

Toponium formation at the Large Hadron Collider

Benjamin Fuks

LPTHE / Sorbonne Université

In collaboration with Kaoru Hagiwara, Kai Ma & Ya-Juan Zheng:

- PRD 104 (2021) 034023

- EPJC 85 (2025) 157

+ Moriond'25 proceedings: 2505.03869

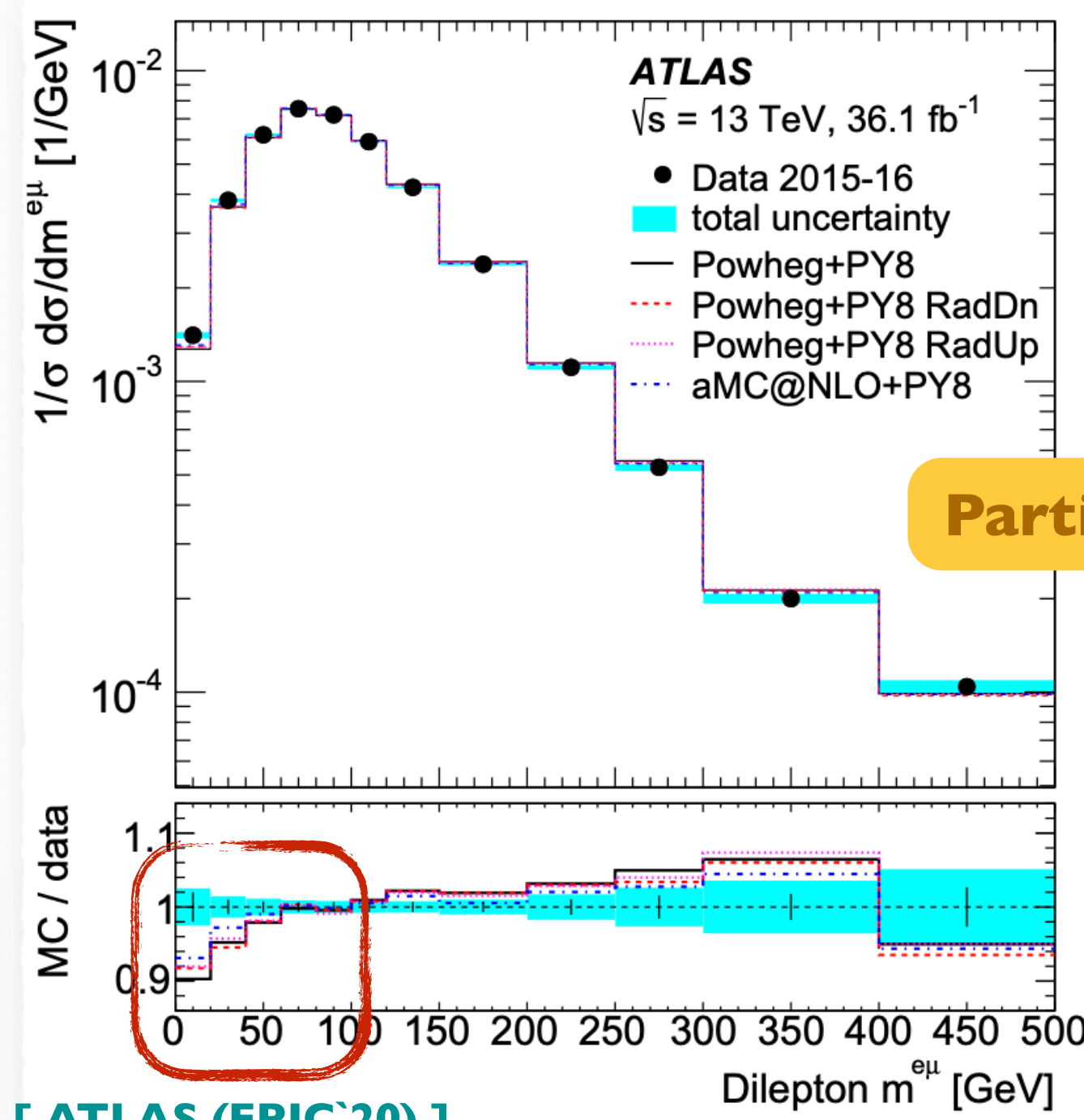
Higgs Hunting 2025

15-17 July 2025, Paris

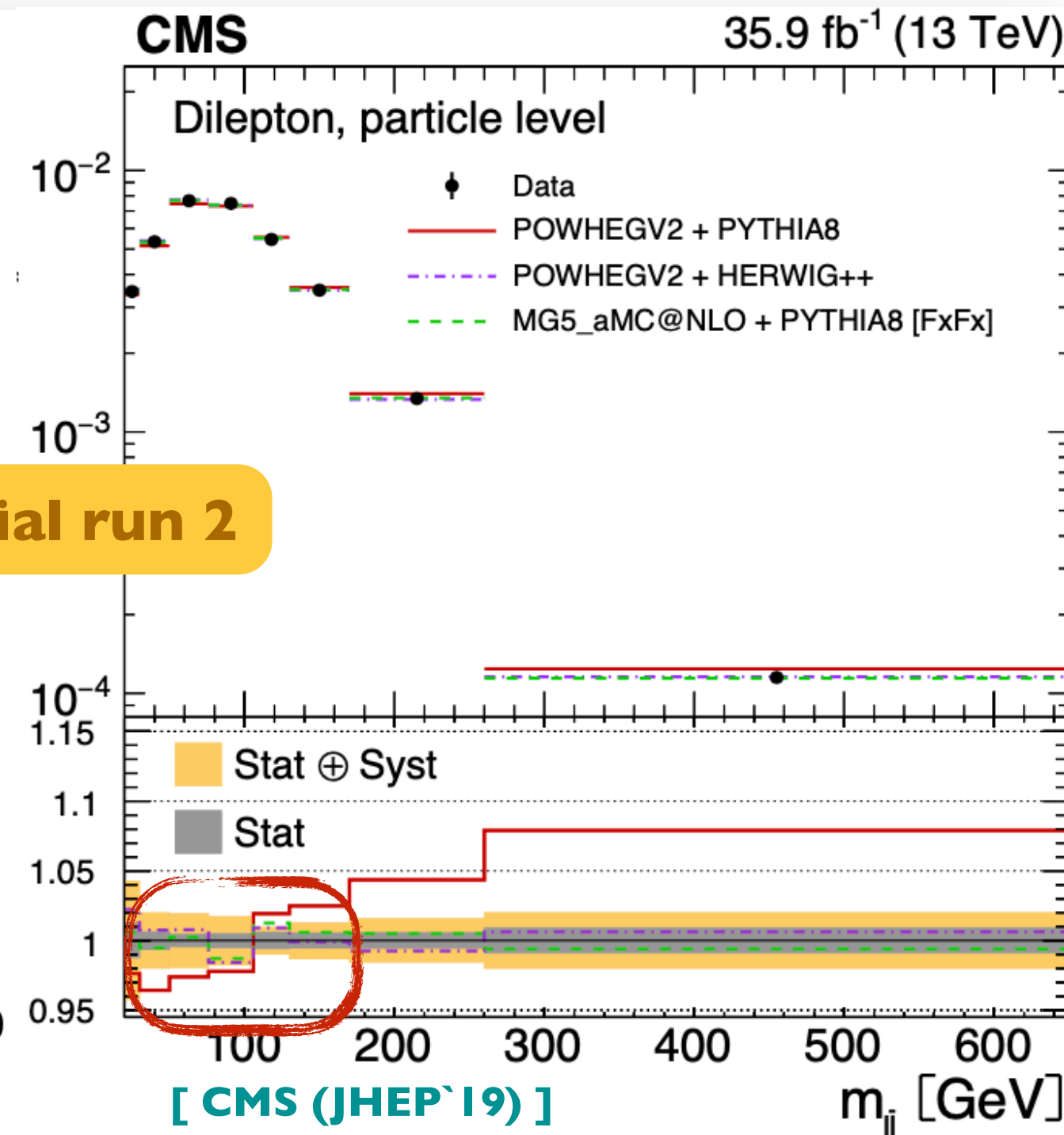
How it all started: excesses...

Copious top quark production at the LHC [$\sigma(13 \text{ TeV}) \sim 810 \text{ pb}$]

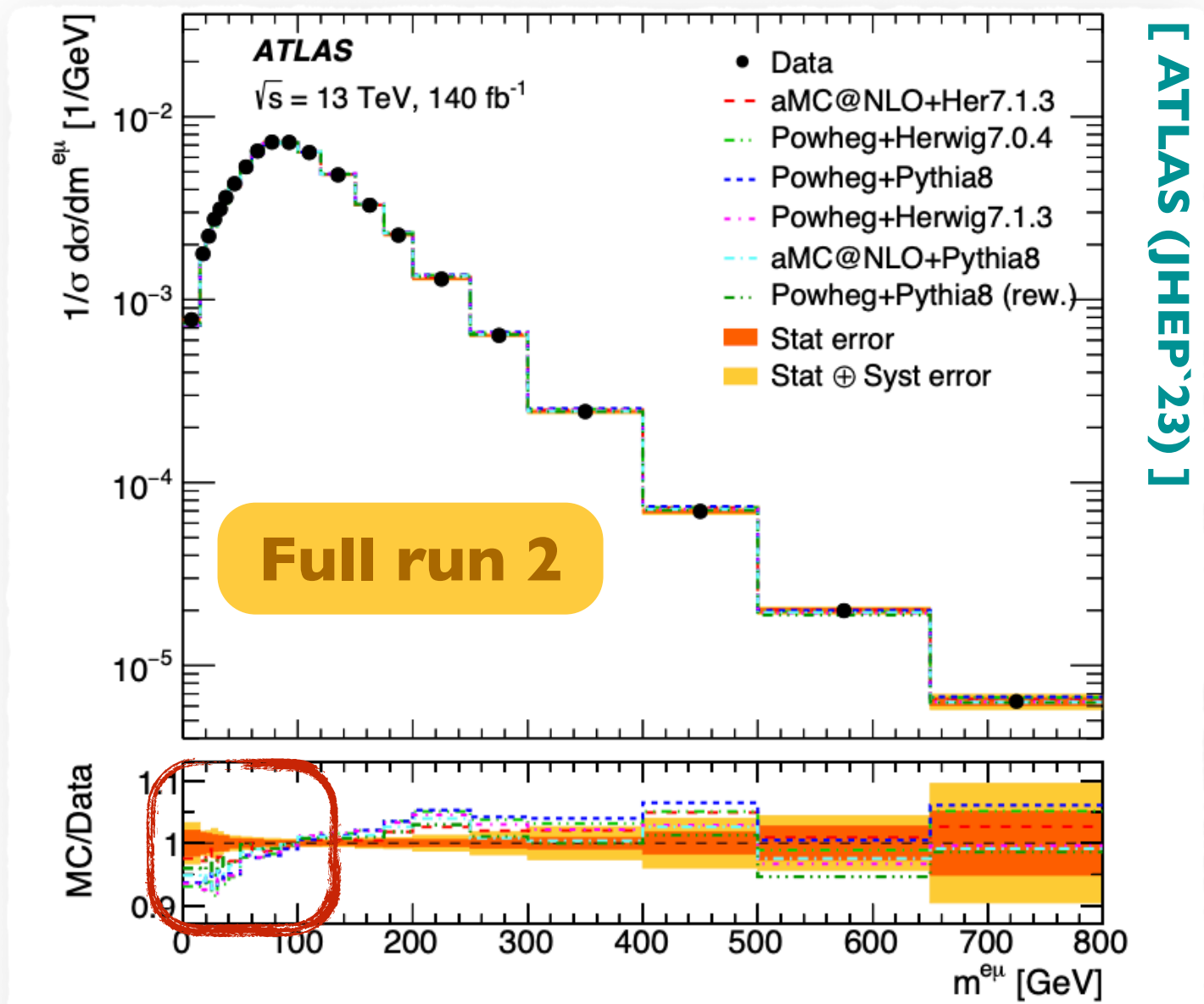
- Detailed analysis of top properties (mass, width, etc.)
 - Many differential distributions precisely measured
- Precision measurements as probes for anomalies
- Issues with SM modelling close to threshold?
 - Bound state effects



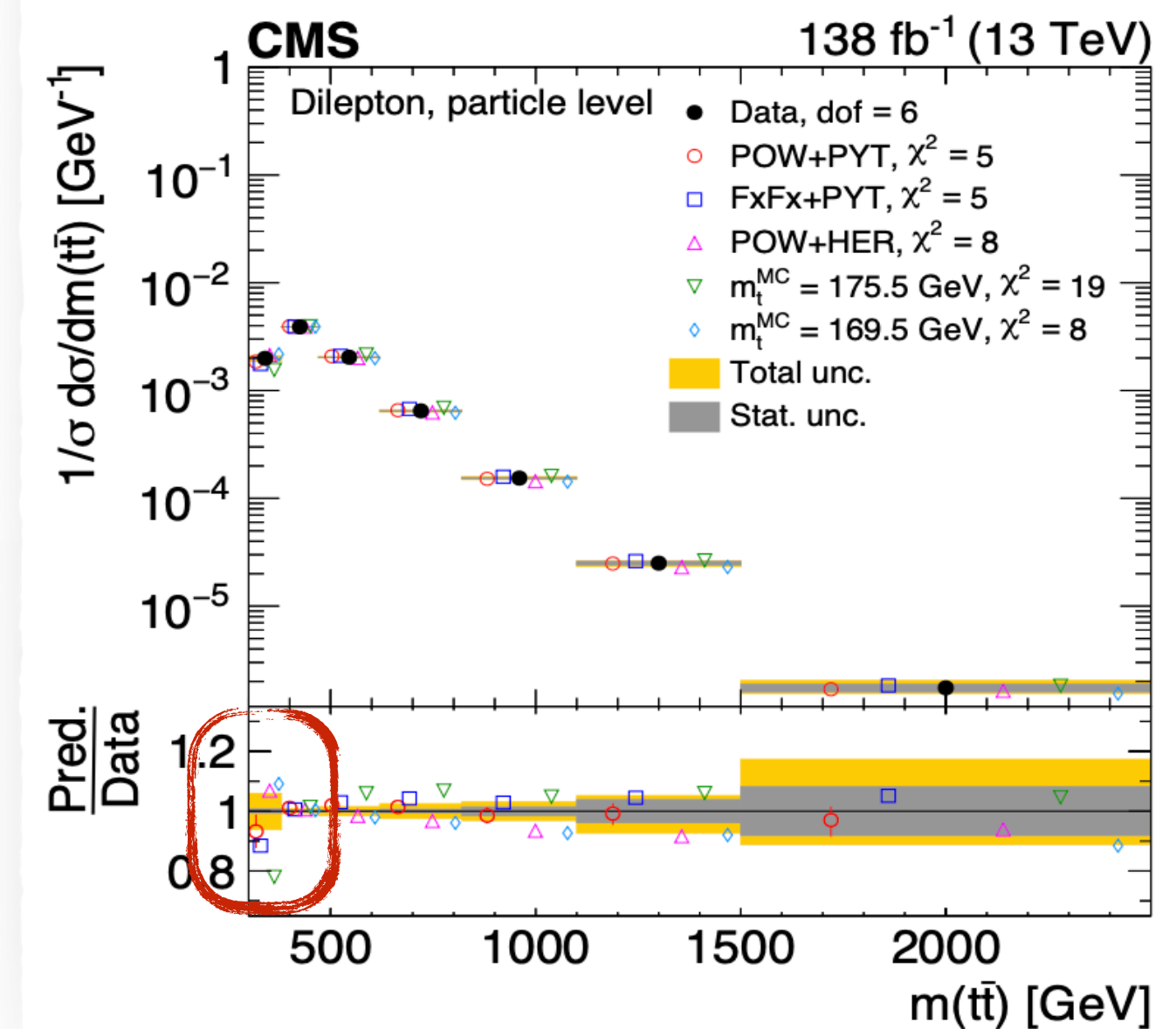
[ATLAS (EPJC'20)]



[CMS (JHEP'19)]



[ATLAS (JHEP'23)]

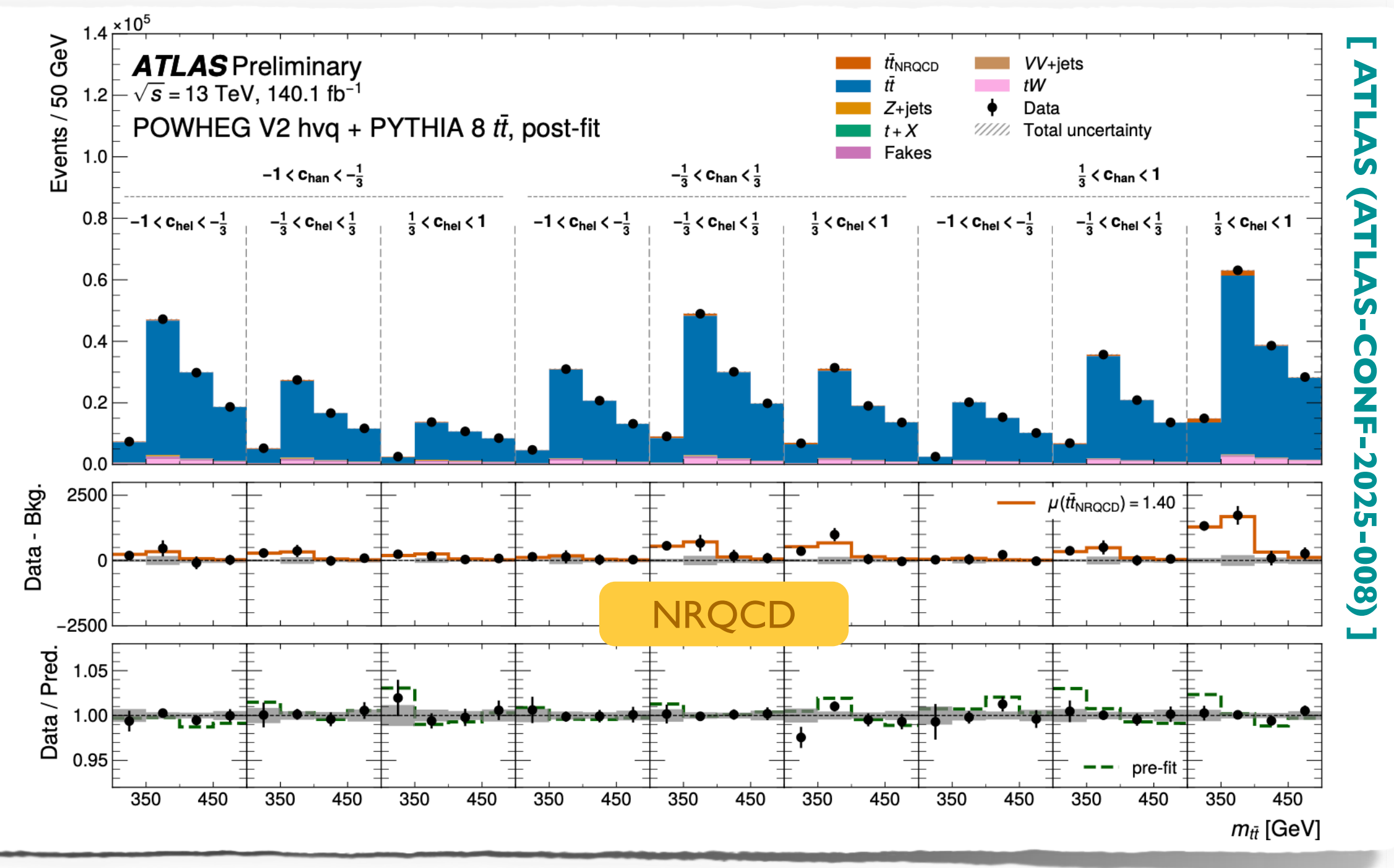
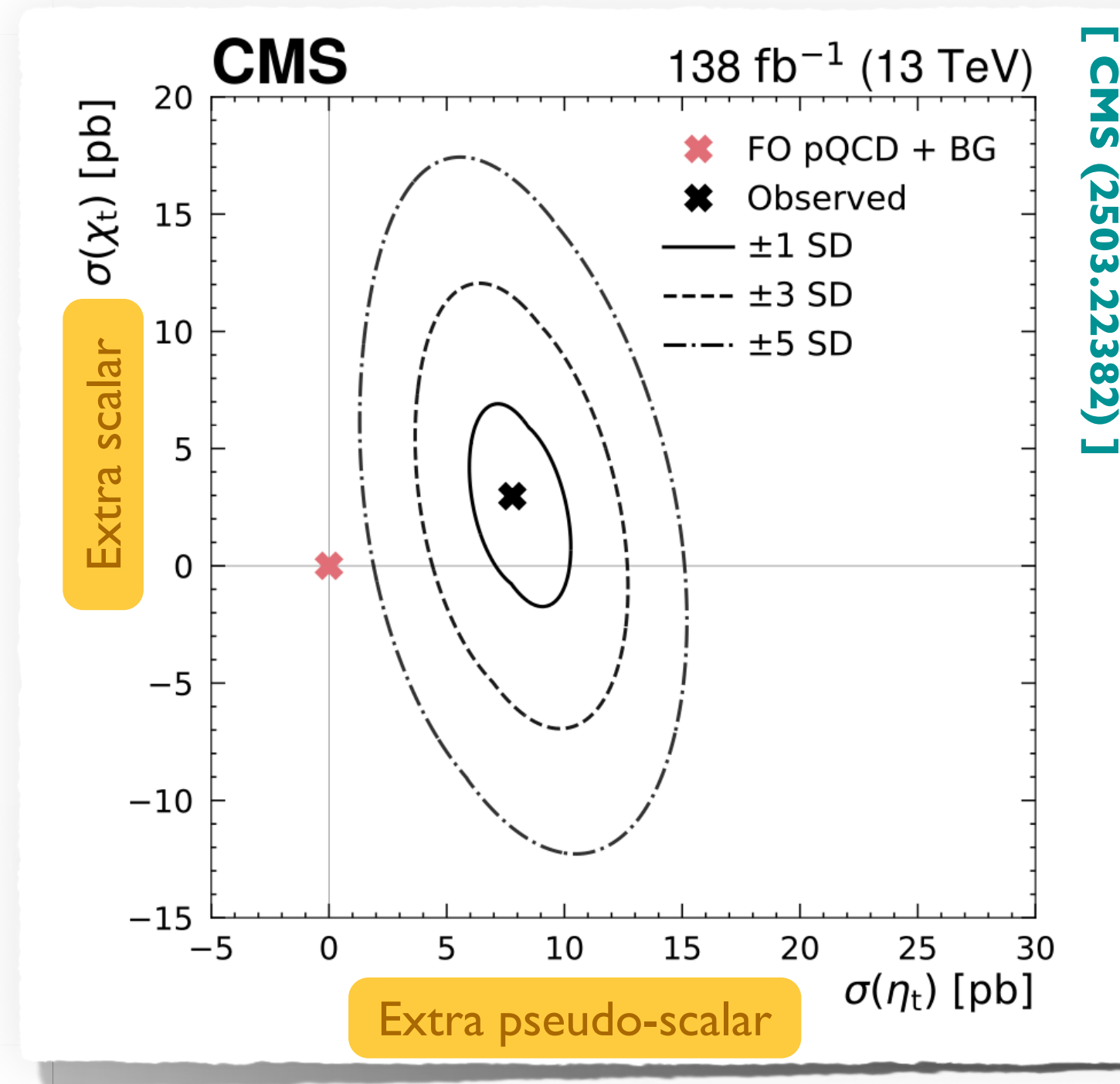


[CMS (JHEP'25)]

Fast forward: Sep 2024 – Jul 2025

ATLAS and CMS confirmations

- CMS: SM + extra scalar χ_t and or pseudo-scalar η_t
 → toponium ground state approximation
- ATLAS: NRQCD predictions for toponium effects

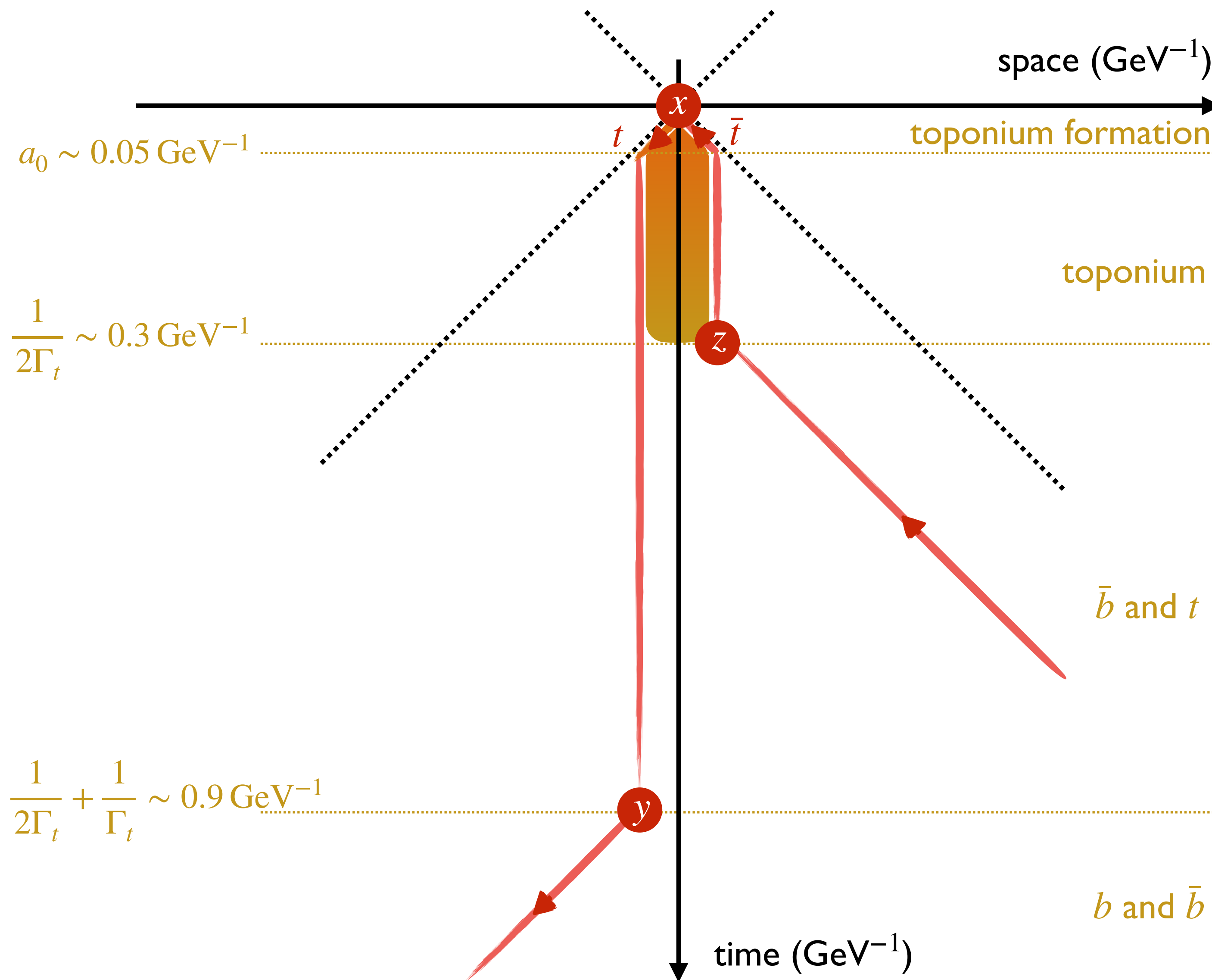


Top-antitop production near threshold

[Fadin & Khoze (JETP Lett`87)]

[Fadin, Khoze & Sjöstrand (Z.Phys.C`90)]

[Sumino, Fujii, Hagiwara, Murayama & Ng (PRD`93)]



A position-space picture

- Two-particle state created at $x = (t_0, \vec{x})$
 $\approx$ wave packet propagation to the V_{QCD} barrier
 $\rightarrow$ Typical scale: the Bohr radius

$$a_0 = \frac{1}{C_F \alpha_s m_t/2} \sim (20 \text{ GeV})^{-1}$$
- Oscillations within the barrier until system decay
 $\rightarrow$ Top [$y = (t_1, \vec{y})$] or antitop [$z = (t_2, \vec{z})$] decay
 $\rightarrow$ Typical scale $\approx (2\Gamma_t)^{-1} \sim (3 \text{ GeV})^{-1}$
- Probe of the QCD potential
 $\rightarrow$ Toponium effects
 $\rightarrow$ Expected to be 0.8% of $\sigma_{t\bar{t}}$
 $\rightarrow$ Currently not included in MC simulations

The toponium Green's function

Three-point correlation function in the non-relativistic limit

$$K_{abcd}(x, y, z) = \langle 0 | T \{ t_c(y) \bar{t}_d(z) : \bar{t}_a(x) t_b(x) : \} | 0 \rangle$$

$$= \frac{(1 + \gamma^0)_{ca}}{2} \frac{(1 - \gamma^0)_{bd}}{2} \int d^3r \left[K_1(y; (z^0, \vec{r})) K_2(z^0, \vec{r}, \vec{z}; x^0, \vec{x}, \vec{x}) + K_1(z; (y^0, \vec{r})) K_2(y^0, \vec{y}, \vec{r}; x^0, \vec{x}, \vec{x}) \right]$$

Non-relativistic spin projection operators

Antitop-decay first

Top-decay first

1-particle-state and 2-particle-state propagators

The toponium Green's function (from K_2)

- Solution to the Lippmann-Schwinger equation
 - Fourier transform of the QCD potential
 - S-wave contributions
- To be solved numerically

$$\widetilde{G}(E; p) = \widetilde{G}_0(E; p) + \int \frac{d^3q}{(2\pi)^3} \widetilde{V}_{\text{QCD}}(\vec{p} - \vec{q}) \widetilde{G}(E; q)$$

Free Green's function

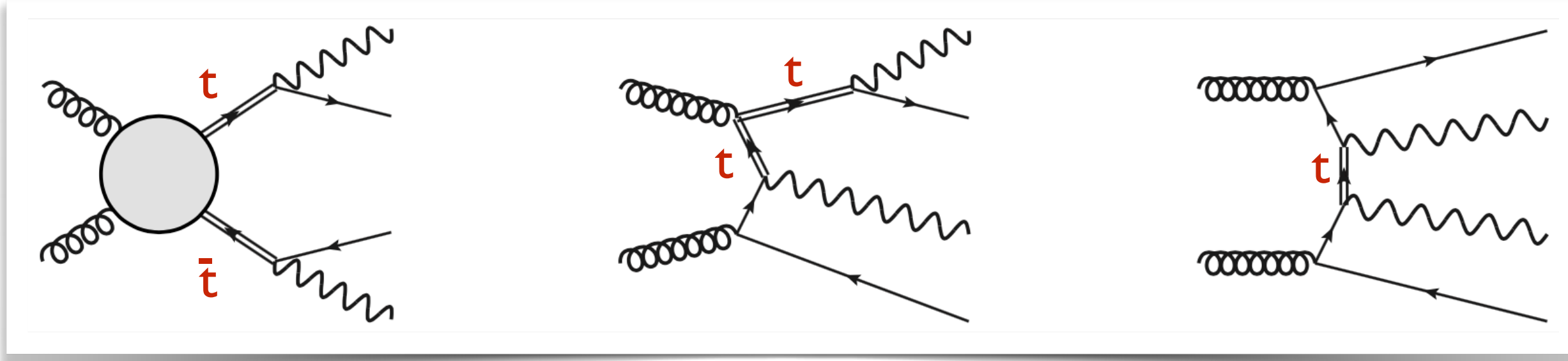
[Jezabek, Kuhn & Teubner (Z.Phys.C`92)]

The QCD Green's function as a seed for toponium modelling

Top-antitop production near threshold

$Wb Wb$ production in the threshold regime

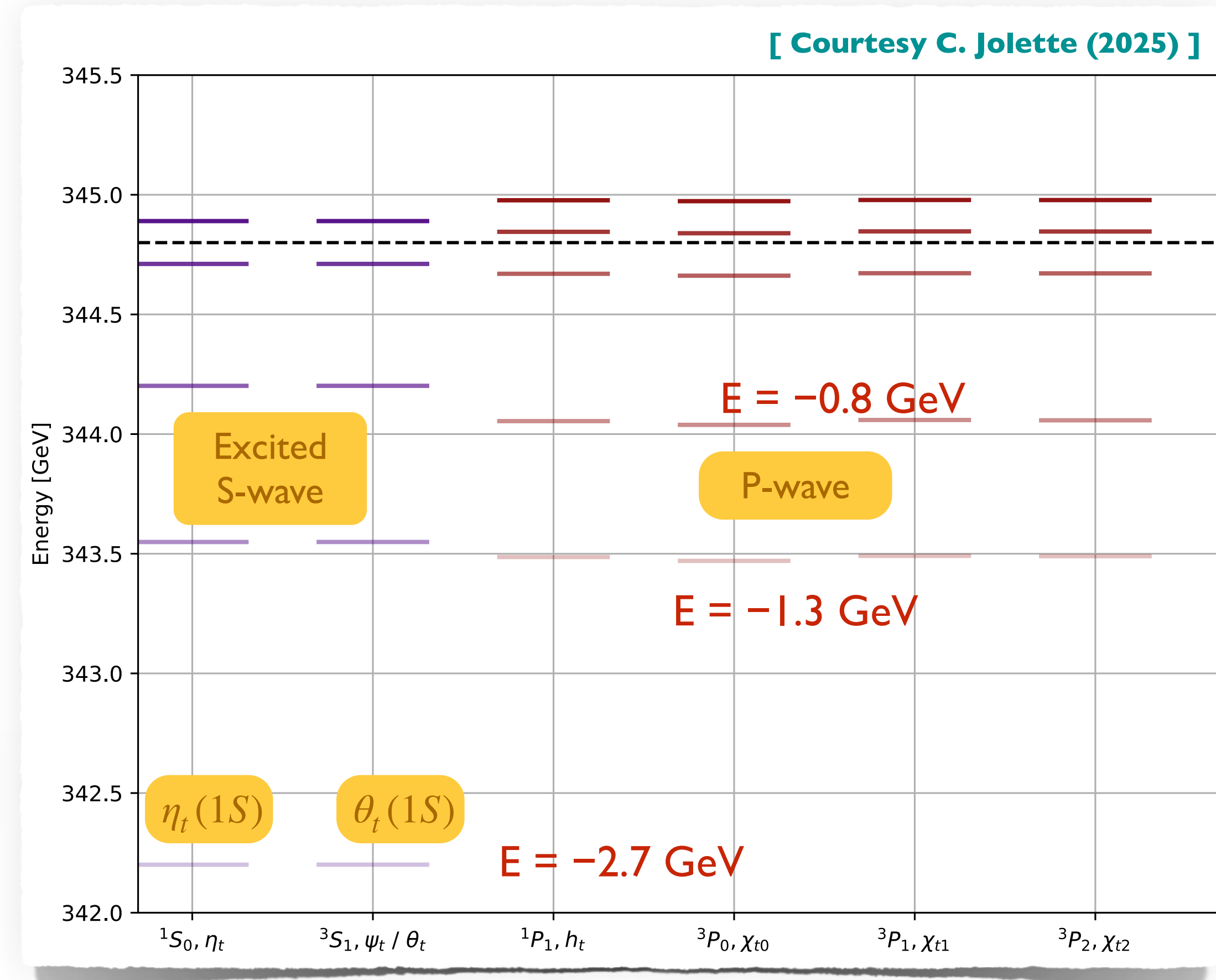
- 3 classes of contributions with 0, 1, 2 (possibly off-shell) resonant tops



- Coulomb gluon exchanges between tops (first class of diagrams)
 - ★ **Scattering states**: perturbative Sommerfeld enhancement
 - Partly included in fixed-order calculations (α_s/v corrections)
 - ★ **Bound states**: non-perturbative origin
- The Green's function of NRQCD in the Coulomb gauge
 - ★ Consistent treatment of the two contributions

Scattering and bound top-antitop states

- Schrödinger equation with the QCD potential
 - Coulomb at small distance $\oplus$ Cornell at large distance
 - NLO contributions included



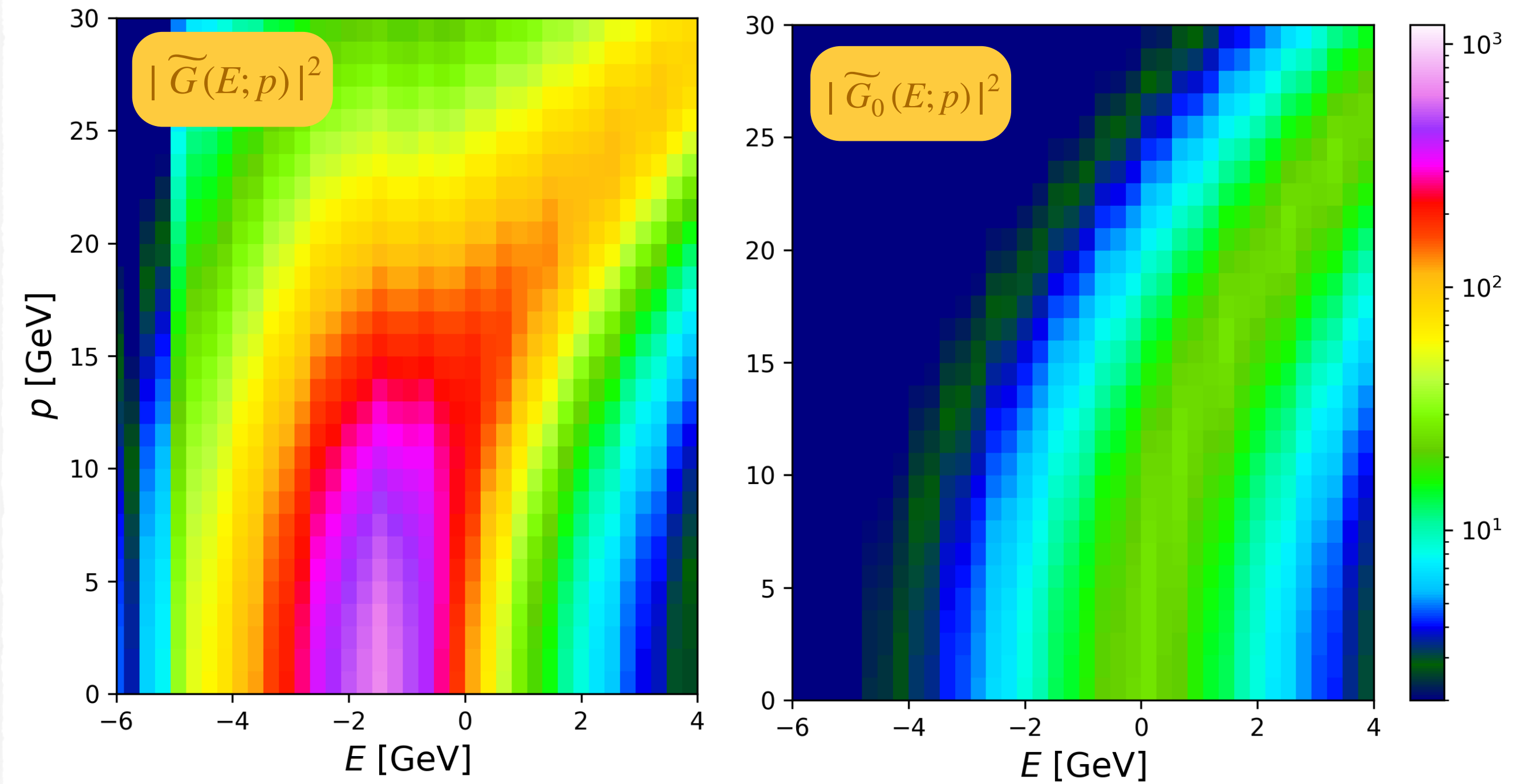
The toponium Green's function

Green's function ratio as a seed for toponium modelling

- LO Coulomb potential
- $|\tilde{G}_0|^2$: Breit-Wigner behaviour (standard top propagator)
 - $\forall E$, peak at $p_{\text{peak}} \simeq \frac{2m_t + E}{2} \sqrt{1 - \frac{4m_t^2}{(2m_t + E)^2}}$
 - $|\tilde{G}|^2 \equiv \forall E \geq 2 \text{ GeV}$
(different normalisation persists at low positive E)
- Significant toponium effects between -4 GeV and 0 GeV
 - Most prominent effect at $E \approx -2 \text{ GeV}$ and low p

Modelling strategy around MC event generators

- Valid near threshold → non-relativistic matrix elements
- Conventional MC generators: standard matrix elements
 - Assumption 1: relativistic effects negligible
 - Assumption 2: excited-state contributions negligible
- Matrix-element re-weighting
 - Colour singlet projection
 - Validity: $E < 4 \text{ GeV}$ and $p^* < 50 \text{ GeV}$



$$i\mathcal{M} \rightarrow i\mathcal{M} \times \frac{\tilde{G}(E; p^*)}{\tilde{G}_0(E; p^*)}$$

$t\bar{t}$ production near threshold with MG5aMC

Process: $gg \rightarrow t\bar{t} \rightarrow b\ell^+\nu_\ell \bar{b}\ell'^-\bar{\nu}_\ell'$

- Six-body final state: spin correlations included
- Projection on the colour singlet
- Matrix-elements with and without re-weighting

Without re-weighting

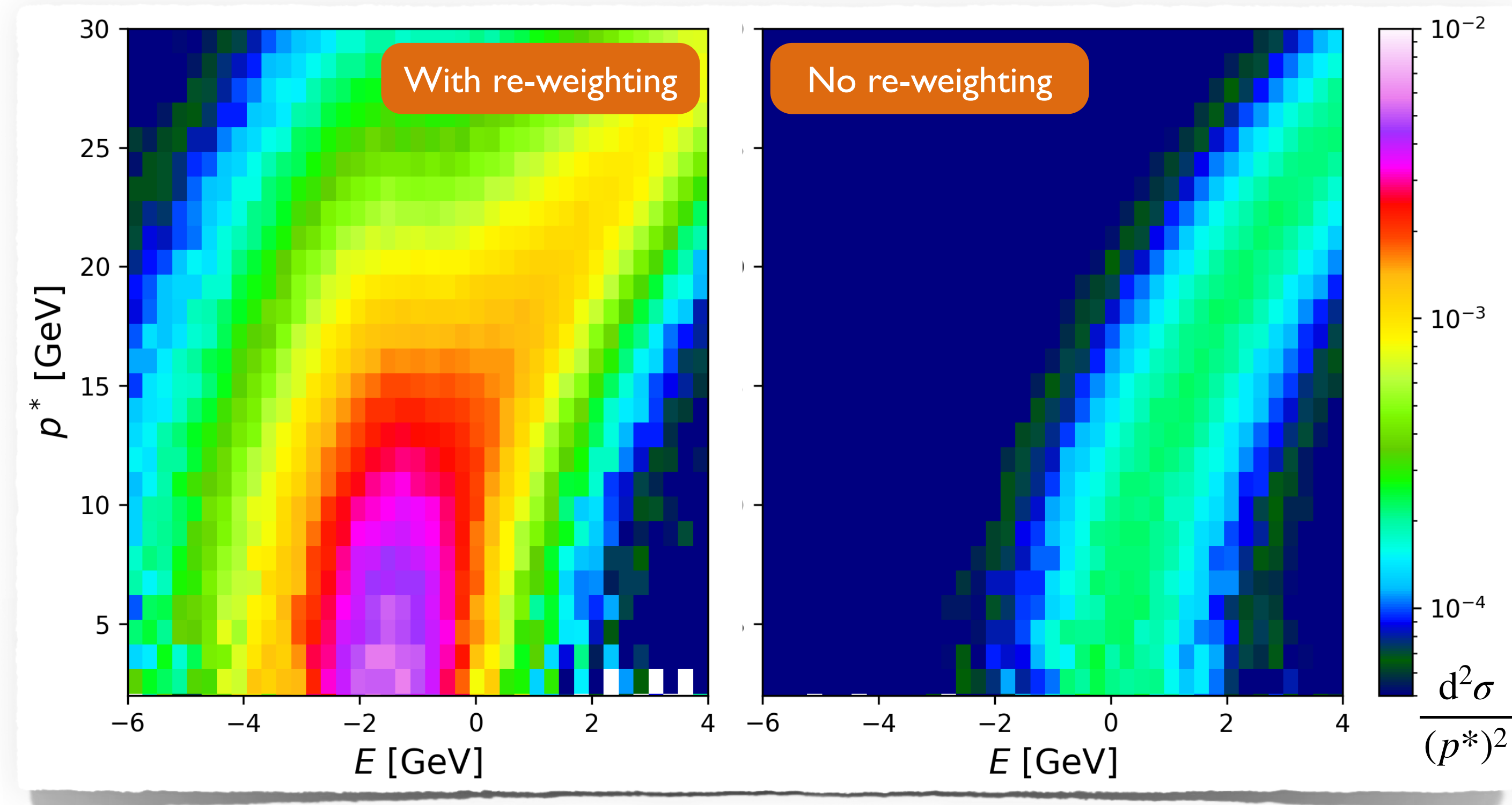
- Rates $\equiv$ phase space $\times$ Breit-Wigner
- Similar heat map as for $\widetilde{G}_0$

With re-weighting

- **Similar heat map as for $\widetilde{G}$**
- Normalisation and shape affected
- Access to the QCD Green's function

Typical top momentum in the toponium rest frame

$$\langle p(E) \rangle = \frac{\int p \, d^3p \, \frac{d\sigma}{p^2 dp dE}}{\int d^3p \, \frac{d\sigma}{p^2 dp dE}} \quad \Leftrightarrow \text{for } E \simeq -2 \text{ GeV: } 20 \text{ GeV} \text{ (the Bohr radius)}$$



[Hagiwara, BF, Ma & Zheng (EPJ C'25)]

Comparison with pseudo-scalar toy models

A toponium toy Lagrangian with a pseudo-scalar state

$$\mathcal{L}_{\eta_t} = \frac{1}{2} \partial_\mu \eta_t \partial^\mu \eta_t - \frac{1}{2} m_{\eta_t}^2 \eta_t^2 - \frac{1}{4} g_{gg} \eta_t G_a^{\mu\nu} \tilde{G}_{\mu\nu}^a - i g_{tt} \eta_t \bar{t} \gamma_5 t$$

- No free parameters

$$\sigma(\eta_t) \equiv \int_{-8 \text{ GeV}}^{4 \text{ GeV}} dE \left(\frac{d\sigma_{\text{full}}}{dE} - \frac{d\sigma_{\text{NLO}}}{dE} \right) \rightarrow \sigma_\eta(13 \text{ TeV}) \approx 6.5 \text{ pb}$$

→ To be compared with $\sigma_{t\bar{t}} \approx 810 \text{ pb}$

(Non-perfect) Breit-Wigner fit → $m_{\eta_t} \simeq 344 \text{ GeV}$; $\Gamma_{\eta_t} \simeq 7 \text{ GeV}$

⊕ Green's functions re-weighting

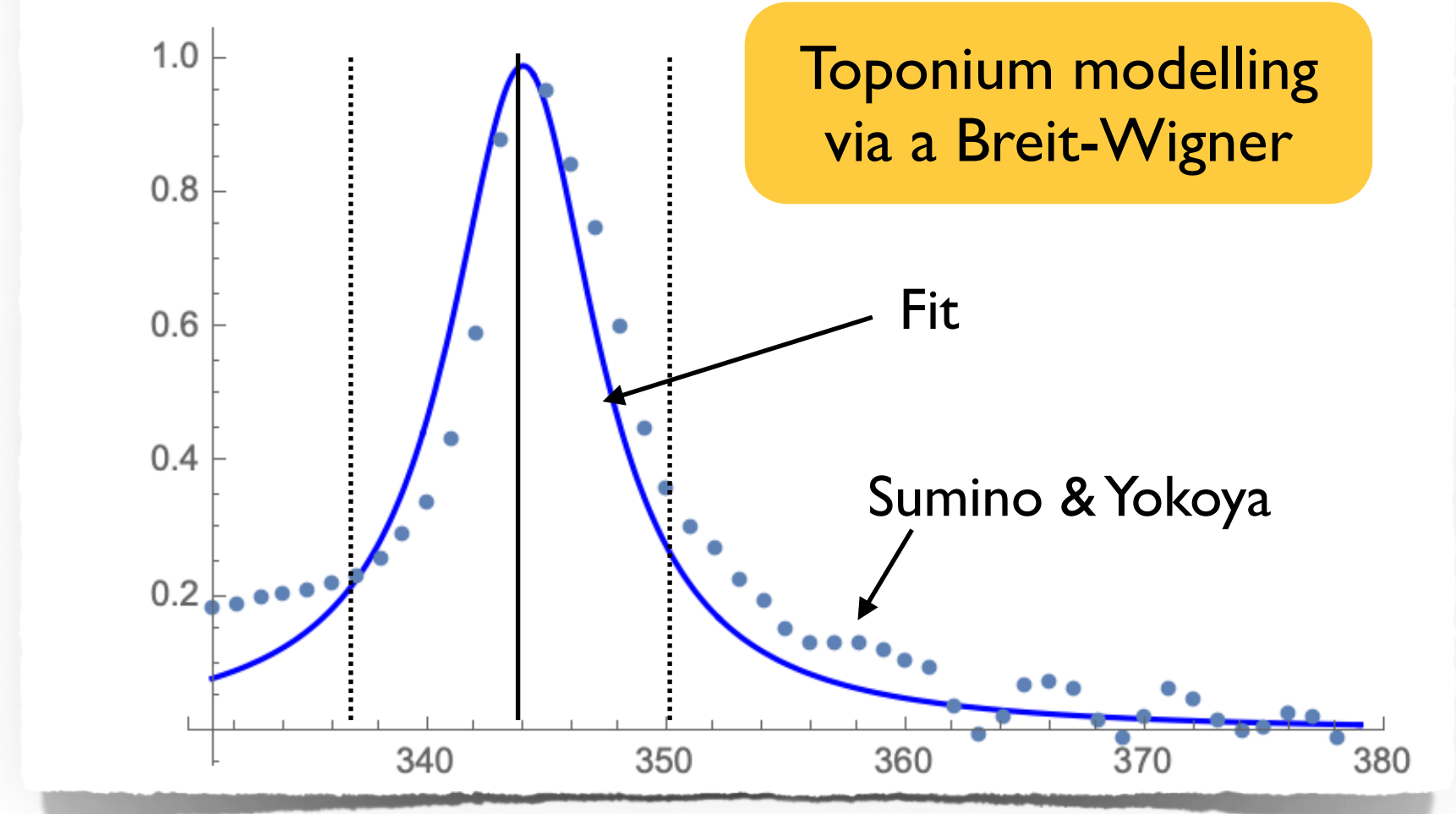
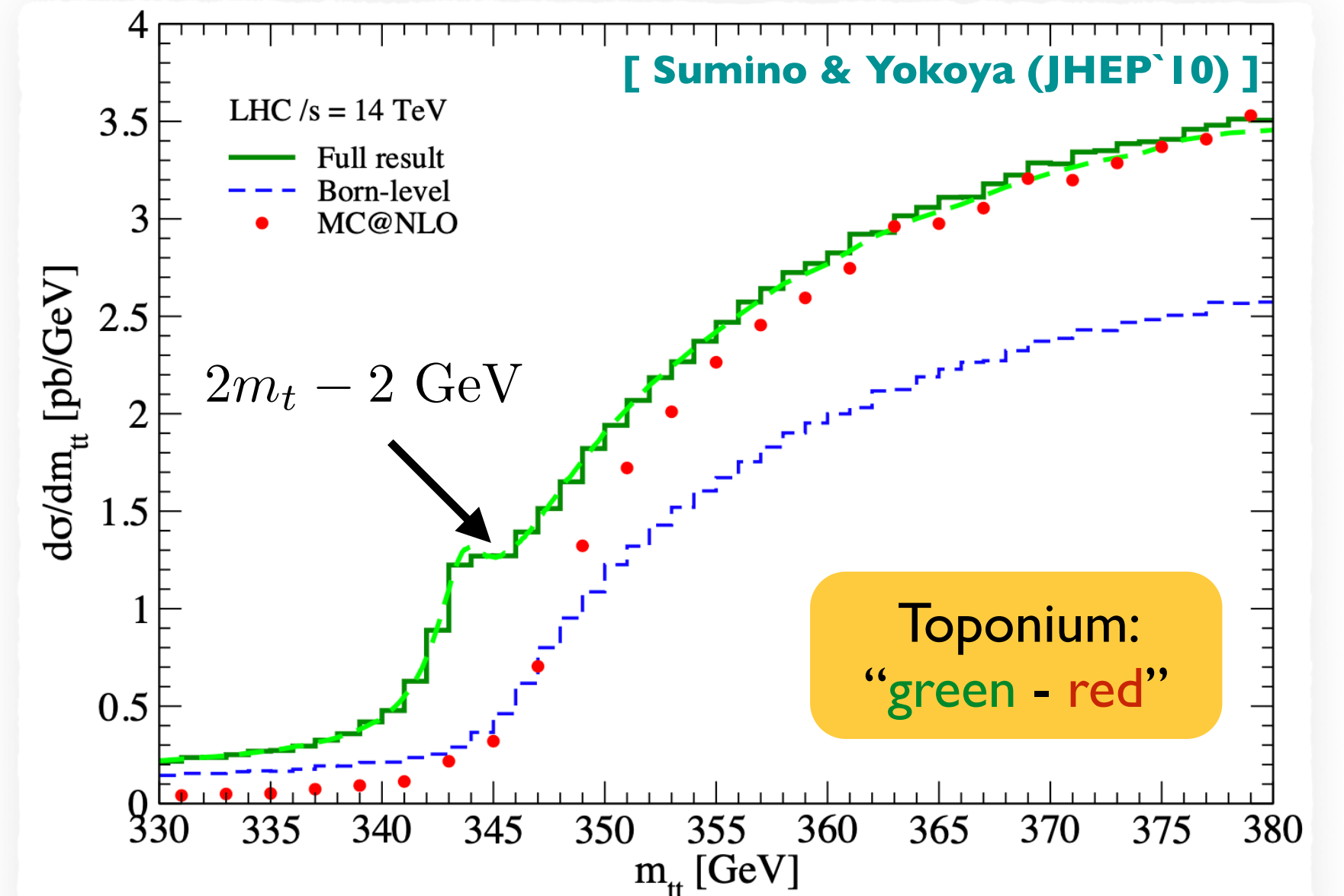
⊕ cut on $m_{t\bar{t}}$

[Hagiwara, BF, Ma & Zheng (PRD'21)]

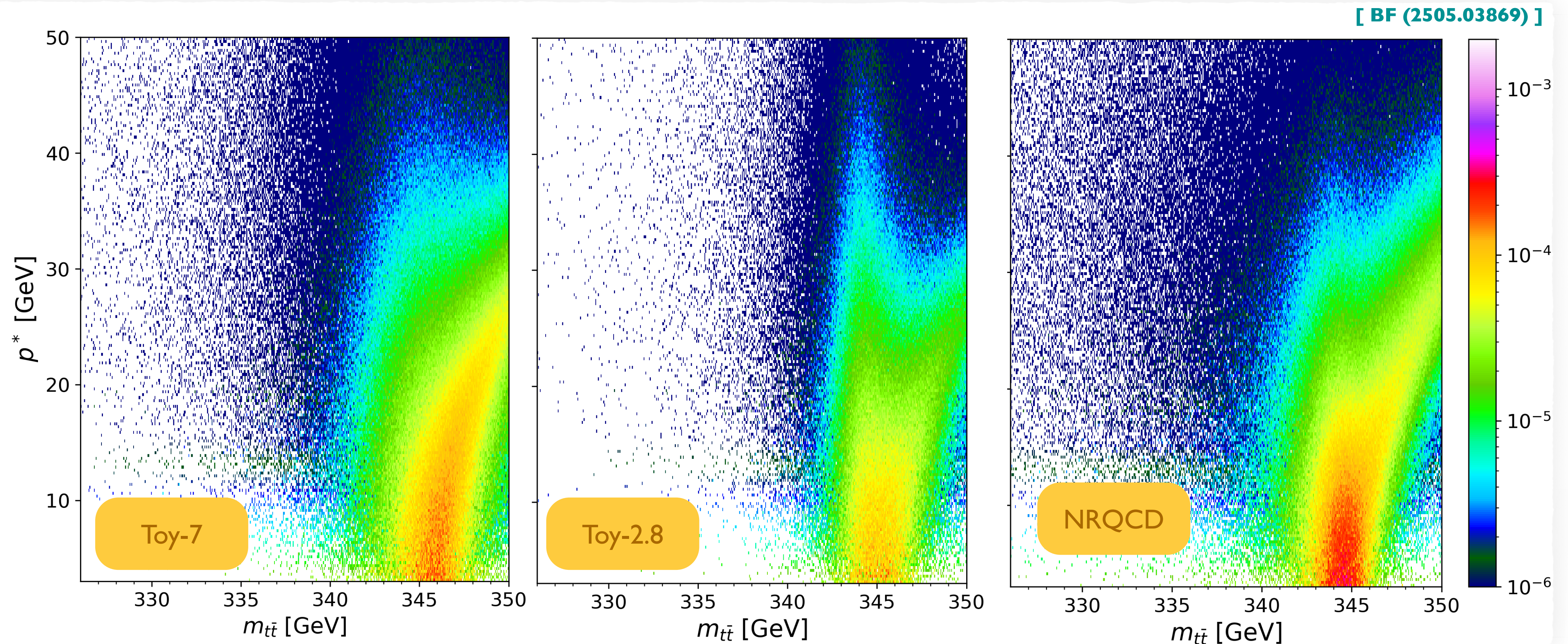
A ground-state toponium toy Lagrangian with a pseudo-scalar state

- Only capturing the vicinity of the ground state
→ OK for $E \lesssim -0.5 \text{ GeV}$
- No effects from excitations in $-0.5 \text{ GeV} \leq E \leq 0 \text{ GeV}$
- Mixed contributions around threshold ($\Gamma_{\eta_t} \simeq 3 \text{ GeV}$)

[Maltoni, Severi, Tentori & Vryonidou (JHEP'24)]



Toy models versus NRQCD: $d^2\sigma/(p^*)^2$



Imperfections of the toy model visible in the key distribution

- The $m_{t\bar{t}}$ spectrum OK in all cases $\rightarrow$ peak more or less reproduced...
- Strong distortion with the p^* distribution

Toy models not to be used anymore!

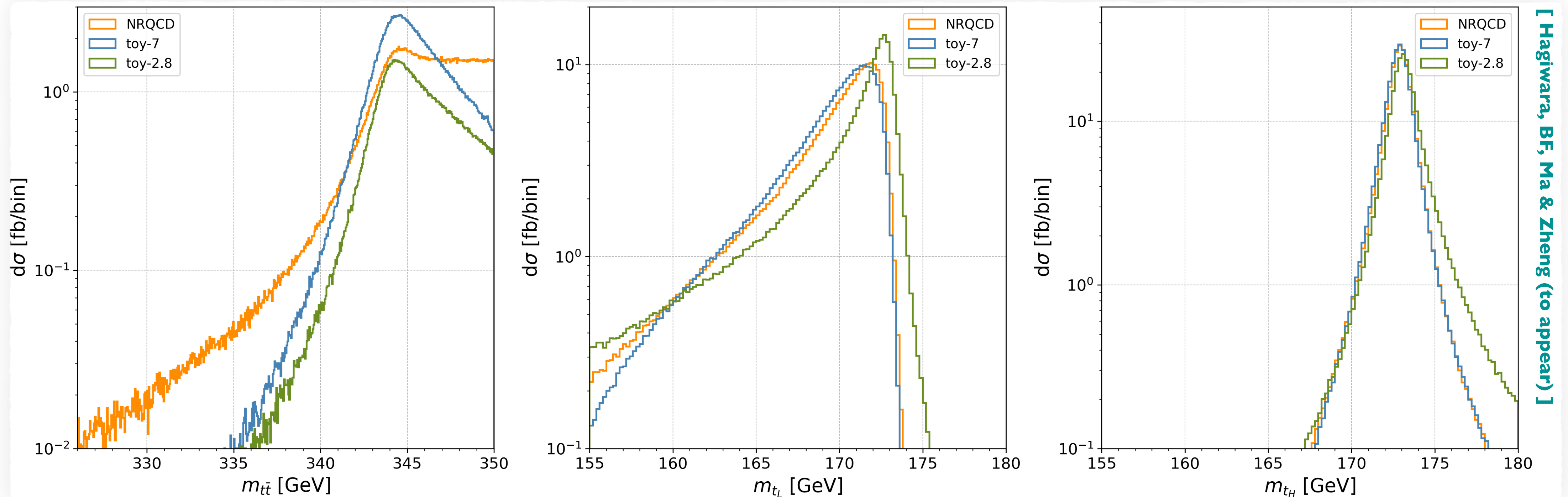
Invariant masses: tops & toponium

$m_{t\bar{t}}$ invariant mass distribution

- Peak at $E \simeq -2$ GeV $\oplus$ extended bound state effects
- The toy models different from NRQCD
- ⚠ Matched calculations in order

Top mass distributions

- Light top $\leftrightarrow$ invariant mass shifted to lower values
 - $\rightarrow$ Governed by the QCD Green's function (distortion of the shape)
 - $\rightarrow$ Bound state effects within the Coulomb potential generated by t_H
- Heavy top $\leftrightarrow$ Breit-Wigner
 - $\rightarrow$ Effectively stable until t_L decays
 - $\rightarrow$ No QCD potential from the decayed top (free propagation)
- Toy models incorrect!



Towards discovery: spin density matrices (and pheno)

Di-leptonic signal properties from $t\bar{t}$ production and $t/\bar{t}$ decay density matrices

- Distribution of the angles of the leptons in their parent top rest frames

$$\sum_{\sigma, \bar{\sigma}, \sigma', \bar{\sigma}'} \rho_{gg \rightarrow (t\bar{t})_1}^{\sigma\bar{\sigma}, \sigma'\bar{\sigma}'} d\rho_{\sigma, \sigma'}^{t \rightarrow b\ell^+\nu_\ell} d\rho_{\bar{\sigma}', \bar{\sigma}}^{\bar{t} \rightarrow \bar{b}\ell'^-\bar{\nu}_\ell} \Rightarrow \frac{d\Gamma}{d\cos\theta d\varphi d\cos\bar{\theta} d\bar{\varphi}} \propto \frac{1 + \sin\bar{\theta} \sin\theta \cos(\bar{\varphi} - \varphi) + \cos\bar{\theta} \cos\theta}{2}$$

[Hagiwara, Yokoya & Zheng (JHEP'18)]

- Toponium characteristics: **small $\Delta\varphi_{\ell\ell'}$, small $|\Delta\eta_{\ell\ell'}|$ and small $m_{\ell\ell'}$**

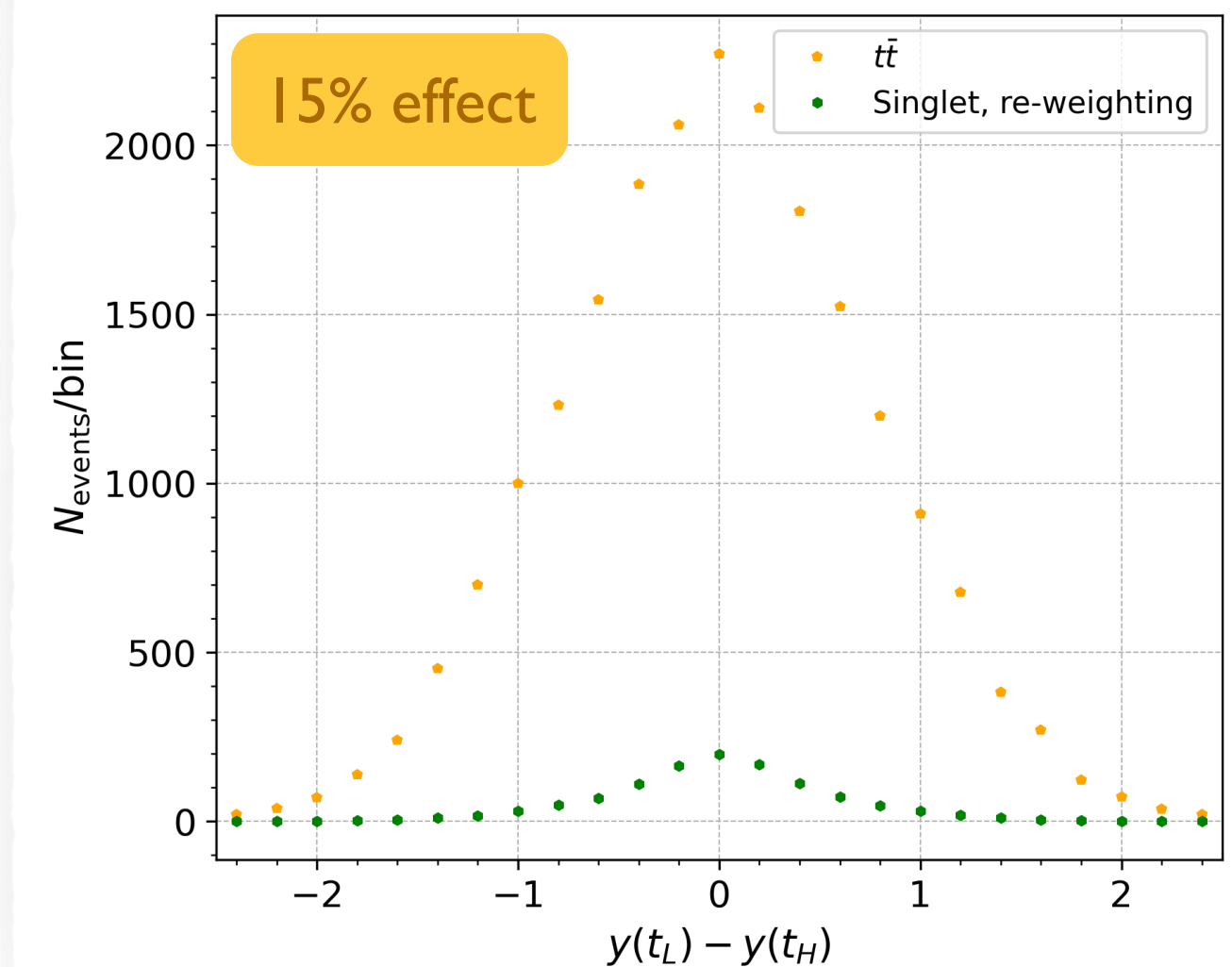
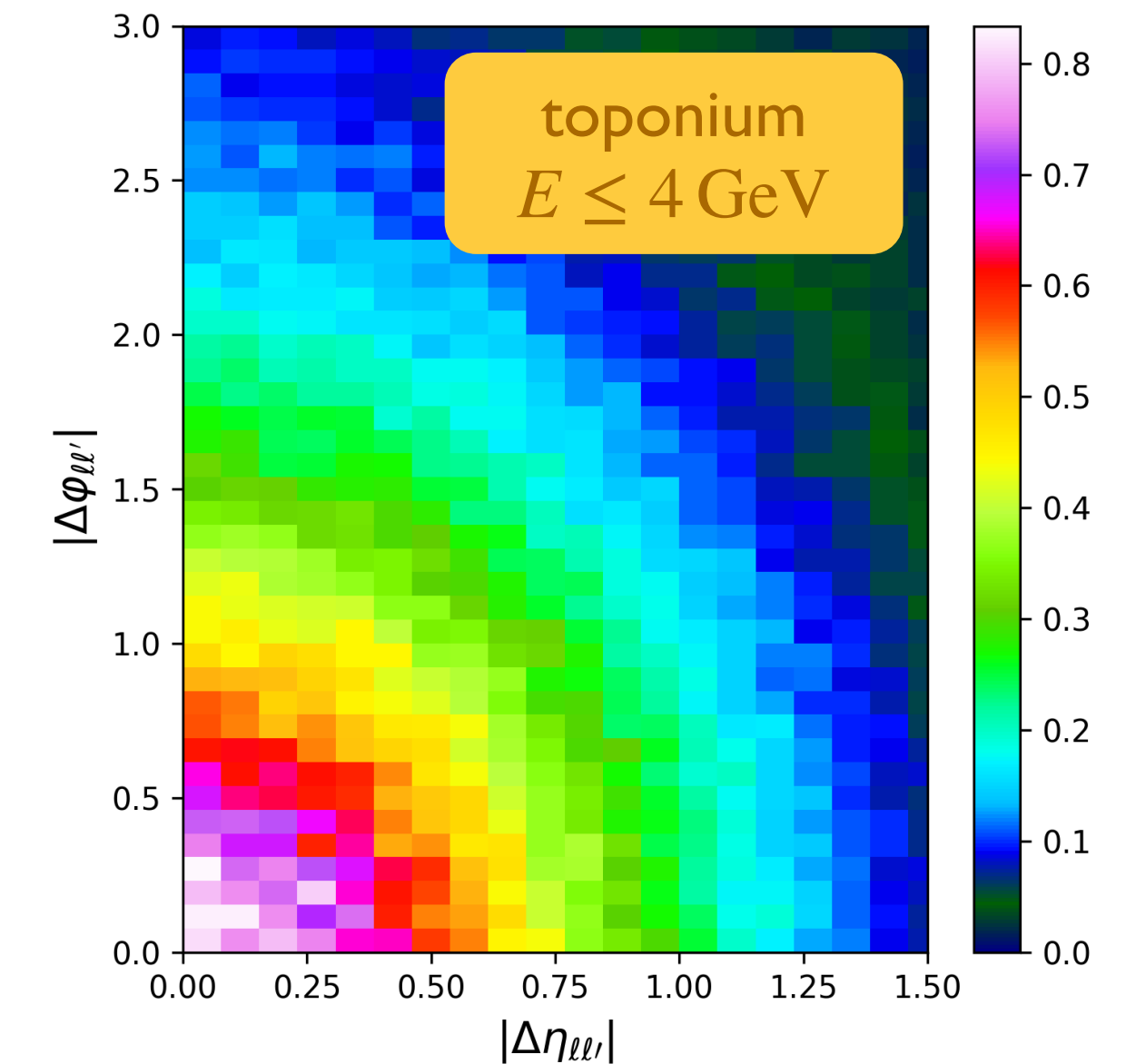
→ Small azimuthal angle separation (survives the lab frame boost)

→ Small di-lepton invariant mass
(cf. tension with data?)

$$m_{\ell\ell'}^2 = 2E_{\bar{\ell}}E_{\ell'} \left(1 - \sin\bar{\theta} \sin\theta \cos(\bar{\varphi} - \varphi) - \cos\bar{\theta} \cos\theta \right)$$

Bulk of toponium events

- **Option A:** $\Delta\varphi_{\ell\ell'} < 0.9$; $\Delta\eta_{\ell\ell'} < 0.9$
→ $N_{t\bar{t}} = 127,000$; $N_{\text{toponium}} = 3,520$ [sensitivity of 9.7σ (S/B $\sim 2.8\%$)]
- **Option B:** $\Delta\varphi_{\ell\ell'} < \pi/5$; $m_{\ell\ell'} < 40$ GeV
→ $N_{t\bar{t}} = 77,100$; $N_{\text{toponium}} = 2,200$ [sensitivity of 7.8σ (S/B $\sim 2.9\%$)]
- **Option C:** $\Delta R_{\ell\ell'}^2 < 0.8$
→ $N_{t\bar{t}} = 99,400$; $N_{\text{toponium}} = 2,980$ [sensitivity of 9.3σ (S/B $\sim 3\%$)]



A few last words...

Top-antitop production near the threshold

- Emergence of a toponium system at a scale of 0.05 GeV^{-1}
- Decay at a time scale of $\sim 0.3 \text{ GeV}^{-1}$
- Occurs well before hadronisation at 5 GeV^{-1}

Possibility to probe the toponium wave function

- 'The smallest hadron in the SM'
- The story just begins...

To-do list

- Single leptonic channel [Courtesy L. Munoz-Aillaud (2025)]
 - 11σ reachable with spin density matrix info
- Better potential modelling
- Excited states
- Colour octet (no resonant enhancement)
- Matching NRQCD to NLO-QCD + parton showers
- Impact on top mass measurements

