Data-driven FF related isospin corrections

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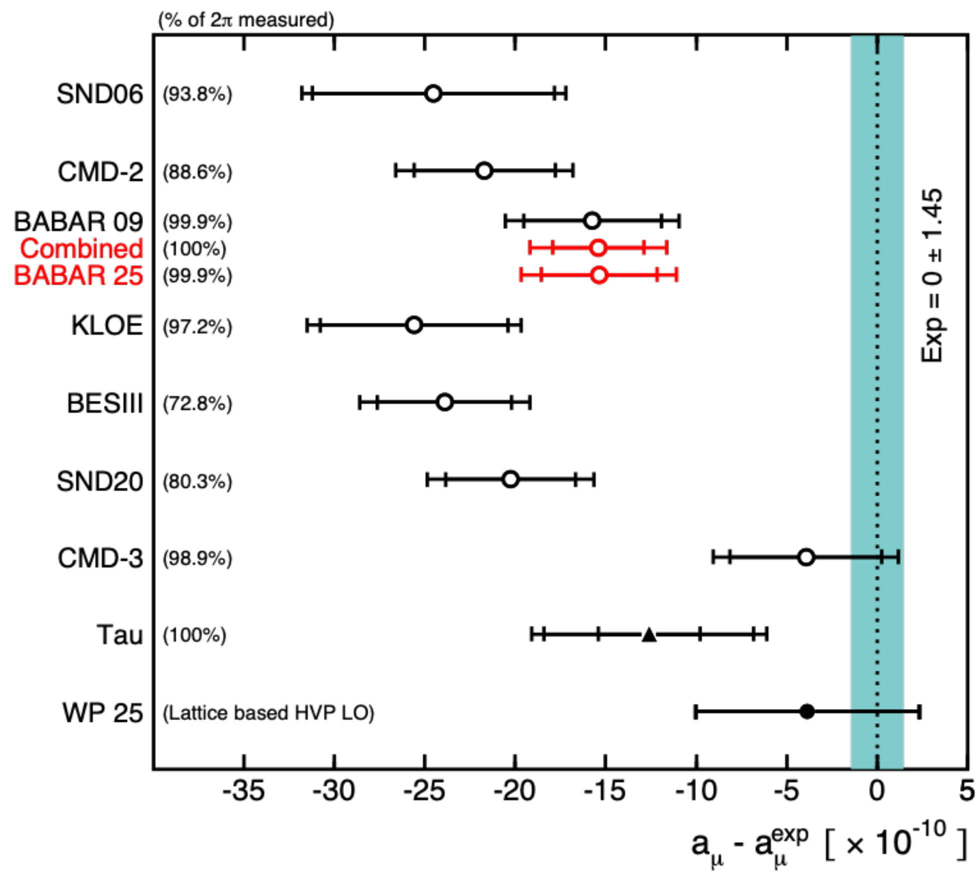
Outline

- Introduction
- Mass and width between charged and neutral rho mesons
- Impact on the isospin correction
- Summary

Based on arXiv:2504.13789 + additional studies

Introduction

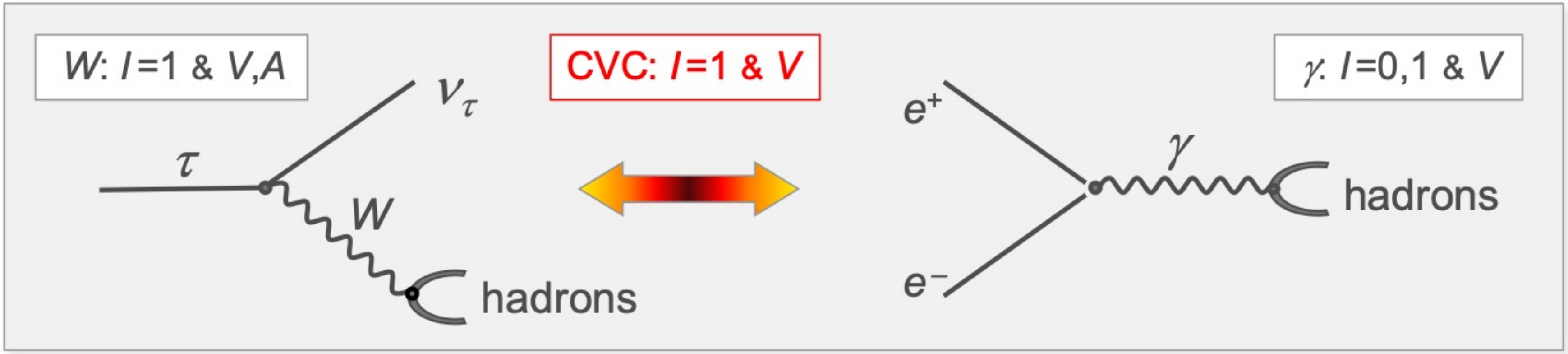
- Large discrepancy among different 2π measurements currently unresolved
- It's important to have independent and precise measurement to clarify the situation
- Tau data provide such an alternative
- Need isospin breaking corrections
- The uncertainties of model dependent isospin breaking corrections were inflated in WP2025 following recommendations from theorists
  ➔ unsatisfactory solution
- In arXiv:2504.13789, we proposed data-driven form-factor related isospin corrections to compare
- Will show some additional studies since then



- Inner error: uncertainty of the 2π channel <1.8 GeV (non-100% part from combined 2π data)
- Outer one: uncertainty of other components
- Intermediated one for tau: inflated uncertainty of isospin breaking corrections

Original proposal to use tau data

Proposed by Alemany-Davier-Hoecker (ADH), EPJC 2 (1998) 123



Hadronic physics factorises in Spectral Functions:
Isospin symmetry connects I=1 e^+e^- cross section to vector τ spectral functions

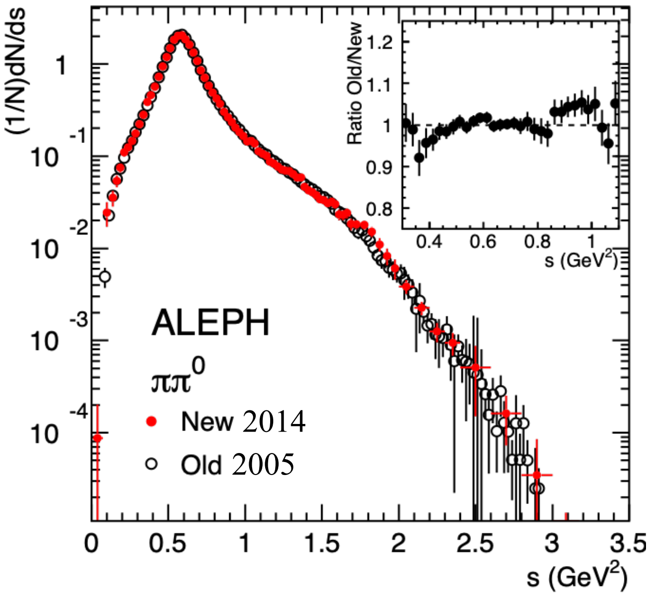
Fundamental ingredient relating long distance (resonances) to short distance description (QCD)

$$\sigma^{(I=1)}\left[e^+e^- \to \pi^+\pi^-\right] = \frac{4\pi\alpha^2}{s}\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]$$

$$\upsilon\left[\tau^- \to \pi^-\pi^0\nu_\tau\right] \propto \frac{\mathrm{BR}\left[\tau^- \to \pi^-\pi^0\nu_\tau\right]}{\mathrm{BR}\left[\tau^- \to e^-\bar{\nu}_e\nu_\tau\right]} \frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds} \frac{m_\tau^2}{\left(1-s/m_\tau^2\right)^2\left(1+s/m_\tau^2\right)}$$

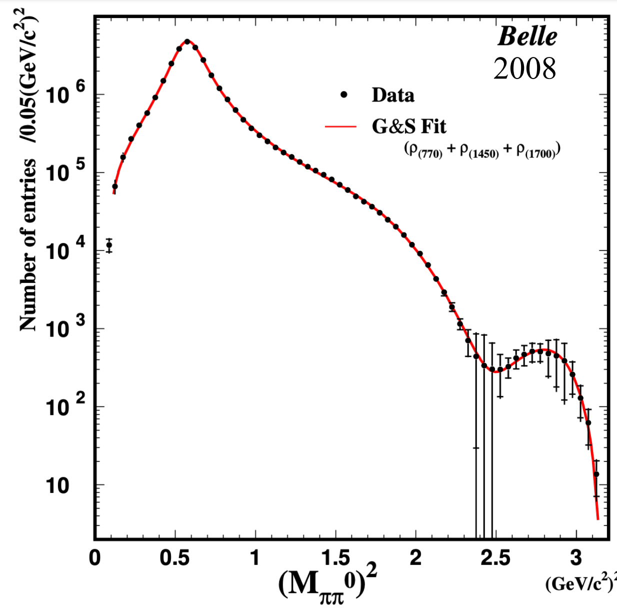
Branching fractions — Mass spectrum — Kinematic factors (PS)

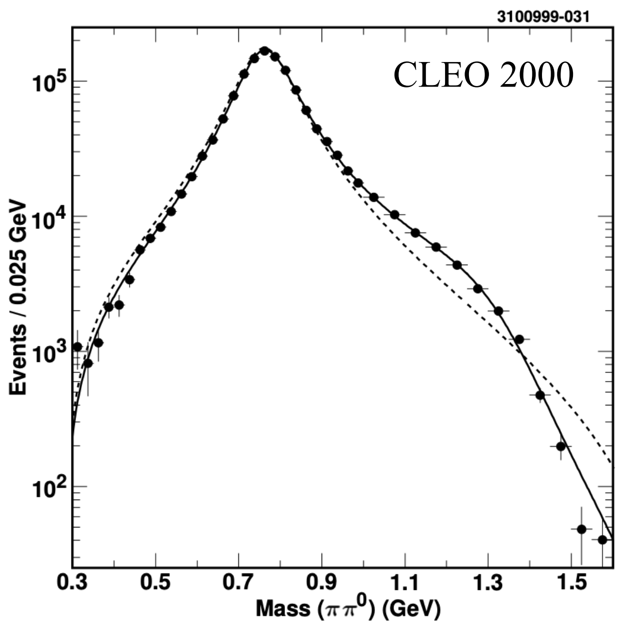
Tau data (ALEPH, Belle, CLEO, OPAL)



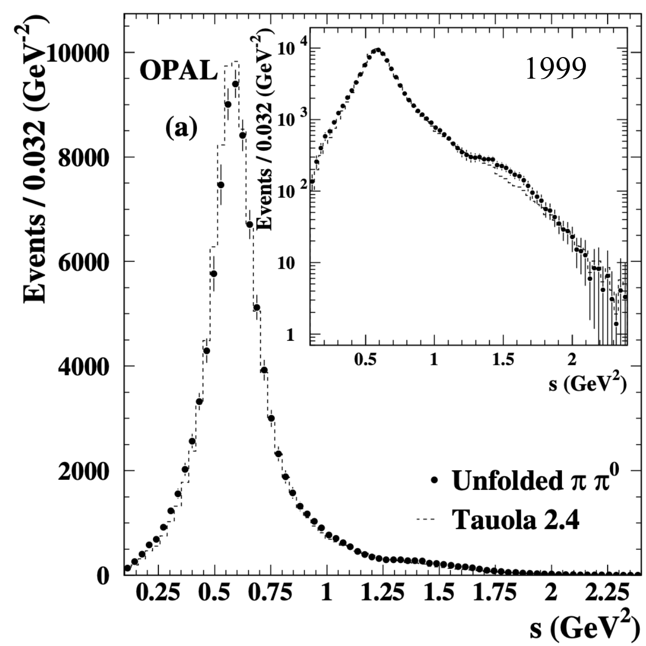
SFs measured under very different exp. conditions

ALEPH had the best branching fraction (normalisation) measurement, while Belle had the best shape measurement

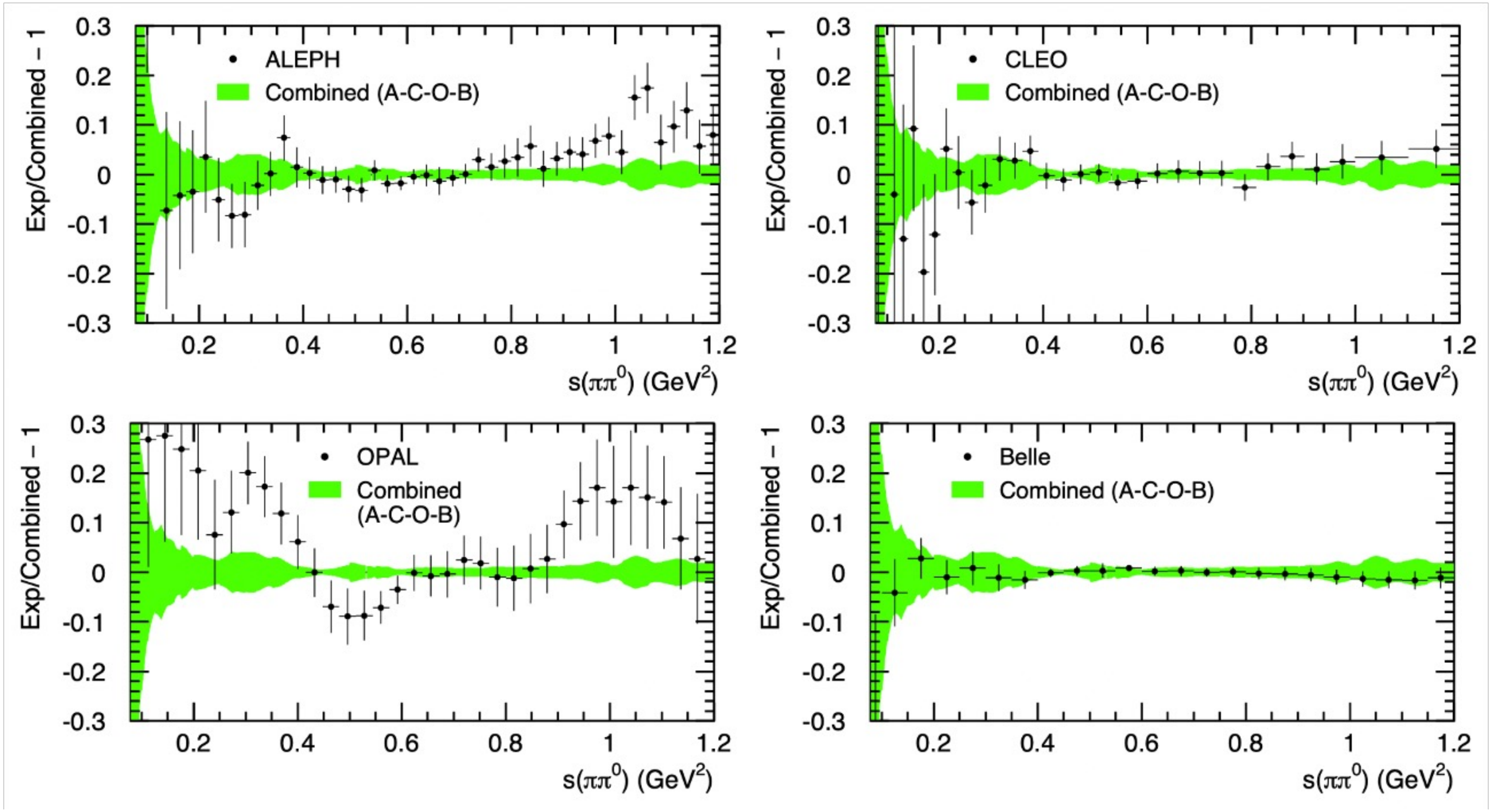




ALEPH: Phys. Rept. 421 (2005) 191, Eur. Phys. J. C 74 (2014) 2803
Belle: Phys. Rev. D 78, 072006 (2008)
CLEO: Phys. Rev. D 61, 112002 (2000)
OPAL: Eur. Phys. J. C 7, 571 (1999)



Comparison of the tau data



➔ Good agreement given the uncertainties, so the measurements can be combined to get the combined shown in the green error band

Isospin Breaking (IB) Corrections

- Before being used for HVP, tau SFs need to be applied with Isospin Breaking (IB) corrections:

$$v_{1,\pi^-\pi^0\nu_\tau} \propto \frac{B_{\pi\pi^0}}{B_e} \frac{dN_{\pi\pi^0}}{N_{\pi\pi^0} ds} \frac{m_\tau^2}{(1-s/m_\tau^2)^2 (1+2s/m_\tau^2)} \frac{R_{\rm IB}(s)}{S_{\rm EW}}$$

$$\frac{R_{\rm IB}(s)}{S_{\rm EW}} \quad \text{with} \quad R_{\rm IB}(s) = \frac{{\rm FSR}(s)}{G_{\rm EM}(s)} \frac{\beta_0^3(s)}{\beta_-^3(s)} \left| \frac{F_0(s)}{F_-(s)} \right|^2$$

- Energy independent short-distance EW correction (leading one) $S_{\rm EW}$
- Energy dependent corrections:
  - (Small) long-distance radiation correction $G_{\rm EM}$(s)
  - Final State Radiation correction FSR(s)
  - Neutral/charged pion mass difference $\beta_{0/-}$ ratio term
  - Neutral/charged rho mass & width differences $F_{0/-}$ ratio term

WA2025 IB corrections and uncertainties

Table 2 taken from WP2025

		Refs. [168, 196]	Ref. [211]	Refs. [239, 249]	Our estimate
Phase space		−7.88	−7.52	–	−7.7(2)
S_{EW}		−12.21(15)	−12.16(15)	–	−12.2(1.3)
G_{EM}		−1.92(90)	$-1.67^{+0.60}_{-1.39}$	–	−2.0(1.4)
FSR		4.67(47)	4.62(46)	4.42(4)	4.5(3)
ρ–ω mixing		4.0(4)	2.87(8)	3.79(19)	3.9(3)
$\frac{F_\pi^V}{f_+}$ (w/o ρ–ω)	ΔM_ρ	$0.20(^{+27}_{-19})(9)$	$1.95^{+1.56}_{-1.55}$	–	
	$\Delta\Gamma_\rho(\Delta M_\pi)$	4.09(0)(7)	3.37	–	
	$\Delta\Gamma_\rho(\pi\pi\gamma)$	−5.91(59)(48)	−6.66(73)	–	
	$\Delta\Gamma_\rho(g_{\rho\pi\pi})$	–	–	–	
	Total	−1.62(65)(63)	$(-1.34)^{+1.72}_{-1.71}$	–	−1.5(4.7)
Sum		−14.9(1.9)	$(-15.20)^{+2.26}_{-2.63}$	–	−15.0(5.1)

The largest uncertainty is from the FF ratio terms

Data-driven Δm_ρ and $\Delta\Gamma_\rho$ determination

- Given that FF ratio may be parameterised with Δm_ρ and $\Delta\Gamma_\rho$, the latter are determined from data
- To avoid the circularity problem with a_μ, m_ρ and Γ_ρ are determined from the shape of the relevant FF by decoupling it from its normalisation
- In arXiv:2504.13785, the nominal fit is based on the GS (Gounaris-Sakurai) parameterisation, the KS (Kuhn-Santama) parameterisation is used for systematic variation
- BABAR data covers the largest energy range 0.3−3.0 GeV, it is used to fit 20 free parameters (see next slide)
- The other ee data below 0.9 GeV are fitted with 7 free parameters by setting the other high mass parameters to those of BABAR
- The tau data has no $\rho-\omega$ interference and the data below 0.9 GeV are fitted with only 3 free parameters by setting the other parameters also to those of BABAR (assuming no IB corrections for the high mass resonances)

Fitted parameters and their χ^2 values (ee)

Table 1: Results of the GS fit to e^+e^- data. The numbers in the parentheses correspond to the fitted uncertainties. The χ^2 value of the fit per number of degrees of freedom (DF) is also shown.

Parameter	BABAR	BES	CMD2	CMD3	KLOE	SND	SND20
Normalization	1.031(14)	0.990(15)	0.996(12)	1.052(8)	1.004(6)	1.013(16)	1.020(15)
m_ρ (MeV)	775.21(33)	775.63(75)	775.70(48)	774.92(9)	774.44(18)	775.26(39)	775.57(32)
Γ_ρ (MeV)	149.45(67)	146.56(2.00)	145.98(1.38)	148.74(21)	149.05(48)	148.47(1.20)	148.41(1.25)
m_ω (MeV)	782.02(15)	782.56(1.23)	782.66(28)	782.32(6)	783.28(41)	782.25(27)	782.21(16)
Γ_ω (MeV)	8.05(24)	10.2(2.7)	8.7(5)	8.45(9)	9.7(7)	8.50(35)	8.40(38)
$\lvert c_\omega\rvert$ (10^{-3})	1.64(5)	1.89(41)	1.63(12)	1.73(2)	1.55(7)	1.69(8)	1.72(8)
ϕ_ω (°)	1.5(2.3)	11(12)	9.6(3.9)	4.5(7)	4.9(2.1)	8.5(3.3)	6.2(2.4)
$m_{\rho'}$ (GeV)	1.464(16)	–	–	–	–	–	–
$\Gamma_{\rho'}$ (GeV)	0.285(22)	–	–	–	–	–	–
$\lvert c_{\rho'}\rvert$	0.15(1)	–	–	–	–	–	–
$\phi_{\rho'}$ (°)	202(6)	–	–	–	–	–	–
$m_{\rho''}$ (GeV)	1.825(18)	–	–	–	–	–	–
$\Gamma_{\rho''}$ (GeV)	0.285(22)	–	–	–	–	–	–
$\lvert c_{\rho''}\rvert$	0.063(6)	–	–	–	–	–	–
$\phi_{\rho''}$ (°)	46(13)	–	–	–	–	–	–
$m_{\rho'''}$ (GeV)	2.232(14)	–	–	–	–	–	–
$\Gamma_{\rho'''}$ (GeV)	0.060(64)	–	–	–	–	–	–
$\lvert c_{\rho'''}\rvert$ (10^{-3})	3.3(1.6)	–	–	–	–	–	–
$\phi_{\rho'''}$ (°)	−9(30)	–	–	–	–	–	–
χ^2/DF	0.99	0.87	0.85	0.92	1.2	0.85	1.5

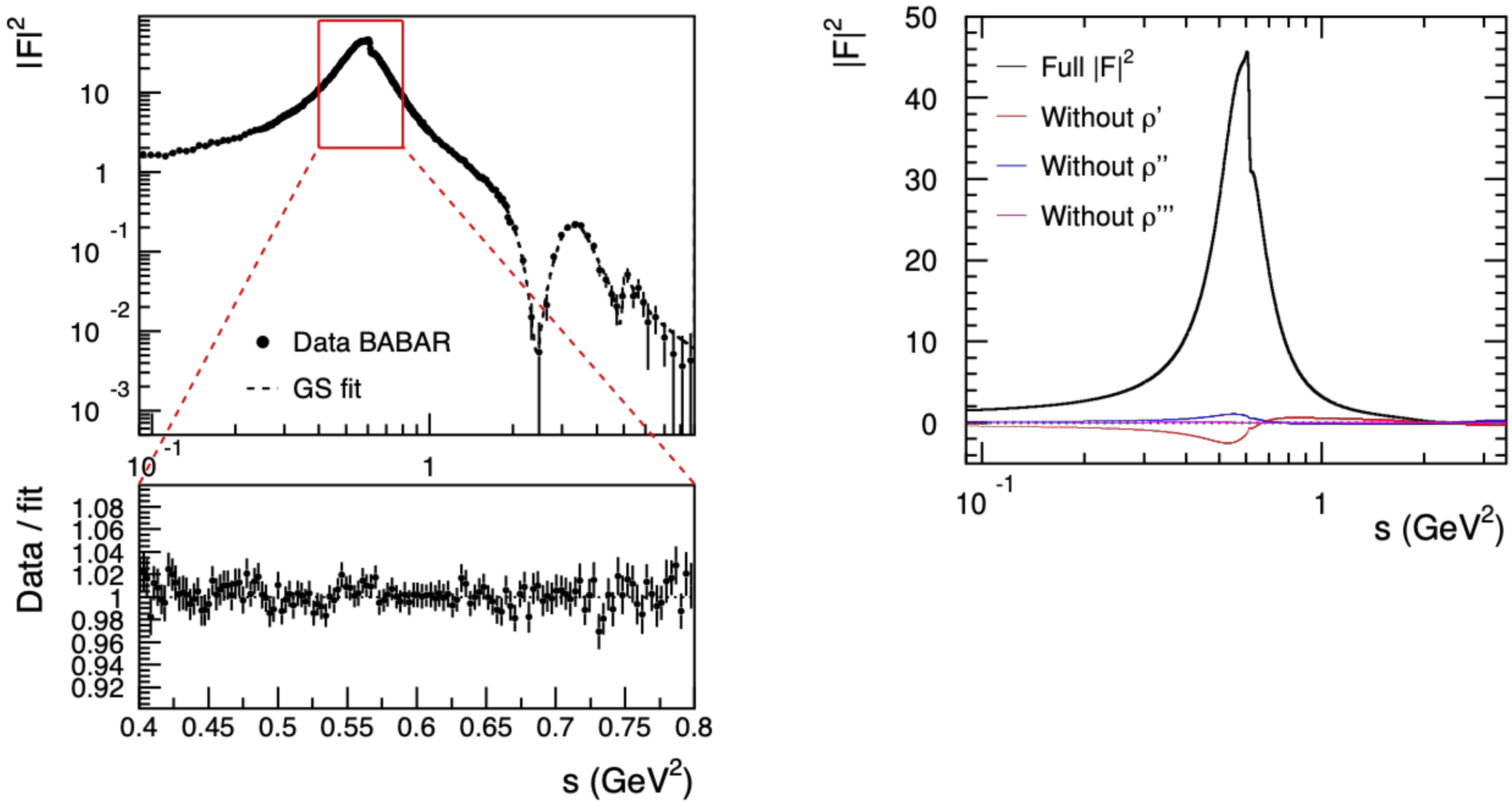
Fitted parameters and their χ^2 values (tau)

Table 2: Results of the GS fit to τ data. The numbers in the parentheses correspond to the fitted uncertainties. For Belle and CLEO, additional calibration and resolution systematic uncertainties on m_ρ and Γ_ρ from their publications are also included. The χ^2 value of the fit per number of degrees of freedom (DF) is also shown.

Parameter	ALEPH	Belle	CLEO
Normalization	1.012(17)	1.002(10)	1.023(10)
m_ρ (MeV)	776.63(1.10)	775.04(59)	775.60(0.99)
Γ_ρ (MeV)	151.62(2.17)	147.45(1.85)	150.04(1.39)
χ^2/DF	0.64	1.8	1.2

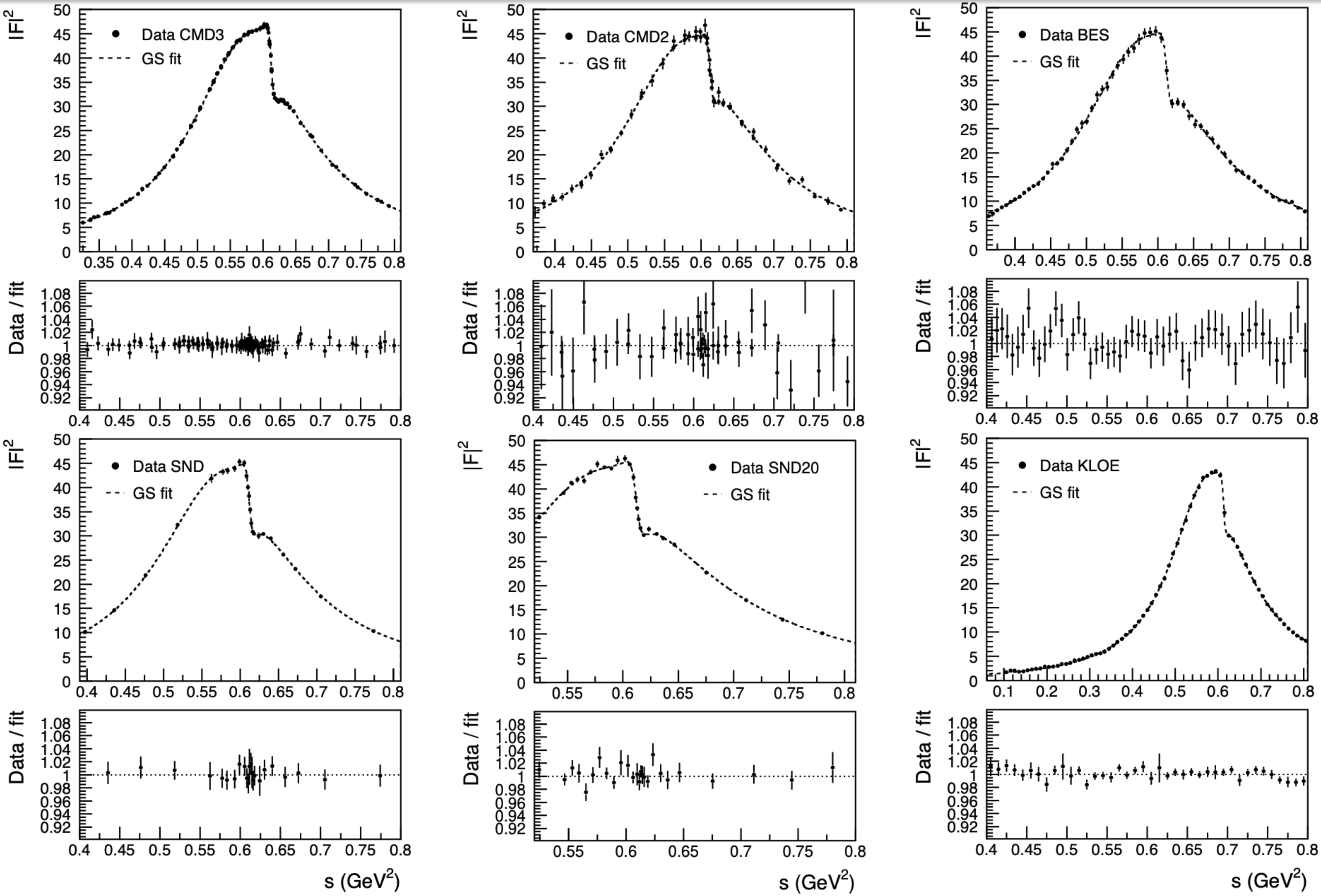
The χ^2 values for Belle and CLEO are a bit large since some of the experimental uncertainties were not included in the error matrices used for the fit. The quoted uncertainties include however these additional uncertainties from their original publications: e.g. for Belle $\delta\Gamma_\rho = 1.85$ MeV corresponds to $0.73_{\rm fit} + 1.70_{\rm pub}$ MeV

Full fit to BABAR

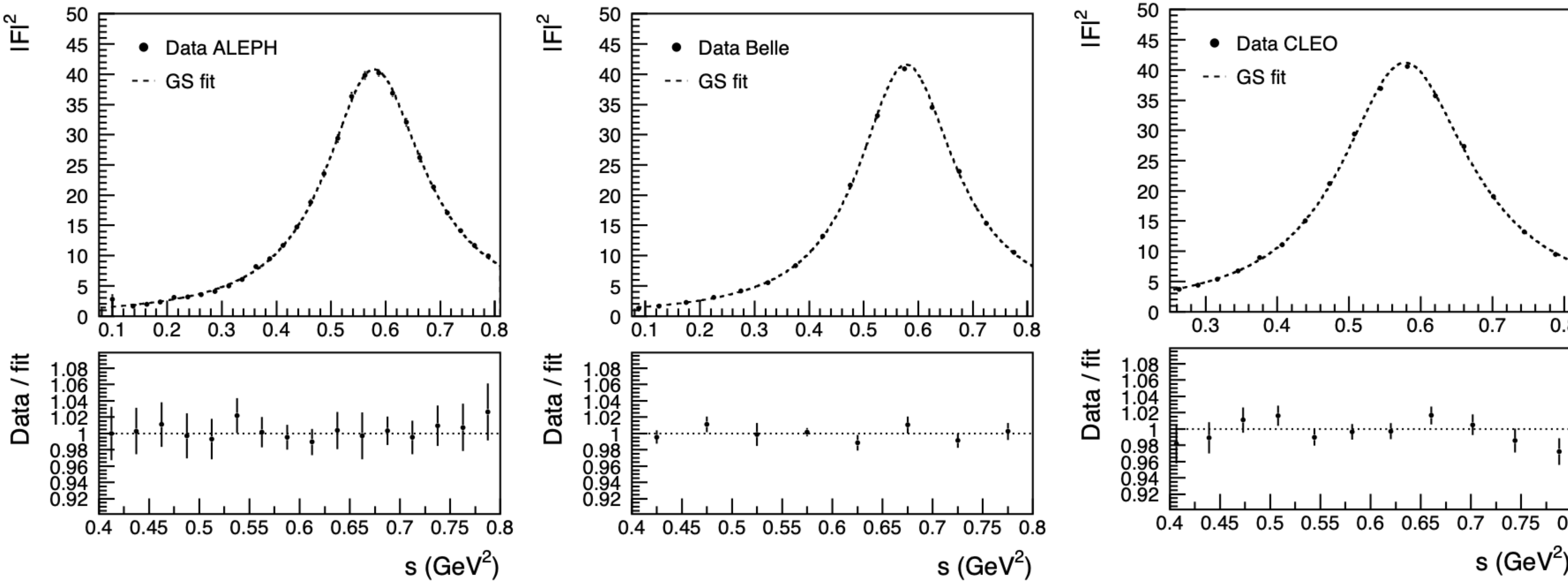


- The data is well fitted by the GS parameterisation
- The high mass resonances have decreasing impact to the fit in the rho peak region

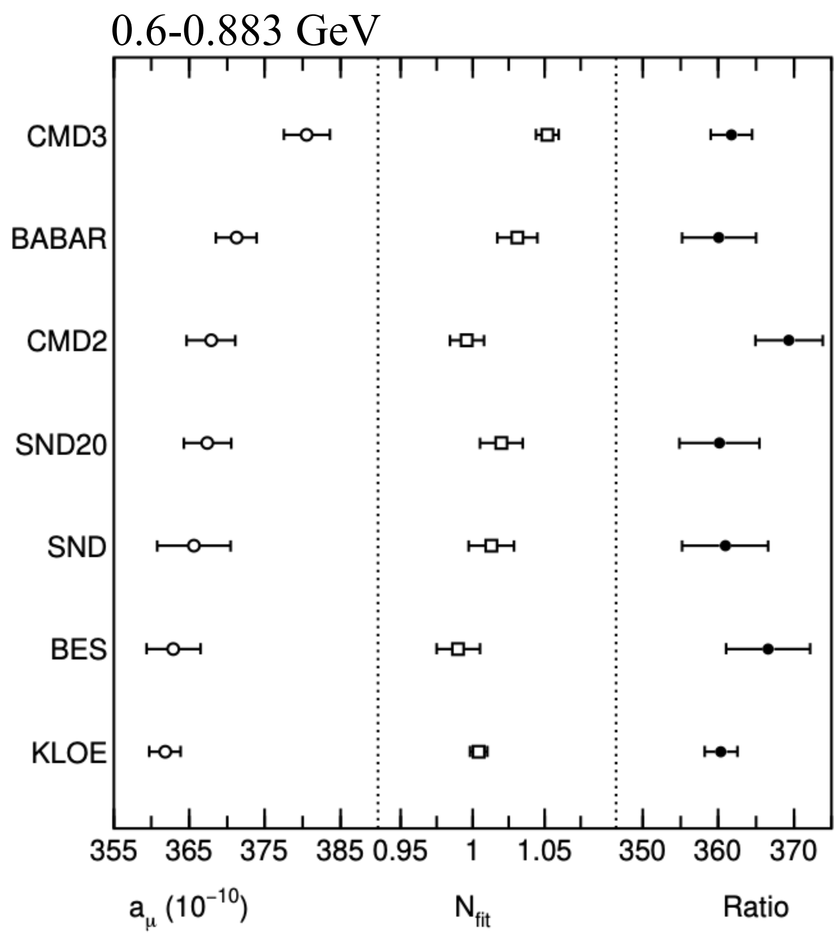
Fits to other ee data



Fits to tau data



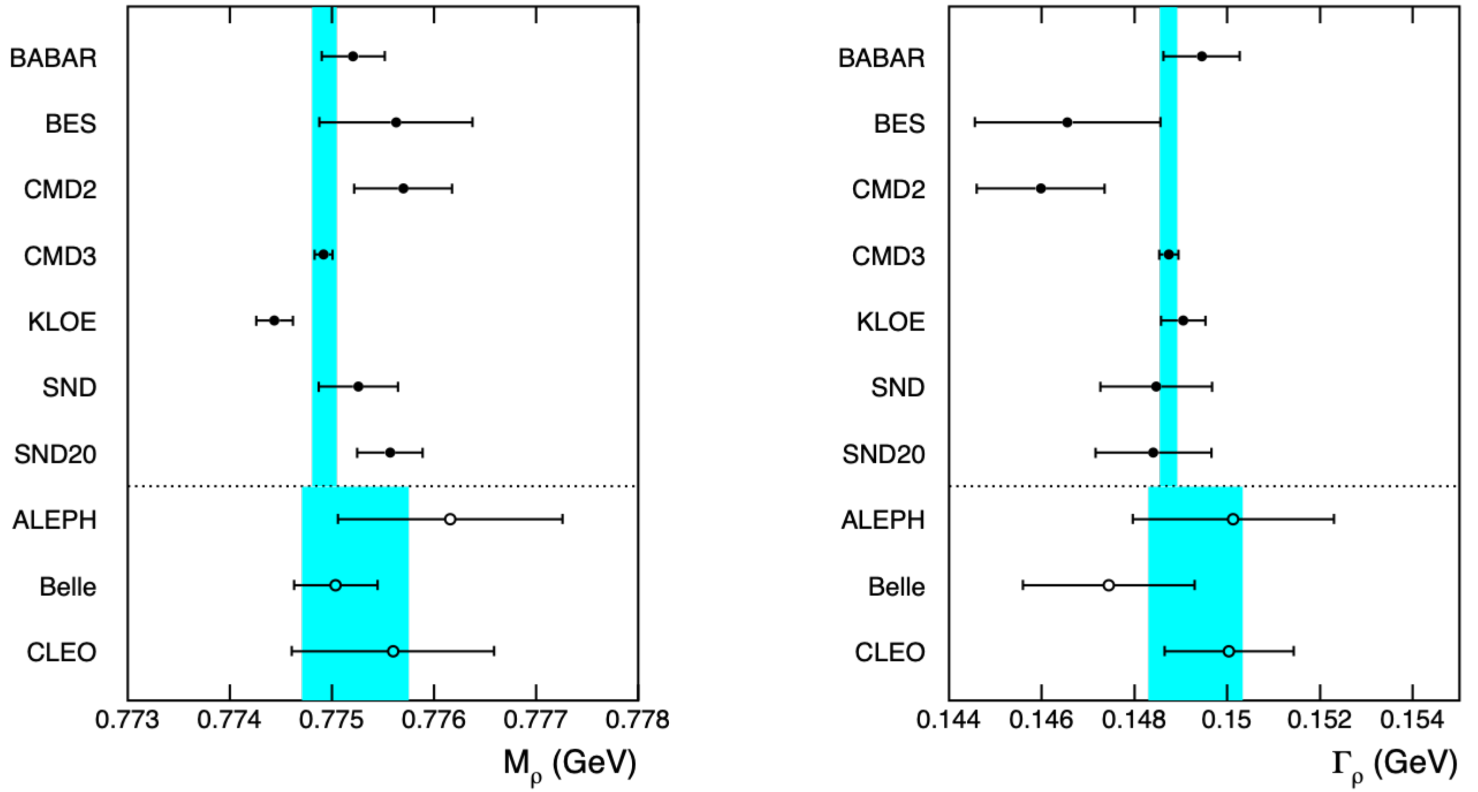
Correlation between the Normalisation and a_μ



The largest discrepancy KLOE-CMD3 of 5.1σ for a_μ reduced to 0.4σ in the ratio

➔ Clear correlation between the normalisation of the FF and a_μ valides our decoupling procedure

Fitted rho mass and width values



$\Delta m_\rho^{\rm GS} = (-0.30 \pm 0.38)\,\text{MeV}$ $\quad$ $\Delta\Gamma_\rho^{\rm GS} = (-0.58 \pm 1.01)\,\text{MeV}$

- CMD3 is by far the most precise data in the rho region which constrains the mass and width values
- Tau data have currently limited precision, big room for future improvement
- The lower mass value of KLOE may be due to the observed slope wrt other data

$$\Delta a_\mu^{m_\rho}\ {}^{\rm GS} = (0.06 \pm 0.10) \times 10^{-10}$$
$$\Delta a_\mu^{\Gamma_\rho}\ {}^{\rm GS} = (1.62 \pm 2.92) \times 10^{-10}$$

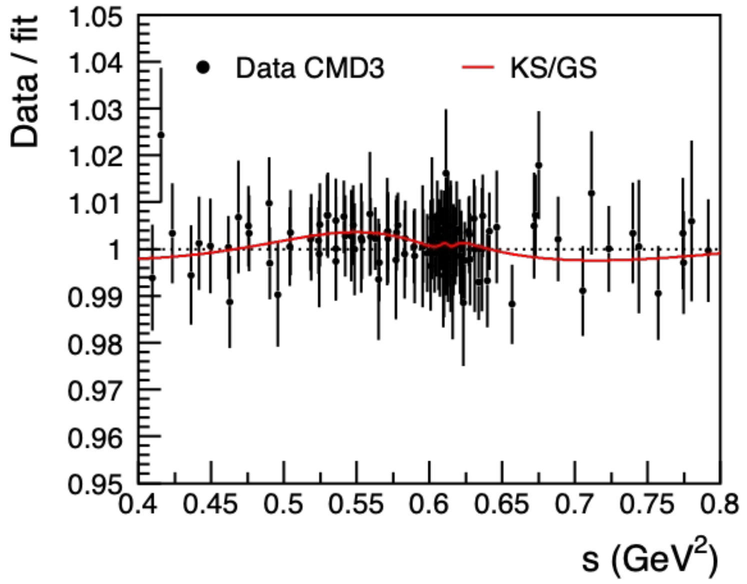
Systematic variations

- Alternative GS fits to tau data allowing the complex amplitude of ρ' and ρ'' be free:
  ➔ m_ρ varied by +0.38 MeV and
  Γ_ρ varied by −0.25 MeV

- GS vs. KS fits

$$\Delta m_\rho^{\mathrm{KS}} = (-0.91 \pm 0.48)\,\mathrm{MeV}$$

$$\Delta \Gamma_\rho^{\mathrm{KS}} = (-0.83 \pm 1.00)\,\mathrm{MeV}$$



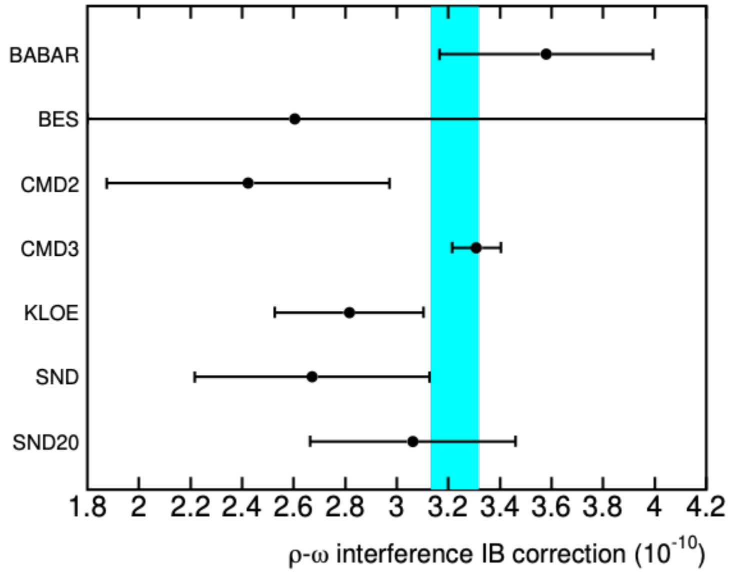
- Resulting variations on a_μ:

$$\Delta a_\mu^{m_\rho\ \mathrm{GS-KS}} = 0.02 \times 10^{-10}$$

$$\Delta a_\mu^{\Gamma_\rho\ \mathrm{GS-KS}} = 1.39 \times 10^{-10}$$

- Model dependence of G_{EM} is checked to be below 0.1 MeV for both m_ρ and Γ_ρ

Byproduct of the fits



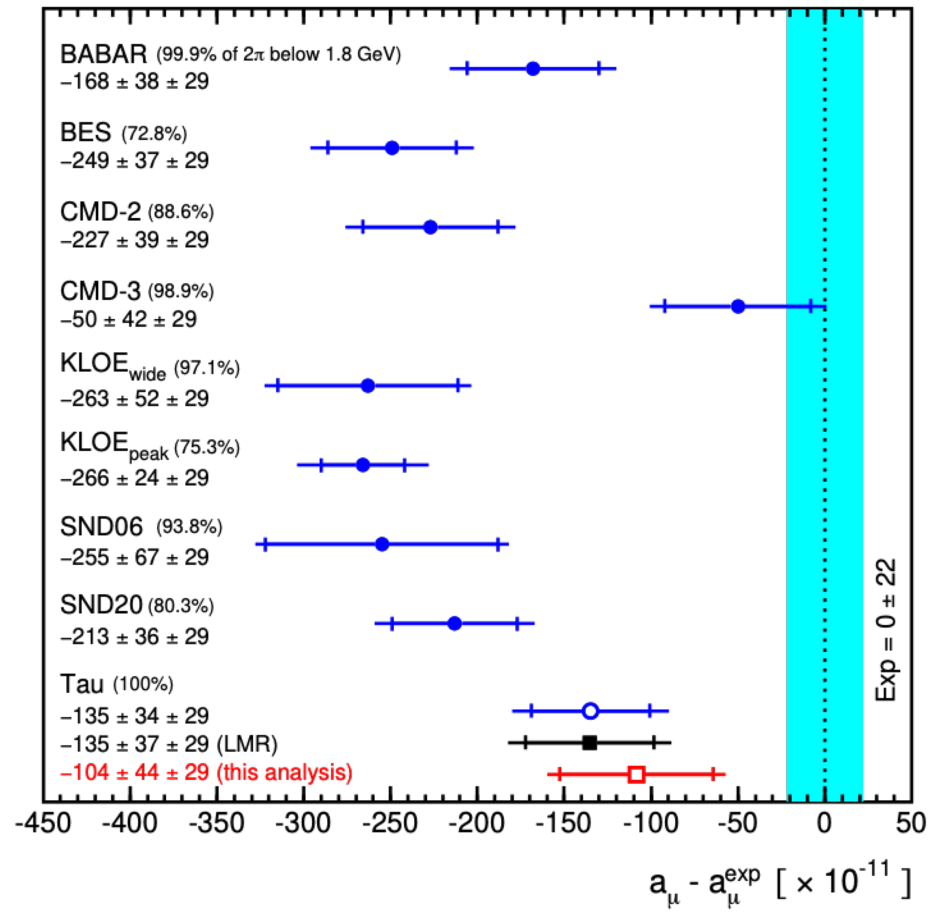
Fitted parameters of $\rho{-}\omega$ interference can be used to obtain the corresponding IB corrections shown above

➔ Weight average: $\Delta a_\mu^{\mathrm{HVP,\,LO},\,\rho-\omega} = +(3.22 \pm 0.09) \times 10^{-10}$ dominated by CMD3

Results shown in arXiv:2504.13789v1

Source of IB	$\Delta a_\mu^{\mathrm{HVP,\,LO}}$ (10^{-10})
S_{EW}	$-12.21(15)$
G_{EM}	$-1.87(96)$
FSR	$+4.65(44)$
β^3 cross-section	-7.69
$\rho-\omega$ interference	$+3.22(9)(8)$
Form factor Δm_ρ	$+0.06(10)(2)$
Form factor $\Delta\Gamma_\rho$	$+1.62(2.92)(1.39)$
Sum	$-12.22(3.41)$

To be compared with WP2025: −15.0(5.1)



➔ Despite limited precision from tau data, the uncertainty from the data-based IB corrections found to be competitive wrt the inflated uncertainty in WP2025

Studies beyond arXiv:2504.13789v1

Tried a dispersive approach using the following formulae:

$$|F_\pi^0|^2 = |G(s) \times J(s)|^2$$

$$G(s) = 1 + \alpha_V s + \frac{\kappa s}{m_\omega^2 - s - i m_\omega \Gamma_\omega}$$

$$J(s) = e^{1-\frac{\delta_1(s_0)}{\pi}} \left(1 - \frac{s}{s_0}\right)^{\left[1-\frac{\delta_1(s_0)}{\pi}\right]\frac{s_0}{s}} \left(1 - \frac{s}{s_0}\right)^{-1} e^{\frac{s}{\pi}\int_{4m_\pi^2}^{s_0} dt \frac{\delta_1(t)}{t(t-s)}}$$

$$\cot\delta_1(s) = \frac{\sqrt{s}}{2k^3}\left(m_\rho^2 - s\right)\left[\frac{2m_\pi^3}{m_\rho^2\sqrt{s}} + B_0 + B_1\omega(s)\right]$$

$$k = \frac{\sqrt{s - 4m_\pi^2}}{2}$$

$$\omega(s) = \frac{\sqrt{s} - \sqrt{s_0 - s}}{\sqrt{s} + \sqrt{s_0 - s}}$$

$$\sqrt{s_0} = 1.05 \text{ GeV}$$

$G(s)$ from [arXiv:1611.09359]
$J(s)$ from [hep-ph/0106025, 0402285]
$\cot\delta_1(s)$ from [arXiv:1102.2183]

Seven free parameters: Normalisation, α_V, κ, m_ω, m_ρ, B_0 and B_1

$$\Gamma_\rho = \frac{2k^3(m_\rho)}{m_\rho^2\left(B_0 + B_1\omega(m_\rho)\right)}$$

Resulting m_ρ and Γ_ρ from Dispersive Approach

Fit CMD3 and tau data up to 0.9 GeV

Parameter	CMD3	ALEPH	Belle	CLEO
m_ρ (MeV)	774.67(37)	774.40(1.06)	775.76(1.35)	774.64(1.25)
Γ_ρ (MeV)	150.93(59)	152.08(1.29)	149.37(2.24)	150.62(2.46)

The uncertainties include systematic variations:

$\sqrt{s_0} = 1.05 \rightarrow 1.3$ GeV

with vs. without B_1 term

Weighted tau $m_\rho = 774.83(69)$ MeV

Weighted tau $\Gamma_\rho = 151.27(1.02)$ MeV

$$\Delta m_\rho^{\text{Dispersive}} = (-0.16 \pm 0.79)\,\text{MeV}$$
$$\Delta \Gamma_\rho^{\text{Dispersive}} = (-0.34 \pm 1.18)\,\text{MeV}$$

Resulting variations on a_μ:

$$\Delta a_\mu^{m_\rho\ \text{Dispersive}} = (0.12^{+0.35}_{-0.60}) \times 10^{-10}$$
$$\Delta a_\mu^{\Gamma_\rho\ \text{Dispersive}} = (0.85^{+3.04}_{-2.99}) \times 10^{-10}$$

fully consistent with arXiv result:

$$\Delta a_\mu^{m_\rho\ \text{GS/KS}} = 0.06(10)(2) \times 10^{-10}$$
$$\Delta a_\mu^{\Gamma_\rho\ \text{GS/KS}} = 1.62(2.92)(1.39) \times 10^{-10}$$

Summary

- Data-based form factor related IB corrections derived using GS (and KS) parameterisation as shown in arXiv:2504.13789

$$\Delta a_\mu^{m_\rho\ \mathrm{GS/KS}} = 0.06(10)(2) \times 10^{-10}$$
$$\Delta a_\mu^{\Gamma_\rho\ \mathrm{GS/KS}} = 1.62(2.92)(1.39) \times 10^{-10}$$

- We revisited the evaluation using a dispersive approach and found fully consistent corrections

$$\Delta a_\mu^{m_\rho\ \mathrm{Dispersive}} = (0.12^{+0.35}_{-0.60}) \times 10^{-10}$$
$$\Delta a_\mu^{\Gamma_\rho\ \mathrm{Dispersive}} = (0.85^{+3.04}_{-2.99}) \times 10^{-10}$$

- The uncertainties are competitive wrt to the current inflated one in WP2025
- Need improved tau data to reduce the uncertainties
- Would be interesting to compare wrt the improved model calculations (see e.g. the talk of Genaro Toledo)