

# Bootstrap the EFT from Glueball Exchanging

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## **EFT Operators Basis**

Standard Model EFT:

$$S_{
m UV}[\Phi] \ = \ \int d^4x \ {\cal L}_{
m UV}igl(\Phi(x), \partial_\mu \Phi(x), \dotsigr) \, .$$

$$\mathcal{L} = \Lambda^4 + \Lambda^2 \mathcal{O}^{(2)} + m \mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda} \mathcal{O}^{(5)} + \frac{1}{\Lambda^2} \mathcal{O}^{(6)} + \frac{1}{\Lambda^3} \mathcal{O}^{(7)} + \frac{1}{\Lambda^4} \mathcal{O}^{(8)} + \dots$$

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-\frac{1}{2}\partial_{\nu}g^a_{\mu}\partial_{\nu}g^a_{\mu} - g_s f^{abc}\partial_{\mu}g^a_{\nu}g^b_{\mu}g^c_{\nu} - \frac{1}{4}g^2_s f^{abc}f^{ade}g^b_{\mu}g^c_{\nu}g^d_{\mu}g^e_{\nu} +
                                                                                              \tfrac{1}{2}ig_s^2(\bar{q}_i^\sigma\gamma^\mu q_i^\sigma)g_\mu^a + \bar{G}^a\partial^2G^a + g_sf^{abc}\partial_\mu\bar{G}^aG^bg_\mu^c - \partial_\nu W_\mu^+\partial_\nu W_\mu^- -
 2 M^2 W_{\mu}^+ W_{\mu}^- - \frac{1}{2} \partial_{\nu} Z_{\mu}^0 \partial_{\nu} Z_{\mu}^0 - \frac{1}{2c^2} M^2 Z_{\mu}^0 Z_{\mu}^0 - \frac{1}{2} \partial_{\mu} A_{\nu} \partial_{\mu} A_{\nu} - \frac{1}{2} \partial_{\mu} H \partial_{\mu} H \partial_{\mu} H - \frac{1}{2} \partial_{\mu} H \partial_{\mu} H \partial_{\mu} H - \frac{1}{2} \partial_{\mu} H \partial_{\mu
                                                  \frac{1}{2}m_{h}^{2}H^{2} - \partial_{\mu}\phi^{+}\partial_{\mu}\phi^{-} - M^{2}\phi^{+}\phi^{-} - \frac{1}{2}\partial_{\mu}\phi^{0}\partial_{\mu}\phi^{0} - \frac{1}{2c_{c}^{2}}M\phi^{0}\phi^{0} - \beta_{h}[\frac{2M^{2}}{g^{2}} + \frac{1}{2}(\frac{2M^{2}}{g^{2}})] + \frac{1}{2}(\frac{2M^{2}}{g^{2}})
                                                                          \tfrac{2M}{g} H + \tfrac{1}{2} (H^2 + \phi^0 \phi^0 + 2\phi^+ \phi^-)] + \tfrac{2M^4}{q^2} \alpha_h - ig c_w [\partial_\nu Z^0_\mu (W^+_\mu W^-_\nu -
                                                                                                                         W_{\nu}^{+}W_{\mu}^{-}) - Z_{\nu}^{0}(W_{\mu}^{+}\partial_{\nu}W_{\mu}^{-} - W_{\mu}^{-}\dot{\partial}_{\nu}W_{\mu}^{+}) + Z_{\mu}^{0}(W_{\nu}^{+}\partial_{\nu}W_{\mu}^{-} -
                                                                             W_{\nu}^{-}\partial_{\nu}W_{\mu}^{+})] - igs_{w}[\partial_{\nu}A_{\mu}(W_{\mu}^{+}W_{\nu}^{-} - W_{\nu}^{+}W_{\mu}^{-}) - A_{\nu}(W_{\mu}^{+}\partial_{\nu}W_{\mu}^{-} - W_{\mu}^{-}W_{\mu}^{-})]
                                                            W_{\mu}^{-}\partial_{\nu}W_{\mu}^{+}) + A_{\mu}(W_{\nu}^{+}\partial_{\nu}W_{\mu}^{-} - W_{\nu}^{-}\partial_{\nu}W_{\mu}^{+})] - \frac{1}{2}g^{2}W_{\mu}^{+}W_{\mu}^{-}W_{\nu}^{+}W_{\nu}^{-} +
                                                                                                               \frac{1}{2}g^2W_{\mu}^{+}W_{\nu}^{-}W_{\mu}^{+}W_{\nu}^{-} + g^2c_w^2(Z_{\mu}^0W_{\mu}^{+}Z_{\nu}^0W_{\nu}^{-} - Z_{\mu}^0Z_{\mu}^0W_{\nu}^{+}W_{\nu}^{-}) +
                                                                                    g^2 s_w^2 (A_\mu W_\mu^+ A_\nu W_\nu^- - A_\mu A_\mu W_\nu^+ W_\nu^-) + g^2 s_w c_w [A_\mu Z_\nu^0 (W_\mu^+ W_\nu^- - W_\mu^-)] + g^2 s_w c_w [A_\mu Z_\nu^0 (W_\mu^+ W_\nu^-)] + g^2 s_w c_w [A_\mu Z_\nu^0 (W_\mu^+ W_\mu^-)] + g^2 s_w c_w [A_\mu Z_\nu^0 (W_\mu^- W_\mu^- W_\mu^-)] + g^2 s_w c_w [A_\mu Z_\nu^0 (W_\mu^- W_\mu^- W_
                                                                                                        W_{\nu}^{+}W_{\mu}^{-}) - 2A_{\mu}Z_{\mu}^{0}W_{\nu}^{+}W_{\nu}^{-}] - g\alpha[H^{3} + H\phi^{0}\phi^{0} + 2H\phi^{+}\phi^{-}] -
                                                  \frac{1}{8}g^2\alpha_h[H^4+(\phi^0)^4+4(\phi^+\phi^-)^2+4(\phi^0)^2\phi^+\phi^-+4H^2\phi^+\phi^-+2(\phi^0)^2H^2]-
                                                                                           gMW_{\mu}^{+}W_{\mu}^{-}H - \frac{1}{2}g\frac{M}{c^{2}}Z_{\mu}^{0}Z_{\mu}^{0}H - \frac{1}{2}ig[W_{\mu}^{+}(\phi^{0}\partial_{\mu}\phi^{-} - \phi^{-}\partial_{\mu}\phi^{0}) -
                                               W_{\mu}^-(\phi^0\partial_{\mu}\phi^+-\phi^+\partial_{\mu}\phi^0)] + \tfrac{1}{2}g[W_{\mu}^+(H\partial_{\mu}\phi^--\phi^-\partial_{\mu}H) - W_{\mu}^-(H\partial_{\mu}\phi^+-\phi^-\partial_{\mu}H)] + W_{\mu}^-(H\partial_{\mu}\phi^+-\phi^-\partial_{\mu}H) + W_{\mu}^-(H\partial_{\mu}\phi^--\phi^-\partial_{\mu}H) + W_{\mu}^-(H\partial_{\mu}H) + W_{\mu}^-(H\partial_{\mu}H) + W_{\mu}^-(H\partial_{\mu}H) + W_{\mu}^-(H\partial_{\mu}H) + 
                                               [\phi^{+}\partial_{\mu}H)] + \frac{1}{2}g\frac{1}{c_{m}}(Z_{\mu}^{0}(H\partial_{\mu}\phi^{0} - \phi^{0}\partial_{\mu}H) - ig\frac{s_{w}^{2}}{c_{m}}MZ_{\mu}^{0}(W_{\mu}^{+}\phi^{-} - W_{\mu}^{-}\phi^{+}) +
                                                                             igs_w MA_{\mu}(W_{\mu}^+\phi^- - W_{\mu}^-\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z_{\mu}^0(\phi^+\partial_{\mu}\phi^- - \phi^-\partial_{\mu}\phi^+) +
                                                         igs_w A_{\mu}(\phi^+\partial_{\mu}\phi^- - \phi^-\partial_{\mu}\phi^+) - \frac{1}{4}g^2 W_{\mu}^+ W_{\mu}^- [H^2 + (\phi^0)^2 + 2\phi^+\phi^-] -
                                                     \tfrac{1}{4}g^2\tfrac{1}{c^2}Z^0_\mu Z^0_\mu [H^2 + (\phi^0)^2 + 2(2s^2_w - 1)^2\phi^+\phi^-] - \tfrac{1}{2}g^2\tfrac{s^2_w}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- +
                                                                                W_{\mu}^{-}\phi^{+}) - \frac{1}{2}ig^{2}\frac{s_{w}^{2}}{c_{w}}Z_{\mu}^{0}H(W_{\mu}^{+}\phi^{-} - W_{\mu}^{-}\phi^{+}) + \frac{1}{2}g^{2}s_{w}A_{\mu}\phi^{0}(W_{\mu}^{+}\phi^{-} + W_{\mu}^{-}\phi^{+})
                                               W_{\mu}^{-}\phi^{+}) + \frac{1}{2}ig^{2}s_{w}A_{\mu}H(W_{\mu}^{+}\phi^{-} - W_{\mu}^{-}\phi^{+}) - g^{2}\frac{s_{w}}{c_{w}}(2c_{w}^{2} - 1)Z_{\mu}^{0}A_{\mu}\phi^{+}\phi^{-} - \frac{1}{2}ig^{2}s_{w}A_{\mu}H(W_{\mu}^{+}\phi^{-} - W_{\mu}^{-}\phi^{-}) - \frac{1}{2}ig^{2}s_{
                                                                      g^1 s_w^2 A_u A_u \phi^+ \phi^- - \bar{e}^{\lambda} (\gamma \partial + m_e^{\lambda}) e^{\lambda} - \bar{\nu}^{\lambda} \gamma \partial \nu^{\lambda} - \bar{u}_i^{\lambda} (\gamma \partial + m_u^{\lambda}) u_i^{\lambda} - \bar
 \bar{d}_{i}^{\lambda}(\gamma\partial + m_{d}^{\lambda})d_{i}^{\lambda} + igs_{w}A_{\mu}[-(\bar{e}^{\lambda}\gamma^{\mu}e^{\lambda}) + \frac{2}{3}(\bar{u}_{i}^{\lambda}\gamma^{\mu}u_{i}^{\lambda}) - \frac{1}{3}(\bar{d}_{i}^{\lambda}\gamma^{\mu}d_{i}^{\lambda})] + 
                                                                      \frac{ig}{4c_w}Z_{\mu}^0[(\bar{\nu}^{\lambda}\gamma^{\mu}(1+\gamma^5)\nu^{\lambda})+(\bar{e}^{\lambda}\gamma^{\mu}(4s_w^2-1-\gamma^5)e^{\lambda})+(\bar{u}_i^{\lambda}\gamma^{\mu}(\frac{4}{3}s_w^2-1)e^{\lambda})
                                               (1 - \gamma^5)u_j^{\lambda}) + (\bar{d}_j^{\lambda}\gamma^{\mu}(1 - \frac{8}{3}s_w^2 - \gamma^5)d_j^{\lambda})] + \frac{ig}{2\sqrt{2}}W_{\mu}^{+}[(\bar{\nu}^{\lambda}\gamma^{\mu}(1 + \gamma^5)e^{\lambda}) + (\bar{d}_j^{\lambda}\gamma^{\mu}(1 + \gamma^5)e^{\lambda})] + (\bar{d}_j^{\lambda}\gamma^{\mu}(1 - \frac{8}{3}s_w^2 - \gamma^5)d_j^{\lambda})] + (\bar{d}_j^{\lambda}\gamma^{\mu}(1 - \frac{8}{3}s_w^2 - \gamma^5)d_j^{\lambda})]
                                                            (\bar{u}_{i}^{\lambda}\gamma^{\mu}(1+\gamma^{5})C_{\lambda\kappa}d_{i}^{\kappa})] + \frac{ig}{2\sqrt{2}}W_{\mu}^{-}[(\bar{e}^{\lambda}\gamma^{\mu}(1+\gamma^{5})\nu^{\lambda}) + (\bar{d}_{i}^{\kappa}C_{\lambda\kappa}^{\dagger}\gamma^{\mu}(1+\gamma^{5})\nu^{\lambda})]
                                                                                                               [\gamma^{5}]u_{j}^{\lambda}] + \frac{ig}{2\sqrt{2}} \frac{m_{e}^{\lambda}}{M} [-\phi^{+}(\bar{\nu}^{\lambda}(1-\gamma^{5})e^{\lambda}) + \phi^{-}(\bar{e}^{\lambda}(1+\gamma^{5})\nu^{\lambda})] - ig
                                                         \frac{g}{2}\frac{m_e^{\lambda}}{M}[H(\bar{e}^{\lambda}e^{\lambda}) + i\phi^0(\bar{e}^{\lambda}\gamma^5e^{\lambda})] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^{\kappa}(\bar{u}_j^{\lambda}C_{\lambda\kappa}(1-\gamma^5)d_j^{\kappa}) +
                                                  m_u^{\lambda}(\bar{u}_j^{\lambda}C_{\lambda\kappa}(1+\gamma^5)d_j^{\kappa}] + \frac{ig}{2M\sqrt{2}}\phi^{-}[m_d^{\lambda}(\bar{d}_j^{\lambda}C_{\lambda\kappa}^{\dagger}(1+\gamma^5)u_j^{\kappa}) - m_u^{\kappa}(\bar{d}_j^{\lambda}C_{\lambda\kappa}^{\dagger}(1-\gamma^5)u_j^{\kappa}) - m_u^{
                                                                                                                  \gamma^5)u_j^\kappa] - \tfrac{q}{2} \tfrac{m_u^\lambda}{M} H(\bar{u}_j^\lambda u_j^\lambda) - \tfrac{q}{2} \tfrac{m_d^\lambda}{M} H(\bar{d}_j^\lambda d_j^\lambda) + \tfrac{iq}{2} \tfrac{m_u^\lambda}{M} \phi^0(\bar{u}_j^\lambda \gamma^5 u_j^\lambda) -
                                                  \frac{ig}{2} \frac{m_d^{\lambda}}{M} \phi^0(\bar{d}_j^{\lambda} \gamma^5 d_j^{\lambda}) + \bar{X}^+(\partial^2 - M^2) X^+ + \bar{X}^-(\partial^2 - M^2) X^- + \bar{X}^0(\partial^2 - M^2) X^- +
\frac{M^{2}}{c^{2}}X^{0} + \bar{Y}\partial^{2}Y + igc_{w}W_{\mu}^{+}(\partial_{\mu}\bar{X}^{0}X^{-} - \partial_{\mu}\bar{X}^{+}X^{0}) + igs_{w}W_{\mu}^{+}(\partial_{\mu}\bar{Y}X^{-} - \partial_{\mu}\bar{X}^{-}X^{0}) + igs_{w}W_{\mu}^{+}(\partial_{\mu}\bar{Y}X^{-} - \partial_{\mu}\bar{Y}X^{-}) + igs_{w}W_{\mu}^{+}(\partial_{\mu}\bar{Y}X^
                                                                      \partial_{\mu}\bar{X}^{+}Y) + igc_{w}W_{\mu}^{-}(\partial_{\mu}\bar{X}^{-}X^{0} - \partial_{\mu}\bar{X}^{0}X^{+}) + igs_{w}W_{\mu}^{-}(\partial_{\mu}\bar{X}^{-}Y - \partial_{\mu}\bar{X}^{0}X^{+})
                                                                      \partial_{\mu}\bar{Y}X^{+}) + igc_{w}Z^{0}_{\mu}(\partial_{\mu}\bar{X}^{+}X^{+} - \partial_{\mu}\bar{X}^{-}X^{-}) + igs_{w}A_{\mu}(\partial_{\mu}\bar{X}^{+}X^{+} - \partial_{\mu}\bar{X}^{-}X^{-})
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 $\begin{array}{l} \partial_{\mu}\bar{X}^{-}X^{-}) - \frac{1}{2}gM[\bar{X}^{+}X^{+}H + \bar{X}^{-}X^{-}H + \frac{1}{c_{w}^{2}}\bar{X}^{0}X^{0}H] + \\ \frac{1-2c_{w}^{2}}{2c_{w}}igM[\bar{X}^{+}X^{0}\phi^{+} - \bar{X}^{-}X^{0}\phi^{-}] + \frac{1}{2c_{w}}igM[\bar{X}^{0}X^{-}\phi^{+} - \bar{X}^{0}X^{+}\phi^{-}] + \\ igMs_{w}[\bar{X}^{0}X^{-}\phi^{+} - \bar{X}^{0}X^{+}\phi^{-}] + \frac{1}{2}igM[\bar{X}^{+}X^{+}\phi^{0} - \bar{X}^{-}X^{-}\phi^{0}] \end{array}$ 

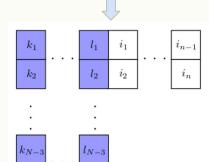
#### On-shell EFT Operator Construction

- EFT from UV massive particles:
  - Warsaw basis by EOM and IBP
  - All operator basis for SMEFT (José's talk)
  - SMEFT Geometry (Mia's talk)
  - 0 ...
- It that all?

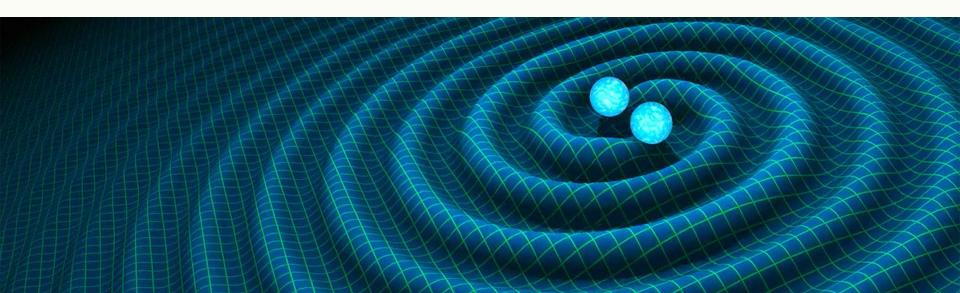
$$[j_1j_2]\dots[j_{\widetilde{n}-1}j_{\widetilde{n}}]\langle i_1i_2\rangle\dots\langle i_{n-1}i_n\rangle,$$



$$[j_1j_2]\epsilon^{j_1j_2k_1..k_{N-2}}\dots[j_{\widetilde{n}-1}j_{\widetilde{n}}]\epsilon^{j_{\widetilde{n}-1}j_{\widetilde{n}}l_1..l_{N-2}}\langle i_1i_2\rangle\dots\langle i_{n-1}i_n\rangle,$$



# Extra Graviton EFT Needed



#### Puzzle from Graviton Scattering

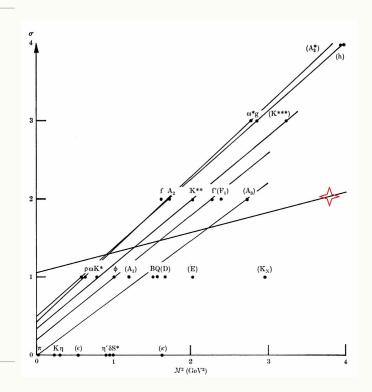
- Dispersion relations link EFT and UV spectrum
- ☐ Froissart Bound

$$M(s,t) \lesssim s \log^2(s)$$
,

Regge High-Energy Behaviour

$$M(s,t) \sim (s/s_0)^{\alpha(t)}$$
,

■ No one allow Amplitude growth faster than s^2



#### Dispersion Relation



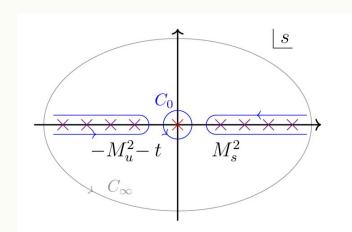
$$\oint_{|s| \to \infty} \frac{ds}{s^3} \mathcal{M}_{hhhh}(s, t) = 0$$

→ 4-graviton Low-Energy EFT:

$$\mathcal{M}_4(1^+, 2^-, 3^-, 4^+) = (\langle 23 \rangle [14])^4 f(s, u),$$

$$f(s,u) = \left(\frac{\kappa}{2}\right)^2 \frac{1}{stu} + |\beta_{R^3}|^2 \frac{su}{t} - |\beta_{\phi}|^2 \frac{1}{t} + \sum_{i=0}^{\infty} f_{2i,i} s^i u^i + \sum_{i=1}^{\infty} \sum_{j=0}^{\lfloor \frac{i}{2} \rfloor} f_{i,j} (s^{i-j} u^j + s^j u^{i-j}),$$

Only graviton-pole survive!



#### Isolate QFT from Gravity

Graviton-pole cancelled by its own Ladder diagrams, even when mix with QFT

$$+ \boxed{\mathsf{EFT}} = \int ds \int dL \operatorname{IPS} \left( \cdots \right) + \boxed{ }$$

☐ Change the spatial size B where scattering occur, we can isolate QFT from Gravity

$$G\Lambda_{\text{QCD}}^2 \log(B/b) \ll 1$$
,

■ i.e. EFT = resonances

$$\boxed{\mathsf{EFT}} = \int \mathsf{d}\mathbf{s} \left( \begin{array}{c} \\ \\ \\ \end{array} \right)$$

Elastic Scattering → Positivity

## Not all EFT from Massive UV particle

- No matching term from massive-particle induced EFT
- Another mechanism to obtain mass: Glueball mass. Glueball (Pomeron) and Meson (Reggeon),

$$\mathcal{M}_{hhhh}^{ ext{QCD-tree}} = \underbrace{\mathbb{R}_{\text{Add}}^{ ext{QCD-tree}}}_{\text{and }} + \underbrace{\mathbb{R}_{\text{Add}}^{ ext{QCD-tree}}}_{\text{and }} + \cdots$$

 $\Box$  In Regge limit  $-t \ll \Lambda_{\rm QCD}^2 \ll s$ ,

$$\mathcal{M}_{1234}^{\text{QCD-tree}}(s \gg t) \sim \beta(t, \varepsilon_1, \varepsilon_3) \cdot \frac{1 + \eta(-1)^{\alpha(t)}}{\sin(\pi\alpha(t))} \cdot \left(\frac{s}{s_0}\right)^{\alpha(t)} \cdot \beta(t, \varepsilon_2, \varepsilon_4).$$

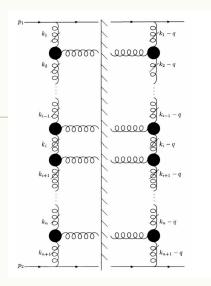
#### Glueball may contribute to the missed EFT

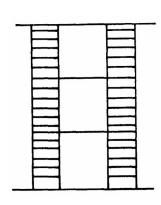
■ Vanished Boundary + Positive s-, u-channel Discontinuity



- Something must be in Co contour! Could we calculate it?
- Feynman ladder diagram only give cuts, only running of αs let cuts become poles.

Too complicate to get low-energy EFT!

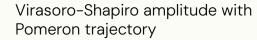






## Glueball Exchanging Amplitude

- 't Hooft double line view:
  Glueball —— Closed QCD string;
  - Meson Open QCD string.
- Uniquely identify the UV Singularity
  - 1. Quantum Number;
  - 2. Even/Odd Spin;
  - 3. Angular momentum selection rule, J≥∆h.
- ☐ String-like behaviour in Lattice QCD



$$\alpha_P(t) = 1.08 + 0.25 \,\text{GeV}^{-2} t$$
,

not Quantum Gravity trajectory

$$\alpha_{\rm QG}(t) = 2 + \frac{\alpha'}{2}t.$$

#### Virasoro-Shapiro amplitude for Glueball

□ Role of each part of V-S amplitude:

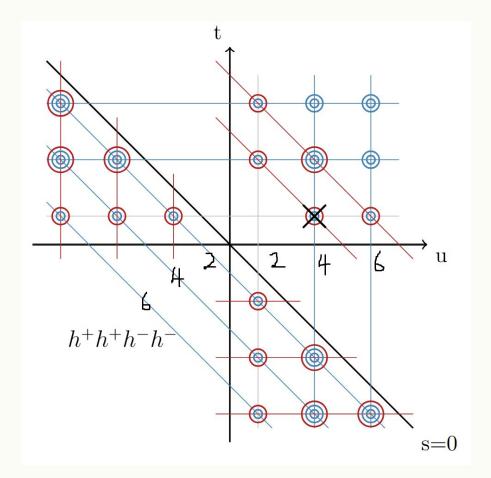
$$M_0(s,t) = 8\pi G \left(\frac{su}{t} + \frac{tu}{s} + \frac{st}{u}\right) \frac{\Gamma(1 - \frac{\alpha'}{4}s)\Gamma(1 - \frac{\alpha'}{4}t)\Gamma(1 - \frac{\alpha'}{4}u)}{\Gamma(1 + \frac{\alpha'}{4}s)\Gamma(1 + \frac{\alpha'}{4}t)\Gamma(1 + \frac{\alpha'}{4}u)},$$

- The prefactor is the low-energy amplitude for  $s, t, u \ll 1/\alpha'$ .
- The  $\Gamma$ -functions generate all massive poles.
- The  $\Gamma$ -functions cancel all double poles and nonsense poles  $(J < \Delta h)$ .
- Regge trajectory uniquely fix all Γ-functions.

☐ Plot all the poles for ++-- 4-graviton scattering

No double pole,

No nonsense-pole.



#### V-S amplitude has new EFT!

After contain all massive poles, we also need to ensure the high-energy Regge behavior of the amplitude



$$\mathcal{M}^{P}_{++--} = \left(\frac{[12]^2 \langle 34 \rangle^2}{\langle 12 \rangle^2 [34]^2} G^2 s^2\right) \frac{\Gamma(\frac{2-\alpha(s)}{2}) \Gamma(\frac{4-\alpha(t)}{2}) \Gamma(\frac{4-\alpha(u)}{2})}{\Gamma(\frac{6-\alpha(t)-\alpha(u)}{2}) \Gamma(\frac{4-\alpha(s)-\alpha(u)}{2}) \Gamma(\frac{4-\alpha(s)-\alpha(t)}{2})}.$$

Give a New EFT ~ s^2 not in Local EFT Construction!

#### What's the new EFT?

Remarkably, the low-energy prefactor is exactly the one-gluon-loop amplitude for four gravitons

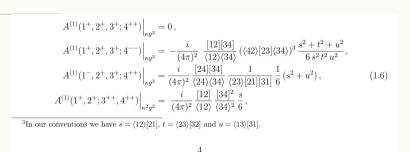
$$\mathcal{M}_{++--}^{1-\text{gluon-loop}} = \begin{cases} h^{+} & \text{for } h^{-} \\ h^{+} & \text{for } h^{-} \end{cases} = \frac{[12]^{2} \langle 34 \rangle^{2}}{\langle 12 \rangle^{2} [34]^{2}} G^{2} s^{2}$$

The log brunch cuts (2, 3, ... loops) are absorbed by the Glueball's Regge behavior, generating an tower of massive glueballs.

$$1 + \alpha_s \log(s/s_0) + \alpha_s^2 \log^2(s/s_0) + \dots = (s/s_0)^{\alpha_s} \sim \frac{\Gamma(s/s_0 + \alpha_s)}{\Gamma(s/s_0)}$$

#### Coincidence?

|                        | QCD V-S  |
|------------------------|--|
| $h^+h^+h^+h^+$         | $\frac{[12]^2[34]^2}{\langle 12\rangle^2\langle 34\rangle^2}(s^2+t^2+u^2)$   |
| $h^+h^+h^+h^-$         | $\frac{[12]^2[23]^2\langle 24\rangle^2}{\langle 12\rangle^2\langle 23\rangle^2[24]^2}(s^2+t^2+u^2)$                      |
| $h^{+}h^{+}h^{-}h^{-}$ | $\frac{[12]^2\langle 34\rangle^2}{\langle 12\rangle^2[34]^2}(s^2)$   |
| $g^+g^+h^+h^+$         | $\frac{[12][34]^2}{\langle 12 \rangle \langle 34 \rangle^2}(s)$  |
| $g^-g^+h^+h^+$         | $\frac{[24][34]^2\langle 14\rangle}{\langle 24\rangle\langle 34\rangle^2[14]}(s)$  |
| $g^+g^+h^+h^-$         | $\frac{[12][13]^2\langle 14\rangle^2}{\langle 12\rangle\langle 13\rangle^2[14]^2} \left(\frac{t^2+u^2}{s}\right)$        |
| $g^-g^-h^+h^+$         | $\frac{[12]\langle 34\rangle^2}{\langle 12\rangle[34]^2}(s)$   |
| $g^+g^-h^+h^-$         | $\frac{[13]^2 \langle 24 \rangle \langle 14 \rangle}{\langle 13 \rangle [24]^2 [14]^2} \left(\frac{t^2 + u^2}{s}\right)$ |
| $gggh \cdots$          |  |
| $gggg \cdots$          | • • •  |

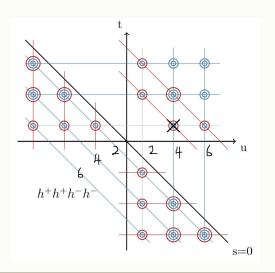


 $M_4^{(1)}(1^+, 2^+, 3^+, 4^+) = -\frac{i}{(4\pi)^2} \frac{1}{60} \left(\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\right)^2 (s^2 + st + t^2),$ 

$$M_4^{(1)}(1^-, 2^+, 3^+, 4^+) = \frac{i}{(4\pi)^2} \frac{1}{180} \left( \frac{\langle 12 \rangle [23][24]}{[12]\langle 23 \rangle \langle 24 \rangle} \right)^2 (s^2 + st + t^2) ,$$

$$\begin{split} A^{(1)}(1^-,2^+;3^{++},4^{++})\Big|_{\kappa^2g^2} &= \frac{i}{(4\pi)^2} \frac{[24]^2[34]^2\langle 14\rangle^2}{\langle 34\rangle^2} \frac{s}{6\,t\,u}\,, \\ A^{(1)}(1^+,2^+;3^{++},4^{--})\Big|_{\kappa^2g^2} &= \frac{i}{(4\pi)^2} \frac{[1\,2][1\,3]^4\langle 1\,4\rangle^4}{\langle 1\,2\rangle} \frac{t^2+u^2}{6\,s\,t^2\,u^2}\,, \\ A^{(1)}(1^\pm;2^{++},3^{++},4^{++})\Big|_{\kappa^3g} &= 0\,, \\ A^{(1)}(1^+;2^{++},3^{++},4^{--})\Big|_{\kappa^3g} &= 0\,. \end{split}$$

#### Sudoku Game or 6 Pages Calculation





#### 4 The (1-2+3+4++) amplitude

Constructing this amplitude is a slightly harder task, hence as an introduction we will first re-derive the four-point gluon amplitude with a single negative-lackity gluon of [22] and then apply a similar procedure to the more complicated EVM case. The form of the four-gluon integrand is also of use for a double-copy based construction of the EVM amplitudes. Warmup. As for the case of the all-plus amplitude derived in the previous section, we work with two particle cuts. Because only glasses are involved, color ordering lowers us with only two channels to consider, see Figure II for the schanzel we have

 $A_4^{(1)}(1^-, 2^+, 3^+, 4^+)$  =  $A(3^+, 4^+, l_{1,i}, l_{3,j}) A(1^-, 2^+, -l_{3,i}, -l_{1,j})$ 

\*The integral functions approxing in (111) and is the rest of the paper are defined in Appendix (2) following conventions of (22) up to a minus sign for the I<sub>j</sub> integrals.

 $D_{\delta} = l_1^2 - \mu^2 = (l - p_1 - p_2)^2 - \mu^2$ ,  $D_{\delta} = l_1^2 - \mu^2 = (l + p_4)^2 - \mu^2$ . (4.10) Note that we have set  $l_1 = -l$ . The l-channel cut of the (1234)-configuration on the other side takes the form

$$A_{(120)}[i] = \frac{1}{2} \int_{-1}^{2} \frac{1}{(2\pi)^{2}(1-2\pi)^{2}} = A(\delta^{+}, 1^{-}, \phi_{+}, \phi_{+}) A(2^{+}, 3^{+}, \phi_{-1}, \phi_{-1})$$
  
 $= -2\mu^{2} e^{\frac{1}{2}} \frac{[2][24]}{(22)[4]} \frac{(10)^{2}}{(24)^{2}} \frac{[(2\pi)^{2}(D_{c})]}{(2D_{c}, D_{c})} \frac{[(2\pi)^{2}(D_{c})]}{(2D_{c}, D_{c})}.$  (4.11)

$$= -2e^{\alpha r} \frac{(21/36)}{(12/36)} \frac{(41)^2}{(41)^2} \frac{1}{(43)} \frac{1}{(43)^2} \frac{1}{(4$$

Using these relations, we observe the identity  

$$\begin{array}{c} \langle 30|4 (10|2)^2 = \frac{12[(34)^2}{s_{21}(22)}(10|4)^4 + \frac{(24)(22)}{s_{21}}s_{21}(10|4)^2 \\ + \frac{(12)(23)}{s_{21}}s_{21}(10|4)(10|4) + (25)(10|2)^4, \end{array}$$
(4.13)

$$+ \frac{1}{s_{B}/22} \gamma_{B} \left( 2||\hat{\theta}||_{2}^{2} \left( 1||\hat{\theta}||_{2}^{2} + ||\hat{\theta}||_{2}^{2} \right) \right), \quad (4.13)$$
Inserting this into the s-cut amplitude  $\Omega_{a}^{2}$ , and rewriting the Mandebtam invariants  $s_{1} = 2(l_{B})$   
as
$$s_{11} = D_{1} - D_{2} = D_{2}|_{\text{invarian}}, \quad s_{2} = D_{2} - D_{1} = -D_{1}|_{\text{invarian}}, \quad (4.14)$$

back as to an experience for the 
$$A_{(10)} = \frac{1}{100} \frac{1}{100}$$

This expression may be straightforwardly reduced to scalar integrals using e.g. FernCalc. As a An alternative representation for A<sub>1720</sub> is obtained if one rewrites the t-cut expression (1.11) in terms of the s-cut one plus  $D_0$  terms, arriving at  $A'_{(3000)} = 2\epsilon^2 \frac{[12][34]}{[12](34)} \int \frac{d^4l}{(2\pi)^4} \frac{d^{-2\epsilon}\mu}{(2\pi)^{-3\epsilon}} \left\{ \frac{(3|l|4)(1|l|2)^2}{[12]^2(34)} + \frac{[24]}{[14][12]\epsilon_{22}} D_0(1|l|4)^2 \right\}$ 

+  $\frac{\langle 13 \rangle [23] \langle 3|l|4 \rangle}{[12] \langle 41|(34)^2 s_{11}} D_6 \left( \langle 34 \rangle \langle 1|l|4 \rangle + \langle 23 \rangle \langle 1|l|2 \rangle \right) \frac{\mu^2}{D_0 D_1 D_2 D_1}$ .

which upon Passarina-Veliman reduction indeed matches 
$$A_{(120)}$$
 of  $(128)$ . The result after reduction reads:  

$$A_{(120)} = \frac{2i}{(4\pi)^{2+\epsilon}} \frac{[2i([2i]]}{(2i([2i]))} \frac{1}{[2i([2i])]} \left[ -\frac{3}{2} v H_1 | e^i, s, \epsilon] - \frac{i^2 \ell^2}{2a} H_2 (p^2, s, \epsilon] \right]$$

$$-\frac{s^2(s+3u)}{(2}I_0[a^4;t] - \frac{s^2(s^2+3ut+3u^2)}{tu}I_0[\mu^2;t] + \frac{t(u-s)}{s}I_0[a^5;s] + \frac{s^2t}{u}I_0[\mu^2;s] + \frac{s^2(2-u) + u^2}{2t}I_1[\mu^2;s] - \frac{s(2s-u)(s+3u)}{2t^2}I_1[\mu^2;t] \right]. \quad (4.17)$$

Diagram (1243). For the (1243)-contribution we have a 
$$\alpha$$
-channel and a  $\alpha$ -channel cut, which read

$$A_{\mathrm{CHO}/s} = \frac{1}{2} \underbrace{\sum_{i} \sum_{k} \frac{2}{1 - (\delta_{i,k} - \delta_{i,k})} A(2^{k}, 4^{k+1}, \phi_{i,k}, \hat{\phi}_{i,k})}_{k}$$

$$= 2\mu^2 \epsilon^2 \frac{(12)[34]}{(12)(34)} \frac{(24)[4(1)]^3}{(24)[43]^2} \frac{(128)[D_D]}{D_1 D_2} \frac{(128)[D_D]}{D_2 D_3},$$

$$A_{\Omega\Omega\Omega}|_{b} = \sum_{j}^{5} \sum_{(i,j) \in \mathcal{A}} \sum_{(j,j) \in \mathcal{A}} e^{-i\phi} = A(1^{-}, 2^{+}, \phi_{a_{1}}, \bar{\phi}_{1}) A(4^{+}, 3^{+}, \phi_{a_{2}}, \bar{\phi}_{1a_{2}})$$

$$= -2p^{3/4} \frac{|D|||3|4}{|D|||2|+|D||} \frac{|D|||2|}{|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||2|+|D|||$$

$$=-2\mu^{j}\epsilon^{2}\frac{([2i]pq]}{(2i-2pq)}\frac{(3iq(q)(11-pq))\epsilon^{2}}{(3iq(q)(2i))}\frac{(2i\pi pq(2i))}{(3iq(q))^{2}},$$
where we have introduced the loop parametrization  $I:=-I_{1}$  along with 
$$D_{0}=I_{1}^{2}-\mu^{2}=I^{2}-\mu^{2}, \qquad D_{1}=I_{1}^{2}-\mu^{2}=(I-pq)^{2}-\mu^{2},$$

$$D_{1}=I_{1}^{2}-\mu^{2}=(I-pq)^{2}-\mu^{2},$$

$$D_0 = l_0^* - \mu^* = l^* - \mu^*$$
,  $D_1 = l_1^* - \mu^* = (l - p_1)^* - \mu^*$ ,  $D_2 = l_2^* - \mu^* = (l - p_1)^* - \mu^2$ ,  $D_3 = l_4^* - \mu^* = (l + p_2)^* - \mu^3$ .

The s-cut expression may now be lifted off the cut by using the identities

The secut expression may now be lifted off the cut by using the identities 
$$|31||30||4| = |12||23||4| + |14||s_{14}, |42||(1|\ell - p_{2}|2) = |34||4||13| + |14||2(\ell - p_{2}) \cdot p_{1}.$$
 (4.21)

Figure 3: The s- and t-channel cuts of the  $A^{(1)}(1^-,2^+,3^+,4^+)$  amplitude in pure Yang-

 $A_4^{(0)}(1^-,2^+,3^+,4^+)\Big| = A(4^+,1^-,l_{2,0},l_{4,0})\,A(2^+,3^+,-l_{4,0},-l_{2,0})$  $= -\frac{(1|l_4|4|^2}{(41)(14)(l_1^2 - \mu^2)} \mu^2 \frac{[23]}{(23)(l_1^2 - \mu^2)}.$ 

The stratery to find the integrand is now to rewrite the t-channel expression in such a way to

 $\langle 1|l_1|4 \rangle = \frac{1}{1241} \left[ \langle 13 \rangle s_{0,1} + \langle 1|l_1|2 \rangle \langle 23 \rangle \right]$ where  $s_{0,1} = \langle 1|l_1|1 \rangle = 2\,l_1 \cdot p_1$ , which in turn may be written as  $s_{21} = (l_1^2 - \mu^2) - (l_2^2 - \mu^2) \doteq (l_1^2 - \mu^2) \Big|_{\rm sn \; total} \, .$ 

This last expression holds on the t-channel cut. Inserting the expression  $\{ \{ \} \}$  for  $\{ [1]i_1 ]i \}$  into the t-channel cut simplifiede,  $A_0^i$  if  $\{ \{ \} \}$  then yields an expression which may straightforwardly be lifted off the cut. Thus we get an integrand  $\{ \}$ 

 $A^{(2)}(1^-,2^+,3^+,4^+) = -\int \frac{d^4l}{(2\pi)^4} \frac{d^{-2r}\mu}{(2\pi)^{-2r}} \left( -\frac{\mu^2}{(34)^2} \right) \left[ (1|l|2)^2 \right.$  $+2\frac{(13)}{(99)}D_0(1|\tilde{\eta}|2) + \frac{(13)^2}{(23)^2}D_0s_3\left[\frac{1}{D_0D_1D_2D_3}\right]$ 

 $D_0 = l^2 - \mu^2$ ,  $D_1 = (l - p_1)^2 - \mu^2$ ,  $D_2 = (l - p_1 - p_2)^2 - \mu^2$ ,  $D_4 = (l + p_4)^2 - \mu^2$ . (4.6)  $^{9}\mathrm{Again},$  the minus sign in front of the following expression arises from two cut propagators.

could have also replaced  $u_{i,j}$  by  $D_0$  as the resulting expression would agree with  $\{ \mathcal{Q} \}$  and  $\{ \mathcal{Q} \}$  on the respective cuts. However, only the choice quoted above does reproduce the result in the liberature. The final step is to now reduce the tensor integrals appearing in  $\{ \mathcal{Q} \}$ , which we do again using the flathesmatical postage PoysCale  $\{ \mathcal{Q} \}$ . Doing this we find

 $A^{(1)}(1^-, 2^+, 3^+, 4^+) = \frac{2i}{(4\pi)^{3/-4}} \frac{(1|4|2)^3}{(4\pi)^{3/-4}} \frac{s}{t_0} \left[I_4[a^4; s, t] + \frac{st}{2u}I_4[a^2; s, t] + \frac{s(u - \ell)}{t_0}I_4[a^2; t]\right]$  (4.7)  $+\frac{t(s-u)}{su}I_2[\mu^2; s] + \frac{u-s}{t^2}I_2[\mu^2; t] + \frac{u-t}{s^2}I_2[\mu^2; s]$ 

This result agrees with the result in the literature 22. Single graviton amplitude. After this warmen let us now consider the EYM amplitude for a Single graviton amplitude. After the varanty for us now consider the IVM angitude for a single graviton and three phones with or a significantly confirmed to the single graviton and three phones with on a significant phone in a single graviton and the single graviton and the single graviton of th

 $A^{(1)}(1^-, 2^+, 3^+; 4^{++}) = A_{(1201)} + A_{(1201)} + A_{(1201)}$ Diagram (1234). Here we encounter an s-channel and a t-channel cut. For the s-channel of the (1234)-confirmation we find

 $A_{(1204)}|_{*} = \frac{2}{2} \left( -A(1^{-}, 2^{+}, \phi_{i_1}, \tilde{\phi}_{i_1}) A(3^{+}, 4^{++}, \phi_{-i_1}, \tilde{\phi}_{-i_2}) \right)$ (4.9)  $=2\mu^2\ell^2\left[\frac{|12||34|}{(12)(34)}\frac{(3l_1|4](1|l_1|2)^2}{[12]^2(34)}\frac{[(2\pi)\delta(D_2)]\left[(2\pi)\delta(D_2)\right]}{D\cdot D\cdot}\right]$ 

where for the diagram (1234) we use the following loop momentum assignments:  $D_0 = l_1^2 - \mu^2 =: l^2 - \mu^2$ ,

"It would be valuable to understand this seeming ambiguity better. Such as ambiguity does not appear in the procedure of merging cuts employed in later sections, which we have used to confirm all calculations of this paper. In the latter procedure, canading integrals are smirted, which may clearure a chadde-supy interpretaof the results.

If field we taken  $D_n^0$  instead of  $D_{n,k_1}$  in the last term of  $\{0\}$  we weak) on tap find a term proportional to  $\{a(t)a(t)|D_n^1a(t)\}$  in the above, in disagreement with [2]. On the s-cut (where  $D_1=D_2=0$ ) we may replace  $s_1=D_2-D_3=-D_6$  as well as  $2(l-p_3)\cdot p_1=D_2-D_1\equiv D_2$ . Using this we arrive at the integrand for the (1243)-type contribution,  $A_{(1240)} = 2i^2 \frac{[12][34]}{(12)(34)} \int \frac{d^4l}{(2\pi)^4} \frac{d^{-2i}\mu}{(2\pi)^{-2i}} \left\{ \frac{(2)[4]}{(24)(31)^2} - \frac{[14]}{(12)(24)(31)^2} D_6(1)(3)^2 \right\}$  $-\frac{(14)}{(12)^2(24)^2(34)}D_4(3|l|4)\left((34)(1|l|3]+(42)(1|l-p_3|2]\right)\left.\right\}\frac{\mu^2}{D_0D_1D_2D_4}.$ 

Again we have an expression in terms of box and triangle tensor integrals amenable to standard

 $D_0 = (l+p_2)^2 - \mu^2$ ,  $D_1 = l^2 - \mu^2$ ,  $D_0 = (l-p_1)^2 - \mu^2$ ,  $D_0 = (l+p_2+p_3)^2 - \mu^2$ . (4.24) Passarino-Veltman reducing (122) or (123), one arrives at  $A_{(1243)} = -\frac{2i}{(4\pi)^{2-i}} \frac{[24][34]}{(24)(34)} \frac{1}{[12](23)[31]} \Big[ -\frac{us}{2} I_4[\mu^4; u, s] + \frac{s^2}{u} I_5[\mu^4; u]$  $+\frac{u^2}{s}I_3[\mu^4; s] - \frac{tu}{2s}I_2[\mu^2; s] - \frac{st}{2u}I_2[\mu^2; u]$ . (4.25)

Diagram (1423). The remaining (1423)-contribution carries a n-channel and a t-channel cut.  $A_{\mathrm{LMS}(b)} = \prod_{i=1}^{n} \sum_{j=1}^{n} \prod_{i=1}^{n} A(3^{+},1^{+},\phi_{1i},\bar{\phi}_{0j}) A(4^{++},2^{+},\phi_{-1i},\bar{\phi}_{-1i})$ 

 $A_{(1420)}|_{\alpha} = \sum_{i=1}^{n} \cdots = A(2^{+}, 3^{+}, \phi_{i_{1}}, \dot{\phi}_{i_{2}}) A(1^{-}, 4^{++}, \phi_{-i_{1}}, \dot{\phi}_{-i_{2}})$ 

 $=-2i^{2}\mu^{2}\frac{(2|t|4](1|t-p_{2}|3]^{2}}{(24)^{3}}\frac{\left[(2\pi)\delta(D_{1})\right]\left[(2\pi)\delta(D_{2})\right]}{D_{2}D_{2}}$ where we identified the loop momentum as  $l := -l_2$  and used the inverse propagators suitable

 $D_2 = l_1^2 - \mu^2 = (l - p_2 - p_3)^2 - \mu^2$ ,  $D_3 = l_4^2 - \mu^2 = (l + p_4)^2 - \mu^2$ . (4.28) However, by inspection we see that  $A_{(1121)}$  may be obtained from the (1234)-configuration by simply swapping 2 ++ 3 (or  $s \leftrightarrow w$ ). Hence we conclude that

> $= -\frac{2i}{(4\pi)^{2-\epsilon}} \frac{[24][34]}{(24)^{(24)}} \frac{1}{(12)(23)[31]} \left[ \frac{3}{2} \pi t I_{\delta}[\mu^{\delta}; u, t] + \frac{u^{2}t^{2}}{2\epsilon} I_{\delta}[\mu^{2}; u, t] \right]$  $+\frac{u^{2}(u+3s)}{2}I_{3}[\mu^{4};t]+\frac{u^{2}(u^{2}+3su+3s^{2})}{2}I_{3}[\mu^{3};t]+\frac{t(u-s)}{2}I_{3}[\mu^{4};u]-\frac{u^{2}t}{2}I_{3}[\mu^{3};u]$

 $-\frac{u^{2}(2\ell - s) + s^{3}}{2ss}I_{2}[\mu^{2}; u] + \frac{u(2u^{2} + 5cs - 3s^{2})}{5cs}I_{2}[\mu^{2}; t]$ 

Final result. Adding all the three contributions  $A_{(1210)} + A_{(1410)} + A_{(1421)}$  leads to the final D-dimensional result:  $A^{(1)}(1^-,2^+,3^+;4^{++}) = -\frac{2i}{(4\pi)^{2+i}} \frac{[24][34]}{(24)(34)} \frac{1}{[12](23)[31]} \Big\{ -\frac{3}{2}stI_4[\mu^4;s,t] - \frac{s^2\delta^2}{2a}I_4[\mu^2;s,t] \Big\}$ 

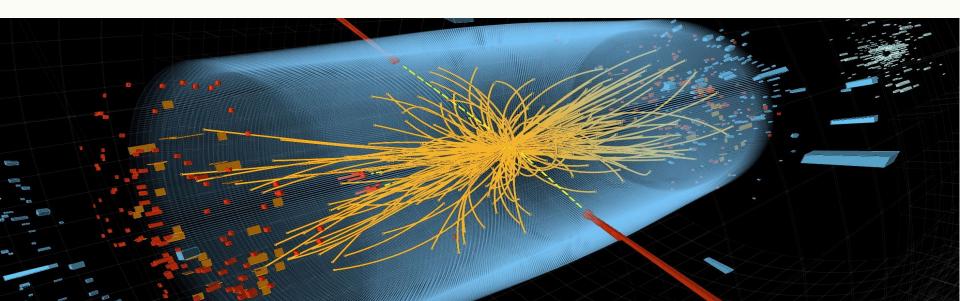
 $+\frac{1}{2}suI_d[\mu^4;u,s] - \frac{3}{2}tuI_4[\mu^4;u,t] - \frac{t^4u^2}{2s}I_4[\mu^2;u,t] - \frac{(-t^2u - 2tu^2 + u^2)}{su}I_2[\mu^4;s]$  $\begin{array}{lll} & 2ss \\ & -\frac{(t^2+3t^2u+3t^2u^2+6u^2)}{3t^2}I_2[\mu^2;v] - \frac{(t^2u+6u^2)}{3t}I_2[\mu^2;t] \\ & -(-t^2-2t^2u-t^2u^2+2tu^2+u^4)}I_2[\mu^2;t] - \frac{(2t^2+6tu+u^2)}{3t}I_2[u^2;u] + \frac{tu^2}{2}I_2[\mu^2;u] \end{array}$  $\frac{t(t+2u)}{I_2[\mu^2;s]}\frac{su}{I_2[\mu^2;s]} + \frac{(t^2+2tu+2\alpha^2)}{I_2[\mu^2;t]}\frac{I_2[\mu^2;t]}{I_2[\mu^2;u]} + \frac{t(t+2u)}{I_2[\mu^2;u]}\frac{u}{I_2[\mu^2;u]}$ 

Toking the four-dimensional limit yields the compact final expression  $A^{(4)}(1^{\circ}, 2^{\circ}, 3^{\circ}; 4^{++}) = \frac{i}{(4\pi)^2} \frac{[24][34]}{(24)(34)} \frac{1}{(23)(24)(34)} \frac{1}{6} (s^2 + u^2)$ .

#### 5 The (1+2+3+4--) amplitude

We now consider the rational one-loop amplitude with a single negative-helicity graviton and three positive-helicity gluons  $A^{(2)}[1^{+},2^{+},3^{+};4^{-})$ . For amplitudes containing progressively more

# Back to EFTs on Collider



#### New EFT Operators in High-J Channel

Gluon loops need not satisfy the usual angular momentum selection rules

$$J_{\min}^{\text{particle}} \ge |h_1 - h_3| \sim \Delta h$$

lacksquare Higher point EFT Operator:  $J_{\min}^{\text{particle}} =$ 

$$J_{\min}^{\text{particle}} = |\vec{s}_1 + \vec{s}_2 + \vec{s}_3| > 2$$

