Fast evaluation of Feynman integrals for MC generators

Pau Petit Rosàs

In collaboration with T. Dave, J. Paltrinieri & W. J. Torres Bobadilla

UNIVERSITY OF LIVERPOOL

LEVERHULME TRUST

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& Phokhara

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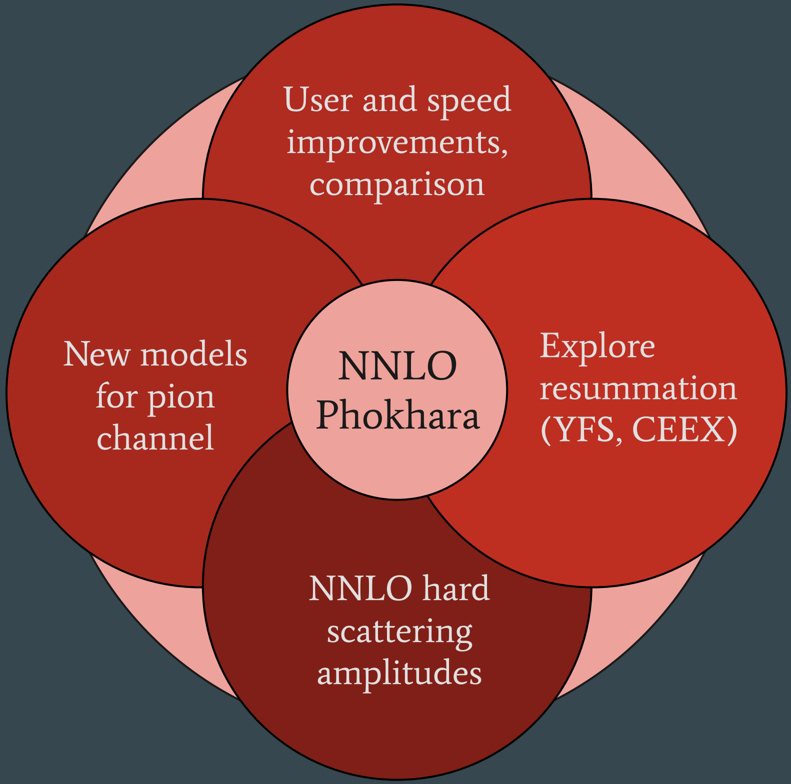
LIVERPOOL THEORETICAL PHYSICS

A tiny introduction

Phokhara, a MC generator for low energy e^+e^- colliders, with +20 years of development.

$e^+e^- \to$	Order	VP	VFF	Extras
$\mu^+\mu^-$	LO	alphaQED, or NSK	-	Narrow resonances of J/ψ and $\psi(2S)$
$\mu^+\mu^-\gamma$	NLO with full mass dependence			
$\pi^+\pi^-$	LO	alphaQED, or NSK	F×sQED choice of 3 VFF	Narrow resonances of J/ψ and $\psi(2S)$ Radiative ϕ decays
$\pi^+\pi^-\gamma$	NLO with full mass dependence			
X	$X \in 2\pi^0\pi^+\pi^-$, $2\pi^+2\pi^-$, $p\bar{p}$, $n\bar{n}$, K^+K^-, $K^0\bar{K}^0$, $\pi^+\pi^-\pi^0$, $\Lambda(\to \pi^-p)\bar{\Lambda}(\to \pi^+\bar{p})$, $\eta\pi^+\pi^-$, $\pi^0\gamma$, $\eta\gamma$, $\eta'\gamma$, $\chi_{c1} \to J/\psi(\to \mu^+\mu^-)\gamma$, $\chi_{c2} \to J/\psi(\to \mu^+\mu^-)\gamma$			

Objectives of the new effort

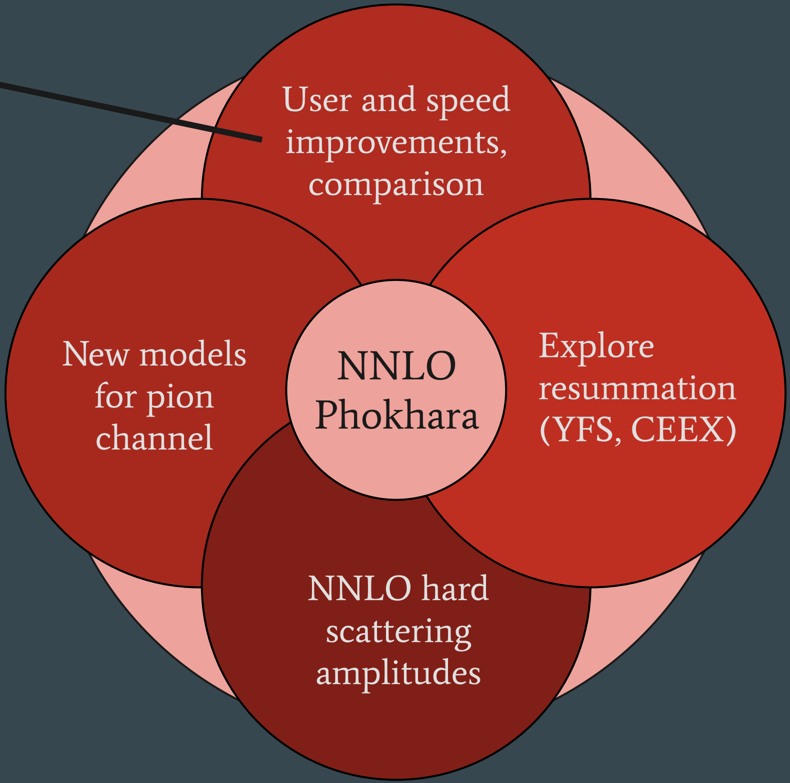


Objectives of the new effort

➔ User defined variables for histograms.

➔ Custom cuts at the generation level.

➔ Comparison in **[SciPost 10.21468]** (See Marco's talk).



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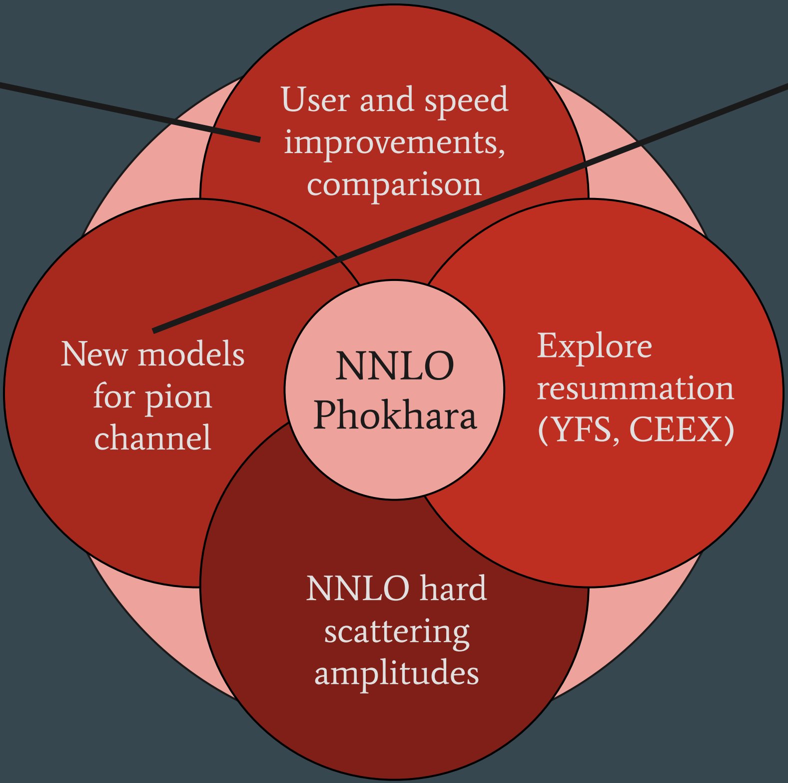

- ➔ Focus on GVMD model for $\pi\pi\gamma$.
- ➔ Amplitudes for ISR, reduced to minimal set of MIs.
- ➔ Integrate Collier library for evaluation of integrals.
- ➔ Add in Phokhara.
- ➔ Expand to FSR.

Progress on resummation

Use of YFS to expand fixed order amplitudes to include resummation of the leading contributions in the soft-photon regime. Able to recover fixed order calculation.

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Multiphoton phase-space

Generator of momenta for multi-photon events. Speed up via VEGAS.

YFS Form Factor

Account for IR poles in CEEX framework. Validated with Mathematica.

Analysis framework

Developed for debugging, to generate plots and histograms.

Muon scan mode

Implemented with LO resummed. Validation via scenarios of RadioMonteCarlow.

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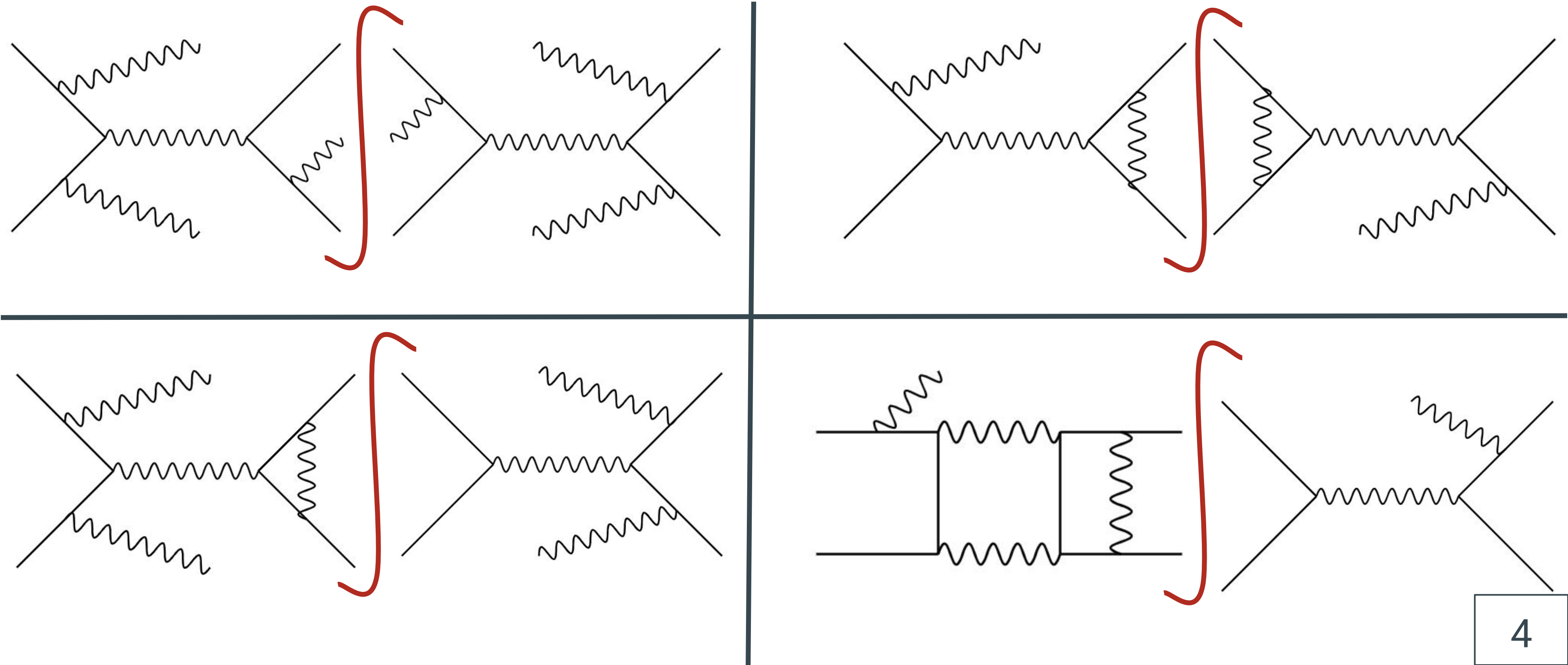
Muon radiative mode

Automation of the evaluation of amplitudes via public tools.

Merge in Phokhara and include higher order amplitudes

NNLO Hard Scattering Amplitudes



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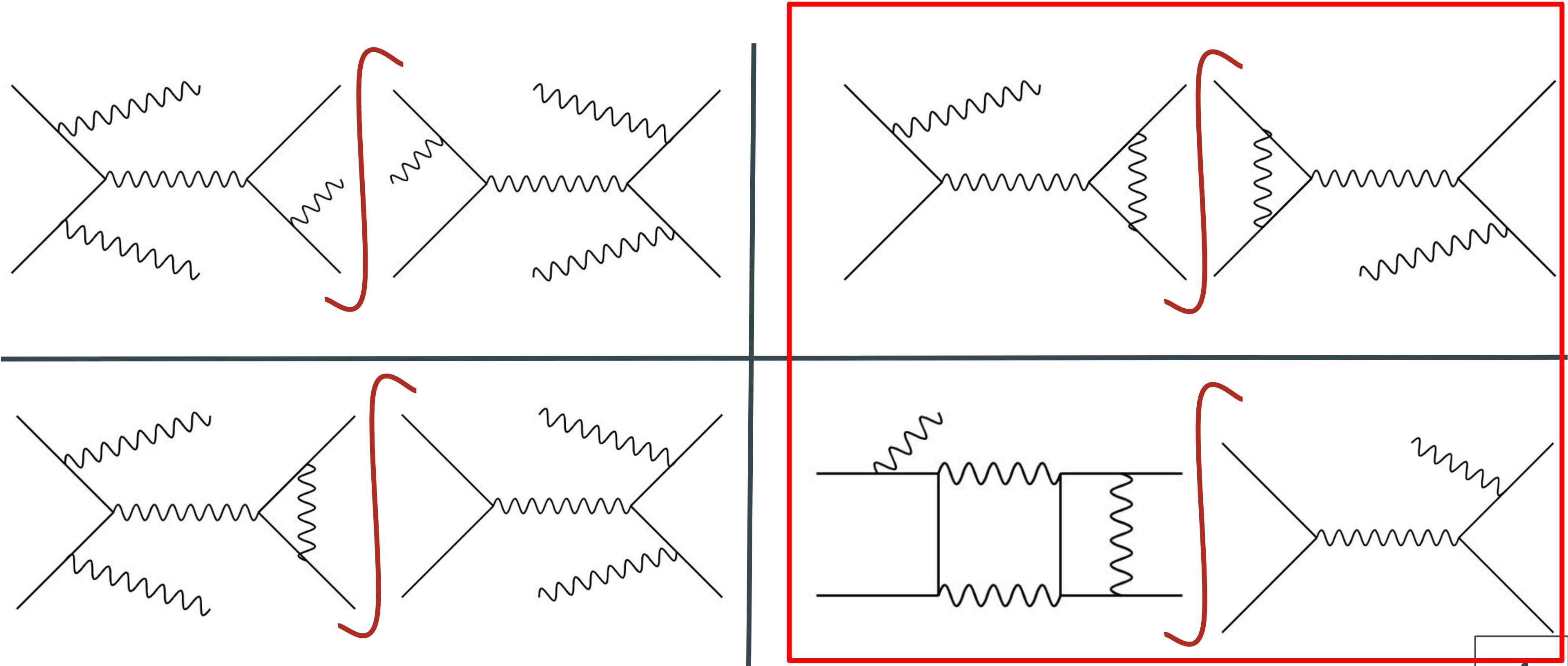

Calculating amplitudes...

Amplitude generation

Generate topologies, dress them with Feynman rules, perform Dirac algebra.

Tensor decomposition

No interference; make use of tensor decomposition techniques to exploit repeated analytic structures.

IBP reduction

Reduce all loop integrals into a minimal set of Masters by means of integration-by-part (IBPs) relations.

Evaluation of loop integrals

Evaluate both the kinematics coefficient of the amplitudes and the integrals themselves to obtain a final number.

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See [2411.07063] & T. Dave poster.

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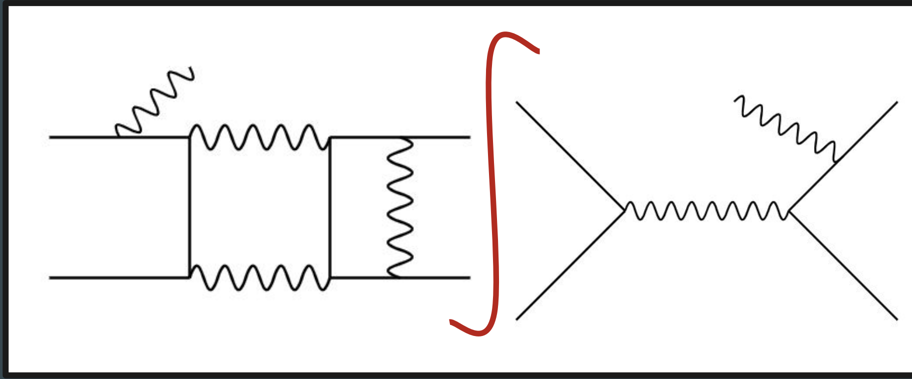
Problem:
How do we evaluate the Feynman integrals?

The Two-loop Monster



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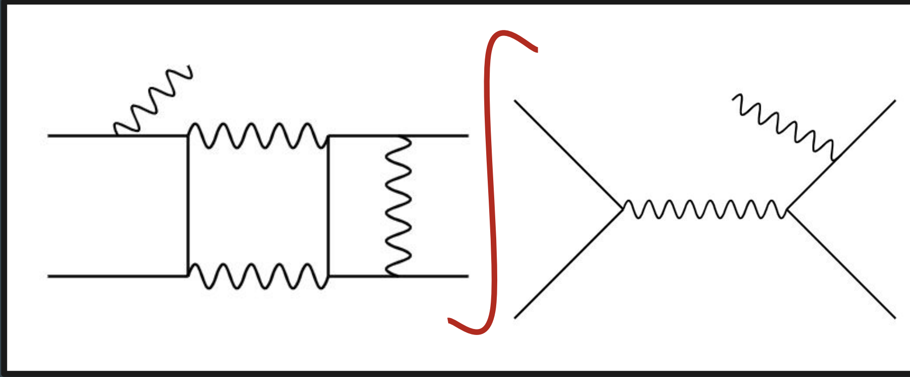
The Two-loop Monster

- State of the art **[JHEP07(2025)001].** Differential equation to evaluate integrals of two-loop pp → ttW.
- However, no initial state masses.
- CPU time ~ O(30 min) ppp!
- Need faster evaluation of integrals!
  - High dimensionality make grid interpolation not feasible.



Problem:
How do we evaluate the Feynman integrals?

A new integrator

- Differential equations solved numerically + new treatment for branch cuts + pathfinding + optimization.
- One- and Two-loop applications.
- Significantly smaller runtimes than other tools.

[2507.12548 in press in JHEP]

Fast evaluation of Feynman integrals for Monte Carlo generators

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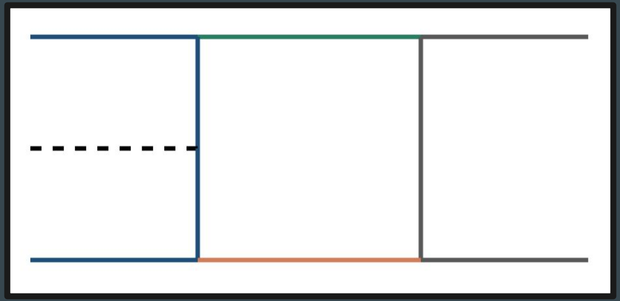
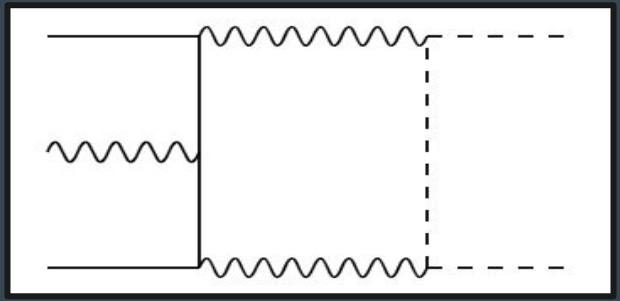
Abstract: Building on the idea of numerically integrating differential equations satisfied by Feynman integrals, we propose a novel strategy for handling branch cuts within a numerical framework. We develop an integrator capable of evaluating a basis of integrals in both double and quadruple precision, achieving significantly reduced computational times compared to existing tools. We demonstrate the performance of our integrator by evaluating one- and two-loop five-point Feynman integrals with up to nine complex kinematic scales. In particular, we apply our method to the radiative return process of massive electron-positron annihilation into pions plus an energetic photon within scalar QED, for which we also build the differential equation, and extend it to the case where virtual photons acquire an auxiliary complex mass under the Generalised Vector-Meson Dominance model. Furthermore, we validate our approach on two integral families relevant for the two-loop production of $t\bar{t}$ + jet. The integrator achieves, in double precision, execution times of the order of milliseconds for one-loop topologies and hundreds of milliseconds for the two-loop families, enabling for on-the-fly computation of Feynman integrals in Monte Carlo generators and a more efficient generation of grids for the topologies with prohibitive computational costs.

One-loop tests



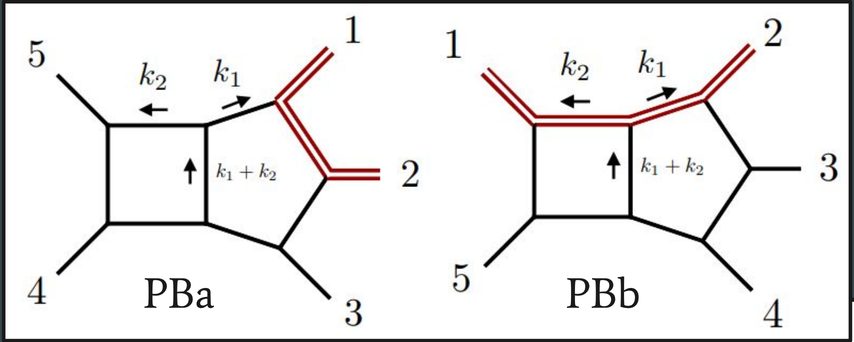
- One-loop topology with up to 9 kinematic scales.
- Runtime O(ms).
- One-loop libraries are faster **[1407.0087]** .



- Pentabox gauge invariant group at $O(\epsilon^2)$.
- Runtime O(ms).
- Faster and more precise than DCT **[2412.21054]** .

Two-loop pp → tt+jet

Test the integrator with something similar to $e^+e^- \to x^+x^-\gamma$.



- Evaluated random benchmark points.
- Big speed up (x1000 in double, x10 in quad), compared with DiffExp.
- Opens the door to on-the-fly evaluation!

Figure from **[2404.12325]**

		$\mathcal{T}_A$, $\mathcal{T}_R$	$\mathcal{R}$	$\langle\tau\rangle$ [s]	DiffExp $\langle\tau\rangle$ [s]
PB_A	double	10^{-12}, 10^{-12}	10	0.0881	580.85
	quad	10^{-28}, 10^{-28}	27	51.588	795.516
PB_B	double	10^{-12}, 10^{-12}	10	0.100	555.438
	quad	10^{-28}, 10^{-28}	27	89.088	826.219

Outlook

- Updates with GVMD and resummation around the corner.
- Integrator available for evaluating two-loop integrals in O(100ms).
- Near completion of tensor decomposition of the one-loop amplitudes.
- Building “nice” differential equations for two-loop integrals of $e^+e^- \to \gamma\gamma^*$.
- Starting the work to include $2 \to 4$ @ one-loop and $2 \to 5$ @ tree-level.

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QUESTIONS?