Bootstrap the EFT from Glueball Exchanging

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EFT Operators Basis

Standard Model EFT:

$$S_{\text{UV}}[\Phi] = \int d^4x\, \mathcal{L}_{\text{UV}}\big(\Phi(x), \partial_\mu\Phi(x), \dots\big)\,.$$

$$\mathcal{L} = \Lambda^4 + \Lambda^2\mathcal{O}^{(2)} + m\mathcal{O}^{(3)} + \mathcal{O}^{(4)} + \frac{1}{\Lambda}\mathcal{O}^{(5)} + \frac{1}{\Lambda^2}\mathcal{O}^{(6)} + \frac{1}{\Lambda^3}\mathcal{O}^{(7)} + \frac{1}{\Lambda^4}\mathcal{O}^{(8)} + \dots$$

1

$$-\frac12\partial_\nu g^a_\mu\partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac14 g_s^2 f^{abc}f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu + \frac12 i g_s^2(\bar q^\sigma_i\gamma^\mu q^\sigma_j) g^a_\mu + \bar G^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar G^a G^b g^c_\mu$$

2

$$- \partial_\nu W^+_\mu \partial_\nu W^-_\mu - M^2 W^+_\mu W^-_\mu - \frac12\partial_\nu Z^0_\mu\partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac12\partial_\mu A_\nu\partial_\mu A_\nu - \frac12\partial_\mu H\partial_\mu H - \frac12 m_h^2 H^2 - \partial_\mu\phi^+\partial_\mu\phi^- - M^2\phi^+\phi^- - \frac12\partial_\mu\phi^0\partial_\mu\phi^0 - \frac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h\left[\frac{2M^2}{g^2} + \frac{2M}{g}H + \frac12(H^2+\phi^0\phi^0+2\phi^+\phi^-)\right] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu\partial_\nu W^-_\mu - W^-_\mu\partial_\nu W^+_\mu) + A_\mu(W^+_\nu\partial_\nu W^-_\mu - W^-_\nu\partial_\nu W^+_\mu)] - \frac12 g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \frac12 g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] - \frac18 g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2] - gMW^+_\mu W^-_\mu H - \frac12 g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac12 ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac12 g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)] + \frac12 g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_wMA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_wA_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac14 g^2W^+_\mu W^-_\mu[H^2+(\phi^0)^2+2\phi^+\phi^-] - \frac14 g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2+(\phi^0)^2+2(2s_w^2-1)^2\phi^+\phi^-] - \frac12 g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \frac12 ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac12 g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) + \frac12 ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2-1)Z^0_\mu A_\mu\phi^+\phi^- - g^1 s_w^2 A_\mu A_\mu\phi^+\phi^-$$

3

$$- \bar e^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar\nu^\lambda\gamma\partial\nu^\lambda - \bar u^\lambda_j(\gamma\partial+m_u^\lambda)u^\lambda_j - \bar d^\lambda_j(\gamma\partial+m_d^\lambda)d^\lambda_j + igs_wA_\mu\left[-(\bar e^\lambda\gamma^\mu e^\lambda) + \frac23(\bar u^\lambda_j\gamma^\mu u^\lambda_j) - \frac13(\bar d^\lambda_j\gamma^\mu d^\lambda_j)\right] + \frac{ig}{4c_w}Z^0_\mu\left[(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar e^\lambda\gamma^\mu(4s_w^2-1-\gamma^5)e^\lambda) + (\bar u^\lambda_j\gamma^\mu(\tfrac43 s_w^2 - 1 - \gamma^5)u^\lambda_j) + (\bar d^\lambda_j\gamma^\mu(1-\tfrac83 s_w^2-\gamma^5)d^\lambda_j)\right] + \frac{ig}{2\sqrt2}W^+_\mu[(\bar\nu^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) + (\bar u^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \frac{ig}{2\sqrt2}W^-_\mu[(\bar e^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar d^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1+\gamma^5)u^\lambda_j)]$$

4

$$+ \frac{ig}{2\sqrt2}\frac{m_e^\lambda}{M}[-\phi^+(\bar\nu^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar e^\lambda(1+\gamma^5)\nu^\lambda)] - \frac g2\frac{m_e^\lambda}{M}[H(\bar e^\lambda e^\lambda) + i\phi^0(\bar e^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt2}\phi^+[-m_d^\kappa(\bar u^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m_u^\lambda(\bar u^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j] + \frac{ig}{2M\sqrt2}\phi^-[m_d^\lambda(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar d^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j] - \frac g2\frac{m_u^\lambda}{M}H(\bar u^\lambda_j u^\lambda_j) - \frac g2\frac{m_d^\lambda}{M}H(\bar d^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar u^\lambda_j\gamma^5 u^\lambda_j) - \frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar d^\lambda_j\gamma^5 d^\lambda_j)$$

5

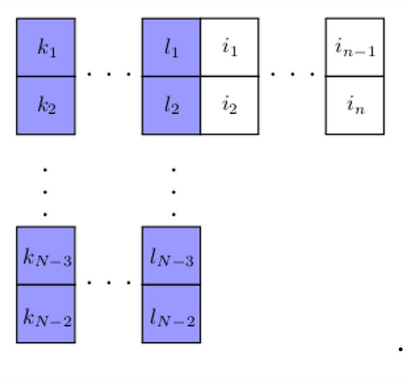
$$+ \bar X^+(\partial^2 - M^2)X^+ + \bar X^-(\partial^2-M^2)X^- + \bar X^0\left(\partial^2 - \frac{M^2}{c_w^2}\right)X^0 + \bar Y\partial^2 Y + igc_wW^+_\mu(\partial_\mu\bar X^0X^- - \partial_\mu\bar X^+X^0) + igs_wW^+_\mu(\partial_\mu\bar YX^- - \partial_\mu\bar X^+Y) + igc_wW^-_\mu(\partial_\mu\bar X^-X^0 - \partial_\mu\bar X^0X^+) + igs_wW^-_\mu(\partial_\mu\bar X^-Y - \partial_\mu\bar YX^+) + igc_wZ^0_\mu(\partial_\mu\bar X^+X^+ - \partial_\mu\bar X^-X^-) + igs_wA_\mu(\partial_\mu\bar X^+X^+ - \partial_\mu\bar X^-X^-) - \frac12 gM\left[\bar X^+X^+H + \bar X^-X^-H + \frac{1}{c_w^2}\bar X^0X^0H\right] + \frac{1-2c_w^2}{2c_w}igM[\bar X^+X^0\phi^+ - \bar X^-X^0\phi^-] + \frac{1}{2c_w}igM[\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-] + igMs_w[\bar X^0X^-\phi^+ - \bar X^0X^+\phi^-] + \frac12 igM[\bar X^+X^+\phi^0 - \bar X^-X^-\phi^0]$$

On-shell EFT Operator Construction

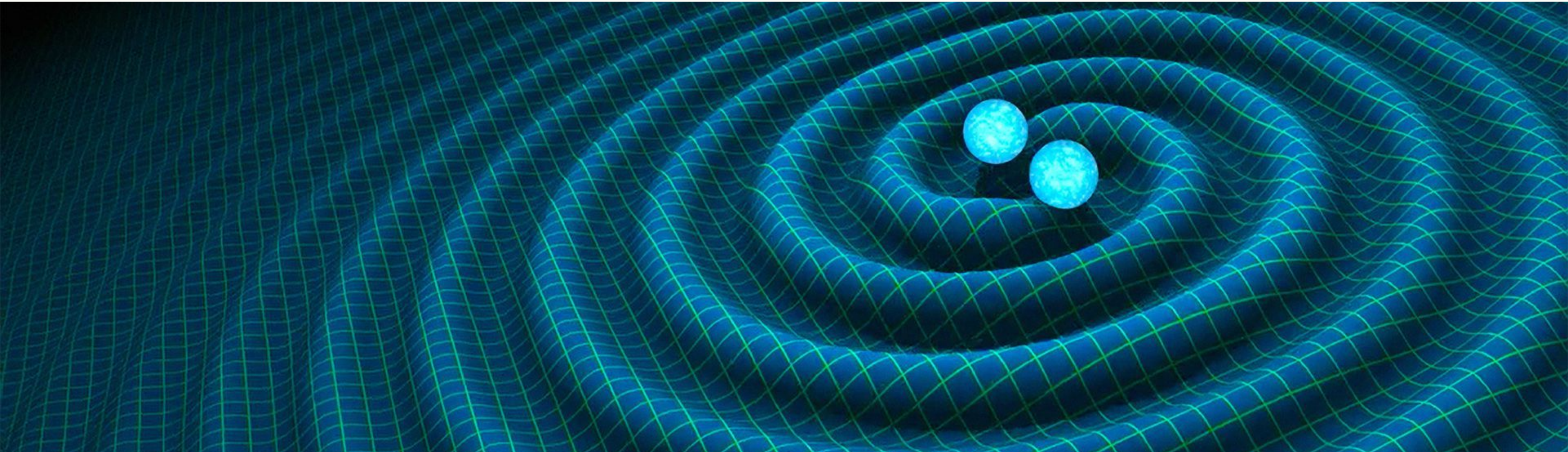
- EFT from UV massive particles:
  - Warsaw basis by EOM and IBP
  - All operator basis for SMEFT (José's talk)
  - SMEFT Geometry (Mia's talk)
  - . . .
- It that all?

$$[j_1 j_2] \ldots [j_{\tilde{n}-1} j_{\tilde{n}}] \langle i_1 i_2 \rangle \ldots \langle i_{n-1} i_n \rangle,$$

$$[j_1 j_2]\epsilon^{j_1 j_2 k_1 .. k_{N-2}} \ldots [j_{\tilde{n}-1} j_{\tilde{n}}]\epsilon^{j_{\tilde{n}-1} j_{\tilde{n}} l_1 .. l_{N-2}} \langle i_1 i_2 \rangle \ldots \langle i_{n-1} i_n \rangle,$$



Extra Graviton EFT Needed



Puzzle from Graviton Scattering

- Dispersion relations link EFT and UV spectrum

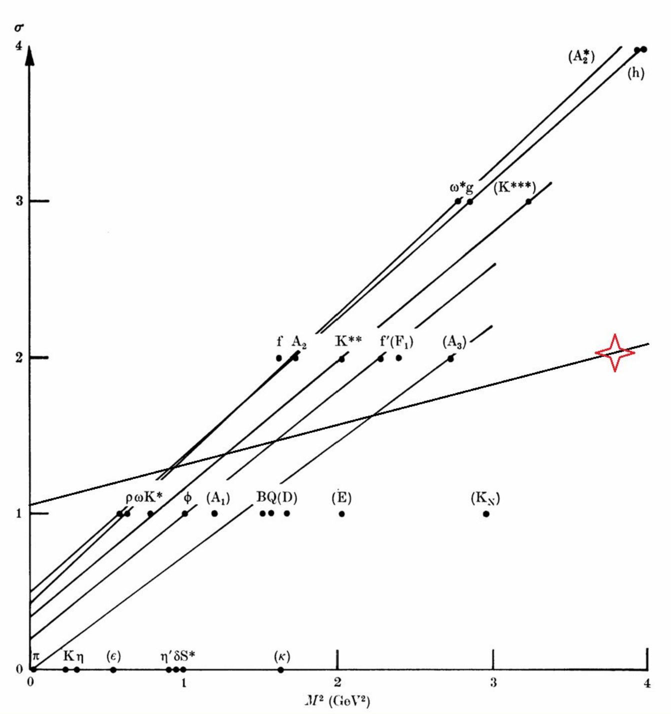
- Froissart Bound

$$M(s,t) \lesssim s\log^2(s)\,,$$

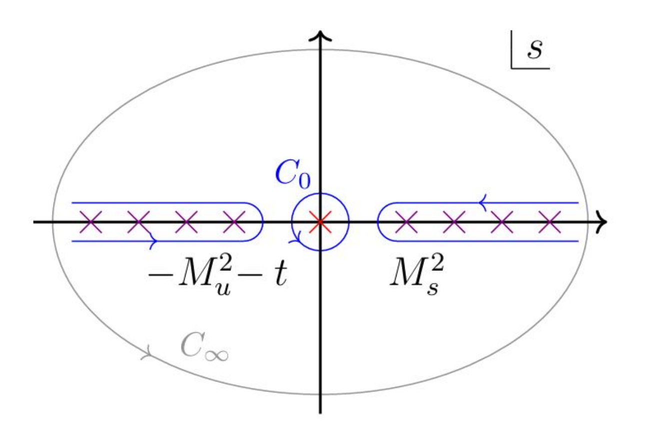
Regge High-Energy Behaviour

$$M(s,t) \sim (s/s_0)^{\alpha(t)}\,,$$

- No one allow Amplitude growth faster than s^2



Dispersion Relation



- Vanishing Boundary term

$$\oint_{|s|\to\infty} \frac{ds}{s^3} \mathcal{M}_{hhhh}(s,t) = 0$$

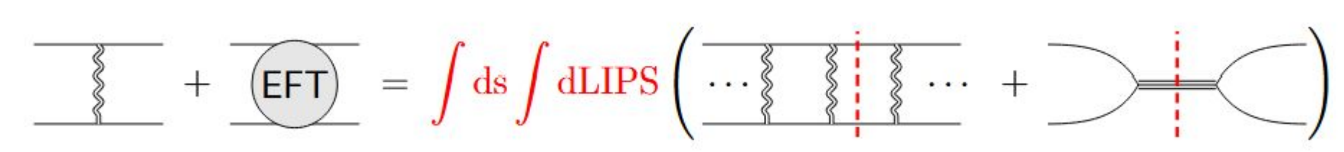
- 4-graviton Low-Energy EFT: $\mathcal{M}_4(1^+, 2^-, 3^-, 4^+) = (\langle 23\rangle[14])^4 f(s,u)\,,$

$$f(s,u) = \left(\frac{\kappa}{2}\right)^2 \frac{1}{stu} + |\beta_{R^3}|^2 \frac{su}{t} - |\beta_\phi|^2 \frac{1}{t} + \sum_{i=0}^{\infty} f_{2i,i} s^i u^i + \sum_{i=1}^{\infty} \sum_{j=0}^{[\frac{i}{2}]} f_{i,j} (s^{i-j} u^j + s^j u^{i-j})\,,$$

- Only graviton-pole survive!

Isolate QFT from Gravity

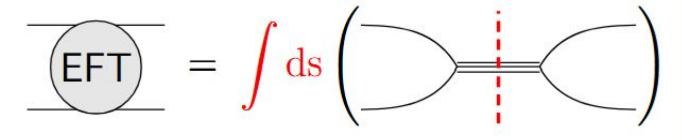
- Graviton-pole cancelled by its own Ladder diagrams, even when mix with QFT



- Change the spatial size B where scattering occur, we can isolate QFT from Gravity

$$G\Lambda_{\mathrm{QCD}}^2 \log(B/b) \ll 1\,,$$

- i.e. EFT = resonances

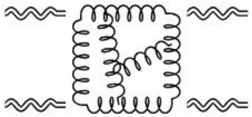


Elastic Scattering → Positivity

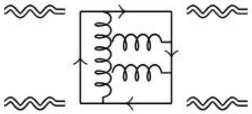
Not all EFT from Massive UV particle

- No matching term from massive-particle induced EFT
- Another mechanism to obtain mass: Glueball mass.
  Glueball (Pomeron) and Meson (Reggeon),

$$\mathcal{M}_{hhhh}^{\mathrm{QCD-tree}} =$$



$$+$$

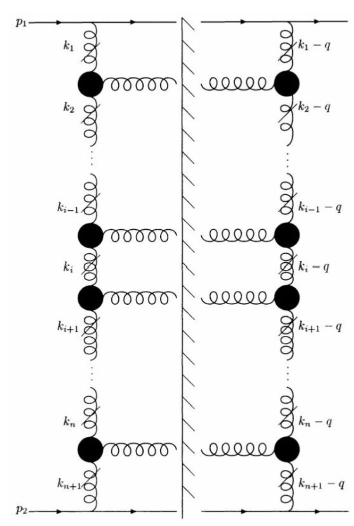


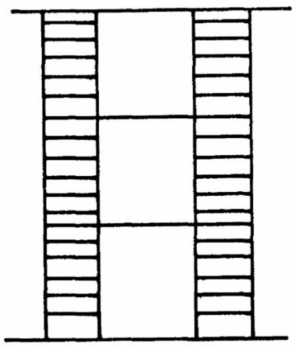
$$+ \cdots ,$$

- In Regge limit $-t \ll \Lambda_{\mathrm{QCD}}^2 \ll s,$

$$\mathcal{M}_{1234}^{\text{QCD-tree}}(s \gg t) \sim \beta(t, \varepsilon_1, \varepsilon_3) \cdot \frac{1 + \eta\,(-1)^{\alpha(t)}}{\sin(\pi\alpha(t))} \cdot \left(\frac{s}{s_0}\right)^{\alpha(t)} \cdot \beta(t, \varepsilon_2, \varepsilon_4)\,.$$

Glueball may contribute to the missed EFT





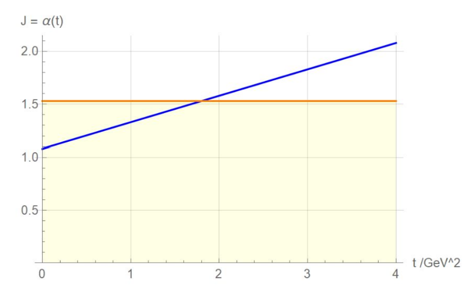
- ❏ Vanished Boundary + Positive s-, u-channel Discontinuity

⇩

- ❏ Something must be in C_0 contour! Could we calculate it?

- ❏ Feynman ladder diagram only give cuts, only running of α_s let cuts become poles.

  Too complicate to get low-energy EFT!



Glueball Exchanging Amplitude

- ’t Hooft double line view:
  Glueball —— Closed QCD string;
  Meson —— Open QCD string.

- Uniquely identify the UV Singularity
  1. Quantum Number;
  2. Even/Odd Spin;
  3. Angular momentum selection rule, J≥Δh.

- String-like behaviour in Lattice QCD

Virasoro-Shapiro amplitude with Pomeron trajectory

$$\alpha_P(t) = 1.08 + 0.25\ \mathrm{GeV}^{-2}\, t\,,$$

not Quantum Gravity trajectory

$$\alpha_{\mathrm{QG}}(t) = 2 + \frac{\alpha'}{2} t.$$

Virasoro-Shapiro amplitude for Glueball

- Role of each part of V-S amplitude:

$$M_0(s,t) = 8\pi G\left(\frac{su}{t} + \frac{tu}{s} + \frac{st}{u}\right) \frac{\Gamma(1-\frac{\alpha'}{4}s)\Gamma(1-\frac{\alpha'}{4}t)\Gamma(1-\frac{\alpha'}{4}u)}{\Gamma(1+\frac{\alpha'}{4}s)\Gamma(1+\frac{\alpha'}{4}t)\Gamma(1+\frac{\alpha'}{4}u)},$$

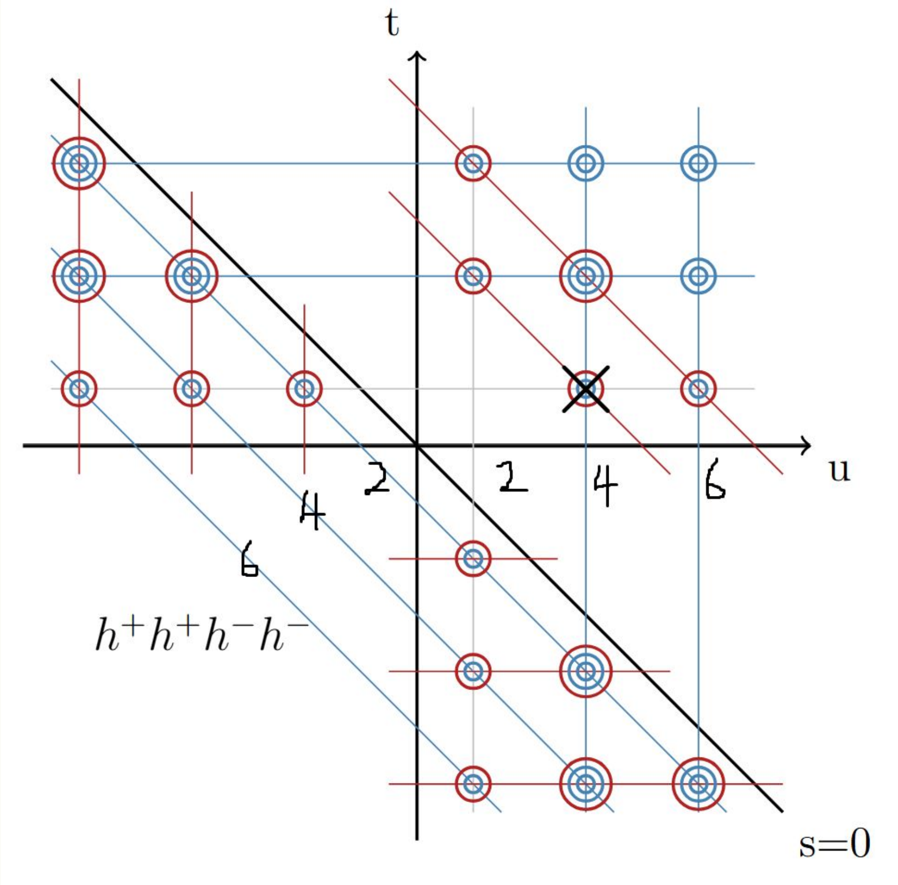
- The prefactor is the low-energy amplitude for $s, t, u \ll 1/\alpha'$.
- The Γ-functions generate all massive poles.
- The Γ-functions cancel all double poles and nonsense poles ($J < \Delta h$).

- Regge trajectory uniquely fix all Γ-functions.

❑ Plot all the poles for ++-- 4-graviton scattering

No double pole,

No nonsense-pole.



V-S amplitude has new EFT!

- After contain all massive poles, we also need to ensure the high-energy Regge behavior of the amplitude

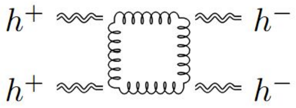
$$\mathcal{M}^{P}_{++--} = \left(\frac{[12]^2\langle 34\rangle^2}{\langle 12\rangle^2[34]^2}G^2 s^2\right)\frac{\Gamma(\frac{2-\alpha(s)}{2})\Gamma(\frac{4-\alpha(t)}{2})\Gamma(\frac{4-\alpha(u)}{2})}{\Gamma(\frac{6-\alpha(t)-\alpha(u)}{2})\Gamma(\frac{4-\alpha(s)-\alpha(u)}{2})\Gamma(\frac{4-\alpha(s)-\alpha(t)}{2})}\,.$$

- Give a New EFT ~ s^2 not in Local EFT Construction!

What’s the new EFT?

- ❏ Remarkably, the low-energy prefactor is exactly the one-gluon-loop amplitude for four gravitons

$$\mathcal{M}^{1-\text{gluon}-\text{loop}}_{++--} =$$



$$= \frac{[12]^2\langle 34\rangle^2}{\langle 12\rangle^2[34]^2} G^2 s^2$$

- ❏ The log brunch cuts (2, 3, … loops) are absorbed by the Glueball’s Regge behavior, generating an tower of massive glueballs.

$$1 + \alpha_s \log(s/s_0) + \alpha_s^2 \log^2(s/s_0) + \cdots = (s/s_0)^{\alpha_s} \sim \frac{\Gamma(s/s_0 + \alpha_s)}{\Gamma(s/s_0)}$$

Coincidence?

	QCD V-S
$h^+h^+h^+h^+$	$\frac{[12]^2[34]^2}{\langle 12\rangle^2\langle 34\rangle^2}(s^2+t^2+u^2)$
$h^+h^+h^+h^-$	$\frac{[12]^2[23]^2\langle 24\rangle^2}{\langle 12\rangle^2\langle 23\rangle^2[24]^2}(s^2+t^2+u^2)$
$h^+h^+h^-h^-$	$\frac{[12]^2\langle 34\rangle^2}{\langle 12\rangle^2[34]^2}(s^2)$
$g^+g^+h^+h^+$	$\frac{[12][34]^2}{\langle 12\rangle\langle 34\rangle^2}(s)$
$g^-g^+h^+h^+$	$\frac{[24][34]^2\langle 14\rangle}{\langle 24\rangle\langle 34\rangle^2[14]}(s)$
$g^+g^+h^+h^-$	$\frac{[12][13]^2\langle 14\rangle^2}{\langle 12\rangle\langle 13\rangle^2[14]^2}(\frac{t^2+u^2}{s})$
$g^-g^-h^+h^+$	$\frac{[12]\langle 34\rangle^2}{\langle 12\rangle[34]^2}(s)$
$g^+g^-h^+h^-$	$\frac{[13]^2\langle 24\rangle\langle 14\rangle}{\langle 13\rangle[24]^2[14]^2}(\frac{t^2+u^2}{s})$
$gggh \cdots$	$\cdots$
$gggg \cdots$	$\cdots$

$$A^{(1)}(1^+,2^+,3^+;4^{++})\Big|_{\kappa g^3} = 0\,,$$

$$A^{(1)}(1^+,2^+,3^+;4^{--})\Big|_{\kappa g^3} = -\frac{i}{(4\pi)^2}\,\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\,(\langle 42\rangle[23]\langle 34\rangle)^3\,\frac{s^2+t^2+u^2}{6\,s^2\,t^2\,u^2},$$

$$A^{(1)}(1^-,2^+,3^+;4^{++})\Big|_{\kappa g^3} = \frac{i}{(4\pi)^2}\,\frac{[24][34]}{\langle 24\rangle\langle 34\rangle}\,\frac{1}{\langle 23\rangle[21][31]}\,\frac{1}{6}\,(s^2+u^2)\,, \quad (1.6)$$

$$A^{(1)}(1^+,2^+;3^{++},4^{++})\Big|_{\kappa^2 g^2} = \frac{i}{(4\pi)^2}\,\frac{[12]}{\langle 12\rangle}\,\frac{[34]^2}{\langle 34\rangle^2}\,\frac{s}{6}\,,$$

[3]In our conventions we have $s=\langle 12\rangle[21]$, $t=\langle 23\rangle[32]$ and $u=\langle 13\rangle[31]$.

4

$$M_4^{(1)}(1^+,2^+,3^+,4^+) = -\frac{i}{(4\pi)^2}\,\frac{1}{60}\left(\frac{[12]\,[34]}{\langle 12\rangle\langle 34\rangle}\right)^2(s^2+st+t^2)\,,$$

$$M_4^{(1)}(1^-,2^+,3^+,4^+) = \frac{i}{(4\pi)^2}\,\frac{1}{180}\left(\frac{\langle 12\rangle[23][24]}{[12]\langle 23\rangle\langle 24\rangle}\right)^2(s^2+st+t^2)\,,$$

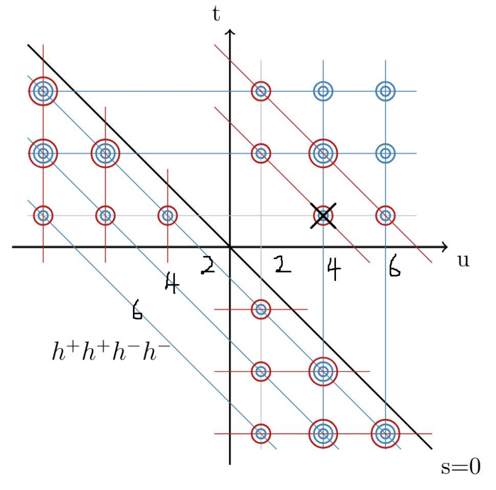
$$A^{(1)}(1^-,2^+;3^{++},4^{++})\Big|_{\kappa^2 g^2} = \frac{i}{(4\pi)^2}\,\frac{[24]^2[34]^2\langle 14\rangle^2}{\langle 34\rangle^2}\,\frac{s}{6\,t\,u}\,,$$

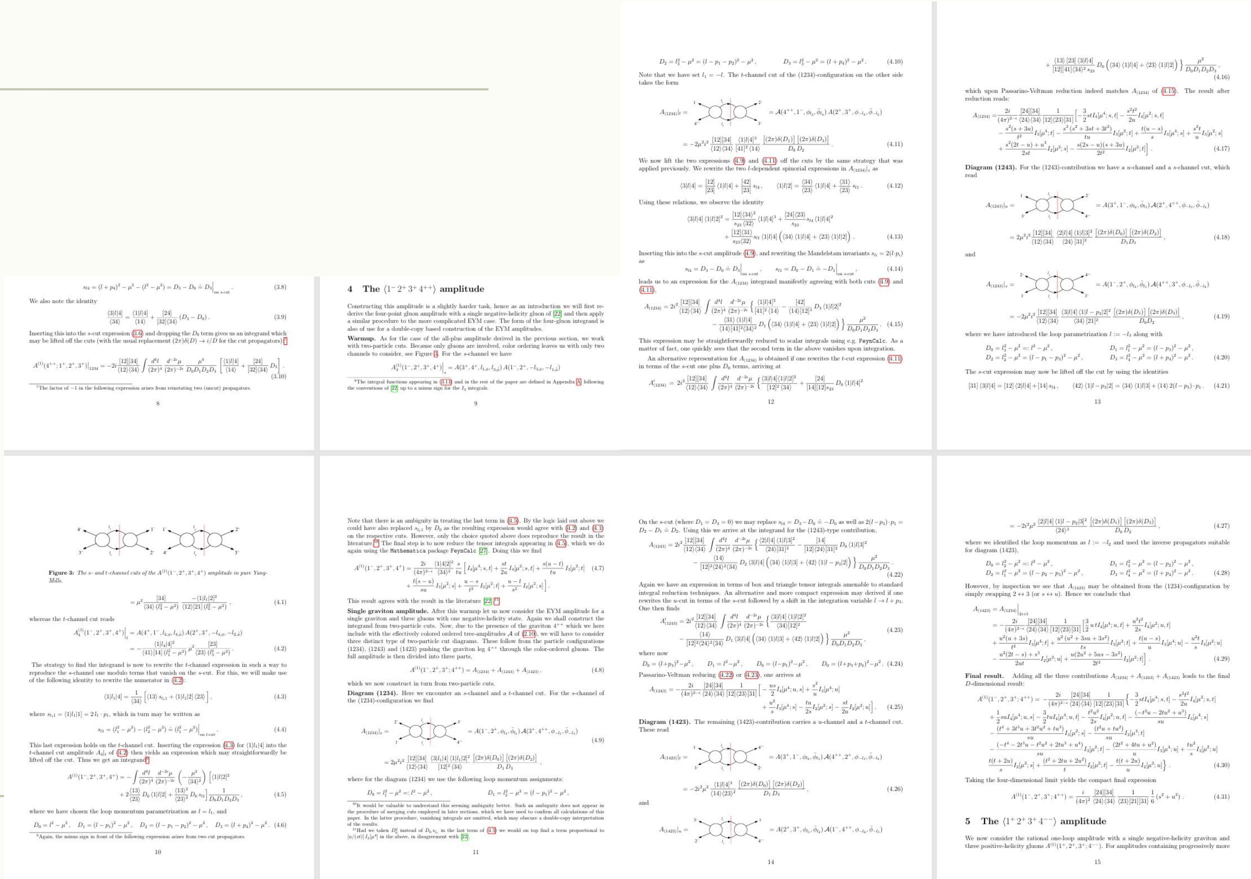
$$A^{(1)}(1^+,2^+;3^{++},4^{--})\Big|_{\kappa^2 g^2} = \frac{i}{(4\pi)^2}\,\frac{[1\,2][1\,3]^4\langle 1\,4\rangle^4}{\langle 1\,2\rangle}\,\frac{t^2+u^2}{6\,s\,t^2\,u^2}\,,$$

$$A^{(1)}(1^\pm;2^{++},3^{++},4^{++})\Big|_{\kappa^3 g} = 0\,,$$

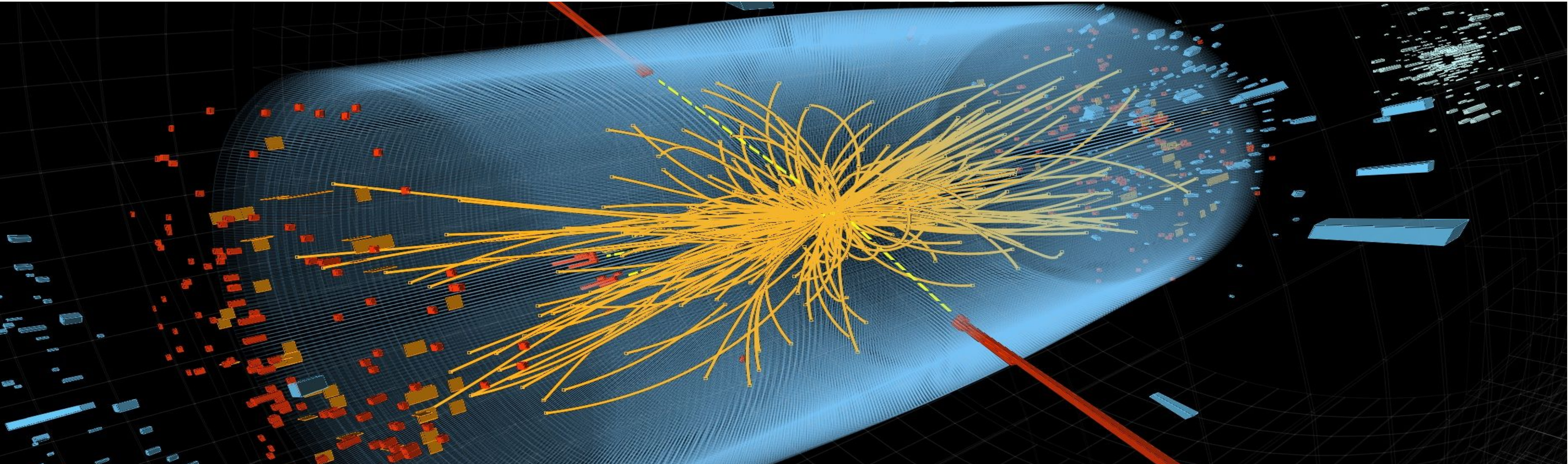
$$A^{(1)}(1^+;2^{++},3^{++},4^{--})\Big|_{\kappa^3 g} = 0\,.$$

Sudoku Game or 6 Pages Calculation





Back to EFTs on Collider



New EFT Operators in High-J Channel

- ❏ Gluon loops need not satisfy the usual angular momentum selection rules

$$J_{\min}^{\text{particle}} \geq |h_1 - h_3| \sim \Delta h$$

- ❏ Higher point EFT Operator: $J_{\min}^{\text{particle}} = |\vec{s}_1 + \vec{s}_2 + \vec{s}_3| > 2$