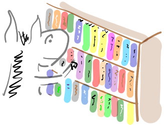
8th Plenary workshop of the Muon $g-2$ Theory Initiative

McMule update

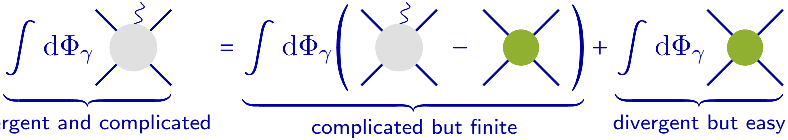
Sophie Kollatzsch

PSI / UZH



Monte **C**arlo for **MU**ons and other **LE**ptons

- integrator (generator WIP) for fixed-order QED up to NNLO
- use QCD methods: FKS$^\ell$ subtraction with massive fermions



- challenge virtual amplitudes with $m \neq 0 \implies$ massification (photonic)
- challenge numerical instabilities $\implies$ next-to-soft stabilisation + **O**pen**L**oops

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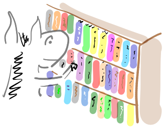
- challenge virtual amplitudes with $m \neq 0 \implies$ massification (photonic)

$$\mathcal{A}(m) = \Big(\prod_j \sqrt{Z(m)}\Big) \times \mathcal{A}(m=0) + \mathcal{O}(m) \quad \text{iff} \quad m^2 \ll \text{all other scales}$$

- challenge numerical instabilities $\implies$ next-to-soft stabilisation + **O**pen**L**oops



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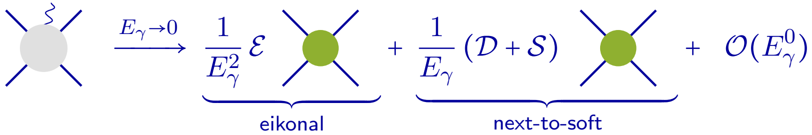

process	experiment	physics motivation	order
$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO
$\ell N \to \ell N$	P2, Muse, A1, MAGIX, ...	proton radius and weak charge	NNLO−
$e\nu \to e\nu$	DUNE	flux & $\sin^2\theta_W$	NNLO−
$e^-e^- \to e^-e^-$	Prad	normalisation	NNLO
	MOLLER, ...	$\sin^2\theta_W$ at low Q^2	
$e^+e^- \to e^+e^-$	any e^+e^- collider	luminosity measurement	NNLO
$ee \to \gamma^*$			NNLO
$ee \to \gamma\gamma^*$			NNLO−
$ee \to \ell\ell$	CMD+SND, BES, KLOE, ...	R-ratio	NNLO+
	Belle	τ properties & $\sin^2\theta_W$	
$ee \to \pi\pi$	CMD+SND, BES, KLOE, ...	R-ratio	NLO+
$ee \to \gamma\gamma$	KLOE	dark searches	NNLO−
	any e^+e^- collider	luminosity measurement	
$\mu \to \nu\bar{\nu}e$	MEG, Mu3e, Pioneer, Mu2e	ALP searches	NNLO+
	DUNE	beam-line profiling	
$\mu \to \nu\bar{\nu}eee$	Mu3e	background	NLO

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$e\mu \to e\mu$	MUonE	HVP to $(g-2)_\mu$	NNLO
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$\mu \to \nu\bar{\nu}eee$	Mu3e	background	NLO

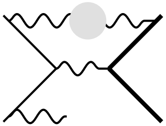
Two major upgrades ahead.

1. **Disperon QED**
   - how to deal with internal (HVP) and external $(\pi, p, \ldots)$ hadrons in a Monte Carlo
   - $e^+e^- \to \pi^+\pi^-(\gamma)$ in FsQED at NLO

2. **Radiative NNLO** for $e^+e^- \to \gamma\gamma^*$ :: initial-state corrections (ISC)
   - today :: preliminary photonic (no VP) results for KLOE-like
   - HVP via Disperon QED WIP

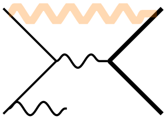
- **challenge** :: (data → loop integral) → pheno
- how
  - (data → loop integral) :: dispersion relations ✓
  - (data → loop integral) → pheno
    goal automatised and efficient as much as possible

Disperon QED
= OpenLoops ⊕ Disperon EFT ⊕ universal threshold subtraction



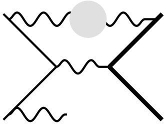
$\in ee \to \mu\mu\gamma$ @ NNLO $\quad \dfrac{\Pi(k^2)}{k^2} = -\dfrac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}S \, \dfrac{\mathrm{Im}\Pi(S)}{S(k^2-S)}$

HVP treatment well established :: also hyperspherical method [Fael 18] (t-channel)

$\Longrightarrow \int \mathrm{d}S \Bigg($  $\Bigg)$

. . . can be described by a massive **dispers**ive phot**on**, a **disperon**

(manual calculation in $[e^+e^- \to e^+e^-]$, $[e^-e^- \to e^-e^-]$, $[e\mu \to e\mu]$, $[ee \to \mu\mu]$)



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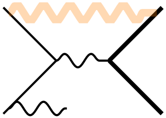
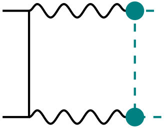


$$- \mathrm{CT}\Bigg) + \int \mathrm{d}S \, \mathrm{CT}$$

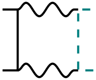
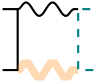
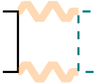
. . . can be described by a massive dispersive photon, a disperon

(manual calculation in $[e^+e^- \to e^+e^-]$, $[e^-e^- \to e^-e^-]$, $[e\mu \to e\mu]$, $[ee \to \mu\mu]$)

example: $ee \to \pi\pi$ @ NLO FsQED [Colangelo et al 22]
(also calculated in [Budassi et al 24] and [Holz, Cottini])



$$\Longrightarrow \frac{F_\pi(k^2)}{k^2} = \frac{1}{k^2} - \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}S\, \frac{\mathrm{Im}F_\pi(S)}{S(k^2 - S)}$$

$\Longrightarrow$  $+ \int \mathrm{d}S \Big($  $\Big)$ $+ \int \mathrm{d}S_1\, \mathrm{d}S_2$ 

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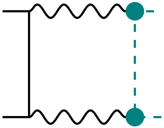

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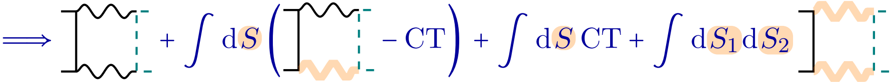


with threshold subtraction via counterterm for amplitude [McMule soon]
. . . can be described by a massive dispersive photon, a disperon

Disperon QED

$=$ **OpenLoops** $\oplus$ **Disperon EFT** $\oplus$ universal threshold subtraction

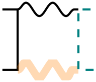
- disperon $=$ massive photon w special Feynman rule
  $\Longrightarrow$ rely on automated tools :: **OpenLoops** (OL)

- $\int^\infty \mathrm{d}S_1 \in$ Monte Carlo requires fast and stable evaluation for $S_1 \to \infty \Longrightarrow$ switch from full to expanded at $S_1 = \Lambda \gg s, 4m_\pi^2$ described by **Disperon EFT**

$$\int_{4m_\pi^2}^{\Lambda} \mathrm{OpenLoops} + \int_{\Lambda}^{\infty} \mathrm{EFT}$$

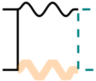
Disperon QED

$= $ OpenLoops $\oplus$ Disperon EFT $\oplus$ **universal threshold subtraction**

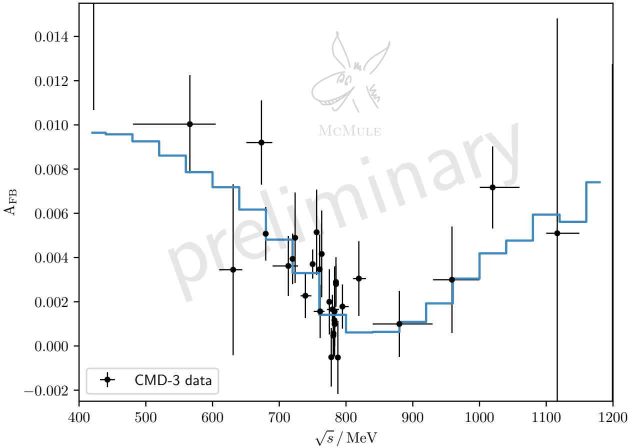
$$\int \mathrm{d}S \Bigg($$

$$- \mathrm{CT}\Bigg) + \int \mathrm{d}S\, \mathrm{CT}$$

goal :: universal description generalisable to more complicated topologies

$\Longrightarrow$ take whole amplitude

$$\mathrm{CT} =$$

$$+ \mathcal{O}(S - \mathrm{threshold})^0 = \underbrace{\frac{1}{S}\left(\frac{\mathrm{threshold}}{S - \mathrm{threshold} - i\delta}\right)^{1+2\epsilon}}_{(\#)} f(s,t)$$

- integral of $(\#)$ regulated with $i\delta$
- form of $f(s,t)$ **universal**, ie. does not change for eg. pentagons



$$A_{FB} = \frac{\frac{d\sigma}{d\theta_{avg}}(\theta_{avg} > \frac{\pi}{2}) - \frac{d\sigma}{d\theta_{avg}}(\theta_{avg} < \frac{\pi}{2})}{\frac{d\sigma}{d\theta_{avg}}(\theta_{avg} > \frac{\pi}{2}) + \frac{d\sigma}{d\theta_{avg}}(\theta_{avg} < \frac{\pi}{2})}$$

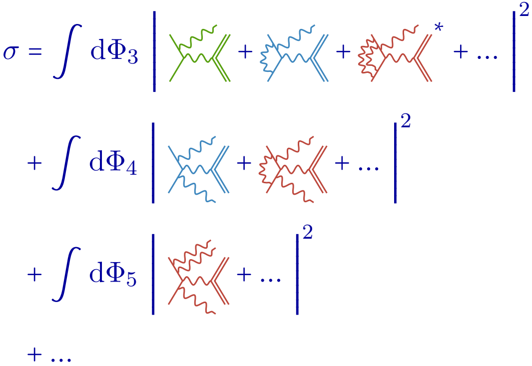
$$\theta_{avg} \equiv \frac{\theta^- - \theta^+ + \pi}{2}$$

A_{FB} for $ee \to \pi\pi$ with

ISC :: NNLO

FsQED :: NLO


Red
Mule

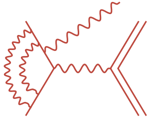


*(virtual)^2 from [Badger et al 23] requires **massification**

(virtual)^2 from [Badger et al 23] calculated with $m_e = 0$

- iff $m_e^2 \ll$ all other scales $\Longrightarrow$ **massification**

$$\mathcal{A}_n(m_e) \xrightarrow{m_e \to 0} \mathcal{A}_n(0) \times Z \times Z$$



- enough for hard detected photon (eg. KLOE-like large angle) ☺
- not enough for collinear emission (eg. KLOE-like small angle) ☹

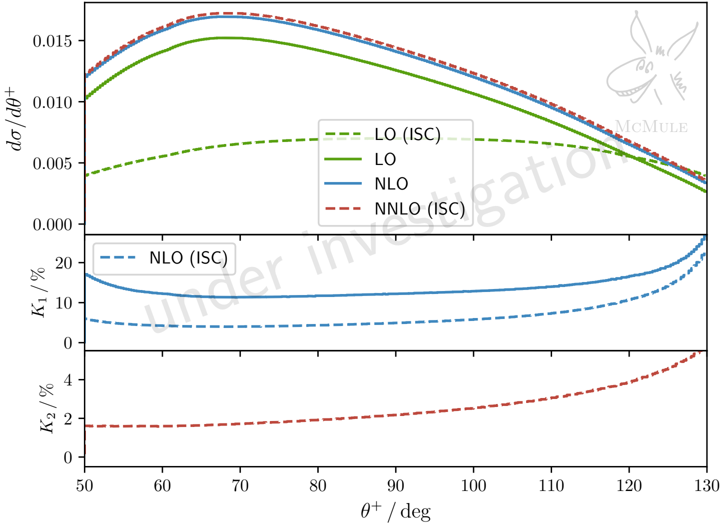
preliminary results for KLOE-like (large angle)

$\sqrt{s} = 1\,\mathrm{GeV}$, $E_\gamma > 20\,\mathrm{MeV}$, $50^\circ \le \theta_\gamma \le 130^\circ$, $0.1\,\mathrm{GeV}^2 \le M^2_{\mu\mu} \le 0.85\,\mathrm{GeV}^2$, $50^\circ \le \theta^\pm \le 130^\circ$, $|p^z_\pm| > 90\,\mathrm{MeV}$ or $p^\perp_\pm > 160\,\mathrm{MeV}$

muons $ee \to \mu\mu\gamma$

$$K_\ell = \frac{\sigma_\ell - \sigma_{\ell-1}}{\sigma_{\ell-1}}$$

without VP (WIP for NNLO)



preliminary results for KLOE-like (small angle)

$\sqrt{s} = 1\,\mathrm{GeV}$, $\theta_{\tilde\gamma} \le 15^\circ$ or $\theta_{\tilde\gamma} \ge 165^\circ$, $0.35\,\mathrm{GeV}^2 \le M^2_{\mu\mu} \le 0.95\,\mathrm{GeV}^2$, $50^\circ \le \theta^\pm \le 130^\circ$, $|p^z_\pm| > 90\,\mathrm{MeV}$ or $p^\perp_\pm > 160\,\mathrm{MeV}$

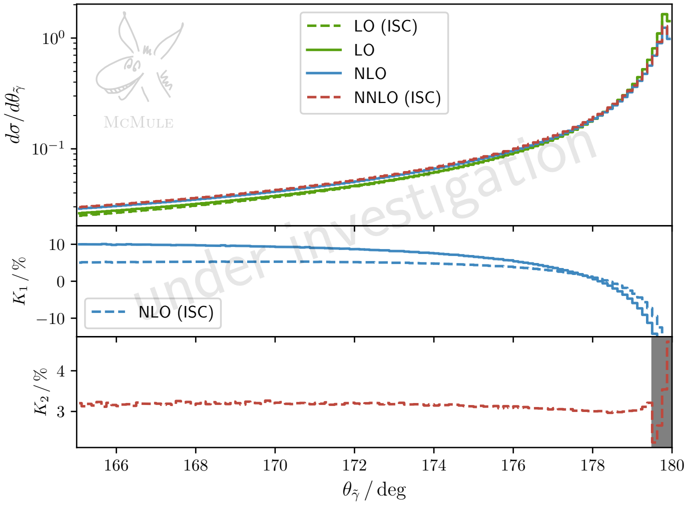
muons $ee \to \mu\mu\gamma$

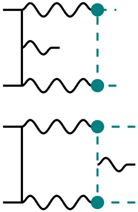
two-loop reliable for $\theta_{\tilde\gamma} \lesssim 179.5^\circ$ for 'untagged' photon

$\vec{p}_{\tilde\gamma} \equiv -(\vec{p}_+ + \vec{p}_-)$

$$K_\ell = \frac{\sigma_\ell - \sigma_{\ell-1}}{\sigma_{\ell-1}}$$

without VP (WIP for NNLO)



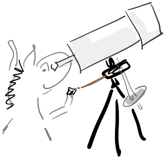


pions

- $q_e^3\, q_\pi^2$ contributions for $ee \to \pi\pi\gamma$ in FsQED in preparation
- RMCL2 plan (Pisa consensus) :: FsQED also beyond $q_e^3\, q_\pi^2$
- rescattering corrections . . . ?

radiative NNLO

- finish $ee \to \gamma\gamma^*$ investigations & add VP
  (to be solved: collinear emission & higher energies)
- full $ee \to \mu\mu\gamma$ with (virtual)^2 from [Badger et al 23]'s $pp \to 2j + \gamma$



f.l.t.r.: S.Kollatzsch (Zurich & PSI), A.Signer (Zurich & PSI), V.Sharkovska (Zurich & Mainz), S.Gündogdu (Zurich & PSI), D.Moreno (PSI), A.Coutinho (IFIC), Y.Ulrich (Liverpool), D.Radic (Zurich & PSI), L.Naterop (Zurich & PSI), M.Rocco (Turin)
not shown: F.Hagelstein (Mainz), N.Schalch (Oxford), P.Banerjee (Cosenza), M.Ronchi (Mainz), Y.Fang (ETH)

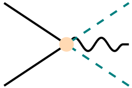
McMule

`mule-tools.gitlab.io`

Why do we need an EFT?

EFT $\ni$ lin. independent* set of **gauge invariant** operators

- matching $ee \to \pi\pi$

  $\Longrightarrow$ operators with ∂_μ
- use gauge invariance $\partial_\mu \to D_\mu$
- get 

  $\Longrightarrow$ no need to redo matching

*not important here

How do we pick Λ?

- similar to 'slicing'
- first guess based on

$$\text{OpenLoops}(S') \approx \text{EFT}(S')$$

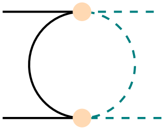
vary it and check obs. dependence

- $\int \frac{\mathrm{d}S}{S} \mathrm{Im} F_\pi^V(S) \dots$

$\Longrightarrow$ allows for early switch
- precision vs. speed
- $\Lambda = \Lambda(s)$

double-dispersive **double-heavy** contributions [MSc thesis Y. Fang]

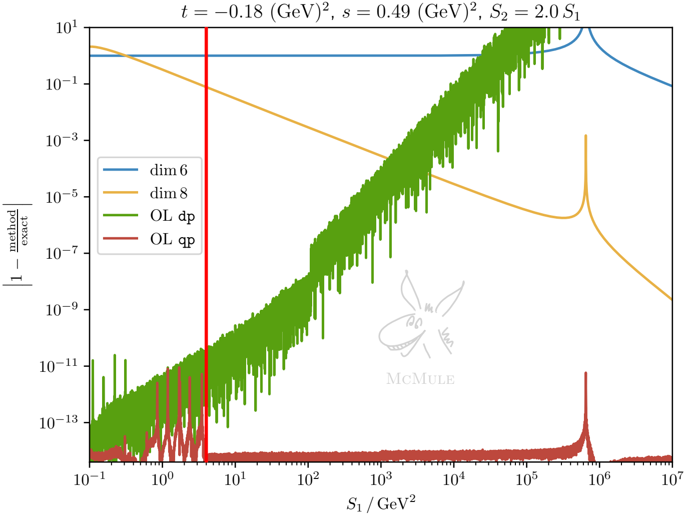
$$\int_{\Lambda}^{\infty}\frac{\mathrm{d}S_1}{S_1}\mathrm{Im}F_\pi^V(S_1)\times$$
$$\int_{\Lambda}^{\infty}\frac{\mathrm{d}S_2}{S_2}\mathrm{Im}F_\pi^V(S_2)$$



$$\frac{\text{time}_{\text{OpenLoops}}}{\text{time}_{\text{EFT}}} \approx 100$$

dim6 only $\sim m_e^2$ (seagull)

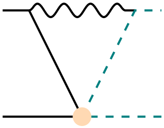
$\Longrightarrow$ dim8 needed



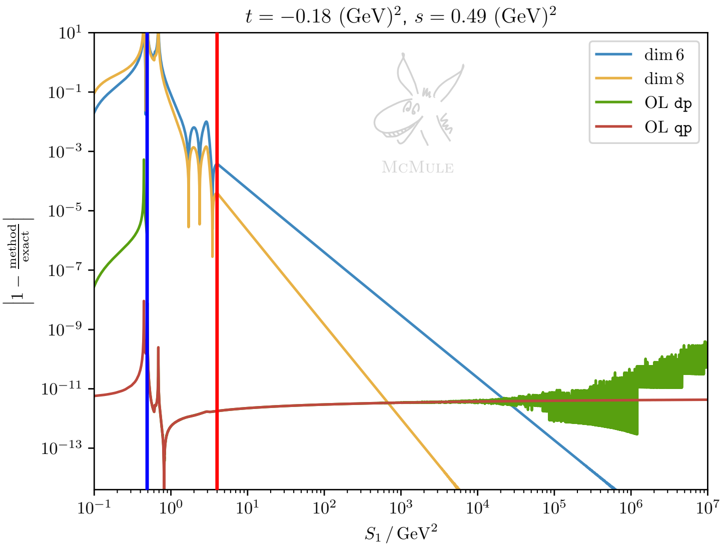
single-dispersive contributions [MSc thesis Y. Fang]

threshold $S_1 = s$

$$\int_{\Lambda}^{\infty} \frac{\mathrm{d}S_1}{S_1} \mathrm{Im} F_\pi^V(S_1)$$



$\Longrightarrow$ dim8 nice to have



preliminary results for KLOE-like (large angle)

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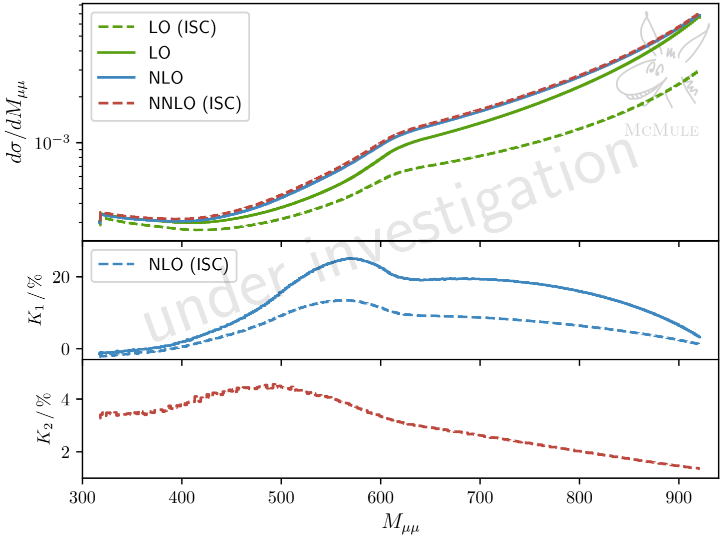

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