

Naturalness, near-criticality and future colliders



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Based on arXiv:2503.03825
in collaboration with

Belén Gavela
and
Thomas Steingasser



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Framework Programme of the
European Union

Higgs Hunting 2025, 15 July, Orsay

Motivation

The Higgs potential,

$$V_{\text{Higgs}}(\Phi) = V_0 - \frac{m_{\text{eff}}^2}{2} H^2 + \frac{\lambda_{\text{eff}}}{4} H^4, \text{ is severely fine tuned:}$$

— If NP, then why $m_{\text{eff}}^2 \ll \Lambda_{\text{NP}}^2$? (EW hierarchy problem)

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Naturalness approach:

An underlying symmetry protects m_{eff}^2 ,
rendering it small

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Other approaches required?

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Motivation

$$\lambda_{\text{eff}}^{\text{LO}} \sim \lambda_H + \frac{1}{4\pi^2}(-y_t^4 + g_2^4 + (g_1^2 + g_2^2)^2)$$

The Higgs potential,

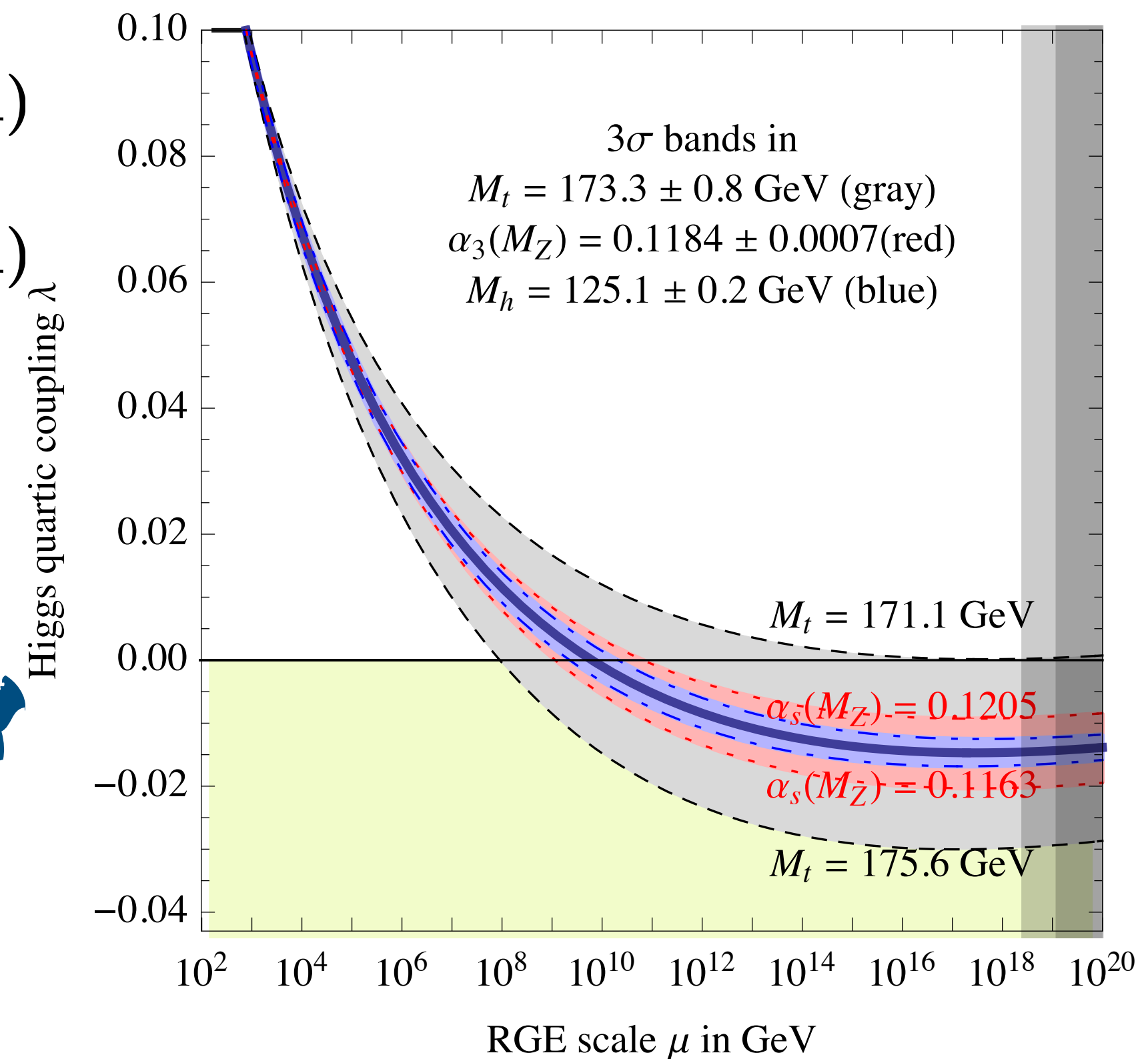
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[Buttazo et al 1307.3536]

- The SM predicts $V_0 \gg \Lambda_{\text{cosmo}}$ (Cosm. const. problem)
- If NP, then why $m_{\text{eff}}^2 \ll \Lambda_{\text{NP}}$? (EW hierarchy problem)
- Measured λ_{eff} and m_t show metastability

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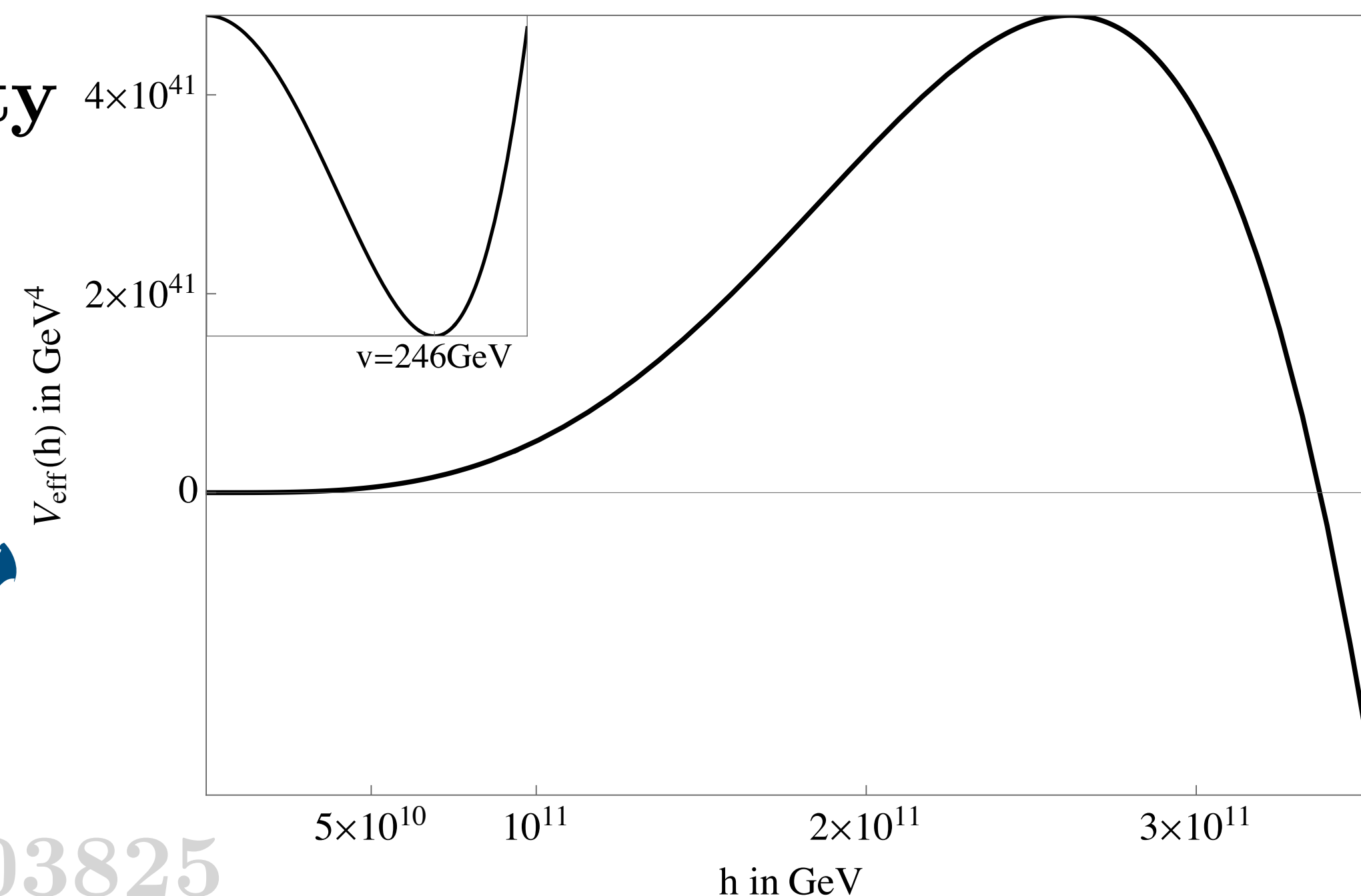
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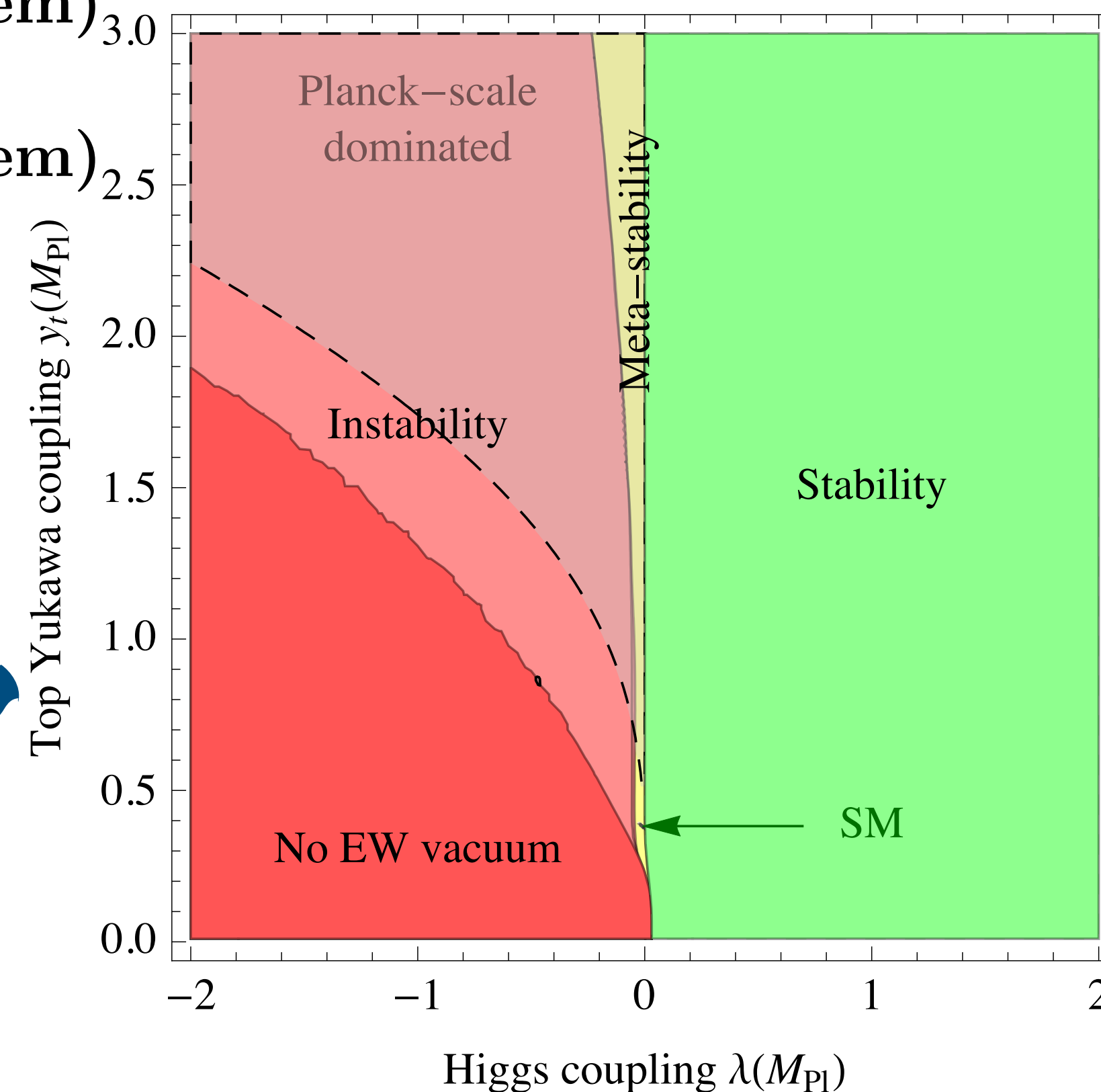
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[Buttazo et al 1307.3536]



Motivation

In the Higgs potential,

$$V_{\text{Higgs}}(\Phi) = \boxed{V_0} - \boxed{\frac{m_{\text{eff}}^2}{2}} H^2 + \boxed{\frac{\lambda_{\text{eff}}}{4}} H^4, \text{ all parameters are near-critical}$$

	$V_0 :$		$\text{dS} \longleftrightarrow \text{AdS}$	
	$m_{\text{eff}}^2 :$	close to	$\text{SSB} \longleftrightarrow \text{no SSB}$	transition
	$\lambda_{\text{eff}} :$		$\text{stable} \longleftrightarrow \text{unstable}$	

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 V_0 :

dS $\longleftrightarrow$ AdS

 m_{eff}^2 :

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 λ_{eff} :

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Naturalness approach:

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Near-criticality alternative:
Some unknown dynamical
mechanism forces the parameters to
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Motivation

Which dynamics?

Naturalness approach:

An underlying symmetry protects m_{eff}^2 ,
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Near-criticality alternative:
Some unknown dynamical
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* Ideas of underlying dynamics : *self-organised criticality*



PerBak, ChaoTang, and KurtWiesenfeld, '87



is a property of some dynamical systems that have a critical point as an attractor —> macroscopic behaviour alike to phase transitions

Often based on cosmological dynamics:

G. F. Giudice, M. McCullough, and T. You, "Self-organised localisation," arXiv:2105.08617

J. Khoury, "Accessibility Measure for Eternal Inflation: Dynamical Criticality and Higgs Metastability" arXiv:1912.06706

J. Khoury and O. Parrikar, "Search Optimization, Funnel Topography, and Dynamical Criticality on the String Landscape, arXiv:1907.07603

G. Kartvelishvili, J. Khoury, and A. Sharma, "The Self-Organized Critical Multiverse, arXiv:2003.12594

J. Khoury and S. S. C. Wong, "Early-time measure in eternal inflation", arXiv:2106.12590,

Motivation

Which dynamics?

Even ignoring the dynamics, the hypothesis has testable consequences

Which consequences?

Naturalness approach:

An underlying symmetry protects m_{eff}^2 , rendering it small

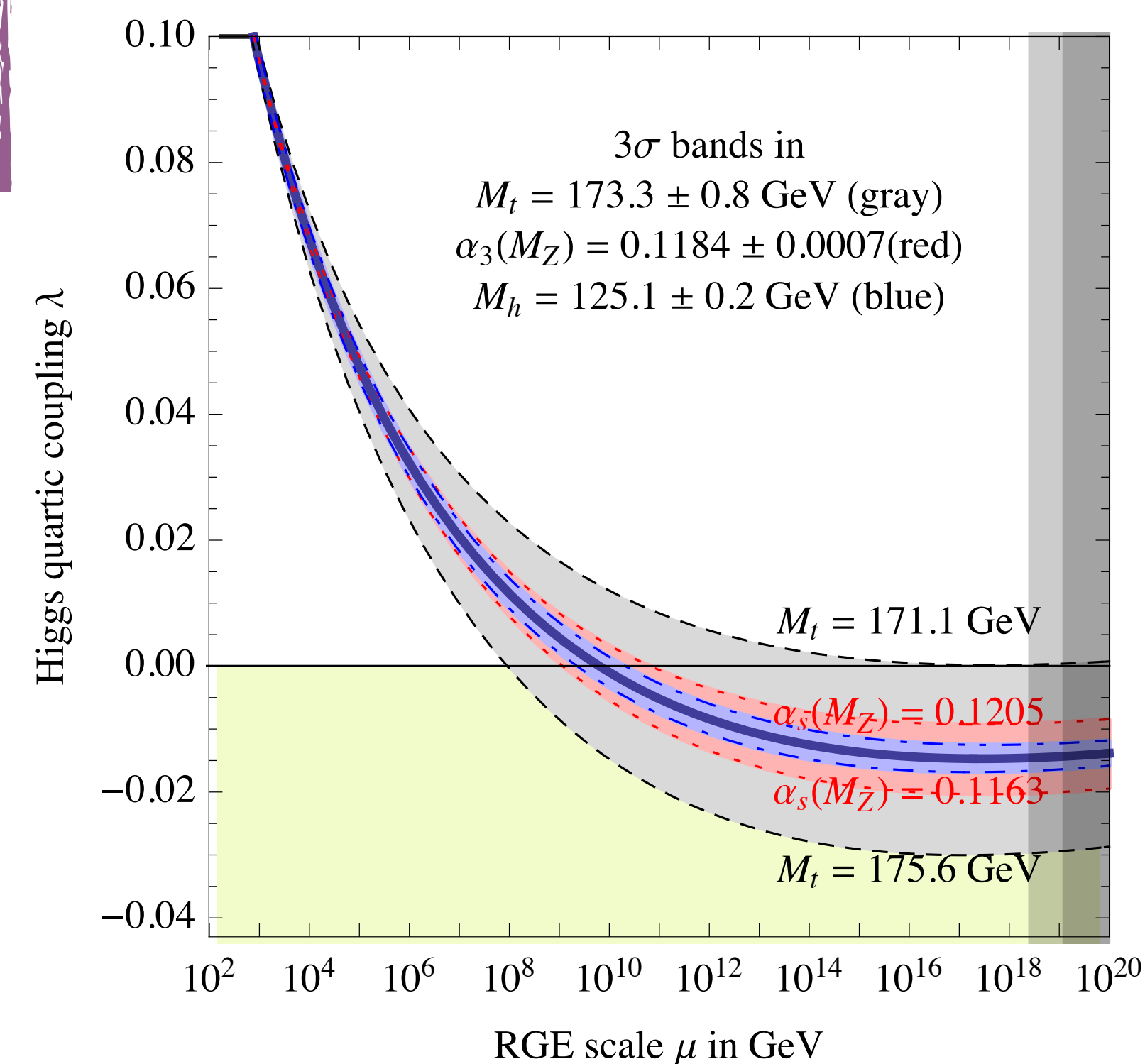
Near-criticality alternative:
Some unknown dynamical mechanism forces the parameters to be near-critical

Higgs metastability bound

The instability scale μ_I , defined by $\lambda_{\text{eff}}(\mu_I) = 0$

Metastability possible only if

$$m_h^2 < |\beta_\lambda(\mu_I)| e^{-3/2} \mu_I^2$$



[Buttazo et al 1307.3536]

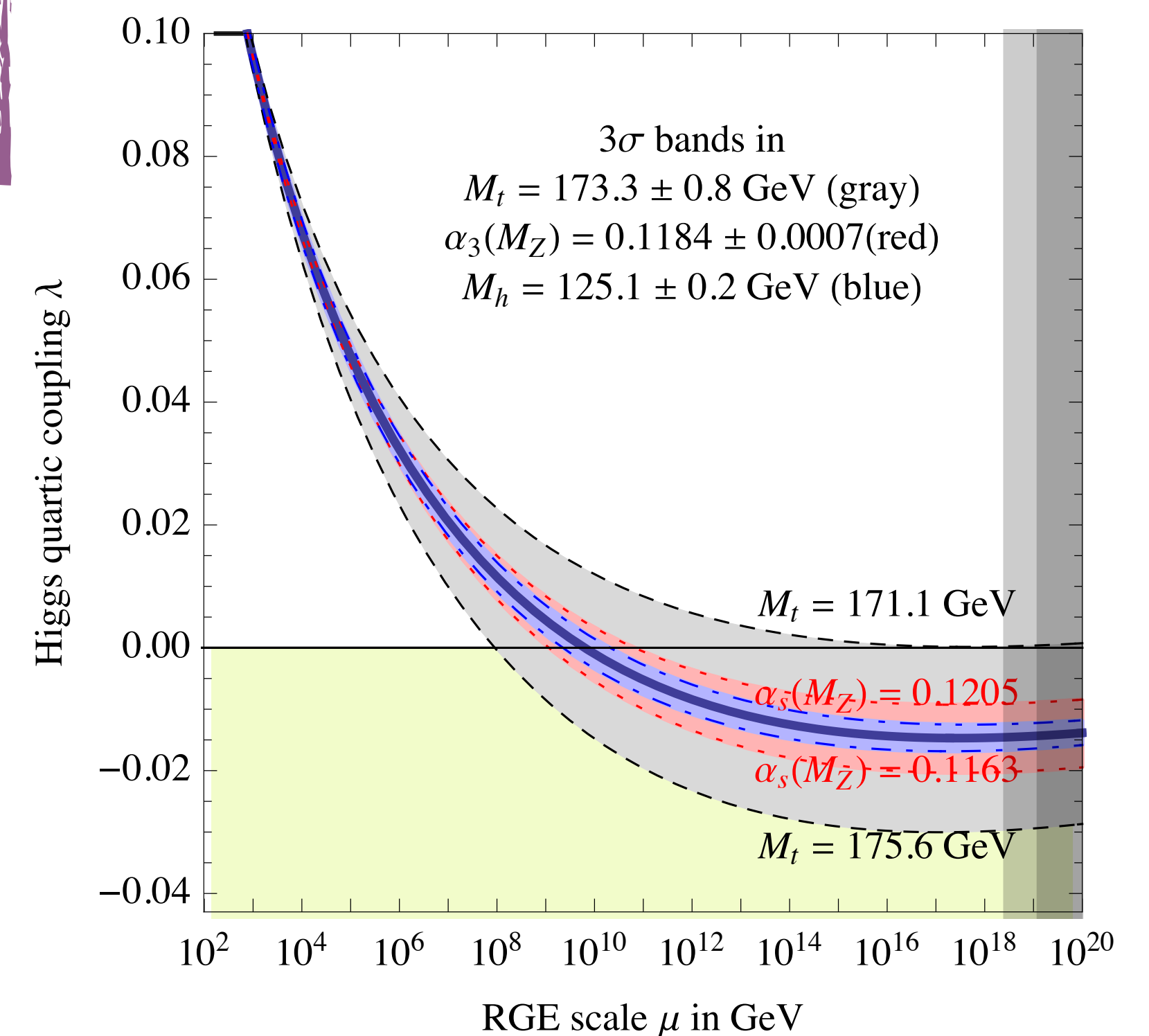
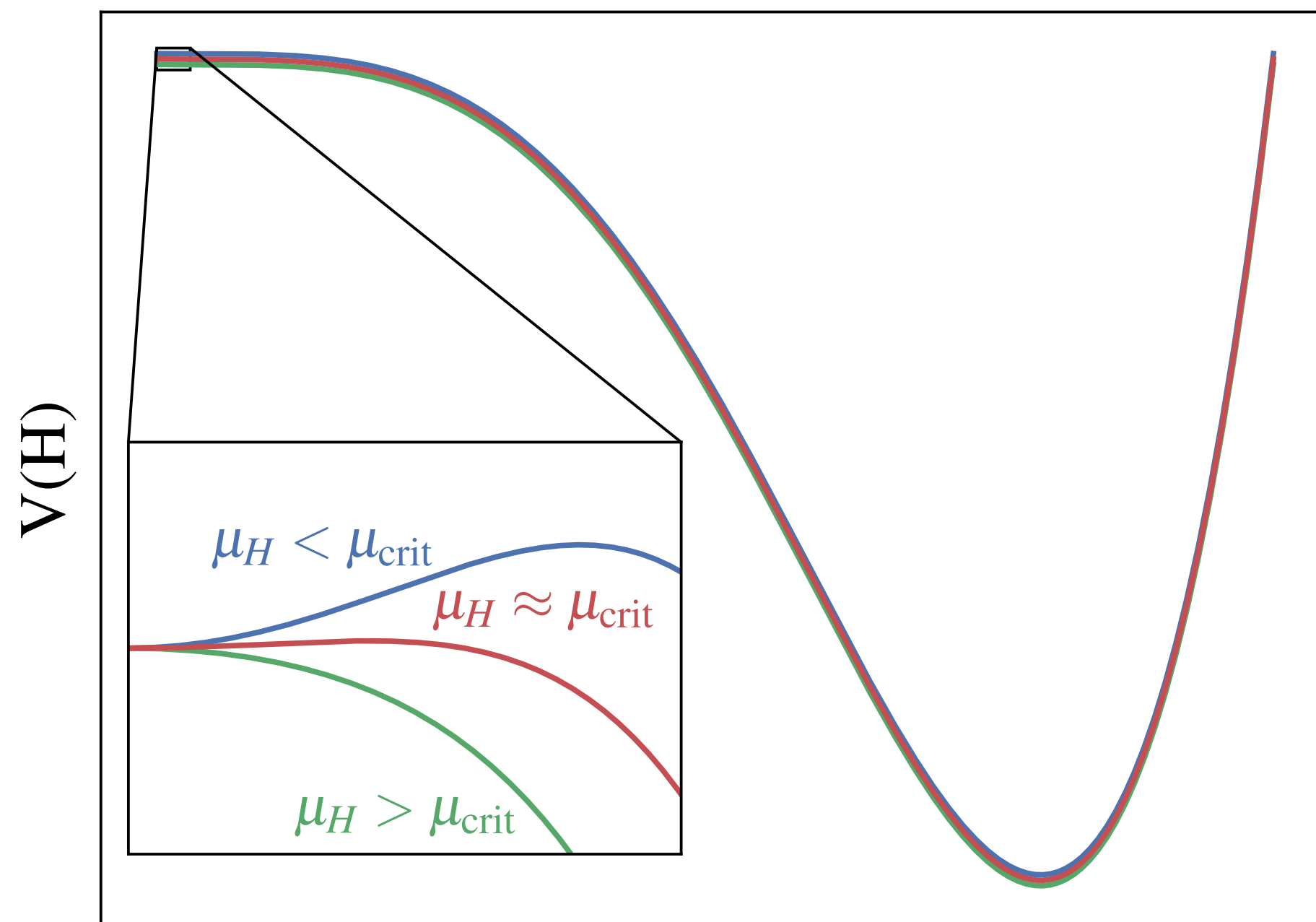
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$\exists$ IR vacuum
only for small
enough m_{eff}^2



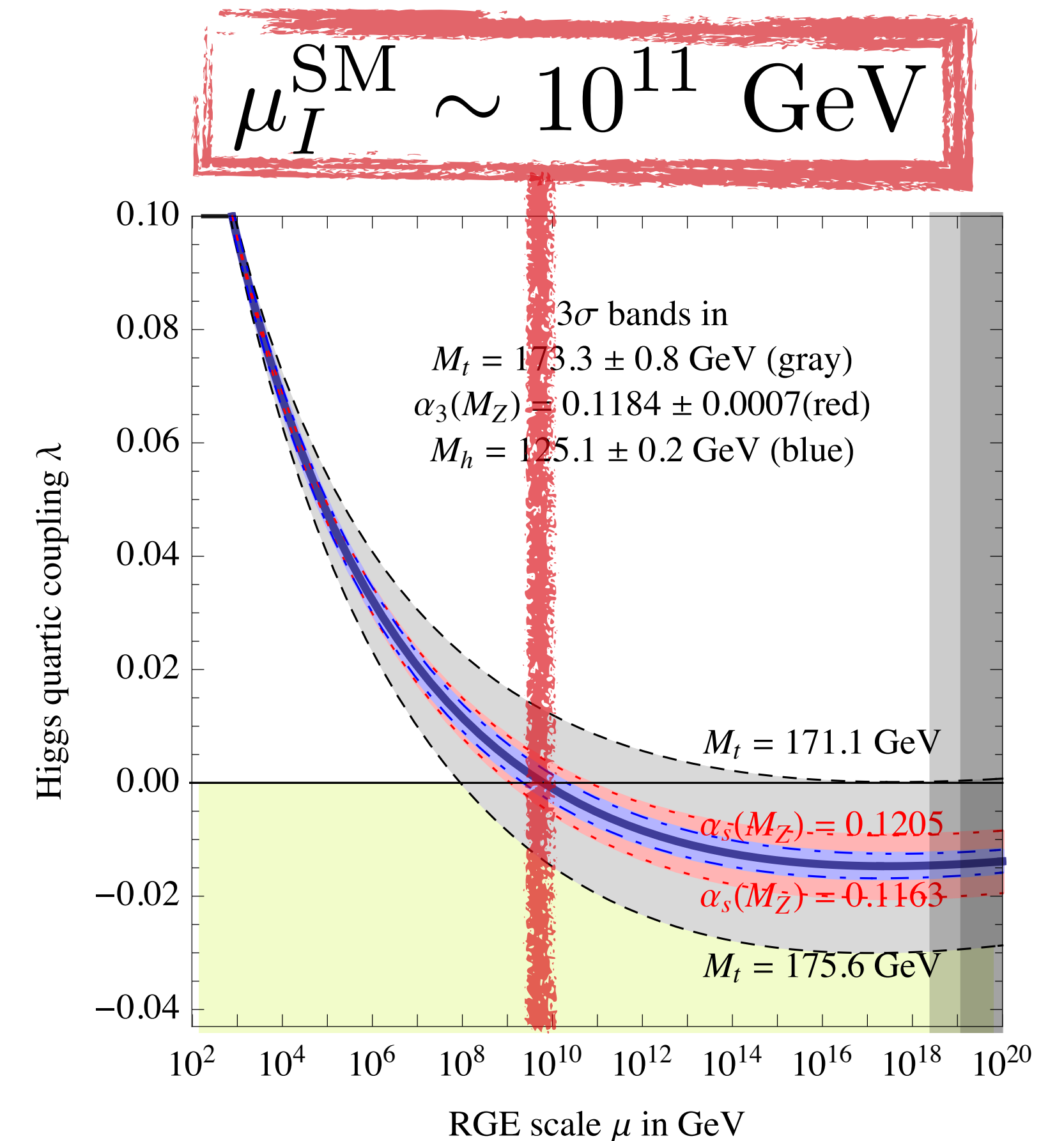
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$\left\{ \begin{array}{ll} \text{In the SM} & \longrightarrow \text{weak bound } m_h^2 \lesssim 10^{10} \text{ GeV} \\ \text{Many BSM extensions} & \longrightarrow \text{stronger bound} \end{array} \right.$



[Buttazo et al 1307.3536]

Higgs metastability bound

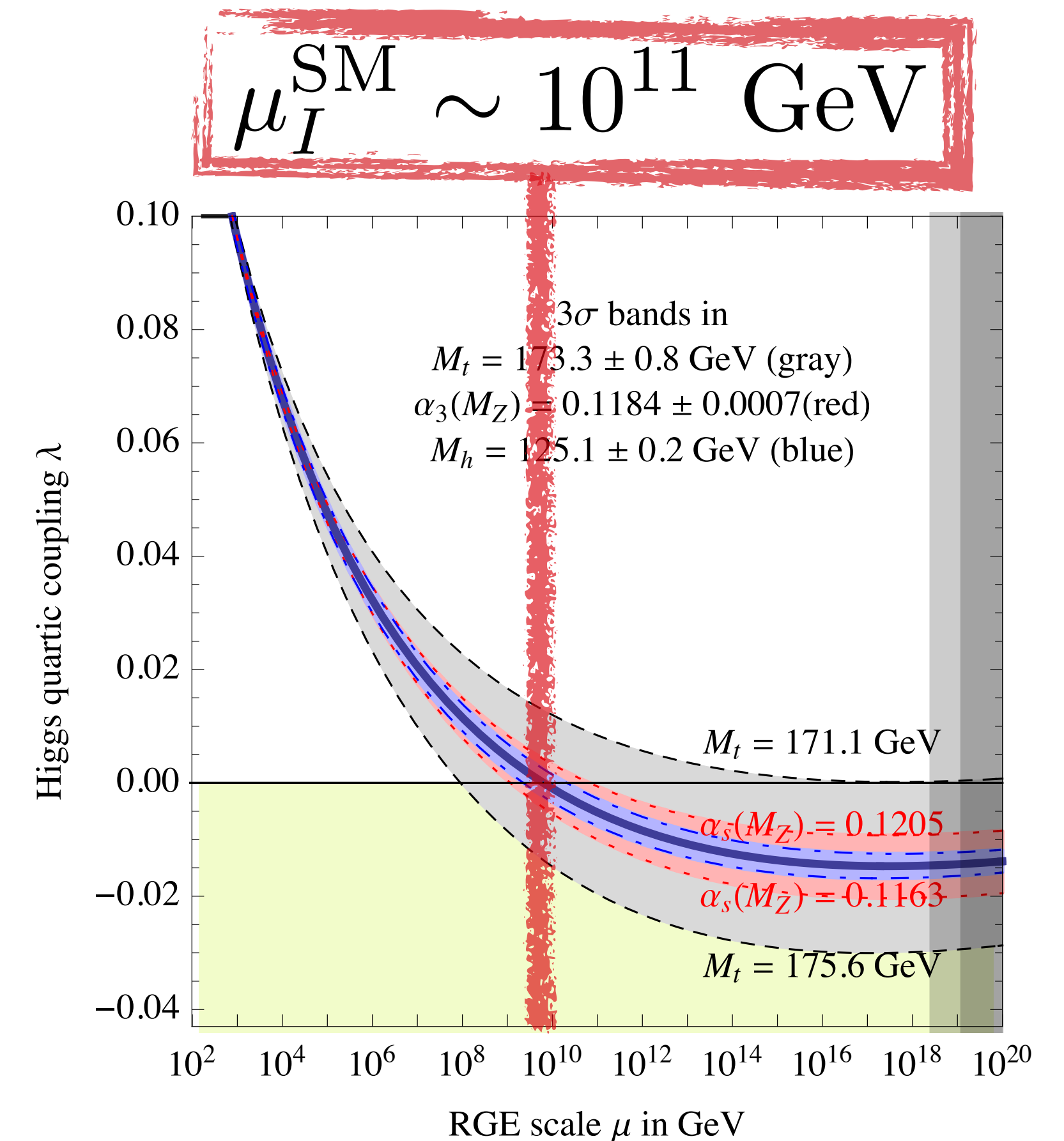
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Can we achieve e.g. $m_h^2 \lesssim \text{TeV}$?



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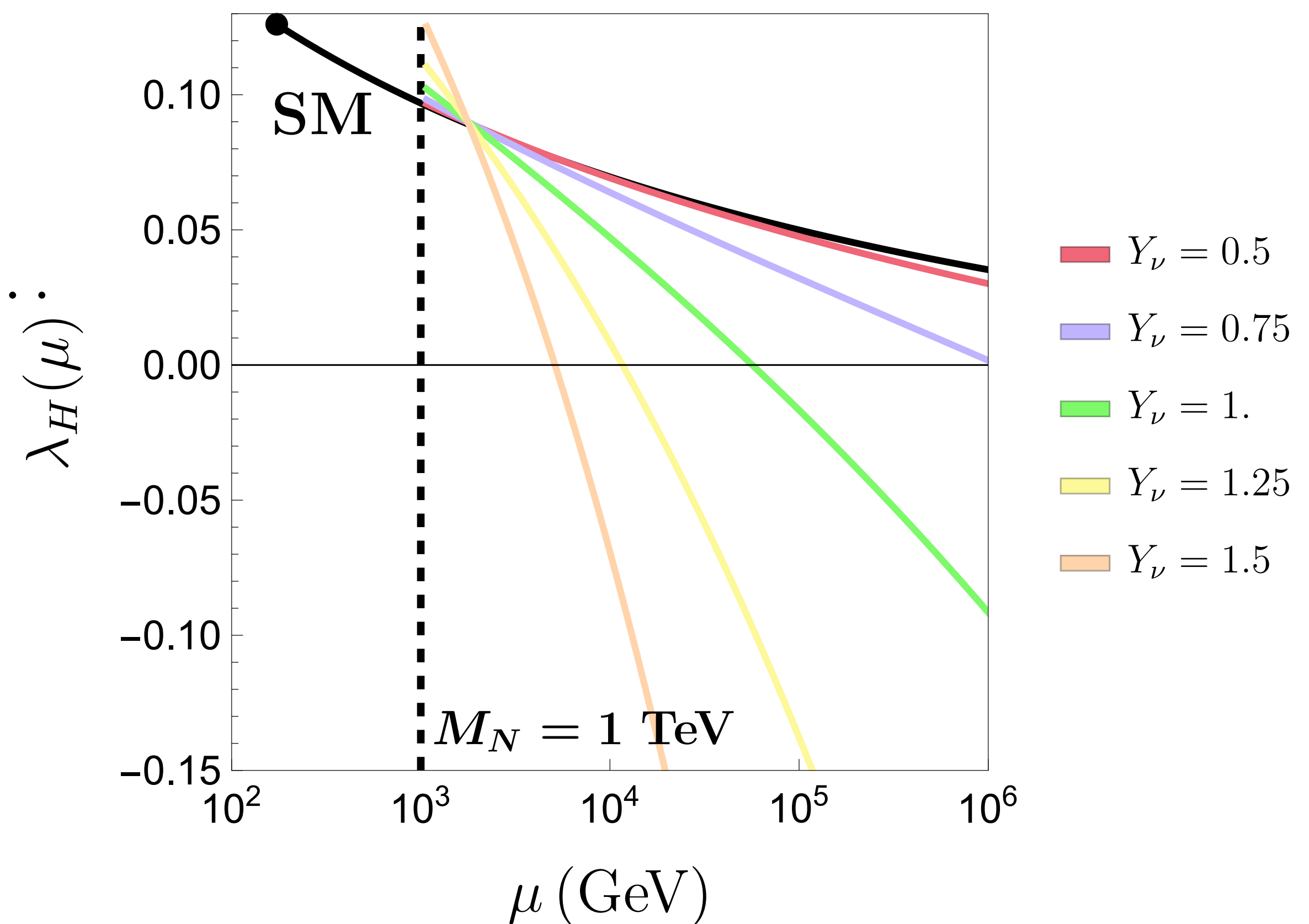
BSM Higgs metastability

Can we achieve e.g. $m_h^2 \lesssim \text{TeV}$?

Adding fermions generically lowers μ_I

e.g. for Right Handed Neutrinos,

$$\beta_\lambda \rightarrow \beta_\lambda + \frac{1}{(4\pi)^2} \left(4\lambda |Y_\nu|^2 - 2|Y_\nu|^4 \right) + \dots$$



BSM Higgs metastability

Can we achieve e.g. $m_h^2 \lesssim \text{TeV}$?

Adding fermions generically lowers μ_I

However, smaller $\mu_I \implies$ shorter vacuum lifetime τ

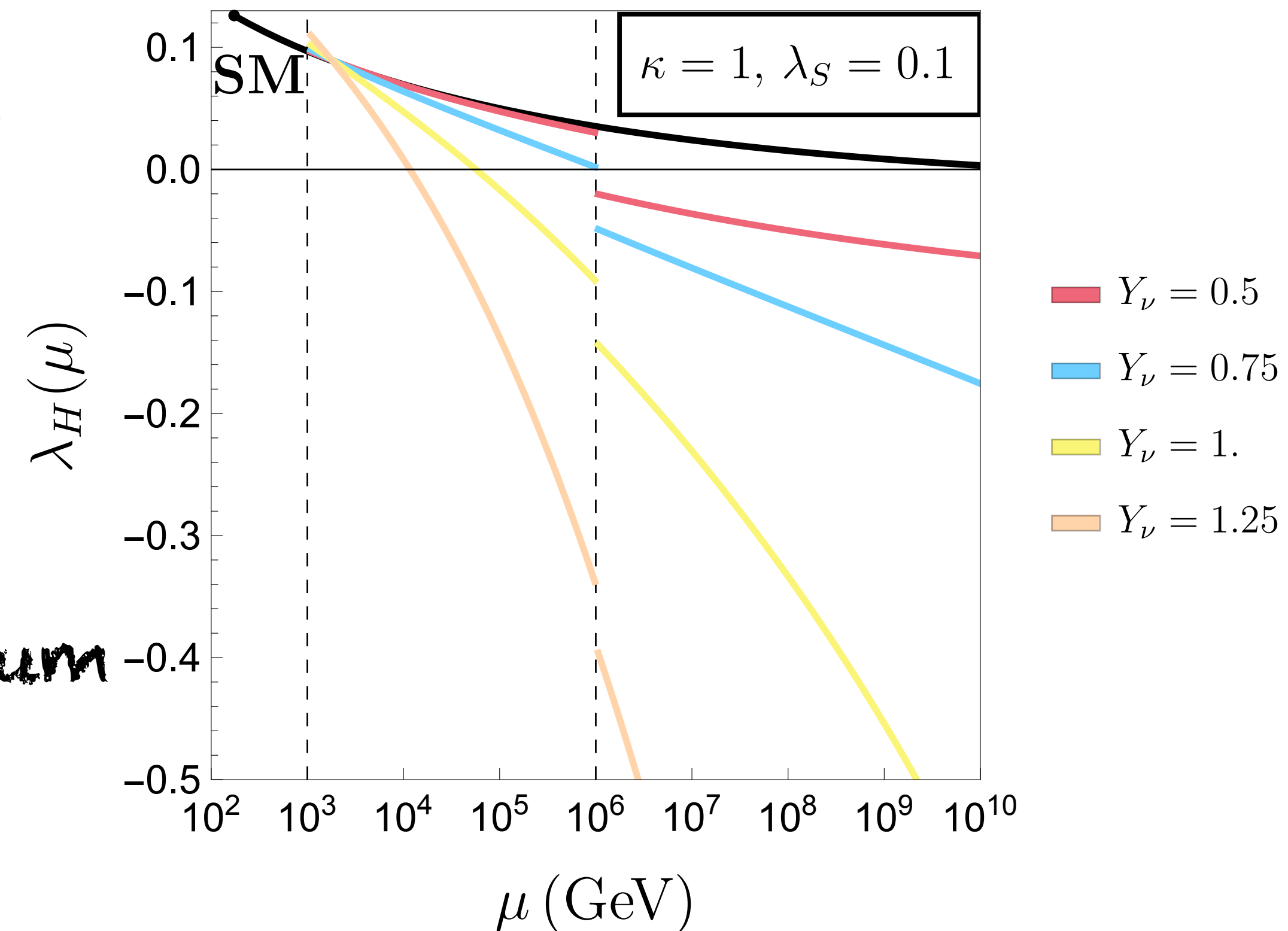
In general, $\mu_I \sim \mathcal{O}(\text{TeV}) \implies \tau < \text{age of the universe}$

BSM Higgs metastability

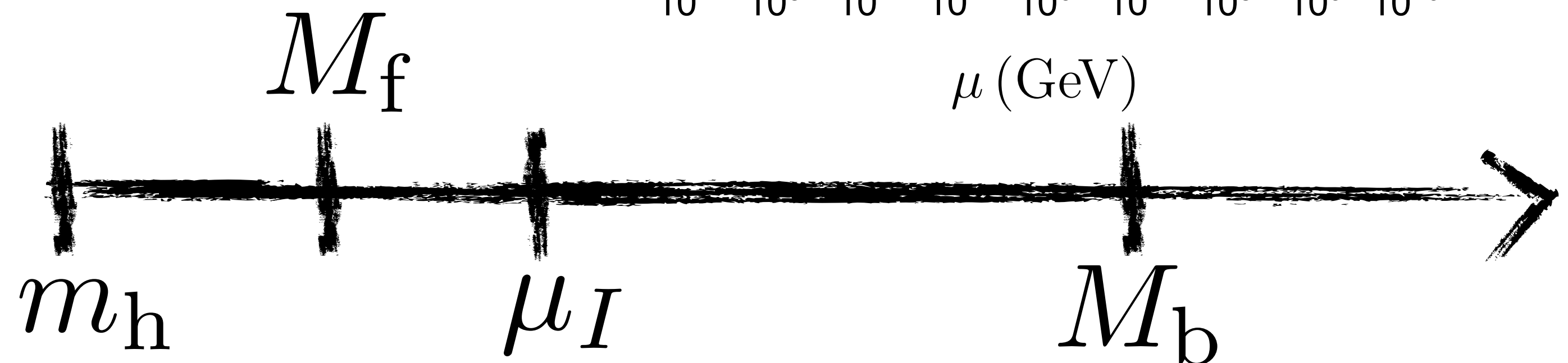
Can we achieve e.g. $m_h^2 \lesssim \text{TeV}^2$?

Adding fermions generically lowers μ_I

Adding new bosons stabilizes the vacuum



Suggests a scale hierarchy



BSM Higgs metastability

Can we achieve e.g. $m_h^2 \lesssim \text{TeV}$?

Adding fermions generically lowers μ_I

- Vector-like fermions [2105.08617, 2502.07876, 2408.10297]
- Heavy neutral leptons [2108.09315]
- **The Majoron model** [VE, B. Gavela, T. Steingasser, 2503.03825]

Adding new bosons stabilizes the vacuum

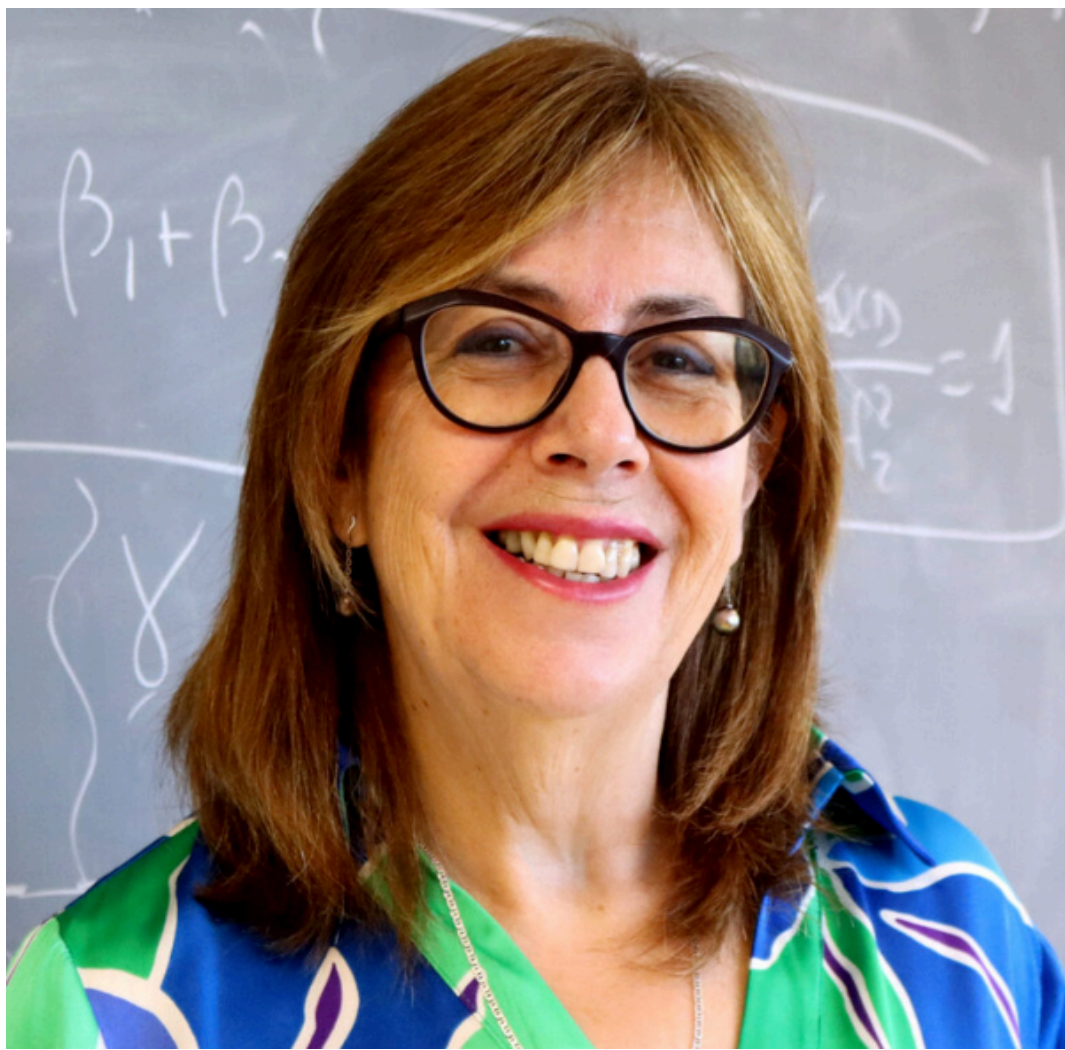
- ALPs [2412.03542]
- **The Majoron model**
[VE, Gavela, Steingasser,
2503.03825]



The low-scale Majoron model...

...contains just the right ingredients: fermions + scalar

Higgs near-criticality at future colliders [2503.03825]



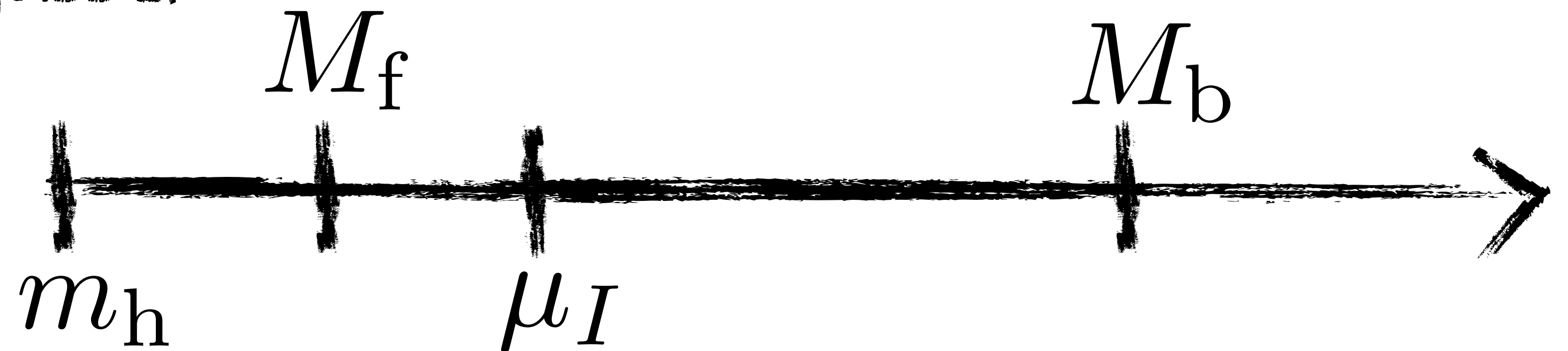
B. Gavela



T. Steingasser

+ well motivated

- Type-I Seesaw mechanism
- Strong coupling with SM allowed



The low-scale Majoron model...

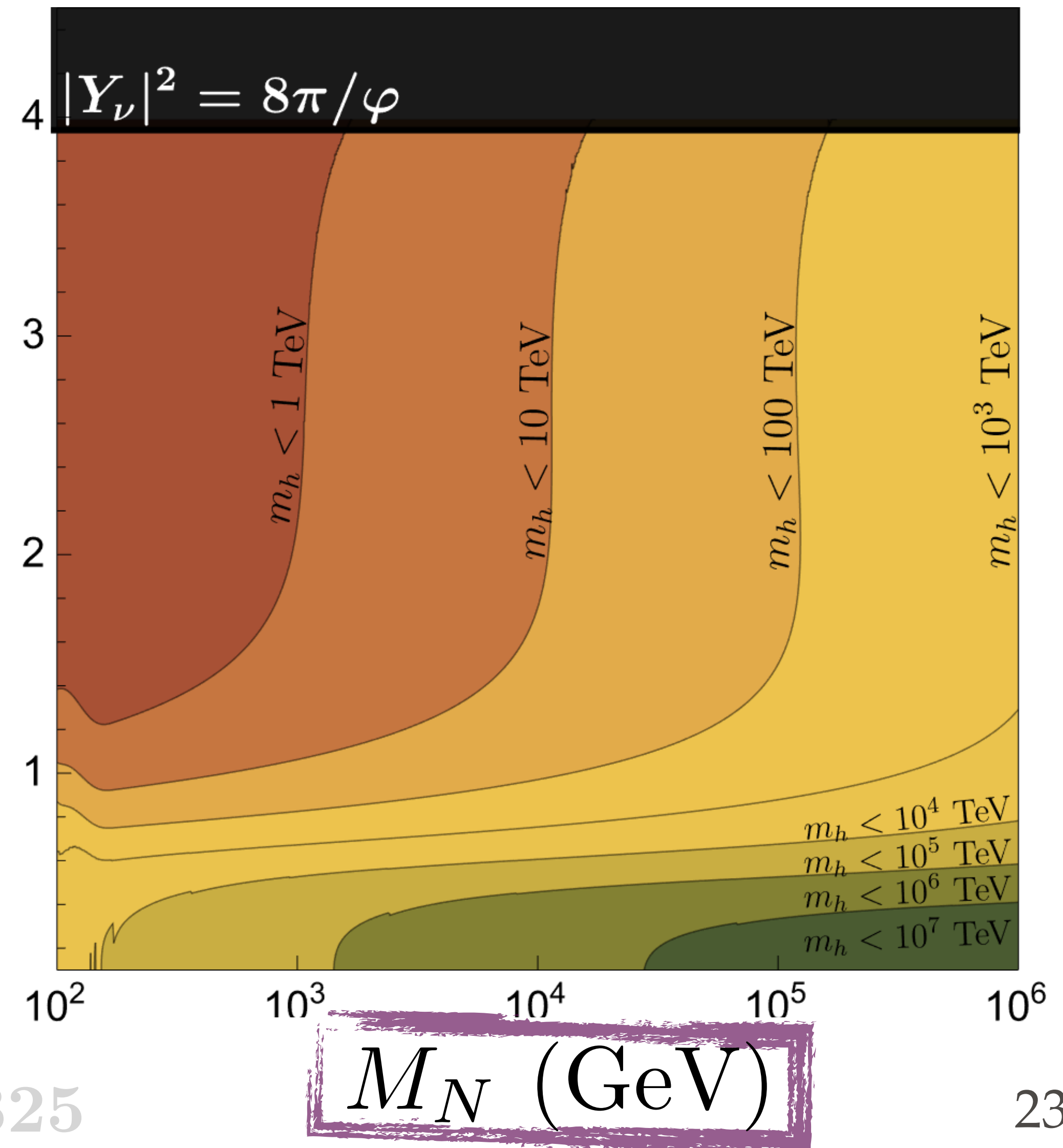
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Fermion content: RHNs

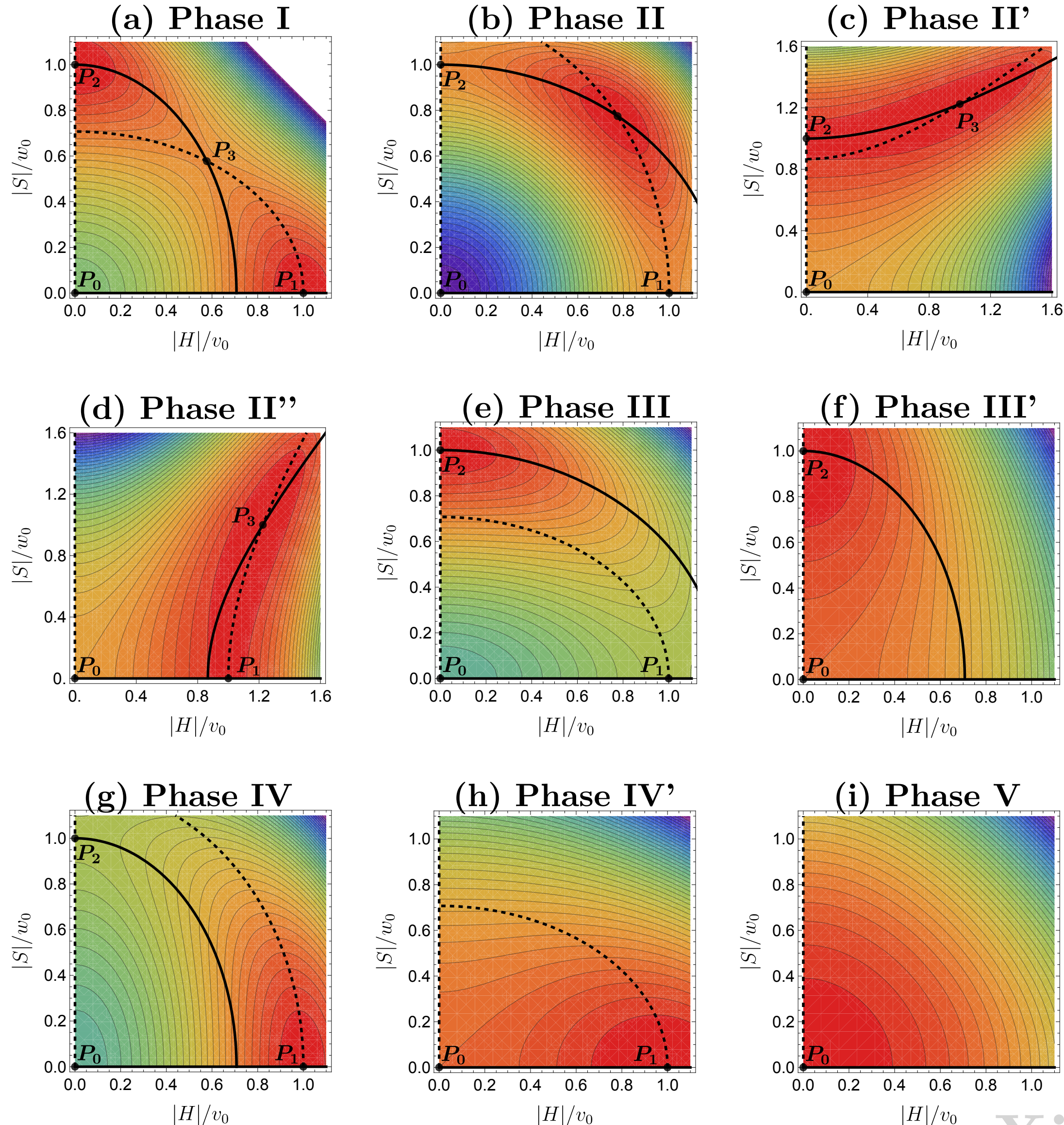
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$$\text{Tr} (Y_{\nu}^{\dagger} Y_{\nu})$$

$$m_h^2 < |\beta_{\lambda}(\mu_I)| e^{-3/2} \mu_I^2$$



The low-scale Majoron model



$$\Omega_H \equiv 2 \frac{\mu_S^2}{\mu_H^2} \cdot \frac{\lambda_H}{\kappa} \quad \text{and} \quad \Omega_S \equiv 2 \frac{\mu_H^2}{\mu_S^2} \cdot \frac{\lambda_S}{\kappa}$$

$$P_0 \equiv (0, 0),$$

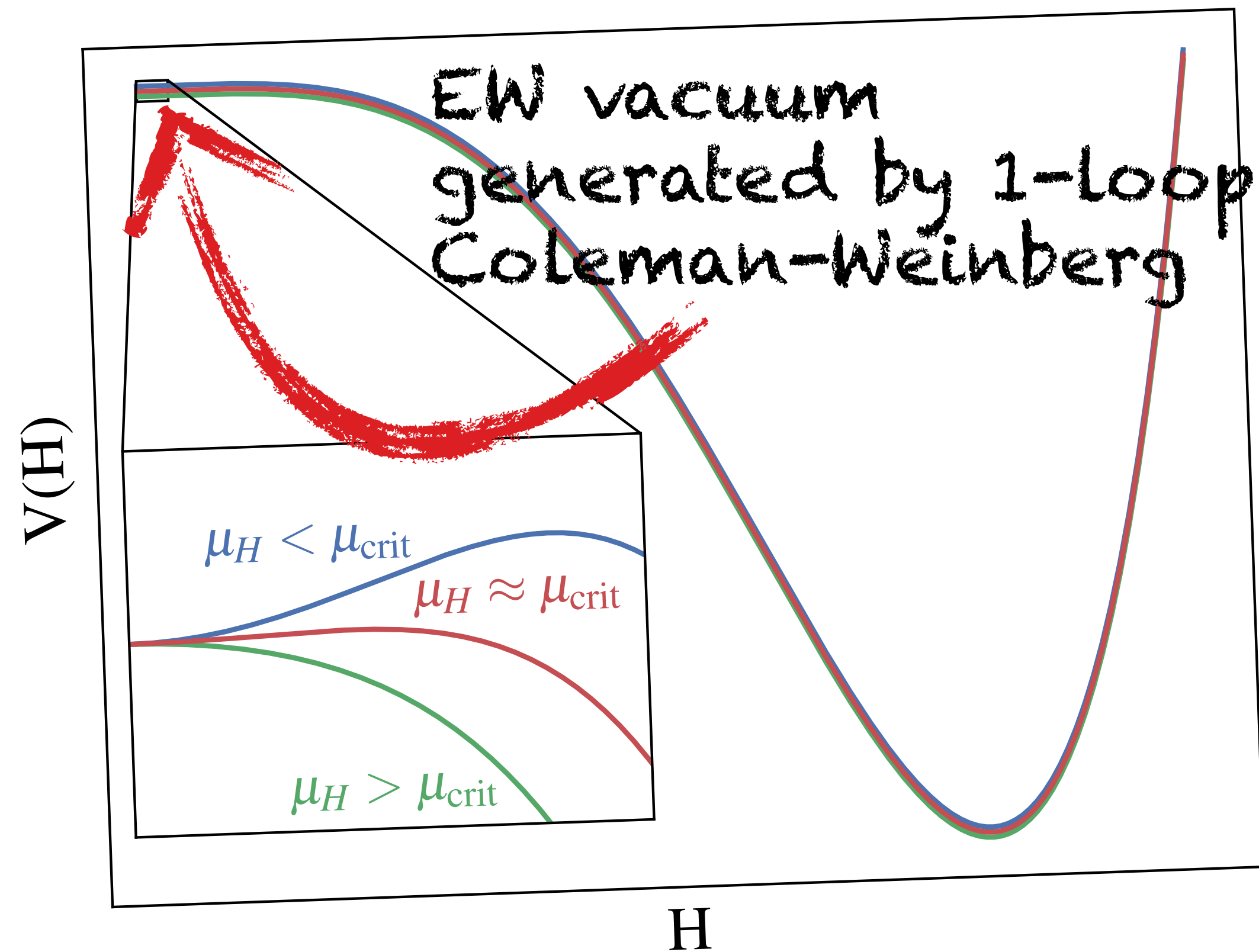
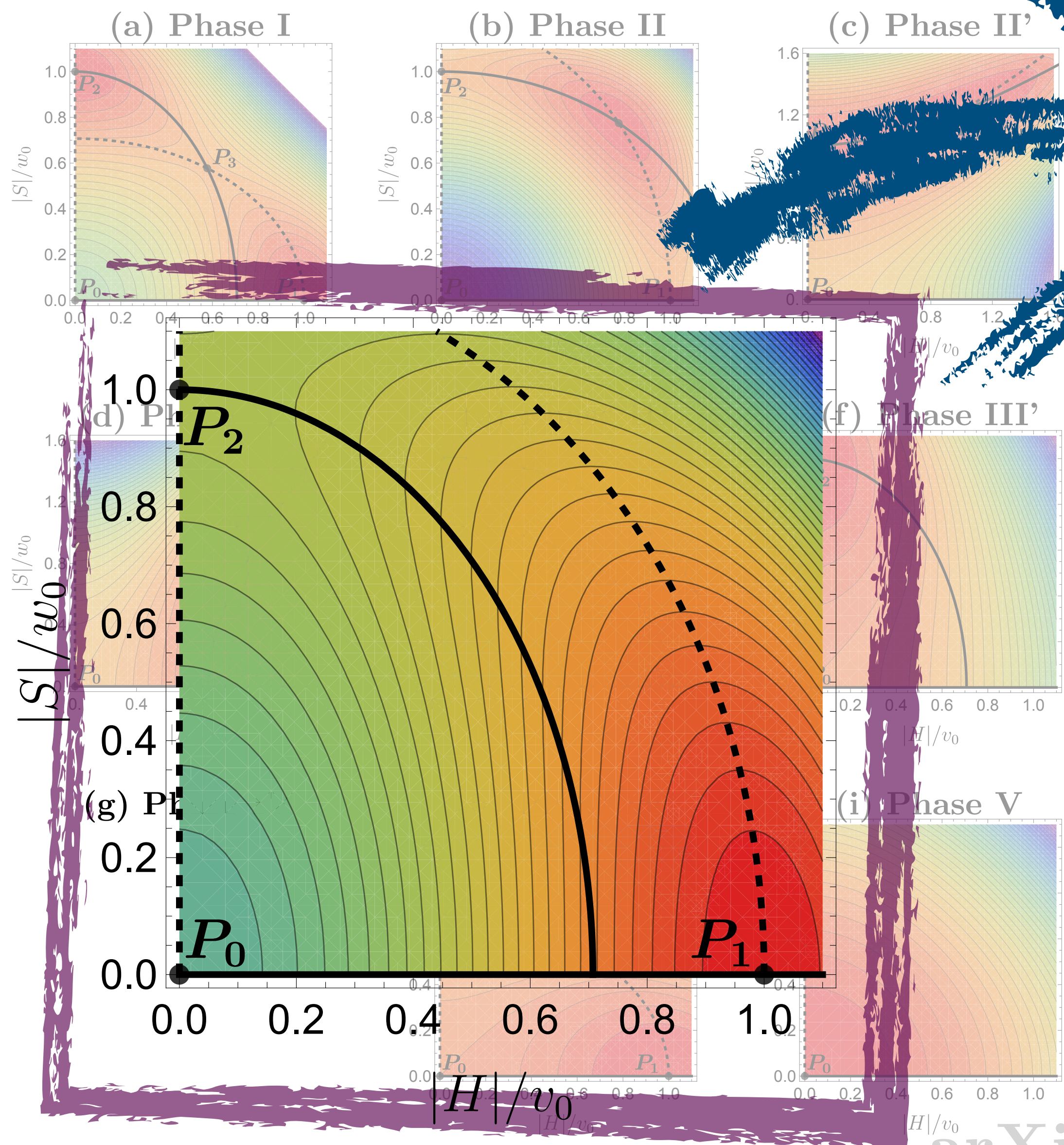
$$P_1 \equiv \left(\mu_H / (2\lambda_H)^{1/2}, 0 \right),$$

$$P_2 \equiv \left(0, \mu_S / (2\lambda_S)^{1/2} \right),$$

$$P_3 = \left(\sqrt{\frac{2\mu_H^2 \lambda_S - \mu_S^2 \kappa}{4\lambda_H \lambda_S - \kappa^2}}, \sqrt{\frac{2\mu_S^2 \lambda_H - \mu_H^2 \kappa}{4\lambda_H \lambda_S - \kappa^2}} \right)$$

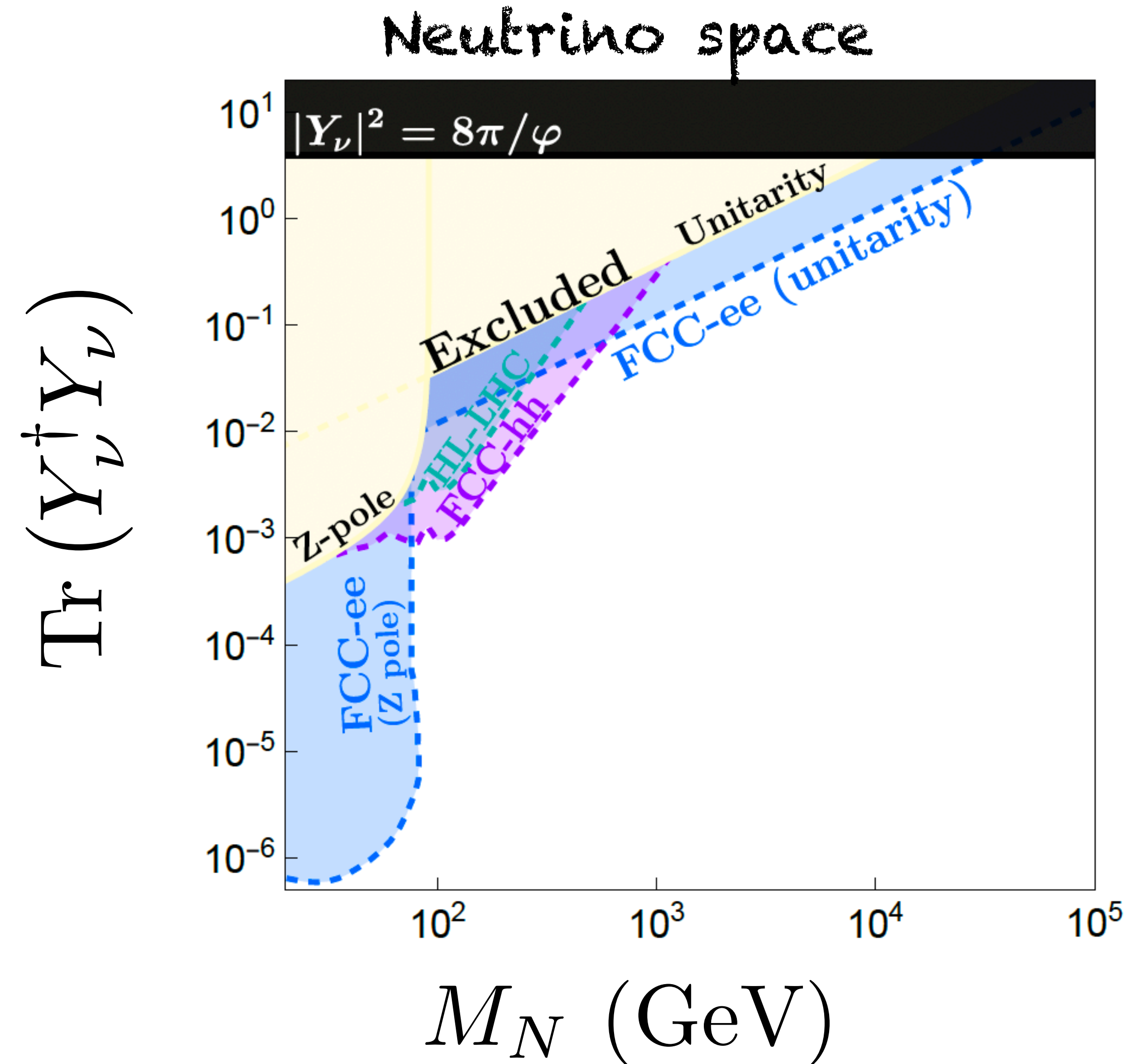
	I	II	II'	II''	III	III'	IV	IV'	V
MAXIMA	P_0	P_0	—	—	P_0	—	P_0	—	—
MINIMA	P_1, P_2	P_3	P_3	P_3	P_2	P_2	P_1	P_1	P_0
SADDLES	P_3	P_1, P_2	P_0, P_2	P_0, P_1	P_1	P_0	P_2	P_0	—

The low-scale Majoron model



	I	II	II'	II''	III	III'	IV	IV'	V
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SADDLES	P_3	P_1, P_2	P_0, P_2	P_0, P_1	P_1	P_0	P_2	P_0	—

Experimental limits



Excluded regions:

- Missing energy at the Z-pole
- PMNS unitarity + flavor violation
[DELPHI Collaboration, Z. Phys. C 74 (1997) 57–71]
[Blennow et al, 2306.01040]
- Flavor sensitive limits -> assume $\mathcal{T}$

Prospects:

[Antusch et al, 1612.02728]

Theoretical:

- Perturbative unitarity
[Urquía-Cardelon et al, 2409.13412]

Experimental limits

Theoretical grounds:

- SMEFT operator analysis

[Jiang, Craig, Shuterland, 1811.08878]

- + pseudo-observable scheme

One-loop analysis:

$$\mathcal{L}_{\text{eff}} \supset \frac{C_H}{\Lambda^2} \mathcal{O}_H + \frac{C_{H\Box}}{\Lambda^2} \mathcal{O}_{H\Box} + \frac{C_{HD}}{\Lambda^2} \mathcal{O}_{HD}$$

Operators:

$$\mathcal{O}_H \equiv (H^\dagger H)^3,$$

$$\mathcal{O}_{H\Box} \equiv (H^\dagger H) \Box (H^\dagger H),$$

$$\mathcal{O}_{HD} \equiv (H^\dagger D^\mu H)^* (H^\dagger D_\mu H)$$

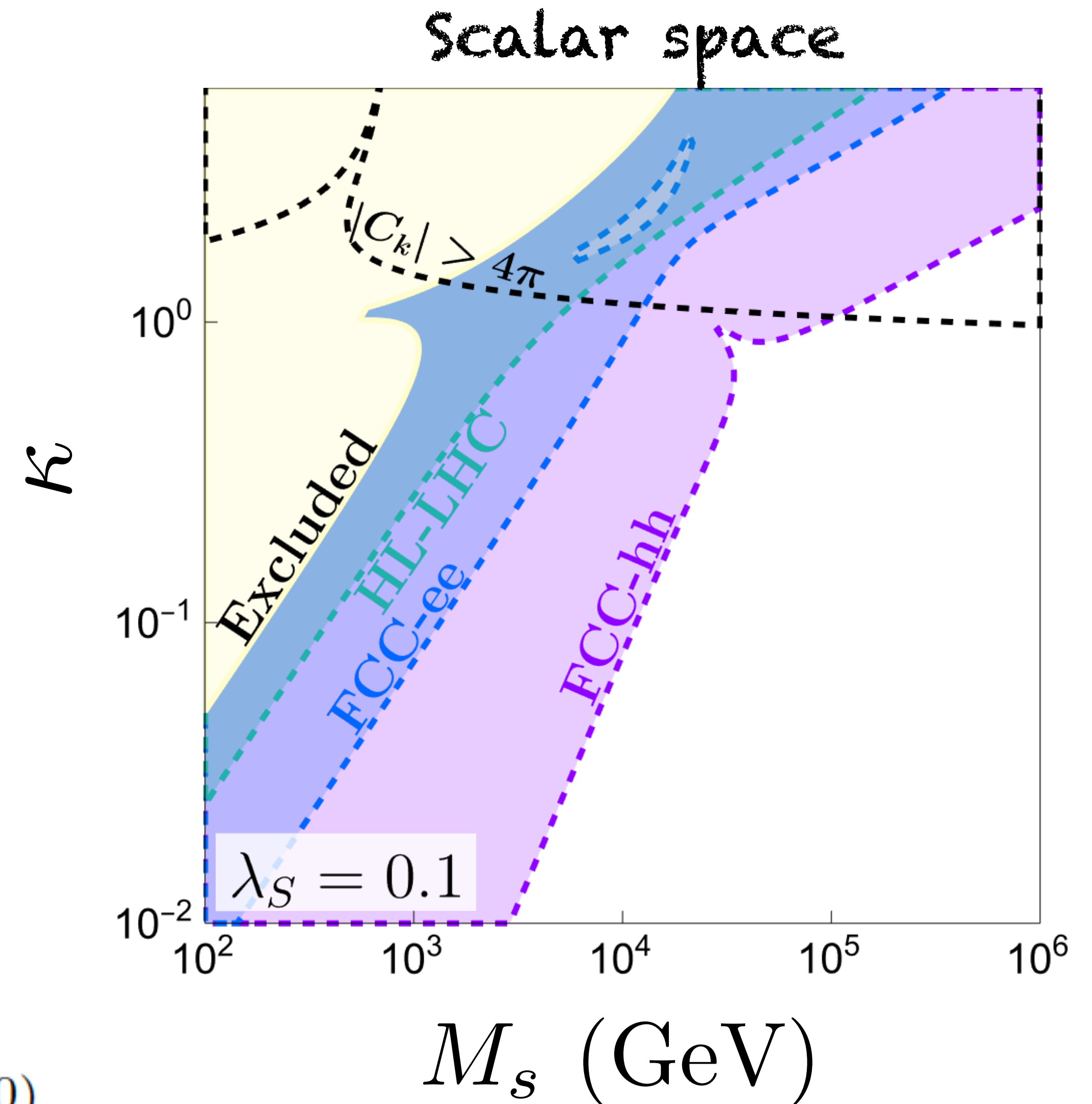
Tree level:

$$C_H^{(0)} = 0,$$

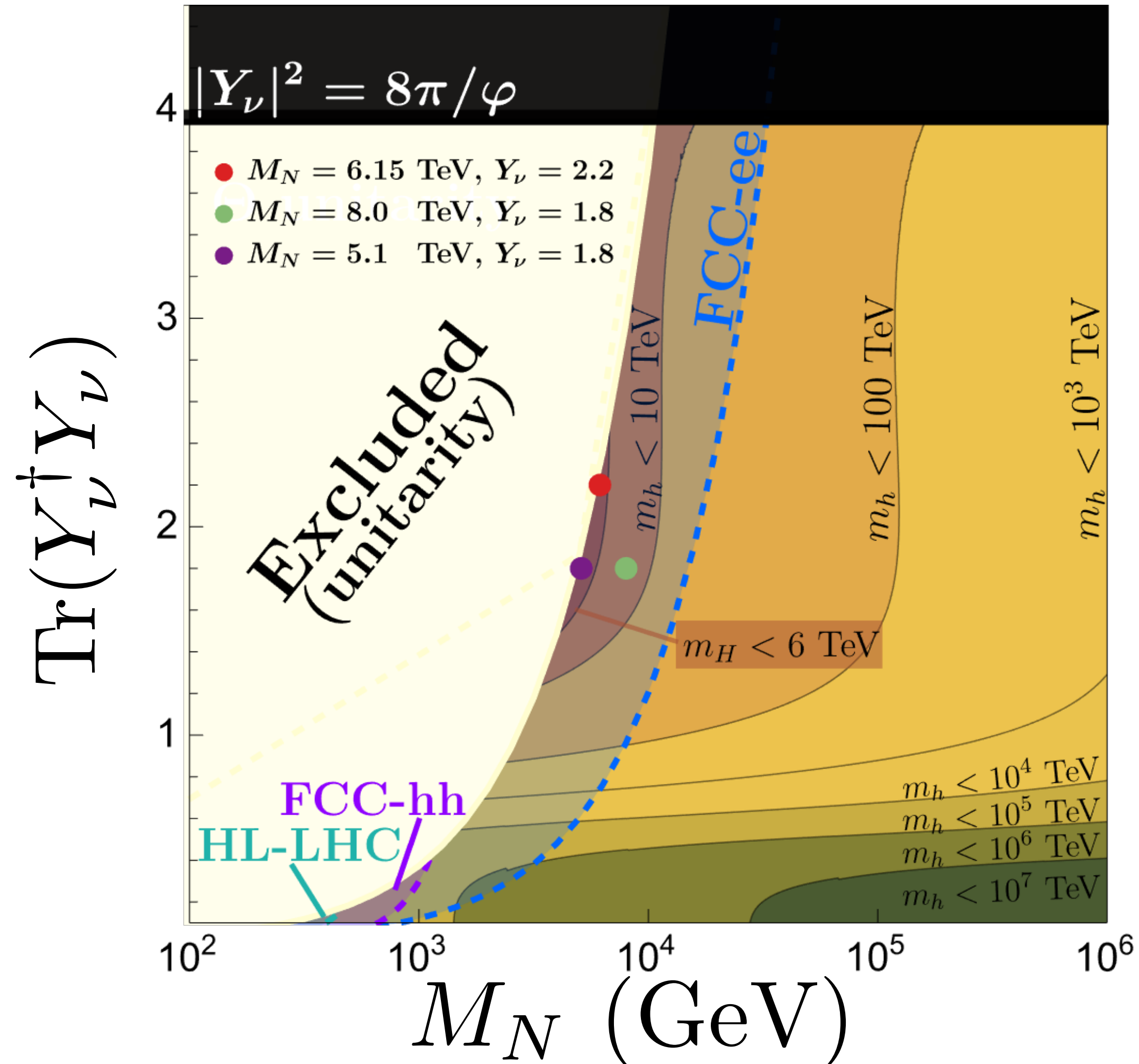
$$C_{H\Box}^{(0)} = -\frac{\kappa^2}{4\lambda_S},$$

$$C_{HD}^{(0)} = 0$$

Trilinear coupling: $g_{hhh} = 1 + \frac{3}{\sqrt{2}} \frac{1}{G_F M_s^2} C_{H\Box}^{(0)}$

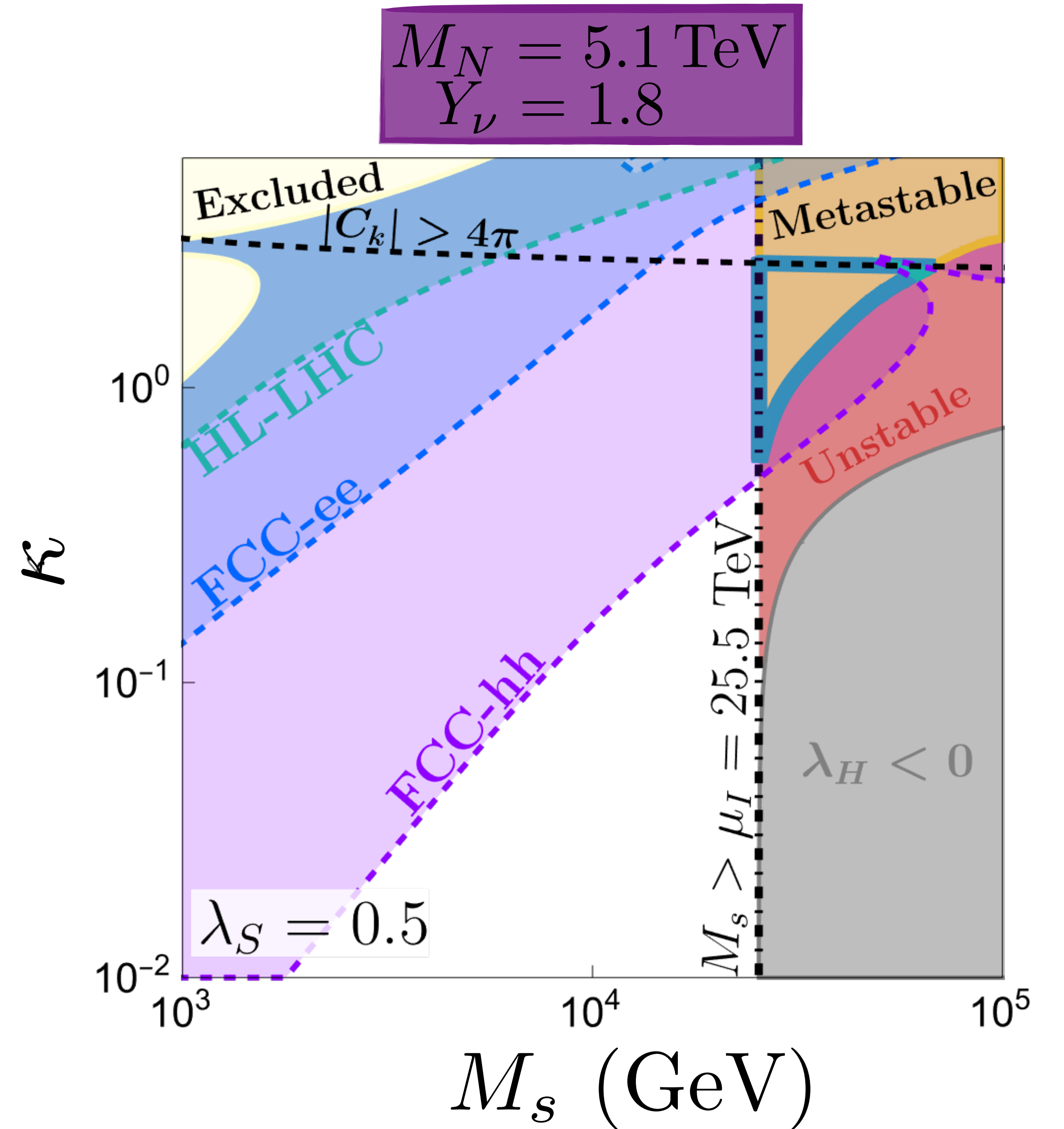


Results

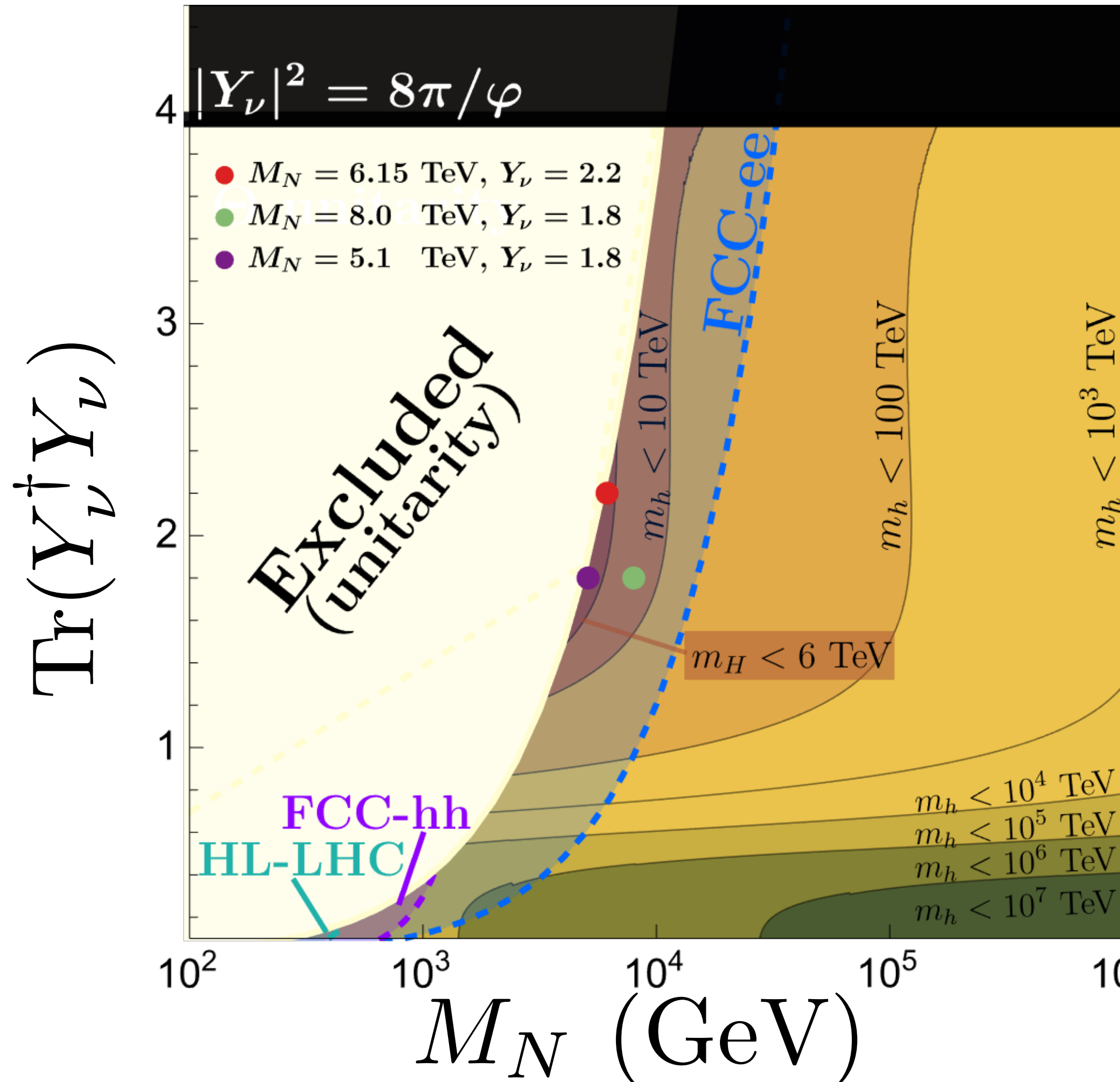


#	M_N	$ Y_\nu $	μ_I	m_{crit}^2
1	5.1 TeV	1.8	25.5 TeV	$(5.2 \text{ TeV})^2$
2	6.15 TeV	2.2	19 TeV	$(5.6 \text{ TeV})^2$
3	8.0 TeV	1.8	39.8 TeV	$(7.8 \text{ TeV})^2$

Benchmark points

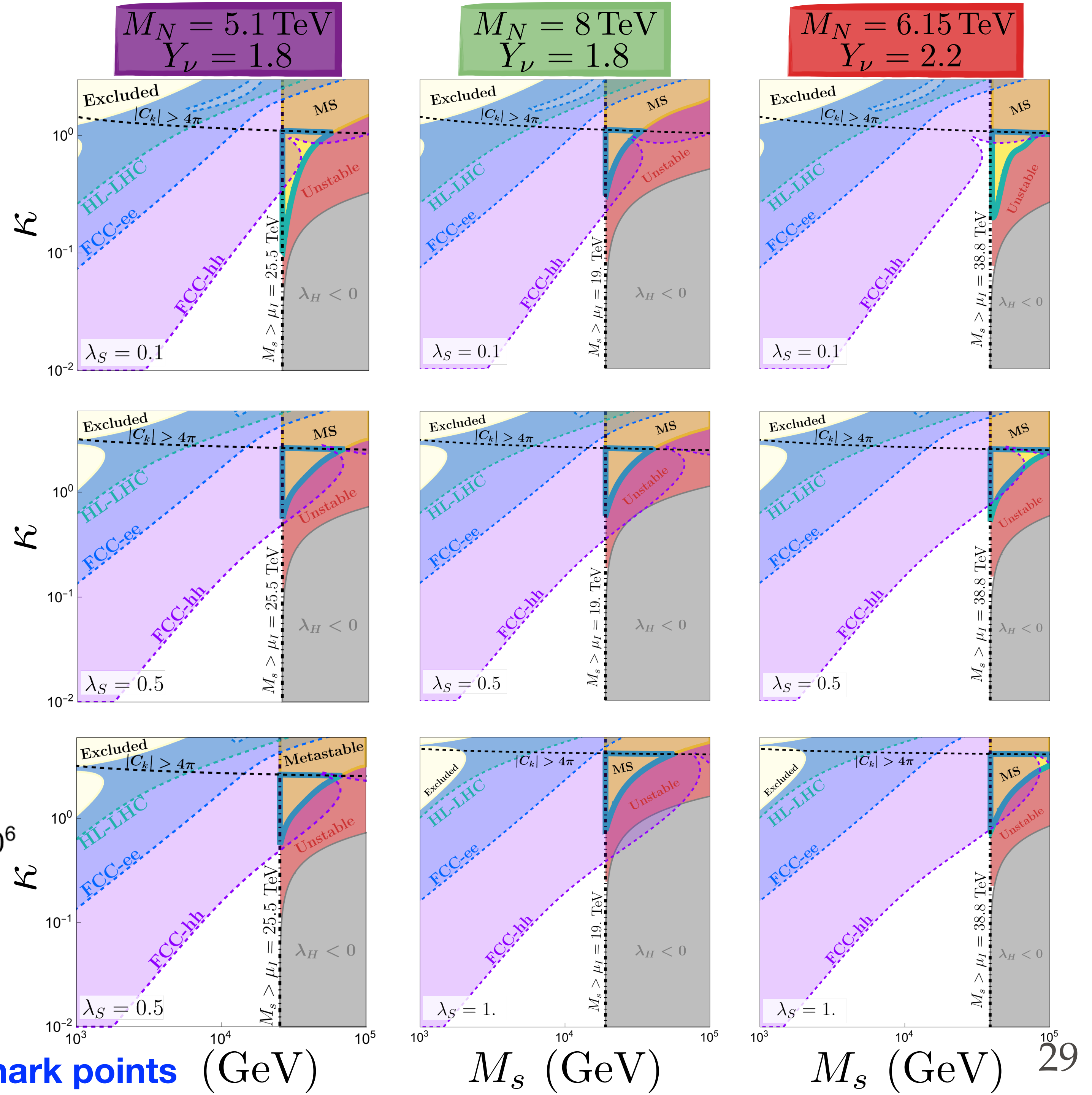


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Benchmark points (GeV)



Summary

The metastability bound is $m_h^2 < |\beta_\lambda(\mu_I)| e^{-3/2} \mu_I^2 \ll \Lambda_{UV}^2$

Lowered by BSM

Stability of the EW vacuum can be spoiled

In the Majaron model:

• RHNS.

• Scalar.



FCC-ee

FCC-hh

Thank you for your attention!
victor.enguita@uam.es

* Ideas of underlying dynamics : **dynamical criticality
on the landscape**

string theory + eternal inflation

- Multiple vacua + tunneling $\longrightarrow$ network of vacua
- Eternal observer “explores” the network
- Accessibility measure $\longrightarrow$ efficient when $\tau_{\text{ideal}} \sim \tau_{\text{Page}} \sim \frac{M_{\text{Pl}}^2}{H_0^3} \simeq 10^{130}$ years
- Implied hierarchy between m_{eff}^2 and μ_I

J. Khoury and O. Parrikar, “Search Optimization, Funnel Topography, and Dynamical Criticality on the String Landscape”, arXiv:1907.07693

J. Khoury and T. Steingasser, “Gauge hierarchy from electroweak vacuum metastability”, arXiv:2108.09315

S. Benevedes, T. Steingasser, and S. Trifinopoulos, “Spontaneous symmetry breaking, gauge hierarchy, and electroweak vacuum metastability”, arXiv:2408.10297

J. Khoury, Accessibility Measure for Eternal Inflation: Dynamical Criticality and Higgs Metastability”, arXiv:1912.06706

J. Khoury and S. S. C. Wong, “Bayesian reasoning in eternal inflation: A solution to the measure problem”, arXiv:2205.11524

Higgs metastability bound: proof

The instability scale μ_I , defined by $\lambda_{\text{eff}}(\mu_I) = 0$

Metastability possible only if

$$m_h^2 < |\beta_\lambda(\mu_I)| e^{-3/2} \mu_I^2$$

[Buttazo et al 1307.3536]

[Steingasser, Khoury, 2108.09315]

[Steingasser, Kaiser, 2307.10361]

Proof:

Stat. point condition is $\frac{\partial V_{\text{eff}}(H)}{\partial |H|} = |H| [-2m_{\text{eff}}^2 - \beta_m^2 + |H|^2 (4\lambda_{\text{eff}} + \beta_\lambda)] = 0$

At leading-log, around μ_I : $\lambda_{\text{eff}} \simeq \underbrace{\lambda_{\text{eff}}(\mu_I)}_0 + \beta_\lambda \ln \frac{v'}{\mu_I} \implies \left[2\lambda_{\text{eff}}(\mu_I) + \frac{\beta_\lambda(\mu_I)}{2} \right] v'^2 = m_{\text{eff}}^2$

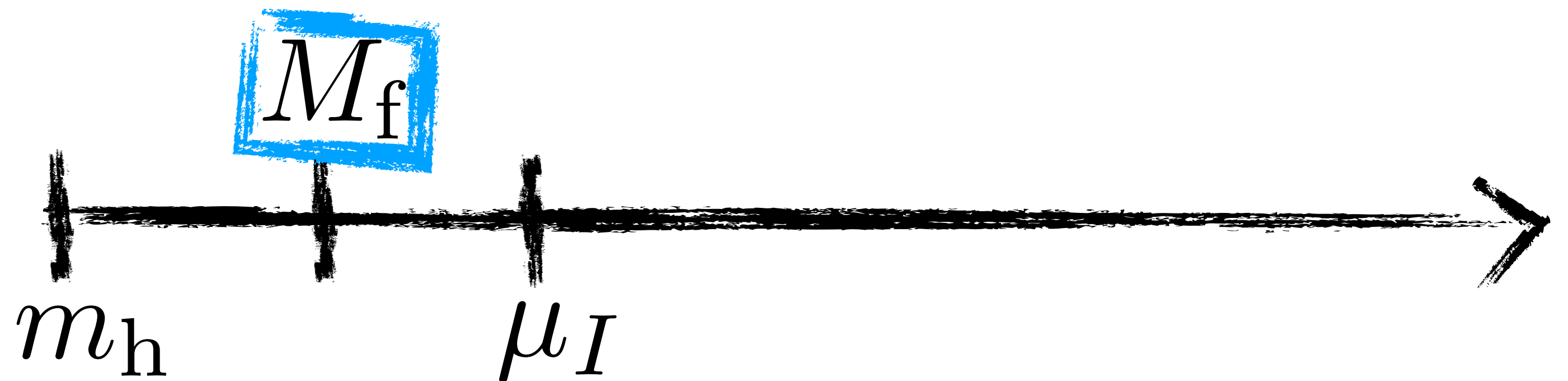
$\implies$ solutions only if $m_{\text{eff}}^2 \leq m_{\text{crit}}^2 \equiv -\beta_{\lambda|\mu_I} e^{-3/2} \mu_I^2 = |\beta_{\lambda|\mu_I}| e^{-3/2} \mu_I^2$

The low-scale Majoron model...

...contains just the right ingredients

Fermion content: RHNs

$$\begin{aligned}\mathcal{L}_{\nu_R} \supset & \sum_i \bar{\nu}_{R,i} \not{\partial} \nu_{R,i} \\ & - \sum_{\nu} Y_{\nu}^{ai} \bar{\ell}_L^a \tilde{H} \nu_{R,j} \\ & - \frac{1}{2} \sum_{i,j} M_N^{ij} \bar{\nu}_{R,i}^c \nu_{R,j}\end{aligned}$$



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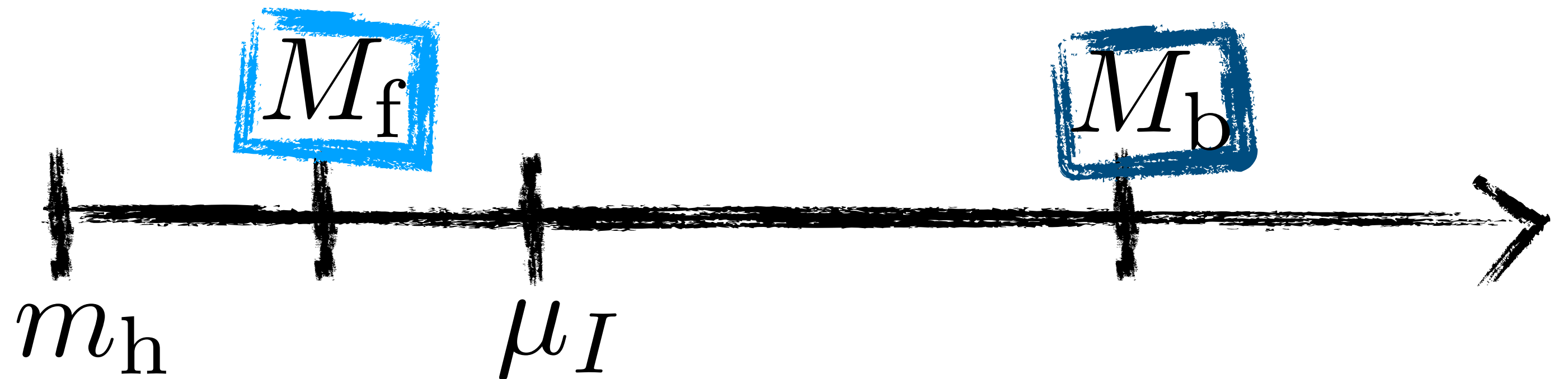
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Scalar content: a complex singlet

Generates masses for the RHNs

$$\mathcal{L}_{\nu_R} \supset -\frac{1}{2} \sum_{i,j} Y_R^{ij} S \overline{\nu_{R,i}^c} \nu_{R,j} + \text{h.c.}$$



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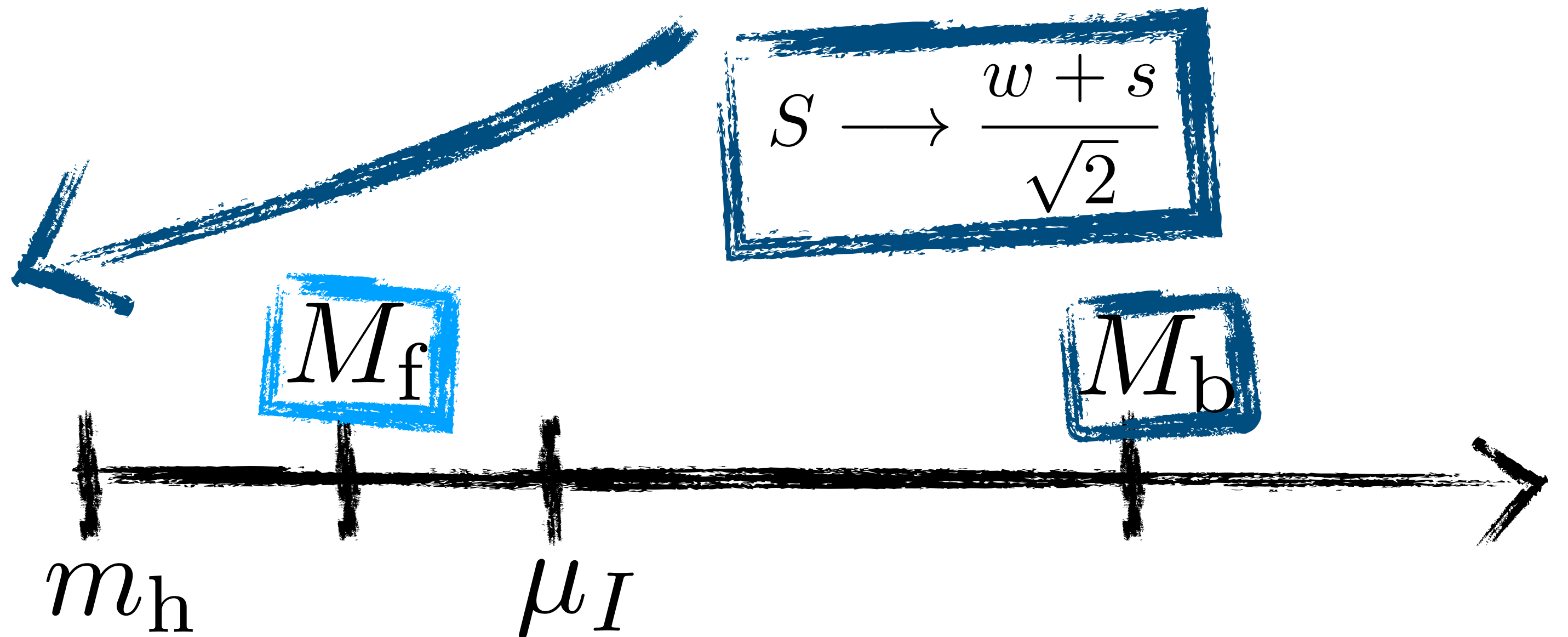
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[Shaposhnikov, 0605047]

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$$\begin{aligned}\mathcal{L}_{\nu_R} \supset & \sum_i \bar{\nu}_{R,i} \not{\partial} \nu_{R,i} \\ & - \sum_{\nu} \mathbf{Y}_{\nu}^{ai} \bar{\ell}_L^a \tilde{H} \nu_{R,j} \\ & - \frac{1}{2} \sum_{i,j} \underbrace{\frac{\mathbf{Y}_R^{ij} w}{\sqrt{2}}}_{\equiv M_N} \bar{\nu}_{R,i}^c \nu_{R,j}\end{aligned}$$

Symmetry Protected Seesaw Scenario (SPSS)

- $M_{\nu} \equiv \frac{v^2}{\sqrt{2}} \mathbf{Y}_{\nu} \frac{1}{M_N} \mathbf{Y}_{\nu}^T \xrightarrow{U(1)_L} 0$
- $\mathbf{Y}_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & Y_R \\ 0 & Y_R & 0 \end{pmatrix}, \quad \mathbf{Y}_{\nu} = \begin{pmatrix} 0 & Y_{12} & 0 \\ 0 & Y_{22} & 0 \\ 0 & Y_{32} & 0 \end{pmatrix}$
 $\longrightarrow \text{Tr} \left(\mathbf{Y}_{\nu}^{\dagger} \mathbf{Y}_{\nu} \mathbf{Y}_{\nu}^{\dagger} \mathbf{Y}_{\nu} \right) = \text{Tr} \left(\mathbf{Y}_{\nu}^{\dagger} \mathbf{Y}_{\nu} \right)^2 = |\mathbf{Y}_{\nu}|^4$
- $\Theta_{\nu}^a = \frac{\mathbf{Y}_{\nu}^{*a2}}{\sqrt{2}} \frac{v}{M_N}$ and $|\Theta_{\nu}|^2 = \frac{\text{Tr} \left(\mathbf{Y}_{\nu}^{\dagger} \mathbf{Y}_{\nu} \right)}{2} \frac{v^2}{M_N^2}$

The low-scale Majoron model...

...contains just the right ingredients

Fermion content: RHNs

$$\begin{aligned}\mathcal{L}_{\nu_R} \supset & \sum_i \bar{\nu}_{R,i} \not{\partial} \nu_{R,i} \\ & - \sum_{\nu} Y_{\nu}^{ai} \bar{\ell}_L^a \tilde{H} \nu_{R,j} \\ & - \frac{1}{2} \sum_{i,j} \underbrace{\frac{Y_R^{ij} w}{\sqrt{2}}}_{\equiv M_N} \bar{\nu}_{R,i}^c \nu_{R,j}\end{aligned}$$

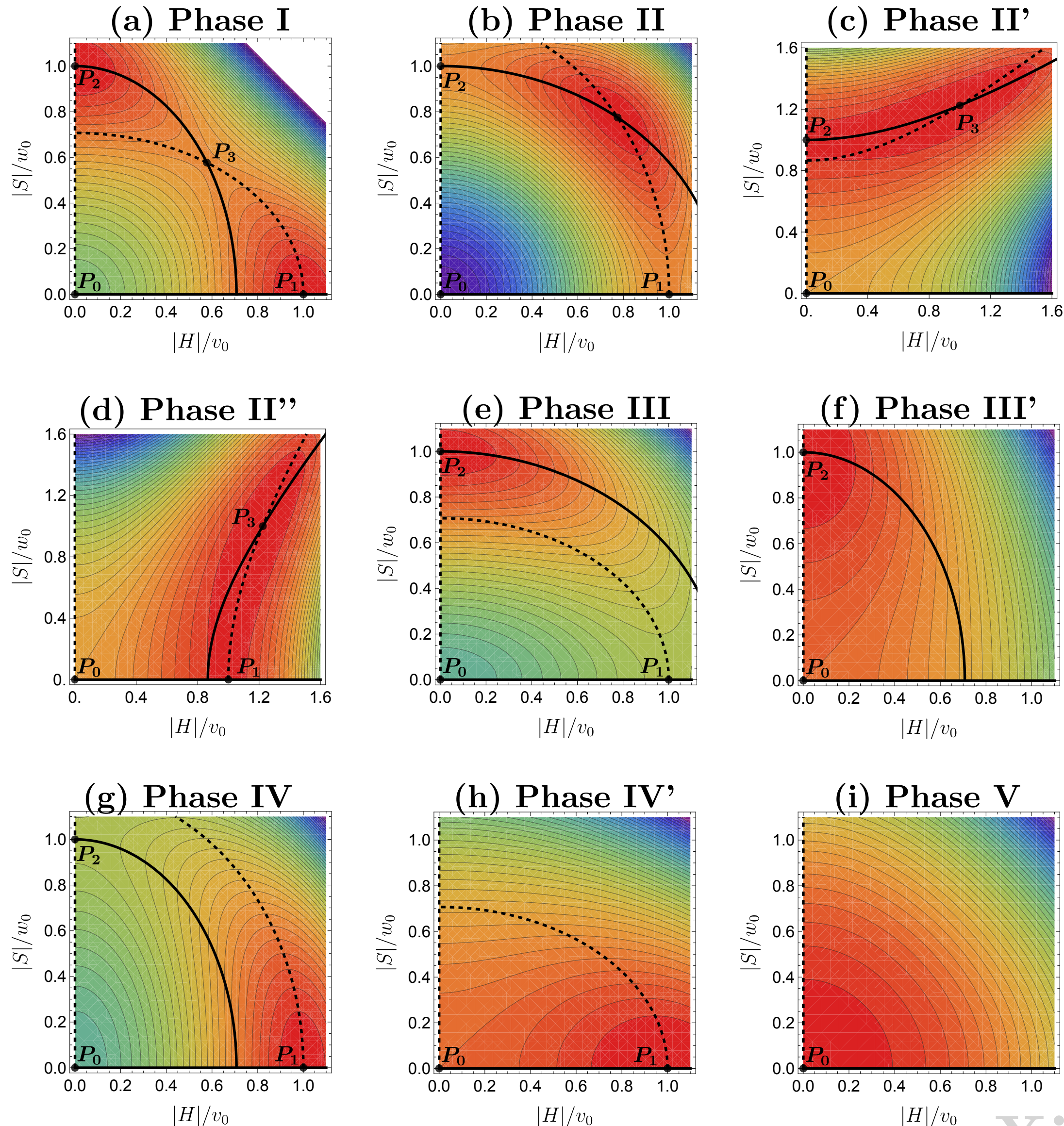
Scalar content: a complex singlet

Implies a two-field potential

$$\begin{aligned}V(H, S) = & -m_H^2 |H|^2 + \lambda_H |H|^4 \\ & + \lambda_S |S|^4 - m_S^2 |S|^2 + \kappa |H|^2 |S|^2\end{aligned}$$



The low-scale Majoron model



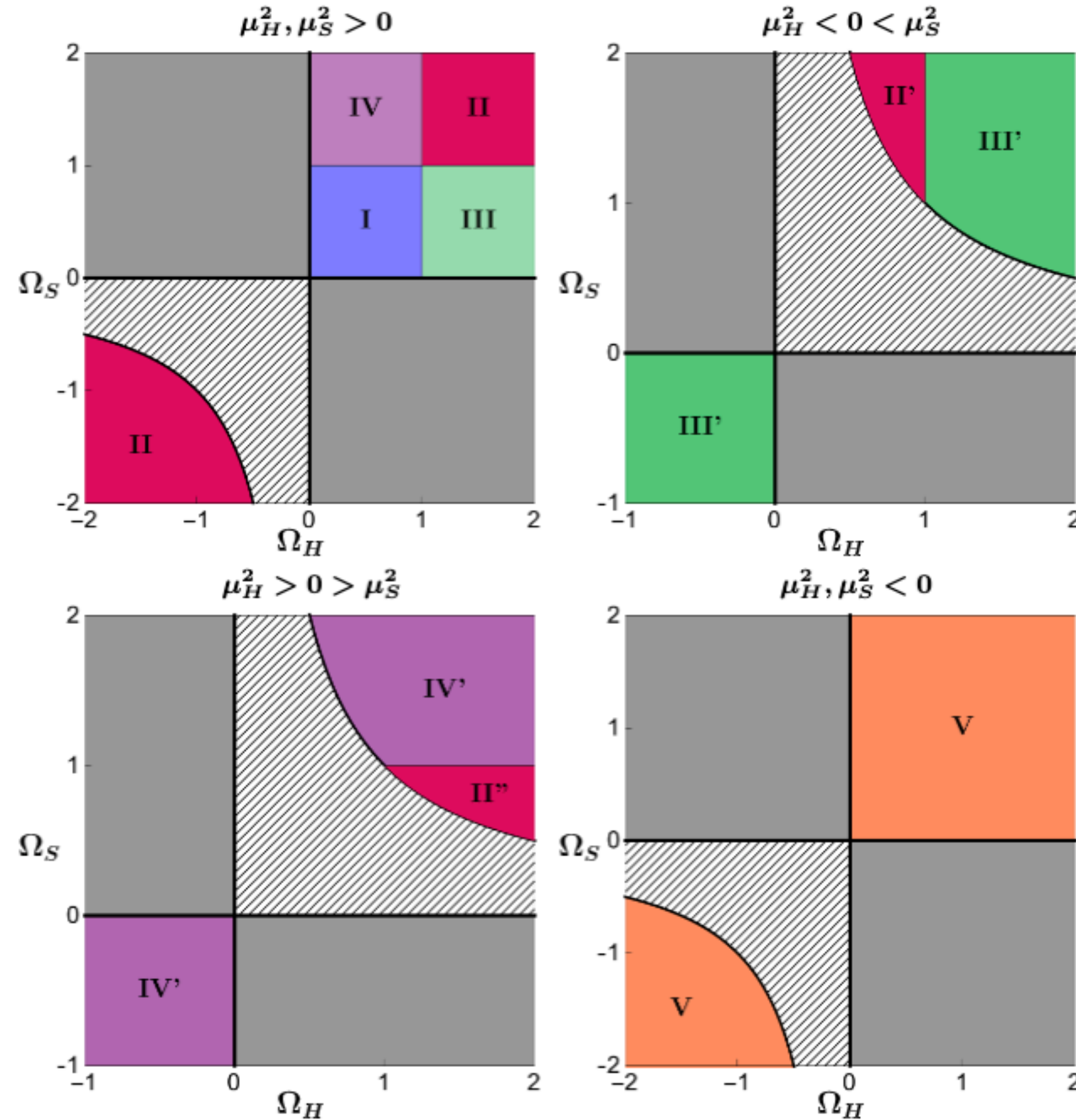
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$$V(H, S) = -m_H^2 |H|^2 + \lambda_H |H|^4 + \lambda_S |S|^4 - m_S^2 |S|^2 + \kappa |H|^2 |S|^2$$



The low-scale Majoron model...



$$\Omega_H \equiv 2 \frac{\mu_S^2}{\mu_H^2} \cdot \frac{\lambda_H}{\kappa} \quad \text{and} \quad \Omega_S \equiv 2 \frac{\mu_H^2}{\mu_S^2} \cdot \frac{\lambda_S}{\kappa}$$

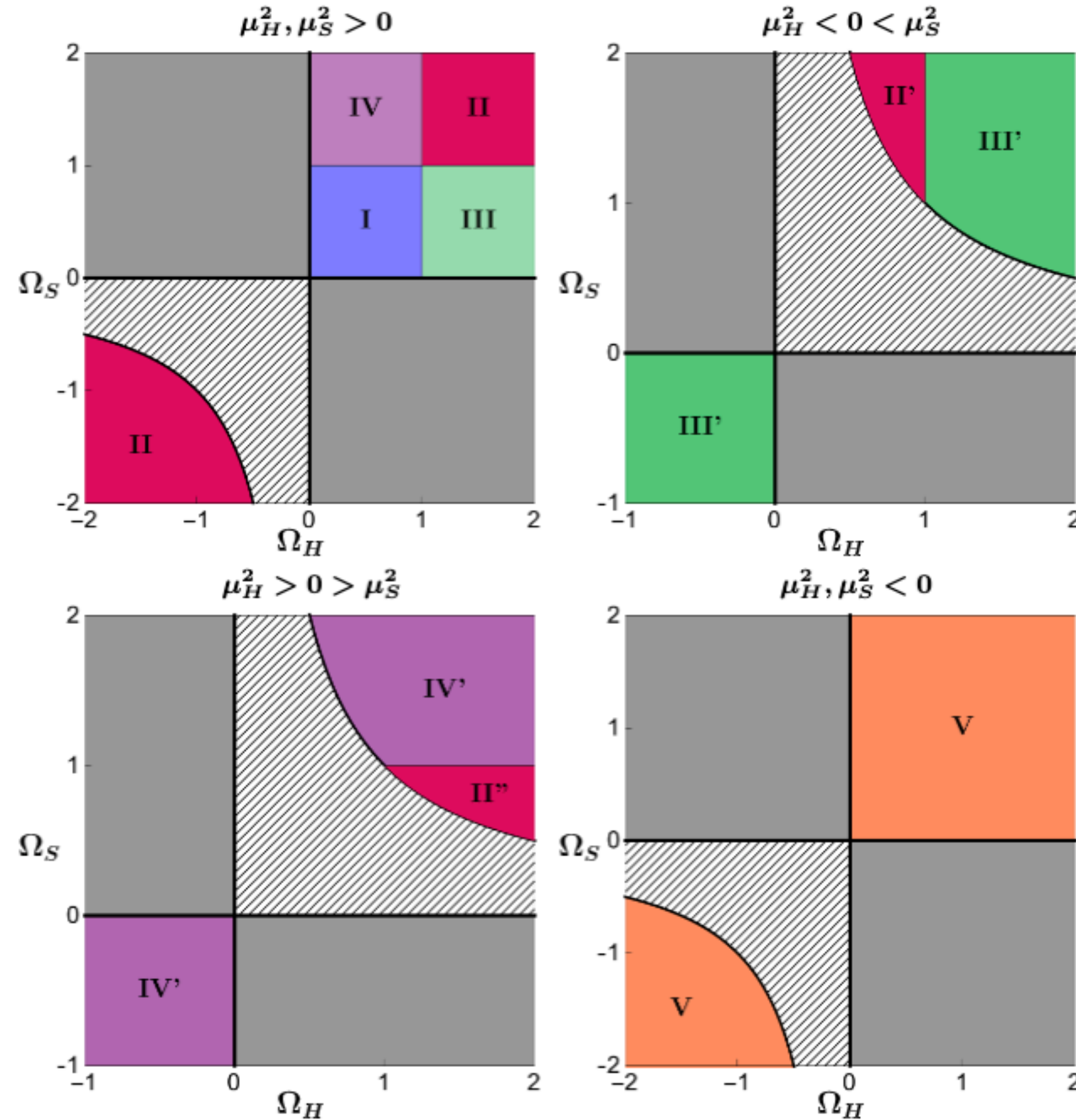
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Implies a two-field potential

$$V(H, S) = -m_H^2 |H|^2 + \lambda_H |H|^4 + \lambda_S |S|^4 - m_S^2 |S|^2 + \kappa |H|^2 |S|^2$$

FIG. 1: Phase space (Ω_H, Ω_S) . The Phases I-V, II'-IV' and II'' correspond to all possible configurations of the stationary points of the potential in Eq. (4) under the assumption of stability. Grey regions are not accessible under that assumption. Hatched regions are unstable, see text.

The low-scale Majoron model



$$\Omega_H \equiv 2 \frac{\mu_S^2}{\mu_H^2} \cdot \frac{\lambda_H}{\kappa} \quad \text{and} \quad \Omega_S \equiv 2 \frac{\mu_H^2}{\mu_S^2} \cdot \frac{\lambda_S}{\kappa}$$

$$P_0 \equiv (0, 0),$$

$$P_1 \equiv \left(\mu_H / (2\lambda_H)^{1/2}, 0 \right),$$

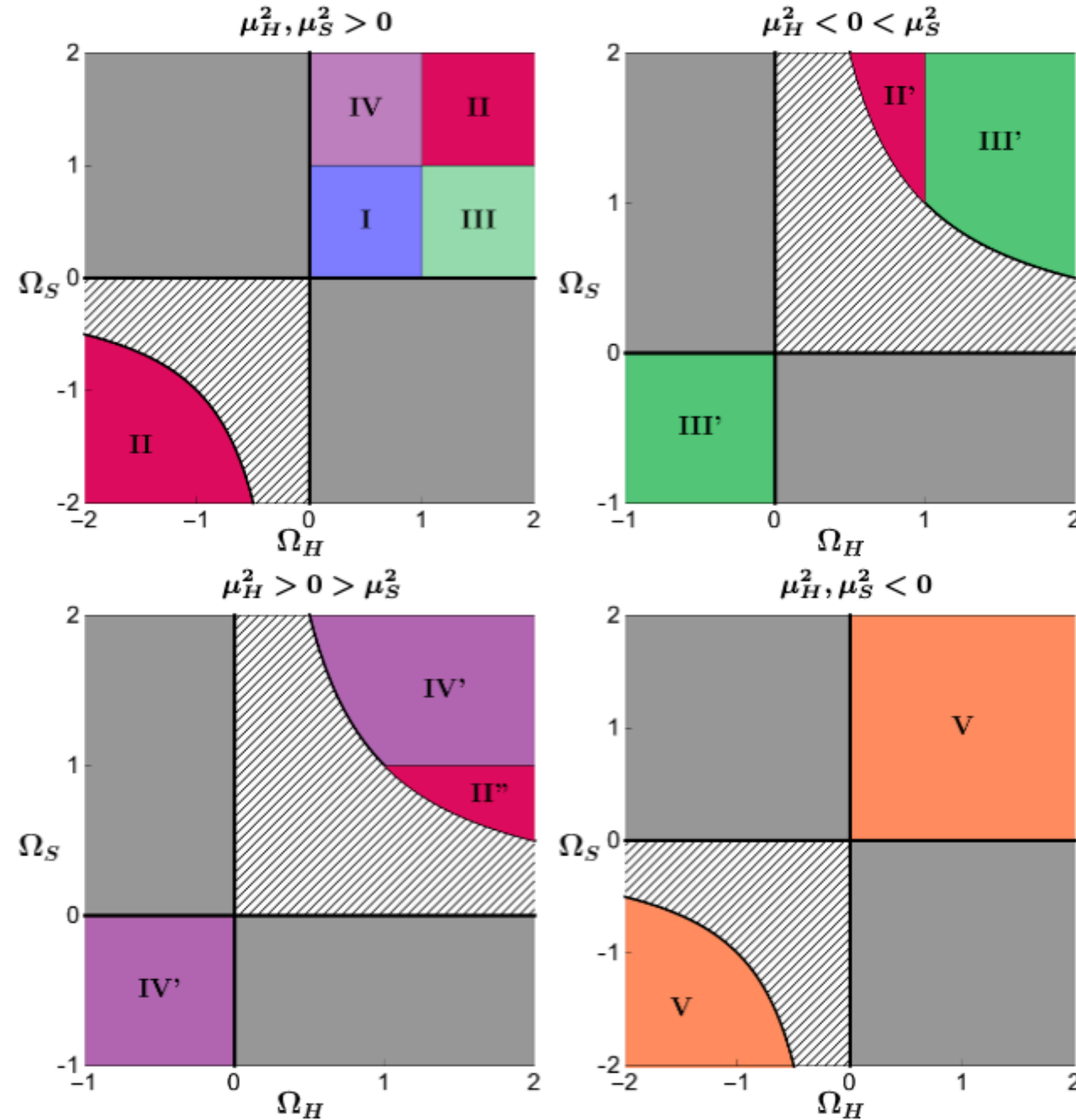
$$P_2 \equiv \left(0, \mu_S / (2\lambda_S)^{1/2} \right),$$

$$P_3 = \left(\sqrt{\frac{2\mu_H^2 \lambda_S - \mu_S^2 \kappa}{4\lambda_H \lambda_S - \kappa^2}}, \sqrt{\frac{2\mu_S^2 \lambda_H - \mu_H^2 \kappa}{4\lambda_H \lambda_S - \kappa^2}} \right)$$

	I	II	II'	II''	III	III'	IV	IV'	V
MAXIMA	P_0	P_0	—	—	P_0	—	P_0	—	—
MINIMA	P_1, P_2	P_3	P_3	P_3	P_2	P_2	P_1	P_1	P_0
SADDLES	P_3	P_1, P_2	P_0, P_2	P_0, P_1	P_1	P_0	P_2	P_0	—

FIG. 1: Phase space (Ω_H, Ω_S) . The Phases I-V, II'-IV' and II'' correspond to all possible configurations of the stationary points of the potential in Eq. (4) under the assumption of stability. Grey regions are not accessible under that assumption. Hatched regions are unstable, see text.

The low-scale Majoron model



$$\Omega_H \equiv 2 \frac{\mu_S^2}{\mu_H^2} \cdot \frac{\lambda_H}{\kappa} \quad \text{and} \quad \Omega_S \equiv 2 \frac{\mu_H^2}{\mu_S^2} \cdot \frac{\lambda_S}{\kappa}$$

$$\bar{P}_0 = (0, 0)$$

$$\bar{P}_1 = \left(\sqrt{\text{sign}[\mu_H^2]}, 0 \right),$$

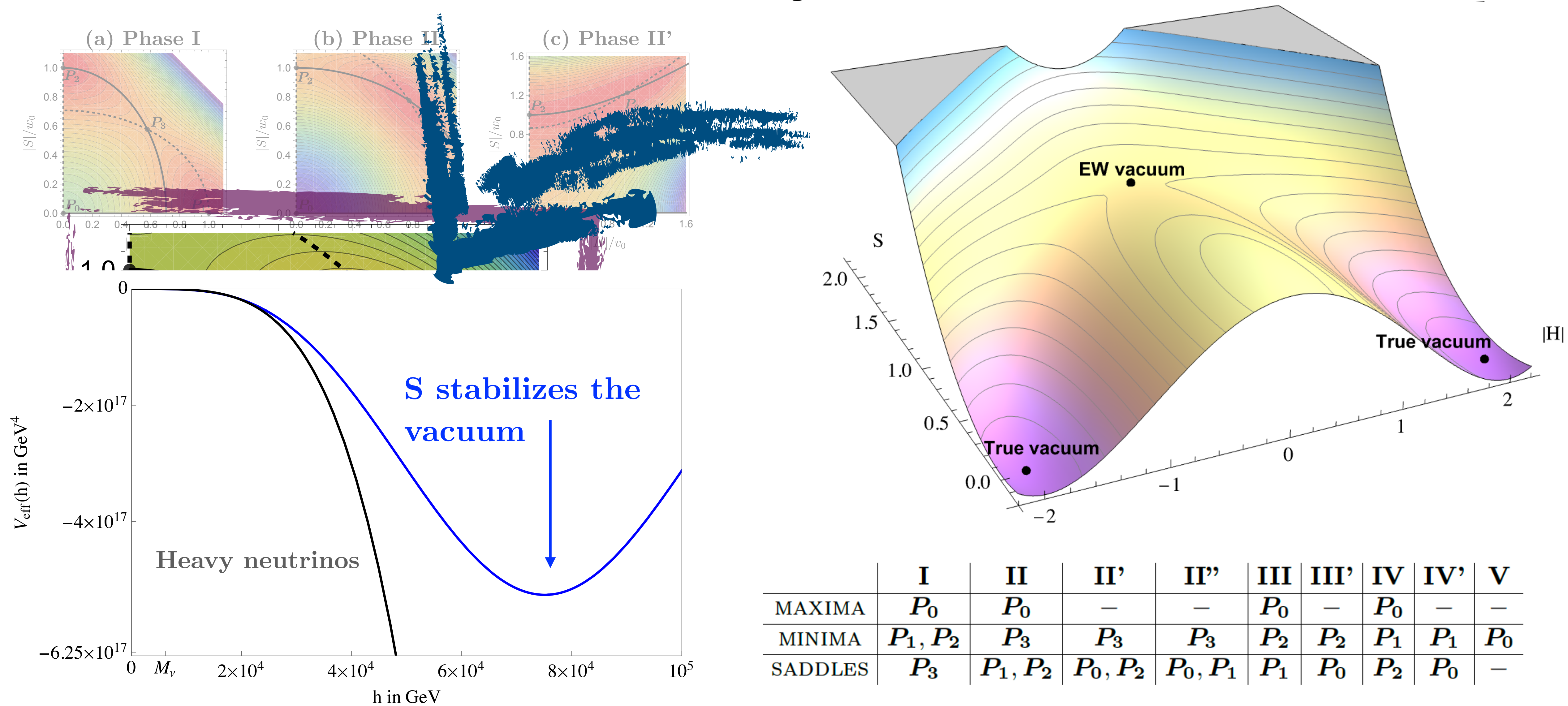
$$\bar{P}_2 = \left(0, \sqrt{\text{sign}[\mu_S^2]} \right),$$

$$\bar{P}_3 = \left(\sqrt{\text{sign}[\mu_H^2]} \frac{\Omega_H (\Omega_S - 1)}{\Omega_H \Omega_S - 1}, \sqrt{\text{sign}[\mu_S^2]} \frac{\Omega_S (\Omega_H - 1)}{\Omega_H \Omega_S - 1} \right)$$

	I	II	II'	II''	III	III'	IV	IV'	V
MAXIMA	P_0	P_0	—	—	P_0	—	P_0	—	—
MINIMA	P_1, P_2	P_3	P_3	P_3	P_2	P_2	P_1	P_1	P_0
SADDLES	P_3	P_1, P_2	P_0, P_2	P_0, P_1	P_1	P_0	P_2	P_0	—

FIG. 1: Phase space (Ω_H, Ω_S) . The Phases I-V, II'-IV' and II'' correspond to all possible configurations of the stationary points of the potential in Eq. (4) under the assumption of stability. Grey regions are not accessible under that assumption. Hatched regions are unstable, see text.

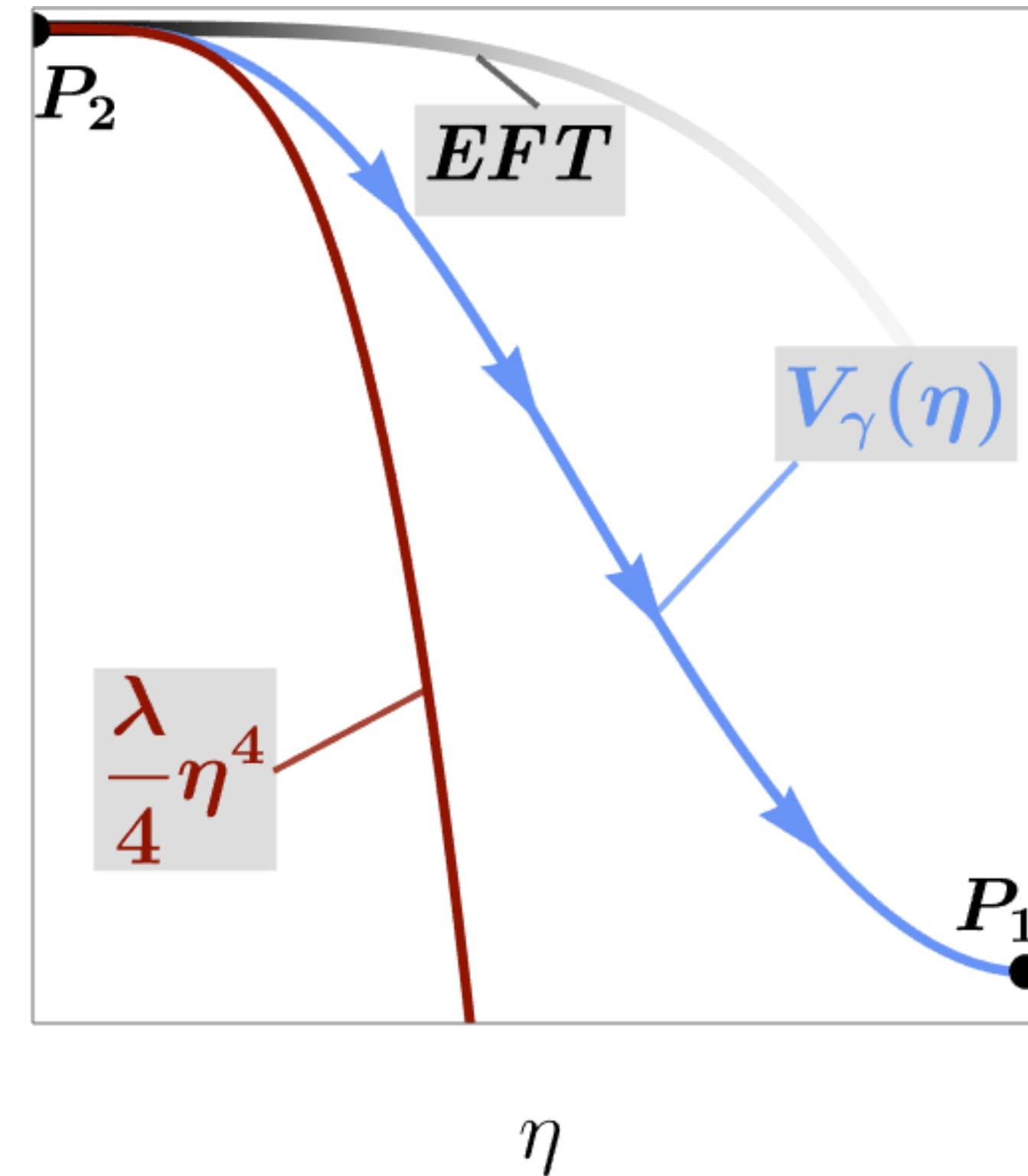
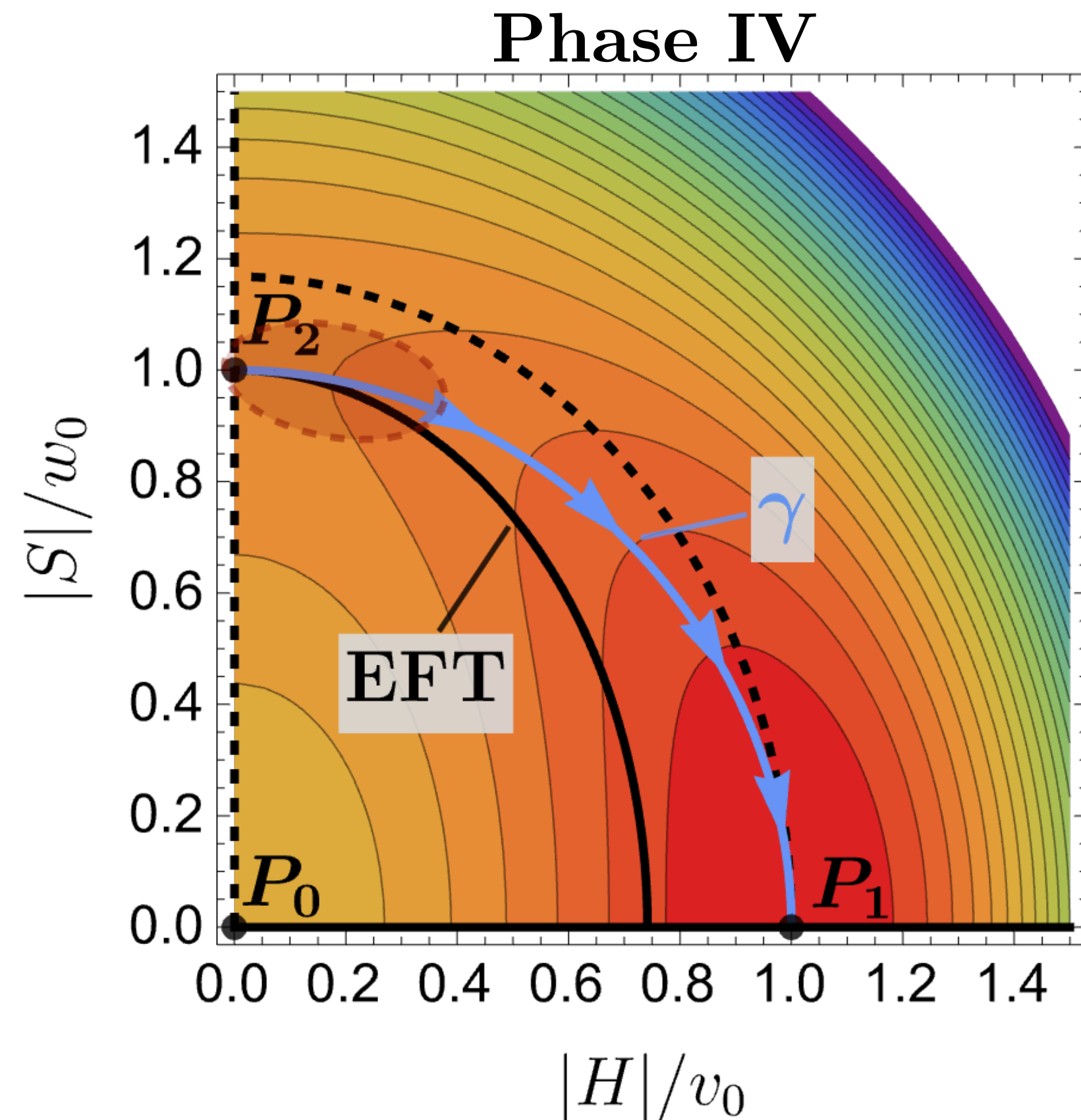
The low-scale Majoron model



Tunneling rate

Tunneling rate calculated in 1-d effective potential

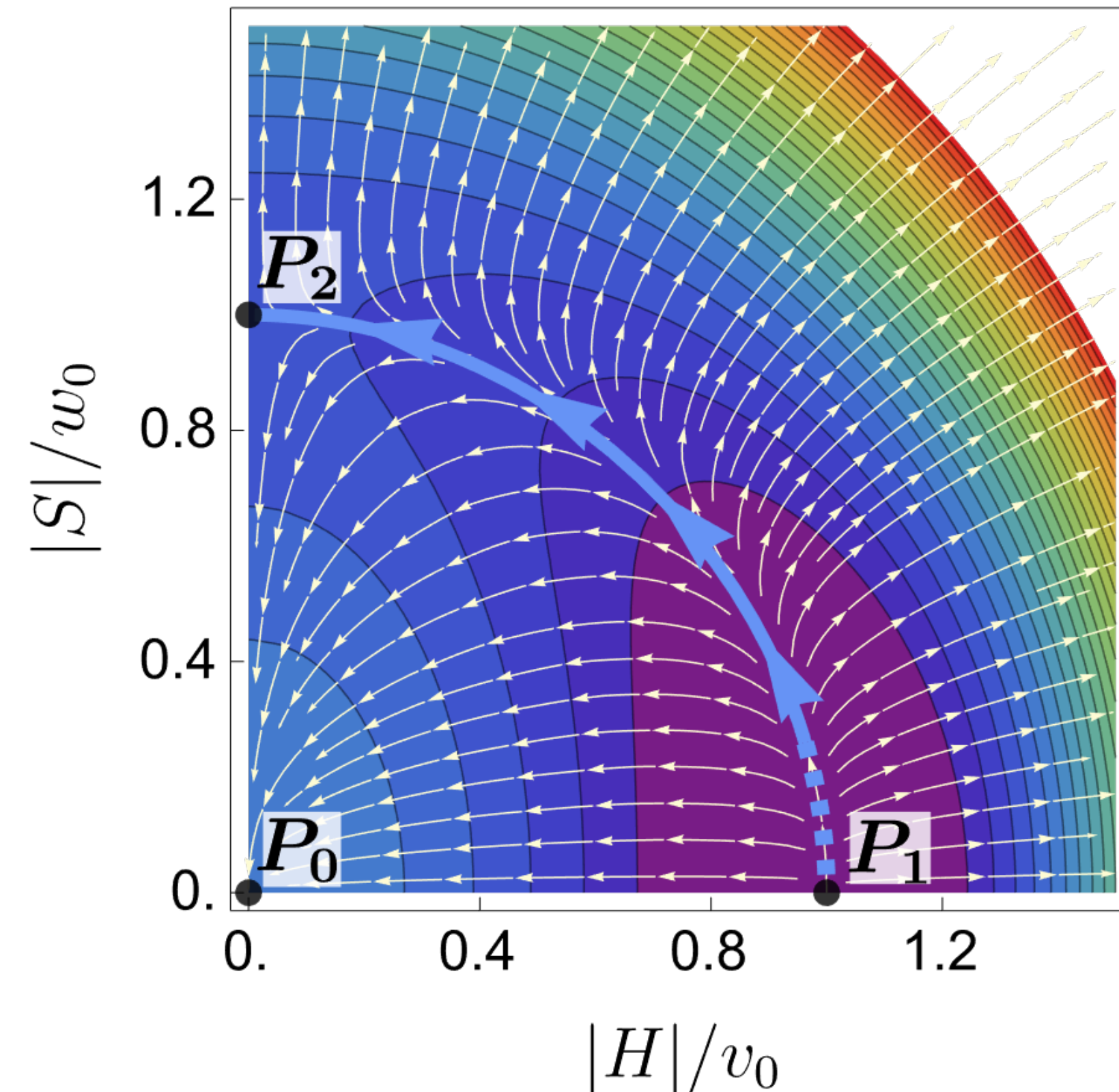
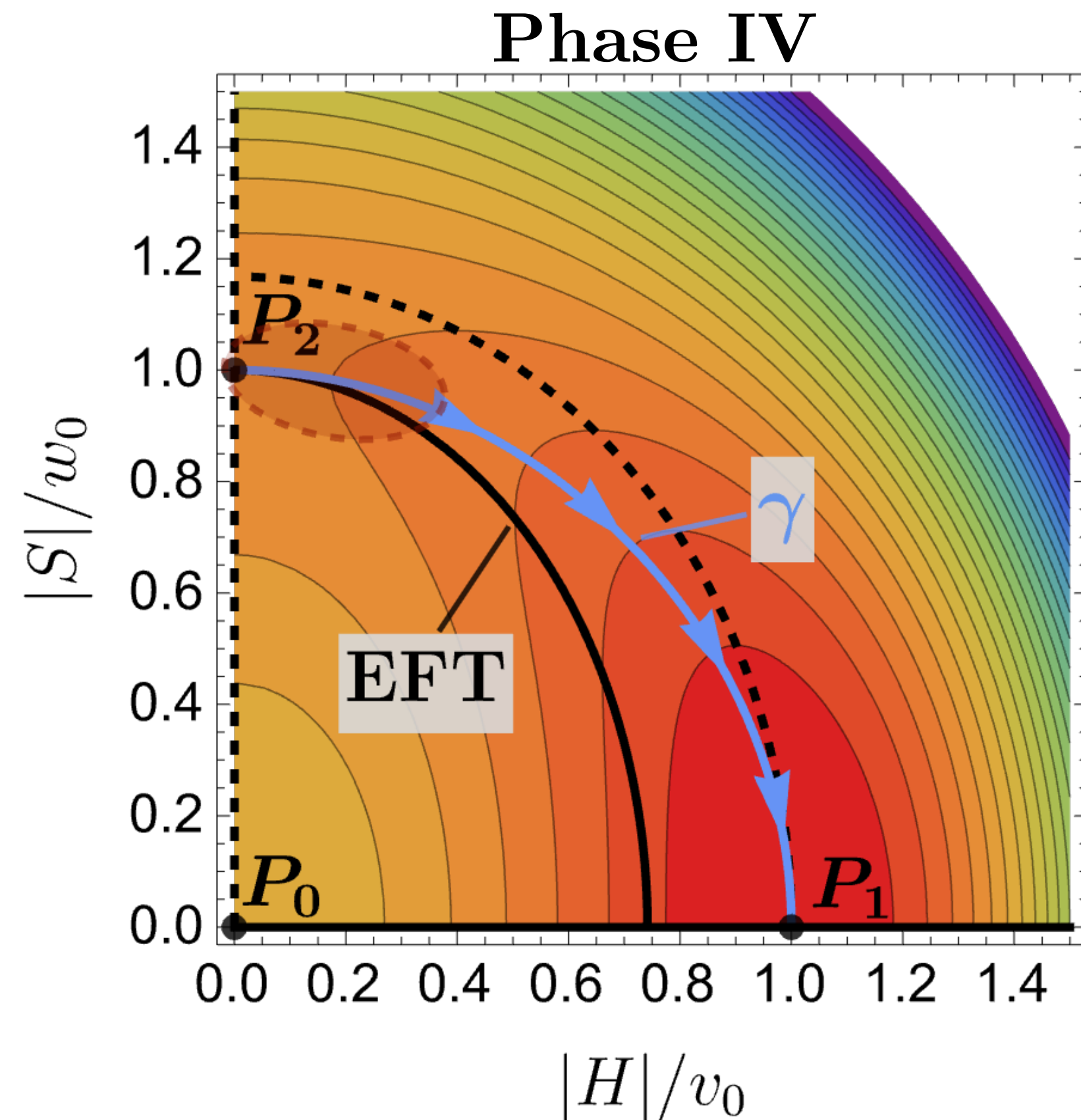
$$\mathcal{L}_\gamma = \frac{1}{2} \partial_\mu \eta \partial^\mu \eta - V_\gamma(\eta), \quad \text{computed through gradient descend}$$



Tunneling rate

Tunneling rate calculated in 1-d effective potential

$$\mathcal{L}_\gamma = \frac{1}{2} \partial_\mu \eta \partial^\mu \eta - V_\gamma(\eta), \quad \text{computed through gradient descend}$$



Tunneling rate

$$V = -m_H^2 |H|^2 + \lambda_H |H|^4 - m_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |H|^2 |S|^2$$

Compute the Euclidean action of the two-field instanton (H_I , S_I)

$$\frac{d^2}{d\rho^2}(H_I, S_I) + \frac{3}{\rho} \frac{d}{d\rho}(H_I, S_I) = \nabla_{H,S} V(H_I, S_I)$$

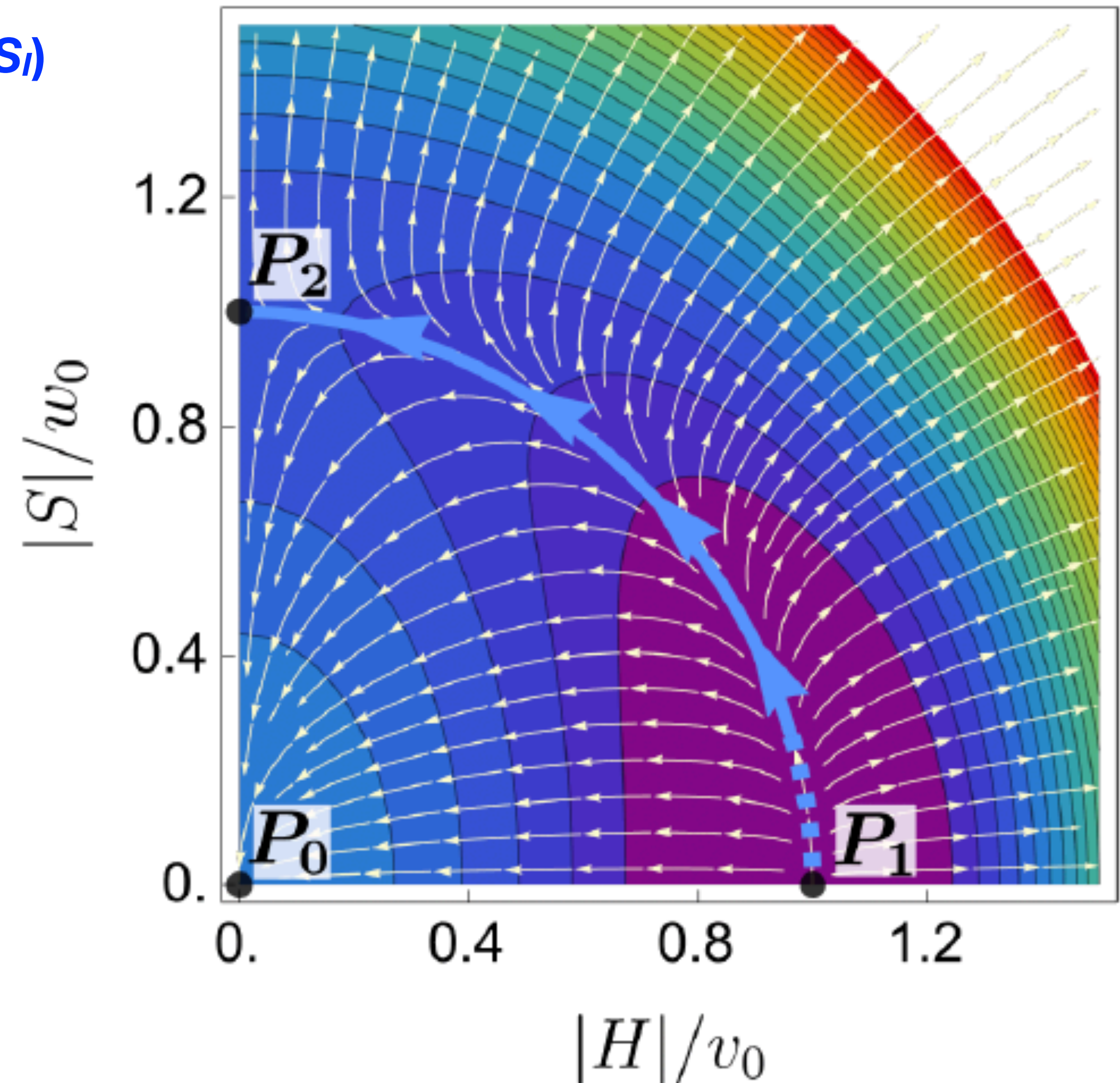
O(4)-symmetric ansatz $\rho^2 = t^2 + \mathbf{x}^2$

Approximate $(H_I(\rho), S_I(\rho) = H_\gamma(\eta_I(\rho)), S_\gamma(\eta_I(\rho)))$

where $\eta_I(\rho)$ is the instant in the effective potential V_γ

$$\frac{d^2}{d\rho^2} \eta_I + \frac{3}{\rho} \frac{d}{d\rho} \eta_I = \frac{d}{d\eta} V_\gamma(\eta_I)$$

$$S_E = \int d^4x \left(\frac{1}{2} \dot{\eta}_I^2 + \frac{1}{2} |\nabla \eta_I|^2 + V_\gamma(\eta_I) \right)$$



Experimental limits

Theoretical grounds:

- SMEFT operator analysis

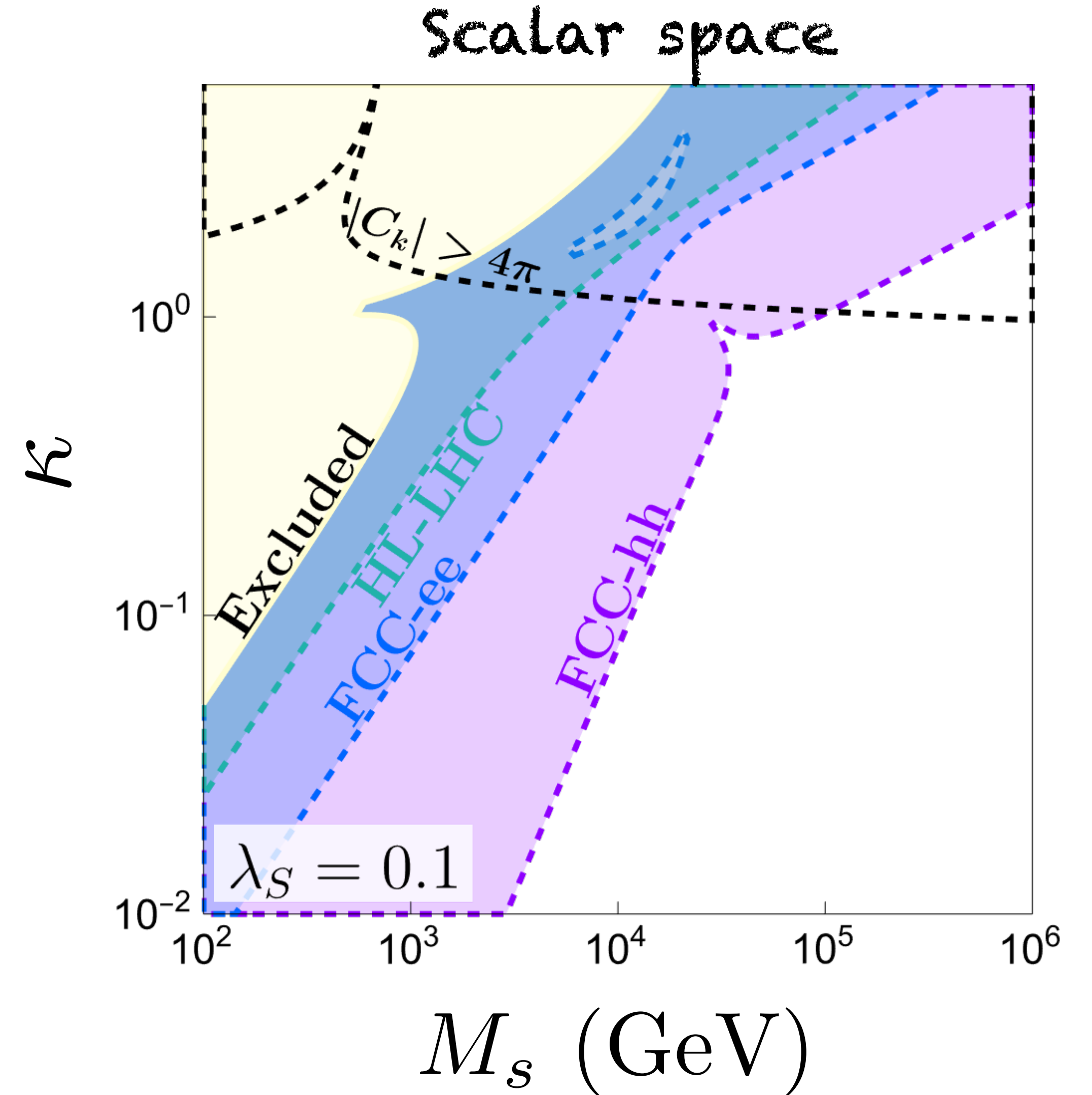
[Jiang, Craig, Shuterland, 1811.08878]

- + pseudo-observable scheme

One-loop analysis:

$\mathcal{O}_H = (H^\dagger H)^3$	$\mathcal{O}_{HW} = (H^\dagger H) W_{\mu\nu}^a W^{a\mu\nu}$
$\mathcal{O}_{H\Box} = (H^\dagger H) \Box (H^\dagger H)$	$\mathcal{O}_{HB} = (H^\dagger H) B_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{HD} = (H^\dagger D^\mu H)^* (H^\dagger D_\mu H)$	$\mathcal{O}_{HWB} = (H^\dagger \sigma^a H) W_{\mu\nu}^a B^{\mu\nu}$
$\mathcal{O}_{uH} = (H^\dagger H) (\bar{q} u \tilde{H})$	$\mathcal{O}_{He} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{e} \gamma^\mu e)$
$\mathcal{O}_{dH} = (H^\dagger H) (\bar{q} d H)$	$\mathcal{O}_{Hq}^{(1)} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{q} \gamma^\mu q)$
$\mathcal{O}_{eH} = (H^\dagger H) (\bar{\ell} e H)$	$\mathcal{O}_{Hq}^{(3)} = (H^\dagger i \overleftrightarrow{D}_\mu^a H) (\bar{q} \gamma^\mu \sigma^a q)$
$\mathcal{O}_{Hu} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{u} \gamma^\mu u)$	$\mathcal{O}_{H\ell}^{(1)} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{\ell} \gamma^\mu \ell)$
$\mathcal{O}_{Hd} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{d} \gamma^\mu d)$	$\mathcal{O}_{H\ell}^{(3)} = (H^\dagger i \overleftrightarrow{D}_\mu^a H) (\bar{\ell} \gamma^\mu \sigma^a \ell)$

$$\mathcal{L}_{\text{eff}} \supset \frac{C_H}{\Lambda^2} \mathcal{O}_H + \frac{C_{H\Box}}{\Lambda^2} \mathcal{O}_{H\Box} + \frac{C_{HD}}{\Lambda^2} \mathcal{O}_{HD}$$



Experimental limits

Theoretical grounds:

- SMEFT operator analysis

[Jiang, Craig, Shuterland, 1811.08878]

- + pseudo-observable scheme

One-loop analysis:

$$\mathcal{L}_{\text{eff}} \supset \frac{C_H}{\Lambda^2} \mathcal{O}_H + \frac{C_{H\Box}}{\Lambda^2} \mathcal{O}_{H\Box} + \frac{C_{HD}}{\Lambda^2} \mathcal{O}_{HD}$$

Operators:

$$\mathcal{O}_H \equiv (H^\dagger H)^3,$$

$$\mathcal{O}_{H\Box} \equiv (H^\dagger H) \Box (H^\dagger H),$$

$$\mathcal{O}_{HD} \equiv (H^\dagger D^\mu H)^* (H^\dagger D_\mu H)$$

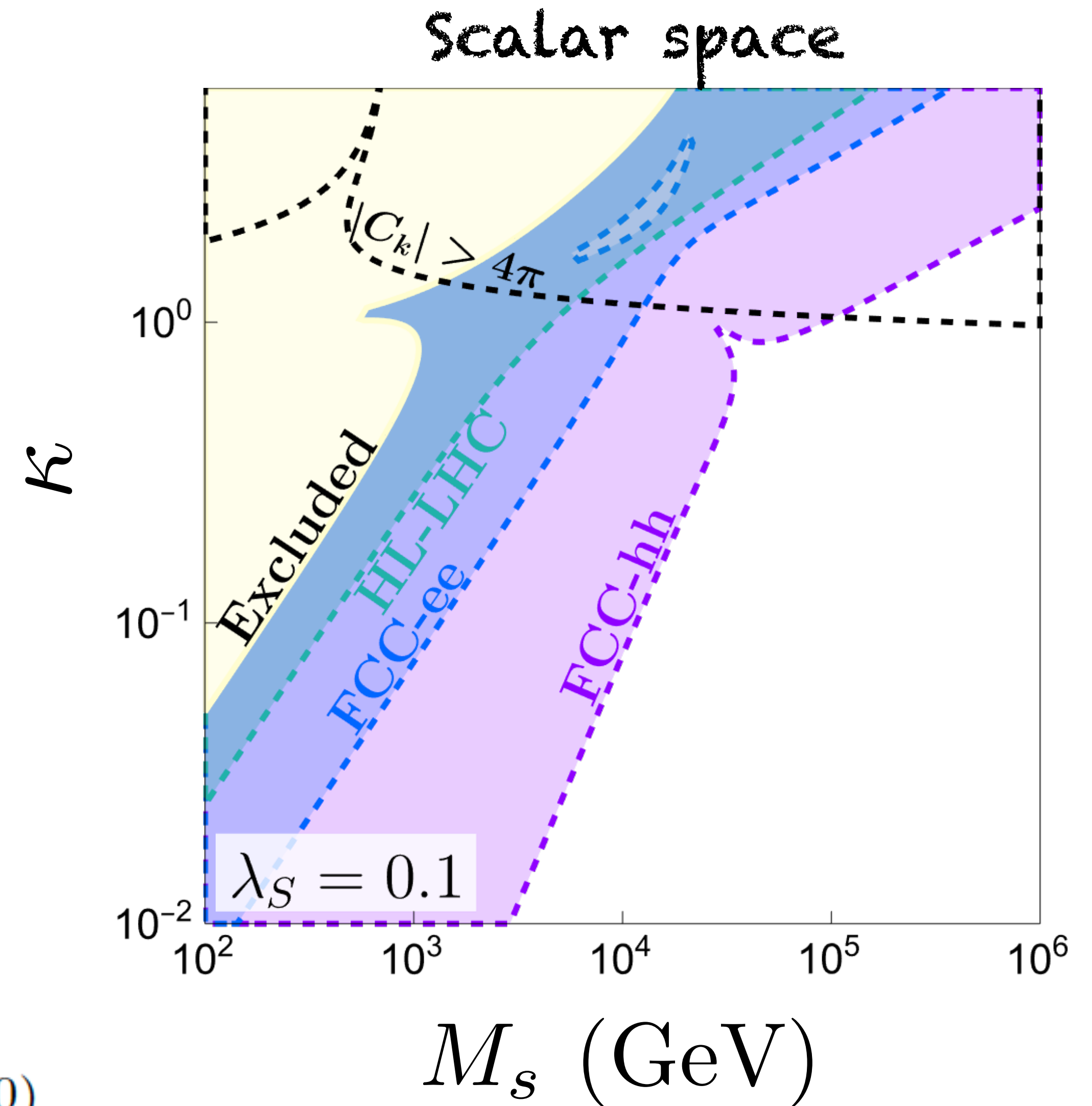
Tree level:

$$C_H^{(0)} = 0,$$

$$C_{H\Box}^{(0)} = -\frac{\kappa^2}{4\lambda_S},$$

$$C_{HD}^{(0)} = 0$$

Trilinear coupling: $g_{hhh} = 1 + \frac{3}{\sqrt{2}} \frac{1}{G_F M_s^2} C_{H\Box}^{(0)}$



Experimental limits

Theoretical grounds:

- SMEFT operator analysis

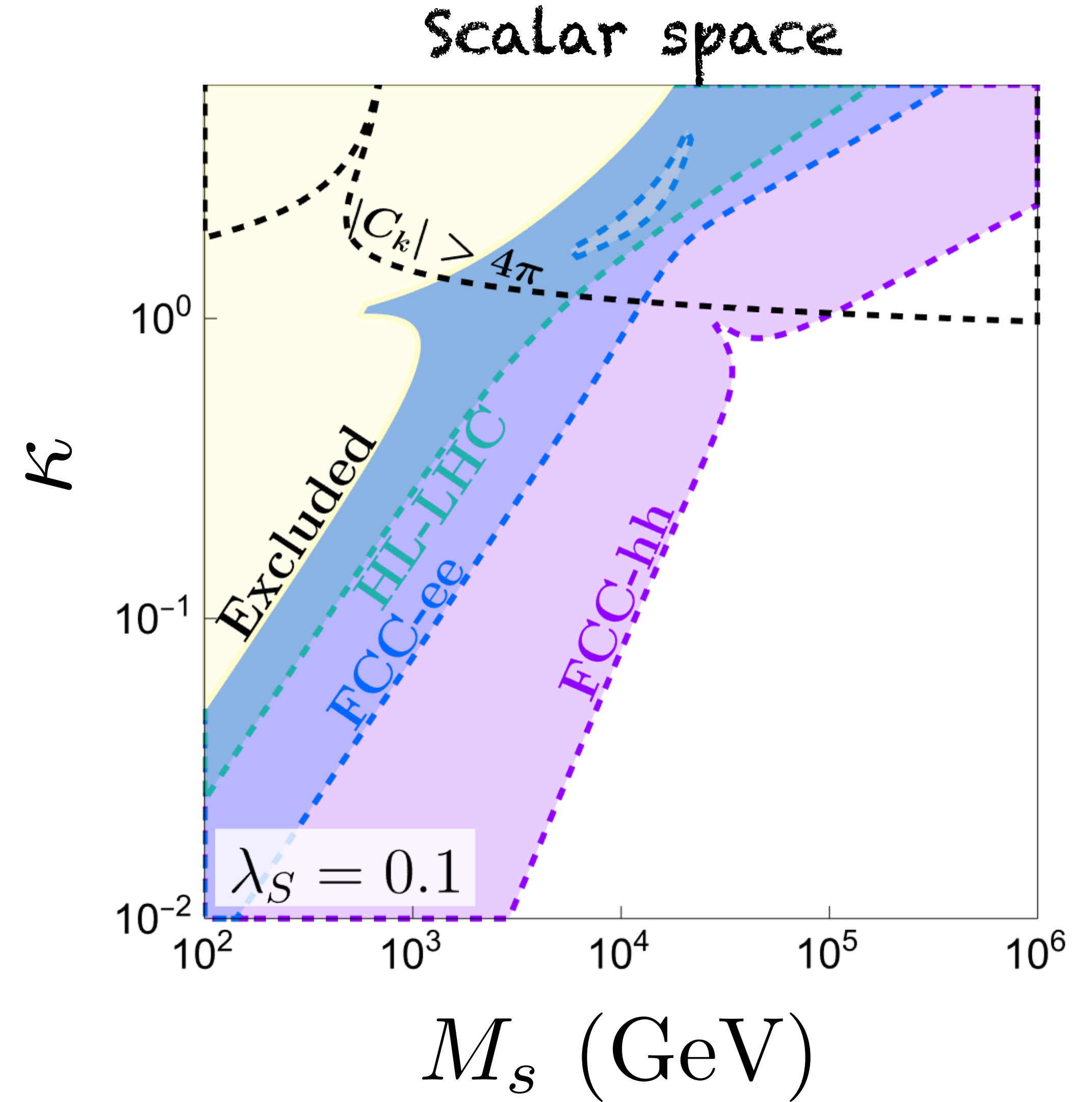
[Jiang, Craig, Shuterland, 1811.08878]

Excluded + HL-LHC + FCC-ee:

- Used results from SMEFT global fit [Celada et al. 2404.12809]

One-loop analysis:

$\mathcal{O}_H = (H^\dagger H)^3$	$\mathcal{O}_{HW} = (H^\dagger H) W_{\mu\nu}^a W^{a\mu\nu}$
$\mathcal{O}_{H\Box} = (H^\dagger H) \Box (H^\dagger H)$	$\mathcal{O}_{HB} = (H^\dagger H) B_{\mu\nu} B^{\mu\nu}$
$\mathcal{O}_{HD} = (H^\dagger D^\mu H)^* (H^\dagger D_\mu H)$	$\mathcal{O}_{HWB} = (H^\dagger \sigma^a H) W_{\mu\nu}^a B^{\mu\nu}$
$\mathcal{O}_{uH} = (H^\dagger H) (\bar{q} u \tilde{H})$	$\mathcal{O}_{He} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{e} \gamma^\mu e)$
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$\mathcal{O}_{Hu} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{u} \gamma^\mu u)$	$\mathcal{O}_{H\ell}^{(1)} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{\ell} \gamma^\mu \ell)$
$\mathcal{O}_{Hd} = (H^\dagger i \overleftrightarrow{D}_\mu H) (\bar{d} \gamma^\mu d)$	$\mathcal{O}_{H\ell}^{(3)} = (H^\dagger i \overleftrightarrow{D}_\mu^a H) (\bar{\ell} \gamma^\mu \sigma^a \ell)$



Experimental limits

Theoretical grounds:

- SMEFT operator analysis
[Jiang, Craig, Shuterland, 1811.08878]

FCC-hh:

- O(3%) prosp. sensitivity on $\kappa_\lambda = \frac{\lambda_3}{\lambda_{3(\text{SM})}}$
[Mangano et al, 2004.03505]

Operators:

$$\mathcal{O}_H \equiv (H^\dagger H)^3,$$

$$\mathcal{O}_{H\Box} \equiv (H^\dagger H) \Box (H^\dagger H),$$

$$\mathcal{O}_{HD} \equiv (H^\dagger D^\mu H)^* (H^\dagger D_\mu H) \quad C_{HD}^{(0)} = 0$$

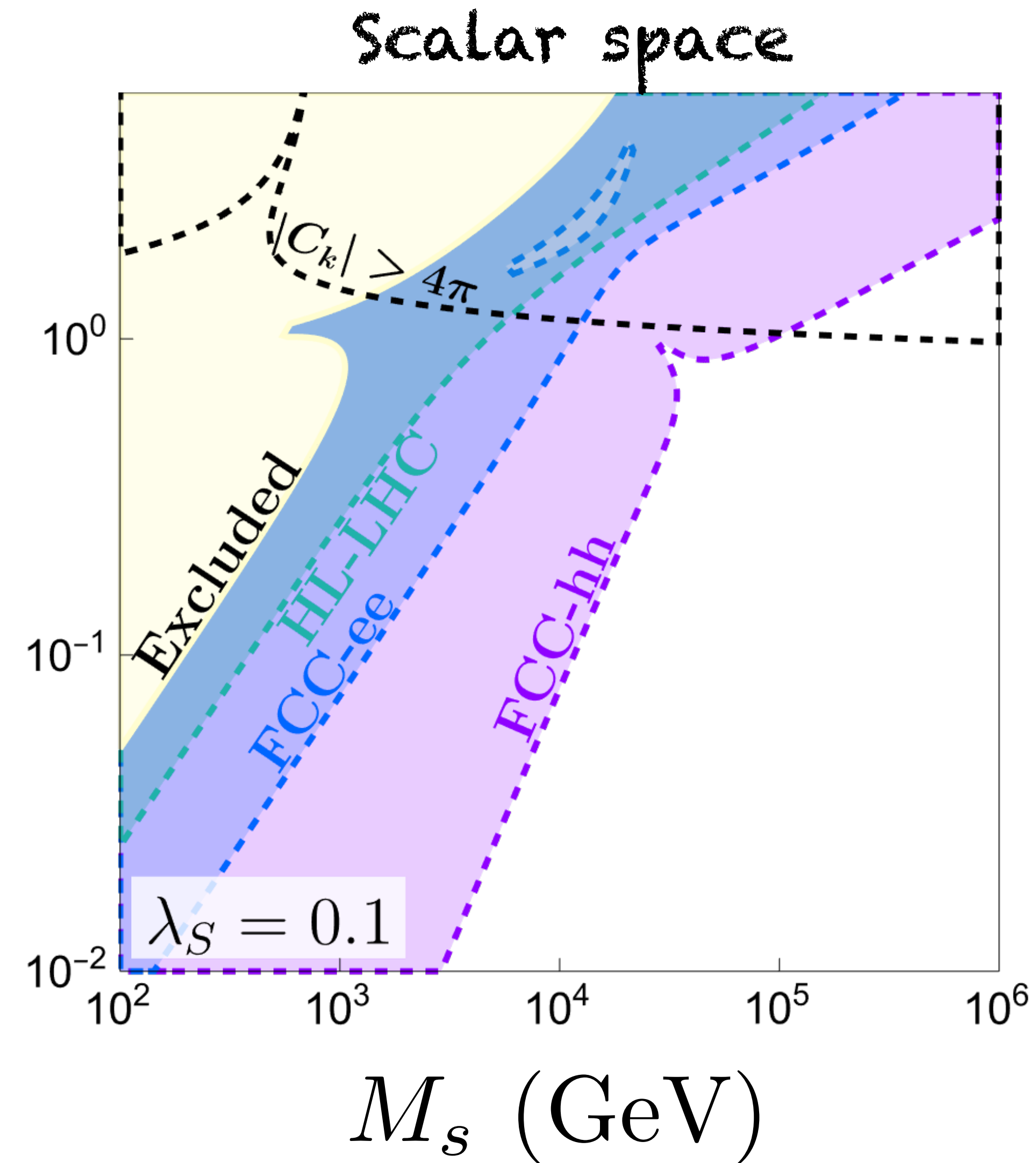
Tree level:

$$C_H^{(0)} = 0,$$

$$C_{H\Box}^{(0)} = -\frac{\kappa^2}{4\lambda_S},$$

$$C_{HD}^{(0)} = 0$$

Trilinear coupling: $g_{hhh} = 1 + \frac{3}{\sqrt{2}} \frac{1}{G_F M_s^2} C_{H\Box}^{(0)}$



Other models: ALPs

An altogether different model : SM+ ALPs: instability scale lowered by only scalars

[Detering-You, 2412.03542]

$$V(H, S) = -\frac{1}{2}m_H^2 H^2 + \frac{1}{4}\lambda H^4 + m_a^2 f^2 \left(1 - \cos\left(\frac{a}{f}\right)\right) - \frac{1}{2}A f (H^2 - v^2) \cos\left(\frac{a}{f} - \delta\right)$$

$$m_h^2 \leq m_{\text{crit}}^2 = -\beta_\lambda(\mu_I) e^{-3/2} \mu_I^2 + \frac{v^2 A^2 \sin^2 \delta}{m_a^2}$$

$$\lambda_{\text{eff}} \equiv \lambda_H - \frac{1}{2} \frac{A^2 \sin^2 \delta}{m_a^2}$$

Other models: ALPs

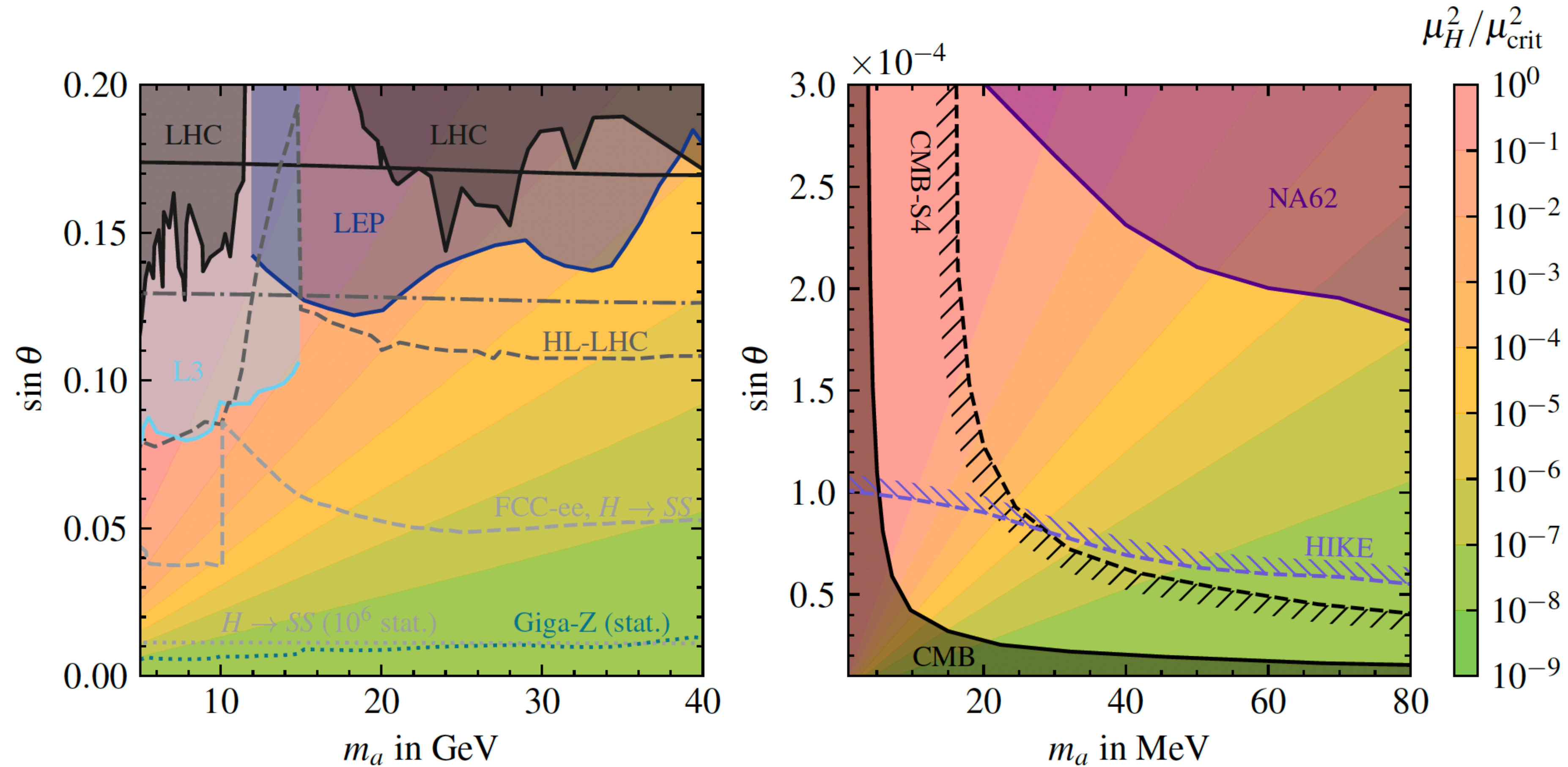
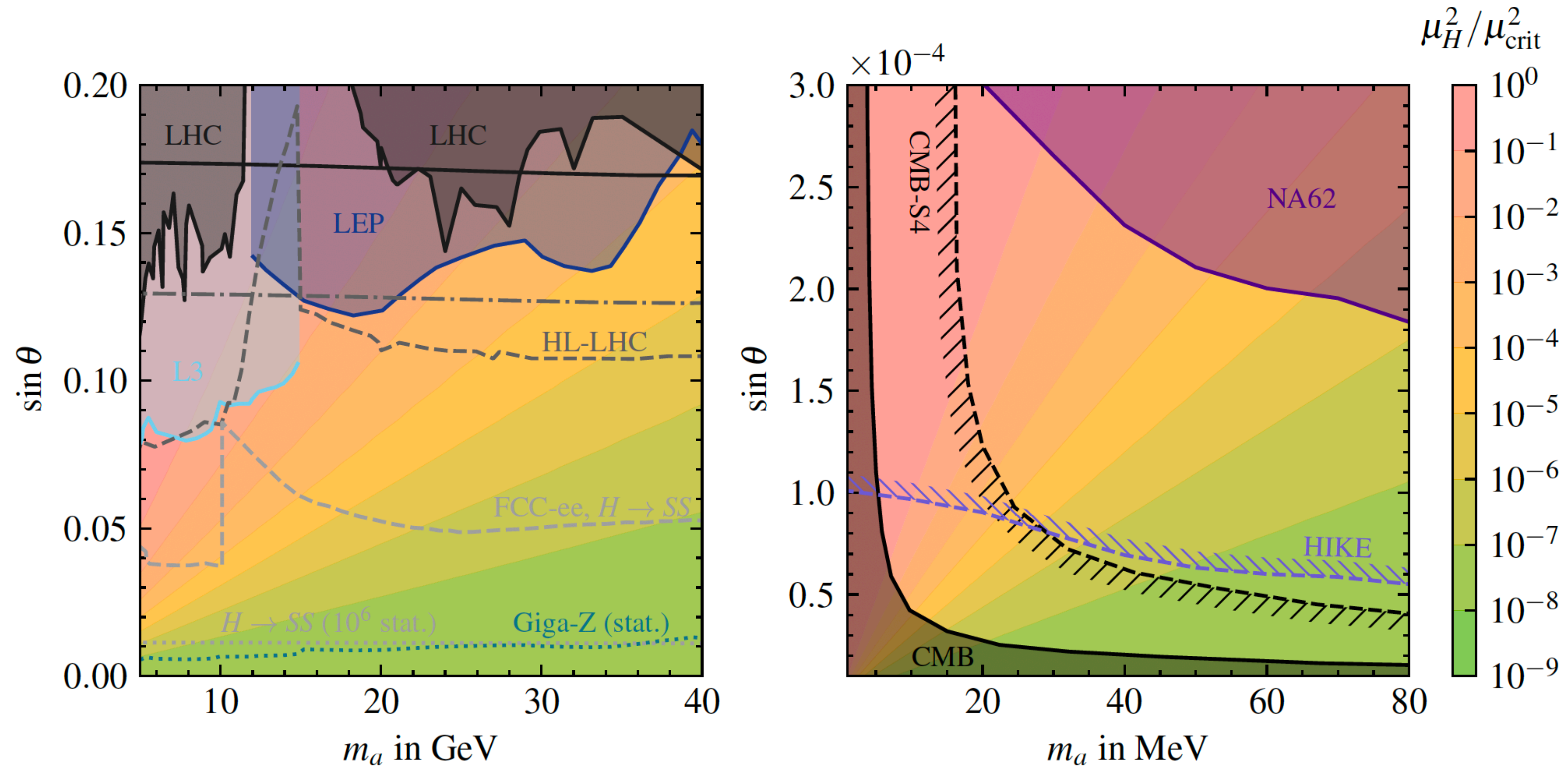


Figure 2: Critical value of the Higgs mass parameter in the Axion-Higgs model. The contours of the remaining hierarchy between the observed Higgs mass parameter in the Axion-Higgs model and the metastability bound are shown from red (small hierarchy) to green (large hierarchy). Existing constraints on the parameter space are shaded and projected experimental sensitivities are indicated by dashed and dotted lines. [9]

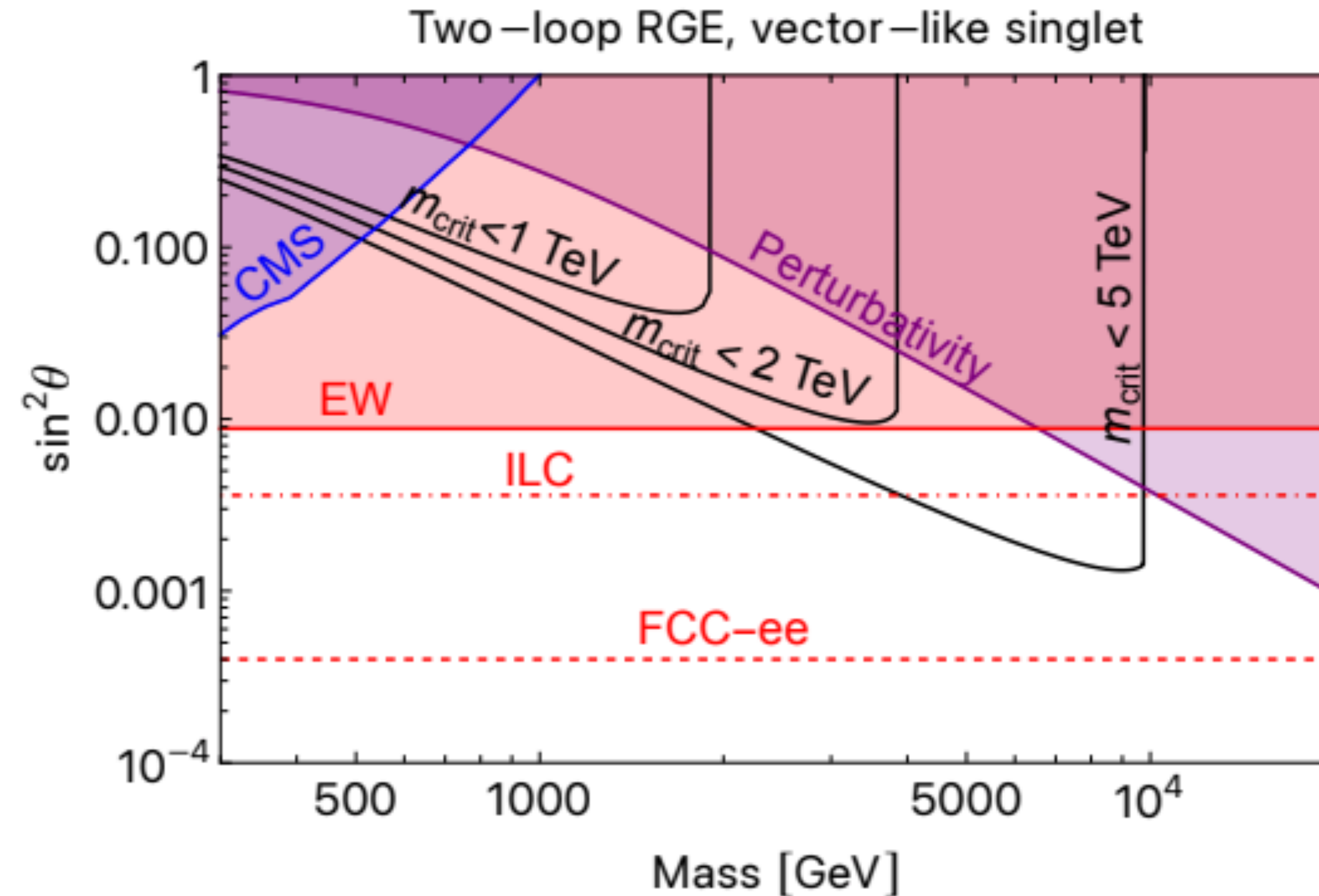
Other models: ALPs



➔ ALP mass in MeV - GeV range: $H \rightarrow aa, Zaa$ vertex, rare decays, CMB ...

Again the entire range to be covered at FCC-ee for GeV ALPs, and rare decays and/or CMB for MeV ALPs

Other models: vector-like quarks



$$\sin \theta = \frac{yv/\sqrt{2}}{\sqrt{y^2v^2/2 + M^2}}.$$

Figure 5: Higgs mass bounds and experimental constraints for the heavy neutral lepton model. The black lines show contours of constant $m_{\text{crit}} = 1, 2, 5$ TeV. The red solid line and shaded area correspond to experimental constraints from EW precision observables [40–43], and the red broken lines are projected reach from EW precision at FCC-ee (dashed) [44, 45] and ILC (dash-dotted) [44]. The blue solid line and shaded region depicts constraints from a CMS direct search [39], and the purple solid line and shaded region correspond to constraints from requiring perturbativity of y .

We worked in the “Majoron scheme”
with physical inputs:

$$\underbrace{\{\alpha_{\text{em}}, G_F, m_Z, m_h\}}_{\text{Z-scheme}} \underbrace{\{M_N, M_s, \kappa, Y_\nu, \lambda_S\}}_{\text{Majoron model}}$$

$$\begin{aligned}
\beta_{\lambda_H}^{(1)} = & +\frac{27}{200}g_1^4 + \frac{9}{20}g_1^2g_2^2 + \frac{9}{8}g_2^4 + \lambda_{H\sigma}^2 - \frac{9}{5}g_1^2\lambda_H - 9g_2^2\lambda_H + 24\lambda_H^2 + 12\lambda_H y_t^2 + 4\lambda_H \text{Tr}\left(Y_\nu Y_\nu^\dagger\right) - 6y_t^4 \\
& - 2\text{Tr}\left(Y_\nu Y_\nu^\dagger Y_\nu Y_\nu^\dagger\right), \tag{C1}
\end{aligned}$$

$$V = -m_H^2 |H|^2 + \lambda_H |H|^4 - m_S^2 |S|^2 + \lambda_S |S|^4 + \kappa |H|^2 |S|^2$$

Tree-level analysis:

$$M_N < E < M_s \quad \mathcal{L}_{\text{eff}} \supset \frac{C_H}{\Lambda^2} \mathcal{O}_H + \frac{C_{H\Box}}{\Lambda^2} \mathcal{O}_{H\Box} + \frac{C_{HD}}{\Lambda^2} \mathcal{O}_{HD}$$

$$\mathcal{O}_H \equiv (H^\dagger H)^3, \quad \mathcal{O}_{H\Box} \equiv (H^\dagger H)\Box(H^\dagger H), \quad \mathcal{O}_{HD} \equiv (H^\dagger D^\mu H)^*(H^\dagger D_\mu H)$$

$$C_H^{(0)} = 0, \quad C_{H\Box}^{(0)} = -\frac{\kappa^2}{4\lambda_S}, \quad C_{HD}^{(0)} = 0,$$

$$g_{hhh} = 1 + \frac{3}{\sqrt{2}} \frac{1}{G_F M_s^2} C_{H\Box}^{(0)}$$

$$g_{hhhh} = 1 + \frac{50}{3\sqrt{2}} \frac{1}{G_F M_s^2} C_{H\Box}^{(0)}$$

ν SMEFT

$$M_N < E < M_s$$

d=5:

$$\mathcal{O}_{NH} = \bar{N}_i N_j^c H^\dagger H + h.c. ,$$

$$\text{with } \frac{C_{NH}^{(0)}}{\Lambda} = -2\sqrt{2} \kappa \frac{M_N}{M_s^2}$$

d=6:

$$\mathcal{O}_{NN} = (\bar{N} N^c)(\bar{N} N^c) ,$$

$$\text{with } \frac{C_{NN}^{(0)}}{\Lambda^2} = 24 \lambda_S \frac{M_N^2}{M_s^4}$$

$$\lambda \rightarrow \lambda(\mu, \eta) \equiv \lambda(\mu) + \delta\lambda(\eta, \mu)$$

$$\begin{aligned} (4\pi)^2 \delta\lambda(\mu, \eta) = & -\frac{15g^4}{32} - \frac{5g^2(g')^2}{16} - \frac{5(g')^4}{32} + \\ & + \frac{9y_t^4}{2} + \frac{3}{8}g^4 \log\left(\frac{g^2}{4} \frac{\eta^2}{\mu^2}\right) + \\ & + \frac{3}{16} (g^2 + (g')^2)^2 \log\left(\frac{g^2 + (g')^2}{4} \frac{\eta^2}{\mu^2}\right) + \\ & - 3y_t^4 \log\left(\frac{y_t^2}{2} \frac{\eta^2}{\mu^2}\right) + \frac{3|Y_\nu|^4}{2} - \\ & - |Y_\nu|^4 \log\left(\frac{|Y_\nu|^2}{2} \frac{\eta^2}{\mu^2}\right) . \end{aligned} \quad (4)$$

$$\beta_\lambda \sim \lambda^2 + \lambda(y_t^2 - \text{gauge terms}) - y_t^4 + \text{gauge terms}.$$

$$\beta_{y_t} \sim -y_t(\text{gauge terms} - y_t^2)$$

$$\frac{\Gamma}{V} = A \cdot e^{-S_E} \simeq \mu_S^4 \cdot e^{-S_E} \quad \text{SM:} \quad S_E = \frac{8\pi^2}{3|\lambda(\mu_S)|},$$

For a given vacuum decay rate per unit volume, the lifetime of the vacuum is defined as the time after which the probability that a vacuum bubble has nucleated within the past lightcone of any observer is ~ 1

$$1 \sim \int_{\mathcal{P}} d^4x \frac{\Gamma}{V}$$

$$V_{\mathcal{P}} = \frac{0.15}{H_0^4} = 2.2 \cdot 10^{163} (\text{GeV})^{-4}.$$

Lower bound on the euclidean action:

$$S_E > 367.104 + 4 \ln \left(\frac{\mu_S}{\text{GeV}} \right)$$