

# $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$ decay at Belle II and STCF

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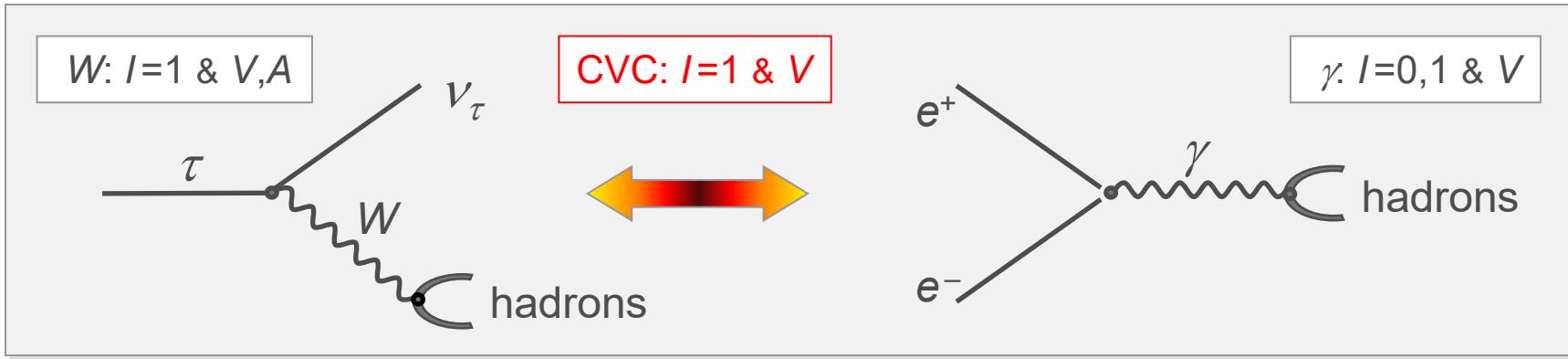
(for the Belle II Collaboration)

IHEP, Beijing

The 8<sup>th</sup> Plenary Workshop of the Muon g-2 Theory Initiative

Orsay, Sept. 8-12, 2025

# CVC implies using $\tau$ decays to calculate HVP



Hadronic physics factorizes in **Spectral Functions (SF)** :

Isospin symmetry connects *Isospin-1*  $e^+e^-$  cross section to vector  $\tau$  spectral functions:

$$\sigma^{(I=1)}[e^+e^- \rightarrow \pi^+\pi^-] = \frac{4\pi\alpha^2}{s} v[\tau^- \rightarrow \pi^-\pi^0\nu_\tau]$$

fundamental ingredient relating long distance (resonances) to short distance description (QCD)

$$v[\tau^- \rightarrow \pi^-\pi^0\nu_\tau] \propto \frac{B[\tau^- \rightarrow \pi^-\pi^0\nu_\tau]}{B[\tau^- \rightarrow e^-\bar{\nu}_e\nu_\tau]} \frac{1}{N_{\pi\pi^0}} \frac{dN_{\pi\pi^0}}{ds} \frac{m_\tau^2}{(1-s/m_\tau^2)^2(1+s/m_\tau^2)}$$

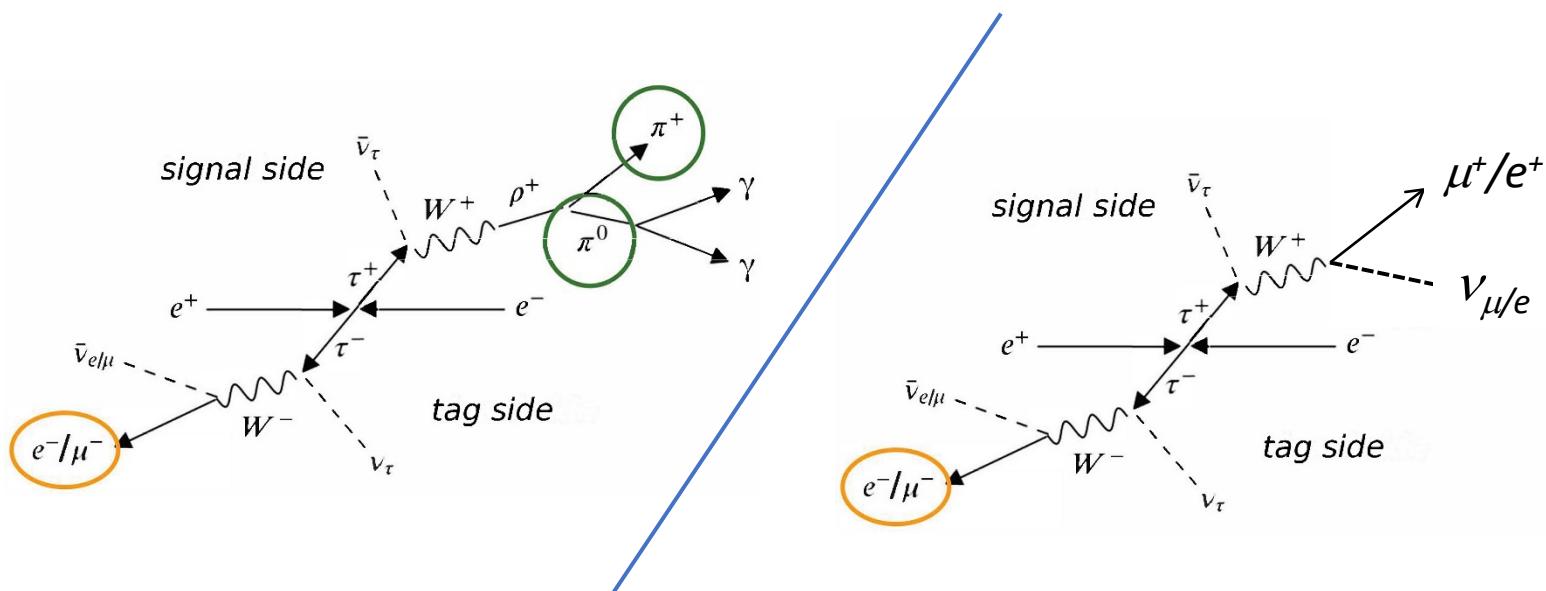
branching fractions   mass spectrum   kinematic factor

# Strategy of spectral function (SF) measurements

Measure the ratio of the BFs directly

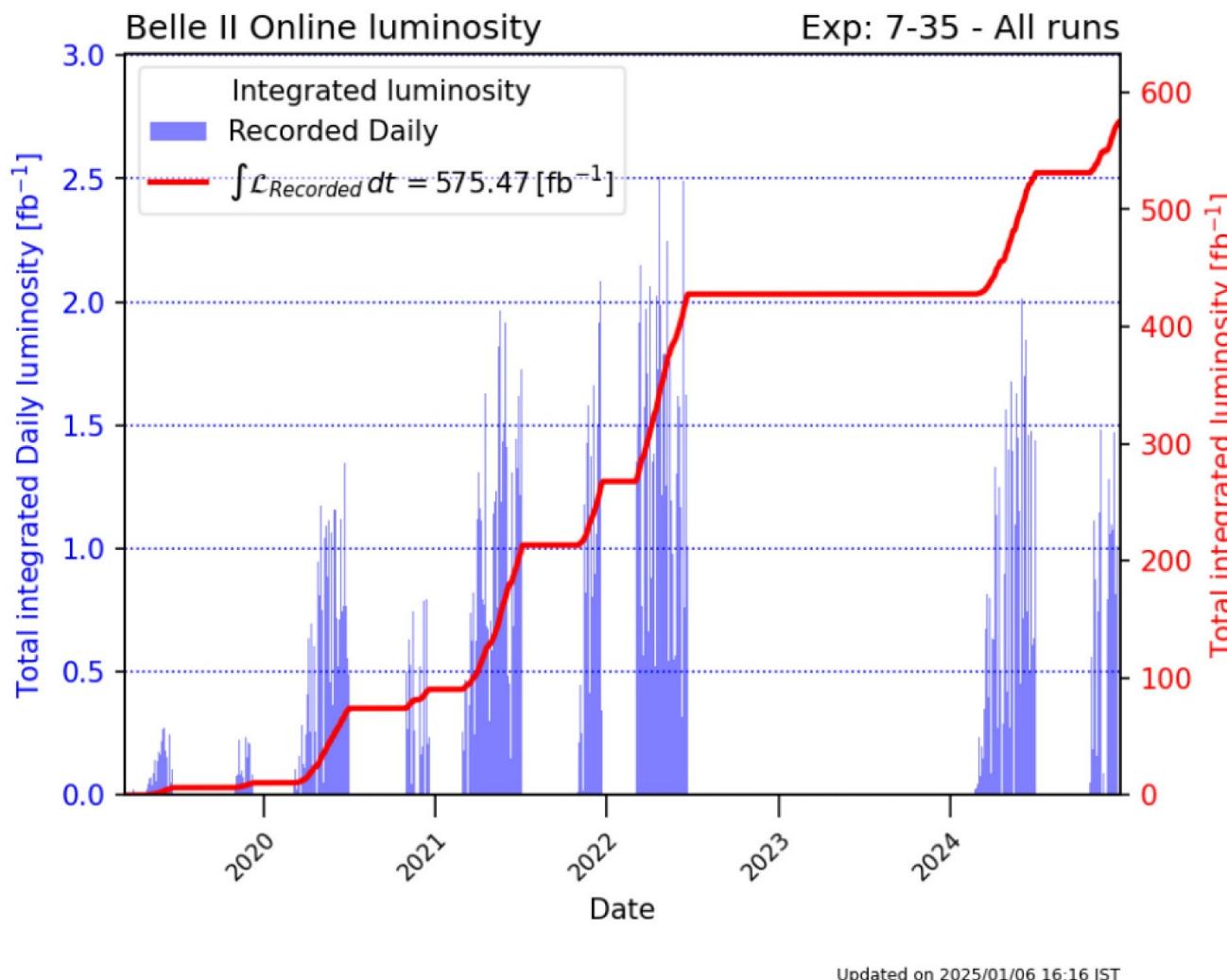
Measure the normalized  $\pi\pi^0$  invariant mass distribution

$$v_1(s) \equiv \frac{m_\tau^2}{6 |V_{ud}|^2 S_{EW}} \frac{B(\tau^- \rightarrow V^- \bar{\nu}_\tau)}{B(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \times \frac{dN_V}{N_V ds} \left[ \left(1 - \frac{s}{m_\tau^2}\right)^2 \left(1 + \frac{2s}{m_\tau^2}\right) \right]^{-1}$$



$\tau^- \rightarrow \pi^-\pi^0\nu_\tau$  decay at Belle II

# Belle II data

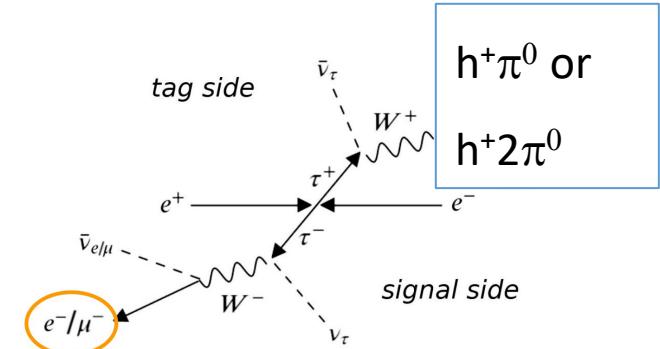
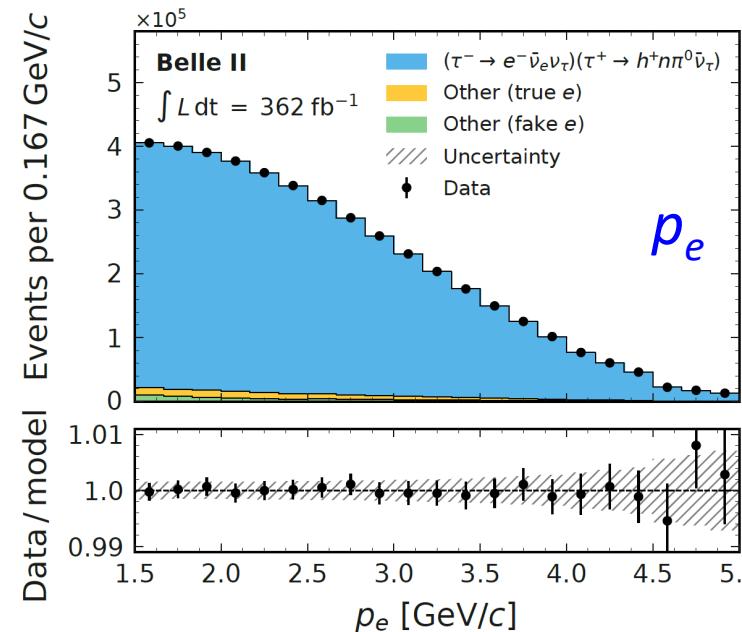
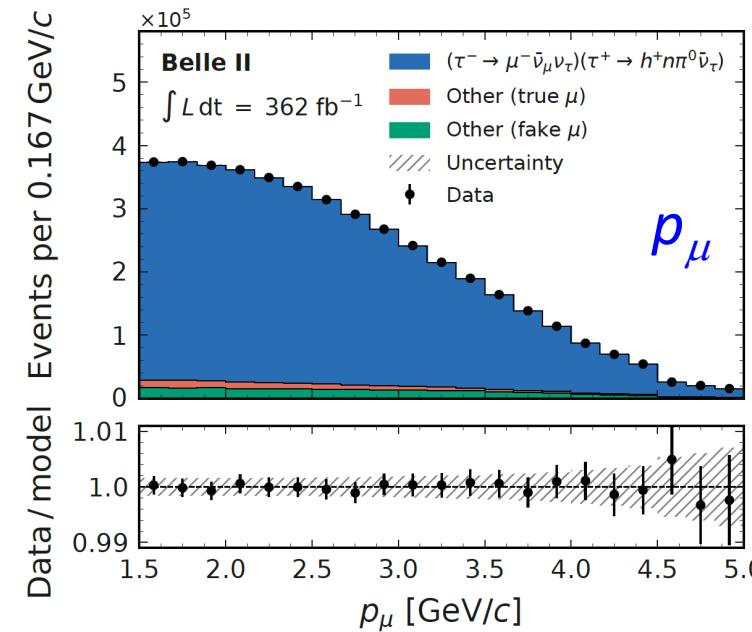


- 575 fb<sup>-1</sup> on tape
- 428 fb<sup>-1</sup> data have been used for physics
- Publications on  $\tau$  physics with tagged  $\tau$ 's
  - Search for the lepton-flavor-violating  $\tau^- \rightarrow e^\mp \ell^\pm \ell^-$  decays at Belle II, [2507.18236](#), 428 fb<sup>-1</sup> (inclusive)
  - **Test of light-lepton universality in  $\tau$  decays with Belle II**, [2405.14625](#), 362 fb<sup>-1</sup> ( $h\pi^0$ ,  $h2\pi^0$  tag)
- Measurement of the  $\tau$ -lepton mass with the Belle II experiment, [2305.19116](#), 190 fb<sup>-1</sup>, ( $e$ ,  $\mu$ ,  $h$ ,  $h\pi^0$  tag)
- Search for lepton-flavor-violating  $\tau$  decays to a lepton and an invisible boson at Belle II, [2212.03634](#), 63 fb<sup>-1</sup> (3h tag)
- Belle II trigger & data quality are suitable for high precision  $\tau$  analyses

# Statistics are large enough in the $362 \text{ fb}^{-1}$ data sample

Test of light-lepton universality in  $\tau$  decays with Belle II, 2405.14625 (*JHEP* 08 (2024) 205),  $362 \text{ fb}^{-1}$  ( $h\pi^0$ ,  $h2\pi^0$  tag)

- The data were recorded between 2019 and 2022 at a center-of-mass energy of 10.58 GeV and correspond to about  $333 \times 10^6 e^+e^- \rightarrow \tau^+\tau^-$  events.



$$R_\mu = 0.9675 \pm 0.0007(\text{stat.}) \pm 0.0036(\text{sys.})$$

World best precision!

$h\pi^0$  is about 2/3 of the  $h\pi^0+h2\pi^0$  tags

Expect  $\sim 5 \times 10^6$   $e/\mu$ -tagged  $h\pi^0$  events

Expect  $\sim 2 \times 10^6$   $e\mu$  events

Statistical uncertainty in the ratio of the BFs is  $\sim 10^{-3}$

The signal yields are ( $h\pi^0+h2\pi^0$  tags):

4,156,500 for  $\tau^- \rightarrow e^- \bar{\nu} \nu$

4,000,190 for  $\tau^- \rightarrow \mu^- \bar{\nu} \nu$

We do similar selection +  $\pi$ -ID and improved  $\pi^0$  reconstruction for  $\tau^- \rightarrow \pi^- \pi^0 \bar{\nu}_\tau$  analysis!

# Photon and $\pi^0$ selection

Belle II simulation, 32 fb<sup>-1</sup>

Category	Fraction (%)
2 photons detectable	76.0
1 photon detectable	14.5
Merged $\pi^0$ cluster	1.2
1 photon converted	2.5
$\pi^0 \rightarrow \gamma e^+ e^-$	0.5
Sum	94.7

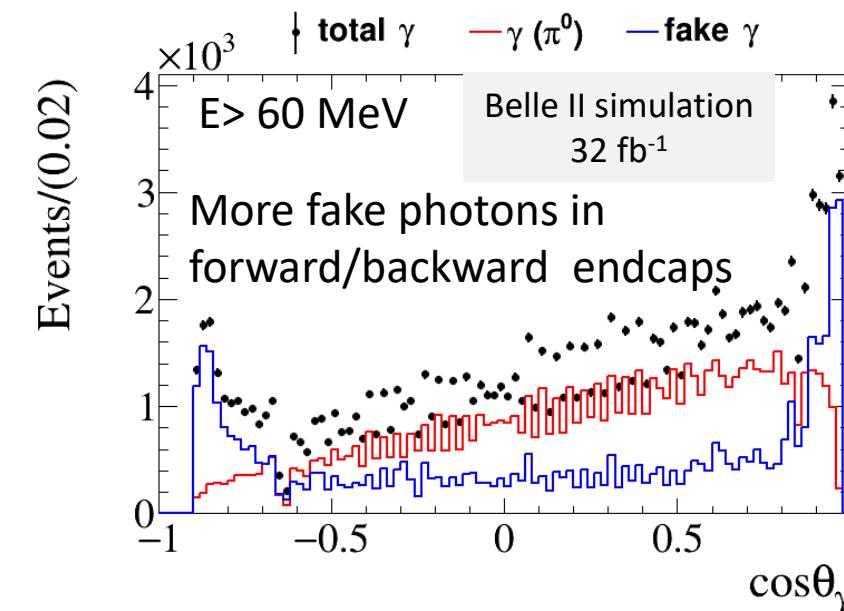
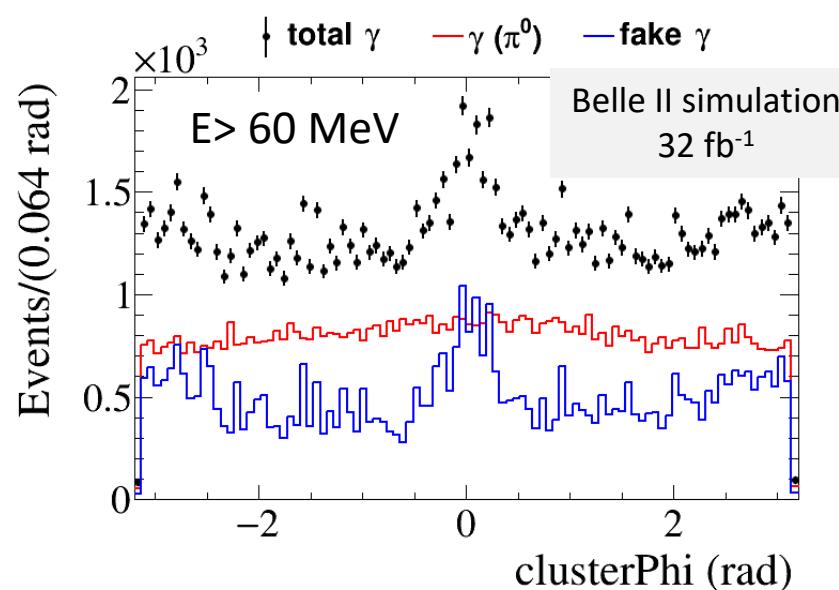
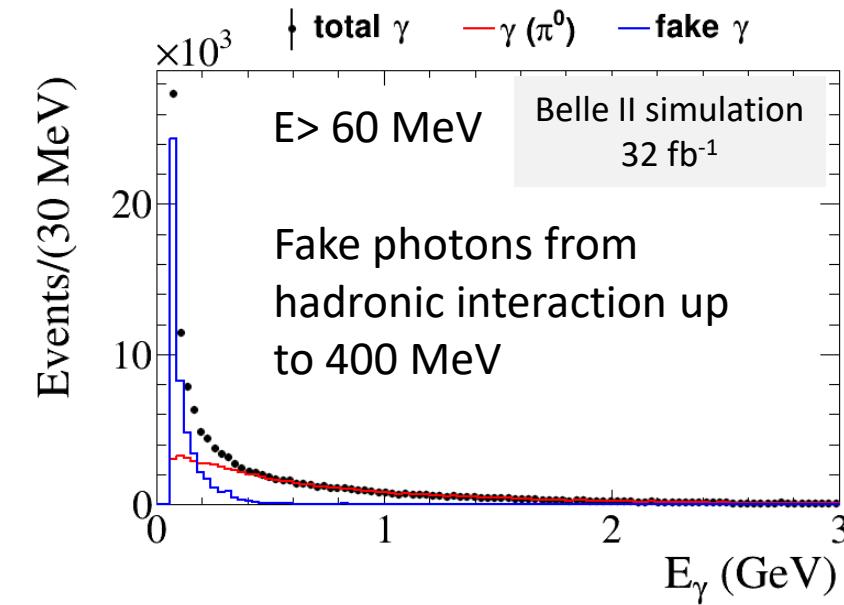
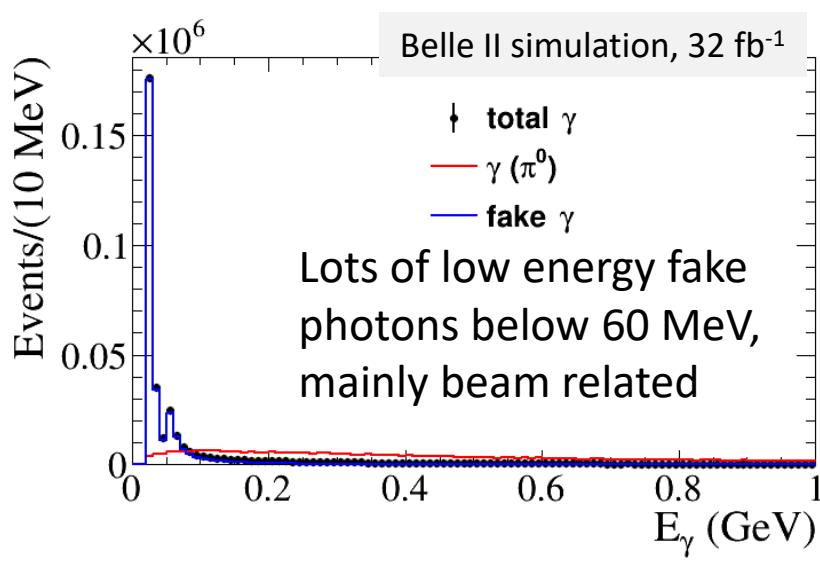
Note:  $\mathcal{B}(\pi^0 \rightarrow \gamma\gamma) \sim 98.82\%$ ,  $\mathcal{B}(\pi^0 \rightarrow \gamma e^+ e^-) \sim 1.17\%$

From  $\tau^- \rightarrow \pi^-\pi^0\nu$  MC sample:

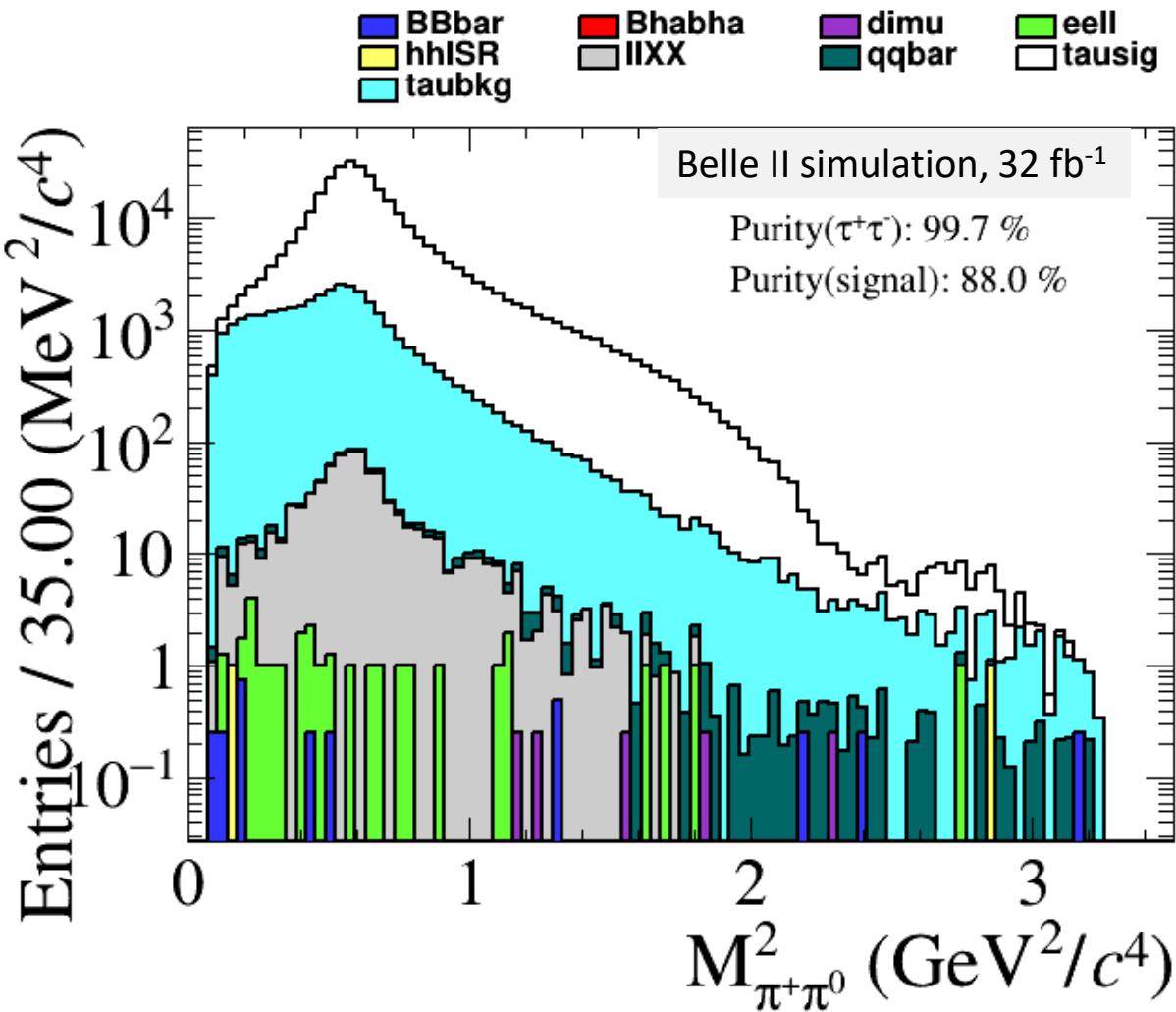
- Two photons can be detected in most of the cases
- Significant fraction of events with only one photon detected
- Small fraction ( $\sim 1\%$ ) of merged  $\pi^0$  (ALEPH: 13%, Phys. rep. 421 (2005) 191)
- $\pi^0 \rightarrow \gamma e^+ e^-$  and  $\gamma$ -conversion (3%) can be added in the sample

Select  $\pi^0 \rightarrow \gamma\gamma$  and  $\gamma$  to improve efficiency and reduce systematic uncertainties

# Photon and $\pi^0$ selection

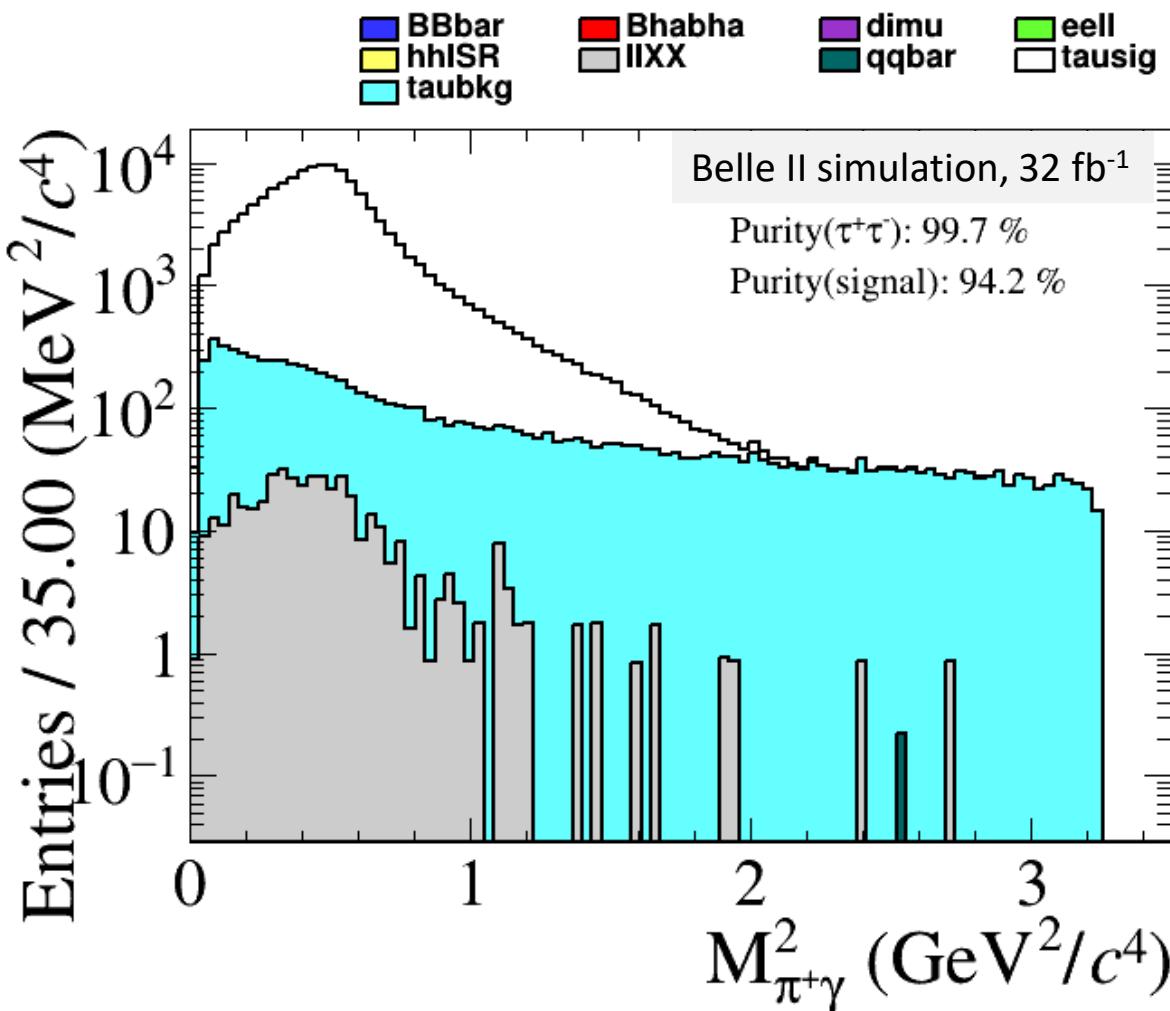


# Very preliminary MC study: $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$ , $\pi^0 \rightarrow \gamma\gamma$



- Purity: 88%
- Efficiency: 8.8% (e-tag), 8.7% ( $\mu$ -tag)
- Background:
  - non- $\tau\tau$ : 0.3%
  - $\tau\tau$  bkg: 12.0% (below are ratio in  $\tau\tau$  mode):
    - $\tau^- \rightarrow \pi^- 2\pi^0\nu_\tau$ : 80.0%
    - $\tau^- \rightarrow \pi^-\nu_\tau$ : 10.4%
    - $\tau^- \rightarrow \pi^-\pi^0 K_L^0\nu_\tau$ : 2.8%
    - $\tau^- \rightarrow \pi^- 3\pi^0\nu_\tau$ : 1.8%
    - $\tau^- \rightarrow \pi^- K_S^0\nu_\tau$ : 1.2%
    - ...

# Very preliminary MC study: $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$ , $\pi^0 \rightarrow \gamma + \text{missing}$



- Purity: 94%
- Efficiency: 5.7% (e-tag), 5.8% ( $\mu$ -tag)
- Background:
  - non- $\tau\tau$ : 0.3%
  - $\tau\tau$  bkg: 5.8% (below are ratio in  $\tau\tau$  mode):
    - $\tau^- \rightarrow \pi^-\pi^0 K_L^0 \nu_\tau$ : 27.7%
    - $\tau^- \rightarrow \pi^-\pi^0 \eta \nu_\tau$ : 22.5%
    - $\tau^- \rightarrow \pi^-\nu_\tau$ : 19.6%
    - $\tau^- \rightarrow \pi^- K_L^0 \nu_\tau$ : 10.5%
    - $\tau^- \rightarrow \pi^-\pi^0 K_S^0 \nu_\tau$ : 8.4%
    - ...

# Efficiencies and systematic uncertainties

- Trigger
- Tracking
- Particle ID
- Photon ID and  $\pi^0$  reconstruction
- Background
- Unfolding
- .....

# Trigger

## Test of light-lepton universality in $\tau$ decays with Belle II, 2405.14625, 362/fb (h $\pi^0$ , h2 $\pi^0$ tag)

The trigger efficiency is measured with a reference sample selected by independent triggers based on the number of particles reconstructed in the CDC. The trigger efficiency in data is 99.8 % for  $\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau$  and 96.6 % for  $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$  decays, which is primarily driven by the tag hemisphere. In simulation, the corresponding efficiencies are 98.6 % and 95.4 %, respectively. To account for imperfection in the simulation of the trigger, we apply correction factors as ratios of efficiencies in data and simulation to our simulated samples.

ECL trigger  
only!

Plan to use both CDC and ECL trigger:

CDC trigger — based on charged objects only:

- At least 1 charged track with  $p>0.7$  GeV/c, or
- At least 2 charged tracks with opening angle larger than 90 degrees

ECL trigger — based on calorimeter objects only:

- total ECL energy above 1 GeV, or
- exactly 1 or  $\geq 1$  cluster with  $E \geq 2$  GeV in different ECL regions

Combined trigger efficiency is very close to 100%, no  $M(\pi^-\pi^0)$  dependence

MC simulates data well, uncertainty < 0.1%

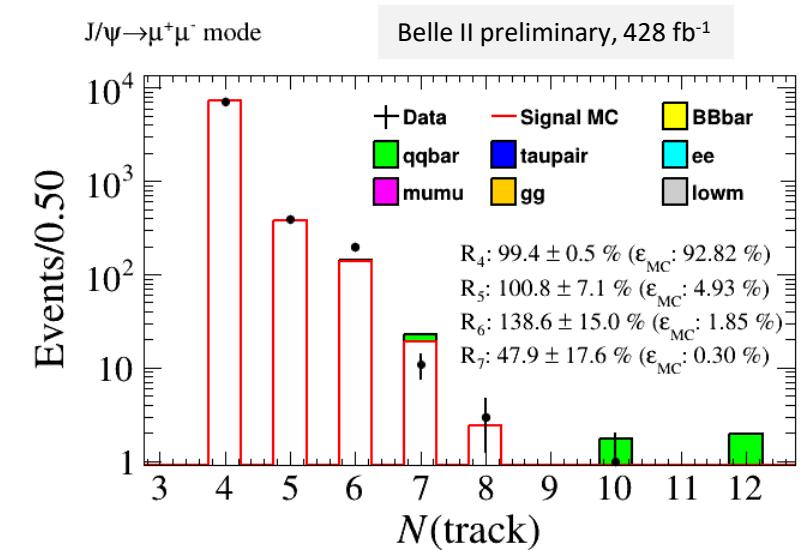
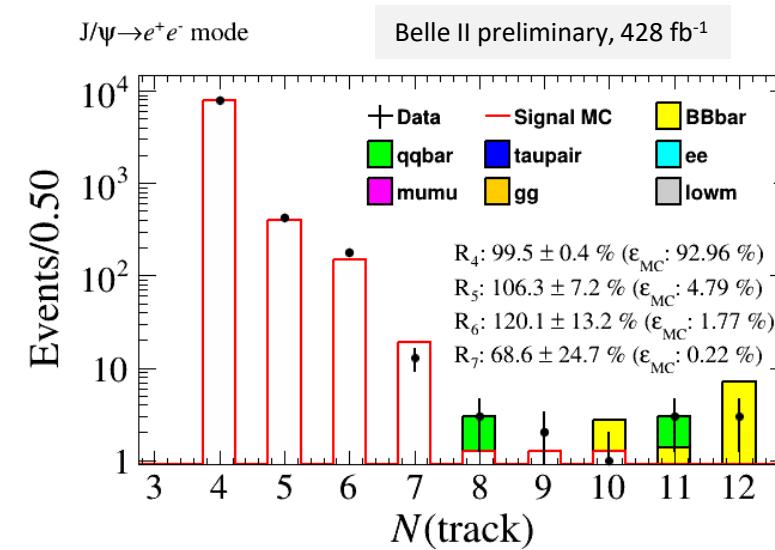
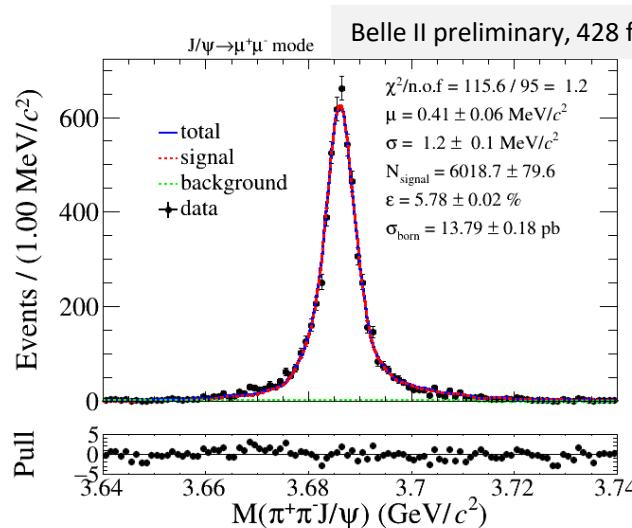
# Tracking efficiency & fake tracks

Test of light-lepton universality in  $\tau$  decays with Belle II, 2405.14625, 362/fb (h $\pi^0$ , h2 $\pi^0$  tag)

Differences between the track finding efficiencies in simulation and data have been measured in  $e^+e^- \rightarrow \tau^+\tau^-$  events with one of the  $\tau$  leptons decaying to three charged hadrons. A per-track systematic uncertainty of 0.24 % is included as a normalisation uncertainty of the templates to account for these differences. The associated systematic uncertainty on  $R_\mu$  is 0.01 %.

Similar effect in measuring  $R_e = B(\pi^-\pi^0)/B(e^-)$ , the uncertainty is  $\sim 0.01\%$ .

Check simulation of fake tracks with ISR  $\psi' \rightarrow \pi^+\pi^-J/\psi$  sample (almost bkg free!): MC simulates data well



# Particle Identification

Charged-hadron identification at Belle II, [2506.04355](#) (EPJC in press)

To identify particles, information from all subdetectors except PXD is used. For each subdetector,  $d$ , and particle species,  $\alpha$ , with momentum  $\vec{p}$ , the local likelihood,  $L_\alpha^d(x^d | \vec{p})$ , for subdetector's measurements  $x^d$ , is defined.

$$\mathcal{L}_\alpha \equiv \prod_d \mathcal{L}_\alpha^d$$

$$\tilde{\mathcal{L}}_\alpha \equiv \exp \left( \sum_d w_{\alpha,d} \log \mathcal{L}_\alpha^d \right)$$

Define likelihood ratio to do PID

$$P_\alpha = \frac{\mathcal{L}_\alpha}{\sum_\gamma \mathcal{L}_\gamma}$$

where the sum in the denominator runs over all species (e,  $\mu$ ,  $\pi$ , K, p, d).

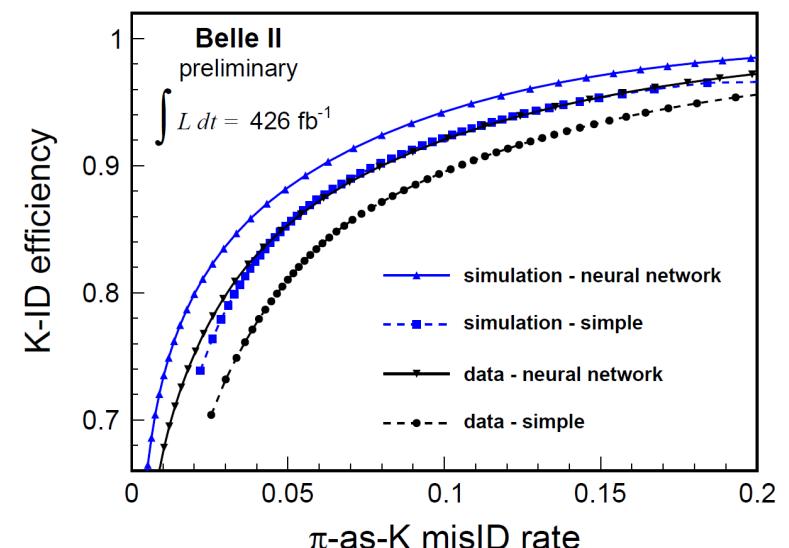
The binary likelihood ratio, restricting  $\gamma$  to two species,

$$P_{\alpha/\beta} \equiv \mathcal{L}_\alpha / (\mathcal{L}_\alpha + \mathcal{L}_\beta)$$

for  $\pi/K$  identification.

	e	$\mu$	$\pi$	K	p
SVD	0.81	1.36	1.06	1.79	1.72
CDC	2.27	1.96	1.43	1.91	1.87
TOP	0.43	0.43	0.43	0.42	0.41
ARICH	0.60	0.61	0.60	0.61	0.65
ECL	2.46	1.96	2.02	1.66	1.96
KLM	0.16	0.48	0.18	0.22	0.22

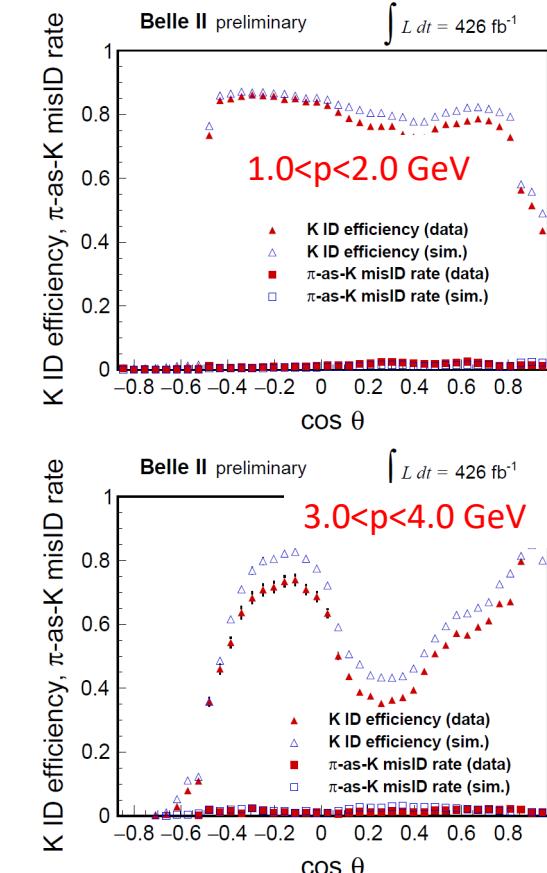
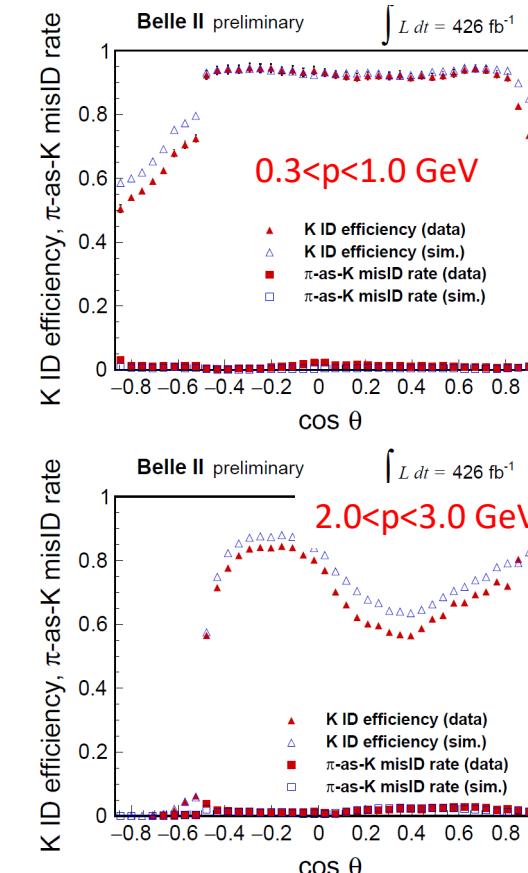
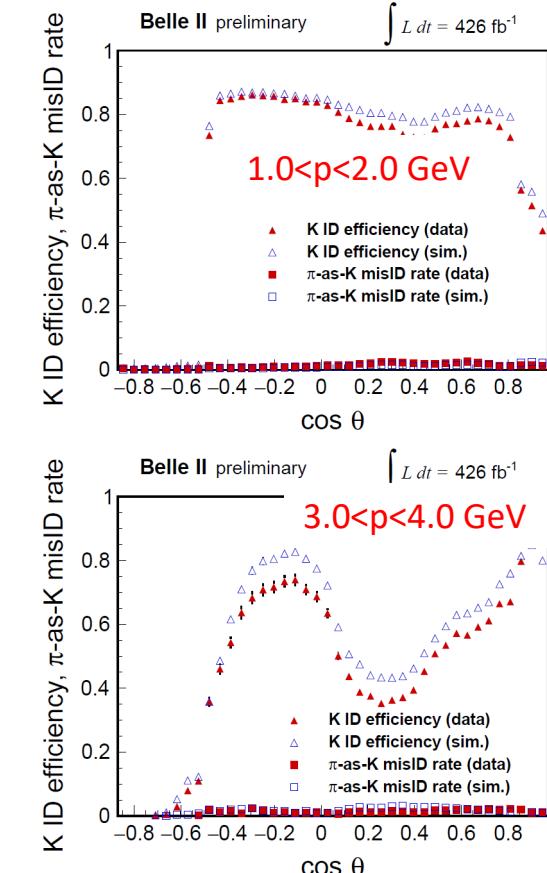
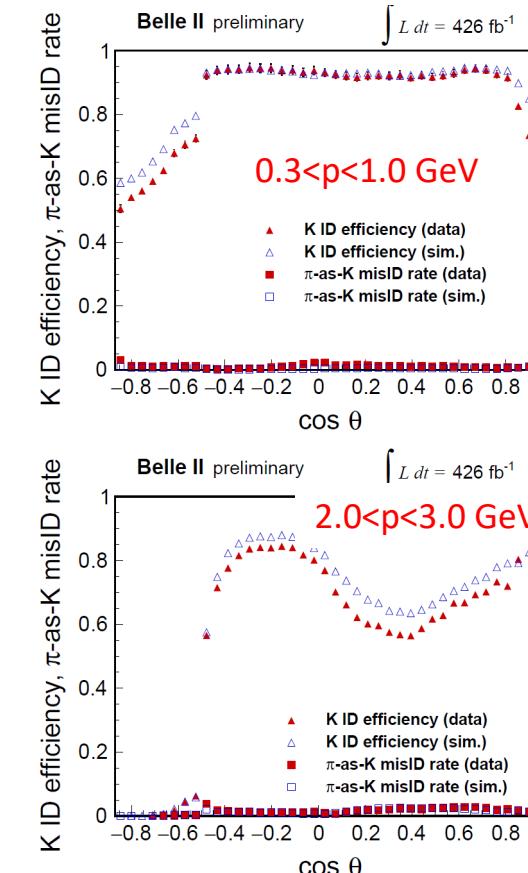
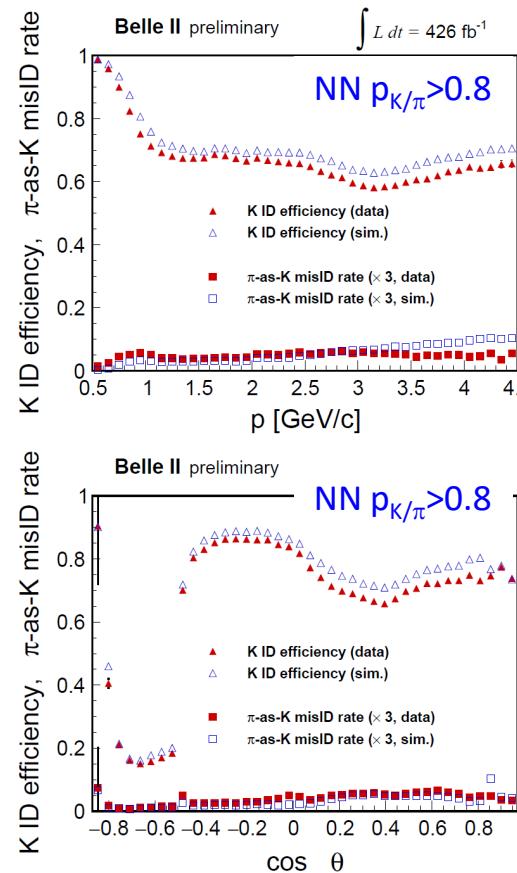
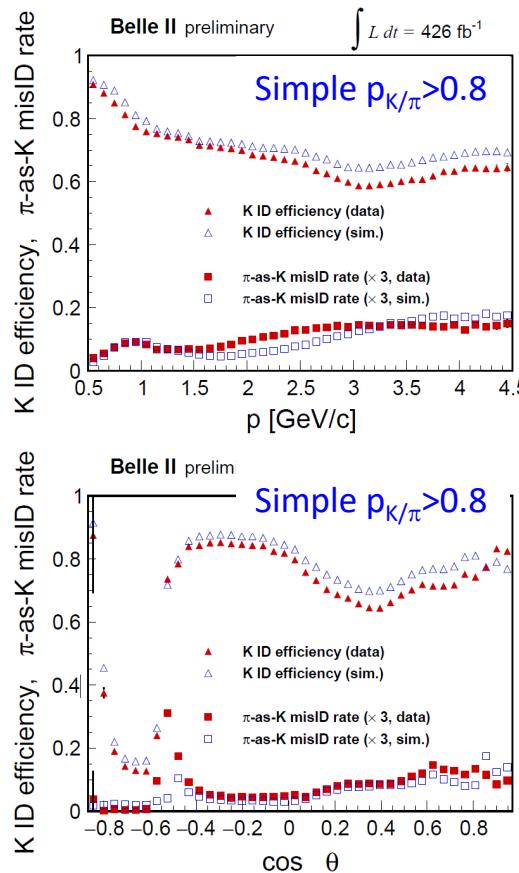
Local-likelihood weights of the reweighted global PID likelihood for particles uniformly distributed in [0.5, 5.0] GeV/c and the angular acceptance of the detector.



# $\pi/K$ separation

Charged-hadron identification at Belle II, [2506.04355](#) (EPJC in press)

Control samples  $K_S \rightarrow \pi^+ \pi^-$  and  $D^* \rightarrow \pi^- D^0 \rightarrow \pi^- K^- \pi^+$  can be used to measure the PID efficiencies in high precision.



**Fig. 5:**  $K$ -ID efficiency and  $\pi\text{-as-}K$  misID rate as functions of  $p$  and  $\cos \theta$  for  $P_{K/\pi} > 0.8$  using the simple (left) and neural-network (right) probabilities. To aid visibility, the  $\pi\text{-as-}K$  misID rates are scaled by a factor of 3.

**Fig. 6:**  $K$ -ID efficiency and  $\pi\text{-as-}K$  misID rate as functions of  $\cos \theta$  for  $P_{K/\pi} > 0.8$  using the neural-network probability in four momentum ranges.

# $\pi^0$ efficiency ( $\pi^0 \rightarrow \gamma\gamma$ ) via ISR $\omega$

MEASUREMENT OF THE  $e^+e^- \rightarrow \pi^+\pi^-\pi^0$  CROSS SECTION IN ...

PHYS. REV. D **110**, 112005 (2024)

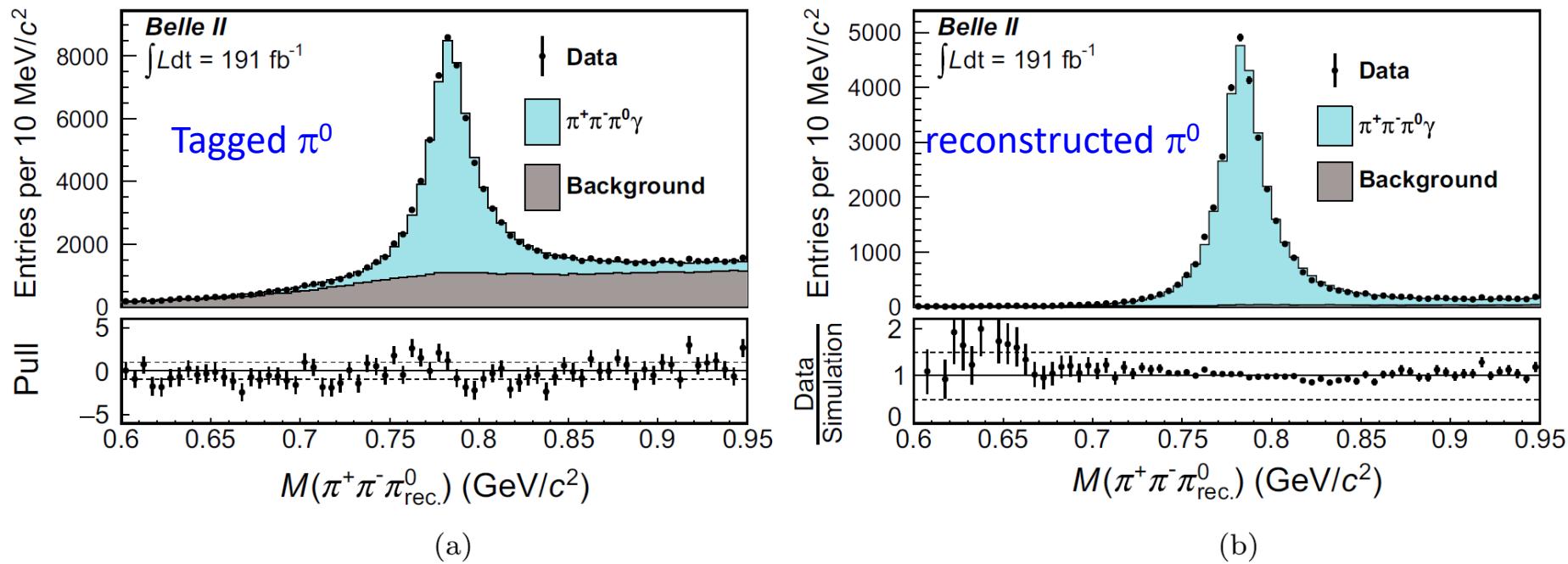
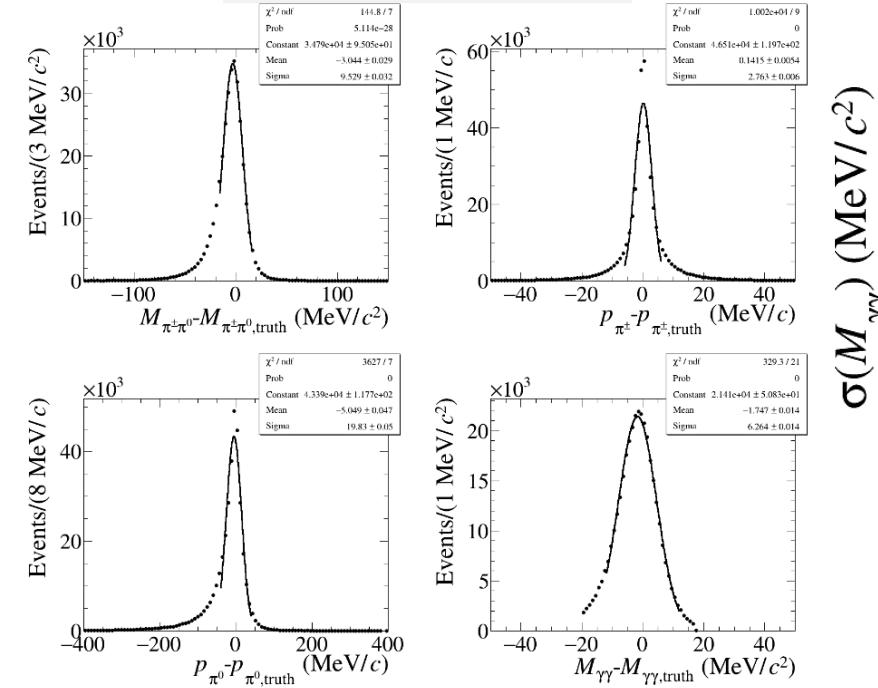


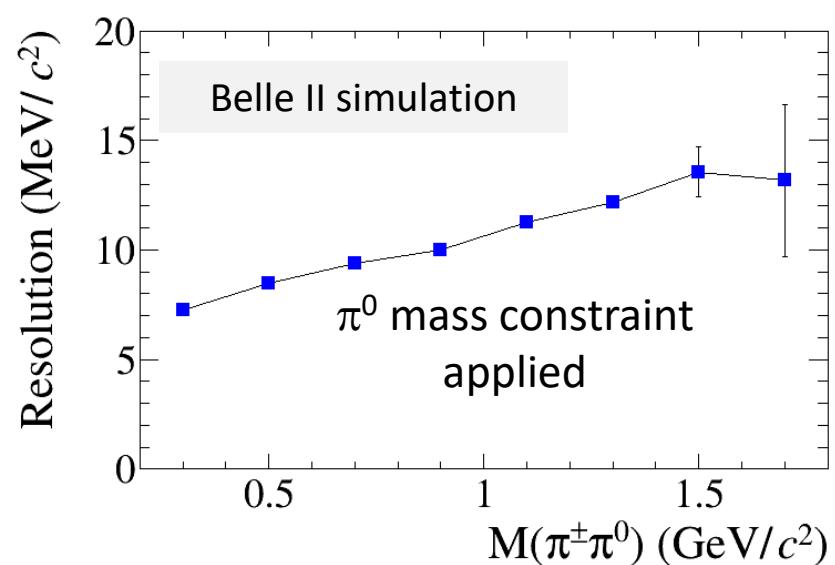
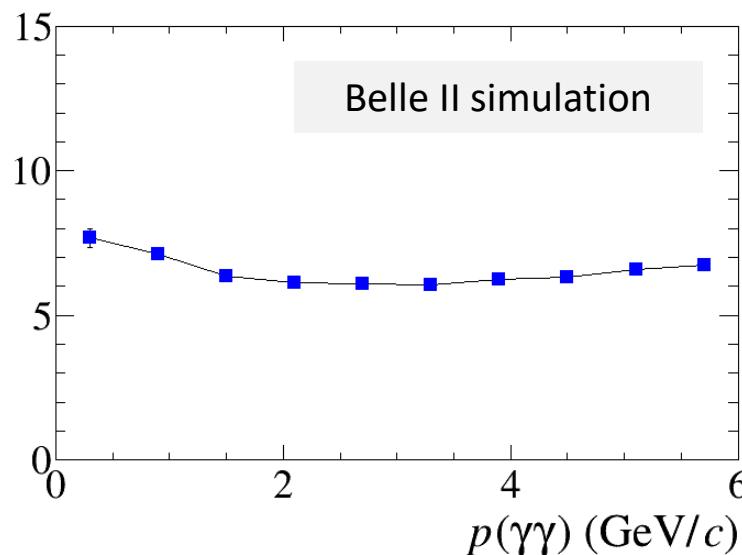
FIG. 16. (a) Three-pion mass  $M(\pi^+\pi^-\pi^0_{\text{rec}})$  distribution for event fully reconstructed using  $\pi^+\pi^-\gamma$  particles. The shaded histograms in the top panel show the result of a  $M(\pi^+\pi^-\pi^0_{\text{rec}})$  fit to the data with signal and background components. The differences between data and fit results divided by the data uncertainties (pull) are shown in the bottom panel. (b) The same  $M(\pi^+\pi^-\pi^0_{\text{rec}})$  distribution for the events fully reconstructed using  $\pi^+\pi^-\pi^0\gamma$  particles. The convention of the figure is the same as (a). (b) The shaded histograms, which show simulated signal and background, are normalized to the data to check the consistency of the signal model. The data-to-simulation ratio is shown in the bottom panel.

Efficiency  $\sim 70\%$ , relative data/MC efficiency difference  $\sim (-1.4 \pm 1.0)\%$ .

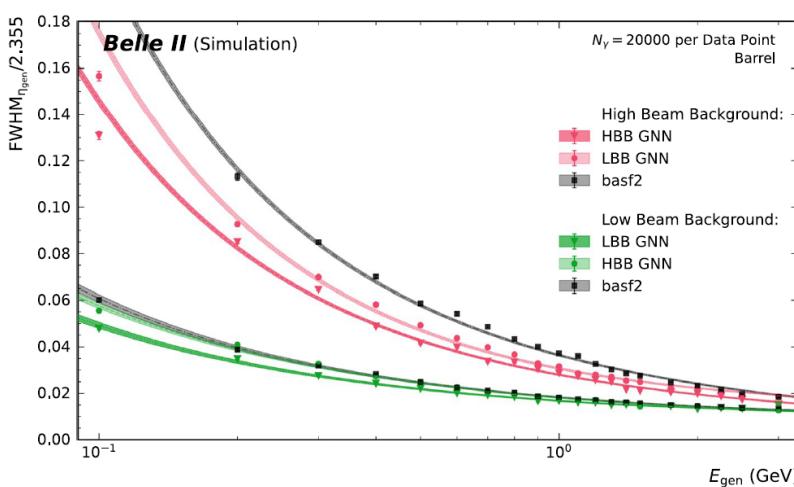
## Belle II simulation



## $\pi^0$ resolution ( $\pi^0 \rightarrow \gamma\gamma$ )



Belle II: Photon Reconstruction in the Belle II Calorimeter Using GNN, <https://doi.org/10.1007/s41781-023-00105-w>



Better understanding the photon selection and  $\pi^0$  reconstruction is essential: fake photon suppression, gamma-conversion reconstruction, merged- $\pi^0$  resolution, ...

The  $\pi^0$  efficiency can be improved with combined control samples ( $\tau$ -3prong and  $\eta$  decays), and better signal simulation

# Backgrounds & unfolding

## ➤ Background:

- Non- $\tau$  background should be studied in a data-driven method
- $\tau\tau$  background, mainly from  $h2\pi^0$ , depends strongly on  $\pi^0$  reconstruction,  $h2\pi^0$  invariant mass distribution and intermediate states, can be studied with data

## ➤ Unfolding:

- $\pi^-$  momentum and resolution calibration
- $\pi^0$  energy and resolution calibration
- A few unfolding softwares are available

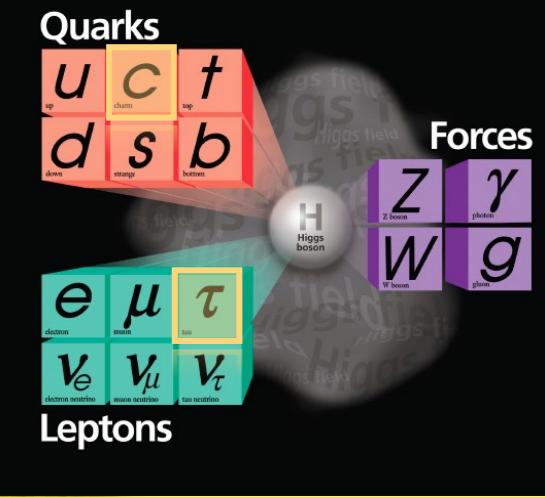
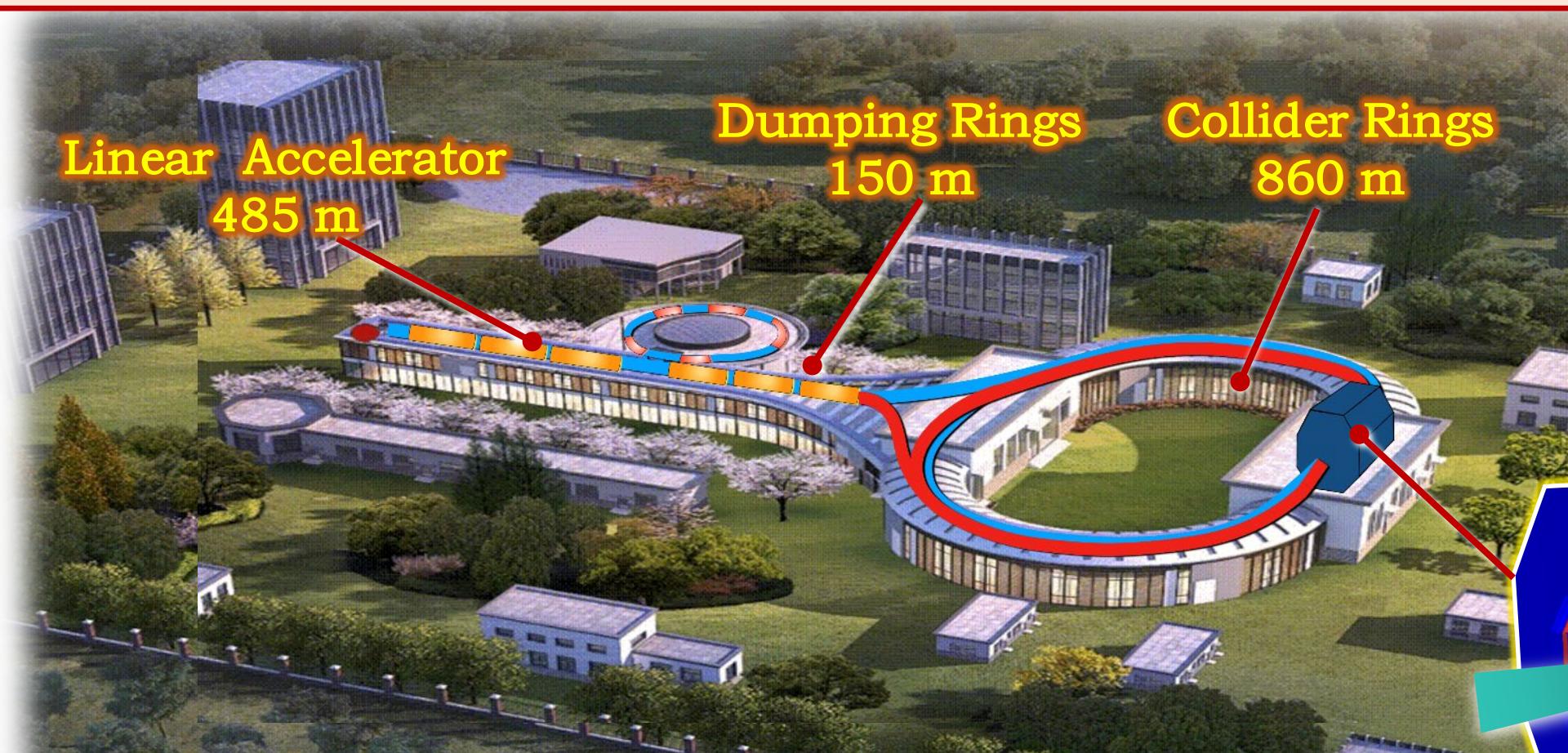
$\tau^- \rightarrow \pi^-\pi^0\nu_\tau$  decay at STCF

(slides from Xiao-Rong Zhou of USTC)

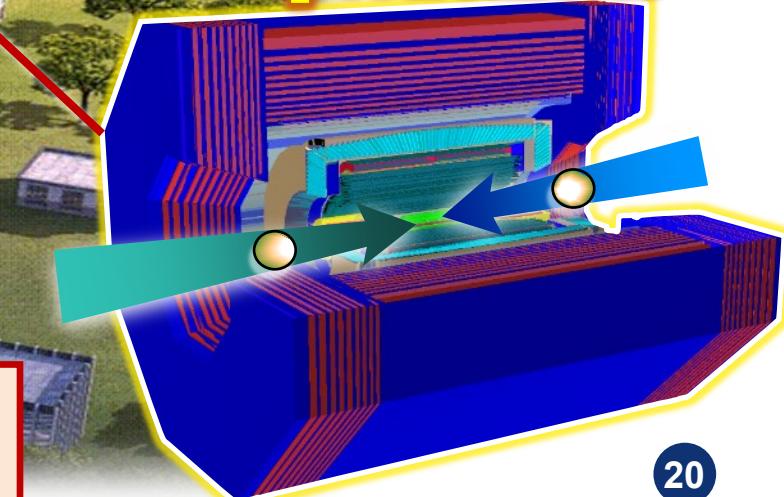
# Super Tau-Charm Facility (STCF) in Hefei, China



A factory producing massive **tau lepton** and **charm hadrons**, to unravel the mystery of how quarks form matter and the symmetries of fundamental interactions



New generation Spectrometer



- $E_{cm} = 2\text{-}7 \text{ GeV}$ ,  $\mathcal{L} > 0.5 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1}$
- Site: Hefei's suburban "Future Big Science City"

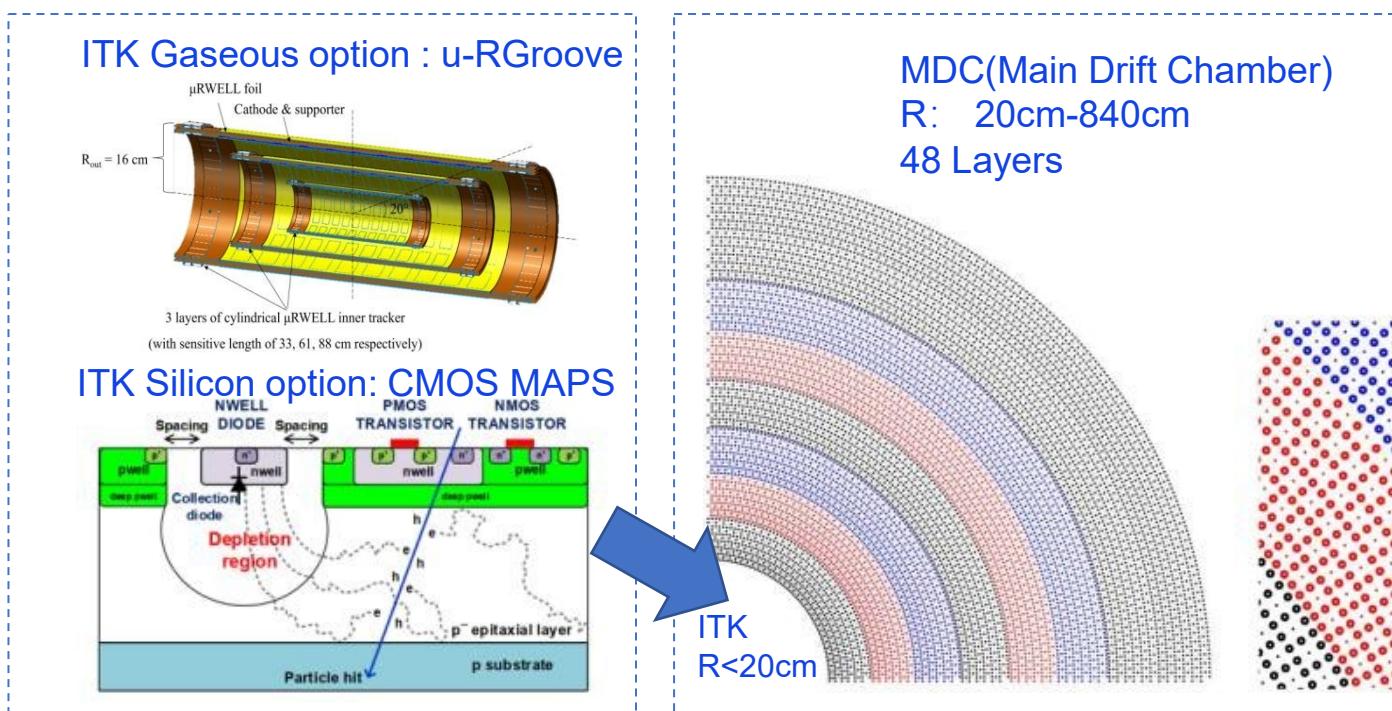
# Expected $\tau$ samples at STCF

- About 1  $\text{ab}^{-1}$  integrated luminosity at STCF per year
- Tau pairs at STCF produced back-to-back in center-of-mass system
- Operating at different energy, from tau pair threshold, to the peak cross section in 4-5 GeV

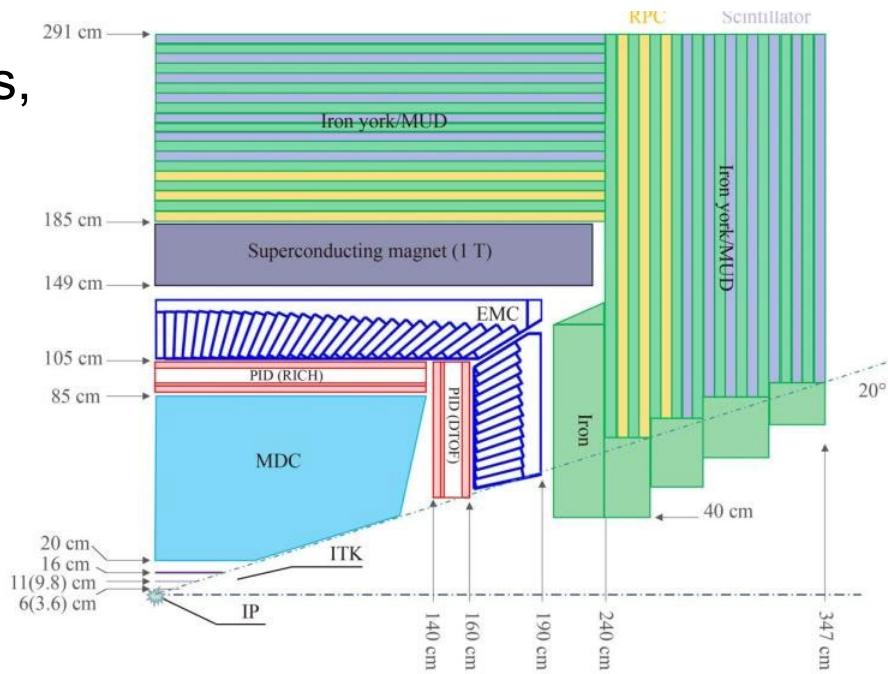
Exp.	Lum.	$\sqrt{q}$	Number of tau pairs
ALEPH	200 $\text{pb}^{-1}$	91.2 GeV	$3.3 \times 10^5$ reconstructed
BaBar	467 $\text{fb}^{-1}$	$\sim 10.58$ GeV	$4.3 \times 10^8$
Belle	988 $\text{fb}^{-1}$	$\sim 10.58$ GeV	$9.1 \times 10^8$
Belle II (by 2040s)	50 $\text{ab}^{-1}$	$\sim 10.58$ GeV	$4.6 \times 10^{10}$
BESIII	$\sim 35 \text{ fb}^{-1}$	threshold to 4.95 GeV	$\sim 1.2 \times 10^8$
STCF (expected)	1 $\text{ab}^{-1}/\text{year}$	4-5 GeV	$3.5 \times 10^9$

# Tracking System: MDC+ITK

- Works in a **1T magnetic field**
- **MDC**: main tracker with large detection volume range 48 layers, 4 stereo super-layers, 4 axial super layers
- **ITK**: 3 layers of detectors with high counting rate capability
  - Placed in the area close to the beam pipe (3-20 cm)
  - Two options: MPGD/MAPS



STCF CDR (arXiv: 2303.15790)

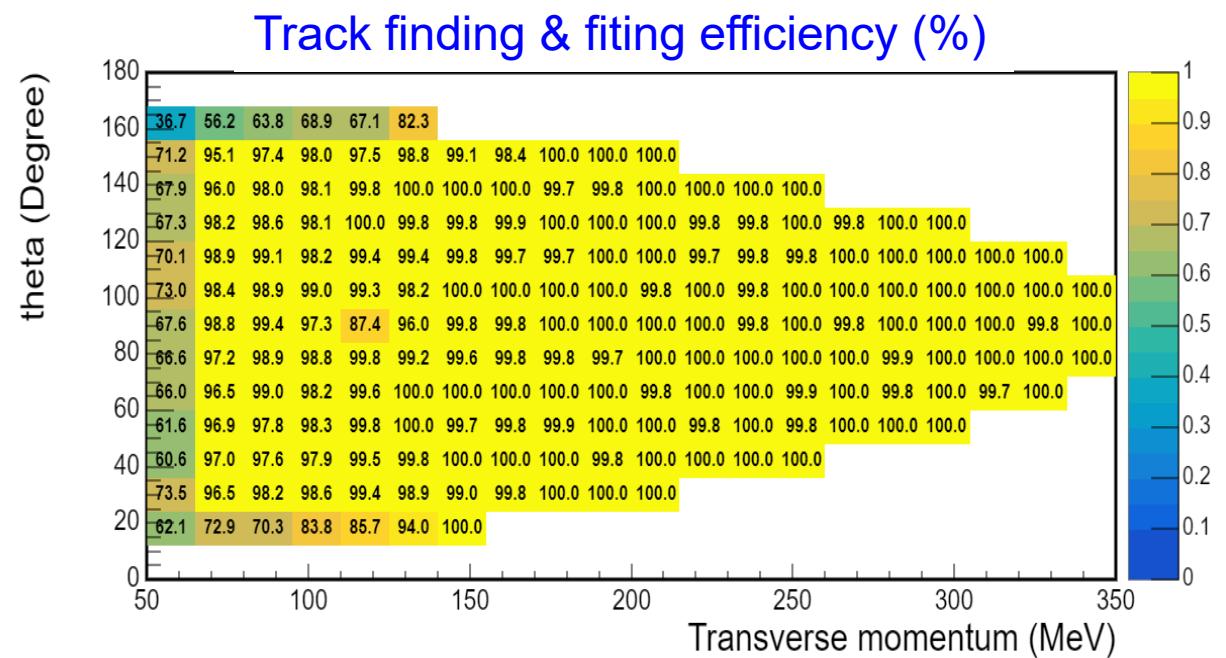
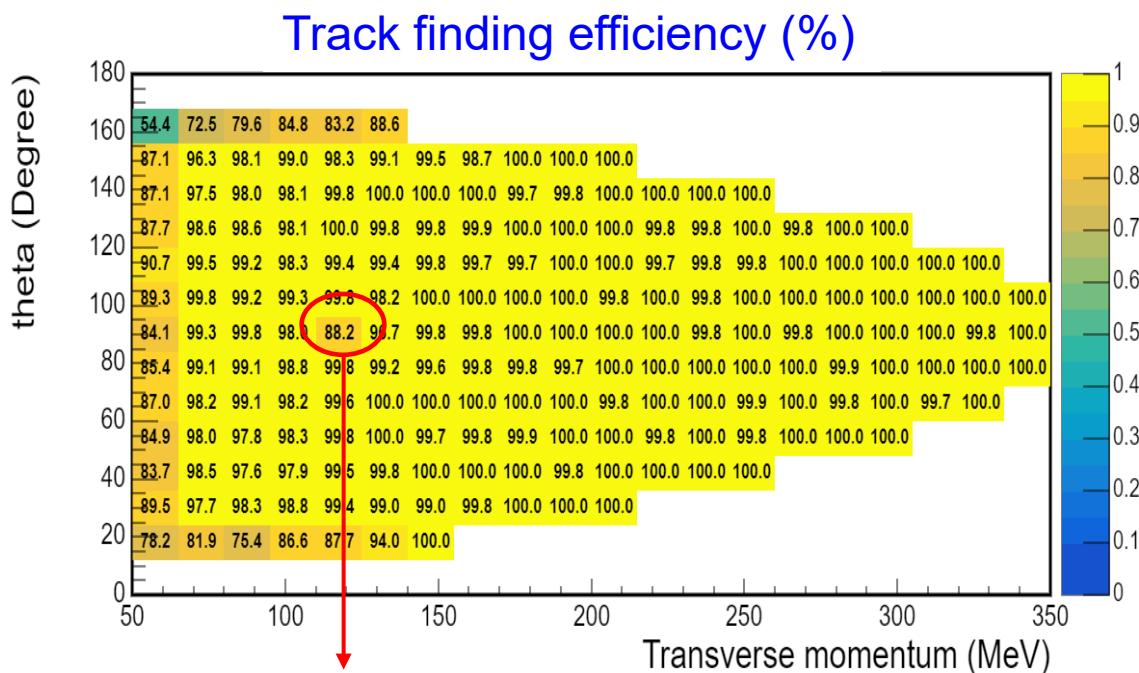
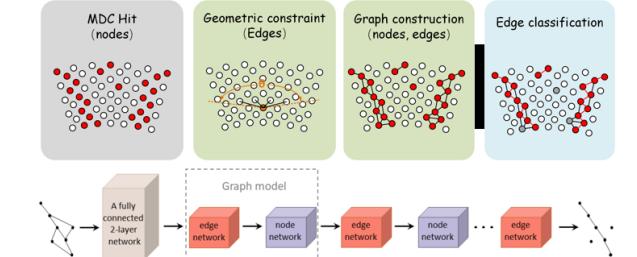
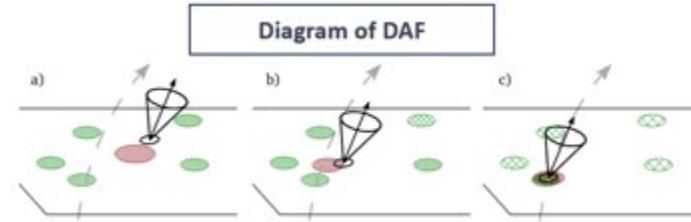
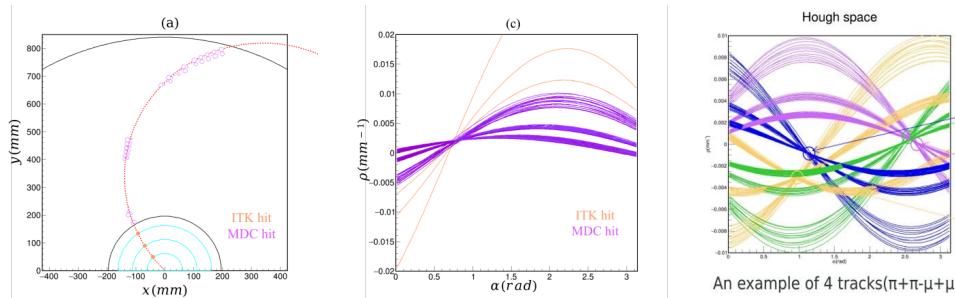


Superlayer	Radius (mm)	Num. of Layers	Stereo angle (mrad)	Num. of Cells	Cell size (mm)
A	200.0	6	0	128	9.8 to 12.5
U	271.6	6	39.3 to 47.6	160	10.7 to 12.9
V	342.2	6	-41.2 to -48.4	192	11.2 to 13.2
A	419.2	6	0	224	11.7 to 13.5
U	499.8	6	50.0 to 56.4	256	12.3 to 13.8
V	578.1	6	-51.3 to -57.2	288	12.6 to 14.0
A	662.0	6	0	320	13.0 to 14.3
A	744.0	6	0	352	13.3 to 14.5
total	200 to 827.3	48		11520	

A further optimization of the tracking system is ongoing.

# Tracking System: MDC+ITK

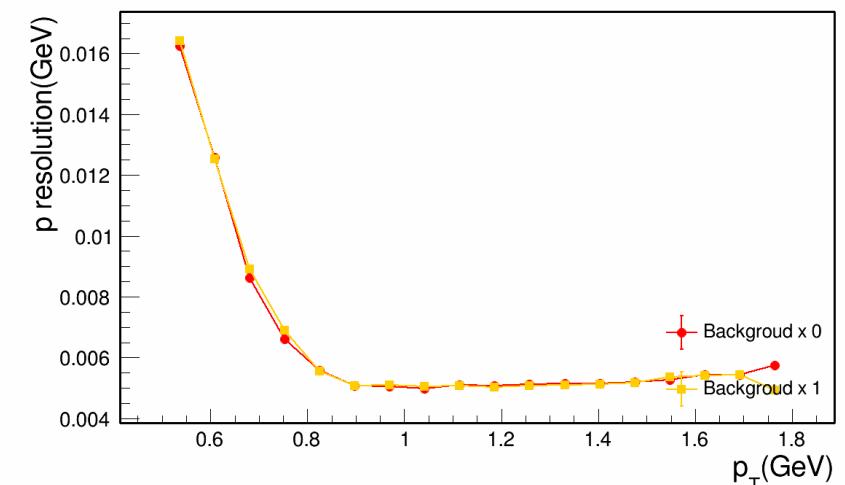
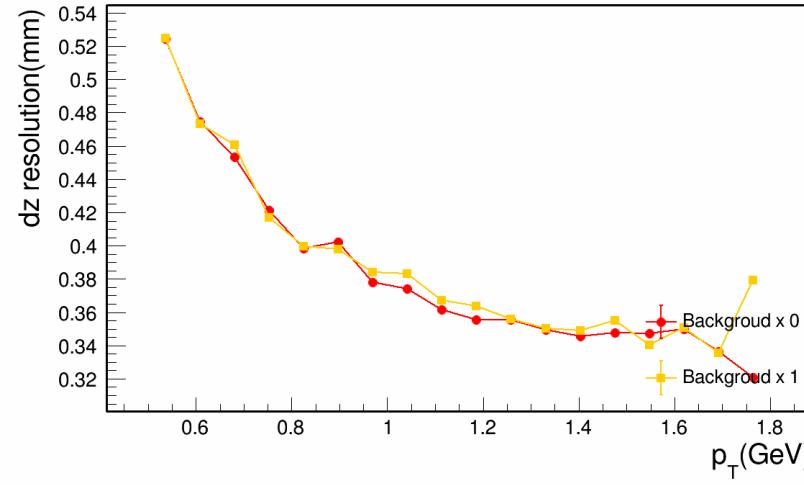
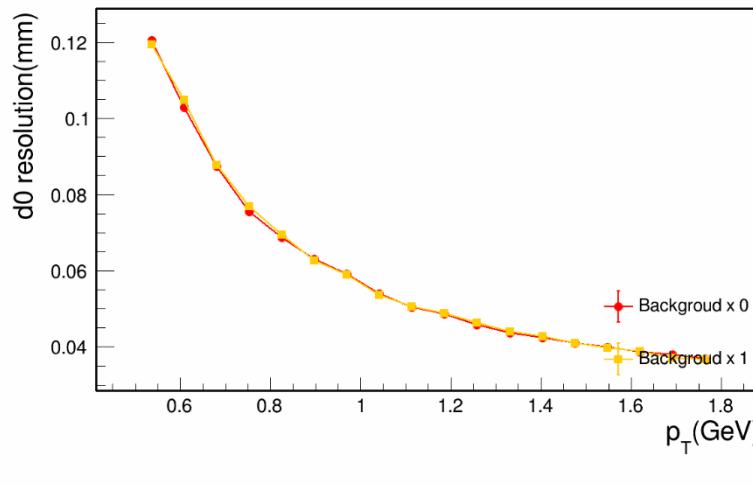
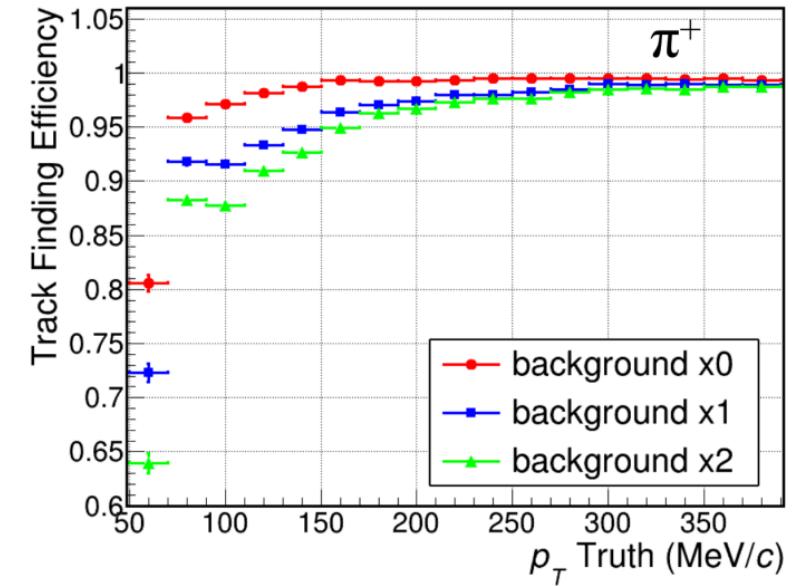
- Tracking reconstruction algorithms: Hough transform + DAF in Genfit2 + GNN



Circling tracking at  $z=0$ , difficult to distinguish the charge

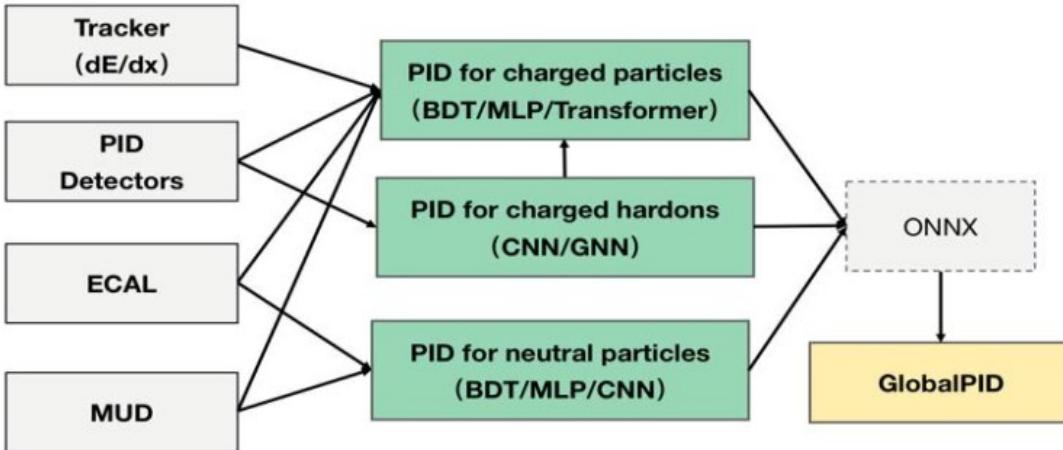
# Tracking System: MDC+ITK

- Beam-induced background from MC simulation: beam-beam interaction, beam-gas interaction, etc.
- Tracking efficiency:
  - ~ 90% @ 100 MeV
  - ~ 99% @ 300 MeV
- Impact parameters and momentum resolution:
  - d0 resolution  $\sim 40 \mu\text{m}$ , dz resolution  $\sim 400 \mu\text{m}$
  - Momentum resolution better than 0.5% at 1 GeV/c



# Particle Identification

- Two methods for PID
- A global PID algorithm based on machine learning



- A combined PID algorithm with weighted likelihood

$$\text{Combined likelihood } \mathcal{L}_h = \prod_{det} \mathcal{L}_{det,h}$$

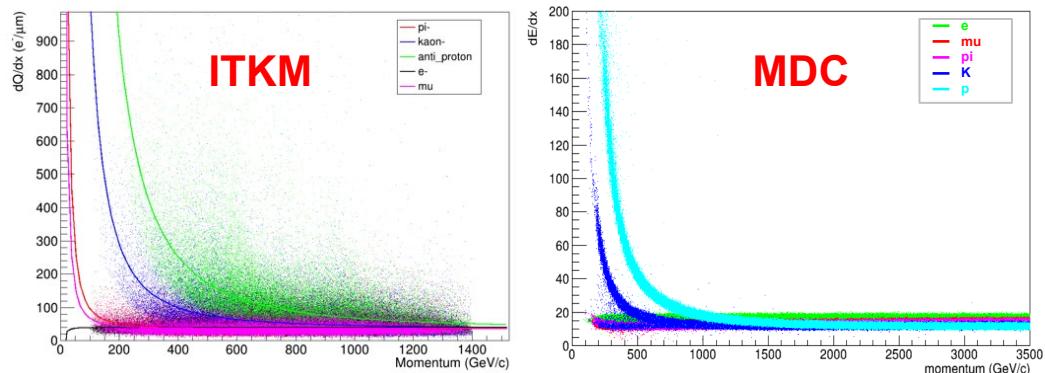
$$\text{Likelihood ratio } p(h) = \frac{P_{\text{prior},h} \mathcal{L}_h}{\sum_h P_{\text{prior},h} \mathcal{L}_h}$$

Detector	Feature	Description
Track	'Charge'	Reconstructed particle charge
	'Mom_x', 'Mom_y', 'Mom_z'	Particle momentum along the x, y, and z directions
	'HelixPar_d0'	Distance between the reference point and the point ( $p_0$ ) on the helix closest to it in the $x-y$ plane
	'HelixPar_phi'	Azimuthal angle of the line connecting the helix circle center and the reference point in the $x-y$ plane
	'HelixPar_cpa'	Inverse of the track transverse momentum, with the sign determined by the charge of the track
	'HelixPar_z0'	$z$ coordinate of the point $p_0$ on the helix closest to the reference point in the $x-y$ plane
	'HelixPar_tanl'	Helix slope ( $\frac{p_z}{p_t}$ )
BTOF	'Likelihood_e'	Likelihood assuming the particle is an electron
	'Likelihood_mu'	Likelihood assuming the particle is a muon
	'Likelihood_k'	Likelihood assuming the particle is a kaon
	'Likelihood_pi'	Likelihood assuming the particle is a pion
	'Likelihood_p'	Likelihood assuming the particle is a proton
dEdX	'dEdXsep'	$\chi^2$ values under five particle hypotheses
ECAL	'NumHits'	Number of hits in the ECAL
	'Energy'	Reconstructed particle energy
	'ESeed'	Energy of the seed crystal
	'E3x3'	Local shower energy in $3 \times 3$ crystal around the seeding crystal
	'E5x5'	Local shower energy in $5 \times 5$ crystal around the seeding crystal
	'Position_x', 'Position_y', 'Position_z'	Spatial coordinates of the shower
	'SecondMoment'	Second matrix
	'LateralMoment'	Lateral matrix
	'ZernikeMoment{2,0}', 'ZernikeMoment{4,2}'	Zernike matrix ( $2 \times 0$ and $4 \times 2$ )
MUD	'phi'	Polar angle
	'phi'	Azimuthal angle in the $x-y$ plane
	'HitNum'	Number of hits in the MUD
	'RPCHitNum'	Number of hits in the RPC
	'PSHitNum'	Number of hits in the plastic scintillator detector
	'MaxHit'	Number of hits in the layer with the maximum occupancy
	'MaxHitLayer'	Index of the layer with the maximum number of hits
	'MudExp'	Probability of being identified as a muon
	'IsMuon'	Muon identification result (1 for muon, 0 otherwise)
DTOF	'logL_e'	Log-likelihood under the electron hypothesis
	'logL_mu'	Log-likelihood under the muon hypothesis
	'logL_pi'	Log-likelihood under the pion hypothesis
	'logL_k'	Log-likelihood under the kaon hypothesis
	'logL_p'	Log-likelihood under the proton hypothesis

Table 1: Detector features employed in global particle identification.

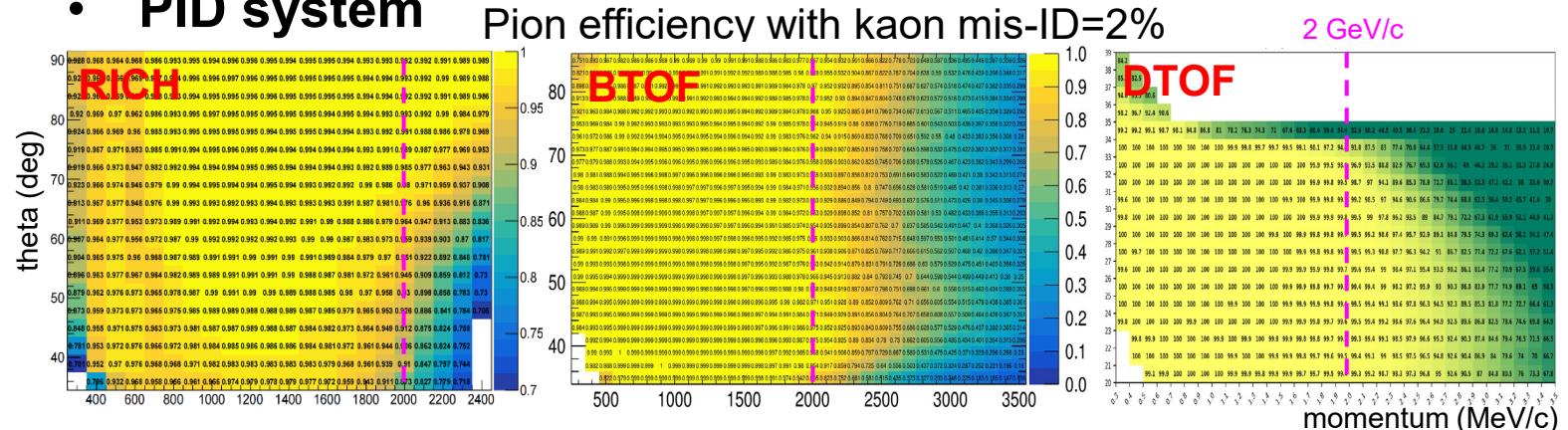
# Subdetector PID performance

- **dE/dx**



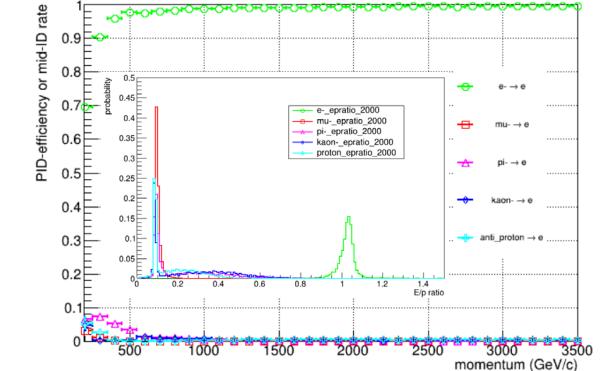
- **pi/K:**  $>3\sigma$  at  $p < 750$  MeV/c,  $>2\sigma$  at  $p > 2$  GeV/c
- **mu/pi:**  $>3\sigma$  at  $p < 150$  MeV/c; **e/pi:**  $>3\sigma$  except 120-250 MeV/c

- **PID system**

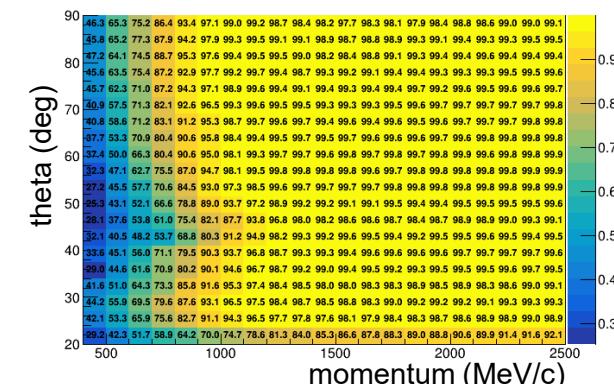


- **>97% pion efficiency with kaon mis-ID=2% at  $p < 2$  GeV/c**

- **Electron ID (ECAL)**

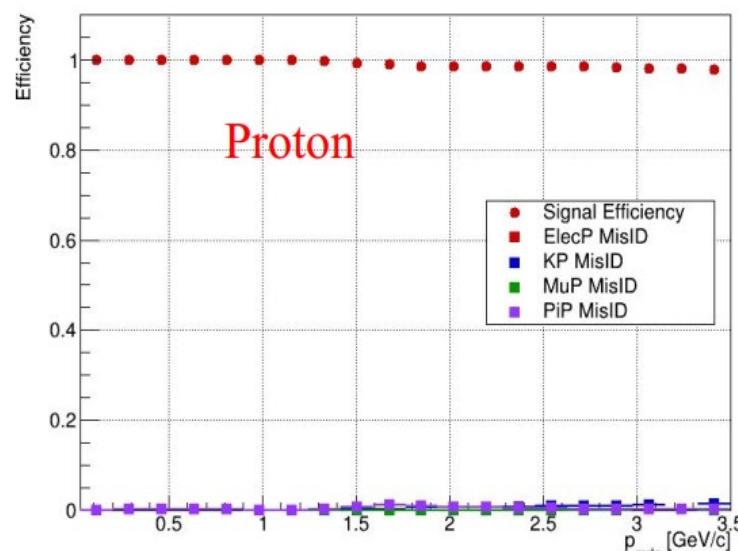
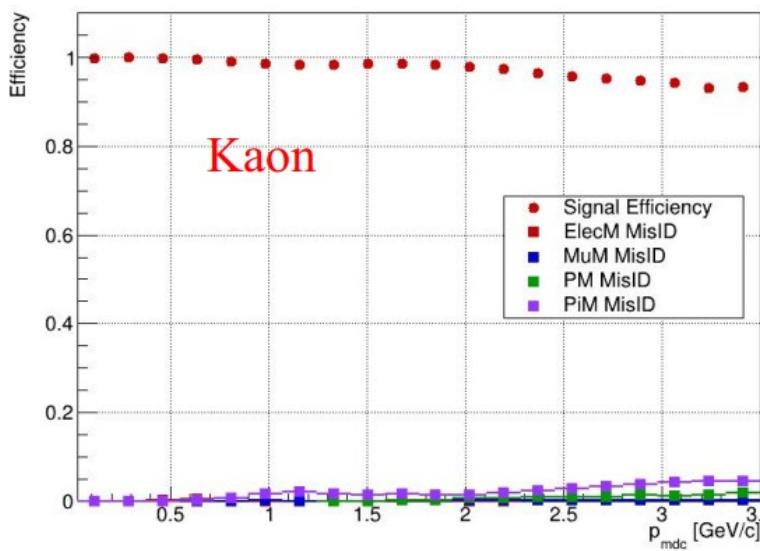
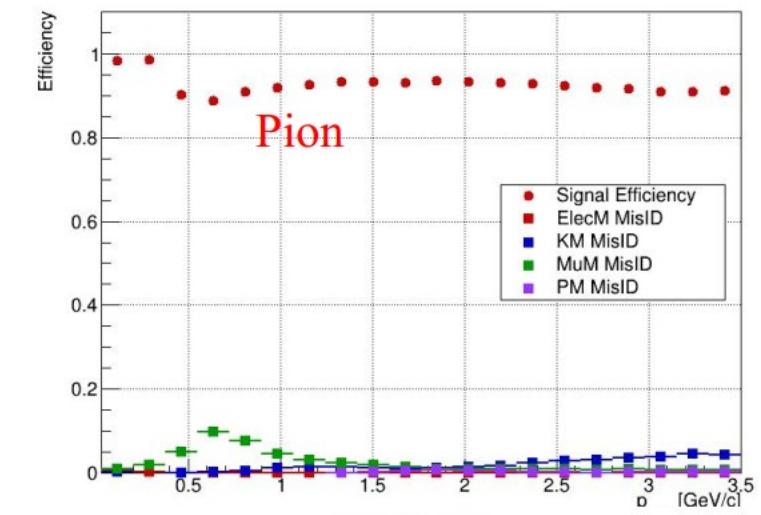
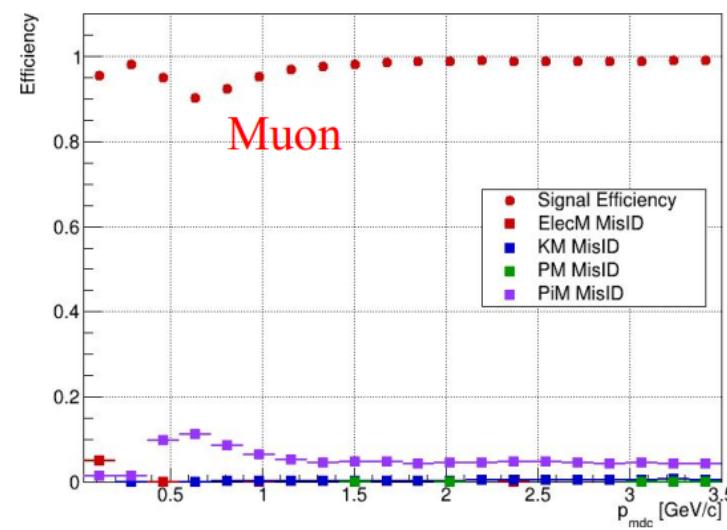
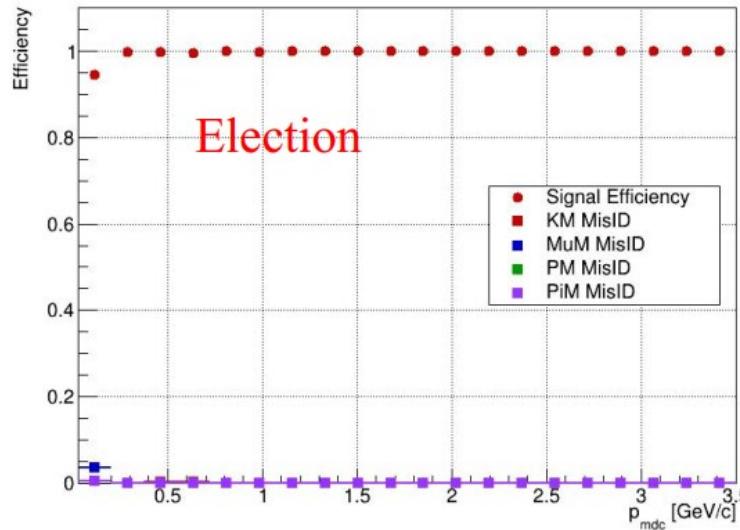


- **Muon ID (MUD&ECAL)**

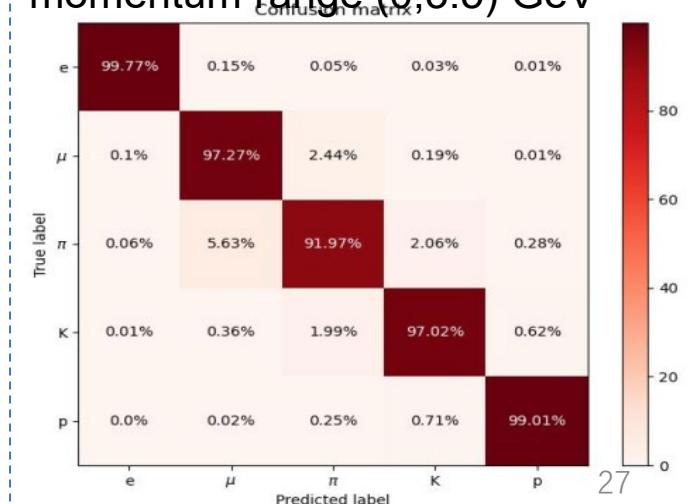


- **>95% muon efficiency with pion mis-ID=3% at  $p > 1$  GeV/c**

# Global PID performance



Overall ID/mis-ID efficiency with momentum range (0,3.5) GeV



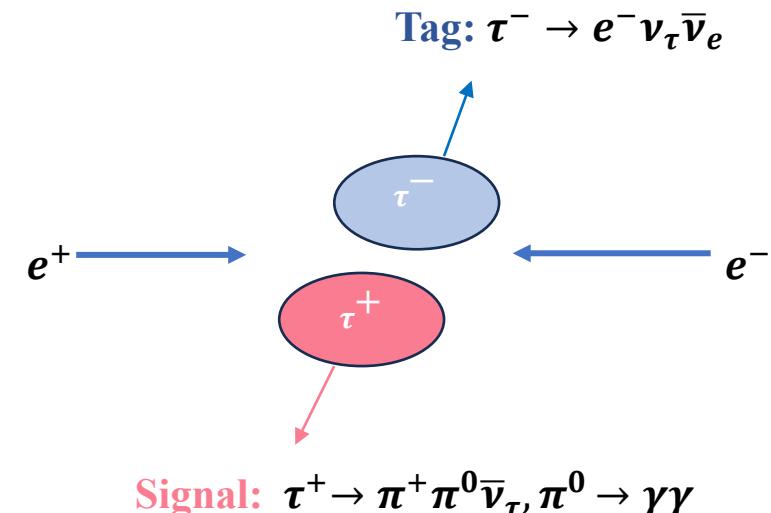
# Study of $\tau^+ \rightarrow \pi^+ \pi^0 \bar{\nu}_\tau$ at STCF

## $\tau^+ \rightarrow \pi^+ \pi^0 \bar{\nu}_\tau$ sample:

- $e^+ e^- \rightarrow \tau^+ \tau^-$  at  $\sqrt{s} = 4.260$  GeV
- Generator: madgraph5 + pythia8
- $3 \times 10^6$  events,  $\tau$  decays according to PDG BFs
- OSCAR version: 2.6.2

## Selection Criteria:

- Charge tracks:  $V_{xy} < 1$  cm,  $|V_z| < 10$  cm,  $|\cos\theta| < 0.93$
- Photon selection:  $E_\gamma > 25$  MeV in barrel;  $E_\gamma > 50$  MeV at end caps,  $0 \leq \text{TDC} \leq 14 (\times 50\text{ns})$
- $\pi^0$  reconstruction:  $\chi^2 < 200$  (1C kinematic fit)
- PID requirement: Global PID
  - e:  $Prob_e > Prob_\pi \& Prob_e > Prob_K$
  - $\pi$ :  $Prob_\pi > Prob_e \& Prob_\pi > Prob_K$

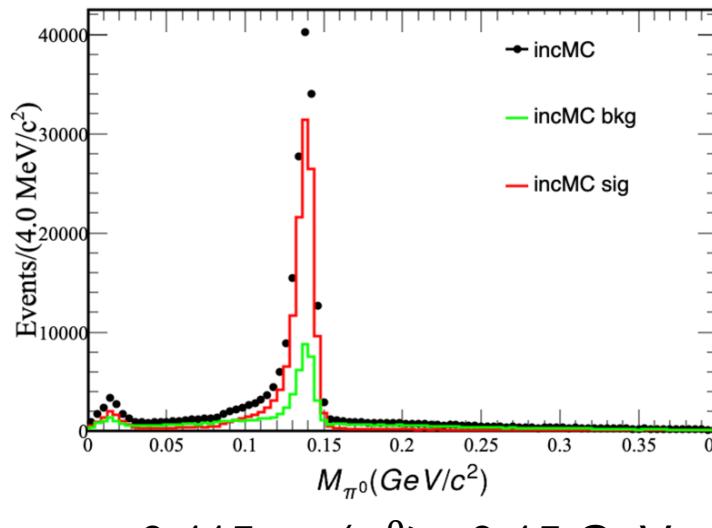


**Signal:**  $\tau^+ \rightarrow \pi^+ \pi^0 \bar{\nu}_\tau, \pi^0 \rightarrow \gamma\gamma$

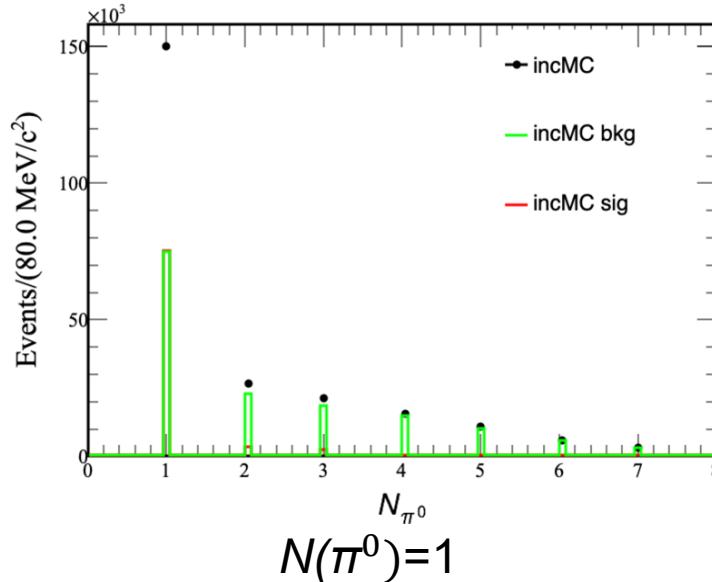
Further selection	Effi.	Bkg. ratio
$N_\pi = 1, N_e = 1, N_\gamma \geq 2$	55.7%	37.4%
$0.115 < m(\pi^0) < 0.15$	42.8%	23.5%
$0.8 < \frac{E}{P}(e) < 1.05$	37.1%	21.8%
$N(\pi^0) = 1, exN(\gamma) = 0$	33.9%	3.5%

\*Stay tuned with more constraints on vertex and ECAL time

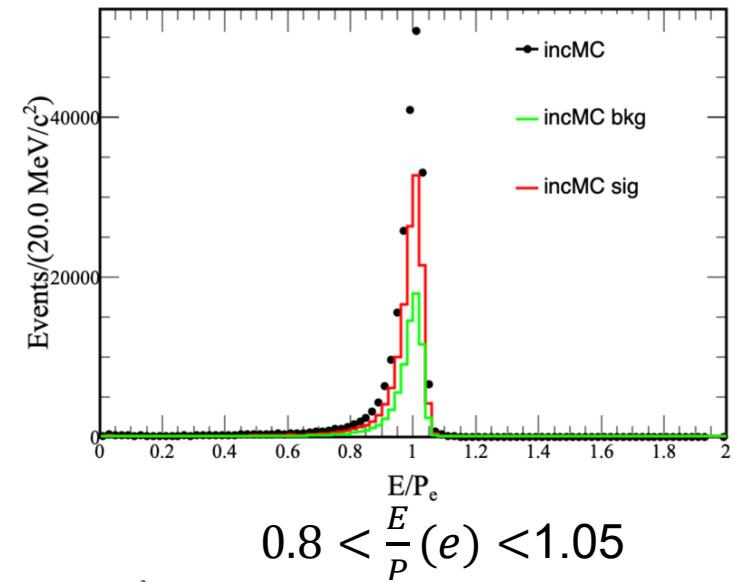
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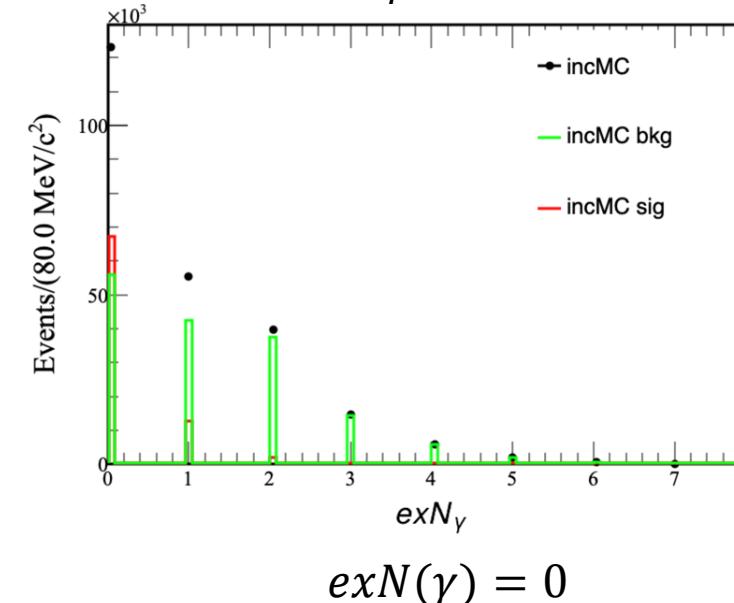
$$0.115 < m(\pi^0) < 0.15 \text{ GeV}$$



$$N(\pi^0)=1$$

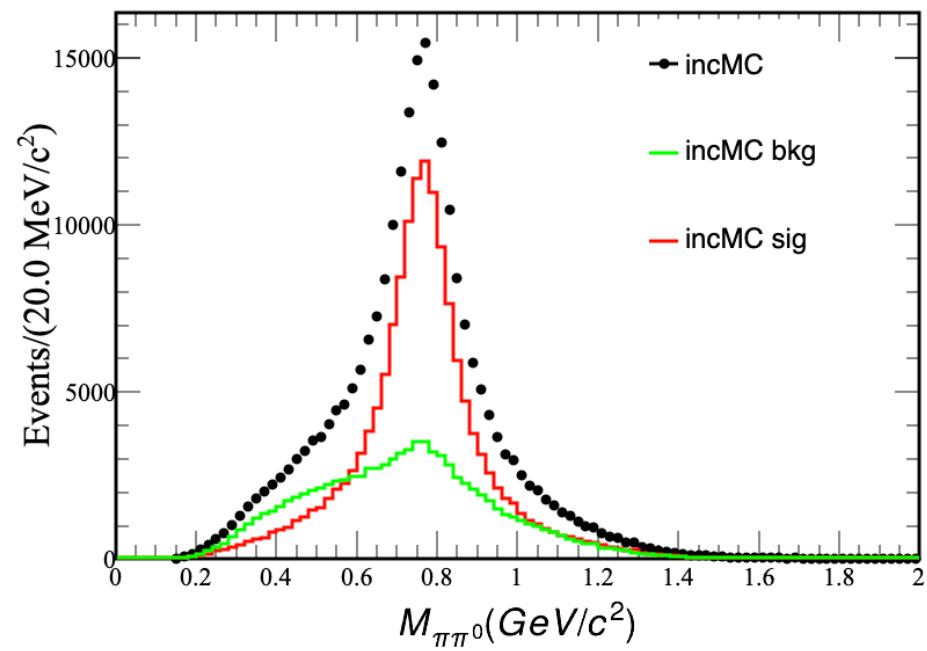


$$0.8 < \frac{E}{P}(e) < 1.05$$

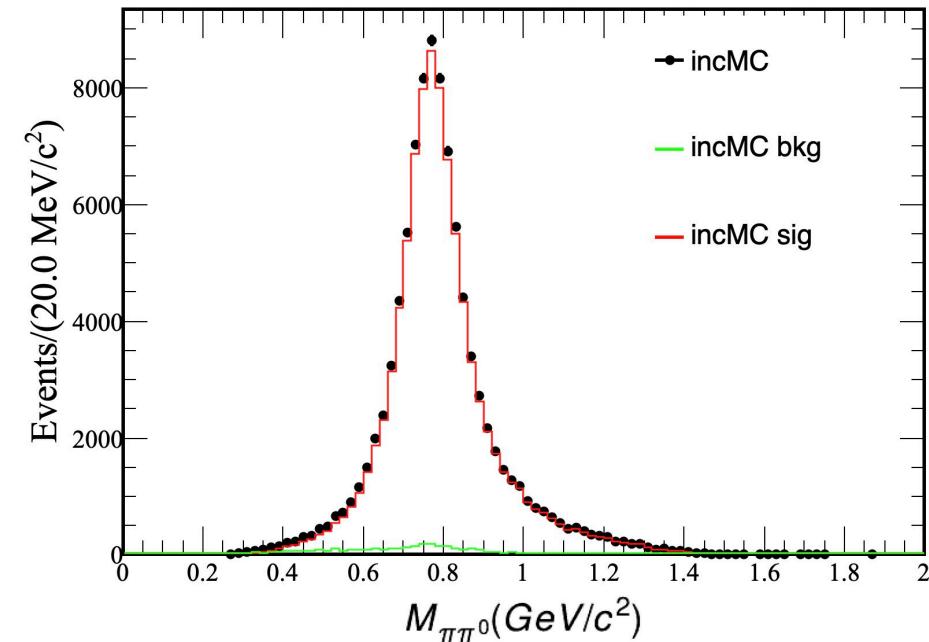


$$exN(\gamma) = 0$$

# Study of $\tau^+ \rightarrow \pi^+ \pi^0 \bar{\nu}_\tau$ at STCF

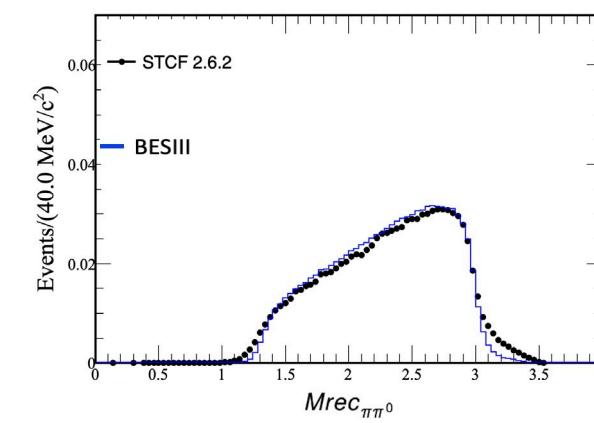
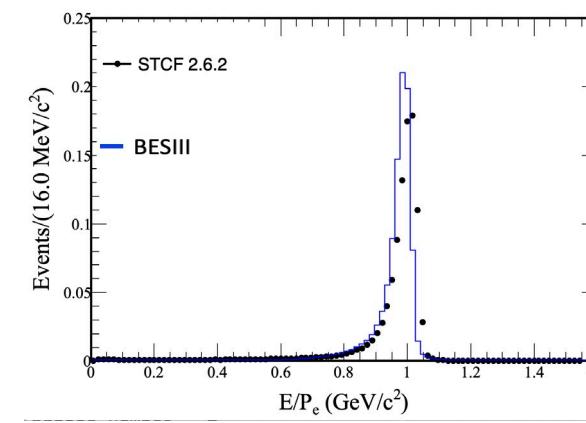
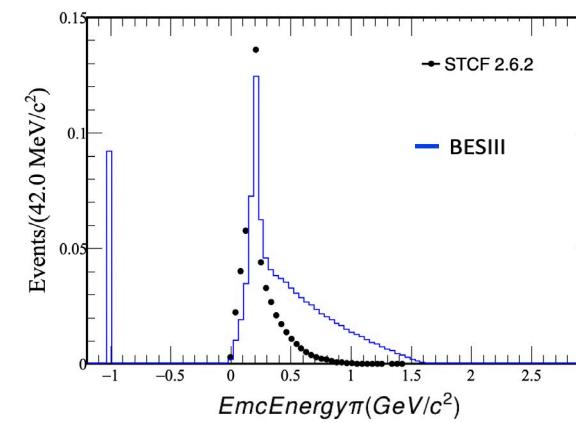
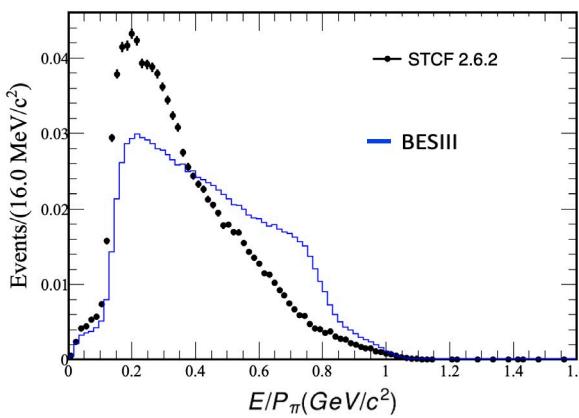
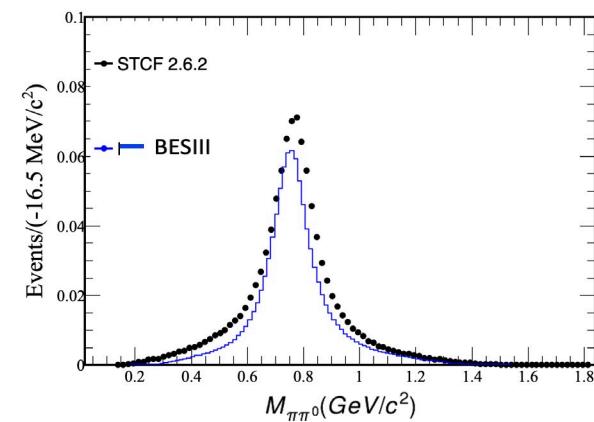
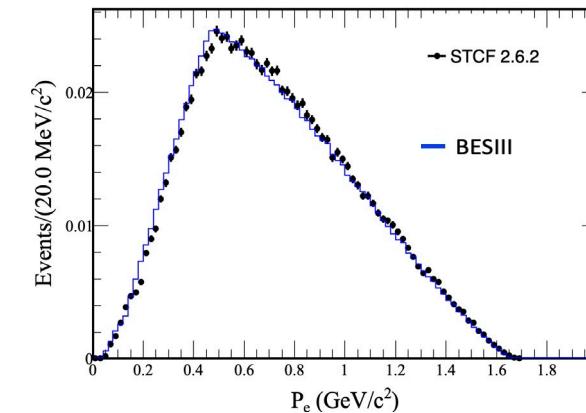
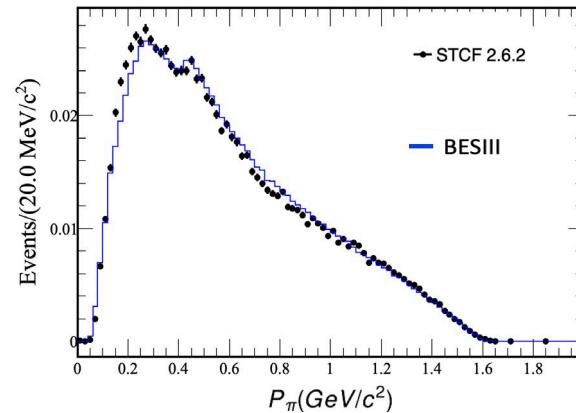
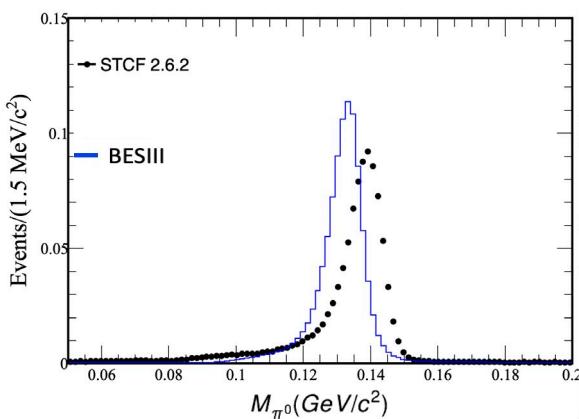


$0.115 < m(\pi^0) < 0.15 \text{ GeV},$   
 $0.8 < \frac{E}{P}(e) < 1.05,$   
 $N(\pi^0)=1, exN(\gamma) = 0$



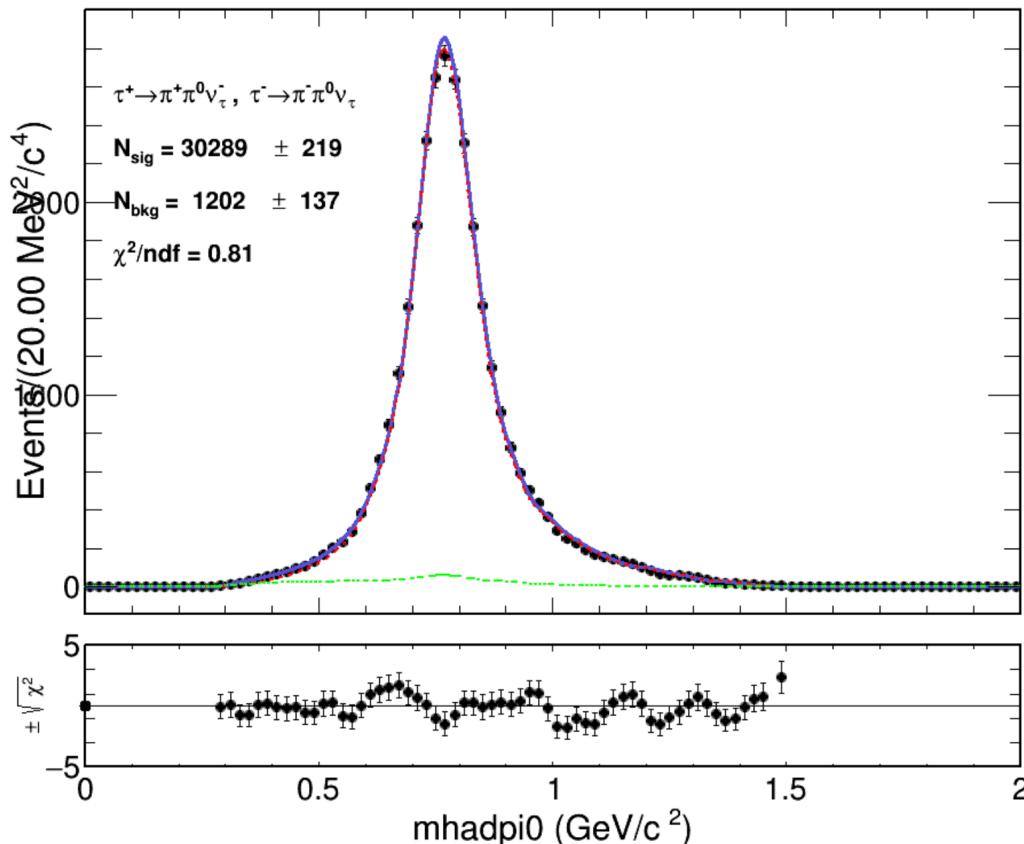
# Study of $\tau^+ \rightarrow \pi^+ \pi^0 \bar{\nu}_\tau$ at STCF

Comparison of distributions between STCF-OSCAR and BESIII-BOSS simulations



# Study of $\tau^+ \rightarrow \pi^+ \pi^0 \bar{\nu}_\tau$ at STCF

- Data:  $1 \times 10^6 \tau\tau$  inclusive MC serve as data
- Signal shape: from  $2 \times 10^6 \tau\tau$  inclusive MC
- Background shape: from  $2 \times 10^6 \tau\tau$  inclusive MC



$$\gg \mathcal{B}(\tau^+ \rightarrow \pi^+ \pi^0 \bar{\nu}_\tau) = \frac{N_{\text{sig}}}{2N_{\tau\tau}\varepsilon B(\tau^- \rightarrow e^- \nu_\tau \bar{\nu}_e)B(\pi^0 \rightarrow \gamma\gamma)}$$

	data volume	stat.
STCF (4.260 GeV)	1,000,000 MC	0.21(0.83%)
	Estimated with 1 ab <sup>-1</sup> luminosity	0.0036(0.01%)
ALEP	--	0.097(0.38%)
Belle	--	0.01(0.04%)

## Plan:

- Detailed background estimation
- Further optimization of the event selection
- Estimation of the systematic uncertainties from calibrations of momentum, energy, resolutions, etc.
- Measure the spectral functions for g-2 input

# Summary

- BF & SF of  $\tau^- \rightarrow \pi^-\pi^0\nu_\tau$  can be measured at Belle II and STCF
- No solid estimation on the final precision Belle II can achieve
  - New techniques may be developed for photon-ID and  $\pi^0$  reconstruction
  - Lots of efforts needed to understand all the sources of systematic uncertainties reliably
  - May extend the analysis to more Belle II data
- Potential for improved measurements at STCF
  - Constraints on detector design
  - Charm and other non-tau backgrounds

Thank you very much!

