP} = \mathcal{O}\left(N\log(V)\left(Nt + \log\left(\frac{1}{\epsilon}\right)\right)\right)$$

$$C_{2nd}^T = \mathcal{O}\left(\frac{t^{3/2}}{\sqrt{\epsilon}}\sqrt{N}V\right) \qquad C_{1st}^T = \mathcal{O}\left(\frac{t^{3/2}}{\sqrt{\epsilon}}N^{7/2}\log(V)\right)$$

We can do even better using Quantum Signal Processing [Low & Chuang (2017)]

$$C_{1st}^{QSP} = \mathcal{O}\left(N\log(V)\left(Nt + \log\left(\frac{1}{\epsilon}\right)\right)\right)$$

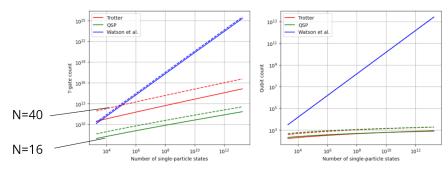


 $\bullet~10^7-10^9$ operations and $\approx 70-500$ qubits [Spagnoli, Lissoni, Roggero (2025)]

$$C_{2nd}^T = \mathcal{O}\left(\frac{t^{3/2}}{\sqrt{\epsilon}}\sqrt{N}V\right) \qquad C_{1st}^T = \mathcal{O}\left(\frac{t^{3/2}}{\sqrt{\epsilon}}N^{7/2}\log(V)\right)$$

We can do even better using Quantum Signal Processing [Low & Chuang (2017)]

$$C_{1st}^{QSP} = \mathcal{O}\left(N\log(V)\left(Nt + \log\left(\frac{1}{\epsilon}\right)\right)\right)$$

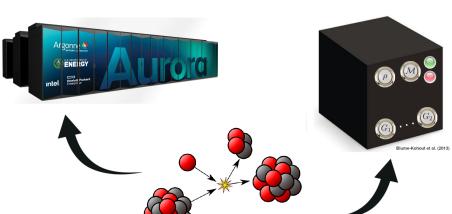


ullet 10^7-10^9 operations and pprox 70-500 qubits [Spagnoli, Lissoni, Roggero (2025)]

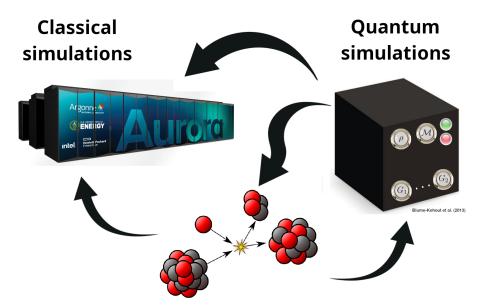
Simulations of nuclear dynamics II

Classical simulations





Simulations of nuclear dynamics II



Quantum inspired simulation of reactions

Since we want Chebyshev moments, why not get them classically instead?

Orthogonal polynomials satisfy recurrence relations, for Chebyshev

$$\mathsf{T}_0(H) = 1 \quad \mathsf{T}_1(H) = H \quad \Rightarrow \quad \mathsf{T}_{n+1}(H) = 2H\mathsf{T}_n(H) - \mathsf{T}_{n-1}(H)$$

To get Chebyshev moments we need a many-body method such that

- ullet we can prepare a good approximation to the ground state $|\Psi_0
 angle$
- we can apply the Hamiltonian efficiently

$$\begin{split} |\phi_0\rangle = |\Psi_0\rangle \quad |\phi_1\rangle = H\, |\Psi_0\rangle \\ |\phi_n\rangle \quad \to \quad |\phi_{n+1}\rangle = 2H\, |\phi_n\rangle - |\phi_{n-1}\rangle \end{split}$$

• we can calculate overlaps efficiently $m_k = \langle \phi_0 | \phi_k \rangle = \langle \phi_k | \phi_0 \rangle$

Once we have the moments, all the post processing is carried out as if we obtained them from a quantum computer

Quantum inspired simulation of reactions with CC-theory

Sobczyk & Roggero (2022), Sobczyk, Jiang, Roggero (2025)

Coupled-cluster theory allows for accurate nuclear ground states to be prepared efficiently. We can use EOM-CC to study excited-states/moments

$$|\Psi_0\rangle = e^T |HF\rangle$$
 $\langle \widetilde{\Psi}_0| = \langle HF|(1+\Lambda)e^{-T}$

the natural construction uses a similarity transformed Hamiltonian

 $\overline{H} = e^{-T}He^T$ in CCSD operator T contains 1p1h and 2p2h

Quantum inspired simulation of reactions with CC-theory

Sobczyk & Roggero (2022), Sobczyk, Jiang, Roggero (2025)

Coupled-cluster theory allows for accurate nuclear ground states to be prepared efficiently. We can use EOM-CC to study excited-states/moments

$$|\Psi_0\rangle = e^T |HF\rangle$$
 $\langle \widetilde{\Psi}_0| = \langle HF|(1+\Lambda)e^{-T}$

the natural construction uses a similarity transformed Hamiltonian

$$\overline{H} = e^{-T}He^T$$
 in CCSD operator T contains 1p1h and 2p2h

Within EOM the excited states are parametrized as

$$|\phi_{n+1}\rangle = 2\overline{H} |\phi_n\rangle - |\phi_{n-1}\rangle = \mathcal{R}_n |\Psi_0\rangle$$

Once we collected the parameters of \mathcal{R}_n we can get moments as

$$m_k = \langle \widetilde{\Psi}_0 | \phi_k \rangle$$

Spin response of bulk neutron matter



Dynamic spin structure factor

$$S_{\sigma}(\vec{q},\omega) \propto \int dt e^{i\omega t} \langle \vec{s}(t,\vec{q}) \cdot \vec{s}(0,\vec{q}) \rangle$$

 νN scattering and ν pair-production emissivity dominated by $S_{\sigma}(\vec{q},\omega)$ for small wave-lenghts $|\vec{q}|\to 0$

Spin response of bulk neutron matter



Dynamic spin structure factor

$$S_{\sigma}(\vec{q},\omega) \propto \int dt e^{i\omega t} \langle \vec{s}(t,\vec{q}) \cdot \vec{s}(0,\vec{q}) \rangle$$

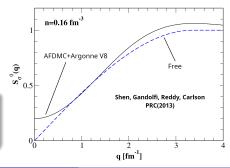
 νN scattering and ν pair-production emissivity dominated by $S_{\sigma}(\vec{q},\omega)$ for small wave-lenghts $|\vec{q}|\to 0$

Total strength given by sum rule

$$S_{\sigma}^{0}(\vec{q}) = \int d\omega S_{\sigma}(\vec{q}, \omega)$$

Tensor & Spin-orbit terms lead to

$$S_{\sigma}^{0}(0) = \frac{4}{3N} \langle \mathsf{S}^{2} \rangle \neq 0$$



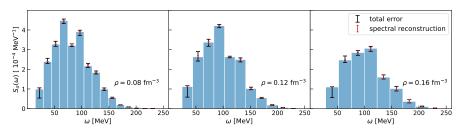
Spin response of bulk neutron matter from CC

Sobczyk & Roggero (2022), Sobczyk, Jiang, Roggero (2025)

With EOM-CC we can get a reasonably good approximation of m_k in a very efficient way: 5k moments for N=114 particles in $\approx 7k$ CPU hours

Sobczyk & Roggero (2022), Sobczyk, Jiang, Roggero (2025)

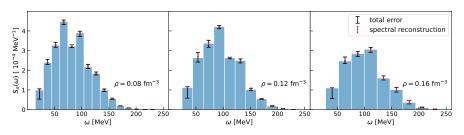
With EOM-CC we can get a reasonably good approximation of m_k in a very efficient way: 5k moments for N=114 particles in $\approx 7k$ CPU hours



First ab-initio calculation of the frequency dependent spin response of neutron matter with **controllable errors**

Sobczyk & Roggero (2022), Sobczyk, Jiang, Roggero (2025)

With EOM-CC we can get a reasonably good approximation of m_k in a very efficient way: 5k moments for N=114 particles in $\approx 7k$ CPU hours

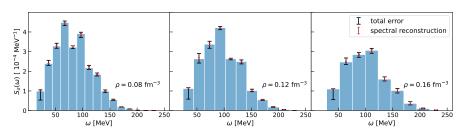


First ab-initio calculation of the frequency dependent spin response of neutron matter with **controllable errors**

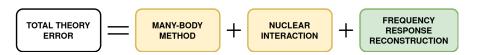


Sobczyk & Roggero (2022), Sobczyk, Jiang, Roggero (2025)

With EOM-CC we can get a reasonably good approximation of m_k in a very efficient way: 5k moments for N=114 particles in $\approx 7k$ CPU hours



First ab-initio calculation of the frequency dependent spin response of neutron matter with **controllable errors**



Summary and perspective

- Ab-initio treatement of nuclear dynamics is important for both terrestrial experiments and extreme astrophysical sites
- Quantum computing is a strong candidate to considerably improve our simulations of nuclear physics, especially dynamical properties
- Substantial advances in the last years in the implementation of time evolution for simple nuclear Hamiltonians. Recent results seem to suggest that a first-quantization formulation could be advantageous
- Classical simulation methods continue to improve also thanks to work on quantum algorithms. Quantum advantage constantly rising bar

Summary and perspective

- Ab-initio treatement of nuclear dynamics is important for both terrestrial experiments and extreme astrophysical sites
- Quantum computing is a strong candidate to considerably improve our simulations of nuclear physics, especially dynamical properties
- Substantial advances in the last years in the implementation of time evolution for simple nuclear Hamiltonians. Recent results seem to suggest that a first-quantization formulation could be advantageous
- Classical simulation methods continue to improve also thanks to work on quantum algorithms. Quantum advantage constantly rising bar

Thanks to my collaborators

- J. Sobczyk (Chalmers)
- W. Jiang (Mainz)
- L. Spagnoli (Trento)
- C. Lissoni (Trento)

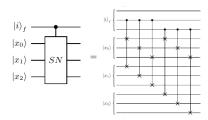




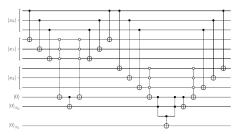


Block encoding of the potential

• we use a SWAP network to place in first place the particle i flagged by ancilla register $|i\rangle$ (which we start in superposition)

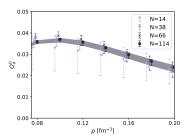


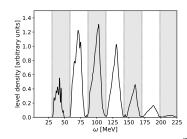
evaluate the inner sum in the parenthesis on the swapped register



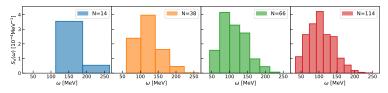
Finite size systematics

We use TABC to minimize finite size effects. This works well for sum rules but residual N dependence in the density of states (and thus the response)





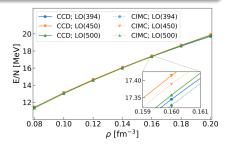
Clustering can be explained as a shell effect: at fixed density $\rho=N/L^3$ so the free single particle energies are $E_n \propto n(2\pi/L)^2$



Despite being a low energy observable, important interaction dependence

We tested three different Chiral potentials at LO using CC and CIMC

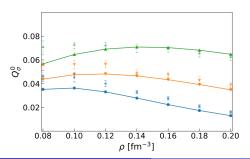
 energy per particle has negligible dependence on method/model

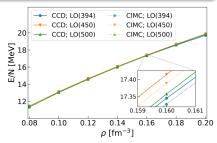


Despite being a low energy observable, important interaction dependence

We tested three different Chiral potentials at LO using CC and CIMC

- energy per particle has negligible dependence on method/model
- large impact on sum rule!

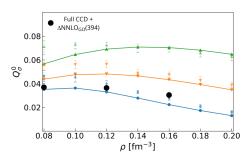


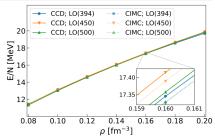


Despite being a low energy observable, important interaction dependence

We tested three different Chiral potentials at LO using CC and CIMC

- energy per particle has negligible dependence on method/model
- large impact on sum rule!



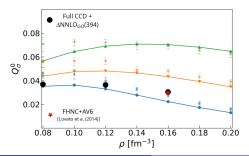


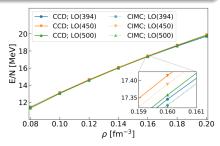
Small effect in going NNLO

Despite being a low energy observable, important interaction dependence

We tested three different Chiral potentials at LO using CC and CIMC

- energy per particle has negligible dependence on method/model
- large impact on sum rule!



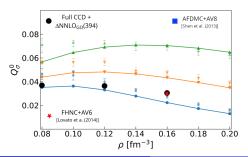


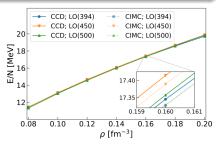
- Small effect in going NNLO
- FHNC+AV6 looks similar

Despite being a low energy observable, important interaction dependence

We tested three different Chiral potentials at LO using CC and CIM

- energy per particle has negligible dependence on method/model
- large impact on sum rule!



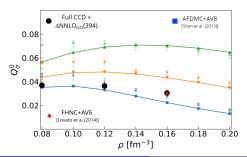


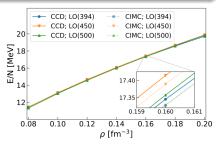
- Small effect in going NNLO
- FHNC+AV6 looks similar
- AFDMC+AV8 not at all

Despite being a low energy observable, important interaction dependence

We tested three different Chiral potentials at LO using CC and CIM

- energy per particle has negligible dependence on method/model
- large impact on sum rule!





- Small effect in going NNLO
- FHNC+AV6 looks similar
- AFDMC+AV8 not at all

What is the value of $\langle S^2 \rangle$ in zero temperature neutron matter?