The Higgs meets the SMEFT at higher orders

Based on JHEP 12 (2025) 220 with R. Gröber, M. K. Mandal

Higgs Hunting, Paris, France

Stefano Di Noi | 16 July 2025

Introduction

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- **Effective Field Theories** (**EFTs**): search for NP with minimal UV assumptions.

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- Parametrise heavy NP effects with a tower of gauge/Lorentz invariant operators:

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_{\mathfrak{D}_i > 4} \frac{\mathcal{C}_i}{\Lambda^{\mathfrak{D}_i - 4}} \mathcal{O}_i, \quad O_i \sim \partial^{n_d^i} \phi^{n_\phi^i} A^{n_A^i} \psi^{n_\psi^i}, \; \mathfrak{D}_i = n_d^i + n_\phi^i + n_A^i + \frac{3}{2} n_\psi^i.$$

- $\phi,\ A,\ \psi$: SM fields, gauge group: $\mathrm{SU}(3)_{\mathrm{C}} \otimes \mathrm{SU}(2)_{\mathrm{W}} \otimes \mathrm{U}(1)_{\mathrm{Y}}$.

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- Dominant effect in collider physics at $\mathfrak{D} = 6$ (**Warsaw basis**, [Grzadkowski,Iskrzynski,Misiak,Rosiek,'10], 2499 operators assuming B, L conservation).

SMEFT: how should we use it?

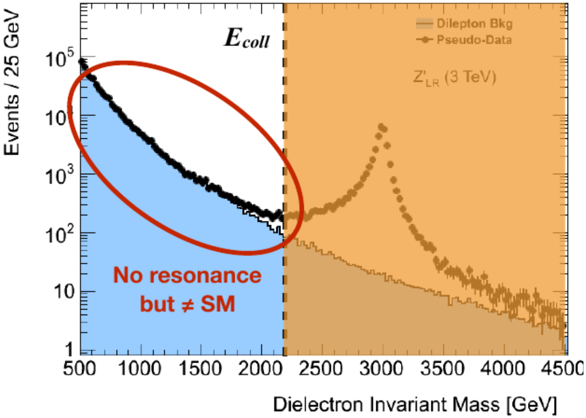

Figure: Courtesy of P. Azzi.

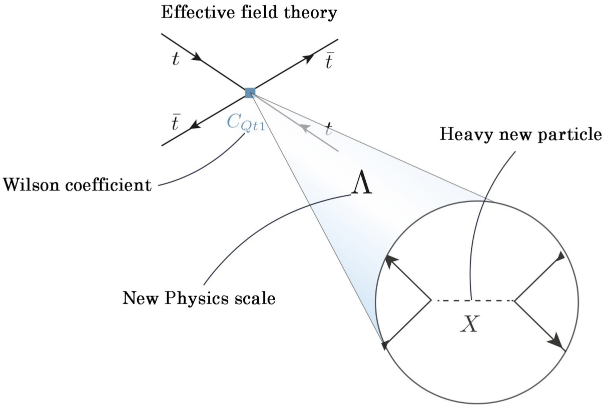

Figure: courtesy of L. Alasfar

Running effects

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- SMEFT dim=6 $\rightarrow$ linear system $\rightarrow$ $\Gamma_{ij}(\mu)$, **Anomalous Dimension Matrix** (**ADM**).
- $\Gamma_{ij}(\mu)$: known @ 1L [(Alonso),Jenkins,Manohar,Trott,'13] and automatized [Fuentes-Martín et al.,'20], [Aebischer et al.,'18], [SDN,Silvestrini,'22].
- Partial results @ 2L [Bern et al.,'19], [Bern et al.,'20], [SDN et al.,'23], [Jenkins et al.,'23], [Born et al.,'24],[Talk by U. Haisch @ EFT WG], [Duhr et al.,'25],[Haisch,'25] **and this talk!**.
- Partial results @ 1L dim8, e.g. [Grojean et al.,'24].

Running effects in the SMEFT

- Several SMEFT studies show the importance of strong running effects: [Grazzini et al.,'18],[Battaglia et al.,'21],[Aoude et al.,'22],[Maltoni et al.,'24],[Heinrich,Lang,'24],[Haisch,'25].

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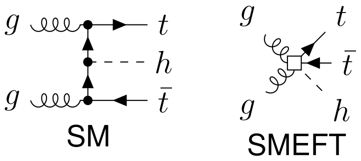
- SM Tree-level, SMEFT tree-level



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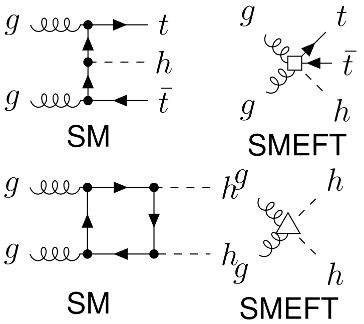
- hh/hj: **SM One-loop**, SMEFT tree-level



Power counting in Higgs production

- Assumption on the UV completion: renormalizable and weakly coupled

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- $\blacktriangle = \mathcal{O}_{tH} \equiv (\bar{Q}_L \tilde{H} t_R)\left(H^\dagger H\right)$: Tree-level generated but enters at one-loop level.

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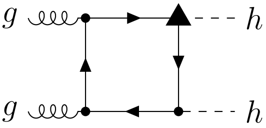

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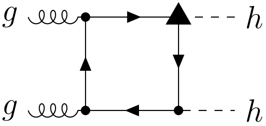

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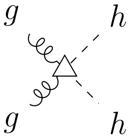

$$\sim 1 \times \frac{\mathcal{C}_{HG}}{\Lambda^2} \sim \frac{1}{16\pi^2} \times \frac{1}{\Lambda^2}$$

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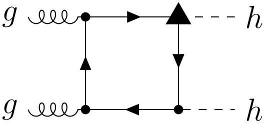

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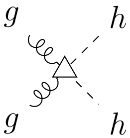

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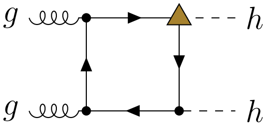
- $\mathcal{O}_{tH}$, $\mathcal{O}_{HG}$ **contribute at the same order to** hh/hj **production!**

Running in Higgs production

- 1L running in $\mathcal{C}_{tH}$:  $\sim \frac{1}{16\pi^2}\frac{1}{16\pi^2}\log\left(\frac{\mu_R^2}{\Lambda^2}\right)\frac{1}{\Lambda^2}$: **2L effect.**

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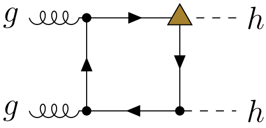
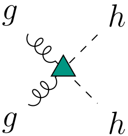
- 1L running in $\mathcal{C}_{tH}$:  $\sim \frac{1}{16\pi^2}\frac{1}{16\pi^2}\log\left(\frac{\mu_R^2}{\Lambda^2}\right)\frac{1}{\Lambda^2}$: **2L effect.**

- We need 2L running operators in $\mathcal{C}_{HG}$:  $\sim \left(\frac{1}{16\pi^2}\right)^2\log\left(\frac{\mu_R^2}{\Lambda^2}\right)\frac{1}{\Lambda^2}$ **from potentially tree-level generated operators.**

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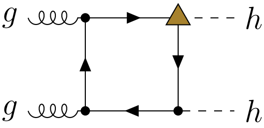
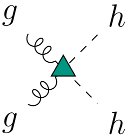
- 1L running in $\mathcal{C}_{tH}$:  $\sim \frac{1}{16\pi^2}\frac{1}{16\pi^2}\log\left(\frac{\mu_R^2}{\Lambda^2}\right)\frac{1}{\Lambda^2}$: **2L effect.**

- We need 2L running operators in $\mathcal{C}_{HG}$:  $\sim \left(\frac{1}{16\pi^2}\right)^2 \log\left(\frac{\mu_R^2}{\Lambda^2}\right)\frac{1}{\Lambda^2}$ **from potentially tree-level generated operators.**

- $\mathcal{C}_{4t} \xrightarrow{2L} \mathcal{C}_{HG}$ computed in [SDN,Gröber,Heinrich,Lang,Vitti,'23], **this work** : $\mathcal{C}_{tH} \xrightarrow{2L} \mathcal{C}_{HG}$.

Two-loop running of Higgs-gluon coupling: method

- Determining the ADM $\equiv$ computing divergent parts: $\delta\mathcal{C}_{HG} \sim \frac{1}{\epsilon}\gamma_{HG,tH}\mathcal{C}_{tH}$.

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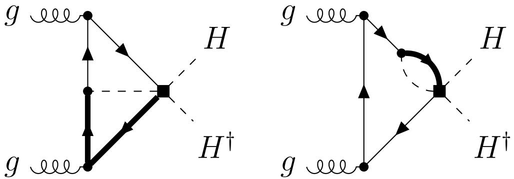
-  = RH field t_R, b_R,  = LH field Q_L.
- Two families, 4 Master Integrals (MIs) each.
- The result does not depend on the γ_5 scheme.



(a) Family 1 (b) Family 2

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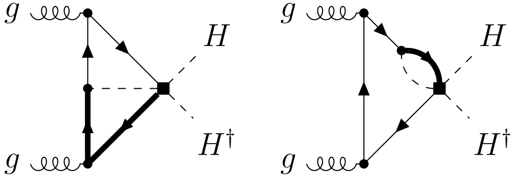
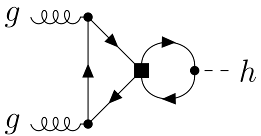


(a) Family 1 (b) Family 2

- Differential eqs for the MIs via the Magnus method.
- Solving the system, the MIs can be expressed in terms of Harmonic PolyLogarithms.
- Result cross-checked numerically with `AMFlow` [Liu, Ma, '22].

Two-loop running of Higgs-gluon coupling: result



- RGE@2L for $\mathcal{C}_{HG}$ combining the result in this work with the result in [SDN,Gröber,Heinrich,Lang,Vitti,'23] we have:

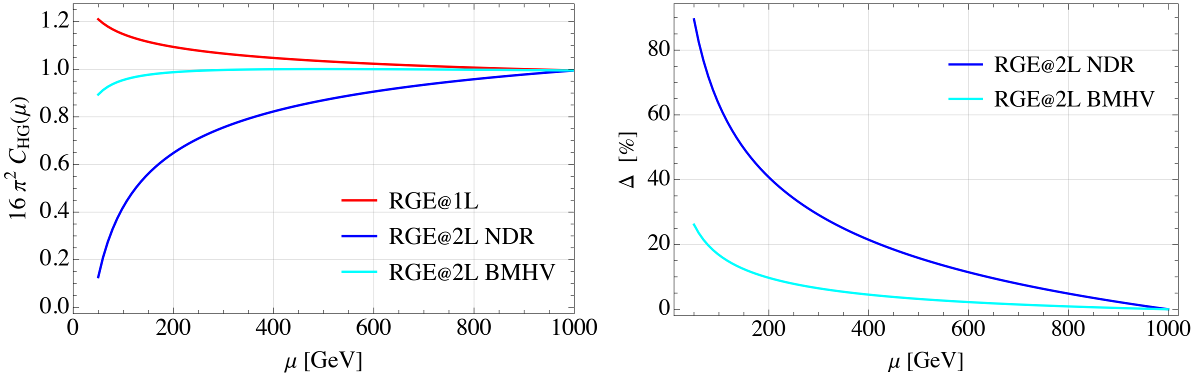
$$\mu\frac{d\mathcal{C}_{HG}}{d\mu} \supset 3\left(\frac{1}{16\pi^2}\right)^2 g_s^2\left[\mathcal{C}_{tH}Y_t + \mathcal{C}_{tH}^*Y_t^* + \mathcal{C}_{bH}Y_b + \mathcal{C}_{bH}^*Y_b^*\right] -4\left(\frac{1}{16\pi^2}\right)^2 g_s^2 Y_t Y_t^* \delta_{\rm X}\left(\mathcal{C}_{Qt}^{(1)} - \frac{1}{6}\mathcal{C}_{Qt}^{(8)}\right),$$

$$\delta_{\rm X} = \begin{cases} 1 & (\text{NDR}), \\ 0 & (\text{BMHV}). \end{cases}$$

- $\mathcal{O}_{tH}$, $\mathcal{O}_{4t}$ potentially **tree-level generated** if the UV completion is renormalizable and weakly coupled [Arzt,Einhorn,Wudka,'95],[Craig,Jiang,Li,Sutherland,'20],[Buchalla,Heinrich,Müller-Salditt,Pandler,'22].
- The four-top contribution depends on the continuation scheme for γ_5!

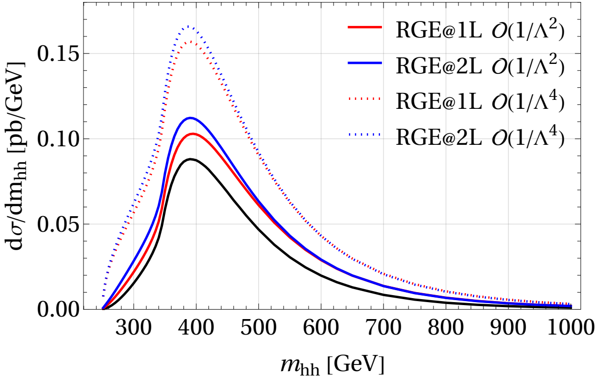
Running of $\mathcal{C}_{HG}$: 1L vs 2L

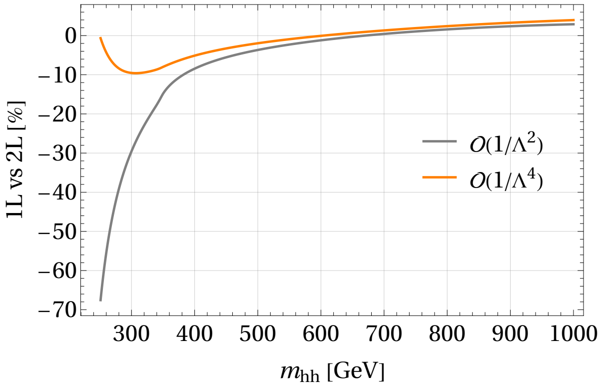
$$\text{S1}: \quad \mathcal{C}_{tH}(\Lambda) = 1,\ \mathcal{C}_{HG}(\Lambda) = 1/16\pi^2,\ \mathcal{C}_{tG}(\Lambda) = -1/16\pi^2,\ \mathcal{C}_{Qt(1,8)}(\Lambda) = -10.$$



- Running computed with a private version of `RGESolver` [SDN,Silvestrini,'22].

Di-Higgs: 1L vs 2L (NDR)





- $\mathcal{O}\left(1/\Lambda^4\right)$ terms are relevant! $\rightarrow$ **higher order EFT may be larger than RG running!**
- 1L-2L **relative** difference is larger in the linear case near the threshold!

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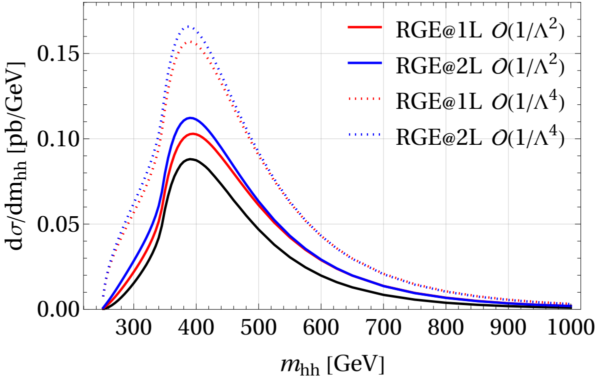
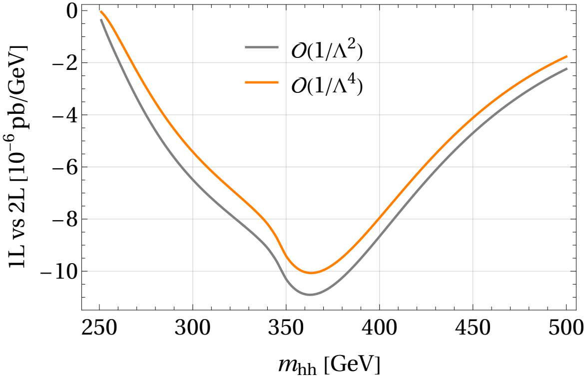


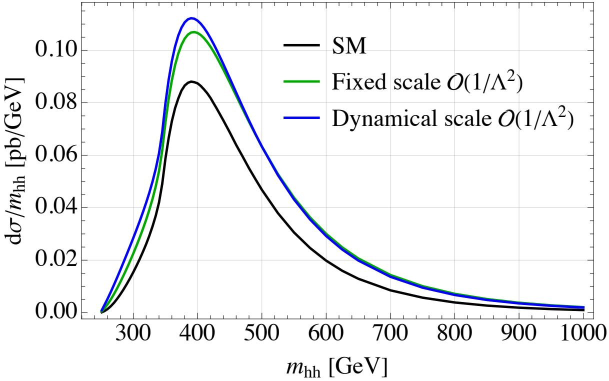


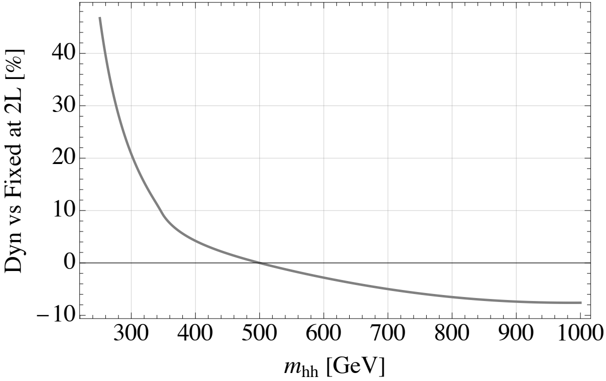
- $\mathcal{O}\left(1/\Lambda^4\right)$ terms are relevant! $\rightarrow$ **higher order EFT may be larger than RG running!**
- 1L-2L **absolute** difference is similar to the linear case near the threshold!

Di-Higgs: fixed vs dynamical renormalization scale

- Estimate the size of running effects by comparing

$$\mu_R = m_{hh}/2 \quad \text{(dynamical)} \quad \text{vs} \quad \mu_R = 2m_h \quad \text{(fixed)}$$





- Same result when $m_{hh}/2 = 2m_h \to m_{hh} = 500\,\text{GeV}$.

Summary

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- Phenomenological impact in di-Higgs production:
  - threshold: up to $\sim 70\%$ difference 1L vs 2L, up to $\sim 40\%$ difference fixed vs dynamical renormalization scale,
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- Outlook: full two-loop RGE for the Higgs-gluon coupling (WIP).

Thanks for your attention!


Back-up slides

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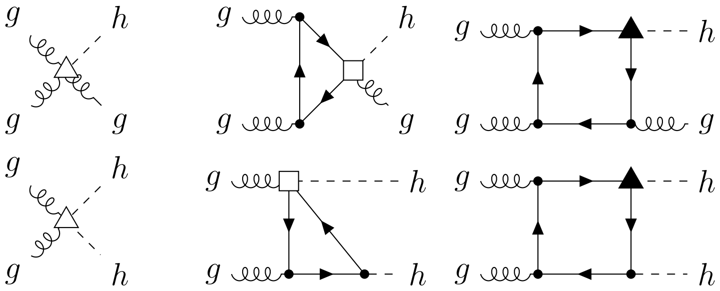
  - **Algebraically consistent.**
  - **Breaks Ward ids/chiral symmetries** ([Larin,'93], [Olgoso,Vecchi,'24]).
  - **Computationally demanding.**

Phenomenological impact: set-up I

- We study the two-loop running effects for:

$\frac{d\sigma}{dp_T}(pp \to hj)$ using the results in [Grazzini,Ilnicka,Spira,'18].

$\frac{d\sigma}{dm_{hh}}(pp \to hh)$ using a private version of hpair [Dawson,Dittmaier,Spira,'98].



$$\Lambda = 1\,\mathrm{TeV}, \quad \mu_R = \mu_F = \begin{cases} \sqrt{m_h^2 + p_T^2}/2 \text{ for } pp \to hj, \\ m_{hh}/2 \text{ for } pp \to hh. \end{cases}$$

Phenomenological impact: set-up II

- Initial conditions at $\Lambda = 1$ TeV.

$$\mathcal{C}_{tH}(\Lambda) = 1,\ \mathcal{C}_{HG}(\Lambda) = \frac{1}{16\pi^2},\ \mathcal{C}_{tG}(\Lambda) = -\frac{1}{16\pi^2},$$
$$\mathcal{C}_{Qt(1,8)}(\Lambda) = -10,\ \mathcal{C}_{H}(\Lambda) = 0.$$

- Full set of operators included in $pp \to hh/hj$

$$\mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H), \quad \mathcal{O}_{HD} = (H^\dagger D_\mu H)^*(H^\dagger D^\mu H), \quad \mathcal{O}_{H} = (H^\dagger H)^3,$$
$$\mathcal{O}_{tH} = (\bar{Q}_L \tilde{H} t_R)\left(H^\dagger H\right), \quad \mathcal{O}_{bH} = (\bar{Q}_L H b_R)\left(H^\dagger H\right),$$
$$\mathcal{O}_{HG} = \left(H^\dagger H\right) G^A_{\mu\nu} G^{\mu\nu,A}, \quad \mathcal{O}_{tG} = \bar{Q}_L \tilde{H} \sigma_{\mu\nu} T^A t_R G^{\mu\nu,A},$$
$$\mathcal{O}_{Qt(1)} = \left(\bar{Q}_L \gamma^\mu Q_L\right)\left(\bar{t}_R \gamma_\mu t_R\right), \quad \mathcal{O}_{Qt(8)} = \left(\bar{Q}_L \gamma^\mu T^A Q_L\right)\left(\bar{t}_R \gamma_\mu T^A Q_R\right).$$

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Karlsruhe Institute of Technology

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- Difference between fixed and dynamical scale $\to$ **importance of running effects!**
- SMEFT evolution is complex: fixed scale faster (orders of magnitude) and simpler of dynamical!

Solving the RGEs

1 **Approximate solution** (first leading log):

$$\mathcal{C}_i(\mu_\mathrm{F}) = \mathcal{C}_i(\mu_\mathrm{I}) + \Gamma_{ij}(\mu_\mathrm{I})\mathcal{C}_j(\mu_\mathrm{I})\frac{\log(\mu_\mathrm{F}/\mu_\mathrm{I})}{16\pi^2}.$$

- Neglects dependence on μ of Γ.
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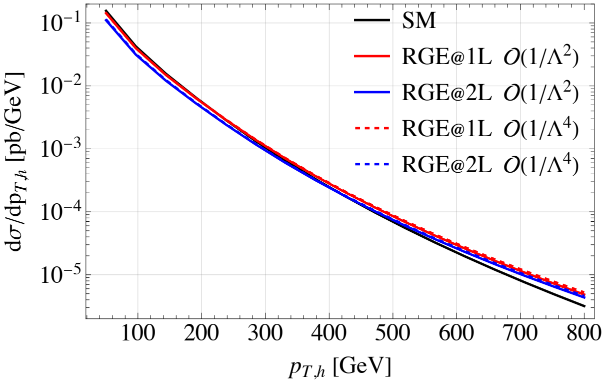
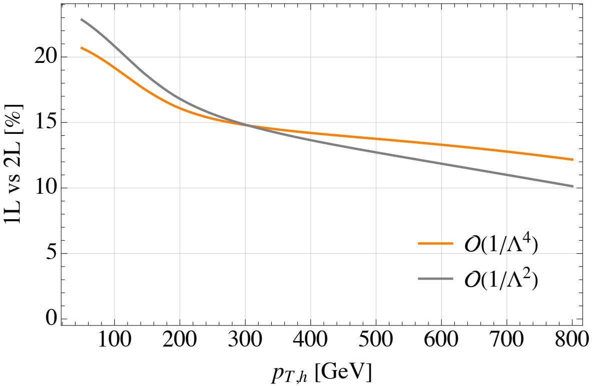
2 **Numeric solution**:

- More precise.
- Slow! Problem for extensive phenomenological analyses.

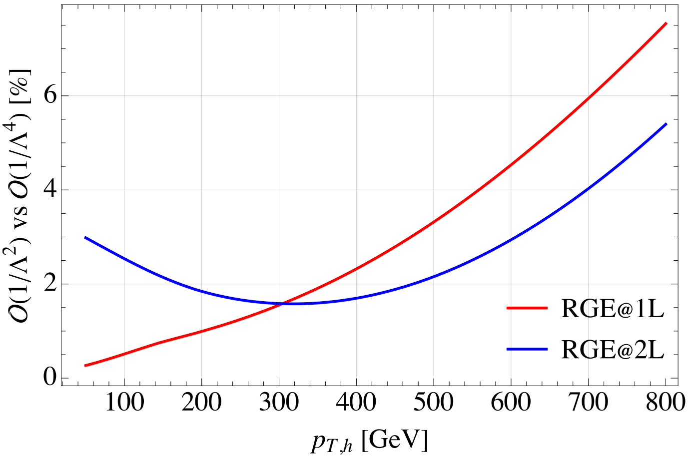
Higgs+jet: 1L vs 2L (NDR)

$$\mu \frac{d\mathcal{C}_{HG}}{d\mu} \supset 3 \left(\frac{1}{16\pi^2}\right)^2 g_s^2 \times 2\mathcal{C}_{tH} Y_t - 4 \left(\frac{1}{16\pi^2}\right)^2 g_s^2 Y_t^2 \left(\mathcal{C}_{Qt}^{(1)} - \frac{1}{6}\mathcal{C}_{Qt}^{(8)}\right)$$



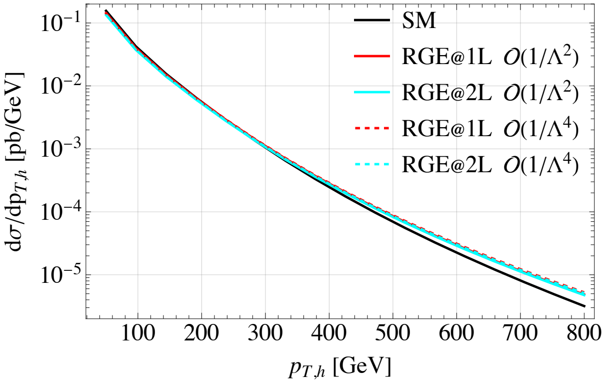


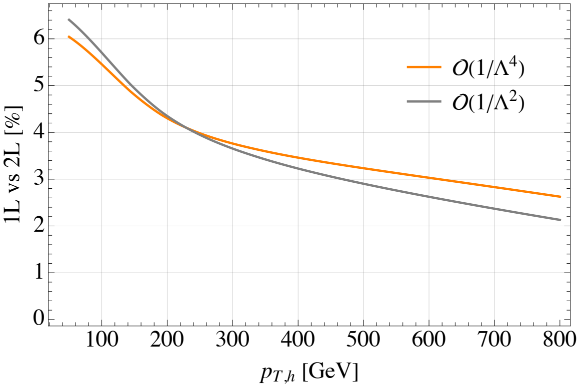
Higgs+jet: linear vs quadratic order in $1/\Lambda^2$



Higgs+jet: 1L vs 2L (BMHV)

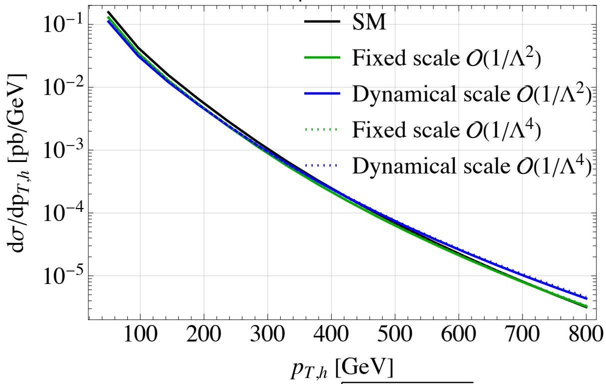
$$\mu\frac{d\mathcal{C}_{HG}}{d\mu} \supset 3\left(\frac{1}{16\pi^2}\right)^2 g_s^2 \times 2\mathcal{C}_{tH}Y_t - 4\left(\frac{1}{16\pi^2}\right)^2 g_s^2 Y_t^2\left(\mathcal{C}_{Qt}^{(1)} - \frac{1}{6}\mathcal{C}_{Qt}^{(8)}\right)$$

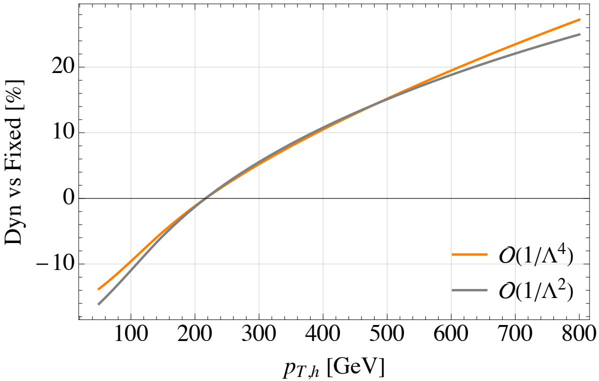




Higgs+jet: fixed vs dynamical scale

$$\mu_R = \sqrt{m_h^2 + p_T^2}/2 \quad \text{(dynamical)} \quad \text{vs} \quad \mu_R = m_h \quad \text{(fixed)}$$





- Same result when $\sqrt{p_{T,h}^2 + m_h^2}\,/2 = m_h \to p_{T,h} \simeq 216.5\,\text{GeV}$.