8^{th} **Plenary Workshop of the Muon $g-2$ Theory Initiative**

RadioMonteCarLow 2 :: An Overview

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WHO a community effort for existing Monte Carlo software to be

- ◇ kept alive
- ◇ made accessible
- ◇ further improved

WHAT theoretical description of $e^+e^- \to \text{stuff}$ at $\sqrt{s} \lesssim$ few GeV

- ◇ scan and radiative return at e^+e^- colliders
- ◇ "other": luminosity, normalisation, cross checks, ...

WHY (not only) because of e^+e^- experiments for $(g-2)_\mu$:

- ◇ improve SM precision tests at low energies
- ◇ implement progress developed at higher energies for LHC

`https://radiomontecarlow2.gitlab.io/`

past, present, future :: 3 PHases

PH.0 inspired by 

Radio MonteCarlo and its outcome: [0912.0749]

Quest for precision in hadronic cross section at low energy:
Monte Carlo tools vs. experimental data

PH.1 culminated in [2410.22882]

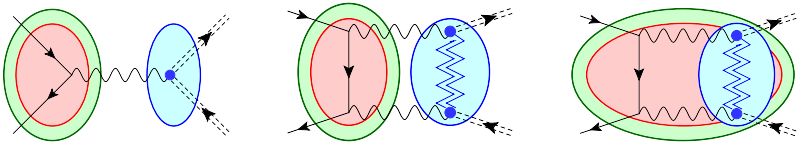
Radiative corrections and Monte Carlo tools
for low-energy hadronic cross sections in e^+e^- collisions

PH.2 ongoing as we speak!

$\hookrightarrow$ new people for new ideas/approaches are welcome!

...

- organisation in Work Packages
  - ◇ WP1 & WP2: fixed-order QED
  - ◇ WP3: beyond fixed-order, involving hadrons (read: pions)
  - ◇ WP4: all-order QED (leading contributions from multiple emissions)
  - ◇ WP5: experimental input



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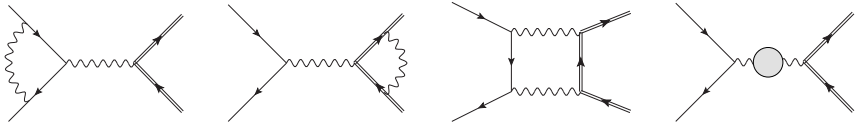
- o QED @ N^{k}LO means every diagram contributing at that order
- o QED @ N^{k}LO for $2 \to 2 \supset$ QED @ N^{k-1}LO for $2 \to 3$
- o analysis by gauge-invariant subsets:



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- o $\pi\pi\gamma^n$ treatment and terminology:

sQED		scalar QED; no form factors
F x sQED	$[n=1]$	sQED $\times$ FFs; loop integrals not affected
FsQED	$[n=2]$	pion-pole contribution to 2-virtual Compton tensor; loop integrals dispersively
GVMD	$[n=2]$	pion FF via Breit-Wigner propagators; loop integrals analytically
FULL	$[n>2$,	beyond pion-pole] not yet available $\Rightarrow$ PH.2

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- o all-order terminology:

PS parton shower, resummation of $\log m_e$, in cases with angular effects

CS collinear structures, resummation of $\log m_e$

CEEX / YFS (coherent) Monte Carlo generation of soft photons

- 3 $e^+e^- \to X^+X^-$ channels
- 5 experimental scenarios
- 7 Monte Carlo codes

- 3 $e^+e^- \to X^+X^-$ channels, in both Scan and Radiative return mode:

$$e^+e^- \to e^+e^- \quad e^+e^- \to \mu^+\mu^- \quad e^+e^- \to \pi^+\pi^-$$
$$e^+e^- \to e^+e^-\gamma \quad e^+e^- \to \mu^+\mu^-\gamma \quad e^+e^- \to \pi^+\pi^-\gamma$$

- 5 experimental scenarios
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- 3 $e^+e^- \to X^+X^-$ channels, in both Scan and Radiative return mode
- 5 experimental scenarios, inspired by

CMD $e^+e^- \to X^+(p_+)X^-(p_-)$ @ $\sqrt{s} = 0.7$ GeV
cuts on $p_\pm$, $\left||\phi^+ - \phi^-| - \pi\right| < 0.15$ rad; $|\theta^+ + \theta^- - \pi| < 0.25$ rad

KLOE-SA (untagged) "small-angle" $e^+e^- \to X^+(p_+)X^-(p_-)\gamma$ @ $\sqrt{s} = 1.02$ GeV
cuts on $p_\pm$, $\theta_\pm$, M_{XX}, if $\vec{p}_{\tilde{\gamma}} \equiv -(\vec{p}_+ + \vec{p}_-)$ then $\theta_{\tilde{\gamma}} \leq 15$ deg or $\theta_{\tilde{\gamma}} \geq 165$ deg

KLOE-LA (tagged) "large-angle" $e^+e^- \to X^+(p_+)X^-(p_-)\gamma$ @ $\sqrt{s} = 1.02$ GeV

BES III $e^+e^- \to X^+(p_+)X^-(p_-)\gamma$ @ $\sqrt{s} = 4$ GeV

B factory $e^+e^- \to X^+(p_+)X^-(p_-)\gamma$ @ $\sqrt{s} = 10$ GeV
cuts on $p_\pm$, $\theta_\pm$, $M_{XX(\gamma)}$, and the photon, for the last three scenarios

- 7 Monte Carlo codes

- 3 $e^+e^- \to X^+X^-$ channels, in both Scan and Radiative return mode
- 5 experimental scenarios
- 7 Monte Carlo codes (@PH.1!)

AfkQed R: LO+ISC with CS, FSC with Photos, $X \in \{\mu, \pi\}$

BabaYaga S: NLO+PS, R: LO + PS, F×sQED for $X = \pi$ (FsQED/GVMD in PH.2)

KKMC S: CEEX, R: CEEX for $X = \mu$

MCGPJ S: NLO+CS, R: LO+CS for $X \in \{e, \mu\}$; GVMD for $X = \pi$

McMule S: NNLO, R: NLO for $X \in \{e, \mu\}$; S: NLO/NNLO ISC, R: NLO ISC for $X = \pi$

Phokhara R: NLO for $X = \mu$, F×sQED for $X = \pi$

Sherpa S: NLO+YFS for $X \in \{e, \mu\}$, YFS sQED for $X = \pi$

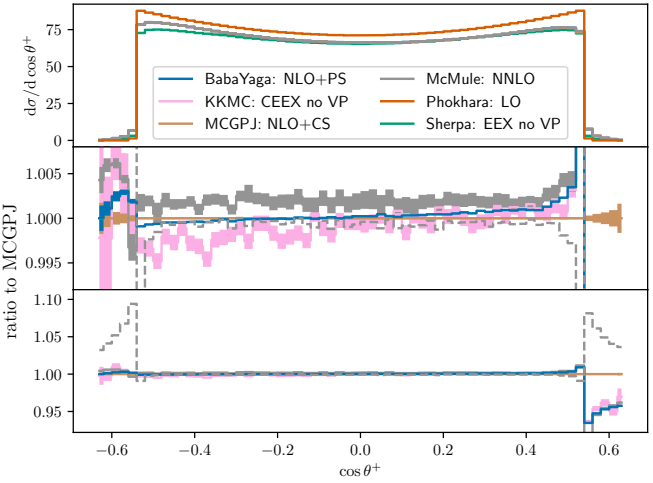
- 3 $e^+e^- \to X^+X^-$ channels, in both Scan and Radiative return mode
- 5 experimental scenarios
- 7 Monte Carlo codes
- 1 disclaimer!
  - this is a theoretical study on MC tools, relying on experimental expertise
  - we use acceptance cuts, no additional selection or detector effects

    $\Rightarrow$ limited experimental conclusions can be drawn

PH.1 :: An Anthology of Plots

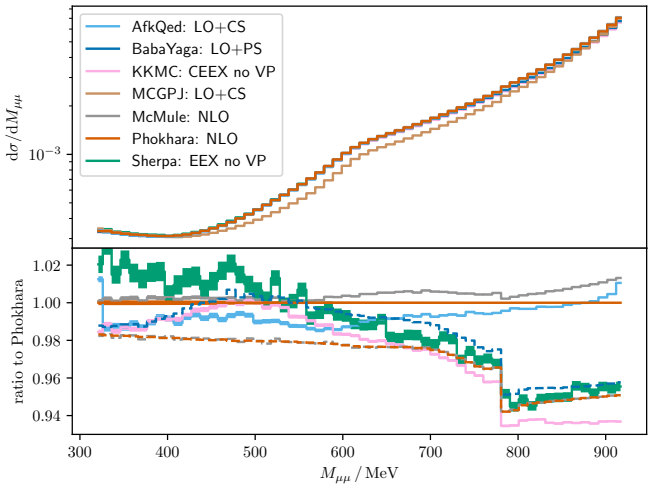
https://radiomontecarlow2.gitlab.io/plots/

https://doi.org/10.5281/zenodo.13928140



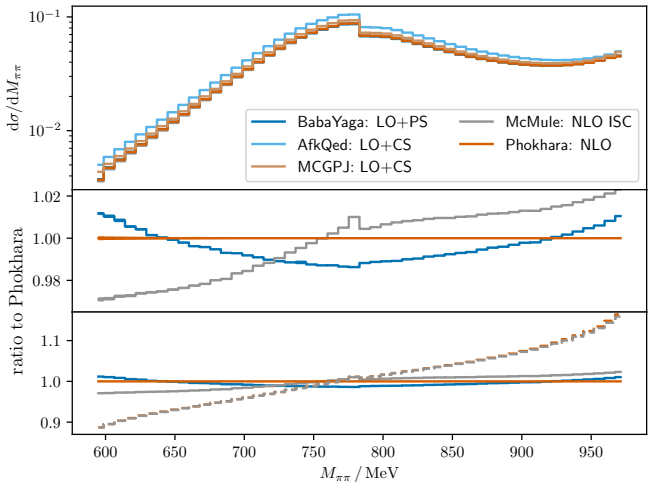
[dashed gray = McMule @ NLO]

- ◇ PHOKHARA not designed for scan setup
- ◇ agreement within 0.2% in the bulk, larger deviations at the edges
- ◇ NLO VPC $\simeq 0$ at this $\sqrt{s}$
- ◇ NNLO $\sim$ a few 0.1% in the bulk, more at edges
  - ↪ CS & PS dominated by one more emission



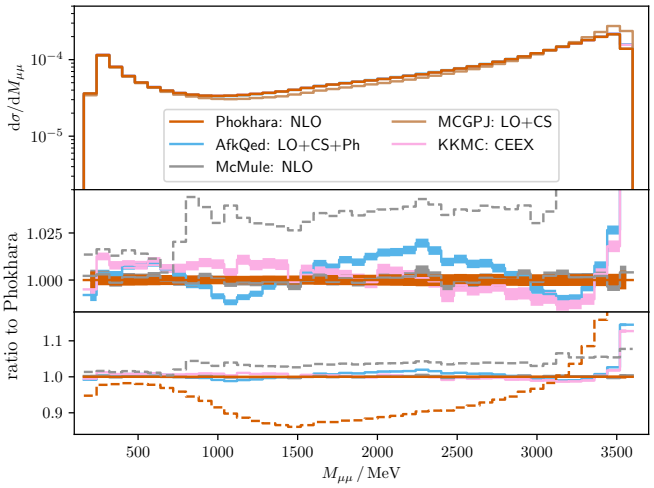
- $50° < \theta_\gamma < 130°$
- PHOKHARA is reference
- MCGPJ not designed for radiative return
- VPC $\sim 1\%$ difference
  - ↪ MCMULE: single VP
  - ↪ PHOKHARA: resummed
- (C)EEX within $2-3\%$ of NLO no VPC

[dashed = NLO no VPC, BABAYAGA no HVPC]



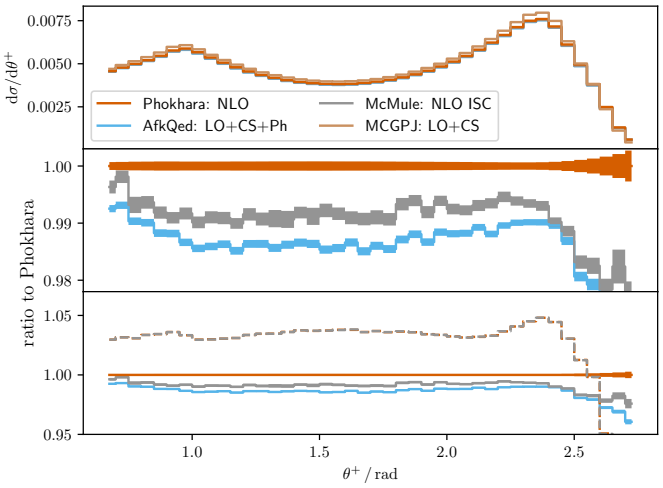
- ◇ PHOKHARA is reference
- ◇ AFKQED not designed for selection cuts
- ◇ LO ISC agrees very well with LO full
  ↪ not at large angle
- ◇ NLO $\sim 10\%$!!
- ◇ 2% difference if no FSC @NLO

[dashed MCMULE = LO ISC, dashed PHOKHARA LO full]



- technical: no VPC
  $\hookrightarrow$ VPC $\gtrsim 3\%$
- agreement within 2%
  in the bulk, larger at edge
- large $\pm 10\%$ corrections
  at NLO

[dashed McMule = NLO with VPC, dashed Phokhara LO]



- LO: FSC $\lesssim 0.01\%$
  $\hookrightarrow$ form-factor suppression
- NLO: FSC $1-2\%$
- 1%-level precision
  is a long way ahead

[dashed McMule = LO ISC, dashed Phokhara LO full]

remarks from PH.1

- fixed-order: NNLO for $2 \to 2$, NLO for $2 \to 3$
  VPC @ $\sim 1\%$ are to be considered carefully
- hadrons: FsQED available in codes but not used here
  FSC impact can deeply vary depending on scenario
- all-order: effects seem dominated by one extra radiation

looking into PH.2

- fixed-order: NNNLO for $2 \to 2$ (through $e^+e^- \to \gamma^*$ @ NNNLO)
  NNLO for $2 \to 3$ (through $e^+e^- \to \gamma^*\gamma$ @ NNLO)
  $\hookrightarrow$ Sophie's and Pau's talks later today
- hadrons: FsQED, GVMD used and compared in [2409.03469]
  $\hookrightarrow$ Fulvio's talk later today
  FSC for $2 \to 3$ needs improvement/better understanding
- all-order: combine higher fixed orders with CS/PS/YFS

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  - Pisa consensus: ISC vs FSC pentagons 
  - beyond pion pole: S- ($\pi\pi$, $\pi\pi\gamma$) and P-wave ($\pi\pi\gamma$) rescattering
- all-order: combine higher fixed orders with CS/PS/YFS

- fixed-order: NNNLO for $2 \to 2$ (through $e^+e^- \to \gamma^*$ @ NNNLO)
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- all-order: combine higher fixed orders with CS/PS/YFS, e.g.
  - NLOPS for $XX\gamma$ (BabaYaga)
  - EEX+NNLO (McMule)
  - improved CEEX+NNLO (Phokhara)
  - NLO+YFS w/ improved VP (Sherpa)

looking into PH.2

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- all-order: combine higher fixed orders with CS/PS/YFS
- general: widen set of processes under scrutiny: luminosity, $3\pi, \ldots$
  widen set of codes (some will not be updated)
  investigate theory error estimate

Thank You!

https://radiomontecarlow2.gitlab.io/

past meetings (Liverpool 11.24, Pisa 05.25)

future meetings (Liverpool 11.25, Torino 06.26)

PH.1 members

Riccardo Aliberti, Paolo Beltrame, Ettore Budassi, Carlo M. Carloni Calame, Gilberto Colangelo, Lorenzo Cotrozzi, Achim Denig, Anna Driutti, Tim Engel, Lois Flower, Andrea Gurgone, Martin Hoferichter, Fedor Ignatov, Sophie Kollatzsch, Bastian Kubis, Andrzej Kupść, Fabian Lange, Alberto Lusiani, Stefan E. Müller, Jérémy Paltrinieri, Pau Petit Rosàs, Fulvio Piccinini, Alan Price, Lorenzo Punzi, Marco Rocco, Olga Shekhovtsova, Andrzej Siódmok, Adrian Signer, Giovanni Stagnitto, Peter Stoffer, Thomas Teubner, William J. Torres Bobadilla, Francesco P. Ucci, Yannick Ulrich, Graziano Venanzoni

feedback & new collaborators are welcome!