Gauge-Higgs Unification and Symmetry Breaking: application to 5D GUTs

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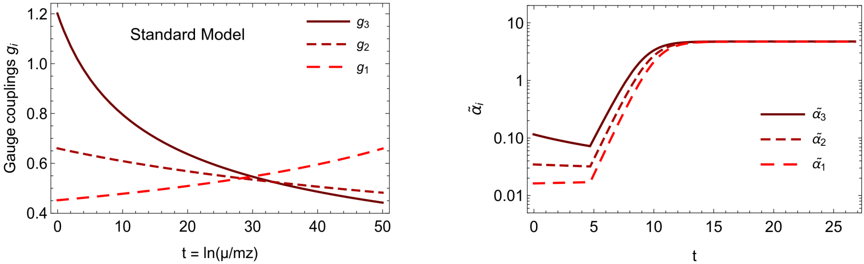
Introduction: Why 5D GUTs?

- The Standard Model relies on local gauge symmetries: $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$
- At $\Lambda_{\mathrm{GUT}} \sim 10^{16}$ GeV, the gauge couplings meet: enlarge gauge structure ($\mathrm{SU}(5)$, $\mathrm{SO}(10)$...)
- Traditional GUTs have limitations (proton decay, doublet-triplet splitting, Landau poles...)
- New paradigm: No exact unification, gauge couplings tend to the same UV fixed point
  $\longrightarrow$ Need power law running of the couplings, introduce 1 **extra-dimension**



- Lower effective unification scale, no proton decay, natural doublet-triplet splitting [Cot 2022]

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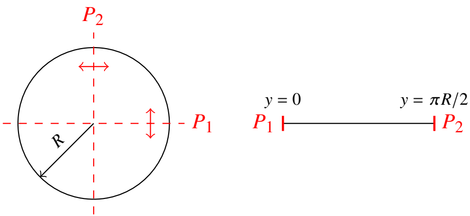
I. Introduction to 5D Orbifolds

Adding 1 extra dimension

- Higher dimensional QFT:
  - Gauge group $\mathcal{G}$
  - $\mathcal{M} = \mathbb{R}^4 \times C$, coordinates $\{x^\mu, y\}$

- Extra dimension compactified on a circle/ interval, radius R



- Define Parity on each boundary (boundary condition for the fields)

- Action of the parity on the fields Φ:

$$P_1 : \; \Phi(x^\mu, y) \sim P_1 \, \Phi(x^\mu, -y), \qquad P_2 : \; \Phi(x^\mu, y') \sim P_2 \, \Phi(x^\mu, -y')$$

- Choose diagonal basis for P_1 and P_2: $\quad P_{1,2} \, \Phi(x^\mu, -y) = \pm\Phi(x^\mu, y)$

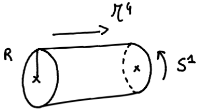
- Classify fields according to their eigenvalues $(P_1, P_2) = (\pm, \pm)$

I. Introduction to 5D Orbifolds

Parities and Symmetry Breaking

- Kaluza Klein (KK) Decomposition:

$$\phi(x,y) = \sum_{n=0}^{+\infty} \phi^{(n)}(x) f_n(y) \quad \text{with} \quad f_n(y) = \begin{cases} (++) & \frac{1}{\sqrt{2\pi R}} + \frac{1}{\sqrt{\pi R}} \cos\left(\frac{ny}{R}\right) & \color{red}{\longrightarrow m_n = \frac{n}{R}} \\ (+-) & \frac{1}{\sqrt{\pi R}} \cos\left(\frac{(n+1/2)y}{R}\right) & \longrightarrow m_n = \frac{n+1/2}{R} \\ (-+) & \frac{1}{\sqrt{\pi R}} \sin\left(\frac{(n+1/2)y}{R}\right) & \longrightarrow m_n = \frac{n+1/2}{R} \\ (--) & \frac{1}{\sqrt{\pi R}} \sin\left(\frac{(n+1)y}{R}\right) & \longrightarrow m_n = \frac{n+1}{R} \end{cases}$$



- Low energy effective theory: KK towers integrated out, only zero modes coming from $(++)$ states remain

- Apply it to gauge fields: each parity breaks the GUT gauge group $\mathcal{G}$ at low energies:

$$\left.\begin{array}{l} P_1 : \mathcal{G} \to \mathcal{H}_1 \\ P_2 : \mathcal{G} \to \mathcal{H}_2 \end{array}\right\} (P_1, P_2) : \mathcal{G} \to \mathcal{H}_1 \cap \mathcal{H}_2$$

- Example: $\mathrm{SU}(N)$, $\quad p + q + r + s = N$

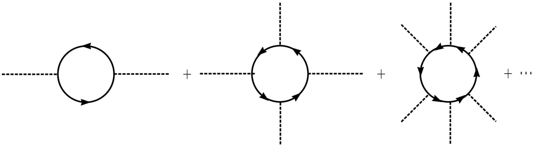
$$\left.\begin{array}{l} P_1 : \mathrm{SU}(N) \to \mathrm{SU}(p+q) \times \mathrm{SU}(r+s) \\ P_2 : \mathrm{SU}(N) \to \mathrm{SU}(p+r) \times \mathrm{SU}(q+s) \end{array}\right\} (P_1, P_2) : \mathrm{SU}(N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{SU}(r) \times \mathrm{SU}(s) \times \mathrm{U}(1)^3$$

II. Gauge Scalar and Symmetry Breaking

Potential for the gauge-scalar

- The 5D gauge field A_M decomposes as a 4D vector field A_μ and a scalar A_5
- Gauge invariance forbids tree level potential for A_5 but consider quantum corrections
  $\rightarrow$ Higgs mechanism, Gauge-Higgs unification [Hosotani 1983]
- 1-loop effective potential (Coleman Weinberg) for I fields in the model:

$$V_{\text{eff}}(A_5) = \frac{1}{2} \sum_{I,n} (-1)^{F_I} \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \log\left(p^2 + m_{I,n}^2\right) \quad \text{with} \quad F_I = \{0, 1\}$$



[Quiros 1999]

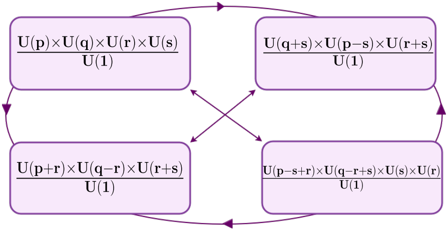
$\rightarrow$ **Finite potential**

II. Gauge Scalar and Symmetry Breaking

Gauge Transformation and VEV

- If A_5 develops a VEV, can use large gauge transformations along the 5th dimension to cancel it
- However, gauge transformations modify the boundary conditions, can lead to different symmetry breaking patterns
- Use gauge transformation to build equivalence class of parities:

Choose P_1 and P_2 such as: $\mathrm{SU}(N) \rightarrow \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{SU}(r) \times \mathrm{SU}(s) \times \mathrm{U}(1)^3$



II. Gauge Scalar and Symmetry Breaking

Orbifold Stability

- Contributions to the effective potential coming from all kind of fields:

$$V_{\text{eff}}(A_5) = V_{\text{eff}}^{\text{gauge}}(A_5) + V_{\text{eff}}^{\text{fermion}}(A_5) + V_{\text{eff}}^{\text{scalar}}(A_5)$$

- $V_{\text{eff}}^{\text{gauge}}(A_5)$ can't lead to the breaking of the gauge group ($\langle A_5 \rangle \neq 0$) as it would mean that the starting gauge group is already broken at high energy

- If it does, one can use large gauge transformation to remove the VEV $\longrightarrow$ modification on the parities, different breaking patterns

Orbifold Vacuum Stability

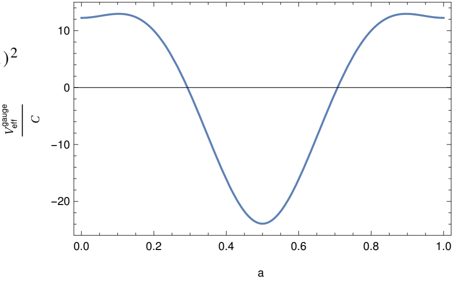
$V_{\text{eff}}^{\text{gauge}}(A_5)$ must have its minimum $\langle A_5 \rangle = 0$

Stability Criteria: example with SU(6)

$(p = 2,\ q = 3,\ s = 1\ )$

$\mathrm{SU}(6) \to \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)^2$

$\phi_{A_5} = (\mathbf{1}, \mathbf{2})_{1/2,3}$



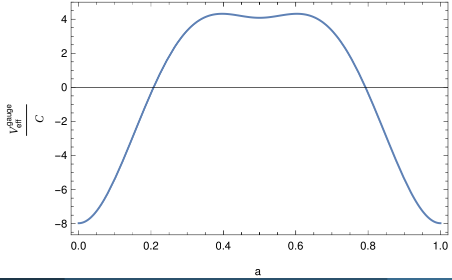
→ **UNSTABLE**

↓ **GAUGE TF**

$(p = 1,\ q = 4,\ r = 1\ )$

$\mathrm{SU}(6) \to \mathrm{SU}(4) \times \mathrm{U}(1) \times \mathrm{U}(1)$

$\phi_{A_5} = \mathbf{4}_{5,1}$



→ **STABLE**

II. Gauge Scalar and Symmetry Breaking

Classification of stable orbifolds

Model	**Breaking pattern**	**Stability criteria**
$SU(N)$	$SU(N) \to SU(A) \times SU(N-A) \times U(1)$	stable $\forall$ A
	$SU(N) \to SU(p) \times SU(q) \times SU(s) \times U(1)^2$	$p \geq N/2$
$Sp(2N)$	$Sp(2N) \to Sp(2A) \times Sp(2(N-A))$	stable $\forall$ A
	$Sp(2N) \to Sp(2p) \times Sp(2q) \times Sp(2s)$	$p \geq N/2$
	$Sp(2N) \to SU(p) \times SU(q) \times U(1)^2$	stable $\forall p, q$
$SO(2N)$	$SO(2N) \to SO(2A) \times SO(2(N-A))$	stable $\forall$ A
	$SO(2N) \to SO(2p) \times SO(2q) \times SO(2s)$	$p \geq N/2$
	$SO(2N) \to SU(p) \times SU(q) \times U(1)^2$	stable $\forall p, q$

III. Example: another SU(6) model

Parity definition and Matter content

- Choice of parities leading to a stable orbifold:

$$P_1 = \mathrm{diag}(+1 \cdots, +1, +1, \cdots, +1, -1, \cdots, -1),$$
$$P_2 = \mathrm{diag}(\underbrace{+1, \cdots, +1}_{p=3}, \underbrace{-1, \cdots, -1}_{q=2}, \underbrace{-1, \cdots, -1}_{s=1}).$$

- Orbifold breaking: $\mathrm{SU}(6) \longrightarrow \mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)^2$

- Gauge-Scalar: $\phi_{A_5} = (\mathbf{3}, \mathbf{1})_{-1/3,3}$

- Fermion representations leading to SM zero modes:

$$\Psi_{\mathbf{15}}^{(+,-)} \supset q_L + l_L^c \quad \text{and} \quad \Psi_{\overline{\mathbf{15}}}^{(-,-)} \supset u_R + e_R + d_R^c$$

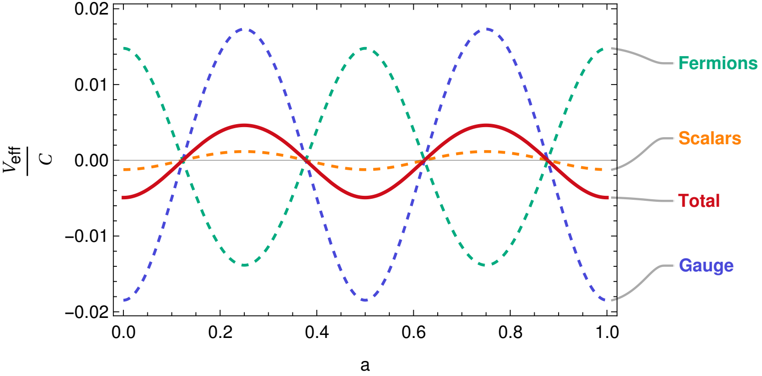
- Scalar that contains the SM Higgs field:

$$\Phi_{\mathbf{15}}^{(-,+)} \supset \phi_h$$

III. Example: another SU(6) model

Effective potential for the Gauge Scalar

- Effective potential:



- Gauge Higgs doesn't break the gauge group, SU(3) × SU(2) × U(1)^2 preserved

III. Example: another SU(6) model

A bit of phenomenology

- Compute mass of the gauge scalar ϕ_{A_5}:

$$m^2_{\phi_{A_5}} = \frac{R^2}{2} \, \frac{\partial^2}{\partial a^2} V_{\text{eff}}(a)\Big|_{a=0} = \frac{3}{16}\zeta(3)\frac{1}{\pi^4 R^2} \,,$$

- Bulk gauge interactions allow leptoquark coupling with the gauge scalar:

$$\mathcal{L} \supset \overline{\Psi}^{(+,-)}_{\mathbf{15}} \, iD_M \Gamma^M \Psi^{(+,-)}_{\mathbf{15}} \supset \bar{q}_L \phi_{A_5} l^c_L$$

- Leptoquark searches at LHC: $m_{\phi_{A_5}} \geq 2$ TeV

- Constraints on the compactification scale:

$$m_{KK} = \frac{1}{R} \geq 50\text{TeV}$$

Conclusion

- aGUTs: alternative to traditional GUTs, can be realized in 5D
- Orbifolds come in handy to build realistic models
- Stable Orbifold: the Gauge-Higgs scalar has to preserve the gauge part of the theory
- Only a few scenario are compatible with the orbifold stability criteria, constrains on aGUTs theories that can be built

Outlook

- More phenomenological aspects of those models still need to be studied: flavour, matching to SM observables, localized anomaly cancellation, KK modes phenomenology and identification of Dark Matter candidates
- Some formal aspects on the asymptotic behaviour of those models need to be clarified: renormalization of 5D theories on orbifolds, non-perturbative contributions

Thank you for your attention

Parities for $\mathcal{G} = \mathrm{SU}(N)$

- Most general parities ($p + q + r + s = N$):

$$
\begin{aligned}
P_1 &= \mathrm{diag}(+1, \cdots, +1, +1, \cdots, +1, -1, \cdots, -1, -1, \cdots, -1), \\
P_2 &= \mathrm{diag}(\underbrace{+1, \cdots, +1}_{p}, \underbrace{-1, \cdots, -1}_{q}, \underbrace{+1, \cdots, +1}_{r}, \underbrace{-1, \cdots, -1}_{s}),
\end{aligned}
$$

- Action of the parities on A_μ:

$$
(P_1, P_2)(A_\mu) = \begin{array}{c} \begin{array}{cccc} p & q & r & s \end{array} \\ \begin{pmatrix} (+,+) & (+,-) & (-,+) & (-,-) \\ (+,-) & (+,+) & (-,-) & (-,+) \\ (-,+) & (-,-) & (+,+) & (+,-) \\ (-,-) & (-,+) & (+,-) & (+,+) \end{pmatrix} \end{array} \begin{array}{c} p \\ q \\ r \\ s \end{array}
$$

- At low energy, only the $(++)$ degrees of freedom remain:

$$
\mathrm{SU}(N) \to \mathrm{SU}(p) \times \mathrm{SU}(q) \times \mathrm{SU}(r) \times \mathrm{SU}(s) \times \mathrm{U}(1)^3
$$

GHU and Gauge transformations

- Gauge Transformation:

$$\Omega(y) = \begin{pmatrix} e^{i\alpha_1 y/R} & & & \\ & e^{i\alpha_2 y/R} & & \\ & & \ddots & \\ & & & e^{i\alpha_N y/R} \end{pmatrix}, \quad \sum_{j=1}^{N} \alpha_j = 0$$

- Under this gauge transformation, an unitary matrix $U \in SU(N)$ transforms:

$$U' = \Omega(y + 2\pi R)\, U \Omega(y)^{-1} = \begin{pmatrix} e^{i(\lambda_1 + 2\pi\alpha_1)} & & & \\ & e^{i(\lambda_2 + 2\pi\alpha_2)} & & \\ & & \ddots & \\ & & & e^{i(\lambda_N + 2\pi\alpha_N)} \end{pmatrix}$$

$\rightarrow$ True also for parity matrices, changed if α_i are not all integers

Asymptotic Unification: α running

- In 5D, α carries a mass dimension, define effective t'Hooft coupling:

$$\tilde{\alpha} = \mu R \alpha \tag{1}$$

- 1-loop 5D Beta function:

$$2\pi \frac{\mathrm{d}\tilde{\alpha}}{\mathrm{d}\ln\mu} = 2\pi\tilde{\alpha} - b_5\tilde{\alpha}^2 \tag{2}$$

$$b_5 = \frac{7}{3}C(\mathcal{G}) - \frac{4}{3}\sum_f T(R_f) - \frac{1}{3}\sum_s T(R_s) \tag{3}$$

- UV fixed point for $b_5 > 0$:

$$\tilde{\alpha}^* = \frac{2\pi}{b_5} \tag{4}$$

- Similar for Yukawa couplings, RGE given by:

$$2\pi \frac{\mathrm{d}\tilde{\alpha}_y}{\mathrm{d}\ln\mu} = 2\pi\tilde{\alpha}_y + c_y\tilde{\alpha}_y^2 - d_y\tilde{\alpha}\tilde{\alpha}_y \tag{5}$$

- Fixed point when $d_y > 0$, $c_y > 0$ and $d_y\tilde{\alpha}^* > 2\pi$:

$$\tilde{\alpha}_y^* = \frac{d_y\tilde{\alpha}^* - 2\pi}{c_y} \tag{6}$$

Asymptotic Unification: SU(6)

- SU(6) Beta function:

$$b_5 = \frac{61 - 16n_g}{3} \tag{7}$$

- $b_5 > 0$ for $n_g \leq 3$
- Yukawa term: $\mathcal{L} \supset -Y_u \overline{\Psi}_{\overline{\mathbf{15}}} \Phi_{\mathbf{15}} \Psi_{\mathbf{15}}$
- We get the following fixed point:

$$d_y = 28, \qquad c_y = 144 \qquad \tilde{\alpha}_y = \frac{23 + 16n_g}{72(61 - 16n_g)}\pi \tag{8}$$

- We can have at most $n_g \leq 3$